U.S. ENVIRONMENTAL PROTECTION AGENCY
Annapolis Field Office
Annapolis Science Center
Annapolis, Maryland 21401

TECHNICAL PAPERS

Volume 21

Table of Contents

Volume 21

11 A Steady State Segmented Estuary Model

12 Simulation of Chloride Concentrations in the Potomac Estuary - March 1968

13 Optimal Release Sequences for Water Quality Control in Multiple-Reservoir Systems - 1968

PUBLICATIONS

U.S. ENVIRONMENTAL PROTECTION AGENCY
REGION III
ANNAPOLIS FIELD OFFICE*

VOLUME 1
Technical Reports

5 A Technical Assessment of Current Water Quality Conditions and Factors Affecting Water Quality in the Upper Potomac Estuary

6 Sanitary Bacteriology of the Upper Potomac Estuary

7 The Potomac Estuary Mathematical Model

9 Nutrients in the Potomac River Basin

11 Optimal Release Sequences for Water Quality Control in Multiple Reservoir Systems

VOLUME 2
Technical Reports

13 Mine Drainage in the North Branch Potomac River Basin

15 Nutrients in the Upper Potomac River Basin

17 Upper Potomac River Basin Water Quality Assessment

VOLUME 3
Technical Reports

19 Potomac-Piscataway Dye Release and Wastewater Assimilation Studies

21 LNEPLT

23 XYPLOT

25 PLOT3D

* Formerly CB-SRBP, U.S. Department of Health, Education, and Welfare; CFS-FWPCA, and CTSL-FWQA, Middle Atlantic Region, U.S. Department of the Interior

VOLUME 3 (continued)

Technical Reports

27 Water Quality and Wastewater Loadings - Upper Potomac Estuary during 1969

VOLUME 4

Technical Reports

29 Step Backward Regression

31 Relative Contributions of Nutrients to the Potomac River Basin from Various Sources

33 Mathematical Model Studies of Water Quality in the Potomac Estuary

35 Water Resource - Water Supply Study of the Potomac Estuary

VOLUME 5

Technical Reports

37 Nutrient Transport and Dissolved Oxygen Budget Studies in the Potomac Estuary

39 Preliminary Analyses of the Wastewater and Assimilation Capacities of the Anacostia Tidal River System

41 Current Water Quality Conditions and Investigations in the Upper Potomac River Tidal System

43 Physical Data of the Potomac River Tidal System Including Mathematical Model Segmentation

45 Nutrient Management in the Potomac Estuary

VOLUME 6

Technical Reports

47 Chesapeake Bay Nutrient Input Study

49 Heavy Metals Analyses of Bottom Sediment in the Potomac River Estuary

VOLUME 6 (continued)

Technical Reports

51	A System of Mathematical Models for Water Quality Management
52	Numerical Method for Groundwater Hydraulics
53	Upper Potomac Estuary Eutrophication Control Requirements
54	AUTØ-QUAL Modelling System
Supplement to 54	AUTØ-QUAL Modelling System: Modification for Non-Point Source Loadings

VOLUME 7

Technical Reports

55	Water Quality Conditions in the Chesapeake Bay System
56	Nutrient Enrichment and Control Requirements in the Upper Chesapeake Bay
57	The Potomac River Estuary in the Washington Metropolitan Area - A History of its Water Quality Problems and their Solution

VOLUME 8

Technical Reports

58	Application of AUTØ-QUAL Modelling System to the Patuxent River Basin
59	Distribution of Metals in Baltimore Harbor Sediments
60	Summary and Conclusions - Nutrient Transport and Accountability in the Lower Susquehanna River Basin

VOLUME 9

Data Reports

Water Quality Survey, James River and Selected Tributaries - October 1969

Water Quality Survey in the North Branch Potomac River between Cumberland and Luke, Maryland - August 1967

VOLUME 9 (continued)

Data Reports

Investigation of Water Quality in Chesapeake Bay and Tributaries at Aberdeen Proving Ground, Department of the Army, Aberdeen, Maryland - October-December 1967

Biological Survey of the Upper Potomac River and Selected Tributaries - 1966-1968

Water Quality Survey of the Eastern Shore Chesapeake Bay, Wicomico River, Pocomoke River, Nanticoke River, Marshall Creek, Bunting Branch, and Chincoteague Bay - Summer 1967

Head of Bay Study - Water Quality Survey of Northeast River, Elk River, C & D Canal, Bohemia River, Sassafras River and Upper Chesapeake Bay - Summer 1968 - Head ot Bay Tributaries

Water Quality Survey of the Potomac Estuary - 1967

Water Quality Survey of the Potomac Estuary - 1968

Wastewater Treatment Plant Nutrient Survey - 1966-1967

Cooperative Bacteriological Study - Upper Chesapeake Bay Dredging Spoil Disposal - Cruise Report No. 11

VOLUME 10

Data Reports

9 Water Quality Survey of the Potomac Estuary - 1965-1966

10 Water Quality Survey of the Annapolis Metro Area - 1967

11 Nutrient Data on Sediment Samples of the Potomac Estuary 1966-1968

12 1969 Head of the Bay Tributaries

13 Water Quality Survey of the Chesapeake Bay in the Vicinity of Sandy Point - 1968

14 Water Quality Survey of the Chesapeake Bay in the Vicinity of Sandy Point - 1969

VOLUME 10(continued)

Data Reports

15 Water Quality Survey of the Patuxent River - 1967

16 Water Quality Survey of the Patuxent River - 1968

17 Water Quality Survey of the Patuxent River - 1969

18 Water Quality of the Potomac Estuary Transects, Intensive and Southeast Water Laboratory Cooperative Study - 1969

19 Water Quality Survey of the Potomac Estuary Phosphate Tracer Study - 1969

VOLUME 11

Data Reports

20 Water Quality of the Potomac Estuary Transport Study 1969-1970

21 Water Quality Survey of the Piscataway Creek Watershed 1968-1970

22 Water Quality Survey of the Chesapeake Bay in the Vicinity of Sandy Point - 1970

23 Water Quality Survey of the Head of the Chesapeake Bay Maryland Tributaries - 1970-1971

24 Water Quality Survey of the Upper Chesapeake Bay 1969-1971

25 Water Quality of the Potomac Estuary Consolidated Survey - 1970

26 Water Quality of the Potomac Estuary Dissolved Oxygen Budget Studies - 1970

27 Potomac Estuary Wastewater Treatment Plants Survey 1970

28 Water Quality Survey of the Potomac Estuary Embayments and Transects - 1970

29 Water Quality of the Upper Potomac Estuary Enforcement Survey - 1970

VOLUME 11 (continued)

Data Reports

30	Water Quality of the Potomac Estuary - Gilbert Swamp and Allen's Fresh and Gunston Cove - 1970
31	Survey Results of the Chesapeake Bay Input Study - 1969-1970
32	Upper Chesapeake Bay Water Quality Studies - Bush River, Spesutie Narrows and Swan Creek, C & D Canal, Chester River, Severn River, Gunpowder, Middle and Bird Rivers - 1968-1971
33	Special Water Quality Surveys of the Potomac River Basin Anacostia Estuary, Wicomico River, St. Clement and Breton Bays, Occoquan Bay - 1970-1971
34	Water Quality Survey of the Patuxent River - 1970

VOLUME 12

Working Documents

1	Biological Survey of the Susquehanna River and its Tributaries between Danville, Pennsylvania and Conowingo, Maryland
Appendix to 1	Tabulation of Bottom Organisms Observed at Sampling Stations during the Biological Survey between Danville, Pennsylvania and Conowingo, Maryland - November 1966
2	Biological Survey of the Susquehanna River and its Tributaries between Cooperstown, New York and Northumberland, Pennsylvnaia - January 1967
Appendix to 2	Tabulation of Bottom Organisms Observed at Sampling Stations during the Biological Survey between Cooperstown, New York and Northumberland, Pennsylvania - November 1966

VOLUME 13

Working Documents

3	Water Quality and Pollution Control Study, Mine Drainage Chesapeake Bay-Delaware River Basins - July 1967
4	Biological Survey of Rock Creek (from Rockville, Maryland to the Potomac River) October 1966

VOLUME 13 (continued)

Working Documents

5 Summary of Water Quality and Waste Outfalls, Rock Creek in Montgomery County, Maryland and the District of Columbia - December 1966

6 Water Pollution Survey - Back River 1965 - February 1967

7 Efficiency Study of the District of Columbia Water Pollution Control Plant - February 1967

VOLUME 14

Working Documents

8 Water Quality and Pollution Control Study - Susquehanna River Basin from Northumberland to West Pittson (Including the Lackawanna River Basin) March 1967

9 Water Quality and Pollution Control Study, Juniata River Basin - March 1967

10 Water Quality and Pollution Control Study, Rappahannock River Basin - March 1967

11 Water Quality and Pollution Control Study, Susquehanna River Basin from Lake Otsego, New York, to Lake Lackawanna River Confluence, Pennsylvania - April 1967

VOLUME 15

Working Documents

12 Water Quality and Pollution Control Study, York River Basin - April 1967

13 Water Quality and Pollution Control Study, West Branch, Susquehanna River Basin - April 1967

14 Water Quality and Pollution Control Study, James River Basin - June 1967

15 Water Quality and Pollution Control Study, Patuxent River Basin - May 1967

VOLUME 16

Working Documents

16 Water Quality and Pollution Control Study, Susquehanna River Basin from Northumberland, Pennsylvania, to Havre de Grace, Maryland - July 1967

17 Water Quality and Pollution Control Study, Potomac River Basin - June 1967

18 Immediate Water Pollution Control Needs, Central Western Shore of Chesapeake Bay Area (Magothy, Severn, South, and West River Drainage Areas) July 1967

19 Immediate Water Pollution Control Needs, Northwest Chesapeake Bay Area (Patapsco to Susquehanna Drainage Basins in Maryland) August 1967

20 Immediate Water Pollution Control Needs - The Eastern Shore of Delaware, Maryland and Virginia - September 1967

VOLUME 17

Working Documents

21 Biological Surveys of the Upper James River Basin Covington, Clifton Forge, Big Island, Lynchburg, and Piney River Areas - January 1968

22 Biological Survey of Antietam Creek and some of its Tributaries from Waynesboro, Pennsylvania to Antietam, Maryland - Potomac River Basin - February 1968

23 Biological Survey of the Monocacy River and Tributaries from Gettysburg, Pennsylvania, to Maryland Rt. 28 Bridge Potomac River Basin - January 1968

24 Water Quality Survey of Chesapeake Bay in the Vicinity of Annapolis, Maryland - Summer 1967

25 Mine Drainage Pollution of the North Branch of Potomac River - Interim Report - August 1968

26 Water Quality Survey in the Shenandoah River of the Potomac River Basin - June 1967

27 Water Quality Survey in the James and Maury Rivers Glasgow, Virginia - September 1967

VOLUME 17 (continued)

Working Documents

28 Selected Biological Surveys in the James River Basin, Gillie Creek in the Richmond Area, Appomattox River in the Petersburg Area, Bailey Creek from Fort Lee to Hopewell - April 1968

VOLUME 18

Working Documents

29 Biological Survey of the Upper and Middle Patuxent River and some of its Tributaries - from Maryland Route 97 Bridge near Roxbury Mills to the Maryland Route 4 Bridge near Wayson's Corner, Maryland - Chesapeake Drainage Basin - June 1968

30 Rock Creek Watershed - A Water Quality Study Report March 1969

31 The Patuxent River - Water Quality Management - Technical Evaluation - September 1969

VOLUME 19

Working Documents

Tabulation, Community and Source Facility Water Data Maryland Portion, Chesapeake Drainage Area - October 1964

Waste Disposal Practices at Federal Installations Patuxent River Basin - October 1964

Waste Disposal Practices at Federal Installations Potomac River Basin below Washington, D.C.- November 1964

Waste Disposal Practices at Federal Installations Chesapeake Bay Area of Maryland Excluding Potomac and Patuxent River Basins - January 1965

The Potomac Estuary - Statistics and Projections - February 1968

Patuxent River - Cross Sections and Mass Travel Velocities - July 1968

VOLUME 19 (continued)

Working Documents

Wastewater Inventory - Potomac River Basin - December 1968

Wastewater Inventory - Upper Potomac River Basin - October 1968

VOLUME 20

Technical Papers

1 A Digital Technique for Calculating and Plotting Dissolved Oxygen Deficits

2 A River-Mile Indexing System for Computer Application in Storing and Retrieving Data (unavailable)

3 Oxygen Relationships in Streams, Methodology to be Applied when Determining the Capacity of a Stream to Assimilate Organic Wastes - October 1964

4 Estimating Diffusion Characteristics of Tidal Waters - May 1965

5 Use of Rhodamine B Dye as a Tracer in Streams of the Susquehanna River Basin - April 1965

6 An In-Situ Benthic Respirometer - December 1965

7 A Study of Tidal Dispersion in the Potomac River February 1966

8 A Mathematical Model for the Potomac River - what it has done and what it can do - December 1966

9 A Discussion and Tabulation of Diffusion Coefficients for Tidal Waters Computed as a Function of Velocity February 1967

10 Evaluation of Coliform Contribution by Pleasure Boats July 1966

VOLUME 21

Technical Papers

11 A Steady State Segmented Estuary Model

12 Simulation of Chloride Concentrations in the Potomac Estuary - March 1968

13 Optimal Release Sequences for Water Quality Control in Multiple-Reservoir Systems - 1968

VOLUME 22

Technical Papers

Summary Report - Pollution of Back River - January 1964

Summary of Water Quality - Potomac River Basin in Maryland - October 1965

The Role of Mathematical Models in the Potomac River Basin Water Quality Management Program - December 1967

Use of Mathematical Models as Aids to Decision Making in Water Quality Control - February 1968

Piscataway Creek Watershed - A Water Quality Study Report - August 1968

VOLUME 23

Ocean Dumping Surveys

Environmental Survey of an Interim Ocean Dumpsite, Middle Atlantic Bight - September 1973

Environmental Survey of Two Interim Dumpsites, Middle Atlantic Bight - January 1974

Environmental Survey of Two Interim Dumpsites Middle Atlantic Bight - Supplemental Report - October 1974

Effects of Ocean Disposal Activities on Mid-continental Shelf Environment off Delaware and Maryland - January 1975

VOLUME 24

1976 Annual

	Current Nutrient Assessment - Upper Potomac Estuary Current Assessment Paper No. 1
	Evaluation of Western Branch Wastewater Treatment Plant Expansion - Phases I and II
	Situation Report - Potomac River
	Sediment Studies in Back River Estuary, Baltimore, Maryland
Technical Report 61	Distribution of Metals in Elizabeth River Sediments
Technical Report 62	A Water Quality Modelling Study of the Delaware Estuary

TABLE OF CONTENTS

		Page
	ACKNOWLEDGMENTS	
I.	INTRODUCTION	I - 1
II.	GENERAL APPROACH TO THE PROBLEM	II - 1
III.	DETAILED DERIVATION	III - 1
IV.	MATHEMATICAL TECHNIQUE EMPLOYED IN THE MODEL FOR MATRIX INVERSION	IV - 1
V.	PREPARATION OF THE COMPUTER PROGRAM	V - 1
VI.	INPUT DATA PREPARATION AND PROGRAM CASE CONTROL	VI - 1
VII.	BIBLIOGRAPHY	VII - 1

APPENDIX I - FLOW DIAGRAMS

APPENDIX II - IBM 360 FORTRAN PROGRAM

APPENDIX III - SAMPLE INPUTS

APPENDIX IV - TYPICAL PROGRAM CASE SOLUTIONS

A. Sample Input and Output Data for Case 1 with Control IMAT=1, INDEX (1)=1. Output αA^{-1}.

B. Sample Input and Output Data for Case 2 with Control IMAT=1, INDEX (2)=1. Output $L\alpha A^{-1}$.

C. Sample Input and Output Data for Case 3 with Control IMAT=1, INDEX (1) and INDEX (2). Both = 1. Output αA^{-1} and $L\alpha A^{-1}$.

D. Sample Input and Output Data for Case 4 with Control IMAT=2, INDEX (3)=1. Output αB^{-1}.

E. Sample Input and Output Data for Case 5 with Control IMAT=2, INDEX (4)=1. Output $F\alpha B^{-1}$.

F. Sample Input and Output Data for Case 6 with Control IMAT=2, INDEX (3) and INDEX (4)=1. Output αB^{-1} and $F\alpha B^{-1}$.

TABLE OF CONTENTS (Continued)

G. Sample Input and Output Data for Case 7 with Control IMAT=3, INDEX (5)=1. Output $\alpha B^{-1} A^{-1}$.

H. Sample Input and Output Data for Case 8 with Control IMAT=3, INDEX (6)=1. Output VD times $\alpha B^{-1} A^{-1}$.

I. Sample Input and Output Data for Case 9 with Control IMAT=3, INDEX (7)=1. Output L (ultimate BOD Vector) times Output Matrix in H. above.

J. Sample Input and Output Data for Case 10 with Control IMAT=3, INDEX (8)=1. Output = Dissolved Oxygen at Saturation plus output from E. above minus Output from I. above.

K. Sample Input and Output Data for Case 11 with Control IMAT=3, INDEX (1) through INDEX (8)=1. All cases above, i.e., A to J.

LIST OF FIGURES

Figure

1 Estuary Segments

2 Advection Into and Out of Segment K

3 Program Deck Structure

ACKNOWLEDGMENTS

Special appreciation and acknowledgment are expressed to Mr. Emanuel Mehr, Research Scientist, New York University, for allowing us to employ his tridiagonal matrix inversion routine. This routine is exceedingly fast and allows rapid inversion of the tridiagonal matrix utilized in the model. His guidance and encouragement was a great contribution during the course of certain phases of this work.

Grateful acknowledgment is made to Mr. Richard L. O'Connell of the Central Pacific River Basins Comprehensive Project, FWPCA, for the assistance he gave in understanding the model and setting up the program while he was Director of the Chesapeake Field Station; to Mr. John M. Jeglic of the Re-entry Systems Department of General Electric for the many programming ideas which were taken directly from his time-dependent version of the model; and to Dr. Robert Thomann of the Delaware Estuary Comprehensive Study Project, both for the basic tneory on which the program is based and for his detailed, patient explanations of it.

I. INTRODUCTION

The "oxygen sag" equation developed in 1925 by Streeter and Phelps[1] has found widespread use in the analysis of a particular type of stream pollution problems. This mathematical model may be applied where the pollutant of concern is biologically degradable organic material which brings about a depression of the natural dissolved oxygen (DO) content of a stream.

The "oxygen sag" formula, however, is not applicable to tidal bodies of water, primarily because of the over-riding importance of turbulent diffusion in estuaries which is not normally taken into consideration in streams. In fresh water streams "plug flow," in effect, is assumed, and this is not an unreasonable assumption in most cases. Although longitudinal diffusion does in fact occur in streams due to horizontal and vertical velocity gradients, the longitudinal exchange which this brings about involves similar material; viz., the material that is "diffused" upstream is very nearly the same with respect to age, concentration, etc., as that which is "diffused" downstream. The net result is usually negligible in the analysis of stream pollution problems. Furthermore, advection, or transport downstream by the stream, produces a transfer of waterborne material which greatly exceeds the transfer brought about by diffusion. In estuaries, however, because of their relatively large cross-section and small net flow downstream, the advective transport is often quite small compared to the transfer of materials by turbulent diffusion. For this reason, the latter effect must be considered.

In 1960, 35 years after the development of the "oxygen sag" formula for streams, O'Connor[2] devised a similar mathematical model for estuaries which added to Streeter and Phelp's formula a term to account for longitudinal diffusion. As with the original stream sag formula, the O'Connor model is given as a differential equation which may be integrated to give a solution for the dissolved oxygen as a continuous function of distance along the longitudinal axis of the tidal estuary.

In 1963 Thomann[3] developed a computational procedure based upon the principles of systems analysis which incorporated the terms of the O'Connor model for estuaries. This method of computation employs an incremental or segmented approach where average conditions in a finite number of connected segments are determined rather than a continuous function solution. The degree of resolution possible in the solution is directly related to the number of segments chosen. This approach simplifies the mathematics involved and allows much greater flexibility in its use. The models previously cited apply to the one dimensional steady state case; whereas, with the segmented approach, a second or third dimension may be added, and the time dependent situation may also be investigated. Also, any pollutant for which the relationships affecting its behavior can be expressed mathematically can be incorporated into the segmented model.

The model has been described previously by Thomann[3], and solutions to the one-dimensional time-dependent model have been

programmed for both the digital and analog computer[4,5]. Applications of the model and methods of estimating required parameters also have been presented[6,7,8,9]. The purpose of this paper is to describe in detail the theory of the "Thomann Model" as applicable to the one-dimensional steady-state case of organic pollution of an estuary and to document a digital computer program developed to solve this case.

II. GENERAL APPROACH TO THE PROBLEM

A first step in the application of the model is to divide the estuary into a satisfactory number of segments. A mass balance for the pollutant is written for each segment. The results from these balance a series of equations equal in number to the number of segments. The only unknowns in these equations are the mean steady-state concentrations of the pollutant in each segment. Thus, a series of n equations with n unknowns is obtained where n is the number of segments. By combining terms, this series of equations may be expressed in matrix notation as:

$$AC = L \qquad 1$$

where A is a collection of known terms such as flow, diffusion constants, deoxygenation constants, and segment volumes in combinations having units of cubic foot/day. C is a vector matrix of unknown concentration of pollutants in pounds/cubic foot. The product of A and C is L, a vector of pollutant loads added directly to each segment expressed as pounds/day. The terms of the vector L have been referred to as forcing functions by Thomann.

The matrix of unknown concentrations, C, may be solved by inverting A, since:

$$C = LA^{-1} \qquad 2$$

where A^{-1}, the "unit loading matrix," is the inverse of A. C may then be obtained by performing the matrix multiplication of L times A^{-1} as indicated.

This is an extremely convenient arrangement since, once having solved for A^{-1}, it can be multiplied by any distribution of loads along the estuary as described by the vector L. Furthermore, because of the nature of A^{-1}, it is possible to determine easily what part of the resulting pollutant concentration in any one segment is due to a given discharge in any other segment. The unit loading matrix is essentially a table having n rows and n columns, the elements of which have the units lbs/cfs per lb/day discharged. Thus, $A_{4,7}$, the element in the fourth row and seventh column of the A^{-1} matrix, would give the numerical value for the concentration in segment 4 resulting from a discharge of one pound per day in segment 7. By multiplying the element, $A_{4,7}$, by the actual load applied to segment 7, the concentration in segment 4 resulting from that discharge will be obtained. If the estuary being studied has been modeled using correctly verified parameters, it will be possible to reproduce known pollutant distributions from known input loads. The verification step must be carried out with satisfactory results if the model is to be useful for predicting altered loading conditions expected in the future. That is to say, the computor model is no substitute for the great quantity of engineering field and office work required for any sound, scientific discussion.

Once a verified unit loading matrix is obtained, the pollutant distribution resulting from any loading pattern may be obtained.

In the analysis of oxygen conditions, it is necessary as a first step to multiply the unit loading matrix by the particular ultimate oxygen demand (UOD) loading conditions being studied. Having the UOD distribution among the segments, a mass balance may then be written for each segment incorporating the terms which affect the oxygen distribution, namely, advection, diffusion, deoxygenation, and reaeration. When applicable, terms describing uptake by benthal loads and the net effect of photosynthesis and respiration can also be included. Just as in the case of the pollutant distribution, these mass balances around each of n segments results in a series of n equations in n unknowns which are the DO concentrations being sought. Through the use of matrices, it is possible then to obtain a series of linear expressions for the DO in any segment as a function of the discharged loads, benthal uptake and photosynthetic effects in all other segments.

III. DETAILED DERIVATION

Having divided the estuary into n sections, a mass balance is written for the oxygen in the k^{th} section (Figure 1). Assume the concentration of oxygen in each section is uniform and equal to C_k, the mean concentration in the section.

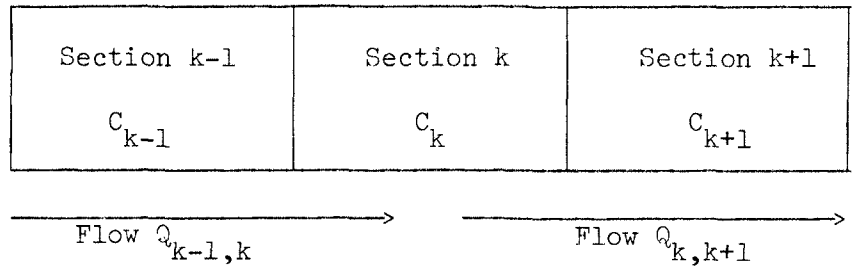


Figure 1

Oxygen can be gained or lost by several ways in Section k.

1. Advection into Section k from Section k-1 and out of Section k to Section k+1.
2. Diffusion into or out of Sections k-1 and k+1.
3. Reaeration of Section k from the atmosphere.
4. Use of oxygen in Section k by oxygen consuming waste (UOD).
5. Production of oxygen in Section k by photosynthesis.
6. Other mechanisms (benthol deposits, immediate oxygen demands, etc.).

Advection

In the system pictured below,

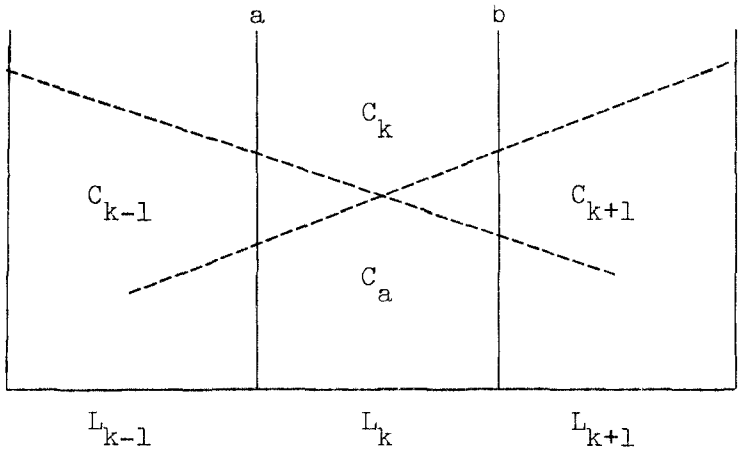


Figure 2

the C's represent mean concentrations in each segment of length, L.

If it is assumed that the concentration gradient through any two adjacent sections is approximated satisfactorily by a straight line, then the concentration at a boundary, a, between the two segments can be shown geometrically to be:

$$C_a = \frac{L_k\ C_{k-1} + L_{k-1}\ C_k}{L_k + L_{k-1}} \qquad 3$$

which can be written as:

$$C_a = \frac{C_{k-1}\ L_{k-1}}{L_{k-1} + L_k} + \frac{C_k\ L_k}{L_{k-1} + L_k} \qquad 4$$

or by letting

$$\xi_{k-1,k} = \frac{L_k}{L_{k-1} + L_k} \qquad 5$$

the concentration at the boundary may be expressed as:

$$C_a = \xi_{k-1,k}\, C_{k-1} + (1 - \xi_{k-1,k})\, C_k \qquad 6$$

The amount of material advected across boundary "a" is then given by:

$$A_{k-1,k} = Q_{k-1,k}\, [\xi_{k-1,k}\, C_{k-1} + (1 - \xi_{k-1,1})\, C_k] \qquad 7$$

where $\xi_{k-1,1}$ is defined as above.

For the case of boundary"b" the concentration gradient is rising and has a sign opposite to that passing boundary "a." In this case it can be shown that $\xi_{k,k+1}$ will be defined as:

$$\xi_{k,k+1} = 1 - \frac{L_{k+1}}{L_k + L_{k+1}} \qquad 8$$

or

$$\xi_{k,k+1} = \frac{L_k}{L_k + L_{k+1}} \qquad 9$$

For a falling gradient, then, ξ is defined by the ratio of the downstream segment length to the sum of the upstream and downstream lengths; whereas, for a rising gradient, ξ is defined by 1 minus this quantity or the ratio of the upstream length to the sum of the upstream and downstream lengths.

The above procedure may be used to find ξ subject to the restrictions that $\xi > 1 - \frac{E}{Q}$.

Following this derivation, the advection from Section k-1 to k becomes:

$$A_{k-1,k} = Q_{k-1,k} \left[\xi_{k-1,k} C_{k-1} + (1 - \xi_{k-1,1}) C_k\right] \quad 10$$

In a similar manner, the advection out of Section k into Section k+1 is:

$$A_{k,k+1} = Q_{k,k+1} \left[\xi_{k,k+1} C_k + (1 - \xi_{k,k+1}) C_{k+1}\right] \quad 11$$

Diffusion

The rate of diffusion from one section to another can be assumed to be proportional to the difference in concentration between the sections or:

$$D_{k-1,k} = E_{k-1,k} (C_{k-1} - C_k) \quad 12$$

$$D_{k,k+1} = E_{k,k+1} (C_{k+1} - C_k) \quad 13$$

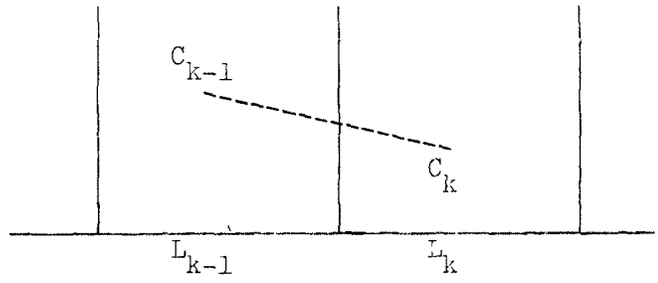
where E is an exchange factor related to the classical diffusion factor (K) by:

$$E_{k,k+1} = \frac{K_{k,k+1} A_{k,k+1}}{0.5 (L_k + L_{k+1})} \quad 14$$

This expression for E can be derived by assuming that diffusion proceeds according to Fick's first law, i.e.,

$$N = -K\frac{\partial C}{\partial X} \quad 15$$

where N is the time rate of transfer of substance per unit area, $\frac{M}{L^2T}$, and K is the coefficient of diffusion for the system shown below:



$$\frac{\partial C}{\partial X} = \frac{C_{k-1} - C_k}{1/2L_{k-1} + 1/2L_k} \qquad 16$$

and

$$N = K \left[\frac{(C_{k-1} - C_k)}{1/2\ (L_{k-1} + L_k)}\right] \qquad 17$$

D, the total time rate of transfer of substance across the boundary between Segment k-1 and K which has a cross-sectional area A is given by:

$$D = AN \qquad 18$$

$$= \frac{AK\ (C_{k-1} - C_k)}{1/2(L_{k-1} + L_k)} \qquad 19$$

or

$$D = E_{k-1,k}\ (C_{k-1} - C_k) \qquad 20$$

where

$$E_{k-1,k} = \frac{KA}{1/2\ (L_{k-1} + L_k)} \qquad 21$$

If E is taken as positive, the direction of the diffusion will be fixed by the relative magnitude of C_{k-1}, C_k, and C_{k+1}.

Reaeration

The rate of reaeration (r_k) into Section k may be expressed as being proportional to the difference in the actual oxygen concentration and the oxygen saturation value C_{sc}.

$$R_k = r_k V_k (C_{sc} - C_k) \qquad 22$$

BOD

The rate of utilization of oxygen by the UOD present is expressed as:

$$B_k = V_k \, d_k(t) \, L_k(t) \qquad 23$$

where $d_k(t)$ = the decay coefficient (k_1) in Section k at time t.

$L_k(t)$ = the UOD in Section k at time t.

Other Sources and Sinks

The effect of algae, benthal deposits, COE, and any other sources or sinks of oxygen in Section k will be expressed as $S_k(t)$. Later it may be desirable to separate these effects and treat them separately; but for the present, in order to keep the analysis tractable, they will be considered together.

If an equation is now written for all the sources and sinks of oxygen in Section k, the following differential equation for the mass rate of change in Section k is obtained.

$$V_k\frac{dC_k}{dt} = Q_{k-1,k}\,[\xi_{k-1,k}\,C_{k-1} + (1 - \xi_{k-1,k})\,C_k]$$
$$- Q_{k,k+1}\,[\xi_{k,k+1}\,C_k + (1 - \xi_{k,k+1})\,C_{k+1}] \qquad 24$$
$$+ E_{k-1,k}\,(C_{k-1} - C_k) + E_{k,k+1}\,(C_{k+1} - C_k)$$
$$+ r_k\,V_k\,[C_{sc}(t) - C_k] - V_k\,d_k(t)\,L_k(t)$$
$$- S_k(t)$$

or rearranging:

$$-V_k\frac{dC_k}{dt} + Q_{k-1,k}\,\xi_{k-1,k}\,C_{k-1} + Q_{k-1,k}\,(1 - \xi_{k-1,k})\,C_k$$
$$-Q_{k,k+1}\,\xi_{k,k+1}\,C_k - Q_{k+1}\,(1 - \xi_{k,k+1})\,C_{k+1} \qquad 25$$
$$+E_{k-1,k}\,C_{k-1} - E_{k-1,k}\,C_k + E_{k,k+1}\,C_{k+1} - E_{k,k+1}\,C_k$$
$$- V_k r_k\,C_k + -r_k V_k C_{sc}(t) + V_k d_k(t)\,L_k(t) + V_k S_k(t)$$

Factoring out C_k's and multiplying through by -1:

$$- [Q_{k-1,k}\,\xi_{k-1,k} + E_{k-1,k}]\,C_{k-1} - [V_k\,\frac{d}{dt} + Q_{k-1,k}\,(1 - \xi_{k-1,k})]$$
$$- Q_{k,k+1}\,\xi_{k,k+1} - E_{k-1,k} - E_{k,k+1} - r_k\,V_k\,C_k \qquad 26$$
$$- C_{k-1}\,[-Q_{k,k+1}\,(1 - \xi_{k,k+1}) + E_{k,k+1}] = r_k V_k C_{sc}(t)$$
$$- V_k d_k(t)\,L_k(t) - S_k(t)$$

Letting

$$a_{k-1,k} = -[Q_{k-1,k}\,\xi_{k-1,k} + E_{k-1,k}] \qquad 27$$

$$a_{kk} = [-Q_{k-1,k}\,(1 - \xi_{k-1,k}) + Q_{k,k+1}\,\xi_{k,k+1} + E_{k-1,k} \qquad 28$$
$$+ E_{k,k+1} + r_k V_k - V_k\,\frac{d}{dt}]$$

$$a_{k,k+1} = [Q_{k,k+1}\,(1 - \xi_{k,k-1}) - E_{k,k+1}] \qquad 29$$

the mass balance equation reduces to:

$$a_{k-1,k}\ C_{k-1} + a_{k,k}\ C_k + a_{k,k+1}\ C_{k+1} =$$

$$-r_k V_k C_{sc}(t) + V_k d_k(t)\ L_k(t) + S_k(t)\ V_k \qquad 30$$

If steady state conditions are assumed:

$$\frac{d}{dt} = 0 \qquad 31$$

$$C_{sc}(t) = C_{sc} \qquad 32$$

$$d_k(t) = d_k \qquad 33$$

$$L_k(t) = L_k \qquad 34$$

$$S_k(t) = S_k \qquad 35$$

therefore:

$$a_{kk} = [Q_{k,k+1}\ \xi_{k,k+1} = Q_{k-1,k}\ (1 - \xi_{k-1,k}) + E_{k-1,k}$$

$$+ E_{k,k+1} + r_k\ V_k] \qquad 36$$

and

$$a_{k-1,k}\ C_{k-1} + a_{kk}\ C_k + a_{k,k+1} - r_k\ V_k\ C_{sc} + V_k\ d_k\ L_k + S_k\ V_k = P_k \qquad 37$$

Assuming appropriate boundary conditions, i.e., C_0 and C_{n+1} and writing mass balance equations for each section, we obtain an equation of the following type:

$$
\begin{aligned}
a_{11}C_1 + a_{12}C_2 &= P_1 \\
a_{12}C_1 + a_{22}C_2 + a_{23}C_3 &= P_2 \\
a_{23}C_2 + a_{33}C_3 + a_{34}C_4 &= P_4 \\
\quad " \qquad\quad " \qquad\quad " \qquad &\quad " \\
\quad " \qquad\quad " \qquad\quad " \qquad &\quad " \\
\quad " \qquad\quad " \qquad\quad " \qquad &\quad " \\
a_{n-1,n}C_{n-1} + a_{n,n}C_n &= P_n
\end{aligned}
\tag{38}
$$

or in a matrix notation

$$
\begin{bmatrix}
a_{11} & a_{12} & 0 & \cdot & \cdot & \cdot & 0 \\
a_{21} & a_{22} & a_{23} & \cdot & \cdot & \cdot & 0 \\
0 & a_{32} & a_{33} & & & & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_{n-1,n} \\
0 & & & a_{n,n-1} & & & a_{nn}
\end{bmatrix}
\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix}
=
\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \cdot \\ \cdot \\ \cdot \\ P_n \end{bmatrix}
\tag{39}
$$

$$
A\,C = F \tag{40}
$$

$$
C = F\,A^{-1} \tag{41}
$$

The system can thus be solved for all values of C by finding the inverse of the matrix A. Matrix inversion, although time-consuming by hand, can be accomplished easily by high speed digital computers.

IV. MATHEMATICAL TECHNIQUE EMPLOYED IN THE MODEL FOR MATRIX INVERSION

The matrix of coefficients we are dealing with in the program is of a particular type. The system of equations is tri-diagonal since the matrix of unknowns is tridiagonal, i.e., each element is zero except the main diagonal and its adjacent diagonals.

The solution to such a system is conveniently handled on the computer by solving for the inverse employing a tridiagonal technique.

This may be illustrated by an example:

$$\begin{bmatrix} C_1 & q_1 & 0 & 0 \\ p_1 & C_2 & q_2 & 0 \\ 0 & p_2 & C_3 & q_3 \\ 0 & 0 & p_3 & C_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \qquad 42$$

Now define an S sequence recursively by:

$$S_1 = C_1 \qquad 43$$

$$S_2 = C_2 - \frac{p_1 q_1}{S_1} \qquad 44$$

$$S_3 = C_3 - \frac{p_2 q_2}{S_2} \qquad 45$$

" "

" "

" "

$$S_k = C_k - \frac{p_{k-1}\, q_{k-1}}{S_{k-1}} \qquad 46$$

" "

" "

" "

$$S_n = C_n - \frac{p_{n-1}\, q_{n-1}}{S_n} \qquad 47$$

In terms of this sequence, define the sequence K by:

$$K_1 = b_1 \qquad 48$$

$$K_2 = b_2 - \frac{p_1\, k_1}{S_1} \qquad 49$$

" "

" "

" "

$$K_i = b_i - \frac{p_{i-1}\, k_{i-1}}{S_{i-1}} \qquad 50$$

" "

" "

" "

$$K_n = b_n - \frac{p_{n-1}\, k_{n-1}}{S_{n-1}} \qquad 51$$

The solution to the set of equations can now be written down by means of a backward sweep as follows:

NOTE: We illustrate by means of a 4x4. The generalization is obvious.

$$X_4 = \frac{K_4}{S_4} \qquad 52$$

$$X_3 = \frac{K_3 - q_3 X_4}{S_3} \tag{53}$$

$$X_2 = \frac{K_2 - q_2 X_3}{S_2} \tag{54}$$

$$X_1 = \frac{K_1 - q_1 X_2}{S_1} \tag{55}$$

The above procedure is well known. In order to prove this, we note that the forward sweep corresponds to taking the i-th row, multiplying it by $\frac{-C_i}{p_i}$, and adding it to the i+i-th row as i advances from 1 to n. This procedure triangularizes the matrix, i.e., it is reduced to:

$$\begin{bmatrix} S_1 & q_1 & 0 & 0 \\ 0 & S_2 & q_2 & 0 \\ 0 & 0 & S_3 & q_3 \\ 0 & 0 & 0 & S_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} \tag{56}$$

The backward sweep is now clear. Obviously now X_3 is determined since X_4 is known, etc.

V. PREPARATION OF THE COMPUTER PROGRAM

The program was written in Fortran IV and originally run on the IBM 7094 at the National Bureau of Standards in Washington, D. C. The more recent version presented here was run on the IBM 360 system at the U. S. Geological Computer Center, Washington, D. C. Though Fortran programs will run on any installation that maintains a Fortran compiler, it usually requires a number of changes to be made when switching from one system to another. Generally, these changes require an understanding of the installation's job control language, together with the particular level of the compiler.

In developing the program and in the choice of mnemonics, an effort was made to make the program as compatible as possible with the time dependent model previously programmed by Jeglic.[4]

The complete program deck structure is shown in Figure 3. The entire package consists of one main program and eight sub-routines. The main program calls these various sub-routines as they are needed and controls the actual operating features and cycling. The sub-routine POLYB reads in coefficients and the powers of polynomian equations when the vectors for reaeration, decay, and dissolved oxygen are to be generated rather than read in as input data. The sub-routine PRELIM reads in various vector quantities and prepares the coefficients of the matrix and other required vectors. Sub-routine PRECAL loads the calculated matrix elements in their proper positions and returns control to the

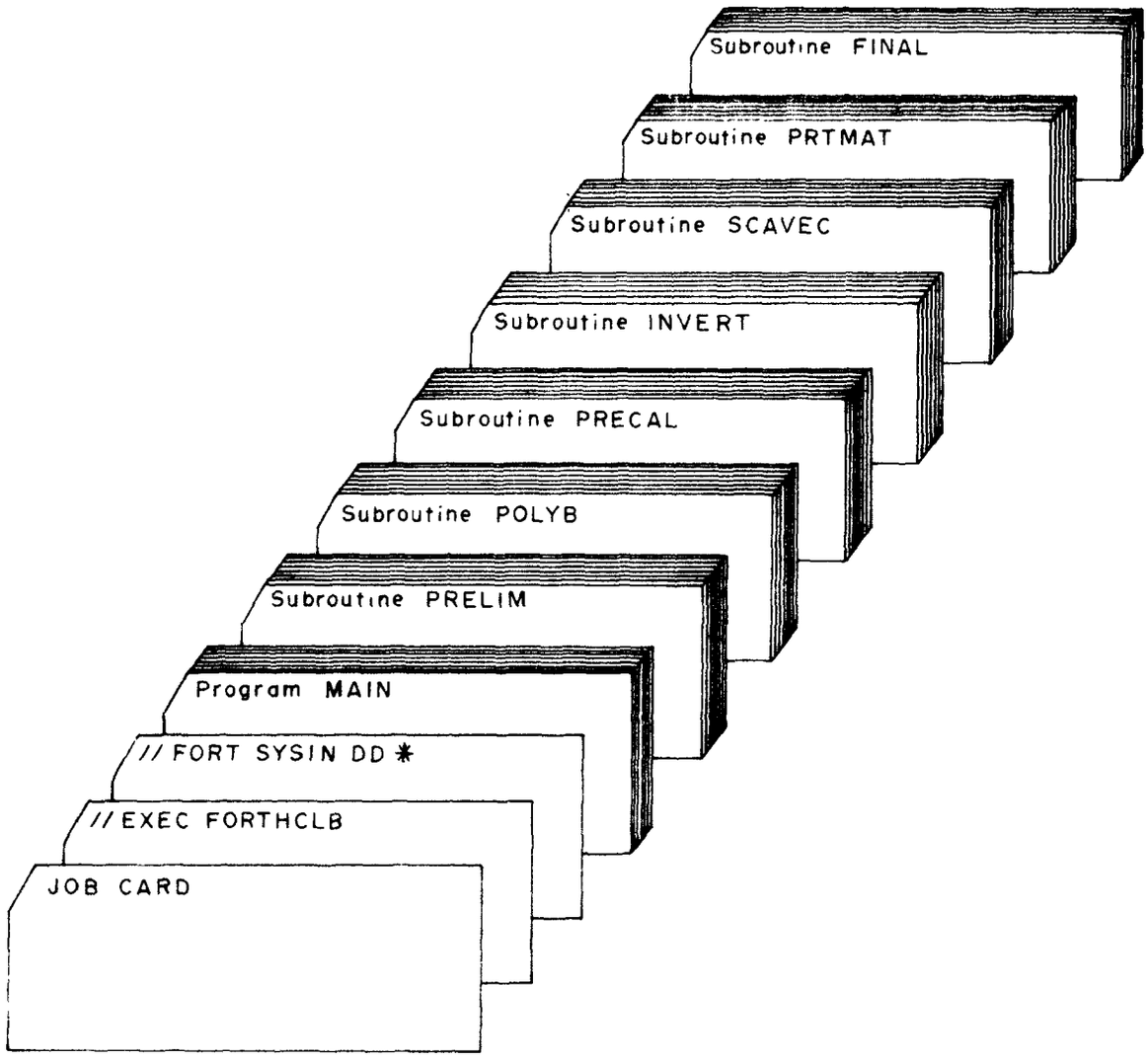


FIGURE 3.

PROGRAM DECK STRUCTURE

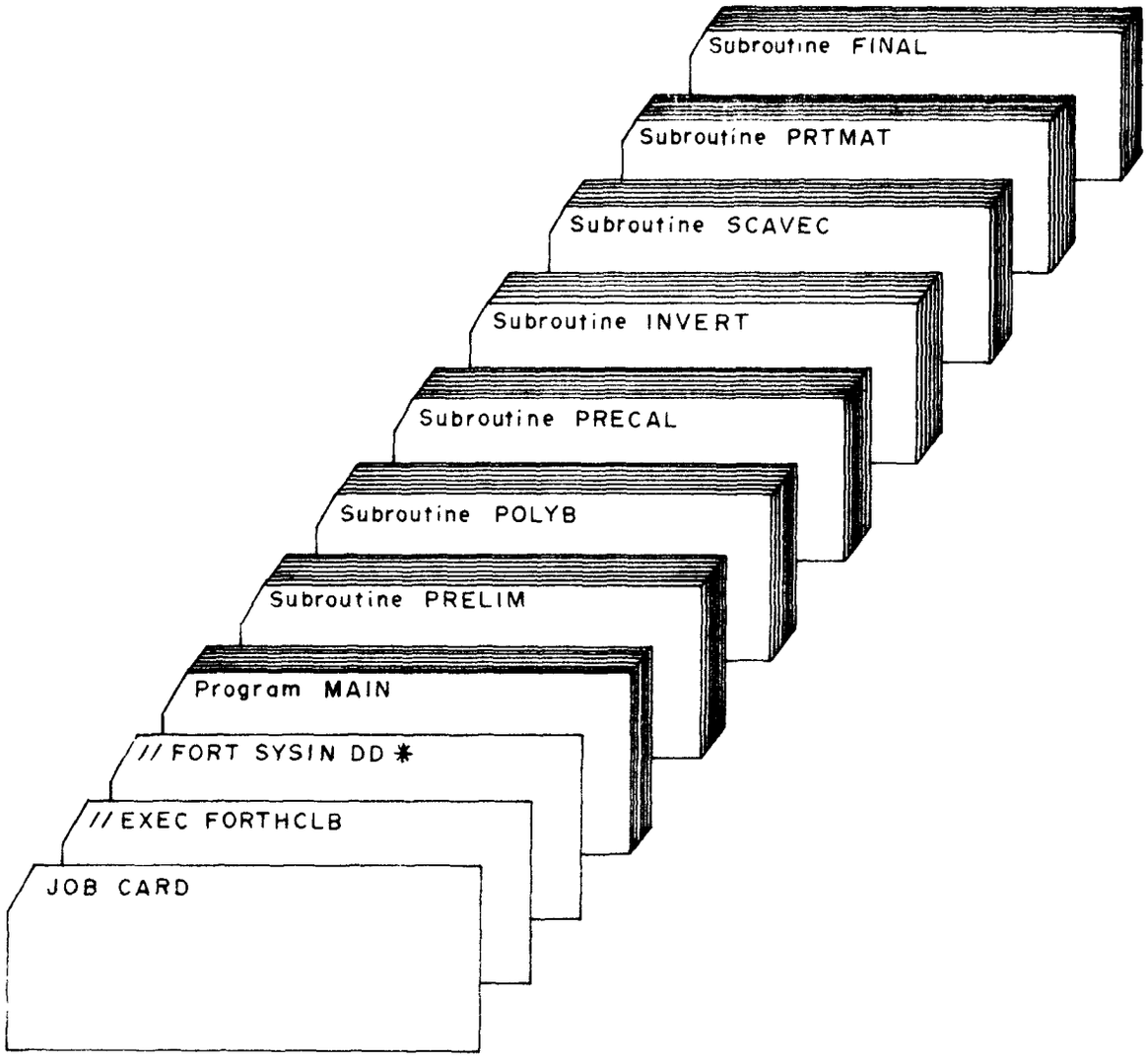


FIGURE 3.

PROGRAM DECK STRUCTURE

main program, which then calls the matrix inversion program. This sub-routine then inverts the tridiagonal matrix. The sub-routine SCAVEC performs the calculations on the inverted matrix that have been specified in the control card. PRTMAT is a general sub-routine used to print the inverted matrix and could be used to print any matrix with five elements printed per row per page. Sub-routine FINAL is used only when both the A and B matrix inversions have been specified for a single case of input data.

PRECFN is employed to adjust the vectors with proper dimensions when it is more convenient to enter the vectors in units other than those employed in the actual calculations. This allows the analyst to enter a conversion constant into the program when the source data set up is in other units.

As shown in Figure 3, the first card of the program deck is the job control card. The card format for this card at the U.S.G.S. Computer Center where the program was run is as follows:

JOB CONTROL CARD

Card Columns	Required Information
1 - 2	//
3	Center Code: For use in conjunction with the job or program number to identify the center originating or assigning the job or program number.
4 - 5	Federal Agency code
6 - 8	User Registration Code: Individual users registration code. To be assigned by Computer Center Division.

JOB CONTROL CARD (Continued)

Card Columns	Required Information
9 - 10	User's ID: This is structly a user's ID to uniquely identify submissions of data to the Computing Center. The only caution to be observed by the user is when two sets of data for the same program are submitted simultaneously, then different alphanumeric characters should be punched in columns 9 - 10.
11	Blank column
12-14	JOB (e.g., the word JOB)
15	Blank column
16	((left parenthesis)
17 - 20	Program Number: Four digit numeric job or program number assigned by the Computer Center Division at each field center and Washington, D. C. When a new number is assigned at any field center, the attached Program Registration Form should be completed in duplicate and transmitted to the Computer Operations Branch, Washington, D. C.
21	, (comma)
22 - 25	User assigned auxiliary account number. Four alpha-numeric characters chosen by the user according to any method he chooses. Accounting data will be sequenced and subtotaled by this number for user information.
26	, (comma)
27 - 30	Estimated execution time in minutes. Requires four numeric digits.
31	, (comma)
32 - 35	Estimated lines of print expressed in thousands of lines. Requires four numeric digits (e.g., 0001 = 1000 lines of print).
36	, (comma)

JOB CONTROL CARD (Continued)

Card Columns	Required Information
37 - 40	Estimated number of cards to be punched. Requires four numeric digits and is an exact number (e.g., 0100 = 100 cards).
41	, (comma)
42	Reserved for future use - must be 1.
43	, (comma)
44	Reserved for future use - must be 1.
45	, (comma)
46	Type run. C = Compile only T = Test of program P = Production use of program D = Data conversion required to convert from prior systems
47	) (right parenthesis)
48	, (comma)
49	' (single quote)
50 - 70	Programmer or user's name. May be from 1 to 20 characters, with or without imbedded blanks. Must be followed by one single quote ('). This field is required and cannot exceed 20 characters. If the user requires additional information in the JOB card, the following rules apply: (e.g., MSGLEVEL - 1 or other comments). A - Place a comma immediately following the trailing quote after name field. B - Punch any alphanumeric character into column 72 - Do Not Use this column for any other purpose.

C - Punch // in columns 1 and 2 of a second card.

D - Columns 3 - 15 MUST BE BLANK.

E - If MSGLEVEL = 1 is desired, it must be punched in columns 16 - 25 and followed by one or more blanks prior to any other comments.

F - If comments only are desired, leave column 16 blank and start the comment in column 17.

G - Column 72 of the second card MUST BE BLANK.

The second card in the program source deck is the EXEC statement card. The EXEC statement indicates the beginning of a job step and describes that job step. For the program here, the EXEC statement should appear as:

```
Columns No.  1  2  3  .  .  .  .  .  .  .  .  .  .  .  .  .  .
             /  /  b  E  X  E  C  b  F  O  R  T  H  C  L  G
```

(b = blank)

The third card in the program contains the following:

```
Columns No.  1  2  3  4  5  .  .  .  .  .  .  .  .  .  .  .  .
             /  /  F  O  R  T  .  S  Y  S  I  N  b  D  D  b  *
```

(b = blank)

The statement specifies the location of the source module(s) or the object module(s) to the control program.

VI. INPUT DATA PREPARATION AND PROGRAM CASE CONTROL

A. General

The model contains the feature of running any number of cases during a production run. The original run would consist of placing the FORTRAN source program before the data deck. Subsequent computer runs would use the binary deck (called the object program) in front of the data.

Appendix III shows a typical data deck structure. The first card in the deck contains the number of cases to be run. This variable* is identified by the name NCASES. The first three columns on the card are used to enter the value. It is possible to run 999 cases on an individual computer run.

The format for this card is:

CARD COLUMNS	1	2	3	4	5	.	.	.	.
			8						

Note that for the sample shown, the program would operate on 8 data sets before termination. In entering data for the number of cases, the value should always be right justified. If, for example, the 8 were inadvertently entered in column 2, the program would try to operate on 80 sets of information rather than the 8 desired.

The second card in the data deck is the date card. This should be given as the date the computer run will be made. Its

---

* The word "variable" is not used in the normal mathematical sense.

main use is to quickly identify the computer run since, in most simulation modeling, input information will be adjusted upon analysis of initial output, and at a later time the analyst may want to refer to prior runs.

The numeric values and special symbols normally used to write the date may be entered as:

5/17/67 (Starting in column 1)
or 05/17/67 (Starting in column 1)

The date may be entered in columns 1 to 24 on the card, but for use of identification, it would be convenient to establish a definite entry technique as shown above.

The third card in the data deck should contain the name of the user. Again 24 columns are employed, and the entry may appear in any of the card columns 1 through 24.

The fourth card in the data deck is the title card. This card is employed to identify the first set of data the program will operate on. Note in Appendix III that each subsequent set of input data contains a title card. Card columns 1 through 22 are employed. The data may be numeric or alphabetic.

The title card and its subsequent set of data may be considered as a separate subdivision from the entire data deck. In practice, the title card and its associated input data is a subset of the input deck; therefore, the following discussion will outline the subset structure.

B. Namelist Input

1. General Description of Namelist Input With Rules

The namelist input mode was chosen in order to avoid the detail necessary to code input data into a definite format structure. Under NAMELIST options, the data to be entered into the computer is loaded in memory without the use of explicit lists.

The general form of the NAMELIST statement is:

$$\text{NAMELIST/name}_1\text{V}_1, \text{V}_2, \text{V}_3, \ldots, \text{V}_n/\text{name}_2/\text{W}_1, \text{W}_2, \ldots, \text{W}_n$$

Here name_i is the name of the list of variables whose names are specified by V_i, W_i. The name of the NAMELIST may be from one to six characters in length.

In order to understand the data entry for a variable associated with a NAMELIST, the following general requirements must be satisfied:

a. Only columns 2-72 of the data cards may be used.

b. Columns 73-80 may be used for card sequencing or identification but will be ignored by the computer.

c. Each NAMELIST begins with an ampersand (&) in column 2, followed by the NAMELIST name, and followed by at least one blank.

d. The NAMELIST is terminated by an ampersand (&) in any column 2-72.

e. All entries are separated by commas; the last entry must also be followed by a comma.

f. Blanks may not be imbeded before or after the = sign. (Ex. A = 5.0, is illegal and should be punched as: A=5.0,)

g. Blanks may be embedded between data values after the first value of vectors or arrays as shown below:

(X=1.2,1.3,1.5 2.2,2.3,2.5,)

h. Exponential notation may be used for large or small values, i.e., 2.153E5 and 2.154E-2 denote 215300 and 0.02154.

i. If the decimal point is omitted, it is assumed to be at the extreme right.

j. No decimal point may be used in the entries of variables typed integer.

k. When an array name appears without subscripts, all elements in the array must be present.

l. To enter the same value in several elements of an array, the data card may be set up as follows:

If X is dimensioned as X (10); then

X-1.4,2.4,6.3,5*6,.2,7.1,8.5,

2. Description of Namelist and Variables Used in This Program

a. NAMELIST CF

This namelist deals with the problem controls and conversion factors.

NSECTS (integer) - Number of estuarine segments

IMAT (integer) - Program Control for A and/ or B matrix

INDEX (i)(integer) - Printing Option Control

ITP (i)(integer) - Control value variable employed by polynomial sub-routine and temperature conversion control

CFQ (real) - Advection conversion constant employed in sub-routine PRECFM

CFLEN (real) - Length conversion constant employed in sub-routine PRECFM

CFK (real) - Diffusion conversion constant employed in sub-routine PRECFM

CFL (real) - UOD conversion constant employed in sub-routine PRECFM

CFAREA (real) - Area conversion constant employed in sub-routine PRECFM

CFVOL (real) - Volume conversion constant employed in sub-routine PRECFM

CFP (real) - Dissolved oxygen sink and source conversion constant employed in sub-routine PRECFM

The above CF namelist is developed for each data set.

INDEX controls the printing of information with regard to a particular state of IMAT. In the event that IMAT would equal 1, two sets of information may be optionally printed, i.e., ALPHA x A^{-1} or ALPHA x VECTOR (L) x A^{-1}. If IMAT would equal 2, then the index would optionally be used to print ALPhA x B^{-1} or ALPHA x F x B^{-1}. In the third state, i.e., IMAT = 3, the INDEX option allows printing of ALPHA x A^{-1}, ALPHA x L x A^{-1}, ALPHA x B^{-1}, ALPHA x F x B^{-1}, ALPHA x A^{-1} x B^{-1}, VD $A^{-1}B^{-1}$, LVD A^{-1} B^{-1}, and D.O. Sat. + ALPHA x F x B^{-1} - LVD A^{-1} B^{-1}. IMAT and the index values will be discussed in greater detail later in this section.

A large number of possible combinations of output may be obtained. A typical output is shown in Appendix III, and each case shown here has been verified. Case K in the Appendix shows the results for IMAT = 3 and all INDEXES 1 through 8 set equal to 1.

The ITP variable is entered on the card following the INDEX control card. Two values should always be given for this variable. If ITP (2) is set equal to 1, the polynomial sub-routine is called. This sub-routine, discussed in Section V, generates the reaeration, dissolved oxygen at saturation, and the decay vector. If ITP (1) is set equal to 1, the temperature vector may be given in degrees Fahrenheit and is converted in the sub-routine to degrees Centigrade.

The conversion constants CFQ, CFLEN, CFK, CFL, CFAREA, CFVOL, and CFP are entered in succeeding cards. These constants are employed to convert their corresponding vectors to the proper units so that the actual vector input data may be entered in any units providing the corresponding conversion factor is given. In the event that the original input vectors are already in the proper units, the corresponding conversion factor would be set equal to 1.

b. NAMELIST RIVER

This namelist contains the parameters which describe the geometry of the river and the waste loadings to the system.

Q (i)(real) - $Q_{i-1,i}$, net flow of the estuary between segments i-1 and i, expressed in cubic feet per second. All boundaries, including the flow into the first and the flow out of the last segment, must be specified.

LENGTH (i)(real) - L_{i-1}, length of the estuary segments. Since a length above the uppermost segment and below the lowermost segment is required (see equations 9 and 19), n + 2 lengths must be specified. The input must be in feet. If other units are to be used, the proper conversion factor CFLEN (Namelist CF) must be used.

VOL (i)(real) - V_i, volume of section i in cubic feet. All sections must be specified. If units other than cubic feet are used, a suitable conversion factor must be specified (CFVOL in namelist CF).

AREA (i)(real) - $A_{i-1,i}$, cross-sectioned area of the interface between segments i-1 and i, in square feet. All interfaces, including the first and last, must be defined. Conversion factor CFAREA may be used.

DIFFCO (i)(real) - $K_{i-1,i}$, longitudinal dispersion coefficient between segments i-1 and i in square miles per day. All segment boundaries, including the first and last, must be specified. Conversion factor CFK is available if different units are used.

L (i)(real) - L_i, ultimate oxygen demand being discharged to segment i in pounds per day. All segments must be defined. Conversion factor CFL may be used if desired.

P (i)(real) - P_i, other sources or sinks of oxygen in segment i, in pounds per day. All segments must be defined even if all values are zero or meaningless. Conversion factor CFP is available.

c. NAMELIST RIVTO

This namelist is used if one wishes to specify definite values for the ultimate oxygen demand decay rate, the reaeration rate and the saturation value of oxygen in each segment.

DISOXS (i)(real) - C_{sc}, the saturation value of dissolved oxygen in segment i. All segments must be defined. The values must be entered in mg/l.

REAERK (i)(real) - r_i, the reaeration coefficient for segment i in units of 1/day. All segments must be defined.

DECAYK (i)(real) - d_i, the ultimate oxygen demand decay rate in segment i also with units of 1/day. All segments must be defined.

d. NAMELIST R R

This namelist is used if one wishes to generate the values of the ultimate oxygen demand decay rate, the reaeration rate and the saturation value of oxygen from the river temperature using polynomials.

T P (i)(real) - The temperature of the water in segment i. The temperature may be given in degrees Centigrade if ITP (2) in namelist CF is set equal to 0 or in degrees Fahrenheit if ITP (2) in namelist CF is set equal to 1.

K D (Integer) - The order of the polynomial which is being used to describe the decay rate.

C D (j)(real) - Polynomial coefficient of the jth term in the polynomial which is being used to describe the decay rate. C D must be defined for all values of $j \leq K D + 1$.

K R (integer) - The order of the polynomial which is being used to describe the reaeration rate.

C R (j)(real) - Polynomial coefficient of the jth term in the polynomial which is being used to describe the reaeration rate. C R must be defined for all values of $j \leq K R + 1$.

K C (integer) - The order of the polynomial which is being used to describe the dissolved oxygen saturation value.

C C (j)(real) - Polynomial coefficient of the jth term in the polynomial which is being used to describe the dissolved oxygen saturation value. C C must be defined for all values of $j \leq K C + 1$.

In order to understand the arrangement of the various NAMELISTS in the input data structure, the following explanation should prove helpful:

Consider the following:

1. CF Namelist (Always First and Present)
2. RIVER Namelist
3. RIVTO Namelist
4. R R Namelist

1. CF is always first and present.

Now, if ITP (2) = 0, NAMELIST R R is second, NAMELIST RIVER is third, and RIVTO is not required.

If ITP (2) = 1, RIVER is second, RIVTO is third, and RR is not required.

VII. BIBLIOGRAPHY

1. Streeter, H. W., and Phelps, E. B., "A Study of the Pollution and Natural Purification of the Ohio River - III, Factors Concerned in the Phenomena of Oxidation and Reaeration," Public Health Bulletin No. 146, U. S. Public Health Service, February 1925.

2. O'Connor, Donald J., "Oxygen Balance of An Estuary," Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Proceedings Paper 2472, Vol. 86, No. SA3, May 1960.

3. Thomann, Robert V., "Mathematical Model for Dissolved Oxygen," Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Proceedings Paper 3680, Vol. 89, No. SA5, October 1963.

4. Jeglic, John M., "Mathematical Simulation of the Estuarine Behavior," Digital Computer Technology and Programming Analysis Memo. No. 1032, Rev. A, General Electric Re-Entry Systems Department, Philadelphia, Pennsylvania, July 1967.

5. "Program for Analog Computer Simulation of Dye Diffusion in the Potomac River," Final Report by: Electronic Associates, Incorporated, Washington Computation Center to the Chesapeake Field Station, Chesapeake Bay-Susquehanna River Basins Project, Federal Water Pollution Control Administration, Annapolis, Maryland, in completion of Contract RO-3-2131-65, October 1965.

6. Hetling, Leo J., and O'Connell, Richard L., "A Study of Tidal Dispersion in the Potomac River," Water Resources Research, Vol. 2, No. 4, Fourth Quarter, 1966.

7. Hetling, Leo J., and O'Connell, Richard L., "An Oxygen Balance for the Potomac Estuary," CB-SRBP Technical Paper No. 13, Federal Water Pollution Control Administration, Middle Atlantic Region, Charlottesville, Virginia (in press).

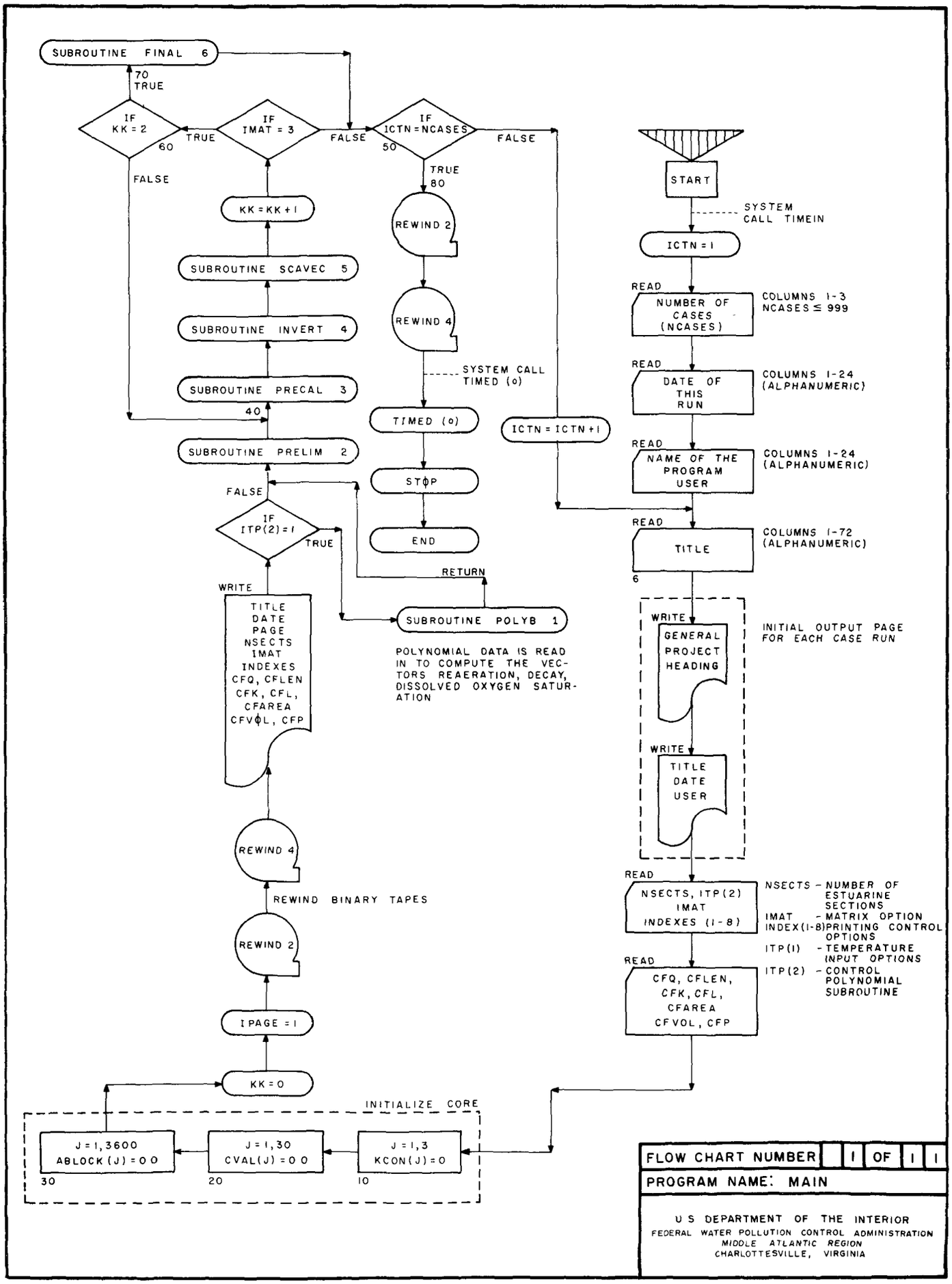
8. Hetling, Leo J., "Simulation of Chloride Concentrations in the Potomac Estuary," CB-SRBP Technical Paper No. 12, Federal Water Pollution Control Administration, Middle Atlantic Region, Charlottesville, Virginia (in press).

9. Hall, Charles W., and Hetling, Leo J., "Use of Mathematical Models as Aids to Decision Making in Water Quality Control," Presented at the Sixty-third National Meeting of the American Institute of Chemical Engineers, St. Louis, Missouri, February 19, 1968.

A P P E N D I X I

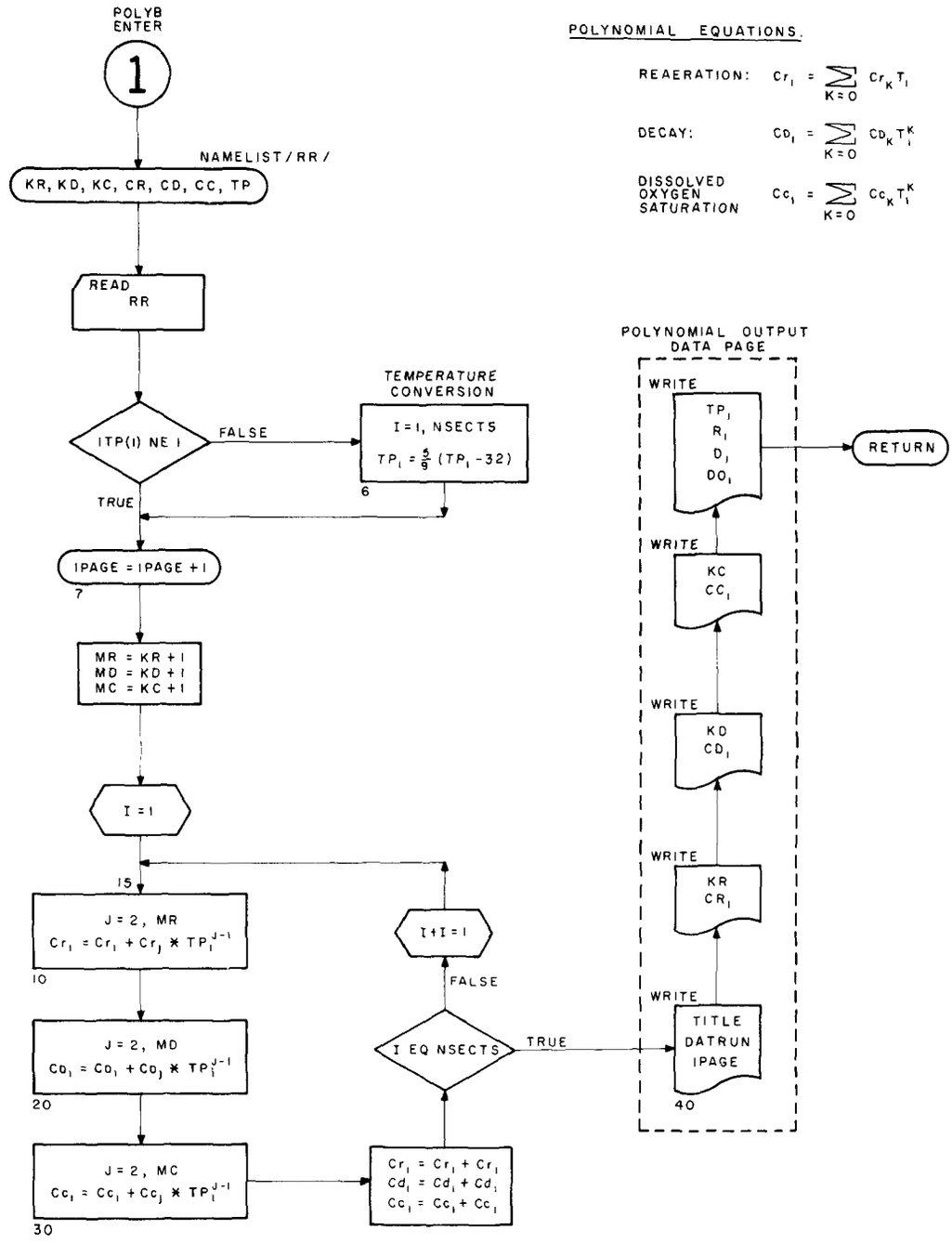
PROGRAM

FLOW DIAGRAMS



FLOW CHART NUMBER		1	OF	1	1
PROGRAM NAME: MAIN					

U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



POLYNOMIAL EQUATIONS.

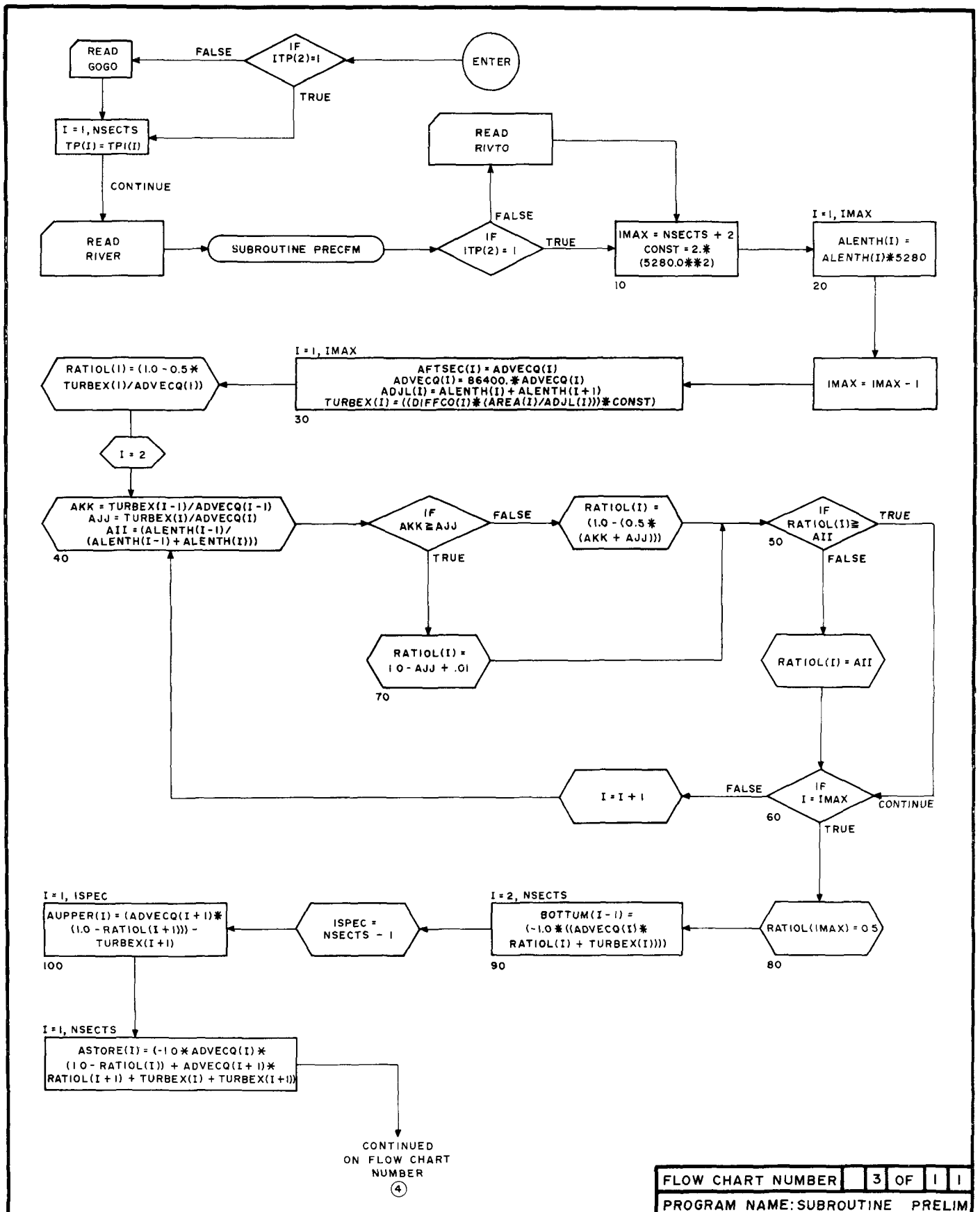
REAERATION: $Cr_I = \sum_{K=0}^{M} Cr_K T_I$

DECAY: $Cd_I = \sum_{K=0}^{M} Cd_K T_I^K$

DISSOLVED OXYGEN SATURATION: $Cc_I = \sum_{K=0}^{M} Cc_K T_I^K$

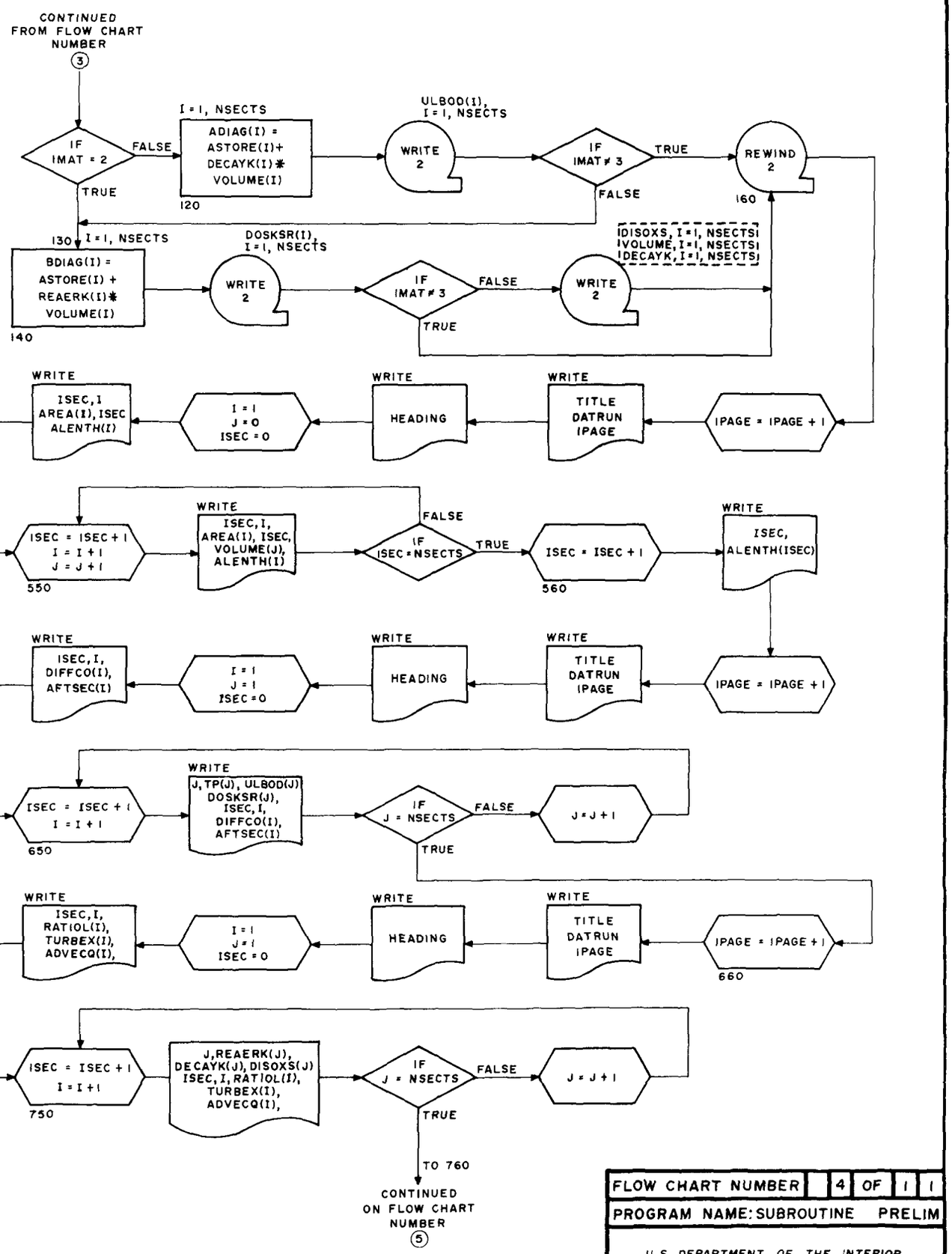
FLOW CHART NUMBER	2 OF 1 1
PROGRAM NAME: SUBROUTINE POLYB	

U S. DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



FLOW CHART NUMBER	3	OF	1	1
PROGRAM NAME: SUBROUTINE PRELIM				

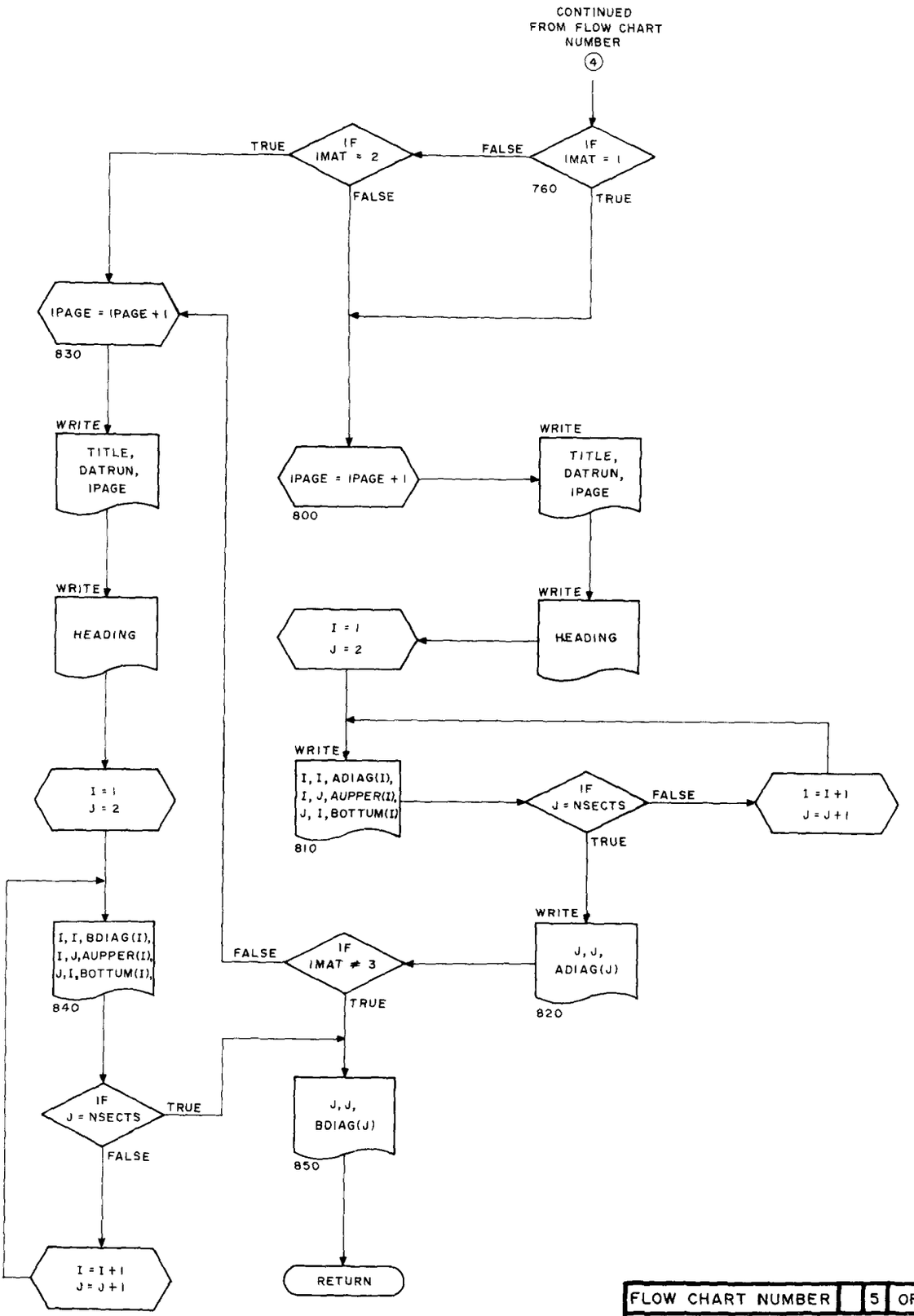
U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



FLOW CHART NUMBER		4	OF	1	1

PROGRAM NAME: SUBROUTINE PRELIM

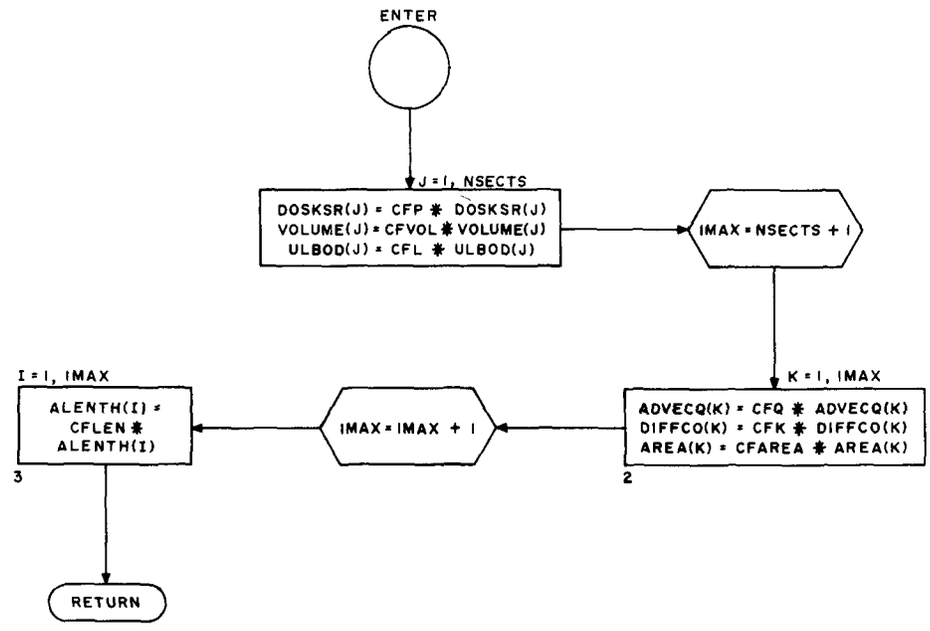
U.S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



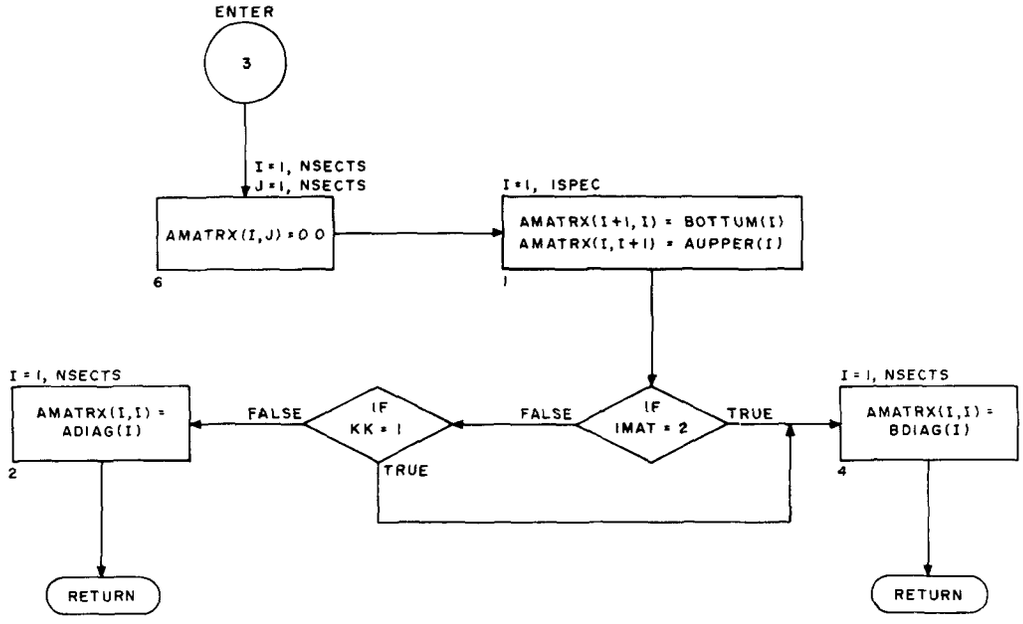
FLOW CHART NUMBER		5	OF	1	1

PROGRAM NAME: SUBROUTINE PRELIM

U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA

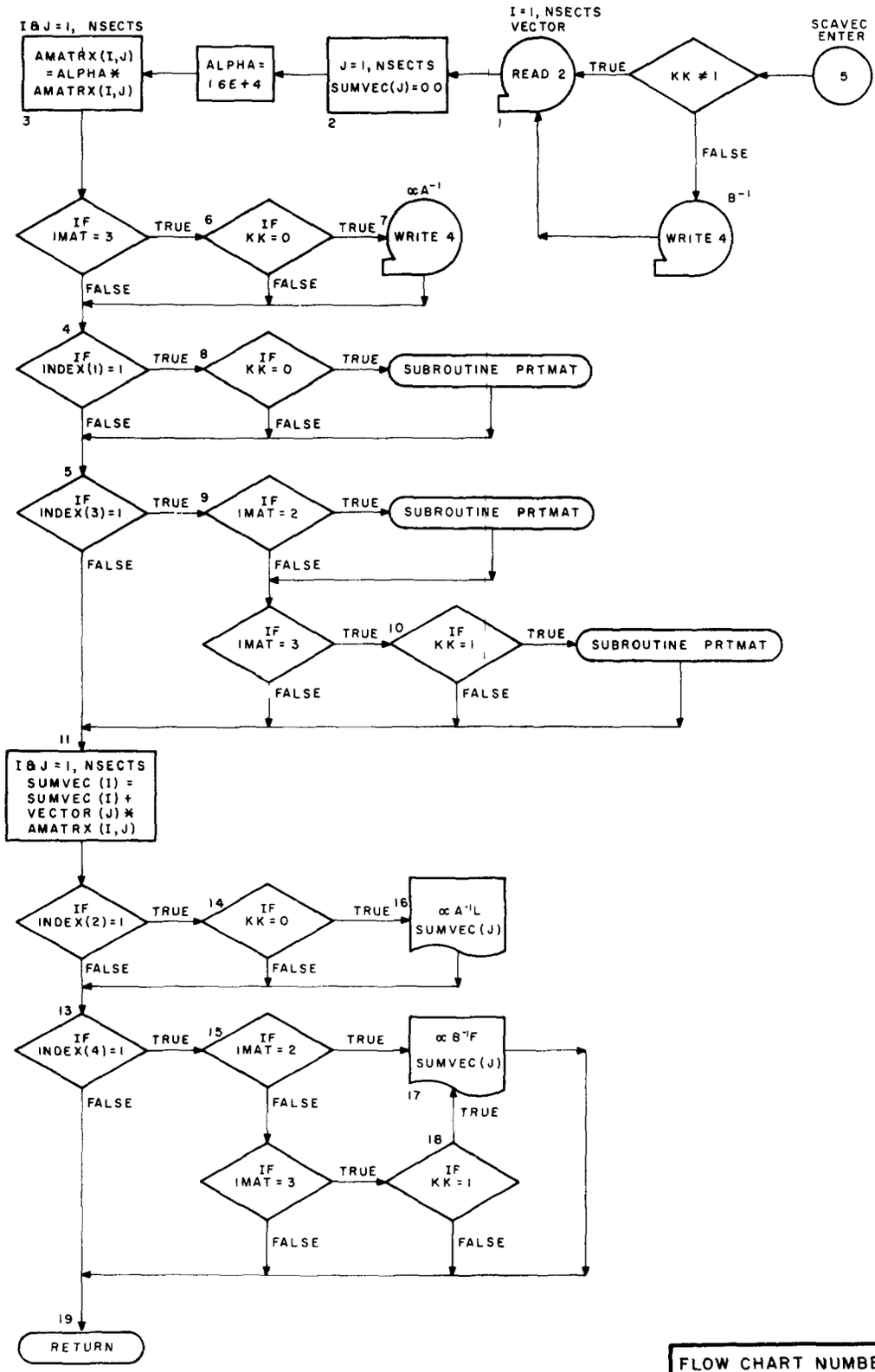


FLOW CHART NUMBER	1	1	OF	1	1
PROGRAM NAME: SUBROUTINE PRECFM					



FLOW CHART NUMBER		6	OF	1	1
PROGRAM NAME: SUBROUTINE PRECAL					

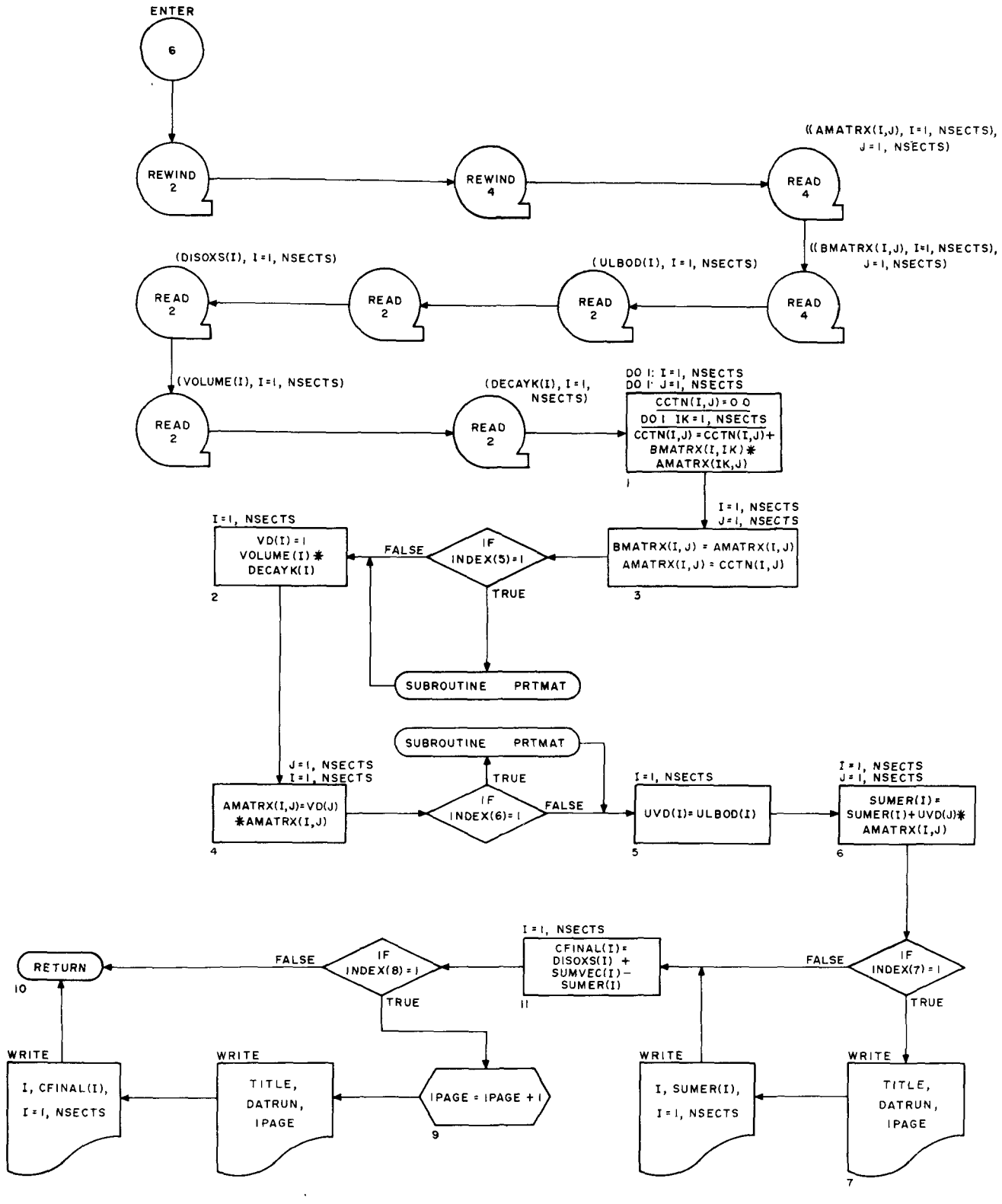
U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



FLOW CHART NUMBER		7	OF	1	1

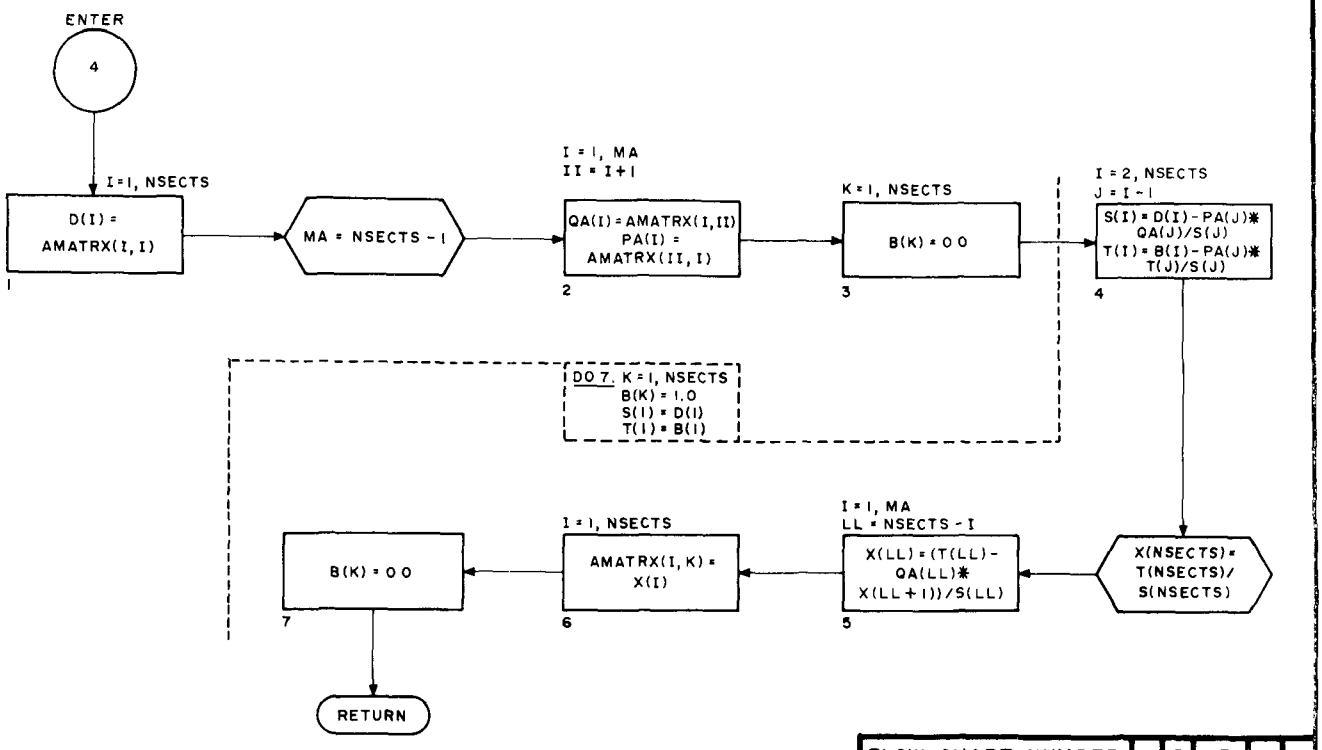
PROGRAM NAME: SUBROUTINE SCAVEC

U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



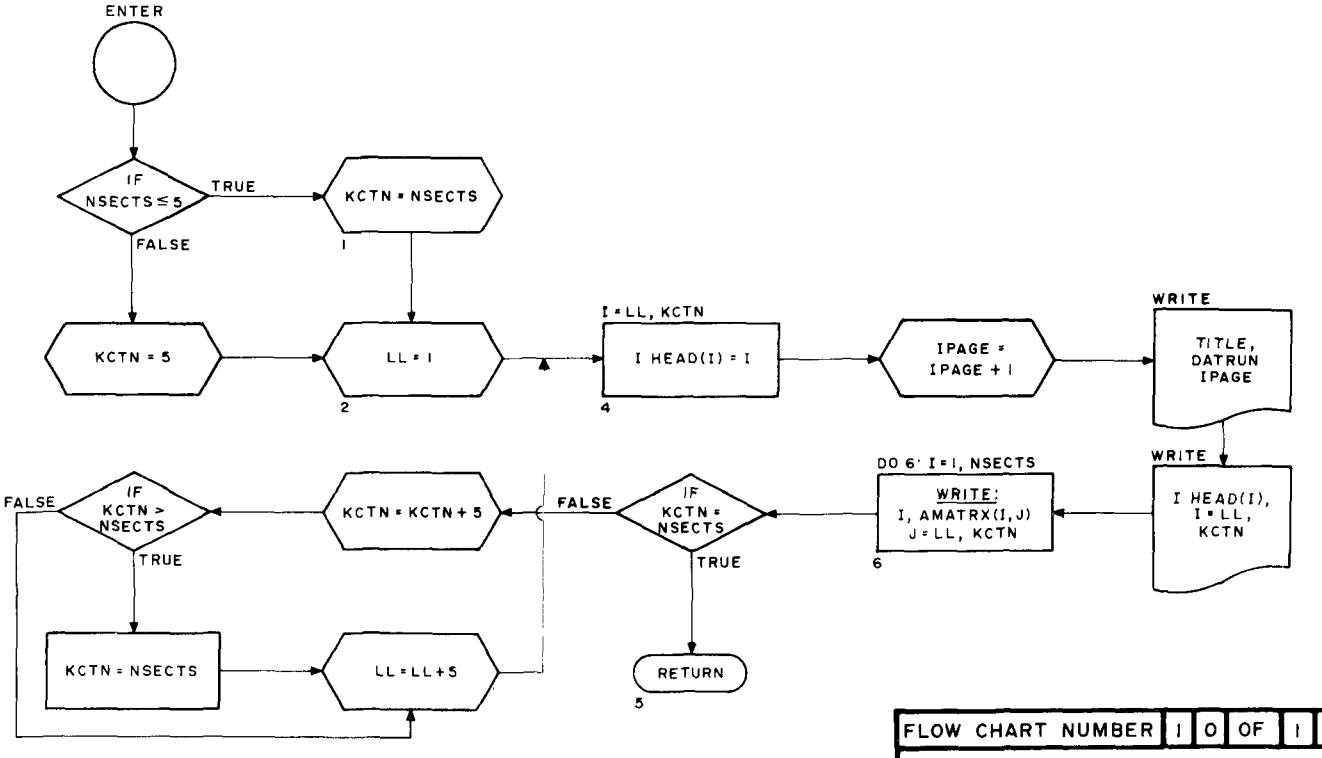
FLOW CHART NUMBER		8	OF	1	1
PROGRAM NAME: SUBROUTINE FINAL					

U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA



FLOW CHART NUMBER 9 OF 11

PROGRAM NAME: SUBROUTINE INVERT



FLOW CHART NUMBER 10 OF 11

PROGRAM NAME: SUBROUTINE PRTMAT

U S DEPARTMENT OF THE INTERIOR
FEDERAL WATER POLLUTION CONTROL ADMINISTRATION
MIDDLE ATLANTIC REGION
CHARLOTTESVILLE, VIRGINIA

A P P E N D I X II

IBM 360 VERSION I

FORTRAN PROGRAM LISTING

```
                C      PROGRAM MAIN
                C      MIDDLE ATLANTIC REGION - IBM 360 VERSION I
                C      STEADY STATE SEGMENTED ESTUARY MODEL
                C      FOR FURTHER INFORMATION CONTACT
                C            PROJECT ENGINEER- DR. L.HETLING
                C            SYSTEMS ANALYST-PROGRAMMER- R.E.BUNCE
                C
                C      STORAGE ALLOCATION-MAIN PROGRAM
                C
ISN 0002              DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0003              COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0004              COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,ITP(
                     12),CFQ,CFLEN,CFK,CFL,CFAREA,CFVOL,CFP
ISN 0005              COMMON/AWORKS/KCON(3),CVAL(30),TP(40),TP1(40)
ISN 0006              COMMON/BWORKS/ABLOCK(3600),CCTN(1600)
ISN 0007              NAMELIST/CF/NSECTS,IMAT,INDEX,ITP,CFQ,CFLEN,CFK,CFL,CFAREA,CFVOL,C
                     1FP
                C
                C      CONTROL INFORMATION
                C
                C      TITLE-72ALPHANUMERIC CHARACTERS
                C      NCASES - NUMBER OF RUNS FOR PRODUCTION       CONTROL CARD 1,COL1-3
                C      DATRUN - DATE FOR THE COMPUTER RUN           CONTROL CARD 2,COL1-24
                C      USER   - NAME OF THE PROGRAM USER            CONTROL CARD 2,COL1-24
                C      TITLE  - INFORMATION FOR THE HEADING         CONTROL CARD 3,COL1-72
                C
                C      DATA INPUT-CONTROL INFORMATION AND HEADING OUTPUT DATA
                C
ISN 0008              CALL TIMEIN
ISN 0009              ICTN=1
ISN 0010              READ (5,100) NCASES
ISN 0011              READ (5,150) (DATRUN(I),I=1,4)
ISN 0012              READ (5,150) (USER(I),I=1,4)
ISN 0013            6 READ (5,200) (TITLE(I),I=1,12)
ISN 0014              WRITE (6,250)
ISN 0015              WRITE(6,300)(TITLE(I),I=1,12),(DATRUN(I),I=1,4),(USER(I),I=1,4)
ISN 0016              READ (5,CF)
                C
                C      INITIALIZE CORE
                C
ISN 0017              DO 10 J=1,3
ISN 0018           10 KCON(J)=0
ISN 0019              DO 20 J=1,30
ISN 0020           20 CVAL(J)=0.0
ISN 0021              DO 30 J=1,3600
ISN 0022           30 ABLOCK(J)=0.0
ISN 0023              KK=0
ISN 0024              IPAGE=1
ISN 0025              REWIND 2
ISN 0026              REWIND 4
ISN 0027              WRITE (6,400) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0028              WRITE (6,450) NSECTS,IMAT,INDEX(1)
ISN 0029              WRITE (6,500) INDEX(2),INDEX(3),INDEX(4)
```

```
ISN 0030            WRITE (6,550) INDEX(5),INDEX(6),INDEX(7),INDEX(8)
ISN 0031            WRITE (6,650) CFQ,CFLEN,CFK,CFL,CFAREA,CFVOL,CFP
ISN 0032            IF(ITP(2).EQ.1) CALL POLYB
ISN 0034            CALL PRELIM
ISN 0035         40 CALL PRECAL
ISN 0036            CALL INVERT
ISN 0037            CALL SCAVEC
ISN 0038            KK=KK+1
ISN 0039            IF (IMAT.EQ.3) GO TO 60
ISN 0041         50 IF (ICTN.EQ.NCASES) GO TO 80
ISN 0043            ICTN=ICTN+1
ISN 0044            GO TO 6
ISN 0045         60 IF (KK.EQ.2) GO TO 70
ISN 0047            GO TO 40
ISN 0048         70 CALL FINAL
ISN 0049            GO TO 50
ISN 0050         80 REWIND 2
ISN 0051            REWIND 4
ISN 0052        100 FORMAT (I3)
ISN 0053        150 FORMAT (4A6)
ISN 0054        200 FORMAT (12A6)
ISN 0055        250 FORMAT (1H1////////////////44X,49(1H*)/44X,49H                MIDDLE
                   1 ALTANTIC REGION             /53X,36HSTEADY STATE SEGMENTED ESTUARY
                   2 MODEL/44X,49(1H*)//////////)
ISN 0056        300 FORMAT (1H0,15X18HPROBLEM TITLE  *  ,12A6/16X18HDATE OF RUN    *
                   1,4A6/16X8HENGINEER,7X,3H*  ,4A6)
ISN 0057        350 FORMAT (I2,I2,8I2,2I2)
ISN 0058        400 FORMAT (1H1,12A6,2X,4A6,2X5HPAGE ,I2///)
ISN 0059        450 FORMAT (1H ,26X20HPROGRAM CASE CONTROL///18X30HNUMBER OF ESTUARINE
                   1 SECTIONS= ,I2//18X34HMATRIX CASE BEING CONSIDERED=IMAT=,I2/21X23H
                   2WHERE= IMAT=1  A MATRIX/28X16HIMAT=2  B MATRIX/28X22HIMAT=3  A AND
                   3 B MATRIX//18X38HPRINTING INDEX OPTIONS   IF INDEX(N)=1/21X23HFOR
                   4THIS CASE INDEX(1)=,I2,2X,22HALPHA A MATRIX INVERSE)
ISN 0060        500 FORMAT (1H ,34X,9HINDEX(2)=,I2,2X24HALPHA A MATRIX INVERSE L/35X,9
                   1HINDEX(3)=,I2,2X,22HALPHA B MATRIX INVERSE/35X,9HINDEX(4)=,I2,2X,2
                   24HALPHA B MATRIX INVERSE F)
ISN 0061        550 FORMAT (1H ,34X,9HINDEX(5)=,I2,2X24HALPHA B A MATRIX INVERSE/35X,9
                   1HINDEX(6)=,I2,2X,70HALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I)
                   2FORMED ON MATRIX COLUMNS/35X,9HINDEX(7)=,I2,2X,31HALPHA B A MATRIX
                   3 INVERSE V D L /35X,9HINDEX(8)=,I2,2X,69HDISOXS + ALPHA B MATRIX
                   4INVERSE F - ALPHA B A MATRIX INVERSE V D L  ///)
ISN 0062        600 FORMAT (1H ,18X68HTHE CONTROL VALUE ITP ALLOWS COMPUTATION BY POLY
                   1NOMIAL APPROXIMATION/19X66HFOR THE VECTORS * REAERATION,DECAY,AND
                   2DISSOLVED OXYGEN SATURATION//35X55HIF ITP(2) IS SET EQUAL TO 1 THE
                   3POLYNOMIALS ARE EMPLOYED/35X51HIF ITP(1) IS SET EQUAL TO 1 TEMPERA
                   4TURE DATA MAY BE/35X61HENTERED IN DEGREES FAHRENHEIT FOR THE POLYN
                  5OMIAL COMPUTATIONS//35X22HFOR THIS CASE ITP(Z1)=,I2/49X,7HITP(2.=,I
                   62)
ISN 0063        650 FORMAT (1H ,////18X42HCONVERSION FACTORS USED ON INPUT VARIABLES/2
                   19X,8HCFQ    =,F6.2/29X,8HCFLEN  =,F6.2/29X,8HCFK    =,F6.2/29X,8HC
                   2FL    =, F6.2/29X,8HCFAREA =,F6.2/29X,8HCFVOL  =,F6.2/29X,8HCFP
                   3 =,F6.2//)
ISN 0064            CALL TIMED(0)
ISN 0065            STOP
ISN 0066            END
```

```
ISN 0002            SUBROUTINE POLYB
                C
                C     STORAGE ALLOCATION
                C
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0005            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0006            COMMON/AWORKS/KR,KD,KC,CR(10),CD(10),CC(10),TP(40),TP1(40)
ISN 0007            COMMON/BWORKS/AMATRX(40,40),REAERK(40),ADVECQ(40),DOSKSR(40),DIFFC
                   10(40),ALENTH(40),AFTSEC(40),ADJL(40),TURBFX(40),RATIOL(40),VECTOR(
                   240),AWASTE(1000),ASTORE(40),ADIAG(40),BDIAG(40),AUPPER(40),BOTTOM(
                   340),DISOXS(40),ULBOD(40),VOLUME(40),DECAYK(40),VD(40),UVD(40),AREA
                   4(40),SUMER(40),CFINAL(40),SUMVEC(40),CCTN(1600)
ISN 0008            DIMENSION Q(40),L(40),P(40),LENGTH(40),VOL(40)
ISN 0009            REAL L,LENGTH
ISN 0010            EQUIVALENCE (Q,ADVECQ),(ULBOD,L),(DOSKSR,P),(ALENTH,LENGTH),(VOLUM
                   1E,VOL)
                C
                C     LOAD INPUT LIST
                C
ISN 0011            NAMELIST/RR/KR,KD,KC,CR,CD,CC,TP
ISN 0012            READ (5,RR)
                C
                C     COMPUTE TEMPERATURE
                C
ISN 0013            IF (ITP(1).NE.1) GO TO 7
ISN 0015            DO 6 I=1,NSECTS
ISN 0016          6 TP(I)=((5./9.)*(TP(I)-32.))
ISN 0017          7 IPAGE=IPAGE + 1
                C
                C     COMPUTE POLYNOMIALS
                C
ISN 0018            MR=KR+1
ISN 0019            MD=KD+1
ISN 0020            MC=KC+1
ISN 0021            I=1
ISN 0022         15 DO 10 J=2,MR
ISN 0023         10 REAERK(I)=REAERK(I) + CR(J) * TP(I)**(J-1)
ISN 0024            DO 20 J=2,MD
ISN 0025         20 DECAYK(I)=DECAYK(I) + CD(J) * TP(I)**(J-1)
ISN 0026            DO 30 J=2,MC
ISN 0027         30 DISOXS(I)=DISOXS(I) + CC(J) * TP(I)**(J-1)
ISN 0028            REAERK(I)= REAERK(I) + CR(1)
ISN 0029            DECAYK(I)= DECAYK(I) + CD(1)
ISN 0030            DISOXS(I)= DISOXS(I) + CC(1)
ISN 0031            IF (I.EQ.NSECTS) GO TO 40
ISN 0033            I=I+1
ISN 0034            GO TO 15
                C
                C     PREPARE OUTPUT CONTROL PAGE
                C
ISN 0035         40 WRITE (6,100) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
```

```
ISN 0036            WRITE (6,150) KR,(CR(I),I=1,MR)
ISN 0037            WRITE (6,200) KD,(CD(I),I=1,MD)
ISN 0038            WRITE (6,250) KC,(CC(I),I=1,MC)
ISN 0039            WRITE (6,300) (I,REAERK(I),DECAYK(I),DISOXS(I),TP(I),I=1,NSECTS)
ISN 0040          100 FORMAT (1H1,12A6,2X,4A6,2X5HPAGE ,I2///)
ISN 0041          150 FORMAT (1H ,18X55HPOLYNOMIAL POWER AND COEFFICIENTS FOR REAERATION
                     1 VALUES//27X3HKR=,I2,16X2HCD/(40X,E15.8))
ISN 0042          200 FORMAT (1H0,18X50HPOLYNOMIAL POWER AND COEFFICIENTS FOR DECAY VALU
                     1ES//27X3HKD=,I2,16X2HCD/(40X,E15.8))
ISN 0043          250 FORMAT (1H0,18X65HPOLYNOMIAL POWER AND COEFFICIENTS FOR DISSOLVED
                     1OXYGEN SATURATION//27X3HKC=,I2,16X2HCC/(40X,E15.8))
ISN 0044          300 FORMAT (1H ,///18X45HCOMPUTED VALUES FROM THE POLYNOMIAL EQUATIONS
                     1//18X2H I,7X10HREAERATION,12X5HDECAY,3X14HD.O.SATURATION,6X11HTEMP
                     2ERATURE/18X19H              VALUES,11X6HVALUES,11X6HVALUES,11X6HVAL
                     3UES//(18XI2,4XE13.6,4XE13.6,4XE13.6,4XE13.6))
ISN 0045            RETURN
ISN 0046            END
```

```
ISN 0002            SUBROUTINE PRELIM
                C
                C     STORAGE ALLOCATION
                C
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0005            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0006            COMMON/AWORKS/KR,KD,KC,CR(10),CD(10),CC(10),TP(40),TP1(40)
ISN 0007            COMMON/BWORKS/AMATRX(40,40),REAERK(40),ADVECQ(40),DOSKSR(40),DIFFC
                   10(40),ALENTH(40),AFTSEC(40),ADJL(40),TURBEX(40),RATIOL(40),VECTOR(
                   240),AWASTE(1000),ASTORE(40),ADIAG(40),BDIAG(40),AUPPER(40),BOTTOM(
                   340),DISOXS(40),ULBOD(40),VOLUME(40),DECAYK(40),VD(40),UVD(40),AREA
                   4(40),SUMER(40),CFINAL(40),SUMVEC(40),CCTN(1600)
ISN 0008            DIMENSION Q(40),L(40),P(40),LENGTH(40),VOL(40)
ISN 0009            REAL L,LENGTH
ISN 0010            EQUIVALENCE (Q,ADVECQ),(ULBOD,L),(DOSKSR,P),(ALENTH,LENGTH),(VOLUM
                   1E,VOL)
                C
                C     NAMELIST FOR INPUT DATA
                C
ISN 0011            NAMELIST/RIVER/Q,L,P,VOL,AREA,DIFFCO,LENGTH
ISN 0012            NAMELIST/RIVTO/DISOXS,REAERK,DECAYK
ISN 0013            NAMELIST/GOGO/TP1
ISN 0014            IF (ITP(2).EQ.1) GO TO 5
ISN 0016            READ (5,GOGO)
ISN 0017            DO 4 I=1,NSECTS
ISN 0018          4 TP(I)=TP1(I)
ISN 0019          5 CONTINUE
ISN 0020            READ (5,RIVER)
ISN 0021            CALL PRECFM
ISN 0022            IF(ITP(2).EQ.1) GO TO 10
ISN 0024            READ (5,RIVTO)
ISN 0025         10 IMAX = NSECTS + 2
ISN 0026            CONST= 2.*(5280.0**2)
ISN 0027            DO 20 I=1,IMAX
ISN 0028         20 ALENTH(I)=ALENTH(I)*5280.
ISN 0029            IMAX=IMAX - 1
ISN 0030            DO 30 I=1,IMAX
ISN 0031            AFTSEC(I)=ADVECQ(I)
ISN 0032            ADVECQ(I) = 86400.*ADVECQ(I)
ISN 0033            ADJL(I)   = ALENTH(I) + ALENTH(I+1)
ISN 0034         30 TURBEX(I)=((DIFFCO(I)*(AREA(I)/ADJL(I)))*CONST)
                C
                C     PREPARATION OF THE RATIOL VALUES
                C
ISN 0035            RATIOL(1)=(1.0-0.5*TURBEX(1)/ADVECQ(1))
ISN 0036            I=2
ISN 0037         40 AKK=TURBEX(I-1)/ADVECQ(I-1)
ISN 0038            AJJ=TURBEX(I)/ADVECQ(I)
ISN 0039            AII=(ALENTH(I-1)/(ALENTH(I-1)+ALENTH(I)))
ISN 0040            IF(AKK.GE.AJJ) GO TO 70
ISN 0042            RATIOL(I)=(1.0 - (0.5*(AKK + AJJ)))
```

```
ISN 0043          50 IF(RATIOL(I).GE.AII) GO TO 60
ISN 0045             RATIOL(I)=AII
ISN 0046          60 CONTINUE
ISN 0047             IF(I.EQ.IMAX) GO TO 80
ISN 0049             I=I+1
ISN 0050             GO TO 40
ISN 0051          70 RATIOL(I)=1.0-AJJ+.01
ISN 0052             GO TO 50
ISN 0053          80 RATIOL(IMAX)=0.5
                  C
                  C     PREPARATION OF THE DIAGONAL MATRIX ELEMENTS
                  C
ISN 0054             DO 90 I=2,NSECTS
ISN 0055          90 BOTTUM(I-1)=(-1.0*((ADVECQ(I)*RATIOL(I)+TURBEX(I))))
ISN 0056             ISPEC = NSECTS - 1
ISN 0057             DO 100 I=1,ISPEC
ISN 0058         100 AUPPER(I)=(ADVECQ(I+1) *(1.0 - RATIOL(I+1)))-TURBEX(I+1)
ISN 0059             DO 110 I =1,NSECTS
ISN 0060         110 ASTORE(I)=(-1.0 *ADVECQ(I)*(1.0 -RATIOL(I))+ADVECQ(I+1)*RATIOL(I+1
                     1) + TURBEX(I) + TURBEX(I+1))
                  C
                  C     PREPARE MAIN DIAGONAL FOR A OR B OR BOTH MATRIX ELEMENTS
                  C
ISN 0061             IF(IMAT.EQ.2) GO TO 130
ISN 0063             DO 120 I=1,NSECTS
ISN 0064         120 ADIAG(I) = ASTORE(I) + DECAYK(I) * VOLUME(I)
ISN 0065             WRITE (2) (ULBOD(I),I=1,NSECTS)
ISN 0066             IF(IMAT.NE.3) GO TO 160
ISN 0068         130 DO 140 I=1,NSECTS
ISN 0069         140 BDIAG(I) = ASTORE(I) + REAERK(I) * VOLUME(I)
ISN 0070             WRITE (2) (DOSKSR(I),I=1,NSECTS)
ISN 0071             IF(IMAT.NE.3) GO TO 160
ISN 0073             WRITE (2) (DISOXS(I),I=1,NSECTS)
ISN 0074             WRITE (2) (VOLUME(I),I=1,NSECTS)
ISN 0075             WRITE (2) (DECAYK(I),I=1,NSECTS)
ISN 0076         160 CONTINUE
ISN 0077             REWIND 2
                  C
                  C     GEOMETRIC INPUT DATA-PREPARATION OUTPUT-DESIGNED AS A UNIT
                  C
ISN 0078             IPAGE=IPAGE+1
ISN 0079             WRITE (6,1) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0080             WRITE (6,500)
ISN 0081             I    =1
ISN 0082             J    =0
ISN 0083             ISEC=0
ISN 0084             WRITE (6,501) ISEC,I,AREA(I),ISEC,ALENTH(I)
ISN 0085         550 ISEC=ISEC+1
ISN 0086             I=I+1
ISN 0087             J=J+1
ISN 0088             WRITE (6,502) ISEC,I,AREA(I),ISEC,VOLUME(J),ALENTH(I)
ISN 0089             IF(ISEC.EQ.NSECTS) GO TO 560
ISN 0091             GO TO 550
ISN 0092         560 ISEC=ISEC+1
```

```
ISN 0093            WRITE (6,503) ISEC,ALENTH(ISEC)
                C
                C     HYDRAULIC-LOAD OUTPUT - DESIGNED AS UNIT
                C
ISN 0094            IPAGE=IPAGE + 1
ISN 0095            WRITE (6,1) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0096            WRITE (6,600)
ISN 0097            I=1
ISN 0098            J=1
ISN 0099            ISEC=0
ISN 0100            WRITE (6,601) ISEC,I,DIFFCO(I),AFTSEC(I)
ISN 0101          650 ISEC=ISEC+1
ISN 0102            I=I+1
ISN 0103            WRITE (6,602) J,TP(J),ULBOD(J),DOSKSR(J),ISEC,I,DIFFCO(I),AFTSEC(I
                   1)
ISN 0104            IF(J.EQ.NSECTS) GO TO 660
ISN 0106            J=J+1
ISN 0107            GO TO 650
ISN 0108          660 CONTINUE
                C
                C     COMPUTED SYSTEM PARAMETERS
                C
ISN 0109            IPAGE=IPAGE+1
ISN 0110            WRITE (6,1) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0111            WRITE (6,700)
ISN 0112            I=1
ISN 0113            J=1
ISN 0114            ISEC=0
ISN 0115            WRITE (6,701) ISEC,I,RATIOL(I),TURBEX(I),ADVECQ(I)
ISN 0116          750 ISEC=ISEC+1
ISN 0117            I=I+1
ISN 0118            WRITE (6,702) J,REAERK(J),DECAYK(J),DISOXS(J),ISEC,I,RATIOL(I),TUR
                   1BEX(I),ADVECQ(I)
ISN 0119            IF(J.EQ.NSECTS) GO TO 760
ISN 0121            J=J+1
ISN 0122            GO TO 750
ISN 0123          760 CONTINUE
                C
                C     MATRIX ELEMENT OUTPUT DATA
                C
                C
ISN 0124            IF(IMAT.EQ.1) GO TO 800
ISN 0126            IF(IMAT.EQ.2) GO TO 830
ISN 0128          800 IPAGE=IPAGE+1
ISN 0129            WRITE (6,1) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0130            WRITE (6,801)
ISN 0131            I=1
ISN 0132            J=2
ISN 0133          810 WRITE (6,802) I,I,ADIAG(I),I,J,AUPPER(I),J,I,BOTTOM(I)
ISN 0134            IF(J.EQ.NSECTS) GO TO 820
ISN 0136            I=I+1
ISN 0137            J=J+1
ISN 0138            GO TO 810
ISN 0139          820 WRITE (6,805)  J,J,ADIAG(J)
```

```
ISN 0140            IF(IMAT.NE.3) GO TO 850
ISN 0142          830 IPAGE=IPAGE+1
ISN 0143            WRITE (6,1) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0144            WRITE (6,803)
ISN 0145            I=1
ISN 0146            J=2
ISN 0147          840 WRITE (6,804) I,I,BDIAG(I),I,J,AUPPER(I),J,I,BOTTUM(I)
ISN 0148            IF(J.EQ.NSECTS) GO TO 850
ISN 0150            I=I+1
ISN 0151            J=J+1
ISN 0152            GO TO 840
ISN 0153          850 WRITE (6,806)  J,J,BDIAG(J)
ISN 0154            1 FORMAT (1H1,12A6,2X4A6,2X5HPAGE ,I2///)
ISN 0155          500 FORMAT (1HO,42X39HG E O M E T R I C   I N P U T   D A T A///11X89H
                     1INTERFACE                    AREA              SECTION                VOLU
                     2ME                LENGTH/13X86HNUMBER                  (SQ.FT)
                     3   NUMBER                (CU.FT.)                (FT.)//)
ISN 0156          501 FORMAT (1H ,10XI2,2X1H-,2XI2,11XF10.0,13XI2,34XF11.1)
ISN 0157          502 FORMAT (1H ,10XI2,2X1H-,2XI2,11XF10.0,13XI2,14XE13.6,7XF11.1)
ISN 0158          503 FORMAT (1H ,53XI2,34XF11.1)
ISN 0159          600 FORMAT (1H ,29X53HH Y D R A U L I C  A N D  L O A D  I N P U T  D
                     1A T A///1X,103HSECTION        T              J                  P
                     2             INTERFACE          K                 •  Q/1X,108HNUMBER
                     3       (DEG C)       (LBS/DAY)          (LBS/DAY)          NUMBER
                     4       (SQ.MI/DAY)        (CU.FT/SEC)//)
ISN 0160          601 FORMAT (1H ,63XI2,3H - ,I2,6XE13.6,6XE13.6)
ISN 0161          602 FORMAT (1H ,2XI2,6XF8.3,6XF13.6,6XE13.6,7XI2,3H - ,I2,6XE13.6,6XE1
                     13.6)
ISN 0162          700 FORMAT (1H ,43X49HC O M P U T E D  S Y S T E M  P A R A M E T E R
                     1S//5X7HSECTION,9X1HR,10X1HD,10X10HSATURATION,9X9HINTERFACE,13X2HXI
                     2,12X1HE,15X1HQ/5X6HNUMBER,7X6H(/DAY),6X6H(/DAY),8X10HD.O.(MG/L),10
                     3X6HNUMBER,24X12H(CU.FT./DAY),5X12H(CU.FT./DAY)///)
ISN 0163          701 FORMAT (1H ,62XI2,3H - ,I2,8XE13.6,2XF13.6,4XE13.6)
ISN 0164          702 FORMAT (1H ,6XI2,8XF7.3,5XF7.3, 9XF7.3,11XI2,3H - ,I2,8XE13.6,2XE1
                     13.6,4XE13.6)
ISN 0165          801 FORMAT (1H ,43X45HD I A G O N A L  M A T R I X  E L E M E N T S//6
                     13X8HA-MATRIX//16X22HMAIN DIAGONAL ELEMENTS,19X23HUPPER DIAGONAL EL
                     2EMENTS,18X23HLOWER DIAGONAL ELEMENTS///)
ISN 0166          802 FORMAT (1H ,14X2HA(,I2,1H,,I2,2H)=1XE15.8,16X2HA(,I2,1H,,I2,2H)=,1
                     1XE15.8,16X2HA(,I2,1H,,I2,2H)=,1XE15.8)
ISN 0167          803 FORMAT (1H ,43X45HD I A G O N A L  M A T R I X  E L E M E N T S//6
                     13X8HB-MATRIX//16X22HMAIN DIAGONAL ELEMENTS,19X23HUPPER DIAGONAL EL
                     2EMENTS,18X23HLOWER DIAGONAL ELEMENTS///)
ISN 0168          804 FORMAT (1H ,14X2HB(,I2,1H,,I2,2H)=1XE15.8,16X2HB(,I2,1H,,I2,2H)=,1
                     1XE15.8,16X2HB(,I2,1H,,I2,2H)=,1XE15.8)
ISN 0169          805 FORMAT(1H ,14X2HA(,I2,1H,,I2,2H)=1XE15.8)
ISN 0170          806 FORMAT(1H ,14X2HB(,I2,1H,,I2,2H)=1XE15.8)
ISN 0171            RETURN
ISN 0172            END
```

```
ISN 0002            SUBROUTINE PRECAL
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0005            COMMON/BWORKS/AMATRX(40,40),REAERK(40),ADVECQ(40),DOSKSR(40),DIFFC
                   10(40),ALENTH(40),AFTSEC(40),ADJL(40),TURBEX(40),RATIOL(40),VECTOR(
                   240),AWASTE(1000),ASTORE(40),ADIAG(40),BDIAG(40),AUPPER(40),BOTTUM(
                   340),DISOXS(40),ULBOD(40),VOLUME(40),DECAYK(40),VD(40),UVD(40),AREA
                   4(40),SUMER(40),CFINAL(40),SUMVEC(40),CCTN(1600)
ISN 0006            DO 6 I=1,NSECTS
ISN 0007            DO 6 J=1,NSECTS
ISN 0008          6 AMATRX(I,J)=0.0
ISN 0009            DO 1 I=1,ISPEC
ISN 0010            AMATRX (I+1,I) = BOTTUM(I)
ISN 0011          1 AMATRX (I,I+1) = AUPPER(I)
ISN 0012            IF(IMAT.EQ.2) GO TO 3
ISN 0014            IF(KK.EQ.1) GO TO 3
ISN 0016            DO 2 I=1,NSECTS
ISN 0017          2 AMATRX(I,I)=ADIAG(I)
ISN 0018            RETURN
ISN 0019          3 DO 4 I=1,NSECTS
ISN 0020          4 AMATRX(I,I)=BDIAG(I)
ISN 0021            RETURN
ISN 0022            END
```

```
ISN 0002            SUBROUTINE SCAVEC
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0005            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0006            COMMON/BWORKS/AMATRX(40,40),REAERK(40),ADVECQ(40),DOSKSR(40),DIFFC
                   1Q(40),ALENTH(40),AFTSEC(40),ADJL(40),TURBEX(40),RATIOL(40),VECTOR(
                   240),AWASTE(1000),ASTORE(40),ADIAG(40),BDIAG(40),AUPPER(40),BOTTOM(
                   340),DISOXS(40),ULBOD(40),VOLUME(40),DECAYK(40),VD(40),UVD(40),AREA
                   4(40),SUMER(40),CFINAL(40),SUMVEC(40),CCTN(1600)
ISN 0007            DIMENSION Q(40),L(40),P(40),LENGTH(40),VOL(40)
ISN 0008            REAL L,LENGTH
ISN 0009            EQUIVALENCE (Q,ADVECQ),(ULBOD,L),(DOSKSR,P),(ALENTH,LENGTH),(VOLUM
                   1E,VOL)
ISN 0010            IF (KK.NE.1) GO TO 1
ISN 0012            WRITE (4) ((AMATRX(I,J),I=1,NSECTS),J=1,NSECTS)
ISN 0013          1 READ (2) (VECTOR(I),I=1,NSECTS)
ISN 0014            DO 2 J=1,NSECTS
ISN 0015          2 SUMVEC(J) = 0.0
                  C
                  C     CONSTANT ALPHA TIMES MATRIX INVERSE
                  C
ISN 0016            ALPHA = 1.6E+4
ISN 0017            DO 3 I=1,NSECTS
ISN 0018            DO 3 J=1,NSECTS
ISN 0019          3 AMATRX(I,J)= ALPHA * AMATRX(I,J)
                  C
                  C     PRINTING OPTION ALPHA INVERSE
                  C
ISN 0020            IF ( IMAT.EQ.3) GO TO 6
ISN 0022          4 IF (INDEX(1).EQ.1) GO TO 8
ISN 0024          5 IF (INDEX(3).EQ.1) GO TO 9
ISN 0026            GO TO 11
ISN 0027          6 IF (KK.EQ.0) GO TO 7
ISN 0029            GO TO 4
ISN 0030          7 WRITE (4) ((AMATRX(I,J),I=1,NSECTS),J=1,NSECTS)
ISN 0031            GO TO 4
ISN 0032          8 IF ( KK.EQ.0) CALL PRTMAT
ISN 0034            GO TO 5
ISN 0035          9 IF (IMAT.EQ.2) CALL PRTMAT
ISN 0037            IF (IMAT.EQ.3) GO TO 10
ISN 0039            GO TO 11
ISN 0040         10 IF (KK.EQ.1) CALL PRTMAT
ISN 0042         11 CONTINUE
                  C
                  C     ALPHA TIMES MATRIX INVERSE TIMES VECTOR
                  C
ISN 0043            DO 12 I=1,NSECTS
ISN 0044            DO 12 J=1,NSECTS
ISN 0045         12 SUMVEC(I) = SUMVEC(I) + VECTOR(J)*AMATRX(I,J)
                  C
                  C     PRINTING OPTION ALPHA * MATRIX INVERSE * VECTOR
                  C
```

```
ISN 0046            IF (INDEX(2).EQ.1) GO TO 14
ISN 0048         13 IF (INDEX(4).EQ.1) GO TO 15
ISN 0050            GO TO 19
ISN 0051         14 IF (KK.EQ.0) GO TO 16
ISN 0053            GO TO 13
ISN 0054         15 IF (IMAT.EQ.2) GO TO 17
ISN 0056            IF (IMAT.EQ.3) GO TO 18
ISN 0058            GO TO 19
ISN 0059         16 IPAGE=IPAGE+1
ISN 0060            WRITE (6,50) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0061            WRITE (6,51)
ISN 0062            WRITE (6,53) (I,SUMVEC(I),I=1,NSECTS)
ISN 0063            GO TO 13
ISN 0064         17 IPAGE=IPAGE+1
ISN 0065            WRITE (6,50) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0066            WRITE (6,52)
ISN 0067            WRITE (6,53) (I,SUMVEC(I),I=1,NSECTS)
ISN 0068            GO TO 19
ISN 0069         18 IF (KK.EQ.1) GO TO 17
ISN 0071         19 CONTINUE
ISN 0072         50 FORMAT (1H1,12A6,2X4A6,2X5HPAGE ,I2//////////)
ISN 0073         51 FORMAT (1H ,33X,24HALPHA A MATRIX INVERSE L/////35X,1HI,6X,13HFINA
                   1L ANSWERS///)
ISN 0074         52 FORMAT (1H ,33X,24HALPHA B MATRIX INVERSE F/////35X,1HI,6X,13HFINA
                   1L ANSWERS///)
ISN 0075         53 FORMAT (1H ,33XI2,6XE15.8)
ISN 0076            RETURN
ISN 0077            END
```

```
ISN 0002            SUBROUTINE FINAL
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0005            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0006            COMMON/BWORKS/AMATRX(40,40),BMATRX(40,40),DISOXS(40),ULBOD(40),VOL
                   1UME(40),DECAYK(40),VD(40),UVD(40),AREA(40),SUMER(40),CFINAL(40),SU
                   2MVEC(40),CCTN(40,40)
ISN 0007            DIMENSION L(40),VOL(40)
ISN 0008            REAL L
ISN 0009            EQUIVALENCE (ULBOD,L),(VOLUME,VOL)
ISN 0010            REWIND 2
ISN 0011            REWIND 4
                C
                C     READ IN BINARY MATRIX DATA ALPHA A-INVERSE AND
                C     B-INVERSE MATRIX, READ IN VECTORS ULBOD,DISOXS
                C     ,VOLUME AND DECAYK
                C
ISN 0012            READ (4) ((AMATRX(I,J),I=1,NSECTS),J=1,NSECTS)
ISN 0013            READ (4) ((BMATRX(I,J),I=1,NSECTS),J=1,NSECTS)
ISN 0014            READ (2) (ULBOD(I),I=1,NSECTS)
ISN 0015            READ (2)
ISN 0016            READ (2) (DISOXS(I),I=1,NSECTS)
ISN 0017            READ (2) (VOLUME(I),I=1,NSECTS)
ISN 0018            READ (2) (DECAYK(I),I=1,NSECTS)
                C
                C     COMPUTE THE PRODUCT MATRIX WHICH IS EQUAL TO THE
                C     ALPHA A INVERSE MATRIX TIMES B INVERSE MATRIX.
                C     PRINT OUT PRODUCT RESULTS ON INDEX(5) OPTION
                C
ISN 0019            DO 1 I=1,NSECTS
ISN 0020            DO 1 J=1,NSECTS
ISN 0021            CCTN(I,J)=0.0
ISN 0022            DO  1 IK=1,NSECTS
ISN 0023          1 CCTN(I,J)=CCTN(I,J)+BMATRX(I,IK)*AMATRX(IK,J)
ISN 0024            DO 3 I=1,NSECTS
ISN 0025            DO 3 J=1,NSECTS
ISN 0026            BMATRX(I,J)=AMATRX(I,J)
ISN 0027          3 AMATRX(I,J)=CCTN(I,J)
ISN 0028            IF(INDEX(5).EQ.1) CALL PRTMAT
                C
                C     COMPUTE THE VOLUME-DECAYK VECTOR PRODUCT AS VD
                C
ISN 0030            DO 2 I=1,NSECTS
ISN 0031          2 VD(I)=VOLUME(I)*DECAYK(I)
                C
                C     FORM SPECIAL MATRIX-EACH ELEMENT OF VD VECTOR
                C     IS MULTIPLIED TIMES COLUMN OF AMATRX,PRIOR TO
                C     THIS THE AMATRX IS SAVED IN BMATRX.
                C
ISN 0032            DO 4 J=1,NSECTS
ISN 0033            DO 4 I=1,NSECTS
ISN 0034          4 AMATRX(I,J)=VD(J)*AMATRX(I,J)
```

```
ISN 0035            IF(INDEX(6).EQ.1) CALL PRTMAT
                C
                C     FORM THE ULBOD VOLUME DECAY VECTOR + AREA VECTOR
                C
ISN 0037            DO 5 I=1,NSECTS
ISN 0038          5 UVD(I)=ULBOD(I)
                C
                C     FORM THE ALPHA A INVERSE B INVERSE ULBOD VOLUME DECAYK PRODUCT
                C
ISN 0039            DO 6 I=1,NSECTS
ISN 0040            DO 6 J=1,NSECTS
ISN 0041          6 SUMER(I)=SUMER(I)+UVD(J)*AMATRX(I,J)
ISN 0042            IF(INDEX(7).EQ.1) GO TO 7
ISN 0044            GO TO 8
ISN 0045          7 WRITE (6,100) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0046            WRITE (6,150) (I,SUMER(I),I=1,NSECTS)
                C
                C
                C     DISOXS + ALPHA B INVERSE F - ALPHA A B INVERSE L V D A
                C
ISN 0047          8 CONTINUE
ISN 0048            DO 11 I=1,NSECTS
ISN 0049         11 CFINAL(I)= DISOXS(I) + SUMVEC(I) - SUMER(I)
ISN 0050            IF (INDEX(8).EQ.1) GO TO 9
ISN 0052            GO TO 10
ISN 0053          9 IPAGE=IPAGE + 1
ISN 0054            WRITE (6,200) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0055            WRITE (6,250) (I,CFINAL(I),I=1,NSECTS)
ISN 0056         10 CONTINUE
ISN 0057        100 FORMAT (1H1,12A6,2X,4A6,2X5HPAGE ,I2///)
ISN 0058        150 FORMAT (1H ,////////////30X38HA L P H A  B A  I N V E R S E  V D L
                   1///35X7HSECTION,10X13HFINAL ANSWERS/35X6HNUMBER,/35X6HNUMBER///(37
                   2X,I2,12X,E13.6))
ISN 0059        200 FORMAT (1H1,12A6,2X,4A6,2X5HPAGE ,I2///)
ISN 0060        250 FORMAT (1H ,////////////20X62HD.O.SATURATION + ALPHA B INVERSE F - A
                   1LPHA B A INVERSE V D L  ///35X7HSECTION,10X13HFINAL ANSWERS/35X6HN
                   2UMBER///(37X,I2,12X,E13.6))
ISN 0061            RETURN
ISN 0062            END
```

```
ISN 0002            SUBROUTINE INVERT
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0005            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(3),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0006            COMMON/BWORKS/AMATRX(40,40),AWASTE(2000)
ISN 0007            DIMENSION PA(50), D(50),QA(50),S(50),T(50),B(50),X(50)
ISN 0008            DO 1 I =1,NSECTS
ISN 0009          1 D(I) = AMATRX(I,I)
ISN 0010            MA = NSECTS - 1
ISN 0011            DO 2 I=1,MA
ISN 0012            II=I+1
ISN 0013            QA(I)=AMATRX(I,II)
ISN 0014          2 PA(I)=AMATRX(II,I)
ISN 0015            DO 3 K =1,NSECTS
ISN 0016          3 B(K)=0.0
ISN 0017            DO 7 K =1,NSECTS
ISN 0018            B(K)=1.0
ISN 0019            S(1)=D(1)
ISN 0020            T(1)=B(1)
ISN 0021            DO 4 I =2,NSECTS
ISN 0022            J=I-1
ISN 0023            S(I)=D(I)-PA(J)*QA(J)/S(J)
ISN 0024          4 T(I)=B(I)-PA(J)*T(J)/S(J)
ISN 0025            X(NSECTS) = T(NSECTS)/S(NSECTS)
ISN 0026            DO 5 I=1,MA
ISN 0027            LL = NSECTS - I
ISN 0028          5 X(LL)=(T(LL)-QA(LL)*X(LL+1))/S(LL)
ISN 0029            DO 6 I=1,NSECTS
ISN 0030          6 AMATRX(I,K)=X(I)
ISN 0031          7 B(K)=0.0
ISN 0032            RETURN
ISN 0033            END
```

```
ISN 0002            SUBROUTINE PRTMAT
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/DATUSE/DATRUN,USER,IPAGE
ISN 0005            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),BC(7)
ISN 0006            COMMON/BWORKS/AMATRX(40,40),AWASTE(2000)
ISN 0007            DIMENSION  IHEAD(50)
ISN 0008          50 FORMAT(1H1,12A6,2X,4A6,2X5HPAGE ,I2///)
ISN 0009          51 FORMAT (1H ,8X6HI     J,7XI2,17XI2,17XI2,17XI2,17XI2)
ISN 0010          52 FORMAT (1H ,7XI2,6XE15.8,4XE15.8,4XE15.8,4XE15.8,4XE15.8)
ISN 0011            IF (NSECTS.LE.5) GO TO 1
ISN 0013            KCTN = 5
ISN 0014            GO TO 2
ISN 0015          1 KCTN = NSECTS
ISN 0016          2 LL = 1
ISN 0017          3 DO 4 I=LL,KCTN
ISN 0018          4 IHEAD(I)=I
ISN 0019            IPAGE=IPAGE+1
ISN 0020            WRITE (6,50) (TITLE(I),I=1,12),(DATRUN(I),I=1,4),IPAGE
ISN 0021            WRITE (6,51) (IHEAD(I),I=LL,KCTN)
ISN 0022            DO 6 I=1,NSECTS
ISN 0023          6 WRITE (6,52)  I,(AMATRX(I,J),J=LL,KCTN)
ISN 0024            IF (KCTN.EQ.NSECTS)  GO TO 5
ISN 0026            KCTN = KCTN + 5
ISN 0027            IF (KCTN.GT.NSECTS)  KCTN = NSECTS
ISN 0029            LL = LL + 5
ISN 0030            GO TO 3
ISN 0031          5 CONTINUE
ISN 0032            RETURN
ISN 0033            END
```

```
ISN 0002            SUBROUTINE PRECFM
ISN 0003            DOUBLE PRECISION DATRUN(4),USER(4),TITLE(12)
ISN 0004            COMMON/CONTRL/TITLE,NCASES,NSECTS,IMAT,INDEX(8),KK,ISPEC,IMAX,
                   1ITP(2),CFQ,CFLEN,CFK,CFL,CFAREA,CFVOL,CFP
ISN 0005            COMMON/BWORKS/AA(1640),ADVECQ(40),DOSKSR(40),DIFFCO(40),ALENTH(40)
                   1,BB(480),ULBOD(40),VOLUME(40),AB(120),AREA(40),AC(1720)
ISN 0006            DIMENSION Q(40),L(40),P(40),LENGTH(40),VOL(40)
ISN 0007            REAL L,LENGTH
ISN 0008            EQUIVALENCE (Q,ADVECQ),(ULBOD,L),(DOSKSR,P),(ALENTH,LENGTH),(VOLUM
                   1E,VOL)
ISN 0009            DO 1 J=I,NSECTS
ISN 0010            DOSKSR(J)=CFP *DOSKSR(J)
ISN 0011            VOLUME(J)=CFVOL*VOLUME(J)
ISN 0012          1 ULBOD(J) =CFL*ULBOD(J)
ISN 0013            IMAX=NSECTS+1
ISN 0014            DO 2 K=I,IMAX
ISN 0015            ADVECQ(K)=CFQ*ADVECQ(K)
ISN 0016            DIFFCO(K)=CFK*DIFFCO(K)
ISN 0017          2 AREA(K)=CFAREA*AREA(K)
ISN 0018            IMAX=IMAX+1
ISN 0019            DO 3 I=1,IMAX
ISN 0020          3 ALENTH(I)=CFLEN*ALENTH(I)
ISN 0021            RETURN
ISN 0022            END
```

A P P E N D I X III

SAMPLE INPUTS

```
  2
 11/1/67
NAME OF THE PROGRAM USER   RONALD BUNCE
NAME OF THE ESTUARY BEING INVESTIGATED
 $CF
 NSECTS=14,
 IMAT=3,
 INDEX=8*1,
 ITP=2*0,
 CFQ=1.,
 CFLEN=1.,
 CFK=1.,
 CFL=1.,
 CFAREA=1.,
 CFVOL=1.,
 CFP=1.,
 $END
 $CCGC
 TP1=6*28.0,4*29.0,4*30.0,
 $END
 $RIVER
 Q=2471,2471,13*2500,
 L=63000,81600,3*0.0,672,22500,7*0.0,
 P=-990,-450,-550,-570,-640,-685,-695,7*-700,
 VOL=19*6,4*7CE6,64E6,74E6,69E6,64E6,67E6,75E6,3*76E6,
 AREA=13E3,4*13250,12100,14000,13050,12100,12680,14200,4*14400,
 DIFFCO=0.0,0.12,0.14,0.15,0.16,0.16,0.17,0.18,0.19,0.20,2*0.21,3*0.23,
 LENGTH=0.3,15*1.0,
 $END
 $RIVIC
 DISOXS=6*7.92,3*7.77,5*7.63,
 REAERK=14*0.19,
 DECAYK=6*0.46,3*0.49,5*0.52,
 $END
```

```
CASE NUMBER 2 NAME OF ESTUARY BEING INVESTIGATED
 $CF
 NSECTS=14,
 IMAT=3,
 INDEX=8*1,
 ITP=2*0,
 CFG=1.,
 CFLEN=1.,
 CFK=1.,
 CFL=1.,
 CFAREA=1.,
 CFVOL=1.,
 CFP=1.,
 $END
 $GOGO
 TP1=6*28.0,4*29.0,4*30.0,
 $END
 $RIVER
 C=1245,1245,13*1300,
 L=63000,81600,3*0.0,672,14200,7*0.0,
 P=-900,-450,-550,-570,-640,-685,-695,7*-700,
 VOL=19E6,4*70E6,64E6,74E6,69E6,64E6,67E6,75E6,3*76E6,
 AREA=13E3,4*13250,12100,14000,13050,12100,12680,14200,4*14400,
 DIFFCO=0.0,0.12,0.14,0.15,0.16,0.16,0.17,0.18,0.19,0.20,2*0.21,3*0.23,
 LENGTH=0.3,15*1.0,
 $END
 $RIVTO
 DISOXS=6*7.92,3*7.77,5*7.63,
 REAERK=14*0.04,
 DECAYK=6*0.46,3*0.49,5*0.52,
 $END
```

A P P E N D I X IV

TYPICAL PROGRAM CASE SOLUTIONS

SECTION A - SAMPLE INPUT & OUTPUT

αA^{-1}

IMAT = 1

INDEX(1) = 1

CONVERSION FACTORS USED ON INPUT VARIABLES

CFQ	=	1.00
CFLEN	=	1.00
CFK	=	1.00
CFL	=	1.00
CFAREA	=	1.00
CFVOL	=	1.00
CFP	=	1.00

****
MIDDLE ATLANTIC REGION
STEADY STATE SEGMENTED ESTUARY MODEL
****

PROBLEM TITLE	*	STEADY STATE TEST CASE 1
DATE OF RUN	*	9/20/67
ENGINEER	*	R.E.BUNCE

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 1
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 1 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

G E O M E T R I C - I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

DIAGONAL MATRIX ELEMENTS

A-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
A(1, 1)= 0.82356864E 08	A(1, 2)= -0.16569040E 07	A(2, 1)= -0.44856880E 08
A(2, 2)= 0.82541184E 08	A(2, 3)= -0.16843470E 07	A(3, 2)= -0.44884320E 08
A(3, 3)= 0.98695408E 08	A(3, 4)= -0.41383280E 07	A(4, 3)= -0.47511120E 08
A(4, 4)= 0.11169682E 09	A(4, 5)= -0.41857280E 07	A(5, 4)= -0.47558512E 08
A(5, 5)= 0.11776109E 09	A(5, 6)= -0.45620800E 06	A(6, 5)= -0.46075392E 08
A(6, 6)= 0.15417074E 09	A(6, 7)= -0.10649600E 07	A(7, 6)= -0.75714544E 08
A(7, 7)= 0.20001643E 09	A(7, 8)= -0.69323200E 07	A(8, 7)= -0.84951504E 08
A(8, 8)= 0.20277424E 09	A(8, 9)= -0.18822752E 08	A(9, 8)= -0.96841936E 08
A(9, 9)= 0.26419011E 09	A(9,10)= -0.48848224E 08	A(10, 9)= -0.12686741E 09
A(10,10)= 0.31100186E 09	A(10,11)= -0.86289104E 08	A(11,10)= -0.16465389E 09
A(11,11)= 0.39201152E 09	A(11,12)= -0.89098464E 08	A(12,11)= -0.16772245E 09
A(12,12)= 0.69926784E 09	A(12,13)= -0.15328653E 09	A(13,12)= -0.23216971E 09
A(13,13)= 0.80363571E 09	A(13,14)= -0.31646618E 09	A(14,13)= -0.39534925E 09
A(14,14)= 0.12109588E 10	A(14,15)= -0.44060954E 09	A(15,14)= -0.51949286E 09
A(15,15)= 0.13651917E 10	A(15,16)= -0.35069901E 09	A(16,15)= -0.42958234E 09
A(16,16)= 0.21093960E 10	A(16,17)= -0.10123144E 10	A(17,16)= -0.10911977E 10
A(17,17)= 0.32019016E 10	A(17,18)= -0.13307044E 10	A(18,17)= -0.14095875E 10
A(18,18)= 0.37194258E 10	A(18,19)= -0.15073385E 10	A(19,18)= -0.15862218E 10
A(19,19)= 0.36371612E 10	A(19,20)= -0.11209403E 10	A(20,19)= -0.11998234E 10
A(20,20)= 0.32107914E 10	A(20,21)= -0.12234683E 10	A(21,20)= -0.13023516E 10
A(21,21)= 0.34772705E 10		
B(21,21)= 0.0		

I	J	1	2	3	4	5
1		0.19644471E-03	0.39809856E-05	0.69193050E-07	0.26030185E-08	0.92629779E-10
2		0.10777620E-03	0.19787614E-03	0.34392615E-05	0.12938375E-06	0.46041890E-08
3		0.49918046E-04	0.91649155E-04	0.16669781E-03	0.62711115E-05	0.22316078E-06
4		0.21559681E-04	0.39583421E-04	0.71997129E-04	0.14815708E-03	0.52722480E-05
5		0.87171229E-05	0.16004575E-04	0.29110248E-04	0.59903687E-04	0.13815780E-03
6		0.26121306E-05	0.47958501E-05	0.87230237E-05	0.17950428E-04	0.41399719E-04
7		0.10039403E-05	0.18432283E-05	0.33525921E-05	0.68990321E-05	0.15911472E-04
8		0.43686208E-06	0.80207593E-06	0.14588713E-05	0.30020974E-05	0.69238331E-05
9		0.17522098E-06	0.32170419E-06	0.58513791E-06	0.12041100E-05	0.27770802E-05
10		0.81580481E-07	0.14978110E-06	0.27243181E-06	0.56061708E-06	0.12929704E-05
11		0.36410608E-07	0.66849566E-07	0.12159057E-06	0.25021183E-06	0.57707234E-06
12		0.94371266E-08	0.17326499E-07	0.31514624E-07	0.64851463E-07	0.14956919E-06
13		0.32110234E-08	0.58954086E-08	0.10722982E-07	0.22065986E-07	0.50891568E-07
14		0.12307055E-08	0.22595656E-08	0.41098538E-08	0.84573450E-08	0.19505478E-07
15		0.50125726E-09	0.92030361E-09	0.16739126E-08	0.34446144E-08	0.79444362E-08
16		0.12822847E-09	0.23542635E-09	0.42820969E-09	0.88117980E-09	0.20322961E-08
17		0.54482502E-10	0.10002936E-09	0.18194034E-09	0.37440073E-09	0.86349439E-09
18		0.25944885E-10	0.47634563E-10	0.86641028E-10	0.17829162E-09	0.41120085E-09
19		0.13070873E-10	0.23997998E-10	0.43649265E-10	0.89822288E-10	0.20716046E-09
20		0.56975024E-11	0.10460552E-10	0.19026405E-10	0.39152931E-10	0.90299782E-10
21		0.21339007E-11	0.39178183E-11	0.71260133E-11	0.14664048E-10	0.33820238E-10

I	J	6	7	8	9	10
1		0.27483105E-12	0.14857055E-14	0.52756640E-16	0.41128065E-17	0.73728810E-18
2		0.13660557E-10	0.73847395E-13	0.26222840E-14	0.20442823E-15	0.36647132E-16
3		0.66211459E-09	0.35793165E-11	0.12709967E-12	0.99084491E-14	0.17762529E-14
4		0.15642680E-07	0.84562607E-10	0.30027716E-11	0.23409047E-12	0.41964563E-13
5		0.40991227E-06	0.22159392E-08	0.78686779E-10	0.61342728E-11	0.10996690E-11
6		0.10418006E-03	0.56318589E-06	0.19998417E-07	0.15590385E-08	0.27948310E-09
7		0.40040381E-04	0.81434730E-04	0.28917020E-05	0.22543151E-06	0.40412296E-07
8		0.17423459E-04	0.35436125E-04	0.83215127E-04	0.64872902E-05	0.11629527E-05
9		0.69883654E-05	0.14213060E-04	0.33376753E-04	0.68869034E-04	0.12345907E-04
10		0.32536891E-05	0.66174016E-05	0.15539757E-04	0.32064505E-04	0.64465887E-04
11		0.14521711E-05	0.29534467E-05	0.69356320E-05	0.14310881E-04	0.28772134E-04
12		0.37638290E-06	0.76549298E-06	0.17976190E-05	0.37091841E-05	0.74573445E-05
13		0.12806584E-06	0.26046206E-06	0.61164781E-06	0.12620649E-05	0.25373911E-05
14		0.49084470E-07	0.99828696E-07	0.23442936E-06	0.48371840E-06	0.97251996E-06
15		0.19991742E-07	0.40659501E-07	0.95481369E-07	0.19701497E-06	0.39610023E-06
16		0.51141598E-08	0.10401248E-07	0.24425447E-07	0.50399098E-07	0.10132783E-06
17		0.21729336E-08	0.44193449E-08	0.10378031E-07	0.21413868E-07	0.43052783E-07
18		0.10347634E-08	0.21045179E-08	0.49420770E-08	0.10197414E-07	0.20501986E-07
19		0.52130789E-09	0.10602439E-08	0.24897895E-08	0.51373945E-08	0.10328783E-07
20		0.22723440E-09	0.46215276E-09	0.10852819E-08	0.22393554E-08	0.45022404E-08
21		0.85106672E-10	0.17309129E-09	0.40647374E-09	0.83871221E-09	0.16862369E-08

I	J	11	12	13	14	15
1		0.17244999E-18	0.23743980E-19	0.53340322E-20	0.16364856E-20	0.56532019E-21
2		0.85716748E-17	0.11802006E-17	0.26512974E-18	0.81341995E-19	0.28099400E-19
3		0.41546119E-15	0.57203258E-16	0.12850599E-16	0.39425719E-17	0.13619521E-17
4		0.98154076E-14	0.13514468E-14	0.30359969E-15	0.93144613E-16	0.32176571E-16
5		0.25721000E-12	0.35414275E-13	0.79557468E-14	0.24408279E-14	0.84317789E-15
6		0.65370445E-10	0.90006154E-11	0.20219694E-11	0.62034172E-12	0.21429538E-12
7		0.94523394E-08	0.13014581E-08	0.29236968E-09	0.89699234E-10	0.30986352E-10
8		0.27201196E-06	0.37452345E-07	0.84135863E-08	0.25812963E-08	0.89170094E-09
9		0.28876793E-05	0.39759391E-06	0.89318576E-07	0.27403036E-07	0.94662909E-08
10		0.15078424E-04	0.20760936E-05	0.46638957E-06	0.14308887E-06	0.49429570E-07
11		0.50099741E-04	0.68980553E-05	0.15496316E-05	0.47542881E-06	0.16423530E-06
12		0.12985171E-04	0.26513138E-04	0.59561135E-05	0.18273422E-05	0.63124924E-06
13		0.44182570E-05	0.90212061E-05	0.25475238E-04	0.78158346E-05	0.26999523E-05
14		0.16934091E-05	0.34576060E-05	0.97640323E-05	0.18506966E-04	0.63931684E-05
15		0.68971315E-06	0.14082561E-05	0.39768183E-05	0.75377566E-05	0.15148203E-04
16		0.17643799E-06	0.36025136E-06	0.10173244E-05	0.19282606E-05	0.38751186E-05
17		0.74965953E-07	0.15306574E-06	0.43224685E-06	0.81929107E-06	0.16464828E-05
18		0.35699234E-07	0.72890771E-07	0.20583849E-06	0.39015123E-06	0.78406458E-06
19		0.17985069E-07	0.36721943E-07	0.10370019E-06	0.19655590E-06	0.39500691E-06
20		0.78395637E-08	0.16006847E-07	0.45202174E-07	0.85677300E-07	0.17218093E-06
21		0.29361737E-08	0.59950871E-08	0.16929693E-07	0.32088955E-07	0.64487324E-07

I	J	16	17	18	19	20
1		0.11806067E-21	0.46536091E-22	0.20920629E-22	0.10015538E-22	0.40786696E-23
2		0.58682390E-20	0.23130900E-20	0.10398660E-20	0.49782500E-21	0.20273152E-21
3		0.28442812E-18	0.11211332E-18	0.50401355E-19	0.24129120E-19	0.98262110E-20
4		0.67197109E-17	0.26487150E-17	0.11907488E-17	0.57005871E-18	0.23214752E-18
5		0.17608813E-15	0.69408818E-16	0.31203225E-16	0.14938220E-16	0.60833633E-17
6		0.44753182E-13	0.17640400E-13	0.79303663E-14	0.37965812E-14	0.15460998E-14
7		0.64711552E-11	0.25507400E-11	0.11467025E-11	0.54897260E-12	0.22356075E-12
8		0.18622186E-09	0.73403186E-10	0.32998909E-10	0.15797905E-10	0.64334588E-11
9		0.19769306E-08	0.77924822E-09	0.35031622E-09	0.16771046E-09	0.68297576E-10
10		0.10322822E-07	0.40689514E-08	0.18292259E-08	0.87572372E-09	0.35662540E-09
11		0.34298729E-07	0.13519553E-07	0.60778085E-08	0.29096914E-08	0.11849279E-08
12		0.13182944E-06	0.51963323E-07	0.23360464E-07	0.11183591E-07	0.45543516E-08
13		0.56385562E-06	0.22225538E-06	0.99916406E-07	0.47834000E-07	0.19479678E-07
14		0.13351428E-05	0.52627433E-06	0.23659027E-06	0.11326523E-06	0.46125589E-07
15		0.31635363E-05	0.12469727E-05	0.56058553E-06	0.26837461E-06	0.10929153E-06
16		0.10337174E-04	0.40746108E-05	0.18317696E-05	0.87694139E-06	0.35712117E-06
17		0.43921236E-05	0.79612546E-05	0.35790363E-05	0.17134280E-05	0.69776803E-06
18		0.20915541E-05	0.37911959E-05	0.71096938E-05	0.34036957E-05	0.13861045E-05
19		0.10537115E-05	0.19099834E-05	0.35818221E-05	0.67964620E-05	0.27677579E-05
20		0.45930545E-06	0.83254866E-06	0.15612914E-05	0.29625317E-05	0.70192100E-05
21		0.17202501E-06	0.31181679E-06	0.58475422E-06	0.11095644E-05	0.26289244E-05

I	J	21
1		0.14350705E-23
2		0.71330621E-22
3		0.34573274E-20
4		0.81680570E-19
5		0.21404159E-17
6		0.54399124E-15
7		0.78659193E-13
8		0.22635921E-11
9		0.24030278E-10
10		0.12547753E-09
11		0.41691339E-09
12		0.16024342E-08
13		0.68538633E-08
14		0.16229141E-07
15		0.38453919E-07
16		0.12565204E-06
17		0.24550758E-06
18		0.48769675E-06
19		0.97382781E-06
20		0.24696892E-05
21		0.55262881E-05

SECTION B - SAMPLE INPUT & OUTPUT

αLA^{-1}

IMAT = 1

INDEX(2) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 1
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 1 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

G E O M E T R I C I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	F (CU.FT./DAY)	O (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

DIAGONAL MATRIX ELEMENTS

A-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
A(1, 1)= 0.82356864E 08	A(1, 2)= -0.16569040E 07	A(2, 1)= -0.44856880E 08
A(2, 2)= 0.82541184E 08	A(2, 3)= -0.16843470E 07	A(3, 2)= -0.44884320E 08
A(3, 3)= 0.98695408E 08	A(3, 4)= -0.41383280E 07	A(4, 3)= -0.47511120E 08
A(4, 4)= 0.11169682E 09	A(4, 5)= -0.41857280E 07	A(5, 4)= -0.47558512E 08
A(5, 5)= 0.11776109E 09	A(5, 6)= -0.45620800E 06	A(6, 5)= -0.46075392E 08
A(6, 6)= 0.15417074E 09	A(6, 7)= -0.10649600E 07	A(7, 6)= -0.75714544E 08
A(7, 7)= 0.20001643E 09	A(7, 8)= -0.69323200E 07	A(8, 7)= -0.84951504E 08
A(8, 8)= 0.20277424E 09	A(8, 9)= -0.18822752E 08	A(9, 8)= -0.96841936E 08
A(9, 9)= 0.26419011E 09	A(9,10)= -0.48848224E 08	A(10, 9)= -0.12686741E 09
A(10,10)= 0.31100186E 09	A(10,11)= -0.86289104E 08	A(11,10)= -0.16465389E 09
A(11,11)= 0.39201152E 09	A(11,12)= -0.89098464E 08	A(12,11)= -0.16772245E 09
A(12,12)= 0.69926784E 09	A(12,13)= -0.15328653E 09	A(13,12)= -0.23216971E 09
A(13,13)= 0.80363571E 09	A(13,14)= -0.31646618E 09	A(14,13)= -0.39534925E 09
A(14,14)= 0.12109588E 10	A(14,15)= -0.44060954E 09	A(15,14)= -0.51949286E 09
A(15,15)= 0.13651917E 10	A(15,16)= -0.35069901E 09	A(16,15)= -0.42958234E 09
A(16,16)= 0.21093960E 10	A(16,17)= -0.10123144E 10	A(17,16)= -0.10911977E 10
A(17,17)= 0.32019016E 10	A(17,18)= -0.13307044E 10	A(18,17)= -0.14095875E 10
A(18,18)= 0.37194258E 10	A(18,19)= -0.15073385E 10	A(19,18)= -0.15862218E 10
A(19,19)= 0.36371612E 10	A(19,20)= -0.11209403E 10	A(20,19)= -0.11998234E 10
A(20,20)= 0.32107914E 10	A(20,21)= -0.12234683E 10	A(21,20)= -0.13023516E 10
A(21,21)= 0.34772705E 10		
B(21,21)= 0.0		

ALPHA A MATRIX INVERSE L

I	FINAL ANSWERS
1	0.45443794E-05
2	0.22587976E-03
3	0.10948177E-01
4	0.11211649E-01
5	0.17491460E 00
6	0.52816373E 00
7	0.20301640E 00
8	0.88995576E-01
9	0.42475488E-01
10	0.28723411E-01
11	0.13817206E-01
12	0.35812324E-02
13	0.12185287E-02
14	0.46703196E-03
15	0.19021875E-03
16	0.48660499E-04
17	0.20675172E-04
18	0.98456394E-05
19	0.49601749E-05
20	0.21621026E-05
21	0.80977918E-06

SECTION C - SAMPLE INPUT & OUTPUT

$\alpha \mathbf{A}^{-1}$ and $\alpha \mathbf{L}\mathbf{A}^{-1}$

IMAT = 1

INDEX(1) and INDEX(2) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 1
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 1 ALPHA A MATRIX INVERSE
INDEX(2)= 1 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

G E O M E T R I C I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

DIAGONAL MATRIX ELEMENTS

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44884320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.46075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84851504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96941936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.16465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80353571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				
B(21,21)=	0.0				

I	J	1	2	3	4	5
1		0.19644471E-03	0.39809856E-05	0.69193050E-07	0.26030185E-08	0.92629779E-10
2		0.10777620E-03	0.19787614E-03	0.34392615E-05	0.12938375E-06	0.46041890E-08
3		0.49918046E-04	0.91649155E-04	0.16669781E-03	0.62711115E-05	0.22316078E-06
4		0.21559681E-04	0.39583421E-04	0.71997129E-04	0.14815708E-03	0.52722480E-05
5		0.87171229E-05	0.16004575E-04	0.29110248E-04	0.59903687E-04	0.13815780E-03
6		0.26121306E-05	0.47958501E-05	0.87230237E-05	0.17950428E-04	0.41399719E-04
7		0.10039403E-05	0.18432283E-05	0.33525921E-05	0.68990321E-05	0.15911472E-04
8		0.43686208E-06	0.80207593E-06	0.14588713E-05	0.30020974E-05	0.69238331E-05
9		0.17522098E-06	0.32170419E-06	0.58513791E-06	0.12041100E-05	0.27770802E-05
10		0.81580481E-07	0.14978110E-06	0.27243181E-06	0.56061708E-06	0.12929704E-05
11		0.36410608E-07	0.66849566E-07	0.12159057E-06	0.25021183E-06	0.57707234E-06
12		0.94371266E-08	0.17326499E-07	0.31514624E-07	0.64851463E-07	0.14956919E-06
13		0.32110234E-08	0.58954086E-08	0.10722982E-07	0.22065986E-07	0.50891568E-07
14		0.12307055E-08	0.22595656E-08	0.41098538E-08	0.84573450E-08	0.19505478E-07
15		0.50125726E-09	0.92030361E-09	0.16739126E-08	0.34446144E-08	0.79444362E-08
16		0.12822847E-09	0.23542635E-09	0.42820969E-09	0.88117980E-09	0.20322961E-08
17		0.54482502E-10	0.10002936E-09	0.18194034E-09	0.37440073E-09	0.86349439E-09
18		0.25944885E-10	0.47634563E-10	0.86641028E-10	0.17829162E-09	0.41120085E-09
19		0.13070873E-10	0.23997998E-10	0.43649265E-10	0.89822288E-10	0.20716046E-09
20		0.56975024E-11	0.10460552E-10	0.19026405E-10	0.39152931E-10	0.90299782E-10
21		0.21339007E-11	0.39178183E-11	0.71260133E-11	0.14664048E-10	0.33820238E-10

I	J	6	7	8	9	10
1		0.27483105E-12	0.14857055E-14	0.52756640E-16	0.41128065E-17	0.73728810E-18
2		0.13660557E-10	0.73847395E-13	0.26222840E-14	0.20442823E-15	0.36647132E-16
3		0.66211459E-09	0.35793165E-11	0.12709967E-12	0.99084491E-14	0.17762529E-14
4		0.15642680E-07	0.84562607E-10	0.30027716E-11	0.23409047E-12	0.41964563E-13
5		0.40991227E-06	0.22159392E-08	0.78686779E-10	0.61342728E-11	0.10996690E-11
6		0.10418006E-03	0.56318589E-06	0.19998417E-07	0.15590385E-08	0.27948310E-09
7		0.40040381E-04	0.81434730E-04	0.28917020E-05	0.22543151E-06	0.40412296E-07
8		0.17423459E-04	0.35436125E-04	0.83215127E-04	0.64872902E-05	0.11629527E-05
9		0.69883654E-05	0.14213060E-04	0.33376753E-04	0.68869034E-04	0.12345907E-04
10		0.32536891E-05	0.66174016E-05	0.15539757E-04	0.32064505E-04	0.64465887E-04
11		0.14521711E-05	0.29534467E-05	0.69356320E-05	0.14310881E-04	0.28772134E-04
12		0.37638290E-06	0.76549298E-06	0.17976190E-05	0.37091841E-05	0.74573445E-05
13		0.12806584E-06	0.26046206E-06	0.61164781E-06	0.12620649E-05	0.25373911E-05
14		0.49084470E-07	0.99828696E-07	0.23442936E-06	0.48371840E-06	0.97251996E-06
15		0.19991742E-07	0.40659501E-07	0.95481369E-07	0.19701497E-06	0.39610023E-06
16		0.51141598E-08	0.10401248E-07	0.24425447E-07	0.50399098E-07	0.10132783E-06
17		0.21729336E-08	0.44193449E-08	0.10378031E-07	0.21413868E-07	0.43052783E-07
18		0.10347634E-08	0.21045179E-08	0.49420770E-08	0.10197414E-07	0.20501986E-07
19		0.52130789E-09	0.10602439E-08	0.24897895E-08	0.51373945E-08	0.10328783E-07
20		0.22723440E-09	0.46215276E-09	0.10852819E-08	0.22393554E-08	0.45022404E-08
21		0.85106672E-10	0.17309129E-09	0.40647374E-09	0.83871221E-09	0.16862369E-08

I	J	11	12	13	14	15
1		0.17244999E-18	0.23743980E-19	0.53340322E-20	0.16364856E-20	0.56532019E-21
2		0.85716748E-17	0.11802006E-17	0.26512974E-18	0.81341995E-19	0.28099400E-19
3		0.41546119E-15	0.57203258E-16	0.12850599E-16	0.39425719E-17	0.13619521E-17
4		0.98154076E-14	0.13514468E-14	0.30359969E-15	0.93144613E-16	0.32176571E-16
5		0.25721000E-12	0.35414275E-13	0.79557468E-14	0.24408279E-14	0.84317789E-15
6		0.65370445E-10	0.90006154E-11	0.20219694E-11	0.62034172E-12	0.21429538E-12
7		0.94523394E-08	0.13014581E-08	0.29236968E-09	0.89699234E-10	0.30986352E-10
8		0.27201196E-06	0.37452345E-07	0.84135863E-08	0.25812963E-08	0.89170094E-09
9		0.28876793E-05	0.39759391E-06	0.89318576E-07	0.27403036E-07	0.94662909E-08
10		0.15078424E-04	0.20760936E-05	0.46638957E-06	0.14308887E-06	0.49429570E-07
11		0.50099741E-04	0.68980553E-05	0.15496316E-05	0.47542881E-06	0.16423530E-06
12		0.12985171E-04	0.26513138E-04	0.59561135E-05	0.18273422E-05	0.63124924E-06
13		0.44182570E-05	0.90212061E-05	0.25475238E-04	0.78158346E-05	0.26999523E-05
14		0.16934091E-05	0.34576060E-05	0.97640323E-05	0.18506966E-04	0.63931684E-05
15		0.68971315E-06	0.14082561E-05	0.39768183E-05	0.75377566E-05	0.15148203E-04
16		0.17643799E-06	0.36025136E-06	0.10173244E-05	0.19282606E-05	0.38751186E-05
17		0.74965953E-07	0.15306574E-06	0.43224685E-06	0.81929107E-06	0.16464828E-05
18		0.35699234E-07	0.72890771E-07	0.20583849E-06	0.39015123E-06	0.78406458E-06
19		0.17985069E-07	0.36721943E-07	0.10370019E-06	0.19655590E-06	0.39500691E-06
20		0.78395637E-08	0.16006847E-07	0.45202174E-07	0.85677300E-07	0.17218099E-06
21		0.29361737E-08	0.59950871E-08	0.16929693E-07	0.32088955E-07	0.64487324E-07

I	J	16	17	18	19	20
1		0.11806067E-21	0.46536091E-22	0.20920629E-22	0.10015538E-22	0.40786696E-23
2		0.58682390E-20	0.23130900E-20	0.10398660E-20	0.49782500E-21	0.20273152E-21
3		0.28442812E-18	0.11211332E-18	0.50401355E-19	0.24129120E-19	0.98262110E-20
4		0.67197109E-17	0.26487150E-17	0.11907488E-17	0.57005871E-18	0.23214752E-18
5		0.17608813E-15	0.69408818E-16	0.31203225E-16	0.14938220E-16	0.60833633E-17
6		0.44753182E-13	0.17640400E-13	0.79303663E-14	0.37965812E-14	0.15460998E-14
7		0.64711552E-11	0.25507400E-11	0.11467025E-11	0.54897260E-12	0.22356075E-12
8		0.18622186E-09	0.73403186E-10	0.32998909E-10	0.15797905E-10	0.64334588E-11
9		0.19769306E-08	0.77924822E-09	0.35031622E-09	0.16771046E-09	0.68297576E-10
10		0.10322822E-07	0.40689514E-08	0.18292259E-08	0.87572372E-09	0.35662540E-09
11		0.34298729E-07	0.13519553E-07	0.60778085E-08	0.29096914E-08	0.11849279E-08
12		0.13182944E-06	0.51963323E-07	0.23360464E-07	0.11183591E-07	0.45543516E-08
13		0.56385562E-06	0.22225538E-06	0.99916406E-07	0.47834000E-07	0.19479678E-07
14		0.13351428E-05	0.52627433E-06	0.23659027E-06	0.11326523E-06	0.46125589E-07
15		0.31635363E-05	0.12469727E-05	0.56058553E-06	0.26837461E-06	0.10929153E-06
16		0.10337174E-04	0.40746108E-05	0.18317696E-05	0.87694139E-06	0.35712117E-06
17		0.43921236E-05	0.79612546E-05	0.35790363E-05	0.17134280E-05	0.69776803E-06
18		0.20915541E-05	0.37911959E-05	0.71096938E-05	0.34036957E-05	0.13861045E-05
19		0.10537115E-05	0.19099834E-05	0.35818221E-05	0.67964620E-05	0.27677579E-05
20		0.45930545E-06	0.83254866E-06	0.15612914E-05	0.29625317E-05	0.70192100E-05
21		0.17202501E-06	0.31181679E-06	0.58475422E-06	0.11095644E-05	0.26289244E-05

I	J 21
1	0.14350705E-23
2	0.71330621E-22
3	0.34573274E-20
4	0.81680570E-19
5	0.21404159E-17
6	0.54399124E-15
7	0.78659193E-13
8	0.22635921E-11
9	0.24030278E-10
10	0.12547753E-09
11	0.41691339E-09
12	0.16024342E-08
13	0.68538633E-08
14	0.16229141E-07
15	0.38453919E-07
16	0.12565204E-06
17	0.24550758E-06
18	0.48769675E-06
19	0.97382781E-06
20	0.24696892E-05
21	0.55262881E-05

ALPHA A MATRIX INVERSE L

I	FINAL ANSWERS
1	0.45443794E-05
2	0.22587976E-03
3	0.10948177E-01
4	0.11211649E-01
5	0.17491460E 00
6	0.52816373E 00
7	0.20301640E 00
8	0.88995576E-01
9	0.42475488E-01
10	0.28723411E-01
11	0.13817206E-01
12	0.35812324E-02
13	0.12185287E-02
14	0.46703196E-03
15	0.19021875E-03
16	0.48660499E-04
17	0.20675172E-04
18	0.98456394E-05
19	0.49601749E-05
20	0.21621026E-05
21	0.80977918E-06

SECTION D - SAMPLE INPUT & OUTPUT

αB^{-1}

IMAT = 2

INDEX(3) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 2
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 1 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864200E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.913000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.913000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.894302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.455192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899442E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137097E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

I	J	1	2	3	4	5
1		0.21644370E-03	0.45101733E-05	0.62929132E-07	0.14266754E-08	0.34782302E-10
2		0.12210278E-03	0.20376421E-03	0.28430622E-05	0.64455435E-07	0.15714217E-08
3		0.45399094E-04	0.75761694E-04	0.13359697E-03	0.30287947E-05	0.73841932E-07
4		0.11816545E-04	0.19719373E-04	0.34772864E-04	0.88441593E-04	0.21562028E-05
5		0.32732660E-05	0.54624034E-05	0.96323156E-05	0.24498935E-04	0.93789989E-04
6		0.53096301E-06	0.88606765E-06	0.15624773E-05	0.39740189E-05	0.15213877E-04
7		0.62869219E-07	0.10491561E-06	0.18500674E-06	0.47054800E-06	0.18014134E-05
8		0.12405692E-07	0.20702537E-07	0.36506545E-07	0.92851110E-07	0.35546475E-06
9		0.21481539E-08	0.35848251E-08	0.63214252E-08	0.16077955E-07	0.61551816E-07
10		0.57269745E-09	0.95571373E-09	0.16852924E-08	0.42863846E-08	0.16409718E-07
11		0.15683337E-09	0.26172220E-09	0.46151727E-09	0.11738266E-08	0.44938062E-08
12		0.20340341E-10	0.33943806E-10	0.59856037E-10	0.15223840E-09	0.58281957E-09
13		0.47838521E-11	0.79832616E-11	0.14077572E-10	0.35804998E-10	0.13707356E-09
14		0.10805159E-11	0.18031583E-11	0.31796649E-11	0.80871854E-11	0.30960456E-10
15		0.27122673E-12	0.45262156E-12	0.79814557E-12	0.20300116E-11	0.77715638E-11
16		0.42250136E-13	0.70506697E-13	0.12433050E-12	0.31622339E-12	0.12106097E-11
17		0.11943741E-13	0.19931624E-13	0.35147100E-13	0.89393499E-13	0.34222842E-12
18		0.38941662E-14	0.64985554E-14	0.11459458E-13	0.29146058E-13	0.11158099E-12
19		0.13423626E-14	0.22401258E-14	0.39502025E-14	0.10046983E-13	0.38463234E-13
20		0.40424330E-15	0.67459822E-15	0.11895750E-14	0.30255775E-14	0.11582932E-13
21		0.10872322E-15	0.18143663E-15	0.31994234E-15	0.81374476E-15	0.31152887E-14

I	J	6	7	8	9	10
1		0.55864455E-13	0.93038628E-16	0.14981440E-17	0.50421767E-19	0.51757822E-20
2		0.25238865E-11	0.42033671E-14	0.67684312E-16	0.22779950E-17	0.23383564E-18
3		0.11859876E-09	0.19751854E-12	0.31805202E-14	0.10704415E-15	0.10988060E-16
4		0.34631134E-08	0.57675930E-11	0.92871999E-13	0.31257161E-14	0.32085417E-15
5		0.15063762E-06	0.25087732E-09	0.40397234E-11	0.13596161E-12	0.13956440E-13
6		0.56353660E-04	0.93853430E-07	0.15112640E-08	0.50863341E-10	0.52211178E-11
7		0.66726179E-05	0.25032670E-04	0.40308601E-06	0.13566318E-07	0.13925820E-08
8		0.13166773E-05	0.49395821E-05	0.37244332E-04	0.12535002E-05	0.12867179E-06
9		0.22799406E-06	0.85533225E-06	0.64491842E-05	0.28825933E-04	0.29589810E-05
10		0.60783236E-07	0.22803152E-06	0.17193524E-05	0.76849919E-05	0.34411350E-04
11		0.16645490E-07	0.62446418E-07	0.47084467E-06	0.21045353E-05	0.94235584E-05
12		0.21588198E-08	0.80989260E-08	0.61065691E-07	0.27294561E-06	0.12221781E-05
13		0.50773385E-09	0.19047888E-08	0.14362065E-07	0.64194182E-07	0.28744472E-06
14		0.11468056E-09	0.43022985E-09	0.32439216E-08	0.14499385E-07	0.64924393E-07
15		0.28786612E-10	0.10799450E-09	0.81427576E-09	0.36395729E-08	0.16297044E-07
16		0.44842125E-11	0.16822752E-10	0.12684319E-09	0.56695182E-09	0.25386611E-08
17		0.12676466E-11	0.47556386E-11	0.35857456E-10	0.16027207E-09	0.71765704E-09
18		0.41330654E-12	0.15505409E-11	0.11691054E-10	0.52255533E-10	0.23398661E-09
19		0.14247148E-12	0.53448901E-12	0.40300333E-11	0.18013063E-10	0.80657800E-10
20		0.42904269E-13	0.16095767E-12	0.12136169E-11	0.54245037E-11	0.24289543E-10
21		0.11539323E-13	0.43290370E-13	0.32640833E-12	0.14589484E-11	0.65327917E-11

I	J	11	12	13	14	15
1		0.74280274E-21	0.51176714E-22	0.79467718E-23	0.14367784E-23	0.30589023E-24
2		0.33558911E-19	0.23121007E-20	0.35902526E-21	0.64911906E-22	0.13819745E-22
3		0.15769504E-17	0.10864676E-18	0.16870778E-19	0.30502451E-20	0.64939716E-21
4		0.46047358E-16	0.31725131E-17	0.49263108E-18	0.89067913E-19	0.18962560E-19
5		0.20029566E-14	0.13799719E-15	0.21428345E-16	0.38742551E-17	0.82482909E-18
6		0.74930757E-12	0.51624869E-13	0.80163605E-14	0.14493625E-14	0.30856902E-15
7		0.19985608E-09	0.13769435E-10	0.21381317E-11	0.38657543E-12	0.82301841E-13
8		0.18466295E-07	0.12722678E-08	0.19755914E-09	0.35718803E-10	0.76045255E-11
9		0.42465740E-06	0.29257524E-07	0.45431392E-08	0.82140206E-09	0.17487643E-09
10		0.49385380E-05	0.34024936E-06	0.52834277E-07	0.95524690E-08	0.20337194E-08
11		0.27963368E-04	0.19265854E-05	0.29916220E-06	0.54088744E-07	0.11515478E-07
12		0.36266856E-05	0.12622109E-04	0.19599738E-05	0.35436449E-06	0.75444120E-07
13		0.85296188E-06	0.29686007E-05	0.16669132E-04	0.30137899E-05	0.64163561E-06
14		0.19265627E-06	0.67050962E-06	0.37650161E-05	0.98216933E-05	0.20910375E-05
15		0.48359745E-07	0.16830853E-06	0.94507845E-06	0.24654009E-05	0.82559836E-05
16		0.75332061E-08	0.26218125E-07	0.14721888E-06	0.38404545E-06	0.12860592E-05
17		0.21295690E-08	0.74116322E-08	0.41617458E-07	0.10856633E-06	0.36356005E-06
18		0.69433037E-09	0.24165070E-08	0.13569064E-07	0.35397225E-07	0.11853615E-06
19		0.23934343E-09	0.83299767E-09	0.46774069E-08	0.12201831E-07	0.40860744E-07
20		0.72076567E-10	0.25085134E-09	0.14085684E-08	0.36744956E-08	0.12304930E-07
21		0.19385368E-10	0.67467670E-10	0.37884162E-09	0.98827369E-09	0.33094740E-08

I	J	16	17	18	19	20
1		0.38900016E-25	0.10201709E-25	0.31400533E-26	0.10285841E-26	0.28938580E-27
2		0.17574543E-23	0.46090061E-24	0.14186381E-24	0.46470172E-25	0.13074103E-25
3		0.82583677E-22	0.21657955E-22	0.66662533E-23	0.21836569E-23	0.61435910E-24
4		0.24114641E-20	0.63241775E-21	0.19465630E-21	0.63763324E-22	0.17939454E-22
5		0.10489327E-18	0.27508751E-19	0.84671112E-20	0.27735622E-20	0.78032576E-21
6		0.39240654E-16	0.10291047E-16	0.31675525E-17	0.10375914E-17	0.29192041E-18
7		0.10466305E-13	0.27448382E-14	0.84485247E-15	0.27674747E-15	0.77861320E-16
8		0.96706497E-12	0.25361749E-12	0.78062665E-13	0.25570905E-13	0.71942303E-14
9		0.22238975E-10	0.58322826E-11	0.17951560E-11	0.58803776E-12	0.16544107E-12
10		0.25862734E-09	0.67826328E-10	0.20876703E-10	0.68385644E-11	0.19239905E-11
11		0.14644197E-08	0.38405168E-09	0.11820969E-09	0.38721845E-10	0.10894148E-10
12		0.95942063E-08	0.25161295E-08	0.77445539E-09	0.25368774E-09	0.71373560E-10
13		0.81596568E-07	0.21399117E-07	0.65865677E-08	0.21575579E-08	0.60701622E-09
14		0.26591658E-06	0.69737951E-07	0.21465091E-07	0.70313035E-08	0.19782160E-08
15		0.10499107E-05	0.27534441E-06	0.84750013E-07	0.27761498E-07	0.78105309E-08
16		0.59654321E-05	0.15644646E-05	0.48153657E-06	0.15773657E-06	0.44378236E-07
17		0.16863751E-05	0.45872957E-05	0.14119532E-05	0.46251222E-06	0.13012504E-06
18		0.54982974E-06	0.14956540E-05	0.41612075E-05	0.13630834E-05	0.38349521E-06
19		0.18953261E-06	0.51556913E-06	0.14344168E-05	0.39469260E-05	0.11104439E-05
20		0.57076413E-07	0.15526001E-06	0.43196468E-06	0.11885886E-05	0.43502296E-05
21		0.15350970E-07	0.41757971E-07	0.11617914E-06	0.31967693E-06	0.11700167E-05

I	J	21
1		0.73117629E-28
2		0.33033612E-26
3		0.15522682E-24
4		0.45326622E-23
5		0.19716059E-21
6		0.73757886E-19
7		0.19672777E-16
8		0.18177251E-14
9		0.41801062E-13
10		0.48612405E-12
11		0.27525681E-11
12		0.18033561E-10
13		0.15337140E-09
14		0.49982463E-09
15		0.19734439E-08
16		0.11212805E-07
17		0.32877978E-07
18		0.96895633E-07
19		0.28056974E-06
20		0.10991498E-05
21		0.35998619E-05

SECTION E - SAMPLE INPUT & OUTPUT

αFB^{-1}

IMAT = 2

INDEX(4) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 2
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 1 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

G E O M E T R I C I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511170E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

ALPHA B MATRIX INVERSE F

I	FINAL ANSWERS
1	-0.38415543E-02
2	-0.17355675E 00
3	-0.15060654E 01
4	-0.26199942E 01
5	-0.30891132E 01
6	-0.26022635E 01
7	-0.18353462E 01
8	-0.18310022E 01
9	-0.16112576E 01
10	-0.15556459E 01
11	-0.13702049E 01
12	-0.11081715E 01
13	-0.99193245E 00
14	-0.88227552E 00
15	-0.94078726E 00
16	-0.82828283E 00
17	-0.80633312E 00
18	-0.78522062E 00
19	-0.58843279E 00
20	-0.62664169E 00
21	-0.49962330E 00

SECTION F - SAMPLE INPUT & OUTPUT

αB^{-1} and αFB^{-1}

IMAT = 2

INDEX(3) and INDEX(4) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 2
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 1 ALPHA B MATRIX INVERSE
INDEX(4)= 1 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

G E O M E T R I C I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

I	J	1	2	3	4	5
1		0.21644370E-03	0.45101733E-05	0.62929132E-07	0.14266754E-08	0.34782302E-10
2		0.12210278E-03	0.20376421E-03	0.28430622E-05	0.64455435E-07	0.15714217E-08
3		0.45399094E-04	0.75761694E-04	0.13359697E-03	0.30287947E-05	0.73841932E-07
4		0.11816545E-04	0.19719373E-04	0.34772864E-04	0.88441593E-04	0.21562028E-05
5		0.32732660E-05	0.54624034E-05	0.96323156E-05	0.24498935E-04	0.93789989E-04
6		0.53096301E-06	0.88606765E-06	0.15624773E-05	0.39740189E-05	0.15213877E-04
7		0.62869219E-07	0.10491561E-06	0.18500674E-06	0.47054800E-06	0.18014134E-05
8		0.12405692E-07	0.20702537E-07	0.36506545E-07	0.92851110E-07	0.35546475E-06
9		0.21481539E-08	0.35848251E-08	0.63214252E-08	0.16077955E-07	0.61551816E-07
10		0.57269745E-09	0.95571373E-09	0.16852924E-08	0.42863846E-08	0.16409718E-07
11		0.15683337E-09	0.26172220E-09	0.46151727E-09	0.11738266E-08	0.44938062E-08
12		0.20340341E-10	0.33943806E-10	0.59856037E-10	0.15223840E-09	0.58281957E-09
13		0.47838521E-11	0.79832616E-11	0.14077572E-10	0.35804998E-10	0.13707356E-09
14		0.10805159E-11	0.18031583E-11	0.31796649E-11	0.80871854E-11	0.30960456E-10
15		0.27122673E-12	0.45262156E-12	0.79814557E-12	0.20300116E-11	0.77715638E-11
16		0.42250136E-13	0.70506697E-13	0.12433050E-12	0.31622339E-12	0.12106097E-11
17		0.11943741E-13	0.19931624E-13	0.35147100E-13	0.89393499E-13	0.34222842E-12
18		0.38941662E-14	0.64985554E-14	0.11459458E-13	0.29146058E-13	0.11158099E-12
19		0.13423626E-14	0.22401258E-14	0.39502025E-14	0.10046983E-13	0.38463234E-13
20		0.40424330E-15	0.67459822E-15	0.11895750E-14	0.30255775E-14	0.11582932E-13
21		0.10872322E-15	0.18143663E-15	0.31994234E-15	0.81374476E-15	0.31152887E-14

I	J	6	7	8	9	10
1		0.55864455E-13	0.93038628E-16	0.14981440E-17	0.50421767E-19	0.51757822E-20
2		0.25238865E-11	0.42033671E-14	0.67684312E-16	0.22779950E-17	0.23383564E-18
3		0.11859876E-09	0.19751854E-12	0.31805202E-14	0.10704415E-15	0.10988060E-16
4		0.34631134E-08	0.57675930E-11	0.92871999E-13	0.31257161E-14	0.32085417E-15
5		0.15063762E-06	0.25087732E-09	0.40397234E-11	0.13596161E-12	0.13956440E-13
6		0.56353660E-04	0.93853430E-07	0.15112640E-08	0.50863341E-10	0.52211178E-11
7		0.66726179E-05	0.25032670E-04	0.40308601E-06	0.13566318E-07	0.13925820E-08
8		0.13166773E-05	0.49395821E-05	0.37244332E-04	0.12535002E-05	0.12867179E-06
9		0.22799406E-06	0.85533225E-06	0.64491842E-05	0.28825933E-04	0.29589810E-05
10		0.60783236E-07	0.22803152E-06	0.17193524E-05	0.76849919E-05	0.34411350E-04
11		0.16645490E-07	0.62446418E-07	0.47084467E-06	0.21045353E-05	0.94235584E-05
12		0.21588198E-08	0.80989260E-08	0.61065691E-07	0.27294561E-06	0.12221781E-05
13		0.50773385E-09	0.19047888E-08	0.14362065E-07	0.64194182E-07	0.28744472E-06
14		0.11468056E-09	0.43022985E-09	0.32439216E-08	0.14499385E-07	0.64924393E-07
15		0.28786612E-10	0.10799450E-09	0.81427576E-09	0.36395729E-08	0.16297044E-07
16		0.44842125E-11	0.16822752E-10	0.12684319E-09	0.56695182E-09	0.25386611E-08
17		0.12676466E-11	0.47556386E-11	0.35857456E-10	0.16027207E-09	0.71765704E-09
18		0.41330654E-12	0.15505409E-11	0.11691054E-10	0.52255533E-10	0.23398661E-09
19		0.14247148E-12	0.53448901E-12	0.40300333E-11	0.18013063E-10	0.80657800E-10
20		0.42904269E-13	0.16095767E-12	0.12136169E-11	0.54245037E-11	0.24289543E-10
21		0.11539323E-13	0.43290370E-13	0.32640833E-12	0.14589484E-11	0.65327917E-11

I	J	11	12	13	14	15
1		0.74280274E-21	0.51176714E-22	0.79467718E-23	0.14367784E-23	0.30589023E-24
2		0.33558911E-19	0.23121007E-20	0.35902526E-21	0.64911906E-22	0.13819745E-22
3		0.15769504E-17	0.10864676E-18	0.16870778E-19	0.30502451E-20	0.64939716E-21
4		0.46047358E-16	0.31725131E-17	0.49263108E-18	0.89067913E-19	0.18962560E-19
5		0.20029566E-14	0.13799719E-15	0.21428345E-16	0.38742551E-17	0.82482909E-18
6		0.74930757E-12	0.51624869E-13	0.80163605E-14	0.14493625E-14	0.30856902E-15
7		0.19985608E-09	0.13769435E-10	0.21381317E-11	0.38557543E-12	0.82301841E-13
8		0.18466295E-07	0.12722678E-08	0.19755914E-09	0.35718803E-10	0.76045255E-11
9		0.42465740E-06	0.29257524E-07	0.45431392E-08	0.82140206E-09	0.17487643E-09
10		0.49385380E-05	0.34024936E-06	0.52834277E-07	0.95524690E-08	0.20337194E-08
11		0.27963368E-04	0.19265854E-05	0.29916220E-06	0.54088744E-07	0.11515478E-07
12		0.36266856E-05	0.12622109E-04	0.19599738E-05	0.35436449E-06	0.75444120E-07
13		0.85296188E-06	0.29686007E-05	0.16669132E-04	0.30137899E-05	0.64163561E-06
14		0.19265627E-06	0.67050962E-06	0.37650161E-05	0.98216933E-05	0.20910375E-05
15		0.48359745E-07	0.16830853E-06	0.94507845E-06	0.24654009E-05	0.82559836E-05
16		0.75332061E-08	0.26218125E-07	0.14721888E-06	0.38404545E-06	0.12860692E-05
17		0.21295690E-08	0.74116322E-08	0.41617458E-07	0.10856633E-06	0.36356005E-06
18		0.69433037E-09	0.24165070E-08	0.13569064E-07	0.35397225E-07	0.11853615E-06
19		0.23934343E-09	0.83299767E-09	0.46774069E-08	0.12201831E-07	0.40860744E-07
20		0.72076567E-10	0.25085134E-09	0.14085684E-08	0.36744956E-08	0.12304930E-07
21		0.19385368E-10	0.67467670E-10	0.37884162E-09	0.98827369E-09	0.33094740E-08

I	J	16	17	18	19	20
1		0.38900016E-25	0.10201709E-25	0.31400533E-26	0.10285841E-26	0.28938580E-27
2		0.17574543E-23	0.46090061E-24	0.14186381E-24	0.46470172E-25	0.13074103E-25
3		0.82583677E-22	0.21657955E-22	0.66662533E-23	0.21836569E-23	0.61435910E-24
4		0.24114641E-20	0.63241775E-21	0.19465630E-21	0.63763324E-22	0.17939454E-22
5		0.10489327E-18	0.27508751E-19	0.84671112E-20	0.27735622E-20	0.78032576E-21
6		0.39240654E-16	0.10291047E-16	0.31675525E-17	0.10375914E-17	0.29192041E-18
7		0.10466305E-13	0.27448382E-14	0.84485247E-15	0.27674747E-15	0.77861320E-16
8		0.96706497E-12	0.25361749E-12	0.78062665E-13	0.25570905E-13	0.71942303E-14
9		0.22238975E-10	0.58322826E-11	0.17951560E-11	0.58803776E-12	0.16544107E-12
10		0.25862734E-09	0.67826328E-10	0.20876703E-10	0.68385644E-11	0.19239905E-11
11		0.14644197E-08	0.38405168E-09	0.11820969E-09	0.38721845E-10	0.10894168E-10
12		0.95942063E-08	0.25161295E-08	0.77445539E-09	0.25368774E-09	0.71373560E-10
13		0.81596568E-07	0.21399117E-07	0.65865677E-08	0.21575579E-08	0.60701622E-09
14		0.26591658E-06	0.69737951E-07	0.21465091E-07	0.70313035E-08	0.19782160E-08
15		0.10499107E-05	0.27534441E-06	0.84750013E-07	0.27761498E-07	0.78105309E-08
16		0.59654321E-05	0.15644646E-05	0.48153657E-06	0.15773657E-06	0.44378236E-07
17		0.16863751E-05	0.45872957E-05	0.14119532E-05	0.46251222E-06	0.13012504E-06
18		0.54982974E-06	0.14956540E-05	0.41612075E-05	0.13630834E-05	0.38349521E-06
19		0.18953261E-06	0.51556913E-06	0.14344168E-05	0.39469260E-05	0.11104439E-05
20		0.57076413E-07	0.15526001E-06	0.43196468E-06	0.11885886E-05	0.43502296E-05
21		0.15350970E-07	0.41757971E-07	0.11617914E-06	0.31967693E-06	0.11700167E-05

I	J 21
1	0.73117629E-28
2	0.33033612E-26
3	0.15522682E-24
4	0.45326622E-23
5	0.19716059E-21
6	0.73757886E-19
7	0.19672777E-16
8	0.18177251E-14
9	0.41801062E-13
10	0.48612405E-12
11	0.27525681E-11
12	0.18033561E-10
13	0.15337140E-09
14	0.49982463E-09
15	0.19734439E-08
16	0.11212805E-07
17	0.32877978E-07
18	0.96895633E-07
19	0.28056974E-06
20	0.10991498E-05
21	0.35998619E-05

ALPHA B MATRIX INVERSE F

I	FINAL ANSWERS
1	-0.38415543E-02
2	-0.17355675E 00
3	-0.15060654E 01
4	-0.26199942E 01
5	-0.30891132E 01
6	-0.26022635E 01
7	-0.18353462E 01
8	-0.18310022E 01
9	-0.16112576E 01
10	-0.15556459E 01
11	-0.13702049E 01
12	-0.11081715E 01
13	-0.99193245E 00
14	-0.88227552E 00
15	-0.94078726E 00
16	-0.82828283E 00
17	-0.80633312E 00
18	-0.78522062E 00
19	-0.58843279E 00
20	-0.62664169E 00
21	-0.49962330E 00

SECTION G - SAMPLE INPUT & OUTPUT

$\alpha B^{-1}A^{-1}$

IMAT = 3

INDEX(5) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 3
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 1 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	F (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44884320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.46075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84951504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96841936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.16465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80363571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				

DIAGONAL MATRIX ELEMENTS

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

I	J	1	2	3	4	5
1		0.26880286E-11	0.10999611E-12	0.25676213E-14	0.10969013E-15	0.41992287E-17
2		0.28806662E-11	0.25668287E-11	0.74241557E-13	0.33846547E-14	0.13381078E-15
3		0.14886582E-11	0.17210781E-11	0.14221376E-11	0.81305242E-13	0.35213711E-14
4		0.50674666E-12	0.66695531E-12	0.76847090E-12	0.84081998E-12	0.48261091E-13
5		0.19116994E-12	0.27801530E-12	0.38250707E-12	0.58199268E-12	0.81846238E-12
6		0.40212215E-13	0.61992403E-13	0.92777023E-13	0.15764272E-12	0.27860928E-12
7		0.63425285E-14	0.10243285E-13	0.16265567E-13	0.29531563E-13	0.58048889E-13
8		0.22806820E-14	0.39107443E-14	0.66463423E-14	0.12899516E-13	0.27765361E-13
9		0.72417484E-15	0.12816909E-14	0.22503978E-14	0.44962881E-14	0.10026183E-13
10		0.37568643E-15	0.67699024E-15	0.12098072E-14	0.24536787E-14	0.55673477E-14
11		0.16458227E-15	0.29867504E-15	0.53734923E-15	0.10959418E-14	0.25025120E-14
12		0.29057373E-16	0.52895699E-16	0.95444983E-16	0.19513417E-15	0.44678892E-15
13		0.10332497E-16	0.18863702E-16	0.34130610E-16	0.69934958E-16	0.16052802E-15
14		0.31000992E-17	0.56676703E-17	0.10268095E-16	0.21062162E-16	0.48403539E-16
15		0.10292873E-17	0.18837169E-17	0.34160301E-17	0.70125899E-17	0.16129943E-16
16		0.21291712E-18	0.38997244E-18	0.70771919E-18	0.14537104E-17	0.33459845E-17
17		0.76760241E-19	0.14066496E-18	0.25540194E-18	0.52482345E-18	0.12085059E-17
18		0.32151580E-19	0.58943081E-19	0.10706328E-18	0.22007331E-18	0.50694012E-18
19		0.14304832E-19	0.26233626E-19	0.47665045E-19	0.98001877E-19	0.22581064E-18
20		0.58731342E-20	0.10774005E-19	0.19581317E-19	0.40269511E-19	0.92810337E-19
21		0.20202925E-20	0.37068128E-20	0.67381196E-20	0.13859030E-19	0.31946197E-19

I	J	6	7	8	9	10
1		0.12822592E-19	0.69793124E-22	0.24860983E-23	0.19403099E-24	0.34803511E-25
2		0.41343935E-18	0.22564952E-20	0.80478542E-22	0.62838703E-23	0.11273990E-23
3		0.11219668E-16	0.61661869E-19	0.22061073E-20	0.17244988E-21	0.30957388E-22
4		0.16572709E-15	0.92538598E-18	0.33342458E-19	0.26129196E-20	0.46966229E-21
5		0.34086810E-14	0.19709464E-16	0.72085906E-18	0.56788431E-19	0.10234829E-19
6		0.36756308E-12	0.24668047E-14	0.95445803E-16	0.76621583E-17	0.13939747E-17
7		0.10658369E-12	0.12854848E-12	0.66586988E-14	0.57814742E-15	0.10908658E-15
8		0.62076863E-13	0.10884447E-12	0.19734800E-12	0.20840792E-13	0.42390713E-14
9		0.23880918E-13	0.45554970E-13	0.96890215E-13	0.13301957E-12	0.35413689E-13
10		0.13650180E-13	0.26958308E-13	0.60617310E-13	0.10724077E-12	0.15375104E-12
11		0.61989056E-14	0.12387359E-13	0.28353015E-13	0.53618761E-13	0.90861184E-13
12		0.11115435E-14	0.22321326E-14	0.51462300E-14	0.99851903E-14	0.17878338E-13
13		0.40095427E-15	0.80875405E-15	0.18767489E-14	0.37234586E-14	0.69693397E-14
14		0.12112619E-15	0.24483127E-15	0.56986873E-15	0.11422076E-14	0.21797114E-14
15		0.40419790E-16	0.81825818E-16	0.19087940E-15	0.38541037E-15	0.74557513E-15
16		0.83934521E-17	0.17011399E-16	0.39749814E-16	0.80703380E-16	0.15769146E-15
17		0.30336369E-17	0.61530971E-17	0.14393326E-16	0.29327020E-16	0.57672197E-16
18		0.12732394E-17	0.25840692E-17	0.60499312E-17	0.12362065E-16	0.24433393E-16
19		0.56739704E-18	0.11520955E-17	0.26991846E-17	0.55276535E-17	0.10968409E-16
20		0.23329802E-18	0.47391798E-18	0.11110111E-17	0.22798545E-17	0.45400048E-17
21		0.80322495E-19	0.16320853E-18	0.38275508E-18	0.78638556E-18	0.15692907E-17

I	J	11	12	13	14	15
1		0.81424990E-26	0.11211938E-26	0.25188544E-27	0.77280455E-28	0.26696555E-28
2		0.26378897E-24	0.36323895E-25	0.81606336E-26	0.25037634E-26	0.86492898E-27
3		0.72452220E-23	0.99774102E-24	0.22416769E-24	0.68777923E-25	0.23759739E-25
4		0.10998020E-21	0.15147896E-22	0.34037281E-23	0.10443580E-23	0.36078721E-24
5		0.23994379E-20	0.33059416E-21	0.74301218E-22	0.22799708E-22	0.78767888E-23
6		0.32811758E-18	0.45261627E-19	0.10180545E-19	0.31249365E-20	0.10797506E-20
7		0.26067015E-16	0.36115615E-17	0.81470042E-18	0.25035903E-18	0.86552509E-19
8		0.10425104E-14	0.14561727E-15	0.33023935E-16	0.10169535E-16	0.35192118E-17
9		0.94559811E-14	0.13497419E-14	0.31037659E-15	0.96092050E-16	0.33336768E-16
10		0.49601206E-13	0.73850736E-14	0.17423205E-14	0.54463837E-15	0.18979699E-15
11		0.98480740E-13	0.16705865E-13	0.42244718E-14	0.13532266E-14	0.47682703E-15
12		0.23383947E-13	0.23833339E-13	0.84434582E-14	0.29648931E-14	0.10855373E-14
13		0.10313449E-13	0.15434146E-13	0.29741583E-13	0.12309161E-13	0.47737083E-14
14		0.33839148E-14	0.56390202E-14	0.12797448E-13	0.14304025E-13	0.66409958E-14
15		0.11949405E-14	0.21209788E-14	0.52046143E-14	0.73667133E-14	0.92557696E-14
16		0.25848947E-15	0.47811579E-15	0.12279047E-14	0.19382369E-14	0.30308222E-14
17		0.95873330E-16	0.18171340E-15	0.47858314E-15	0.79710480E-15	0.13575545E-14
18		0.41061709E-16	0.79261995E-16	0.21256154E-15	0.36702467E-15	0.65792352E-15
19		0.18587569E-16	0.36374043E-16	0.98833313E-16	0.17496666E-15	0.32415845E-15
20		0.77513631E-17	0.15351519E-16	0.42181817E-16	0.76228995E-16	0.14492319E-15
21		0.26911329E-17	0.53669447E-17	0.14841248E-16	0.27129009E-16	0.52295417E-16

I	J	16	17	18	19	20
1		0.55753069E-29	0.21976230E-29	0.98795692E-30	0.47297666E-30	0.19261213E-30
2		0.18063201E-27	0.71200041E-28	0.32088486E-28	0.15323722E-28	0.62403764E-29
3		0.49619875E-26	0.19558891E-26	0.87928601E-27	0.42094858E-27	0.17142575E-27
4		0.75347754E-25	0.29700259E-25	0.13352029E-25	0.63921522E-26	0.26031223E-26
5		0.16450440E-23	0.64844297E-24	0.29151536E-24	0.13956064E-24	0.56834076E-25
6		0.22551920E-21	0.88898200E-22	0.39966028E-22	0.19133651E-22	0.77920391E-23
7		0.18082617E-19	0.71288784E-20	0.32051837E-20	0.15345491E-20	0.62495232E-21
8		0.73560045E-18	0.29006784E-18	0.13043310E-18	0.62457055E-19	0.25435862E-19
9		0.69770054E-17	0.27527934E-17	0.12382546E-17	0.59301617E-18	0.24156254E-18
10		0.39811118E-16	0.15723314E-16	0.70769322E-17	0.33904711E-17	0.13814818E-17
11		0.10056644E-15	0.39815856E-16	0.17946855E-16	0.86058630E-17	0.35089366E-17
12		0.23316234E-15	0.93056337E-16	0.42144693E-16	0.20267725E-16	0.82821277E-17
13		0.10518860E-14	0.42440438E-15	0.19343584E-15	0.93382696E-16	0.38270060E-16
14		0.15659670E-14	0.64913683E-15	0.30043285E-15	0.14636246E-15	0.60390577E-16
15		0.26399728E-14	0.11664673E-14	0.55838402E-15	0.27732155E-15	0.11404120E-15
16		0.46500381E-14	0.25480765E-14	0.13386350E-14	0.69768113E-15	0.30176819E-15
17		0.26503676E-14	0.31416912E-14	0.19786599E-14	0.11138799E-14	0.50535828E-15
18		0.14384701E-14	0.20653829E-14	0.25974403E-14	0.17345386E-14	0.85887214E-15
19		0.75558494E-15	0.11827504E-14	0.17782466E-14	0.22731664E-14	0.13673113E-14
20		0.35372879E-15	0.58491204E-15	0.96445628E-15	0.14984758E-14	0.23402184E-14
21		0.13066298E-15	0.22170995E-15	0.38015601E-15	0.63216441E-15	0.11723262E-14

I	J	21
1		0.67769882E-31
2		0.21956572E-29
3		0.60315672E-28
4		0.91590256E-27
5		0.19997094E-25
6		0.27416261E-23
7		0.21989252E-21
8		0.89500361E-20
9		0.85004854E-19
10		0.48621123E-18
11		0.12353992E-17
12		0.29192230E-17
13		0.13509328E-16
14		0.21391963E-16
15		0.41396260E-16
16		0.10940096E-15
17		0.18726414E-15
18		0.33005676E-15
19		0.56177089E-15
20		0.11394944E-14
21		0.14477343E-14

SECTION H - SAMPLE INPUT & OUTPUT

VD $\alpha B^{-1}A^{-1}$

IMAT = 3

INDEX(6) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 3
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 1 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

G E O M E T R I C I N P U T D A T A

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	F (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44884320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.48075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84951504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96841936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.14465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80363571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				

DIAGONAL MATRIX ELEMENTS

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

I	J	1	2	3	4	5
1		0.10080103E-03	0.39598572E-05	0.12709717E-06	0.65814056E-08	0.28344771E-09
2		0.10802493E-03	0.92405782E-04	0.36749543E-05	0.20307920E-06	0.90322203E-08
3		0.55824654E-04	0.61958781E-04	0.70395763E-04	0.48783131E-05	0.23769240E-06
4		0.19002982E-04	0.24010369E-04	0.38039274E-04	0.50449176E-04	0.32576218E-05
5		0.71688692E-05	0.10008545E-04	0.18934079E-04	0.34919547E-04	0.55246172E-04
6		0.15079568E-05	0.22317254E-05	0.45924589E-05	0.94585603E-05	0.18806109E-04
7		0.23784469E-06	0.36875804E-06	0.80514502E-06	0.17718930E-05	0.39182978E-05
8		0.85525528E-07	0.14078671E-06	0.32899368E-06	0.77397073E-06	0.18741603E-05
9		0.27156542E-07	0.46140848E-07	0.11139457E-06	0.26977716E-06	0.67676700E-06
10		0.14088233E-07	0.24371637E-07	0.59885394E-07	0.14722065E-06	0.37579576E-06
11		0.61718310E-08	0.10752295E-07	0.26598769E-07	0.65756467E-07	0.16891943E-06
12		0.10896508E-08	0.19042441E-08	0.47245230E-08	0.11708046E-07	0.30158237E-07
13		0.38746828E-09	0.67909278E-09	0.16894639E-08	0.41960959E-08	0.10835635E-07
14		0.11625366E-09	0.20403604E-09	0.50827031E-09	0.12637293E-08	0.32672371E-08
15		0.38598250E-10	0.67813768E-10	0.16909338E-09	0.42075521E-09	0.10887704E-08
16		0.79843883E-11	0.14039001E-10	0.35032074E-10	0.87222590E-10	0.22585384E-09
17		0.28785073E-11	0.50639354E-11	0.12642387E-10	0.31489394E-10	0.81574109E-10
18		0.12056831E-11	0.21219493E-11	0.52996288E-11	0.13204395E-10	0.34218434E-10
19		0.53643093E-12	0.94440948E-12	0.23594173E-11	0.58801106E-11	0.15242210E-10
20		0.22024239E-12	0.38786395E-12	0.96927414E-12	0.24161697E-11	0.62646945E-11
21		0.75760904E-13	0.13344518E-12	0.33353669E-12	0.83154154E-12	0.21563671E-11

I	J	6	7	8	9	10
1		0.10001618E-11	0.79564109E-14	0.24612363E-15	0.22992656E-16	0.33933405E-17
2		0.32248260E-10	0.25724036E-12	0.79673715E-14	0.74463831E-15	0.10992134E-15
3		0.87513374E-09	0.70294508E-11	0.21840456E-12	0.20435300E-13	0.30183437E-14
4		0.12926709E-07	0.10549396E-09	0.33009021E-11	0.30963079E-12	0.45792048E-13
5		0.26587702E-06	0.22468782E-08	0.71365025E-10	0.67294260E-11	0.99789448E-12
6		0.28669907E-04	0.28121565E-06	0.94491313E-08	0.90796526E-09	0.13591246E-09
7		0.83135255E-05	0.14654523E-04	0.65921103E-06	0.68510417E-07	0.10635933E-07
8		0.48419943E-05	0.12408265E-04	0.19537445E-04	0.24696319E-05	0.41330924E-06
9		0.18627106E-05	0.51932648E-05	0.95921296E-05	0.15762809E-04	0.24528321E-05
10		0.10647136E-05	0.30732454E-05	0.60011125E-05	0.12708026E-04	0.14990718E-04
11		0.48351450E-06	0.14121579E-05	0.28069480E-05	0.63538200E-05	0.88589604E-05
12		0.86700368E-07	0.25446303E-06	0.50947665E-06	0.11832444E-05	0.17431366E-05
13		0.31274425E-07	0.92197922E-07	0.18579806E-06	0.44122964E-06	0.67951026E-06
14		0.94478381E-08	0.27910755E-07	0.56416994E-07	0.13535151E-06	0.21252174E-06
15		0.31527430E-08	0.93281400E-08	0.18897055E-07	0.45671108E-07	0.72693524E-07
16		0.65468897E-09	0.19392989E-08	0.39352308E-08	0.95633439E-08	0.15374908E-07
17		0.23662361E-09	0.70145267E-09	0.14249391E-08	0.34752503E-08	0.56230363E-08
18		0.99312641E-10	0.29458369E-09	0.59894290E-09	0.14649040E-08	0.23822546E-08
19		0.44256959E-10	0.13133884E-09	0.26721914E-09	0.65502648E-09	0.10694192E-08
20		0.18197235E-10	0.54026630E-10	0.10999007E-09	0.27016256E-09	0.44265014E-09
21		0.62651525E-11	0.18605756E-10	0.37892744E-10	0.93186639E-10	0.15300576E-09

I	J	11	12	13	14	15
1		0.11236645E-17	0.42381111E-18	0.64230730E-19	0.28980153E-19	0.13214789E-19
2		0.36402869E-16	0.13730427E-16	0.20809606E-17	0.93891038E-18	0.42813964E-18
3		0.99984023E-15	0.37714587E-15	0.57162746E-16	0.25791703E-16	0.11761066E-16
4		0.15177262E-13	0.57259021E-14	0.86795043E-15	0.39163394E-15	0.17858959E-15
5		0.33112234E-12	0.12496455E-12	0.18946806E-13	0.85498862E-14	0.38990079E-14
6		0.45280210E-10	0.17108884E-10	0.25960380E-11	0.11718499E-11	0.53447633E-12
7		0.35972472E-08	0.13651698E-08	0.20774857E-09	0.93884581E-10	0.42843465E-10
8		0.14386637E-06	0.55043309E-07	0.84211003E-08	0.38135717E-08	0.17420090E-08
9		0.13049248E-05	0.51020226E-06	0.79145991E-07	0.36034500E-07	0.16501691E-07
10		0.68449644E-05	0.27915567E-05	0.44429163E-06	0.20423926E-06	0.93949439E-07
11		0.13590338E-04	0.63148145E-05	0.10772401E-05	0.50745967E-06	0.23602928E-06
12		0.32269836E-05	0.90089989E-05	0.21530814E-05	0.11118336E-05	0.53734072E-06
13		0.14232555E-05	0.58341047E-05	0.75841017E-05	0.46159321E-05	0.23629846E-05
14		0.46698011E-06	0.21315482E-05	0.32633479E-05	0.53640060E-05	0.32872913E-05
15		0.16490173E-06	0.80172970E-06	0.13271756E-05	0.27625156E-05	0.45816041E-05
16		0.35671537E-07	0.18072768E-06	0.31311561E-06	0.72683844E-06	0.15002561E-05
17		0.13230515E-07	0.68687598E-07	0.12203867E-06	0.29891413E-06	0.67198920E-06
18		0.56665144E-08	0.29961022E-07	0.54203181E-07	0.13763417E-06	0.32567198E-06
19		0.25650837E-08	0.13749382E-07	0.25202489E-07	0.65612426E-07	0.16045834E-06
20		0.10696877E-08	0.58028711E-08	0.10756359E-07	0.28585855E-07	0.71736906E-07
21		0.37137604E-09	0.20287043E-08	0.37845140E-08	0.10173373E-07	0.25886219E-07

I	J	16	17	18	19	20
1		0.37215114E-20	0.17141446E-20	0.79283485E-21	0.43986783E-21	0.15168196E-21
2		0.12057171E-18	0.55535976E-19	0.25686795E-19	0.14251046E-19	0.49142929E-20
3		0.33121230E-17	0.15255924E-17	0.70562663E-18	0.39148178E-18	0.13499769E-18
4		0.50294566E-16	0.23166179E-16	0.10714997E-16	0.59446956E-17	0.20499570E-17
5		0.10980656E-14	0.50578518E-15	0.23394076E-15	0.12979126E-15	0.44756797E-16
6		0.15053388E-12	0.69340529E-13	0.32072720E-13	0.17794275E-13	0.61362252E-14
7		0.12070133E-10	0.55605208E-11	0.25721578E-11	0.14271292E-11	0.49214962E-12
8		0.49101256E-09	0.22625275E-09	0.10467249E-09	0.58080957E-10	0.20030727E-10
9		0.46571458E-08	0.21471773E-08	0.99369868E-09	0.55150440E-09	0.19023037E-09
10		0.26573890E-07	0.12264174E-07	0.56792331E-08	0.31531349E-08	0.10879162E-08
11		0.67127985E-07	0.31056345E-07	0.14402342E-07	0.80034432E-08	0.27632858E-08
12		0.15563563E-06	0.72583873E-07	0.33821099E-07	0.18848965E-07	0.65221712E-08
13		0.70213309E-06	0.33103515E-06	0.15523216E-06	0.86845773E-07	0.30137652E-07
14		0.10452814E-05	0.50632639E-06	0.24109721E-06	0.13611691E-06	0.47557549E-07
15		0.17621796E-05	0.90984383E-06	0.44810292E-06	0.25790877E-06	0.91382333E-07
16		0.31038962E-05	0.19874979E-05	0.10742533E-05	0.64884279E-06	0.23764227E-06
17		0.17691182E-05	0.24505171E-05	0.15878732E-05	0.10359072E-05	0.39796936E-06
18		0.96017720E-06	0.16109971E-05	0.20844445E-05	0.16131189E-05	0.67636137E-06
19		0.50435239E-06	0.92254470E-06	0.14270418E-05	0.21140422E-05	0.10767562E-05
20		0.23611369E-06	0.45623108E-06	0.77397573E-06	0.13935805E-05	0.18429200E-05
21		0.87217416E-07	0.17293360E-06	0.30507499E-06	0.58791233E-06	0.92320630E-06

I	J	21
1		0.66075595E-22
2		0.21407646E-20
3		0.58807734E-19
4		0.89300434E-18
5		0.19497150E-16
6		0.26730842E-14
7		0.21439507E-12
8		0.87262810E-11
9		0.82879689E-10
10		0.47405568E-09
11		0.12045136E-08
12		0.28462410E-08
13		0.13171586E-07
14		0.20857151E-07
15		0.40361332E-07
16		0.10666588E-06
17		0.18258243E-06
18		0.32180515E-06
19		0.54772636E-06
20		0.11110060E-05
21		0.14115394E-05

SECTION I - SAMPLE INPUT & OUTPUT

LVD $\alpha B^{-1}A^{-1}$

IMAT = 3

INDEX(7) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 3
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 1 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 0 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479994E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

D I A G O N A L M A T R I X E L E M E N T S

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44984320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.46075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84951504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96841936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.16465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80363571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				

A L P H A B A I N V E R S E V D L

SECTION NUMBER NUMBER	FINAL ANSWERS
1	0.848998E-05
2	0.246535E-03
3	0.480383E-02
4	0.652980E-02
5	0.708773E-01
6	0.154674E 00
7	0.429225E-01
8	0.247300E-01
9	0.110817E-01
10	0.865148E-02
11	0.451300E-02
12	0.853267E-03
13	0.322235E-03
14	0.994087E-04
15	0.336782E-04
16	0.707312E-05
17	0.257525E-05
18	0.108718E-05
19	0.486707E-06
20	0.200956E-06
21	0.693598E-07

D I A G O N A L M A T R I X E L E M E N T S

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534975E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

SECTION J - SAMPLE INPUT & OUTPUT

$$D.O._{SAT.} + FB^{-1} - LVD\ \alpha B^{-1}A^{-1}$$

IMAT = 3

INDEX(8) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 3
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 0 ALPHA A MATRIX INVERSE
INDEX(2)= 0 ALPHA A MATRIX INVERSE L
INDEX(3)= 0 ALPHA B MATRIX INVERSE
INDEX(4)= 0 ALPHA B MATRIX INVERSE F
INDEX(5)= 0 ALPHA B A MATRIX INVERSE
INDEX(6)= 0 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VD(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 0 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 1 DISOXS + ALPHA B MATRIX INVERSE F - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFP = 1.00

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

DIAGONAL MATRIX ELEMENTS

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44884320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.46075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84951504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96841936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.16465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80363571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				

DIAGONAL MATRIX ELEMENTS

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

D.O.SATURATION + ALPHA B INVERSE F - ALPHA B A INVERSE V D L

SECTION NUMBER	FINAL ANSWERS
1	0.776615E 01
2	0.759620E 01
3	0.625913E 01
4	0.514348E 01
5	0.461001E 01
6	0.501306E 01
7	0.589173E 01
8	0.591427E 01
9	0.614766E 01
10	0.620570E 01
11	0.639528E 01
12	0.666097E 01
13	0.677774E 01
14	0.688762E 01
15	0.682918E 01
16	0.694171E 01
17	0.696366E 01
18	0.698478E 01
19	0.718157E 01
20	0.714336E 01
21	0.727038E 01

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

SECTION K - SAMPLE INPUT & OUTPUT

(COMPOSITE OUTPUT)

IMAT = 3

INDEX(1) through INDEX(8) = 1

PROGRAM CASE CONTROL

NUMBER OF ESTUARINE SECTIONS= 21

MATRIX CASE BEING CONSIDERED=IMAT= 3
WHERE= IMAT=1 A MATRIX
IMAT=2 B MATRIX
IMAT=3 A AND B MATRIX

PRINTING INDEX OPTIONS IF INDEX(N)=1
FOR THIS CASE INDEX(1)= 1 ALPHA A MATRIX INVERSE
INDEX(2)= 1 ALPHA A MATRIX INVERSE L
INDEX(3)= 1 ALPHA B MATRIX INVERSE
INDEX(4)= 1 ALPHA B MATRIX INVERSE E
INDEX(5)= 1 ALPHA B A MATRIX INVERSE
INDEX(6)= 1 ALPHA B A MATRIX INVERSE TIMES ELEMENTS VO(I) FORMED ON MATRIX COLUMNS
INDEX(7)= 1 ALPHA B A MATRIX INVERSE V D L
INDEX(8)= 1 DISPXS + ALPHA B MATRIX INVERSE E - ALPHA B A MATRIX INVERSE V D L

CONVERSION FACTORS USED ON INPUT VARIABLES
CFQ = 1.00
CFLEN = 1.00
CFK = 1.00
CFL = 1.00
CFAREA = 1.00
CFVOL = 1.00
CFD = 1.00

GEOMETRIC INPUT DATA

INTERFACE NUMBER	AREA (SQ.FT)	SECTION NUMBER	VOLUME (CU.FT.)	LENGTH (FT.)
0 - 1	4605.	0		14203.2
1 - 2	30000.	1	0.250000E 09	14203.2
2 - 3	24300.	2	0.240000E 09	11035.2
3 - 4	28950.	3	0.330000E 09	9240.0
4 - 5	34260.	4	0.400000E 09	6916.8
5 - 6	32700.	5	0.450000E 09	9873.6
6 - 7	47640.	6	0.520000E 09	11140.8
7 - 8	49860.	7	0.760000E 09	13569.6
8 - 9	43620.	8	0.660000E 09	11563.2
9 - 10	58980.	9	0.790000E 09	12883.2
10 - 11	66480.	10	0.650000E 09	10876.8
11 - 12	84900.	11	0.920000E 09	11880.0
12 - 13	110670.	12	0.252000E 10	20856.0
13 - 14	129750.	13	0.170000E 10	15523.2
14 - 15	171090.	14	0.250000E 10	15470.4
15 - 16	116730.	15	0.330000E 10	23284.8
16 - 17	221526.	16	0.445000E 10	22492.8
17 - 18	220200.	17	0.520000E 10	23337.6
18 - 19	242520.	18	0.535000E 10	24129.6
19 - 20	193920.	19	0.620000E 10	28300.8
20 - 21	216960.	20	0.525000E 10	27456.0
21 - 22	222600.	21	0.650000E 10	30043.2
		22		30043.2

HYDRAULIC AND LOAD INPUT DATA

SECTION NUMBER	T (DEG C)	J (LBS/DAY)	P (LBS/DAY)	INTERFACE NUMBER	K (SQ.MI/DAY)	Q (CU.FT/SEC)
				0 - 1	0.0	0.500000E 03
1	20.000	0.0	0.0	1 - 2	0.500000E-01	0.500000E 03
2	20.000	0.0	-0.700000E 03	2 - 3	0.100000E 00	0.500000E 03
3	20.000	0.640000E 02	-0.103000E 05	3 - 4	0.150000E 00	0.502000E 03
4	20.000	0.0	-0.248000E 05	4 - 5	0.200000E 00	0.502000E 03
5	20.000	0.123900E 04	-0.253000E 05	5 - 6	0.250000E 00	0.528000E 03
6	20.000	0.457200E 04	-0.372000E 05	6 - 7	0.350000E 00	0.864000E 03
7	20.000	0.0	-0.604000E 05	7 - 8	0.450000E 00	0.903000E 03
8	20.000	0.0	-0.380000E 05	8 - 9	0.550000E 00	0.903000E 03
9	20.000	0.750000E 02	-0.418000E 05	9 - 10	0.650000E 00	0.903000E 03
10	20.000	0.147000E 03	-0.284000E 05	10 - 11	0.750000E 00	0.907000E 03
11	20.000	0.230000E 02	-0.303000E 05	11 - 12	0.900000E 00	0.910000E 03
12	20.000	0.0	-0.682000E 05	12 - 13	0.120000E 01	0.913000E 03
13	20.000	0.0	-0.321000E 05	13 - 14	0.150000E 01	0.913000E 03
14	20.000	0.0	-0.522000E 05	14 - 15	0.195000E 01	0.913000E 03
15	20.000	0.0	-0.781000E 05	15 - 16	0.280000E 01	0.913000E 03
16	20.000	0.0	-0.813000E 05	16 - 17	0.390000E 01	0.913000E 03
17	20.000	0.0	-0.946000E 05	17 - 18	0.530000E 01	0.913000E 03
18	20.000	0.0	-0.111200E 06	18 - 19	0.600000E 01	0.913000E 03
19	20.000	0.0	-0.599000E 05	19 - 20	0.600000E 01	0.913000E 03
20	20.000	0.0	-0.866000E 05	20 - 21	0.600000E 01	0.913000E 03
21	20.000	0.0	-0.100200E 06	21 - 22	0.600000E 01	0.913000E 03

COMPUTED SYSTEM PARAMETERS

SECTION NUMBER	R (/DAY)	D (/DAY)	SATURATION D.O.(MG/L)	INTERFACE NUMBER	XI	E (CU.FT./DAY)	Q (CU.FT./DAY)
				0 - 1	0.100000E 01	0.0	0.432000E 08
1	0.120	0.150	7.770	1 - 2	0.961646E 00	0.331381E 07	0.432000E 08
2	0.140	0.150	7.770	2 - 3	0.884302E 00	0.668249E 07	0.432000E 08
3	0.220	0.150	7.770	3 - 4	0.749900E 00	0.149859E 08	0.433728E 08
4	0.330	0.150	7.770	4 - 5	0.571895E 00	0.227538E 08	0.433728E 08
5	0.270	0.150	7.770	5 - 6	0.534532E 00	0.216904E 08	0.456192E 08
6	0.400	0.150	7.770	6 - 7	0.510267E 00	0.376234E 08	0.746496E 08
7	0.730	0.150	7.770	7 - 8	0.450855E 00	0.497762E 08	0.780192E 08
8	0.500	0.150	7.770	8 - 9	0.539916E 00	0.547181E 08	0.780192E 08
9	0.540	0.150	7.770	9 - 10	0.473002E 00	0.899642E 08	0.780192E 08
10	0.440	0.150	7.770	10 - 11	0.542222E 00	0.122163E 09	0.783648E 08
11	0.390	0.150	7.770	11 - 12	0.477958E 00	0.130143E 09	0.786240E 08
12	0.400	0.150	7.770	12 - 13	0.362903E 00	0.203543E 09	0.788832E 08
13	0.300	0.150	7.770	13 - 14	0.573295E 00	0.350126E 09	0.788832E 08
14	0.410	0.150	7.770	14 - 15	0.500852E 00	0.479984E 09	0.788832E 08
15	0.380	0.150	7.770	15 - 16	0.399182E 00	0.398094E 09	0.788832E 08
16	0.360	0.150	7.770	16 - 17	0.508651E 00	0.105107E 10	0.788832E 08
17	0.360	0.150	7.770	17 - 18	0.490783E 00	0.137087E 10	0.788832E 08
18	0.360	0.150	7.770	18 - 19	0.491657E 00	0.154744E 10	0.788832E 08
19	0.360	0.150	7.770	19 - 20	0.460222E 00	0.116352E 10	0.788832E 08
20	0.360	0.150	7.770	20 - 21	0.507576E 00	0.126231E 10	0.788832E 08
21	0.360	0.150	7.770	21 - 22	0.500000E 00	0.123936E 10	0.788832E 08

DIAGONAL MATRIX ELEMENTS

A-MATRIX

MAIN DIAGONAL ELEMENTS		UPPER DIAGONAL ELEMENTS		LOWER DIAGONAL ELEMENTS	
A(1, 1)=	0.82356864E 08	A(1, 2)=	-0.16569040E 07	A(2, 1)=	-0.44856880E 08
A(2, 2)=	0.82541184E 08	A(2, 3)=	-0.16843470E 07	A(3, 2)=	-0.44884320E 08
A(3, 3)=	0.98695408E 08	A(3, 4)=	-0.41383280E 07	A(4, 3)=	-0.47511120E 08
A(4, 4)=	0.11169682E 09	A(4, 5)=	-0.41857280E 07	A(5, 4)=	-0.47558512E 08
A(5, 5)=	0.11776109E 09	A(5, 6)=	-0.45620800E 06	A(6, 5)=	-0.46075392E 08
A(6, 6)=	0.15417074E 09	A(6, 7)=	-0.10649600E 07	A(7, 6)=	-0.75714544E 08
A(7, 7)=	0.20001643E 09	A(7, 8)=	-0.69323200E 07	A(8, 7)=	-0.84951504E 08
A(8, 8)=	0.20277424E 09	A(8, 9)=	-0.18822752E 08	A(9, 8)=	-0.96841936E 08
A(9, 9)=	0.26419011E 09	A(9,10)=	-0.48848224E 08	A(10, 9)=	-0.12686741E 09
A(10,10)=	0.31100186E 09	A(10,11)=	-0.86289104E 08	A(11,10)=	-0.16465389E 09
A(11,11)=	0.39201152E 09	A(11,12)=	-0.89098464E 08	A(12,11)=	-0.16772245E 09
A(12,12)=	0.69926784E 09	A(12,13)=	-0.15328653E 09	A(13,12)=	-0.23216971E 09
A(13,13)=	0.80363571E 09	A(13,14)=	-0.31646618E 09	A(14,13)=	-0.39534925E 09
A(14,14)=	0.12109588E 10	A(14,15)=	-0.44060954E 09	A(15,14)=	-0.51949286E 09
A(15,15)=	0.13651917E 10	A(15,16)=	-0.35069901E 09	A(16,15)=	-0.42958234E 09
A(16,16)=	0.21093960E 10	A(16,17)=	-0.10123144E 10	A(17,16)=	-0.10911977E 10
A(17,17)=	0.32019016E 10	A(17,18)=	-0.13307044E 10	A(18,17)=	-0.14095875E 10
A(18,18)=	0.37194258E 10	A(18,19)=	-0.15073385E 10	A(19,18)=	-0.15862218E 10
A(19,19)=	0.36371612E 10	A(19,20)=	-0.11209403E 10	A(20,19)=	-0.11998234E 10
A(20,20)=	0.32107914E 10	A(20,21)=	-0.12234683E 10	A(21,20)=	-0.13023516E 10
A(21,21)=	0.34772705E 10				

DIAGONAL MATRIX ELEMENTS

B-MATRIX

MAIN DIAGONAL ELEMENTS	UPPER DIAGONAL ELEMENTS	LOWER DIAGONAL ELEMENTS
B(1, 1)= 0.74856864E 08	B(1, 2)= -0.16569040E 07	B(2, 1)= -0.44856880E 08
B(2, 2)= 0.80141184E 08	B(2, 3)= -0.16843470E 07	B(3, 2)= -0.44884320E 08
B(3, 3)= 0.12179539E 09	B(3, 4)= -0.41383280E 07	B(4, 3)= -0.47511120E 08
B(4, 4)= 0.18369682E 09	B(4, 5)= -0.41857280E 07	B(5, 4)= -0.47558512E 08
B(5, 5)= 0.17176107E 09	B(5, 6)= -0.45620800E 06	B(6, 5)= -0.46075392E 08
B(6, 6)= 0.28417050E 09	B(6, 7)= -0.10649600E 07	B(7, 6)= -0.75714544E 08
B(7, 7)= 0.64081613E 09	B(7, 8)= -0.69323200E 07	B(8, 7)= -0.84951504E 08
B(8, 8)= 0.43377408E 09	B(8, 9)= -0.18822752E 08	B(9, 8)= -0.96841936E 08
B(9, 9)= 0.57228979E 09	B(9,10)= -0.48848224E 08	B(10, 9)= -0.12686741E 09
B(10,10)= 0.49950182E 09	B(10,11)= -0.86289104E 08	B(11,10)= -0.16465389E 09
B(11,11)= 0.61281126E 09	B(11,12)= -0.89098464E 08	B(12,11)= -0.16772245E 09
B(12,12)= 0.13292677E 10	B(12,13)= -0.15328653E 09	B(13,12)= -0.23216971E 09
B(13,13)= 0.10586358E 10	B(13,14)= -0.31646618E 09	B(14,13)= -0.39534925E 09
B(14,14)= 0.18609587E 10	B(14,15)= -0.44060954E 09	B(15,14)= -0.51949286E 09
B(15,15)= 0.21241917E 10	B(15,16)= -0.35069901E 09	B(16,15)= -0.42958234E 09
B(16,16)= 0.30438953E 10	B(16,17)= -0.10123144E 10	B(17,16)= -0.10911977E 10
B(17,17)= 0.42939013E 10	B(17,18)= -0.13307044E 10	B(18,17)= -0.14095875E 10
B(18,18)= 0.48429220E 10	B(18,19)= -0.15073385E 10	B(19,18)= -0.15862218E 10
B(19,19)= 0.49391575E 10	B(19,20)= -0.11209403E 10	B(20,19)= -0.11998234E 10
B(20,20)= 0.43132887E 10	B(20,21)= -0.12234683E 10	B(21,20)= -0.13023516E 10
B(21,21)= 0.48422666E 10		

I	J	1	2	3	4	5
1		0.19644471E-03	0.39809856E-05	0.69193050E-07	0.26030185E-08	0.92629779E-10
2		0.10777620E-03	0.19787614E-03	0.34392615E-05	0.12938375E-06	0.46041890E-08
3		0.49918046E-04	0.91649155E-04	0.16669781E-03	0.62711115E-05	0.22316078E-06
4		0.21559681E-04	0.39583421E-04	0.71997129E-04	0.14815708E-03	0.52722480E-05
5		0.87171229E-05	0.16004575E-04	0.29110248E-04	0.59903687E-04	0.13815780E-03
6		0.26121306E-05	0.47958501E-05	0.87230237E-05	0.17950428E-04	0.41399719E-04
7		0.10039403E-05	0.18432283E-05	0.33525921E-05	0.68990321E-05	0.15911472E-04
8		0.43686208E-06	0.80207593E-06	0.14588713E-05	0.30020974E-05	0.69238331E-05
9		0.17522098E-06	0.32170419E-06	0.58513791E-06	0.12041100E-05	0.27770802E-05
10		0.81580481E-07	0.14978110E-06	0.27243181E-06	0.56061708E-06	0.12929704E-05
11		0.36410608E-07	0.66849566E-07	0.12159057E-06	0.25021183E-06	0.57707234E-06
12		0.94371266E-08	0.17326499E-07	0.31514624E-07	0.64851463E-07	0.14956919E-06
13		0.32110234E-08	0.58954086E-08	0.10722982E-07	0.22065986E-07	0.50891568E-07
14		0.12307055E-08	0.22595656E-08	0.41098538E-08	0.84573450E-08	0.19505478E-07
15		0.50125726E-09	0.92030361E-09	0.16739126E-08	0.34446144E-08	0.79444362E-08
16		0.12822847E-09	0.23542635E-09	0.42820969E-09	0.88117980E-09	0.20322961E-08
17		0.54482502E-10	0.10002936E-09	0.18194034E-09	0.37440073E-09	0.86349439E-09
18		0.25944885E-10	0.47634563E-10	0.86641028E-10	0.17829162E-09	0.41120085E-09
19		0.13070873E-10	0.23997998E-10	0.43649265E-10	0.89822288E-10	0.20716046E-09
20		0.56975024E-11	0.10460552E-10	0.19026405E-10	0.39152931E-10	0.90299782E-10
21		0.21339007E-11	0.39178183E-11	0.71260133E-11	0.14664048E-10	0.33820238E-10

I	J	6	7	8	9	10
1		0.27483105E-12	0.14857055E-14	0.52756640E-16	0.41128065E-17	0.73728810E-18
2		0.13660557E-10	0.73847395E-13	0.26222840E-14	0.20442823E-15	0.36647132E-16
3		0.66211459E-09	0.35793165E-11	0.12709967E-12	0.99084491E-14	0.17762529E-14
4		0.15642680E-07	0.84562607E-10	0.30027716E-11	0.23409047E-12	0.41964563E-13
5		0.40991227E-06	0.22159392E-08	0.78686779E-10	0.61342728E-11	0.10996690E-11
6		0.10418006E-03	0.56318589E-06	0.19998417E-07	0.15590385E-08	0.27948310E-09
7		0.40040381E-04	0.81434730E-04	0.28917020E-05	0.22543151E-06	0.40412296E-07
8		0.17423459E-04	0.35436125E-04	0.83215127E-04	0.64872902E-05	0.11629527E-05
9		0.69883654E-05	0.14213060E-04	0.33376753E-04	0.68869034E-04	0.12345907E-04
10		0.32536891E-05	0.66174016E-05	0.15539757E-04	0.32064505E-04	0.64465887E-04
11		0.14521711E-05	0.29534467E-05	0.69356320E-05	0.14310881E-04	0.28772134E-04
12		0.37638290E-06	0.76549298E-06	0.17976190E-05	0.37091841E-05	0.74573445E-05
13		0.12806584E-06	0.26046206E-06	0.61164781E-06	0.12620649E-05	0.25373911E-05
14		0.49084470E-07	0.99828696E-07	0.23442936E-06	0.48371840E-06	0.97251996E-06
15		0.19991742E-07	0.40659501E-07	0.95481369E-07	0.19701497E-06	0.39610023E-06
16		0.51141598E-08	0.10401248E-07	0.24425447E-07	0.50399098E-07	0.10132783E-06
17		0.21729336E-08	0.44193449E-08	0.10378031E-07	0.21413868E-07	0.43052783E-07
18		0.10347634E-08	0.21045179E-08	0.49420770E-08	0.10197414E-07	0.20501986E-07
19		0.52130789E-09	0.10602439E-08	0.24897895E-08	0.51373945E-08	0.10328783E-07
20		0.22723440E-09	0.46215276E-09	0.10852819E-08	0.22393554E-08	0.45022404E-08
21		0.85106672E-10	0.17309129E-09	0.40647374E-09	0.83871221E-09	0.16862369E-08

I	J	11	12	13	14	15
1		0.17244999E-18	0.23743980E-19	0.53340322E-20	0.16364856E-20	0.56532019E-21
2		0.85716748E-17	0.11802006E-17	0.26512974E-18	0.81341995E-19	0.28099400E-19
3		0.41546119E-15	0.57203258E-16	0.12850599E-16	0.39425719E-17	0.13619521E-17
4		0.98154076E-14	0.13514468E-14	0.30359969E-15	0.93144613E-16	0.32176571E-16
5		0.25721000E-12	0.35414275E-13	0.79557468E-14	0.24408279E-14	0.84317789E-15
6		0.65370445E-10	0.90006154E-11	0.20219694E-11	0.62034172E-12	0.21429538E-12
7		0.94523394E-08	0.13014581E-08	0.29236968E-09	0.89699234E-10	0.30986352E-10
8		0.27201196E-06	0.37452345E-07	0.84135863E-08	0.25812963E-08	0.89170094E-09
9		0.28876793E-05	0.39759391E-06	0.89318576E-07	0.27403036E-07	0.94662909E-08
10		0.15078424E-04	0.20760936E-05	0.46638957E-06	0.14308887E-06	0.49429570E-07
11		0.50099741E-04	0.68980553E-05	0.15496316E-05	0.47542881E-06	0.16423530E-06
12		0.12985171E-04	0.26513138E-04	0.59561135E-05	0.18273422E-05	0.63124924E-06
13		0.44182570E-05	0.90212061E-05	0.25475238E-04	0.78158346E-05	0.26999523E-05
14		0.16934091E-05	0.34576060E-05	0.97640323E-05	0.18506966E-04	0.63931684E-05
15		0.68971315E-06	0.14082561E-05	0.39768183E-05	0.75377566E-05	0.15148203E-04
16		0.17643799E-06	0.36025136E-06	0.10173244E-05	0.19282606E-05	0.38751186E-05
17		0.74965953E-07	0.15306574E-06	0.43224685E-06	0.81929107E-06	0.16464828E-05
18		0.35699234E-07	0.72890771E-07	0.20583849E-06	0.39015123E-06	0.78406458E-06
19		0.17985069E-07	0.36721943E-07	0.10370019E-06	0.19655590E-06	0.39500691E-06
20		0.78395637E-08	0.16006847E-07	0.45202174E-07	0.85677300E-07	0.17218093E-06
21		0.29361737E-08	0.59950871E-08	0.16929693E-07	0.32088955E-07	0.64487324E-07

I	J	16	17	18	19	20
1		0.11806067E-21	0.46536091E-22	0.20920629E-22	0.10015538E-22	0.40786696E-23
2		0.58682390E-20	0.23130900E-20	0.10398660E-20	0.49782500E-21	0.20273152E-21
3		0.28442812E-18	0.11211332E-18	0.50401355E-19	0.24129120E-19	0.98262110E-20
4		0.67197109E-17	0.26487150E-17	0.11907488E-17	0.57005871E-18	0.23214752E-18
5		0.17608813E-15	0.69408818E-16	0.31203225E-16	0.14938220E-16	0.60833633E-17
6		0.44753182E-13	0.17640400E-13	0.79303663E-14	0.37965812E-14	0.15460998E-14
7		0.64711552E-11	0.25507400E-11	0.11467025E-11	0.54897260E-12	0.22356075E-12
8		0.18622186E-09	0.73403186E-10	0.32998909E-10	0.15797905E-10	0.64334588E-11
9		0.19769306E-08	0.77924822E-09	0.35031622E-09	0.16771046E-09	0.68297576E-10
10		0.10322822E-07	0.40689514E-08	0.18292259E-08	0.87572372E-09	0.35662540E-09
11		0.34298729E-07	0.13519553E-07	0.60778085E-08	0.29096914E-08	0.11849279E-08
12		0.13182944E-06	0.51963323E-07	0.23360464E-07	0.11183591E-07	0.45543516E-08
13		0.56385562E-06	0.22225538E-06	0.99916406E-07	0.47834000E-07	0.19479678E-07
14		0.13351428E-05	0.52627433E-06	0.23659027E-06	0.11326523E-06	0.46125589E-07
15		0.31635363E-05	0.12469727E-05	0.56058553E-06	0.26837461E-06	0.10929153E-06
16		0.10337174E-04	0.40746108E-05	0.18317696E-05	0.87694139E-06	0.35712117E-06
17		0.43921236E-05	0.79612546E-05	0.35790363E-05	0.17134280E-05	0.69776803E-06
18		0.20915541E-05	0.37911959E-05	0.71096938E-05	0.34036957E-05	0.13861045E-05
19		0.10537115E-05	0.19099834E-05	0.35818221E-05	0.67964620E-05	0.27677579E-05
20		0.45930545E-06	0.83254866E-06	0.15612914E-05	0.29625317E-05	0.70192100E-05
21		0.17202501E-06	0.31181679E-06	0.58475422E-06	0.11095644E-05	0.26289244E-05

I	J 21
1	0.14350705E-23
2	0.71330621E-22
3	0.34573274E-20
4	0.81680570E-19
5	0.21404159E-17
6	0.54399124E-15
7	0.78659193E-13
8	0.22635921E-11
9	0.24030278E-10
10	0.12547753E-09
11	0.41691339E-09
12	0.16024342E-08
13	0.68538633E-08
14	0.16229141E-07
15	0.38453919E-07
16	0.12565204E-06
17	0.24550758E-06
18	0.48769675E-06
19	0.97382781E-06
20	0.24696892E-05
21	0.55262881E-05

ALPHA A MATRIX INVERSE L

I	FINAL ANSWERS
1	0.45443794E-05
2	0.22587976E-03
3	0.10948177E-01
4	0.11211649E-01
5	0.17491460E 00
6	0.52816373E 00
7	0.20301640E 00
8	0.88995576E-01
9	0.42475488E-01
10	0.28723411E-01
11	0.13817206E-01
12	0.35812324E-02
13	0.12185287E-02
14	0.46703196E-03
15	0.19021875E-03
16	0.48660499E-04
17	0.20675172E-04
18	0.98456394E-05
19	0.49601749E-05
20	0.21621026E-05
21	0.80977918E-06

I	J	1	2	3	4	5
1		0.21644370E-03	0.45101733E-05	0.62929132E-07	0.14266754E-08	0.34782302E-10
2		0.12210278E-03	0.20376421E-03	0.28430622E-05	0.64455435E-07	0.15714217E-08
3		0.45399094E-04	0.75761694E-04	0.13359697E-03	0.30287947E-05	0.73841932E-07
4		0.11816545E-04	0.19719373E-04	0.34772864E-04	0.88441593E-04	0.21562028E-05
5		0.32732660E-05	0.54624034E-05	0.96323156E-05	0.24498935E-04	0.93789989E-04
6		0.53096301E-06	0.88606765E-06	0.15624773E-05	0.39740189E-05	0.15213877E-04
7		0.62869219E-07	0.10491561E-06	0.18500674E-06	0.47054800E-06	0.18014134E-05
8		0.12405692E-07	0.20702537E-07	0.36506545E-07	0.92851110E-07	0.35546475E-06
9		0.21481539E-08	0.35848251E-08	0.63214252E-08	0.16077955E-07	0.61551816E-07
10		0.57269745E-09	0.95571373E-09	0.16852924E-08	0.42863846E-08	0.16409718E-07
11		0.15683337E-09	0.26172220E-09	0.46151727E-09	0.11738266E-08	0.44938062E-08
12		0.20340341E-10	0.33943806E-10	0.59856037E-10	0.15223840E-09	0.58281957E-09
13		0.47838521E-11	0.79832616E-11	0.14077572E-10	0.35804998E-10	0.13707356E-09
14		0.10805159E-11	0.18031583E-11	0.31796649E-11	0.80871854E-11	0.30960456E-10
15		0.27122673E-12	0.45262156E-12	0.79814557E-12	0.20300116E-11	0.77715638E-11
16		0.42250136E-13	0.70506697E-13	0.12433050E-12	0.31622339E-12	0.12106097E-11
17		0.11943741E-13	0.19931624E-13	0.35147100E-13	0.89393499E-13	0.34222842E-12
18		0.38941662E-14	0.64985554E-14	0.11459458E-13	0.29146058E-13	0.11158099E-12
19		0.13423626E-14	0.22401258E-14	0.39502025E-14	0.10046983E-13	0.38463234E-13
20		0.40424330E-15	0.67459822E-15	0.11895750E-14	0.30255775E-14	0.11582932E-13
21		0.10872322E-15	0.18143663E-15	0.31994234E-15	0.81374476E-15	0.31152887E-14

I	J	6	7	8	9	10
1		0.55864455E-13	0.93038628E-16	0.14981440E-17	0.50421767E-19	0.51757822E-20
2		0.25238865E-11	0.42033671E-14	0.67684312E-16	0.22779950E-17	0.23383564E-18
3		0.11859876E-09	0.19751854E-12	0.31805202E-14	0.10704415E-15	0.10988060E-16
4		0.34631134E-08	0.57675930E-11	0.92871999E-13	0.31257161E-14	0.32085417E-15
5		0.15063762E-06	0.25087732E-09	0.40397234E-11	0.13596161E-12	0.13956440E-13
6		0.56353660E-04	0.93853430E-07	0.15112640E-08	0.50863341E-10	0.52211178E-11
7		0.66726179E-05	0.25032670E-04	0.40308601E-06	0.13566318E-07	0.13925820E-08
8		0.13166773E-05	0.49395821E-05	0.37244332E-04	0.12535002E-05	0.12867179E-06
9		0.22799406E-06	0.85533225E-06	0.64491842E-05	0.28825933E-04	0.29589810E-05
10		0.60783236E-07	0.22803152E-06	0.17193524E-05	0.76849919E-05	0.34411350E-04
11		0.16645490E-07	0.62446418E-07	0.47084467E-06	0.21045353E-05	0.94235584E-05
12		0.21588198E-08	0.80989260E-08	0.61065691E-07	0.27294561E-06	0.12221781E-05
13		0.50773385E-09	0.19047888E-08	0.14362065E-07	0.64194182E-07	0.28744472E-06
14		0.11468056E-09	0.43022985E-09	0.32439216E-08	0.14499385E-07	0.64924393E-07
15		0.28786612E-10	0.10799450E-09	0.81427576E-09	0.36395729E-08	0.16297044E-07
16		0.44842125E-11	0.16822752E-10	0.12684319E-09	0.56695182E-09	0.25386611E-08
17		0.12676466E-11	0.47556386E-11	0.35857456E-10	0.16027207E-09	0.71765704E-09
18		0.41330654E-12	0.15505409E-11	0.11691054E-10	0.52255533E-10	0.23398661E-09
19		0.14247148E-12	0.53448901E-12	0.40300333E-11	0.18013063E-10	0.80657800E-10
20		0.42904259E-13	0.16095767E-12	0.12136169E-11	0.54245037E-11	0.24289543E-10
21		0.11539323E-13	0.43290370E-13	0.32640833E-12	0.14589484E-11	0.65327917E-11

I	J	11	12	13	14	15
1		0.74280274E-21	0.51176714E-22	0.79467718E-23	0.14367784E-23	0.30589023E-24
2		0.33558911E-19	0.23121007E-20	0.35902526E-21	0.64911906E-22	0.13819745E-22
3		0.15769504E-17	0.10864676E-18	0.16870778E-19	0.30502451E-20	0.64939716E-21
4		0.46047358E-16	0.31725131E-17	0.49263108E-18	0.89067913E-19	0.18962560E-19
5		0.20029566E-14	0.13799719E-15	0.21428345E-16	0.38742551E-17	0.82482909E-18
6		0.74930757E-12	0.51624869E-13	0.80163605E-14	0.14493625E-14	0.30856902E-15
7		0.19985608E-09	0.13769435E-10	0.21381317E-11	0.38657543E-12	0.82301841E-13
8		0.18466295E-07	0.12722678E-08	0.19755914E-09	0.35718803E-10	0.76045255E-11
9		0.42465740E-06	0.29257524E-07	0.45431392E-08	0.82140206E-09	0.17487643E-09
10		0.49385380E-05	0.34024936E-06	0.52834277E-07	0.95524690E-08	0.20337194E-08
11		0.27963368E-04	0.19265854E-05	0.29916220E-06	0.54088744E-07	0.11515478E-07
12		0.36266856E-05	0.12622109E-04	0.19599738E-05	0.35436449E-06	0.75444120E-07
13		0.85296188E-06	0.29686007E-05	0.16669132E-04	0.30137899E-05	0.64163561E-06
14		0.19265627E-06	0.67050962E-06	0.37650161E-05	0.98216933E-05	0.20910375E-05
15		0.48359745E-07	0.16830853E-06	0.94507845E-06	0.24654009E-05	0.82559836E-05
16		0.75332061E-08	0.26218125E-07	0.14721888E-06	0.38404545E-06	0.12860692E-05
17		0.21295690E-08	0.74116322E-08	0.41617458E-07	0.10856633E-06	0.36356005E-06
18		0.69433037E-09	0.24165070E-08	0.13569064E-07	0.35397225E-07	0.11853615E-06
19		0.23934343E-09	0.83299767E-09	0.46774069E-08	0.12201831E-07	0.40860744E-07
20		0.72076567E-10	0.25085134E-09	0.14085684E-08	0.36744956E-08	0.12304930E-07
21		0.19385368E-10	0.67467670E-10	0.37884162E-09	0.98827369E-09	0.33094740E-08

I	J	16	17	18	19	20
1		0.38900016E-25	0.10201709E-25	0.31400533E-26	0.10285841E-26	0.28938580E-27
2		0.17574543E-23	0.46090061E-24	0.14186381E-24	0.46470172E-25	0.13074103E-25
3		0.82583677E-22	0.21657955E-22	0.66662533E-23	0.21836569E-23	0.61435910E-24
4		0.24114641E-20	0.63241775E-21	0.19465630E-21	0.63763324E-22	0.17939454E-22
5		0.10489327E-18	0.27508751E-19	0.84671112E-20	0.27735622E-20	0.78032576E-21
6		0.39240654E-16	0.10291047E-16	0.31675525E-17	0.10375914E-17	0.29192041E-18
7		0.10466305E-13	0.27448382E-14	0.84485247E-15	0.27674747E-15	0.77861320E-16
8		0.96706497E-12	0.25361749E-12	0.78062665E-13	0.25570905E-13	0.71942303E-14
9		0.22238975E-10	0.58322826E-11	0.17951560E-11	0.58803776E-12	0.16544107E-12
10		0.25862734E-09	0.67826328E-10	0.20876703E-10	0.68385644E-11	0.19239905E-11
11		0.14644197E-08	0.38405168E-09	0.11820969E-09	0.38721845E-10	0.10894168E-10
12		0.95942063E-08	0.25161295E-08	0.77445539E-09	0.25368774E-09	0.71373560E-10
13		0.81596568E-07	0.21399117E-07	0.65865677E-08	0.21575579E-08	0.60701622E-09
14		0.26591658E-06	0.69737951E-07	0.21465091E-07	0.70313035E-08	0.19782160E-08
15		0.10499107E-05	0.27534441E-06	0.84750013E-07	0.27761498E-07	0.78105309E-08
16		0.59654321E-05	0.15644646E-05	0.48153657E-06	0.15773657E-06	0.44378236E-07
17		0.16863751E-05	0.45872957E-05	0.14119532E-05	0.46251222E-06	0.13012504E-06
18		0.54982974E-06	0.14956540E-05	0.41612075E-05	0.13630834E-05	0.38349521E-06
19		0.18953261E-06	0.51556913E-06	0.14344168E-05	0.39469260E-05	0.11104439E-05
20		0.57076413E-07	0.15526001E-06	0.43196468E-06	0.11885886E-05	0.43502296E-05
21		0.15350970E-07	0.41757971E-07	0.11617914E-06	0.31967693E-06	0.11700167E-05

I	J	21
1		0.73117629E-28
2		0.33033612E-26
3		0.15522682E-24
4		0.45326622E-23
5		0.19716059E-21
6		0.73757886E-19
7		0.19672777E-16
8		0.18177251E-14
9		0.41801062E-13
10		0.48612405E-12
11		0.27525681E-11
12		0.18033561E-10
13		0.15337140E-09
14		0.49982463E-09
15		0.19734439E-08
16		0.11212805E-07
17		0.32877978E-07
18		0.96895633E-07
19		0.28056974E-06
20		0.10991498E-05
21		0.35998619E-05

ALPHA B MATRIX INVERSE F

I	FINAL ANSWERS
1	-0.38415543E-02
2	-0.17355675E 00
3	-0.15060654E 01
4	-0.26199942E 01
5	-0.30891132E 01
6	-0.26022635E 01
7	-0.18353462E 01
8	-0.18310022E 01
9	-0.16112576E 01
10	-0.15556459E 01
11	-0.13702049E 01
12	-0.11081715E 01
13	-0.99193245E 00
14	-0.88227552E 00
15	-0.94078726E 00
16	-0.82828283E 00
17	-0.80633312E 00
18	-0.78522062E 00
19	-0.58843279E 00
20	-0.62664169E 00
21	-0.49962330E 00

I	J	1	2	3	4	5
1		0.26880286E-11	0.10999611E-12	0.25676213E-14	0.10969013E-15	0.41992287E-17
2		0.28806662E-11	0.25668287E-11	0.74241557E-13	0.33846547E-14	0.13381078E-15
3		0.14886582E-11	0.17210781E-11	0.14221376E-11	0.81305242E-13	0.35213711E-14
4		0.50674666E-12	0.66695531E-12	0.76847090E-12	0.84081998E-12	0.48261091E-13
5		0.19116994E-12	0.27801530E-12	0.38250707E-12	0.58199268E-12	0.81846238E-12
6		0.40212215E-13	0.61992403E-13	0.92777023E-13	0.15764272E-12	0.27860928E-12
7		0.63425285E-14	0.10243285E-13	0.16265567E-13	0.29531563E-13	0.58048889E-13
8		0.22806820E-14	0.39107443E-14	0.66463423E-14	0.12899516E-13	0.27765361E-13
9		0.72417484E-15	0.12816909E-14	0.22503978E-14	0.44962881E-14	0.10026183E-13
10		0.37568643E-15	0.67699024E-15	0.12098072E-14	0.24536787E-14	0.55673477E-14
11		0.16458227E-15	0.29867504E-15	0.53734923E-15	0.10959418E-14	0.25025120E-14
12		0.29057373E-16	0.52895699E-16	0.95444983E-16	0.19513417E-15	0.44678892E-15
13		0.10332497E-16	0.18863702E-16	0.34130610E-16	0.69934958E-16	0.16052802E-15
14		0.31000992E-17	0.56676703E-17	0.10268095E-16	0.21062162E-16	0.48403539E-16
15		0.10292873E-17	0.18837169E-17	0.34160301E-17	0.70125899E-17	0.16129943E-16
16		0.21291712E-18	0.38997244E-18	0.70771919E-18	0.14537104E-17	0.33459845E-17
17		0.76760241E-19	0.14066496E-18	0.25540194E-18	0.52482345E-18	0.12085059E-17
18		0.32151580E-19	0.58943081E-19	0.10706328E-18	0.22007331E-18	0.50694012E-18
19		0.14304832E-19	0.26233626E-19	0.47665045E-19	0.98001877E-19	0.22581064E-18
20		0.58731342E-20	0.10774005E-19	0.19581317E-19	0.40269511E-19	0.92810337E-19
21		0.20202925E-20	0.37068128E-20	0.67381196E-20	0.13859030E-19	0.31946197E-19

I	J	6	7	8	9	10
1		0.12822592E-19	0.69793124E-22	0.24860983E-23	0.19403099E-24	0.34803511E-25
2		0.41343935E-18	0.22564952E-20	0.80478542E-22	0.62838703E-23	0.11273990E-23
3		0.11219668E-16	0.61661869E-19	0.22061073E-20	0.17244988E-21	0.30957388E-22
4		0.16572709E-15	0.92538598E-18	0.33342458E-19	0.26129196E-20	0.46966229E-21
5		0.34086810E-14	0.19709464E-16	0.72085906E-18	0.56788431E-19	0.10234829E-19
6		0.36756308E-12	0.24668047E-14	0.95445803E-16	0.76621583E-17	0.13939747E-17
7		0.10658369E-12	0.12854848E-12	0.66586988E-14	0.57814742E-15	0.10908658E-15
8		0.62076863E-13	0.10884447E-12	0.19734800E-12	0.20840792E-13	0.42390713E-14
9		0.23880918E-13	0.45554970E-13	0.96890215E-13	0.13301957E-12	0.35413689E-13
10		0.13650180E-13	0.26958308E-13	0.60617310E-13	0.10724077E-12	0.15375104E-12
11		0.61989056E-14	0.12387359E-13	0.28353015E-13	0.53618761E-13	0.90861184E-13
12		0.11115435E-14	0.22321326E-14	0.51462300E-14	0.99851903E-14	0.17878338E-13
13		0.40095427E-15	0.80875405E-15	0.18767489E-14	0.37234586E-14	0.69693397E-14
14		0.12112619E-15	0.24483127E-15	0.56986873E-15	0.11422076E-14	0.21797114E-14
15		0.40419790E-16	0.81825818E-16	0.19087940E-15	0.38541037E-15	0.74557513E-15
16		0.83934521E-17	0.17011399E-16	0.39749814E-16	0.80703380E-16	0.15769146E-15
17		0.30336369E-17	0.61530971E-17	0.14393326E-16	0.29327020E-16	0.57672197E-16
18		0.12732394E-17	0.25840692E-17	0.60499312E-17	0.12362065E-16	0.24433393E-16
19		0.56739704E-18	0.11520955E-17	0.26991846E-17	0.55276535E-17	0.10968409E-16
20		0.23329802E-18	0.47391798E-18	0.11110111E-17	0.22798545E-17	0.45400048E-17
21		0.80322495E-19	0.16320853E-18	0.38275508E-18	0.78638556E-18	0.15692907E-17

I	J	11	12	13	14	15
1		0.81424990E-26	0.11211938E-26	0.25188544E-27	0.77280455E-28	0.26696555E-28
2		0.26378897E-24	0.36323895E-25	0.81606336E-26	0.25037634E-26	0.86492898E-27
3		0.72452220E-23	0.99774102E-24	0.22416769E-24	0.68777923E-25	0.23759739E-25
4		0.10998020E-21	0.15147896E-22	0.34037281E-23	0.10443580E-23	0.36078721E-24
5		0.23994379E-20	0.33059416E-21	0.74301218E-22	0.22799708E-22	0.78767888E-23
6		0.32811758E-18	0.45261627E-19	0.10180545E-19	0.31249365E-20	0.10797506E-20
7		0.26067015E-16	0.36115615E-17	0.81470042E-18	0.25035903E-18	0.86552509E-19
8		0.10425104E-14	0.14561727E-15	0.33023935E-16	0.10169535E-16	0.35192118E-17
9		0.94559811E-14	0.13497419E-14	0.31037659E-15	0.96092050E-16	0.33336768E-16
10		0.49601206E-13	0.73850736E-14	0.17423205E-14	0.54463837E-15	0.18979699E-15
11		0.98480740E-13	0.16705865E-13	0.42244718E-14	0.13532266E-14	0.47682703E-15
12		0.23383947E-13	0.23833339E-13	0.84434582E-14	0.29648931E-14	0.10855373E-14
13		0.10313449E-13	0.15434146E-13	0.29741583E-13	0.12309161E-13	0.47737083E-14
14		0.23839148E-14	0.56390202E-14	0.12797448E-13	0.14304025E-13	0.66409958E-14
15		0.11949405E-14	0.21209788E-14	0.52046143E-14	0.73667133E-14	0.92557696E-14
16		0.25848947E-15	0.47811579E-15	0.12279047E-14	0.19382369E-14	0.30308222E-14
17		0.95873330E-16	0.18171340E-15	0.47858314E-15	0.79710480E-15	0.13575545E-14
18		0.41061709E-16	0.79261995E-16	0.21256154E-15	0.36702467E-15	0.65792352E-15
19		0.18587569E-16	0.36374043E-16	0.98833313E-16	0.17496666E-15	0.32415845E-15
20		0.77513631E-17	0.15351519E-16	0.42181817E-16	0.76228995E-16	0.14492319E-15
21		0.26911329E-17	0.53669447E-17	0.14841248E-16	0.27129009E-16	0.52295417E-16

I	J	16	17	18	19	20
1		0.55753069E-29	0.21976230E-29	0.98795692E-30	0.47297666E-30	0.19261213E-30
2		0.18063201E-27	0.71200041E-28	0.32008486E-28	0.15323722E-28	0.62403764E-29
3		0.49619875E-26	0.19558891E-26	0.87928601E-27	0.42094858E-27	0.17142575E-27
4		0.75347754E-25	0.29700259E-25	0.13352029E-25	0.63921522E-26	0.26031223E-26
5		0.16450440E-23	0.64844297E-24	0.29151536E-24	0.13956064E-24	0.56834076E-25
6		0.22551920E-21	0.88898200E-22	0.39966028E-22	0.19133651E-22	0.77920391E-23
7		0.18082617E-19	0.71288784E-20	0.32051837E-20	0.15345491E-20	0.62495232E-21
8		0.73560045E-18	0.29006784E-18	0.13043310E-18	0.62452705E-19	0.25435862E-19
9		0.69770054E-17	0.27527934E-17	0.12382546E-17	0.59301617E-18	0.24156254E-18
10		0.39811118E-16	0.15723314E-16	0.70769322E-17	0.33904711E-17	0.13814818E-17
11		0.10056644E-15	0.39815856E-16	0.17946855E-16	0.86058630E-17	0.35089366E-17
12		0.23316234E-15	0.93056337E-16	0.42144693E-16	0.20267725E-16	0.82821277E-17
13		0.10518860E-14	0.42440438E-15	0.19343584E-15	0.93382696E-16	0.38270060E-16
14		0.15659670E-14	0.64913683E-15	0.30043285E-15	0.14636246E-15	0.60390577E-16
15		0.26399728E-14	0.11664673E-14	0.55838402E-15	0.27732155E-15	0.11604120E-15
16		0.46500381E-14	0.25480765E-14	0.13386350E-14	0.69768113E-15	0.30176819E-15
17		0.26503676E-14	0.31416912E-14	0.19786599E-14	0.11138799E-14	0.50535828E-15
18		0.14384701E-14	0.20653829E-14	0.25974403E-14	0.17345386E-14	0.85887214E-15
19		0.75558494E-15	0.11827504E-14	0.17782466E-14	0.22731664E-14	0.13673113E-14
20		0.35372879E-15	0.58491204E-15	0.96445628E-15	0.14984758E-14	0.23402184E-14
21		0.13066298E-15	0.22170995E-15	0.38015601E-15	0.63216441E-15	0.11723262E-14

I	J 21
1	0.67769882E-31
2	0.21956572E-29
3	0.60315672E-28
4	0.91590256E-27
5	0.19997094E-25
6	0.27416261E-23
7	0.21989252E-21
8	0.89500361E-20
9	0.85004854E-19
10	0.48621123E-18
11	0.12353992E-17
12	0.29192230E-17
13	0.13509328E-16
14	0.21391963E-16
15	0.41396260E-16
16	0.10940096E-15
17	0.18726414E-15
18	0.33005676E-15
19	0.56177089E-15
20	0.11394944E-14
21	0.14477343E-14

I	J	1	2	3	4	5
1		0.10080103E-03	0.39598572E-05	0.12709717E-06	0.65814056E-08	0.28344771E-09
2		0.10802493E-03	0.92405782E-04	0.36749543E-05	0.20307920E-06	0.90322203E-08
3		0.55824654E-04	0.61958781E-04	0.70395763E-04	0.48783131E-05	0.23769240E-06
4		0.19002982E-04	0.24010369E-04	0.38039274E-04	0.50449176E-04	0.32576218E-05
5		0.71688692E-05	0.10008545E-04	0.18934079E-04	0.34919547E-04	0.55246172E-04
6		0.15079568E-05	0.22317254E-05	0.45924589E-05	0.94585603E-05	0.18806109E-04
7		0.23784469E-06	0.36875804E-06	0.80514502E-06	0.17718930E-05	0.39182978E-05
8		0.85525528E-07	0.14078671E-06	0.32899368E-06	0.77397073E-06	0.18741603E-05
9		0.27156542E-07	0.46140848E-07	0.11139457E-06	0.26977716E-06	0.67676700E-06
10		0.14088233E-07	0.24371637E-07	0.59885394E-07	0.14722065E-06	0.37579576E-06
11		0.61718310E-08	0.10752295E-07	0.26598769E-07	0.65756467E-07	0.16891943E-06
12		0.10896508E-08	0.19042441E-08	0.47245230E-08	0.11708046E-07	0.30158237E-07
13		0.38746828E-09	0.67909278E-09	0.16894639E-08	0.41960959E-08	0.10835635E-07
14		0.11625366E-09	0.20403604E-09	0.50827031E-09	0.12637293E-08	0.32672371E-08
15		0.38598250E-10	0.67813768E-10	0.16909338E-09	0.42075521E-09	0.10887704E-08
16		0.79843883E-11	0.14039001E-10	0.35032074E-10	0.87222590E-10	0.22585384E-09
17		0.28785073E-11	0.50639354E-11	0.12642387E-10	0.31489394E-10	0.81574109E-10
18		0.12056831E-11	0.21219493E-11	0.52996288E-11	0.13204395E-10	0.34218434E-10
19		0.53643093E-12	0.94440948E-12	0.23594173E-11	0.58801106E-11	0.15242210E-10
20		0.22024239E-12	0.38786395E-12	0.96927414E-12	0.24161697E-11	0.62646945E-11
21		0.75760904E-13	0.13344518E-12	0.33353669E-12	0.83154154E-12	0.21563671E-11

I	J	6	7	8	9	10
1		0.10001618E-11	0.79564109E-14	0.24612363E-15	0.22992656E-16	0.33933405E-17
2		0.32248260E-10	0.25724036E-12	0.79673715E-14	0.74463831E-15	0.10992134E-15
3		0.87513374E-09	0.70294508E-11	0.21840456E-12	0.20435300E-13	0.30183437E-14
4		0.12926709E-07	0.10549396E-09	0.33009021E-11	0.30963079E-12	0.45792048E-13
5		0.26587702E-06	0.22468782E-08	0.71365025E-10	0.67294260E-11	0.99789448E-12
6		0.28669907E-04	0.28121565E-06	0.94491313E-08	0.90796526E-09	0.13591246E-09
7		0.83135255E-05	0.14654523E-04	0.65921103E-06	0.68510417E-07	0.10635933E-07
8		0.48419943E-05	0.12408265E-04	0.19537445E-04	0.24696319E-05	0.41330924E-06
9		0.18627106E-05	0.51932648E-05	0.95921296E-05	0.15762809E-04	0.34528321E-05
10		0.10647136E-05	0.30732454E-05	0.60011125E-05	0.12708026E-04	0.14990718E-04
11		0.48351450E-06	0.14121579E-05	0.28069480E-05	0.63538200E-05	0.88589604E-05
12		0.86700368E-07	0.25446303E-06	0.50947665E-06	0.11832444E-05	0.17431366E-05
13		0.31274425E-07	0.92197922E-07	0.18579806E-06	0.44122964E-06	0.67951026E-06
14		0.94478381E-08	0.27910755E-07	0.56416994E-07	0.13535151E-06	0.21252174E-06
15		0.31527430E-08	0.93281400E-08	0.18897055E-07	0.45671108E-07	0.72693524E-07
16		0.65468897E-09	0.19392989E-08	0.39352308E-08	0.95633439E-08	0.15374908E-07
17		0.23662361E-09	0.70145267E-09	0.14249391E-08	0.34752503E-08	0.56230363E-08
18		0.99312641E-10	0.29458369E-09	0.59894290E-09	0.14649040E-08	0.23822546E-08
19		0.44256959E-10	0.13133884E-09	0.26721914E-09	0.65502648E-09	0.10694192E-08
20		0.18197235E-10	0.54026630E-10	0.10999007E-09	0.27016256E-09	0.44265014E-09
21		0.62651525E-11	0.18605756E-10	0.37892744E-10	0.93186639E-10	0.15300576E-09

I	J	11	12	13	14	15
1		0.11236645E-17	0.42381111E-18	0.64230730E-19	0.28980153E-19	0.13214789E-19
2		0.36402869E-16	0.13730427E-16	0.20809606E-17	0.93891038E-18	0.42813964E-18
3		0.99984023E-15	0.37714587E-15	0.57162746E-16	0.25791703E-16	0.11761066E-16
4		0.15177262E-13	0.57259021E-14	0.86795043E-15	0.39163394E-15	0.17858959E-15
5		0.33112234E-12	0.12496455E-12	0.18946806E-13	0.85498862E-14	0.38990079E-14
6		0.45280210E-10	0.17108884E-10	0.25960380E-11	0.11718499E-11	0.53447633E-12
7		0.35972472E-08	0.13651698E-08	0.20774857E-09	0.93884581E-10	0.42843465E-10
8		0.14386637E-06	0.55043309E-07	0.84211003E-08	0.38135717E-08	0.17420090E-08
9		0.13049248E-05	0.51020226E-06	0.79145991E-07	0.36034500E-07	0.16501691E-07
10		0.68449644E-05	0.27915567E-05	0.44429163E-06	0.20423926E-06	0.93949438E-07
11		0.13590338E-04	0.63148145E-05	0.10772401E-05	0.50745967E-06	0.23602928E-06
12		0.32269836E-05	0.90089989E-05	0.21530814E-05	0.11118336E-05	0.53734072E-06
13		0.14232555E-05	0.58341047E-05	0.75841017E-05	0.46159321E-05	0.23629846E-05
14		0.46698011E-06	0.21315482E-05	0.32633479E-05	0.53640060E-05	0.32872913E-05
15		0.16490173E-06	0.80172970E-06	0.13271756E-05	0.27625156E-05	0.45816041E-05
16		0.35671537E-07	0.18072768E-06	0.31311561E-06	0.72683844E-06	0.15002561E-05
17		0.13230515E-07	0.68687598E-07	0.12203867E-06	0.29891413E-06	0.67198920E-06
18		0.56665144E-08	0.29961022E-07	0.54203181E-07	0.13763417E-06	0.32567198E-06
19		0.25650837E-08	0.13749382E-07	0.25202489E-07	0.65612426E-07	0.16045834E-06
20		0.10696877E-08	0.58028711E-08	0.10756359E-07	0.28585855E-07	0.71736906E-07
21		0.37137604E-09	0.20287043E-08	0.37845140E-08	0.10173373E-07	0.25885219E-07

I	J	16	17	18	19	20
1		0.37215114E-20	0.17141446E-20	0.79283485E-21	0.43986783E-21	0.15168196E-21
2		0.12057171E-18	0.55535976E-19	0.25686795E-19	0.14251046E-19	0.49142929E-20
3		0.33121230E-17	0.15255924E-17	0.70562663E-18	0.39148178E-18	0.13499769E-18
4		0.50294566E-16	0.23166179E-16	0.10714997E-16	0.59446956E-17	0.20499570E-17
5		0.10980656E-14	0.50578518E-15	0.23394076E-15	0.12979126E-15	0.44756797E-16
6		0.15053388E-12	0.69340529E-13	0.32072720E-13	0.17794275E-13	0.61362252E-14
7		0.12070133E-10	0.55605208E-11	0.25721578E-11	0.14271292E-11	0.49214962E-12
8		0.49101256E-09	0.22625275E-09	0.10467249E-09	0.58080957E-10	0.20030727E-10
9		0.46571458E-08	0.21471773E-08	0.99369868E-09	0.55150440E-09	0.19023037E-09
10		0.26573890E-07	0.12264174E-07	0.56792331E-08	0.31531349E-08	0.10879162E-08
11		0.67127985E-07	0.31056345E-07	0.14402342E-07	0.80034432E-08	0.27632858E-08
12		0.15563563E-06	0.72583873E-07	0.33821099E-07	0.18848965E-07	0.65221712E-08
13		0.70213309E-06	0.33103515E-06	0.15523216E-06	0.86845773E-07	0.30137652E-07
14		0.10452814E-05	0.50632639E-06	0.24109721E-06	0.13611691E-06	0.47557549E-07
15		0.17621796E-05	0.90984383E-06	0.44810292E-06	0.25790877E-06	0.91382333E-07
16		0.31038962E-05	0.19874979E-05	0.10742533E-05	0.64884279E-06	0.23764227E-06
17		0.17691182E-05	0.24505171E-05	0.15878732E-05	0.10359072E-05	0.39796936E-06
18		0.96017720E-06	0.16109971E-05	0.20844445E-05	0.16131189E-05	0.67536137E-06
19		0.50435239E-06	0.92254470E-06	0.14270418E-05	0.21140422E-05	0.10767562E-05
20		0.23611369E-06	0.45623108E-06	0.77397573E-06	0.13935805E-05	0.18429200E-05
21		0.87217416E-07	0.17293360E-06	0.30507499E-06	0.58791233E-06	0.92320630E-06

I	J 21
1	0.66075595E-22
2	0.21407646E-20
3	0.58807734E-19
4	0.89300434E-18
5	0.19497150E-16
6	0.26730842E-14
7	0.21439507E-12
8	0.87262810E-11
9	0.82879689E-10
10	0.47405568E-09
11	0.12045136E-08
12	0.28462410E-08
13	0.13171586E-07
14	0.20857151E-07
15	0.40361332E-07
16	0.10666588E-06
17	0.18258243E-06
18	0.32180515E-06
19	0.54772636E-06
20	0.11110060E-05
21	0.14115394E-05

A L P H A B A I N V E R S E V D L

SECTION NUMBER NUMBER	FINAL ANSWERS
1	0.848998E-05
2	0.246535E-03
3	0.480383E-02
4	0.652980E-02
5	0.708773E-01
6	0.154674E 00
7	0.429225E-01
8	0.247300E-01
9	0.110817E-01
10	0.865148E-02
11	0.451300E-02
12	0.853267E-03
13	0.322235E-03
14	0.994087E-04
15	0.336782E-04
16	0.707312E-05
17	0.257525E-05
18	0.108718E-05
19	0.486707E-06
20	0.200956E-06
21	0.693598E-07

D.O.SATURATION + ALPHA B INVERSE F - ALPHA B A INVERSE V D L

SECTION NUMBER	FINAL ANSWERS
1	0.776615E 01
2	0.759620E 01
3	0.625913E 01
4	0.514348E 01
5	0.461001E 01
6	0.501306E 01
7	0.589173E 01
8	0.591427E 01
9	0.614766E 01
10	0.620570E 01
11	0.639528E 01
12	0.666097E 01
13	0.677774E 01
14	0.688762E 01
15	0.682918E 01
16	0.694171E 01
17	0.696366E 01
18	0.698478E 01
19	0.718157E 01
20	0.714336E 01
21	0.727038E 01

SIMULATION OF CHLORIDE CONCENTRATIONS IN THE POTOMAC ESTUARY

BY

Leo J. Hetling*

* Now employed as Director of Research Unit, Environmental Health Services, New York State Department of Health, 84 Holland Avenue, Albany, New York.

TABLE OF CONTENTS

		Page
I.	INTRODUCTION	I - 1
II.	THE POTOMAC ESTUARY	II - 1
III.	PRESENT AND FUTURE WATER SUPPLY REQUIREMENTS	III - 1
IV.	THE SEGMENTED ESTUARY MODEL	IV - 1
	The Pumping Problem	IV - 2
V.	EVALUATION OF MODEL PARAMETERS	V - 1
	Segmentation	V - 1
	Segment Volumes	V - 1
	Net River Flow	V - 3
	Boundary Conditions	V - 4
	External Sources of Chlorides	V - 6
	Turbulent Exchange Factor	V - 6
	Proportionality Factor	V - 10
VI.	VERIFICATION RESULTS	VI - 1
VII.	SIMULATION	VII - 1
VIII.	DISCUSSION OF RESULTS	VIII - 1
	Validity of the Model and Parametric Analyses	VIII - 1
	Simulation Results	VIII - 3
IX.	CONCLUSION	IX - 1
X.	BIBLIOGRAPHY	X - 1

LIST OF TABLES

Number		Page
1	Present and Proposed Major Waste Water Discharges to the Upper Potomac Estuary	II - 2
2	Present Sources of Water Supply--Washington Metropolitan Area	III - 2
3	Monthly Distribution of Annual Water Usage	III - 3
4	Projected Water Demand--Washington Metropolitan Area	III - 4
5	Potomac Estuary Chloride Model--Physical Parameters	V - 2
6	Potomac Estuary Chloride Model--Interface Parameter	V - 9
7	Hydrologic Conditions Simulated	VII - 2

LIST OF FIGURES

Number	
1	Study Area Map
2	Washington Metropolitan Area Water Supply Needs
3	The Pumping Problem
4	1930-1965-1966 Potomac River Hydrographs
5	1965 Chlorides at Great Falls
6	Relationship Between Chlorides and River Flow at Great Falls
7	Assumed 1965 Lower Boundary Conditions
8	Assumed 1930 Lower Boundary Conditions
9	Potomac Estuary Chloride Model - 1965 Chlorides at Possum Point
10	Potomac Estuary Chloride Model - 1965 Chlorides at Maryland Point
11	Potomac Estuary Chloride Model - 1965 Chlorides October 8, 1965
12	Potomac Estuary Chloride Model - 1930 Chlorides at Indian Head
13	Potomac Estuary Chloride Model - 1930 River Flows--2010 Water Use--Waste Discharge Out of Basin
14	Potomac Estuary Chloride Model - 1966 River Flows--2010 Water Use
15	Potomac Estuary Chloride Model - 1930 River Flows--2010 Water Use--Parametric Analysis I
16	Potomac Estuary Chloride Model - 1930 River Flows--2010 Water Use--Parametric Analysis II

I. INTRODUCTION

A significant water resource problem in the Potomac River Basin is the provision of an adequate water supply to meet the projected needs of the rapidly growing Washington Metropolitan Area. The conventional approach to this problem would be the construction of upstream reservoirs to supply the required water. However, rapid changes in technology and state of our social-economic outlook are forcing planners to look at an ever-enlarging matrix of solutions to water resource problems.[1] In this case, conflicting interests are exerting significant pressure against the construction of these reservoirs.[2]

Use of the upper Potomac Estuary as a water supply source has been proposed as an alternative to the upstream reservoirs. This proposal has been discarded previously because waste water treatment technology was lacking [It is projected that the rate of waste water being discharged to the upper Estuary will increase from its present rate of 270 million gallons per day (mgd) to over 800 mgd by the year 2010.], and because of uncertainty concerning the possibility of salinity (chloride) intrusion from the Chesapeake Bay if large withdrawals of fresh water from the upper Estuary were made. Advances in the technology of waste water treatment are rapidly eliminating the first objection, except for the buildup of chlorides and total dissolved solids. The purpose of this paper is to present the results of a simulation

model of chloride concentrations in the upper Estuary. It is hoped that these simulation results will prove useful to water resource planners in this critical area.

II. THE POTOMAC ESTUARY

The Potomac River is the second largest river in the Middle Atlantic states. Its tidal section begins in the Washington, D. C., Metropolitan Area at Little Falls and extends 116 miles southeastward to the Chesapeake Bay (Figure 1). The Estuary is several hundred feet in width at the head and broadens to nearly six miles at its mouth. A channel with a minimum depth of 24 feet is maintained in the Estuary up to Washington, D. C. Except for the channel and depths up to 80 feet just below Chain Bridge, the Estuary is relatively shallow.

The Estuary is bounded by the large metropolitan areas of Washington, D. C., and Arlington and Alexandria, Virginia, in its upper part and by forest and farmland in the lower portion.

Several waste water treatment plants presently discharge treated waste to the Estuary, and several more are projected in the future. These are shown in Figure 1. Additional information on these plants is given in Table 1.

TABLE 1

PRESENT AND PROPOSED MAJOR WASTE WATER DISCHARGES TO THE UPPER POTOMAC ESTUARY

Plant Name	Discharge Point (Miles from Chain Bridge)	Present*		1985		2010	
		Population Served x 1000	Flow (mgd)	Population Served x 1000	Flow (mgd)	Population Served x 1000	Flow (mgd)
Arlington County Sewage Treatment Plant	8.9	191	17.3	244	39.0	282	52.2
District of Columbia Water Pollution Control Plant	10.0	1,400	217.1	2,056	328.9	3,085	570.7
Alexandria Sewage Treatment Plant	12.2	138	16.0	220	35.2	291	53.8
Fairfax County Westgate Sewage Treatment Plant	12.3	120	10.2	90	14.4	92	17.0
Washington Suburban Sanitary Commission, Piscataway Creek Sewage Treatment Plant	18.0	**	**	189	30.2	310	57.4
Fairfax County Little Hunting Creek Sewage Treatment Plant	19.6	3.3	2.6	28	5.0	32	5.9
Fairfax County Dogue Creek Sewage Treatment Plant	22.0	3.9	1.4	31	5.0	46	8.5
Fairfax County Accotink Creek Sewage Treatment Plant	24.0	**	**	183	29.3	300	55.5
Charles County Pomonkey-Marbury Sewage Treatment Plant	28.0	**	**	40	6.4	91	16.8

* 1966 Average

** Plant presently not in existence or not of significant size

III. PRESENT AND FUTURE WATER SUPPLY REQUIREMENTS

The present status of the major water supply systems in the Washington Metropolitan Region is given in Table 2. It can be seen from this table that presently 72 per cent of the area's water needs are supplied by the Potomac River.

In estimating future water supply demands, the data presented in Volume 5 of the Corps of Engineers 1962 Potomac River Basin Report[3] was used. The report's conclusion that in the future only 92 mgd would be available from sources other than the Potomac River was also accepted. A more recent report[4] indicates that this is a conservative assumption, and that local sources other than the Potomac will provide a safe yield of 110 mgd in 1985 and 150 mgd in 2010.

Although more recent, possibly more exacting, demand data could be developed, they should not change the major conclusions of the simulations.

Because of the time-dependent nature of the system, it was necessary to estimate the distribution of the annual demands by month. An analysis of the 1960 to 1965 records of the Corps of Engineers Washington Aqueduct Division gave the distribution of filtered water pumped to the District of Columbia shown in Table 3.

Using the above monthly distributions and the base data from the Corps of Engineers report, the water demands on the Potomac River shown in Table 4 were developed. These data,

shown graphically in Figure 2, clearly indicate the need for the development of additional supplies.

TABLE 2

PRESENT SOURCES OF WATER SUPPLY
WASHINGTON METROPOLITAN AREA

Water System	Source	Use (mgd)
District of Columbia	Potomac River	148
Arlington County	Potomac River	20
City of Falls Church - Fairfax County Water Authority	Potomac River	11
Fairfax County Water Authority	Wells	2
Fairfax City	Goose Creek	3
Washington Suburban Sanitary Commission	Patuxent River	47
Washington Suburban Sanitary Commission	Potomac River	43
Rockville City	Potomac River	4
Alexandria Water Company	Occoquan Creek	30
Other Miscellaneous Small Systems	---	9
	Sub-total Potomac River	230
	Sub-total Other Sources	90
	Total All Systems	320

TABLE 3

MONTHLY DISTRIBUTION OF ANNUAL WATER USAGE

Month	Mean Monthly Use*	Mean Monthly Use / Average Annual Use
January	148.5	0.883
February	148.2	0.881
March	148.8	0.885
April	154.1	0.916
May	168.9	1.004
June	189.7	1.128
July	206.1	1.225
August	204.7	1.217
September	183.9	1.093
October	161.9	0.962
November	153.2	0.911
December	150.6	0.895
Average	168.2	1.000

* Mean value of filtered water pumped from 1960 to 1965.

TABLE 4

PROJECTED WATER DEMAND--WASHINGTON METROPOLITAN AREA

Month	1985 Gross Demand (mgd)	1985 Available from Other Sources (mgd)	1985 Net Demand from Potomac River (mgd)	2010 Gross Demand (mgd)	2010 Available from Other Sources (mgd)	2010 Net Demand from Potomac River (mgd)
January	513	92	421	907	92	815
February	512	92	420	905	92	813
March	514	92	422	909	92	817
April	532	92	440	941	92	849
May	583	92	491	1,031	92	939
June	655	92	563	1,158	92	1,066
July	712	92	620	1,258	92	1,166
August	707	92	615	1,250	92	1,158
September	635	92	543	1,123	92	1,031
October	559	92	467	988	92	896
November	529	92	437	936	92	844
December	520	92	428	919	92	827
Average Annual	581	92	489	1,027	92	935

IV. THE SEGMENTED ESTUARY MODEL

The mathematical model employed in the simulation study was a modification of the model developed by Thomann.[5] This model consists of a system of "n" equations, each describing a mass balance of the material being studied for each of the "n" segments of an estuary. For an estuary where good vertical and lateral mixing may be assumed, these segments are selected along the longitudinal axis of the estuary.

The mass balance over each of the segments includes terms describing changes in chloride concentration caused by advection, dispersion, and, in the case of the segment to which sewage was added, a chloride source. A mass balance constructed for the "i"th segment takes the form:

$$V_i \frac{dC_i}{dt} = Q_i [\xi_i C_{i-1} + (1 - \xi_i) C_i]$$

$$- Q_{i+1} [\xi_{i+1} C_i + (1 - \xi_{i+1}) C_{i+1}]$$

$$+ E_i (C_{i-1} - C_i) + E_{i+1} (C_{i+1} - C_i) \qquad (1)$$

$$+ J_i$$

where

V_i = volume of "i"th segment, cubic feet (cf)

C_i = mean chloride concentration in "i"th segment (lb/cf)

Q_i = net waterflow across the upstream boundary of the "i"th segment (cf/day)

ξ_i = a dimensionless proportionality factor used to estimate concentration at upper boundary of the "i"th segment

E_i = turbulent exchange factor for upstream boundary of the "i"th segment (cf/day)

J_i = rate of chloride addition from external source (lb/day)

t = time (days)

Since for this study 28 estuary segments were employed, a like number of expressions similar to Equation 1 were obtained. This system of 28 linear first order, non-homogeneous, ordinary differential equations may be solved simultaneously by numerical methods using a digital computer or by programming the equations on a relatively large analog computer.[6,7,8] In either case, all terms of the equations must be known in order to simulate the chloride concentration of each segment.

The Pumping Problem

In simulating the future use of the upper estuary as a source of water, it was necessary to modify the above basic model in order to account for the transfer of relatively large quantities of water (and its chloride content) from the uppermost section of the estuary through the city's water supply system and back into section six of the estuary as sewage. This transfer is shown schematically in Figure 3.

To handle this transfer theoretically would mean an extension of the above model into the second dimension. However, a simpler approach was sought and found to be adequately accurate. The simpler approach taken consists of the following sequence of operations:

1. Integrate over a period of time, Δ t (normally 1 day), ignoring the transfers, J_1 and J_2.
2. Stop the integrations.
3. Modify the concentration in segments one and six according to the effect of J_1 and J_6.
4. Resume the integration over the next time period of time, Δ t, again ignoring the flow exchanges and continue in this manner.

A detailed description of the modified simulation model and the computer program has been published.[6] Although the computer program has the capability of distributing the pumped flow to different segments, it was assumed that all wastes were discharged into segment 6. It is projected that over 70 per cent of the waste water will be discharged to this segment. It did not appear that the additional precision in the final results justified the additional office and computer time that would have been required to spacially redistribute the effluent.

V. EVALUATION OF MODEL PARAMETERS

Segmentation

The size of the segments selected represents a compromise between accuracy and computational efficiency. In this case the segments were made small (approximately two miles in length) in the upper critical portion of the estuary and relatively large (six miles in length) near the mouth. The final segmentation chosen is shown in Figure 1.

Segment Volumes (V_i)

The segment volumes were determined from the latest (1965) U. S. Coast and Geodetic Survey navigation charts for the Potomac Estuary which give soundings at mean low water. The water volumes determined by planimetering these charts were increased by an appropriate amount to give estimates of mean tide level volumes. No corrections were made for the small deviations of the mean tidal elevations from the long term mean values upon which the volume calculations were based. The segment volumes used are given in Table 5.

TABLE 5

POTOMAC ESTUARY CHLORIDE MODEL - PHYSICAL PARAMETERS

Segment	Length (Miles)	Volume* (Cubic Feet) x 10^7
1	2.69	25
2	2.09	24
3	1.75	33
4	1.31	40
5	1.87	45
6	2.11	52
7	2.57	76
8	2.19	66
9	2.44	79
10	2.06	65
11	2.25	92
12	3.95	252
13	2.94	170
14	2.93	250
15	4.41	330
16	4.26	445
17	4.42	520
18	4.57	535
19	5.36	620
20	5.20	525
21	5.69	650
22	6.00	1,160
23	6.00	1,593
24	6.00	1,957
25	5.40	1,620
26	8.60	2,165
27	6.00	3,055
28	11.00	5,655

* Mean Tide Level

Net River Flow (Q_i)

The term "Q_i" refers to the net flow into the "i"th segment from the adjacent upstream segment, i.e., the net flow across the i-1,i or upstream boundary of the "i"th segment.

In a dye diffusion study of the estuary,[9] it was found upon examination that the inflows to the estuary, except for the gaged Potomac River flow and the District of Columbia waste discharge, were approximately balanced by losses due to evaporation. Therefore, in order to reduce the number of flow input functions, Q_1 was taken as the gaged Potomac River flow reported for the U. S. Geological Survey gaging station (Leiter Gage) at Washington, D. C.; Q_2 through Q_6 were taken as the gaged River flow minus the quantity of water pumped for water supply purposes; and Q_7 through Q_{28} were set equal to Q_6 plus the sum of all the waste water discharges. In the simulation studies, this was assumed to be equal to 90 per cent of the water pumped.

The net tidal flow into and out of each segment was assumed to be zero over a complete tidal cycle. No consideration was given to the small changes in tidal elevation and, therefore, tidal flow which occur on succeeding days during a lunar tidal cycle.

Hydrographs showing the 1930, 1965, and 1966 River flows used are given in Figure 4.

Boundary Conditions (C_0 and C_{n+1})

In order to solve the system of differential equations, the time variation in chloride concentrations at the upper and lower boundary must be known.

At Great Falls approximately ten miles above the upper boundary, there is a National Water Quality Network Station of the Federal Water Pollution Control Administration. Chlorides are measured at this station at weekly intervals. Since there are no known sources of chlorides between the station and the upper boundary of the model, these values were used in the verification studies. The values measured at this station for 1965 are shown in Figure 5. Upper boundary conditions for 1966 were determined in a similar manner.

No measurements of the chloride concentration of the incoming River flow were made in 1930. In order to construct a 1930 upper boundary, a regression of the log River flow on the log chloride concentration using the 1957 to 1965 data collected at Great Falls Water Quality Network Station was run. This regression gave the following equation.

$$\text{Log Cl} = 3.654 - 0.712 \text{ Log Q} \qquad 2$$

A graph showing this equation is shown in Figure 6. Daily chloride values for 1930 were calculated using the 1930 gaged flows and Equation 2.

Obtaining boundary conditions at the lower boundary proved more difficult. There were no regular measurements of chlorides made at the mouth of the Potomac Estuary. A search of all known sources of data produced the following salinity measurements.

1. Daily surface water salinity measurements were taken by the U. S. Coast and Geodetic Survey at Fort McHenry, Baltimore, from 1914 to the present, and at Solomons, Maryland, since 1954.[10]

2. In an atlas of salinity and temperature by Stroup and Lynn,[11] graphical summaries of the distributions of salinity in Chesapeake Bay measured on various cruises from 1952 to 1961 are given. Seasonal averages for the period 1944 to 1961 are also given.

3. In conjunction with some oyster studies, sporadic salinity measurements were taken approximately seven miles up from the mouth of the Potomac,[12] at Bean's Pier in Smith Creek and at Jones Shore approximately four miles above Point Lookout.

4. Salinity measurements were taken at the mouth of the Potomac River by Chesapeake Bay Institute in conjunction with a Potomac River nutrient study.[13]

All of the above measurements taken in 1965 are plotted in Figure 9. Using all of the data shown in Figure 9, the solid line labeled "assumed 1965 boundary conditions" was drawn. Similarly, the 1963 to 1966 boundary conditions were determined.

For 1930 only (the year of the worst drought on record), the measurements for Baltimore Harbor were available. Using these values and judgment obtained from the study of the 1963 to 1966 records, the solid line marked "assumed 1930 boundary conditions" was drawn as shown in Figure 8. The resultant salinities were then converted to chloride concentrations using the classical relationship:

$$S = 0.03 + 1.805\ Cl$$

where

S = Salinity in parts per thousand

Cl = Chloride concentration in parts per thousand

External Sources of Chlorides (J_i)

Since there are no major chloride producing industries either present or planned in the Washington Metropolitan Area, the only external source of chlorides considered in the simulation is that which is added to the waste water by people. Fair and Geyer[14] report that wastes from the human body contain from 5 to 9 grams of chloride a day. In all of the simulation studies made, the higher value of 9 grams per capita per day was used.

Turbulent Exchange Factor (E_i)

The turbulent exchange factor "E" is calculated for each segment boundary from the expression:

$$E_i\ (cf/day) = \frac{K_i\ A_i}{0.5\ (L_i + L_{i-1})}$$

3

where the "i" subscript refers to the boundary between segments i-1 and i, "K" is the longitudinal dispersion coefficient (analogous to the classical Fickian diffusion coefficient, expressed in areal units per day), "A" is the cross-sectional area of the boundary plane, and "L" is the segment length.

The above expression is based on Fick's first law of diffusion, i.e.,

$$N_i \text{ (lb/sf/day)} = K \frac{dC}{dx} \qquad 4$$

where "N" is the rate of mass transfer of substance per unit area across a boundary where the spatial gradient of the substance is "dC/dx" ("C" is in lbs/cf) and "K" is defined as above.

If the mean concentration in two adjacent segments is assumed to occur at their midpoints, and the gradient between these midpoints is linear, it can be shown by geometry that:

$$\frac{dC}{dx} = \frac{C_{i-1} - C_i}{0.5\ (L_{i-1} + L_i)} \qquad 5$$

Substituting Equation 5 into Equation 4 gives:

$$N_i \text{ (lbs/sf/day)} = \frac{K(C_{i-1} - C_i)}{0.5\ (L_{i-1} + L_i)} \qquad 6$$

Multiplying Equation 6 by the cross-sectional area across which the turbulent exchange takes place yields the mass flow rate "D_i:"

$$D_i \text{ (lbs/day)} = \frac{A_i\ K_i\ (C_{i-1} - C_i)}{0.5\ (L_{i-1} + L_i)} \qquad 7$$

Substituting Equation 3 into Equation 7 yields:

$$D_i = E_i (C_{i-1} - C_i) \qquad 8$$

which is the expression used in Equation 1 to describe the mass flow of substance across a boundary due to turbulent exchange.

The "E" term, or more specifically "K," was the unknown parameter which the verification study was designed to evaluate. The "K" term as used herein is defined as the coefficient of longitudinal dispersion. This term applied to net longitudinal mass transport resulting not only from turbulent diffusion but also from velocity and concentration variations in a cross-section. The latter effect has been shown to be of greater significance in estuary type flows.[15] It has been assumed that dispersion is analogous to Fickian diffusion with the diffusion coefficient replaced by a dispersion coefficient.

The values of "K" for the upper 12 boundaries were obtained from a large-scale dye diffusion study described elsewhere.[9] For the lower 17 boundaries, initial values of "K" were first assumed on an empirical basis.[16] These values were then adjusted to obtain good agreement of the model output with the salinity concentrations observed in the estuary.

A list of the values which gave the best matching between observed and computed values is given in Table 6. These values were used in the simulation studies which follow.

TABLE 6

POTOMAC ESTUARY CHLORIDE MODEL - INTERFACE PARAMETER

Interface	Area (A_i) (Sq. Ft.)	Longitudinal Dispersion Coefficient (K_j) (Sq. Mi/day)	Turbulent Exchange Factor (E_i) (Cu. Ft/day) x 10^8
0-1	4,610	0.00	0.000
1-2	30,000	0.05	0.033
2-3	24,300	0.10	0.067
3-4	29,000	0.15	0.150
4-5	34,300	0.20	0.228
5-6	32,700	0.25	0.217
6-7	47,600	0.35	0.376
7-8	49,900	0.45	0.498
8-9	43,600	0.55	0.546
9-10	59,000	0.65	0.900
10-11	66,500	0.75	1.222
11-12	84,900	0.90	1.301
12-13	111,000	1.20	2.041
13-14	130,000	1.50	3.508
14-15	171,000	1.95	4.797
15-16	117,000	2.80	3.990
16-17	221,000	3.90	10.486
17-18	220,000	5.30	13.696
18-19	243,000	6.00	15.505
19-20	193,000	6.00	11.580
20-21	217,000	6.00	12.625
21-22	245,000	6.00	13.279
22-23	490,000	10.00	43.120
23-24	576,000	10.00	50.688
24-25	509,000	10.00	47.149
25-26	604,000	10.00	45.559
26-27	753,000	10.00	54.463
27-28	1,160,000	10.00	72.056
28-29	1,700,000	10.00	81.600

Proportionality Factor (ξ_i)

The proportionality factor "ξ" is used in the first two terms of Equation 1 that describe the advective movement into and out of a segment. This mass movement across the i-1,i boundary due to advection may be expressed simply as the product of the net advective flow "Q_i" and the concentration at the boundary. However, since only the average concentration of the adjacent segments is considered in the model, the concentration at the boundary must be calculated. It can be shown by geometry that, for the case of a linear gradient between the midpoints of two adjacent segments, the concentration at the boundary is given by the first two terms in brackets in Equation 1 where:

$$\xi_i = \frac{L_i}{L_{i-1} + L_i} \qquad 9$$

Equation 9 gives a satisfactory first approximation for the proportioning factor applicable to the i-1,i boundary. To assure a realistic solution, however, the following relationship must be observed:

$$\xi > 1 - \frac{E}{Q} \qquad 10$$

To simplify computations where "Q" was changed frequently, the inequality given by Equation 10 was treated as an equality. So long as the proportionality factor "ξ" remains within the lower limit defined by Equation 10 and the upper limit of unity, the solutions obtained by the model are relatively insensitive to the value chosen for "ξ."

VI. VERIFICATION RESULTS

Various sources of chloride data were used in the verification of the model. The most significant were the data provided by the D. C. Department of Sanitary Engineering and by the Virginia Electric and Power Company's Possum Point Power Station. The D. C. Department of Sanitary Engineering made bi-weekly sampling runs down the Estuary and also obtained daily values at Woodrow Wilson Bridge (12 miles downstream from Chain Bridge), while the Virginia Electric and Power Company runs daily chloride analyses at Possum Point (approximately 40 miles downstream from Chain Bridge).

As stated previously, knowing values of the river inflow and the boundary conditions, the values of "K" were adjusted to obtain good agreement of the model output with observed chloride concentrations. The final results of these adjustments are shown in Figures 9, 10, and 11 for 1965. Similar results were obtained for 1963 and 1964 using the same "K" values.

In attempting to reproduce the 1930 conditions, only the results of one set of 11 grab samples could be found.[17] These data and the computed 1930 conditions are shown in Figure 12.

VII. SIMULATION

With the values of "K" obtained in the verification studies, it was possible to simulate the effects of future diversions into and out of the Estuary system on chloride concentration. In order to investigate the question posed in the introduction and to examine the sensitivity of the results to the assumption made in formulating the model, the simulations shown in Table 7 were made. The results of these simulations are shown in Figures 13 to 16.

TABLE 7

HYDROLOGIC CONDITIONS SIMULATED

River Flow and Boundary Conditions	Water Supply Use and Waste Water Flows	Waste Discharge Point	Other Imposed Conditions
1930	2010	Out of Estuary	None
1930	2010	To present sites	None
1966	2010	To present sites	None
1930	2010	To present sites	Consumptive water use raised by a factor of 6 for last 15 days in October.
1930	2010	To present sites	Dispersion coefficients in upper eight segments increased by one order of magnitude.
1930	2010	To present sites	Expected chloride loading from people doubled.
1930	2010	To present sites	All of above conditions.

VIII. DISCUSSION OF RESULTS

Validity of the Model and Parametric Analyses

The mathematical model utilized in this study is actually a numerical approximation of the partial differential equation which describes the temporal and spatial relationships of a conservative substance in an estuary. Two problems arise in its use here.

1. The model does not completely explain the integrated mechanisms which control the hydrograph of an estuary.

2. The lower portion of the estuary is highly stratified, and the assumption of vertical homogeneity is not actually valid.

The validity of both of the above statements is strongest near the mouth and weakest in the upper estuary. From an engineering and planning standpoint, it was assumed for this study that since the model matched historical data with reasonable accuracy, it has sufficient integrity to predict future effects as long as the simulated conditions are reasonably similar to the verified conditions.

For example, recent studies indicate that the exchange coefficient is not a fixed physical parameter which depends only on tidal velocity and geometry, but that it is a time-varying parameter which depends on density currents set up by chemical concentration (salinity) gradients. Therefore, it must be realized that if there are drastic changes in salinity within the

system caused by diversions, the results of the model must be interpreted with caution.

The most significant case in the simulation study where this (drastic salinity changes) occurred was the one where the waste effluents were pumped out of the system and the chloride concentration rose rapidly to levels never experienced in the system during the verification studies. In this case since the higher chlorides would probably cause an increase in "K" and, therefore, an increase in the rate at which the chlorides were diffused into the upper segments of the estuary, this effect would not change the obvious conclusions of this result.

Another way in which the simulated condition varied significantly from the verified condition is in the direction of river flow in the upper segments. Such a change might also affect the values of "K."

The effect of an increased turbulent exchange factor which might be caused by the changes in the tidal and flow regimes forced on the system in simulations were investigated by varying the value of "K" in the upper eight segments. The results of this study are shown in Figure 15. From Figure 15 it was concluded that the effect of such changes, should they occur, would not change the study conclusions.

The effect of an error in the assumption of a per capita chloride contribution of 9 gm/day to the system was also checked. This rate of chloride addition was doubled, with the results shown

in Figure 15. This shows that major errors in this estimate would have only a small effect on the results.

The effects of a large consumptive withdrawal for a short period (e.g., as might be caused by a major conflagration) was also investigated, and the water withdrawal rate was increased by 50 per cent for the last 15 days of October. None of this increase was returned to the estuary. This had the effect of increasing consumptive use by a factor of six. The results of this simulator, also shown in Figure 16, indicate that such an action would only increase the chloride concentration from 61 to 64 mg/l.

In order to check the possible cumulative effect of all the above conditions, a final simulation run was made with all of the above conditions imposed. This run, also shown in Figure 16, showed an increase in the maximum chloride concentration by 25 per cent should this occur.

Simulation Results

The simulation results shown in Figure 13 indicate that if the water of the upper estuary is used as a water supply source, and the waste water is transferred out of the upper estuary, salinity intrusion will become a significant problem. In such a case, provision for desalting equipment must be made. However, if the present plans to discharge waste to the upper estuary are carried out, the results shown in Figures 15 and 16 show that

it can be used as a source of water supply without a significant problem of chloride buildup occurring.

Even in the extreme event of the water use rate doubling during a critical period, the chloride concentration of the upper estuary did not approach the Public Health Service Standard for drinking water of 250 mg/l.

IX. CONCLUSIONS

1. The results of computations of the temporal and spatial variation in chlorides in the Potomac Estuary using the segmented estuary model developed by Thomann are in reasonable agreement with observed data.

2. Simulation results using this model indicate that if the Washington Metropolitan Area used the upper Potomac Estuary as an auxiliary source of water, the chlorides in the system would not even begin to approach the Public Health Service limits; and thus, desalting equipment would not be required.

X. BIBLIOGRAPHY

1. Davis, Robert K., "The Range of Choice in Water Resource Management: A Study of the Potomac Estuary," Resources for the Future, Inc., Washington, D. C. (in press).

2. Fox, Irving K., "The Potomac Puzzle--Is There a Reasonable Solution?" Atlantic Naturalist, April-June 1962.

3. U. S. Army Corps of Engineers, "Potomac River Basin Study," Water Supply and Water Quality Control, Vol 5, Prepared by Division of Water Supply and Pollution Control, U. S. Public Health Service, Charlottesville, Virginia, 1962.

4. Hazen, Richard, et al, "Future Water Supply, Metropolitan Washington Region, Report No. 1, Requirements and Sources," Metropolitan Washington Council of Governments, Washington, D. C., 1967.

5. Thomann, Robert V., "Mathematical Model for Dissolved Oxygen," Journal of the Sanitary Engineering Division, American Society of Civil Engineers, Vol. 84, No. SA5, Proceedings Paper 3680, Oct. 1963, pp. 1-30.

6. Jeglic, John M., "Mathematical Simulation of the Estuarine Chloride Distribution," Digital Computer Technology and Programming Analysis Memo No. 1033, General Electric Re-Entry Systems Department, Philadelphia, Pa., Feb. 1967.

7. Hetling, Leo J., and Bunce, Ronald E., "A Digital Computer Program for the Steady-State Segmented Estuary Model," (in press).

8. Jeglic, John M., "Mathematical Simulation of the Estuarine Behavior," Digital Computer Technology and Programming Analysis Memo No. 1032, Rev. A, General Electric Re-Entry Systems Department, Philadelphia, Pa., July 1967.

9. Hetling, Leo J., and O'Connell, R. L., "A Study of Tidal Dispersion in the Potomac River," Water Resources Research, Vol. 2, No. 4, Fourth Quarter 1966, pp. 825-841.

10. U. S. Department of Commerce, "Surface Water Temperature and Salinity, Atlantic Coast, North and South America," Coast and Geodetic Survey Publication 31-1, U. S. Government Printing Office, Washington, D. C., 1965.

11. Stroup, E. D., and Lynn, R. J., "Atlas of Salinity and Temperature Distributions in Chesapeake Bay 1952-1961 and Seasonal Averages 1949-1961," Chesapeake Bay Institute, Graphical Summary Report 2, Johns Hopkins University, Baltimore, Md., Feb. 1963.

12. Dunnington, Elgin, Private Communication, Research Biologist, Chesapeake Biological Laboratory, Natural Resources Institute, University of Maryland.

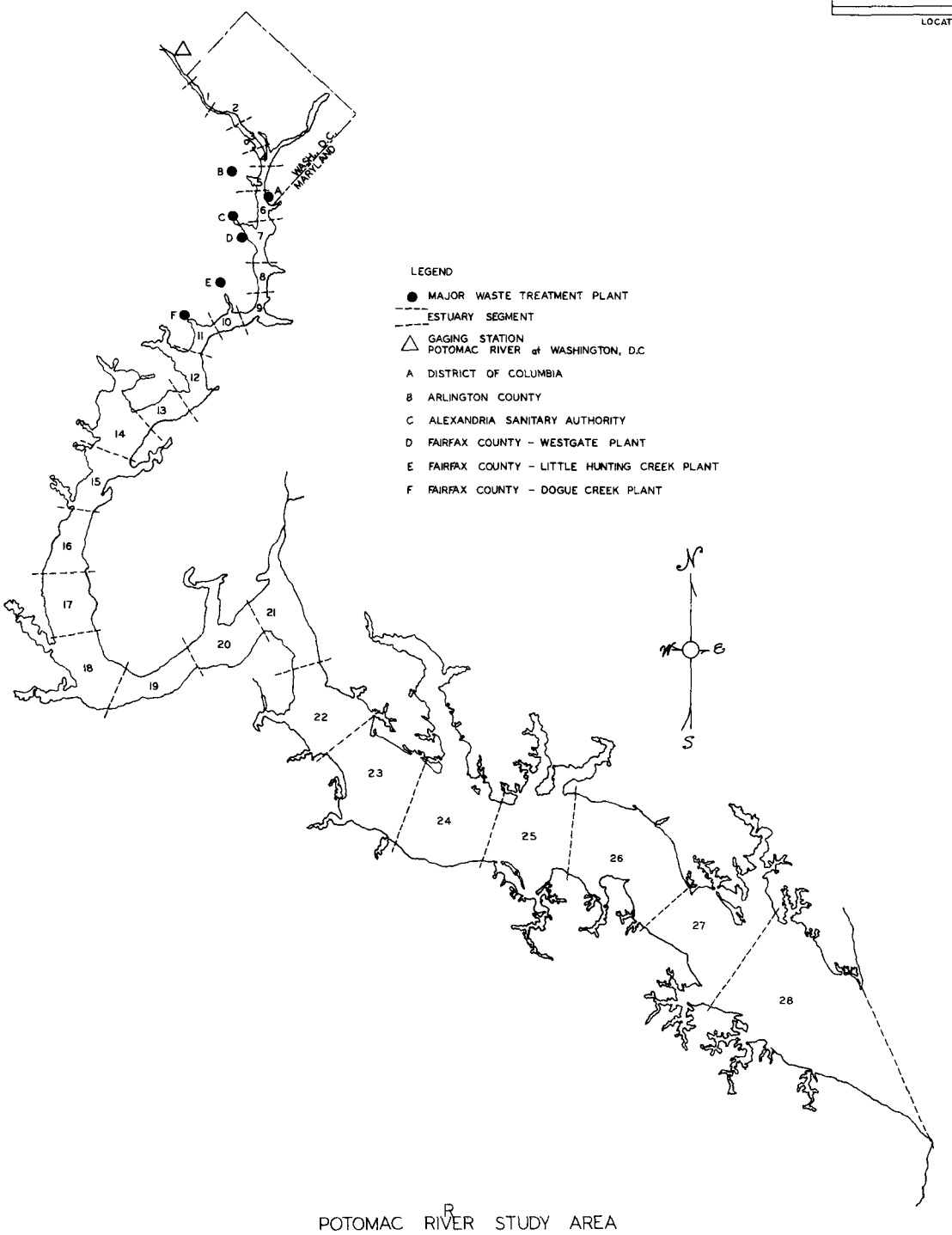
13. Whaley, R. C., Carpenter, J. H., and Baker, R. L., "Data Summary Potomac River Nutrient Cruises 1965-1966," Chesapeake Bay Institute, Special Report 11, Johns Hopkins University, Baltimore, Md., Aug. 1966.

14. Fair, Gordon M., and Geyer, John C., "Water Supply and Waste Water Disposal," John Wiley and Sons, Inc., New York, 1954.

15. Holley, E. R., Jr., and Harleman, D. F., "Dispersion of Estuary-Type Flows," M.I.T. Hydrodynamics Laboratory Report No. 74, Cambridge, Mass., Jan. 1965.

16. Hetling, Leo J., and O'Connell, Richard L., "Estimating Diffusion Characteristics in Tidal Waters," Water and Sewage Works, Vol. 112, No. 10, Oct. 1965.

17. Durford, C. N., "Water Quality and Hydrology in the Fort Belvoir Area, Virginia, 1954-1955," Geological Survey Water Supply Paper 1586-A, U. S. Government Printing Office, Washington, D. C., 1961.



POTOMAC RIVER STUDY AREA

FIGURE 1

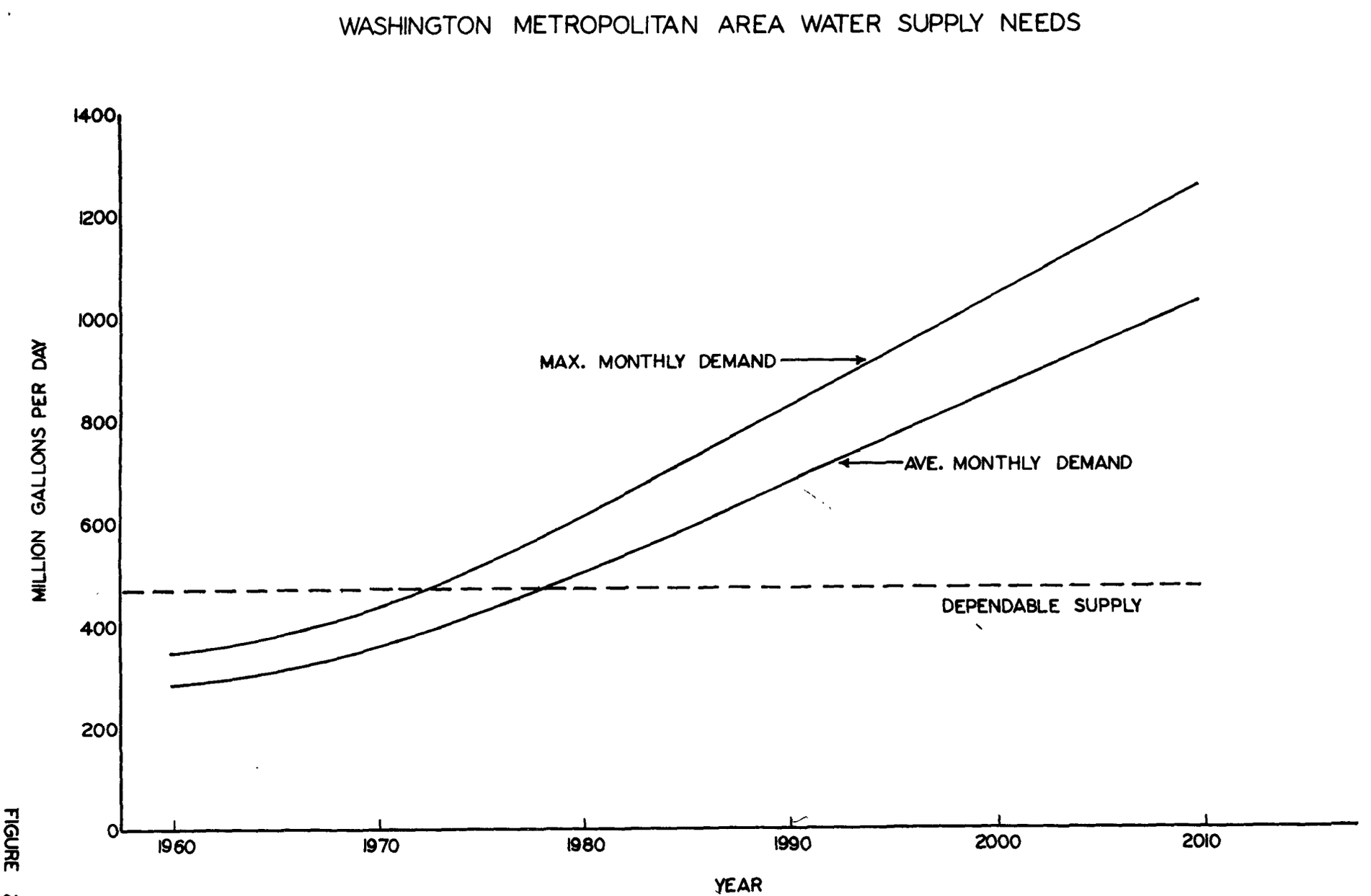

WASHINGTON METROPOLITAN AREA WATER SUPPLY NEEDS
1400
1200
1000
800
600
400
200
0
MILLION GALLONS PER DAY
MAX. MONTHLY DEMAND
AVE. MONTHLY DEMAND
DEPENDABLE SUPPLY
1960
1970
1980
1990
2000
2010
YEAR

FIGURE 2

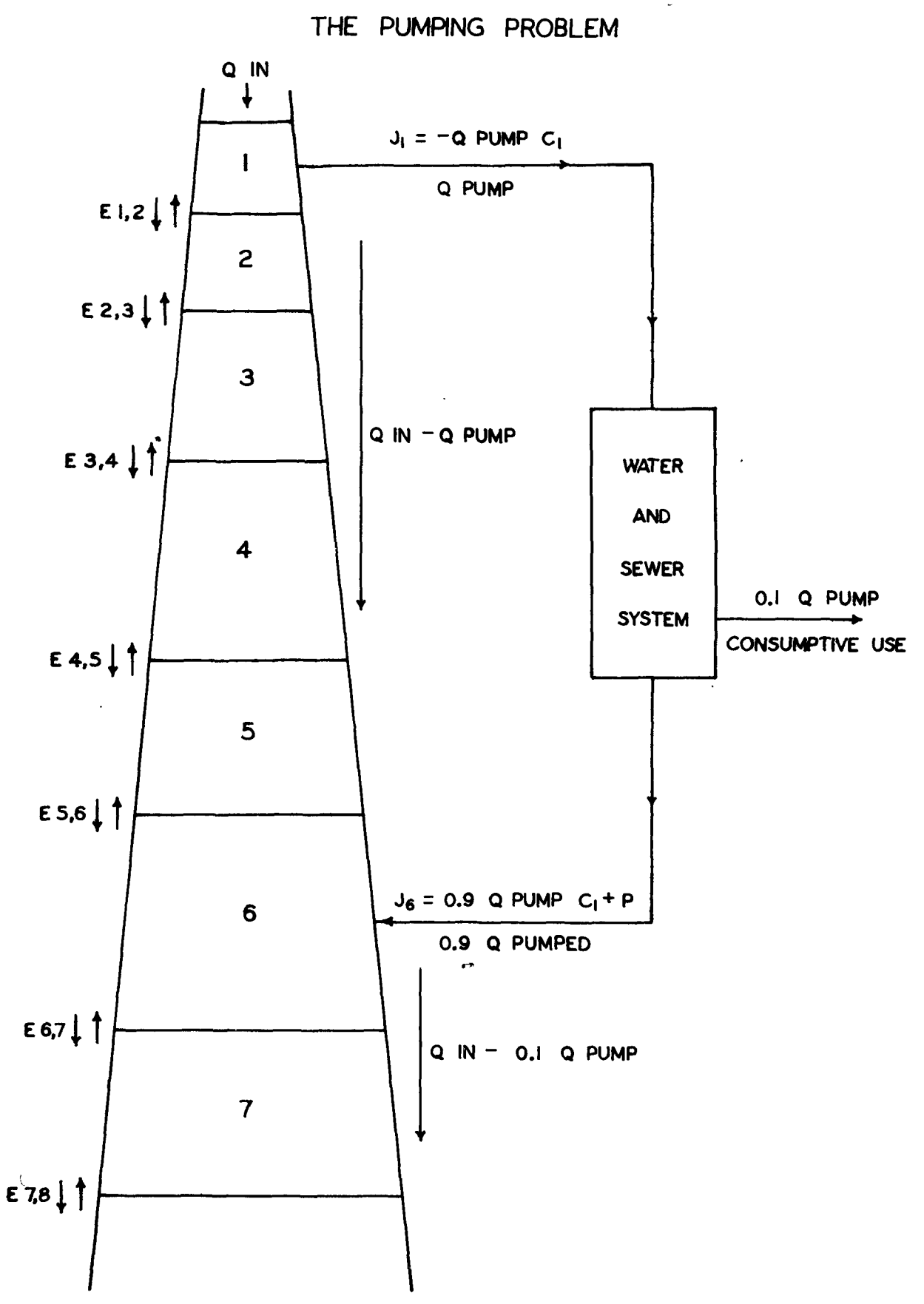

THE PUMPING PROBLEM
Q IN
1
2
3
4
5
6
7
E 1,2
E 2,3
E 3,4
E 4,5
E 5,6
E 6,7
E 7,8
J1 = −Q PUMP C1
Q PUMP
Q IN − Q PUMP
WATER
AND
SEWER
SYSTEM
0.1 Q PUMP
CONSUMPTIVE USE
J6 = 0.9 Q PUMP C1 + P
0.9 Q PUMPED
Q IN − 0.1 Q PUMP

FIGURE 3

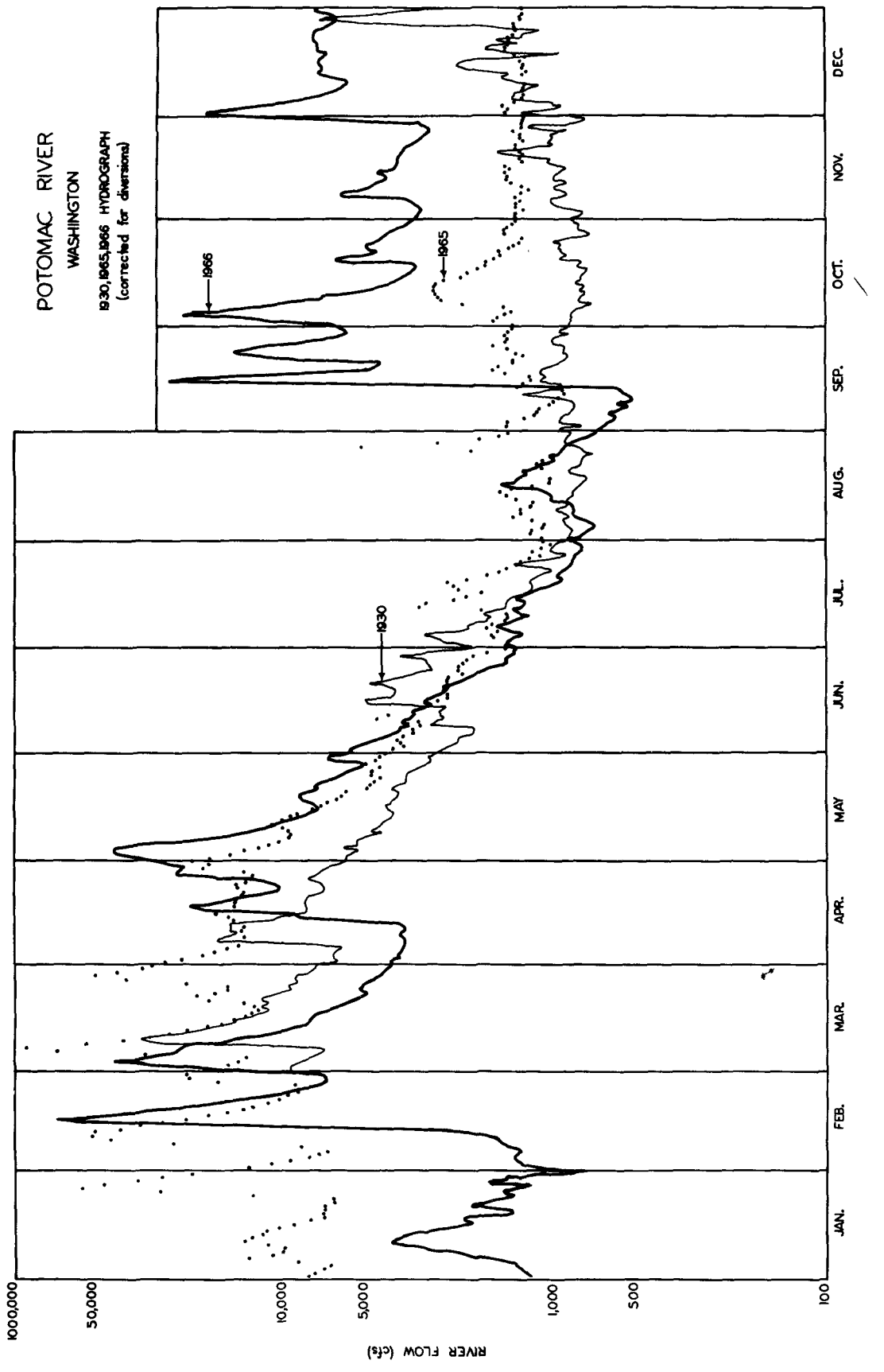

POTOMAC RIVER
WASHINGTON
1930, 1965, 1966 HYDROGRAPH
(corrected for diversions)
1966
1965
1930
RIVER FLOW (cfs)
100,000
50,000
10,000
5,000
1,000
500
100
JAN.
FEB.
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE 4

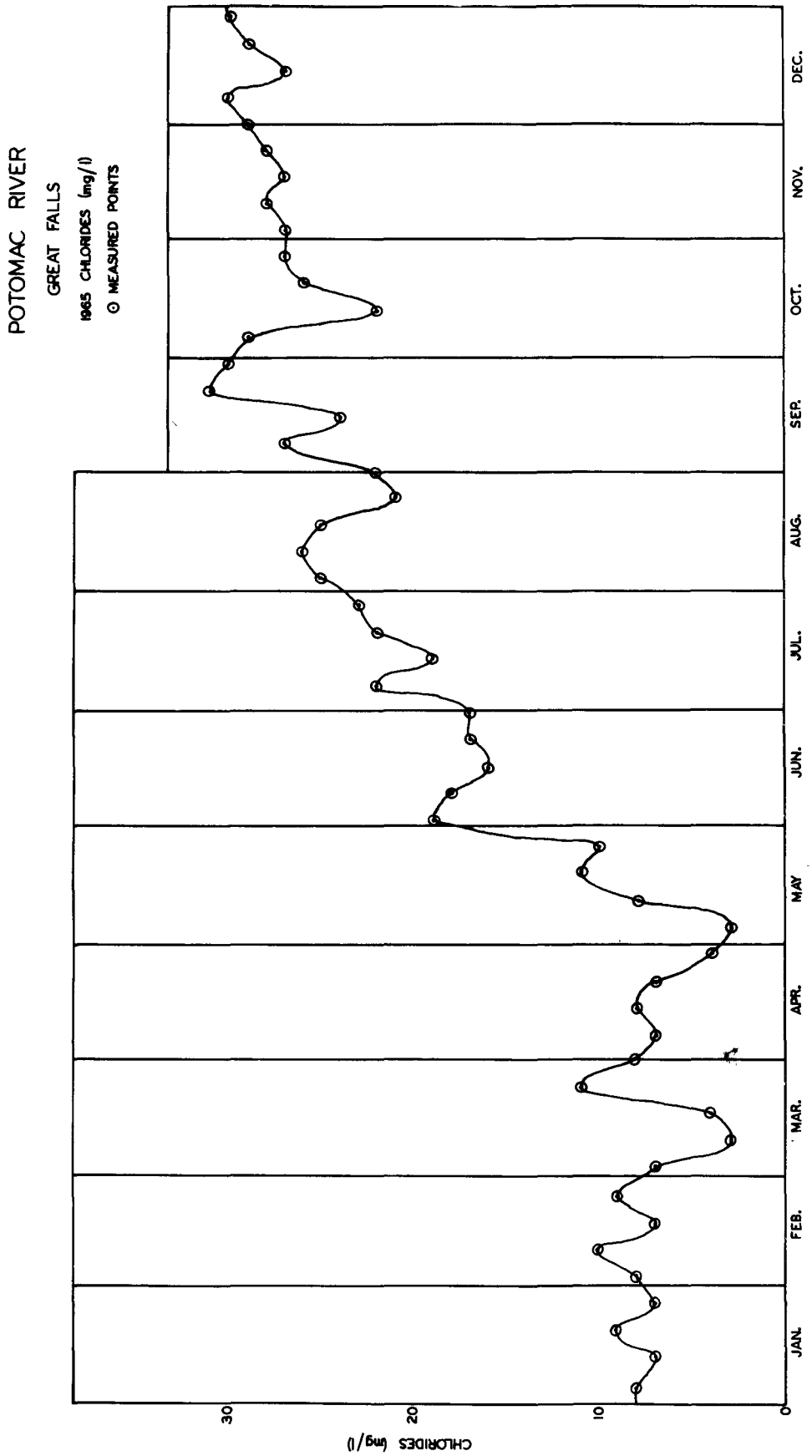

POTOMAC RIVER
GREAT FALLS
1965 CHLORIDES (mg/l)
⊙ MEASURED POINTS
CHLORIDES (mg/l)
0
10
20
30
JAN.
FEB.
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE 5

POTOMAC ESTUARY

CHLORIDE MODEL
UPPER BOUNDARY CONDITIONS

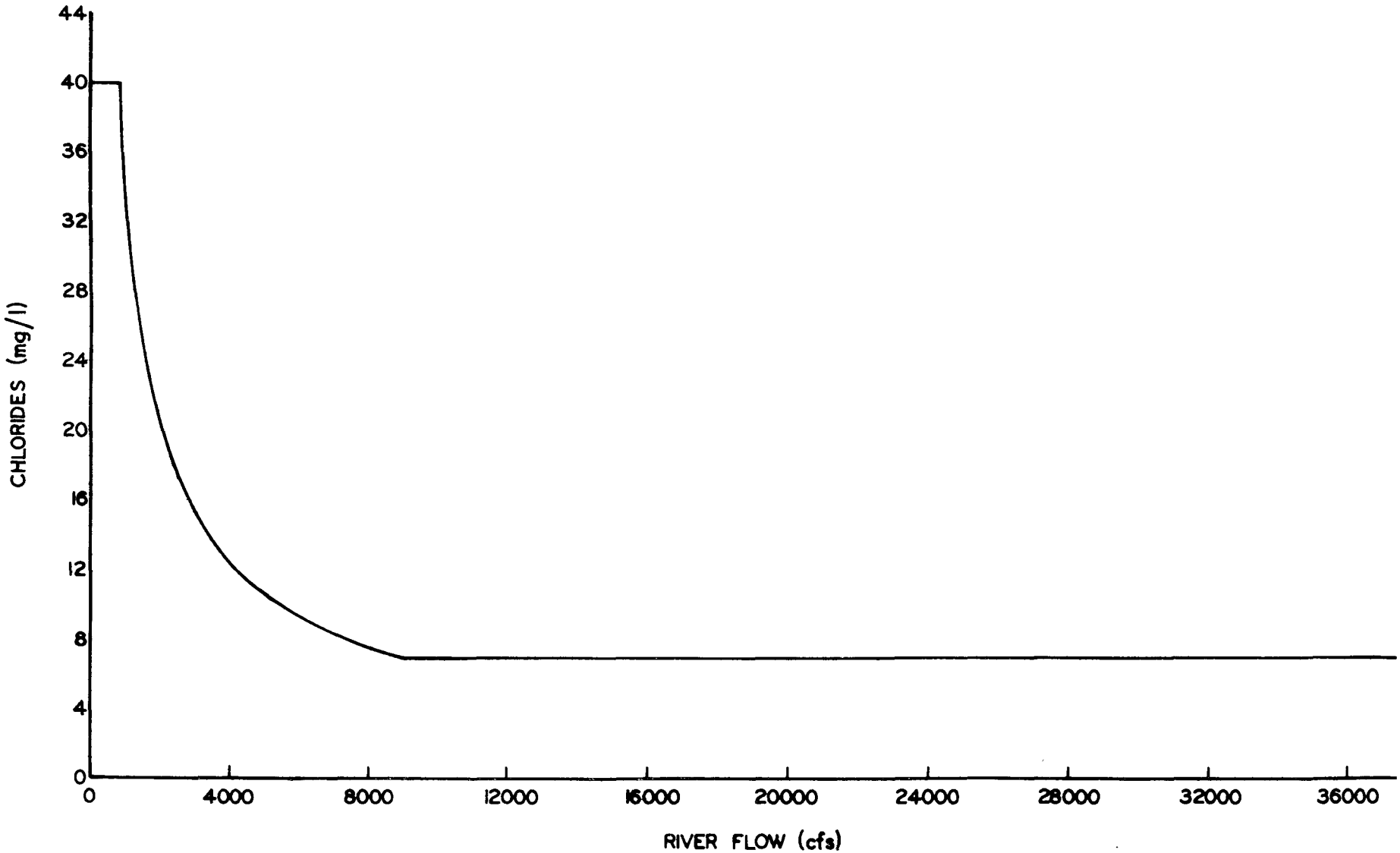


FIGURE 9

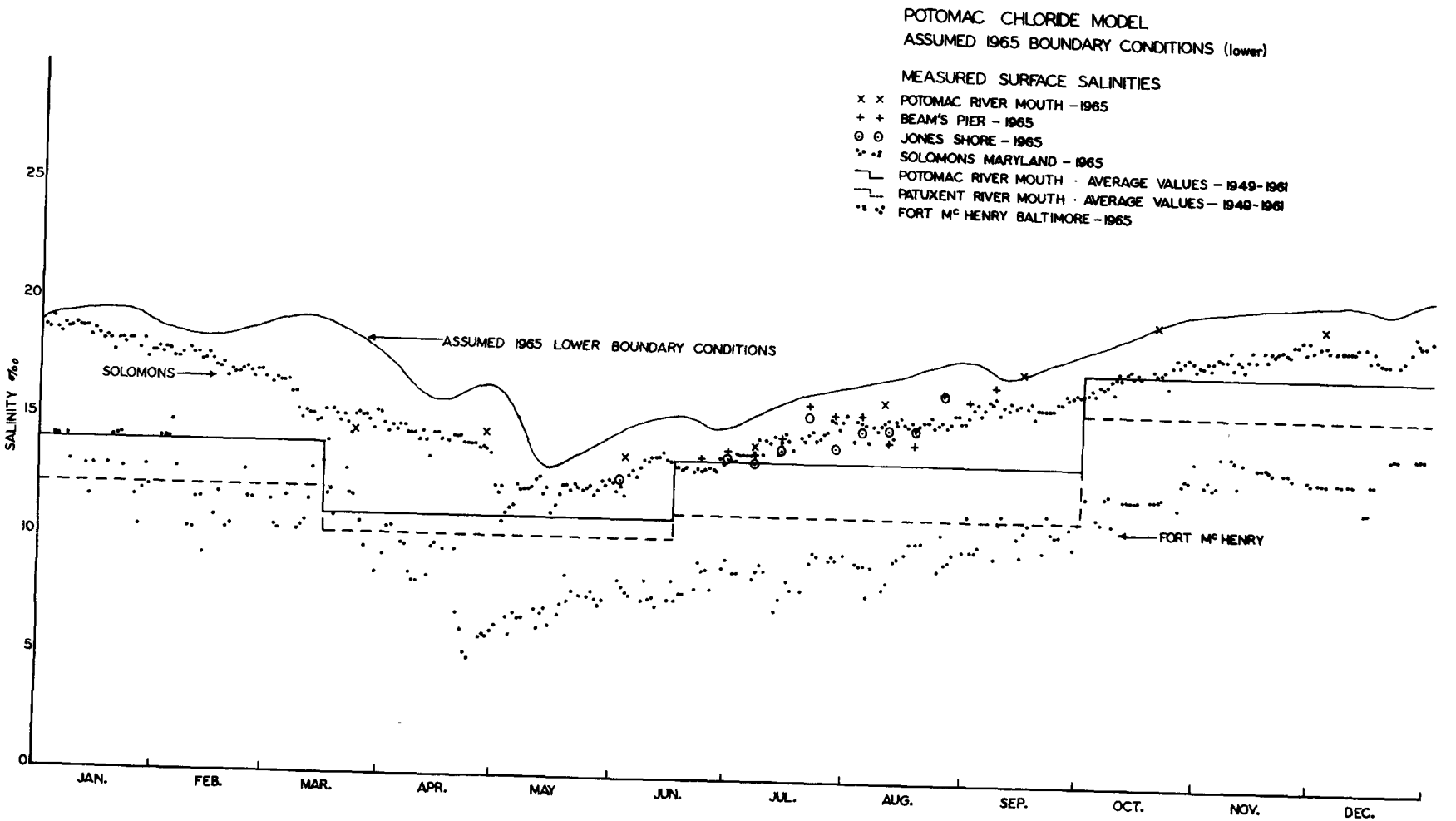
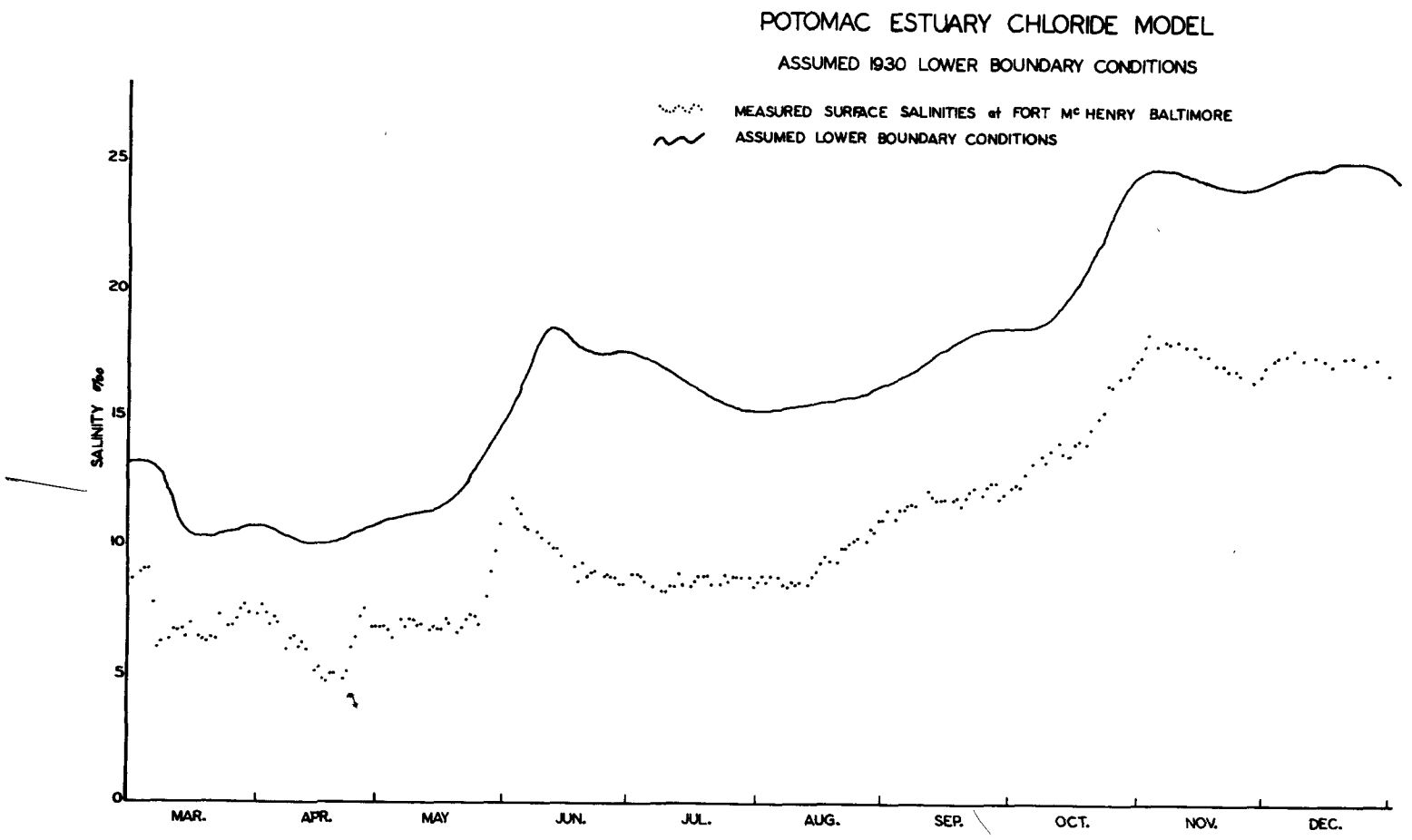

POTOMAC CHLORIDE MODEL
ASSUMED 1965 BOUNDARY CONDITIONS (lower)
MEASURED SURFACE SALINITIES
POTOMAC RIVER MOUTH - 1965
BEAM'S PIER - 1965
JONES SHORE - 1965
SOLOMONS MARYLAND - 1965
POTOMAC RIVER MOUTH · AVERAGE VALUES - 1949-1961
PATUXENT RIVER MOUTH · AVERAGE VALUES - 1949-1961
FORT Mc HENRY BALTIMORE - 1965
ASSUMED 1965 LOWER BOUNDARY CONDITIONS
SOLOMONS
FORT Mc HENRY
SALINITY ‰
0
5
10
15
20
25
JAN.
FEB.
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE 7


POTOMAC ESTUARY CHLORIDE MODEL
ASSUMED 1930 LOWER BOUNDARY CONDITIONS
MEASURED SURFACE SALINITIES at FORT Mc HENRY BALTIMORE
ASSUMED LOWER BOUNDARY CONDITIONS
SALINITY ‰
0
5
10
15
20
25
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE

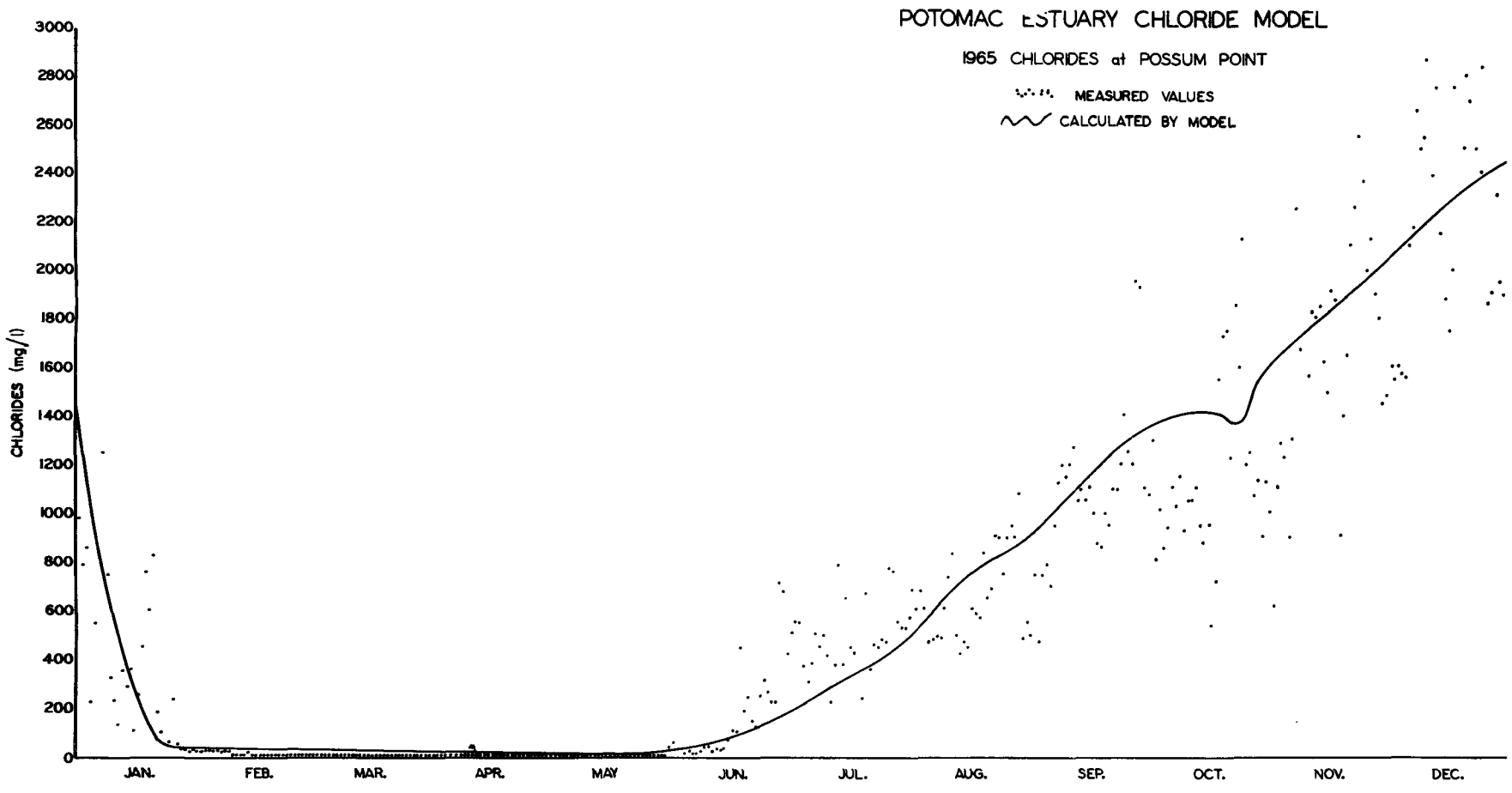

POTOMAC ESTUARY CHLORIDE MODEL
1965 CHLORIDES at POSSUM POINT
MEASURED VALUES
CALCULATED BY MODEL
CHLORIDES (mg/l)
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
JAN.
FEB.
MAR.
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE 6

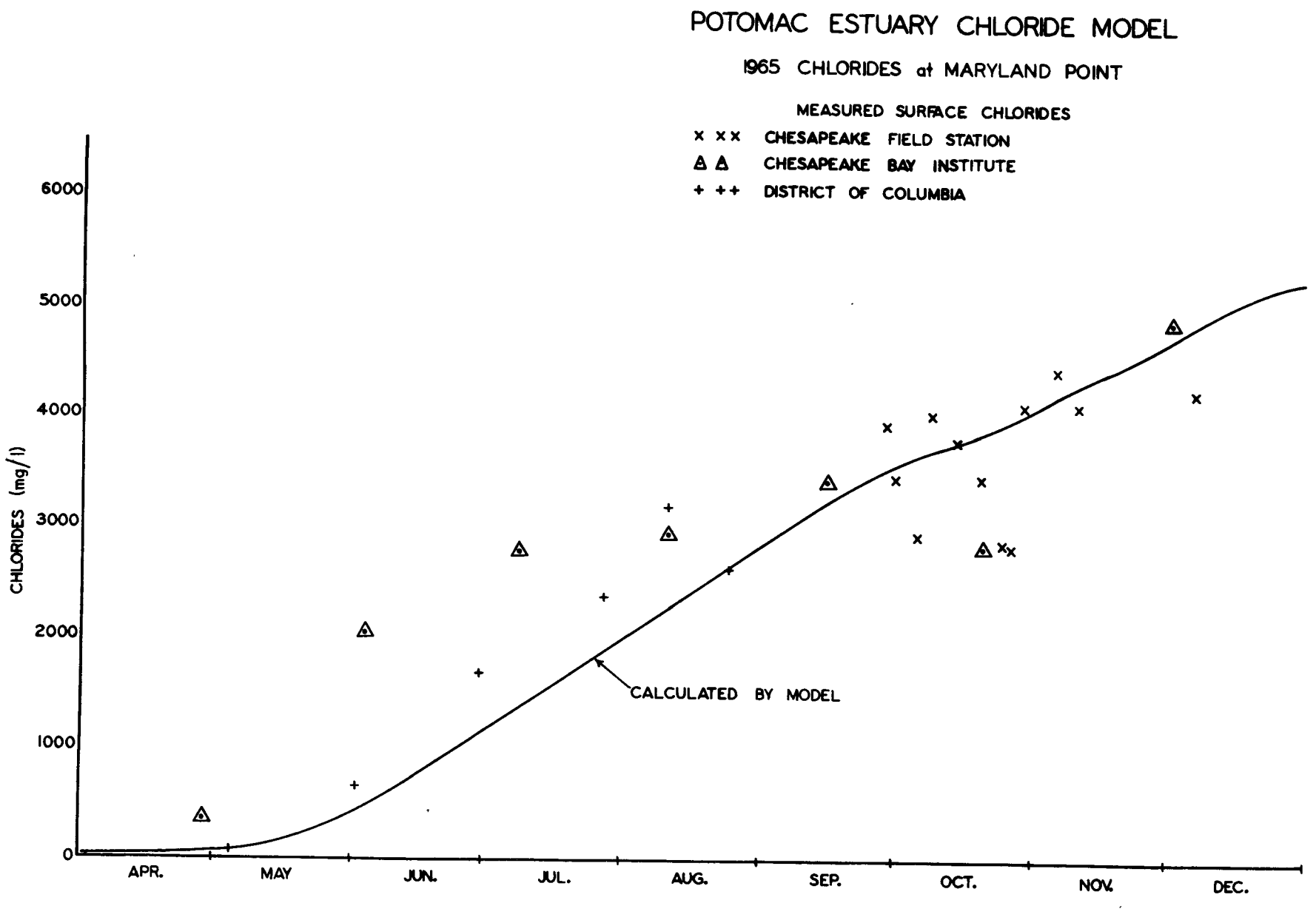

POTOMAC ESTUARY CHLORIDE MODEL
1965 CHLORIDES at MARYLAND POINT
MEASURED SURFACE CHLORIDES
× ×× CHESAPEAKE FIELD STATION
Δ Δ CHESAPEAKE BAY INSTITUTE
+ ++ DISTRICT OF COLUMBIA
CHLORIDES (mg/l)
6000
5000
4000
3000
2000
1000
0
CALCULATED BY MODEL
APR.
MAY
JUN.
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.

FIGURE 10

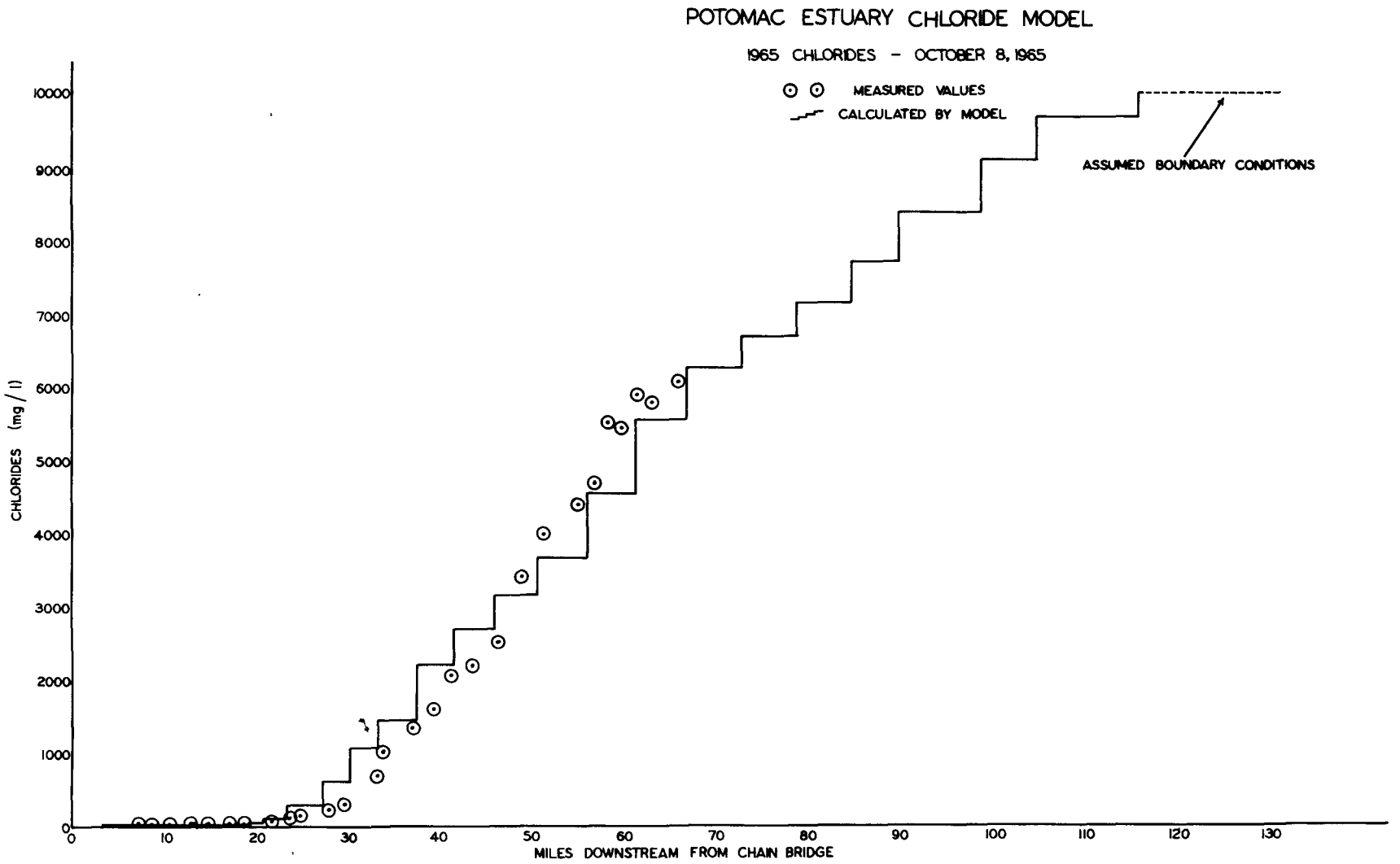

POTOMAC ESTUARY CHLORIDE MODEL
1965 CHLORIDES - OCTOBER 8, 1965
MEASURED VALUES
CALCULATED BY MODEL
ASSUMED BOUNDARY CONDITIONS
CHLORIDES (mg/l)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
MILES DOWNSTREAM FROM CHAIN BRIDGE

FIGURE 11

POTOMAC ESTUARY CHLORIDE MODEL

1930 CHLORIDES at INDIAN HEAD

xxx MEASURED VALUES

I I DAILY RANGE OF CHLORIDES FOUND IN ESTUARY OVER A TIDAL CYCLE

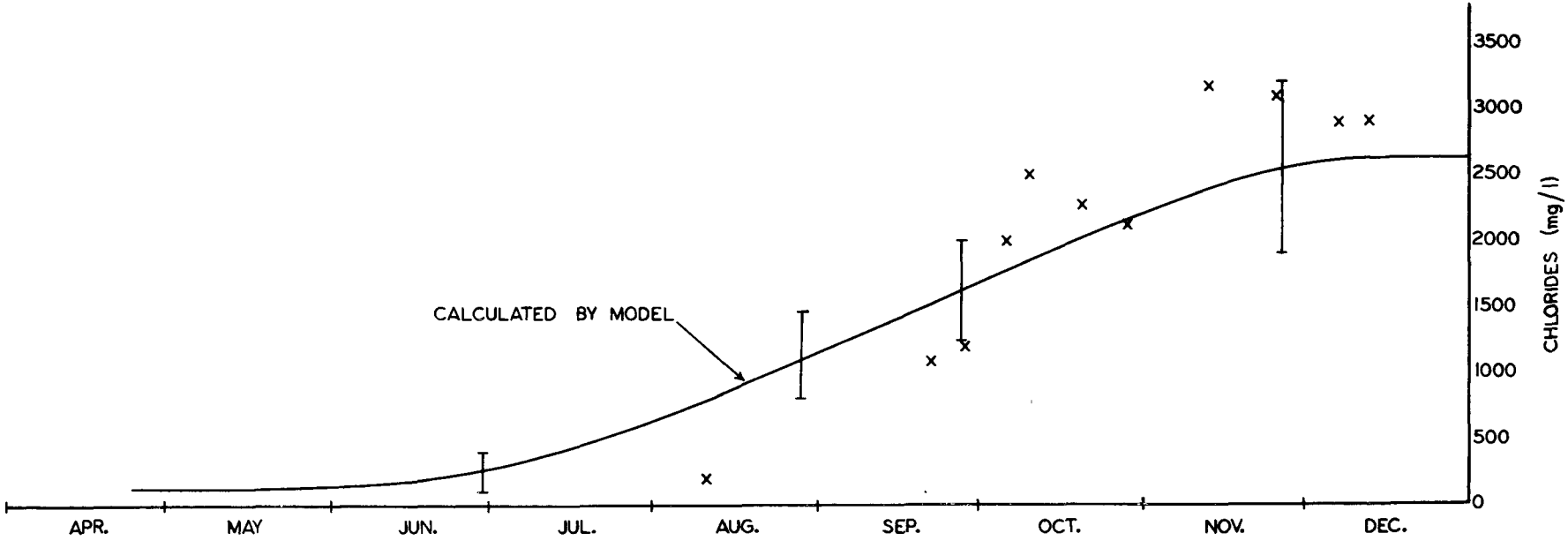


FIGURE 12

POTOMAC ESTUARY CHLORIDE MODEL

CALCULATED CHLORIDE CONCENTRATION at WATER INTAKE BASED ON;

1930 RIVER FLOWS

1930 BOUNDARY CONDITIONS

2010 WATER USE

WATER SUPPLY TAKEN FROM ESTUARY OPPOSITE GEORGETOWN RESERVOIR AND DISCHARGED TO CHESAPEAKE BAY

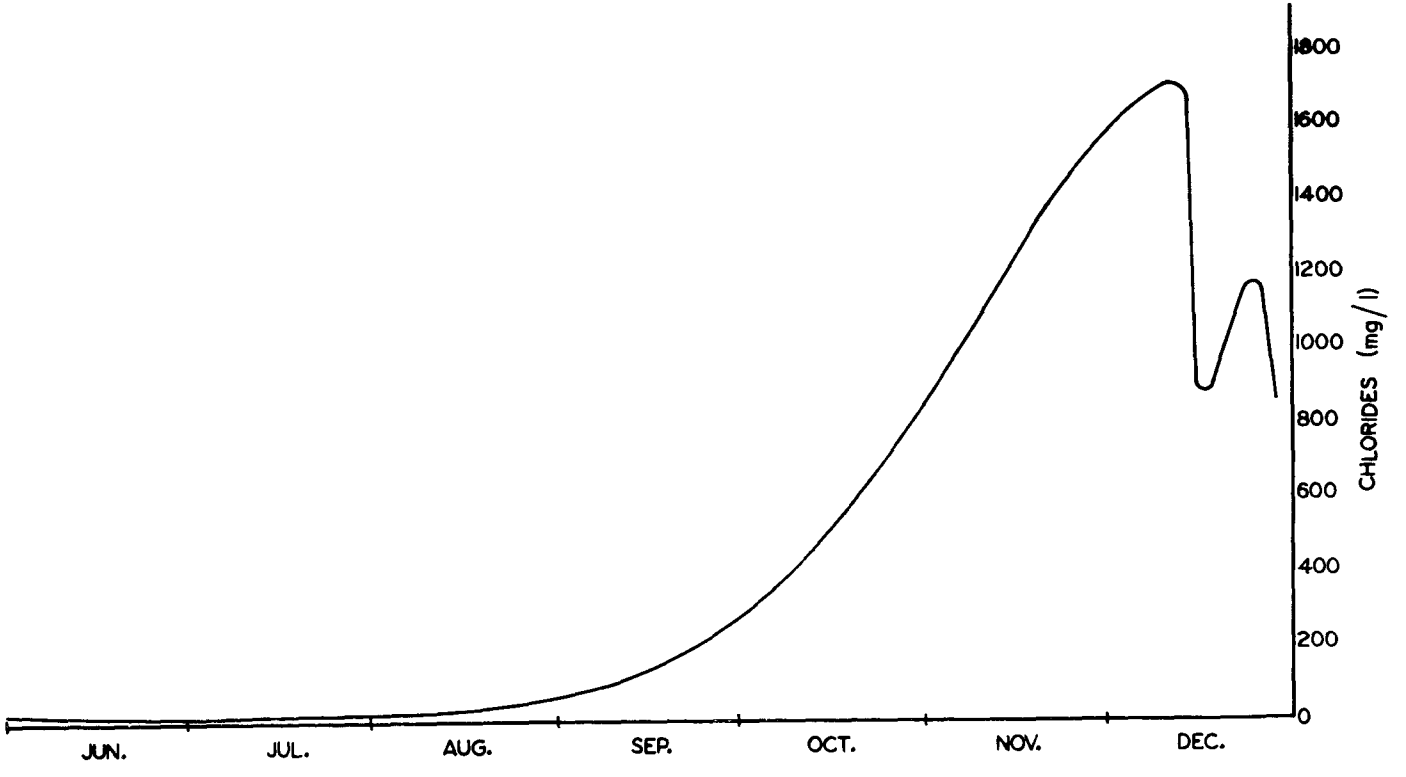


FIGURE 13

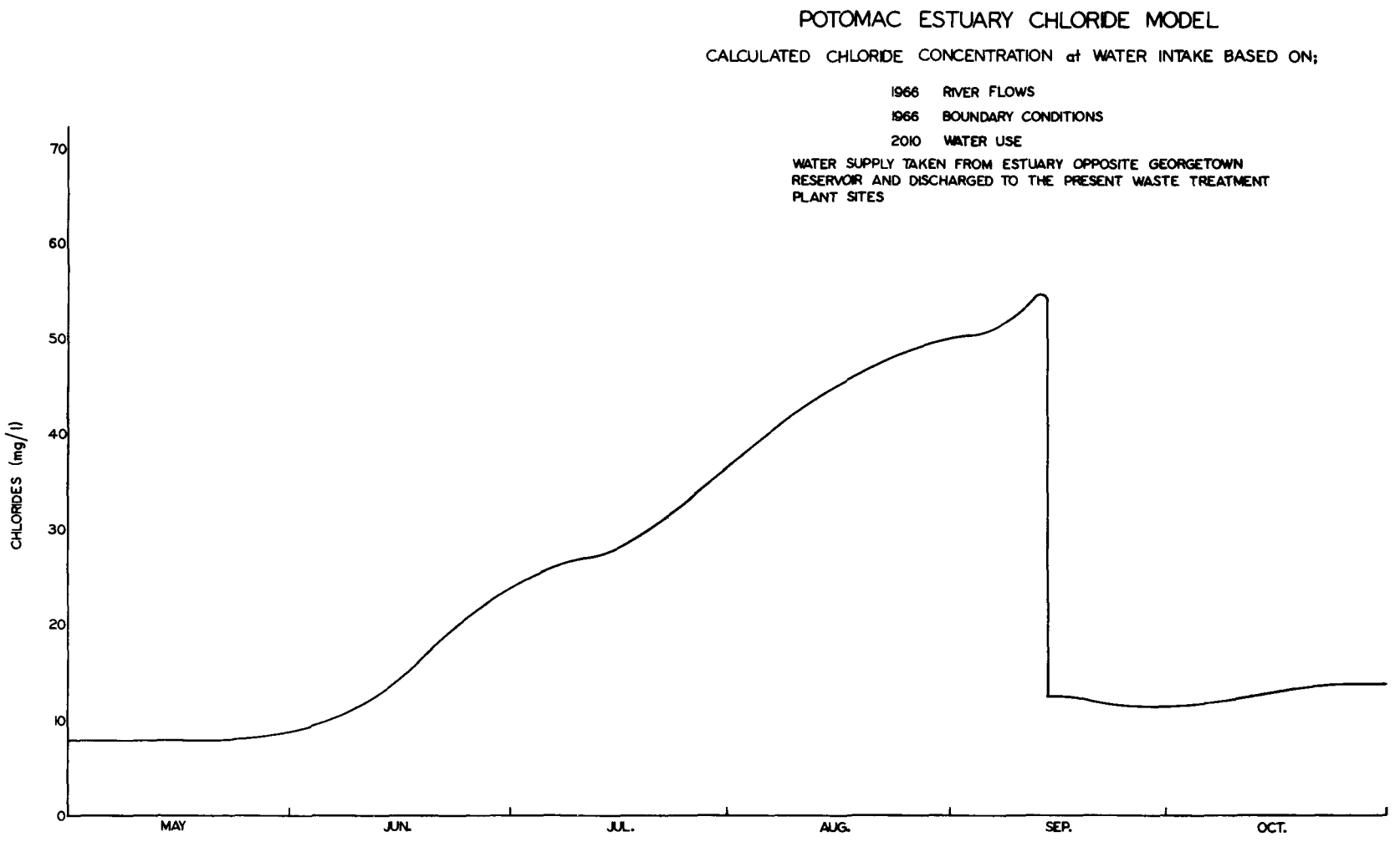

POTOMAC ESTUARY CHLORIDE MODEL
CALCULATED CHLORIDE CONCENTRATION at WATER INTAKE BASED ON;
1966 RIVER FLOWS
1966 BOUNDARY CONDITIONS
2010 WATER USE
WATER SUPPLY TAKEN FROM ESTUARY OPPOSITE GEORGETOWN RESERVOIR AND DISCHARGED TO THE PRESENT WASTE TREATMENT PLANT SITES
CHLORIDES (mg/l)
0
10
20
30
40
50
60
70
MAY
JUN.
JUL.
AUG.
SEP.
OCT.

FIGURE 14

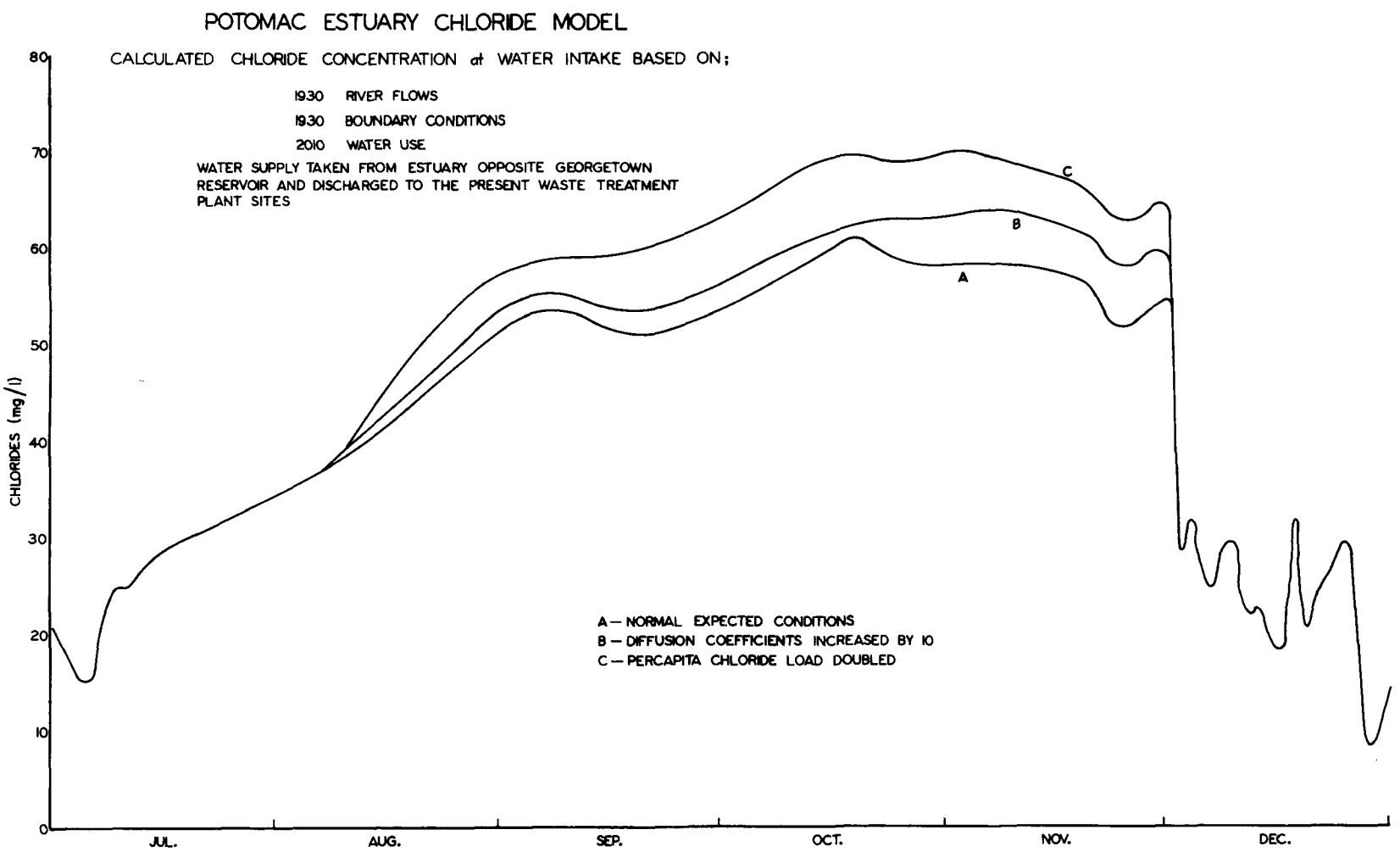

POTOMAC ESTUARY CHLORIDE MODEL
CALCULATED CHLORIDE CONCENTRATION at WATER INTAKE BASED ON;
1930 RIVER FLOWS
1930 BOUNDARY CONDITIONS
2010 WATER USE
WATER SUPPLY TAKEN FROM ESTUARY OPPOSITE GEORGETOWN RESERVOIR AND DISCHARGED TO THE PRESENT WASTE TREATMENT PLANT SITES
CHLORIDES (mg/l)
80
70
60
50
40
30
20
10
0
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.
A
B
C
A — NORMAL EXPECTED CONDITIONS
B — DIFFUSION COEFFICIENTS INCREASED BY 10
C — PERCAPITA CHLORIDE LOAD DOUBLED

FIGURE 15

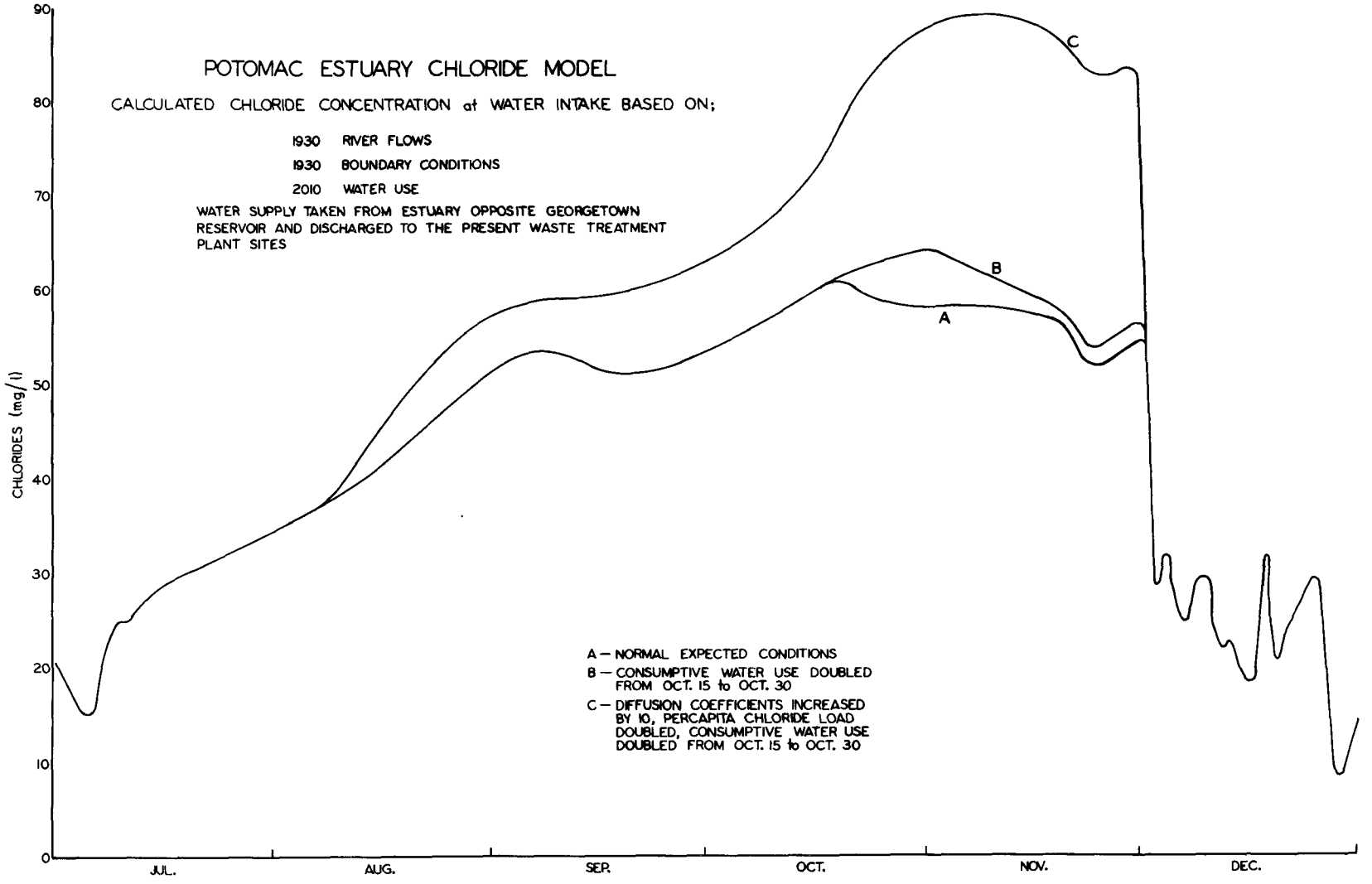

POTOMAC ESTUARY CHLORIDE MODEL
CALCULATED CHLORIDE CONCENTRATION at WATER INTAKE BASED ON;
1930 RIVER FLOWS
1930 BOUNDARY CONDITIONS
2010 WATER USE
WATER SUPPLY TAKEN FROM ESTUARY OPPOSITE GEORGETOWN RESERVOIR AND DISCHARGED TO THE PRESENT WASTE TREATMENT PLANT SITES
A — NORMAL EXPECTED CONDITIONS
B — CONSUMPTIVE WATER USE DOUBLED FROM OCT. 15 to OCT. 30
C — DIFFUSION COEFFICIENTS INCREASED BY 10, PERCAPITA CHLORIDE LOAD DOUBLED, CONSUMPTIVE WATER USE DOUBLED FROM OCT. 15 to OCT. 30
CHLORIDES (mg/l)
0
10
20
30
40
50
60
70
80
90
JUL.
AUG.
SEP.
OCT.
NOV.
DEC.
A
B
C

FIGURE 16

TABLE OF CONTENTS

	Page
PREFACE	ii
LIST OF TABLES	vii
LIST OF FIGURES	viii
LIST OF APPENDICES	x
Chapter	
I. INTRODUCTION	1
II. REVIEW OF LITERATURE	5
WATER QUALITY FORMULATIONS	5
MATHEMATICAL MODELS AND METHODS FOR ANALYSIS	16
Descriptive Analysis	16
Statistical Models	17
River Flow Simulation	19
River Basin Management	21
Flow Regulation Models	21
Treatment Requirements	23
Case Study, The Potomac Estuary	25
III. FLOW RELEASE MODEL	27
THE PROBLEM	27
GENERAL APPROACH	29
QUALITY FORMULATIONS	29
Reaeration	30
Temperature	31
Physical Parameters	33
Reservoir Quality	33
DESCRIPTIVE PARAGON	34
NODAL OPERATIONS	35
DYNAMIC PROGRAMMING SOLUTION	36
OVERALL ALGORITHM AND COMPUTER PROGRAMMING	41

TABLE OF CONTENTS (continued)

Chapter		Page
IV.	PATUXENT AND POTOMAC RIVER BASIN SYSTEMS	45
	THE PATUXENT SYSTEM	45
	THE POTOMAC SYSTEM	46
	Source of Data	46
	Description of Basin	47
V.	DEVELOPMENT AND INTERPRETATION OF TABLEAUX AND THE FORMATION OF RESERVOIR RELEASE PATTERNS	52
	DEVELOPMENT AND INTERPRETATION OF TABLEAUX	52
	Development of Tableaux	54
	Incremental Flow Tableaux	66
	Dynamic Programming Tableaux	67
	FLOW RELEASE PATTERNS	68
	WATER QUALITY CONTROL CONSIDERATIONS IN RESERVOIR DESIGN	70
VI.	SENSITIVITY ANALYSES	73
	BIOCHEMICAL AND PHYSICAL PARAMETERS	73
	Reaeration	74
	Deaeration and Minimum BOD Concentrations	75
	DESIGN PARAMETERS	77
	Temperature	77
	BOD-DO Concentrations in the Reservoirs	78
	Wastewater Loadings	79
	DO Concentration in the Wastewater Effluents	81
	SOCIO-ECONOMIC PARAMETERS	82
	Water Quality Objectives	82
	Imposed Wastewater Loadings	82

TABLE OF CONTENTS (continued)

Chapter		Page
VII.	FURTHER DEVELOPMENTS OF THE FLOW RELEASE MODEL	86
	LEAST-COST SOLUTIONS	86
	COMPARISON OF OPTIMIZATION PARAMETERS	88
	OTHER POSSIBLE WATER QUALITY MEASUREMENTS	90
	LINKAGE TO RIVER FLOW SIMULATION MODELS	92
	Use of Yield Curves	92
	River Flow Simulation Models	94
	LINKAGE TO ESTUARY AND RIVER BASIN QUALITY - SIMULATION MODELS	95
	IN-STREAM IMPOUNDMENTS	98
VIII.	DISCUSSION OF RESULTS	99
	QUALITY FORMULATIONS	99
	OPTIMIZATION CONCEPTS AND ADAPTATIONS	100
	USE OF OPTIMAL RELEASE SEQUENCES	104
	USE OF WATER QUALITY MANAGEMENT MODELS WITHIN THE FRAMEWORK OF CURRENT TECHNOLOGICAL AND INSTITUTIONAL PRACTICES	105
IX.	SUMMARY AND CONCLUSIONS	110

Appendix

A		115
B		137
C		157
D		188
E		200

BIBLIOGRAPHY 225

LIST OF TABLES

Number	Title	Page
1	Inventory of the Potomac River Basin	47
2	Inventory of Flow Release Model for the Potomac River Basin	48
3	Reservoir Data - Potomac River Basin	51
4	Incremental Flow Tableau - Nodes 570, 562, and 494 . .	57
5	Incremental Flow Tableau - Nodes 462 and 458	58
6	Incremental Flow Tableau - Node 460	59
7	Incremental Flow Tableau - Node 436	60
8	Dynamic Programming Tableau - Node 560	61
9	Dynamic Programming Tableau - Node 460	62
10	Dynamic Programming Tableau - Node 456	63
11	Dynamic Programming Tableau (Churchill K_2 Formulation) - Node 456	64
12	Flow Range Data - Upper Potomac River Basin	65
13	Example of an Optimal Release Sequence	69
14	Optimization Criteria of the Flow Release Model - Version II	87
15	Comparison of Reaeration Rates	123
16	Stream Flow Data for Flow Release Model - Potomac River Basin	150
17	Surface Water Supply and Wastewater Inventory - Potomac River Basin	153
18	Cost of Field Studies	156
19	Card Formats for Verification Link and Flow Release Model	160
20	Array Notation for Verification Link and Flow Release Model	168

LIST OF FIGURES

Number	Title	Page
1	Factors Affecting Stream Dissolved Oxygen in a Non-Tidal System	15
2	A Schematic Representation of Proposed Reservoir System - Potomac River Basin	37
3	Simplified Flow Chart for Flow Release Model	44
4	Potomac River Basin Map Showing Proposed Reservoirs	50
5	Schematic of Upper Potomac River Basin Flow Release Model	53
6	Reservoir Release Sequences for Various Flows at the Estuary	71
7	Comparison of Reservoir Release Sequences with Varying Biochemical and Physical Parameters for 1500 cfs at the Estuary	76
8	Comparison of Reservoir Release Sequences with Varying Engineering Design Parameters for 1500 cfs at the Estuary	80
9	Comparison of Reservoir Release Sequences with Varying Social-Economic Parameters for 1500 cfs at the Estuary	83
10	Comparison of Reservoir Release Sequences for Various Optimization Parameters for 1500 cfs at the Estuary	89
11	West Branch of Conococheague Creek, Site No. 5, Uniform Use Rate Versus Storage	93
12	Linkage for River Basin Water Quality Simulation	97
13	Nomograph for Reaeration Rates, O'Connor and Dobbins	119
14	Nomograph for Reaeration Rates, TVA	120
15	Nomograph for Reaeration Rates, USGS	121
16	Computed Temperatures, DO and BOD Profiles and Stream Survey Data for the North Branch Potomac River	122
17	Depth and Reaeration Variations, North Branch Potomac River	128

LIST OF FIGURES (Continued)

Number	Title	Page
18	Computed Phosphate Profiles and Stream Survey Data for the Patuxent River Basin	130
19	Velocity Versus Flow and Depth Versus Flow for the Patuxent River at Hardesty, Md.	133
20	Lower Potomac and Monocacy River	139
21	Lower-Middle Potomac and Antietam Creek	140
22	Lower-Main Stem Shenandoah River	141
23	North Fork Shenandoah River	142
24	South Fork Shenandoah River	143
25	North, Middle and South Rivers of South Fork Shenandoah River	144
26	Upper-Middle Potomac River and Lower Conococheague and Opequon Creeks	145
27	Upper Conocheague and Opequon Creeks	146
28	Upper-Main Stem and South Branch Potomac River	147
29	North Branch Potomac River	148
30	Over-all Potomac River Basin System	149
31	Formats for Data Cards	174
32	Typical Data Compilation for Verification Link	175
33	Typical Data Compilation for Flow Release Model	176
34	Patuxent River Basin	177

LIST OF APPENDICES

Appendix		Page
A	SOME COMMENTS ON MODEL VERIFICATION	115
	REAERATION COEFFICIENT	115
	Calculated from Observed Reaeration Rates	115
	Predictive Formulations	116
	Gaseous Tracer Techniques	117
	Comparison of Methods	117
	USE OF PREDICTIVE FORMULATIONS	118
	CROSS-SECTION DATA	125
	BOD MECHANISMS AND FACTORS	127
	A PREDICTIVE MODEL FOR PHOSPHATES	129
	TIME OF TRAVEL	131
	A PROCEDURE FOR VERIFICATION OF THE QUALITY FORMULATIONS	134
	Defining the Physical System	134
	Establishing Stream Channel Characteristics	134
	Basin Partitioning and Parameter Determinations	135
	Verification of Quality Formulations	135
B	MODEL DATA FOR THE POTOMAC RIVER BASIN	137
	COST OF DATA	156
C	COMPUTER PROGRAM DOCUMENTATION AND USAGE WITH SAMPLE PROBLEM	157
	GENERAL	157
	DATA FORMATS AND TYPICAL DECK COMPUTATIONS	157
	PROGRAM USAGE WITH SAMPLE PROBLEM	158
	SAMPLE OF INPUT DATA FOR FLOW RELEASE MODEL - PATUXENT RIVER BASIN	178
D	VERIFICATION LINK COMPUTER PROGRAM	188
E	OPTIMAL FLOW RELEASE MODEL COMPUTER PROGRAM	200

CHAPTER I

INTRODUCTION

To better control, protect, and manage the water quality of our rivers, water resources engineers have directed increased attention to development of analytical techniques for determination of procedures and policies by which optimal operation and management of entire river basins may be realized. Analytical approaches to the design and management of water resources systems have as their fundamental objective the provision of water at a pre-set quality for the least cost. The analyses also include investigation of alternatives, determination of inadequacy probabilities, and establishment of optimal operational procedures.

There are essentially three methods for controlling water quality within a given river basin:

1. Regulation of the total waste load. This approach requires establishment of specifications for the degree of treatment of wastewaters to be effected prior to discharge, or, in the case of some industrial processes, for certain in-plant modifications or process changes.
2. Regulation of river flow, either by reservoirs, pumping from well fields, or by pumped storage, in order to provide appropriate dilution of waste loads when required.
3. Installation of in-stream treatment devices which either provide an integral means for removal of pollutants or which act to enhance the rate of natural removal of pollutants; stream aerators, for example, function in the latter capacity.

The engineering approach to analysis and solution of water quality control problems is greatly influenced by the degree of river basin development. Two general types of situations which are encountered in the regulation of flow for quality control are:

1. Operational Situation

   For a river basin in which the reservoir system is fully developed, the problem is one of determining that flow release sequence which will provide best quality for a given probability of inadequacy. Other water requirements may or may not be included in the approach.

2. Design and Operational Situation

   For a basin in which flow regulation possibilities are either limited or nonexistent, but for which reservoir development is proposed, the problem becomes one of making decisions regarding both design and operation. Not only must storage requirements for quality control be determined, but also a flow release sequence to meet the prescribed quality control criteria for a given probability of inadequacy must be established.

The two problems are closely related, although the data required for proper design are usually less readily available than that needed for establishing operational patterns. Many parameters cannot be rigidly defined, and thus often are not included in reservoir design for quality control. The problem is further complicated by the dependence of the operational pattern on the proposed storage requirements and possible operating sequences.

The purposes of the present study have been to:

1. develop a method for determination of an optimal flow regulation sequence for quality control from multiple reservoir sites;

2. investigate various physical, biochemical, engineering design, and socio-economic parameters which may influence the optimal flow release sequence;
3. demonstrate the response sensitivity of the method to these parameters in an actual basin; and
4. investigate the significance of various definitions of optimal such as "best" quality, minimum flow, and least-cost on the reservoir release sequences.

Chapter II presents a brief review of the literature pertaining to formulations and relationships for description of water quality transformations and for determining flow regulation and wastewater treatment requirements. Included in the review are models which have been proposed or are currently being used in the area of water pollution control for least-cost solutions for wastewater treatment, flow regulation, estuary analysis, and generation of synthetic hydrology.

Chapter III is devoted primarily to development of the flow release model. The basic concepts of the model and the development of the descriptive paragon are presented. A brief description of the Patuxent and Potomac River Basin Systems which were used as test basins for the model are presented in Chapter IV.

In Chapter V, the development and interpretation of the incremental-flow and the dynamic-programming tableaux are explained. Formulation of flow release patterns is presented in the latter part of Chapter V.

Chapter VI is devoted to a sensitivity analysis of certain physical, biochemical, engineering design, and socio-economic parameters.

Various adaptations of the flow release model, including that providing for least-cost solution, are presented in Chapter VII. A comparison is made of the reservoir release sequence as developed using various optimization parameters. Other possible model expansions are proposed, including river basin water quality simulation with linkage to estuarine models.

In Chapter VIII, the results of the flow release model are discussed. The summary and conclusions of the study are presented in Chapter IX.

Some typical problems and possible solutions encountered in model verification are presented in Appendix A. A proposed model for predicting phosphate concentration in flowing streams is also presented therein.

Detailed schematics of the Potomac River Basin showing wastewater discharges, dams, water intakes, etc. are exhibited in Appendix B, along with the basic inventory and stream flow data used in the flow release model.

Data formats, array notation and a brief description of the use of the verification link and flow release model are contained in Appendix C. The computer programs for the verification link and flow release model are given in Appendices D and E, respectively.

CHAPTER II

REVIEW OF LITERATURE

Before one can attempt to evaluate the effects of wastewater discharge or flow regulation on water quality in a stream, it is necessary to understand the relationships between the geophysical characteristics of the drainage area and the biochemical and physical environment of the stream. The first part of this chapter presents the historical development of the relationships between dissolved oxygen (DO), biochemical oxygen demand (BOD), stream flow, temperature, velocity, depth, and certain other parameters which may affect water quality. The basic quality parameters considered in this study are DO, BOD, and temperature. The latter part of the chapter presents various models which are currently being used for determining treatment requirements and flow regulation policies.

WATER QUALITY FORMULATIONS

The first quantitative description of deoxygenation and reaeration relationships in streams was presented in the classical studies of Streeter and Phelps[1] in 1925. These authors noted that the net rate of change ($\frac{dD}{dt}$) in the oxygen deficit (D) in a stream at any time was equal to the algebraic sum of the opposing rates of deoxygenation and reaeration. This is expressed mathematically as:

$$\frac{dD}{dt} = K_1 L - K_2 D \qquad \text{- - - - - (2-1)}$$

in which

K_1 = Rate coefficient for deoxygenation, reflects the availability of organic matter and the activity of the organisms present

L = Oxygen demand of the organic matter, in mg/l

K_2 = Rate coefficient for reaeration, reflects the degree of turbulence and the transfer-efficiency at the air-water interface

D = Dissolved oxygen deficit of the water in mg/l

The expression given by Streeter and Phelps is a linear differential equation of the first order. The first term, K_1L, expresses the rate of oxygen utilization or deaeration resulting from oxidation of organic waste matter. The term K_2D is the rate of reaeration, which is a function of the difference between the dissolved oxygen concentration at any time and the concentration corresponding to a condition of oxygen saturation.

Streeter and Phelps further stated that "The rate of biochemical oxidation of organic matter is proportional to the remaining concentration of the unoxidized substance, measured in terms of oxidizability." This can be expressed mathematically as a first order reaction with the integrated form

$$L_t = L_a e^{-K_1 t} \qquad \text{- - - - - (2-2)}$$

in which

L_a = The initial concentration of oxidizable organic matter, or ultimate BOD

L_t = The concentration of oxidizable organic matter at time (t)

t = Time elapsed, in days

The integrated form of the Streeter-Phelps linear differential equation for $\frac{dD}{dt}$ defines the dissolved oxygen deficit, D_t, at any time in terms K_1, K_2, L_a, and the initial deficit, D_a, as

$$D_t = \frac{K_1 L_a}{K_2 - K_1} \left(e^{-K_1 t} - e^{-K_2 t} \right) + D_a e^{-K_2 t} \qquad \text{- - - - - (2-3)}$$

In their studies of the Ohio River, Streeter and Phelps also investigated the effect of temperature on the deoxygenation and the reaeration coefficients and formulated appropriate empirical relationships. The basic formulation for the effect of temperatures on the deoxygenation rate coefficient is

$$\frac{K'}{K} = \theta^{(T' - T)} \qquad \text{- - - - - (2-4)}$$

where T' and T are two known temperatures, K' and K are the corresponding values of the deoxygenation coefficient, and θ is the thermal coefficient, a constant for the reaction.

The reaction rate constant for reaeration was determined experimentally for the Ohio River, and was found to vary with flow and depth according to the relationship

$$K_2 = \frac{C V^n}{H} \qquad \text{- - - - - (2-5)}$$

in which

V = Velocity of flow in feet/sec

H = Mean river depth in feet

C = Empirical constant

n = Empirical exponent

The original formulation proposed by Streeter and Phelps has been expanded over the years. However, the dissolved oxygen and BOD formulations, temperature corrections, and reaeration formulation have held true with slight modifications.

In 1939, Fair[2] simplified the use of the oxygen sag relationship developed by Streeter and Phelps. Fair introduced a new constant called the Self-Purification Factor, "f".

$$f = \frac{K_2}{K_1} \qquad \text{------ (2-6)}$$

By setting $\frac{dD}{dt} = 0$ in Equation 2-1, one then obtains

$$L = \frac{K_2}{K_1} D \qquad \text{------ (2-7)}$$

By solving for the critical deficit, D_c, and incorporating the "f" factor

$$D_c = L_a (f)^{\frac{f}{1-f}} \qquad \text{------ (2-8)}$$

where D_c is the maximum dissolved oxygen deficit at the critical deficit point, in mg/l.

To further facilitate the use of the oxygen sag equation, Fair developed a nomogram and a table of "f" values for various types of streams. The use of this equation eliminates the need for determining a separate reaeration coefficient. However, it must be recognized that the formulation is based on the assumptions that the reach is homogeneous. The use of this self-purification factor is quite popular today, primarily for rough evaluation or approximation.

In 1948, Thomas[3] expanded the oxygen sag relationship to include a third term, K_3. The factor K_3 was used to reflect the

composition of the waste and receiving water and the relative quiescence of the stream at the point of interest.

The equation developed by Thomas has the form

$$D_t = \frac{L_a K_1}{K_2-(K_1+K_3)} \left[e^{-(K_1 + K_3)t} - e^{-K_2 t} \right] + D_a e^{-K_2 t} \quad ----- (2\text{-}9)$$

in which K_3 is a constant of proportionality reflecting the composition of the waste and receiving stream, and the quiescence of the stream.

Thomas indicated that the time-average value of K_3 is zero, with fluctuations occurring with changes in temperature, flow, and channel conditions. The range of values of K_3 given by Thomas was -0.36 and +0.36. Thomas also developed a nomogram for the solution of the expanded equation.

Another variation of the oxygen sag equation for establishing oxygen relationships in streams was proposed by Velz[4,5,6,7]. This variation, called the "Oxygen Budget," is an accounting stepwise method for determining oxygen balance.

On the asset side of the accounting ledger, Velz includes dissolved oxygen present in the stream, that provided by reaeration, and that contributed by photosynthetic activity. On the liability side of the ledger, Velz includes exertion of the BOD, demand of sludge deposits, biological extraction, etc. The DO deficit is calculated for a given reach and stream flow by summing the sources and sinks of oxygen and applying proper conversion factors.

In 1961, Thomas[8] simplified the oxygen sag relationship and made it more amenable to stream sampling data. An approximate

solution to the basic linear differential equation proposed by Streeter and Phelps was developed. The equation for the approximate solution or "step-method" of Thomas is

$$D_t = Y_t e^{-K_2 t/2} + e^{-K_2 t} D_a \qquad \text{- - - - - (2-10)}$$

in which Y_t is the oxygen uptake in the reach, mg/l or #/day, and all other terms are as previously defined. The principal advantage of the step method is that only 5-day BOD analyses need to be determined. The time-consuming deoxygenation coefficient (K_1) and ultimate BOD (L_a) analyses are not required.

Thomas developed nomograms to reduce the number of calculations required to determine the necessary coefficients and constants. Once approximate values of K_1 and K_2 are obtained from statistical analysis by interpolation and extrapolation of observed stream survey data, the equation can then be used to determine the oxygen balance in the stream at different stream conditions, loadings or temperatures.

In order to more realistically represent the BOD reaction in the stream, Thomas proposed using a second-order formulation rather than the first-order formulation of Streeter and Phelps. This can easily be incorporated into the step method and the nomograms are also available for its solution.

The step method as originally proposed by Thomas does not include any benthic or photosynthetic components; however, these can be readily added to the equation. The approximate solution is accurate if the change in deficit is kept to less than 3 mg/l within a section.

In considering the stabilization of organic matter in a natural stream, O'Connor[9] distinguishes between factors that affect the

overall removal of oxygen-utilizing material and those that affect the direct oxidation of organic material. The total rate of removal is expressed by the coefficient "K_r", while the rate of oxidation is described by the coefficient "K_d". The coefficients "K_d" and "K_r" may be quite different from that determined in the standard laboratory test "K_1". This is primarily due to the difference in physical and chemical characteristics of the laboratory and stream environments.

The relationship for the oxygen balance in the stream as proposed by O'Connor is

$$D_t = \frac{K_d L_a}{K_2 - K_r} \left[e^{-K_r t} - e^{-K_2 t} \right] + D_a e^{-K_2 t} \quad ---- (2\text{-}11)$$

The above equation, which is similar to the formulation developed by Thomas, also provides for factors like biological extraction and sludge deposits as proposed by Velz.

A time-dependent dissolved oxygen sag relationship, developed by Li [10], incorporates non-uniform stream channel cross sections and temporal variations of BOD and DO in the wastewaters. Li obtained the following equation for the oxygen sag

$$D_t = e^{-K_2 t} \left[C_s - F(\xi) + f(\xi) \int \frac{1}{v} K_1 e^{(K_2 - K_1)t} \, dx \right] \quad ---- (2\text{-}12)$$

where $t(x) = \int \frac{dx}{v}$ is the time of travel from outfall to point x

V = Velocity of stream flow

C = Saturation value of DO

$F(\xi)$ = DO of stream at waste outfall

$f(\xi)$ = BOD of stream at waste outfall

ξ = Variable time element indicating the net effect of time of day and travel time downstream.

overall removal of oxygen-utilizing material and those that affect the direct oxidation of organic material. The total rate of removal is expressed by the coefficient "K_r", while the rate of oxidation is described by the coefficient "K_d". The coefficients "K_d" and "K_r" may be quite different from that determined in the standard laboratory BOD test "K_1". This is primarily due to the difference in physical and chemical characteristics of the laboratory and stream environments.

The relationship for the oxygen balance in the stream as proposed by O'Connor is

$$D_t = \frac{K_d L_a}{K_2 - K_r} \left[e^{-K_r t} - e^{-K_2 t} \right] + D_a e^{-K_2 t} \quad ---- (2\text{-}11)$$

The above equation, which is similar to the formulation developed by Thomas, also provides for factors like biological extraction and sludge deposits as proposed by Velz.

A time-dependent dissolved oxygen sag relationship, developed by Li[10], incorporates non-uniform stream channel cross sections and temporal variations of BOD and DO in the wastewaters. Li obtained the following equation for the oxygen sag

$$D_t = e^{-K_2 t} \left[C_s - F(\xi) + f(\xi) \int \frac{1}{v} K_1 e^{(K_2 - K_1)t} \, dx \right] \quad ---- (2\text{-}12)$$

Where $t(x) = \int \frac{dx}{v}$ is the time of travel from outfall to point x

V = Velocity of stream flow

C = Saturation value of DO

$F(\xi)$ = DO of stream at waste outfall

$f(\xi)$ = BOD of stream at waste outfall

ξ = Variable time element indicating the net effect of time of day and travel time downstream.

The non-uniform channel aspect investigated by Li is important. However, modern high-speed computers permit the incorporation of many small streams in the basic formulation, thus reducing the significance of the non-uniform channel solution given by Li. The temporal fluctuations in an effluent can be significant in some areas, especially for industrial waste discharges.

Frankel[11,12] expanded the formulations of Li and developed a dynamic oxygen sag analysis which also incorporates photosynthetic effects. A coliform and detergent model was also advanced by Frankel.

Expanded BOD and DO formulations which include the effects of sedimentation, absorption, additions of BOD along the stretch of a stream, removal of oxygen by benthal demand or plant respiration, and the addition of oxygen by photosynthesis have been developed by Dobbins[13,14]. The equation for the dissolved oxygen profile as proposed by Dobbins is given below:

$$D_t = \frac{K_1 \left[L_a - \frac{L_A}{K_1 + K_3}\right] \left[e^{-(K_1 + K_3)t} - e^{-K_2 t}\right]}{K_2 - (K_1 + K_3)} + D_a e^{-K_2 t} + \left[\frac{D_b}{K_2} + \frac{K_1 L_A}{K_2(K_1+K_3)}\right] (1-e^{-K_2 t}) \quad ------ (2\text{-}13)$$

where

D_b = Denotes the net rate of oxygen removal by benthal demand and the effects of plants

L_A = Rate of addition of BOD along the reach

The other terms in the equation are similar to those in the previous equations.

Dobbins also investigated the effect of longitudinal dispersion on BOD and DO profiles, and concluded that the effect is negligible in most fresh-water streams.

In field surveys of the Truckee River, O'Connell, et al.[15,16], measured the effects of benthic algae and other attached plants. A modified form of the oxygen relationship which includes the net oxygen change caused by aquatic plants was used successfully for predicting daily minimum DO concentrations in the Truckee River. The DO concentrations were calculated using the following expression:

$$D_t = \frac{K_1 L_a}{K_2 - K_1} \left(e^{-K_1 t} - e^{-K_2 t}\right) - \frac{(P-R)}{K_2} \left(1 - e^{-K_2 t}\right) + D_a e^{-K_2 t} \quad \text{---(2-14)}$$

where (P-R) is the net contribution (photosynthesis minus respiration) and all other terms are as previously defined.

O'Connell and Thomas concluded that the oxygen produced by the algae and other attached plants generally has little net effect on the oxygen balance of a stream. However, nighttime respiration may add a large oxygen demand.

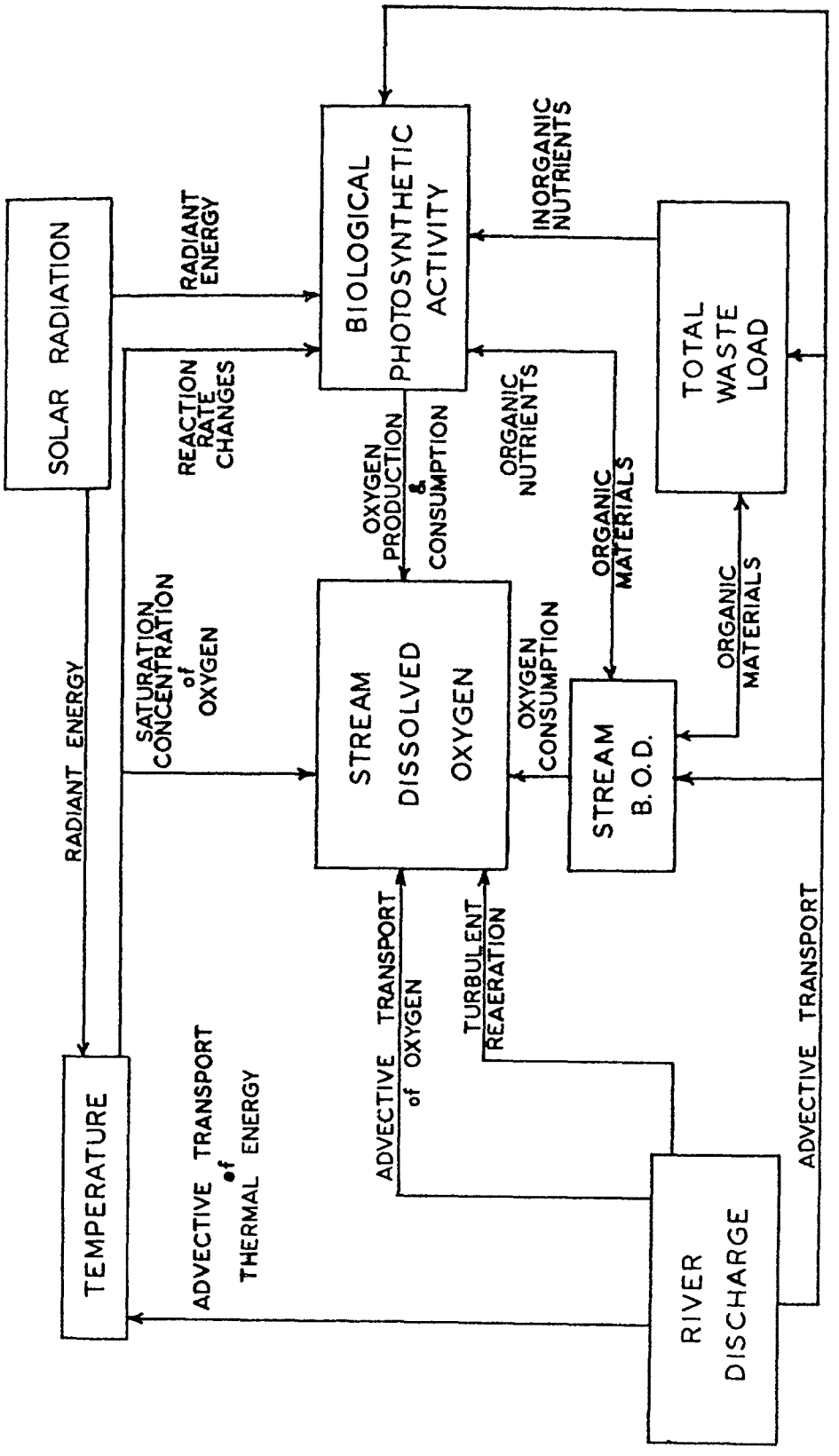
In a study of the Merrimack River, Camp[17] developed an oxygen balance formulation which included the addition of BOD from bottom deposits, removal of BOD by settling, DO production by photosynthesis, and the effects of longitudinal mixing by tides. Camp reported that the amount of DO supplied by photosynthetic activity in the Merrimack River is considerably higher than that by atmospheric reaeration, and that BOD removal by settling is greater than that by oxidation.

A general form which describes the temporal and spatial distribution of either a conservative or a non-conservative substance in a one-dimensional stream has been described by O'Connor[18] as follows:

$$\frac{\partial c}{\partial t} = \frac{1}{A_{(x,t)}} \frac{\partial}{\partial x} (Q_{(x,t)} c) \pm \Sigma^{S}_{(c,x,t)} \quad \text{- - - - - - (2-15)}$$

in which c is the concentration of a substance of interest, A is the cross-sectional area of the stream, Q is the flow rate, and S represents the appropriate sources and sinks. The above formulation accounts for variations in the fresh-water flow and cross-sectional area, various sources and sinks of oxygen, natural and artificial reaeration, the photosynthetic contribution, bacterial and algal respiration, carbonatious and nitrogenous oxidation, and benthic deposits.

Thus, the formulation for oxygen balance in a stream originally proposed by Streeter and Phelps has evolved gradually into the more elaborate relationship given by O'Connor. A schematic summary of factors which affect the dissolved oxygen content of a stream is shown in Figure 1.



FACTORS AFFECTING STREAM DISSOLVED OXYGEN IN A NON-TIDAL SYSTEM

Figure 1

One of the major challenges facing the water resource engineer today is the selection and implementation of a quality algorithm or model which is best suited for a particular application. Judgments have to be made regarding the significance of various parameters in a particular situation. For example, an untreated waste discharge may currently be resulting in deposition of a sludge bed in a particular section of stream; if the waste is treated, will the sludge bed be a significant parameter in the BOD and DO formulations?

Another problem associated with the implementations of a quality algorithm is the lack of systematically collected data. One of the most important contributions of the systematic approach to water resource problems has been to point out the need for well-planned field studies.

MATHEMATICAL MODELS AND METHODS FOR ANALYSIS

This section presents a review of the current literature on mathematical methods and models commonly used in water resources for analysis and control of pollution. For ease of presentation, the developments have been grouped into four categories. These are:

1. descriptive analysis;
2. statistical models;
3. river flow simulation; and
4. basin management models.

Descriptive Analysis

A descriptive analysis provides for mathematical representation of a process or processes by algorithms such as the "oxygen sag" relationship. The analysis is non-statistical and non-optimal seeking.

This type of analysis is used primarily to demonstrate cause and effect relationships of various parameters. A descriptive algorithm may be an integral part of another type of analysis, as discussed later in this chapter.

The computer programming of the "oxygen budget" by Gannon and Down[19] and the dynamic "oxygen sag" model as developed by Frankel[11] are examples of descriptive analysis for non-tidal waters. Thomann's[20] segmented and O'Connor's[21] estuary models are two examples of descriptive analysis for estuarine waters.

Statistical Models

Interpretation of observed data can often be measurably improved by use of statistical models. Graybill[22] suggests a three-step approach to the use of statistical models:

1. selection of a model to represent the physical situation;
2. mathematical treatment of the model to obtain information about the various components; and,
3. use of the model to make decisions regarding the physical situation.

Various statistical models have been used for predicting water pollution control needs. In 1950, LeBosquet and Isiveglu[23,24] used a simple regression analysis to relate dissolved oxygen deficit to stream flow. This relationship provides a simplified approach to calculation of permissible BOD loadings for a given reach of stream.

In 1956, Churchill and Buckingham[25] presented a multiple-regression method for determining BOD loadings. In this analysis, dissolved oxygen was related to the BOD in the stream, the stream temperature, and the stream flow. The Churchill-Buckingham method was

used by the Virginia State Water Control Board for estimation of the assimilation capacity of the Chickahominy River[26]. Okum, et al.[27], also reported the use of multiple-regression techniques for analysis of non-tidal waters. Wolman and Geyer[28], and more recently Durum and Langbein[29], have attempted regressive analysis in the Potomac Estuary.

Because regression models are not valid outside of the range of the original data, their use for flow regulation for a system of multiple-reservoirs is rather limited. Further, after analyzing various regression equations used for determining pollution control needs, Thomas[8] has suggested that this technique is more suitable for data reduction for use in the Streeter and Phelps formulation than it is for development of a predictive model.

Thomas[30] developed a queueing model employing probability theory and finite Markov chains for pollution transport in streams. The object of the model was to predict the quantity and quality of water at given control points for any pattern of flow, temperature, solar radiation, channel configuration, dam location, and pattern or type of pollution. The model includes a stochastic formulation of hydrology with lag-one serial correlation between flows; pollutants with both lag-one serial correlation and a decay mechanism. The model is limited to a single set of stream reaches. Although many of the parameters for self-purification in the oxygen budget are not included, the model represents the first attempt to incorporate two interdependent stochastic parameters, stream flow and wastewater.

Loucks[31,32] developed a mathematical model to predict the probability distribution of the minimum dissolved oxygen concentration

which occurs downstream from any wastewater treatment facility. The model has utility for determination of stream standards and treatment requirements that maximize net benefits, for a given economic benefit loss function.

Wastler[33] studied interactions of tides, solar radiation, river flow, and waste loads in the Potomac Estuary with the aid of spectral analysis, including the computation of cross-spectra. The cross-spectra were used to gain quantitative information on the response of a change in DO as a function of a change in BOD. Spectral and cross-spectrum analysis was used very successfully as a statistical tool in the Potomac River and more recently in the Delaware River Estuarine Studies[34].

River Flow Simulation

The use of stream flow simulation to evaluate a given wastewater treatment policy or for design and operation of a system of dams in water resources management, makes maximum use of information available from a given hydrologic record.

A model for synthetically extending a given historical, hydrological record has been advanced largely through the efforts of Thomas and Fiering[37,48]. The model, a lag-one Markov process, may be represented by the equation

$$X_{i+1} = \mu + \beta (X_i - \mu) + t_{i+1} \sigma(1 - \rho^2)^{\frac{1}{2}} \qquad - - - - - (2\text{-}16)$$

in which

X_{i+1} = The flow in the i+1 interval, a linear function of X_i

X_i = The flow in the "i"th interval

t_{i+1} = The standardized random deviate

μ = Population mean

σ = Population standard deviation

β = Regression coefficient for the values of flows in the i+1 and "i"th interval

ρ = Correlation coefficient between flows in successive time periods.

The Markov model as given above was expanded to simulate the stream flows in an entire river basin by Fiering[37]. The model uses principal components to maintain first and second moments, serial, and spatial correlation for all stream gaging stations in the basin.

Matalas[39] has proposed a more sophisticated simulation model which maintains the third moment, and spatial and serial correlation more effectively. Other developments in flow generation have been provided by Chow[40], Beard[41], Crawford[42], and ABT Associates[43], and, more recently, on the "regional" basis by the United States Geological Survey[44].

Simulation is currently being used quite extensively in the United States in water resource planning and evaluation. The Harvard Water Resources Group[36] has designed a simulation model for the Lehigh River to aid in identification of the particular combination of flow-regulating structures, treatment plants, and other hydraulic works which most nearly achieves the objectives of the Delaware Comprehensive Plan.

Simulation was also used by the United States Corps of Engineers and lately by Davis of Resources for the Future[45] in the study of the proposed reservoir system in the Potomac River Basin. Stream flow simulation is also currently being used by the Chesapeake Field Station

of the Federal Water Pollution Control Administration for evaluating water quality needs in the James, Potomac, and Patuxent River Basins.

River Basin Management

Management models are algorithms which may incorporate some of the techniques mentioned above, plus some type of decision-making mechanism. Most of the past river basin developments in water resources have been primarily devoted to water quantity, and not directly towards water quality management. In water pollution control activities there have been two general types of decision models: (1) wastewater-treatment requirement models, and (2) flow-regulation models. Included in the ensuing review of these two types of models is a case study of the Potomac Estuary in which three alternatives have been considered.

A. Flow Regulation Models

Thomas and Watermeyer[46], and more recently Young[47], have summarized the research on that aspect of flow regulation relating to the probabilistic characteristics of storage systems and dynamic decision making. Thomas describes the flow regulation problem as one of choosing:

1. the optimal operative policies;
2. the optimal level of development; and,
3. the optimal reservoir capacities.

The optimal conditions are defined in economic terms as the expected net benefits of the reservoir system.

Dorfman[48] has studied the relationships between design and operation decisions. A mathematical model for analyzing a hypothetical system involving two dams for the development of an irrigation and hydroelectric project has been proposed. Dorfman obtained an optimal design deduced from operating considerations.

Thomas and Watermeyer[46], utilizing linear programming, proposed a model for the reservoir operation problem. The model optimizes the operating policy and the levels of development and, by sampling, the reservoir capacity. It has the further advantage over the Dorfman model that the stream flow is treated as a stochastic variable. The model, limited to a single reservoir, has no mechanism for incorporating quality considerations.

Hall[49] has applied dynamic programming for allocating water for various purposes such as hydroelectric power generation, consumptive use, and water quality control. The model proposed by Hall is limited to the economic considerations involved in the design of a single reservoir. However, the application of dynamic programming for designing multiple-purpose water projects was shown.

In order to evaluate the need for, and the value of, storage for quality control according to United States Public Law 660, Worley[50] has developed a general river basin model capable of determining and adjusting flows necessary to maintain a minimum allowable dissolved oxygen concentration in a stream. The model was not an optimum-seeking model; that is, there was no mechanism incorporated that would provide for the best operation of a reservoir system.

The model does have the advantage that reservoir quality is taken into consideration. Another advantage of the Worley model is that for a given flow rate, the effect of the physical parameters of the basin is incorporated into the quality determinations. There are no economic considerations in the model.

Young[47] has developed two Monte Carlo techniques for finding optimal annual operating policies for single reservoirs such that

the economic loss as a function of draft rate is minimized. Hydrologic simulation and a forward-looking dynamic programming algorithm is used in the solution. The techniques are limited to three reservoirs. No consideration is given to the water quality in the system.

B. Treatment Requirements

Various methods for formulation of water pollution control policies with regard to treatment requirements have been developed. In recent years, considerable effort has been directed toward the problems involved in finding the minimum cost of waste treatment to meet a set of stream quality standards. Thomann and Sobel[51] have presented some of the first formulations of this problem in their studies of the Delaware Estuary.

Deininger[52] has investigated alternative means for determining waste treatment policies. Least-cost linear programming, chance-constrained, and integer-programs models were formulated to determine optimal solutions.

Sobel[53] has also proposed a linear programming solution, and a mixed integer formulation for the water quality improvement problem. The maximization of the benefit-cost ratio in water resources planning has been transformed into a linear programming problem by Sobel.

Thomann[54] has utilized the results of the steady-state segmented estuary model and a linear programming formulation to determine a least-cost water pollution control policy for the Delaware Estuary.

In order to overcome non-linear cost functions, Kerri[55] has used a dissolved oxygen cost matrix to transform the least-cost formulation into a linear programming problem. Liebman[56] has used dynamic

ogramming for determining waste treatment requirements for problems volving non-linear cost functions.

While the least-cost solutions are of great interest in water resources planning and management, there are some inherent technical fficulties in the use of this technique:

1. Except for some industrial waste treatment facilities, there is a poor relationship between effectiveness of treatment and cost of the waste treatment facility.
2. It is difficult to estimate with any accuracy the cost of a proposed wastewater treatment plant.
3. The least-cost solutions are sensitive to physical parameters of the stream and to DO constraints.

r example, in the Delaware Estuary, Thomann[57] reported that, for ven flow conditions, if the reaeration coefficient was overestimated 100 percent the least-cost solution increased about 350 percent. e increase, although contrary to engineering judgment, was easily plained by Thomann. The allocation of treatment cost was also anged quite drastically. A significant change in treatment cost alcation was also reported by Liebman[56] in his studies of the lamette River when the DO constraint was changed by 0.1 mg/l.

In the opinion of the author, the least-cost solutions have indited two things:

1. the great sensitivity of the solution to the physical parameters in the stream; and
2. the ability to adequately describe and predict the water quality in a stream by the descriptive formulations is greatly exceeded by the accuracy maintained in the optimization processes.

C. Case Study, The Potomac Estuary

The water quality problems in the Potomac River Basin are unique in that the quality in the estuary is not greatly affected by upstream wastewater discharges. This independence, a result of the geological "fall-line" above Washington, D.C., simplifies the quality investigation for the estuary.

A large municipal wastewater load from the Washington, D.C., area is causing a serious quality problem in the estuary. The deterioration is not only due to low dissolved oxygen but also to high nutrient levels, resulting in large algal populations.

Three methods for alleviating water quality problems in the estuary are as follows:

1. a high degree of wastewater treatment including nutrient removal;
2. flow regulation from upstream reservoirs to provide dilution of wastewater; and,
3. diversion of the wastewater farther down the estuary.

In studying the various alternatives, Hetling[58], using the segmented estuary model, investigated all possible solutions and combinations of solutions. By ranking the alternatives, least-cost solutions were obtained to meet the various levels of water quality constraints.

Davis[59,60], using a sampling strategy consisting of systematically sampling the trade-off relationships among the various alternatives, also developed a method for cost minimization in the Potomac Estuary. In later studies, Davis used river flow simulation to evaluate the various alternatives.

The Potomac Estuary is an ideal case study in that there are three alternative solutions, two basic quality considerations, cost data available, and the quality for a given condition can be predicted by the segmented mathematical model.

CHAPTER III

FLOW RELEASE MODEL

THE PROBLEM

The problem pursued in this study is as follows: given a basin with a developed and/or proposed reservoir system, develop a method for formulating reservoir operating policies for quality control in some optimal manner. An optimal release sequence for the initial phase of this study is defined as providing the "best" quality of water for a given flow rate at a point in the system while maintaining a minimum preset dissolved oxygen level in all reaches of the basin above this point. The "best" quality of water is further defined as the minimum DO deficit for a given level of BOD. The method for determining the optimal release sequence should also incorporate:

1. Physical and biochemical parameters of the stream for a given section:
   a. Flow
   b. Temperature
   c. Velocity
   d. Depth
   e. Biological activity
2. Wastewater discharge parameters:
   a. Rate of discharge
   b. Concentration and characteristics of the pollutants in the wastewater
   c. Treatment policy

3. Regulated and unregulated stream flow parameters:
   a. Reservoir quality
   b. Change in quality with a change in flow
   c. Stochastic nature of the stream flow
4. Economic considerations
5. The necessary formulation to investigate various parameters which may alter spatial release patterns:
   a. Changes in reservoir quantity
   b. Changes in quality constraints
   c. Physical and biochemical parameters of the stream channel
6. A mechanism for intermodel linkage to other formulations such as the segmented estuary model or simulation
7. Facility of adaptation to computer programming.

The problem as presented above is an expansion of Worley's model to include optimal operation and cost. In contrast to Young's approach, the solutions sought are in physical space and not in time. The study approach is operationally oriented, that is, primary emphasis is placed on investigating parameters that affect the solution rather than on how the solution affects socio-economic considerations.

The study is limited to steady-state conditions and a deterministic flow system. That is, the final solution is for a given season and a prescribed flow range. A mechanism for overcoming the possible dependence on flow range is given in Chapter VI.

GENERAL APPROACH

Various methods for solving the proposed problem have been investigated, such as linear programming, application of influence lines, and dynamic programming. Due to the nature of the stream flow network in the basin, the use of dynamic programming to obtain an optimal solution was found to be an ideal approach. The general approach to the flow regulation problem has been divided into four phases:

1. the adaption of existing formulations to describe the appropriate water quality parameters in the river basin;
2. development of a paragon which describes the physical basin and is readily adaptable to computer programming;
3. utilization of the dynamic programming optimization technique; and,
4. overall algorithm formulation and computer programming of the model.

QUALITY FORMULATIONS

Quality considerations in the formulation have been limited to temperature, DO, and BOD, with the main parameter being DO. As illustrated in Figure 1, there are many factors which influence dissolved oxygen in a river. Control of flow directly affects advective transport of oxygen and reaeration. Temperature, advective transport of waste loads, and biological activity are indirectly affected by change in flow.

In order to make the problem more manageable initially, only the direct effects have been included in the study. However, provisions are made in the formulations to add the indirect effects at a later time.

The basic water quality algorithm which has been used in the present work is Thomas's modification[8] of the Streeter-Phelps equation. The integrated form of the equation may be written as

$$D(1) = Y * P + D(0) * P^2 \qquad \text{- - - - - (3-1)}$$

where

$Y = L_o (1 - e^{-K_1 t})$, oxygen sink

$P = e^{-(K_2 t/2)}$, reaeration parameter

D(1) = Dissolved oxygen deficit at lower node

D(0) = Dissolved oxygen deficit at upper node

The advantages of this formulation over others are:

1. the formulation can easily incorporate other than first order reactions;
2. solutions for the integrated forms exist for all ranges of reaeration and deaeration coefficients; and
3. if the oxygen deficit is kept to less than three mg/l indirect effects can be readily added or subtracted to the oxygen sink (Y) in the reach.

In solving the algorithm, four states are carried forward. These are DO, BOD, temperature, and the deaeration coefficient. The fifth state, which is flow, is used to calculate the reaeration coefficient and the time of travel.

Reaeration

As indicated in Figure 1, there are three main sources of dissolved oxygen. Reaeration is usually the most important factor in the dissolved oxygen budget. Various formulations are available for relating the reaeration coefficient to the depth, slope, velocity, and

longitudinal diffusion in the stream channel. In this study, the reaeration formulations employed have been the non-isotrophic relationship developed by O'Connor[61]:

$$K_2 = \frac{(D_L V)^{1/2}}{H^{3/2}} \qquad \text{- - - - - (3-2)}$$

and the Churchill formulation[62]:

$$K_2 = \frac{5.026 \quad V^{0.969}}{H^{1.673}} \qquad \text{- - - - - (3-3)}$$

where D_L is the coefficient of diffusion of oxygen in water and all other terms are as previously defined.

Other formulations, such as those of Dobbins[13,14], Streeter and Phelps[1], and Krenkel[63], can easily be substituted into the system. The effects of the different formulations for the reaeration coefficient on the verification of the system are discussed in Appendix A.

Temperature

Since the reaeration and deaeration coefficients are temperature dependent it has been necessary to incorporate a mechanism for adding and subtracting various temperature sources and sinks. Various types of stream temperature models, as summarized by Zeller[64], have been investigated. To maintain simplicity, the exponential decay temperature model of Duttweiler[65] has been incorporated into the overall algorithm. The formulation is given below:

$$T_1 = Ts_1 + [Ts_0 - To]\, e^{-\Phi X/V} \qquad \text{- - - - - (3-4)}$$

where

φ = X/H

X = $C_1 + C_2 U_2$

C_1 = 1.35

C_2 = A constant ranging in value from 0.182 to 0.24

U_2 = The velocity of the wind

X = The distance downstream from the heat source

Ts_1 = The steady-state temperature at the downstream end of the stream reach

Ts_0 = The steady-state temperature at the upstream end of the reach

T_1 = The temperature at the downstream end of the reach

To = The temperature at the upstream end of the reach.

The modified version of the Duttweiler model provides the necessary formulation to incorporate heated discharges, varying reservoir temperatures and physical conditions of the stream. The steady-state temperatures are developed from historical traces.

The corrections for the effects of temperature on the deaeration and reaeration coefficients[62,66] are given below:

$$K_t = K_{20} * \theta^{(T-20)} \qquad \text{- - - - -} \quad (3\text{-}5)$$

where

K_t = The coefficient at temperature T

K_{20} = The coefficient at T 20°C

θ = A temperature coefficient

In the formulation, temperature coefficients of 1.0471 and 1.0241 have been used in the deaeration and reaeration corrections, respectively, at selected standard temperature (20°C).

A third order polynomial for determining the solubility of dissolved oxygen in water at a given temperature (T) has been incorporated into the model. The polynomial coefficients have been those developed by the TVA[67], yielding

$$\text{Sat DO} = 14.652 - 0.41022T + 0.00799107T^2 - 0.0000777779T^3 \quad \text{- - - - - (3-6)}$$

where "Sat DO" represents the saturation concentration of dissolved oxygen at temperature (T).

Physical Parameters

In the above dissolved oxygen, reaeration, and temperature formulations, physical parameters of the stream such as width, W, velocity, V, and depth, D, are required for various ranges of flow, Q. Leopold and Maddox[68] have carried out extensive study of the changes in velocity, depth, and width for a change in flow rate at a given stream cross section. The characteristic relationships developed by these investigators for the mean values for these parameters are:

$$D = aQ^b \quad \text{- - - - - (3-7)}$$

$$V = cQ^d \quad \text{- - - - - (3-8)}$$

$$W = eQ^f \quad \text{- - - - - (3-9)}$$

where a,c,e and b,d,f are characteristic coefficients and exponents for correlation with Q. Operational problems involving the use of these formulations are discussed in Appendix A.

Reservoir Quality

To provide for possible BOD, DO, and temperature variations with flow from unregulated and regulated stream sources, a linear relationship has been assumed and incorporated into the flow release model.

Extensive review of stream survey data for the unpolluted, unregulated reaches of the Potomac River Basin revealed that the linear assumption was a fair approximation for DO, BOD, and temperature. However, a recent study by Churchill and Nicholas[83] indicated that the linear assumption for a regulated system was an oversimplification. Therefore, in the developing of the optimal solutions as presented in Chapters V and VI, the water quality of all regulated and unregulated sources was assumed to be constant (a preset level) for all ranges of flow.

DESCRIPTIVE PARAGON

In order to formulate a general flow release model, it has been necessary to develop a descriptive paragon which can be used for data storage and retrieval, which adequately describes the branching of a physical basin and relates it to waste discharge points, impoundments, hydrologic conditions, etc., and which is easy to program on the computer. Various types of descriptive river basin systems were investigated such as the Storet System[69], the method used by Worley[50], and the numbering system of the FWPCA stream flow simulation model[70]. A simplified method similar to that of an arrow-and-line diagram[71] of a critical path network was adapted.

For a given segment of stream the following restrictions are to be maintained:

1. stream flow in segment remains constant;
2. all tributaries, waste discharges, or water intakes are indexed at the upper node of the segment;
3. the relationships among velocity, width, depth, and with flow in the segment remain constant;

4. only one wastewater discharge, tributary, or intake is allowed per node (if two or more above entities are close together, a greater number of shorter segments are established); and,
5. the numerical value of the upper node should be greater than the lower node (not absolutely necessary; however, it simplifies some of the coding and sorting procedures).

To aid in the computation procedures and yet maintain flexibility in data analysis, storage and retrieval, the stream nodes, wastewater discharges, and stream flow additions have been indexed as given in Appendix C. The ability to add or subtract any waste source, stream flow, or add more sections to the system is maintained if the numbering of nodes is in units of 2 or greater.

NODAL OPERATIONS

All mathematical operations are at the upper node of a given reach. For nodes at which there is an addition of flow due to a waste load or confluence with another tributary, a mass balance is made with respect to flow, pounds of BOD, and pounds of dissolved oxygen deficit. The deaeration coefficient is prorated according to the pounds of BOD of the respective contributions. The temperature of the combined system is prorated according to the flow values of the two components.

The mass balance of BOD and the prorating of the deaeration coefficient treats the system linearly and does not take into consideration any antagonistic or synergistic actions or reactions. The treatment of this entire system non-linearly is beyond the scope of this study.

DYNAMIC PROGRAMMING SOLUTION

The solution to the multiple-reservoir release problem is very amenable to dynamic programming technique. The complex problem can readily be decomposed, called staging, into a series of smaller problems. The technique* relies upon decision-making at each stage rather than trying to solve the entire N-stage optimization problem simultaneously.

There are two general approaches to the reservoir problem:

1. starting at mouth of the basin, proceeding upstream, and treating the regulated confluence points as diverging branches; or
2. starting at the uppermost point of the basin, proceeding downstream, and treating the regulated confluence point as converging branches.

Since BOD, DO, deaeration, and temperature are flow dependent, the problem is significantly simplified by the latter approach.

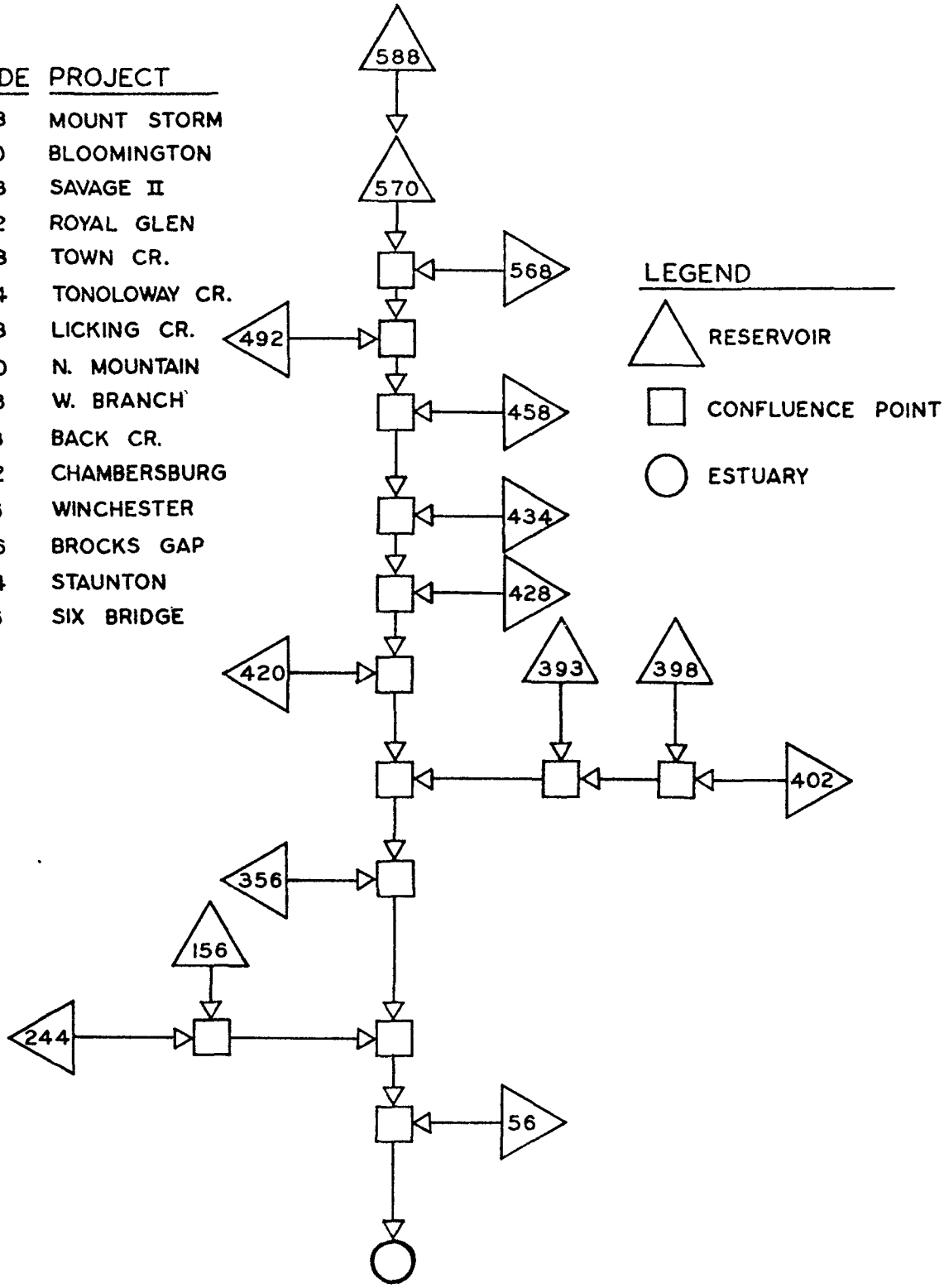
The overall schematic of the proposed reservoir system for the Potomac River Basin shown in Figure 2 typifies this converging structure. The reservoirs (triangles) are the controlling mechanisms, and the regulated confluences (squares) are the decision points or the stages. In the Potomac system, the converging branches structure results in a 13-multistage optimization problem.

A general solution to a multistage converging dynamic programming problem has been structured by Nemhauser[72]. Represented below,

---

*For a complete discussion of the dynamic programming technique, see Nemhauser[72].

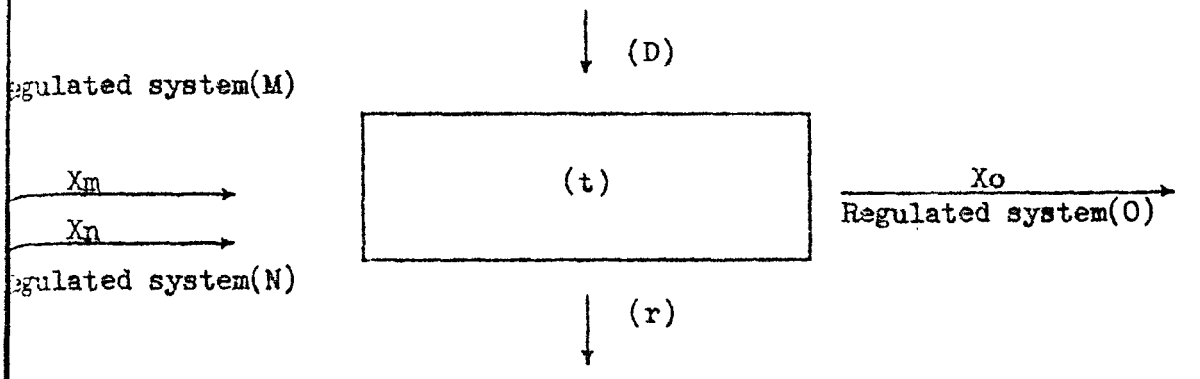
NODE	PROJECT
588	MOUNT STORM
570	BLOOMINGTON
568	SAVAGE II
492	ROYAL GLEN
458	TOWN CR.
434	TONOLOWAY CR.
428	LICKING CR.
420	N. MOUNTAIN
393	W. BRANCH
398	BACK CR.
402	CHAMBERSBURG
356	WINCHESTER
156	BROCKS GAP
244	STAUNTON
56	SIX BRIDGE



A SCHEMATIC REPRESENTATION
OF
PROPOSED RESERVOIR SYSTEM – POTOMAC RIVER BASIN

Figure 2

stage of a regulated river system is characterized by six factors:



1. An input state (Xm), flow from a regulated system (M), the dependent states being BOD, DO, deaeration, and temperature.
2. An input state (Xn), flow from a regulated system (N), the dependent states being BOD, DO, deaeration, and temperature.
3. An output state (Xo), flow from the combined regulated systems (M) and (N), the values of the dependent states being a function of a combination of (Xm) and (Xn).
4. A decision variable (D), dictates combination of (Xm) and (Xn) for a given state of (Xo).
5. A stage return (r), for a given (Xo), the best water quality measured in terms of minimum dissolved oxygen deficit (DOD for a given BOD state).
6. A stage transformation (t), couples (Xm) and (Xn) linearly, as for the dependent states described earlier in the section on nodal operations.

The flow release problem is different from the problems structured by Nemhauser in three respects:

1. each stage consists of two input states;

2. flow release optimization is required for all output states in order to provide for regulation for quality control downstream if needed; and
3. the stage return is a dual-valued function.

The latter requires a minimization decision procedure for two dependent states. In the decision process, the object is, for a given flow state, Xo, and within a given BOD grid, to determine that combination of Xn and Xm which will yield the minimum DOD. Combinations of Xm and Xn with greater DOD are deleted.

For a given stage or decision point, the range of Xo is dependent on the regulation capabilities of Xm and Xn. For example, if the regulation capability of Xm is from 100 to 300 cfs and of Xn, from 200 to 500 cfs, the range of Xo would be from 300 (200 + 100) to 800 (300 + 500) cfs. To reduce the infinite number of input and output states, reservoir releases are made in discrete increments. In the above example, if a 20 cfs increase is employed, there would be 26 states (800/20 - 300/20 + 1) of Xo. The number of feasible solutions per Xo is mainly a function of the BOD grid size.

The number of combinations of Xm and Xn which will equal a given Xo state is dependent on the value of Xo and the number of feasible solutions per states of Xm and Xn. The maximum number of combinations will be near the mean of the Xo range with a decreasing number towards the extremes of the range.

For each state of Xo, starting with the minimum and progressing to the maximum regulation capability, the optimization algorithm, which is an efficient enumeration process, consists of the following steps:

1. calculate all dependent variables for all combinations of Xm and Xn which equal the given Xo state;
2. sort and rank upward all feasible solutions according to value of BOD parameter;
3. determine BOD grid size by subtracting minimum BOD value from the maximum, comparing BOD difference to various input grids, and selecting proper BOD increment size;
4. for the given Xo and BOD values within the first increment range, select the combination of Xm and Xn which has the minimum DOD;
5. increment to the next BOD state and repeat step 4 until all feasible solutions are exhausted.

With the converging branches approach the flow regulation needs of reaches downstream from the decision point under investigation are unknown. Therefore, for a given stage, a range of feasible solutions for all states of Xo must be carried downstream.

In the initial development of the flow release model, the dual-value return function is not cumulative. The optimization process for all output states for a given stage is expressed mathematically as follows:

$$F\,(Xo(k,p)) = \text{Min}\,[DOD(k,p) \;\big|\; BOD(k,p) \;\big|\; Xo(k,p) = Xm(i) + Xn(j)] \qquad \text{- - - - - (3-10)}$$

where

$F\,(Xo(k,p))$ = Optimal operation or return measured by the minimum DOD for a given BOD and flow state

Xo(k,p) = States of output Xo, k = 1,2, ... (M+N)

p = 1,2, ... P

Xm(i) = States of input Xm, i = 1,2, ... M

Xn(j) = States of input Xn, j = 1,2, ... N

as measured in

P = Maximum number of increments in BOD grid

M = Maximum number of input states for Xm

N = Maximum number of input states for Xn

BOD(k,p) = State of the first dependent variable for a given Xo state and DOD value

DOD(k,p) = State of the second dependent variable for a given Xo and BOD state.

This enumeration process makes maximum use of the natural assimilation capacity of the stream, and yet maintains the principle of optimality. If an accumulation stage return is substituted for the DOD parameter, such as a total cost parameter, the optimization procedure can easily be modified to include a minimum of cost; this is presented in Chapter VII.

OVERALL ALGORITHM AND COMPUTER PROGRAMMING

The quality and descriptive formulations adopted in the flow release model are general in nature and applicable to most river basins. The following assumptions are made in the overall formulation:

1. there is complete lateral mixing in the stream;
2. all flows are steady-state with no longitudinal diffusion;
3. the wastewater discharges are uniform in quality and quantity for a given time period;

4. all major impoundments are in the headwaters of the basin. If two or more are in series they are treated as one unit; and
5. the quality formulation as presented earlier adequately described the process in the stream.

The first three assumptions could possibly be eliminated if temporal aspects were incorporated into the model. However, other technical limitations make these refinements impractical at the present time.

In the overall computer program for the flow release model, the description paragon mathematically links the water quality formulations, multistaging process and optimization procedure to the stream network of the river basin. The paragon also selects stream flow routing patterns and sequences the multistaging process in the dynamic programming technique.

The basic computational steps of the flow release model are given below.

1. Read in and display input data.
2. Determine minimum and maximum stream flow ranges for each node in the basin. Required in the stream flow routing and multistaging process.
3. Route the unregulated stream flows to regulated sections of the basin.
4. Starting at uppermost decision point, route the stream flows and develop optimal solutions for each stage of the converging branch systems. (In Chapter V, a more detailed description of the routing, staging, and optimization process is presented along with an example problem for the upper Potomac Basin.)

5. Print out all flow routing data and optimal solutions for each stage.

A simplified flow chart* of the release model is given in Figure 3. The formulations have been programmed in FORTRAN IV, Subset E. Complete listings of the variables and computer programs are given in Appendices C, D, and E.

The input data formats are flexible to allow for various input options depending on refinements needed in the calculations (see Appendix C for data input formats and a compilation of data decks).

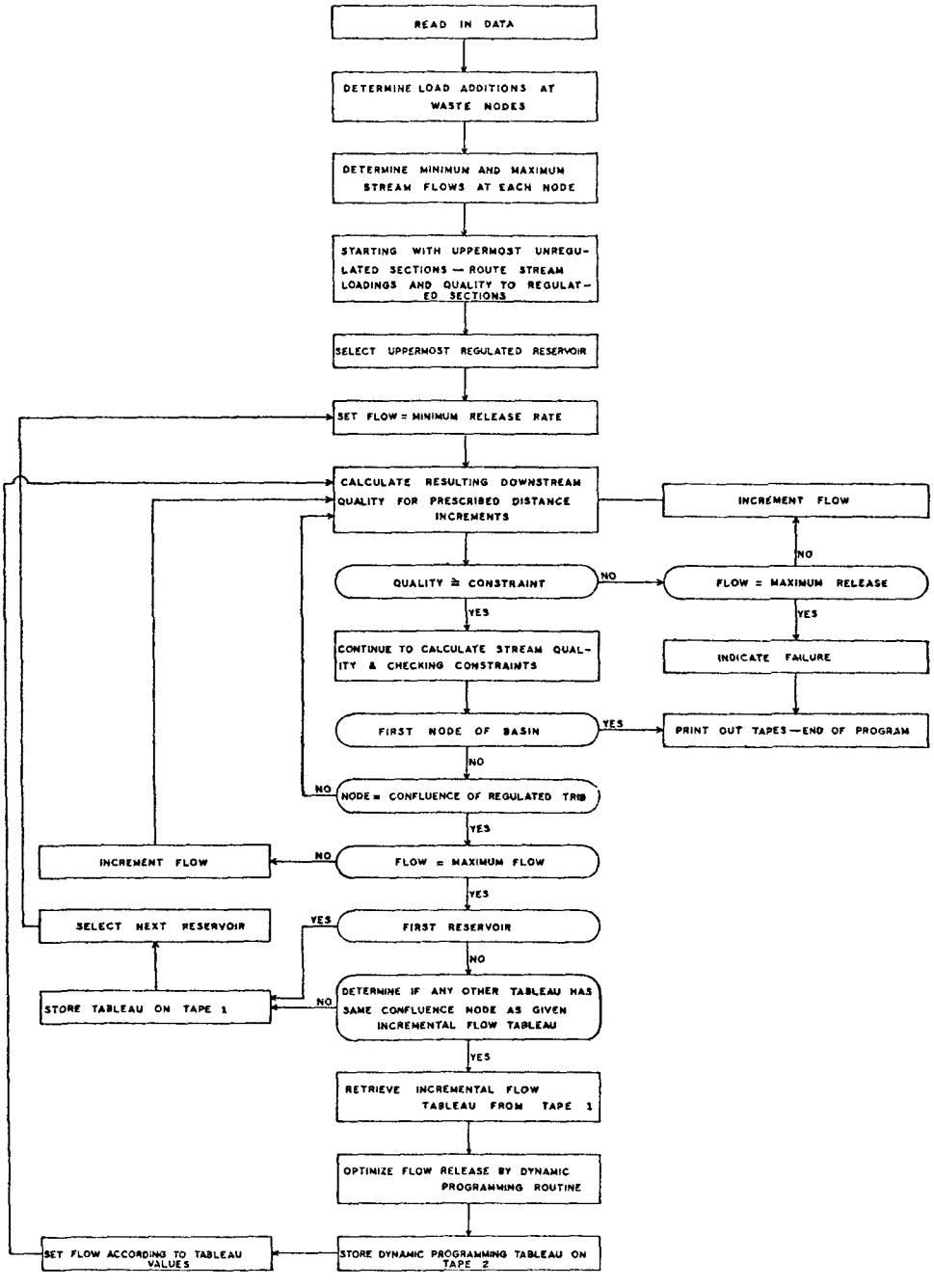
The output of the computer program is a series of incremental flow and dynamic programming tableaux. The incremental flow tableaux contain all pertinent information required for the optimization process, while the dynamic programming tableaux consist of the results of the process for the given decision point. The interpretation of the tableaux, development of optimal reservoir release sequence, and analysis of results are presented in Chapters V and VI.

A computer program employing the same formulations as in the flow release model was written to aid in verifying the concepts of the model. The program is also used to tabulate all input data, aid in coefficient modification, and display of specific quality profiles. A listing of the program is given in Appendix E.

---

*Since flow charting capabilities via the computer are now standard subroutines at most computing centers, a detailed flow can be readily obtained and therefore was not included in the presentation.

SIMPLIFIED FLOW CHART FOR FLOW RELEASE MODEL



Figure

CHAPTER IV

PATUXENT AND POTOMAC RIVER BASIN SYSTEMS

Most of the emphases in the river basin models reviewed in Chapter II have been focused on the consequence of a feasible solution. Little attention has been given to physical and biochemical parameters that dictated a given solution. The lack of good field data has been the major factor limiting a comprehensive sensitivity analysis of the formulations.

To fully evaluate the flow release model and to analyze some of the controlling parameters, the formulations have been applied to two river basins in the present study, the Patuxent and the Potomac, in the Middle Atlantic Region. A brief description of the two river basin systems is presented in this chapter.

THE PATUXENT SYSTEM

The Patuxent River Basin, which has two existing reservoirs in series and a drainage area of 930 square miles, has been used for a pilot study in the testing of the quality formulations and developing methods for data analyses. There are six major sources of wastewater and two water intakes in the non-tidal portion of the basin. See Appendix C for a further description, including a schematic of the basin.

Although small in scope, the pilot study of the Patuxent River revealed the following:

1. large amounts of data are required to describe the physical and biological systems of a river basin;

2. a systematic method of data analysis and reduction is needed to implement the flow release model; and
3. a procedure for the adjustment of the various coefficients is required for model verification.

See Appendix A for a detailed description of a procedure for verification of the quality formulations.

THE POTOMAC SYSTEM

The final testing and analyses of the flow release model were done on the non-tidal portion of the Potomac River Basin, which has 15 proposed reservoirs and a drainage area of over 10,000 square miles.

The Potomac River, being located in the political center of our nation, has been studied almost continuously by various agencies since the early 1940's. The availability of data, proposed reservoir development, and current water quality problems have been decided assets in testing the formulation in the flow release model.

Source of Data

The data required for the field testing have been obtained from numerous sources, including:

1. Chesapeake Field Station, FWPCA, U. S. Department of the Interior;
2. Department of Water Resources, State of Maryland;
3. Department of Health, State of Maryland;
4. State Water Control Board, Commonwealth of Virginia;

5. Division of Sanitary Engineering, Commonwealth of Pennsylvania;
6. Division of Water Resources, State of West Virginia;
7. U. S. Geological Survey; and
8. U. S. Army Corps of Engineers.

Data also have been extracted from various technical and non-technical reports on many diverse topics concerning the Potomac Basin. Of the approximately 450 reports written concerning the Potomac, the major sources of data are references [73] through [79].

Description of Basin

The non-tidal portion of the Potomac River Basin, which contains most of the typical water quality problems, provided diverse conditions for testing the flow release model. Some of the pertinent data describing the basin is given in Table 1.

Table 1

INVENTORY OF THE POTOMAC RIVER BASIN

Entity	Quantity
Square Miles of Drainage Area	11,500
1960 Population - (Excluding Washington, D.C.)	1,000,000
Stream Miles - (Above River Mile 116.0)	2,750*
Wastewater Discharges to Streams	173
Surface Water Intakes	74

*Does not include minor tributaries

For the model development and evaluation phase of this study, only the stream reaches receiving significant wastewater discharges or those subject to flow regulation have been incorporated into the system. (See Table 2.) Included in the wastewater inventory are all discharges with a flow equal to or greater than 0.5 mgd, or a population equivalent equal to or greater than 1,000.

Table 2

INVENTORY OF FLOW RELEASE MODEL
FOR THE POTOMAC RIVER BASIN

Entity	Quantity
Wastewater Discharges	
Organic	64
Thermal	8
Stream Flow Addition Points	
Regulated	14
Unregulated	11
Increments	56
Surface Water Supplies	26
Stream Segments	307
Stream Miles	894
Existing Impoundments	2
Proposed Impoundments	14

The wastewater discharges included in the model represent over 90 percent of the total BOD load to the stream, and all major water

supply intakes in the non-tidal portion of the Potomac Basin have been included.

Figure 4 is a general map of the Potomac River Basin, including the proposed reservoir system. In Table 3 are presented data on 14 proposed impoundments for the Potomac system. Appendix B contains detailed schematics of the individual stream reaches, showing water intakes, wastewater discharges, low level dams, gaging stations, stream segments, and reservoirs. Basin data used in the flow release model are also presented in Appendix B.

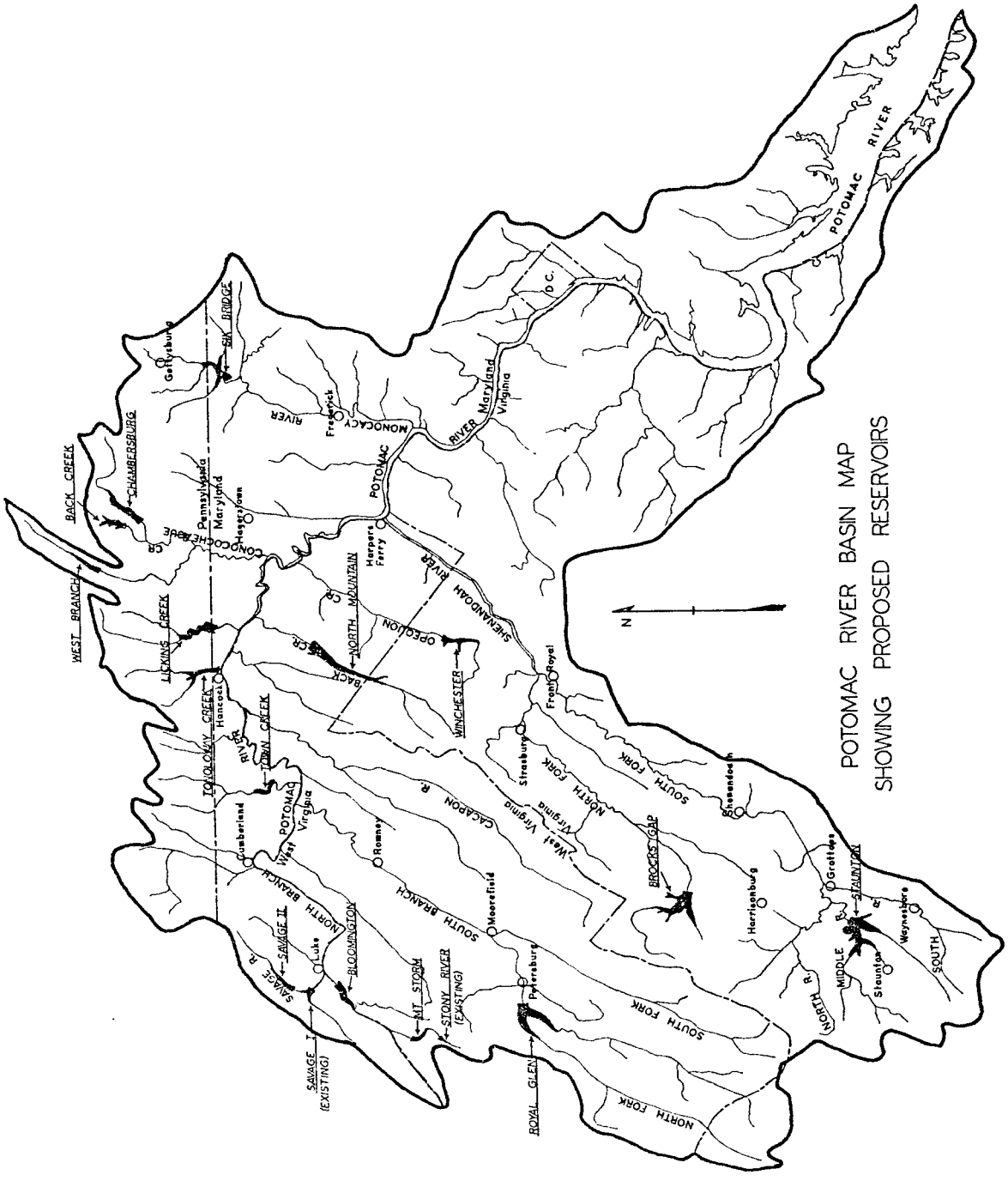

POTOMAC RIVER BASIN MAP
SHOWING PROPOSED RESERVOIRS
N
WEST BRANCH
BACK CREEK
CHAMBERSBURG
Gettysburg
SIX BRIDGE
Pennsylvania
Maryland
Hagerstown
CONOCOCHEAGUE CR
LICKING CREEK
Hancock
IOKOLOMOY CREEK
TOWN CREEK
RIVER
POTOMAC
Cumberland
West Virginia
NORTH BRANCH
SAVAGE R
SAVAGE II
Luke
BLOOMINGTON
SAVAGE I (EXISTING)
Romney
MT STORM
STONY RIVER (EXISTING)
Moorefield
SOUTH BRANCH
CACAPON R
Petersburg
ROYAL GLEN
Harpers Ferry
BACK CR
NORTH MOUNTAIN
OPEQUON CR
WINCHESTER
SHENANDOAH RIVER
Front Royal
Strasburg
Frederick
MONOCACY RIVER
POTOMAC RIVER
Maryland
Virginia
D.C.
NORTH FORK
SOUTH FORK
West Virginia
Virginia
BROCKS GAP
Harrisonburg
Grottoes
STAUNTON
Staunton
Waynesboro
NORTH R.
MIDDLE R.
SOUTH
Rosendale

Figure 4

Table 3

RESERVOIR DATA

Potomac River Basin

Name	Node	Total Storage (Acre-Ft)	Storage for Water Quality Control (Acre-Ft)	Total Cost ($1000)	Water Quality Control Cost ($1000)	Average Stream Flow (cfs)	Increase in Dependable Flow (cfs)
Bloomington	570	137500	92000	50865	22703	476	212
Royal Glen	492	338000	228000	30953	15744	684	484
Chambersburg	402	42800	28000	12595	9468	163	63
Staunton	244	143000	104000	24363	16485	276	163
Six Bridge	56	103000	65000	14601	6487	350	104
W. Branch	393	77500	45000	11893	5717	90	65
Brocks Gap	156	187000	120000	16641	9477	185	147
Winchester	356	77000	28300	11132	5631	77	45
Licking Creek	428	120300	82100	11528	5065	166	138
Town Creek	458	96800	57500	10009	3066	150	97
N. Mountain	420	195000	95500	16655	4655	183	137
Savage	566	50000	39000	13233	4018	73	48
Back Creek	398	46900	19700	7587	3711	72	28
Tonoloway Creek	434	88000	50000	14086	4461	118	84

CHAPTER V

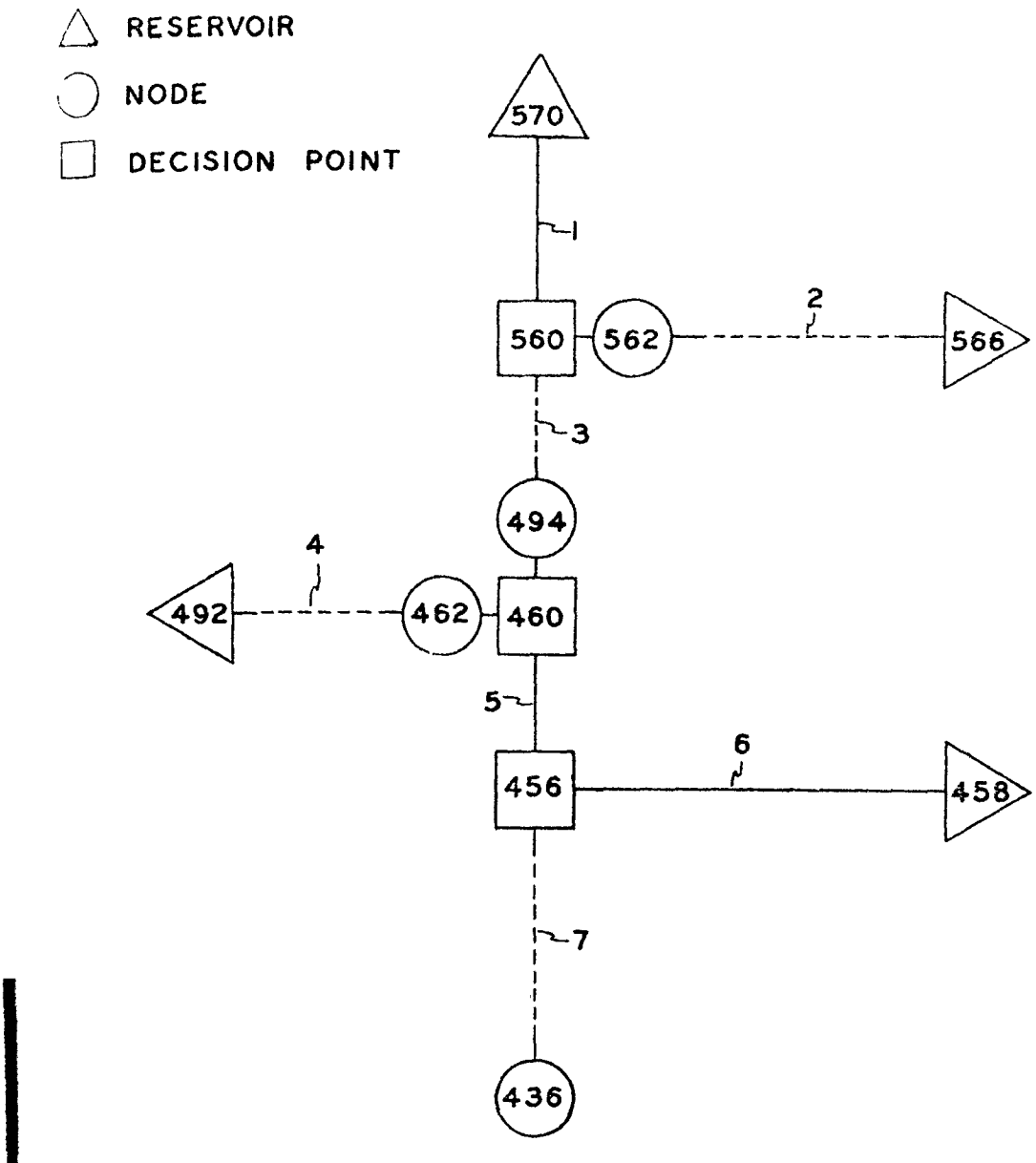
DEVELOPMENT AND INTERPRETATION OF TABLEAUX AND THE FORMATION OF RESERVOIR RELEASE PATTERNS

The most current data available have been used for testing the various quality algorithms in the computer runs of the flow release model for the Potomac River Basin. Although it has not been the main purpose of this study to obtain a completely verified model of the entire Potomac Basin, a great deal of effort, especially in the North Branch sub-basin, has been spent in data analysis so that the development, interpretation, and sensitivity analysis reported are a true indication of natural stream conditions.

The first part of this chapter is devoted to the development and interpretation of the Incremental Flow Tableaux (IFT) and the resulting Dynamic Programming Tableaux (DPT). In the latter part of this chapter, the formation of the release sequences for water quality control from multiple-reservoir systems from the tableaux is presented.

DEVELOPMENT AND INTERPRETATION OF TABLEAUX

For ease of presentation, the development and interpretation of the tableaux are limited to the upper portion of Potomac River Basin. (See Figure 5.) The entire watershed is included in the formation of general release patterns for various flow requirements at the estuary. Appendix B includes detailed schematics of the various sub-basins and a listing of the input data.



SCHEMATIC OF
UPPER POTOMAC RIVER BASIN
FLOW RELEASE MODEL

Figure 5

Development of Tableaux

A simplified flow chart of the entire model including the development of the tableaux within the general basin paragon is presented in Figure 3. An itemized description of the basic computational steps of the model specifically relating the formation of the various tableaux to the paragon of the upper portion of the Potomac River Basin is given below. The pertinent nodes and reaches are shown in Figure 5.

1. For reach 1 containing the Bloomington Project (node 570), set the reservoir release rate to the minimum discharge value.
2. Route flow from reservoir and resulting water quality to first decision point (node 560) incorporating into the system changes in quality or quantity resulting from any downstream waste loads, unregulated tributaries, water intakes, etc. During the routing process, the water quality is monitored at prescribed increments of distance to determine if the DO constraint is being met. If the constraint is violated, the release rate is increased by a fixed flow increment and the routing process is repeated.
3. Continue routing process from the reservoir by increasing the discharge rate by the fixed flow increment until the maximum release rate from the impoundment is obtained, thereby completing the development of the IFT for reach 1. The tableaux for reach 1 are indexed* by node 570. (See Table 4.)

---

*In the model, the IFT for a given reach is indexed by the first node upstream from the decision point.

4. For reach 2 containing Savage II project (node 566), repeat the operations similar to those for the Bloomington Reservoir in steps 1, 2, and 3. The IFT for this reach is indexed by node 562. (See Table 4.)
5. Utilizing the converging branch system of dynamic programming and the enumeration process, develop for the first decision point from the IFT's of nodes 570 and 562 the optimal flow release sequences from the Bloomington and Savage II Reservoirs. The end product of the enumeration process is a DPT which contains a listing of all feasible solutions for all flow states within the minimum and maximum flow regulation capabilities of reaches 1 and 2. (See Table 8 for a DPT for node 560.)
6. Similar to the operations in step 2, in reach 3 route all feasible solutions of the above decision point to the next downstream decision point (node 460). The IFT for reach 3 is indexed by node 494. (See Table 4.)
7. For reach 4 containing the Royal Glen Project (node 492), repeat the operations similar to those for the Bloomington Reservoir in steps 1, 2, and 3. The IFT for this reach is indexed by node 462. (See Table 5.)
8. As described in step 5, develop for second decision point (node 460) all optimal release sequences from Royal Glen Project and the feasible solutions in reach 3 from IFT's of nodes 462 and 494, respectively. (See Table 9 for DPT for node 460.)

9. Similar to the operations in step 2, route in reach 5 all feasible solutions of the above decision point to the next downstream decision point (node 456). The IFT for reach 5 is indexed by node 460. (See Table 6.)

10. For reach 6 containing the Town Creek Project (node 458), repeat the operations similar to those for the Bloomington Reservoir in steps 1, 2, and 3. The IFT for this reach is indexed by node 458. (See Table 5.)

11. As described in step 5, develop for the third decision point (node 456) all optimal release sequences from the Town Creek Project and the feasible solutions in reach 5 from IFT's of nodes 458 and 460, respectively. (See Table 10 for DPT for node 456.)

12. For the final reach of the upper portion of the Potomac River Basin, the terminal IFT is developed by routing all feasible solutions from the above decision point to node 436. (See Table 7.)

In summary the following aggregation of tableaux has been computed for the nodes below:

Incremental Flow Tableaux			Dynamic Programming Tableaux	
Nodes	Reaches		Nodes	Reaches
570	1	←→	560	3
562	2			
494	3	←→	460	5
462	4			
460	5	←→	456	7
458	6			
436	7	←→	Terminal IFT	

Table 4

INCREMENTAL FLOW TABLEAU

NODE 570

Flow (cfs)	BOD (mg/l)	DO Deficit (mg/l)	Sat DO (mg/l)	Temp (°C)
31.20	2.74	0.33	8.51	22.95
51.20	2.78	0.64	8.53	22.78
71.20	2.81	0.93	8.57	22.55
91.20	2.83	1.18	8.62	22.29
111.20	2.85	1.40	8.66	22.04
131.20	2.86	1.59	8.70	21.79
151.20	2.87	1.76	8.74	21.56
171.20	2.87	1.91	8.78	21.34
191.20	2.88	2.05	8.82	21.14
211.20	2.89	2.17	8.85	20.95
231.20	2.89	2.27	8.88	20.77
		NODE 562		
61.80	1.84	0.35	8.56	22.65
81.80	1.87	0.45	8.61	22.33
101.80	1.89	0.54	8.67	21.99
		NODE 494		
243.33	4.30	0.86	8.50	23.00
263.33	4.16	0.87	8.50	23.00
283.33	4.04	0.88	8.50	23.00
303.33	3.93	0.89	8.50	23.00
323.33	3.83	0.90	8.50	23.00
343.33	3.74	0.91	8.50	23.00
363.33	3.67	0.92	8.50	23.00
383.33	3.59	0.93	8.50	23.00

Table 5

INCREMENTAL FLOW TABLEAU

NODE 462

Flow (cfs)	BOD (mg/l)	DOD (mg/l)	Sat DO (mg/l)	Temp (°C)
81.10	0.73	0.10	8.53	22.78
101.10	0.79	0.09	8.53	22.82
121.10	0.84	0.09	8.52	22.85
141.10	0.89	0.09	8.52	22.87
161.10	0.93	0.09	8.52	22.89
181.10	0.96	0.09	8.51	22.90
201.10	0.99	0.10	8.51	22.91
221.10	1.02	0.10	8.51	22.92
241.10	1.05	0.10	8.51	22.93
261.10	1.07	0.10	8.51	22.93
281.10	1.09	0.11	8.51	22.94
301.10	1.11	0.11	8.51	22.94
321.10	1.13	0.11	8.51	22.94
341.10	1.15	0.12	8.51	22.95
361.10	1.16	0.12	8.51	22.95
381.10	1.18	0.12	8.51	22.95
401.10	1.19	0.13	8.51	22.96
421.10	1.21	0.13	8.51	22.96
441.10	1.22	0.13	8.50	22.96
461.10	1.23	0.14	8.50	22.96
481.10	1.25	0.14	8.50	22.96
501.10	1.26	0.14	8.50	22.96
521.10	1.27	0.15	8.50	22.97
541.10	1.28	0.15	8.50	22.97
561.10	1.29	0.15	8.50	22.97

NODE 458

5.00	1.80	0.01	8.50	23.00
25.00	1.86	0.05	8.51	22.95
45.00	1.88	0.11	8.53	22.82
65.00	1.90	0.18	8.55	22.67
85.00	1.90	0.24	8.58	22.53
105.00	1.91	0.29	8.60	22.40

Table 6

INCREMENTAL FLOW TABLEAU

NODE 460

Flow (cfs)	BOD (mg/l)	DOD (mg/l)	Sat DO (mg/l)	Temp (°C)
324.42	3.19	0.41	8.50	22.99
344.42	3.07	0.40	8.50	22.99
364.42	2.97	0.39	8.50	22.99
384.42	2.88	0.39	8.50	22.99
404.42	2.79	0.38	8.50	22.99
424.42	2.72	0.38	8.50	22.99
444.42	2.66	0.37	8.50	22.99
464.42	2.60	0.37	8.50	22.99
484.42	2.55	0.36	8.50	22.99
504.42	2.50	0.36	8.50	22.99
524.42	2.46	0.35	8.50	22.99
544.42	2.42	0.35	8.50	22.99
564.22	2.38	0.35	8.50	22.99
584.42	2.35	0.34	8.50	22.99
604.42	2.32	0.34	8.50	22.99
624.42	2.29	0.34	8.50	22.99
644.42	2.27	0.33	8.50	22.99
664.42	2.24	0.33	8.50	22.99
684.42	2.22	0.33	8.50	22.99
704.42	2.20	0.33	8.50	22.99
724.42	2.18	0.33	8.50	22.99
744.42	2.17	0.32	8.50	22.99
764.42	2.15	0.32	8.50	22.99
784.42	2.13	0.32	8.50	22.99
804.42	2.12	0.32	8.50	22.99
824.42	2.13	0.33	8.50	22.99
844.42	2.13	0.35	8.50	22.99
864.42	2.14	0.36	8.50	22.99
884.42	2.14	0.37	8.50	22.99
904.42	2.14	0.38	8.50	22.99
924.42	2.15	0.39	8.50	22.99
944.42	2.15	0.40	8.50	22.99

Table 7

INCREMENTAL FLOW TABLEAU

NODE 436

Flow (cfs)	BOD (mg/l)	DOD (mg/l)	Sat DO (mg/l)	Temp (°C)
386.42	1.15	0.11	8.50	23.00
406.42	1.16	0.12	8.50	23.00
426.42	1.18	0.12	8.50	23.00
446.42	1.17	0.12	8.50	23.00
466.42	1.16	0.12	8.50	22.99
486.42	1.16	0.12	8.50	22.99
506.42	1.15	0.13	8.50	22.99
526.42	1.15	0.13	8.50	22.99
546.42	1.15	0.13	8.50	22.99
566.42	1.15	0.13	8.50	22.99
586.42	1.15	0.13	8.50	22.99
606.42	1.15	0.14	8.50	22.99
626.42	1.15	0.14	8.50	22.99
646.42	1.15	0.14	8.50	22.99
666.42	1.15	0.14	8.50	22.99
686.42	1.15	0.14	8.50	22.99
706.42	1.15	0.14	8.50	22.99
726.42	1.15	0.15	8.50	22.99
746.42	1.15	0.15	8.50	22.99
766.42	1.15	0.15	8.50	22.99
786.42	1.16	0.15	8.50	22.99
806.42	1.16	0.15	8.50	22.99
826.42	1.16	0.16	8.50	22.99
846.42	1.16	0.16	8.50	22.98
866.42	1.16	0.16	8.50	22.98
886.42	1.16	0.16	8.50	22.98
906.42	1.17	0.16	8.50	22.98
926.42	1.17	0.16	8.50	22.98
946.42	1.18	0.17	8.50	22.98
966.42	1.19	0.17	8.50	22.98
986.42	1.19	0.17	8.50	22.98
1006.42	1.20	0.17	8.50	22.98
1026.42	1.21	0.18	8.50	22.98
1046.42	1.21	0.18	8.50	22.98
1066.42	1.22	0.18	8.50	22.98
1086.42	1.22	0.18	8.50	22.98
1106.42	1.23	0.19	8.50	22.98

Table 8

DYNAMIC PROGRAMMING TABLEAU

NODE 560

Total Flow (cfs)	Flow at 562 (cfs)	Flow at 570 (cfs)	BOD (mg/l)	DOD (mg/l)
93.00	61.80	31.20	2.14	0.34
113.00	81.80	31.20	2.11	0.42
133.00	101.80	31.20	2.09	0.49
153.00	101.80	51.20	2.19	0.57
173.00	101.80	71.20	2.27	0.70
193.00	101.80	91.20	2.33	0.84
213.00	101.80	111.20	2.39	0.99
233.00	101.80	131.20	2.43	1.13
253.00	101.80	151.20	2.47	1.27
273.00	101.80	171.20	2.51	1.40
293.00	101.80	191.20	2.54	1.52
313.00	101.80	211.20	2.56	1.64
333.00	101.80	231.20	2.58	1.74

Table 9

DYNAMIC PROGRAMMING TABLEAU

NODE 460

Flow (cfs)	Flow at 462 (cfs)	Flow at 494 (cfs)	BOD (mg/l)	DOD (mg/l)
.42	81.10	243.33	3.41	0.67
.42	101.10	243.33	3.27	0.63
.42	121.10	243.33	3.15	0.50
.42	141.10	243.33	3.05	0.58
42	161.10	243.33	2.96	0.55
.42	181.10	243.33	2.88	0.53
.42	201.10	243.33	2.80	0.51
.42	221.10	243.33	2.74	0.50
.42	241.10	243.33	2.68	0.48
42	261.10	243.33	2.63	0.47
.42	281.10	243.33	2.58	0.46
.42	301.10	243.33	2.54	0.44
.42	321.10	243.33	2.50	0.43
.42	341.10	243.33	2.46	0.43
42	361.10	243.33	2.43	0.42
.42	381.10	243.33	2.40	0.41
.42	401.10	243.33	2.37	0.40
.42	421.10	243.33	2.34	0.40
42	441.10	243.33	2.32	0.39
.42	461.10	243.33	2.29	0.39
42	481.10	243.33	2.27	0.38
.42	501.10	243.33	2.25	0.38
.42	521.10	243.33	2.23	0.37
.42	541.10	243.33	2.22	0.37
.42	561.10	243.33	2.20	0.36
42	561.10	263.33	2 21	0.38
.42	561.10	283.33	2.21	0.40
.42	561.10	303.33	2.22	0.41
42	561.10	323.33	2.22	0.43
.42	561.10	343.33	2.22	0.44
.42	561.10	363.33	2.22	0.46
.42	561.10	383.33	2.23	0.47

Table 10

DYNAMIC PROGRAMMING TABLEAU

NODE 456

Total Flow (cfs)	Flow at 458 (cfs)	Flow at 460 (cfs)	BOD (mg/l)	DOD (mg/l)
329.42	5.00	324.42	3.17	0.40
349.42	25.00	324.42	3.10	0.38
369.42	45.00	324.42	3.04	0.37
389.42	45.00	344.42	2.94	0.37
409.42	45.00	364.42	2.85	0.36
429.42	45.00	384.42	2.77	0.36
449.42	45.00	404.42	2.70	0.35
469.42	45.00	424.42	2.64	0.35
489.42	45.00	444.42	2.59	0.35
509.42	45.00	464.42	2.54	0.34
529.42	45.00	484.42	2.49	0.34
549.42	45.00	504.42	2.45	0.34
569.42	45.00	524.42	2.41	0.33
589.42	45.00	544.42	2.38	0.33
609.42	45.00	564.42	2.35	0.33
629.42	45.00	584.42	2.32	0.33
649.42	45.00	604.42	2.29	0.32
669.42	45.00	624.42	2.27	0.32
689.42	45.00	644.42	2.24	0.32
709.42	45.00	664.42	2.22	0.32
729.42	45.00	684.42	2.18	0.32
749.42	45.00	704.42	2.18	0.31
769.42	45.00	724.42	2.17	0.31
789.42	45.00	744.42	2.15	0.31
809.42	45.00	764.42	2.13	0.31
829.42	45.00	784.42	2.12	0.31
849.42	45.00	804.42	2.11	0.31
869.42	65.00	804.42	2.10	0.31
889.42	85.00	804.42	2.10	0.31
909.42	105.00	804.42	2.10	0.32
929.42	105.00	824.42	2.10	0.33
949.42	105.00	844.42	2.11	0.34
969.42	105.00	864.42	2.11	0.35
989.42	105.00	884.42	2.12	0.36
1009.42	105.00	904.42	2.12	0.37
1029.42	105.00	924.42	2.12	0.38
1049.42	105.00	944.42	2.12	0.39

Table 11

DYNAMIC PROGRAMMING TABLEAU
(Churchill's K_2 Formulation)

NODE 456

Total Flow (cfs)	Flow at 458 (cfs)	Flow at 460 (cfs)	BOD (mg/l)	DOD (mg/l
349.42	5.00	344.42	3.13	0.62
369.42	25.00	344.42	3.06	0.59
389.42	45.00	344.42	3.00	0.57
409.42	65.00	344.42	2.95	0.55
429.42	65.00	364.42	2.86	0.54
449.42	65.00	384.42	2.79	0.53
469.42	65.00	404.42	2.72	0.52
489.42	65.00	424.42	2.66	0.51
509.42	65.00	444.42	2.60	0.50
529.42	65.00	464.42	2.55	0.50
549.42	65.00	484.42	2.51	0.49
569.42	65.00	504.42	2.47	0.48
589.42	65.00	524.42	2.43	0.47
609.42	65.00	544.42	2.40	0.47
629.42	65.00	564.42	2.36	0.46
649.42	65.00	584.42	2.34	0.45
669.42	45.00	624.42	2.29	0.45
689.42	45.00	644.42	2.27	0.44
709.42	45.00	664.42	2.25	0.44
729.42	45.00	684.42	2.23	0.43
749.42	45.00	704.42	2.21	0.43
769.42	45.00	724.42	2.19	0.42
789.42	45.00	744.42	2.17	0.42
809.42	45.00	764.42	2.16	0.41
829.42	45.00	784.42	2.14	0.41
849.42	45.00	804.42	2.13	0.41
869.42	45.00	824.42	2.11	0.40
889.42	65.00	824.42	2.11	0.40
909.42	85.00	824.42	2.11	0.40
929.42	105.00	824.42	2.10	0.40
949.42	105.00	844.42	2.11	0.41
969.42	105.00	864.42	2.11	0.42
989.42	105.00	884.42	2.12	0.44
1009.42	105.00	904.42	2.12	0.45
1029.42	105.00	924.42	2.12	0.46
1049.42	105.00	944.42	2.12	0.47

Table 12

FLOW RANGE DATA

Upper Potomac River Basin

Node	Minimum Flow* (cfs)	Maximum Flow** (cfs)
570***	31.20	231.20
566***	61.80	101.80
562	61.80	101.80
560	93.00	333.00
494	143.33	393.33
492***	40.00	520.00
462	81.10	561.09
460	224.42	944.42
458***	5.00	105.00
456	229.42	1049.42
436	286.42	1106.42

* minimum flow for this study computed as 7-day low flow with recurrence interval of once in ten years.

** maximum flow equals minimum flow plus regulation capacity.

*** denotes reservoir sites.

Partial listing of the tableaux for conditions typical for the month of July are given in Tables 4 through 10. A reduced form of the minimum-maximum flow range listing for the above nodes is presented in Table 12.

Incremental Flow Tableaux

As an integral part of the model output, the IFT's are used for developing the release patterns and are also very useful for establishing the effects of flow regulation on the various water quality parameters. In IFT's for nodes 570, 562, 462, and 450, the effects of increasing reservoir release rates (assuming the quality in the impoundment remains constant with flow) are clearly demonstrated. In these reaches, the water quality is primarily controlled by the conditions in the impoundment and the stream reaeration capacity.

For the reservoir at nodes 570 and 568, the BOD was assumed to be 3.0 mg/l and DO concentration 5.0 mg/l, and 2.0 mg/l of BOD and 8.0 mg/l DO, respectively. With the quality remaining constant with flow, the IFT for this node clearly demonstrates how the increase flow rates lessen the recovery of initial DO deficits. Similar response is shown for the reservoir at node 568, although in this case the response is not so drastic because of the longer stream reach.

The effects of the wastewater loads in reach 3, the quality constraints, and flow augmentation can be readily seen in IFT's for nodes 494 and 460. In the downstream end of reach 3, as indexed by the IFT for node 494, the flow range is from 143 to 383 cfs. (See Table 12.) However, due to large wastewater loads in reach 3, a minimum of 243 cfs is required to meet the DO quality constraint of 4.0 mg/l. This results in a feasible range of flow from 243 to 383 cfs instead

of the maximum regulatable range of 143 to 383 cfs. A similar reduction is also imposed on downstream nodes affected by the release range in reach 3.

Dynamic Programming Tableaux

As can be seen in the DPT's for nodes 560, 460, and 456 in Tables 8, 9, and 10, respectively, the tableaux contains for each flow state an optimal combination of the two contributing tributaries or reaches. The tableaux also contains the flow contributions of the IFT's indexed by the first upstream node and the numerical values of the two return variables, BOD and DOD. Not included in the partial listing are two remaining state variables, temperature and deoxygenation rate. Appendix C includes a complete display of computer output of the flow release model for the Patuxent Basin.

The indexing as described above and a listing of minimum-maximum flow ranges for each node as given in Table 12 provides a simple linkage* between the DPT's and IFT's. This linkage is necessary in developing the reservoir release sequences which is discussed in the next two sections of this chapter.

Since there are no major waste sources above the first decision point (node 560), the major factor controlling the optimization process is the water quality in the proposed reservoir sites. In the dynamic programming optimization routine for node 560, 33 feasible solutions have been investigated using a flow grid of 20 cfs. With the selection

---

*Linkage in this study is defined as providing a mechanism, either manual or via the computer, for data transmission between the output of a series of calculations and/or decisions and the required input for a succeeding series of calculations and/or decisions.

of the minimum BOD increment of 0.5 mg/l, the number of feasible solutions was reduced to thirteen. Since the BOD range was never greater than 0.5 mg/l per flow state, only one feasible solution was retained per flow state. Hence, the optimal solution for the given flow state is that combination which yields the minimum DO deficit.

For the second decision point (node 460), the release pattern is from the predominantly better quality of reach 4. The 200 feasible solutions were reduced optimally to 32, corresponding to the number of flow states. Similar to node 560, there is only one solution per flow state.

The DPT of the third decision point (node 456) is made of IFT's of nodes 458 and 460, which are quite different in quality. At this node, the 192 feasible solutions were reduced to 37 optimal solutions. As presented in Table 10, there is a slight vacillation in release pattern as the flow state is increased. The vacillation is even more pronounced for this node when the Churchill reaeration is used, as can be seen in Table 11. This vacillation is due to the following factors: (1) a coarse BOD grid, (2) the water quality in the reservoir at node 458, and (3) the DOD return parameter. In Chapter VI, these and other factors will be discussed in greater detail.

FLOW RELEASE PATTERNS

From the IFT's and DPT's, it is possible to determine two types of optimal reservoir release sequences: (1) "best" water quality for a given flow requirement, or (2) minimum release rates for a given water quality requirement. Assuming that all reservoirs in the upper part of the Potomac Basin are operational, the development of a release pattern is best illustrated by an example. Using the above

test runs and tableaux for July, the optimal release sequences given in Table 13 would be required to provide the best water quality (minimum DOD for a given BOD state) for a flow target of 706 cfs at node 436.

Table 13

EXAMPLE OF AN OPTIMAL RELEASE SEQUENCE

Point	Node	Total Flow (cfs)	IFT Node	IFT Flow (cfs)	IFT Node	IFT Flow (cfs)
Terminal	436	706	--	--	--	--
3rd Decision	456	649	458*	45	460	604
2nd Decision	460	644	462*	401	494	243
1st Decision	560	193	562*	102	570*	91

*IFT's downstream from reservoirs

The above release sequences for the decision points have been developed in reverse order from which the tableaux were formed. That is, by starting with the terminal node 436, proceeding upstream, and diverging at each decision point, the flow requirements for each contributing IFT of each stage have been determined as indicated in Table 13.

From the above Table 13, Table 12, and by subtracting the flow contributions of the unregulated drainage areas, the following reservoir operating scheme has been derived for the imposed conditions:

1. Reservoir at node 570, a release rate of 91 cfs.
2. Reservoir at node 568, a release rate of 102 cfs.
3. Reservoir at node 492, a release rate of 360 cfs.
4. Reservoir at node 458, a release rate of 45 cfs.

Since the reservoir at node 568 is drawn to its maximum capacity, the draw down of the reservoir at node 570 is necessary to meet quality constraints in reach 3.

It is also possible to develop a flow release pattern which is dictated by desired quality at a decision point. For example, at node 456 a total of 529 cfs or greater would be required to maintain a BOD level of 2.5 mg/l or less. A release pattern can be obtained to maintain the BOD objective similar to the sequence developed for a given flow rate at node 436.

WATER QUALITY CONTROL CONSIDERATIONS IN RESERVOIR DESIGN

As indicated in the previous section, the flow pattern may vary for a given decision point, depending on the total flow. While it is not feasible to construct a reservoir to meet these vacillations, the existence of a trend in release patterns is as important as are absolute release sequences.

In studies at the Chesapeake Field Station, Hetling[79] determined effects of flow regulation and waste treatment on the water quality in the Potomac Estuary. Using a DO objective of 4.0 mg/l, 91, 92, and 93 percent removal of the 5-day BOD is required for 2000, 1500, and 1000 cfs of flow, respectively. The optimal flow release sequences for the proposed reservoir system to meet the three flow targets are shown in Figure 6.

These release patterns have been developed employing the following conditions:

1. Wastewater loadings and water supply usage as given in Appendix B.

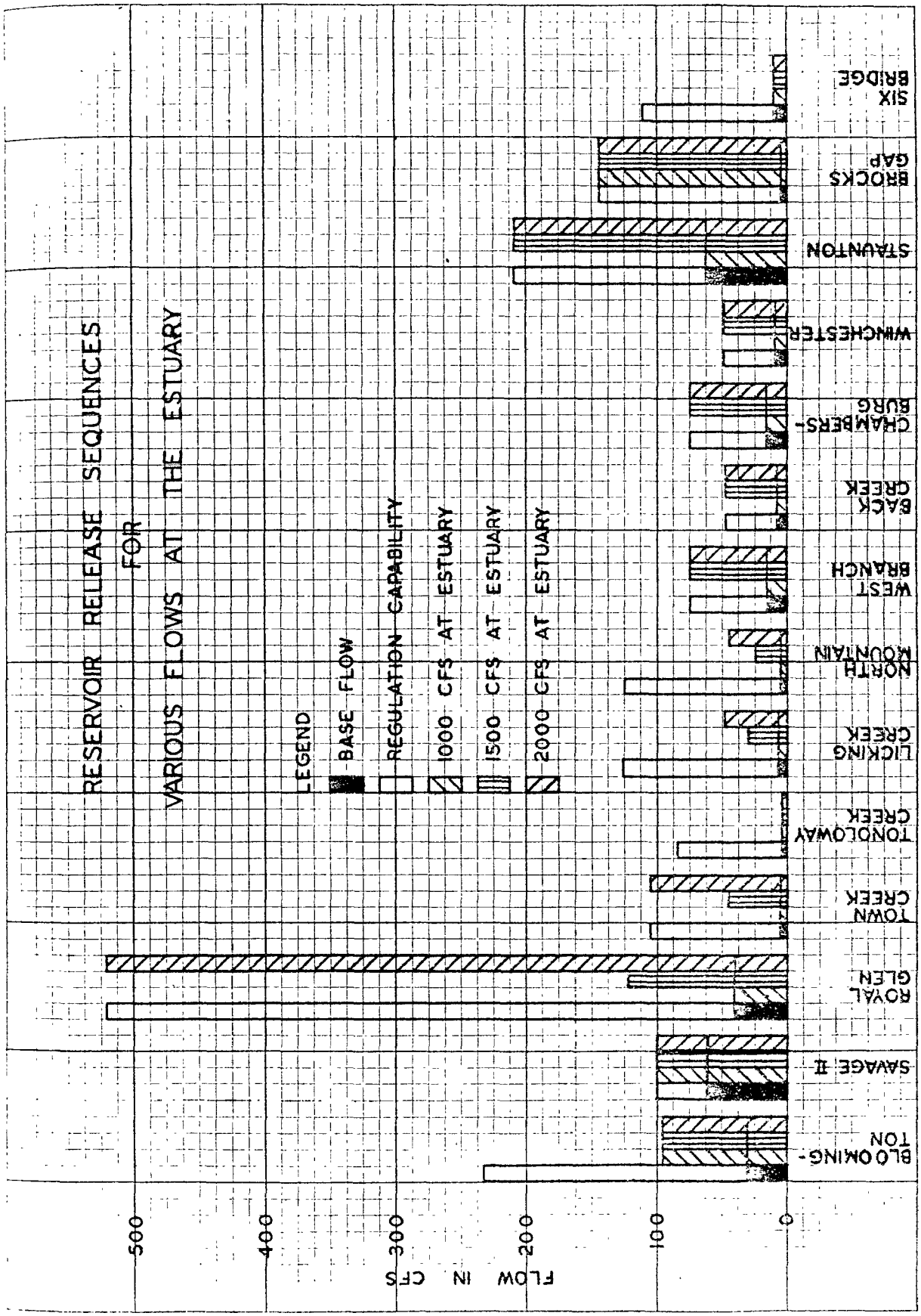

RESERVOIR RELEASE SEQUENCES
FOR
VARIOUS FLOWS AT THE ESTUARY
LEGEND
BASE FLOW
REGULATION CAPABILITY
1000 CFS AT ESTUARY
1500 CFS AT ESTUARY
2000 CFS AT ESTUARY
FLOW IN CFS
500
400
300
200
100
0
BLOOMING-TON
SAVAGE II
ROYAL GLEN
TOWN CREEK
TONOLOWAY CREEK
LICKING CREEK
NORTH MOUNTAIN
WEST BRANCH
BACK CREEK
CHAMBERS-BURG
WINCHESTER
STAUNTON
BROCKS GAP
SIX BRIDGE

Figure 6

2. Water quality of all stream flow addition points (including reservoirs) remains constant with flow. BOD and DO are assumed to be 2.0 and 8.0 mg/l, respectively.
3. O'Connor's K_2 formulation is used to determine reaeration rate.
4. Steady-state temperature for entire river system is 23° C.
5. Base flow and regulation capacity of the reservoirs are as indicated in Figure 6.
6. DO constraint is 4.0 mg/l in all reaches.

The above have been used also as standard test conditions for the sensitivity analysis in conjunction with 1500 cfs requirement in the estuary.

For the preceding conditions, it can be seen that the Savage II and Brocks Gap Reservoirs at nodes 104 and 568 are at their maximum release rate for all three target flows at the estuary while the Tonoloway Creek and Six Bridge Reservoirs at nodes 56 and 434 are at their base flow rates. The two reservoirs which are at their maximum rates are of primary importance in the optimal flow release pattern. The Staunton, Winchester, Chambersburg, Back Creek, and West Branch Reservoirs at nodes 244, 356, 393, 398, and 402, respectively, are utilized to their capacity at 1500 and 2000 cfs target, indicating a secondary importance in the release pattern.

In the entire Potomac River Basin affected by the proposed reservoir system, the North Branch is the only section of the Potomac that requires additional flow regulation to meet the 4.0 mg/l DO objective. This is reflected in the constant release rates from Savage II and Bloomington Reservoirs for all targets and levels at the estuary.

CHAPTER VI

SENSITIVITY ANALYSES

In this chapter the spatial sensitivity of the reservoir release patterns are related to changes in (1) biochemical and physical, (2) design, and (3) socio-economic parameters. Comparisons of the various release sequences in response to a change in a given parameter are made to the standard test run which was described in the previous chapter and is presented in Figure 6. The biochemical and physical parameters investigated were time of travel, reaeration, minimum BOD concentrations and deaeration; the design parameters were stream temperature, BOD-DO concentrations in the proposed reservoirs, wastewater loadings, and DO concentration in the wastewater effluent; and the socio-economic parameters were water quality objectives and imposed waste loadings.

BIOCHEMICAL AND PHYSICAL PARAMETERS

The three basic biochemical and physical parameters required in the quality formulations are (1) time of travel, (2) reaeration rate, and (3) deaeration rate. Problems in determining these parameters and suggested steps in simplifying the verification of the quality formulations are given in Appendix A.

Based on verification studies in the North Branch of the Potomac River and in the Patuxent Basin, the most important parameter appears to be time of travel. An example of the importance of travel time is given in Appendix A, where a 65 percent over-estimation in time of travel resulted in a 300 percent error in calculating the assimilative capacity

of the North Branch. Not only is the travel time an integral part of the water quality formulations, but also a constituent parameter in the predictive reaeration formulations and used in calculating the deaeration rate of the stream. Therefore, any analysis of the sensitivity of the flow release pattern to this parameter would also reflect changes in the temperature profile, reaeration rates, etc., and would not yield any definitive information. Also, with the use of tracers the time of travel measurement is becoming more exact and less costly.

Reaeration

Of the remaining two parameters, the reaeration rate appears to be next in importance. Even with good information on deaeration rates and times of travel, considerable engineering judgment is required in the selection and use of the reaeration formulations. In Table 15 and in Figures 13, 14, and 15, it can readily be seen that even with the same velocity and depth data, more than a threefold difference in reaeration rates can be observed depending on which formulation is used.

A test run has been made using the Churchill K_2 formulation and with all the remaining parameters being the same as in the standard test conditions. When the release pattern is compared to the standard test run (O'Connor's K_2 formulation), differences in the release rates from four reservoirs are observed. (See Figure 7.) Three of these changes are minor, 20 cfs, with the remaining being more significant, 60 cfs.

The small number of changes in the entire release pattern is primarily due to the location of the wastewater loads in the Potomac Basin. In only one section of the basin, reach 3 in the North Branch watershed, flow augmentation is required to meet the DO quality objective. The

constraining section of this area (River Mile 305-310) is in waters ranging from 6 to 12 feet in depth. As can be seen in Figure 16, the two computed DO profiles are essentially the same at this critical reach, hence there is only a slight difference in the flow requirements. However, if the constraining reach had been downstream about 10 miles, the change in release pattern would have been significant.

Deaeration and Minimum BOD Concentrations

The deaeration rates of the wastewaters used in the standard test runs were either determined as outlined in Appendix A or obtained from previous water quality studies. For all stream flow addition points, a deaeration rate of 0.1 (base 10 at 20°C) was assumed.

An additional computer run was made using the standard test conditions, except that the deaeration rate of all wastewaters was set at 0.15 (base 10 at 20°C). The reservoir release sequence to maintain 1500 cfs at the estuary was very similar to the standard test run. See Figure 7 for comparison to standard test run.

The insensitivity is primarily due to the small changes in the overall stream deaeration rates. In setting all values at 0.15, some wastewater rates are increased while others are decreased, with an overall slight increase of less than 0.02 for the entire basin.

Of equal importance is the assumption of a minimum equilibrium level for BOD between the waters of the stream and the stream bed. Field studies conducted by the Chesapeake Field Station and by others indicate that there is a background BOD of 1.0 to 3.0 mg/l with a corresponding DO level of about 80 to 90 percent of saturation in long reaches of the Potomac containing no point-source pollution. With the first-order BOD decay equation used in the model, the resulting BOD

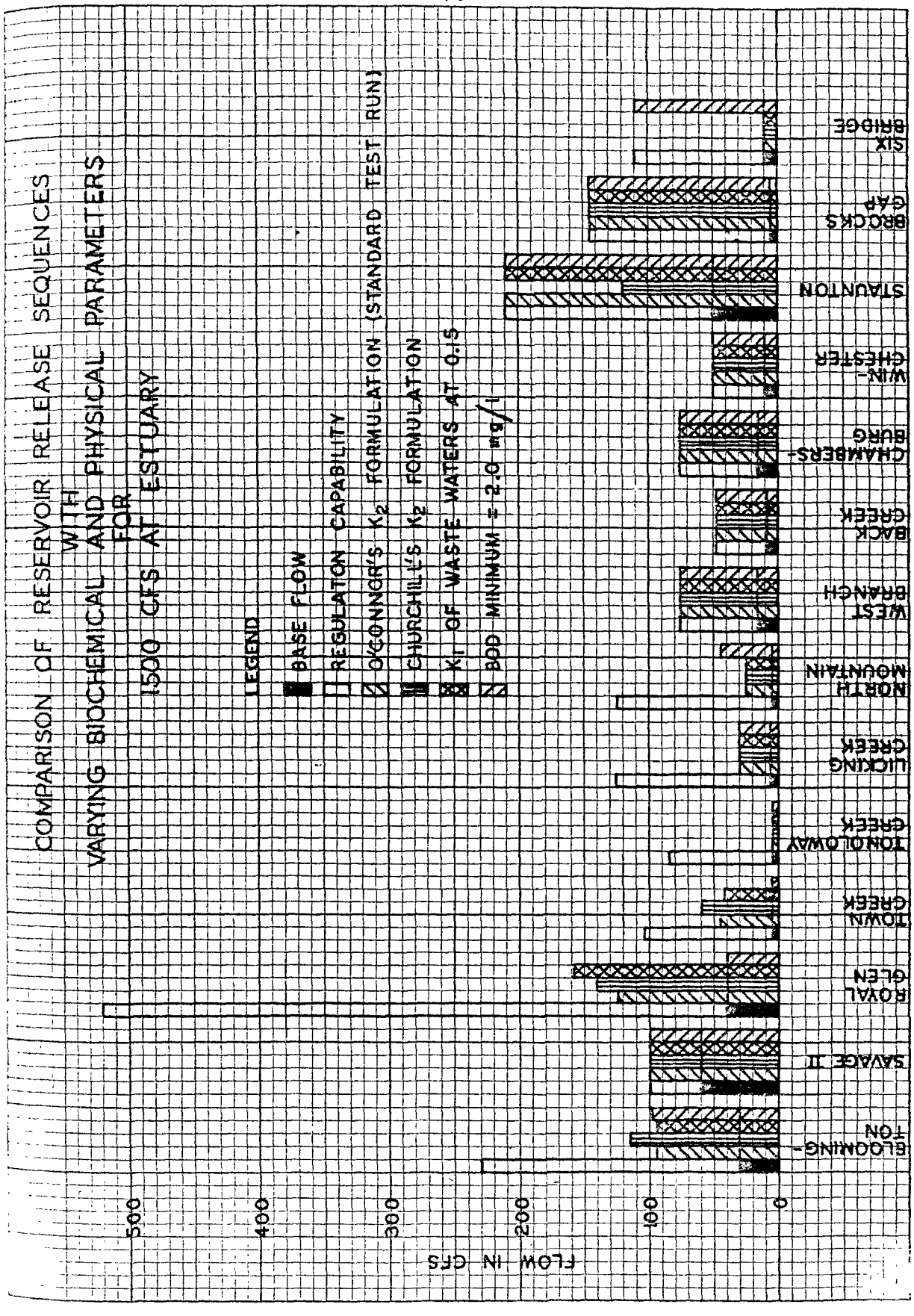

COMPARISON OF RESERVOIR RELEASE SEQUENCES WITH VARYING BIOCHEMICAL AND PHYSICAL PARAMETERS FOR 1500 CFS AT ESTUARY
LEGEND
BASE FLOW
REGULATON CAPABILITY
O'CONNOR'S K_2 FORMULATION (STANDARD TEST RUN)
CHURCHILL'S K_2 FORMULATION
K_T OF WASTE WATERS AT 0.5
BOD MINIMUM = 2.0 mg/l
FLOW IN CFS
500
400
300
200
100
0
BLOOMING-TON
SAVAGE II
ROYAL GLEN
TOWN GREEK
TONOLOWAY GREEK
LICKING GREEK
NORTH MOUNTAIN
WEST BRANCH
BACK GREEK
CHAMBERS-BURG
WIN-CHESTER
STAUNTON
BROCK'S GAP
SIX BRIDGE

Figure 7

concentrations approach 0.0, and the DO values are near saturation in these reaches. As can be seen in Table 7 for Node 436 (River Mile 238.0), the BOD is about 1.0 mg/l with the DO near saturation. Since there is no major point-source of pollution between Node 436 and the estuary, the BOD approaches 0.0 mg/l and the DO remains near saturation for the succeeding downstream reaches.

Under standard test conditions, a computer run was made in which the dynamic equilibrium was limited to a minimum level of 2.0 mg/l. In the run, whenever the BOD dropped below 2.0 mg/l, it was reset to 2.0 mg/l.

Major changes occurred in the release rates from Six Bridge and Royal Glen Reservoirs when compared to standard test runs. The limiting of the BOD to 2.0 mg/l causes all the reservoirs in the lower portion of the basin to draw down first. (See Figure 7.) If the existence of dynamic equilibrium level can be firmly established for various flow conditions and temperatures, the effect of this equilibrium could be significant for developing release rates for water quality control in the Potomac.

DESIGN PARAMETERS

The effects of four engineering design parameters have been investigated in this study; these are (1) temperature, (2) BOD-DO levels in the reservoir, (3) wastewater loadings, and (4) DO concentrations in the wastewater effluents.

Temperature

In the flow-release model, the temperature algorithm exponentially delays an increase or decrease in temperature to a steady-state temperature. For the standard test run a steady-state temperature of 23°C

has been used, with the temperatures of the waters from the reservoirs being taken as 20°C.

The steady-state temperature was determined from a statistical analysis of the temperature data by months. The value of 23°C is an average of all the mean temperatures for the month of July for all water quality stations in the Potomac Basin.

To test the effect of temperature on the release rates, a computer run was made with the temperatures set at 28°C, with other parameters being the same as for standard test conditions. As can be seen in Figure 8, only a slight change, 20 cfs, occurred in the release sequence from four reservoirs.

With an increase in temperature, the deaeration and the reaeration rates both increase. Therefore, there is only a small net change in the maximum DO deficit which is attributed to the differences in reaction constants. The major effect of temperature is on the DO saturation concentration. This effect is most pronounced in the North Branch area which receives wastewater high in BOD and large volumes of cooling water, as can be seen in Figure 16 and where flow regulation is required to meet the DO quality constraints.

BOD-DO Concentrations in the Reservoirs

A computer run was made to investigate the effect of water quality in the reservoirs on the flow release pattern. In this run the BOD and DO were set at 3.0 and 5.0 mg/l, respectively, as compared to 2.0 and 8.0 mg/l for the standard test run.

The optimal reservoir release sequence to meet the 1500 cfs flow requirement is almost similar to that for the standard test run, except for the Royal Glen project. (See Figure 8.) The small sensitivity of

the release sequence to a change in reservoir quality in the Potomac is primarily due to the location of the proposed upstream impoundments.

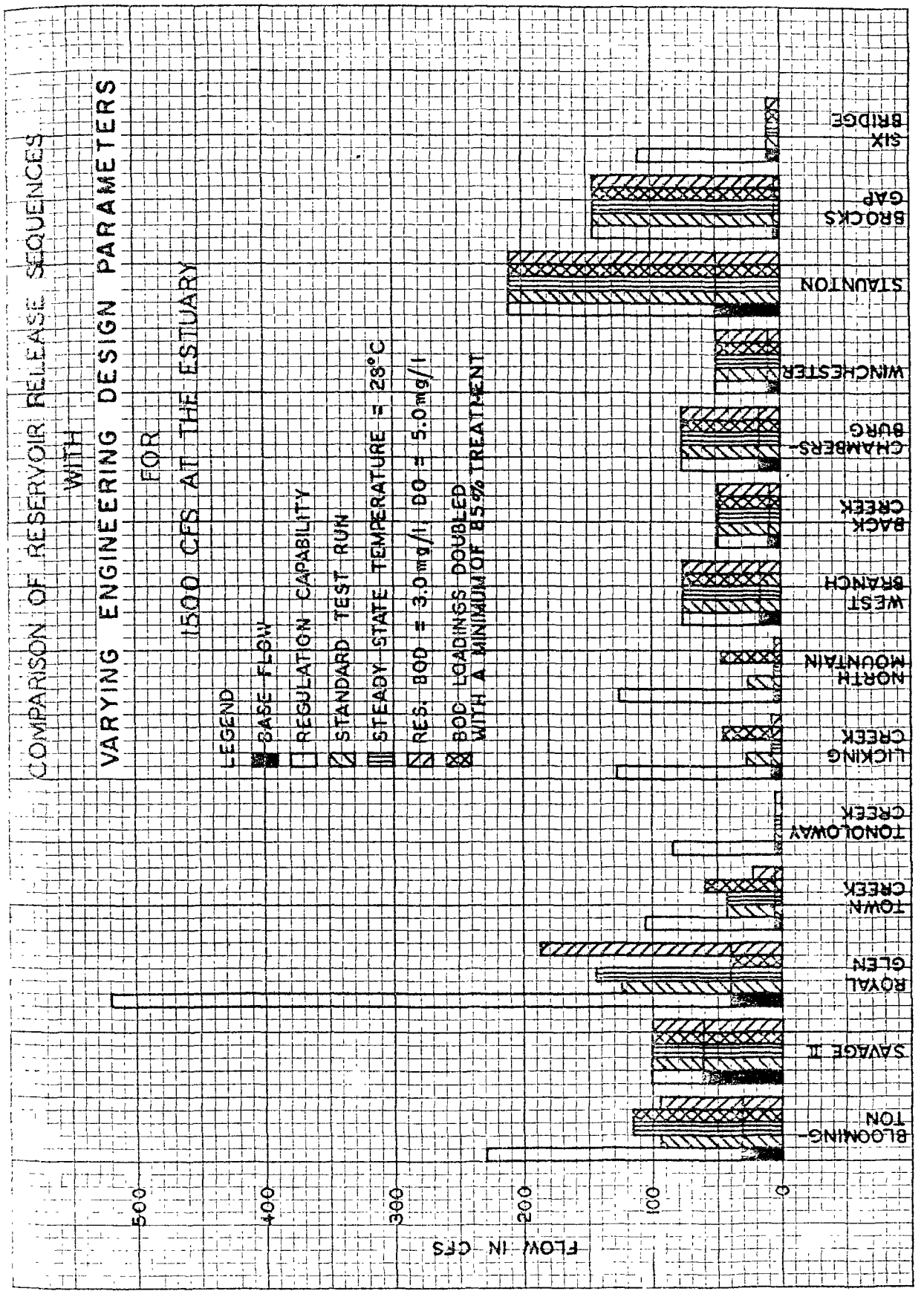
The distance of most impoundments from the decision points is ample to allow significant recovery of initial DO deficit imposed in the reservoirs. The extent of the recovery can be readily seen when the IFT's are examined in detail. (See Tables 4, 5, and 6.) Similar recoveries resulted for the runs when the BOD and DO were 3.0 mg/l and 5.0 mg/l, respectively.

Wastewater Loadings

To test the sensitivity of the release sequence to changes in waste loadings, the current BOD loadings before treatment were doubled for all discharges. The release pattern developed for the doubled loading for which the minimum treatment was set at 85 percent removal of BOD is presented in Figure 8.

When compared to the standard test run, release rates from five reservoirs are changed for a doubled waste load, with Royal Glen having the greatest change of about 120 cfs. The increase in release rates from North Mountain, Licking Creek, and Town Creek follows naturally since there are no waste loads in these sub-basins.

The doubling of waste load resulted in flow requirements above the base flow in the North Fork and stem of the Shenandoah River. However, due to quality differences at the confluence point with the Potomac River, the release rates from two reservoirs in the Shenandoah sub-basin are at their maximum to meet the 1500 cfs requirements for the estuary as determined optimally using the flow release model.


COMPARISON OF RESERVOIR RELEASE SEQUENCES
WITH
VARYING ENGINEERING DESIGN PARAMETERS
FOR
1500 CFS AT THE ESTUARY
LEGEND
BASE FLOW
REGULATION CAPABILITY
STANDARD TEST RUN
STEADY-STATE TEMPERATURE = 28°C
RES. BOD = 3.0mg/l, DO = 5.0mg/l
BOD LOADINGS DOUBLED
WITH A MINIMUM OF 85% TREATMENT
FLOW IN CFS
500
400
300
200
100
0
SIX BRIDGE
BROCKS GAP
STAUNTON
WINCHESTER
CHAMBERS-BURG
BACK CREEK
WEST BRANCH
NORTH MOUNTAIN
LICKING CREEK
TONOLOWAY CREEK
TOWN CREEK
ROYAL GLEN
SAVAGE II
BLOOMING-TON

Figure

Concentration in the Wastewater Effluents

One of the parameters often overlooked or minimized is the DO concentration in the wastewater effluents. The effects of low DO wastewater discharges are most pronounced in reaches where the ratio of wastewater to stream flow is greater than 0.5, such as the North Branch. In Figure 16, the large drops in the DO at River Mile 338 and 312 are the result of cooling water and wastewater discharges which are low in dissolved oxygen and/or have a high immediate dissolved oxygen demand.

In the model, the concentration of BOD in thermal discharges is set equal to that of intake water plus any BOD added by the industrial facility. The DO and heat content can be set at any prescribed level by input data. For the standard test runs, the DO of the large waste discharges, including cooling water, was assumed to be 0.0 mg/l.

To meet a 4.0 mg/l DO requirement in the North Branch for the BOD loading used in the standard test, approximately 193 cfs is required by code 560. If the BOD loading is doubled with the waste volume held constant, the flow requirement is about 213 cfs. However, if the DO of the cooling water is set at 2.0 mg/l instead of 0.0 mg/l, the flow requirement is about 173 cfs. This is a 20 cfs decrease in flow requirement even with the BOD loading being doubled. Although not shown in Table 8, the effect of a 2.0 mg/l change in DO in the effluent can be greater than the effect of doubling the waste loads, as illustrated here for the North Branch sub-basin, indicating the importance of this parameter in design.

SOCIO-ECONOMIC PARAMETERS

To determine the effect of selected socio-economic parameters on the release sequences, computer runs have been made for an increased water quality objective to 4.5 mg/l of DO and for imposed BOD loadings at two different reaches. The remaining parameters have been held constant as stated for the standard test runs.

Water Quality Objectives

The effect of selecting a given water quality objective can be readily observed when the sequences for the doubled BOD loadings in Figures 8 and 9 are compared. In Figure 8, the DO constraint was at 4.0 mg/l while in Figure 9, a DO constraint of 4.5 mg/l was used. The major changes in the release rates are for the Royal Glen and Bloomington Reservoirs, with minor changes for the Town Creek, Licking Creek and North Mountain projects.

The 0.5 mg/l increase in DO constraint increases the flow requirements in the North Branch by about 60 cfs. The release rates for the reservoirs or tributaries entering the main stem in the downstream portion of the Potomac, except for Six Bridge, are at their maximum rates and therefore are not affected by the increase in DO constraint.

Imposed Wastewater Loadings

To simulate the effect of an unexpected rapid growth of a community or a wastewater-producing industry in the basin, two computer runs were made with 10,000-pound BOD loadings imposed at nodes 448 and 281. The load at node 281 is on the main stem below Opequon Creek tributary. The load at node 448 is on the main stem between the tributaries of Town Creek and Tonoloway Creek. See Appendix B for further details on node locations.

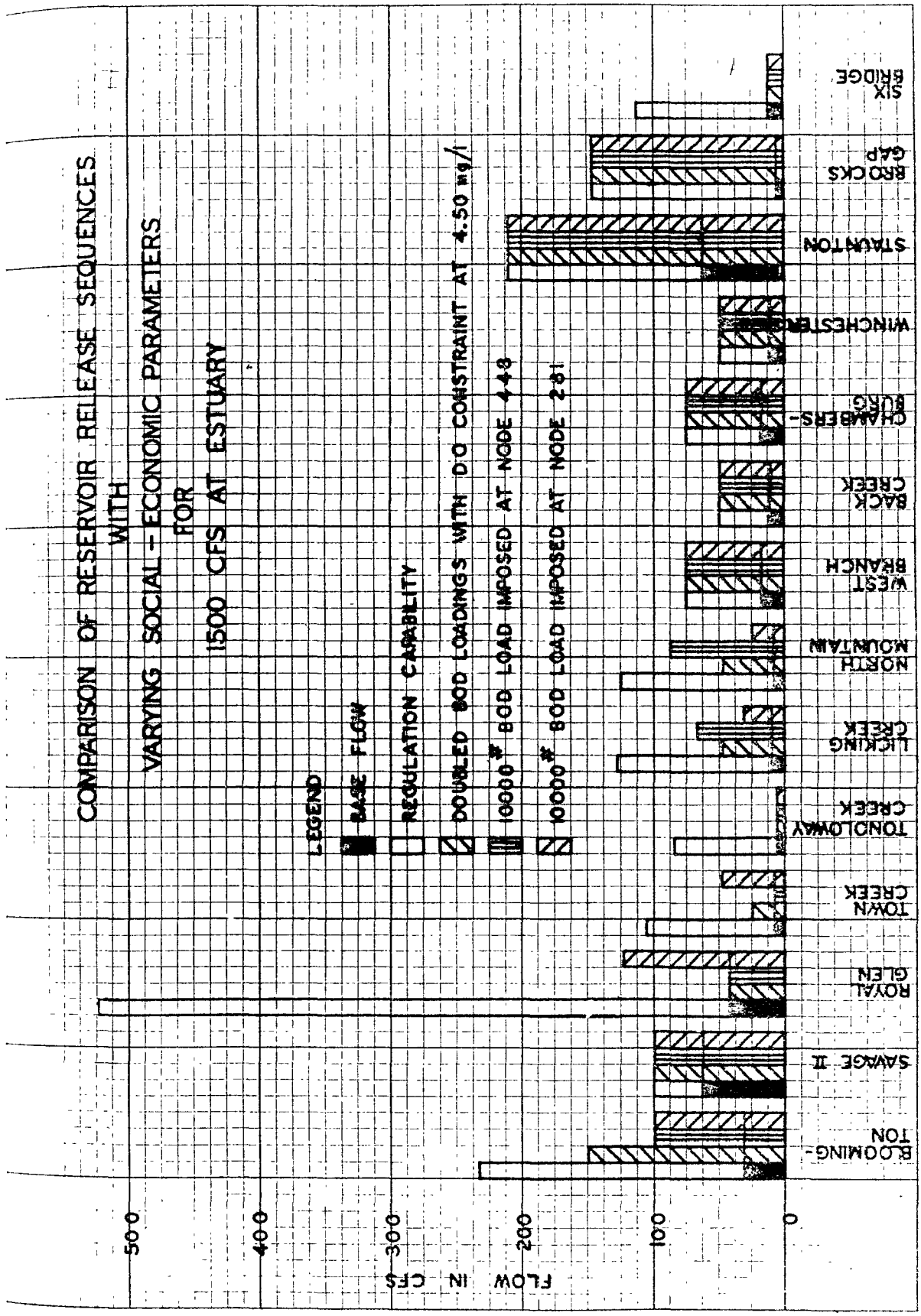

COMPARISON OF RESERVOIR RELEASE SEQUENCES
WITH
VARYING SOCIAL - ECONOMIC PARAMETERS
FOR
1500 CFS AT ESTUARY
LEGEND
BASE FLOW
REGULATION CAPABILITY
DOUBLED BOD LOADINGS WITH DO CONSTRAINT AT 4.50 mg/l
10000 # BOD LOAD IMPOSED AT NODE 448
10000 # BOD LOAD IMPOSED AT NODE 281
FLOW IN CFS
500
400
300
200
100
0
ROOMING-TON
SAVAGE II
ROYAL GLEN
TOWN CREEK
TONDLOWAY CREEK
LICKING CREEK
NORTH MOUNTAIN
WEST BRANCH
BACK CREEK
CHAMBERS-BURG
WINCHESTER
STAUNTON
BROCKS GAP
SIX BRIDGE

Figure 9

As also can be seen in Figure 9, the release sequence is sensitive to the imposed loadings at node 448, especially in the upstream portion of the basin. The release rates from four upstream reservoirs are changed with the greatest change being at the North Mountain Reservoir, about 60 cfs. However, the release sequence is not sensitive to the imposed load at node 281; in fact, the release rates are similar to those for the standard test run.

In summary, it has been demonstrated for 1500 cfs flow target at the estuary, the spatial reservoir release sequences are sensitive to changes in reaeration rates, stream temperature, wastewater loading, etc., when compared to standard test run. The greatest changes in release rates, as shown in Figures 7 to 9, were in the Bloomington, Royal Glen, Town Creek, Licking Creek, and North Mountain Reservoirs, all of which have an effect on the pollution problem originating in the North Branch. The insensitivity of the release sequences from the West Branch, Back Creek, Chambersburg, Staunton, Brocks Gap, Winchester, and Six Bridge Reservoirs is mainly due to two causes (1) relatively good water quality in these areas and (2) the seven projects and major sources of pollution are located in headwaters which allow ample time and distance for recovery of any degradation in water quality in these localized areas.

The above sensitivity analysis has been made for only one target flow at the estuary and was limited to a single parameter per test run. While other target flows and combinations of parameters may show more or less drastic changes in the release patterns, it has been well established that the release sequences are sensitive to changes of all the categories of parameters.

The grouping of the parameters as presented in Figures 7, 8, and 9 gives the engineer a great insight as to which parameter in a particular group may have the greatest effect on a given solution. The insight is even more meaningful when coupled to the ability to predict a given parameter in the future is considered. For example, of the physical and biochemical parameters, velocity and the reaeration rates for a given flow and temperature condition should remain fairly constant in the future while the deaeration rates are very dependent on the future wastewater characteristics. Fortunately, in the Potomac system the deaeration rates appear to have the least effect on the optimal flow release patterns and thereby minimizing any possible change in the release sequence due to any future changes in wastewater characteristics. The above example demonstrates how with the use of the flow release model effective planning can be realized even when all parameters cannot be accurately predicted.

CHAPTER VII

FURTHER DEVELOPMENTS OF THE FLOW RELEASE MODEL

An expanded version of the flow release model (Version II), which incorporates the cost of reservoir construction and operation, is presented in this chapter. Least-cost solutions are compared to those in Chapters V and VI, which were primarily concerned with water quality. Other possible expansions of the model, such as minimizing the deficit miles, inclusion of nutrient considerations, etc., are also proposed in this chapter.

Methods for overcoming the deterministic flow system are presented in the latter part of this chapter. Linkage to water quality, stream flow, and estuary model is also presented.

LEAST-COST SOLUTIONS

From the interpretation of DPT's in Chapter V and the sensitivity analysis in Chapter VI, it can be generally concluded that the second return variable, DOD, is not too significant in the optimization procedure. This is especially true in the Potomac Basin or when the waste loads are not overlapping, when reservoir quality is similar, or when the DO quality constraint is high. Based on this finding, the model has been expanded to incorporate the cost of reservoir construction and operation.

The cost of reservoir storage for a given release rate was substituted for the DOD parameter as the second return variable in the optimization algorithm. To provide for non-linear cost data, the cost

information is read in discrete units amenable to the flow-increment grid. This is explained in greater detail later in this chapter.

For the least-cost solution, the problem is expressed mathematically as:

$$Fo(Xo(k,p)) = \text{Min} \left[Cost(k,p) \,\middle|\, BOD(k,p) \,\middle|\, Xo(k,p) = Xm(i) + Xn(j) \right] \quad ------ (7\text{-}1)$$

where

Cost(k,p) = State of the second dependent variable for a given Xo and BOD state

Fo(Xo(k,p)) = Minimum cost of flow regulation for a given flow and BOD state; other variables as defined in Chapter III.

Since the cost state is additive, an accounting mechanism is also incorporated into the model.

With Version II of the flow release model, it is possible to obtain a flow release sequence for the optimization criteria listed in Table 14.

Table 14

OPTIMIZATION CRITERIA OF THE FLOW RELEASE MODEL-VERSION II

Optimization Index	First Return Variable	Second Return Variable	Remarks
I	BOD	DOD	Standard test runs
II	---	DOD	Use large BOD grid
III	BOD	COST	Least-cost solutions
IV	---	COST	Use large BOD grid
V	BOD	----	Read in all cost data as zeros

Version II also has greater flexibility in choice of reaeration prediction formulations, temperature coefficients and BOD algorithm parameters. See Appendices C, D, and E.

COMPARISON OF OPTIMIZATION PARAMETERS

Using Version II of the flow release model, a series of computer runs were made to determine the effect of various optimization parameters. As can be seen in Figure 10, release sequences are greatly affected by the choice of the optimization criterion.

In the DO deficit and BOD optimization procedure (Optimization Index 1) as discussed in Chapter V, only one solution was normally retained per flow increment even when a BOD grid as small as 0.5 mg/l was employed. The retaining of one solution per flow increment is essentially reducing the optimization to a single return variable. (Optimization Index II.)

With a smaller BOD grid, more solutions could be retained in upstream reaches of the Potomac. However, these additional feasible solutions would be eliminated at the lower decision points due to location of wastewater loadings, assumed reservoir quality, and smooth response surfaces of the BOD and DO profiles.

Similar to the above optimization parameters, the least-cost/BOD (Index III) and least-cost (Index IV) methods yield identical release sequences for the prepared reservoir system of the Potomac River Basin. The release sequences for least-BOD optimization parameter (Index V) is also given in Figure 10.

While the release sequences are very dependent on the choice of the optimization criteria, the effect on the water quality entering

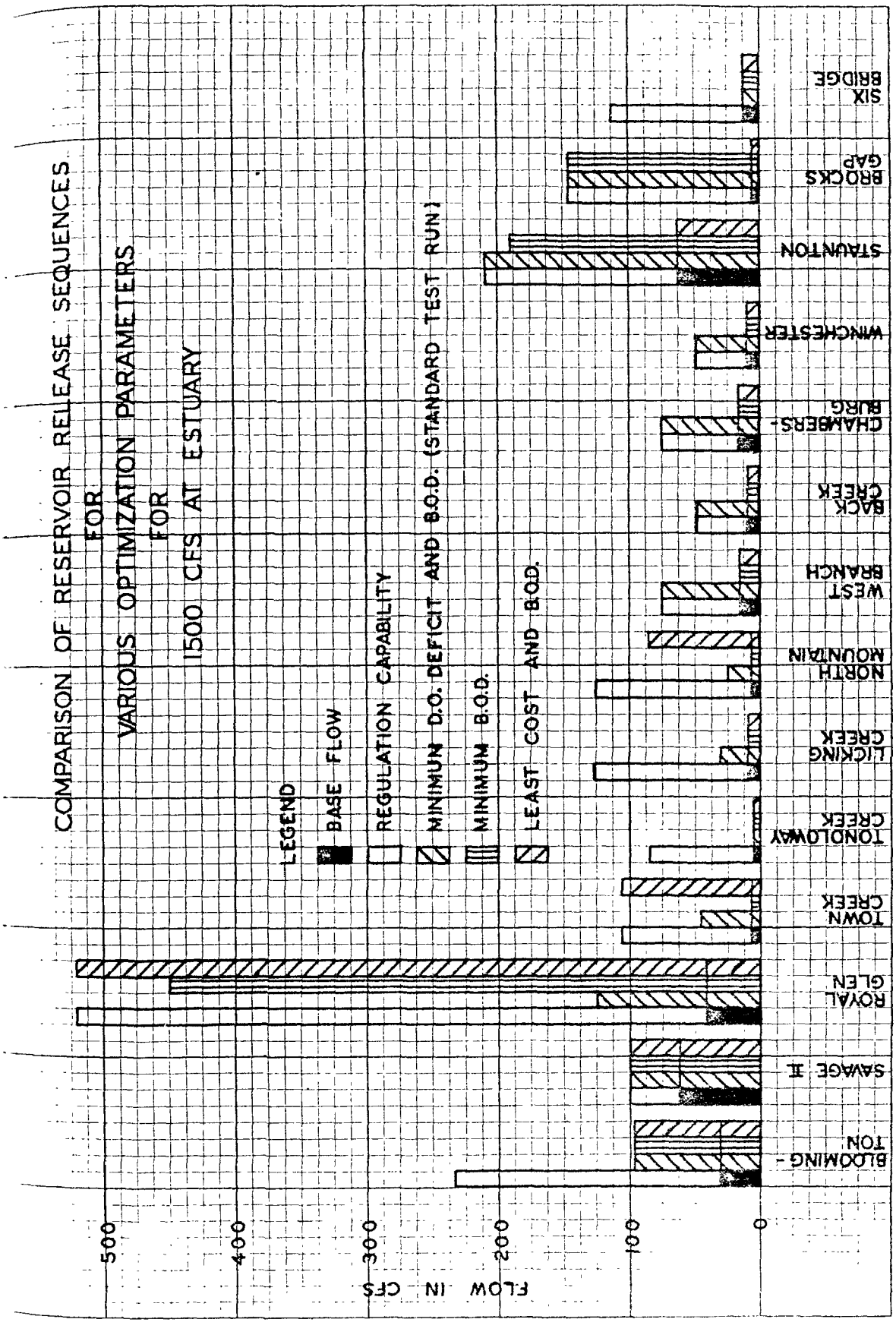

COMPARISON OF RESERVOIR RELEASE SEQUENCES FOR VARIOUS OPTIMIZATION PARAMETERS FOR 1500 CFS AT ESTUARY
LEGEND
BASE FLOW
REGULATION CAPABILITY
MINIMUM D.O. DEFICIT AND B.O.D. (STANDARD TEST RUN)
MINIMUM B.O.D.
LEAST COST AND B.O.D.
FLOW IN CFS
500
400
300
200
100
0
BLOOMING-TON
SAVAGE II
ROYAL GLEN
TOWN CREEK
TONOLOWAY CREEK
LICKING CREEK
NORTH MOUNTAIN
WEST BRANCH
BACK CREEK
CHAMBERS-BURG
WINCHESTER
STAUNTON
BROCKS GAP
SIX BRIDGE

Figure 10

the estuary is minimal. Excluding the runs using a dynamic equilibrium BOD level and imposed BOD loads, the BOD and DO concentrations for 1500 cfs at the estuary vary only about 0.5 mg/l. Since the greatest difference in water quality is in the upstream tributaries of the Potomac, the release sequences for the various optimization criteria are more indicative of upstream conditions and reservoir cost for least-cost solution than they are of conditions in the main stream.

A detailed study on the cost of alternative systems for DO management in the Potomac Estuary has been made by Davis[38]. The effect of developing optimal release patterns for the non-tidal portion of basin on the various alternatives as presented by Davis is beyond the scope of this study.

OTHER POSSIBLE WATER QUALITY MEASUREMENTS

In the above flow release model, primary emphasis is given to the water quality at the decision points. While the constraints guarantee an acceptable water quality in the reach, the entire optimization procedure is based on the quality of the two contributing streams at the decision point.

With judicial use of the quality constraint and well-defined quality algorithms, one can obtain sufficient information for development of a flow release pattern; however, the solution is nonencompassing in regards to the number of stream miles at a given quality level. Three possible measurements indicative of water quality in the entire reach are as follows:

1. Number of stream miles at a prescribed water quality level, e.g., at or above a DO of 6.0 mg/l or at or above a BOD of 3.0 mg/l.
2. Number of mg/l-miles of the wastewater constituent profiles, e.g., the area under the BOD or DOD quality profiles.
3. Similar to Number 2, with the area calculation being prorated according to a predetermined scale, based on the degree of the deterioration of the water quality.

Before any of the above measurements can be applied realistically in flow regulation, it is necessary to assess wastewater treatment requirements in a similar manner. A method for financing waste treatment facilities using the constituent profile has been advanced by the State of Ohio[80].

Another water quality parameter which is receiving much attention today and which is quite indicative of stream conditions is that of nutrient level. High concentrations of nutrients, mainly phosphates and nitrates, usually result in algal blooms, which may result in further deterioration in water quality. An exponential-loss model for predicting the concentration of phosphates in flowing streams was developed as part of this study and is currently being tested at the Chesapeake Field Station. If it is possible to express the loss in phosphates mathematically, the phosphate level could be used as the second return variable, instead of the DOD parameter. See Appendix A for a description of the phosphate model.

LINKAGE TO RIVER FLOW SIMULATION MODELS

The flow release model optimizes in space, i.e., within the physical dimensions of the basin. The solutions are developed for independent time units such as months, seasons, etc. Since the solutions are independent, it is hydrologically possible that reservoir storage required for a given time unit may not be sufficient to operate optimally. The deterministic tableaux can be transformed into statistical solutions with the use of yield curves and validated with linkage to river flow simulation models.

Use of Yield Curves

Yield curves similar to that for the West Branch project can be developed for all given reservoir sites by routing historical or synthetically generated stream flows through the impoundments. (See Figure 11.)

Thirty-six years of historical data were routed through the proposed reservoir site on West Branch of the Conococheague Creek for various reservoir sizes and yield targets using the River Basin Simulation Program[81]. The probability of deficiency in months for each routing and storage capacity was determined for a prescribed uniform use rate.

The probability of failure can be introduced into the flow release sequence simply by developing such curves for each proposed site. The minimum and maximum flow regulation ranges (required input for the flow release model) can be obtained from these curves for a probability of failure. For example, for a maximum of 50,000 acre-feet of storage at West Branch site, the flow range is from a minimum of 20 cfs to a maximum of 120 cfs for 1 percent probability of failure.

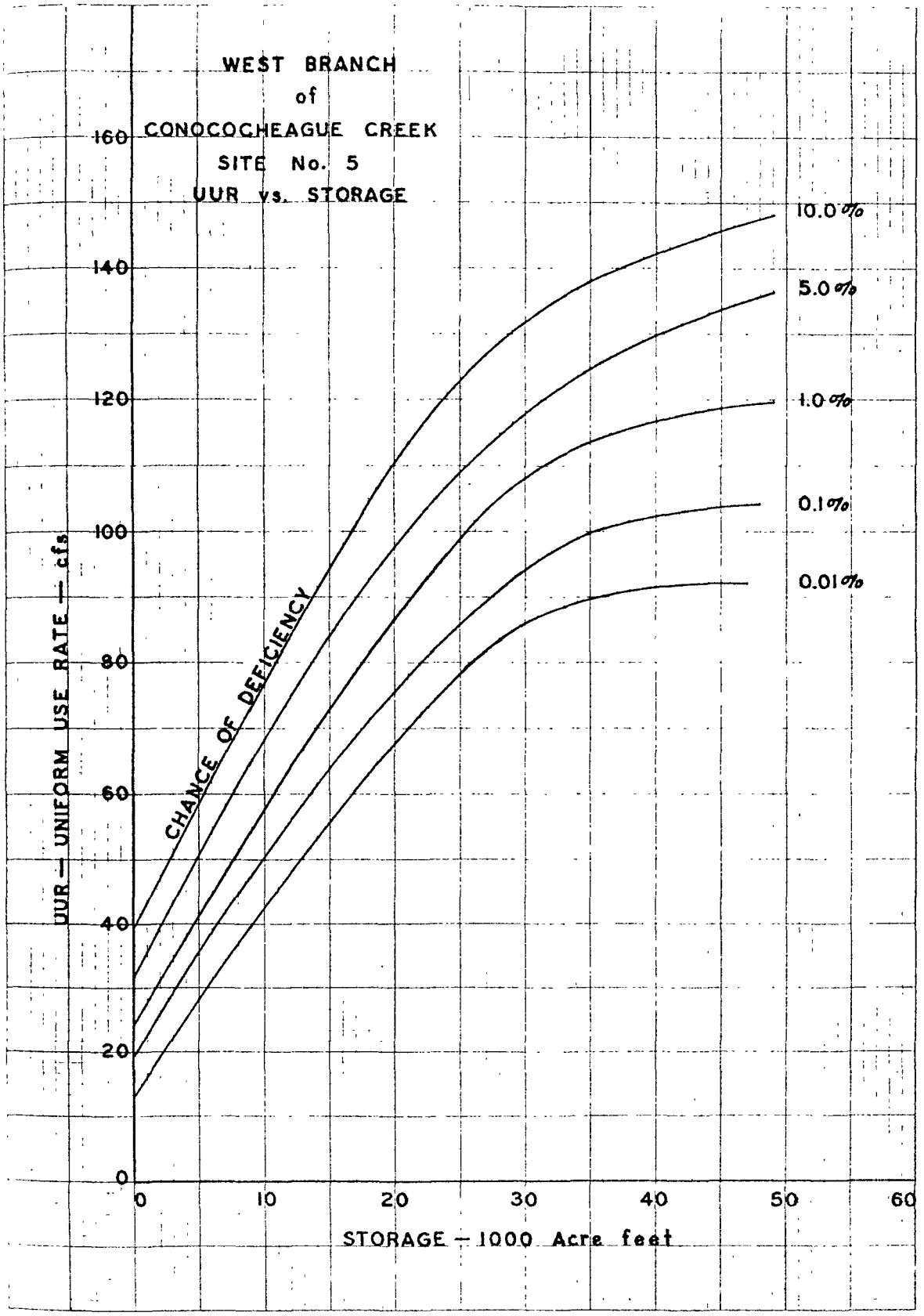

WEST BRANCH
of
CONOCOCHEAGUE CREEK
SITE No. 5
UUR vs. STORAGE
UUR — UNIFORM USE RATE — c.f.s.
STORAGE — 1000 Acre feet
CHANCE OF DEFICIENCY
10.0 %
5.0 %
1.0 %
0.1 %
0.01 %
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60

Figure 11

Using the typical reservoir cost-storage curves as developed by the Corps of Engineers[73], it is possible to transform the Figure 11 into a uniform use rate (UUR) versus storage cost relationship for the various probabilities of deficiency. This transformation is required in the least-cost solutions.

River Flow Simulation Models

In a basin as large as the Potomac, there are great differences in rainfall patterns, ground water, terrain, etc. These differences introduce a joint probability of deficiency for each decision point.

The probability of failure in the flow release sequence developed in conjunction with the yield curves can be further evaluated by using option B of River Basin Simulation Program[81].

The operation patterns as developed from the flow release model are read in as input controls in the simulation model. The option B program attempts to maintain the prescribed releases for each reservoir for each time unit.

A more sophisticated model for river flow simulation has been developed by the Corps of Engineers and greatly expanded by Resources for the Future[82]. The simulation program has the advantage of incorporating an operating policy (release rules) that is consistent with the multiple purpose uses served by the proposed reservoirs. The release rules accomplish the objectives of maintaining reservoir pools for reaeration and flood control, as well as meeting downstream flow requirements for water supply, quality control, etc.

The feature of being able to prescribe preference release patterns for the various control points (or decision points) makes the simulation program very amenable to the solutions generated by flow release

models. From the IFT's and DFT's as described in Chapter V, it is possible to determine optimal release patterns. Further, from the tableaux, it is also possible to determine for each decision point the next best level of operation. Thus, from the flow release model, one can not only determine the specific release sequence needed to operate the system optimally, but also determine the next best operational pattern in case there is insufficient storage to meet the optimal release sequence.

By using the output data of the flow release model as input data for the river flow simulation program, a linkage in determining flow requirements including the probability of failure can be realized. This linkage also can be used to statistically evaluate the significance of various physical, biochemical, design, and socio-economic parameters in regard to failure frequencies.

LINKAGE TO ESTUARY AND RIVER BASIN QUALITY - SIMULATION MODELS

In the application of the Thomann steady-state, Thomann time-dependent, or the O'Connor estuarine models, boundary conditions of flow, BOD, DO, and temperature are required. The final tableaux from flow release program contain these states, and also costs for regulation, optimally determined for each incremental flow state. By using the results of the tableaux as input for the estuary models linkage can be obtained.

A complete simulation program would be required to completely evaluate a water quality control program for an entire river basin, including its estuary. The following considerations should be incorporated in such a basin model.

1. A mechanism for stream flow generation.
2. A mechanism for stream temperature generation.
3. A model for determining the optimal reservoir release sequences conditional on the status of the reservoirs.
4. A routing of stream flows according to optimal release sequences while maintaining given prescribed conditions in the individual reservoirs.
5. A routing of the water quality in non-tidal portion of the basin to develop the boundary conditions at the estuary.
6. A routing of the water quality in the estuary.

The simulation could be either steady-state or time-dependent.

Algorithms for many of the above steps currently exist. Since the overall program would be enormous, it probably would have to be accomplished in various links. A proposed flow chart showing the linkage of the various algorithms is presented in Figure 12.

Boundary conditions for the upper stream terminal points and imposed wastewater loads would be read in as input data. The upper boundary conditions of the estuary model would be generated from the upstream portion by the verification link.

The development of an overall basin water quality simulation model is the next logical step in evaluating any proposed water quality management program.

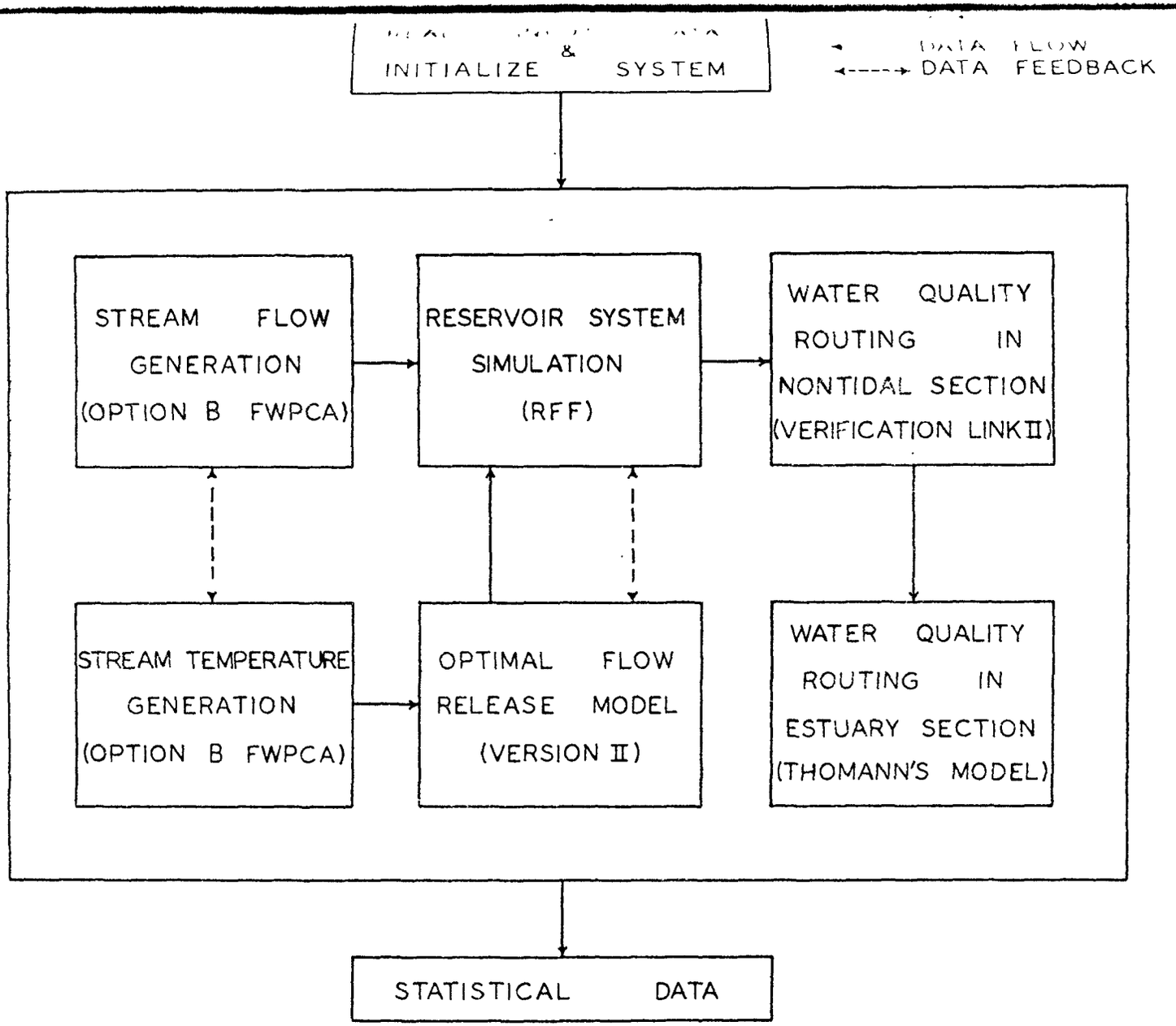


Figure 12

LINKAGE FOR RIVER BASIN WATER QUALITY SIMULATION

IN-STREAM IMPOUNDMENTS

In the flow release model, it has been assumed that there are no in-stream impoundments capable of flow regulation. The imposition of in-stream impoundments adds additional decision points in the flow regulation system. Further, in these impoundments the quality formulations as given in Chapter III are invalid.

One possible solution is to develop the release patterns in reaches bounded by the up-stream reservoirs and in-stream impoundments. The problem would be to (1) maintain a prescribed quality in the reach, and (2) to optimize the water quality entering the in-stream impoundments. For the lower reaches, assumptions would have to be made on the water quality leaving the in-stream impoundments. Models for predicting water quality from impoundments are currently being developed by Churchill and Nicholas[83] and Symons et al.[84].

Another possible solution is to expand the flow release model to include the in-stream impoundment decision points. The structuring of the problem would be very similar to that of the converging branch network proposed by Nemhauser[72] and more recently by Meier and Beightler[85]

In expanding the model, it would be necessary not only to add an algorithm for optimization at the in-stream impoundments, but also to incorporate quality formulations through the impoundments. The quality formulation and coefficients will probably be specific for a given impoundment, thus it is probable that additional input data will also be required. Since the quality formulations are not fully developed and it is possible to develop the release patterns in the bounded sections, a model which incorporates in-stream impoundments would not yield any additional beneficial use at the present time.

CHAPTER VIII

DISCUSSION OF RESULTS

The adequacy and the compatability of the water quality formulations with the optimization criteria, and optimization concepts and adaptation are discussed in this chapter. Specific and general uses of the optimal release sequence are discussed in the latter part of this chapter.

QUALITY FORMULATIONS

In Chapter VI, the spatial sensitivity of the reservoir release patterns to various biochemical and physical, design, and socio-economic parameters is presented. From the release sequences presented in Chapter VI and the quality profiles shown in Figure 16, it appears (1) that the release patterns are sensitive to changes in quality parameters such as reaeration, velocity, temperature, etc. in the reaches receiving large volumes of wastewater, demonstrating the need for well verified quality formulations; and (2) that in relatively unpolluted reaches, the quality formulations, especially the BOD algorithm, used in this study were not adequate.

Two possible solutions to the inadequacy problem are (1) the inclusion of all sources and sinks of oxygen in the formulation, as reviewed in Chapter II, or (2) the assumption of a minimum dynamic equilibrium BOD level by reaches. A major factor as to which approach is taken in selecting a given formulation should be the compatability of the quality algorithms with the optimization criteria, including the indicators or measurements of overall water quality in the basin.

For the optimization criteria of BOD and DOD concentrations at th decision points, the formulation should include all sources and sinks. If a minimum dynamic equilibrium level for BOD is used, the optimization criteria should be coupled to more encompassing indicators of water quality, such as the area under the constituent profiles as suggested in Chapter VII. The ability to take either approach with only slight modification in the algorithm as presented in the previous chapter and as discussed in the next section of this chapter makes the flow release model developed in this study a very flexible and powerful planning tool.

OPTIMIZATION CONCEPTS AND ADAPTATIONS

As described in Chapter III, the optimization procedure used in this study was an enumeration process. In conjunction with the descri tive paragon and the converging branches multistage system of dynamic programming, the enumeration process was very effective in developing the optimal flow release patterns for the proposed reservoirs in the Potomac Basin.

With the converging concept, the number of feasible solutions retained for each stage or decision point was a function of the numerical range of the first return variable (BOD), the distribution of the values of BOD within this range, and the size of the flow increment. Since a uniform flow increment was used for a given test run, the number of solutions retained per flow state was mainly dependent on the range and to a lesser extent on the distribution of BOD at each decision point.

In the optimization procedure, the range of first return variable controls the size of the BOD increment per flow state. The selection of one of five possible increment sizes depending on the range provides for maximum variation in water quality without a corresponding loss in sensitivity when the ranges of BOD are small.

During the enumeration process, only one feasible solution is retained per BOD increment. If a large BOD grid is used or if the range of BOD is very small, only one feasible solution is retained per flow increment.

For most of the test runs excluding the least-cost solutions in the Potomac Basin, the range of BOD per flow state was usually less than 0.5 mg/l. For the stages with a BOD range greater than 0.5 mg/l (the minimum increment used in the test runs), more than one solution was retained per flow state. However, in succeeding downstream decision points the range of the BOD values converged and the addition of feasible solutions were eliminated in the enumeration process.

Upon close examination of the optimal solutions for many of the stages, it was observed for a given flow state that when the release pattern yielded the minimum BOD it usually yielded the minimum DO deficit. While the two minimums do not necessarily have to occur simultaneously, this observation indicates that the second return variable (DOD) is not significant in the optimization process. This observation led to the development of the least-cost solutions as presented in Chapter VII.

The ability to control the number of feasible solutions which are carried forward in the converging concept makes the enumeration process as described above a very powerful optimization procedure. Moreover,

eliminates the need for either a complete enumeration of all feasible solutions or some other complex or time consuming optimization process such as steepest descent.

The enumeration process can be used with both non-cumulative and cumulative return variables. The need for continuous and smooth return functions is completely eliminated.

In general, the optimization technique used in this study is simple yet very flexible. Maximum use is made of water quality ranges at each decision point; and, therefore, the optimization procedure can be considered forward looking. Also, with the proper use of the input DO grids and increments, a balance between the accuracy of water quality predictive formulations and optimization capabilities can be easily realized.

One of the disadvantages of the dynamic programming technique such as employed by Liebman[56] in comparison to linear programming is the lack of sensitivity analysis associated with the dual variables in the simplex solution. The enumeration process and resulting IFT's and IRT's developed for this study overcome this disadvantage.

For all decision points in the converging branch system, it is possible to investigate the sensitivity of various controlling parameters and to determine the effect of selecting a sub-optimal solution in any particular reach on the overall release sequence pattern. These added features of the model produce a mechanism for possible "trade-off" within the system.

The optimal reservoir release sequence as presented in Figures 7 through 10 has been developed for 1500 cfs of flow at the Potomac gage. While the "principle of optimality"[72] is maintained for

all upstream decision points or stages, the optimal solution for the estuary may not always provide the "best" solution for a particular local situation. The preference of a sub-optimal solution can occur when the quality difference at decision points is small and the reaches below it receive an insignificant amount of biodegradable wastewater.

An example of this condition for the standard test run is at the decision point for node 460. The optimal solution requires a total of about 200 cfs from the Bloomington and Savage Reservoirs, with about 120 cfs from Royal Glen impoundment. If the 60 cfs above the base flow Royal Glen were to be released from Bloomington, the minimum DO level in the North Branch would be increased by 0.5 mg/l to 4.5 mg/l. The additional release from Royal Glen contributes very little towards increasing the DO level in the South Branch, since its minimum DO level is greater than 5.0 mg/l.

The release of the additional 60 cfs from Bloomington will increase the BOD and DO deficit at node 460 to about 0.5 mg/l. However, this small increase will be greatly attenuated in reaches below node 460. This can readily be seen when the BOD and DOD concentration of the standard test runs for nodes 460 and 436 in Tables 6 and 7 are compared.

If a return variable more indicative of overall water quality as suggested in the previous section were employed, the additional 60 cfs of flow would be from Bloomington instead of Royal Glen. In lieu of using another measurement of water quality, local situations can be resolved simply by making a series of computer runs of the flow release model at different treatment levels. For decision points at which the quality difference is minimal and there are not any critical

reaches below it, a trade-off between reservoirs can readily be made when the release rates are established from the IFT's and DPT's.

This type of a trade-off can also be made when economic considerations are incorporated into the optimization procedure as in the least-cost solutions. The development and use of release sequences including least-cost solutions for flow regulation and wastewater treatment are discussed in the next section.

USE OF OPTIMAL FLOW RELEASE SEQUENCES

As listed in Chapter VI, five flow release sequences can be developed, depending on the type of parameter employed in the optimization routine. The reservoir release rates, which are usually developed for monthly units of time, are also a function of temperature, treatment levels, reservoir conditions, etc. If monthly variations of the above are significant, the release rates have to be determined for each individual month.

From the individual IFT's, one can also determine the minimum flow requirements to maintain a given water quality objective. By comparing a series of tableaux at various treatment levels, the effect of wastewater treatment policies or flow regulation requirements can be readily observed. A similar approach can be taken in studying the effect of various water quality objectives.

Another valuable use of the output of the flow release model is its ability to investigate alternative methods and use trade-offs. For example, at node 494, the total cost of reservoir storage for water quality control in North Branch from Bloomington and Savage II will be about $10,000,000 for a DO objective of 4.0 mg/l. If the

quality constraint is raised to 4.5 mg/l the cost of storage is increased about $16,000,000.

The additional 60 cfs of flow needed at node 460 to meet the requirement at the estuary can be obtained from Royal Glen at a cost of about $2,600,000. If additional wastewater treatment in North Branch can result in an increase of the minimum DO level to 4.5 mg/l for less than $3,400,000 ($6,000,000 minus $2,600,000) a more economical solution can be obtained by the trade-off.

Least-cost solutions for various combinations of flow regulation and wastewater treatment levels can be readily added to the flow release model. This additional feature is not needed in the Potomac basin as most of the waste loads are not overlapping and trade-offs can be readily made outside of the model as illustrated in the above example. If the need arises for the inclusion of the additional stages in each waste discharge, consideration should also be given to the incorporation of other measurements of water quality as return variables.

USE OF WATER QUALITY MANAGEMENT MODELS WITHIN THE FRAMEWORK OF CURRENT TECHNOLOGICAL AND INSTITUTIONAL PRACTICES

The model as developed in this study represents the first known attempt to determine an optimal flow regulation scheme for an entire river basin based on water quality considerations. One of the significant findings of the study is that most of the optimal solutions for proposed reservoir systems in the Potomac Basin are predicated on BOD and DOD concentration differentials of less than 0.5 mg/l. That is, the selection of the optimal solution for a given flow state is from a combination of release sequences which has a resulting BOD and DOD concentration differentials of less than 0.5 mg/l.

While it is possible mathematically to define an optimal solution (meaning either best quality of water for a given flow state, or minimal flow or storage requirement for a given quality objective), the reduction in storage requirement or the increase in water quality level based on these solutions for the proposed Potomac reservoir system appears to be small. The location of proposed reservoirs, low incidence of overlapping of pollution loads, and the small difference in quality of the two contributing tributaries at the decision point tend to make the quality problems in the various sub-basin areas somewhat independent of one another. Moreover, least-cost waste treatment solutions as reviewed in Chapter II would appear to have limited use in the non-tidal portion of the Potomac Basin.

While it has been clearly demonstrated that the reservoir release patterns are sensitive to various biochemical, physical, design, and socio-economic parameters, the magnitudes of the release rates appear to be dictated by quality constraints in the critical reaches, and/or other non-quality needs such as water supply in the various sub-basins. Once the quality constraints are satisfied in the critical reaches by either flow regulation or waste treatment, the effects of increased treatment or regulation on water quality at the decision point are minimal. This effect will be further reduced when the full effect of the 1965 Water Quality Act is felt.

Intensive stream surveys have been conducted, primarily for formulation verification, in 1967 by the Chesapeake Field Station. For 30 stations sampled around-the-clock, considerable diurnal variations in T and DO deficit have been observed. The mean standard deviations of T and DO deficit were 2.06 and 2.15 mg/l, respectively. While it can

be argued that mean values represent a net balance without utilizing time-dependent formulations, the BOD and DO ranges used in the optimization were greatly exceeded by the diurnal fluctuations.

With time-dependent quality formulations including all sources and sinks of oxygen, a more sensitive model could be realized. However, will the additional cost of data collection, analysis, and interpretation increase the significance of an optimal solution within the current institutional practices of the water resources planning agencies?

Based on expenditures for the Potomac data presented in Appendix B, it will cost approximately $600,000 to obtain sufficient information for a time-dependent model. Assuming about equal expenditures for an engineering evaluation of the data, the total cost could easily exceed $1,200,000.

For compatability purposes, the time-dependent quality formulations should be coupled to new "yardsticks" for overall basin quality. Until the methodology is developed and necessary data collected for assessing the monetary value of a given water quality and quantity level at a given failure rate, the present water quality control models appear to be adequate.

For an overall water resource management model, the problem can be expressed as maximizing the benefit-cost ratio. Benefits per water resource use per stream reach would be a function of quality, quantity, and failure rate. Such a concept could be easily incorporated into the framework of the flow release model as presented in the previous chapters.

Even with overall optimal planning models, will the institutional arrangements be adequate to act to implement the optimal solutions? Appropriations of funds for water resource planning by the U. S. Congress have not always been dictated by monetary considerations.

Haveman[90] in an analysis of Federal expenditures in ten southern states concluded the following:

1. "Although projects with the highest benefit-cost ratios tend to be chosen before projects with low ratios, no consistent or persistent pattern is manifested" and

2. "A somewhat stronger and more consistent motive seems to be present in both force of aid to low income and depressed areas and the drive to exploit development potential in areas of substantial opportunities to productive resources investments."

This appears to be the case in the Potomac River Basin. From the initial studies in the 1940's up to the present, the major construction activities have been in the Appalachia Region. This region is currently defined as a socially and economically "depressed" area.

While at the Chesapeake Field Station, the writer also had the opportunity to study and field test other water resources management models. In the opinion of the author, the most urgent research needs in the area of water quality management are as follows:

1. Inexpensive methods for easily obtaining large quantities of field information such as cross-sectioning, time-of-travel, chemical parameters, etc.;

2. More precise and readily adaptable water quality formulations especially for low BOD stream reaches and in low level impoundments;

. Continued development of methods for projecting industrial water supply requirements and resulting wastewater loadings;

.. Methodology and necessary information for assessing, by individual water user, the monetary value of given water quality and quantity level for a given failure rate; and

. Socio-economic significance of optimal river basin planning within the framework of current institutional practices.

... operational viewpoint, the research areas delineated above ... of the next barriers to overcome in effective river basin ... ment studies.

CHAPTER IX

SUMMARY AND CONCLUSIONS

This study has been concerned with the development and field testing of a method for determining optimal flow release sequences for water quality control from multiple reservoir systems. It has also been the intent of this study to investigate, using an actual river basin, the sensitivity of the release sequence to changes in various physical and biochemical, design, and socio-economic parameters.

The flow release problem was structured as a converging branch, multistage dynamic programming decision system. An optimal flow release model has been developed with the decision-making procedure at each stage (the confluence of two regulated streams) being an efficient enumeration process. Using both single and dual return variables, the model has five options for determining optimal release flow sequences for a given flow state at a decision point. These are: (1) minimum DO deficit for a given BOD state; (2) a minimum DO deficit; (3) least-cost of reservoir storage for a given BOD state; (4) least-cost of reservoir storage; and (5) a minimum BOD.

Predictive algorithms for temperature, BOD, and DO have been incorporated in the flow release model. Ability to vary stream velocity and depth with flow has also been incorporated into the model.

A general descriptive mathematical paragon capable of representing the stream flow system of a river and its network of

tributaries including its hydraulic characteristics and location of all impoundments, waste discharges, water intakes, etc., has been developed. In the flow release model, the paragon provides the required mathematical linkage for both the quality formulations and converging branch multistage decision system to the physical features of the basin.

Optimal reservoir-release sequences from the proposed impoundments in the Potomac River Basin have been developed for the control of water quality at mile point 116.0 (the start of the Potomac estuary). In developing the various optimal release sequences, a minimum preset dissolved oxygen level had to be met in all regulated systems.

The sensitivity of the release sequences developed by the flow release model to various physical and biochemical, design, and socio-economic parameters has been investigated for the proposed reservoir system in the Potomac River Basin. Comparisons have been made to a standard test run for a flow target of 1500 cfs at the Potomac estuary. The conclusions of the sensitivity analysis and comparisons are:

1. Verification of the predictive quality formulations is essential. This requires well-established time of travel coefficients, which appear to be the most sensitive of the physical and biochemical parameters investigated. A method including a specially designed mathematical model has been developed to aid in verifying the predictive quality formulations.
2. Of the remaining physical and biochemical parameters, the reaeration coefficient was the most difficult to define and appears also to have a great effect on the release patterns.

3. Changing the quality of the water in the reservoir from 8.0 to 5.0 mg/l and 2.0 mg/l to 3.0 mg/l for DO and BOD, respectively, had only a small effect on the release pattern.
4. The release rates and spatial sequence can be affected by changes in waste loadings, stream temperatures, water quality objectives, DO concentrations in waste effluents, etc.
5. The location of wastewater discharges, impoundments, etc., and the orientation and geomorphology of the river basin play very important roles in determining release sequences. Therefore, no general conclusion can be made as to which parameter has the greatest effect unless specific conditions are stipulated.
6. The choice of the optimization criterion has a great effect on the release sequence for a given flow rate at the estuary; however, the resulting release pattern has only a minimal effect on water quality entering the estuary.
7. When dual return variables (DO deficit and BOD) were used in the optimization routine, the second return variable, DOD, was significant only for some of the upstream decision points, thus indicating a need for only one return state indicative of water quality.
8. The predictive algorithms used in the model for BOD and DO for relatively unpolluted reaches appear to be inadequate. The imposition of a dynamic-equilibrium for BOD overcomes some of the shortcomings; however, the effects of the imposition on the release sequence are significant.
9. Even with better predictive formulations, the use of concentrations, BOD and DOD, as return variables at the decision

points are not encompassing indicators of water quality conditions in the contributing reaches.

10. A need exists for a new indicator of water quality, such as the area under the constituent profile of a pollutant. The employment of this indicator for the first return variable, with cost of reservoir storage being the second return variable, would provide for a more meaningful approach in forming optimal release patterns for water quality control.
11. One of the most difficult aspects of implementing the flow release model is the obtaining of adequate, systematically collected field data. There is a definite need for better, faster, and less costly methods for collecting and analyzing field data.

A method for overcoming the deterministic nature of release rates from reservoirs has been suggested. Included in the method are the use of yield curves and linkage to river flow simulation models.

In summary, the optimal flow release model with the efficient enumeration process which was designed to make maximum use of the ranges of the return variables and to provide the compatability between the quality formulations and the optimization criteria has utility for:

1. developing an optimal flow release sequence from multiple-reservoir sites to maintain a given water quality objective, flow requirements, or for the least-cost of reservoir storage;
2. determining flow regulation needs to meet a given water quality objective for a particular stream reach in the basin;
3. investigating the sensitivity of the release sequence to various parameters;

4. demonstrating the effects of flow regulation on water quality;
5. providing a mechanism for "trade-off" between flow regulation cost and wastewater treatment costs; and
6. locating sites for possible industrial plants which would allow maximum flexibility with minimum effects on flow requirements.

A river basin model for simulating water quality is proposed in the latter part of the study. The simulation model, a logical next step, would include linkage to synthetic hydrology, river flow routing, optimal flow release, non-tidal quality routing, and estuarine models.

The model developed in this study has been used to determine optimal reservoir release sequences for water quality control under various conditions in the Potomac River Basin. The small ranges of BOD and DO deficit concentration differentials for a given flow state in the optimization procedure has been one of the most significant findings of the application phase of the study. This finding indicates that the solutions of water quality problems in most of the sub-basin are independent of one another, thus optimal reservoir release sequences based on quality control or even least-cost wastewater treatment solutions have limited significance unless a new measure of overall basin quality is utilized.

Future developments in modeling should not only include better quality formulation and measure of water quality, but also include better methods for quantizing the monetary value of water per stream reach per use based on quality, quantity, and failure rate. To avoid unnecessary expenditures and redundancies, there also exists a need for a balance of technical competency, data collection, benefit analysis, and project evaluation.

APPENDIX A

SOME COMMENTS ON MODEL VERIFICATION

In the dissolved oxygen and BOD formulations, the three most difficult and costly parameters to evaluate are the deaeration coefficient, reaeration coefficient, and time-of-travel. Some of the problems which have been encountered in the study, possible solutions and a delineation of areas in which more field data and research are needed to fully cope with the problems are presented in this appendix. A proposed model for predicting phosphates in flowing streams is also presented.

A fully detailed analysis of each of the problems is beyond the scope of this study; however, the author is of the opinion that the particular problems presented in this appendix are basic in model verification.

REAERATION COEFFICIENT

The three methods currently being used for determining reaeration rates in a given stream reach are as follows:

1. calculated from observed data;
2. prediction formulations, either empirical or theoretical; and
3. gaseous tracer technique.

Each of the methods has merit, depending on the situation, available forces, and projected use. For the flow release model, a method was used which could be used to determine the reaeration rate for all reaches of the basin for various ranges of stream flow.

Calculated from Observed Reaeration Rates

In establishing the reaeration rates from observed field data, two surveys are required for a minimum of two different stream flows with deaeration rates and time-of-travel well established. One of

the major disadvantages of this method is the great dependence of the reaeration calculation on the accuracy of the measured deaeration rate and time-of-travel, and the ability to identify and quantize other sources and sinks of oxygen. An error in determining the time-of-travel is reflected in the deaeration rate and must be compensated for in establishing the reaeration coefficient.

For example, in 1954 while studying the North Branch of Potomac River, Fuhrman[78] over-estimated the travel time below Keyser by about 65 percent, depending on the discharge rate. This over-estimation resulted in a large deaeration coefficient which was artificially compensated for by a large reaeration rate in order to verify the stream data. The coefficients and data as developed by Fuhrman could be used to reflect the conditions in the stream for the given conditions; however, the rates as calculated were not a true measure of the reaeration capability. Calculations of the waste treatment required on the observed rates overestimated the assimilation capacity by about 300 percent below Keyser, West Virginia.

Another disadvantage of this method is that the stream must be significantly polluted so that there is measurable dissolved oxygen deficit in the stream in all sections for which reaeration rates are required. For predictive algorithms of the flow release model and verification link, the observed method of determining reaeration rates is the most useful as a check on the other two methods.

Predictive Formulations

Streeter and Phelps[1] empirically related the reaeration rate to the depth and velocity stream parameters. Velz[4,5], expanding the concepts of Phelps[66], also developed an empirical method which relates the reaeration rate to the depth and "time of mix" in the stream channel.

In the past decade a considerable amount of research effort has been exerted in developing various formulations for computing the reaeration coefficient based on the physical properties of the stream channel. Using various gas transfer theories, predictive formulations have been proposed by O'Connor and Dobbins[86], Krenkel and Orlob[63], and more recently by Dobbins[13,14]. The formulations relate the reaeration rate to the physical parameters of the stream, such as velocity, depth, slope, longitudinal diffusion, etc.

A statistical approach was taken by Churchill et.al.[63] of the Tennessee Valley Authority (TVA) in investigating various formulations for determining the reaeration rates in the Tennessee Valley. In their studies, they concluded that a simple equation relating the reaeration rate to velocity and depth was adequate to describe the process, and that parameters such as slope and roughness are automatically included in the formulation. A similar approach was taken by Langbein and Durum[87] of the U. S. Geological Survey (USGS) in which the reaeration rate was determined to be primarily a function of velocity and depth.

Gaseous Tracer Techniques

A method for accurately evaluating atmospheric reaeration rate using a radio-active gas tracer has been developed by Tsivoglou et.al.[88]. The technique and the theoretical concepts have been developed in a laboratory with limited field testings. Until the method is more fully proved, its value is rather limited, and the need for a complete evaluation of predictive formulations remains.

Comparison of Methods

To gain insight as to which predictive formulation in general most accurately describes the reaeration process for various streams, a

...rison has been made for four different stream reaches. A complete ...luation is beyond the scope of this study; however, due to the sensi... ...ity of the flow release model to the reaeration parameter, the limited ...parison is sufficient to illustrate the variations.

The basic data of the four reaches and the reaeration rates calcu...ted by the various methods are presented in Table 15. As can be seen ...Table 15, excluding the tracer technique, no one method adequately ...edicts the reaeration rate in all four of the reaches when compared to ...e rates calculated from observed data.

The divergence of the O'Connor and Dobbins, TVA, and USGS formula...ons is readily seen in Figures 13, 14 and 15. In the O'Connor and ...bbins formulation, velocity is raised to the 0.5 power, thereby reduc...g the sensitivity of the reaeration rate to the velocity term. The ...A and USGS formulations, with velocity raised to the 0.969 and 1.0 ...wers, respectively, yield a greater sensitivity, especially at the ...lower depths.

Computed DO, BOD and temperature profiles for the North Branch of ... Potomac River are presented in Figure 16. The DO profiles were com...ed using the TVA and O'Connor reaeration formulations. The effect of ...ss-section transformation and a depth-factor is also shown. This will ...vered in detail in the next two sections of the appendix.

...F PREDICTIVE FORMULATIONS

...efore any one of the prediction formulas can be used basin-wide, ... probably be necessary to adjust the reaeration coefficients to ...them similar to the values computed from observed data. The adjust... ...an be applied to over-all formulations or to one of the constituent ...ders such as velocity or depth.

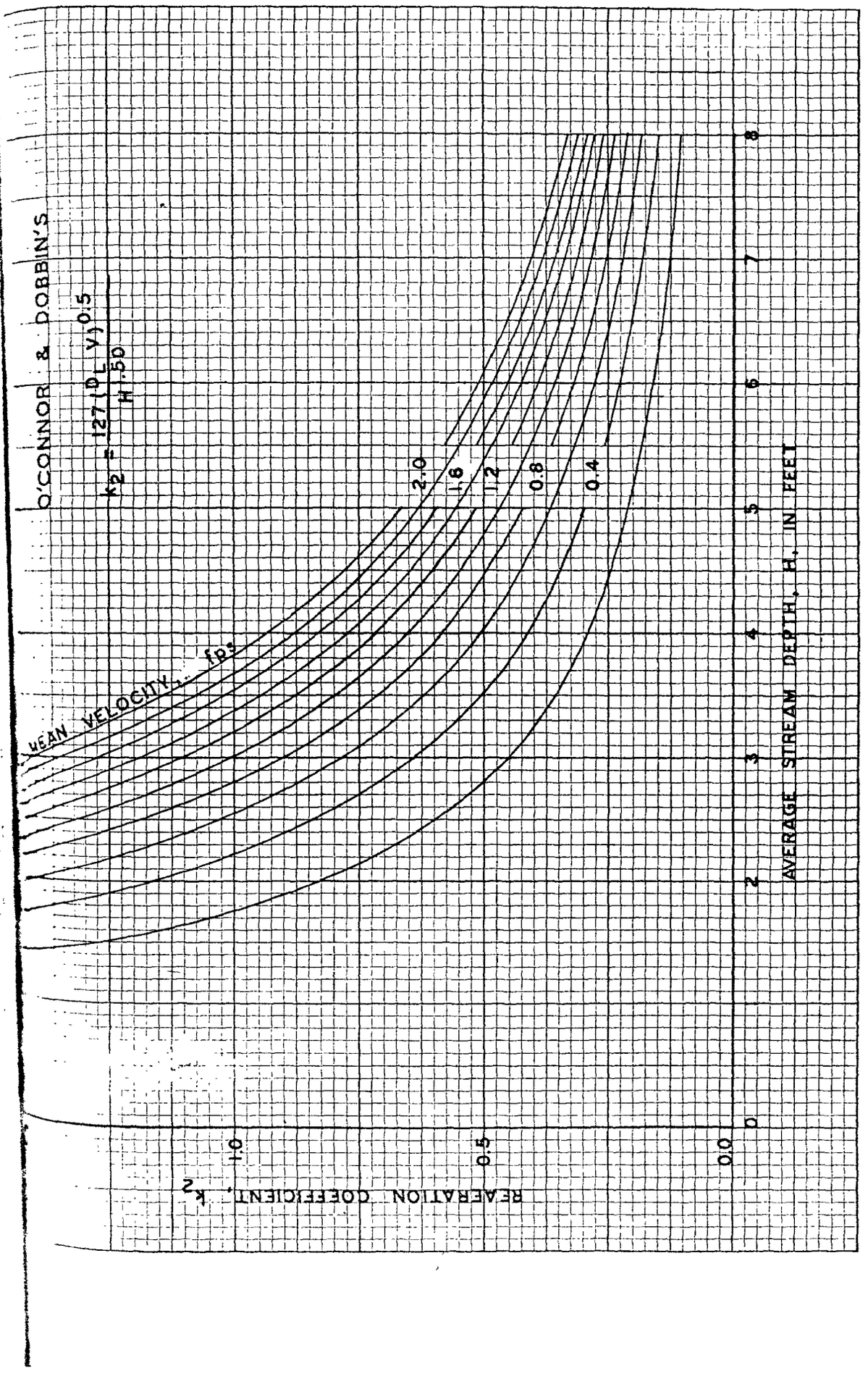

O'CONNOR & DOBBIN'S
$k_2 = \frac{12.7(D_L V)^{0.5}}{H^{1.50}}$
MEAN VELOCITY, fps
2.0
1.6
1.2
0.8
0.4
AVERAGE STREAM DEPTH, H, IN FEET
REAERATION COEFFICIENT, k_2
1.0
0.5
0.0
0
2
3
4
5
6
7
8

Figure 13

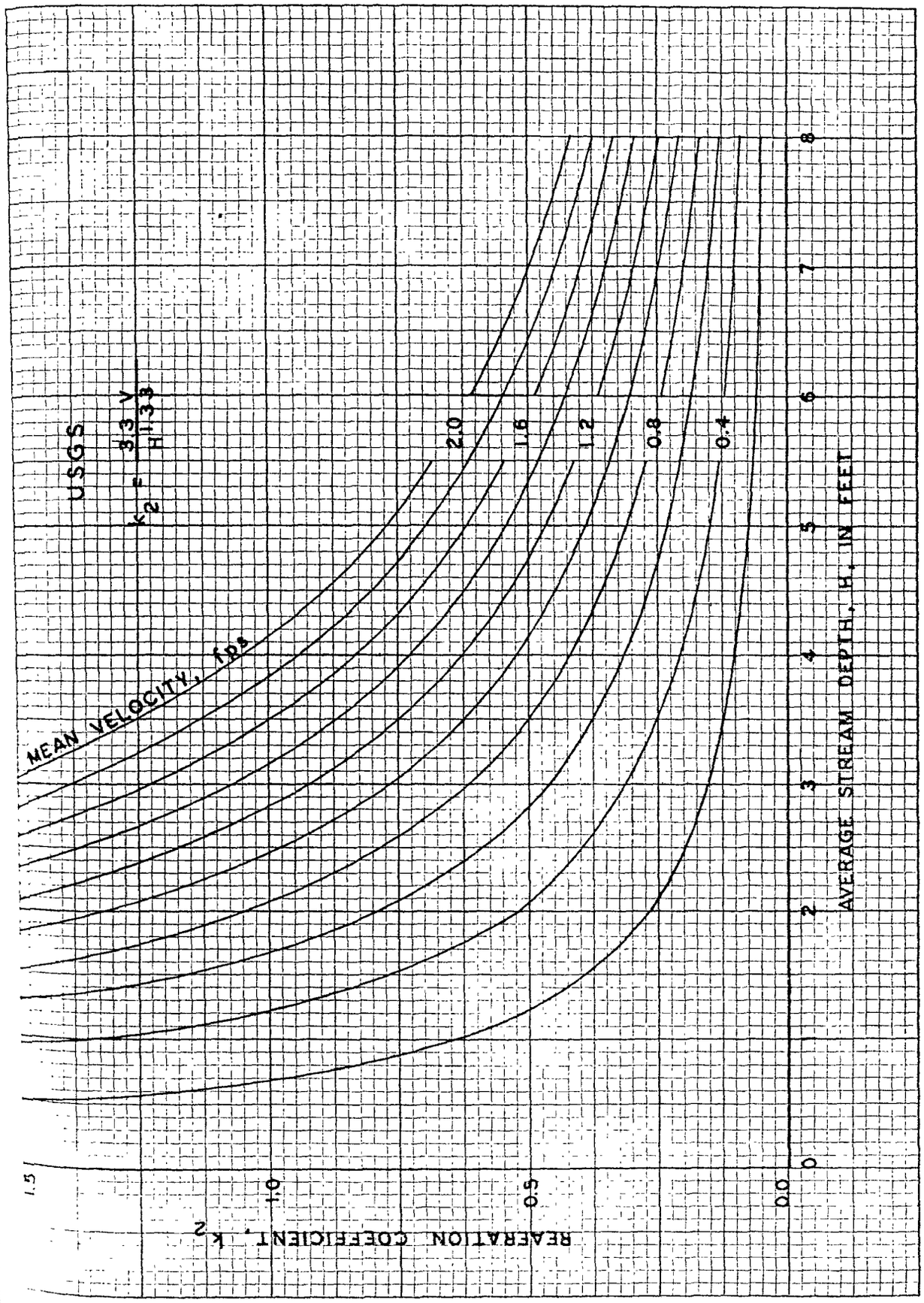

USGS
$K_2 = \frac{3.3\ V}{H^{1.33}}$
MEAN VELOCITY, fps
2.0
1.6
1.2
0.8
0.4
AVERAGE STREAM DEPTH, H, IN FEET
0
1
2
3
4
5
6
7
8
0.0
0.5
1.0
1.5
REAERATION COEFFICIENT, k_2

Figure 15

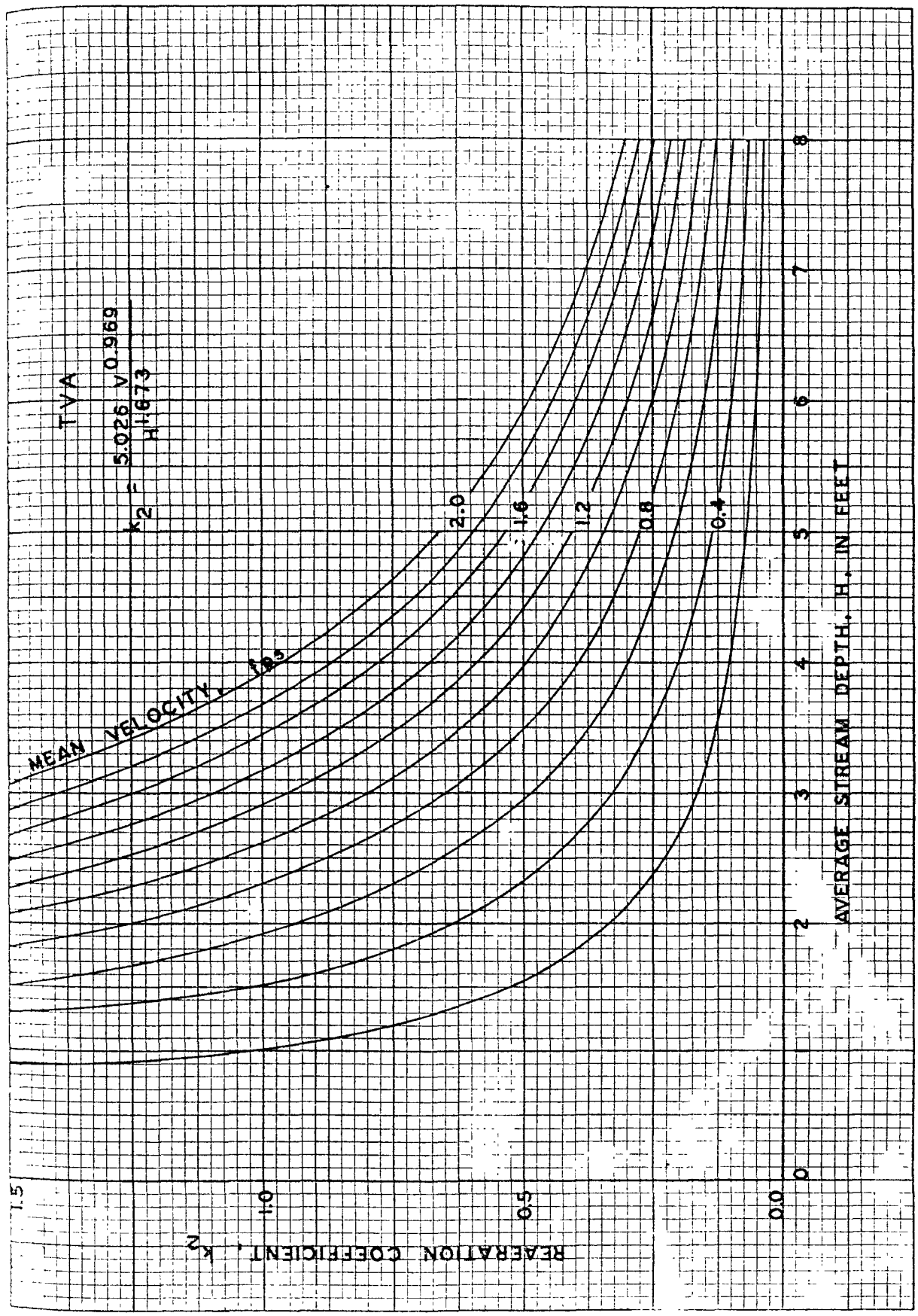

TVA
$k_2 = 5.026\ V^{0.969}\ H^{-1.673}$
MEAN VELOCITY, V
2.0
1.6
1.2
0.8
0.4
REAERATION COEFFICIENT, k_2
AVERAGE STREAM DEPTH, H, IN FEET
0.0
0.5
1.0
1.5
0
1
2
3
4
5
6
7
8

Figure 14

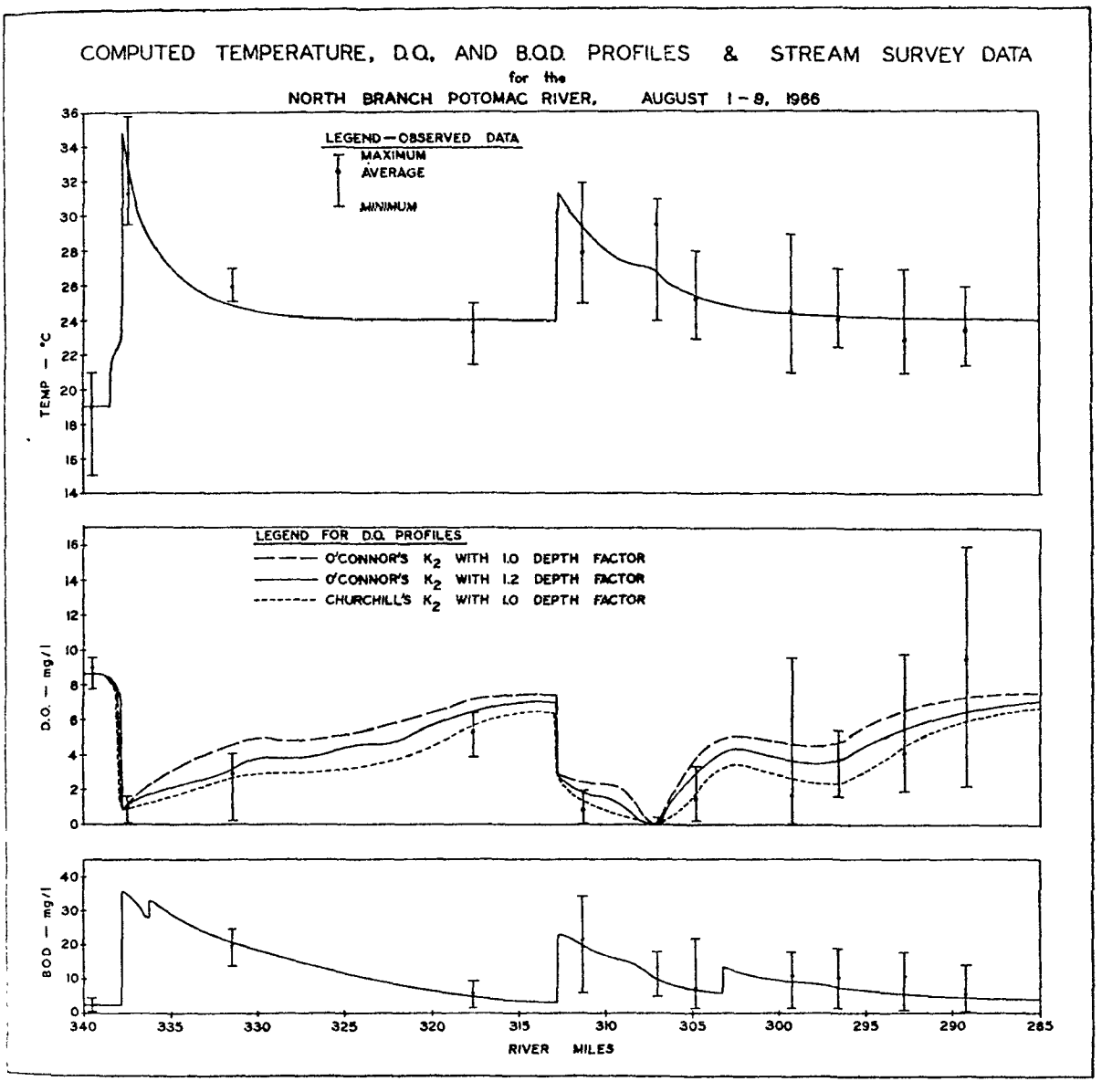

COMPUTED TEMPERATURE, D.O. AND B.O.D. PROFILES & STREAM SURVEY DATA
for the
NORTH BRANCH POTOMAC RIVER, AUGUST 1 - 8, 1966
LEGEND — OBSERVED DATA
MAXIMUM
AVERAGE
MINIMUM
TEMP — °C
LEGEND FOR D.O. PROFILES
O'CONNOR'S K_2 WITH 1.0 DEPTH FACTOR
O'CONNOR'S K_2 WITH 1.2 DEPTH FACTOR
CHURCHILL'S K_2 WITH 1.0 DEPTH FACTOR
D.O. — mg/l
B.O.D. — mg/l
RIVER MILES

Figure 16

Table 15

COMPARISON OF REAERATION RATES

Basic Reach Data

Area	Depth (ft)	Flow (cfs)	Velocity (fps)	Slope (ft/ft)	Observed k_2 (1/day)
Upper Patuxent River (Wildlife Refuge)	1.34	11.0	0.24	0.00038	0.28
Lower Patuxent River (Near Hardesty)	1.89	25.0	0.24	0.00019	0.20
North Branch Potomac (Above Pinto)	1.98	100.0	0.41	0.00128	0.94
Jackson River (Below Covington, Va.)	2.00	100.0	0.54	0.00189	1.70

Reaeration Rates*

Method	Upper Patuxent	Lower Patuxent	North Branch	Jackson River
O'Connor and Dobbins	1.58	0.89	1.20	1.27
Churchill	0.77	0.44	0.65	0.80
USGS	0.53	0.34	0.55	0.65
Dobbins	1.50	0.90	2.00	2.15
Gas	ND**	ND	ND	1.50
Observed (Calculated)	0.28	0.20	0.94	1.70

* All values base ten

** Not determined

Of the formulations investigated in the pilot study of the Patuxent Basin, the TVA relationship appeared to describe the reaeration process best. However, it was still necessary to adjust the coefficient to make the computed DO profiles similar to the observed profile.

Various methods for adjusting the TVA formulation were investigated. Assuming the constants and exponents of the TVA formulation to be valid, and establishing the time-of-travel by dye studies, the only parameter left to adjust was the depth. Further, after examining the cross-section data from the Patuxent Basin, the rectangular transformation appeared to yield a too low mean stream depth. (See next section.)

A series of computer runs was made in which the mean depth was increased linearly by a depth-factor until the computed profiles compared favorably to the observed data. The depth-factor was then used in verifying observed data from other independent stream surveys, and the calculated profiles were very similar to the observed profiles. A similar approach was used for the Potomac Basin.

As can be seen in Figure 16, the DO profile computed using the Churchill formulation with no depth-factor appears to match the observed data. With a depth-factor of 1.2, the DO profile computed using the Connor formulation also appears to be satisfactory. The choice as to which formulation to use is dictated primarily by which formulation in general best describes the reaeration process for the entire basin.

Prior to the use of any factor, it is necessary to analyze the field data carefully and delineate, manually or, if sufficient data is available by spectral analysis, any periodicity in the survey data. For example, a tremendous diurnal DO fluctuation was observed in the lower

reaches of the North Branch, as also can be seen in Figure 16. Average I values were used for model verification.

More research effort is required to either develop a more general formulation and/or methods for systematically adjusting the known methods.

CROSS-SECTION DATA

As indicated in the previous sections, most of the formulations for predicting the reaeration process in streams are dependent upon a depth-of-stream parameter. The depth of a stream cross-section is usually defined as area/top width. Implicit in this definition is a rectangular transformation of the stream cross-section.

From over 100 cross-sections of the Patuxent and Potomac Rivers, it appears that the stream is consistently more parabolic than rectangular. The problem is, "what is the depth of a non-rectangular section?" Furthermore, the velocity of the stream is usually greater in the deeper parts of the cross-section.

Three methods for cross-section transformation, excluding inspection, are given below:

I Rectangular transformation

$$\text{depth} = \frac{\Sigma\ \omega\ *\ d}{\Sigma\ \omega} = \frac{A}{W}$$

II Moments of depth proportional to discharge

$$\text{depth} = \frac{\Sigma\ \omega\ *\ d\ *\ v\ *\ d}{\Sigma\ \omega\ *\ d\ *\ v} = \frac{\Sigma\ q*d}{Q}$$

III Moments of depth proportional to area

assuming $v_1 = v_2 = v_3 \ldots v_n$

$$\text{depth} = \frac{v*\Sigma\ \omega*d*d}{v*\Sigma\ \omega\ *\ d} = \frac{\Sigma\ \omega*d^2}{\Sigma\ \omega*i} = \frac{\Sigma\ \omega\ *\ d^2}{A}$$

...re

... = width of an increment of the cross-section

... = average depth of an increment of the cross-section

... = mean velocity of an increment of the cross-section

... = discharge of an increment of the cross-section

... = total width of cross-section

... = total discharge of cross-section

... = total area of cross-section

A comparison of the three transformations was made for a limited ...ber of cross-sections in the coastal area of the Patuxent Basin and ...stream portions of the Potomac Basin. From cross-sectioning data ...e Patuxent Basin, the mean depths as computed by method III were ...t 15 to 25 percent greater than method I; while in the Potomac Basin, ...thod III yielded a mean depth about 10 to 15 percent greater than ...od I. At USGS gaging stations, method II and III were both about 10 ...nt greater than method I.

...ith the limited number of cross-sections with velocity measurements, ...istical comparison cannot be made; however, results tend to indi- ... that a rectangular transformation does not yield a representative ...epth, especially in coastal streams such as the Patuxent. More- ... the area transformation is similar to the discharge transforma- ... onsiderable saving in field work can be realized.

... other problem in interpreting cross-section data is the variability ... along the longitudinal profile and the possibility of bias on ... of the field personnel in selecting cross-section points. For ... geographically homogeneous reach of stream channel in the Patuxent ... Rivers, the distribution of depth has been found to be normal.

See Figure 17 for variations in depth in the North Branch of the Potomac. O'Connor[9] also reported a normal distribution of depth in his studies of the Wabash Clarion, and Codorus Basins.

Assuming a normal distribution, it is possible to determine statistically the number of cross-sections needed to assure a given probability that the measured mean value will represent the true mean. While it is possible to determine the number of cross-sections required, the reaeration rate, which is a function of depth to a power, may not be normally distributed and statistically less significant.

In a ten-mile reach of the North Branch of the Potomac River below Keyser, the reaeration rate was calculated from fifty-two cross-section points using the O'Connor and Dobbins formulations. As also can be seen in Figure 17, the reaeration rate for this reach has a skewed distribution.

While the point calculations introduce an additional bias in the velocity term, the pronounced skewness in the reaeration rate for the reach is indicative of some of the problems in data interpretation. Some of the variability in the predictive formulations may be due to the interpretation of the cross-section data and the aggregation of sections for a given stream reach. Incorporated in the suggested research effort in the reaeration study should be comprehensive field testing, including a detailed analysis of cross-section transformations.

MECHANISMS AND FACTORS

Using a first order reaction, the calculated BOD concentrations for long, relatively unpolluted stretches of the Potomac River tend to approach 0.0 mg/l of BOD, with resulting DO values near 100 percent of saturation. Field BOD measurements in these reaches normally range from 2.0 to 3.0 mg/l, with DO levels about 80 to 90 percent of saturation.

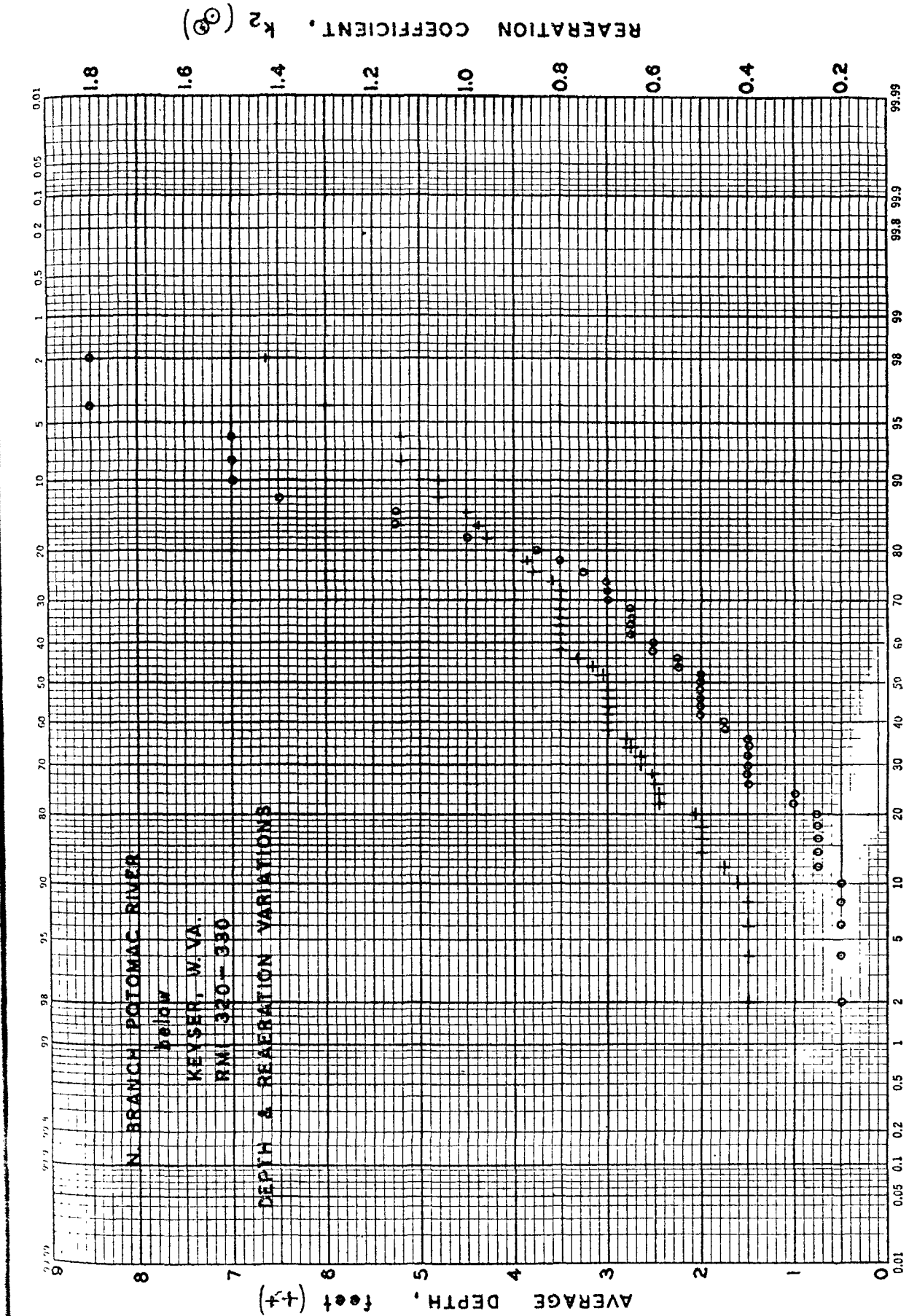

N. BRANCH POTOMAC RIVER
below
KEYSER, W.VA.
RM 320-330
DEPTH & REAERATION VARIATIONS
AVERAGE DEPTH, feet (++)
REAERATION COEFFICIENT, k_2 (⊙)

Figure 17

Similar observations were made by Hall[89] in his work on the James River, a sister basin to the Potomac.

It appears that various not-readily-quantifiable sources and sinks of oxygen such as bank loads, nitrification, scouring, etc. result in a dynamic-equilibrium level for BOD of about 2.0 mg/l. Associated with this 2.0 mg/l of BOD dynamic-equilibrium is a depressed DO level of about 80 to 90 percent of saturation.

Two methods for coping with this problem are (1) inclusion of all sources and sinks of oxygen in the quality algorithms, or (2) the setting of a minimum value for the BOD dynamic-equilibrium level. Since technological developments at the present time are either not adequate or too costly to completely quantify all sources and sinks, the latter approach has been used in the quality algorithms.

In the verification link and flow release models, whenever the BOD concentration decays below a prescribed level (read in as input data), it is reset to the prescribed level after the oxygen uptake for the given section has been computed. BOD and DO profiles obtained from computer runs of the verification link appear to be similar to observed profiles developed from field data.

PREDICTIVE MODEL FOR PHOSPHATES

As indicated in Chapter VI, high concentrations of nutrients and the resulting algal blooms are becoming an increasing problem in water quality management. One method for reducing the algal problem is to maintain a phosphate concentration in the stream below the minimum threshold limits for an algal bloom.

Studies in the Patuxent River Basin indicate a large portion of phosphates coming from waste treatment plants are being lost to plants,

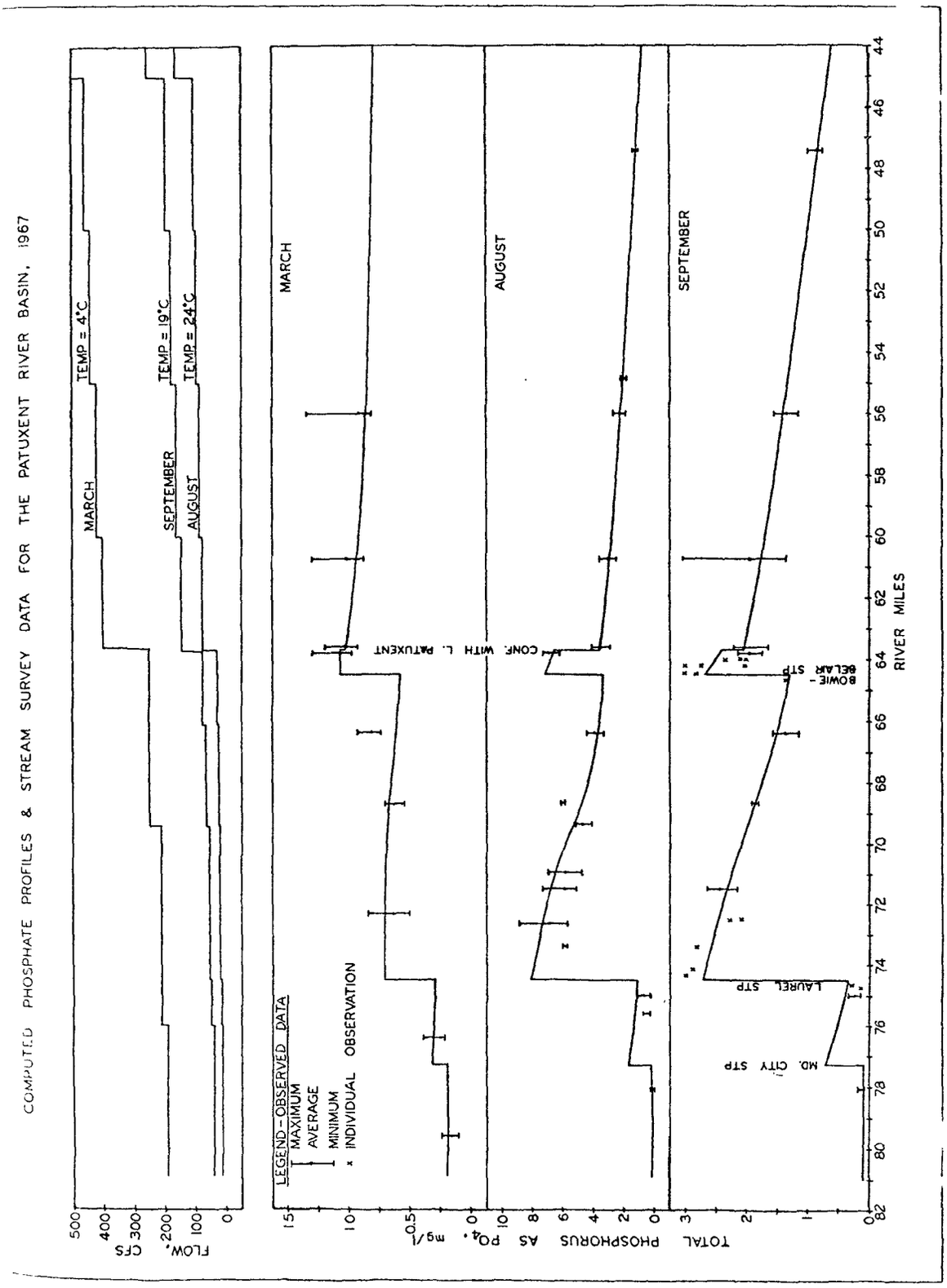

COMPUTED PHOSPHATE PROFILES & STREAM SURVEY DATA FOR THE PATUXENT RIVER BASIN, 1967
FLOW, CFS
500
400
300
200
100
0
TEMP = 4°C
TEMP = 19°C
TEMP = 24°C
MARCH
SEPTEMBER
AUGUST
TOTAL PHOSPHORUS AS PO4, mg/l
LEGEND-OBSERVED DATA
MAXIMUM
AVERAGE
MINIMUM
INDIVIDUAL OBSERVATION
MARCH
AUGUST
SEPTEMBER
CONF. WITH L. PATUXENT
LAUREL STP
MD. CITY STP
BOWIE-BELAIR STP
RIVER MILES
82
80
78
76
74
72
70
68
66
64
62
60
58
56
54
52
50
48
46
44

Figure 18

-eam bed, etc. In analyzing data from the Patuxent, it appears that
e loss of phosphates can be mathematically approximated by a first order
-tion similar to that of a biologically degradable waste. An algorithm
.roveling the loss of phosphates based on the first order reaction is
.ded in the verification link, and provisions are made for inclusion
. the flow release model.

Computed and observed profiles for three studies are presented in
.re 18. The loss rate appears to be a function of temperature and
., with values ranging from 0.1 to 0.2.

More field studies are required to determine how stationary the loss
phosphates is throughout an annual cycle. If the loss rate is predict-
e, the model could be used to determine phosphate treatment requirements
r incorporate into the flow release model as indicated in Chapter VII.

OF TRAVEL

If all the parameters required in the model, the most basic and sig-
'ant is the travel time. As indicated in Appendix B, the cost of the
-f-travel studies range from about $50 to $100 per mile. Preliminary
.es indicate that gaging station data in conjunction with limited dye
.es can reduce this cost considerably.

As presented in Chapter III, the velocity at a given cross-section
e expressed mathematically as:

$$\text{Velocity} = CC^{x} (\text{Flow})^{DD}$$

t gaging stations in the Potomac Basin, the value of exponent
.es from .300 to .500 with constant "CC" fluctuating more, depend-
.ow.

quality relationship being time dependent requires the time-of-
.etween two points rather than the velocity at a given point. By

assuming (1) that time-of-travel and flow can be expressed similarly to the velocity at a cross-section, and (2) that exponents are equal, a considerable amount of effort and money can be saved.

For a 7.8 mile of geographically homogeneous reach of the Patuxent River, four time-of-travel studies were conducted at four different flow rates. As can be seen in Figure 19, the velocity versus flow and time-of-travel versus flow relationships are essentially parallel.

If, in general, the two slopes are parallel, only one time-of-travel study will be required in reaches where velocity versus flow relationship is known. More field studies will be needed in various geographical situations, especially in impoundments, before general relationships can be statistically extracted.

Analogous to time-of-travel and velocity relationships, a similar approach can be taken for the mean depth at given cross-section sites and average depth relationships. Here again, more field work is required to establish the limits of the relationships.

The cost of conducting one time-of-travel study and obtaining cross-sections for the 894 miles on the Potomac River Basin model is about $30,000. If the general relationships can be firmly established, considerable expenditures of money can be saved in some of the field studies required in model verification.

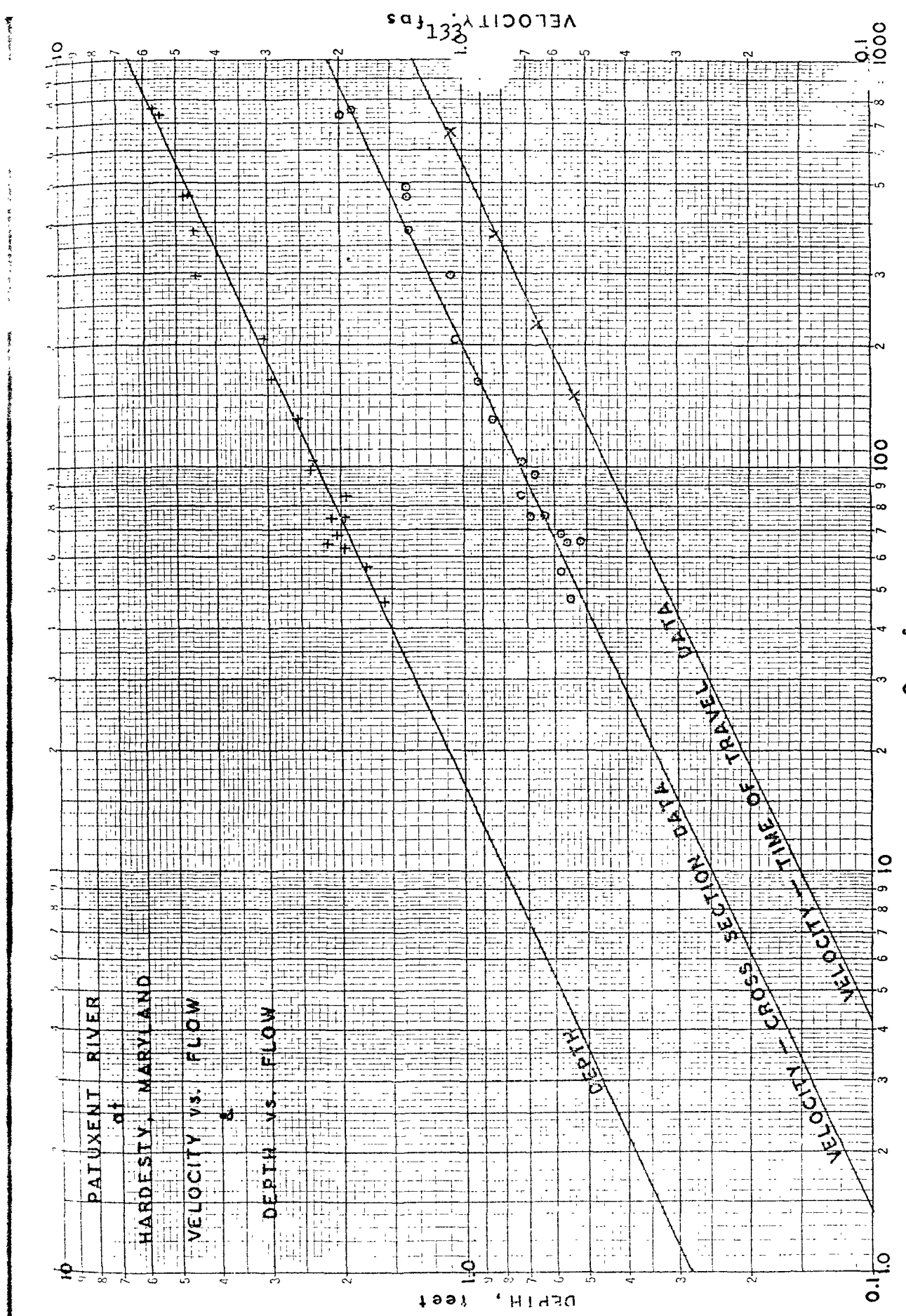

PATUXENT RIVER
at
HARDESTY, MARYLAND
VELOCITY vs. FLOW
&
DEPTH vs. FLOW
DEPTH
VELOCITY
CROSS SECTION DATA
TRAVEL TIME DATA
VELOCITY
DEPTH, feet
VELOCITY, fps
Q, cfs

Figure 19

A PROCEDURE FOR VERIFICATION OF THE QUALITY FORMULATIONS

Based on the experience in the Patuxent Basin, a method for implementing the flow release model has been developed according to the following format.

1. Define the physical system including an inventory of all wastewater discharges and water intakes.
2. Establish the stream channel characteristics.
3. Partition the river basin and determine all parameters, constants, etc.
4. Verify the physical and quality formulations.

The above format provides for a systematic development of the data, thus reducing the chance for error; it can also be used to indicate data voids.

Defining the Physical System

A detailed inventory of wastewater discharges, water intakes, dams, and other topographic features is necessary to define the system. Included in the data requirements are wastewater quality and quantity, water withdrawals, stream flow quality and quantity, river mile indexing, drainage area, etc. Knowledge of the basin area and engineering judgment are essential for definition of the system.

Establishing Stream Channel Characteristics

For all stream flow gaging stations, data required to establish velocity-flow and depth-flow relationship can be obtained from the United States Geological Survey. The data are obtained each time a station is rated by the Survey.

Depth relationships for additional points can be obtained by cross-sectioning the stream at two different stream flows. Similarly,

a greater number of sections can be rated and more velocity relationships established. A method of relating time-of-travel to velocity and statistical analysis of cross-section data is given in this appendix.

Basin Partitioning and Parameter Determinations

Partitioning of the basin and the determination of various parameters, constants, etc., are two of the most important steps for implementation of the model. To expedite the study, the following procedure has been established for a given reach of stream.

1. Plot the longitudinal profile of the stream.
2. Section the reach into segments according to instructions given in the section on the descriptive paragon.
3. Partition similar segments, according to channel characteristics.
4. By segments, plot the drainage area profile.
5. By segments, plot the base stream flow (such as 7-day low flow with a recurrence interval once in 10 years).
6. From time-of-travel data, plot by segments a velocity profile for the base flow.
7. From cross-section data, plot by segments a depth profile for the base flow.
8. Number the nodes and evaluate the constants and exponents for each segment.

Verification of Quality Formulations

Good stream survey data are vital to validation of the quality formulations. By plotting the isopleths of the quality data, diurnal fluctuations can be observed and quantized. These plots are also

useful in reducing the survey data to steady-state conditions and in checking the time-of-travel determinations.

Having reduced the data and plotted the stream quality profiles, the coefficients of the formulations have to be adjusted until the calculated and observed profiles are similar. The procedure given below has been developed for the verification process.

1. Check and correct if necessary the calculated depth and velocity profiles.
2. Compare the calculated temperature profiles with the field data, and if necessary adjust the temperature coefficient.
3. From existing quality data and isopleth plots determine the deaeration coefficients for the various stream reaches.
4. Compare the calculated BOD profiles with the survey data, and where necessary adjust the deaeration coefficient.
5. Whenever necessary the depth-factor constants for each segment are adjusted so that calculated and observed DO profiles are similar.

Due to the dependence of the parameters and coefficients upon those in the preceding step, the order of development and verification as given in the above procedure is important. For example, both the BOD and DO quality formulations are temperature dependent. Therefore, it is a necessity to have a well defined temperature algorithm before one attempts to verify the BOD and DO formulations.

APPENDIX B

MODEL DATA FOR THE POTOMAC RIVER BASIN

Reduced listings of the Potomac River Basin basic data used in the flow release model are exhibited in this appendix. Since numerous computer runs have been made under various conditions, it is not feasible to include all of the input data. The various data inputs are linked by the descriptive paragon as described in Chapter III.

Detailed schematics of individual stream reaches of the Basin, showing all major waste discharges, water intakes, gaging stations, impoundments, etc., are displayed in Figures 20 through 29. An overall general schematic of the Potomac River Basin system is presented in Figure 30.

For 81 nodes, including the 14 proposed reservoirs, stream flows were accrued as summarized in Table 16. In stream reaches below a terminal source where there are no major tributaries, the incremental flow based on the increase in drainage area has been introduced into the system at add points. Unregulated tributaries not receiving any significant waste loads or used for water supply have been indexed similar to add points.

In the non-tidal portion of the Basin, there are over 200 surface-water supplies and waste-water discharges. For this study all surface-water supplies with an intake rate greater than 0.5 mgd and all waste-water discharges with a flow greater than 0.5 mgd or population equivalent of 1,000 or greater were included in the model. The surface-water supplies and waste-water discharges used in this study are presented in Table 17.

Current waste treatment levels have been used except in the North Branch area. For the North Branch area, in order to stay within the regulation capacity of the impoundments, the treatment levels for West

...a Pulp and Paper Company and Celanese Corporation have been ... and 90 percent, respectively. For discharges currently entering ... system above the proposed impoundments and for the South River of the ... the Shenandoah River, the waste loads have been routed as shown in ... detailed schematics.

The basic data for the 14 proposed reservoirs are presented in ...ble 3. Not included are the Seneca Project which has been removed ...om the recommended plan and the Stony Creek Project which is in series ...ith the Bloomington impoundment. Since there are no significant organic ...aste loads between the impoundments, and Stony Creek is small when compared ... Bloomington, the omission of Stony Creek due to model limitations is ...t too significant.

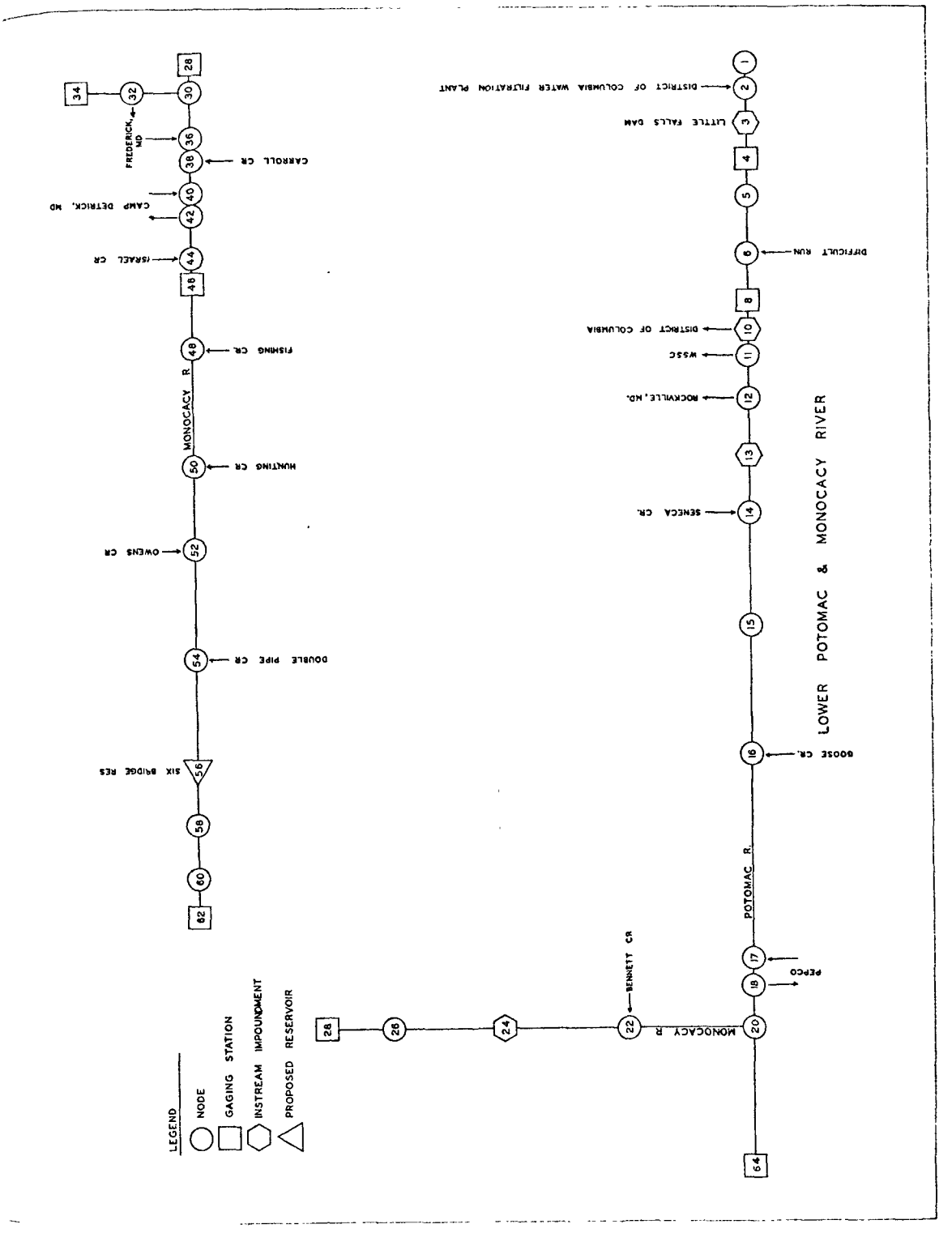

LOWER POTOMAC & MONOCACY RIVER
DISTRICT OF COLUMBIA WATER FILTRATION PLANT
LITTLE FALLS DAM
DIFFICULT RUN
DISTRICT OF COLUMBIA
WSSC
ROCKVILLE, MD.
SENECA CR.
GOOSE CR.
POTOMAC R.
PEPCO
BENNETT CR
MONOCACY R
FREDERICK, MD
CARROLL CR
CAMP DETRICK, MD
ISRAEL CR
FISHING CR.
MONOCACY R.
HUNTING CR
OWENS CR
DOUBLE PIPE CR
SIX BRIDGE RES
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR

Figure 20

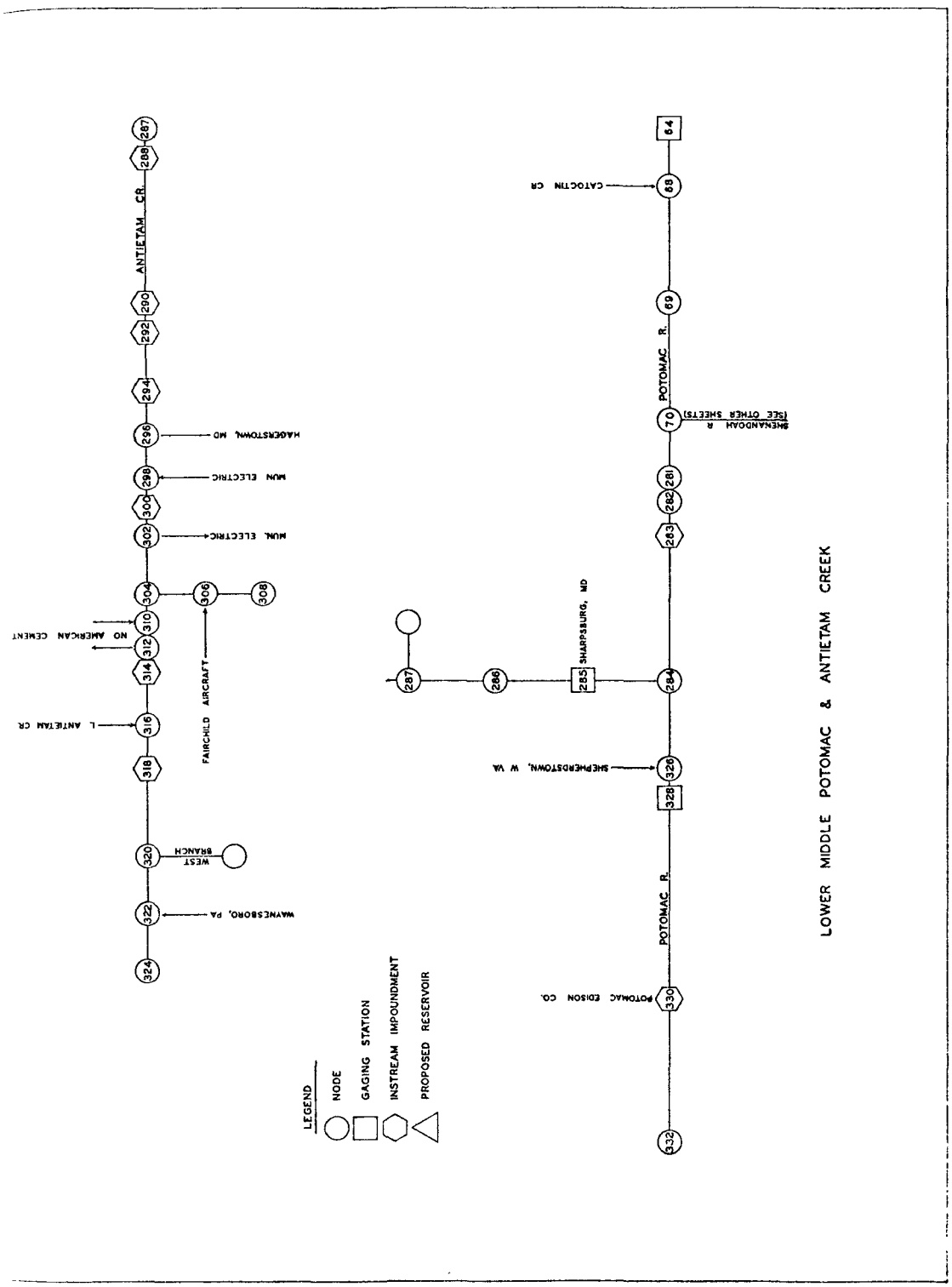

LOWER MIDDLE POTOMAC & ANTIETAM CREEK
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
ANTIETAM CR.
HAGERSTOWN, MD
MUN. ELECTRIC
MUN. ELECTRIC
NO. AMERICAN CEMENT
FAIRCHILD AIRCRAFT
L. ANTIETAM CR.
WEST BRANCH
WAYNESBORO, PA.
CATOCTIN CR.
POTOMAC R.
SHENANDOAH R. (SEE OTHER SHEETS)
SHARPSBURG, MD.
SHEPHERDSTOWN, W VA.
POTOMAC R.
POTOMAC EDISON CO.

Figure 21

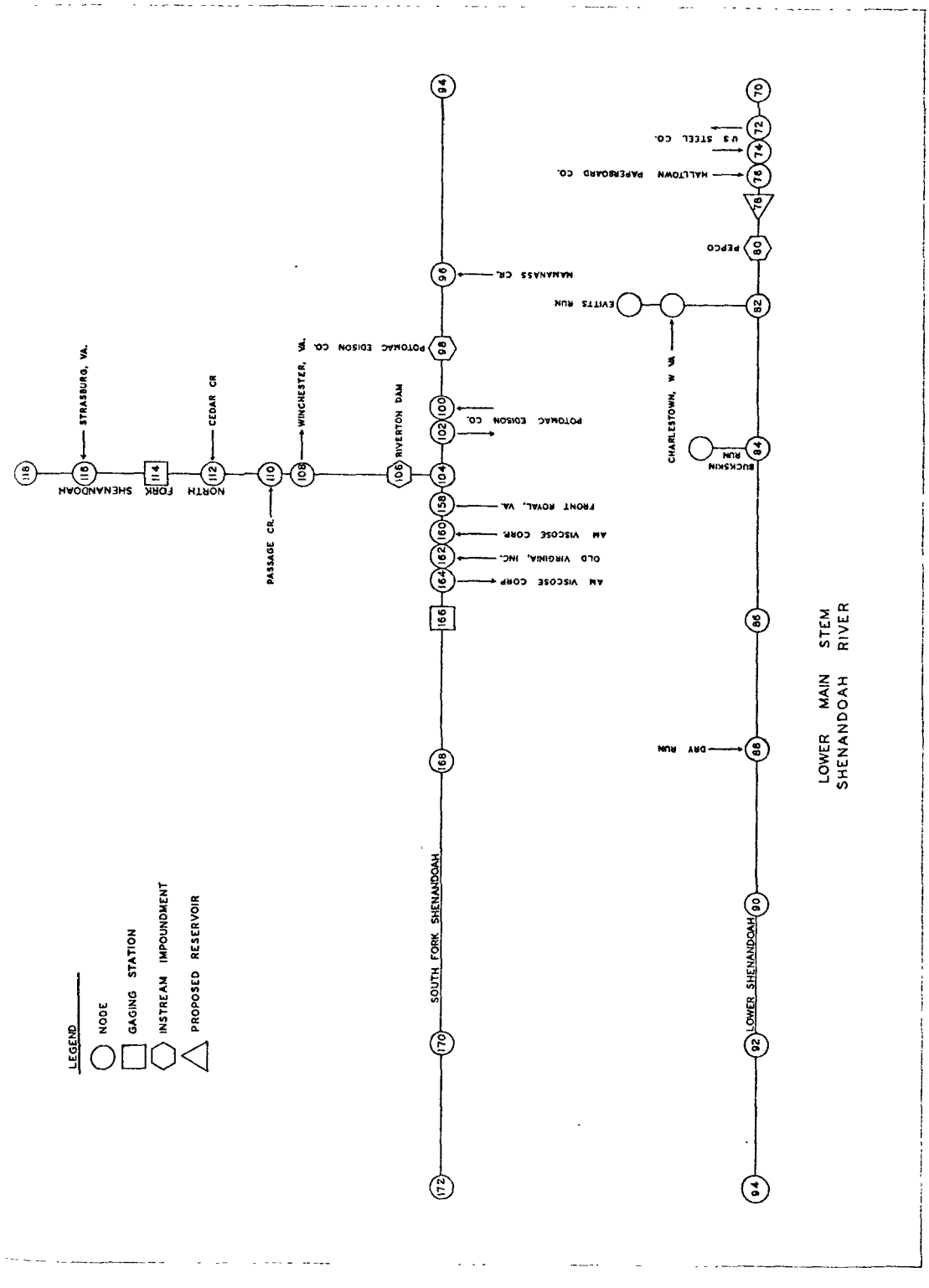

LOWER MAIN STEM
SHENANDOAH RIVER
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
SOUTH FORK SHENANDOAH
LOWER SHENANDOAH
NORTH FORK SHENANDOAH
STRASBURG, VA.
CEDAR CR
WINCHESTER, VA.
RIVERTON DAM
POTOMAC EDISON CO.
PASSAGE CR.
FRONT ROYAL, VA.
AM VISCOSE CORP.
OLD VIRGINIA, INC.
AM VISCOSE CORP
MANANAS CR.
POTOMAC EDISON CO.
EVITTS RUN
CHARLESTOWN, W VA.
BUCKSKIN RUN
DRY RUN
HALLTOWN PAPERBOARD CO.
U S STEEL CO.
PEPCO

Figure 22

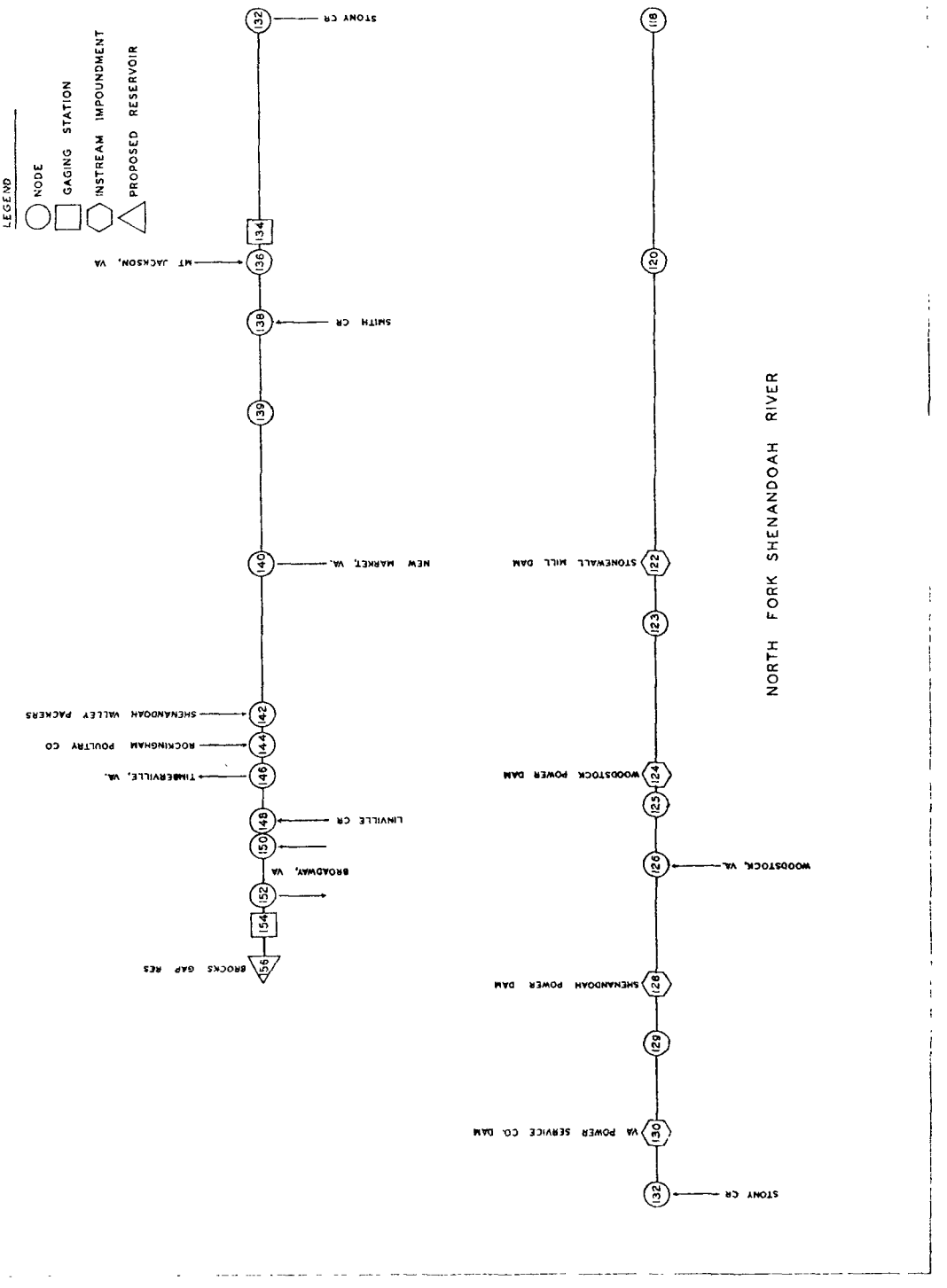

LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
NORTH FORK SHENANDOAH RIVER
STONY CR
MT JACKSON, VA
SMITH CR
NEW MARKET, VA.
SHENANDOAH VALLEY PACKERS
ROCKINGHAM POULTRY CO
TIMBERVILLE, VA.
LINVILLE CR
BROADWAY, VA
BROCKS GAP RES
STONEWALL MILL DAM
WOODSTOCK POWER DAM
WOODSTOCK, VA.
SHENANDOAH POWER DAM
VA POWER SERVICE CO. DAM
STONY CR

Figure 23

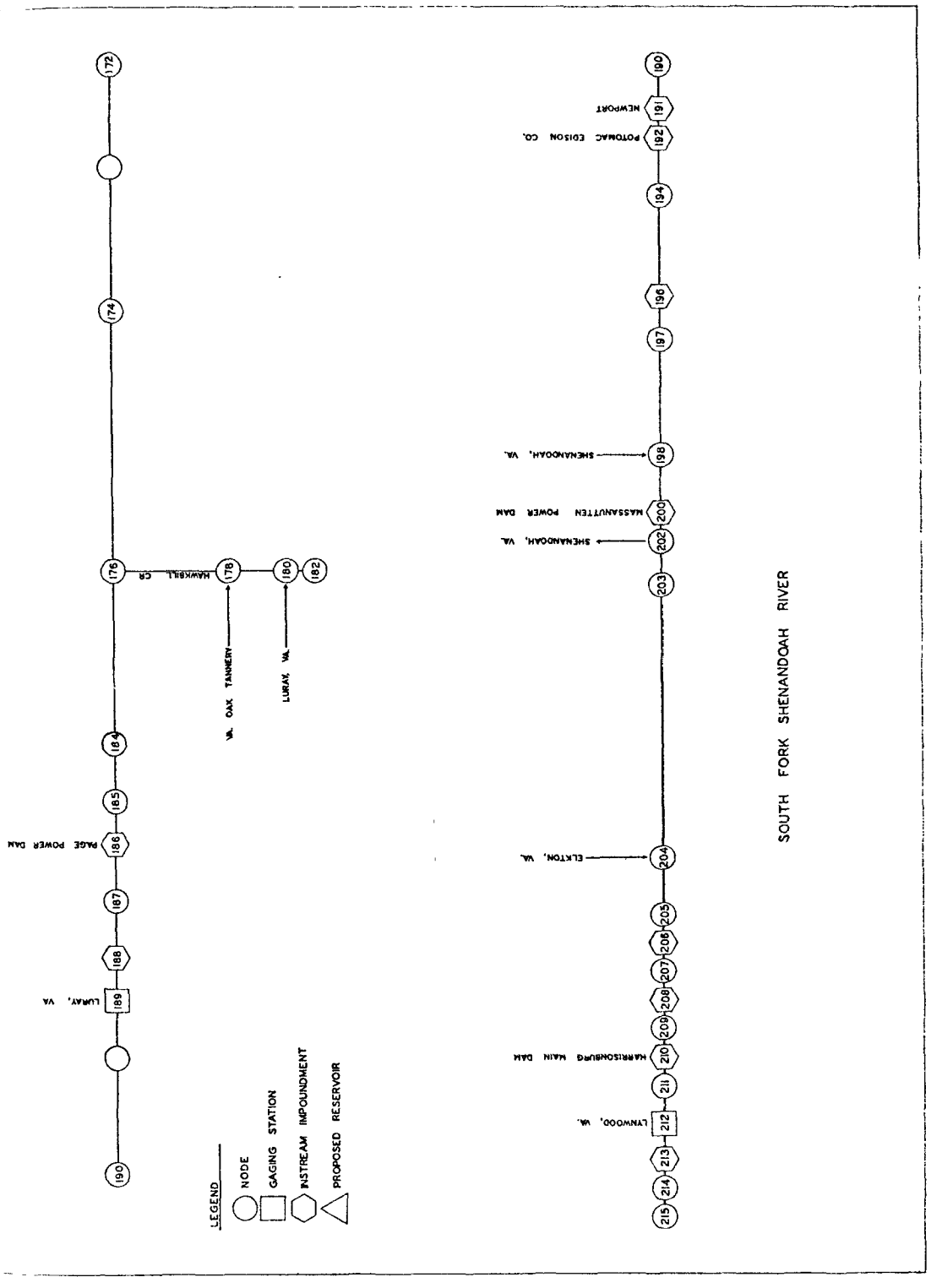

SOUTH FORK SHENANDOAH RIVER
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
LURAY, VA
PAGE POWER DAM
VA. OAK TANNERY
LURAY, VA
HAWKBILL CR
SHENANDOAH, VA.
MASSANUTTEN POWER DAM
SHENANDOAH, VA.
ELKTON, VA.
HARRISONBURG MAIN DAM
LYNWOOD, VA.
POTOMAC EDISON CO.
NEWPORT

Figure 24

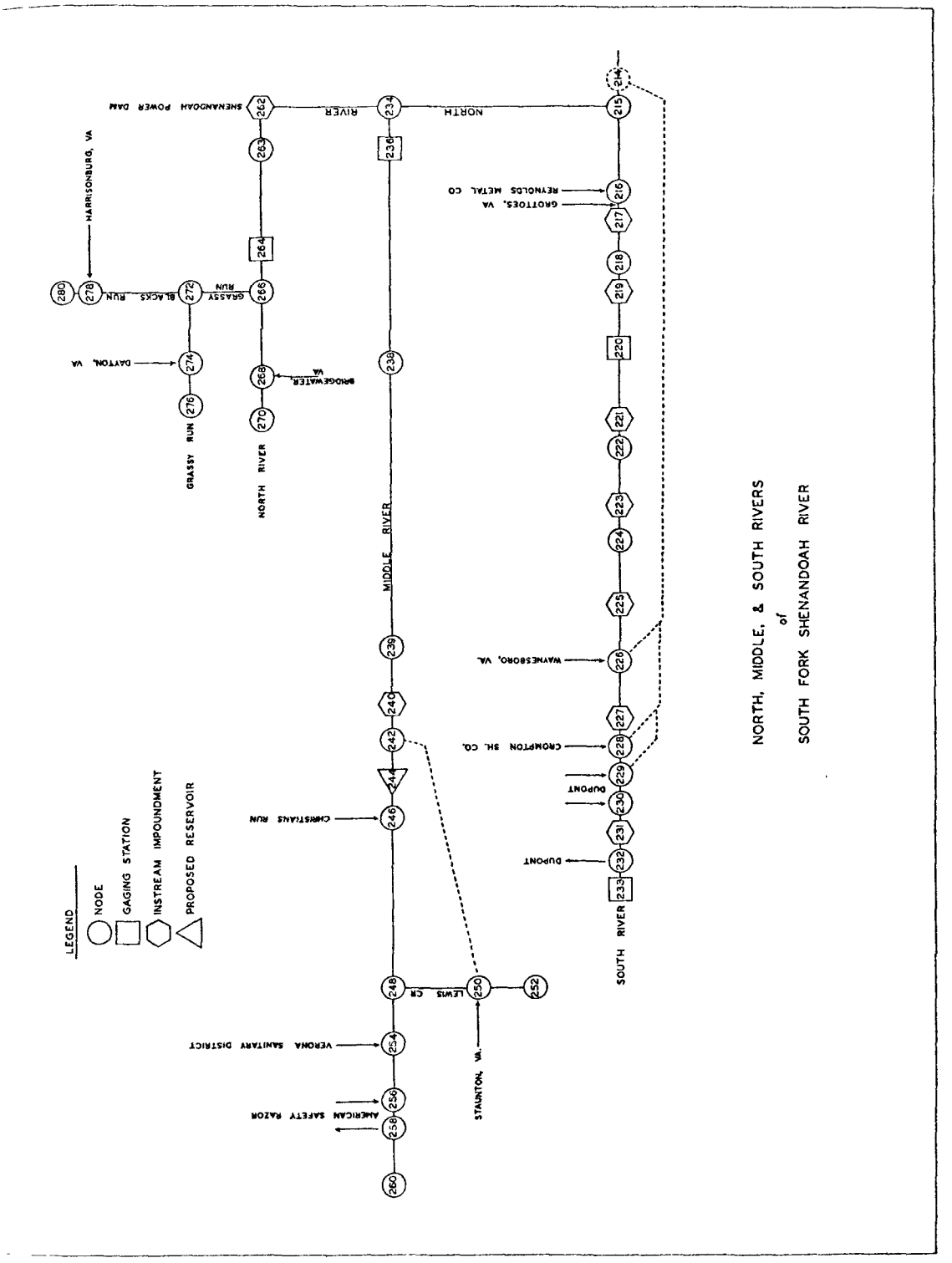

NORTH, MIDDLE, & SOUTH RIVERS
of
SOUTH FORK SHENANDOAH RIVER
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
SHENANDOAH POWER DAM
HARRISONBURG, VA
BLACKS RUN
GRASSY RUN
DAYTON, VA
BRIDGEWATER, VA
GRASSY RUN
NORTH RIVER
MIDDLE RIVER
NORTH RIVER
REYNOLDS METAL CO
GROTTOES, VA
WAYNESBORO, VA.
CROMPTON SH. CO.
DUPONT
DUPONT
SOUTH RIVER
CHRISTIANS RUN
LEWIS CR
VERONA SANITARY DISTRICT
AMERICAN SAFETY RAZOR
STAUNTON, VA.

Figure 25

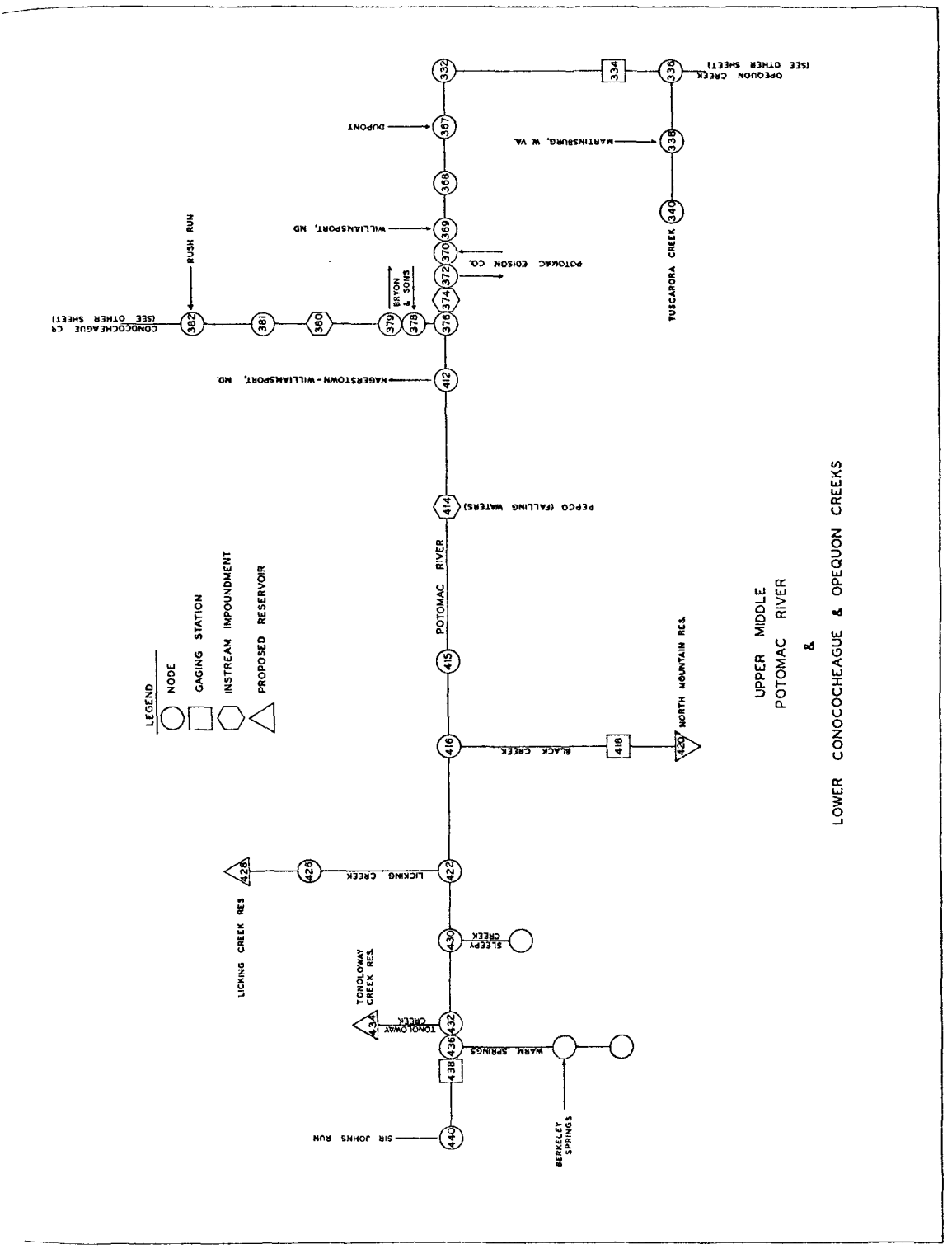

UPPER MIDDLE
POTOMAC RIVER
&
LOWER CONOCOCHEAGUE & OPEQUON CREEKS
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
POTOMAC RIVER
PEPCO (FALLING WATERS)
HAGERSTOWN - WILLIAMSPORT, MD.
DUPONT
MARTINSBURG, W. VA.
OPEQUON CREEK (SEE OTHER SHEET)
TUSCARORA CREEK
WILLIAMSPORT, MD
POTOMAC EDISON CO.
RUSH RUN
BRYON & SONS
CONOCOCHEAGUE CR (SEE OTHER SHEET)
BLACK CREEK
NORTH MOUNTAIN RES.
LICKING CREEK
LICKING CREEK RES
SLEEPY CREEK
TONOLOWAY CREEK
TONOLOWAY CREEK RES.
WARM SPRINGS
BERKELEY SPRINGS
SIR JOHNS RUN

Figure 26

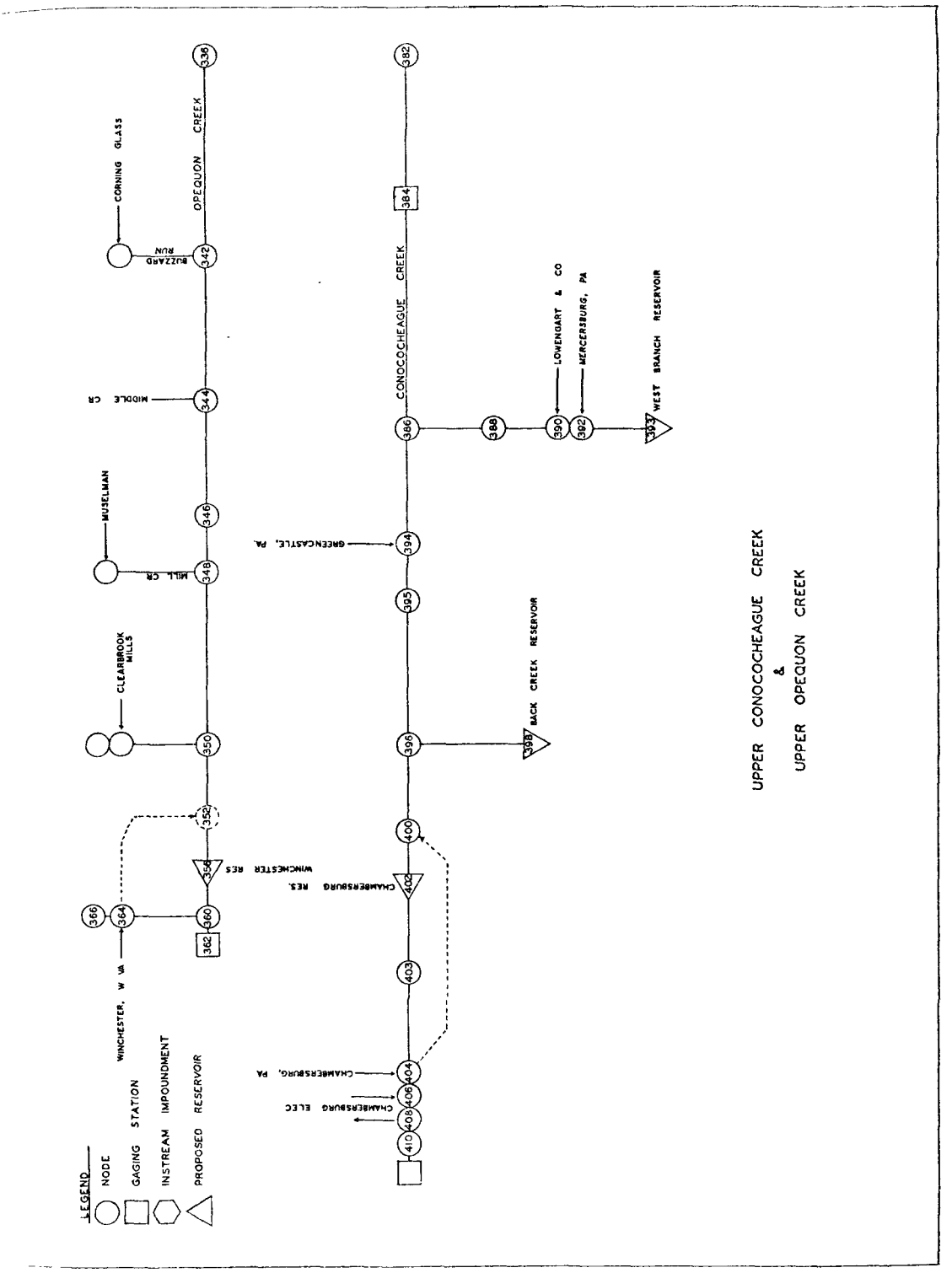

UPPER CONOCOCHEAGUE CREEK
&
UPPER OPEQUON CREEK
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
WINCHESTER, W VA
CHAMBERSBURG, PA
CHAMBERSBURG ELEC
WINCHESTER RES
CHAMBERSBURG RES.
GREENCASTLE, PA.
BACK CREEK RESERVOIR
LOWENGART & CO
MERCERSBURG, PA
WEST BRANCH RESERVOIR
CONOCOCHEAGUE CREEK
OPEQUON CREEK
CORNING GLASS
BUZZARD RUN
MIDDLE CR
MUSELMAN
MILL CR
CLEARBROOK MILLS

Figure 27

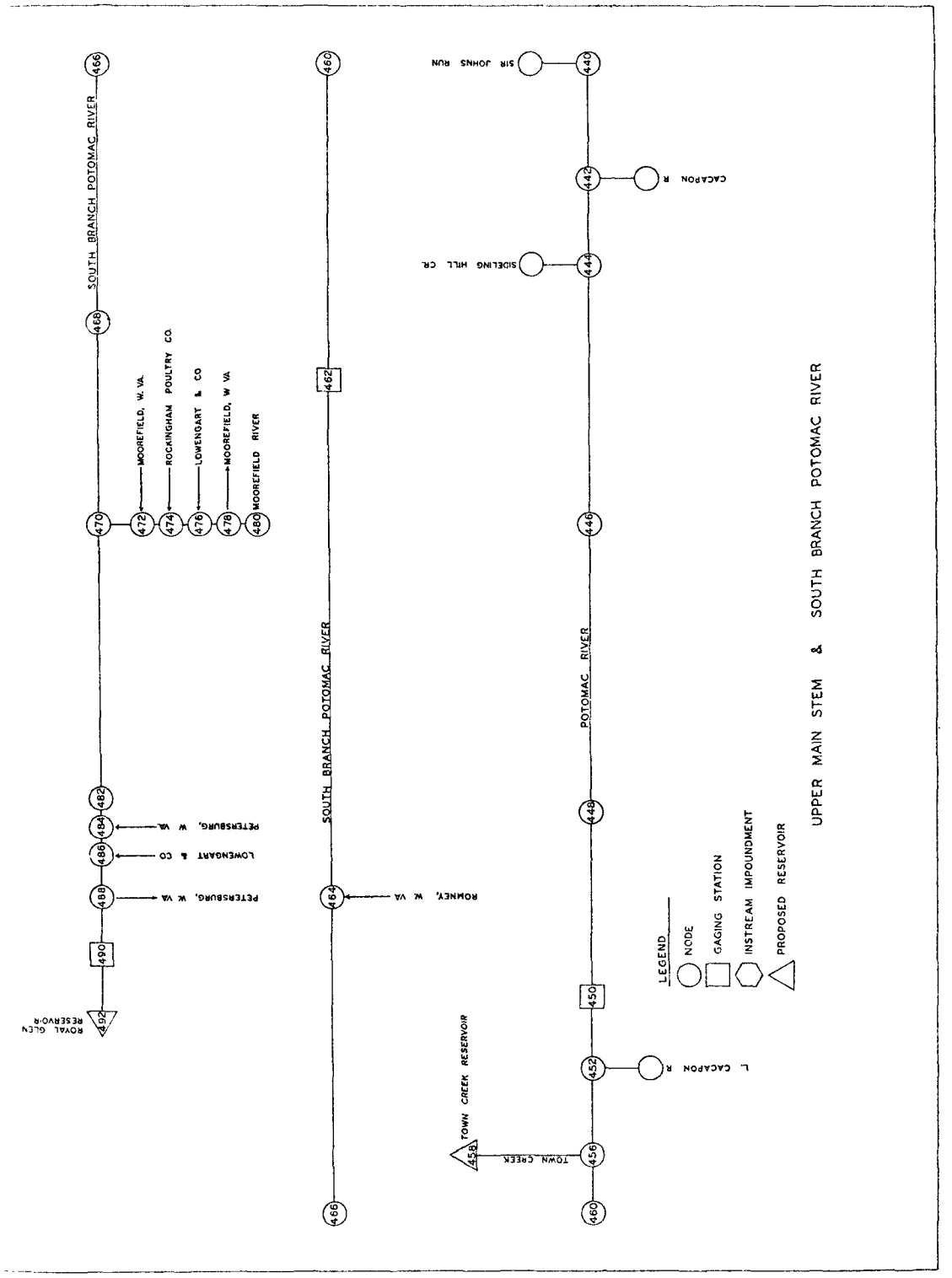

UPPER MAIN STEM & SOUTH BRANCH POTOMAC RIVER
SOUTH BRANCH POTOMAC RIVER
POTOMAC RIVER
SIR JOHNS RUN
CACAPON R
SIDELING HILL CR
L. CACAPON R
TOWN CREEK
TOWN CREEK RESERVOIR
ROWNEY, W VA
PETERSBURG, W VA
LOWENGART & CO
PETERSBURG, W VA
ROYAL GLEN RESERVOIR
MOOREFIELD, W. VA.
ROCKINGHAM POULTRY CO.
LOWENGART & CO
MOOREFIELD, W VA.
MOOREFIELD RIVER
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR

Figure 28

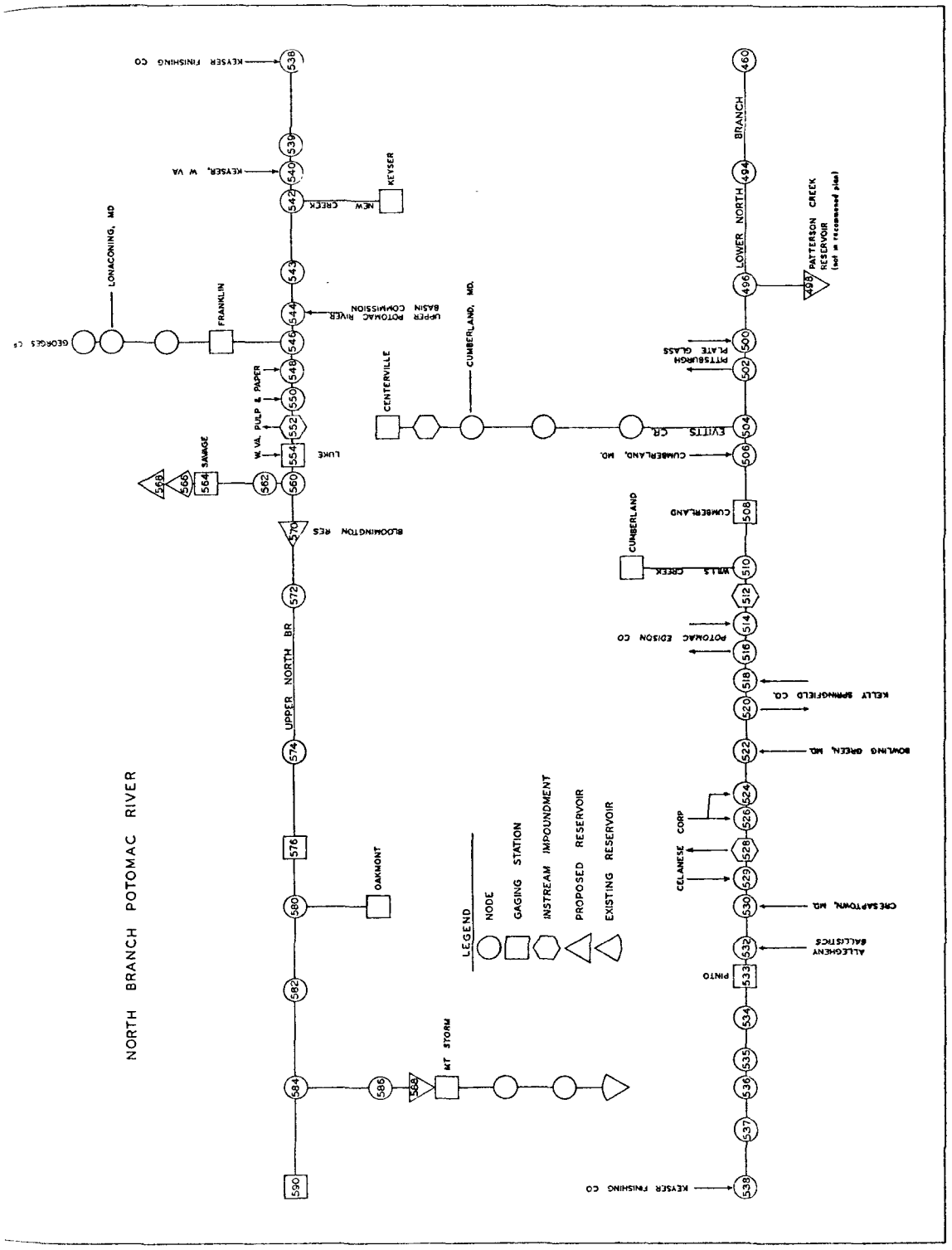
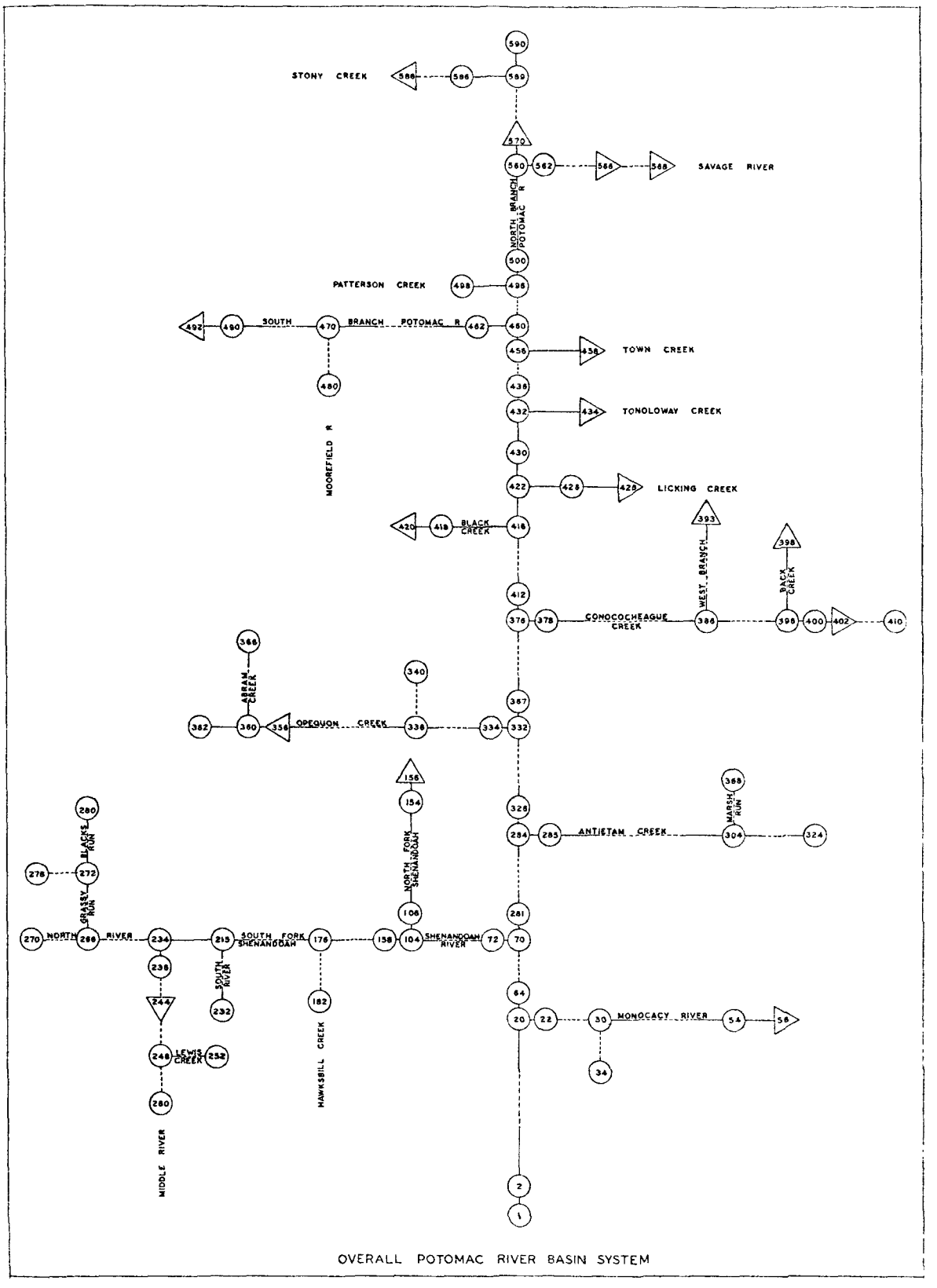

NORTH BRANCH POTOMAC RIVER
LEGEND
NODE
GAGING STATION
INSTREAM IMPOUNDMENT
PROPOSED RESERVOIR
EXISTING RESERVOIR
KEYSER FINISHING CO
KEYSER, W VA
NEW CREEK
KEYSER
LONACONING, MD
FRANKLIN
GEORGES CR
UPPER POTOMAC RIVER BASIN COMMISSION
W. VA. PULP & PAPER
LUKE
SAVAGE
CENTERVILLE
CUMBERLAND, MD.
BLOOMINGTON RES
UPPER NORTH BR
OAKMONT
MT STORM
PINTO
CUMBERLAND
WILLS CREEK
CUMBERLAND
EVITTS CR
CUMBERLAND, MD.
PITTSBURGH PLATE GLASS
LOWER NORTH BRANCH
PATTERSON CREEK RESERVOIR
(not a recognized plan)
POTOMAC EDISON CO
KELLY SPRINGFIELD CO.
BOWLING GREEN, MD.
CELANESE CORP
CHESAPTOWN, MD.
ALLEGHENY BALLISTICS
KEYSER FINISHING CO

Figure 29



OVERALL POTOMAC RIVER BASIN SYSTEM

Figure 30

STREAM FLOW DATA FOR FLOW RELEASE MODEL

Potomac River Basin

Number	Node	Index	Drainage Area (sq. mi.)	Base Flow*** (cfs)	Stream	Tributary To
1	6	0	100	5	Difficult Run	Potomac River
2	14	0	460	10	Seneca Creek	Potomac River
3	16	0	400	2	Goose Creek	Potomac River
4	22	0	80	5	Bennett Creek	Monocacy River
5	26	0	70	4	Add Point	Monocacy River
6	34	1	100	10	Linganore Creek	Monocacy River
7	38	0	30	3	Carroll Creek	Monocacy River
8	44	0	25	3	Israel Creek	Monocacy River
9	48	0	65	4	Fishing Creek	Monocacy River
10	50	0	35	2	Hunting Creek	Monocacy River
11	52	0	65	3	Owens Creek	Monocacy River
12	54	0	192	15	Double Pipe Creek	Monocacy River
13	56	2	308	10	Six Bridge Reservoir*	Monocacy River
14	68	0	350	4	Catoctin Creek	Potomac River
15	82	0	54	7	Evitts Run	Shenandoah River
16	88	0	100	12	Dry Run	Shenandoah River
17	92	0	100	12	Add Point	Shenandoah River
18	96	0	111	12	Manassas Creek	Shenandoah River
19	110	0	146	12	Passage Creek	N. Fk. Shenandoah
20	112	0	118	10	Cedar Creek	N. Fk. Shenandoah
21	120	0	123	11	Add Point	N. Fk. Shenandoah
22	132	0	140	11	Stony Creek	N. Fk. Shenandoah
23	134	0	109	10	Mill Creek	N. Fk. Shenandoah
24	138	0	130	11	Smiths Creek	N. Fk. Shenandoah
25	148	0	57	19	Linville Creek	N. Fk. Shenandoah
26	156	2	213	4	Brocks Gap Reservoir*	N. Fk. Shenandoah

Number	Node	Index	Drainage Area (sq. mi.)	Base Flow*** (cfs)	Stream	Tributary To
27	168	0	43	6	Add Point	S. Fk. Shenandoah
28	172	0	132	20	Add Point	S. Fk. Shenandoah
29	182	1	78	13	Hawkbill Creek	S. Fk. Shenandoah
30	194	0	100	17	Add Point	S. Fk. Shenandoah
31	200	0	115	17	Add Point	S. Fk. Shenandoah
32	210	0	135	23	Add Point	S. Fk. Shenandoah
33	220	0	70	13	Add Point	S.R.S. Fk. Shenandoah
34	224	0	64	11	Add Point	S.R.S. Fk. Shenandoah
35	233	1	136	26	South River	S. Fk. Shenandoah
36	239	0	45	9	Add Point	M.R.S. Fk. Shenandoah
37	244	2	325	50	Middle River*	S. Fk. Shenandoah
38	270	1	380	38	North River	S. Fk. Shenandoah
39	276	1	20	2	Grassy Run	N.R.S. Fk. Shenandoah
40	280	1	20	2	Blacks Run	N.R.S. Fk. Shenandoah
41	286	0	68	17	Add Point	Antietam Creek
42	287	0	70	17	Add Point	Antietam Creek
43	308	1	32	8	Add Point	Antietam Creek
44	316	0	35	18	Little Antietam	Antietam Creek
45	320	0	40	12	West Branch	Antietam Creek
46	324	1	50	12	Add Point	Antietam Creek
47	330	0	68	5	Add Point	Potomac River
48	334	0	73	7	Add Point	Opequon Creek
49	340	1	22	2	Tuscarora Creek	Opequon Creek
50	342	0	50	6	Add Point	Opequon Creek
51	348	0	50	6	Mill Creek	Opequon Creek
52	350	0	39	4	Add Point	Opequon Creek
53	356	2	111	10	Winchester Reservoir*	Opequon Creek
54	382	0	69	9	Add Point	Conococheague Creek
55	388	0	100	10	Add Point	W. Br. Conococheague

Number	Node	Index	Drainage Area (sq mi)	Basin Flow*** (cfs)	Stream	Tributary To
56	394	2	134	16	West Branch Reservoir	Conococheague Creek
57	396	0	38	4	Add Point	Conococheague Creek
58	398	2	80	8	Back Creek Reservoir*	Conococheague Creek
59	402	2	142	16	Chambers Reservoir	Conococheague Creek
60	428	2	273	5	North Mt. Reservoir*	Black Creek
61	428	2	214	7	Licking Creek Reservoir*	Licking Creek
62	430	0	273	6	Sleepy Creek	Potomac River
63	434	2	127	4	Tonoloway Creek Reservoir	Potomac River
64	440	0	90	4	Sir Johns Run	Potomac River
65	442	0	683	41	Cacapon Run	Potomac River
66	444	0	100	4	Sideling Hill Creek	Potomac River
67	446	0	91	4	Add Point	Potomac River
68	452	0	108	4	Little Cacapon River	Potomac River
69	458	2	180	5	Town Creek Reservoir*	Potomac River
70	462	0	93	6	South Bridge	Potomac River
71	466	0	200	11	Add Point	S. Br. Potomac River
72	480	1	300	10	Moorefield River	S. Br. Potomac River
73	482	0	260	13	Add Point	S. Br. Potomac River
74	492	2	640	40	Royal Glen Reservoir*	S. Br. Potomac River
75	498	1	278	5	Patterson Creek	N. Br. Potomac River
76	504	0	175	2	Evitts Creek	N. Br. Potomac River
77	510	0	275	14	Wills Creek	N. Br. Potomac River
78	542	0	120	2	New Creek	N. Br. Potomac River
79	546	0	80	3	Georges Creek	N. Br. Potomac River
80	566	2	112	62	Savage River and **	N. Br. Potomac River
81	570	2	288	32	Bloomington Reservoir*	N. Br. Potomac River

* Denotes proposed reservoirs

** Regulated by Savage I reservoir

*** Minimum 7-day low flow with a recurrence interval of once in 10 years

Table 17

SURFACE WATER SUPPLY & WASTE WATER INVENTORY

Potomac River Basin

Name	Type	Node	Flow (cfs)	Receiving Stream
District of Columbia	2	2	2.20	Potomac River
District of Columbia	5	10	167.00	Potomac River
Washington Suburban Sanitary Commission	5	11	5.50	Potomac River
Rockville, Maryland	5	12	2.20	Potomac River
Potomac Electric and Power Company	3	17	355.00	Potomac River
Potomac Electric and Power Company	5	18	355.00	Potomac River
Frederick, Maryland	5	32	1.10	Monocacy River
Frederick, Maryland	2	36	3.80	Monocacy River
Camp Detrick, Maryland	2	40	0.65	Monocacy River
Camp Detrick, Maryland	5	42	0.80	Monocacy River
U. S. Steel Company	5	72	0.72	Shenandoah River
U. S. Steel Company	3	74	0.72	Shenandoah River
Millstown Board Company	2	76	1.00	Shenandoah River
Potomac Edison Company	3	100	10.64	Shenandoah River
Potomac Edison Company	5	102	10.64	Shenandoah River
Winchester, Virginia	5	108	2.80	N. Fk. Shenandoah River
Strasburg, Virginia	2	116	0.30	N. Fk. Shenandoah River
Woodstock, Virginia	2	126	0.14	N. Fk. Shenandoah River
Mt. Jackson, Virginia	2	136	0.11	N. Fk. Shenandoah River
New Market, Virginia	2	140	0.35	N. Fk. Shenandoah River
Valley Packers	2	142	0.18	N. Fk. Shenandoah River
Harrisonburg Poultry	2	144	0.71	N. Fk. Shenandoah River
Timberville, Virginia	5	146	1.00	N. Fk. Shenandoah River
Broadway, Virginia	2	150	0.13	N. Fk. Shenandoah River
Broadway, Virginia	5	152	0.10	N. Fk. Shenandoah River
Front Royal, Virginia	2	158	0.85	S. Fk. Shenandoah River
American Viscose Company	2	160	13.45	S. Fk. Shenandoah River
Va Virginia, Inc.	2	162	0.10	S. Fk. Shenandoah River
American Viscose Company	5	164	13.50	S. Fk. Shenandoah River
Virginia Oak Tannery	2	178	0.28	S. Fk. Shenandoah River
Luray, Virginia	2	180	0.28	S. Fk. Shenandoah River
Shenandoah, Virginia	2	198	0.71	S. Fk. Shenandoah River
Shenandoah, Virginia	5	202	0.50	S. Fk. Shenandoah River
Elkton, Virginia	2	204	0.18	S. Fk. Shenandoah River
Marlow and Company	2	205	7.86	S. Fk. Shenandoah River
Scottsville, Virginia	2	216	0.09	S.R.S. Fk. Shenandoah
Reynolds Metal Company	2	216	2.33	S.R.S. Fk. Shenandoah
Waynesboro, Virginia	2	226	2.50	S.R.S. Fk. Shenandoah
Crompton-Shenandoah Company	2	228	2.21	S.R.S. Fk. Shenandoah
E. I. Dupont Company	2	229	10.80	S.R.S. Fk. Shenandoah
E. I. Dupont Company	3	230	16.27	S.R.S. Fk. Shenandoah

Table 17 (Continued)

Name	Type	Node	Flow (cfs)	Reach
E. I. Dupont Company	5	232	14.40	S.F.S. Fk. Shenan...
Staunton, Virginia	2	250	3.30	M.F.S. Fk. Shenan...
Verona Sanitary District, Virginia	2	254	0.19	M.F.S. Fk. Shenan...
American Safety Razor Company	2	256	0.50	M.F.S. Fk. Shenan...
American Safety Razor Company	5	258	0.50	M.F.S. Fk. Shenan...
Bridgewater, Virginia	2	268	0.26	M.F.S. Fk. Shenan...
Dayton, Virginia	2	274	0.16	M.F.S. Fk. Shenandoah
Harrisonburg, Virginia	2	278	2.25	M.F.S. Fk. Shenandoah
Hagerstown, Maryland	2	296	5.75	Antietam Creek
Hagerstown Electric	3	298	0.33	Antietam Creek
Hagerstown Electric	5	302	0.33	Antietam Creek
Fairchild Aircraft Company	2	306	0.50	Antietam Creek
North American Cement Company	3	310	7.83	Antietam Creek
North American Cement Company	5	312	8.12	Antietam Creek
Greencastle, Pennsylvania	2	322	1.20	Antietam Creek
Shepherdstown, West Virginia	2	326	0.14	Potomac River
Martinsburg, West Virginia	2	338	2.00	Opequon Creek
Chesapeake M & M Company	2	346	0.45	Opequon Creek
Winchester, West Virginia	2	364	2.53	Opequon Creek
E. I. Dupont Company	2	[illegible]	0.40	Potomac River
Williamsport, Maryland	2	399	0.10	Potomac River
Potomac Edison Company	3	[illegible]	72.42	Potomac River
Potomac Edison Company	5	[illegible]	72.42	Potomac River
Mead and Sons Company	2	[illegible]	0.40	[illegible]
Mead and Sons Company	5	[illegible]	0.42	[illegible]
Kimberly-Clark Company	2	[illegible]	0.42	W. Br. Conococheague Creek
Hagerstown, Pennsylvania	2	[illegible]	0.72	W. Br. Conococheague Creek
Chambersburg, Pennsylvania	2	394	0.13	Conococheague Creek
Mercersburg, Pennsylvania	2	404	3.00	Conococheague Creek
Mercersburg Electric	3	406	2.50	Conococheague Creek
Mercersburg Electric	5	408	2.50	Conococheague Creek
Williamsport, Maryland	5	412	5.13	Potomac River
Kearneysville, West Virginia	2	464	0.30	[illegible] Potomac River
Ranson, West Virginia	2	472	0.17	[illegible] Br. Potomac River
Perdue Poultry Company	2	474	0.15	[illegible] Br. Potomac River
Mead and Company	2	476	0.13	S. Br. Potomac River
Shepherdstown, West Virginia	5	476	[illegible]	[illegible] Potomac River
Charles Town, West Virginia	2	484	0.34	[illegible] Potomac River
Mead and Company	2	486	0.20	S. Br. Potomac River
Harpers Ferry, West Virginia	5	488	2.00	[illegible] Potomac River

Table 1, (Continued)

Name	Type	Node	Flow (cfs)	Location
[illegible]sburgh Plate Glass	2	500	2.00	N. Br. Potomac River
[illegible]sburgh Plate Glass	2	502	0.26	N. Br. Potomac River
[illegible]erland, Maryland	2	506	5.00	N. Br. Potomac River
[illegible]mac Edison Company	3	514	0.69	N. Br. Potomac River
[illegible]mac Edison Company	5	516	0.67	N. Br. Potomac River
[illegible]y Springfield Company	3	518	3.60	N. Br. Potomac River
[illegible]y Springfield Company	5	520	2.97	N. Br. Potomac River
[illegible]ling Green, Maryland	2	522	0.18	N. Br. Potomac River
[illegible]lanese Corporation	2	524	2.80	N. Br. Potomac River
[illegible]lanese Corporation	3	526	58.00	N. Br. Potomac River
[illegible]lanese Corporation	5	528	58.00	N. Br. Potomac River
[illegible]lanese Corporation	2	529	2.60	N. Br. Potomac River
[illegible]esaptown, Maryland	2	530	0.28	N. Br. Potomac River
[illegible]legheny Ballistics	2	532	0.27	N. Br. Potomac River
[illegible]yser Finishing Company	2	538	0.25	N. Br. Potomac River
[illegible]yser, West Virginia	2	540	1.00	N. Br. Potomac River
Upper Potomac River [illegible]sin Commission	2	544	19.[illegible]	N. Br. Potomac River
[illegible]st Virginia Pulp and Paper Company	2	548	7.50	N. Br. Potomac River
West Virginia Pulp and [illegible]aper Company	3	550	29.00	N. Br. Potomac River
[illegible]st Virginia Pulp and [illegible]aper Company	5	552	75.50	N. Br. Potomac River
[illegible]st Virginia Pulp and [illegible]aper Company	3	554	15.00	N. Br. Potomac River

COST OF DATA

As indicated in Appendix A, a large amount of field work including stream and waste treatment surveys is required to implement the flow release model. In order to determine the cost of collecting the necessary information required to implement the model, cost figures have been kept on some of the major field activities at the Chesapeake Field Station.

A summary of the cost data is given in Table 18.

Table 18

COST OF FIELD STUDIES

Activity	Unit of Measurement	Cost Range/Unit
Time of travel	per mile	$ 50 - $ 100
Cross-sectioning	per section	$ 4 - $ 10
Waste Treatment Plant Survey	per 24-hour survey	$2000 - $3000
Stream Survey	per sample	$ 25 - $ 50

The above cost figures include personnel time plus a 33 percent overhead charge. The waste treatment plant and stream survey consists of the sanitary analyses such as BOD, DO, solids, and nutrients. The large cost range reflects the additional expense of per diem and travel when the activities are in remote areas of the basin.

APPENDIX C

COMPUTER PROGRAM DOCUMENTATION AND USAGE WITH SAMPLE PROBLEM

GENERAL

The computer programs for the verification link and flow release ...el have been coded in FORTRAN IV Subset E. The same algorithms, nota... and data inputs are common to both programs

For easy reading and understanding of the programs, symbols for the ...iables are usually abbreviations of the algorithm parameters. Allow... have been made for possible expansion of the model concepts, inclu... input formats.

In the verification link, all calculations are made in the concentra... mode (mg/l), while in the flow release model, the mode is pounds/day. ...se of concentration in the verification link aids in displaying of ...rious profiles. For example, the computed BOD profile is listed ...ng BOD values, which simplifies comparison to field data.

FORMATS AND TYPICAL DECK COMPUTATIONS

...ailed listings of the ten-input formats and array notation are ... in Tables 19 and 20, respectively As can be seen in Figure ... input formats are fixed fie... Card format 6, which currently ...ual, has been designed for ... sible expansion of the BOD, DO and ...are formulations and to allow for greater flexibility in the ...gorithms.

... data deck compilations for the verification link and flow ... are displayed in Figures ... and 33, respectively For a ... ation, the order of computation is as follows

... all levels of waste treatment for given flow module;

... all flow sets for a given waste water module; and,

3. all waste water sets for a typical computation.

If many waste water sets are included for compilation, each waste [illegible] must contain same number of flow sets.

Since there are no data sorting routines designed into program, all [illegible]rmats must be placed in proper sequence. Consequently, Format 5, which [illegible]sed to link the various modes, must be ranked upward by J(N) parameter.

[illegible]GRAM USAGE WITH SAMPLE PROBLEM

To illustrate the data inputs and computer outputs formed for the [illegible]rification link and flow release model, a sample set of data for the [illegible]uxent River Basin is presented in this appendix.

For demonstration purposes, regulation capability not currently [illegible]ting has been introduced in the Little Patuxent River. Cost con-[illegible]erations are not included in the sample problem. A schematic of the [illegible]uxent River Basin model is given in Figure 34.

In the flow release model, the distance increment, flow increment, [illegible] test grid, and BOD increments are read as input data, with the latter [illegible]ot required in the verification link. The latter three parameters [illegible]trolled the number of feasible solutions with first, the frequency [illegible]umber of water quality monitoring points in the basin.

For the sample problem, a flow increment of 5 cfs has been used. In [illegible]tomac Basin, a 20 cfs increment appeared to be adequate. To prevent [illegible]rational problems, the regulation range must be a multiple of the [illegible]crement or vice versa. This also applies to the reservoir cost

allow for maximum flexibility at a decision point (or stage), [illegible] increment is selected by the computer according to BOD range for [illegible] state under investigation. As can be seen in the Patuxent

er basin data, if the BOD range is between 1.0 and 0.0 mg/l, an incre- [illegible] 0.2 mg/l is selected; between 2.0 and 0.0 mg/l, 0.4 mg/l; between and 3.0 mg/l, 0.6 mg/l; etc. In the Potomac Basin, as indicated [illegible], the BOD range was usually about 0.5 mg/l or less.

For the temperature model the decay coefficient appears to vary [illegible] according to season and stream. Verification studies of Potomac [illegible]uxent have indicated a coefficient range of 0.2 to 0.80, with a [illegible]cal value of about 0.3.

Table 19

CARD FORMATS FOR VERIFICATION LINK AND FLOW RELEASE MODEL

Main Parameter Card (No. 1)

Symbol	Description	Field
NOTALN	Number of stream sections	1-5
NOTALW	Number of water intakes and waste water discharge	6-10
NOTALR	Number of streamflow add points	11-15
NRES	Number of reservoirs	16-20
NFNCTI	First node of basin or terminal node of data sets	21-25
NDS	Number of data sets	26-30
NQS	Number of flow sets per data set	31-35
NRMSOD	Number of treatment levels per data set	36-40
ITBOD	Index for waste water BOD loading 1. in pounds/day 2. in mg/l	41-45
ITYPE	Index for optimization 1. BOD-DOD routine 2. BOD-COST routine	46-50
KAIM	Index for reaeration formulation 1. O'Connor 2. Churchill 3. U.S.G.S.	51-55
QINCA	Flow increment from reservoirs in cfs	56-60
EMBOD	Minimum BOD removal level of all waste waters	61-65
(FIRST)	River mileage of first node	66-70
	Distance increment of BOD-DO quality formulations in miles	71-75
EMP	Minimum phosphorus removal level of all waste waters	76-80

Table 19 (Continued)

Five Waste Treatment Levels (No 2)

Symbol	Description	Field
[illegible](1)	First minimal BOD removal level of all waste waters	1-10
[illegible](2)	Second minimal BOD removal level of all waste waters	11-20
[illegible](3)	Third minimal BOD removal level of all waste waters	21-30
[illegible] 4)	Fourth minimal BOD removal level of all waste waters	31-40
[illegible](5)	Fifth minimal BOD removal level of all waste waters	41-50

Dynamic Programming Data - Grid Increment (No. 3)

Symbol	Description	Field
[illegible]	BOD grid increment for Test 1 grid	1-10
[illegible]	BOD grid increment for Test 2 grid	11-20
[illegible]	BOD grid increment for Test 3 grid	21-30
[illegible]	BOD grid increment for Test 4 grid	31-40
	BOD grid increment for Test 5 grid	41-50

Dynamic Programming Data - Test Grid (No 4)

Symbol	Description	Field
	First BOD test grid size	1-10
	Second BOD test grid size	11-20
	Third BOD test grid size	21-30
	Fourth BOD test grid size	31-40
	Fifth BOD test grid size	41-50

Table 19 (Continued)

Physical Data of Stream by Section (No. 5)

Symbol	Description	Field
[illegible](N)	Lower node of section	1-5
[illegible](N)	Upper node of section	6-10
[illegible]S(N)	River mile distance at upper node	11-20
DAREA(N)	Drainage of basin at upper node	21-30
[illegible]A(N)	Constant of depth-flow relationship for section	31-40
[illegible]B(N)	Exponent of depth-flow relationship for section	41-50
[illegible]M(N)	Constant of velocity-flow relationship for section	51-60
[illegible]N(N)	Exponent of velocity-flow relationship for section	61-70
[illegible]ITYPE(N)	Index of upper node	71-75

1. Node where there is a physical discontinuity
2. Confluence of two tributaries
3. Waste water discharge
4. Water intake
5. Cooling water discharge
6. Regulated terminal stream flow source
7. Unregulated terminal stream flow source

Table 19 (Continued)

Continuation of Card 5 - Optional (No. 6)

Symbol	Description	Field
[illegible])	Lower node of section	1-5
[illegible]N)	Upper node of section	6-10
[illegible]S(N)	Slope of section (ft/ft)	11-20
[illegible]FAC(N)	Depth factor of section	21-25
[illegible]NST(N)	DO constraint of section	26-30
[illegible]ECAYT(N)	Temperature decay coefficient of section	31-35
[illegible]TEMP(N)	Steady-state temperature of section	36-40
[illegible]ECAYP(N)	Phosphorus decay coefficient of section	41-45
[illegible]PHOS(N)	Dynamic-equilibrium level of phosphorus	46-50
[illegible]HOTOS(N)	Photosynthesis DO contribution (future use)	51-55
[illegible]LDGE(N)	BOD contribution of sludge deposits (future use)	56-60
[illegible]XTBOD(N)	BOD loss due to extraction (future use)	61-65
[illegible]EBOD(N)	Dynamic-equilibrium level of BOD	66-70

Table 19 (Continued)

Reservoir Information (No. 7)

Symbol	Description	Field
NRES(N)	Reservoir Number	1-5
NRN(N)	Node of reservoir	6-10
MRES(N)	Number of cost cards	11-15
IRES(N)	Flow increment for cost increment	16-20
ACREFT(N)	Total storage of reservoir in acre-feet	21-30
WQ ACFT(N)	Total storage for water quality control in acre-feet	31-40
TCOST(N)	Total cost of reservoir	41-50
WQ COST(N)	Cost of storage for water quality control	51-60
AVRESQ(N)	Average stream flow at reservoir	61-70
INRESQ(N)	Increase in dependable flow by reservoir	71-80

Reservoir Cost (No. 8)

Symbol	Description	Field
NRES(N)	Reservoir number	1-4
CARD	Number of cost card	5-8
COST	Reservoir cost for given flow increments (fields of six)	9-80

Table 19 (Continued)

Waste Water Discharge Data (No. 9)
(Including water intakes)

Symbol	Description	Field
WJ(N)	Node of discharge or intake	1-4
WTYPE(N)	Index of discharge or intake	5-10
	1. Biological waste water discharge in cfs	
	2. Biological waste water discharge in mgd	
	3. Thermal waste water discharge in mgd	
	4. Conservative waste water discharge in mgd	
	5. Surface water supply intake in mgd	
WWFLOW(N)	Discharge or intake flow (For units see WTYPE(N) index)	11-20
WK(N)	Deaeration rate (base 10 at 20°C)	21-30
WBOD(N)	BOD loading - untreated (For units see ITBOD - Card 1)	31-40
WDO(N)	DO content of waste discharge in mg/l	41-50
WTEMP(N)	Temperature of waste discharge in °C	51-60
WPHOS(N)	Phosphorus content of waste discharge in mg/l	61-70
WREM(N)	Percent BOD removal by treatment facility	71-80

Table 19 (Continued)

Secondary Parameter Card (No. 10)

Symbol	Description	Field
IT2	Index of steady-state temperature 1. Uniform basin temperature 2. Temperature by sections from Card 6	1-5
TEMP1	Uniform basin steady-state temperature in °C	6-10
IT3	Index of DO constraint 1. Uniform basin DO constraint 2. DO constraint by sections from Card 6	11-15
DNST1	Uniform DO constraint in mg/l	16-20
IT4	Index of depth factor 1. Uniform basin depth factor 2. Depth factor by sections from Card 6	21-25
DPFAC1	Uniform depth factor	26-30
IT5	Index of temperature decay coefficient 1. Uniform basin decay coefficient 2. Decay coefficient by sections from Card 6	31-35
KTEMP	Uniform temperature decay coefficient	36-40
IT6	Index of phosphorus decay coefficient 1. Uniform basin decay coefficient 2. Decay coefficient by sections from Card 6	41-45
KPHOS	Uniform phosphorus decay coefficient (base 10)	46-50
IT7	Index of dynamic-equilibrium of phosphorus 1. Uniform basin equilibrium level 2. Equilibrium level by sections from Card 6	51-55
SPHOS	Uniform dynamic-equilibrium level of phosphorus in mg/l	56-60
IT8	Index of dynamic-equilibrium level of BOD 1. Uniform basin equilibrium level 2. Equilibrium level by sections from Card 6	61-65
SBOD	Uniform dynamic-equilibrium level of BOD in mg/l	66-70

Table 19 (Continued)

Data at Streamflow Addition Points (No. 11)

Symbol	Description	Field
PJ(N)	Node of addition point	1-4
PTYPE(N)	Index of addition point 0. Increment flow addition (not a terminal source) 1. Unregulated terminal source 2. Regulated terminal source	5-8
PMINQ(N)	Minimum flow at addition point in cfs	9-15
PMAXQ(N)	Maximum flow at addition point in cfs	16-21
PBOD(N)	BOD of addition point in mg/l	22-27
PDO(N)	DO of addition point in mg/l	28-32
PPHOS(N)	Phosphorus content of addition point in mg/l	33-38
PK(N)	Deaeration rate of BOD of addition point (base 10 at 20°C)	39-45
PTEMP(N)	Temperature of addition point	46-51
PPTEMP(N)	Slope of temperature-flow relationship	52-57
PPDO(N)	Slope of DO-flow relationship	58-63
PPBOD(N)	Slope of BOD-flow relationship	64-68
PPPHOS(N)	Slope of phosphorus-flow relationship	69-75

Table 20

ARRAY NOTATION FOR VERIFICATION LINK AND FLOW RELEASE MODEL

A(N)	=	Constant for the log-log depth versus flow relationship for a given section
CAFT(N)	=	Acre-feet of storage in a given reservoir
CAVQ(N)	=	Average flow of stream at given reservoir site
B(N)	=	Exponent for the log-log depth versus flow relationship for a given section
BOD(N)	=	BOD at node in mg/l
C	=	Constant for the log-log velocity versus flow relationship for a given section
DOC(N)	=	Dissolved oxygen constraint for a given section in mg/l
DREA(N)	=	Drainage area at upper node of section in square miles
E	=	Exponent for the log-log velocity versus flow relationship for a given section
KDP(N)	=	Decay rate for phosphorus (base 10) for a given section
KDT(N)	=	Decay rate for temperature (base e) for a given section
DT	=	Difference between steady-state and computed temperatures at a given point
DEP(N)	=	Average water depth in feet for a given section
DIST(N)	=	Distance in miles of given node from confluence of basin
DMAN(T)	=	Distance in miles of given point from confluence of basin
DO	=	Dissolved oxygen concentration at given point in mg/l
DOD	=	Dissolved oxygen deficit at given point in mg/l
F(N)	=	Depth factor for given section
FLOW(N)	=	For expansion to include extraction in the BOD model (Future use)

Table 20 (Continued)

; N)	=	Lower node of section
¥FESQ(N)	=	Increase in stream flow at given reservoir site by regulation
.ES(N)	=	Cost-flow increment for given reservoir site
· N'	=	Upper node of section
.OD(JJ)	=	Pounds of BOD added or deleted at a given node in #/day
_OD(JJ)	=	Pounds of dissolved oxygen deficit added or deleted at a given node in #/day
_LOW(JJ)	=	Flow in cfs added or deleted at a given node
:.(JJ)	=	Deoxygenation coefficient (base 10)
_HOS(JJ)	=	Pounds of phosphorus added or deleted at a given node
.EMP(JJ)	=	Temperature in degrees centigrade
¥AXM(E)	=	Maximum number of increment in a given incremental flow tableau
¥AXRQ(JJ)	=	Maximum flow in cfs at a given node
¥BOD(JJ)	=	BOD in #/day for given grid unit in the incremental flow tableau
¥COST(JJ)	=	Cost of regulation for a given grid unit in the incremental flow tableau
¥DOD(JJ)	=	Dissolved oxygen deficit in #/day for a given grid unit in the incremental flow tableau
¥FLOW(X)	=	Flow in cfs for given grid unit in incremental flow tableau
¥INRQ(JJ)	=	Minimum flow in cfs for a given node
¥(E)	=	Upper node of the section for which the incremental flow tableau is calculated
¥.JJ)	=	Decxygenation coefficient in days (base 10) for given grid unit of the incremental flow tableau
¥'E)	=	Section for which incremental flow tableaus are calculated

Table 20 (Continued)

MRES(N)	=	Number of cards which contain cost data for a given reservoir site
MTEMP(JJ)	=	Temperature of given grid unit in the incremental flow tableau
NBOD(JJ)	=	Counter-part to MBOD(JJ) in dynamic programming solution
NCOST(JJ)	=	Similar to MCOST(JJ)
NDOD(JJ)	=	Counter-part to MDOD(JJ) in dynamic programming solution
NFLOW(JJ)	=	Counter-part to MFLOW(JJ) in dynamic programming solution
NK(JJ)	=	Counter-part to MK(JJ) in dynamic programming solution
NN(JJ)	=	Section number for given upper node
NNTYPE(N)	=	Index of upper node of a given section (data input)
NRES(N)	=	Number of given reservoir site
NRJ(N)	=	Node of given reservoir site
NTEMP(N)	=	Steady-state temperature of a given section
NRN(JJ)	=	Reservoir number for a given node
NTYPE(N)	=	Index type of upper node (reassemble by program)
OBOD(S)	=	BOD in mg/l of feasible solution from dynamic programming tableau
OCOST(S)	=	Reservoir cost of a feasible solution in the dynamic programming tableau
ODOD(S)	=	DO deficit of a feasible solution in the dynamic programming tableau
OFLOW(S)	=	Flow in cfs of a feasible solution in the dynamic programming tableau
OK(S)	=	Deoxygenation coefficient of feasible solution in the dynamic programming tableau (base 10)
OTEMP(S)	=	Temperature in degrees centigrade of feasible solution in the dynamic programming tableau

Table 20 (Continued)

OXFLOW(S)	=	Flow in cfs from M-arrays of feasible solution in the dynamic programming tableau
OYFLOW(S)	=	Flow in cfs from N-arrays of feasible solution in the dynamic programming tableau
PHOS(T)	=	Phosphorus in mg/l at a given point
PHOTOS(N)	=	For expansion to include photosynthesis in BOD model (Future use)
PPHOS(N)	=	Steady-state limiting value of phosphorus for a given section
PBOD(N)	=	BOD in mg/l at a given stream flow addition point
PCOST(NRA,ICR)	=	Cost data for a given reservoir for a given release rate
PDO(N)	=	DO in given reservoir at a given flow
REAIR(JJ)	=	Average reaeration coefficient for a given section (base 10)
SBOD(N)	=	Slope of BOD versus flow relationship for stream flow addition point
SDO(N)	=	Slope of DO versus flow relationship for stream flow addition point
SFLOW(N)	=	Flow in cfs for a given node
SPHOS(N)	=	Slope of phosphorus versus flow relationship at a stream flow addition point
STEMP(N)	=	Slope of temperature versus flow relationship at a stream flow addition point
N)	=	Node of given stream flow addition point
T	=	Deoxygenation coefficient in 1/days at a stream flow addition point (base 10)
XG N)	=	Maximum flow rate in cfs at a given stream flow addition point
NG(N)	=	Minimum flow rate in cfs at a given stream flow addition point

Table 20 (Continued)

RPHOS(N)	=	Phosphorus in mg/l at a given stream flow addition point
RTEMP(N)	=	Temperature in degrees centigrade at a given stream flow addition point
RTYPE(N)	=	Index of a stream flow addition node
SATDO(T)	=	Dissolved oxygen saturation concentration in mg/l for a given point
SBOD(JJ)	=	BOD in #/days at a given node
SDELL(JJ)	=	Difference between steady-state and calculated temperature at a given node
SDOD(JJ)	=	DO deficit in #/days at a given node
SEDBOD(N)	=	For expansion to include sedimentation in the model (Future use)
SFLOW(JJ)	=	Flow in cfs at a given node
SK(JJ)	=	Deaeration coefficient at a given node (base 10)
SLDBOD(N)	=	For expansion to include sludge deposit in the BOD model (Future use)
SLOPE(N)	=	Slope of stream in ft/ft for a given section
SQCOST(N)	=	Cost storage in given reservoir for water quality control
SSBOD(T)	=	BOD in #/days at a given point
SSDOD(T)	=	DO deficit in #/days at a given point
SSTEMP(T)	=	Temperature in degrees centigrade at given point
STEMP(JJ)	=	Temperature in degrees centigrade at given node
SXFLOW(X)	=	Flow in cfs for given solution in the dynamic programming routine, M-arrays
SYFLOW(Y)	=	Flow in cfs for a given solution in the dynamic programming routine, N-arrays
TCOST(N)	=	Total cost of a given reservoir

Table 20 (Continued)

TDIS(N)	=	Distance in miles from the confluence for given node
TEMP(T)	=	Steady-state temperature in degrees centigrade for a given point
TRET(N)	=	For expansion of reservoir site data
TTEMP(N)	=	Steady-state temperature in degrees centigrade for a given section
VEL(JJ)	=	Average velocity for section in ft/sec indexed by upper node
WBOD(N)	=	Untreated BOD in #/day or mg/l of a waste load for a given node
WDO(N)	=	DO in mg/l in the waste load for given node
WFLOW(N)	=	Flow in cfs for a given node
WIDTH(JJ)	=	Average width in feet for section indexed by upper node
WJ(N)	=	Node of waste water discharge
WK(N)	=	Deaeration coefficient of waste load in 1/days (base 10)
WPHOS(N)	=	Phosphorus in mg/l in waste load for given node
WQACFT(Y)	=	Reservoir storage in acre-feet for water quality control
WREM(N)	=	Percent removal of BOD of given waste load
WSTP(IRM)	=	Minimum BOD treatment level
WTEMP(N)	=	Temperature in degrees centigrade of waste load for given node
WTYPE(N)	=	Index of waste load for a given node
WWFLOW(N)	=	Flow in mgd of waste load for a given node

FORMATS FOR DATA CARDS

CARD 1
MAIN PARAMETER CARD

TOTALN	TOTALW	TOTALR	TRES	FIRSTI	ITO	IT1	IRMBOD	ITBOD	OPTYPE	IRSET	DELTA	REMBOD	DIS(FIRST)	INC	REMPHS
XXX	XX	XX	XX	X	X	X	X	X	X	X	XX.	XX.	XXX.	XX.	XX.

CARD 2
FIVE WASTE TREATMENT LEVELS

WSTP(1)	WSTP(2)	WSTP(3)	WSTP(4)	WSTP(5)
XX.X	XX.X	XX.X	XX.X	XX.X

CARD 3
DYNAMIC PROGRAMMING DATA - GRID INCREMENTS

ADD1	ADD2	ADD3	ADD 4	ADD 5
XX.	XX.	XX.	XX.	XX.

CARD 4
DYNAMIC PROGRAMMING DATA - TEST GRID

TEST 1	TEST 2	TEST 3	TEST 4	TEST 5
XX.	XX.	XX.	XX.	XX.

CARD 5
PHYSICAL DATA OF STREAM BY SECTIONS

I(N)	J(N)	TDIS(N)	DAREA(N)	AA(N)	BB(N)	CC(N)	DD(N)	NNTYPE(N)
X	X	XXX.XX	XXX.X	XX.XXX	XXX.XXX	XXX.XXX	XXX.XXX	X

CARD 6
CONTINUATION OF CARD 5 (OPTIONAL)

I(N)	J(N)	SLOPE(N)	DPFAC(N)	CONST(N)	DECAYT(N)	TTEMP(N)	DECAYP(N)	PPHOS(N)	PHOTOS(N)	SLUDGE(N)	EXTBOD(N)	SEDBOD(N)
XXX	XXX	.XXXXX	X.X	X.X	.XX	X.X	.XX	.XXX	.XX	.XX	.XX	.XX

CARD 7
RESERVOIR INFORMATION

NRES(N)	NRN(N)	MES(N)	IRES(N)	ACREFT(N)	WQACFT(N)	TCOST(N)	WQCOST(N)	AVERESQ(N)	INRESQ(N)
XX	XX	XX	XX	XXXX.	XXXX.	XXXX.	XXXX.	XXX.X	XXX.X

CARD 8
RESERVOIR COST

NRES(N)	CARD	COST	COST	COST	COST	COST	COST	COST	COST	COST	COST	COST	COST
XX	XX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX

CARD 9
WASTE WATER DISCHARGE DATA

WJ(N)	WTYPE(N)	WWFLOW(N)	WK(N)	WBOD(N)	WDO(N)	WTEMP(N)	WPHOS(N)	WREM(N)
X	X	XX.XXX	X.XX	XXX.X	XX.X	XX.X	X.XXX	.XX

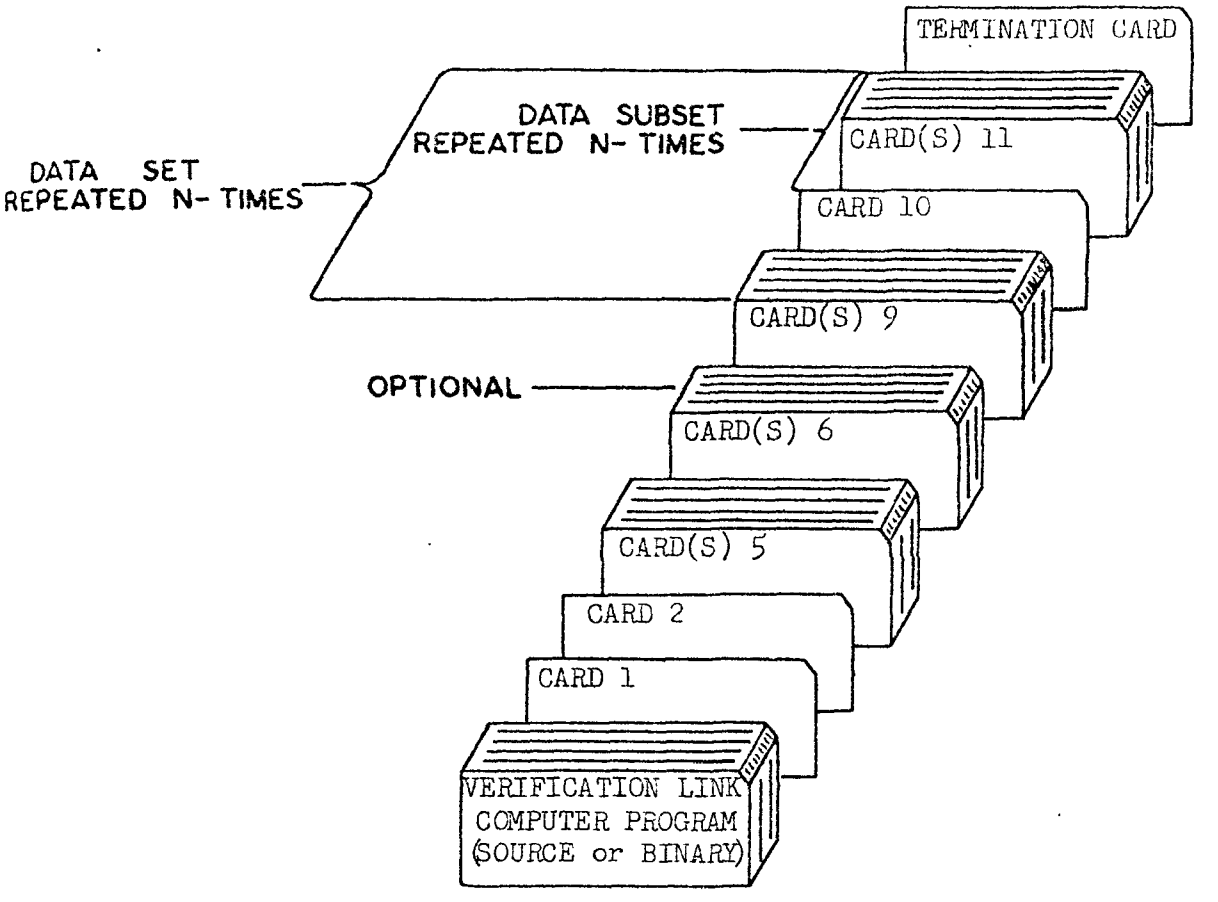
CARD 10
SECONDARY PARAMETER CARD

IT2	TTEMP1	IT3	CONST1	IT 4	DDFAC1	IT5	DKTEMP	IT6	DKPHOS	IT 7	SSPHOS	IT 8	BODSS
X	XX.	X	X.X	X	X.X	X	I.XX	X	I.XX	X	.XXX	X	X.X

CARD 11
DATA AT STREAM FLOW ADDITION POINTS

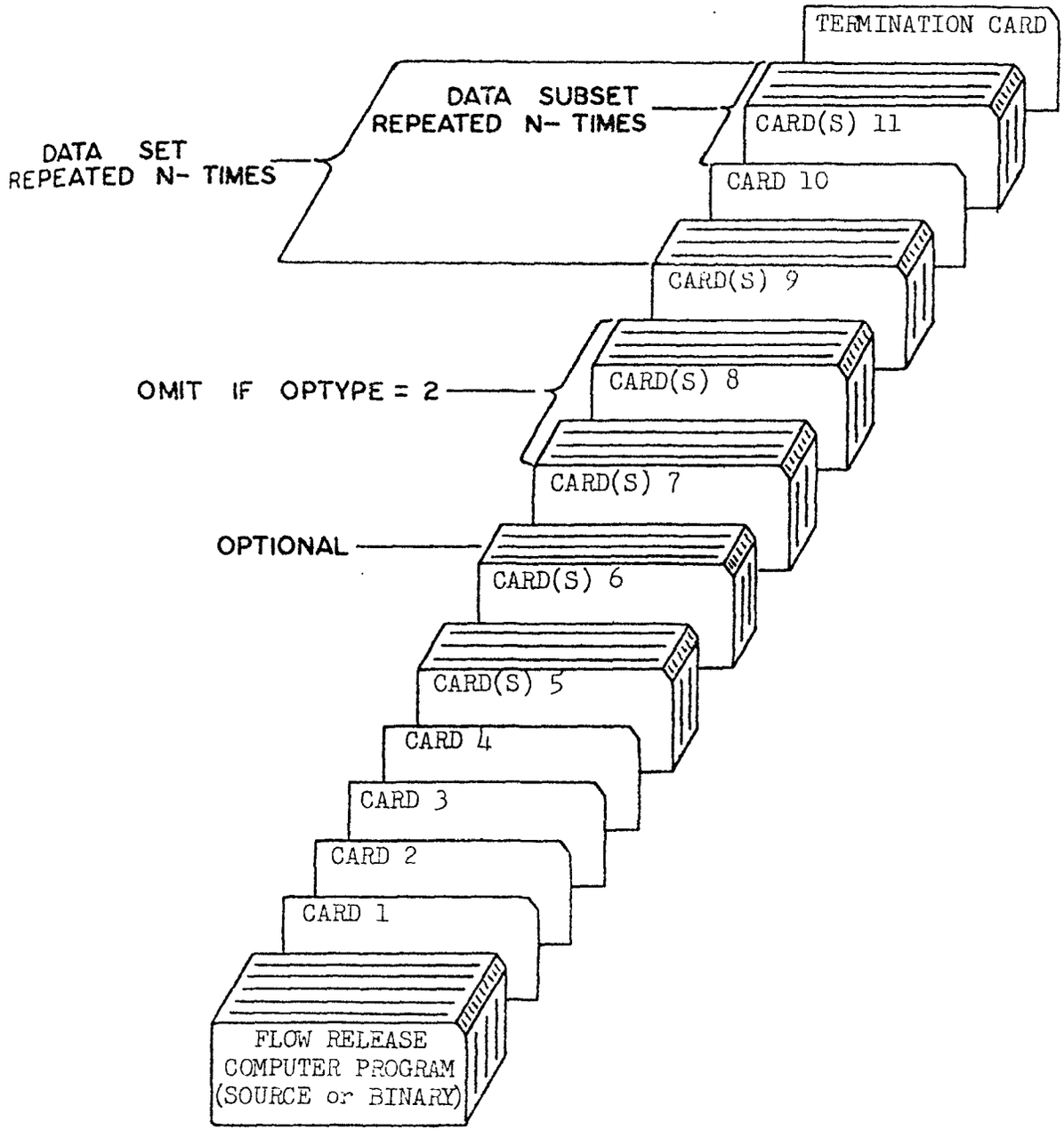
RJ(N)	RTYPE(N)	RMNQ(N)	RMAXQ(N)	RBOD(N)	RDO(N)	RPHOS(N)	RK(N)	RTEMP(N)	RFTEMP(N)	RFDOD(N)	RFBOD(N)	RFPHOS
X	X	XXX.X	XXX.X	XX.X	XX.XX	.XXX	X.XX	XX.X	X.XXXXX	.XXXXX	.XXXXX	.XXX

Figure 31



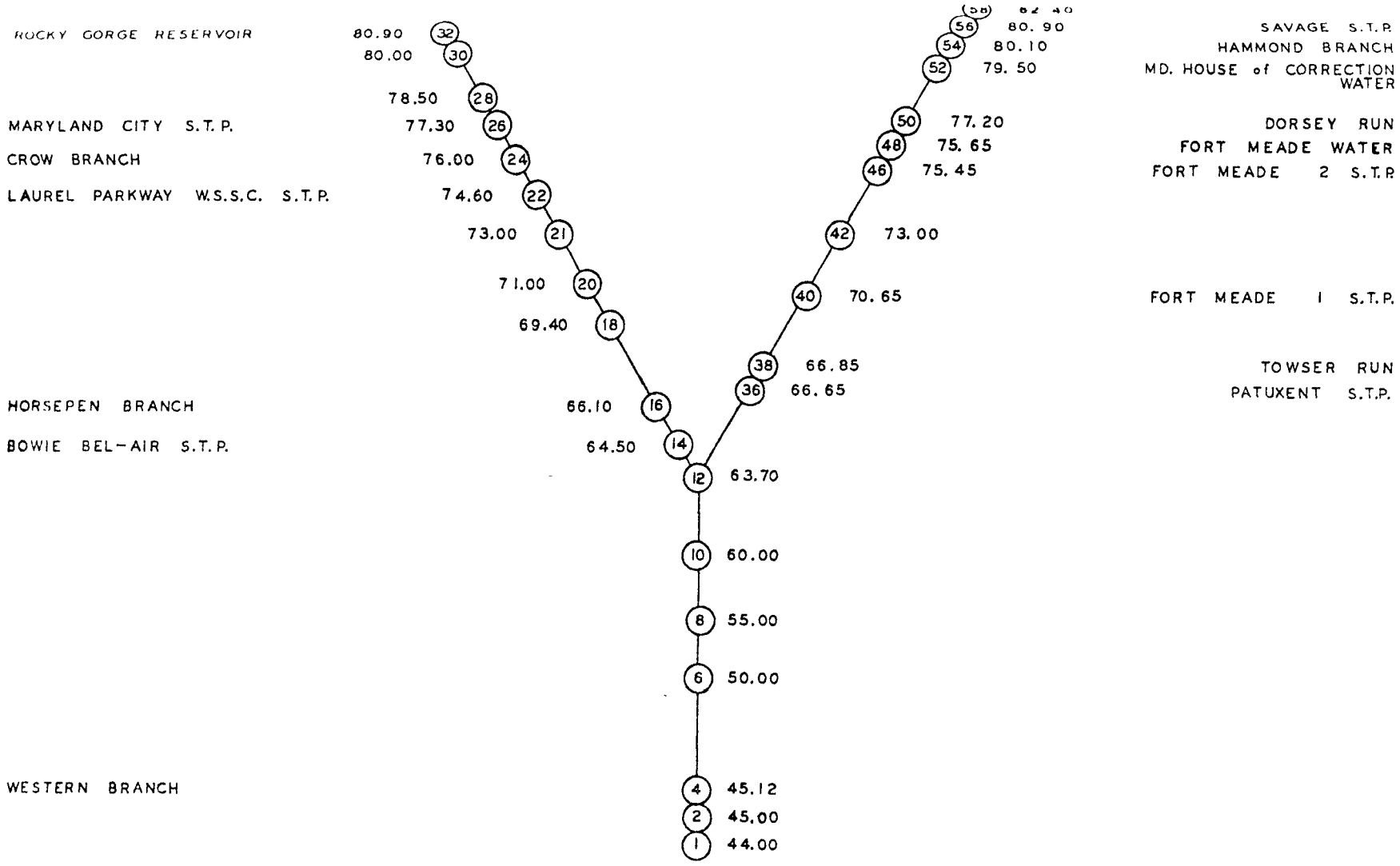
TYPICAL DATA COMPILATION
FOR
VERIFICATION LINK

Figure 32



TYPICAL DATA COMPILATION
FOR
FLOW RELEASE MODEL

Figure 33



PATUXENT RIVER BASIN

Figure 34

INPUT DATA FOR FLOW RELEASE MODEL --- PATUXENT RIVER BASIN

28	9	13	2	1	1	1	1	1	1	2	5.	00.0	44.	2.0	0.0
	85.														
	1.00		0.80		0.60		0.40		0.2						
	4.00		3.00		2.00		1.00		0.00						
1	2		44.990		524.27		0.285		0.440		.050		.483		1
2	4		45.000		524.27		0.285		0.440		.050		.483		3
4	6		50.000		414.77		0.285		0.440		.050		.483		3
6	8		55.000		386.00		0.285		0.440		.050		.483		3
8	10		60.00		362.00		0.375		0.440		0.047		0.525		3
10	12		63.70		342.60		0.360		0.480		0.056		0.490		2
12	14		64.50		181.20		0.630		0.480		0.075		0.490		3
14	16		66.10		181.20		0.620		0.480		0.075		0.490		3
16	18		69.40		181.20		0.600		0.480		0.075		0.490		3
18	20		71.00		157.0		1.000		.285		.062		.622		1
20	21		73.00		157.0		.850		.285		.062		.622		1
21	22		74.60		157.0		.709		.285		.056		.622		3
22	24		76.00		157.10		0.660		0.285		0.070		0.622		3
24	26		77.30		133.00		0.660		0.285		0.076		0.622		3
26	28		78.50		133.00		0.660		0.285		0.080		0.622		1
28	30		80.00		133.00		0.580		0.273		0.360		0.253		1
30	32		80.90		133.00		0.480		0.273		0.400		0.253		6
32	36		66.65		161.10		0.660		0.320		0.038		0.625		3
36	38		66.85		161.10		0.660		0.320		0.039		0.625		3
38	40		70.65		140.00		0.550		0.320		0.041		0.625		3
40	42		73.00		140.00		0.550		0.320		0.042		0.625		3
42	46		75.45		125.00		0.530		0.343		0.240		0.360		3
46	48		75.65		125.00		0.500		0.343		0.240		0.360		4
48	50		77.20		125.00		0.580		0.285		0.060		0.622		3

CONTINUED

INPUT DATA FOR FLOW RELEASE MODEL --- PATUXENT RIVER BASIN

50	52	78.50	110.00	0.580	0.285	0.060	0.622	4	
52	54	80.10	110.00	0.580	0.285	0.060	0.622	3	
54	56	80.90	110.00	0.480	0.285	0.100	0.622	3	
56	58	82.40	98.48	0.330	0.340	0.135	0.525	6	
14	2	1.90	.15	185.0	4.0	25.0	45.0		93.0
22	2	2.50	.15	136.0	4.0	25.0	31.0		87.0
26	2	.440	.15	207.0	4.0	25.0	39.0		95.0
36	2	.700	.15	185.0	4.0	25.0	30.0		95.0
40	2	1.6	.15	210.0	4.0	25.0	25.0		95.0
46	2	1.40	.15	310.0	4.0	25.0	27.0		85.0
48	5	4.000							
52	5	1.000							
56	2	0.300	.15	200.0	4.0	25.0	22.0		85.0
1	25.0	1 5.0	1 1.6	1 .25	1 .20	1 .05	1 2.0		
4	0	4.10 4.10	6.50 8.4	.050	.10	25.0			
6	0	.91 .91	3.00 6.0	.050	.10	25.0			
8	0	.91 .91	3.00 6.0	.050	.10	25.0			
10	0	.91 .91	3.00 6.0	.050	.10	25.0			
16	0	1.70 1.70	3.00 6.0	.050	.10	22.0			
18	0	1.70 1.70	3.00 6.0	.050	.10	22.0			
24	0	3.40 3.40	3.00 6.0	.050	.10	22.0			
32	2	15.00 55.00	3.30 8.9	.050	.10	17.0			
38	0	3.46 3.46	3.00 6.0	.050	.10	22.0			
42	0	2.45 2.45	3.00 6.0	.050	.10	22.0			
50	0	2.45 2.45	11.4 3.2	3.0	.10	22.0			
54	0	1.95 1.95	3.00 6.0	.050	.10	22.0			
58	2	15.00 55.00	3.00 6.0	.050	.10	22.0			

SAMPLE OUTPUT OF FLOW RELEASE MODEL

INPUT DATA

STREAM SEGEMENTS	WASTE DISCHARGES	FLOW ADD POINTS	RESERVOIRS
28	9	13	2

FIRST NODE	REAIR	D-SETS	F-SETS	T-SETS	I-BOD	O-TYPE	INC	DELTA	REMBOD	REMPHS	D-FIRST NODE
1	2	1	1	1	1	1	2.00	5.00	0.0	0.0	44.00

STREAM DATA

SECTION	I-NODE	J-NODE	DISTANCE (MILES)	AREA SQ.MILES)	AA	BB	CC	DD	TYPE
1	1	2	44.990	524.270	0.285	0.440	0.050	0.483	1
2	2	4	45.000	524.270	0.285	0.440	0.050	0.483	3
3	4	6	50.000	414.770	0.285	0.440	0.050	0.483	3
4	6	8	55.000	386.000	0.285	0.440	0.050	0.483	3
5	8	10	60.000	362.000	0.375	0.440	0.047	0.525	3
6	10	12	63.700	342.600	0.360	0.480	0.056	0.490	2
7	12	14	64.500	181.200	0.680	0.480	0.075	0.490	3
8	14	16	66.100	181.200	0.620	0.480	0.075	0.490	3
9	16	18	69.400	181.200	0.600	0.480	0.075	0.490	3
10	18	20	71.000	157.000	1.000	0.285	0.062	0.622	1
11	20	21	73.000	157.000	0.850	0.285	0.062	0.622	1
12	21	22	74.600	157.000	0.700	0.285	0.056	0.622	3
13	22	24	76.000	157.100	0.660	0.285	0.070	0.622	3
14	24	26	77.300	133.000	0.660	0.285	0.076	0.622	3
15	26	28	78.500	133.000	0.660	0.285	0.080	0.622	1
16	28	30	80.000	133.000	0.580	0.273	0.360	0.253	1
17	30	32	80.900	133.000	0.480	0.273	0.400	0.253	6
18	12	36	66.650	161.100	0.660	0.320	0.038	0.625	3
19	36	38	66.850	161.100	0.660	0.320	0.039	0.625	3
20	38	40	70.650	140.000	0.550	0.320	0.041	0.625	3
21	40	42	73.000	140.000	0.550	0.320	0.042	0.625	3
22	42	46	75.450	125.000	0.530	0.343	0.240	0.360	3
23	46	48	75.650	125.000	0.500	0.343	0.240	0.360	4
24	48	50	77.200	125.000	0.580	0.285	0.060	0.622	3
25	50	52	78.500	110.000	0.580	0.285	0.060	0.622	4
26	[illegible]	[illegible]	80.100	110.000	0.580	0.285	0.060	0.622	3
27	[illegible]	[illegible]	[illegible]	[illegible]	0.480	0.285	0.100	0.622	2

DYNAMIC PROGRAMMING BASIC DATA

TEST 1	TEST 2	TEST 3	TEST 4	TEST 5	ADD 1	ADD 2	ADD 3	ADD 4	ADD 5
4.00	3.00	2.00	1.00	0.0	1.00	0.80	0.60	0.40	0.20

WASTE DATA

NUMBER	NODE	TYPE	FLOW (MGD)	DEOX (1/DAY)	BOD (MG/L OR #/DAY)	DO (MG/L)	TEMP (DEG.C)	PHOS (MG/L)	REMOVAL (%)
1	14	2	1.900	0.150	185.000	4.000	25.000	45.000	93.000
2	22	2	2.500	0.150	136.000	4.000	25.000	31.000	87.000
3	26	2	0.440	0.150	207.000	4.000	25.000	39.000	95.000
4	36	2	0.700	0.150	185.000	4.000	25.000	30.000	95.000
5	40	2	1.600	0.150	210.000	4.000	25.000	25.000	85.000
6	46	2	1.400	0.150	310.000	4.000	25.000	27.000	85.000
7	48	5	4.000	0.0	0.0	0.0	0.0	0.0	0.0
8	52	5	1.000	0.0	0.0	0.0	0.0	0.0	0.0
9	56	2	0.300	0.150	200.000	4.000	25.000	22.000	85.000

ADDITIONAL DATA

IT2	S.S. TEMP.	IT3	DO-CONSTR.	IT4	DEPTH FACT	IT5	DECAY TEMP	IT6	DECAY PHOS	IT7	S.S. PHOS.	IT8	DY EQ BOD.
1	25.00	1	5.00	1	1.60	1	0.25	1	0.20	1	0.05	1	2.00

STREAM FLOW DATA

NUMBER	NODE	TYPE	MIN-FLOW (CFS)	MAX-FLOW (CFS)	BOD (MG/L)	DO (MG/L)	PHOS (MG/L)	DEOX (1/DAY)	TEMP (DEG.C)	F-TEMP	F-DO	F-BOD	F-PHOS
1	4	0	4.10	4.10	6.50	8.40	0.05	0.10	25.00	0.0	0.0	0.0	0.0
2	6	0	0.91	0.91	3.00	6.00	0.05	0.10	25.00	0.0	0.0	0.0	0.0
3	8	0	0.91	0.91	3.00	6.00	0.05	0.10	25.00	0.0	0.0	0.0	0.0
4	10	0	0.91	0.91	3.00	6.00	0.05	0.10	25.00	0.0	0.0	0.0	0.0
5	16	0	1.70	1.70	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
6	18	0	1.70	1.70	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
7	24	0	3.40	3.40	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
8	32	2	15.00	55.00	3.30	8.90	0.05	0.10	17.00	0.0	0.0	0.0	0.0
9	38	0	3.46	3.46	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
10	42	0	2.45	2.45	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
11	50	0	2.45	2.45	11.40	3.20	3.00	0.10	22.00	0.0	0.0	0.0	0.0
12	54	0	1.95	1.95	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0
13	58	2	15.00	55.00	3.00	6.00	0.05	0.10	22.00	0.0	0.0	0.0	0.0

MINIMUM TREATMENT

85.00

CALCULATED MIN- & MAX- FLOW DATA

JJ	RFLOW(JJ)	MINRQ(JJ)	MAXRQ(JJ)
58	15.00	15.00	55.00
56	15.46	15.46	55.46
54	17.41	17.41	57.41
52	15.87	15.87	55.87
50	18.32	18.32	58.32
48	12.13	12.13	52.13
46	14.29	14.29	54.29
42	16.74	16.74	56.74
40	19.22	19.22	59.22
38	22.68	22.68	62.68
36	23.76	23.76	63.76
32	15.00	15.00	55.00
30	15.00	15.00	55.00
28	15.00	15.00	55.00
26	15.68	15.68	55.68
24	19.08	19.08	59.08
22	22.95	22.95	62.95
21	22.95	22.95	62.95
20	22.95	22.95	62.95
18	24.65	24.65	64.65
16	26.35	26.35	66.35
14	29.29	29.29	69.29
12	53.05	53.05	133.05
10	53.96	53.96	133.96
8	54.87	54.87	134.87
6	55.78	55.78	135.78
4	59.88	59.88	139.88
2	59.88	59.88	139.88

INCREMENTAL FLOW TABLEAU

U-NODE = 36

FLOW (CFS)	BOD (MG/L)	DOD (MG/L)	SAT-DO (MG/L)	DEAIR (1/DAY)	TEMP (DEG.C)	COST (ANNUAL)
33.76	4.55	2.93	8.19	0.14	24.89	0.0
38.76	4.41	2.69	8.19	0.13	24.89	0.0
43.76	4.29	2.51	8.19	0.13	24.88	0.0
48.76	4.18	2.36	8.20	0.13	24.87	0.0
53.76	4.09	2.24	8.20	0.13	24.86	0.0
58.76	4.00	2.14	8.20	0.13	24.84	0.0
63.76	3.93	2.06	8.20	0.13	24.82	0.0

INCREMENTAL FLOW TABLEAU

U-NODE = 14

LOW FS)	BOD (MG/L)	DOD (MG/L)	SAT-DO (MG/L)	DEAIR (1/DAY)	TEMP (DEG.C)	COST (ANNUAL)
9.29	3.78	2.82	8.21	0.13	24.81	0.0
4.29	3.66	2.61	8.21	0.13	24.77	0.0
9.29	3.57	2.45	8.22	0.13	24.70	0.0
4.29	3.50	2.31	8.24	0.13	24.61	0.0
9.29	3.44	2.19	8.25	0.13	24.51	0.0
4.29	3.40	2.10	8.27	0.12	24.39	0.0
9.29	3.36	2.01	8.29	0.12	24.27	0.0
4.29	3.33	1.94	8.31	0.12	24.14	0.0
9.29	3.31	1.87	8.33	0.12	24.01	0.0

INCREMENTAL FLOW TABLEAU

U-NODE = 2

LOW (CFS)	BOD (MG/L)	DOD (MG/L)	SAT-DO (MG/L)	DEAIR (1/DAY)	TEMP (DEG.C)	COST (ANNUAL)
9.88	2.14	1.24	8.18	0.13	25.00	0.0
4.88	2.12	1.28	8.18	0.13	25.00	0.0
9.88	2.11	1.32	8.18	0.13	25.00	0.0
4.88	2.10	1.35	8.18	0.13	25.00	0.0
0.88	2.10	1.37	8.18	0.13	25.00	0.0
5.88	2.09	1.40	8.18	0.13	25.00	0.0
1.88	2.08	1.42	8.18	0.13	25.00	0.0
6.88	2.08	1.44	8.18	0.13	25.00	0.0
2.88	2.07	1.46	8.18	0.12	25.00	0.0
7.88	2.07	1.48	8.18	0.12	24.99	0.0
3.88	2.06	1.49	8.18	0.12	24.98	0.0
8.88	2.06	1.50	8.18	0.12	24.98	0.0
4.88	2.06	1.52	8.18	0.12	24.98	0.0
9.88	2.05	1.53	8.18	0.12	24.98	0.0
5.88	2.05	1.54	8.18	0.12	24.97	0.0

DYNAMIC PROGRAMMING TABLEAU 1

FOR NODE = 12

AL FLOW CFS)	FLOW AT 14 (CFS)	FLOW AT = 36 (CFS)	BOD (MG/L)	DOD (MG/L)	SAT-DO (MG/L)	DEAIR (1/DAY)	TEMP (DEG.C)	COST (ANNUAL)
3.05	29.29	33.76	4.19	2.88	8.20	0.14	24.86	0.0
3.05	29.29	38.76	4.14	2.75	8.20	0.13	24.86	0.0
3.05	29.29	43.76	4.09	2.63	8.20	0.13	24.85	0.0
3.05	29.29	48.76	4.03	2.53	8.20	0.13	24.85	0.0
3.05	29.29	53.76	3.98	2.44	8.20	0.13	24.84	0.0
3.05	29.29	58.76	3.93	2.37	8.20	0.13	24.83	0.0
3.05	29.29	63.76	3.88	2.30	8.20	0.13	24.82	0.0
8.05	34.29	63.76	3.83	2.25	8.21	0.13	24.80	0.0
3.05	39.29	63.76	3.79	2.21	8.21	0.13	24.78	0.0
8.05	44.29	63.76	3.75	2.16	8.22	0.13	24.74	0.0
3.05	69.29	43.76	3.69	2.12	8.28	0.13	24.35	0.0
3.05	69.29	48.76	3.67	2.07	8.28	0.13	24.36	0.0
3.05	69.29	53.76	3.65	2.03	8.27	0.13	24.38	0.0
8.05	69.29	58.76	3.63	2.00	8.27	0.12	24.39	0.0
3.05	69.29	63.76	3.61	1.96	8.27	0.12	24.40	0.0

APPENDIX D

VERIFICATION LINK COMPUTER PROGRAM

```
C                    VERIFICATION LINK - VERSION II
C                              INCLUDES
C          BOD - DO - TEMPERATURE - PHOSPHOROUS ALGORITHMS
C                              DEVELOPED AT
C                          UNIVERSITY OF MICHIGAN
C                           ANN ARBOR, MICHIGAN
C                             FIELD TESTED AT
C                         CHESAPEAKE FIELD STATION
C                                 FWPCA
C                           ANNAPOLIS, MARYLAND
C
      COMMON        AA(350), BB(350), BOD(100), CC(350), CONST(350)
     A             , DAREA(350), DD(350), DECAYP(350), DECAYT(350)
     B             , DEL(100), DEPTH(600), DIS(600), DISTAN(100), DO1(100)
     C             , DOD(100), DPFAC(350), EXTBOD(350), I(350), J(350)
     D             , LBOD(600), LDOD(600), LFLOW(600), LK(600), LPHOS(600)
     E             , LTEMP(600), MAXRQ(600), MINRQ(600), NNTYPE(350)
     F             , PHOS(100), PHOTOS(350), PPHOS(350), RBOD(150)
     G             , RDO(150), REAIR(600), RFBOD(150), RFDO(150)
     H             , RFLOW(600), RFPHOS(150), RFTEMP(150), RJ(150)
     I             , RK(150), RMAXQ(150), RMINQ(150), RPHOS(150)
     J             , RTEMP(150), RTYPE(150), SATDO(100), SBOD(600)
     K             , SEDBOD(350), SFLOW(600), SK(600), SLDBOD(350)
     L             , SLOPE(350), SSBOD(100), SSDOD(100), SSTEMP(100)
     M             , STEMP(600), SXFLOW(100), SYFLOW(100), TDIS(350)
      DIMENSION     TEMP(100), TRET(30), TTEMP(350), VEL(600), WBOD(150)
     A             , WDO(150), WFLOW(600), WIDTH(600), WJ(150), WK(150)
     B             , WPHOS(150), WREM(150), WSTP(5), WTEMP(150)
     C             , WTYPE(150), WWFLOW(150),NN(600),NTYPE(600)
C
C
      INTEGER       BCC, C, D, E, EE, EEE, FIRSTI, RJ, RTYPE, S, SS, T
     A             , TAPE1, TAPE2, TOTALN, TOTALR, TOTALW, TT, W, WJ
     B             , WTYPE, X, XNODE, Y, YBASE, YNODE, YYBASE, Z , TRES
C
      REAL          INC, K, LBOD, LDOD, LFLOW, LK, LPHOS, LTEMP, MAXRQ
     A             , MBOD, MDOD, MFLOW, MINRQ, MK, MPHOS, MTEMP, NBOD
     B             , NDOD, NFLOW, NK, NPHOS, NTEMP
C
      CALL TIMEIN
  001 READ (1,1001) TOTALN, TOTALW, TOTALR, TRES, FIRSTI, ITO, IT1
     B             , IRMBOD, ITBOD, OPTYPE, IRSET, DELTA, REMBOD
     C             , DIS(FIRSTI), INC, REMPHS
      READ (1,1005) WSTP(1), WSTP(2), WSTP(3), WSTP(4), WSTP(5)
      READ(1,1002) (I(N), J(N), TDIS(N), DAREA(N),          AA(N)
```

```
     1              ,BB(N), CC(N), DD(N), NNTYPE(N),          N=1,TOTALN)
      WRITE  (3,3050) TOTALN, TOTALW, TOTALR , TRES
      WRITE (3,3060) FIRSTI, IRSET, ITO, ITI , IRMBOD,   ITBOD, OPTYPE
     A            , INC, DELTA, REMBOD, REMPHS, DIS(FIRSTI)
      WRITE(3,3002) (N, I(N), J(N), TDIS(N), DAREA(N), AA(N), BB(N)
     B             , CC(N), DD(N), NNTYPE(N), N= 1, TOTALN)
  75  READ(1,1003) (WJ(N), WTYPE(N), WWFLOW(N), WK(N), WBOD(N), WDO(N)
     1             ,WTEMP(N), WPHOS(N), WREM(N), N=1,TOTALW)
      WRITE (3,3003)  ( N, WJ(N), WTYPE(N), WWFLOW(N), WK(N), WBOD(N)
     A              , WDO(N), WTEMP(N), WPHOS(N), WREM(N), N=1,TOTALW)
C
      IRM        =  1
      IQM        = 1
      ERROR      =  .005
  78  READ (1,1008) IT2, TTEMP1, IT3, CONST1, IT4, DPFAC1, IT5, DKTEMP,
     A     IT6, DKPHOS, IT7, SSPHOS,IT8,BODSS
      WRITE(3,4010) IT2, TTEMP1, IT3, CONST1, IT4, DPFAC1, IT5, DKTEMP,
     A     IT6, DKPHOS, IT7, SSPHOS,IT8,BODSS
      IF(IRMBOD) 79, 79, 80
  79  WSTP(IRM) = 0.
  80  GO TO (81,83), IT2
  81  DO 82 IJK = 1, TOTALN
  82  TTEMP(IJK)= TTEMP1
  83  GO TO (84,86), IT3
  84  DO 85 IJK = 1, TOTALN
  85  CONST(IJK)= CONST1
  86  GO TO (87,89), IT4
  87  DO 88 IJK = 1, TOTALN
  88  DPFAC(IJK)= DPFAC1
  89  GO TO (90,92), IT5
  90  DO 91 IJK = 1, TOTALN
  91  DECAYT(IJK)= DKTEMP
  92  GO TO (93,95), IT6
  93  DO 94 IJK = 1, TOTALN
  94  DECAYP(IJK)= DKPHOS
  95  GO TO (96,98), IT7
  96  DO 97 IJK = 1, TOTALN
  97  PPHOS(IJK) = SSPHOS
  98  CONTINUE
      GO TO ( 100,102),IT8
 100  DO 101 IJK= 1, TOTALN
 101  SEDBOD( IJK) = BODSS
 102  DO 103 N = 1, TOTALN
      JJ         = J(N)
      NN(JJ)     = N
      DIS(JJ)    = TDIS(N)
```

```
  103 NTYPE( JJ ) = NNTYPE( N)
      DO  104    IJK = 1, 600
      RFLOW(IJK) = 0.
      SFLOW(IJK) = 0.
      LFLOW(IJK) = 0.
      WFLOW(IJK) = 0.
      MINRQ(IJK) = 0.
  104 MAXRQ( IJK ) = 0.
      READ(1,1004) (RJ(N), RTYPE(N), RMINQ(N), RMAXQ(N), RBOD(N),RDO(N)
     A              ,RPHOS(N), RK(N), RTEMP(N), RFTEMP(N), RFDO(N)
     B              ,RFBOD(N), RFPHOS(N), N=1,TOTALR)
      WRITE (3,3004) ( N, RJ(N), RTYPE(N), RMINQ(N), RMAXQ(N), RBOD(N)
     A               , RDO(N), RPHOS(N), RK(N), RTEMP(N), RFTEMP(N)
     B               , RFDO(N), RFBOD(N), RFPHOS(N), N = 1, TOTALR )
C
      IRM         =  1
  105 DO 109 N    = 1, TOTALR
      ITR1        =  RTYPE(N) + 1
      JJ          =  RJ(N)
      GO TO (107, 106, 106), ITR1
  106 MINRQ(JJ) = RMINQ(N)
      MAXRQ(JJ) = RMAXQ(N)
      GO TO 109
  107 WFLOW(JJ) = RMINQ(N)
      WW5         =  RMINQ(N) * 5.39
      LFLOW(JJ) =  WFLOW(JJ)
      LBOD(JJ)  =  RBOD(N) * WW5
      LTEMP(JJ) =  RTEMP(N)
      LK(JJ)    =  RK(N)
      LPHOS(JJ) =  RPHOS(N) * WW5
      LDOD(JJ)  = (QSATDO(LTEMP(JJ)) - RDO(N)) * WW5
  109 CONTINUE
  110 DO 120  N = 1, TOTALW
      WSTPX = WREM(N)
      JJ          =  WJ(N)
      IWTP        =  WTYPE(N)
      GO TO (115, 116, 116, 116, 119), IWTP
  115 WW5         =  WWFLOW(N) * 5.39
      LFLOW(JJ) =  WWFLOW(N)
      WFLOW(JJ) =  LFLOW(JJ)
      LBOD(JJ)    =  WBOD(N) * WW5
      LTEMP(JJ) =  WTEMP(N)
      LK(JJ)    =  WK(N)
      LPHOS(JJ) =  WPHOS(N) * WW5
      LDOD(JJ)  = (QSATDO(LTEMP(JJ)) - WDO(N)) * WW5
      GO TO 120
```

```
  116 IF (WREM(N)- WSTP(IRM))117, 118, 118
  117 WREM(N)    =  WSTP(IRM)
  118 WW8        =  WWFLOW(N) * 8.345
      LFLOW(JJ) =  WWFLOW(N) * 1.547
      WFLOW(JJ) =  LFLOW(JJ)
      GO TO (112, 113), ITBOD
  113 LBOD(JJ)   =  WBOD(N) * (100. - WREM(N)) / 100.
      GO TO 114
  112 LBOD(JJ)   =  WBOD(N) * (100. - WREM(N)) * WW8 / 100.
  114 LTEMP(JJ)  =  WTEMP(N)
      LK(JJ)     =  WK(N)
      LPHOS(JJ) = WPHOS(N) * WW8 * (100.-REMPHS)/100.
      LDOD(JJ)  = (QSATDO(LTEMP(JJ))- WDO(N)) * WW8
      WREM(N)    =  WSTPX
      GO TO 120
  119 LFLOW(JJ) = -WWFLOW(N) * 1.547
      WFLOW(JJ) =  LFLOW(JJ)
      LBOD(JJ)  =  0.0
      LTEMP(JJ) =  0.0
      LK(JJ)    =  0.0
      LPHOS(JJ) =  0.0
      LDOD(JJ)  =  0.0
  120 CONTINUE
      IF ( IRM -1 ) 136, 136, 137
  136 WRITE ( 3, 2007 )
      DO 140 IJK = 1, TOTALN
      N = TOTALN - IJK + 1
      JJ = J(N)
      II =  I(N)
      MINRQ(JJ) =  MINRQ(JJ) + WFLOW(JJ)
      MAXRQ(JJ) =  MAXRQ(JJ) + WFLOW(JJ)
      MINRQ(II) =  MINRQ(JJ) + MINRQ(II)
      MAXRQ(II) =  MAXRQ(JJ) + MAXRQ(II)
      RFLOW(JJ) =  MINRQ(JJ)
  140 WRITE (3,1007) JJ, RFLOW(JJ), MINRQ(JJ), MAXRQ(JJ)
  137 WRITE ( 3, 5001 ) WSTP(IRM)
      WRITE (3,3015)
C
      L  =  TOTALR
C
   54 IF (RTYPE(L)) 152, 151, 152
  151 L          =  L - 1
      GO TO 54
  152 JJ         =  RJ(L)
      N  =  NN(JJ)
      II =  I(N)
```

```
      T  =  1
      TEMP(T)    =  RTEMP(L)
      BOD(T)     =  RBOD(L)
      DO1(T)     =  RDO(L)
      PHOS(T)    =  RPHOS(L)
      SATDO(T)   = QSATDO(TEMP(T))
      DOD(T) = SATDO(T) - DO1(T)
      DEL(T)     = TEMP(T) - TTEMP(N)
      SK(JJ)     = RK(L)
      DISTAN(T) = DIS(JJ)
      R5 = RFLOW(JJ) * 5.39
      LPHOS(JJ) = PHOS(T)*R5
      LFLOW(JJ) = RFLOW(JJ)
      LTEMP(JJ) = TEMP(T)
      LDOD(JJ)  = DOD(T) * R5
      LBOD(JJ)  = BOD(T) * R5
      LK(JJ) = SK(JJ)
C
   51 DEPTH(JJ) = AA(N) * RFLOW(JJ) ** BB(N)
      VEL(JJ)   = CC(N) * RFLOW(JJ) ** DD(N)
      DIST      = DIS(JJ)
      GO TO (191, 192, 193, 194), IRSET
  191 REAIR(JJ) = ((146.8 * VEL(JJ))**.5)/(2.303*(DEPTH(JJ)*DPFAC(N))
     A            ** 1.50 )
      GO TO  50
  192 REAIR(JJ) = (5.026 * VEL(JJ) **.969) /(DPFAC(N)*DEPTH(JJ))**1.673
      GO TO  50
  193 REAIR(JJ) =(3.3 * VEL(JJ))/(( DEPTH(JJ) * DPFAC(N))**1.33)
      GO TO 50
  194 GO TO 9999
C
   50 K         = SK(JJ)
      IF (DIST - DIS(II) - INC) 750, 750, 760
  750 BCC       = 0
      TDIST     = DIST - DIS(II)
      DIST      =  DIS(II)
      GO TO 770
  760 BCC       = 1
      TDIST     = INC
      DIST      = DIST - INC
C
  770 TIME      =  TDIST / (VEL(JJ) * 16.36364)
      T         =  T + 1
      DISTAN(T) = DIST
      QQQQ      = -TDIST * DECAYT(N)/ (VEL(JJ) * DEPTH(JJ))
  772 TEMP(T) = TTEMP(N) + DEL(T-1) * EXP(QQQQ)
```

```
      DEL(T)     =  TEMP(T) - TTEMP(N)
      R          =  REAIR(JJ) * 1.0241** (TEMP(T)-20.)
      K          =  K * 1.0471 ** (TEMP(T) - 20.)
      A          =  1. / 10. ** (K * TIME)
      P          =  1. / 10. ** (R * TIME/2.)
      BOD(T)     =  A * BOD(T-1)
      UBOD       = 1. / ( 1.- 10.**(-K * 5.))
      DOD(T)      = P * UBOD *(BOD(T-1)-BOD(T)) + P * P * DOD(T-1)
      SATDO(T)   = QSATDO(TEMP(T))
      DO1(T)     =  SATDO(T) - DOD(T)
      IF(BOD(T) -SEDBOD(N))222,223,223
  222 BOD(T)     = SEDBOD(N)
  223 IF ( DO1( T) ) 224,224,225
  224 DOD(T)     = SATDO(T)
      DO1(T)     = 0.
  225 PHOS(T)    = PHOS(T-1) * 10. ** (-DECAYP(N) * TIME)
      IF (PHOS(T)- PPHOS(N)) 226, 227, 227
  226 PHOS(T)    =  PPHOS(N)
  227 CONTINUE
C
      IF (BCC) 50, 780, 50
C
  780 WRITE (3,3021) (JJ, KKK, RFLOW(JJ), DISTAN(KKK), BOD(KKK)
     A              , TEMP(KKK), SATDO(KKK), DO1(KKK), PHOS(KKK)
     B              , KKK = 1, T )
C
      IF ( I(N) - FIRSTI ) 790, 1260, 790
  790 IF (I(N) - J(N-1)) 800,  53, 800
C
  800 R5         =  RFLOW(JJ) * 5.39
      LBOD(II)   =  BOD(T)  * R5
      LDOD(II)   =  DOD(T)  * R5
      LPHOS(II)  =  PHOS(T) * R5
      LK(II)     = K / (1.047 ** (TEMP(T) - 20.))
      LTEMP(II)  =  TEMP(T)
      LFLOW(II)  =  RFLOW(JJ)
      L          =  L - 1
C     WRITE (3,1009) LFLOW(II), LBOD(II), LDOD(II), LPHOS(II), LTEMP(II)
C    A           , LK(II), II
      GO TO 54
C
   53 TT         =  T
      T          =  1
      III        =  JJ
      JJ         =  II
      N          =  NN(JJ)
```

```
      II        = I(N)
      NT = NTYPE(JJ)
      GO TO (55, 1250, 1250, 55, 1240), NT
      GO TO 51
C
 1240 R5        = RFLOW(III) * 5.39
      K         = K / (1.0471 ** (TEMP(TT)- 20.))
      LK(JJ)    = K
      LBOD(JJ)  = LBOD(JJ) + R5 * BOD(TT) +  LFLOW(JJ) * 5.39 * BOD(TT)
      LPHOS(JJ) = LPHOS(JJ) + R5 * PHOS(TT) + LFLOW(JJ) * 5.39*PHOS(TT)
      LDOD(JJ)  = LDOD(JJ) + R5 * DOD(TT)
      LTEMP(JJ) = (RFLOW(III)*TEMP(TT)+LTEMP(JJ)*LFLOW(JJ))/RFLOW(JJ)
      LFLOW(JJ) = RFLOW(JJ)
      TEMP(T)   = LTEMP(JJ)
      DEL(T)    = TEMP(T) - TTEMP(N)
      SATDO(T)  = QSATDO(TEMP(T))
      R5D       = 1.0 / (RFLOW(JJ) * 5.39)
      BOD(T)    = LBOD(JJ)  * R5D
      DOD(T)    = LDOD(JJ)  * R5D
      PHOS(T)   = LPHOS(JJ) * R5D
      DO1(T)    = SATDO(T) - DOD(T)
      DISTAN(T) = DIS(JJ)
      SK(JJ)    = LK(JJ)
      GO TO 51
C
   55 TEMP(T)   = TEMP(TT)
      DEL(T)    = DEL(TT)
      SATDO(T)  = SATDO(TT)
      BOD(T)    = BOD(TT)
      DOD(T)    = DOD(TT)
      DO1(T)    = DO1(TT)
      PHOS(T)   = PHOS(TT)
      DISTAN(T) = DIS(JJ)
      SK(JJ)    = SK(III)
      R5        = RFLOW(JJ) * 5.39
      LPHOS(JJ) = PHOS(T)*R5
      LFLOW(JJ) = RFLOW(JJ)
      LTEMP(JJ) = TEMP(T)
      LK(JJ)    = K / (1.0471 ** (TEMP(T) - 20. ))
      LDOD(JJ)  = DOD(T) * R5
      LBOD(JJ)  = BOD(T) * R5
      GO TO 51
C
 1250 R5        =  RFLOW(III)  * 5.39
      K         =  K / (1.0471 ** (TEMP(TT) - 20.))
```

```
      LBOD(JJ)  =  LBOD(JJ)  + R5 * BOD(TT)
      LDOD(JJ)  =  LDOD(JJ)  + R5 * DOD(TT)
      LPHOS(JJ) =  LPHOS(JJ) + R5 * PHOS(TT)
      LTEMP(JJ) =  (RFLOW(III) * TEMP(TT) + LTEMP(JJ) * LFLOW(JJ)) /
     A             RFLOW(JJ)
      LFLOW(JJ) = RFLOW(JJ)
      TEMP(T)   =  LTEMP(JJ)
      DEL(T)     =  TEMP(T) - TTEMP(N)
      SATDO(T)  = QSATDO(TEMP(T))
      R5D       =  1.0 / (RFLOW(JJ) * 5.39)
      BOD(T)    =  LBOD(JJ)  * R5D
      DOD(T)    =  LDOD(JJ)  * R5D
      PHOS(T)   =  LPHOS(JJ) * R5D
      DO1(T)    =  SATDO(T) - DOD(T)
      DISTAN(T)  =  DIS(JJ)
      SK(JJ)    =  LK(JJ)
      GO TO 51
C
 1260 WRITE (3,3016)
C
      DO  1265  KK = 1, TOTALN
      N         =  TOTALN - KK + 1
      JJ        =  J(N)
 1265 WRITE (3,3006) JJ, LFLOW(JJ), LBOD(JJ), LDOD(JJ), LPHOS(JJ)
     A             , LTEMP(JJ), LK(JJ)
C
      IF (IRMBOD-IRM) 1320, 1320, 1318
 1318 IRM       = IRM + 1
      DO 1319 JJ= 1, 600
      LFLOW(JJ) = 0.
      LDOD(JJ)  = 0.
      LBOD(JJ)  = 0.
      LTEMP(JJ) = 0.
      LK(JJ)    = 0.
 1319 LPHOS(JJ) = 0.
      GO TO 105
 1320 WRITE (3,3007)
      DO  1321 KK = 1, TOTALN
      N  =  TOTALN - KK+ 1
      JJ = J(N)
      IY    =    I(N)
      IX        =  J(N)
      DIST      = DIS(IX)-DIS(IY)
      TIME      = DIST /(VEL(JJ)* 16.36364)
 1321 WRITE (3,3008) JJ, RFLOW(JJ), SK(JJ), REAIR(JJ), VEL(JJ)
     A              , DEPTH(JJ), TIME
```

```
      IF( IT1-IQM) 1400, 1400, 1325
 1325 DO 1330 JJ= 1, 600
      LFLOW(JJ) = 0.
      LBOD(JJ)  = 0.
      LDOD(JJ)  = 0.
      LTEMP(JJ) = 0.
      LK(JJ)    = 0.
      RFLOW(JJ) = 0.
      MINRQ(JJ) = 0.
      MAXRQ(JJ) = 0.
 1330 WFLOW(JJ) = 0.
      IQM       = IQM + 1
      GO TO 78
 1400 IF(ITO-1)   1500, 1500, 1450
 1450  ITO      = ITO - 1
      GO TO 75
 1500 WRITE (3,3000)
C     FORMAT STATEMENTS
 1001 FORMAT ( 11I5, 5F5.0)
C
 1002 FORMAT ( 2I5, 6F10.0, I5)
 1003 FORMAT ( 2I5, 7F10.0    )
 1004 FORMAT ( 2I4, 1X, 11F6.2)
 1005 FORMAT ( 8F10.2 )
 1006 FORMAT( 6F12.2)
 1007 FORMAT ( ' ' 34X I3, 3F20.2 )
 1008 FORMAT ( 7( I5, F5.0))
 1009 FORMAT ( '0           LFLOW(II)            LBOD(II)            LDO
     AD(II)          LPHOS(II)          LTEMP(II)              LK(II)
     B  II ' / 6F20.2, I5 )
C
 1050 FORMAT ( 4I5,6F10.0)
 1090 FORMAT ( 10X, 7F5.0 )
 2002 FORMAT ( 3I5, 6F10.2, I5 )
 2003 FORMAT ( 3I5, 7F10.2     )
 2004 FORMAT ( 3I4, 1X, 11F6.2 )
 2005 FORMAT ( ' END OF JOB ' )
 2007 FORMAT ( '1' 49X 'CALCULATED MIN- & MAX- FLOW DATA' /
     A          '0' 35X 'JJ' 12X ' RFLOW(JJ)' 11X 'MINRQ(JJ)' 11X
     B          'MAXRQ(JJ)' // )
C
 3000 FORMAT('1' 10X 'END OF JOB')
C
 3002 FORMAT ( '1' 60X 'STREAM DATA' / '0' 4X 'SECTION' 3X 'I-NODE' 3X
     A          'J-NODE' 6X 'DISTANCE' 9X 'AREA' 12X 'AA' 14X 'BB' 13X
     B          'CC' 13X 'DD' 8X 'TYPE' / ' ' 35X '(MILES)' 7X
```

```
     C          'ISQ.MILES)' // (' ' 3I9, 6F15.3, I9))
 3003 FORMAT ( '1' 60X ' WASTE DATA' / '0' 5X 'NUMBER' 4X 'NODE' 5X
     A          'TYPE' 9X 'FLOW' 11X 'DEOX' 11X 'BOD' 13X 'OO' 12X 'TEMP'
     B          11X 'PHOS' 9X 'REMOVAL' /38X '(MGD)' 8X '(1/DAY)' 5X
     C          '(MG/L OR #/DAY)' 4X '(MG/L)' 8X '(DEG.C)' 9X '(MG/L)'
     D          10X '(%)' // (1H 3I9, 7F15.3))
 3004 FORMAT ( '1' 57X 'STREAM FLOW DATA' / '0' '   NUMBER  NODE   TYPE'
     A           '   MIN-FLOW  MAX-FLOW    BOD' 8X 'DO' 7X 'PHOS' 6X
     B           'DEOX' 6X 'TEMP     F-TEMP     F-DO      F-BOD      '
     C           'F-PHOS' / 27X '(CFS)     (CFS)     (MG/L)    (MG/L)'
     D           '    (MG/L)   (1/DAY)   (DEG.C)' // (' ' 3I7, 11F10.2) )
 3005 FORMAT ( ' ' / (2I10, 7F15.2))
 3006 FORMAT (  I10, 6F20.2)
 3007 FORMAT ( '1' 49X 'CALCULATED STREAM CHARACTERISTICS'   /
     A          '0' 6X 'NODE' 14X 'FLOW' 16X 'DEOX' 16X 'REAIR' 13X
     B         'VELOCITY' 14X 'DEPTH'  14X     'D-TIME'//
     C         25X '(CFS)' 14X '(1/DAY)' 13X '(1/DAY)' 14X  '(FPS)'
     D         15X '(FEET)'  14X '(DAYS)'//)
 3008 FORMAT  ( I10, 6F20.3)
 3011 FORMAT ( '1' 49X 'CALCULATED MIN- & MAX- FLOW DATA' / '0' 35X
     A          'JJ' 13X 'RFLOW(JJ)' 11X 'MINRQ(JJ)' 11X 'MAXRQ(JJ)' // )
 3012 FORMAT ( ' ' 32X I5, 3F20.2)
 3015 FORMAT ( '0' 50X 'CALCULATED STREAM QUALITY DATA'  /
     A          '0', 6X 'NODE' 8X 'T' 10X 'FLOW' 9X 'DISTANCE' 10X 'BOD'
     B          11X 'TEMP' 11X 'SATDO' 11X 'DO' 12X 'PHOS' / 30X '(CFS)'
     C          9X '(MILES)' 8X '(MG/L)' 9X '(DEG.C)' 8X '(MG/L)' 9X
     D         '(MG/L)' 9X '(MG/L)' )
 3016 FORMAT ( '1' 56X 'CALCULATED NODE DATA'  /
     A          '0' 6X 'NODE' 14X 'FLOW' 16X 'BOD' 17X 'DOD' 16X 'PHOS'
     A          17X 'TEMP' 16X 'DEOX' / 25X '(CFS)' 13X '(#/DAY)' 13X
     B          '(#/DAY)' 13X '(#/DAY)' 14X '(DEG.C)' 12X '(1/DAY)' // )
 3021 FORMAT ('0' / (I10, I10, 2F15.2, F14.2, 4F15.2 ))
C
 3050 FORMAT ('1' 58X 'INPUT DATA' /// 10X 'STREAM SEGEMENTS' 15X
     A         'WASTE DISCHARGES' 15X 'FLOW ADD POINTS' 17X
     B         'RESERVOIRS' // I20, 3I30)
 3060 FORMAT ('0'/ ' ' 3X 'FIRST NODE' 3X 'REAIR' 5X 'D-SETS' 4X'F-SETS'
     A          4X 'T-SETS' 4X 'I-BOD' 4X 'O-TYPE' 4X 'INC' 6X 'DELTA' 5X
     B         'REMBOD' 4X 'REMPHS' 2X 'D-FIRST NODE'// 7I10, 5F10.2 )
 4001 FORMAT ( 3I10, F15.3, I10 )
 4010 FORMAT ('1' 60X 'ADDITIONAL DATA'//'0' 1X 'IT2' 3X 'S.S. TEMP.'
     A        3X 'IT3' 3X 'DO-CONSTR.' 3X 'IT4'  3X  'DEPTH FACT' 3X
     B        'IT5' 3X 'DECAY TEMP' 3X 'IT6' 3X 'DECAY PHOS' 3X 'IT7'
     C        3X 'S.S. PHOS.' 3X 'IT8' 3X 'DY EQ BOD.' //'0'
     D        2X, I1,  F11.2, 6%I8, F11.2<<
C
```

```
 5001 FORMAT ('1' 54X 'MINIMUM % TREATMENT' // ( 62X F5.2 ))
 8000 FORMAT ( 5X, F5.0, 7F10.0 )
C
      CALL TIMED
 9999 STOP
      END
      FUNCTION QSATDO(TEMP)
C
      QSATDO    =  14.652 - (0.41022    * TEMP)
     A                    + (.007991    * TEMP * TEMP)
     B                    - (.77774E-4 * TEMP * TEMP * TEMP)
C
      RETURN
      END
```

APPENDIX E

OPTIMAL FLOW RELEASE MODEL COMPUTER PROGRAM

```
C
C                OPTIMAL FLOW RELEASE MODEL - VERSION II
C                          INCLUDES
C                   BOD - DO - TEMPERATURE ALGORITHMS
C                                WITH
C                          COST OF RESERVOIRS
C                            DEVELOPED AT
C                         UNIVERSITY OF MICHIGAN
C                           ANN ARBOR, MICHIGAN
C                            FIELD TESTED AT
C                        CHESAPEAKE FIELD STATION
C                                FWPCA
C                           ANNAPOLIS, MARYLAND
C
      COMMON      AA(320), ACREFT(20), AVRESQ(20), BB(320), BOD(100)
     A          , CC(320), CONST(320), DAREA(320), DD(320), DECAYP(350)
     B          , DECAYT(350), DEL(100), DEPTH(600), DIS(600)
     C          , DISTAN(100), DO1(100), DOD(100), DPFAC(320)
     D          , EXTBOD(350), I(320), INRESQ(20), IRES(20), J(320)
     E          , LBOD(600), LDOD(600), LFLOW(600), LK(600), LPHOS(600)
     F          , LTEMP(600), MAXM(30), MAXRQ(600), MBOD(200)
     G          , MCOST(200), MDOD(200), MFLOW(200), MINRQ(600), MJ(30)
     H          , MK(200), MN(30), MRES(20), MTEMP(200), NBOD(200)
     I          , NCOST(200), NDOD(200), NFLOW(200), NK(200), NN(600)
     J          , NNTYPE(320), NRES(20), NRJ(600), NTEMP(200), NRN(20)
     K          , NTYPE(600), OBOD(200), OCOST(200), ODOD(200)
     L          , OFLOW(200), OK(200), OTEMP(200), OXFLOW(200)
      DIMENSION   OYFLOW(200), PHOS(100), PHOTOS(350), PPHOS(350)
     A          , RBOD(100), RCOST(20,100), RDO(100), REAIR(600)
     B          , REBOD(100), REDO(100), RFLOW(600), REPHOS(100)
     C          , RETEMP(100), RJ(100), RK(100), RMAXQ(100), RMINQ(100)
     D          , RPHOS(100), RTEMP(100), RTYPE(100), SATDO(100)
     E          , SBOD(600), SCOST(200), SDELL(600), SDOD(600)
     F          , SEDBOD(350), SFLOW(600), SK(600), SLOBOD(350)
     G          , SLOPE(350), SSBOD(100), SSDOD(100), SSTEMP(100)
     H          , STEMP(600), SXFLOW(100), SYFLOW(100), TCOST(20)
     I          , TDIS(320), TEMP(100), TRET(30), TTEMP(320), VEL(600)
     J          , WBOD(100), WDO(100), WFLOW(600), WIDTH(600), WJ(100)
     K          , WK(100), WPHOS(100), WQACFT(20), WQCOST(20), WSTP(5)
     L          , WREM(100), WTEMP(100), WTYPE(100), WWFLOW(100)
C
      INTEGER     BCC, C, D, E, EE, EEE, FIRSTI, RJ, RTYPE, S, SS, T
     A          , TAPE1, TAPE2, TOTALN, TOTALR, TOTALW, TT, W, WJ
     B          , WTYPE, X, XNODE, Y, YBASE, YNODE, YYBASE, Z,TRES
     C          , OPTYPE, OPT
C
```

```
      REAL   .       INC, K, LBOD, LDOD, LFLOW, LK, LPHOS, LTEMP, MAXRQ
     A             , MBOD, MDOD, MFLOW, MINRQ, MK, MPHOS, MTEMP, NBOD
     B             , NDOD, NFLOW, NK, NPHOS, NTEMP
     C             , MCOST, NCOST, OCOST , INRESQ
  001 CALL TIMEIN
      IRM       =1
      TAPE1     =  7
      TAPE2     =  9
      ERROR      = .005
      NE        =  0
      IOM       = 1
      READ (1,1001) TOTALN, TOTALW, TOTALR, TRES, FIRSTI, ITO, IT1   ,
     A      IRMBOD,ITBOD, OPTYPE, IRSET, DELTA, REMBOD, DIS(FIRSTI),
     B      INC, REMPHS
      READ (1,1005) WSTP(1), WSTP(2), WSTP(3), WSTP(4), WSTP(5)
      READ (1,1005) ADD1, ADD2, ADD3, ADD4, ADD5
      READ (1,1005) TEST1, TEST2, TEST3, TEST4,TEST5
      READ(1,1002) (I(N), J(N), TDIS(N), DAREA(N),          AA(N)
     1             ,BB(N), CC(N), DD(N), NNTYPE(N),         N=1,TOTALN)
C
      WRITE  (3,3050) TOTALN, TOTALW, TOTALR , TRES
      WRITE (3,3060) FIRSTI, IRSET, ITO, IT1 , IRMBOD,   ITBOD, OPTYPE,
     A       INC, DELTA, REMBOD, REMPHS, DIS(FIRSTI)
      WRITE(3,3002) (N, I(N), J(N), TDIS(N), DAREA(N), AA(N), BB(N)
     B            , CC(N), DD(N), NNTYPE(N), N= 1, TOTALN)
C
      WRITE (3,3013) TEST1, TEST2, TEST3, TEST4, TEST5, ADD1, ADD2,
     A               ADD3, ADD4, ADD5
      IF(OPTYPE-1)75,75,72
   72 READ (1,4050) (NRES(N), NRN(N), MRES(N), IRES(N), ACREFT(N),
     A     WQACFT(N), TCOST(N), WQCOST(N), AVRESQ(N), INRESQ(N),
     B     N = 1, TRES)
      WRITE (3,3070) ( NRES(N), NRN(N), IRES(N), MRES(N), ACREFT(N),
     A       WQACFT(N), TCOST(N), WQCOST(N), AVRESQ(N), INRESQ(N),
     B       N= 1, TRES)
      DO 73 IXY  = 1, TRES
      IXP       = NRN(IXY)
      NRJ(IXP)  = IXY
      IZY       = 1
      IZT       = 12
      MCP       = MRES(IXY)
      DO 73 IZX  = 1, MCP
      READ(1,8005) (RCOST(IXY,IZZ), IZZ = IZY, IZT)
      IZY       =  IZY + 12
      IZT       =  IZT + 12
   73 CONTINUE
```

```
 75 READ(1,1003) (WJ(N), WTYPE(N), WWFLOW(N), WK(N), WBOD(N), WDO(N)
   1              ,WTEMP(N), WPHOS(N), WREM(N), N=1,TOTALW)
    WRITE (3,3003)  ( N, WJ(N), WTYPE(N), WWFLOW(N), WK(N), WBOD(N)
   A               , WDO(N), WTEMP(N), WPHOS(N), WREM(N), N=1,TOTALW)
    IQM        = 1
 78 READ (1,1008) IT2, TTEMP1, IT3, CONST1, IT4, DPFAC1, IT5, DKTEMP,
   A      IT6, DKPHOS, IT7, SSPHOS,IT8,BODSS
    WRITE(3,4010) IT2, TTEMP1, IT3, CONST1, IT4, DPFAC1, IT5, DKTEMP,
   A      IT6, DKPHOS, IT7, SSPHOS,IT8,BODSS
    IF(IRMBOD) 79, 79, 80
 79 WSTP(IRM) = 0.
 80 GO TO (81,83), IT2
 81 DO 82 IJK = 1, TOTALN
 82 TTEMP(IJK)= TTEMP1
 83 GO TO (84,86), IT3
 84 DO 85 IJK = 1, TOTALN
 85 CONST(IJK)= CONST1
 86 GO TO (87,89), IT4
 87 DO 88 IJK = 1, TOTALN
 88 DPFAC(IJK)= DPFAC1
 89 GO TO (90,92), IT5
 90 DO 91 IJK = 1, TOTALN
 91 DECAYT(IJK)= DKTEMP
 92 GO TO (93,95), IT6
 93 DO 94 IJK = 1, TOTALN
 94 DECAYP(IJK)= DKPHOS
 95 GO TO (96,98), IT7
 96 DO 97 IJK = 1, TOTALN
 97 PPHOS(IJK) = SSPHOS
 98 GO TO ( 100,102),IT8
100 DO 101 IJK= 1, TOTALN
101 SEDBOD( IJK) = BODSS
102 DO 103 N = 1, TOTALN
    JJ         = J(N)
    NN(JJ)     = N
    DIS(JJ)    = TDIS(N)
103 NTYPE( JJ ) = NNTYPE( N)
    DO  104    IJK = 1, 600
    RFLOW(IJK) = 0.
    SFLOW(IJK) = 0.
    LFLOW(IJK) = 0.
    WFLOW(IJK) = 0.
    MINRQ(IJK) = 0.
104 MAXRQ( IJK ) = 0.
    READ(1,1004) (RJ(N), RTYPE(N), RMINQ(N), RMAXQ(N), RBOD(N),RDO(N)
   A              ,RPHOS(N), RK(N), RTEMP(N), RFTEMP(N), RFDO(N)
```

```
     2              ,RFBOD(N), RFPHOS(N), N=1,TOTALR)
      WRITE (3,3004) ( N, RJ(N), RTYPE(N), RMINQ(N), RMAXQ(N), RBOD(N)
     A               , RDO(N), RPHOS(N), RK(N), RTEMP(N), RFTEMP(N)
     B               , RFDO(N), RFBOD(N), RFPHOS(N), N = 1, TOTALR )
C
      IRM        = 1
  105 DO 109 N   = 1, TOTALR
      JJ         =  RJ(N)
      ITR1       =  RTYPE(N) + 1
      GO TO (107, 106, 106), ITR1
  106 MINRQ(JJ) = RMINQ(N)
      MAXRQ(JJ) = RMAXQ(N)
      GO TO 109
  107 WFLOW(JJ) = RMINQ(N)
      WW5        =  RMINQ(N) * 5.39
      LFLOW(JJ) =  WFLOW(JJ)
      LBOD(JJ)   =  RBOD(N) * WW5
      LTEMP(JJ) =  RTEMP(N)
      LK(JJ)     =  RK(N)
      LPHOS(JJ) =  RPHOS(N) * WW5
      LDOD(JJ)  = (QSATDO(LTEMP(JJ)) - RDO(N)) * WW5
  109 CONTINUE
  110 WRITE (3,5001) WSTP(IRM)
      NE         = 0
      REWIND TAPE1
      REWIND TAPE2
      DO 120  N = 1, TOTALW
      WSTPX = WREM(N)
      JJ         =  WJ(N)
      IWTP       =  WTYPE(N)
      GO TO (115, 116, 116, 116, 119), IWTP
  115 WW5        =  WWFLOW(N) * 5.39
      LFLOW(JJ) =  WWFLOW(N)
      WFLOW(JJ) =  LFLOW(JJ)
      LBOD(JJ)   =  WBOD(N) * WW5
      LTEMP(JJ) =  WTEMP(N)
      LK(JJ)     =  WK(N)
      LPHOS(JJ) =  WPHOS(N) * WW5
      LDOD(JJ)  = (QSATDO(LTEMP(JJ)) - WDO(N)) * WW5
      GO TO 120
  116 IF (WREM(N)- WSTP(IRM))117, 118, 118
  117 WREM(N)     =  WSTP(IRM)
  118 WW8        =  WWFLOW(N) * 8.345
      LFLOW(JJ) =  WWFLOW(N) * 1.547
      WFLOW(JJ) =  LFLOW(JJ)
      GO TO (112, 113), ITBOD
```

```
113 LBOD(JJ) =  WBOD(N) * (100. - WREM(N))/ 100.
    GO TO 114
112 LBOD(JJ) =  WBOD(N) * (100. - WREM(N)) * WW8 / 100.
114 LTEMP(JJ)  =  WTEMP(N)
    LK(JJ)     =  WK(N)
    LPHOS(JJ) = WPHOS(N) * WW8 * (100.-REMPHS)/100.
    LDOD(JJ)  = (QSATDO(LTEMP(JJ))- WDO(N)) * WW8
    WREM(N) = WSTPX
    GO TO 120
119 LFLOW(JJ) = -WWFLOW(N) * 1.547
    WFLOW(JJ) =  LFLOW(JJ)
    LBOD(JJ)  =  0.0
    LTEMP(JJ) =  0.0
    LK(JJ)    =  0.0
    LPHOS(JJ) =  0.0
    LDOD(JJ)  =  0.0
120 CONTINUE
    DO 160 IJK = 1, TOTALN
    N = TOTALN - IJK + 1
    JJ = J(N)
    II = I(N)
    MAXRQ(JJ) = MAXRQ(JJ) + WFLOW(JJ)
    MAXRQ(II) = MAXRQ(II) + MAXRQ(JJ)
    MINRQ(JJ) = MINRQ(JJ) + WFLOW(JJ)
    MINRQ(II) = MINRQ(II) + MINRQ(JJ)
160 RFLOW(JJ) = MINRQ(JJ)
    IF( IRM-1) 161,161,163
161 WRITE (3,2007)
    DO 162 IJK = 1, TOTALN
    N = TOTALN - IJK + 1
    JJ = J(N)
162 WRITE (3,1007) JJ, RFLOW(JJ), MINRQ(JJ), MAXRQ(JJ)
C
163 L          =  TOTALR + 1
  4 L          =  L - 1
    IF (L) 5, 5, 189
189 IF (RTYPE(L) - 1) 4, 190, 4
C
190 JJ = RJ(L)
    N  =  NN(JJ)
    II =  I(N)
    T  = 1
    TEMP(T)  = RTEMP(L)
    BOD(T)     = RBOD(L)
    DO1(T)   = RDO(L)
    PHOS(T)  = RPHOS(L)
```

```
      SATDO(T) = QSATDO(TEMP(T))
      DOD(T)   = SATDO(T) - DO1(T)
      DEL(T)   = TEMP(T) - TTEMP(N)
      SK(JJ)   = RK(L)
      DISTAN(T) = DIS(JJ)
C
   51 DEPTH(JJ) = AA(N) * RFLOW(JJ) ** BB(N)
      VEL(JJ)   = CC(N) * RFLOW(JJ) ** DD(N)
      DIST      = DIS(JJ)
      GO TO (191, 192, 193, 194), IRSET
  191 REAIR(JJ) = ((146.8 * VEL(JJ))**.5)/(2.303*(DEPTH(JJ)*DPFAC(N))
     A            ** 1.50 )
  192 REAIR(JJ) = (5.026 * VEL(JJ) **.969) /(DPFAC(N)*DEPTH(JJ))**1.673
      GO TO  50
  193 REAIR(JJ) =(3.3 * VEL(JJ))/(( DEPTH(JJ) * DPFAC(N))**1.33)
      GO TO 50
  194 GO TO 9999
C
   50 K         = SK(JJ)
      IF (DIST - DIS(II) - INC) 200, 200, 210
  200 BCC       = 0
      TDIST     = DIST - DIS(II)
      DIST      =  DIS(II)
      GO TO 220
  210 BCC       = 1
      TDIST     = INC
      DIST      = DIST - INC
C
  220 TIME      =  TDIST / (VEL(JJ) * 16.36364)
      T         =  T + 1
      DISTAN(T) = DIST
      QQQQ      = -TDIST * DECAYT(N) / (VEL(JJ) * DEPTH(JJ))
      TEMP(T) = TTEMP(N) + DEL(T-1)* EXP(QQQQ)
      DEL(T)    =  TEMP(T) - TTEMP(N)
      R         =  REAIR(JJ) * 1.0241** (TEMP(T)-20.)
      K         =  K * 1.0471 ** (TEMP(T) - 20.)
      A         =  1. / 10. ** (K * TIME)
      P         =  1. / 10. ** (R * TIME/2.)
      BOD(T)    =  A * BOD(T-1)
      UBOD      = 1. / ( 1.- 10.**(-K * 5.))
      DOD(T)    = P * UBOD * (BOD(T-1) - BOD(T)) + P * P * DOD(T-1)
      SATDO(T)  = QSATDO(TEMP(T))
      IF(BOD(T) -SEDBOD(N))222,223,223
  222 BOD(T)   = SEDBOD(N)
  223 IF ( DO1( T) ) 224,224,225
  224 DOD(T)    = SATDO(T)
```

```
      DO1(T)    = 0.
  225 PHOS(T)   = PHOS(T-1) * 10. ** (-DECAYP(N) * TIME)
      IF (PHOS(T)- PPHOS(N)) 226, 227, 227
  226 PHOS(T)   =  PPHOS(N)
  227 CONTINUE
C
      IF (BCC) 50, 230, 50
C 230 WRITE (3,5002) JJ, RFLOW(JJ), SK(JJ), REAIR(JJ), VEL(JJ)
C    A             , DEPTH(JJ), ( KKK, DISTAN(KKK), BOD(KKK), TEMP(KKK)
C    B             , SATDO(KKK), DO1(KKK), PHOS(KKK), KKK = 1, T)
  230 IF ( I(N) - FIRSTI ) 240, 1260, 240
  240 IF ( I(N) - J(N-1)) 260, 250, 260
  250 IF ( MAXRQ(II) - MINRQ(II) - ERROR) 53, 53, 260
C
  260 R5        =  RFLOW(JJ) * 5.39
      LBOD(II)  =  BOD(T)  * R5
      LDOD(II)  =  DOD(T)  * R5
      LPHOS(II) =  PHOS(T) * R5
      LK(II)    = K / (1.047 ** (TEMP(T) - 20.))
      LTEMP(II) =  TEMP(T)
      LFLOW(II) =  RFLOW(JJ)
      L         =  L - 1
  270 IF(L) 189, 5, 189
C
   53 TT = T
      T  = 1
      III= JJ
      JJ = II
      N  = NN(JJ)
      NTP1      =  NTYPE(JJ)
      II = I(N)
      GO TO (55, 290, 290, 55, 280), NTP1
      GO TO 51
  280 R5        = RFLOW(JJ) * 5.39
      K         = K / (1.0471 ** (TEMP(TT)- 20.))
      LK(JJ)    = (LK(JJ)*LBOD(JJ) + (R5 + 5.39*LFLOW(JJ))*BOD(TT)*K)/
     A          (LBOD(JJ) + (R5 + 5.39*LFLOW(JJ))*BOD(TT))
      LBOD(JJ)  = LBOD(JJ) + R5 * BOD(TT) +  LFLOW(JJ) * 5.39 * BOD(TT)
      LPHOS(JJ) = LPHOS(JJ) + R5 * PHOS(TT) + LFLOW(JJ) * 5.39*PHOS(TT)
      LDOD(JJ)  = LDOD(JJ) + R5 * DOD(TT)
      LTEMP(JJ) = (RFLOW(III)*TEMP(TT)+LTEMP(JJ)*LFLOW(JJ))/RFLOW(JJ)
      TEMP(T)   = LTEMP(JJ)
      DEL(T)    = TEMP(T) - TTEMP(N)
      SATDO(T)  = QSATDO(TEMP(T))
      R5D       = 1.0 / (RFLOW(JJ) * 5.39)
      BOD(T)    = LBOD(JJ)  * R5D
```

```
      DOD(T).    = LDOD(JJ)  * R5D
      PHOS(T)    = LPHOS(JJ) * R5D
      DO1(T)     = SATDO(T) - DOD(T)
      DISTAN(T) = DIS(JJ)
      SK(JJ)     = LK(JJ)
      GO TO 51
C
C         TO BE FORMULATED LATER
C
   55 TEMP(T)    =  TEMP(TT)
      DEL(T)     =  DEL(TT)
      SATDO(T)   =  SATDO(TT)
      BOD(T)     =  BOD(TT)
      DOD(T)     =  DOD(TT)
      DO1(T)     =  DO1(TT)
      PHOS(T)    =  PHOS(TT)
      DISTAN(T)  =  DIS(JJ)
      SK(JJ)     = SK(III)
      GO TO 51
C
  290 R5         =  RFLOW(III)  * 5.39
      K          =  K / (1.0471 ** (TEMP(TT) - 20.))
      LK(JJ)     = (LK(JJ)*LBOD(JJ) + R5*BOD(TT)*K)/(LBOD(JJ)+R5*BOD(TT))
      LBOD(JJ)   =  LBOD(JJ)  + R5 * BOD(TT)
      LDOD(JJ)   =  LDOD(JJ)  + R5 * DOD(TT)
      LPHOS(JJ)  =  LPHOS(JJ) + R5 * PHOS(TT)
      LTEMP(JJ)  = (RFLOW(III) * TEMP(TT) + LTEMP(JJ) *  LFLOW(JJ)
     A                                    ) / RFLOW(JJ)
      TEMP(T)    =  LTEMP(JJ)
      DEL(T)     =  TEMP(T) - TTEMP(N)
      SATDO(T)   = QSATDO(TEMP(T))
      R5D        =  1.0 / (RFLOW(JJ) * 5.39)
      BOD(T)     =  LBOD(JJ)  * R5D
      DOD(T)     =  LDOD(JJ)  * R5D
      PHOS(T)    =  LPHOS(JJ) * R5D
      DO1(T)     =  SATDO(T) - DOD(T)
      DISTAN(T)  =  DIS(JJ)
      SK(JJ) = LK(JJ)
      GO TO 51
C
    5 E = 0
      L = TOTALR
  380 IF (RTYPE( L) - 2) 390, 400, 390
  390 L  = L-1
      IF (L) 30, 30, 380
  400 E  = E + 1
```

```
      D  = 1
      C = 0
      EE = 0
      LLL= RJ( L)
C
  410 JJ = RJ(L)
      N  = NN(JJ)
      II = I(N)
      T  = 1
      EED        =  EE * DELTA
      SFLOW(JJ)  =  MINRQ(JJ) + EED
      SF5        =  SFLOW(JJ) * 5.39
      SSBOD(T)   = (RBOD(L) + EED * RFBOD(L)) * SF5
      SSTEMP(T)  =  RTEMP(L) + EED * RFTEMP(L)
      DO1(T)     =  RDO(L) + EED * RFDO(L)
      SATDO(T)   = QSATDO(SSTEMP(T))
      DOD(T)     = SATDO(T) - DO1(T)
      SSDOD(T)   =  DOD(T) * SF5
      DEL(T)     = SSTEMP(T) - TTEMP(N)
      SK(JJ)     =  RK(L)
      IF (OPTYPE - 2) 405, 406, 405
  405 XCOST=0.
      GO TO 14
  406 CONTINUE
      NRA        = NRJ(JJ)
      ICR        = (EE*DELTA)/IRES(NRA)
      IF (ICR) 408, 408, 407
  407 XCOST      =  RCOST(NRA,ICR)
      GO TO 14
  408 XCOST      = 0.
C
   14 DEPTH(JJ) = AA(N) * SFLOW(JJ) ** BB(N)
      VEL(JJ)   = CC(N) * SFLOW(JJ) ** DD(N)
      DIST      = DIS(JJ)
      GO TO ( 401, 402, 403, 404 ), IRSET
  401 REAIR(JJ) = ((146.8 * VEL(JJ))**.5)/(2.303*(DEPTH(JJ)*DPFAC(N))
     A             ** 1.50 )
      GO TO  6
  402 REAIR(JJ) = (5.026 * VEL(JJ)**.969) / (DPFAC(N)*DEPTH(JJ))** 1.63
      GO TO  6
  403 REAIR(JJ) =(3.3 * VEL(JJ))/(( DEPTH(JJ) * DPFAC(N))**1.33)
      GO TO 6
  404 GO TO 9999
C
    6 IF (DIST - DIS(II) - INC) 420, 420, 430
C
```

```
  420 BCC = 0
      TIME      = (DIST - DIS(II)) / (VEL(JJ) * 16.36364)
      TDIST =  DIST - DIS(II)
      GO TO 440
C
  430 BCC   = 1
      TIME      =  INC / (VEL(JJ) * 16.36364)
      TDIST = INC
      DIST  = DIST - INC
C
  440 T = T + 1
      QQQQ      = -TDIST * DECAYT(N)/ (VEL(JJ) * DEPTH(JJ))
      SSTEMP(T) = TTEMP(N) + DEL(T-1)* EXP(QQQQ)
      DEL(T) =  SSTEMP(T) - TTEMP(N)
      R  =  REAIR(JJ)  *  1.0241 **(SSTEMP(T) - 20.)
      K  =   SK(JJ) *  1.0471 **(SSTEMP(T) - 20.)
      A  =   1. / 10. **  (K * TIME)
      P  =   1. / 10. **  (R * TIME / 2.)
      SSBOD(T) = A * SSBOD(T-1)
      UBOD      = 1. / ( 1.- 10.**(- K    * 5.))
      SSDOD(T)  = P * UBOD * (SSBOD(T-1)-SSBOD(T)) + P*P *SSDOD(T-1)
      SF8       =  (SFLOW(JJ) * 5.39)
      IF ( SSBOD(T)/SF8-SEDBOD(N))447,447,448
  447 SSBOD(T)  =  SEDBOD(N) * SF8
  448 SATDO(T)  = QSATDO(SSTEMP(T) )
      DOD(T)  = SSDOD(T) / (SFLOW(JJ) * 5.39)
      DO1(T)  = SATDO(T) - DOD(T)
C
      IF (DO1(T) - CONST(N)) 450, 460, 460
C
  450 IF (C) 8, 9, 8
C
  460 IF (BCC) 6, 7, 6
C
C 460 IF (BCC) 6, 461, 6
C 461 IF (D-2) 462, 462, 7
C 462 WRITE (3,6001) JJ, SFLOW(JJ), SK(JJ), REAIR(JJ), VEL(JJ),
C    A           DEPTH(JJ), (KKK, SSTEMP(KKK), SSBOD(KKK), SATDO(KKK),
C    B           DO1(KKK), KKK = 1, T)
C     GO TO 7
C6001 FORMAT ( '1' I3, 5F20.2///(I3, 4F20.2))
C
    8 IF (EE + 1 - S )      470, 470, 12
  470 EE = EE + 1
C
  480 JJ = JJJJ
```

```
      N  = NN(JJ)
      II = I(N)
      T  = 1
      OF5        =  OFLOW(EE) * 5.39
      SSBOD(T)   =  OBOD(EE) * OF5
      SSDOD(T)   =  ODOD(EE) * OF5
      SSTEMP(T)  = OTEMP(EE)
      SK(JJ)     = OK(EE)
      SFLOW(JJ)  = OFLOW(EE)
      SATDO(T)   = QSATDO(SSTEMP(T))
      DOD(T)     = ODOD(EE)
      DO1(T)     = SATDO(T) - DOD(T)
      DEL(T)     = SSTEMP(T) - TTEMP(N)
      XCOST=OCOST(EE)
      GO TO 14
C
    9 IF (MINRQ(LLL)+(EE+1)*DELTA-MAXRQ(LLL)) 490, 490, 12
C
  490 EE = EE + 1
      GO TO 410
C
   10 IF (MINRQ(LLL)+(EE+1)*DELTA-MAXRQ(LLL)) 500, 500, 12
C
  500 EE = EE + 1
      D  = D  + 1
      GO TO 410
C
   11 IF (EE+1- S )      510, 510, 12
C
  510 EE = EE + 1
      D  = D  + 1
      GO TO 480
C
   20 JJJJ = JJ
      AFLOW = OFLOW(1) - MINRQ(JJ)
      C     = 1
      E     = E + 1
      D     = 1
      EE    =  1
      GO TO 480
C
    7 IF (II - FIRSTI) 530, 520, 530
  520 IF (MAXRQ(JJ)-SFLOW(JJ)-ERROR) 553, 553, 554
  530 IF (II - J(N-1)) 540, 560, 540
  540 IF (MAXRQ(II)-MINRQ(II)-MAXRQ(JJ)+MINRQ(JJ)-ERROR) 17, 17, 550
C
```

```
554 III   = JJ
    JJ    = II
    NNN   = N
    GO TO 18
553 MJ(E) = LASTJ
    K  = K / ( 1.047 ** ( SSTEMP(T) - 20.))
    MFLOW(D) = SFLOW(III)
     MBOD(D) = SSBOD(T)
     MDOD(D) = SSDOD(T)
    MTEMP(D) =SSTEMP(T)
    MK(D)   = K
    MCOST(D)   = XCOST
    MN(E) = LASTN
    MAXM(E) = D
    WRITE(TAPE1) E, MJ(E), MAXM(E), MN(E), (MFLOW(IJK), MBOD(IJK),
   A       MDOD(IJK), MK(IJK), MTEMP(IJK), MCOST(IJK), IJK = 1, D)
    GO TO 30
550 III = JJ
    JJ  = II
    NNN = N
    N   = NN(JJ)
    II = I(N)
    GO TO 18
C
560 NTP       =  NTYPE(II)
    GO TO (570, 15, 17, 600, 605), NTP
570 III = JJ
    SFLOW(II) = SFLOW(JJ)
    JJ  = II
    SFLOW(JJ) = SFLOW(II)
    N = NN(JJ)
    II = I(N)
    SK(JJ) = SK(III)
    SDELL(JJ) = DEL(T)
    STEMP(JJ) = SSTEMP(T)
    SBOD(JJ) = SSBOD(T)
    SDOD(JJ) = SSDOD(T)
    T = 1
    SSBOD(T) = SBOD(JJ)
    SSDOD(T) = SDOD(JJ)
    SSTEMP(T) = STEMP(JJ)
    DEL(T) = SDELL(JJ)
    SATDO(T) = QSATDO(SSTEMP(T))
    DOD(T) = SSDOD(T) /(SFLOW(JJ) * 5.39)
    DOI(T) = SATDO(T) - DOD(T)
    GO TO 14
```

```
C
  600 III = JJ
      JJ  = II
      N   = NN(JJ)
      II = I(N)
      IF (C) 610, 620, 610
  610 SFLOW(JJ) = MINRQ(JJ) + OFLOW(EE) - OFLOW(1) + AFLOW
      GO TO 630
  620 SFLOW(JJ) = MINRQ(JJ) + (EE * DELTA)
  630 SK(JJ) = SK(III)
      SFR       =  SFLOW(JJ) /  SFLOW( III )
      SBOD(JJ)  =  SSBOD(T) * SFR
      SDOD(JJ)  =  SSDOD(T) * SFR
      SDELL(JJ) = DEL(T)
      STEMP(JJ)= SSTEMP(T)
      T = 1
      SSBOD(T) = SBOD(JJ)
      SSDOD(T) = SDOD(JJ)
      SSTEMP(T)= STEMP(JJ)
      DEL(T) = SSTEMP(T) - TTEMP(N)
      DOD(T) = SSDOD(T) / (SFLOW(JJ) * 5.39)
      SATDO(T) = QSATDO(SSTEMP(T))
      DO1(T)   = SATDO(T) - DOD(T)
      GO TO 14
C
   17 III = JJ
      JJ  = II
      N   = NN(JJ)
      II  = I(N)
C
   16 IF (C) 640, 650, 640
  640 SFLOW(JJ) = MINRQ(JJ) + OFLOW(EE) - OFLOW(1) + AFLOW
      GO TO 660
  650 SFLOW(JJ) = MINRQ(JJ) + (EE * DELTA)
C
  660 K = K / 1.047 ** (SSTEMP(T) - 20.)
      SBOD(JJ)  = LBOD(JJ) + SSBOD(T)
      SDOD(JJ)  = LDOD(JJ) + SSDOD(T)
      SK(JJ)    = (LK(JJ)*LBOD(JJ) + SSBOD(T)*K) / SBOD(JJ)
      STEMP(JJ) = (LTEMP(JJ)*LFLOW(JJ) + SSTEMP(T)*SFLOW(III))/SFLOW(JJ)
      SDELL(JJ) = DEL(T)
      T  =  1
      SSBOD(T)  = SBOD(JJ)
      SSDOD(T)  = SDOD(JJ)
```

```
      SATDO(T) = QSATDO(SSTEMP(T))
      DOD(T)   = SSDOD(T) / (SFLOW(JJ)*5.39)
      DO1(T)   = SATDO(T) - DOD(T)
C
      IF (DO1(T)-CONST(N))670, 14, 14
  670 IF (C) 8, 9, 8
C
  605 III      = JJ
      JJ       = II
      N        = NN(JJ)
      II       = I(N)
      IF (C) 607, 606, 607
  607 SFLOW(JJ) = MINRQ(JJ) + OFLOW(EE) - OFLOW(1) + AFLOW
      GO TO 608
  606 SFLOW(JJ) = MINRQ(JJ) + (EE * DELTA)
  608 K         = K / 1.047 ** (SSTEMP(T) - 20.)
      SBOD(JJ)  = LBOD(JJ) + SSBOD(T) + SSBOD(T) * (LFLOW(JJ)        /
     A            SFLOW(III))
      SDOD(JJ)  = LDOD(JJ) + SSDOD(T)
      SK(JJ)    =   K
      STEMP(JJ) = (LTEMP(JJ)*LFLOW(JJ) + SSTEMP(T)*SFLOW(III))/SFLOW(JJ)
      SDELL(JJ) = DEL(T)
      T         = 1
      SSBOD(T)  = SBOD(JJ)
      SSDOD(T)  = SDOD(JJ)
      SSTEMP(T) = STEMP(JJ)
      DEL(T)    = SSTEMP(T) - TTEMP(N)
      SATDO(T)  = QSATDO(SSTEMP(T))
      DOD(T)    = SSDOD(T) / (SFLOW(JJ)*5.39 )
      DO1(T)    = SATDO(T) - DOD(T)
      IF (DO1(T)-CONST(N))609, 14, 14
  609 IF (C) 8, 9, 8
   15 III = JJ
      JJ  = II
      NNN = N
      N   = NN(JJ)
      II  =I(N)
C
      IF ( MAXRQ(JJ ) - MINRQ(JJ ) - MAXRQ(III) + MINRQ(III) - ERROR)
     A      16, 16, 18
C
   18 K  = K / ( 1.047 ** ( SSTEMP(T) - 20.))
      MFLOW(D) = SFLOW(III)
       MBOD(D) = SSBOD(T)
       MDOD(D) = SSDOD(T)
      MTEMP(D) =SSTEMP(T)
```

```
      MK(D)  = K
      MCOST(D)   = XCOST
      LASTJ   = JJ
      LASTN   = NNN
      IF(C) 11, 10, 11
C
   12 IF (D-1) 671, 671, 672
  671 WRITE (3,3011)
      E # E -1
      GO TO 30
  672 MJ(E)      = LASTJ
C                                                                    C
      MAXM(E)    = D
      MN(E)      = LASTN
      MMAXM      = D
      MMN        = LASTN
C
      WRITE(TAPE1) E, MJ(E), MAXM(E), MN(E), (MFLOW(IJK), MBOD(IJK),
     A       MDOD(IJK), MK(IJK), MTEMP(IJK), MCOST(IJK), IJK = 1, D)
      EEE        =  E
  680 IF (EEE-1) 999, 390, 685
  685 IF (MJ(E) - MJ(EEE-1)) 690, 19, 690
  690 EEE        =  EEE - 1
      IF (EEE-1) 999, 390, 685
   19 REWIND TAPE1
      IE         =  EEE - 2
      IF (IE) 693, 693, 691
  691 DO 692 IJK = 1, IE
  692 READ (TAPE1)
  693 READ (TAPE1) IXE, LASTJ, NMAXM, NMN, (NFLOW(IJK), NBOD(IJK)
     A      ,NDOD(IJK), NK(IJK), NTEMP(IJK), NCOST(IJK), IJK = 1, NMAXM)
      IF (E - EEE + 1) 999, 696, 694
  694 DO  695 IJK = EEE, E
  695 READ (TAPE1)
C
  696 S  = 1
      SS = 1
      X  = 1
      Y  = 1
      OFLOW(S)   = MFLOW(X) + NFLOW(Y)
      OF15       =  1. / (OFLOW(S) * 5.39)
      OBOD(S)    = (MBOD(X) + NBOD(Y)) * OF15
      ODOD(S)    = (MDOD(X) + NDOD(Y)) * OF15
      OK(S)      = (MK(X)*MBOD(X) + NK(Y)*NBOD(Y)) / (MBOD(X)+NBOD(Y))
      OTEMP(S)   = (MTEMP(X)*MFLOW(X) + NTEMP(Y)*NFLOW(Y)) / OFLOW(S)
      OXFLOW(S)  = MFLOW(X)
```

```
      OYFLOW(S) = NFLOW(Y)
      OCOST(S)  = MCOST(X) + NCOST(Y)
      RFLOW(JJ) = OFLOW(S)
C
   28 RFLOW(JJ) = RFLOW(JJ) + DELTA
      SS        = 1
      IF (MAXRQ(JJ) - RFLOW(JJ) - ERROR) 711, 711, 712
C
  711 S         = S + 1
      X         =  MMAXM
      Y         =  NMAXM
      OFLOW(S)  = MFLOW(X) + NFLOW(Y)
      OF15      =  1. / (OFLOW(S) * 5.39)
      OBOD(S)   = (MBOD(X) + NBOD(Y)) * OF15
      ODOD(S)   = (MDOD(X) + NDOD(Y)) * OF15
      OK(S)     = (MK(X)*MBOD(X) + NK(Y)*NBOD(Y)) / (MBOD(X) + NBOD(Y))
      OTEMP(S)  = (MTEMP(X)*MFLOW(X) + NTEMP(Y)*NFLOW(Y)) / OFLOW(S)
      OXFLOW(S) = MFLOW(X)
      OYFLOW(S) = NFLOW(Y)
      OCOST(S)  = MCOST(X) + NCOST(Y)
C
      XNODE =  J(MMN)
      YNODE =  J(NMN)
      WRITE (TAPE2) LASTJ, S, XNODE, YNODE, (OFLOW(IJK), OBOD(IJK)
     A           , ODOD(IJK), OK(IJK), OTEMP(IJK), OXFLOW(IJK)
     B           , OYFLOW(IJK), OCOST(IJK), IJK = 1, S)
      NE        =  NE + 1
      GO TO 20
C
  712 X         =  1
      Y         = NMAXM
      GO TO 21
C
C
   21 IF (MFLOW(X) + NFLOW(Y) - RFLOW(JJ)+ERROR ) 810, 830, 830
  810 IF (X - MMAXM) 820, 830, 820
C
  820 X = X + 1
      GO TO 21
C
  830 SFLOW(SS) = MFLOW(X) + NFLOW(Y)
      IF (SFLOW(SS) - RFLOW(JJ) - ERROR) 840, 840, 850
C
  840 YBASE     = Y
      GO TO 22
C
```

```
  850 IF (NFLOW(Y-1) + MFLOW(X) - RFLOW(JJ) + ERROR ) 22, 860, 860
C
  860 Y          = Y-1
      YBASE      = Y
      GO TO 850
C
   22 SFLOW(SS) = MFLOW(X) + NFLOW(Y)
      SXFLOW(SS)= MFLOW(X)
      SYFLOW(SS)=  NFLOW(Y)
      SF15       =  1. / (SFLOW(SS) * 5.39)
      SBOD(SS)   = (MBOD(X) + NBOD(Y)) * SF15
      SDOD(SS)   = (MDOD(X) + NDOD(Y)) * SF15
      SK(SS)     = (MK(X)*MBOD(X) + NK(Y)*NBOD(Y)) / (MBOD(X) + NBOD(Y))
      STEMP(SS) = (MTEMP(X)*MFLOW(X) + NTEMP(Y)*NFLOW(Y)) / SFLOW(SS)
      SCOST(SS) = MCOST(X) + NCOST(Y)
C
      IF (Y - NMAXM) 870, 890, 870
  870 IF (NFLOW(Y+1)-NFLOW(Y)-ERROR) 880, 880, 890
  880 Y = Y+1
      SS = SS+1
      GO TO 22
C
  890 IF (X - MMAXM) 900, 23, 900
  900 IF (MFLOW(X+1)-MFLOW(X)-ERROR) 910, 910, 23
  910 X = X+1
      SS = SS+1
      Y = YBASE
      GO TO 22
C
   23 X  = X + 1
      Y  = YBASE - 1
      SS = SS + 1
C
  920 SFLOW(SS)  =  MFLOW(X) + NFLOW(Y)
      IF (ABS(SFLOW(SS)-RFLOW(JJ))-ERROR) 24, 24, 930
  930 IF (X+1-MMAXM) 940, 940, 931
  931 SS         =  SS - 1
      GO TO 25
  940 IF (MFLOW(X+1)-MFLOW(X)-ERROR) 950, 950, 960
C
  950 X = X + 1
      Y = YBASE - 1
      GO TO 920
C
  960 Y     = YYBASE
      X     = X + 1
```

C

```
      YBASE = Y + 1
      GO TO 920
C
   24 SXFLOW(SS) =  MFLOW(X)
      SYFLOW(SS) =  NFLOW(Y)
      SF15       =  1. / (SFLOW(SS) * 5.39)
      SBOD(SS)   = (MBOD(X) + NBOD(Y)) * SF15
      SDOD(SS)   = (MDOD(X) + NDOD(Y)) * SF15
      SK(SS)     = (MK(X)*MBOD(X) + NK(Y)*NBOD(Y)) / (MBOD(X) + NBOD(Y))
      STEMP(SS)  = (MTEMP(X)*MFLOW(X)+NTEMP(Y)*NFLOW(Y)) / SFLOW(SS)
      SCOST(SS) = MCOST(X) + NCOST(Y)
C
      IF (Y - 1) 980, 980, 970
C
  970 Y      =  Y - 1
      SS     =  SS + 1
      YYBASE=   Y
      GO TO 920
C
  980 IF (X + 1 - MMAXM) 990, 990, 25
C
  990 IF (MFLOW(X+1) - MFLOW(X) - ERROR) 1000, 1000, 25
  999 STOP 9999
C
 1000 X  =  X + 1
      SS =  SS + 1
      GO TO 24
C
   25 MAXSS       =  SS
 1010 MMMMM       =  MAXSS - 1
      DO 1030  W = 1, MMMMM
      MWMWM       =  MAXSS - W  + 1
      DO   26  Z = 2, MWMWM
      IF (SBOD(Z) - SBOD(Z-1)) 1020, 26, 26
 1020 SF          = SFLOW(Z)
      SFLOW(Z)    = SFLOW(Z-1)
      SFLOW(Z-1) = SF
      SB          = SBOD(Z)
      SBOD(Z)     = SBOD(Z-1)
      SBOD(Z-1)  = SB
      SD          = SDOD(Z)
      SDOD(Z)     = SDOD(Z-1)
      SDOD(Z-1)  = SD
      SSK         = SK(Z)
      SK(Z)       = SK(Z-1)
      SK(Z-1)     = SSK
```

```
      ST          = STEMP(Z)
      STEMP(Z)    = STEMP(Z-1)
      STEMP(Z-1)  = ST
      SX          = SXFLOW(Z)
      SXFLOW(Z)   = SXFLOW(Z-1)
      SXFLOW(Z-1)= SX
      SY          = SYFLOW(Z)
      SYFLOW(Z)   = SYFLOW(Z-1)
      SYFLOW(Z-1) = SY
      SC         = SCOST(Z)
      SCOST(Z)    = SCOST(Z-1)
      SCOST(Z-1)= SC
   26 CONTINUE
 1030 CONTINUE
C
C
      OBMAX  =  SBOD(SS)
      OBMIN  =  SBOD(1)
      TEST   =  OBMAX - OBMIN
C
      IF (TEST - TEST1) 1050, 1050, 1040
 1040 ADD         =  ADD1
      GO TO 27
C
 1050 IF (TEST - TEST2) 1070, 1070, 1060
 1060 ADD         =  ADD2
      GO TO 27
C
 1070 IF (TEST - TEST3) 1090, 1090, 1080
 1080 ADD         =  ADD3
      GO TO 27
C
 1090 IF (TEST - TEST4) 1110, 1110, 1100
 1100 ADD         =  ADD4
      GO TO 27
C
 1110 ADD         =  ADD5
C
   27 IXC  =  1
      SS   =  1
      S    =  S + 1
      COMPAR      =  SBOD(1)
 1120 OFLOW(S)    =  SFLOW(SS)
      OBOD(S)     =  SBOD(SS)
      ODOD(S)     =  SDOD(SS)
      OK(S)       =  SK(SS)
```

```
      OTEMP(S)    =  STEMP(SS)
      OYFLOW(S)   =  SYFLOW(SS)
      OXFLOW(S)   =  SXFLOW(SS)
      OCOST(S)  = SCOST(SS)
      S  =  S + 1
      IF (S - 200) 29, 29, 1200
   29 SS =  SS + 1
      IF (SS - MAXSS) 1140, 1140, 1130
 1130 S  =  S - 1
      GO TO 28
 1140 IF (SBOD(SS) - COMPAR - ADD) 1160, 1150, 1150
 1150 IXC  =  0
      COMPAR  =  COMPAR + ADD
      GO TO 1140
 1160 IF (IXC - 1) 1170, 1180, 1170
 1170 IXC  =  1
      GO TO 1120
 1180 OPT= OPTYPE
      GO TO (1185, 1186, 1187), OPT
 1185 IF(SDOD(SS)-ODOD(S-1 ))1190, 29, 29
 1186 IF(SCOST(SS)-OCOST(S-1 )) 1190, 29, 29
 1187 STOP 8888
 1190 S  =  S - 1
      GO TO 1120
C
 1200 WRITE (3,3017)
      GO  TO 30
C
C
C
C                                                                    C
 1260 CONTINUE
      WRITE (3,8010)
   30 REWIND TAPE1
      DO 1300 IJK = 1, E
      READ (TAPE1) EEE, MJ(EEE), D, MN(EEE), (MFLOW(JKL),  MBOD(JKL)
     A           , MDOD(JKL), MK(JKL), MTEMP(JKL), MCOST(JKL), JKL= 1, D)
      KLM         =  MN(EEE)
      DO 1290  JKL =1,D
      RD5    =1./(5.39*MFLOW(JKL))
      MBOD(JKL)=MBOD(JKL) *RD5
      MDOD(JKL)=MDOD(JKL) *RD5
 1290 SATDO(JKL)= QSATDO(MTEMP(JKL))
 1300 WRITE (3,3014)  J(KLM), (MFLOW(JKL), MBOD(JKL), MDOD(JKL),
     A       SATDO(JKL), MK(JKL), MTEMP(JKL), MCOST(JKL), JKL = 1, D)
C
```

```
      IF(NE)1316,1316,1304
 1304 REWIND TAPE2
C
      DO 1310 IJK = 1, NE
      READ (TAPE2) LASTJ, S, XNODE, YNODE, (QFLOW(JKL), OBOD(JKL)
     A           , ODOD(JKL), OK(JKL), OTEMP(JKL), OXFLOW(JKL)
     B       ,OYFLOW(JKL), OCOST(JKL), JKL = 1, S)
      DO 1305 IJT = 1, S
 1305 SATDO(IJT)  = QSATDO(OTEMP(IJT))
 1310  WRITE (3,3015) IJK, LASTJ, XNODE, YNODE, (QFLOW(JKL), QXFLOW(JKL)
     A           , QYFLOW(JKL), OBOD(JKL), ODOD(JKL), SATDO(JKL)
     B           , QK(JKL), QTEMP(JKL), QCOST(JKL), JKL = 1, S)
 1316 IF (IRMBOD-IRM) 1320, 1320, 1318
 1318 IRM        = IRM + 1
      DO 1319 JJ= 1, 600
      LFLOW(JJ) = 0.
      LDOD(JJ)  = 0.
      LBOD(JJ)  = 0.
      LTEMP(JJ) = 0.
      LK(JJ)    = 0.
      RFLOW(JJ) = 0.
      MINRQ(JJ) = 0.
      MAXRQ(JJ) = 0.
      WFLOW(JJ) = 0.
 1319 LPHOS(JJ) = 0.
      GO TO 105
 1320 IF( IT1-IQM) 1400, 1400, 1325
 1325 DO 1330 JJ= 1, 600
      LFLOW(JJ) = 0.
      LBOD(JJ)  = 0.
      LDOD(JJ)  = 0.
      LTEMP(JJ) = 0.
      LK(JJ)    = 0.
      RFLOW(JJ) = 0.
      MINRQ(JJ) = 0.
      MAXRQ(JJ) = 0.
 1330 WFLOW(JJ) = 0.
      IQM       = IQM+1
      GO TO 78
 1400 IF (ITO-1) 1500, 1500, 1450
 1450 ITO       = ITO-1
      GO TO 75
 1500 WRITE (3,3000)
C
C     FORMAT STATEMENTS
C
```

```
 1001 FORMAT ( 11I5, 5F5.0)
 1002 FORMAT ( 2I5, 6F10.0, I5)
 1003 FORMAT ( 2I5, 7F10.0 )
 1004 FORMAT ( 2I4, 1X, 11F6.2)
 1005 FORMAT ( 8F10.2 )
 1007 FORMAT ( ' ' 34X I3, 3F20.2 )
 1008 FORMAT ( 7( I5, F5.0))
C
 2002 FORMAT ( 3I5, 6F10.2, I5 )
 2003 FORMAT ( 3I5, 7F10.2     )
C
 2004 FORMAT ( 3I4, 1X, 11F6.2 )
C
 2005 FORMAT  ('0' 4I5,(7F10.3))
C
 2007 FORMAT ( '1' 49X 'CALCULATED MIN- & MAX- FLOW DATA' /
     A          '0' 35X 'JJ' 12X ' RFLOW(JJ)' 11X 'MINRQ(JJ)' 11X
     B          'MAXRQ(JJ)' // )
C
 3000 FORMAT('1' 10X 'END OF JOB')
C
C
 3002 FORMAT ( '1' 60X 'STREAM DATA' / '0' 4X 'SECTION' 3X 'I-NODE' 3X
     A          'J-NODE' 6X 'DISTANCE' 9X 'AREA' 12X 'AA' 14X 'BB' 13X
     B          'CC' 13X 'DD' 8X 'TYPE' / ' ' 35X '(MILES)' 7X
     C          'SQ.MILES)' // (' ' 3I9, 6F15.3, I9))
C
 3003 FORMAT ( '1' 60X ' WASTE DATA' / '0' 5X 'NUMBER' 4X 'NODE' 5X
     A          'TYPE' 9X 'FLOW' 11X 'DEOX' 11X 'BOD' 13X 'DO' 12X 'TEMP'
     B          11X 'PHOS' 9X 'REMOVAL' /38X '(MGD)' 8X '(1/DAY)' 5X
     C          '(MG/L OR #/DAY)' 4X '(MG/L)' 8X '(DEG.C)' 9X '(MG/L)'
     D          10X '(%)' // (1H 3I9, 7F15.3))
C
 3004 FORMAT ( '1' 57X 'STREAM FLOW DATA' / '0' '   NUMBER  NODE   TYPE'
     A           '   MIN-FLOW  MAX-FLOW    BOD' 8X 'DO' 7X 'PHOS' 6X
     B           'DEOX' 6X 'TEMP     F-TEMP     F-DO     F-BOD     '
     C           'F-PHOS' / 27X '(CFS)     (CFS)     (MG/L)    (MG/L)'
     D           '    (MG/L)   (1/DAY)   (DEG.C)' // (' ' 3I7, 11F10.2) )
C
 3005 FORMAT ( '1', 6X 'NODE' 8X 'T' 10X 'FLOW' 9X 'DISTANCE' 10X 'BOD'
     A          11X 'TEMP' 11X 'SATDO' 11X 'DO' 12X 'PHOS' / 30X '(CFS)'
     B          9X '(MILES)' 8X '(MG/L)' 9X '(DEG.C)' 8X '(MG/L)' 9X
     C          '(MG/L)' 9X '(MG/L)' // (2I10, 7F15.2) )
C
 3006 FORMAT (  I10, 6F20.2)
C
```

```
 3007 FORMAT ( '1' 6X 'NODE' 14X 'FLOW' 16X 'DEOX' 16X 'REAIR' 13X
     A          'VELOCITY' 14X 'DEPTH' / 25X '(CFS)' 14X '(1/DAY)' 13X
     B          '(1/DAY)' 14X '(FEET)' )
C
 3008 FORMAT ( / ( I10, 5F20.2 ))
C
C
C
 3011 FORMAT ( '1' '  CONSTRAINT TOO LARGE - - - - JOB CANCELLED ')
C
 3012 FORMAT ( I5, 2F10.3 )
C
 3013 FORMAT ('1' 51X 'DYNAMIC PROGRAMMING BASIC DATA'//'0' 17X
     A         'TEST 1' 4X 'TEST 2' 4X 'TEST 3' 4X 'TEST 4' 4X 'TEST 5'
     B          5X 'ADD 1' 5X 'ADD 2' 5X 'ADD 3' 5X 'ADD 4' 5X 'ADD 5'//
     C         (14X 10F10.2))
C
 3014 FORMAT ('1' 53X 'INCREMENTAL FLOW TABLEAU' / '0' 59X
     A        'U-NODE ='I4/'0' 2X 'FLOW' 16X 'BOD' 17X 'DOD' 16X 'SAT-DO'
     B        15X 'DEAIR' 15X 'TEMP' 13X' COST' / 2X '(CFS)' 15X '(MG/L)'
     C        14X '(MG/L)' 14X '(MG/L)' 14X'(1/DAY)'12X '(DEG.C)' 11X
     D        '(ANNUAL)' // ( F8.2 , 6F20.2 ))
C
 3015 FORMAT ('1' 50X 'DYNAMIC PROGRAMMING TABLEAU' I3/'0' 59X
     A        'FOR NODE = ' I3 / '0TOTAL FLOW' 3X 'FLOW AT ' I3, 4X
     B        'FLOW AT = ' I3, 8X 'BOD' 12X 'DOD' 10X 'SAT-DO' 9X
     C        'DEAIR' 10X 'TEMP' 11X 'COST' /'   (CFS)' 9X '(CFS)' 10X
     D        '(CFS)' 12X '(MG/L)' 9X '(MG/L)' 9X '(MG/L)' 8X '(1/DAY)'
     E         7X '(DEG.C)' 8X '(ANNUAL)' // ( F8.2, 8F15.2 ))
C
 3016 FORMAT ( '1' 6X 'NODE' 14X 'FLOW' 16X 'BOD' 17X 'DOD' 16X 'PHOS'
     A          17X 'TEMP' 16X 'DEOX' / 25X '(CFS)' 13X '(#/DAY)' 13X
     B          '(#/DAY)' 13X '(#/DAY)' 14X '(DEG.C)' 12X '(1/DAY)' // )
C
 3017 FORMAT ( 50H1 O-VALUES UNDERDIMENSIONED - - - - JOB CANCELLED )
 3050 FORMAT ('1' 58X 'INPUT DATA' /// 10X 'STREAM SEGEMENTS' 15X
     A         'WASTE DISCHARGES' 15X 'FLOW ADD POINTS' 17X
     B         'RESERVOIRS' // I20, 3I30)
 3060 FORMAT ('0'/ ' ' 3X 'FIRST NODE' 3X 'REAIR' 5X 'D-SETS' 4X'F-SETS'
     A          4X 'T-SETS' 4X 'I-BOD' 4X 'D-TYPE' 4X 'INC' 6X 'DELTA' 5X
     B         'REMBOD' 4X 'REMPHS' 2X 'D-FIRST NODE'// 7I10, 5F10.2 )
 3070 FORMAT ('1' 57X 'RESERVOIR DATA' // 13X 'NO.' 5X 'NODE' 5X
     A        'COST-INC' 5X 'T-INC' 2X 'T-ACRE-FT' 2X 'WQ-ACRE-FT'
     B        2X 'T-COST' 4X 'WQ-COST' 2X 'AVE-FLOW' 2X 'INCREASE-Q'
     C        // (4X, I11,I9,2I11,2X,2F10.0,1X,2F10.0,F9.0,F12.0))
C
```

```
 4001 FORMAT.( 3I10, F15.3, I10 )
C
 4008 FORMAT ( 2I5, 2F20.3)
 4010 FORMAT ('1' 60X 'ADDITIONAL DATA'//'0' 1X 'IT2' 3X 'S.S. TEMP.'
     A       3X 'IT3' 3X 'DO-CONSTR.' 3X 'IT4'  3X  'DEPTH FACT' 3X
     B       'IT5' 3X 'DECAY TEMP' 3X 'IT6' 3X 'DECAY PHOS' 3X 'IT7'
     C       3X 'S.S. PHOS.' 3X 'IT8' 3X 'DY EQ BOD.' //'0'
     D       2X, I1,  F11.2, 6%I8, F11.2<<
 4050 FORMAT ( 4I5,6F10.0)
C
 5001 FORMAT ('1' 54X 'MINIMUM   TREATMENT' // ( 62X F5.2 ))
C
C
 6000 FORMAT ( 4I5 // ( 5F15.2) )
 8000 FORMAT ( 5X, F5.0, 7F10.0 )
 8005 FORMAT (8X, 12F6.0)
 8010 FORMAT ( 'ERROR 1260' )
 C
       CALL TIMED
 9999  STOP
       END
       FUNCTION QSATDO(TEMP)
       QSATDO    =  14.652 - (0.41022    * TEMP)
                           + (.007991    * TEMP * TEMP)
      A                    - (.77774E-4 * TEMP * TEMP * TEMP)
      B
       RETURN
       END
```

BIBLIOGRAPHY

treeter, H. W., and Phelps, Earle b., "A Study of the Pollution and atural Purification of the Ohio River," Public Health Bulletin, No. 46, U. S. Public Health Service, U. S. Government Printing Office 1925).

air, G. M., "The Dissolved Oxygen Sag - An Analysis," SWJ, 11, 445 1939).

nomas, H. A., Jr., "Pollution Load Capacity of Streams," WSW, 95 1948).

elz, C. J., "Deoxygenation and Reoxygenation," Trans. ASCE, Vol. 04, pp. 560-578 (1939).

elz, C. J., "Factors Influencing Self-Purification of Streams and neir Relation to Sewage and Industrial Wastes Disposal--Part II," WJ, 3, pp. 309-319 (1949).

elz, C. J., "Factors Influencing Self-Purification and Their Relation to Pollution Abatement--Part II," SWJ, 19, 4, pp. 629-644 July 1947).

elz, C. J., "Stream Reccvery," WSW, Vol. 100, No. 12, pp. 495-501 December 1953).

nomas, H. A., Jr., "The Dissolved Oxygen Balance in Streams," eminar Papers on Waste Water Treatment and Disposal, the Boston ociety of Civil Engineers, Boston, Mass. (1961).

'Connor, D. J., "Oxygen Relationships in Streams: Robert A. Taft anitary Engineering Center, Technical Report W58-2, Cincinnati, nio" (1958).

, W. H., "Unsteady Dissolved-Oxygen Sag in a Stream," ASCE-SA, Vol. 8, No. SA3 (May 1962).

rankel, R. J., "Water Quality Management: An Engineering and conomic Model for Domestic Waste Disposal," Doctoral Thesis, The niversity of California, Berkeley, California (1965).

rankel, R. J., "Water Quality Management: Engineering Economic actors in Municipal Waste Disposal," WRR, Vol. 1, No. 2, p. 173 1965).

obbins, W. E., "BOD and Oxygen Relationships in Streams," ASCE-SA, ol. 90, No. SA3, pp. 53-73 (June 1964).

obbins, W. E., closure to "BOD and Oxygen Relationships in Streams," SCE-SA, Vol. 91, No. SA5, pp. 49-55 (October 1965).

O'Connell, R. L., Thomas, N. E., Godsil, P. I., and Hirth, C. R., "Report of Survey of the Truckee River, July 1962," U. S. Public Health Service, Cincinnati, Ohio (1962).

O'Connell, R. L., and Thomas, N. A., "Effects of Benthic Algae on Stream Dissolved Oxygen," ASCE-SA, Vol. 91, No. SA3, pp. 1-16 (June 1965).

Camp, T. R., "Field Estimates of Oxygen Balance Parameters," ASCE-SA, Vol. 91, No. SA5, pp. 1-16 (October 1965).

O'Connor, D. J., "The Temporal and Spatial Distribution of Dissolved Oxygen in Streams," WRR, Vol. 3, No. 1, pp. 65-77 (January 1967).

Gannon, John J., and Down, Thomas D., "Programming River DO Calculations--Part I," WSW, 110, 3, 114 (March 1963).

Thomann, R. V., "Mathematical Model for Dissolved Oxygen," ASCE-SA, Vol. 89, No. SA5, pp. 1-30 (October 1963).

O'Connor, D. J., "Oxygen Balance of an Estuary," ASCE-SA, Vol. 86, No. SA3 (May 1960).

Graybill, F. A., An Introduction to Linear Statistical Model, McGraw-Hill Book Company, Inc., New York (1961).

LeBosquet, M., Jr., and Tsivoglou, E. C., "Simplified Dissolved Oxygen Computations," SIW, Vol. 22, pp. 1054-1061 (August 1950).

LeBosquet, M., Jr., "Oxygen Relationships in Streams," Robert A. Taft Sanitary Engineering Center, Technical Report W58-2, Cincinnati, Ohio (1958).

Churchill, M. A., "Analysis of a Stream's Capacity for Assimilation Pollution," SIW, Vol. 26, pp. 887-904 (July 1954).

Sutherland, E. R., "A Water Quality Study of the Chickahominy River," Virginia State Water Control Board, Richmond, Virginia (1966).

Okum, D. A., Lamb, J. C., and Wells, C. C., "A Waste Control Program for a River with Highly Variable Flow," JWPCF, Vol. 35, No. 8, p. 1025 (August 1963).

Wolman, A., and Geyer, J. C., "Report on Sanitary Sewers and Waste Water Disposal in the Washington Metropolitan Region," Baltimore (1962).

Durum, W. H., and Langbein, W. B., "Water Quality of the Potomac River Estuary at Washington, D. C.," U. S. Geological Survey Circular No. 529-A, U. S. Government Printing Office, Washington, D. C. (1966).

30. Thomas, H. A., and Fiering, M. B., Operation Research in Water Quality Control, Report of Harvard Water Resources Corporation to U. S. Public Health Service (1963).

31. Loucks, D. P., "A Probabilistic Analysis of Wastewater Treatment Systems," Doctoral Thesis, Cornell University, Ithaca, New York (1965).

32. Loucks, D. P., and Lynn, W. R., "Probabilistic Models for Predicting Stream Quality," WRR, Vol. 2, No. 3, pp. 593-605 (1966).

33. Wastler, T. A., "Application of Spectral Analysis to Stream and Estuary Field Surveys," U. S. Public Health Service Publication No. 999-WP-7 (1963).

34. Thomann, R. V., "Time Series Analyses of Water Quality Data," ASCE-SA, Vol. 93, No. SA1, pp. 1-23 (February 1967).

35. Maass, A., Hufschmidt, M. M., Dorfman, R., Thomas, H. A., Margin, S. A., and Fair, G. M., Design of Water Resource Systems, Cambridge, Mass., Harvard University Press (1962).

36. Hufschmidt, M. M., and Fiering, M. B., Simulation Techniques for Design of Water-Resource Systems, Cambridge, Mass., Harvard University Press (1966).

37. Fiering, M. B., "Multivariate Techniques for Synthetic Hydrology," ASCE-HY, Vol. 90, No. HY5, p. 43 (1964).

38. Davis, R. K., The Range of Choice in Water Resources Management: A Study of the Potomac Estuary, Resources for the Future (in press) (1967).

39. Matalas, N., U. S. Geological Survey, private communication.

40. Chow, V. T., and Ramaseshan, S., "Sequential Generation of Rainfall and Runoff Data," ASCE-HY, Vol. 91, No. HY4, p. 205 (1965).

41. Beard, L. R., "Use of Interrelated Records to Simulate Stream Flow," ASCE-HY, Vol. 91, No. HY5, p. 13 (1965).

42. Crawford, N. H., and Linsley, R. K., "The Synthesis of Continuous Streamflow Hydrographs on a Digital Computer," Technical Report No. 12, Department of Civil Engineering, Stanford University (July 1962).

43. ABT Associates, Inc., "Fitting Time Series Models to Hydrologic Data," A Report to Federal Water Pollution Control Administration, U. S. Department of the Interior (June 1967).

44. Benson, M., U. S. Geological Survey, private communication.

45. Davis, passim.

46. Maass, Chapter 14.

47. Young, K. G. "Techniques for Finding Reservoir Operating Rates," Doctoral Thesis, Harvard University, Cambridge, Mass. (November 1966).

48. Maass, Chapter 13.

49. Hall, W. A., "Optimum Design of a Multi-Purpose Reservoir," ASCE-HY, Vol. 90, No. HY4, pp. 141-149 (July 1964).

50. Worley, J. L., "A Systems Analysis Method for Water Quality Management by Flow Augmentation in a Complex River Basin," Doctoral Thesis, Columbia River Basin Project, Division of Water Supply and Pollution Control, U. S. Public Health Service, Region IX, Portland, Oregon (June 1963).

51. Thomann, R. V., and Sobel, M. J., "Estuarine Water Quality Management and Forecasting," ASCE-SA, Vol. 90, No. SA5, pp. 9-37 (1964).

52. Deininger, R. A., "Water Quality Management: The Planning of Economically Optimal Pollution Control Systems," Doctoral Thesis, Northwestern University (1965).

53. Sobel, M. J., "Water Quality Improvement Programming Problems," WRR, Vol. 1, No. 4, p. 477 (1965).

54. Thomann, R. V., "Use of Systems Analysis in Estuarine Water Pollution Control," Western Resources Conference, 1964, University of Colorado Press, Boulder, Colo., p. 47 (1965).

55. Kerri, K. D., "An Investigation of Alternative Means of Achieving Water Quality Objectives," Doctoral Thesis, Oregon State University, Corvallis, Ore. (1966).

56. Liebman, J. C., "The Optimal Allocation of Stream Dissolved Oxygen Resources," Doctoral Thesis, Cornell University, Ithaca, New York (1965).

57. Thomann, R. V., "Recent Results from a Mathematical Model of Water Pollution Control in the Delaware Estuary," WRR, Vol. 1, pp.349, 1965, Public Health Abstract, Vol. 45, pp. 1907 (1965).

58. Hetling, Leo J., "A Mathematical Model for the Potomac River--What It has Done and What It Can Do," Proceedings of the Fall Meeting of the Interstate Commission on the Potomac River Basin, Washington, D. C. (November 1966).

59. Kneese, A. V., and Smith, S. C., editors, Water Research, Johns Hopkins Press (1966).

60. Davis, passim.

61. O'Connor, D. J., "The Effect of Stream Flow on Waste Assimilation Capacities," Proceedings 17th Purdue Indiana Waste Conference (May 1962).

62. Churchill, M. A., Elmore, H. L., and Buckingham, R. A., "The Prediction of Stream Reaeration Rates," ASCE-SA, Vol. 88, No. SA4, pp. 1-46 (July 1962).

63. Krenkel, Peter A., and Orlob, Gerald T., "Turbulent Diffusion and the Reaeration Coefficient," ASCE-SA, Vol. 88, No. SA2, Part I (March 1962).

64. Zeller, Robert, "Water Temperature, Influences, Effects and Controls," Proceedings of the 12th Pacific Northwest Symposium on Water Pollution Research, Corvallis, Oregon (November 1963).

65. Duttweiler, D. W., "A Mathematical Model of Stream Temperature," Doctoral Thesis, The Johns Hopkins University, Baltimore, Md. (1963).

66. Phelps, Earle B., Stream Sanitation, John Wiley and Sons, New York (1944).

67. Tennessee Valley Authority, "The Prediction of Stream Reaeration Rates," Division of Health and Safety, Chattanooga, Tennessee (July 1962).

68. Leopold, L. B., and Maddock, T., Jr., "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," U. S. Geological Survey Professional Paper 252, Washington, D. C. (1953).

69. U. S. Public Health Service, "The Storage and Retrieval of Data for Water Quality Control," U. S. Department of Health, Education, and Welfare, Washington, D. C. (1964).

70. Federal Water Pollution Control Administration, "River Basin Simulation Program," U. S. Department of the Interior, Washington, D. C. (1967).

71. Moder, J. T., and Philips, C. R., Project Management with CPM and Pert, Reinhold Publishing Corp., London (1964).

72. Nemhauser, G. L., Introduction to Dynamic Programming, John Wiley and Sons, Inc., New York (1966).

73. U. S. Corps of Engineers, "Potomac River Basin Study, Volumes I to VII," U. S. Department of the Army, Baltimore District Office, Maryland (1962).

74. Chesapeake Field Station, "A Summary of Water Quality, Potomac River Basin in Maryland," Federal Water Pollution Control Administration, U. S. Department of the Interior, Annapolis, Md. (1965).

75. U. S. Corps of Engineers, "Survey Report on the Potomac River," U. S. Department of the Army, Washington, D. C. (April 1948).

76. U. S. Public Health Service, "Investigation of the North Branch Potomac River," U. S. Department of Health, Education, and Welfare, Cincinnati, Ohio (August 1957).

77. U. S. Public Health Service, "Investigation of the Shenandoah River Basin," U. S. Department of Health, Education, and Welfare, Cincinnatti, Ohio (April 1961).

78. Fuhrman, R. E., "Upper Potomac River Industrial Wastes Investigation," Doctoral Thesis, The Johns Hopkins University, Baltimore, Md. (May 1954).

79. Hetling, Leo J., Federal Water Pollution Control Administration, U. S. Department of the Interior, Deputy Director of Chesapeake Field Station, Annapolis, Maryland, private communications.

80. Burgess and Niple, "Report on the Financing of Low Flow Augmentation Projects," Prepared for the Ohio Department of Health, Columbus, Ohio (1966).

81. Federal Water Pollution Control Administration, "River Basin Simulation Program," Division of Technical Control, U. S. Department of the Interior, Washington, D. C. (March 1967).

82. Davis, Appendix A.

83. Churchill, M. A., and Nicholas, W. R., "Effects of Impoundments on Water Quality," Proceedings of National Symposium on Quality Standards for Natural Water, University of Michigan, Ann Arbor, Michigan (July 1966).

84. Symons, J. M., Irwin, W. H., Clark, R. M., and Roebuck, G. G., "Management and Measurement of DO in Impoundments," Proceedings of National Symposium on Quality Standards for Natural Water, University of Michigan, Ann Arbor (July 1966).

85. Meir, W. L., and Beightler, C. S., "An Optimization Method for Branching Multistage Water Resources Systems," WRR, Vol. 3, No. 2 (December 1967).

86. O'Connor, D. J., and Dobbins, W. E., "The Mechanism of Reaeration in Natural Streams," ASCE-SA, Vol. 82, No. SA6 (1956).

87. Langbein, W. B., and Durum, W. H., "The Aeration Capacity of Streams," U. S. Geological Survey Circular 542, U. S. Department of the Interior, Washington, D. C. (1967).

88. Tsivoglou, E. C., O'Connell, R. L., Walter, C. M., Godsil, P. J., and Logsdon, G. S., "Tracer Measurements of Atmospheric Reaeration - I. Laboratory Studies," JWPCF, Vol. 37, No. 11 (October 1965).

89. Hall, Charles, Sanitary Engineer, Federal Water Pollution Control Administration, U. S. Department of the Interior, Chesapeake Field Station, Annapolis, Maryland, private communications.

90. Haveman, R. H., Water Resource Investment and the Public Interest, Vanderbilt University Press, Nashville, Tennessee (1965).

KEY TO ABBREVIATIONS

ASCE	American Society of Civil Engineers
ASCE-HY	American Society of Civil Engineers, Hydrology Division
ASCE-SA	American Society of Civil Engineers, Sanitary Division
JWPCF	Journal of Water Pollution Control Federation
SIW	Sewage and Industrial Waste
SWJ	Sewage Works Journal
WRR	Water Resources Research
WSW	Water and Sewage Works