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Assessment Of Statistical Tests In 40 CFR 75.21 (FR, 12/3/91) Alternative Monitoring Systems

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Project Summary

Title: Assessment of Statistical Tests in 40 CFR 75.21 (FR, 12/3/91): Alternative Monitoring Systems

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Abstract

This report assesses the three statistical tests that were included in the proposed acid rain regulations (40 CFR 75.21) for determining whether an alternative monitoring system provides information with the same precision as a continuous emissions monitoring system as required under Section 412(a), Title IV, of the Clean Air Act Amendments of 1990. The report presents the results of applying the statistical tests to six databases. Also discussed is a procedure for insuring the applicability of the proposed tests when the alternative monitoring system (AMS) measurements or the continuous emissions monitoring system (CEMS) measurements are autocorrelated.

The analysis found fifty-four subsets of paired CEMS/CEMS data and three subsets of paired AMS/CEMS data that passed all three tests, leading to the conclusion that the proposed tests are stringent but not preclusive. The report presents a procedure for inflating the variance used in the F-test and the standard error of the mean used in the bias test to compensate for possible underestimation of these statistics when measurement data are autocorrelated.

1. Introduction

Section 412(a) of Title IV of the Clean Air Act Amendments of 1990 requires that alternative monitoring systems (AMS) provide information with "the same precision, reliability, accessibility, and timeliness as that provided by CEMS" (continuous emissions monitoring system). To give an explicit and objective basis for determining whether an AMS satisfies the precision requirement, the proposed acid rain regulations (40 CFR 75.21), published in the Federal Register on December 3, 1991, set forth three statistical tests for evaluating the equivalency of an AMS and a CEMS. The tests are designed to gauge the extent of systematic error (bias test), random error (F-test), and correlation between the AMS and CEMS.

To appraise the stringency and workability of the tests and to evaluate comments submitted on the proposed tests, EPA first assembled six databases that approximate a range of potential AMS/CEMS behavior and then applied the proposed statistical tests to these databases. Since none of the databases was originally intended to demonstrate AMS/CEMS equivalency, there are disparities between these databases and the specifications that would have to be met by data submitted to EPA under the AMS provisions. However, despite these disparities, analysis of these data allowed EPA to evaluate whether the tests were serving their intended purpose and whether modifications were needed to enhance the applicability of the tests in cases where time dependencies (also called "autocorrelation") are discernable in the data.

2. <u>Databases</u>

Table 1 summarizes the characteristics of the databases analyzed in this study. To increase the effective number of datasets that were available for analysis, wherever feasible, the three statistical tests were applied to appropriate subsets of a database as well as the whole database. To qualify for analysis, a subset had to meet conditions similar to those contained in §75.21(a)(1)(vi) of the proposed regulations, which require owners or operators who petition EPA for use of an AMS to provide paired hourly observations from both a CEMS and the proposed AMS for a minimum of 30 successive unit operating days (720 hours). No more than 10 percent of the observations are allowed to be missing. In qualifying subsets, observations without concurrent values in its paired dataset were flagged. Then, any observation preceded by a flagged observation was also flagged. Finally, all flagged observations were discarded. This constituted the "lagging" procedure necessary for performing in the autocorrelation analysis described below. Thus, in the end, qualifying subsets may have fewer than 90% of the observations for the required 30-day period of coverage.

The specific features of the datasets and subsets analyzed in this study are described below:

UARG: In comments (Docket A-90-51, IV-D-185) on the proposed statistical tests for alternative monitoring system, the Utility Air Regulatory Group provided 22 hours of paired hourly data obtained from two test teams concurrently performing EPA Test Method 6 for SO_2 (40 CFR Part 60) at an unidentified power plant. While the number of observations falls short

Table 1: Characteristics of Databases Examined in This Study

Name	Monitoring Method	Monitoring Frequency	Monitoring Duration
Utility Air Regulatory Group (unidentified source)	2 Reference Method 6	Hourly	24 hours
Virginia Power Co. Chesapeake Energy Center Unit #4	5 CEMS	Hourly	63 days
Pennsylvania Electric Co. Homer City Unit #1	Coal Sampling and Analysis CEMS	Daily Daily	730 days
Pennsylvania Electric Co. Homer City Unit #3	Coal Sampling and Analysis CEMS	Daily Daily	730 days
Northern States Power Co. Sherburne County Unit #3	Coal Sampling and Analysis CEMS	Daily Hourly	730 days
Niagara Mohawk Oswego Unit #6	Oil Sampling and Analysis CEMS	Weekly hourly	455 days

of the 720 hours required under the proposed regulations, these data were included in the analysis since they had been submitted to EPA as part of comments on the alternative monitoring system statistical tests. Due to the small sample size of the whole database, no subsets of the data were analyzed.

Virginia Power Co. Chesapeake Unit #4: To provide a baseline assessment of the proposed statistical tests for alternative monitoring systems, EPA performed the tests on 63 days of hourly data from five CEMS installed on a common duct at this unit. The concurrent measurements from two CEMS were paired, one being designated the AMS. This enabled EPA to assess the claim that the statistical tests were so restrictive that not even two CEMS could pass them.

The Chesapeake unit data were made available to EPA by Virginia Power, which was testing competing brands of CEMS in anticipation of making a purchasing decision. The five CEMS were not required to meet EPA's certification requirements during installation, so the results on the statistical tests for AMS would not be expected to be fully comparable to results that would be obtained from a certified CEMS. Nevertheless, it was anticipated that analysis of these data would indicate whether the tests were preclusive, as some commenters had maintained.

EPA contracted Entropy Environmentalists, Inc. to collect the data and perform a preliminary analysis of the data. To maintain the anonymity of the manufacturers of the CEMS that were tested, they were designated by letters A-E. The data and Entropy's report appears under the title "Study of Proposed Acid Rain Regulations Section 75.21, Alternative Monitoring Systems" (EPA Contract No. 68-02-4462; Work Assignment No. 91-156). A more comprehensive analysis of the data was subsequently performed for EPA by The Cadmus Group, Inc. and is reported here.

There were two rounds of subsetting the data from the five CEMS. In the first round, monitor A was arbitrarily designated the "official" CEMS. Its measurements were paired, in turn, with concurrent values from monitors B-E, which were each considered to represent an AMS. The subsetting process described below was then applied to the paired data from CEMS A-B, A-C, A-D, and A-E. This produced 37 qualifying subsets in addition to the four full datasets.

In the second round of subsetting, no CEMS was designated a priori as the AMS. Instead, the subsetting process was applied to all possible pairs of CEMS measurements, i.e., A-

B, A-C, A-D, A-E, B-C, B-D, B-E, C-D, C-E, and D-E. Then, the data were analyzed to reveal the number of passes on each of the three statistical tests regardless of which of the paired CEMS was designated as the AMS. This produced 72 qualifying subsets in addition to the ten full datasets.

Data subsets were created by rolling a 720-observation window through the original paired data stream in increments of five observations. Thus, the first subset was created by keeping only the first 720 observations in the data stream. The second subset was created by omitting the first 5 observations in the data stream and then keeping next 720 observations. Therefore, data Subset 1 contains the original observations 1 through 720, whereas data Subset 2 contains the original observations 6 through 725. This process of subsetting was continued throughout the data stream until no subset that contained 720 observations could be created. Since the proposed rule specifies that a minimum of 90% of the data stream must contain nonmissing values, all data subsets which contained less than 648 (90% of 720) paired CEMS observations, where one or both CEMS values were missing, were discarded. Lagging, the process of pairing an observation with the observation immediately preceding it, was necessary for calculation of autocorrelation coefficients. Further reduction in the number of observations may have occurred in some subsets when the data were lagged, potentially pairing missing values to non-missing values. However, these subsets were not discarded (e.g., Subset 1 of a-d contains 613 observations).

Homer City Units #1 and #3: The Homer City datasets contained daily CEMS and AMS readings. In order to test the same time span as 720 hourly readings would cover, data subsets were created in groups of 30 observations (30 days = 720 hours). Unlike the Chesapeake data, the original data stream was "blocked" into non-overlapping and contiguous subsets of 30 observations. Those subsets which contained less than 27 observations (90% of 30) were discarded. Further reduction in the number of observations may have occurred in some subsets when the data were lagged, potentially pairing missing values to non-missing values. However, these subsets were not discarded (e.g. Subset 3 of Homer City #1 contains 26 observations). Subsets from Homer City #1 were created independently of the Homer City #3 subsets.

Northern States Power Company: Although the AMS data from Northern States were recorded daily, the CEMS data were recorded hourly, with AMS data repeated for each hour of a given day. As with the Homer City data, the original data stream was "blocked" into non-overlapping and contiguous subsets of 720 observations. Those subsets which contained less than 648 observations (90% of 720) were discarded. Further reduction in the number of observations may have occurred in some subsets when the data were lagged, potentially pairing missing values to non-missing values (none occurred for Northern States).

Niagara Mohawk: Although the AMS data from Niagara Mohawk were recorded weekly, the CEMS data were recorded hourly. The AMS data were repeated for each hour of a given week. Similar to the Northern States data, the original data stream was "blocked" into non-overlapping and contiguous subsets of 720 observations. Those subsets which contained less than 648 observations (90% of 720) were discarded. Further reduction in the number of observations may have occurred in some subsets when the data were lagged, potentially pairing missing values to non-missing values (none occurred for Niagara Mohawk).

3. Statistical Procedures

The three statistical tests for precision specified in §75.21 are the

F-test:

$$Pr\left[\frac{S^2_{AMS}}{S^2_{CEMS}} \geq F_{.05}\right] \qquad \text{Eq. (1)}$$

where S^2_{AMS} is the sample variance of the AMS measurements, S^2_{CEMS} is the sample variance of the CEMS measurements, and $F_{.05}$ is the critical value of the F statistic at $\alpha = 0.05$. If the ratio of the sample variances exceeds $F_{.05}$, we reject the hypothesis that the variances of the AMS and CEMS are equal with a 95% level of confidence (i.e., $\alpha = 0.05$).

Bias test:

$$Pr\left[\frac{\bar{d}}{S_{\bar{d}}} \geq t_{.025}\right] \qquad \text{Eq. (2)}$$

where $\bar{d}$ is the mean difference between the CEMS and the AMS, $S_{\bar{d}}$ is the standard error of the mean difference, and $t_{0.025}$ is the critical value of the t statistic for rejecting the hypothesis that, with 97.5% confidence (i.e., $\alpha = 0.025$), there is no systematic error in the AMS measurements when compared to the CEMS measurements.

Correlation test:

$$Pr\left[r_{(AMS,CEMS)} \geq 0.80\right] \qquad \text{Eq. (3)}$$

where $r_{(AMS,CEMS)}$ is the correlation coefficient between the CEMS and AMS emissions measurements.

4. Autocorrelation Analysis

Long-term, systematically collected sequential data, like that required under §75.21 of the proposed rule, often display a significantly high level of correlation between successive measurements, known as autocorrelation. The variance calculated from autocorrelated data tends be underestimated. In the parlance of statistics, the variance is said to be downwardly biased, i.e., it underestimates the population variance.[1] If an underestimated variance is used in tests of hypotheses, it can affect the likelihood of committing Type I error: rejecting a hypothesis that is actually true (Cochran 1977: 220; Wolter, 1984; Magee, 1989; Box, et al, 1978, p. 89; Gujarati, 1988, p. 364). Since both the proposed bias test and F-test are variants of hypothesis testing it is necessary to explore ways to compensate for underestimated variances.

This section presents a procedure for adjusting variances when data are autocorrelated. In particular, this section discusses the variance inflation factor required for an unbiased estimation of a sample variance (VIF), the inflation factor required for an unbiased estimation of a standard error of a sample mean (SEIF), and the inflation factor for the best, unbiased estimation of a standard error using an unbiased estimate of the sample variance (SEVIF). The discussion concludes with a comparison of the test results obtained when the bias test and F-test are applied to the datasets described above, first, not using and, then, using the variance and standard error adjustments presented in this section.

Statistical Background. A population consists of an unknown, potentially infinite number of measurements which can be characterized by a relative frequency distribution. Two numerical

[1]The appearance of the term "bias" here should not be confused with the "bias test," where "bias" refers to the systematic error between CEMS and AMS measurements.

measures used to describe a frequency distribution are: the mean, μ, a measure of the population's central tendency and the variance, σ^2, a measure of the population's dispersion, expressed as a function of the deviations of the measurements from their mean.

Consider a sample of size n taken from a population of measurements which are independent and identically distributed. The population variance is σ^2 and the mean of these n measurements has the variance σ^2/n (Steel and Torrie, 1980 [p. 26]; Rawlings, 1988 [p. 11]). The variance of a population is usually unknown, but can be estimated from a sample of the population by:

$$S^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} \qquad \text{Eq. (4)}$$

where S^2 is the sample variance, y_i is the i^{th} measurement, and $\bar{y}$ is the mean of the n measurements sampled (Steel and Torrie, 1980 [p. 23]).

When a population consists of measurements that are closely related in time such that the value of one measurement is to some degree dependent on the value of the previous measurement, the data are said to be autocorrelated. Autocorrelated data contain redundancies in information which reduce the effective sample size, resulting in an estimate of variance that is smaller than expected. An unbiased estimate of the sample variance of an autocorrelated population, $S^2_{unbiased}$, can be calculated by:

$$S^2_{unbiased} = \frac{S^2}{1 - \frac{2\rho}{(n-1)(1-\rho)} + \frac{2\rho(1-\rho^n)}{n(n-1)(1-\rho)^2}} \qquad \text{Eq. (5)}$$

where S^2 is the sample variance obtained by Eq. (4), n is the sample size and ρ is the degree of autocorrelation (Wolter, 1984).

As with the population mean and variance, the population autocorrelation, ρ, is usually unknown. The best unbiased estimate of ρ is the sample autocorrelation, r, which is defined by:

$$r = \frac{\sum\left[(O_i - \bar{O})(L_i - \bar{L})\right]}{\sum(O_i - \bar{O})\sum(L_i - \bar{L})} \qquad \text{Eq. (6)}$$

where O_i is the measurement at time i (original), L_i is the measurement at time i-1 (LAG1), and $\bar{O}$ and $\bar{L}$ are the means of the original measurements and the lagged measurements, respectively (Steel and Torrie, 1980 [p.272]). Substituting r for ρ in Eq. (5) will result in the best, unbiased estimate of the sample variance:

$$S^2_{unbiased} = \frac{S^2}{1 - \frac{2r}{(n-1)(1-r)} + \frac{2r(1-r^n)}{n(n-1)(1-r)^2}} \qquad \text{Eq. (7)}$$

The standard error of the mean of n samples drawn from an autocorrelated population can be denoted by $\sigma_{\bar{x}}$ and defined as:

$$\sigma_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}\left[\frac{(1+\rho)}{(1-\rho)} - \frac{2\rho(1-\rho^n)}{n(1-\rho)^2}\right]} \qquad \text{Eq. (8)}$$

where σ^2 is the population variance obtained from an autocorrelated population of size n, and ρ is the degree of autocorrelation (Box and Jenkins, 1976 [p.194]). Note that for large sample sizes a portion of Eq. (5):

$$\frac{2\rho(1-\rho^n)}{n(n-1)(1-\rho)^2}$$

and of Eq. (8):

$$\frac{2\rho(1-\rho^n)}{n(1-\rho)^2}$$

will be nearly equal to zero. Figures 1 and 2 show the ranges of sensitivity of Eq. (5) and Eq. (8) to changes in ρ and n. Replacing σ^2 and ρ in Eq. (8) with $S^2_{unbiased}$ from Eq. (5) and r as defined in Eq. (6), respectively, will result in an unbiased estimate of the standard error of the sample mean, $S_{\bar{x}_{unbiased}}$:

$$S_{\bar{x}_{unbiased}} = \sqrt{\frac{\frac{S^2}{n}\left[\frac{(1+r)}{(1-r)} - \frac{2r(1-r^n)}{n(1-r)^2}\right]}{1 - \frac{2r}{(n-1)(1-r)} + \frac{2r(1-r^n)}{n(n-1)(1-r)^2}}} \qquad \text{Eq. (9)}$$

Figure 3 shows the ranges of sensitivity of Eq. (9) to changes in r and n.

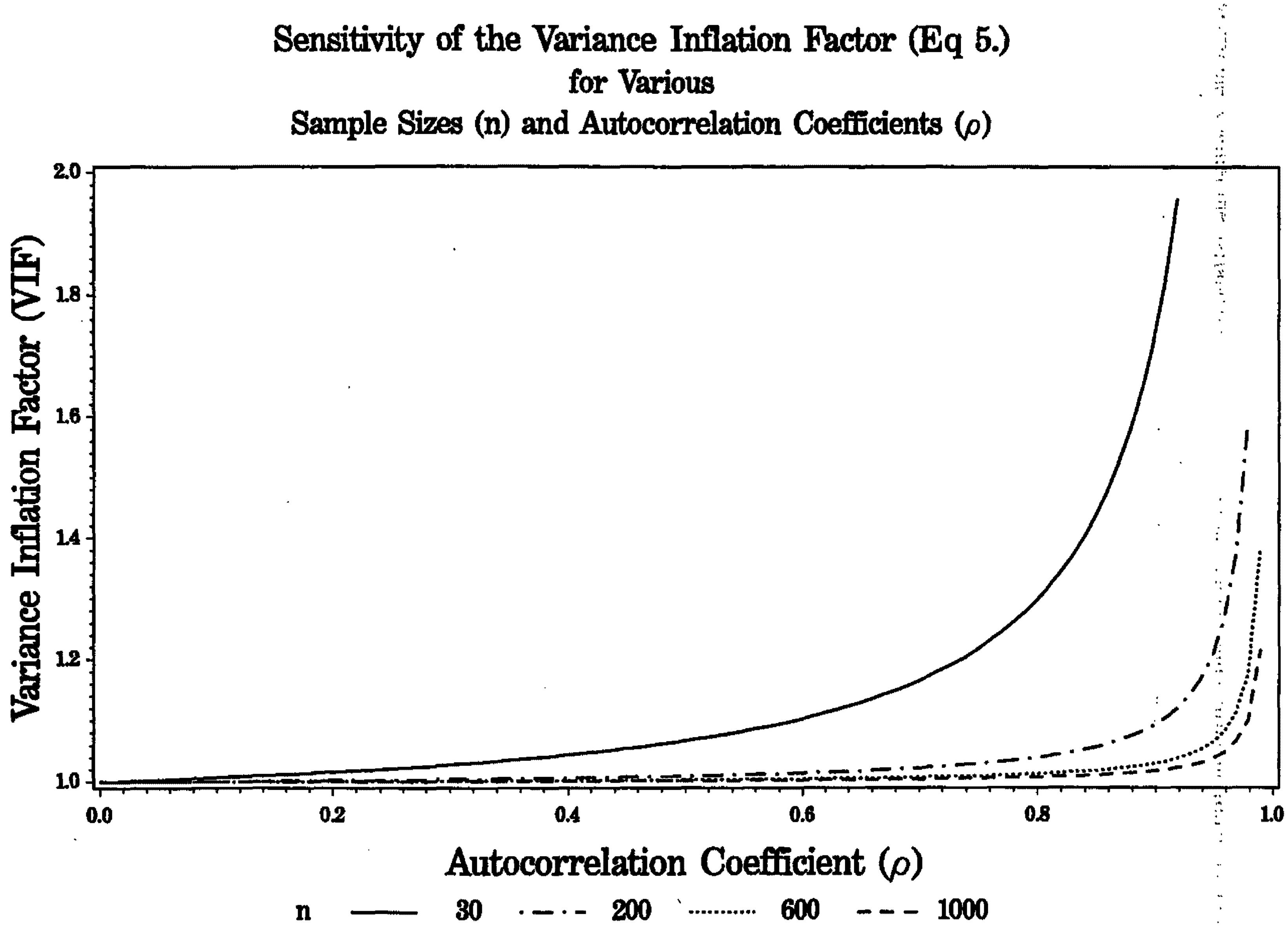
Sensitivity of the Variance Inflation Factor (Eq 5.)
for Various
Sample Sizes (n) and Autocorrelation Coefficients (ρ)
Variance Inflation Factor (VIF)
2.0
1.8
1.6
1.4
1.2
1.0
0.0
0.2
0.4
0.6
0.8
1.0
Autocorrelation Coefficient (ρ)
n
30
200
600
1000

Figure 1:

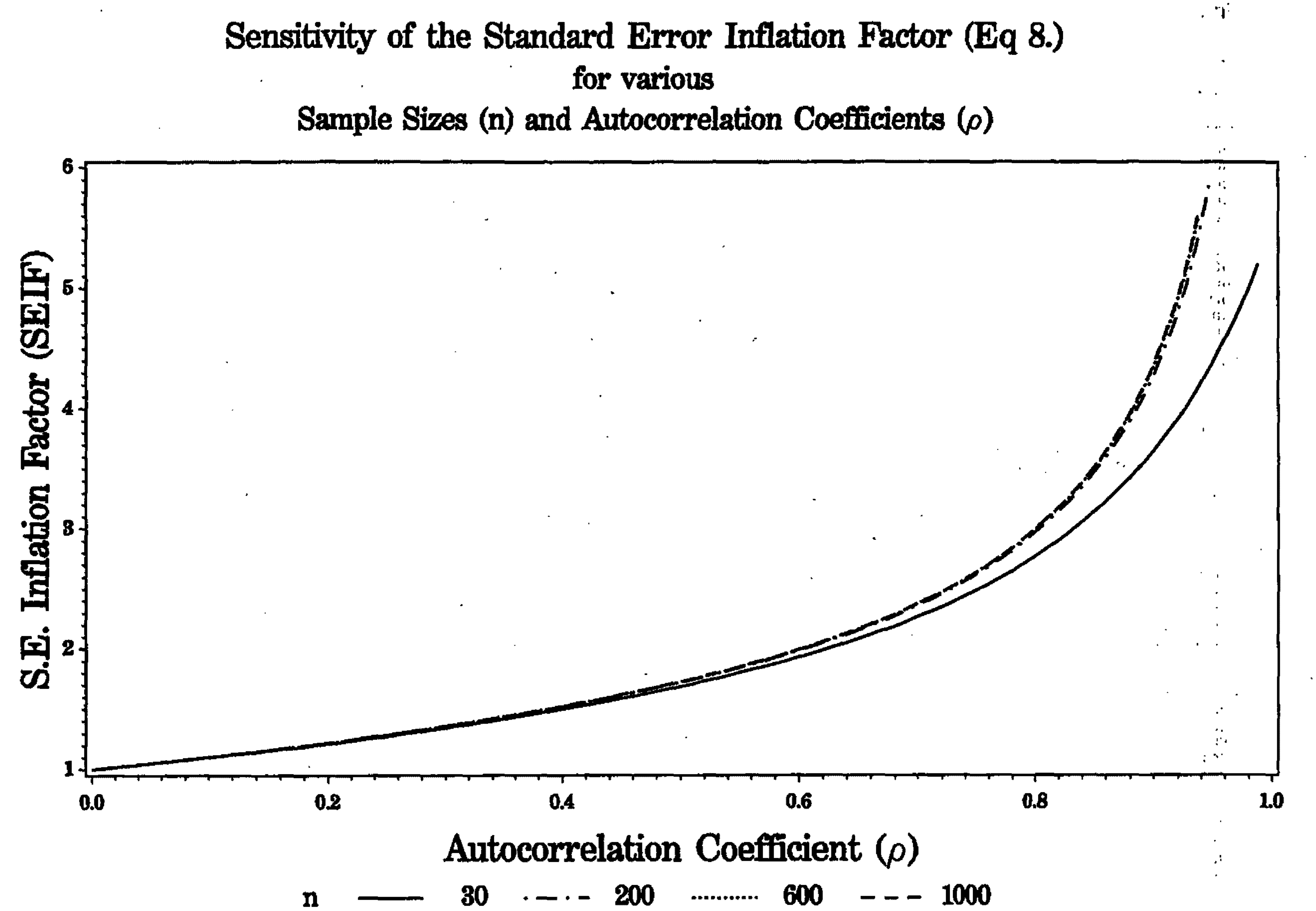
Sensitivity of the Standard Error Inflation Factor (Eq 8.)
for various
Sample Sizes (n) and Autocorrelation Coefficients (ρ)
S.E. Inflation Factor (SEIF)
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
Autocorrelation Coefficient (ρ)
n 30 200 600 1000

Figure 2:

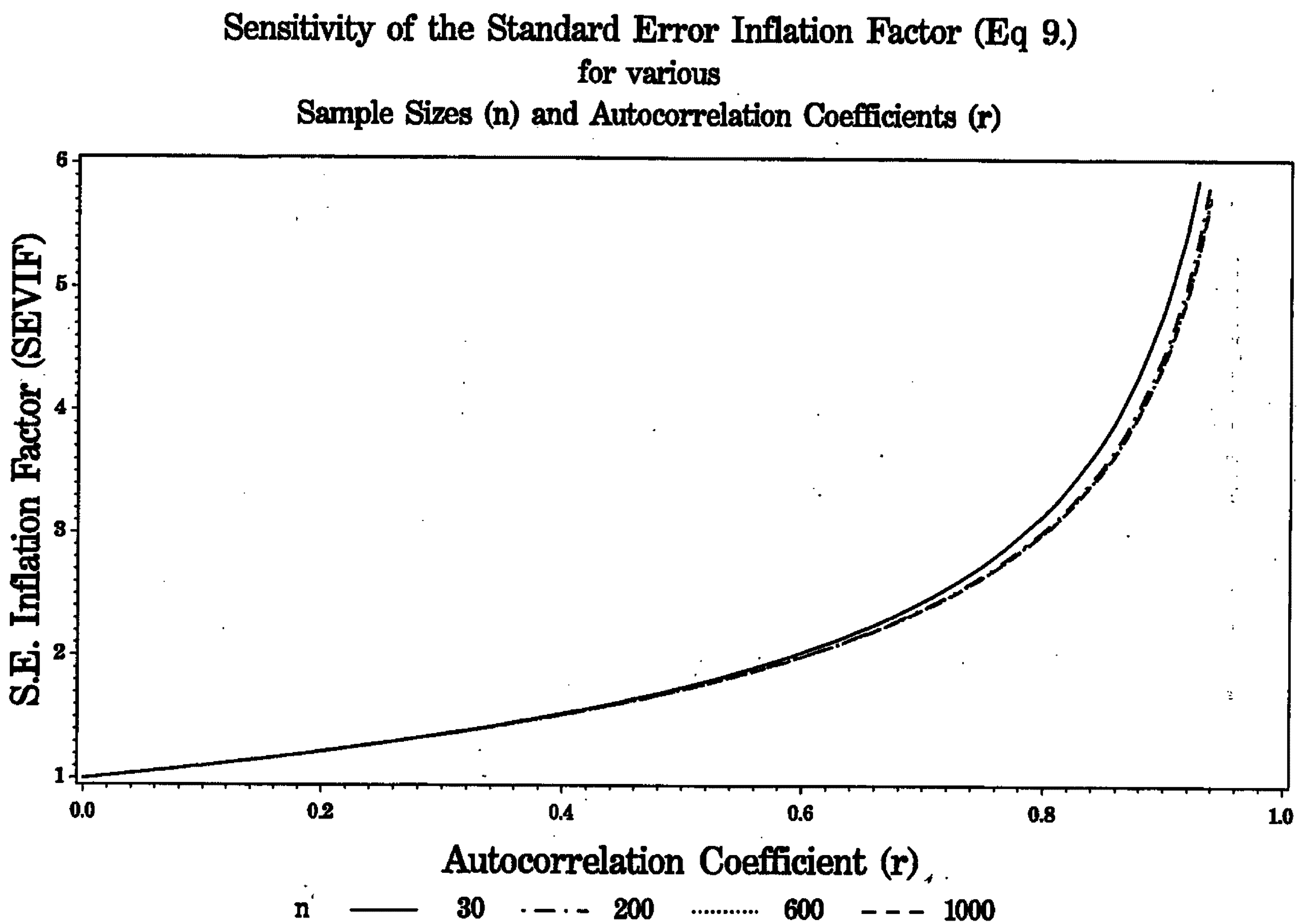
Sensitivity of the Standard Error Inflation Factor (Eq 9.)
for various
Sample Sizes (n) and Autocorrelation Coefficients (r)
S.E. Inflation Factor (SEVIF)
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
Autocorrelation Coefficient (r)
n
30
200
600
1000

Figure 3:

Proposed Tests. Two of the three statistical tests used to evaluate the precision of the AMS could be affected by autocorrelated measurements. The proposed F-test, which is used to determine whether the variances of two populations are significantly different, relies on unbiased estimates of the sample variance for the AMS and CEMS measurements. Thus, Eq. (1) can be restated as follows:

$$Pr\left[\frac{S^2_{AMS_{unbiased}}}{S^2_{CEMS_{unbiased}}} \geq F_{.05}\right] \qquad \text{Eq. (10)}$$

where $S^2_{AMS_{unbiased}}$ is the best, unbiased estimate of the sample variance of the AMS measurements and $S^2_{CEMS_{unbiased}}$ is the best, unbiased estimate of the sample variance of the CEMS measurements. Unbiased estimates of the sample variance of the AMS and CEMS measurements that are used in this test can be derived from Eq.(7) where r and S^2 are calculated separately for the CEMS and AMS measurements.

Similarly, the bias test in the proposed regulations relies on an unbiased estimate of the standard error of mean. Thus, Eq. (2) can be restated as:

$$Pr\left[\frac{\bar{d}}{S_{\bar{d}_{unbiased}}} \geq t_{.025}\right] \qquad \text{Eq. (11)}$$

An unbiased estimate of the standard error of the mean difference can be obtained from Eq. (9).

Comparative Analysis. The F-test was performed on each of the datasets described above in section 2 by taking the ratio of each paired AMS variance and CEMS variance and comparing it to the critical F-value at α=.05 for the appropriate sample size. Of the sixty-eight subsets tested, 43 passed the F-test.

Autocorrelation coefficients for both the CEMS and AMS were calculated using Eq. (6) and are listed in Table 2. Also shown are the p-values achieved on the F-test. The autocorrelation coefficients for AMS data subsets ranged from r=-0.01 (Homer City 3, subset 3) to r=0.99 (Niagara Mohawk, subset 5). Similarly, autocorrelation coefficients for CEMS subsets ranged from r=0.01 (Homer City 1, subset 24) to r=0.98 (Northern States, subset 7). Variance inflation factors (VIFs) for each data subset were calculated from r and n, using Eq (7). The VIFs derived from the AMS and CEMS measurements for each dataset are shown in Table 2. In most cases, the VIF of the CEMS was slightly larger than the VIF of the corresponding AMS. Consequently many F-test statistics decreased slightly when inflation factors were applied. However, only two F-test results changed from passes to failures (Northern States, subsets 1 and 23). In both cases the AMS autocorrelation coefficient was larger than the CEMS autocorrelation.

For the bias test, the difference between the CEMS and AMS measurements was calculated for each observation pair. Mean differences, variances of the differences using Eq (4), and autocorrelation of the differences using Eq (6) were calculated for each data subset. These statistics and the results of the bias tests (using unadjusted standard errors) for each subset are listed in Table 3. The autocorrelation coefficients ranged from r=0.04 (Homer City, subset 7) to r=0.94 (Northern States, subsets 7 and 11).

Table 2: Summary of results of the F-test (α=.05) using original variance (Original) and using inflated variance (Inflated).

SOURCE[1]	Subset ID[2]	N[3]	r AMS[4]	Variance Inflation Factor AMS[5]	r CEMS[4]	Variance Inflation Factor CEMS[5]	Variance AMS	Variance CEMS	Original F-test	Original p-value	Original Pass Fail	Inflated F-test	Inflated p-value	Inflated Pass Fail
Chesapeake a-c	20	648	0.78	1.01082	0.92	1.03603	10277.59	6532.35	1.57334	0.00000	F	1.53505	0.00000	F
Chesapeake a-c	21	648	0.88	1.02360	0.93	1.04241	21762.05	4909.50	4.43264	0.00000	F	4.35267	0.00000	F
Chesapeake a-c	23	648	0.78	1.01072	0.92	1.03549	10240.95	6512.33	1.57255	0.00000	F	1.53493	0.00000	F
Chesapeake a-c	24	648	0.87	1.02183	0.93	1.04308	22120.89	4893.35	4.52060	0.00000	F	4.42850	0.00000	F
Chesapeake a-c	26	648	0.78	1.01070	0.92	1.03534	10249.70	6537.77	1.56777	0.00000	F	1.53045	0.00000	F
Chesapeake a-c	27	648	0.87	1.02029	0.93	1.04229	21841.75	4741.87	4.60615	0.00000	F	4.50889	0.00000	F
Chesapeake a-c	29	648	0.77	1.01066	0.92	1.03483	10232.73	6525.92	1.56801	0.00000	F	1.53139	0.00000	F
Chesapeake a-c	30	648	0.87	1.02028	0.93	1.04415	21796.11	4792.74	4.54774	0.00000	F	4.44379	0.00000	F
Chesapeake a-c	32	648	0.77	1.01060	0.92	1.03469	10228.26	6539.09	1.56417	0.00000	F	1.52776	0.00000	F
Chesapeake a-c	33	648	0.87	1.02020	0.93	1.04446	21663.59	4776.94	4.53503	0.00000	F	4.42970	0.00000	F
Chesapeake a-c	35	648	0.77	1.01063	0.92	1.03452	10291.26	6618.55	1.55491	0.00000	F	1.51900	0.00000	F
Chesapeake a-c	36	648	0.86	1.01991	0.93	1.04285	21367.32	4575.85	4.66958	0.00000	F	4.56685	0.00000	F
Chesapeake a-d	40	613	0.96	1.08429	0.91	1.03348	6485.28	6173.57	1.05049	0.27126	P	1.10214	0.11464	P
Chesapeake a-d	41	629	0.93	1.04492	0.94	1.04915	4394.40	4798.81	0.91573	0.86487	P	0.91203	0.87557	P
Chesapeake a-d	43	613	0.96	1.08608	0.91	1.03288	6466.82	6146.17	1.05217	0.26476	P	1.10637	0.10573	P
Chesapeake a-d	44	629	0.93	1.04486	0.94	1.05009	4386.32	4782.60	0.91714	0.86063	P	0.91257	0.87403	P
Chesapeake a-d	46	613	0.96	1.08232	0.91	1.03276	6479.52	6174.45	1.04941	0.27550	P	1.09977	0.11987	P
Chesapeake a-d	47	629	0.93	1.04501	0.94	1.04922	4265.53	4627.40	0.92180	0.84608	P	0.91810	0.85770	P
Chesapeake a-d	49	613	0.96	1.08163	0.91	1.03227	6478.13	6157.99	1.05199	0.26546	P	1.10229	0.11431	P
Chesapeake a-d	50	629	0.93	1.04652	0.94	1.05173	4275.69	4680.23	0.91356	0.87120	P	0.90904	0.88380	P
Chesapeake a-d	52	613	0.96	1.08030	0.91	1.03213	6476.09	6172.56	1.04917	0.27643	P	1.09814	0.12358	P
Chesapeake a-d	53	629	0.94	1.04745	0.94	1.05216	4325.02	4664.33	0.92725	0.82789	P	0.92311	0.84183	P
Chesapeake a-d	55	613	0.96	1.07822	0.91	1.03205	6524.23	6260.24	1.04217	0.30478	P	1.08879	0.14647	P
Chesapeake a-d	56	629	0.94	1.04682	0.94	1.05022	4194.84	4457.25	0.94113	0.77635	P	0.93808	0.78830	P
Chesapeake a-e	60	629	0.82	1.01440	0.91	1.03373	5794.88	6446.98	0.89885	0.90911	P	0.88205	0.94196	P
Chesapeake a-e	61	625	0.87	1.02163	0.94	1.05053	5106.79	5010.28	1.01926	0.40587	P	0.99122	0.54384	P
Chesapeake a-e	63	629	0.82	1.01424	0.91	1.03318	5783.34	6425.55	0.90005	0.90635	P	0.88356	0.93944	P
Chesapeake a-e	64	625	0.87	1.02178	0.94	1.05149	5064.25	4994.82	1.01390	0.43159	P	0.98525	0.57361	P
Chesapeake a-e	66	629	0.82	1.01444	0.91	1.03307	5879.71	6451.94	0.91131	0.87759	P	0.89488	0.91785	P
Chesapeake a-e	67	625	0.87	1.02075	0.94	1.05066	4807.79	4840.71	0.99320	0.53394	P	0.96492	0.67213	P
Chesapeake a-e	69	629	0.82	1.01419	0.91	1.03260	5870.66	6439.22	0.91170	0.87649	P	0.89545	0.91664	P
Chesapeake a-e	70	625	0.86	1.02071	0.94	1.05322	4825.93	4895.21	0.98585	0.57063	P	0.95542	0.71545	P
Chesapeake a-e	72	629	0.82	1.01436	0.91	1.03247	5873.43	6452.88	0.91020	0.88064	P	0.89423	0.91921	P
Chesapeake a-e	73	625	0.86	1.02061	0.94	1.05367	4827.09	4881.07	0.98894	0.55521	P	0.95792	0.70429	P
Chesapeake a-e	75	629	0.82	1.01439	0.91	1.03236	5926.72	6535.22	0.90689	0.88950	P	0.89111	0.92557	P
Chesapeake a-e	76	625	0.86	1.01999	0.94	1.05177	4678.03	4674.07	1.00085	0.49579	P	0.97060	0.64524	P
Homer City 1	6	30	0.19	1.01613	0.30	1.02838	0.03	0.02	2.22741	0.01741	F	2.20088	0.01878	F
Homer City 1	10	26	0.07	1.00583	0.36	1.04410	0.05	0.02	2.43931	0.01491	F	2.34990	0.01858	F
Homer City 1	21	26	0.12	1.01073	0.15	1.01372	0.01	0.01	0.76707	0.74389	P	0.76481	0.74623	P
Homer City 1	24	29	0.12	1.00947	0.01	1.00041	0.03	0.02	1.65669	0.09394	P	1.67170	0.09009	P
Homer City 3	3	25	-0.01	0.99918	0.03	1.00211	0.01	0.02	0.49862	0.95260	P	0.49717	0.95327	P
Homer City 3	5	30	0.56	1.08716	0.61	1.10788	0.01	0.01	0.83016	0.69024	P	0.81464	0.70774	P
Homer City 3	7	26	0.68	1.17452	0.86	1.53944	0.05	0.05	0.92664	0.57478	P	0.70698	0.80404	P
Homer City 3	8	28	0.72	1.20108	0.68	1.16503	0.01	0.01	1.10830	0.39565	P	1.14259	0.36580	P
Homer City 3	16	30	-0.26	0.98614	0.11	1.00810	0.00	0.01	0.42265	0.98825	P	0.41344	0.98987	P
Homer City 3	17	30	0.78	1.26717	0.78	1.26941	0.01	0.01	0.65332	0.87122	P	0.65216	0.87220	P
Homer City 3	20	29	0.46	1.06086	0.60	1.10902	0.01	0.01	0.95115	0.55224	P	0.90985	0.59781	P
Homer City 3	21	29	0.06	1.00419	0.15	1.01191	0.01	0.00	1.40980	0.18448	P	1.39905	0.18984	P

Table 2: Continued

SOURCE[1]	Subset ID[2]	N[3]	r AMS[4]	Variance Inflation Factor AMS[5]	r CEMS[4]	Variance Inflation Factor CEMS[5]	Variance AMS	Variance CEMS	Original F-test	Original p-value	Original Pass Fail	Inflated F-test	Inflated p-value	Inflated Pass Fail
Niagara Mohawk	2	686	0.55	1.00355	0.57	1.00388	0.00	0.01	0.40410	1.00000	P	0.40396	1.00000	P
Niagara Mohawk	5	668	0.99	1.44638	0.51	1.00318	0.00	0.00	0.29828	1.00000	P	0.43006	1.00000	P
Northern States	1	713	0.97	1.09510	0.78	1.01015	0.02	0.02	1.12909	0.05275	P	1.22404	0.00353	F
Northern States	4	671	0.97	1.09043	0.92	1.03665	0.04	0.02	2.49713	0.00000	F	2.62668	0.00000	F
Northern States	6	720	0.96	1.06271	0.86	1.01682	0.02	0.01	2.41168	0.00000	F	2.52052	0.00000	F
Northern States	7	720	0.98	1.17992	0.98	1.12580	0.06	0.04	1.55537	0.00000	F	1.63014	0.00000	F
Northern States	8	679	0.97	1.09700	0.71	1.00712	0.01	0.00	3.26712	0.00000	F	3.55869	0.00000	F
Northern States	9	720	0.96	1.07389	0.88	1.01980	0.01	0.01	0.60136	1.00000	P	0.63325	1.00000	P
Northern States	10	720	0.99	1.20570	0.89	1.02168	0.03	0.01	1.90502	0.00000	F	2.24813	0.00000	F
Northern States	11	695	0.96	1.07155	0.95	1.05436	0.01	0.00	1.39075	0.00001	F	1.41343	0.00000	F
Northern States	12	720	0.97	1.08792	0.55	1.00346	0.01	0.00	2.02432	0.00000	F	2.19471	0.00000	F
Northern States	13	720	0.97	1.09515	0.86	1.01740	0.02	0.01	1.13125	0.04923	F	1.21770	0.00417	F
Northern States	14	720	0.96	1.06805	0.61	1.00429	0.01	0.01	0.58660	1.00000	P	0.62384	1.00000	P
Northern States	15	720	0.97	1.08679	0.55	1.00343	0.00	0.01	0.38046	1.00000	P	0.41207	1.00000	P
Northern States	16	697	0.96	1.07659	0.68	1.00599	0.01	0.01	1.14229	0.03975	F	1.22245	0.00407	F
Northern States	18	720	0.97	1.08587	0.73	1.00764	0.01	0.03	0.50634	1.00000	P	0.54565	1.00000	P
Northern States	21	703	0.96	1.06416	0.52	1.00313	0.01	0.01	1.20930	0.00595	F	1.28287	0.00049	F
Northern States	22	720	0.97	1.08626	0.89	1.02289	0.01	0.00	1.80955	0.00000	F	1.92166	0.00000	F
Northern States	23	720	0.98	1.12065	0.94	1.04189	0.02	0.02	1.10602	0.08847	P	1.18963	0.01002	F
UARG	1	19	-0.03	0.99679	-0.36	0.97274	2281.84	1348.50	1.69214	0.13692	P	1.73397	0.12622	P
Total Number (out of 68) passing the F-test											43			41

1 o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). One reference CEM (A) and four alternative CEMs (B-E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.
o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
o Homer City Unit #1 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a 730 day period.
o Homer City Unit #3 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days.
o UARG data from Attachment E of Public Comments (Table 1, page 4). There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.

2 Subset identification: Data were subset into smaller datasets of 30 days each.

3 Number of observations after removal of observations containing missing differences or missing LAG1 differences.

4 Autocorrelation of the AMS or CEMS.

5 ***Variance Inflation Factor*** $= \dfrac{1}{1-\dfrac{2r}{(n-1)(1-r)}+\dfrac{2r(1-r^{n})}{n(n-1)(1-r)^{2}}}$

Variance inflation factors were calculated from the number of observations and estimated autocorrelation coefficient of each data subset, using Eq. (7). These were then applied to each subset's estimate of variance. Variance adjustments ranged from 1.00025 (for Chesapeake a-e, subset 76) to 1.16892 (Homer City, subset 8) with a mean VIF of 1.02133. The VIFs for variances of the differences are listed in Table 3.

Inflation factors for the standard errors of the differences were calculated by two methods. The first method used Eq (8), substituting the unadjusted variance of the sample differences, S^2 for σ^2. This inflation factor is labelled SEIF. The second method uses Eq (9), which simultaneously incorporates the inflated sample variance, shown in Eq. (7), and an adjustment for underestimation in the standard error of the sample means, represented by Eq. (8). When applied, this dual inflation factor results in the best, unbiased estimate of the standard error of the differences and is labelled SEVIF to denote that both VIF and SEIF have been applied. (See Appendix B for a fuller treatment of the statistical basis for this procedure.) The purpose for calculating both standard error inflation factors is to illustrate the impact of obtaining an unbiased estimate of the sample variance before applying an adjustment to the standard error of the bias test.

Forty-eight (48) of the 68 subsets tested passed the bias test without applying either inflation factor. An additional two subsets (Homer City 1, subset 10; Homer City 3, subset 8) passed when SEIF was applied. Table 3 shows the magnitude of each inflation factor for each data subset. Using SEVIF compared to SEIF reduced all t-values slightly. However, the additional adjustment was very small in most cases (averaging 1.02133) and none of the pass/fail

Table 3: Summary of results of the 1-tailed bias test (α=.025) using original variance and standard error (original), using inflated standard error (SEIF), and using inflated standard error from an inflated estimate of variance (SEVIF).

SOURCE[1]	Subset[2] ID	N[3]	r[4]	Mean Difference	Variance of Difference	VIF[5]	VIF Adjusted Variance	Standard Error of Difference	SEIF[6]	SEIF Adjusted Standard Error	SEVIF[7] Adjusted Standard Error	Original t-value	Original p-value	Original Pass Fail	SEIF applied t-value	SEIF applied p-value	SEIF applied Pass Fail	SEVIF applied t-value	SEVIF applied p-value	SEVIF applied Pass Fail
Chesapeake a-c	20	648	0.40	25.4366	4358.13	1.00209	4367.23	2.59336	1.53384	3.9778	3.9819	9.8083	0.0000	F	6.3947	0.0000	F	6.3880	0.0000	F
Chesapeake a-c	21	648	0.87	59.0179	17472.77	1.02147	17847.91	5.19270	3.84274	19.9542	20.1673	11.3656	0.0000	F	2.9577	0.0016	F	2.9264	0.0018	F
Chesapeake a-c	23	648	0.40	25.4520	4358.24	1.00209	4367.34	2.59339	1.53377	3.9777	3.9818	9.8142	0.0000	F	6.3988	0.0000	F	6.3921	0.0000	F
Chesapeake a-c	24	648	0.86	59.9091	17783.78	1.01980	18135.92	5.23871	3.70225	19.3950	19.5861	11.4358	0.0000	F	3.0889	0.0010	F	3.0588	0.0012	F
Chesapeake a-c	26	648	0.40	25.5579	4358.19	1.00209	4367.30	2.59338	1.53374	3.9776	3.9817	9.8550	0.0000	F	6.4255	0.0000	F	6.4188	0.0000	F
Chesapeake a-c	27	648	0.86	59.8744	17785.92	1.01837	18112.62	5.23903	3.57711	18.7406	18.9119	11.4285	0.0000	F	3.1949	0.0007	F	3.1660	0.0008	F
Chesapeake a-c	29	648	0.40	25.6296	4358.02	1.00209	4367.12	2.59333	1.53365	3.9772	3.9814	9.8829	0.0000	F	6.4441	0.0000	F	6.4373	0.0000	F
Chesapeake a-c	30	648	0.86	59.8449	17787.78	1.01837	18114.59	5.23930	3.57747	18.7434	18.9148	11.4223	0.0000	F	3.1928	0.0007	F	3.1639	0.0008	F
Chesapeake a-c	32	648	0.40	25.8309	4358.15	1.00209	4367.24	2.59336	1.53335	3.9765	3.9807	9.9604	0.0000	F	6.4958	0.0000	F	6.4891	0.0000	F
Chesapeake a-c	33	648	0.86	59.7348	17793.44	1.01839	18120.64	5.24014	3.57890	18.7539	18.9255	11.3995	0.0000	F	3.1852	0.0008	F	3.1563	0.0008	F
Chesapeake a-c	35	648	0.40	25.9272	4357.13	1.00209	4366.22	2.59306	1.53312	3.9755	3.9796	9.9987	0.0000	F	6.5218	0.0000	F	6.5150	0.0000	F
Chesapeake a-c	36	648	0.86	59.7066	17795.19	1.01839	18122.48	5.24039	3.57920	18.7564	18.9281	11.3935	0.0000	F	3.1833	0.0008	F	3.1544	0.0008	F
Chesapeake a-d	40	613	0.73	-4.4245	3207.51	1.00875	3235.58	2.28746	2.51801	5.7599	5.7850	-1.9343	0.9732	P	-0.7682	0.7787	P	-0.7648	0.7777	P
Chesapeake a-d	41	629	0.25	-0.4053	488.27	1.00108	488.80	0.88106	1.29503	1.1410	1.1416	-0.4600	0.6772	P	-0.3552	0.6387	P	-0.3550	0.6387	P
Chesapeake a-d	43	613	0.73	-4.3880	3220.33	1.00864	3248.17	2.29203	2.50531	5.7422	5.7670	-1.9145	0.9720	P	-0.7642	0.7775	P	-0.7609	0.7765	P
Chesapeake a-d	44	629	0.26	-0.5622	476.67	1.00114	477.21	0.87053	1.30938	1.1399	1.1405	-0.6458	0.7407	P	-0.4932	0.6890	P	-0.4929	0.6889	P
Chesapeake a-d	46	613	0.72	-4.2963	3228.30	1.00858	3256.00	2.29486	2.49756	5.7316	5.7561	-1.8722	0.9692	P	-0.7496	0.7731	P	-0.7464	0.7721	P
Chesapeake a-d	47	629	0.27	-0.7834	468.54	1.00116	469.08	0.86307	1.31520	1.1351	1.1358	-0.9077	0.8178	P	-0.6902	0.7548	P	-0.6898	0.7547	P
Chesapeake a-d	49	613	0.72	-4.3495	3246.40	1.00850	3273.98	2.30129	2.48722	5.7238	5.7481	-1.8900	0.9704	P	-0.7599	0.7762	P	-0.7567	0.7752	P
Chesapeake a-d	50	629	0.27	-0.7745	448.76	1.00117	449.29	0.84466	1.31627	1.1118	1.1125	-0.9169	0.8202	P	-0.6966	0.7568	P	-0.6962	0.7567	P
Chesapeake a-d	52	613	0.72	-4.0593	3263.14	1.00852	3290.92	2.30721	2.48957	5.7440	5.7684	-1.7594	0.9605	P	-0.7067	0.7600	P	-0.7037	0.7591	P
Chesapeake a-d	53	629	0.25	-1.1791	434.32	1.00108	434.78	0.83096	1.29486	1.0760	1.0766	-1.4189	0.9218	P	-1.0958	0.8632	P	-1.0952	0.8631	P
Chesapeake a-d	55	613	0.72	-3.9291	3271.45	1.00854	3299.38	2.31015	2.49214	5.7572	5.7817	-1.7008	0.9553	P	-0.6825	0.7524	P	-0.6796	0.7515	P
Chesapeake a-d	56	629	0.24	-1.3438	427.40	1.00100	427.82	0.82431	1.27541	1.0513	1.0519	-1.6303	0.9482	P	-1.2782	0.8992	P	-1.2776	0.8991	P
Chesapeake a-e	60	629	0.10	-1.4839	1365.35	1.00036	1365.85	1.47332	1.10767	1.6319	1.6322	-1.0072	0.8429	P	-0.9093	0.8182	P	-0.9091	0.8182	P
Chesapeake a-e	61	625	0.12	-2.4210	431.17	1.00043	431.36	0.83059	1.12604	0.9353	0.9355	-2.9148	0.9982	P	-2.5885	0.9951	P	-2.5879	0.9951	P
Chesapeake a-e	63	629	0.10	-1.7633	1368.74	1.00037	1369.24	1.47515	1.10902	1.6360	1.6363	-1.1953	0.8838	P	-1.0778	0.8592	P	-1.0776	0.8592	P
Chesapeake a-e	64	625	0.11	-2.2154	426.90	1.00040	427.08	0.82647	1.11826	0.9242	0.9244	-2.6805	0.9962	P	-2.3970	0.9916	P	-2.3966	0.9916	P
Chesapeake a-e	66	629	0.11	-2.1072	1379.12	1.00039	1379.66	1.48073	1.11516	1.6512	1.6516	-1.4231	0.9224	P	-1.2761	0.8988	P	-1.2759	0.8988	P
Chesapeake a-e	67	625	0.09	-1.8752	416.46	1.00032	416.60	0.81630	1.09411	0.8931	0.8933	-2.2972	0.9890	P	-2.0996	0.9819	P	-2.0993	0.9819	P
Chesapeake a-e	69	629	0.11	-2.3060	1381.75	1.00039	1382.29	1.48214	1.11694	1.6555	1.6558	-1.5559	0.9399	P	-1.3930	0.9179	P	-1.3927	0.9179	P
Chesapeake a-e	70	625	0.08	-1.8664	437.01	1.00027	437.13	0.83619	1.08104	0.9040	0.9041	-2.2320	0.9870	P	-2.0647	0.9803	P	-2.0644	0.9803	P
Chesapeake a-e	72	629	0.11	-2.1536	1378.33	1.00039	1378.87	1.48030	1.11652	1.6528	1.6531	-1.4548	0.9269	P	-1.3030	0.9035	P	-1.3027	0.9034	P
Chesapeake a-e	73	625	0.08	-1.8950	429.53	1.00028	429.65	0.82900	1.08397	0.8986	0.8987	-2.2859	0.9887	P	-2.1088	0.9823	P	-2.1085	0.9823	P
Chesapeake a-e	75	629	0.11	-2.0625	1381.36	1.00040	1381.91	1.48193	1.11838	1.6574	1.6577	-1.3918	0.9178	P	-1.2444	0.8931	P	-1.2442	0.8931	P
Chesapeake a-e	76	625	0.07	-2.0082	426.31	1.00026	426.42	0.82589	1.07706	0.8895	0.8897	-2.4315	0.9923	P	-2.2575	0.9878	P	-2.2572	0.9878	P
Homer City 1	6	30	0.16	-0.3190	0.02	1.01237	0.02	0.02243	1.17005	0.0262	0.0264	-14.2225	1.0000	P	-12.1555	1.0000	P	-12.0810	1.0000	P
Homer City 1	10	26	0.05	0.0950	0.05	1.00414	0.05	0.04509	1.05234	0.0475	0.0476	2.1068	0.0227	F	2.0020	0.0281	P	1.9979	0.0284	P
Homer City 1	21	26	0.54	-0.1835	0.01	1.09498	0.02	0.02393	1.83506	0.0439	0.0459	-7.6669	1.0000	P	-4.1780	0.9998	P	-3.9927	0.9997	P
Homer City 1	24	29	0.68	-0.0910	0.01	1.15445	0.01	0.01965	2.27903	0.0448	0.0481	-4.6318	1.0000	P	-2.0324	0.9741	P	-1.8915	0.9655	P
Homer City 3	3	25	0.58	0.1508	0.00	1.11465	0.00	0.01075	1.93036	0.0208	0.0219	14.0232	0.0000	F	7.2646	0.0000	F	6.8808	0.0000	F
Homer City 3	5	30	0.20	0.0543	0.00	1.01629	0.00	0.00857	1.21857	0.0104	0.0105	6.3380	0.0000	F	5.2012	0.0000	F	5.1593	0.0000	F
Homer City 3	7	26	0.04	0.0912	0.00	1.00311	0.00	0.01169	1.03964	0.0122	0.0122	7.7959	0.0000	F	7.4986	0.0000	F	7.4870	0.0000	F
Homer City 3	8	28	0.69	0.0357	0.00	1.16893	0.00	0.00816	2.32504	0.0190	0.0205	4.3791	0.0001	F	1.8834	0.0352	P	1.7420	0.0464	P
Homer City 3	16	30	0.63	0.1443	0.00	1.11846	0.00	0.01003	2.09132	0.0210	0.0222	14.3890	0.0000	F	6.8803	0.0000	F	6.5058	0.0000	F
Homer City 3	17	30	0.66	0.0587	0.00	1.13585	0.00	0.00969	2.20059	0.0213	0.0227	6.0522	0.0000	F	2.7503	0.0051	F	2.5806	0.0076	F
Homer City 3	20	29	0.44	-0.1028	0.01	1.05531	0.01	0.01352	1.60109	0.0216	0.0222	-7.6017	1.0000	P	-4.7478	1.0000	P	-4.6217	1.0000	P
Homer City 3	21	29	0.22	-0.0734	0.00	1.01911	0.00	0.01108	1.24471	0.0138	0.0139	-6.6299	1.0000	P	-5.3265	1.0000	P	-5.2763	1.0000	P
Niagara Mohawk	2	686	0.70	-0.1009	0.00	1.00692	0.00	0.00259	2.39409	0.0062	0.0062	-39.0089	1.0000	P	-16.2939	1.0000	P	-16.2378	1.0000	P

Table 3: continued.

SOURCE[1]	Subset[2] ID	N[3]	r[4]	Mean Difference	Variance of Difference	VIF[5]	VIF Adjusted Variance	Standard Error of Difference	SEIF[6]	SEIF Adjusted Standard Error	SEVIF[7] Adjusted Standard Error	Original t-value	Original p-value	Original Pass Fail	SEIF applied t-value	SEIF applied p-value	SEIF applied Pass Fail	SEVIF applied t-value	SEVIF applied p-value	SEVIF applied Pass Fail
Niagara Mohawk	5	668	0.43	-0.0746	0.00	1.00230	0.00	0.00131	1.59124	0.0021	0.0021	-56.9446	1.0000	P	-35.7862	1.0000	P	-35.7452	1.0000	P
Northern States	1	713	0.72	-0.1485	0.01	1.00740	0.01	0.00413	2.50168	0.0103	0.0104	-35.9751	1.0000	P	-14.3804	1.0000	P	-14.3274	1.0000	P
Northern States	4	671	0.94	-0.3095	0.04	1.05091	0.05	0.00804	5.85943	0.0471	0.0483	-38.5074	1.0000	P	-6.5719	1.0000	P	-6.4107	1.0000	P
Northern States	6	720	0.91	-0.2391	0.02	1.02792	0.02	0.00533	4.56419	0.0243	0.0247	-44.8710	1.0000	P	-9.8311	1.0000	P	-9.6967	1.0000	P
Northern States	7	720	0.94	-0.3013	0.03	1.04296	0.03	0.00648	5.59429	0.0363	0.0370	-46.4839	1.0000	P	-8.3092	1.0000	P	-8.1362	1.0000	P
Northern States	8	679	0.91	-0.3473	0.02	1.03052	0.02	0.00473	4.62801	0.0219	0.0222	-73.4922	1.0000	P	-15.8799	1.0000	P	-15.6429	1.0000	P
Northern States	9	720	0.92	-0.2337	0.02	1.03259	0.02	0.00539	4.90893	0.0265	0.0269	-43.3286	1.0000	P	-8.8265	1.0000	P	-8.6861	1.0000	P
Northern States	10	720	0.85	-0.2070	0.01	1.01558	0.01	0.00444	3.48355	0.0155	0.0156	-46.6279	1.0000	P	-13.3852	1.0000	P	-13.2821	1.0000	P
Northern States	11	695	0.94	-0.1521	0.01	1.04272	0.01	0.00326	5.48498	0.0179	0.0183	-46.6910	1.0000	P	-8.5125	1.0000	P	-8.3363	1.0000	P
Northern States	12	720	0.80	-0.1862	0.01	1.01102	0.01	0.00415	2.98123	0.0124	0.0124	-44.8607	1.0000	P	-15.0477	1.0000	P	-14.9655	1.0000	P
Northern States	13	720	0.89	-0.1241	0.02	1.02257	0.02	0.00532	4.13186	0.0220	0.0222	-23.3378	1.0000	P	-5.6483	1.0000	P	-5.5856	1.0000	P
Northern States	14	720	0.72	-0.1355	0.02	1.00715	0.02	0.00513	2.47585	0.0127	0.0128	-26.3925	1.0000	P	-10.6600	1.0000	P	-10.6221	1.0000	P
Northern States	15	720	0.59	-0.1255	0.01	1.00405	0.01	0.00444	1.97691	0.0088	0.0088	-28.2411	1.0000	P	-14.2855	1.0000	P	-14.2566	1.0000	P
Northern States	16	697	0.74	-0.1588	0.02	1.00833	0.02	0.00482	2.60377	0.0125	0.0126	-32.9787	1.0000	P	-12.6657	1.0000	P	-12.6133	1.0000	P
Northern States	18	720	0.77	-0.1478	0.03	1.00929	0.03	0.00677	2.76743	0.0187	0.0188	-21.8460	1.0000	P	-7.8940	1.0000	P	-7.8575	1.0000	P
Northern States	21	703	0.62	-0.2097	0.01	1.00457	0.01	0.00380	2.05144	0.0078	0.0078	-55.1863	1.0000	P	-26.9013	1.0000	P	-26.8400	1.0000	P
Northern States	22	720	0.91	-0.1604	0.01	1.02903	0.01	0.00306	4.64862	0.0142	0.0144	-52.4769	1.0000	P	-11.2887	1.0000	P	-11.1283	1.0000	P
Northern States	23	720	0.90	-0.0696	0.01	1.02548	0.01	0.00435	4.37224	0.0190	0.0192	-16.0226	1.0000	P	-3.6646	0.9999	P	-3.6188	0.9998	P
UARG Attch. E	1	19	0.13	15.2632	731.43	1.01631	743.35	6.20453	1.14346	7.0946	7.1522	2.4600	0.0121	F	2.1514	0.0226	F	2.1340	0.0234	F
Total Number (out of 68) passing the bias test.														48			50			50

[1] o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). One reference CEM (A) and four alternative CEMs (B-E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.
o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
o Homer City Unit #1 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a 730 day period.
o Homer City Unit #3 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days.
o UARG data from Attachment E of Public Comments (Table 1, page 4). There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.

[2] Subset identification: Data were subset into smaller datasets of 30 days each.

[3] Number of observations after removal of observations containing missing differences or missing LAG1 differences.

[4] Autocorrelation of the differences.

[5] ***Variance Inflation Factor*** $= \dfrac{1}{1-\dfrac{2r}{(n-1)(1-r)}+\dfrac{2r(1-r^n)}{n(n-1)(1-r)^2}}$

[6] ***Standard Error Inflation Factor*** $= \sqrt{\dfrac{1+r}{1-r}-\dfrac{2r(1-r^n)}{n(1-r)^2}}$

[7] ***Standard Error& Variance Inflation Factor*** $= \sqrt{\dfrac{\left[\dfrac{(1+r)}{(1-r)}-\dfrac{2r(1-r^n)}{n(1-r)^2}\right]}{1-\dfrac{2r}{(n-1)(1-r)}+\dfrac{2r(1-r^n)}{n(n-1)(1-r)^2}}}$

results changed when SEVIF was applied compared to the pass/fail results using SEIF. Table 4 shows the ranges and averages of each inflation factor used in the bias test and the ranges of bias test t-values.

Summary of Findings. The effect of using underestimated variance is most prominent in the bias test, where the underestimation may result in detection of significant systematic error when in fact the systematic error may not be statistically significant. In contrast, underestimation of variance may affect the F-test from two different directions. Since this test is the ratio of the AMS variance to the CEMS variance, it is possible that different degrees of underestimation may be introduced into the estimated variance of the CEMS and AMS measurements. Adjusting for the underestimated variance of each system individually could increase or decrease the resulting F-value depending on which variance requires greater adjustment. (More highly correlated data will have more downwardly biased estimates of variance.) In the case where the autocorrelations of the CEMS and the AMS are equal, the variance adjustments would cancel and the adjusted F-test results would be equal to that of the unadjusted F-test.

However with large sample sizes, as would be required under the proposed regulations, the underestimation of the variances and standard error can be expected to be minimal except for cases where the autocorrelation coefficient is very high ($r \geq 0.90$). Therefore, applying variance inflation factors are not expected to substantially change the pass/fail rates on either the F-test or the bias test.

Table 4: Summary of inflation factors and bias test t-values.

	Inflation Factor			t-value		
Using Equation	Minimum	Maximum	Mean	Minimum	Maximum	Mean
NONE				-73.49	14.39	-9.89
Eq 7. (VIF)	1.00025	1.16892	1.02133			
Eq 8. (SEIF)	1.03964	5.85943	2.33171	-35.79	7.50	-3.07
Eq 9. (SEVIF)	1.04126	6.00672	2.36186	-35.75	7.49	-3.05

5. Test Results

Table 5 shows the results of applying the three statistical tests as specified in §75.21 of the proposed regulations to:

- the data reported by UARG;
- the Chesapeake data subsetted as described above with monitor A designated as the "official" CEMS and each of the other monitors representing an AMS;
- the Homer City data subsetted as described above;
- the Northern States data analyzed at the level of refinement of the CEMS. That is, hourly CEMS measurements were paired with daily CSA measurements for corresponding periods of coverage.
- the Niagara Mohawk data analyzed at the level of refinement of the CEMS. That is, hourly CEMS measurements were paired with weekly OSA measurements for corresponding periods of coverage.

The paired Reference Method 6 data reported by UARG failed the bias test but passed the F-test and the correlation test. For the paired CEMS/CEMS measurements from the Chesapeake unit, 26 subsets passed the bias test, 26 passed the F-test, 20 passed the correlation test, and 20 passed all three tests. For the paired CEMS/CSA measurements from the Homer City and Northern States units, 24 subsets passed the bias test, 18 passed the F-test, 5 passed the correlation test, and none passed all three tests. For the CEMS/OSA measurements from the Niagara Mohawk unit, all three subsets passed the bias and F-test, none passed the correlation test, and, consequently, none passed all three test.

Table 5: Results on Bias Test, F-test, and Correlation Test Analyzed at Refinement of CEMS Without Inflation of Variances.

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
UARG Attachment E	ALL	19	2.46	.0121	F	1.69	.1369	P	.83	P	F
Chesapeake CEMS a-b	ALL	1167	4.44	.0000	F	1.30	.0000	F	.75	F	F
Chesapeake CEMS a-b	N/A	< 648	----	----	-	----	----	-	----	-	-
Chesapeake CEMS a-c	ALL	1383	14.98	.0000	F	2.45	.0000	F	.68	F	F
Chesapeake CEMS a-c	1	648	9.80	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	2	648	11.36	.0000	F	4.43	.0000	F	.44	F	F
Chesapeake CEMS a-c	3	648	9.81	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	4	648	11.43	.0000	F	4.52	.0000	F	.44	F	F
Chesapeake CEMS a-c	5	648	9.85	.0000	F	1.51	.0000	F	.76	F	F
Chesapeake CEMS a-c	6	648	11.43	.0000	F	4.61	.0000	F	.43	F	F
Chesapeake CEMS a-c	7	648	9.88	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	8	648	11.42	.0000	F	4.55	.0000	F	.43	F	F
Chesapeake CEMS a-c	9	648	9.96	.0000	F	1.56	.0000	F	.76	F	F
Chesapeake CEMS a-c	10	648	11.40	.0000	F	4.54	.0000	F	.43	F	F
Chesapeake CEMS a-c	11	648	9.99	.0000	F	1.55	.0000	F	.76	F	F
Chesapeake CEMS a-c	12	648	11.39	.0000	F	4.66	.0000	F	.41	F	F

Table 5: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
Chesapeake CEMS a-d	ALL	1329	-1.96	.9749	P	1.01	.4343	P	.88	P	P
Chesapeake CEMS a-d	1	613	-1.93	.9732	P	1.05	.2713	P	.75	F	F
Chesapeake CEMS a-d	2	629	-0.46	.6772	P	0.91	.8650	P	.95	P	P
Chesapeake CEMS a-d	3	613	-1.91	.9720	P	1.05	.2648	P	.74	F	F
Chesapeake CEMS a-d	4	629	-0.64	.7407	P	0.91	.8606	P	.95	P	P
Chesapeake CEMS a-d	5	613	-1.87	.9692	P	1.05	.2755	P	.75	F	F
Chesapeake CEMS a-d	6	629	-0.90	.8178	P	0.92	.8461	P	.95	P	P
Chesapeake CEMS a-d	7	613	-1.89	.9704	P	1.05	.2655	P	.74	F	F
Chesapeake CEMS a-d	8	629	-0.92	.8202	P	0.91	.8712	P	.95	P	P
Chesapeake CEMS a-d	9	613	-1.75	.9605	P	1.05	.2764	P	.74	F	F
Chesapeake CEMS a-d	10	629	-1.42	.9218	P	0.93	.8279	P	.95	P	P
Chesapeake CEMS a-d	11	613	-1.70	.9553	P	1.04	.3048	P	.74	F	F
Chesapeake CEMS a-d	12	629	-1.63	.9482	P	0.94	.7763	P	.95	P	P
Chesapeake CEMS a-e	ALL	1337	-2.51	.9940	P	0.96	.7843	P	.94	P	P
Chesapeake CEMS a-e	1	629	-1.01	.8429	P	0.89	.9091	P	.88	P	P
Chesapeake CEMS a-e	2	625	-2.91	.9982	P	1.02	.4058	P	.96	P	P

Table 5: Continued.

Data Source[1]	Data Subset[2]	N	1-tailed Bias Test $\alpha<.025$ Computed value	p-value	Pass/Fail	1-tailed F-test $\alpha<.05$ Computed value	p-value	Pass/Fail	Correlation $\rho>.80$ Computed value	Pass/Fail	Overall Pass/Fail
Chesapeake CEMS a-e	3	629	-1.19	.8838	P	0.90	.9063	P	.89	P	P
Chesapeake CEMS a-e	4	625	-2.68	.9962	P	1.01	.4316	P	.96	P	P
Chesapeake CEMS a-e	5	629	-1.42	.9224	P	0.91	.8776	P	.89	P	P
Chesapeake CEMS a-e	6	625	-2.29	.9890	P	0.99	.5339	P	.96	P	P
Chesapeake CEMS a-e	7	629	-1.55	.9399	P	0.91	.8765	P	.89	P	P
Chesapeake CEMS a-e	8	625	-2.23	.9870	P	0.98	.5706	P	.96	P	P
Chesapeake CEMS a-e	9	629	-1.45	.9269	P	0.91	.8806	P	.89	P	P
Chesapeake CEMS a-e	10	625	-2.28	.9887	P	0.98	.5552	P	.96	P	P
Chesapeake CEMS a-e	11	629	-1.39	.9178	P	0.91	.8895	P	.89	P	P
Chesapeake CEMS a-e	12	625	-2.43	.9923	P	1.00	.4958	P	.95	P	P
Summary of CEMS number of tests passed total number of tests=41					26			27		21	20
Homer City #1	ALL	496	-10.94	.9999	P	1.77	.0000	F	.62	F	F
Homer City #1	1	30	-14.22	.9999	P	2.23	.0174	F	.76	F	F
Homer City #1	2	26	2.11	.0227	F	2.44	.0149	F	.30	F	F
Homer City #1	3	26	-7.66	.9999	P	0.77	.7439	P	.18	F	F
Homer City #1	4	29	-4.63	.9999	P	1.66	.0939	P	.76	F	F

Table 5: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
Homer City #3	All	496	10.46	.0000	F	0.77	.9981	P	.77	F	F
Homer City #3	1	25	14.02	.0000	F	0.50	.9526	P	.92	P	F
Homer City #3	2	30	6.34	.0000	F	0.83	.6902	P	.89	P	F
Homer City #3	3	26	7.79	.0000	F	0.93	.5748	P	.96	P	F
Homer City #3	4	28	4.38	.0001	F	1.11	.3956	P	.90	P	F
Homer City #3	5	30	14.39	.0000	F	0.42	.9882	P	.69	F	F
Homer City #3	6	30	6.05	.0000	F	0.65	.8712	P	.84	P	F
Homer City #3	7	29	-7.60	.9999	P	0.95	.5522	P	.79	F	F
Homer City #3	8	29	-6.63	.9999	P	1.41	.1845	P	.65	F	F
Northern States	All	15830	-143.83	.9999	P	.94	.9999	P	.36	F	F
Northern States	1	713	-35.98	.9999	P	1.13	.0528	P	.63	F	F
Northern States	2	671	-38.51	.9999	P	2.50	.0000	F	.30	F	F
Northern States	3	720	-44.87	.9999	P	2.41	.0000	F	.22	F	F
Northern States	4	720	-46.48	.9999	P	1.56	.0000	F	.71	F	F
Northern States	5	679	-73.49	.9999	P	3.26	.0000	F	.04	F	F
Northern States	6	720	-43.33	.9999	P	0.60	.9999	P	-.13	F	F

Table 5: Continued

Data Source[1]	Data Subset[2]	N	1-tailed Bias Test $\alpha<.025$ Computed value	p-value	Pass/Fail	1-tailed F-test $\alpha<.05$ Computed value	p-value	Pass/Fail	Correlation $\rho>.80$ Computed value	Pass/Fail	Overall Pass/Fail
Northern States	7	720	-46.63	.9999	P	1.90	.0000	F	.71	F	F
Northern States	8	695	-46.69	.9999	P	1.39	.0000	F	.37	F	F
Northern States	9	720	-44.86	.9999	P	2.02	.0000	F	.18	F	F
Northern States	10	720	-23.33	.9999	P	1.13	.0492	F	.30	F	F
Northern States	11	720	-26.39	.9999	P	0.59	.9999	P	.06	F	F
Northern States	12	720	-28.24	.9999	P	0.38	.9999	P	.20	F	F
Northern States	13	697	-32.97	.9999	P	1.14	.0398	F	.33	F	F
Northern States	14	720	-21.85	.9999	P	0.51	.9999	P	.20	F	F
Northern States	15	703	-55.19	.9999	P	1.21	.0059	F	.39	F	F
Northern States	16	720	-52.48	.9999	P	1.81	.0000	F	.32	F	F
Northern States	17	720	-16.02	.9999	P	1.11	.0885	P	.60	F	F
Summary of CSA number of tests passed total number of tests=32					24			18		5	0
Niagara Mohawk	All	6801	-124.33	.9999	P	0.28	.9999	P	.30	F	F
Niagara Mohawk	1	686	-39.01	.9999	P	0.40	.9999	P	.42	F	F
Niagara Mohawk	2	668	-56.95	.9999	P	0.30	.9999	P	.41	F	F
Summary of OSA number of tests passed total number of tests=3					3			3		0	0

Footnotes to Table 5:

[1]
- o UARG data from Attachment E (Table 1, page 4) in Correspondence IV-D-185 to EPA Docket No. A-90-51. There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.
- o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). One reference CEM (A) and four alternative CEMs (B-E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.
- o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
- o Homer City Unit #1 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a 730 day period.
- o Homer City Unit #3 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
- o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days.

[2] Data were subset into smaller datasets of 30 days each. ALL = entire dataset used; N/A = less than 90% data available; all other values are set identifiers.

Table 6 analyzes the same datasets as Table 5 but uses the variance inflation factors as described above in the bias test and F-test to compensate for autocorrelation in the data. The test results for the data reported by UARG remain unchanged. For the CEMS/CEMS data from the Chesapeake unit, 27 rather than 26 subsets pass the bias test. For the CEMS/CSA data 26 rather than 24 subsets pass the bias test, 16 rather than 18 pass the F-test, and 1 subset passes all three test. The test results for the CEMS/OSA data remain unchanged.

In a effort to determine whether on not the refinement of the AMS affects the 3 statistical tests, Northern States and Niagara Mohawk data were analyzed at the level of refinement of the AMS. That is, for the Northern States data daily CSA measurements are paired with the daily average CEMS measurements for periods of coverage, and for the Niagara Mohawk data weekly OSA measurements are paired with the average weekly CEMS measurements for the period of coverage. Table 7 and Table 8 report test results for Northern States CEMS/CSA data and the Niagara Mohawk CEMS/OSA data analyzed at the level of refinement of the AMS. Both without (Table 7) and with (Table 8) the variance inflation estimate two of the CEMS/OSA subsets pass all three tests while none of the CEMS/CSA pass all three tests.

Table 9 and Table 10 report test results for the CEMS/CEMS data from the Chesapeake unit using all possible combinations of CEMS. This analysis differs from that reported in Table 5 and Table 6, where monitor A was designated as the "official" CEMS, while each of the other monitors was considered a separate AMS. These tables reveal the number of subsets that will pass a statistical test regardless of which monitor has been designated the "official" CEMS. Here, the one-tail t-test and F-test were applied to data from all possible pairings of CEMS. For each CEMS pair, the tests were performed twice: once with one monitor designated as the

Table 6: Results on Bias Test, F-test, and Correlation Test Analyzed at Refinement of CEMS Using Inflated Variances Estimate[1].

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
UARG Attachment E	ALL	19	2.17	.0219	F	1.65	.1493	P	.83	P	F
Chesapeake CEMS a-b	ALL	1167	1.28	.0996	P	1.31	.0000	F	.75	F	F
Chesapeake CEMS a-b	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS a-c	ALL	1383	5.41	.0000	F	2.42	.0000	F	.68	F	F
Chesapeake CEMS a-c	1	648	6.39	.0000	F	1.54	.0000	F	.76	F	F
Chesapeake CEMS a-c	2	648	2.93	.0018	F	4.35	.0000	F	.44	F	F
Chesapeake CEMS a-c	3	648	6.39	.0000	F	1.53	.0000	F	.76	F	F
Chesapeake CEMS a-c	4	648	3.06	.0012	F	4.43	.0000	F	.44	F	F
Chesapeake CEMS a-c	5	648	6.42	.0000	F	1.53	.0000	F	.76	F	F
Chesapeake CEMS a-c	6	648	3.17	.0008	F	4.51	.0000	F	.43	F	F
Chesapeake CEMS a-c	7	648	6.44	.0000	F	1.53	.0000	F	.76	F	F
Chesapeake CEMS a-c	8	648	3.16	.0008	F	4.44	.0000	F	.43	F	F
Chesapeake CEMS a-c	9	648	6.49	.0000	F	1.53	.0000	F	.76	F	F
Chesapeake CEMS a-c	10	648	3.16	.0008	F	4.43	.0000	F	.43	F	F
Chesapeake CEMS a-c	11	648	6.49	.0000	F	1.52	.0000	F	.76	F	F
Chesapeake CEMS a-c	12	648	3.15	.0008	F	4.57	.0000	F	.41	F	F

Table 6: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
Homer City #3	All	496	3.46	.0003	F	0.74	.9996	P	.77	F	F
Homer City #3	1	25	6.88	.0000	F	0.50	.9533	P	.92	P	F
Homer City #3	2	30	5.15	.0000	F	0.81	.7077	P	.89	P	F
Homer City #3	3	26	7.48	.0000	F	0.71	.8040	P	.96	P	F
Homer City #3	4	28	1.74	.0464	P	1.14	.3658	P	.90	P	P
Homer City #3	5	30	6.51	.0000	F	0.41	.9898	P	.69	F	F
Homer City #3	6	30	2.58	.0076	F	0.65	.8722	P	.84	P	F
Homer City #3	7	29	-4.62	.9999	P	0.91	.5978	P	.79	F	F
Homer City #3	8	29	-5.27	.9999	P	1.40	.1898	P	.65	F	F
Northern States	All	15830	-40.70	.9999	P	0.94	.9999	P	.36	F	F
Northern States	1	713	-14.32	.9999	P	1.22	.0035	F	.63	F	F
Northern States	2	671	-6.41	.9999	P	2.63	.0000	F	.30	F	F
Northern States	3	720	-9.69	.9999	P	2.52	.0000	F	.22	F	F
Northern States	4	720	-8.13	.9999	P	1.63	.0000	F	.71	F	F
Northern States	5	679	-15.64	.9999	P	3.56	.0000	F	.04	F	F
Northern States	6	720	-8.68	.9999	P	0.63	.9999	P	-.13	F	F

Table 6: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
Chesapeake CEMS a-e	3	629	-1.08	.8592	P	0.88	.9394	P	.89	P	P
Chesapeake CEMS a-e	4	625	-2.39	.9916	P	0.98	.5136	P	.96	P	P
Chesapeake CEMS a-e	5	629	-1.27	.8988	P	0.89	.9178	P	.89	P	P
Chesapeake CEMS a-e	6	625	-2.09	.9819	P	0.96	.6721	P	.96	P	P
Chesapeake CEMS a-e	7	629	-1.39	.9179	P	0.89	.9165	P	.89	P	P
Chesapeake CEMS a-e	8	625	-2.06	.9803	P	0.95	.7154	P	.96	P	P
Chesapeake CEMS a-e	9	629	-1.30	.9034	P	0.89	.9192	P	.89	P	P
Chesapeake CEMS a-e	10	625	-2.11	.9823	P	0.96	.7043	P	.96	P	P
Chesapeake CEMS a-e	11	629	-1.24	.8931	P	0.89	.9256	P	.89	P	P
Chesapeake CEMS a-e	12	625	-2.26	.9878	P	0.97	.6452	P	.95	P	P
Summary of CEMS number of tests passed total number of tests=41					27			27		21	20
Homer City #1	ALL	496	-4.05	.9999	P	1.72	.0000	F	.62	F	F
Homer City #1	1	30	-12.08	.9999	P	2.20	.0188	F	.76	F	F
Homer City #1	2	26	1.99	.0284	P	2.35	.0185	F	.30	F	F
Homer City #1	3	26	-3.99	.9997	P	0.76	.7462	P	.18	F	F
Homer City #1	4	29	-1.89	.9655	P	1.67	.0900	P	.76	F	F

Table 6: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS a-d	ALL	1329	-0.88	.8100	P	1.04	.2183	P	.88	P	P
Chesapeake CEMS a-d	1	613	-0.76	.7777	P	1.10	.1146	P	.75	F	F
Chesapeake CEMS a-d	2	629	0.36	.6387	P	0.91	.8356	P	.95	P	P
Chesapeake CEMS a-d	3	613	-0.76	.7765	P	1.11	.1057	P	.74	F	F
Chesapeake CEMS a-d	4	629	-0.49	.6889	P	0.91	.8740	P	.95	P	P
Chesapeake CEMS a-d	5	613	-0.74	.7721	P	1.10	.1199	P	.75	F	F
Chesapeake CEMS a-d	6	629	-0.69	.7547	P	0.92	.8577	P	.95	P	P
Chesapeake CEMS a-d	7	613	-0.76	.7752	P	1.10	.1143	P	.74	F	F
Chesapeake CEMS a-d	8	629	-0.69	.7591	P	0.91	.8838	P	.95	P	P
Chesapeake CEMS a-d	9	613	-0.70	.8631	P	1.10	.1236	P	.74	F	F
Chesapeake CEMS a-d	10	629	-1.09	.7515	P	0.92	.8418	P	.95	P	P
Chesapeake CEMS a-d	11	613	-0.67	.8991	P	1.10	.1465	P	.74	F	F
Chesapeake CEMS a-d	12	629	-1.28	.8182	P	0.94	.7883	P	.95	P	P
Chesapeake CEMS a-e	ALL	1337	-2.25	.9878	P	0.95	.8301	P	.94	P	P
Chesapeake CEMS a-e	1	629	-0.91	.8182	P	0.88	.9420	P	.88	P	P
Chesapeake CEMS a-e	2	625	-2.59	.9951	P	0.99	.5438	P	.96	P	P

Table 6: Continued

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/Fail	Computed value	p-value	Pass/Fail	Computed value	Pass/Fail	Overall Pass/Fail
Northern States	7	720	-13.28	.9999	P	2.25	.0000	F	.71	F	F
Northern States	8	695	-8.34	.9999	P	1.41	.0000	F	.37	F	F
Northern States	9	720	-14.96	.9999	P	2.19	.0000	F	.18	F	F
Northern States	10	720	-5.58	.9999	P	1.22	.0049	F	.30	F	F
Northern States	11	720	-10.62	.9999	P	0.62	.9999	P	.06	F	F
Northern States	12	720	-14.25	.9999	P	0.41	.9999	P	.20	F	F
Northern States	13	697	-12.61	.9999	P	1.22	.0041	F	.33	F	F
Northern States	14	720	-7.85	.9999	P	0.55	.9999	P	.20	F	F
Northern States	15	703	-26.84	.9999	P	1.29	.0005	F	.39	F	F
Northern States	16	720	-11.12	.9999	P	1.92	.0000	F	.32	F	F
Northern States	17	720	-3.62	.9998	P	1.19	.0100	F	.60	F	F
Summary of CSA number of tests passed total number of tests = 32					26			16		5	1
Niagara Mohawk	All	6801	-44.21	.9999	P	0.30	.9999	P	.30	F	F
Niagara Mohawk	1	686	-16.24	.9999	P	0.40	.9999	P	.42	F	F
Niagara Mohawk	2	668	-35.74	.9999	P	0.43	.9999	P	.41	F	F
Summary of OSA number of tests passed total number of tests = 3					3			3		0	0

Footnotes to Table 6:

[1] F-test inflation factor: $\dfrac{1}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^{2}}}$; Bias test inflation factor: $\sqrt{\dfrac{\dfrac{(1+\rho)}{(1-\rho)}-\dfrac{2\rho(1-\rho^{n})}{n(1-\rho)^{2}}}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^{2}}}}$

where ρ is computed from the first order autoregression of the CEMS (or AM) on its Lag, and n id the number of observations in the data stream.

[2]
- o UARG data from Attachment E (Table 1, page 4) in Correspondence IV-D-185 to EPA Docket No. A-90-51. There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.
- o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). One reference CEM (A) and four alternative CEMs (B-E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.
- o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
- o Homer City Unit #1 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a 730 day period.
- o Homer City Unit #3 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.
- o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days.

[3] Data were subset into smaller datasets of 30 days each. ALL=entire data set used; N/A=less than 90% data available; all other values are set identifiers.

Table 7: Results on Bias Test, F-test, and Correlation Test Analyzed at Refinement of Alternative Monitoring System Without Inflation of Variances. (CEMS measurement have been averaged to correspond to the refinement of the alternative method.)

			1-tailed Bias Test $\alpha < .025$			1-tailed F-test $\alpha < .05$			Correlation $\rho > .80$		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Northern States	ALL	664	-34.74	.9999	P	1.47	.0000	F	.44	F	F
Northern States	1	29	-9.31	.9999	P	1.67	.0892	P	.77	F	F
Northern States	2	27	-8.00	.9999	P	3.49	.0011	F	.41	F	F
Northern States	3	30	-9.77	.9999	P	4.05	.0002	F	.29	F	F
Northern States	4	30	-10.76	.9999	P	1.93	.0411	F	.79	F	F
Northern States	5	28	-14.63	.9999	P	16.95	.0000	F	.10	F	F
Northern States	6	30	-9.31	.9999	P	0.78	.7488	P	-.15	F	F
Northern States	7	30	-11.98	.9999	P	3.03	.0019	F	.89	P	F
Northern States	8	28	-9.84	.9999	P	2.32	.0164	F	.11	F	F
Northern States	9	30	-10.41	.9999	P	5.47	.0000	F	.30	F	F
Northern States	10	30	-5.86	.9999	P	2.44	.0097	F	.45	F	F
Northern States	11	30	-8.21	.9999	P	4.64	.0000	F	.16	F	F
Northern States	12	30	-8.82	.9999	P	1.13	.3720	P	.35	F	F
Northern States	13	30	-8.54	.9999	P	2.46	.0089	F	.51	F	F
Northern States	14	30	-6.79	.9999	P	1.77	.0656	P	.37	F	F
Northern States	15	30	-14.21	.9999	P	3.50	.0006	F	.61	F	F
Northern States	16	30	-12.05	.9999	P	3.32	.0009	F	.44	F	F

Table 7: Continued

			1-tailed Bias Test $\alpha < .025$			1-tailed F-test $\alpha < .05$			Correlation $\rho > .80$		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Northern States	17	30	-3.78	.9996	P	1.43	.1665	P	.68	F	F
Summary of CSA number of tests passed total number of tests = 18					18			5		1	0
Niagara Mohawk	ALL	62	-17.95	.9999	P	0.50	.9956	P	.33	F	F
Niagara Mohawk	1	4	-10.07	.9999	P	0.96	.5089	P	.99	P	P
Niagara Mohawk	2	5	-12.99	.9999	P	1.09	.4667	P	.85	P	P
Summary of OSA Number of tests passed Total number of tests = 3					3			3		2	2

[1] o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days. The average CEM value for each day was used in all calculations.
o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days. The average CEM value for each week was used in all calculations.

[2] Data were subset into smaller datasets of 30 days each. ALL = entire dataset used; N/A = less than 90% data available; *all other values are set identifiers.*

Table 8: Results on Bias Test, F-test, and Correlation Test Analyzed at Refinement of Alternative Monitoring System Using Inflated Variances Estimate[1]. (CEMS measurement have been averaged to correspond to the refinement of the alternative method.)

			1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Northern States	ALL	664	-22.15	.9999	P	1.45	.0000	P	.44	F	F
Northern States	1	29	-9.05	.9999	P	1.88	.0496	F	.77	F	F
Northern States	2	27	-5.89	.9999	P	4.21	.0002	F	.41	F	F
Northern States	3	30	-9.77	.9999	P	4.46	.0000	F	.29	F	F
Northern States	4	30	-10.31	.9999	P	2.78	.0037	F	.79	F	F
Northern States	5	28	-6.86	.9999	P	19.92	.0000	F	.10	F	F
Northern States	6	30	-5.25	.9999	P	0.97	.5305	P	-.15	F	F
Northern States	7	30	-8.88	.9999	P	4.54	.0000	F	.89	P	F
Northern States	8	28	-8.33	.9999	P	2.97	.0003	F	.11	F	F
Northern States	9	30	-8.79	.9999	P	5.81	.0000	F	.30	F	F
Northern States	10	30	-5.18	.9999	P	2.98	.0022	F	.45	F	F
Northern States	11	30	-7.76	.9999	P	4.72	.0000	F	.16	F	F
Northern States	12	30	-8.08	.9999	P	1.21	.3040	P	.35	F	F
Northern States	13	30	-7.60	.9999	P	2.58	.0063	F	.51	F	F
Northern States	14	30	-6.61	.9999	P	1.89	.0466	F	.37	F	F
Northern States	15	30	-17.76	.9999	P	3.52	.0006	F	.61	F	F
Northern States	16	30	-10.62	.9999	P	3.54	.0005	F	.44	F	F

Table 8: Continued

Data Source[2]	Data Subset[3]	N	1-tailed Bias Test $\alpha<.025$			1-tailed F-test $\alpha<.05$			Correlation $\rho>.80$		
			Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Northern States	17	30	-2.89	.9965	P	2.01	.0324	F	.68	F	F
Summary of CSA number of tests passed total number of tests = 18					18			3		1	0
Niagara Mohawk	ALL	62	-13.34	.9999	P	0.52	.9947	P	.33	F	F
Niagara Mohawk	1	4	-7.04	.9971	P	0.59	.6287	P	.99	P	P
Niagara Mohawk	2	5	-42.89	.9999	P	4.47	.0879	P	.85	P	P
Summary of OSA Number of tests passed Total number of tests = 3					3			3		2	2

[1] F-test inflation factor: $\dfrac{1}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^2}}$; Bias test inflation factor: $\sqrt{\dfrac{\dfrac{(1+\rho)}{(1-\rho)}-\dfrac{2\rho(1-\rho^{n})}{n(1-\rho)^2}}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^2}}}$

where ρ is computed from the first order autoregression of the CEMS (or AM) on its Lag, and n id the number of observations in the data stream.

[2] o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days. The average CEM value for each day was used in all calculations.
o Niagara Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days. The average CEM value for each week was used in all calculations.

[3] Data were subset into smaller datasets of 30 days each. ALL = entire dataset used; N/A = less than 90% data available; all other values are set identifiers.

"official" CEMS, and then with the other monitor designated as the "official" CEMS.

Table 9 indicates that without using variance inflation factors, 114 subsets pass the bias test, 117 pass the F-test, 68 pass the correlation test, and 53 pass all three tests. As indicated in Table 10, using variance inflation factors results in 119 subsets passing the bias test, 116 passing the F-test, 68 passing the correlation test, and 54 passing all three tests.

Table 9: Results on Bias Test, F-Test, and Correlation Test Using All Possible Combinations of CEMS at Chesapeake Unit (Without Inflation of Variances).

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation p>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
UARG Attachment E	ALL	19	2.46	.0121	F	1.69	.1369	P	.83	P	F
UARG Attachment E	ALL	19	-2.46	.9879	P	0.59	.8631	P	.83	P	P
Chesapeake CEMS a-b	ALL	1167	4.44	.0000	F	1.30	.0000	F	.75	F	F
Chesapeake CEMS a-b	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS b-a	ALL	1167	-4.44	.9999	P	0.77	.9999	P	.75	F	F
Chesapeake CEMS b-a	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS a-c	ALL	1383	14.98	.0000	F	2.45	.0000	F	.68	F	F
Chesapeake CEMS a-c	1	648	9.80	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	2	648	11.36	.0000	F	4.43	.0000	F	.44	F	F
Chesapeake CEMS a-c	3	648	9.81	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	4	648	11.43	.0000	F	4.52	.0000	F	.44	F	F
Chesapeake CEMS a-c	5	648	9.85	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	6	648	11.43	.0000	F	4.61	.0000	F	.43	F	F
Chesapeake CEMS a-c	7	648	9.88	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	8	648	11.42	.0000	F	4.55	.0000	F	.43	F	F
Chesapeake CEMS a-c	9	648	9.96	.0000	F	1.56	.0000	F	.76	F	F
Chesapeake CEMS a-c	10	648	11.40	.0000	F	4.54	.0000	F	.43	F	F

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS a-c	11	648	9.99	.0000	F	1.55	.0000	F	.76	F	F
Chesapeake CEMS a-c	12	648	11.39	.0000	F	4.66	.0000	F	.41	F	F
Chesapeake CEMS c-a	ALL	1383	-14.98	.9999	P	0.41	.9999	P	.68	F	F
Chesapeake CEMS c-a	1	648	-9.80	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	2	648	-11.36	.9999	P	0.23	.9999	P	.44	F	F
Chesapeake CEMS c-a	3	648	-9.81	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	4	648	-11.43	.9999	P	0.22	.9999	P	.44	F	F
Chesapeake CEMS c-a	5	648	-9.85	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	6	648	-11.43	.9999	P	0.22	.9999	P	.43	F	F
Chesapeake CEMS c-a	7	648	-9.88	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	8	648	-11.42	.9999	P	0.22	.9999	P	.43	F	F
Chesapeake CEMS c-a	9	648	-9.96	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	10	648	-11.40	.9999	P	0.22	.9999	P	.43	F	F
Chesapeake CEMS c-a	11	648	-9.99	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	12	648	-11.39	.9999	P	0.21	.9999	P	.41	F	F
Chesapeake CEMS a-d	ALL	1329	-1.96	.9749	P	1.01	.4343	P	.88	P	P
Chesapeake CEMS a-d	1	613	-1.93	.9732	P	1.05	.2713	P	.75	F	F
Chesapeake CEMS a-d	2	629	-0.46	.6772	P	0.91	.8650	P	.95	P	P
Chesapeake CEMS a-d	3	613	-1.91	.9720	P	1.05	.2648	P	.74	F	F

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS a-d	4	629	-0.64	.7407	P	0.91	.8606	P	.95	P	P
Chesapeake CEMS a-d	5	613	-1.87	.9692	P	1.05	.2755	P	.75	F	F
Chesapeake CEMS a-d	6	629	-0.90	.8178	P	0.92	.8461	P	.95	P	P
Chesapeake CEMS a-d	7	613	-1.89	.9704	P	1.05	.2655	P	.74	F	F
Chesapeake CEMS a-d	8	629	-0.92	.8202	P	0.91	.8712	P	.95	P	P
Chesapeake CEMS a-d	9	613	-1.75	.9605	P	1.05	.2764	P	.74	F	F
Chesapeake CEMS a-d	10	629	-1.42	.9218	P	0.93	.8279	P	.95	P	P
Chesapeake CEMS a-d	11	613	-1.70	.9553	P	1.04	.3048	P	.74	F	F
Chesapeake CEMS a-d	12	629	-1.63	.9482	P	0.94	.7763	P	.95	P	P
Chesapeake CEMS d-a	ALL	1329	1.96	.0251	P	0.99	.5657	P	.88	P	P
Chesapeake CEMS d-a	1	613	1.93	.0268	P	0.95	.7287	P	.75	F	F
Chesapeake CEMS d-a	2	629	0.46	.3228	P	1.09	.1351	P	.95	P	P
Chesapeake CEMS d-a	3	613	1.91	.0280	P	0.95	.7352	P	.74	F	F
Chesapeake CEMS d-a	4	629	0.64	.2593	P	1.09	.1394	P	.95	P	P
Chesapeake CEMS d-a	5	613	1.87	.0308	P	0.95	.7245	P	.75	F	F
Chesapeake CEMS d-a	6	629	0.90	.1822	P	1.08	.1539	P	.95	P	P
Chesapeake CEMS d-a	7	613	1.89	.0296	P	0.95	.7345	P	.74	F	F
Chesapeake CEMS d-a	8	629	0.92	.1798	P	1.09	.1288	P	.95	P	P
Chesapeake CEMS d-a	9	613	1.75	.0395	P	0.95	.7236	P	.74	F	F
Chesapeake CEMS d-a	10	629	1.42	.0782	P	1.08	.1721	P	.95	P	P

Table 9: Continued

Data Source[1]	Data Subset[2]	N	1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		Overall Pass/ Fail
			Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	
Chesapeake CEMS d-a	11	613	1.70	.0447	P	0.96	.6952	P	.74	F	F
Chesapeake CEMS d-a	12	629	1.63	.0518	P	1.06	.2237	P	.95	P	P
Chesapeake CEMS a-e	ALL	1337	-2.51	.9940	P	0.96	.7843	P	.94	P	P
Chesapeake CEMS a-e	1	629	-1.01	.8429	P	0.89	.9091	P	.88	P	P
Chesapeake CEMS a-e	2	625	-2.91	.9982	P	1.02	.4058	P	.96	P	P
Chesapeake CEMS a-e	3	629	-1.19	.8838	P	0.90	.9063	P	.89	P	P
Chesapeake CEMS a-e	4	625	-2.68	.9962	P	1.01	.4316	P	.96	P	P
Chesapeake CEMS a-e	5	629	-1.42	.9224	P	0.91	.8776	P	.89	P	P
Chesapeake CEMS a-e	6	625	-2.29	.9890	P	0.99	.5339	P	.96	P	P
Chesapeake CEMS a-e	7	629	-1.55	.9399	P	0.91	.8765	P	.89	P	P
Chesapeake CEMS a-e	8	625	-2.23	.9870	P	0.98	.5706	P	.96	P	P
Chesapeake CEMS a-e	9	629	-1.45	.9269	P	0.91	.8806	P	.89	P	P
Chesapeake CEMS a-e	10	625	-2.28	.9887	P	0.98	.5552	P	.96	P	P
Chesapeake CEMS a-e	11	629	-1.39	.9178	P	0.91	.8895	P	.89	P	P
Chesapeake CEMS a-e	12	625	-2.43	.9923	P	1.00	.4958	P	.95	P	P
Chesapeake CEMS e-a	ALL	1337	2.51	.0060	F	1.04	.2157	P	.94	P	F
Chesapeake CEMS e-a	1	629	1.01	.1571	P	1.11	.0909	P	.88	P	P
Chesapeake CEMS e-a	2	625	2.91	.0018	F	0.98	.5941	P	.96	P	F
Chesapeake CEMS e-a	3	629	1.19	.1162	P	1.11	.0937	P	.89	P	P

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS e-a	4	625	2.68	.0038	F	0.99	.5684	P	.96	P	F
Chesapeake CEMS e-a	5	629	1.42	.0776	P	1.10	.1224	P	.89	P	P
Chesapeake CEMS e-a	6	625	2.29	.0110	F	1.01	.4661	P	.96	P	F
Chesapeake CEMS e-a	7	629	1.55	.0601	P	1.10	.1235	P	.89	P	P
Chesapeake CEMS e-a	8	625	2.23	.0130	F	1.01	.4294	P	.96	P	F
Chesapeake CEMS e-a	9	629	1.45	.0731	P	1.10	.1194	P	.89	P	P
Chesapeake CEMS e-a	10	625	2.28	.0113	F	1.01	.4448	P	.96	P	F
Chesapeake CEMS e-a	11	629	1.39	.0822	P	1.10	.1105	P	.89	P	P
Chesapeake CEMS e-a	12	625	2.43	.0077	F	1.00	.5042	P	.95	P	F
Chesapeake CEMS b-c	ALL	1072	8.55	.0000	F	1.90	.0000	F	.53	F	F
Chesapeake CEMS b-c	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS c-b	ALL	1072	-8.55	.9999	P	0.53	.9999	P	.53	F	F
Chesapeake CEMS c-b	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS b-d	ALL	993	-5.09	.9999	P	0.71	.9999	P	.64	F	F
Chesapeake CEMS b-d	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS d-b	ALL	993	5.09	.0000	F	1.41	.0000	F	.64	F	F
Chesapeake CEMS d-b	N/A	< 648	---	---	-	---	---	-	---	-	-

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation p>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS b-e	ALL	1041	-4.54	.9999	P	0.70	.9999	P	.67	F	F
Chesapeake CEMS b-e	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS e-b	ALL	1041	4.54	.0000	F	1.44	.0000	F	.67	F	F
Chesapeake CEMS e-b	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS c-d	ALL	1368	-14.45	.9999	P	.35	.9999	P	.50	F	F
Chesapeake CEMS c-d	1	625	-8.93	.9999	P	.55	.9999	P	.53	F	F
Chesapeake CEMS c-d	2	638	-11.04	.9999	P	.20	.9999	P	.39	F	F
Chesapeake CEMS c-d	3	626	-8.93	.9999	P	.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	4	638	-11.13	.9999	P	.20	.9999	P	.39	F	F
Chesapeake CEMS c-d	5	626	-8.93	.9999	P	.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	6	638	-11.17	.9999	P	.19	.9999	P	.37	F	F
Chesapeake CEMS c-d	7	626	-8.95	.9999	P	.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	8	638	-11.17	.9999	P	.19	.9999	P	.37	F	F
Chesapeake CEMS c-d	9	626	-8.93	.9999	P	.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	10	638	-11.23	.9999	P	.20	.9999	P	.37	F	F
Chesapeake CEMS c-d	11	626	-8.92	.9999	P	.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	12	638	-11.26	.9999	P	.19	.9999	P	.35	F	F
Chesapeake CEMS d-c	ALL	1368	14.45	.0000	F	2.84	.0000	F	.50	F	F
Chesapeake CEMS d-c	1	625	8.93	.0000	F	1.83	.0000	F	.53	F	F

Table 9: Continued

Data Source[1]	Data Subset[2]	N	1-tailed Bias Test α=.025 Computed value	p-value	Pass/ Fail	1-tailed F-test α=.05 Computed value	p-value	Pass/ Fail	Correlation ρ>.80 Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS d-c	2	638	11.04	.0000	F	5.04	.0000	F	.39	F	F
Chesapeake CEMS d-c	3	626	8.93	.0000	F	1.83	.0000	F	.52	F	F
Chesapeake CEMS d-c	4	638	11.13	.0000	F	5.13	.0000	F	.39	F	F
Chesapeake CEMS d-c	5	626	8.93	.0000	F	1.82	.0000	F	.52	F	F
Chesapeake CEMS d-c	6	638	11.17	.0000	F	5.20	.0000	F	.37	F	F
Chesapeake CEMS d-c	7	626	8.95	.0000	F	1.82	.0000	F	.52	F	F
Chesapeake CEMS d-c	8	638	11.17	.0000	F	5.18	.0000	F	.37	F	F
Chesapeake CEMS d-c	9	626	8.93	.0000	F	1.82	.0000	F	.52	F	F
Chesapeake CEMS d-c	10	638	11.23	.0000	F	5.09	.0000	F	.37	F	F
Chesapeake CEMS d-c	11	626	8.92	.0000	F	1.82	.0000	F	.52	F	F
Chesapeake CEMS d-c	12	638	11.26	.0000	F	5.17	.0000	F	.35	F	F
Chesapeake CEMS c-e	ALL	1502	-15.36	.9999	P	0.36	.9999	P	.57	F	F
Chesapeake CEMS c-e	1	698	-11.37	.9999	P	0.72	.9999	P	.80	P	P
Chesapeake CEMS c-e	2	694	-12.18	.9999	P	0.23	.9999	P	.42	F	F
Chesapeake CEMS c-e	3	697	-11.50	.9999	P	0.72	.9999	P	.80	P	P
Chesapeake CEMS c-e	4	694	-12.21	.9999	P	0.22	.9999	P	.42	F	F
Chesapeake CEMS c-e	5	697	-11.67	.9999	P	0.73	.9999	P	.80	P	P
Chesapeake CEMS c-e	6	694	-12.14	.9999	P	0.22	.9999	P	.41	F	F
Chesapeake CEMS c-e	7	697	-11.78	.9999	P	0.73	.9999	P	.80	P	P

Table 9: Continued

Data Source[1]	Data Subset[2]	N	1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		Overall Pass/ Fail
			Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	
Chesapeake CEMS c-e	8	694	-12.13	.9999	P	0.22	.9999	P	.41	F	F
Chesapeake CEMS c-e	9	697	-11.80	.9999	P	0.74	.9999	P	.80	P	P
Chesapeake CEMS c-e	10	694	-12.12	.9999	P	0.22	.9999	P	.40	F	F
Chesapeake CEMS c-e	11	697	-11.80	.9999	P	0.74	.9999	P	.80	P	P
Chesapeake CEMS c-e	12	694	-12.14	.9999	P	0.22	.9999	P	.38	F	F
Chesapeake CEMS e-c	ALL	1502	15.36	.0000	F	2.76	.0000	F	.57	F	F
Chesapeake CEMS e-c	1	698	11.37	.0000	F	1.38	.0000	F	.80	P	F
Chesapeake CEMS e-c	2	694	12.18	.0000	F	4.36	.0000	F	.42	F	F
Chesapeake CEMS e-c	3	697	11.50	.0000	F	1.38	.0000	F	.80	P	F
Chesapeake CEMS e-c	4	694	12.21	.0000	F	4.45	.0000	F	.42	F	F
Chesapeake CEMS e-c	5	697	11.67	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	6	694	12.14	.0000	F	4.60	.0000	F	.41	F	F
Chesapeake CEMS e-c	7	697	11.78	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	8	694	12.13	.0000	F	4.57	.0000	F	.41	F	F
Chesapeake CEMS e-c	9	697	11.80	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	10	694	12.12	.0000	F	4.54	.0000	F	.40	F	F
Chesapeake CEMS e-c	11	697	11.80	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	12	694	12.14	.0000	F	4.62	.0000	F	.38	F	F
Chesapeake CEMS d-e	ALL	1317	0.31	.3795	P	0.94	0.8630	P	.87	P	P

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS d-e	1	602	1.69	.0460	P	0.83	.9882	P	.73	F	F
Chesapeake CEMS d-e	2	617	-2.01	.9776	P	1.13	.0686	P	.91	P	P
Chesapeake CEMS d-e	3	602	1.54	.0618	P	0.83	.9882	P	.72	F	F
Chesapeake CEMS d-e	4	617	-1.72	.9574	P	1.12	.0800	P	.91	P	P
Chesapeake CEMS d-e	5	602	1.34	.0898	P	0.85	.9801	P	.72	F	F
Chesapeake CEMS d-e	6	617	-1.26	.8965	P	1.09	.1356	P	.91	P	P
Chesapeake CEMS d-e	7	602	1.28	.1013	P	0.84	.9813	P	.72	F	F
Chesapeake CEMS d-e	8	617	-1.26	.8958	P	1.09	.1319	P	.91	P	P
Chesapeake CEMS d-e	9	602	1.22	.1119	P	0.84	.9809	P	.72	F	F
Chesapeake CEMS d-e	10	617	-0.94	.8258	P	1.08	.1631	P	.91	P	P
Chesapeake CEMS d-e	11	602	1.20	.1149	P	0.85	.9791	P	.72	F	F
Chesapeake CEMS d-e	12	617	-0.89	.8138	P	1.08	.1680	P	.91	P	P
Chesapeake CEMS e-d	ALL	1317	-0.31	.6205	P	1.06	.1370	P	.87	P	P
Chesapeake CEMS e-d	1	602	-1.69	.9540	P	1.20	.0118	F	.73	F	F
Chesapeake CEMS e-d	2	617	2.01	.0224	F	0.89	.9314	P	.91	P	F
Chesapeake CEMS e-d	3	602	-1.54	.9382	P	1.20	.0118	F	.72	F	F
Chesapeake CEMS e-d	4	617	1.72	.0426	P	0.89	.9200	P	.91	P	P
Chesapeake CEMS e-d	5	602	-1.34	.9102	P	1.18	.0199	F	.72	F	F
Chesapeake CEMS e-d	6	617	1.26	.1035	P	0.92	.8644	P	.91	P	P
Chesapeake CEMS e-d	7	602	-1.28	.8987	P	1.19	.0187	F	.72	F	F

Table 9: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[1]	Data Subset[2]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS e-d	8	617	1.26	.1042	P	0.91	.8681	P	.91	P	P
Chesapeake CEMS e-d	9	602	-1.22	.8881	P	1.18	.0191	F	.72	F	F
Chesapeake CEMS e-d	10	617	0.94	.1742	P	0.92	.8369	P	.91	P	P
Chesapeake CEMS e-d	11	602	-1.20	.8851	P	1.18	.0209	F	.72	F	F
Chesapeake CEMS e-d	12	617	0.89	.1862	P	0.93	.8320	P	.91	P	P
Summary of CEMS number of tests passed Total number of tests=166					114			117		68	53

Footnotes to Table 9:

[1]
- o UARG data from Attachment E of Public Comments (Table 1, page 4) in Correspondence IV-D-185 to EPA Docket No. A-90-51. There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.
- o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). Five CEMS (A through E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.

[2] Data were subset into smaller datasets of 30 days each. ALL=entire dataset used; N/A=less than 90% data available; all other values are set identifiers.

Table 10: Results on Bias Test, F-Test, and Correlation Test Using All Possible Combinations of CEMS at Chesapeake Unit (With Inflated Variances Estimates[1]).

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
UARG Attachment E	ALL	19	2.17	.0218	F	1.65	.1493	P	.83	P	F
UARG Attachment E	ALL	19	-2.17	.9781	P	0.61	.8507	P	.83	P	P
Chesapeake CEMS a-b	ALL	1167	1.34	.0899	P	1.31	.0000	F	.75	F	F
Chesapeake CEMS a-b	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS b-a	ALL	1167	-1.34	.9101	P	0.76	.9999	P	.75	F	F
Chesapeake CEMS b-a	N/A	< 648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS a-c	ALL	1383	5.58	.0000	F	2.42	.0000	F	.68	F	F
Chesapeake CEMS a-c	1	648	6.39	.0000	F	1.59	.0000	F	.76	F	F
Chesapeake CEMS a-c	2	648	2.93	.0018	F	5.39	.0000	F	.44	F	F
Chesapeake CEMS a-c	3	648	6.39	.0000	F	1.58	.0000	F	.76	F	F
Chesapeake CEMS a-c	4	648	3.06	.0012	F	5.53	.0000	F	.44	F	F
Chesapeake CEMS a-c	5	648	6.42	.0000	F	1.59	.0000	F	.76	F	F
Chesapeake CEMS a-c	6	648	3.17	.0008	F	5.66	.0000	F	.43	F	F
Chesapeake CEMS a-c	7	648	6.44	.0000	F	1.59	.0000	F	.76	F	F
Chesapeake CEMS a-c	8	648	3.16	.0008	F	5.63	.0000	F	.43	F	F
Chesapeake CEMS a-c	9	648	6.49	.0000	F	1.59	.0000	F	.76	F	F
Chesapeake CEMS a-c	10	648	3.16	.0008	F	5.62	.0000	F	.43	F	F

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS a-c	11	648	6.51	.0000	F	1.57	.0000	F	.76	F	F
Chesapeake CEMS a-c	12	648	3.15	.0008	F	5.79	.0000	F	.41	F	F
Chesapeake CEMS c-a	ALL	1383	-5.58	.9999	P	0.41	.9999	P	.68	F	F
Chesapeake CEMS c-a	1	648	-6.39	.9999	P	0.63	.9999	P	.76	F	F
Chesapeake CEMS c-a	2	648	-2.93	.9982	P	0.19	.9999	P	.44	F	F
Chesapeake CEMS c-a	3	648	-6.39	.9999	P	0.63	.9999	P	.76	F	F
Chesapeake CEMS c-a	4	648	-3.06	.9988	P	0.18	.9999	P	.44	F	F
Chesapeake CEMS c-a	5	648	-6.42	.9999	P	0.63	.9999	P	.76	F	F
Chesapeake CEMS c-a	6	648	-3.17	.9992	P	0.18	.9999	P	.43	F	F
Chesapeake CEMS c-a	7	648	-6.44	.9999	P	0.63	.9999	P	.76	F	F
Chesapeake CEMS c-a	8	648	-3.16	.9992	P	0.18	.9999	P	.43	F	F
Chesapeake CEMS c-a	9	648	-6.49	.9999	P	0.63	.9999	P	.76	F	F
Chesapeake CEMS c-a	10	648	-3.16	.9992	P	0.18	.9999	P	.43	F	F
Chesapeake CEMS c-a	11	648	-6.49	.9999	P	0.64	.9999	P	.76	F	F
Chesapeake CEMS c-a	12	648	-3.15	.9992	P	0.17	.9999	P	.41	F	F
Chesapeake CEMS a-d	ALL	1329	-0.90	.8163	P	1.04	.2183	P	.88	P	P
Chesapeake CEMS a-d	1	613	-0.76	.7777	P	1.10	.1146	P	.75	F	F
Chesapeake CEMS a-d	2	629	-0.36	.6387	P	0.91	.8797	P	.95	P	P
Chesapeake CEMS a-d	3	613	-0.76	.7765	P	1.05	.2656	P	.74	F	F

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS a-d	4	629	-0.49	.6889	P	0.91	.8740	P	.95	P	P
Chesapeake CEMS a-d	5	613	-0.74	.7721	P	1.05	.2766	P	.75	F	F
Chesapeake CEMS a-d	6	629	-0.69	.7547	P	0.92	.8577	P	.95	P	P
Chesapeake CEMS a-d	7	613	-0.76	.7752	P	1.05	.2663	P	.74	F	F
Chesapeake CEMS a-d	8	629	-0.69	.7567	P	0.91	.8838	P	.95	P	P
Chesapeake CEMS a-d	9	613	-0.70	.7591	P	1.05	.2774	P	.74	F	F
Chesapeake CEMS a-d	10	629	-1.09	.8631	P	0.92	.8418	P	.95	P	P
Chesapeake CEMS a-d	11	613	-0.67	.7515	P	1.04	.3065	P	.74	F	F
Chesapeake CEMS a-d	12	629	-1.28	.8991	P	0.94	.7883	P	.95	P	P
Chesapeake CEMS d-a	ALL	1329	0.90	.1837	P	0.96	.7817	P	.89	P	P
Chesapeake CEMS d-a	1	613	0.76	.2223	P	0.95	.7277	P	.75	F	F
Chesapeake CEMS d-a	2	629	0.36	.3613	P	1.10	.1203	P	.95	P	P
Chesapeake CEMS d-a	3	613	0.76	.2235	P	0.95	.7344	P	.74	F	F
Chesapeake CEMS d-a	4	629	0.49	.3111	P	1.10	.1239	P	.95	P	P
Chesapeake CEMS d-a	5	613	0.75	.2279	P	0.95	.7234	P	.75	F	F
Chesapeake CEMS d-a	6	629	0.69	.2453	P	1.09	.1388	P	.95	P	P
Chesapeake CEMS d-a	7	613	0.76	.2248	P	0.95	.7337	P	.74	F	F
Chesapeake CEMS d-a	8	629	0.70	.2433	P	1.10	.1132	P	.95	P	P
Chesapeake CEMS d-a	9	613	0.70	.2409	P	0.95	.7226	P	.74	F	F
Chesapeake CEMS d-a	10	629	1.10	.1369	P	1.08	.1550	P	.95	P	P

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS d-a	11	613	0.68	.2485	P	0.96	.6935	P	.74	F	F
Chesapeake CEMS d-a	12	629	1.28	.1009	P	1.07	.2083	P	.95	P	P
Chesapeake CEMS a-e	ALL	1337	-2.36	.9909	P	0.95	.8301	P	.94	P	P
Chesapeake CEMS a-e	1	629	-0.91	.8182	P	0.90	.9171	P	.88	P	P
Chesapeake CEMS a-e	2	625	-2.59	.9951	P	1.02	.4003	P	.96	P	P
Chesapeake CEMS a-e	3	629	-1.08	.8592	P	0.90	.9144	P	.89	P	P
Chesapeake CEMS a-e	4	625	-2.39	.9916	P	1.01	.4270	P	.96	P	P
Chesapeake CEMS a-e	5	629	-1.27	.8988	P	0.91	.8867	P	.89	P	P
Chesapeake CEMS a-e	6	625	-2.09	.9819	P	0.99	.5356	P	.96	P	P
Chesapeake CEMS a-e	7	629	-1.39	.9179	P	0.91	.8855	P	.89	P	P
Chesapeake CEMS a-e	8	625	-2.06	.9803	P	0.99	.5746	P	.96	P	P
Chesapeake CEMS a-e	9	629	-1.30	.9034	P	0.91	.8894	P	.89	P	P
Chesapeake CEMS a-e	10	625	-2.11	.9823	P	0.99	.5597	P	.96	P	P
Chesapeake CEMS a-e	11	629	-1.24	.8931	P	0.90	.8980	P	.89	P	P
Chesapeake CEMS a-e	12	625	-2.26	.9878	P	1.00	.4966	P	.95	P	P
Chesapeake CEMS e-a	ALL	1337	2.36	.0091	P	0.95	.8301	P	.94	P	P
Chesapeake CEMS e-a	1	629	0.91	.1818	P	1.12	.0829	P	.88	P	P
Chesapeake CEMS e-a	2	625	2.59	.0049	F	0.98	.5997	P	.96	P	F
Chesapeake CEMS e-a	3	629	1.08	.1408	P	1.12	.0856	P	.89	P	P

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS e-a	4	625	2.39	.0084	F	0.99	.5730	P	.96	P	F
Chesapeake CEMS e-a	5	629	1.27	.1012	P	1.10	.1133	P	.89	P	P
Chesapeake CEMS e-a	6	625	2.09	.0181	F	1.01	.4644	P	.96	P	F
Chesapeake CEMS e-a	7	629	1.39	.0821	P	1.10	.1145	P	.89	P	P
Chesapeake CEMS e-a	8	625	2.06	.0197	F	1.02	.4254	P	.96	P	F
Chesapeake CEMS e-a	9	629	1.30	.0966	P	1.10	.1106	P	.89	P	P
Chesapeake CEMS e-a	10	625	2.11	.0177	F	1.01	.4403	P	.96	P	F
Chesapeake CEMS e-a	11	629	1.24	.1069	P	1.11	.1020	P	.89	P	P
Chesapeake CEMS e-a	12	625	2.26	.0122	F	1.00	.5034	P	.95	P	F
Chesapeake CEMS b-c	ALL	1072	2.97	.0015	F	1.86	.0000	F	.53	F	F
Chesapeake CEMS b-c	N/A	<648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS c-b	ALL	1072	-2.97	.9985	P	0.54	.9999	P	.53	F	F
Chesapeake CEMS c-b	N/A	<648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS b-d	ALL	993	-1.03	.8481	P	0.73	.9999	P	.64	F	F
Chesapeake CEMS b-d	N/A	<648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS d-b	ALL	993	1.03	.1519	P	1.38	.0000	F	.64	F	F
Chesapeake CEMS d-b	N/A	<648	---	---	-	---	---	-	---	-	-

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS b-e	ALL	1041	-1.29	.9018	P	0.67	.9999	P	.67	F	F
Chesapeake CEMS b-e	N/A	<648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS e-b	ALL	1041	1.29	.0982	P	1.48	.0000	F	.67	F	F
Chesapeake CEMS e-b	N/A	<648	---	---	-	---	---	-	---	-	-
Chesapeake CEMS c-d	ALL	1368	-4.30	.9999	P	0.37	.9999	P	.50	F	F
Chesapeake CEMS c-d	1	625	-3.21	.9993	P	0.54	.9999	P	.53	F	F
Chesapeake CEMS c-d	2	638	-3.41	.9997	P	0.20	.9999	P	.39	F	F
Chesapeake CEMS c-d	3	625	-3.53	.9998	P	0.54	.9999	P	.53	F	F
Chesapeake CEMS c-d	4	638	-3.53	.9998	P	0.19	.9999	P	.39	F	F
Chesapeake CEMS c-d	5	626	-3.54	.9998	P	0.54	.9999	P	.52	F	F
Chesapeake CEMS c-d	6	638	-3.62	.9998	P	0.19	.9999	P	.37	F	F
Chesapeake CEMS c-d	7	626	-3.57	.9998	P	0.54	.9999	P	.52	F	F
Chesapeake CEMS c-d	8	638	-3.62	.9998	P	0.19	.9999	P	.37	F	F
Chesapeake CEMS c-d	9	626	-3.57	.9998	P	0.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	10	638	-3.64	.9999	P	0.19	.9999	P	.37	F	F
Chesapeake CEMS c-d	11	626	-3.56	.9998	P	0.55	.9999	P	.52	F	F
Chesapeake CEMS c-d	12	638	-3.65	.9999	P	0.19	.9999	P	.35	F	F
Chesapeake CEMS d-c	ALL	1368	4.30	.0000	F	2.71	.0000	F	.50	F	F
Chesapeake CEMS d-c	1	625	3.21	.0007	F	1.85	.0000	F	.53	F	F

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS d-c	2	638	3.41	.0003	F	5.13	.0000	F	.39	F	F
Chesapeake CEMS d-c	3	625	3.53	.0002	F	1.84	.0000	F	.53	F	F
Chesapeake CEMS d-c	4	638	3.53	.0002	F	5.21	.0000	F	.39	F	F
Chesapeake CEMS d-c	5	626	3.54	.0002	F	1.84	.0000	F	.52	F	F
Chesapeake CEMS d-c	6	638	3.62	.0002	F	5.28	.0000	F	.37	F	F
Chesapeake CEMS d-c	7	626	3.57	.0002	F	1.83	.0000	F	.52	F	F
Chesapeake CEMS d-c	8	638	3.62	.0002	F	5.26	.0000	F	.37	F	F
Chesapeake CEMS d-c	9	626	3.57	.0002	F	1.83	.0000	F	.52	F	F
Chesapeake CEMS d-c	10	638	3.64	.0001	F	5.17	.0000	F	.37	F	F
Chesapeake CEMS d-c	11	626	3.56	.0002	F	1.83	.0000	F	.52	F	F
Chesapeake CEMS d-c	12	638	3.65	.0001	F	5.25	.0000	F	.35	F	F
Chesapeake CEMS c-e	ALL	1502	-5.30	.9999	P	0.36	.9999	P	.57	F	F
Chesapeake CEMS c-e	1	698	-9.24	.9999	P	0.72	.9999	P	.80	P	P
Chesapeake CEMS c-e	2	694	-4.02	.9999	P	0.23	.9999	P	.42	F	F
Chesapeake CEMS c-e	3	697	-8.98	.9999	P	0.72	.9999	P	.80	P	P
Chesapeake CEMS c-e	4	694	-4.11	.9999	P	0.22	.9999	P	.42	F	F
Chesapeake CEMS c-e	5	697	-9.07	.9999	P	0.73	.9999	P	.80	P	P
Chesapeake CEMS c-e	6	694	-4.17	.9999	P	0.21	.9999	P	.41	F	F
Chesapeake CEMS c-e	7	697	-9.15	.9999	P	0.73	.9999	P	.80	P	P

Table 10: Continued

Data Source[2]	Data Subset[3]	N	1-tailed Bias Test α=.025 Computed value	p-value	Pass/ Fail	1-tailed F-test α=.05 Computed value	p-value	Pass/ Fail	Correlation ρ>.80 Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS c-e	8	694	-4.17	.9999	P	0.22	.9999	P	.41	F	F
Chesapeake CEMS c-e	9	697	-9.17	.9999	P	0.73	.9999	P	.80	P	P
Chesapeake CEMS c-e	10	694	-4.16	.9999	P	0.22	.9999	P	.40	F	F
Chesapeake CEMS c-e	11	697	-9.17	.9999	P	0.73	.9999	P	.80	P	P
Chesapeake CEMS c-e	12	694	-4.17	.9999	P	0.21	.9999	P	.38	F	F
Chesapeake CEMS e-c	ALL	1502	5.30	.0000	F	2.75	.0000	F	.57	F	F
Chesapeake CEMS e-c	1	698	9.24	.0000	F	1.39	.0000	F	.80	P	F
Chesapeake CEMS e-c	2	694	4.02	.0000	F	4.42	.0000	F	.42	F	F
Chesapeake CEMS e-c	3	697	8.98	.0000	F	1.39	.0000	F	.80	P	F
Chesapeake CEMS e-c	4	694	4.11	.0000	F	4.51	.0000	F	.42	F	F
Chesapeake CEMS e-c	5	697	9.07	.0000	F	1.37	.0000	F	.80	P	F
Chesapeake CEMS e-c	6	694	4.17	.0000	F	4.66	.0000	F	.41	F	F
Chesapeake CEMS e-c	7	697	9.15	.0000	F	1.37	.0000	F	.80	P	F
Chesapeake CEMS e-c	8	694	4.17	.0000	F	4.63	.0000	F	.41	F	F
Chesapeake CEMS e-c	9	697	9.17	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	10	694	4.16	.0000	F	4.60	.0000	F	.40	F	F
Chesapeake CEMS e-c	11	697	9.17	.0000	F	1.36	.0000	F	.80	P	F
Chesapeake CEMS e-c	12	694	4.17	.0000	F	4.68	.0000	F	.38	F	F
Chesapeake CEMS d-e	ALL	1317	0.16	.4374	P	0.90	.9701	P	.87	P	P

Table 10: Continued

Data Source[2]	Data Subset[3]	N	1-tailed Bias Test α=.025 Computed value	p-value	Pass/ Fail	1-tailed F-test α=.05 Computed value	p-value	Pass/ Fail	Correlation p>.80 Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS d-e	1	602	0.67	.2506	P	0.81	.9940	P	.73	F	F
Chesapeake CEMS d-e	2	617	-1.63	.9487	P	1.14	.0558	P	.91	P	P
Chesapeake CEMS d-e	3	602	0.62	.2685	P	0.81	.9940	P	.72	F	F
Chesapeake CEMS d-e	4	617	-1.40	.9187	P	1.13	.0660	P	.91	P	P
Chesapeake CEMS d-e	5	602	0.54	.2957	P	0.83	.9886	P	.72	F	F
Chesapeake CEMS d-e	6	617	-1.05	.8528	P	1.10	.1192	P	.91	P	P
Chesapeake CEMS d-e	7	602	0.51	.3043	P	0.83	.9890	P	.72	F	F
Chesapeake CEMS d-e	8	617	-1.05	.8525	P	1.10	.1149	P	.91	P	P
Chesapeake CEMS d-e	9	602	0.49	.3135	P	0.83	.9888	P	.72	F	F
Chesapeake CEMS d-e	10	617	-0.79	.7858	P	1.09	.1453	P	.91	P	P
Chesapeake CEMS d-e	11	602	0.48	.3157	P	0.83	.9875	P	.72	F	F
Chesapeake CEMS d-e	12	617	-0.75	.7747	P	1.09	.1511	P	.91	P	P
Chesapeake CEMS e-d	ALL	1317	-0.16	.5626	P	1.11	.0299	F	.87	P	F
Chesapeake CEMS e-d	1	602	-0.67	.7494	P	1.23	.0060	F	.73	F	F
Chesapeake CEMS e-d	2	617	1.63	.0513	P	0.88	.9442	P	.91	P	P
Chesapeake CEMS e-d	3	602	-0.62	.7315	P	1.23	.0060	F	.72	F	F
Chesapeake CEMS e-d	4	617	1.40	.0813	P	0.89	.9340	P	.91	P	P
Chesapeake CEMS e-d	5	602	-0.54	.7043	P	1.20	.0114	F	.72	F	F
Chesapeake CEMS e-d	6	617	1.05	.1472	P	0.91	.8808	P	.91	P	P
Chesapeake CEMS e-d	7	602	-0.51	.6957	P	1.21	.0110	F	.72	F	F

Table 10: Continued

			1-tailed Bias Test α=.025			1-tailed F-test α=.05			Correlation ρ>.80		
Data Source[2]	Data Subset[3]	N	Computed value	p-value	Pass/ Fail	Computed value	p-value	Pass/ Fail	Computed value	Pass/ Fail	Overall Pass/ Fail
Chesapeake CEMS e-d	8	617	1.05	.1475	P	0.91	.8851	P	.91	P	P
Chesapeake CEMS e-d	9	602	-0.49	.6865	P	1.20	.0112	F	.72	F	F
Chesapeake CEMS e-d	10	617	0.79	.2142	P	0.92	.8547	P	.91	P	P
Chesapeake CEMS e-d	11	602	-0.48	.6843	P	1.20	.0125	F	.72	F	F
Chesapeake CEMS e-d	12	617	0.75	.2253	P	0.92	.8489	P	.91	P	P
Summary of CEMS number of tests passed Total number of tests=166					119			116		68	54

Footnotes to Table 10:

1 F-test inflation factor: $\dfrac{1}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^2}}$; Bias test inflation factor: $\sqrt{\dfrac{\dfrac{(1+\rho)}{(1-\rho)}-\dfrac{2\rho(1-\rho^{n})}{n(1-\rho)^2}}{1-\dfrac{2\rho}{(n-1)(1-\rho)}+\dfrac{2\rho(1-\rho^{n})}{n(n-1)(1-\rho)^2}}}$

where ρ is computed from the first order autoregression of the CEMS (or AM) on its Lag, and n is the number of observations in the data stream.

2
- o UARG data from Attachment E of Public Comments (Table 1, page 4) in Correspondence IV-D-185 to EPA Docket No. A-90-51. There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.
- o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). Five CEMS (A through E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.

3 Data were subset into smaller datasets of 30 days each. ALL=entire data set used; N/A=less than 90% data available; all other values are set identifiers.

6. Conclusions

The analysis performed in this report indicates that statistical tests for alternative monitoring systems, are stringent but not preclusive, particularly if augmented by a procedure that compensates for variance underestimation due to autocorrelation. Despite the absence of strict QA/QC procedures in the field tests at the Chesapeake unit, a substantial number of subsets of the paired CEMS/CEMS data passed the three prescribed statistical test, whether or not a variance inflation estimate was used (Table 5, Table 6, Table 9, and Table 10). Applying variance inflation estimates to the available CSA/CEMS data, one subset passed all three statistical tests (Table 6). Two OSA/CEMS subsets passed all three tests when the data were analyzed at the level of refinement of the alternative monitoring system (Table 7 and Table 8). The latter results suggest that under-performance on the correlation test may have been due to limitations in the data rather than to the stringency of the test. Having to pair hourly CEMS measurements with daily AMS values for the Northern States Power Co. database and with weekly AMS values for the Niagara Mohawk database is likely to have had a detrimental impact on correlation test results. Under the proposed regulations, which require hourly measurements for both the CEMS and AMS, this confounding factor should not be present.

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Appendices

Three appendices supplement this report.

Appendix A summarizes the results of screening each of the databases used in this study for normality and autocorrelation.

Appendix B is a paper by Dr. David A. Dickey, Professor of Statistics at North Carolina State University, entitled "Effects of Autocorrelation on Statistical Analysis." It provides a theoretical background for the discussion in Section 4 ("Autocorrelation Analysis") of this report.

Appendix C provides documentation on the data subsets analyzed in this report.

Appendix A

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May 14, 1992

To: Elliot Lieberman
Emissions Monitoring Section, ARD

From: William Warren-Hicks
Susan E. Spruill
Jane E. Mudano
The Cadmus Group, Inc.

Subject: Statistical Analysis of Alternative Monitoring Systems

Please find enclosed analyses and summaries for parts 2(a-f), 3, and 4(a-e) of your memo dated April 21, 1992, requesting testing of alternative monitoring (AM) systems. The following data were analyzed:

- o UARG data from Attachment E of Public Comments (Table 1, page 4). There are 24 hours of CEM (A) and AM (B) data, but two hours are missing. All data were recorded in ppm.

- o Chesapeake data from Entropy (Section 75.21; EPA Contract No. 68-02-4462; Work assignment No. 91-156). One reference CEM (A) and four alternative CEMs (B-E) were monitored hourly for approximately 63 days. All SO_2 data were recorded in ppm.

- o Homer City Unit #1 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a 730 day period.

- o Homer City Unit #3 (from KEA), recorded as daily CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.

- o Niagra Mohawk (from KEA), recorded as hourly CEM (lbs/MMBtu) and using weekly oil sampling (OSA). Sampling covered a period of approximately 455 days.

- o Northern States Power Company (from KEA), recorded as hourly CEM (lbs/MMBtu) and using daily coal sampling (CSA). Sampling covered a period of approximately 730 days.

The following summaries are labeled to correspond to the memorandum dated April 21, 1992:

2. <u>Screen data to determine whether it is normally distributed</u>.

The SAS procedure UNIVARIATE was applied to all CEMs and AMs in order to:

a) determine the mean
b) determine the standard deviation of the mean
c) compute the Shapiro-Wilks test (or Kolmogorov test) for normality
d) graph normality (Q-Q) plots, and
e) graph frequency distribution histograms of the data.

Table 1 summarizes the univariate results: 2a, 2b, and 2c, above. For the test of normality, note that the UNIVARIATE procedure will use the Shapiro-Wilks test whenever there are less than 2000 observations, and will automatically use the Kolmogorov test whenever there are 2000 or more observations. Please also note that there are a few problems with these tests which will be discussed in part 3.

Normality plots (2d) and frequency histograms (2e) for each CEM and AM are also enclosed. In addition, time-series plots were produced for the CEMs, AMs, and their differences (CEM-AM).

3. <u>Screen data, which is not normally distributed, to determine whether it is lognormally distributed.</u>

It should be noted that nearly all variables failed the test for normality, based on 95% probability ($\alpha = .05$). This is because the normality test is quite sensitive to large sample size. Due to this sensitivity, the test for normality will generally reject the hypothesis that the data are normally distributed. In addition, there are a few "outlier" observations in the data sets we used, which may "skew" their distributions. Classical statistics theory which assumes large sample populations to be normally distributed. Therefore, we do not recommend the use of these tests for determining normality.

Instead, we recommend that you observe only the values of the normality statistics (Shapiro-Wilk's W, and Kolmogorov's D), ignoring the associated probability, and visually inspect the frequency histograms and Q-Q plots. Both statistics (W and D) have a range between 0 and 1: a $W=1$ (or $D=0$) would result if the data were perfectly normally distributed; values approaching $W=0$ (or $D=1$) increase the probability that the data are not normally distributed. Note that most statistics are very close to the extreme of the range which denotes normality. In addition, nearly all frequency distributions demonstrate symmetric curves with the mean approximately equal to the median. The Q-Q plots demonstrate the straight diagonal alignment of the residuals which is typical of the normal distribution. Based on these observations, we determined that all data are actually normally distributed, except for the AM data (CEM B) from the UARG table.

The Shapiro-Wilks test is appropriate for the UARG dataset because of its small sample size ($N=22$). The CEM (CEM A) data were found to be normally distributed. Both UARG CEM A and CEM B were transformed using the natural log and univariate analyses were rerun to determine if the transformation normalized their distributions. This transformation was unsuccessful. Because CEM A and the CEM B do not appear to come from the same distribution, it is not appropriate to compute the difference between them. However, for consistency, statistical summaries of these differences were reported.

There was no justification for testing for normality of the differences (CEM-AM). Differences between normally distributed variables are also normally distributed, and all variables analyzed are assumed to be normally distributed.

4. Autoregression analysis using SAS AUTOREG procedure.

The AUTOREG procedure is not available on our SAS contract. However, determination of autocorrelation of CEMs and AMs could be determined by a number of other methods:

a) Pearson correlation of CEM (or AM) values and the first order lag of those values (ie. $\rho_{CEM,\ lag1(CEM)}$).
b) Regression of the CEM (or AM) values on the first order lag of those values and observing the slope (β).
c) Time series regression of CEM (or AM) over time and computing the Durbin-Watson statistic (D) for first order autocorrelation. This test is available in the AUTOREG procedure. As D approaches zero, the probability of significant autocorrelation increases. A table of critical values for D can be found in most statistics texts. For large samples ($N>100$), the critical value is usually around $D=1.5$ at $\alpha=.05$.

Table 2 summarizes the results of the above tests for CEM and AM data. Due to the high autocorrelation which existed in nearly all CEM and AM data sets, differences between CEM and AM were computed from the residuals of the regressions of these values on their Lag1 (part b, above). Residuals from such analyses are independent, therefore differences between the residuals of the CEM and the residuals of the AM are corrected for the autocorrelation of the CEM and AM data. As a check, we ran a time series analysis of these residual differences and the computed the Durbin-Watson autocorrelation statistic (given in Table 2). All residual differences were uncorrelated.

Table 1. Summary of univariate analysis results.

Data Source	N[1]	Mean	Standard Deviation	Normal Statistic[2]	Normal
UARG					
CEM A	22	484.00	36.77	W=0.9356	Yes
CEM B[3]	22	468.64	46.13	W=0.8825	No
Chesapeake					
CEM A	1617	649.81	95.08	W=0.9707	Yes
CEM B[3]	1342	641.92	114.91	W=0.8915	Yes
CEM C[3]	1560	593.36	152.13	W=0.7617	Yes
CEM D[3]	1448	642.13	89.04	W=0.9548	Yes
CEM E[3]	1521	644.38	89.00	W=0.9606	Yes
Homer City					
Unit 1 CEM	497	2.42	0.19	W=0.9716	Yes
Unit 1 CSA[3]	572	2.53	0.26	W=0.9944	Yes
Unit 3 CEM	496	1.49	0.15	W=0.9583	Yes
Unit 3 CSA[3]	578	1.44	0.12	W=0.9534	Yes
Niagara Mohawk					
CEM	6801	0.61	0.08	D=0.0637	Yes
OSA[3]	62	0.73	0.04	W=0.9390	Yes
Northern States					
CEM	16081	1.28	0.22	D=0.1644	Yes
CSA[3]	667	1.46	0.14	W=0.9785	Yes

1 Homer City alternative monitoring (AM) measured daily, Niagara Mohawk AM measured weekly, Northern States AM measured daily, all others measured hourly.

2 W=Shapiro-Wilks test, range: $0 \leq W \leq 1$
D=Kolmogorov test, range: $0 \leq D \leq 1$
As W approaches 0 (D approaches 1) the probability of rejecting $N \sim (\mu, \sigma^2)$ increases.

3 alternative monitor

Table 2. Summary of autoregression analysis results.

Data Source	Pearson[1] Correlation	Regression[2] Coefficient (β)	Durbin-Watson Autocorrelation[3]	D Statistic[4]
UARG				
CEM A	0.6841	0.6358	0.268	1.385
CEM B[5]	0.6554	0.6659	0.023	1.925
CEM A - CEM B[6]			-0.376	2.694
Chesapeake				
CEM A	0.9347	0.9233	0.885	0.229
CEM B[5]	0.9449	0.9401	0.892	0.214
CEM C[5]	0.8631	0.8668	0.838	0.316
CEM D[5]	0.9739	0.9759	0.960	0.071
CEM E[5]	0.9099	0.9095	0.884	0.224
CEM A - CEM B[6]			-0.391	2.783
CEM A - CEM C[6]			-0.218	2.436
CEM A - CEM D[6]			-0.304	2.608
CEM A - CEM E[6]			-0.400	2.789
Homer City Unit 1				
CEM	0.7431	0.7348	0.735	0.529
CSA[5]	0.7376	0.7348	0.737	0.524
CEM - CSA[6]			-0.108	2.212
Homer City Unit 3				
CEM	0.8507	0.8421	0.817	0.363
CSA[5]	0.7991	0.7998	0.792	0.412
CEM - CSA[6]			-0.155	2.305
Niagara Mohawk				
CEM	0.7834	0.7782	0.760	0.479
OSA[5]	0.9964	0.9964	0.996	0.007
CEM - OSA[6]			-0.217	2.434

Table 2 continued

Data Source	Pearson[1] Correlation	Regression[2] Coefficient (β)	Durbin-Watson Autocorrelation[3]	D Statistic[4]
Northern States				
CEM	0.8129	0.8088	0.754	0.492
CSA[5]	0.9754	0.9757	0.975	0.050
CEM - CSA[6]			-0.091	2.181

[1] Pearson correlation of value (CEM or AM) to its Lag1 value

[2] Simple regression of original value (CEM or AM) on its Lag1 value

[3] First-order autocorrelation from time-series regression

[4] Durbin-Watson statistic (for $N > 100$, critical $D = 1.5$ at $\alpha = 0.05$)

[5] Alternative monitor

[6] Differences computed from residuals in order to remove autocorrelation within CEMS and AMS

UNIVARIATE PROCEDURE

Variable=CEM_A

```
Stem Leaf                     #          Boxplot
  56 3                        1             0
  54 35                       2             |
  52 5                        1             |
  50 1                        1             |
  48 991366                   6          +--+--+
  46 8959                     4          |     |
  44 24937                    5          +-----+
  42 29                       2             |
     ----+----+----+----+
 Multiply Stem.Leaf by 10**+1
```

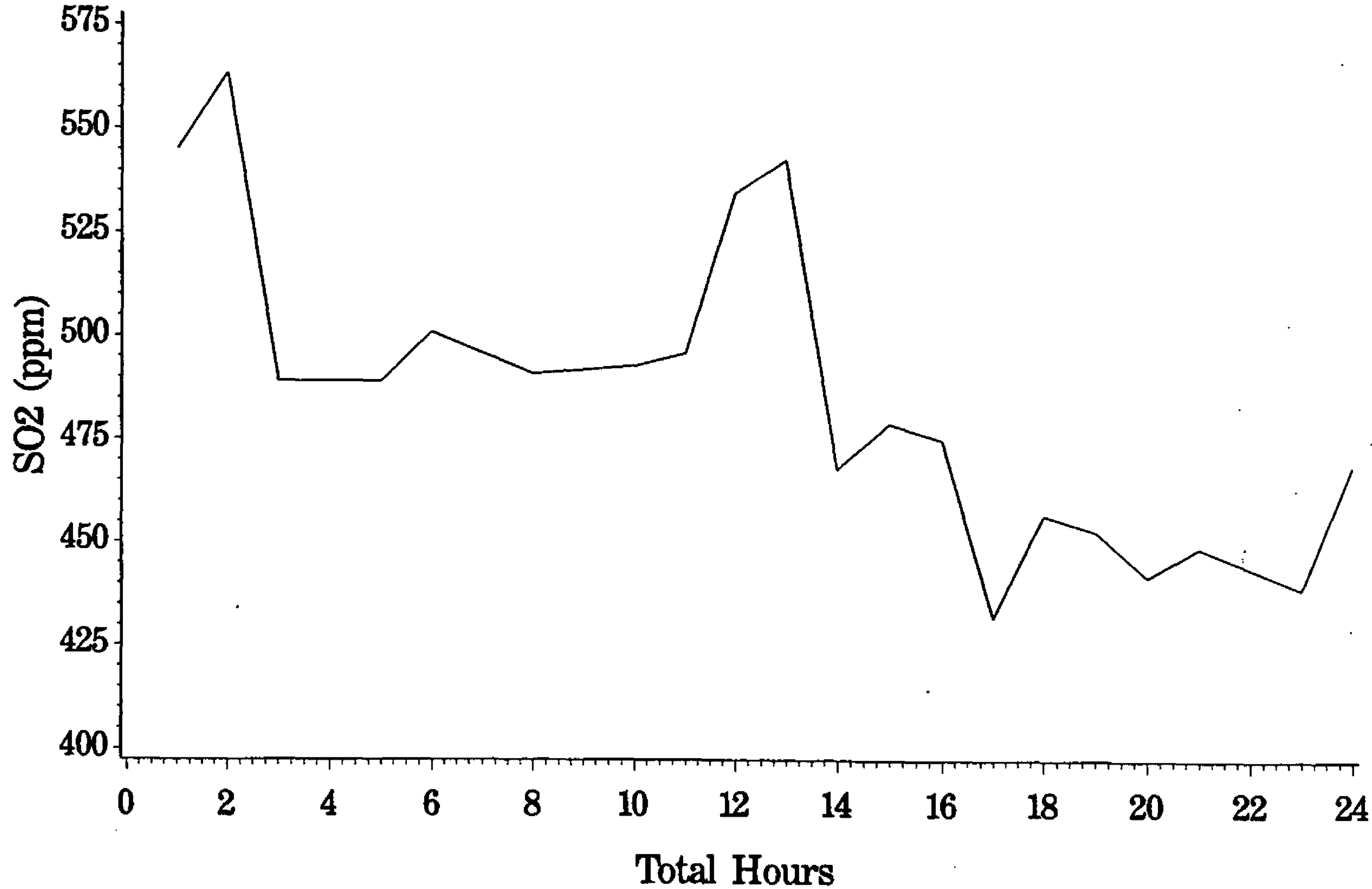
```
                          Normal Probability Plot
 570+                                                    * +++++
    |                                             *  ++++++
    |                                           *+++++
    |                                      ++++*+
    |                                 ++*****
    |                             ++****
    |                        *+*+** *
 430+           *  ++*+++
     +----+----+----+----+----+----+----+----+----+----+
         -2        -1         0        +1        +2
```

UNIVARIATE PROCEDURE

Variable=CEM_B

```
Stem Leaf                       #             Boxplot
  52 250                        3                |
  50 19067                      5             +-----+
  48 955                        3             *-----*
  46 20                         2             |  +  |
  44 6                          1             |     |
  42 464                        3             +-----+
  40 35001                      5                |
     ----+----+----+----+
 Multiply Stem.Leaf by 10**+1


                       Normal Probability Plot
530+                                            *++*    *
   |                                       ** **+*+
   |                                    *+++++
470+                                 **+++
   |                             +++*+
   |                         +++* **
410+      *    *+++*+    *
   +----+----+----+----+----+----+----+----+----+----+
        -2        -1         0        +1        +2
```

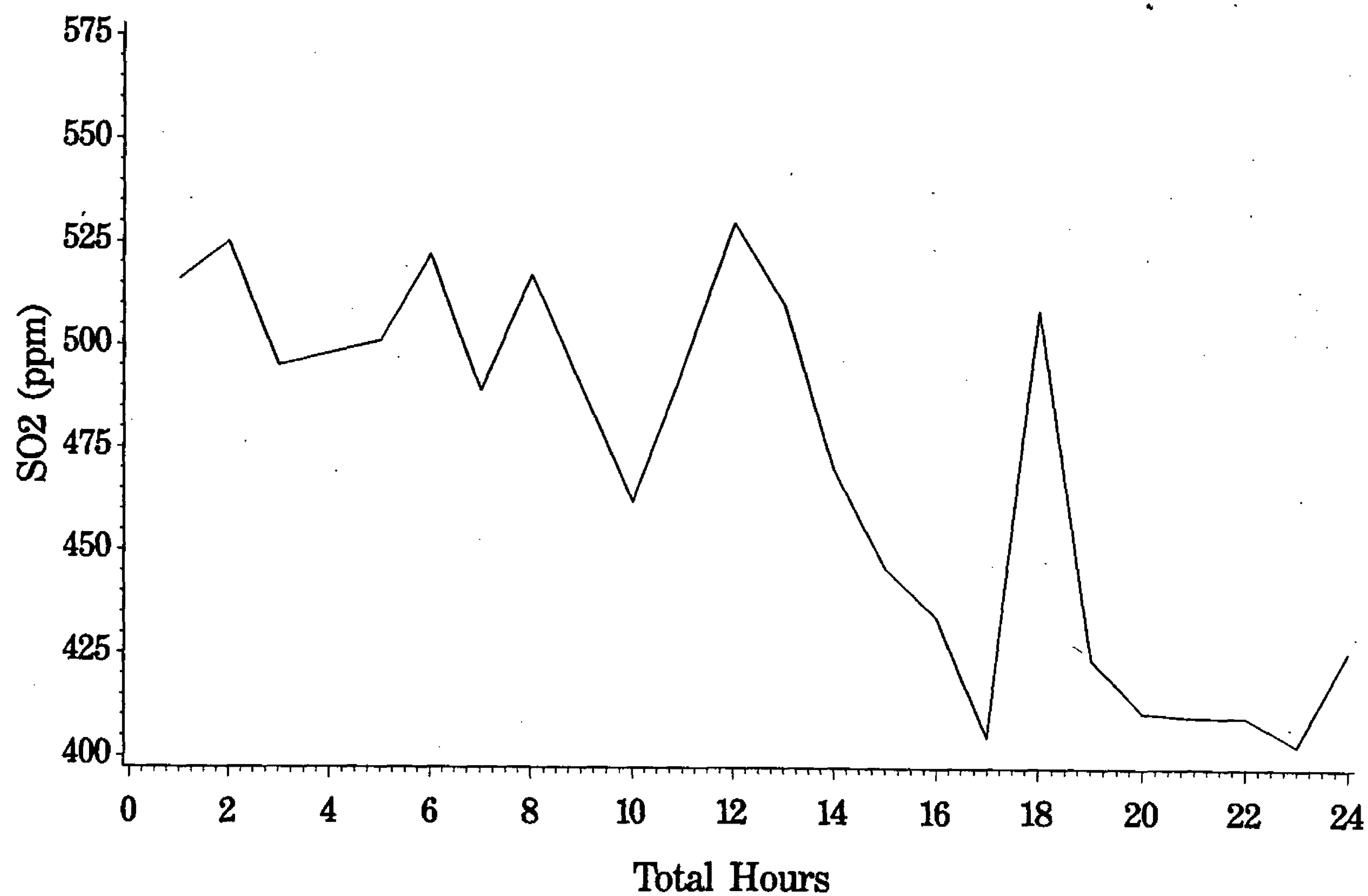
Data From Attachment E
UARG Comments
CEM A



Data From Attachment E

UARG Comments

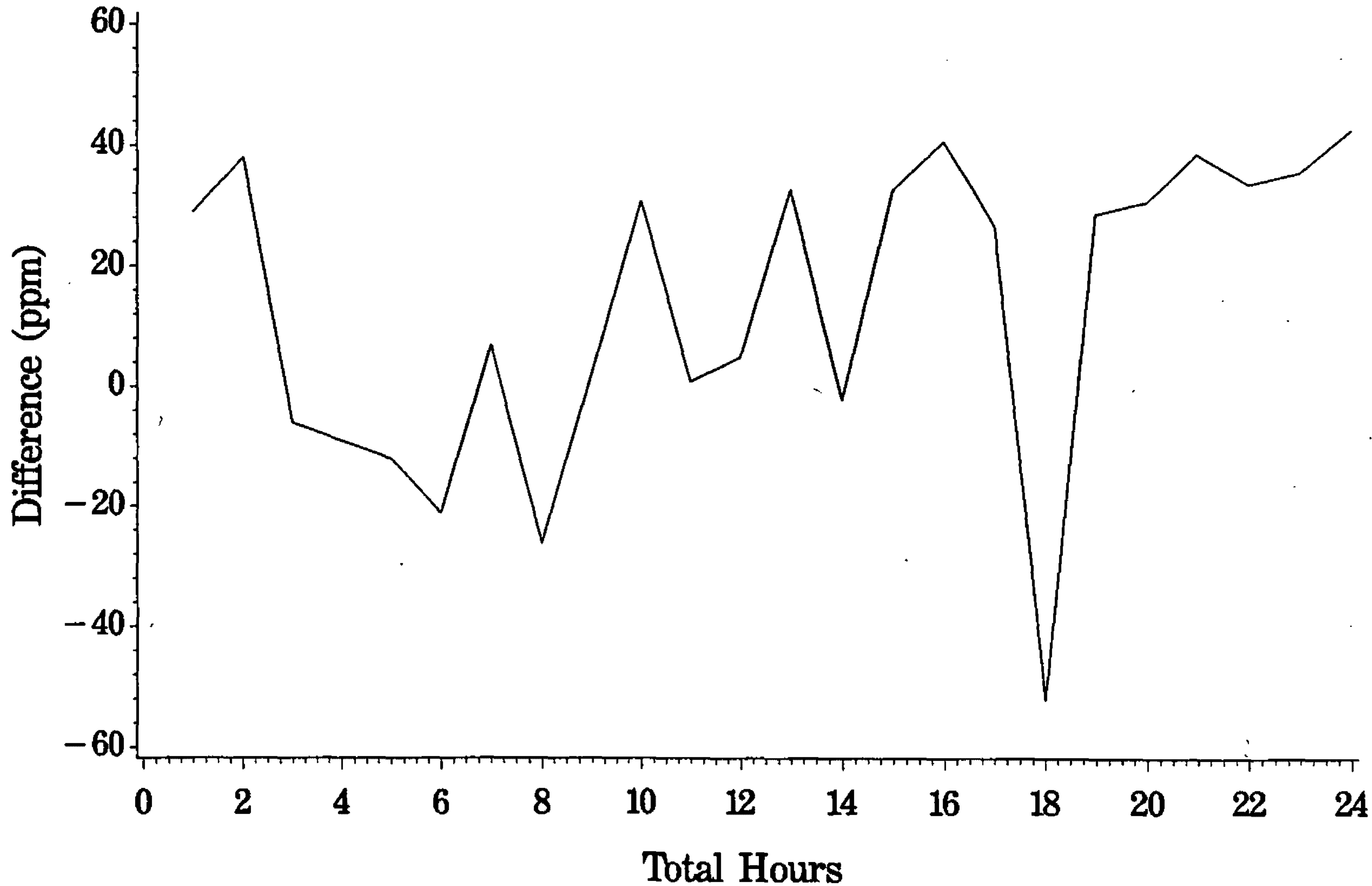
CEM B



Data From Attachment E

UARG Comments

Difference A – B



UARG attachment E data
17:27 Wednesday, May 13, 19

Model: MODEL1
Dependent Variable: CEM_A

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	19302.18052	19302.18052	42.451	0.0001
Error	20	9093.81948	454.69097		
C Total	21	28396.00000			

Root MSE	21.32348	R-square	0.6797
Dep Mean	484.00000	Adj R-sq	0.6637
C.V.	4.40568		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	176.008727	47.48895471	3.706	0.0014
CEM_B	1	0.657207	0.10086893	6.515	0.0001

UNIVARIATE PROCEDURE

Variable=CEM_A

```
                         Histogram                               #            Boxplot
    925+*                                                        3               0
       .******                                                  57               0
       .********                                                79               |
       .***********                                            109               |
       .******************                                     173            +-----+
       .*****************************                          281            |     |
       .*********************************************          441            *--+--*
       .****************************                           275            +-----+
       .**************                                         137               |
    475+******                                                  53               |
       .*                                                        6               0
       .*                                                        2               0
       .
       .
       .
       .
       .
       .
     25+*                                                        1               *
        ----+----+----+----+----+----+----+----+----+
        * may represent up to 10 counts


                         Normal Probability Plot
             925+                                                     *
                |                                             ********
                |                                         *******+++
                |                                     *****+
                |                                 *****
                |                            +*****
                |                       ********
                |                  ********
                |             *******+
             475+*********++
                |****
                |*
                |
                |
                |
              25+*
                 +----+----+----+----+----+----+----+----+----+----+
                     -2        -1         0        +1        +2
```

UNIVARIATE PROCEDURE

Variable=CEM_B

```
                 Histogram                          #             Boxplot
 925+*                                              2                0
    .*****                                         35                |
    .**********                                    65                |
    .**************                                96                |
    .**************************                   180             +-----+
    .********************************             224             |     |
    .*********************************************314             *--+--*
    .***********************************          242             +-----+
    .****************                             107                |
 475+*******                                       45                |
    .*                                              3                |
    .**                                            14                0
    .*                                              1                0
    .
    .*                                              1                0
    .
    .*                                              1                *
    .
  25+**                                            12                *
    ----+----+----+----+----+----+----+----+----+
    * may represent up to 7 counts
```

```
                         Normal Probability Plot
     925+                                                   +++*
        |                                               *******
        |                                           ******
        |                                       *****
        |                                   *****
        |                              ******
        |                         *******
        |                    ********+
        |               *********+++
     475+           *******+++
        |       +*+++
        |  ++*****
        |     *
        |
        |     *
        |
        |     *
        |
      25+**
         +----+----+----+----+----+----+----+----+----+----+
              -2        -1         0        +1        +2
```

UNIVARIATE PROCEDURE

Variable=CEM_C

```
                     Histogram                              #             Boxplot
 875+***                                                   22                0
    .*******                                               54                0
    .*********                                             65                |
    .************                                          92                |
    .******************************                       238             +-----+
    .***********************************************      380             *-----*
    .*****************************************            330             +--+--+
    .***********************                              180                |
    .**************                                       109                |
    .****                                                  25                |
    .
    .
    .
    .
    .
    .
    .
  25+*********                                             65                *
     ----+----+----+----+----+----+----+----+----+---
     * may represent up to 8 counts
```

```
                         Normal Probability Plot
  875+                                                  ++++ ****
     |                                               +++*******
     |                                            ++******
     |                                         ++*****
     |                                     +******
     |                                *******
     |                          *******+
     |                      ****++++
     |                 ****** +++
     |                ** +++
     |                 +++
     |             ++++
     |          +++
     |       +++
     |
     |
   25+*********
      +----+----+----+----+----+----+----+----+----+----+
           -2        -1         0        +1        +2
```

UNIVARIATE PROCEDURE

Variable=CEM_D

```
                   Histogram                          #             Boxplot
925+*                                                  1                0
   .*****                                             41                0
   .*******                                           58                0
   .**********                                        86                |
   .***************                                  134                |
675+*****************************                    256             +-----+
   .********************************************     388             *--+--*
   .*********************************                295             +-----+
   .****************                                 142                |
   .*****                                             45                |
425+*                                                  2                |
    ----+----+----+----+----+----+----+----+----
    * may represent up to 9 counts
```

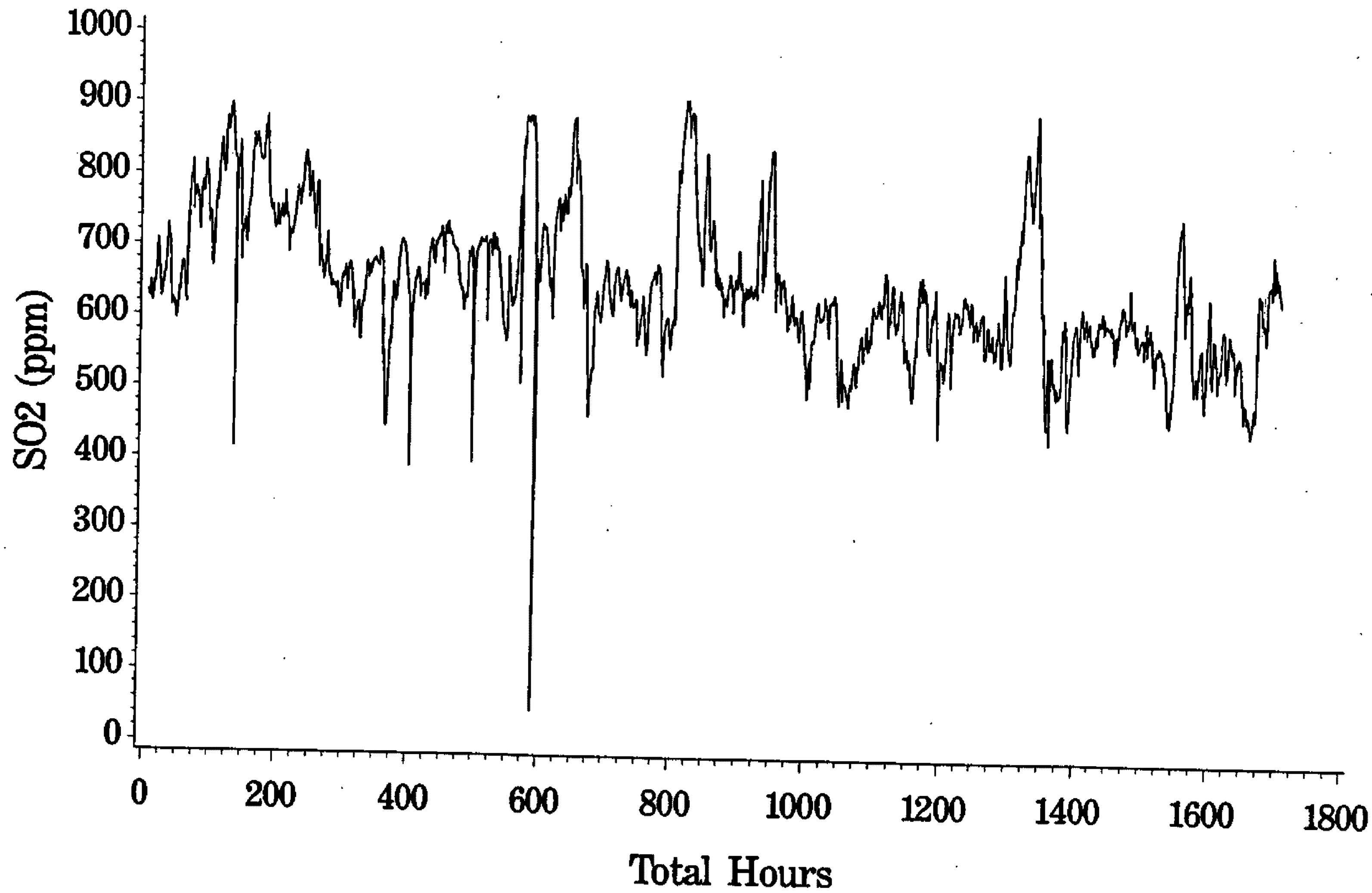
```
                        Normal Probability Plot
          925+                                                  *
             |                                           *******
             |                                       *********++++
             |                                   ******++
             |                               +*****+
          675+                          ++*******
             |                      ********
             |                 ********
             |            ********
             |********++
          425+*+++
              +----+----+----+----+----+----+----+----+----+----+
                  -2        -1         0        +1        +2
```

UNIVARIATE PROCEDURE

Variable=CEM_E

```
                           Histogram                            #             Boxplot
   925+*                                                         5                0
      .*****                                                    38                0
      .*******                                                  61                0
      .**********                                               86                |
      .***************                                         128                |
   675+**************************************                  336             +-----+
      .********************************************            394             *--+--*
      .*******************************                         279             +-----+
      .****************                                        136                |
      .******                                                   54                |
   425+*                                                         4                |
       ----+----+----+----+----+----+----+----+----
       * may represent up to 9 counts


                           Normal Probability Plot
        925+                                                   *
           |                                             *******
           |                                         *********++
           |                                      *****++
           |                                 +*****
        675+                           +********
           |                     ********
           |               *******
           |        ********
           |********+
        425+*+++
            +----+----+----+----+----+----+----+----+----+----+
                -2        -1         0        +1        +2
```

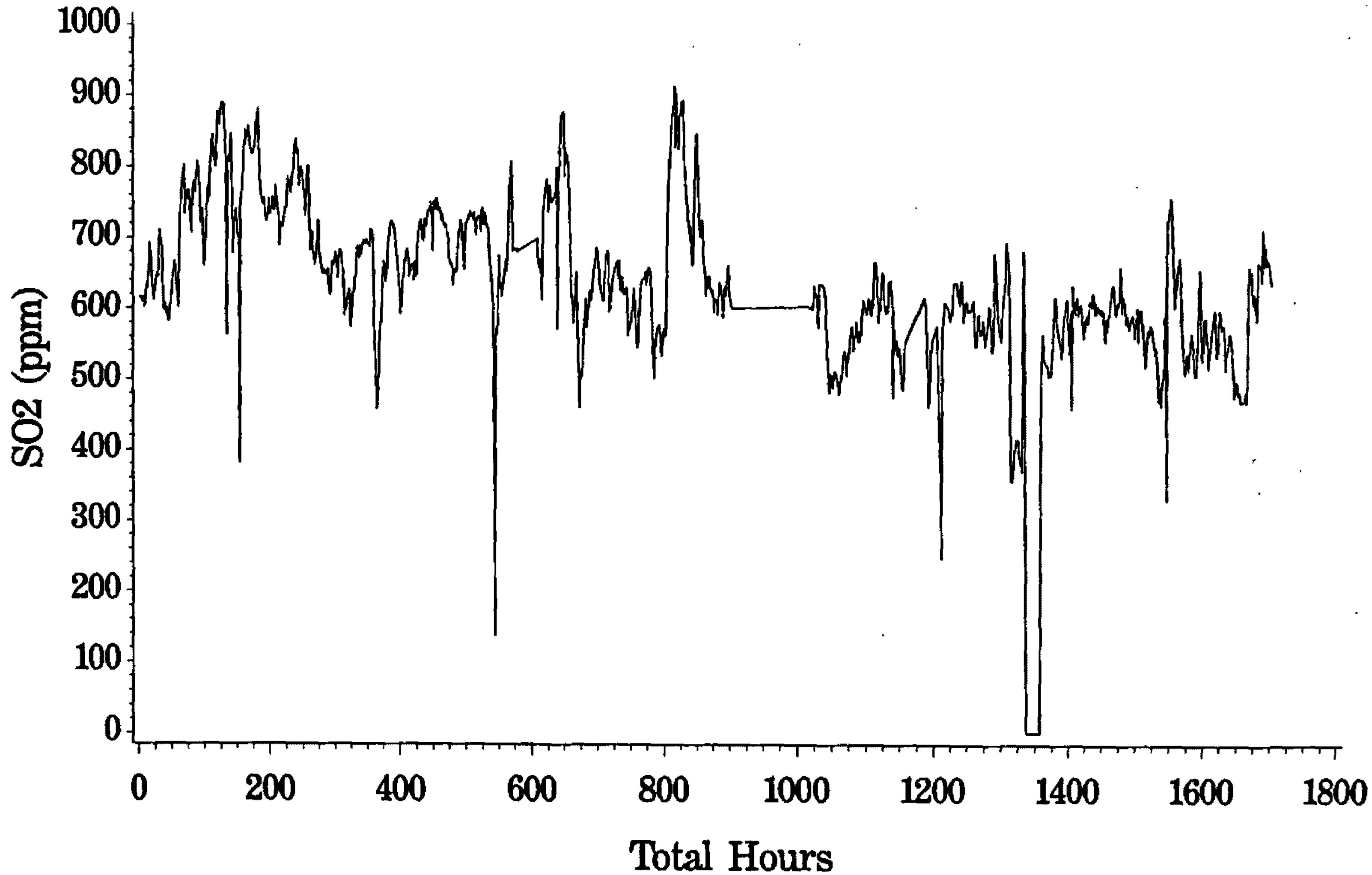
Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
CEM A



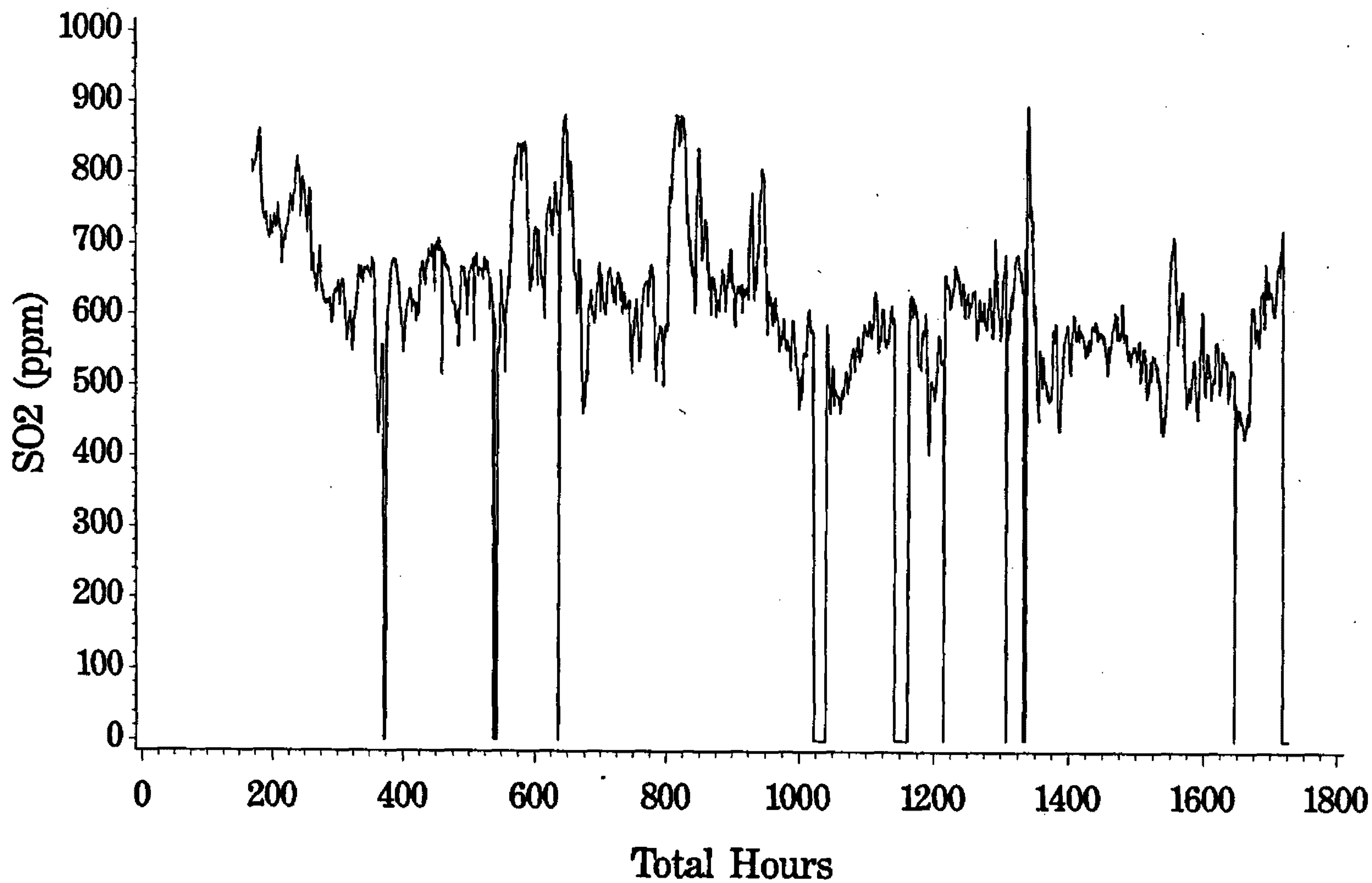
Chesapeake Energy Center – Unit 4

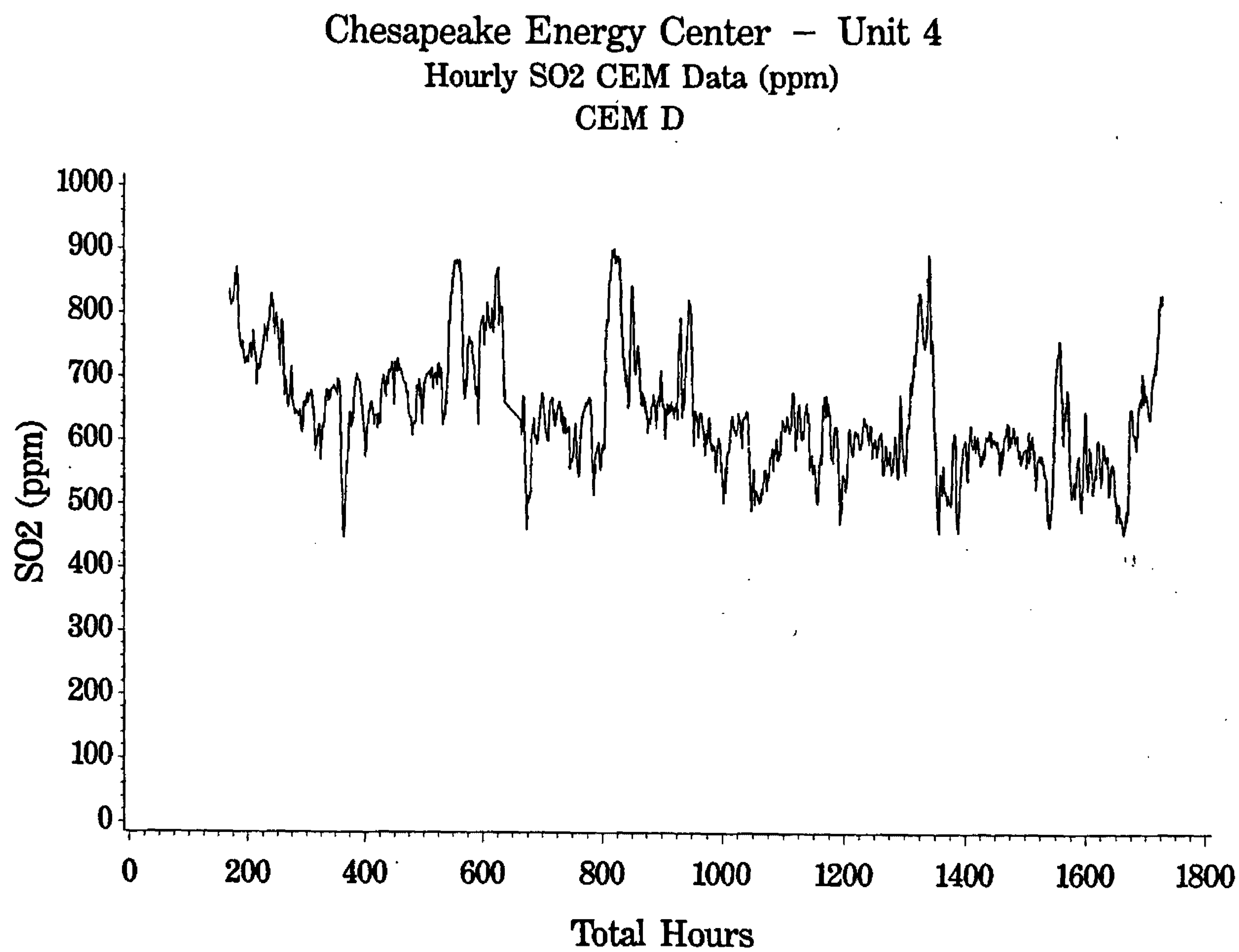
Hourly SO2 CEM Data (ppm)

CEM B



Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
CEM C



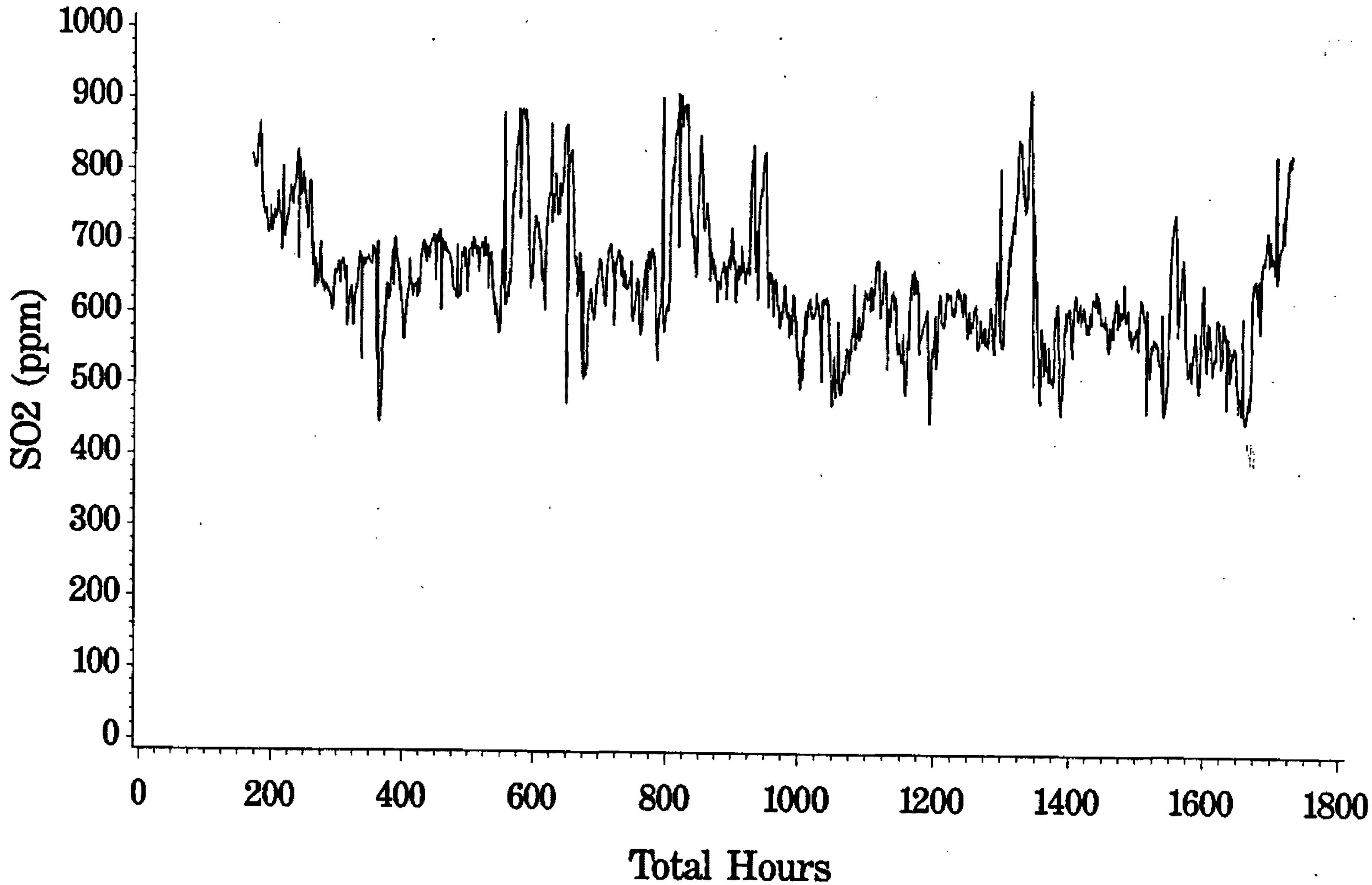


Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
CEM D
SO2 (ppm)
1000
900
800
700
600
500
400
300
200
100
0
0
200
400
600
800
1000
1200
1400
1600
1800
Total Hours

Chesapeake Energy Center – Unit 4

Hourly SO2 CEM Data (ppm)

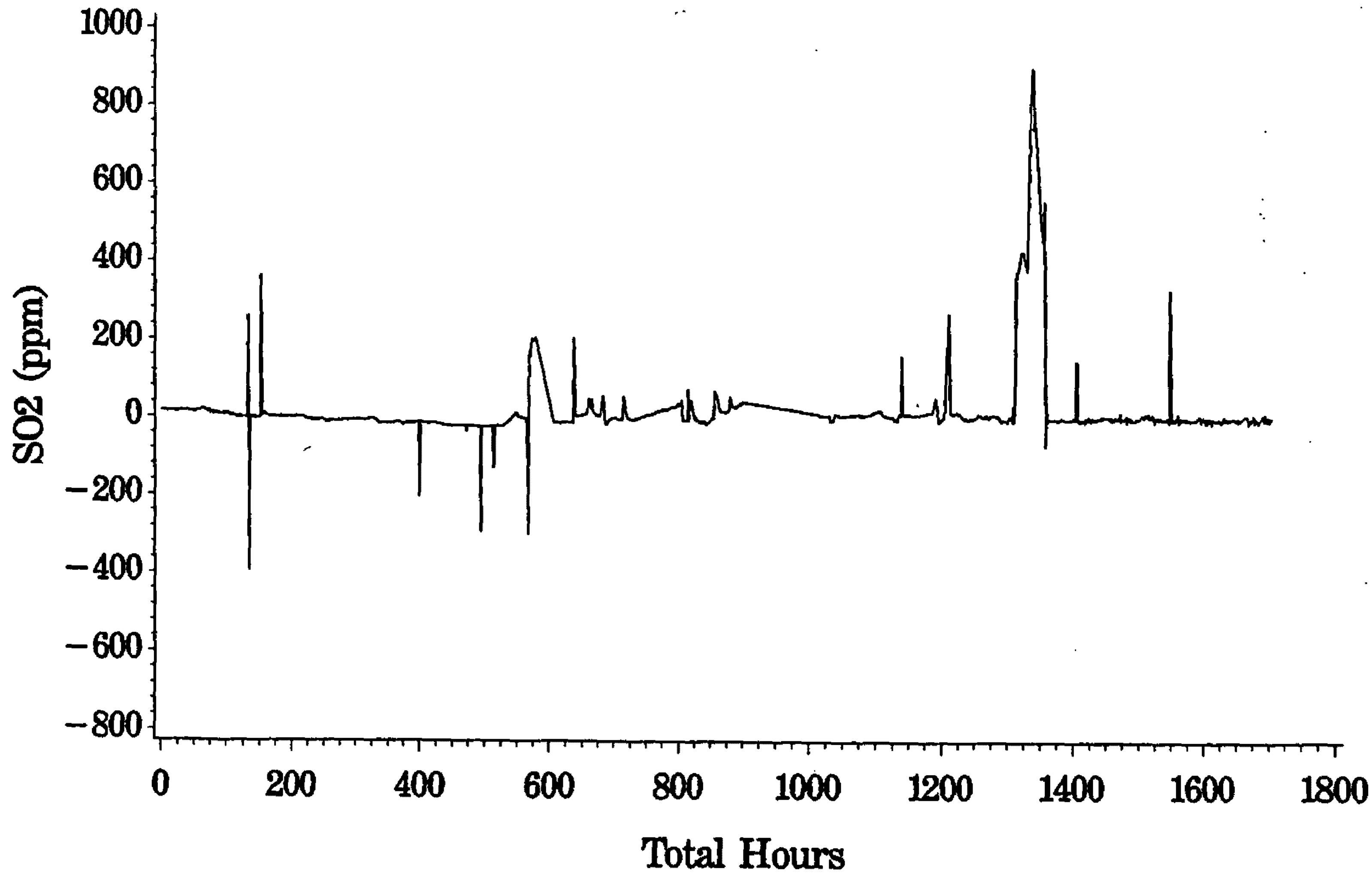
CEM E

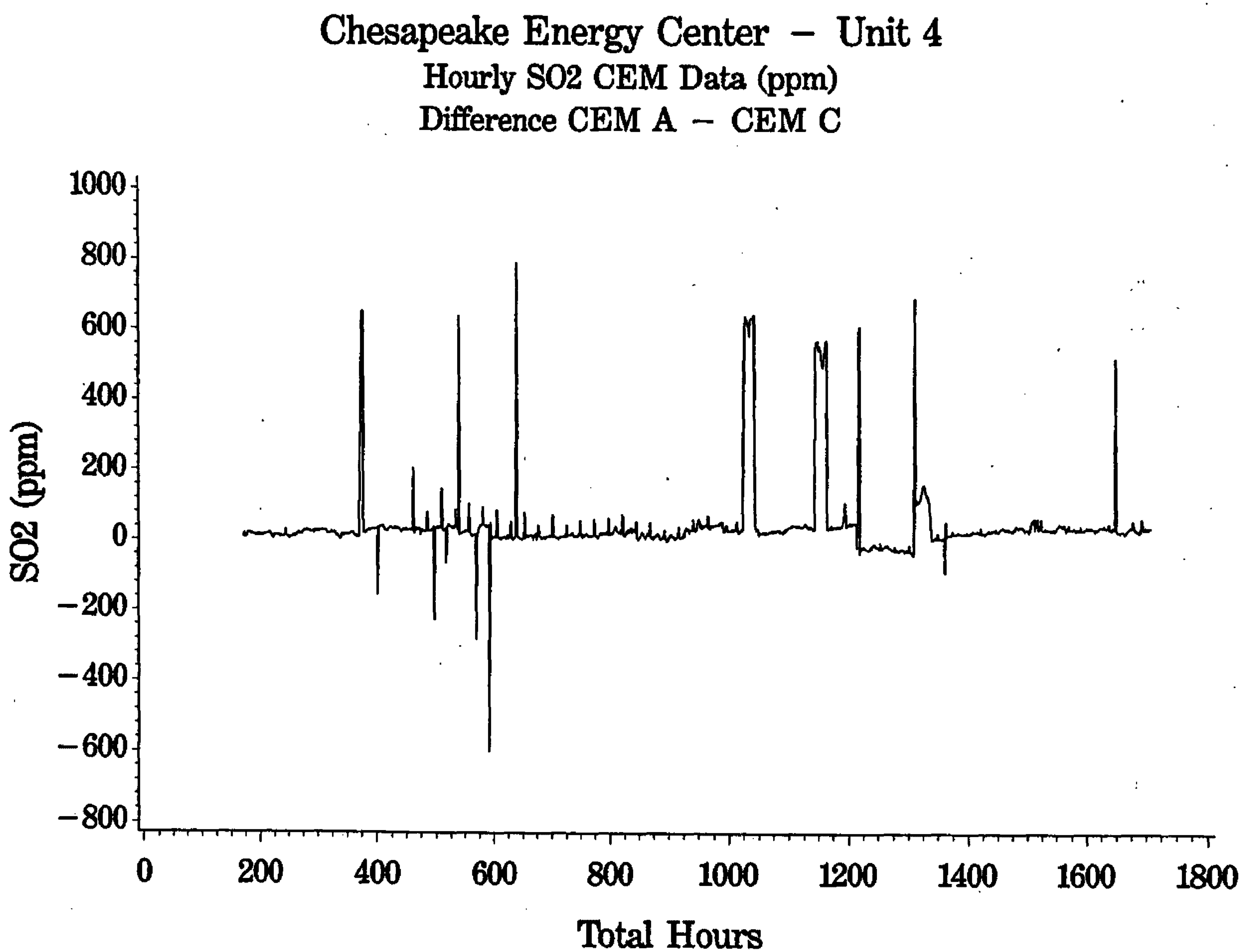


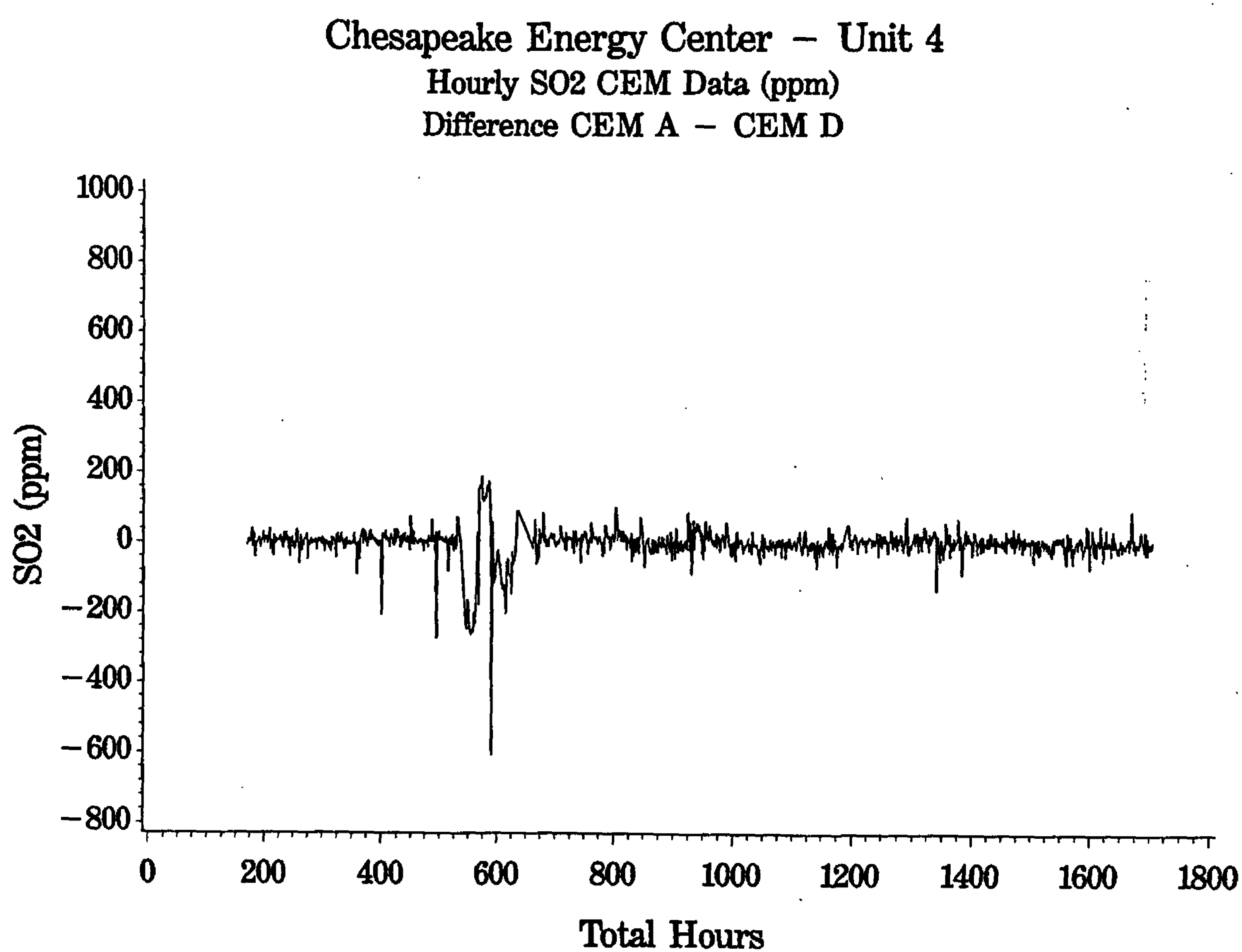
Chesapeake Energy Center – Unit 4

Hourly SO2 CEM Data (ppm)

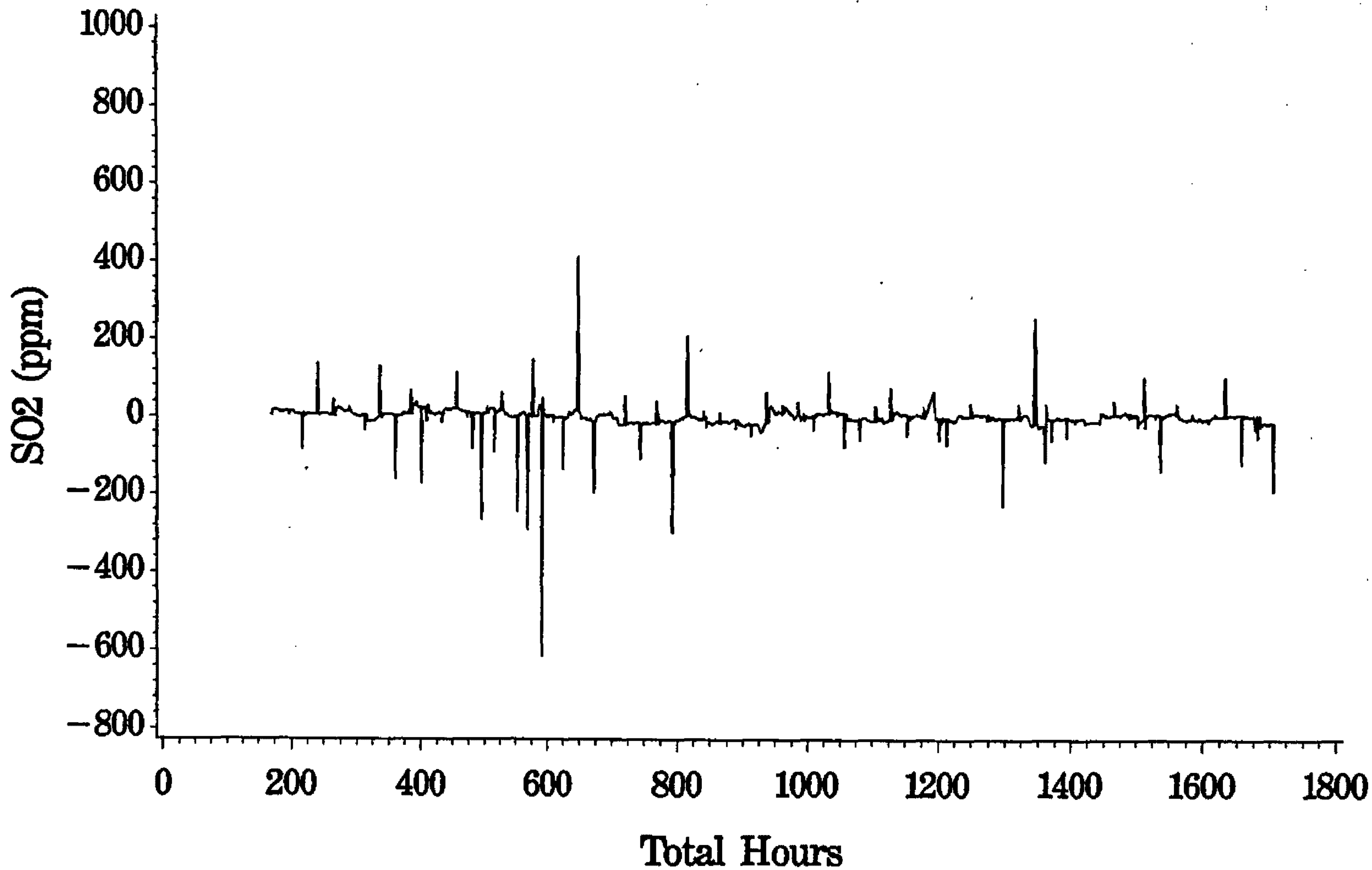
Difference CEM A – CEM B




Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
Difference CEM A – CEM C
SO2 (ppm)
1000
800
600
400
200
0
–200
–400
–600
–800
0
200
400
600
800
1000
1200
1400
1600
1800
Total Hours


Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
Difference CEM A – CEM D
SO2 (ppm)
1000
800
600
400
200
0
−200
−400
−600
−800
0
200
400
600
800
1000
1200
1400
1600
1800
Total Hours

Chesapeake Energy Center – Unit 4
Hourly SO2 CEM Data (ppm)
Difference CEM A – CEM E



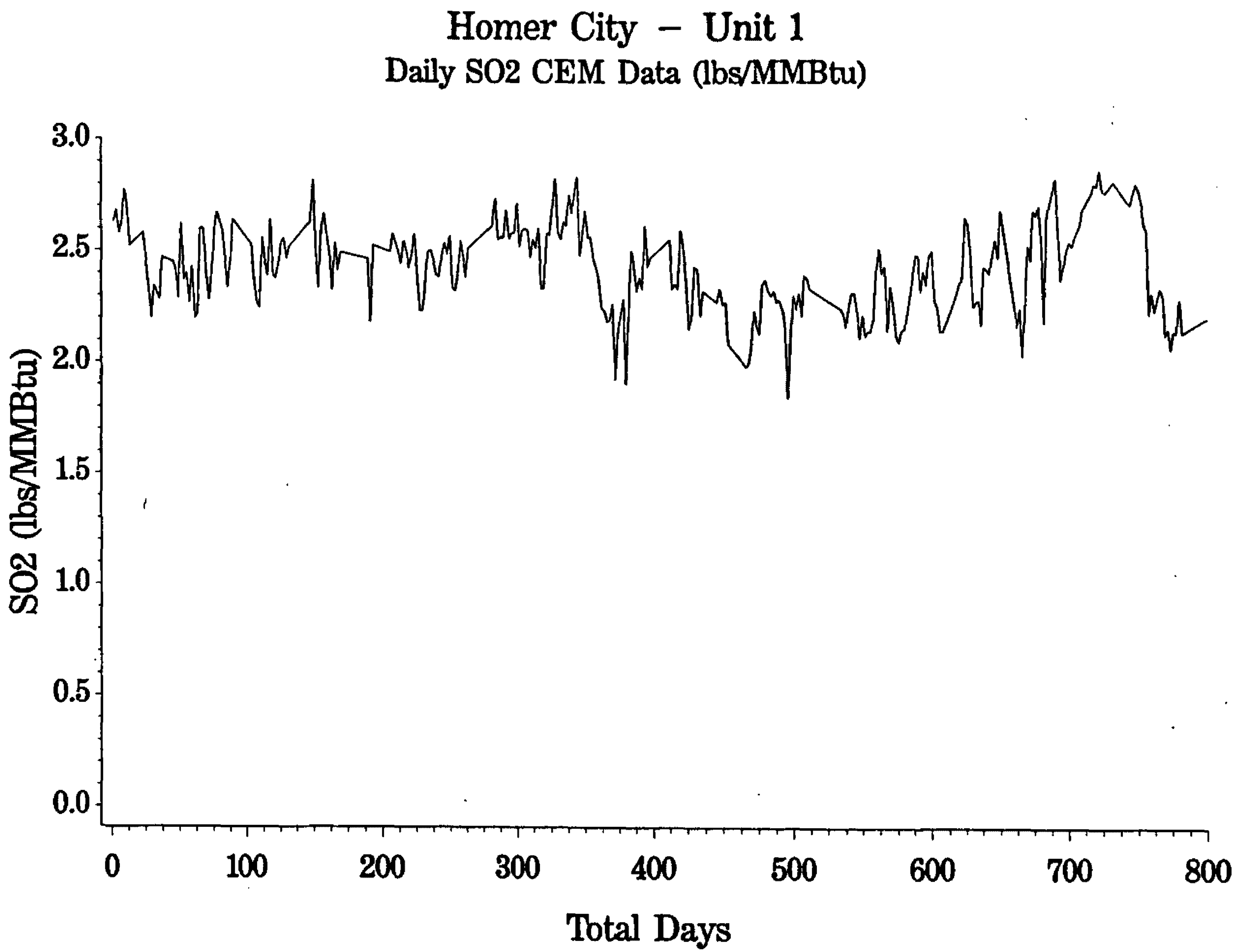
----------------------------------- UNIT=1 -----------------------------------

UNIVARIATE PROCEDURE

Variable=SO2CEM

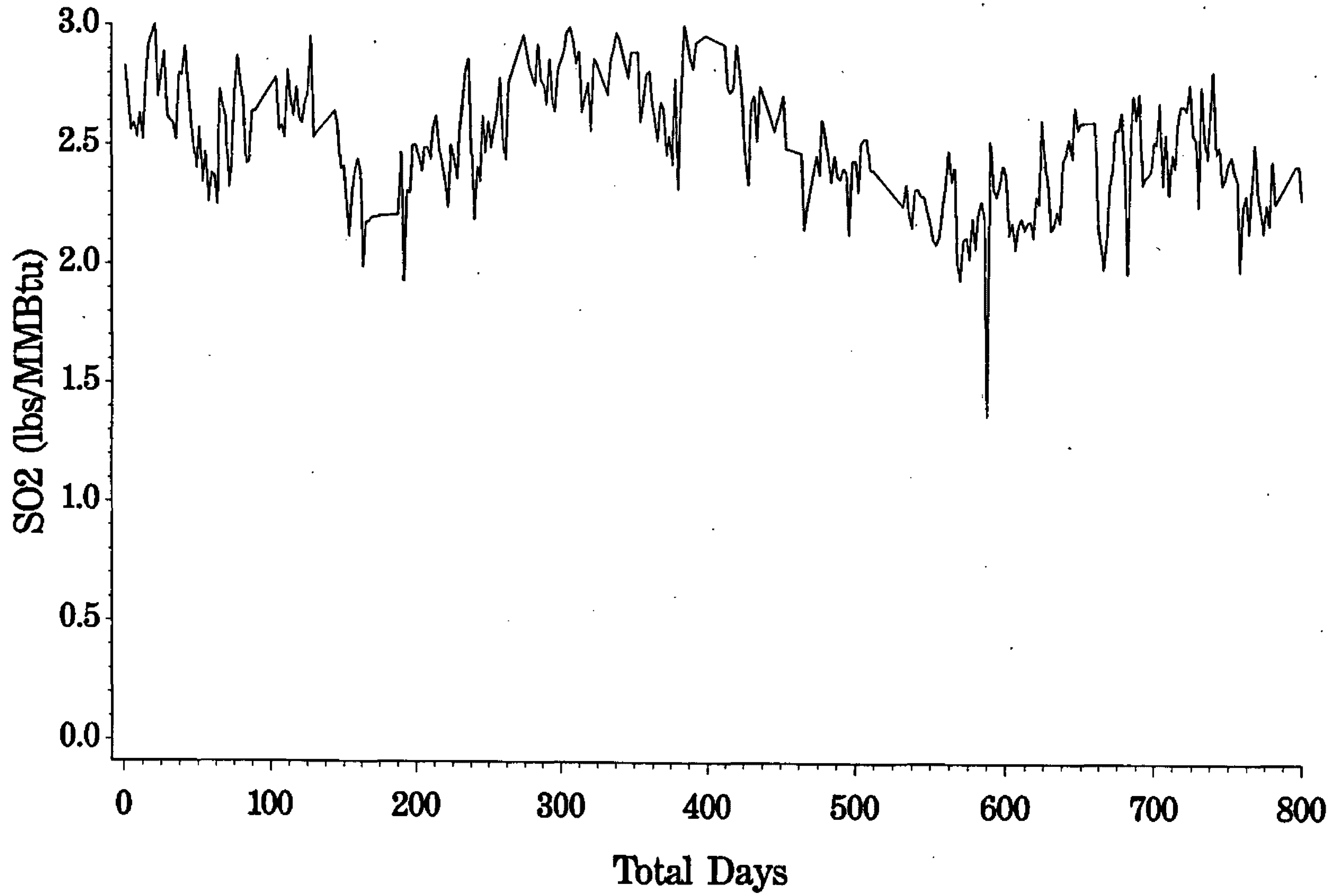
```
                  Histogram                              #             Boxplot
    2.85+***                                             9                |
        .********                                       23                |
        .*****************                              50                |
        .*********************************              98             +-----+
        .**********************************            101             *--+--*
        .*******************************                91             |     |
        .**********************                         65             +-----+
        .************                                   36                |
        .****                                           11                |
        .**                                              6                |
        .**                                              5                0
    1.75+*                                               2                0
         ----+----+----+----+----+----+----
         * may represent up to 3 counts


                             Normal Probability Plot
         2.85+                                                     +******
             |                                                ******
             |                                           ******
             |                                     *******
             |                               ******+
             |                          ******+
             |                     ******
             |                 *****
             |            ++***
             |   +++***
             |** *
         1.75+*
              +----+----+----+----+----+----+----+----+----+----+
                  -2        -1         0        +1        +2
```

```
----------------------------------- UNIT=1 -----------------------------------

                              UNIVARIATE PROCEDURE

Variable=AACS (CSA)

                  Histogram                                  #          Boxplot
  3.35+*                                                     1             0
      .*                                                     2             0
  3.15+***                                                   5             |
      .******                                               11             |
  2.95+**********                                           20             |
      .************************                             48             |
  2.75+******************************                       60          +-----+
      .*******************************************          85          |     |
  2.55+*****************************************            81          *--+--*
      .*********************************************        90          |     |
  2.35+*********************************                    65          +-----+
      .***********************                              46             |
  2.15+******************                                   36             |
      .******                                               12             |
  1.95+*****                                                 9             |
      .
  1.75+
      .
  1.55+
      .
  1.35+*                                                     1             0
       ----+----+----+----+----+----+----+----+----
       * may represent up to 2 counts


                          Normal Probability Plot
           3.35+                                                  *
               |                                                  *
           3.15+                                               ****
               |                                           *****
           2.95+                                        ****
               |                                   ******
           2.75+                                ****
               |                            *****
           2.55+                         ****
               |                     *****
           2.35+                 *****
               |             *****
           2.15+        ******
               |     ****+
           1.95+*+***+
               |+
           1.75+
               |
           1.55+
               |
           1.35+*
                +----+----+----+----+----+----+----+----+----+----+
                    -2        -1         0        +1        +2
```


Homer City – Unit 1
Daily SO2 CEM Data (lbs/MMBtu)
SO2 (lbs/MMBtu)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300
400
500
600
700
800
Total Days

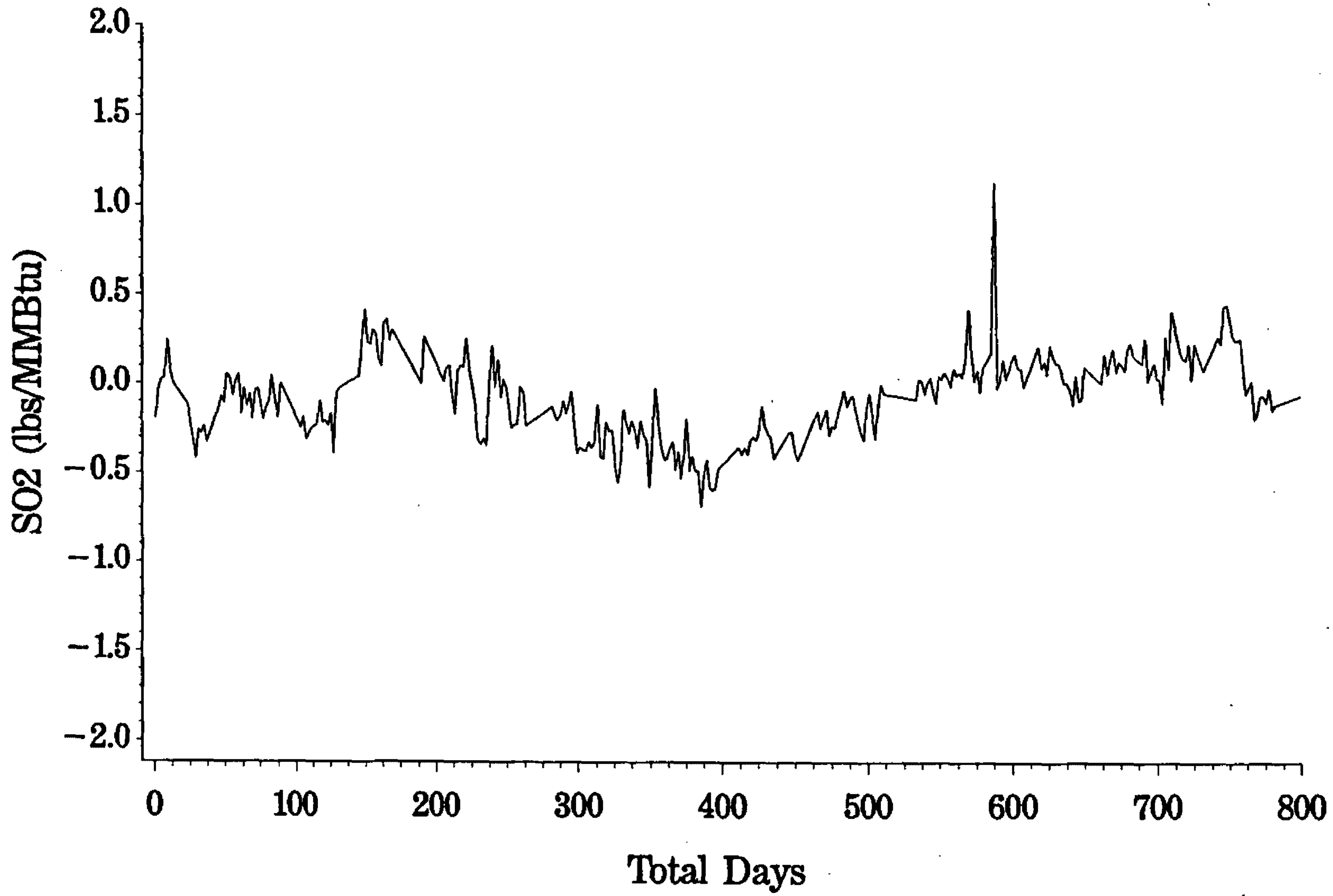
Homer City – Unit 1

Daily Coal Sampling Analysis (CSA) SO2 Data (lbs/MMBtu)



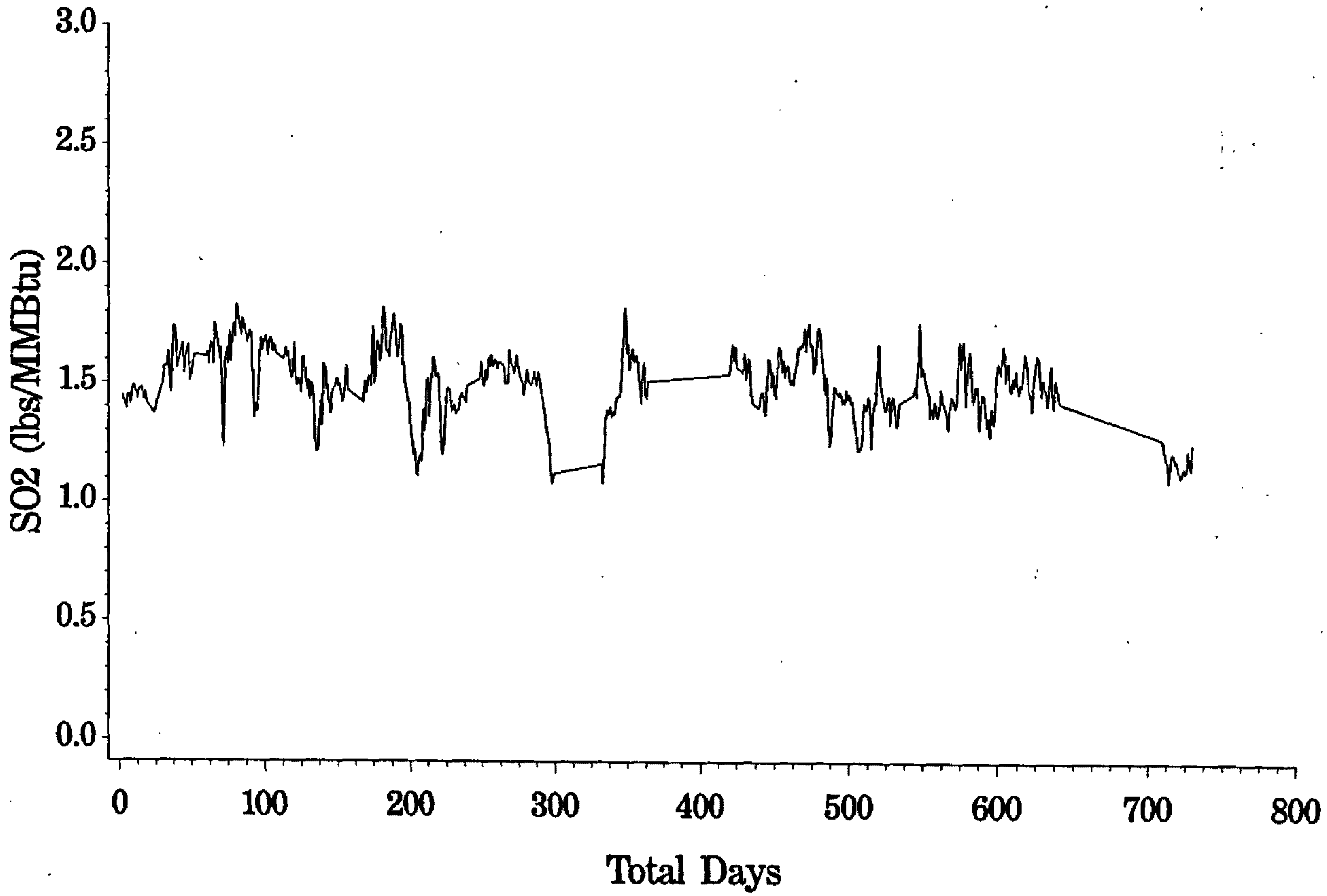
Homer City – Unit 1

Difference CEM – CSA

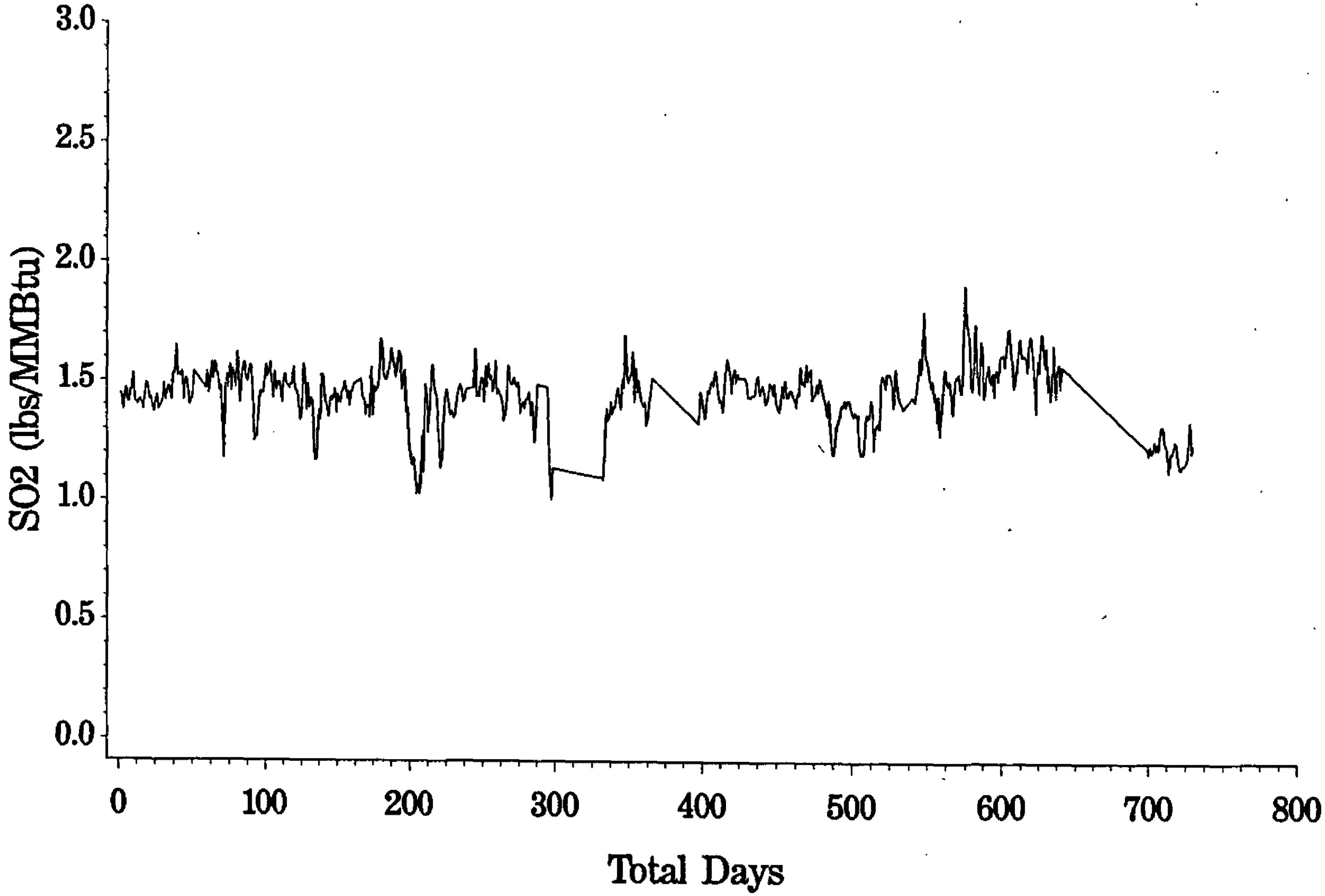


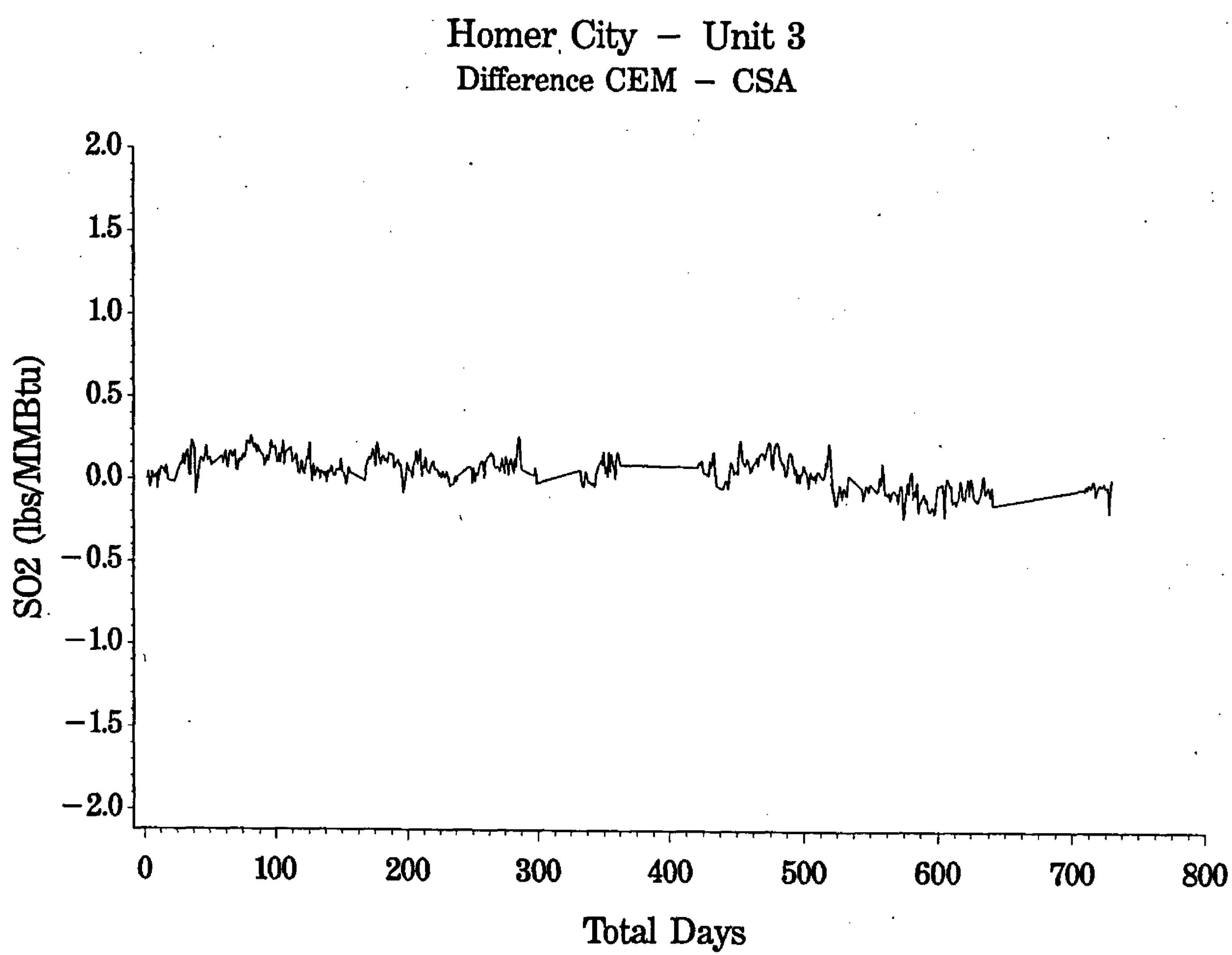
Homer City – Unit 3

Daily SO2 CEM Data (lbs/MMBtu)



Homer City – Unit 3
Daily Coal Sampling Analysis (CSA) SO2 Data (lbs/MMBtu)




Homer City – Unit 3
Difference CEM – CSA
SO2 (lbs/MMBtu)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
100
200
300
400
500
600
700
800
Total Days

UNIVARIATE PROCEDURE

Variable=SO2CEM

```
                         Histogram                                  #        Boxplot
  0.925+*                                                            1          0
       .*                                                           10          0
       .**                                                          65          0
       .******                                                     244          |
       .**************                                             574          |
       .******************************                            1229       +-----+
  0.625+******************************************                1753       *--+--*
       .***********************************************           1971       +-----+
       .***************                                            627          |
       .****                                                       167          |
       .***                                                        115          0
       .**                                                          43          0
  0.325+*                                                            2          0
        ----+----+----+----+----+----+----+----+----+--
        * may represent up to 42 counts


                        Normal Probability Plot
            0.925+                                                  *
                 |                                                  *
                 |                                                ***
                 |                                          *******+
                 |                                    *******+
                 |                              *******
            0.625+                        ********
                 |                  **********
                 |            *******++
                 |        +*****++
                 | ******
                 |*
            0.325+*
                  +----+----+----+----+----+----+----+----+----+----+
                      -2        -1         0        +1        +2
```

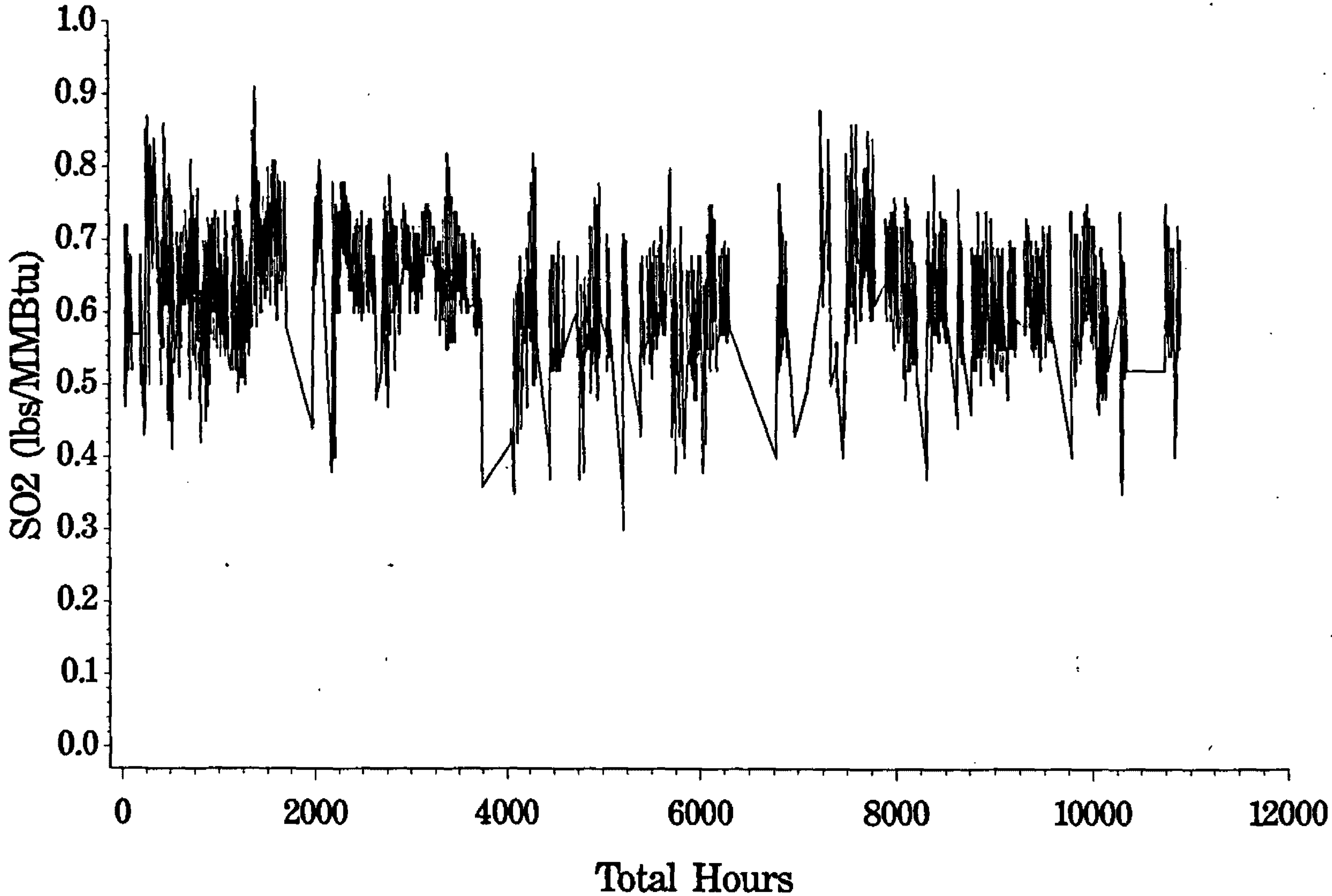
Niagara Mohawk OSA by week
7:20 Thursday, May 14, 199

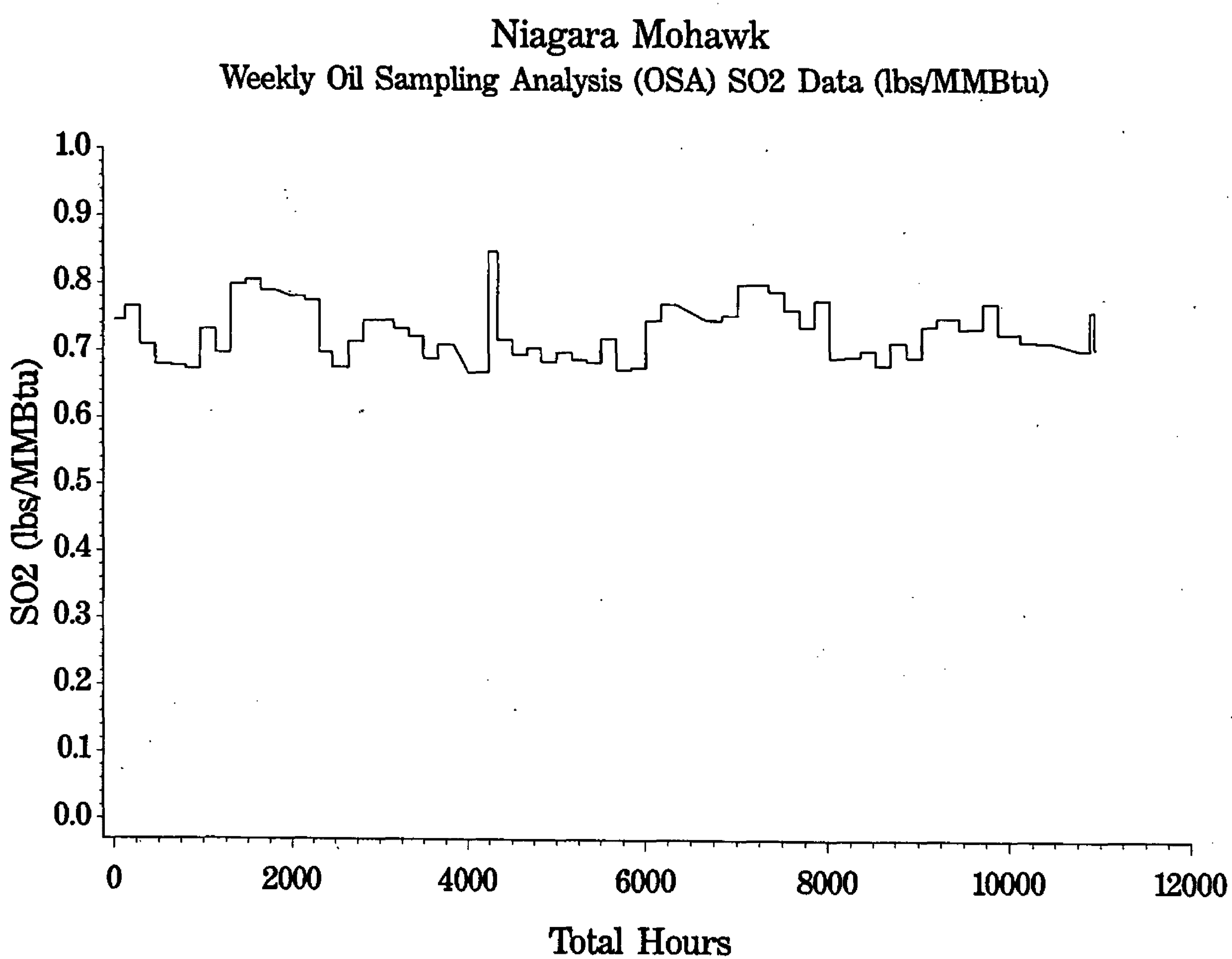
UNIVARIATE PROCEDURE

Variable=SO2OSA

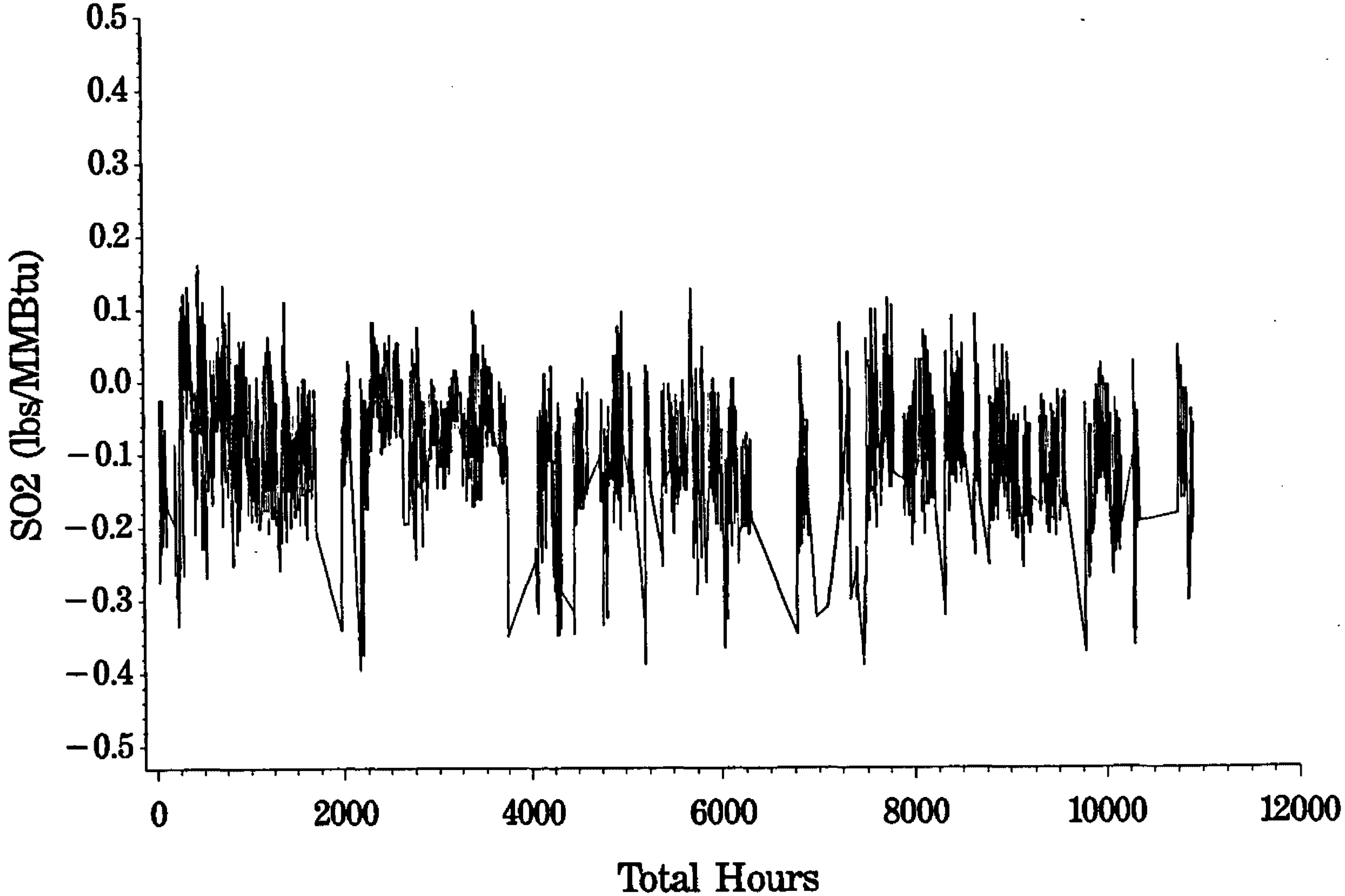
```
Stem Leaf                     #             Boxplot
  80 006                      3                |
  79 09                       2                |
  78 19                       2                |
  77 01256                    5                |
  76 025                      3                |
  75 004                      3             +-----+
  74 56667                    5             |     |
  73 234577                   6             |     |
  72 277                      3             *--+--*
  71 0344579                  7             |     |
  70 1259                     4             |     |
  69 1225579                  7             +-----+
  68 4489                     4                |
  67 3356799                  7                |
  66 8                        1                |
     ----+----+----+----+
Multiply Stem.Leaf by 10**-2


                     Normal Probability Plot
0.805+                                                   * *++ *
     |                                                 *++
     |                                             *****+
     |                                          ****+
     |                                         *+++
     |                                       **+
     |                                    ***
0.735+                                  ***
     |                                +**
     |                             +***
     |                          ++***
     |                        +****
     |                     +***
     |         * ** ***
0.665+   *        +++
      +----+----+----+----+----+----+----+----+----+----+
          -2        -1         0        +1        +2
```

Niagara Mohawk
Hourly SO2 CEM Data (lbs/MMBtu)




Niagara Mohawk
Weekly Oil Sampling Analysis (OSA) SO2 Data (lbs/MMBtu)
SO2 (lbs/MMBtu)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
2000
4000
6000
8000
10000
12000
Total Hours

Niagara Mohawk
Difference CEM – OSA



Northern States Power Co.

```
                                   SAS     10:50 Wednesday, May 13, 1992 12

                             UNIVARIATE PROCEDURE

Variable=SO2CEM

                     Histogram                                  #          Boxplo
   9.25+*                                                        1            *
       .
       .
       .
       .*                                                        1            *
       .
       .*                                                        3            *
       .
       .*                                                        7            *
   4.75+
       .*                                                        3            *
       .*                                                        2            *
       .*                                                        1            *
       .*                                                        3            *
       .*                                                       13            *
       .**                                                     613            0
       .********************************************** 15005         +--+--
       .*                                                      238            0
   0.25+*                                                      191            *
        ----+----+----+----+----+----+----+----+----+---
        * may represent up to 313 counts

                             Normal Probability Plot
              9.25+                                                    *
                  |
                  |                                                    *
                  |                                                    *
                  |                                                    *
              4.75+
                  |                                                    *
                  |                                                    *
                  |                                                    *
                  |                                                    *
                  |                                                    *
                  |                                      +++++++*********
                  |      *************************************
                  |++*****+++++++
              0.25+***
                   +----+----+----+----+----+----+----+----+----+----+
                       -2        -1         0        +1        +2
```

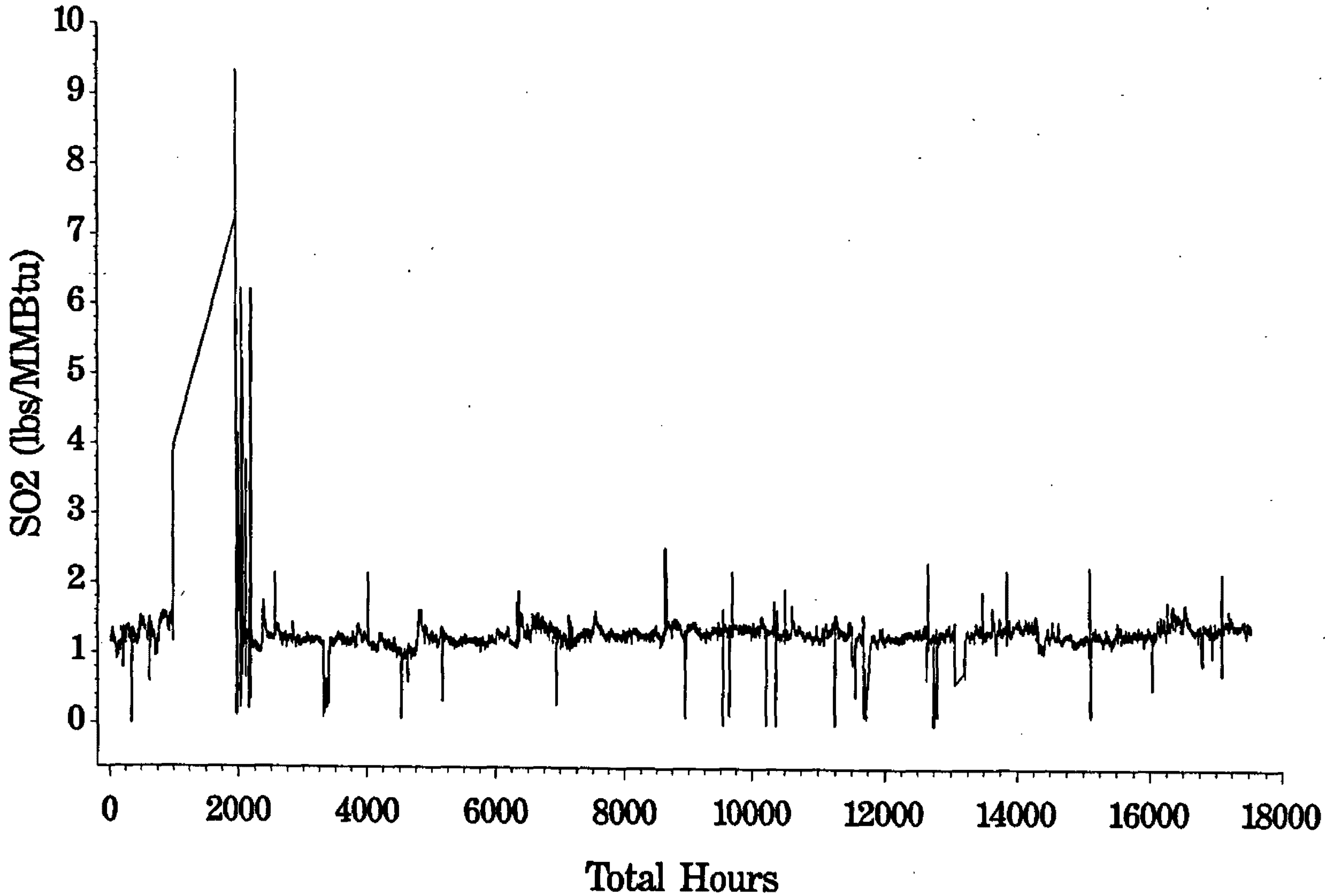
UNIVARIATE PROCEDURE

Variable=SO2CSA

```
                         Histogram                          #            Boxplot
2.15+*                                                      1               *
    .*                                                      1               *
    .*                                                      1               0
    .**                                                     8               0
    .*****                                                 21               0
    .************                                          60               |
    .*****************************                        141            +-----+
    .***********************************************      235            *--+--*
    .****************************                         140            +-----+
    .********                                              40               |
    .***                                                   14               0
1.05+*                                                      5               0
     ----+----+----+----+----+----+----+----+----+--
     * may represent up to 5 counts
```

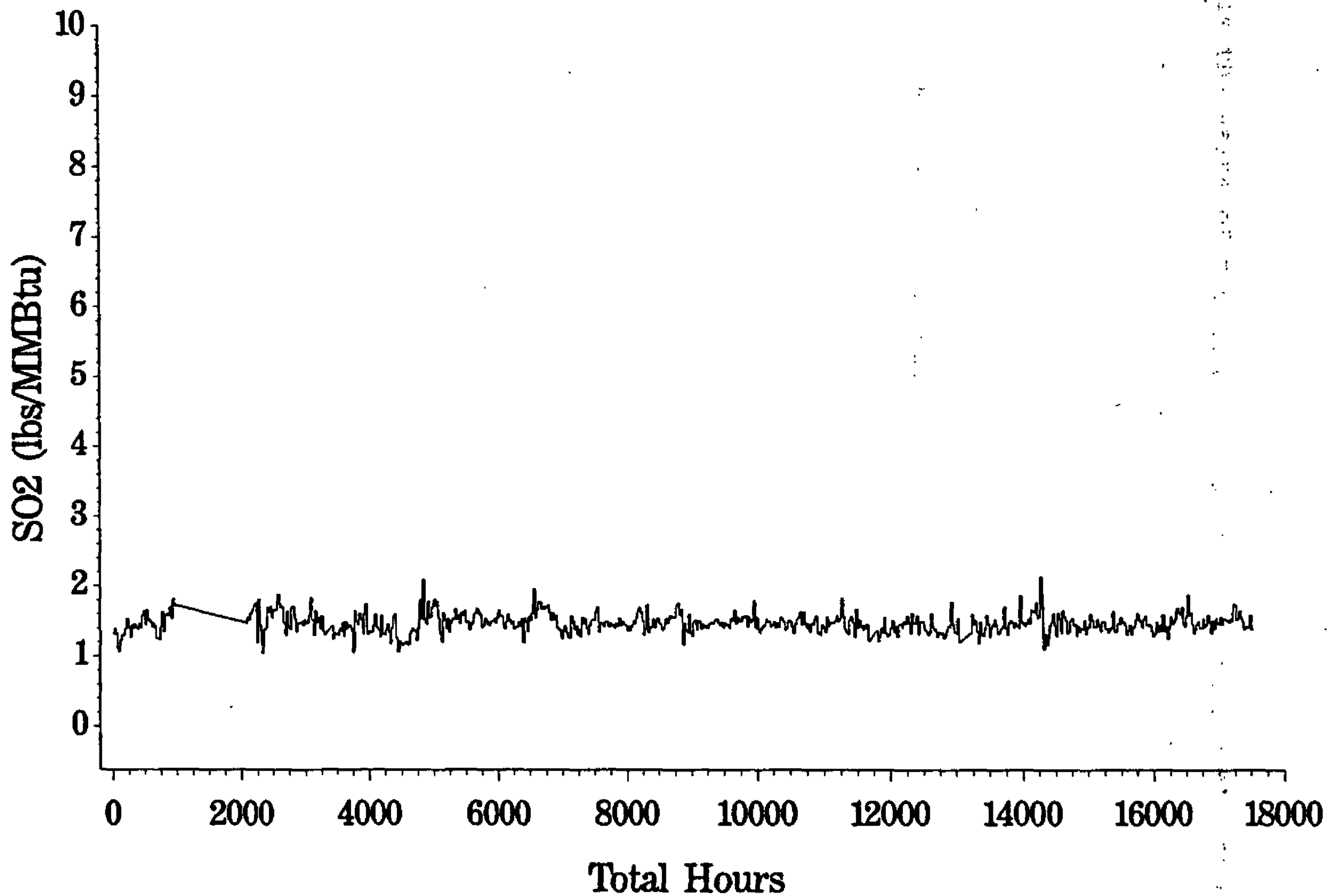
```
                      Normal Probability Plot
2.15+                                                   *
    |                                                   *
    |                                                   *
    |                                               *****
    |                                           *******++
    |                                    +*******
    |                             +*********
    |                     **********
    |             *********
    |       *******++
    |+*******
1.05+*
     +----+----+----+----+----+----+----+----+----+----+
         -2        -1         0        +1        +2
```

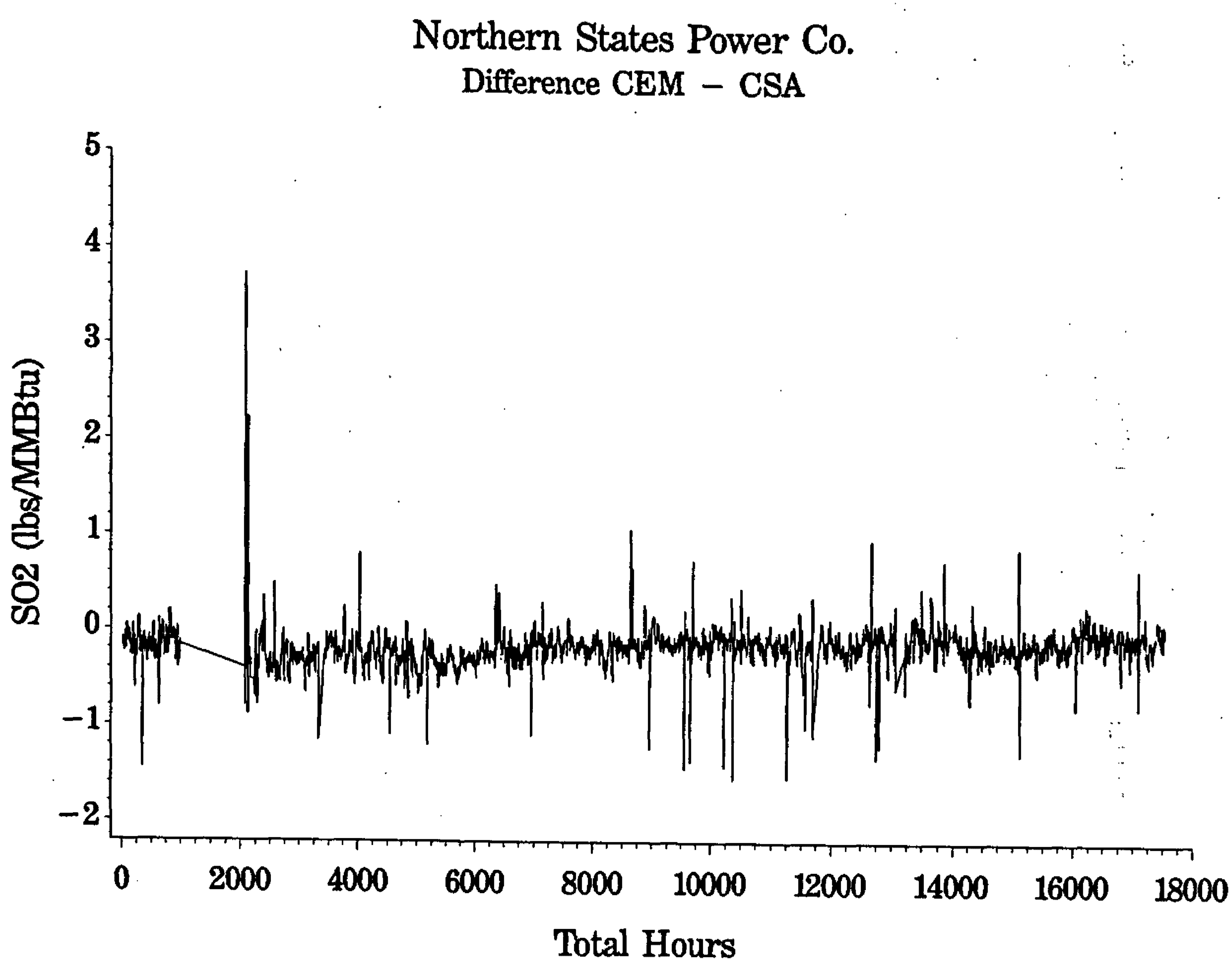
Northern States Power Co.
Hourly SO2 CEM Data (lbs/MMBtu)



Northern States Power Co.

Daily Coal Sampling Analysis (CSA) SO2 Data (lbs/MMBtu)




Northern States Power Co.
Difference CEM – CSA
SO2 (lbs/MMBtu)
5
4
3
2
1
0
−1
−2
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Total Hours

Appendix B

EFFECTS OF AUTOCORRELATION ON STATISTICAL ANALYSES

D. A. DICKEY
Prepared July 28, 1992 for the Cadmus Group

INTRODUCTION

In this paper, we review the concept of autocorrelation, explain how to look for it, and explain how to adjust for it in standard statistical formulas. Formula (7) shows the effect on the variance of a sample mean and formula (10) shows the effect on the estimate of individual variance. The square root of a variance is called a standard deviation and is needed to decide if data points or means are unusual. A data point more than 1.96 standard deviations from the mean will occur by chance only 5% of the time and hence is considered unusual.

Likewise, a sample mean more than 1.96 standard deviations from a hypothesized long run mean casts doubt on that long run mean. Here, of course, standard deviation refers to the standard deviation of a sample mean. When the standard deviation of the mean is estimated from the data, it is referred to as the standard error of the mean. We will consider a sample mean more than 2 standard errors from a hypothesized long term mean as significant evidence against that long term mean but the normal or t tables could be used to provide a slightly more accurate number than 2, depending on the sample size.

1. AUTOCORRELATION

We use statistics to deal with variation. For example, a certain type of automobile may get on average 28 MPG (miles per gallon), but individual mileages will vary around this mean, some particular cars doing better and some worse. If automobile types are to be compared by sampling, this variability must somehow be accounted for.

Most statistical texts concern independent data. For example, if I take a random sample of automobiles, the fact that car 7 is over the mean MPG does not lead me to expect anything in particular about the MPG of car 8 or car 6. When one deviation from the mean tells us nothing about any other, the data are said to be independent. To look at an example where this independence obviously would not hold, consider measurements of flow rate in a stream taken every hour. If the stream is flowing much faster than average now, we would expect it to be flowing faster than average one hour from now, that is, the stream is high now and an hour is not enough time to clear the excess water from the stream.

Pollutants in a stream, in the air, etc. may also exhibit this failure of independence. When data taken over time fail to be independent, we say they are autocorrelated. The most common type of autocorrelation is positive, that is, positive deviations from the mean tend to be followed by positive and negative deviations by negative. In order to adjust standard statistical formulas to deal with autocorrelation, it becomes necessary to pin down the nature of the autocorrelation more precisely. This is the role of time series modeling.

In 1976, a book Time Series Analysis: Forecasting and Control by G. E. P. Box and G. M. Jenkins (Holden-Day publishers) popularized time series modeling. The authors stressed models called AutoRegressive Integrated

Moving Averages, or ARIMA models. A subset of these models, autoregressive or AR models, is discussed below. This subset forms a relatively simple yet powerful class of models.

The autoregressive model of order 1, AR(1), is written as

$$Y(t) = M + r(Y(t-1) - M) + e(t) \quad , t=1,2,3,... \qquad (1)$$

where $Y(t)$ is the value of the data at time t, for example the flow rate of a river at hour t. M is the process long term mean and r is a number strictly between -1 and 1. We interpret r as a proportion when $r>0$. Finally $e(t)$ is an unanticipated error or "shock" to the system as it is sometimes called. This $e(t)$ series is assumed to be an independent sequence with mean 0 and constant variance.

Model (1) expresses the deviation of $Y(t)$ from the mean M as a proportion r of the previous deviation plus an unanticipated shock $e(t)$ and hence is quite realistic for many economic and physical situations.

The AR(1) model can be extended to a general AR(k) model in which $Y(t)$ depends on k previous values as

$$\begin{aligned} Y(t) = M + P1 (Y(t-1) - M) + P2 (Y(t-2) - M) + ... \\ + Pk (Y(t-k) - M) + e(t) \quad , t=1,2,3,... \end{aligned} \qquad (2)$$

where P1, P2, ..., Pk are numbers called autoregressive coefficients and the previous values $Y(t-1)$, $Y(t-2)$, etc. are referred to as lags of Y.

2. REGRESSION ESTIMATES

In section 1, the class of ARIMA models popularized by Box and Jenkins was introduced and one model, the autoregressive order 1 or AR(1) was singled out. In this and the next section, we look at how we can tell if AR(1) is appropriate for our data. The main tools here will be least squares regression and the autocorrelation function.

Least squares regression is a topic covered in standard statistical textbooks. The application of regression to time series is covered in detail in chapter 8 of the book Introduction to Statistical Time Series by Wayne Fuller (Wiley 1976). In an example on page 341 and 342, Fuller shows how to determine the necessary number of lags in a model by running a regression on many lags then using standard tests statistics, t and F produced by most regression programs, to decide how many lags can be omitted. If we can omit all but $Y(t-1)$ from our model, then the AR(1) is appropriate.

Alternatively, we could look at the partial autocorrelation function which is computed by most time series packages, such as PROC ARIMA in the SAS computer package (SAS is the registered trademark of SAS Institute, Cary, N. Carolina). The jth partial autocorrelation coefficient is essentially the lag j coefficient in the multiple regression of $Y(t)$ on $Y(t-1)$, ..., $Y(t-j)$ as explained in The SAS System for Forecasting Time Series by J. C. Brocklebank and D. A. Dickey (SAS Institute publishers). Only the lag 1 partial autocorrelation would estimate a nonzero value for an AR(1).

3. AUTOCORRELATION FUNCTION

Another way to determine if an AR(1) is appropriate is to look at the autocorrelation function R(j). R(j) is the correlation between Y(t) and Y(t-j) where j is called the lag number. This function is produced by most time series computer programs, for example PROC ARIMA and PROC AUTOREG in the SAS computer package .

Specifically, R(j) = G(j)/G(0) where G(j) is called the autocovariance function and is defined as the covariance between Y(t) and Y(t-j). Letting the variance of e be denoted V(e), we find from Fuller (page 37 equation 2.3.5) that the covariance at lag j for an AR(1) model is

Autocovariance = G(j) = r**j V(e)/(1-r**2) (3)

where ** denotes exponentiation and * denotes multiplication, e.g. 3*5=15, 3**2 = 3*3=9, 2**3 = 2*2*2=8. Now the variance of Y is G(0), that is, the variance of Y is, for an AR(1) model,

variance of Y = G(0) = V(e)/(1-r**2)

so we see that the variance of the shocks, V(e), can be quite different than the variance, G(0), of the data if r is near 1.

Using R(j) = G(j)/G(0) and equation (3) it is easily seen that

R(j) = r**j (4)

for the AR(1) model. It is important to note that these formulas are only appropriate for autoregressive order 1 models, not for moving average models, general ARIMA models, or general autoregressive order k models. Equation (4) shows that the autocorrelation function of an AR(1) decays exponentially.

4. EXAMPLE

A dataset of 60 observations using model (1) with r=.7 and M=100 is generated in SAS and analyzed with PROC ARIMA. Here is the program and part of the output;

PROGRAM

```
ata epa; y=100 + 10*normal(1827651)/sqrt(1-.7**2); output;
 do i=2 to 60; y=100 + .7*(y-100) + 10*normal(1827631); output;
nd;

roc arima; identify var=y nlag=10;
```

OUTPUT

```
Name of variable = Y.
Mean of working series = 91.87989
Standard deviation     = 9.979986
Number of observations =       60
```

Notice that the autocorrelations die off in approximately an exponential manner at least for the first few lags. The dots represent two standard errors so lines of asterisks extending beyond the two standard errors indicate statistically significant (non zero) autocorrelations. We are saying that if the autocorrelation were truly 0, an estimated autocorrelation more than 2 standard errors from 0 would be unusual and hence we reject the idea of 0 autocorrelation based on our estimate. Here, of course, standard error refers to a standard error appropriate for autocorrelation estimates.

```
                          Autocorrelations

Lag Covariance Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  0     99.600     1.00000 |                    |********************|
  1  49.884765     0.50085 |               .    |**********          |
  2  25.278038     0.25380 |              .     |*****.              |
  3   7.336610     0.07366 |             .      |*     .             |
  4   7.443015     0.07473 |             .      |*     .             |
  5  11.986170     0.12034 |             .      |**    .             |
  6  -1.278828    -0.01284 |             .      |      .             |
  7   1.552965     0.01559 |             .      |      .             |
  8   0.862817     0.00866 |             .      |      .             |
  9  -6.552627    -0.06579 |             .     *|      .             |
 10 -11.546358    -0.11593 |             .    **|      .             |
                           "." marks two standard errors
```

The partial autocorrelations show one nonzero lag value, .50085, and the rest are insignificant, being within the two standard error bounds:

```
                   Partial Autocorrelations

Lag Correlation -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.50085 |               .    |**********          |
  2     0.00393 |               .    |    .               |
  3    -0.07331 |               .   *|    .               |
  4     0.08643 |               .    |**  .               |
  5     0.08939 |               .    |**  .               |
  6    -0.16725 |               . ***|    .               |
  7     0.08753 |               .    |**  .               |
  8     0.01601 |               .    |    .               |
  9    -0.14884 |               . ***|    .               |
 10    -0.05415 |               .   *|    .               |
```

The estimated autocorrelations and partial autocorrelations seem to be in line with what would be expected for an autoregressive order 1 series.

5. VARIANCE OF THE MEAN, INDEPENDENT SAMPLES

This section deals with sample means. Returning to the example of MPG in cars, suppose a particular brand of car has mean 28 MPG (for the entire fleet of all such cars ever to be produced). If I take a random sample of 10 cars from the production line and measure MPG, I might get a sample average 25.0 MPG. Another sample of 10 might have a sample average 27.2 MPG and another 28.3. Is this much variation in means of samples of 10 cars reasonable? It depends on the individual car-to-car variation, V, in MPG.

If I know the variance V of MPG from car to car, I can compute the variance among sample means from samples of size 10 to see if 25.0, 27.2, and 28.3 are reasonable numbers. The formula when the data are independent is

$$\text{variance of means} = V/n \tag{5}$$

where V is the individual car-to-car variance for these independent cars, and n is the number of cars in each sample. This formula is very well known, for example, see Snedecor and Cochran's Statistical Methods, eighth edition, page 43 (Iowa State University Press, publisher). For a time series, of course, formula (5) becomes

$$\text{variance of means} = G(0)/n \tag{5a}$$

If the estimated variance in MPG is 14.4 then the estimated variance associated with a mean of 10 would be 14.4/10 = 1.44 and the corresponding standard error of the mean (square root of this variance) is 1.2 so none of our sample means is more than 2 standard deviations from the fleet average 28 MPG. If, instead of 1.2, the standard error were 0.5, then the sample mean 25 would be quite unusual since it would now be 4 standard errors away from the fleet average. Clearly, the decision of whether a sample mean is statistically significantly far from any stated value depends on having the correct standard error available. Recall that the cutoff value of 2 standard errors can be refined by referring to tables of the t distribution.

6. EFFECT OF AUTOCORRELATION ON THE VARIANCE OF THE MEAN.

For autocorrelated data, the variance of a sample mean is no longer given by formula (5). A formula giving an approximation to the variance is on page 194, (6.3.17) of Box and Jenkins. This same formula in a different form is given in Corollary 6.1.1.2, page 232 of Fuller. The formula, while approximate, holds for a large subset of ARIMA models including the AR(1) we are discussing here.

We can do better than an approximation if we restrict ourselves to the AR(1) case. In particular, the Fuller text page 232 line 10 shows that the variance of a sample mean of n consecutive values of a time series Y(t) is exactly given by

$$[n\,G(0) + 2(n-1)\,G(1) + 2(n-2)\,G(2) + 2(n-3)\,G(3) + \ldots + 2\,G(n-1)]/(n*n) \tag{6}$$

where G(j) is the autocovariance of the time series in question. In the AR(1) case, G(j) is given by (3) and is seen to be G(j) = G(0) r**j so we can plug G(j) = G(0) r**j into expression (6) and do some algebraic simplification to get the desired variance. Note that formulas (3) and (6) are all we really need. One could write a computer program to evaluate (3) and use it in (6) for every case, however, algebraic reduction will give us a nice formula, (7), for the AR(1) case that will be easy to use.

I now show the algebra just to provide a technical reference. Let

$$S = [n + 2(n-1)\,r + 2(n-2)\,r**2 + \ldots + 2\,r**(n-1)]$$

so that expression (6) is S*G(0)/(n*n). Notice that, for independent data, r=0 so S=n and expression (6) reduces to expression (5a). All we need to do is algebraically reduce S then calculate S*G(0)/(n*n). Now write S and then multiply S by r getting

```
 S =  [n + 2(n-1) r + 2(n-2) r**2 +  ... + 2 r**(n-1)]
rS =  [          n r + 2(n-1) r**2 + 2(n-2) r**3 +  ... + 2 r**n]
```

and subtracting,

```
(1-r)S = n + (n-2)r - 2[r**2 + r**3 + r**4 + ... + r**n]

       = n(1+r) - 2[r + r**2 + r**3 + r**4 + ... + r**n]

       = n(1+r) - 2r[1 + r + r**2 + r**3 + ... + r**(n-1)]

       = n(1+r) - 2rD
```

where D = [1 + r + r**2 + r**3 + ... + r**(n-1)]. Computing D and rD we have

```
D = [1 + r + r**2 + r**3 + ... + r**(n-1)]
rD =   [ r + r**2 + r**3 + ... + r**(n-1) + r**n]
```

and subtracting

(1-r)D = 1-r**n so D = (1-r**n)/(1-r).

Now that we have D and (1-r)S = n(1+r) - 2rD we solve for S as

```
S = n(1+r)/(1-r) -2r[(1-r**n)/(1-r)]/(1-r)

  = n(1+r)/(1-r) -2r(1-r**n)/(1-r)**2.
```

Remember that expression (6), the variance of the mean in the AR(1) case, is S G(0)/(n*n). Thus expression (6) becomes, for the AR(1) case,

```
 ( G(0)/n ) [ (1+r)/(1-r) -2(r/n)(1-r**n)/(1-r)**2 ]          **(7)**.
```

This is our target formula. Notice that if the data are independent, r=0 and the expression in square brackets becomes 1. Since G(0) is the variance of Y this gives the well known formula (5a) for the variance of the mean of n independent observations. The expression in square brackets is thus seen to be an inflater for the usual variance formula. This expression does not approach 1 when n gets large and hence is an important adjustment even in very large samples.

As an example, with 50 observations, autocorrelation r=.8 (first order autocorrelation to be exact), and process variance G(0) = 128, the usual formula for the variance of the mean in independent data would give G(0)/n=128/50 = 2.56 and the standard error would be 1.6. This is incorrect, however since formula (7) shows that 2.56 should be multiplied by

```
    [(1+r)/(1-r) -2(r/n)(1-r**n)/(1-r)**2] =
     (1.8)/.2 - 2(.016)(1-.8**50)/(.04) =
      9 - 0.032(1-0.0000143)/(.04) = 8.2
```

giving the proper variance 20.99 and standard error 4.58. Notice that this is quite an adjustment.

7. EFFECT ON THE ESTIMATE OF INDIVIDUAL VARIANCE

In the above example, the variance of the data, G(0), was assumed to be the known value 128. Of course, the true variance is never known and it must be estimated from the data, using the sample variance. The usual formula for the sample variance is

$$S^2 = (Y(1) - \bar{Y})**2 + (Y(2) - \bar{Y})**2 + ... + (Y(n) - \bar{Y})**2]/(n-1)$$

$$= [Y(1)**2 + Y(2)**2 + ... + Y(n)**2 - n\ \bar{Y}**2\ ]/(n-1) \qquad (8)$$

where $\bar{Y}$ = [Y(1) + Y(2) + ... + Y(n)]/n is the sample mean. For the numbers 10, 12, 13, 7, 8 we get

$$\bar{Y} = 10, \quad S^2 = [0 + 4 + 9 + 9 + 4]/4 = [100+144+169+49+64-2500/5]/4 = 26/4 = 6.5.$$

Why is n-1 instead of n used in the denominator of S^2 ? To answer this, use lower case letters and let y(t) = Y(t)-M where, as before, M is the theoretical mean. The first part of expression (8) shows that y(t) can be substituted for Y(t) and we get

$$(n-1)S^2 = (y(1) - \bar{y})**2 + (y(2) - \bar{y})**2 + ... + (y(n) - \bar{y})**2]$$

$$= [y(1)**2 + y(2)**2 + ... + y(n)**2 - n\ \bar{y}**2\ ] \qquad (9).$$

where $\bar{y} = \bar{Y}$-M is the mean of the y(t) values (y(t)-$\bar{y}$ = Y(t)-M - $\bar{Y}$+M).

Now the last part of expression (9) is the numerator of S^2 and if we take its expected value (the mean value in repeated sampling) assuming independent observations, we get

$$G(0) + G(0) + G(0) + ... + G(0) - n\ (G(0)/n) = (n-1)G(0) \qquad (10)$$

because the expected value of each y(t)**2 is by definition the variance of Y (this being G(0)) and the last term is by definition n times the variance of the sample mean. This shows that division by n-1 is required to get an unbiased estimator, that is, one whose expected value is the quantity to be estimated. Notice that if the data are from an AR(1) process, the n(G(0)/n) in formula (10) would need to be replaced by

$$n\ G(0)/n\ [(1+r)/(1-r)\ -2(r/n)(1-r**n)/(1-r)**2]$$

so that the numerator of S^2 would now estimate

$$G(0)[\ n - (1+r)/(1-r)\ +2(r/n)(1-r**n)/(1-r)**2]$$

and S^2 would estimate this divided by (n-1), namely

$$G(0)[\ 1 + 1/(n-1)[-2r/(1-r)]\ \ +2(r/n)(1-r**n)/(1-r)**2/(n-1)\ ]$$

so that if we divided S^2 by

[1 - 2r/[(n-1)(1-r)] +2(r/n)(1-r**n)/(1-r)**2/(n-1)] **(10)**

we would have an unbiased estimate of G(0). In the example from section 6, the sample variance estimates [(50)(128) - (50)(20.74)]/49 = .853 G(0), in other words we would multiply the sample variance by 1/0.853 = 1.172 to get an unbiased estimate.

Expression (10) approaches 1 as n gets large and hence becomes less important as the sample size increases although for fixed n, the value of r also plays an important role.

8. SUMMARY

In this paper two formulas were developed for dealing with autocorrelation. Autocorrelation is assumed to arise from a first order autoregressive process with autocorrelation parameter r and process variance G(0).

(G(0)/n) [(1+r)/(1-r) -2(r/n)(1-r**n)/(1-r)**2] **(7)**.

[1 - 2r/[(n-1)(1-r)] +2(r/n)(1-r**n)/(1-r)**2/(n-1)] **(10)**

Formula (7) shows a multiplicative adjustment to the usual formula, G(0)/n, for the variance of a sample mean. For r>0 we conclude that autocorrelation increases the variability of sample means around the long run mean. The sample variance estimates a multiple of the true variance of individuals. Formula (10) gives that multiple. If r=0, the multiple is 1 so the sample variance is unbiased. If r>0, the multiple is less than 1 and we say the sample variance is biased downward.

Since the sample variance is defined in terms of deviations from the sample mean, the interpretation is that the points vary less around the sample mean under autocorrelation than they do for independent data. Dividing the sample variance by expression (10) provides an unbiased estimator. As n gets large, expression (10) approaches 1 for any fixed r and hence is less important for large samples.

The formulas involve r which is an unknown, but estimable, quantity. Plugging in an estimated r obviously produces an approximate adjustment.

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SYMBOLS

M	theoretical, or long term, mean
V	theoretical variance
G(0)	theoretical variance of time series
V(e)	theoretical variance of shocks in time series
G(j)	covariance at lag j
R(j)	autocorrelation at lag j. R(j) = G(j)/G(0)
AR(1)	autoregressive order 1 model
r	lag 1 autocorrelation in an AR(1)
n	sample size, number of observations
$\bar{Y}$	sample mean
S^2	standard formula for sample variance

Appendix C

Virginia Power Co., Chesapeake Energy Center, Unit #4
Alternative Monitoring System Study --- Subsets Summary

OBS	LABEL (in	Setno dataset)	Subset (in tables)	Start Date	Start Time	End Date	End Time	n (before lagging)
1	a-c	20	1	10992	7	20892	7	683
2	a-c	21	2	20892	8	30992	8	682
3	a-c	23	3	10992	12	20892	12	683
4	a-c	24	4	20892	13	30992	13	682
5	a-c	26	5	10992	17	20892	17	683
6	a-c	27	6	20892	18	30992	18	682
7	a-c	29	7	10992	22	20892	22	683
8	a-c	30	8	20892	23	30992	23	682
9	a-c	32	9	11092	3	20992	3	683
10	a-c	33	10	20992	4	31092	4	682
11	a-c	35	11	11092	8	20992	8	683
12	a-c	36	12	20992	9	31092	9	682
13	a-d	40	1	10992	7	20892	7	653
14	a-d	41	2	20892	8	30992	8	670
15	a-d	43	3	10992	12	20892	12	653
16	a-d	44	4	20892	13	30992	13	670
17	a-d	46	5	10992	17	20892	17	653
18	a-d	47	6	20892	18	30992	18	670
19	a-d	49	7	10992	22	20892	22	653
20	a-d	50	8	20892	23	30992	23	670
21	a-d	52	9	11092	3	20992	3	653
22	a-d	53	10	20992	4	31092	4	670
23	a-d	55	11	11092	8	20992	8	653
24	a-d	56	12	20992	9	31092	9	670
25	a-e	60	1	10992	7	20892	7	672
26	a-e	61	2	20892	8	30992	8	663
27	a-e	63	3	10992	12	20892	12	672
28	a-e	64	4	20892	13	30992	13	663
29	a-e	66	5	10992	17	20892	17	672
30	a-e	67	6	20892	18	30992	18	663
31	a-e	69	7	10992	22	20892	22	672
32	a-e	70	8	20892	23	30992	23	663
33	a-e	72	9	11092	3	20992	3	672
34	a-e	73	10	20992	4	31092	4	663
35	a-e	75	11	11092	8	20992	8	672
36	a-e	76	12	20992	9	31092	9	663
37	c-d	140	1	10992	7	20892	7	661
38	c-d	141	2	20892	8	30992	8	676
39	c-d	143	3	10992	12	20892	12	661
40	c-d	144	4	20892	13	30992	13	676
41	c-d	146	5	10992	17	20892	17	661
42	c-d	147	6	20892	18	30992	18	676
43	c-d	149	7	10992	22	20892	22	661
44	c-d	150	8	20892	23	30992	23	676
45	c-d	152	9	11092	3	20992	3	661
46	c-d	153	10	20992	4	31092	4	676
47	c-d	155	11	11092	8	20992	8	661
48	c-d	156	12	20992	9	31092	9	676
49	c-e	160	1	10992	7	20892	7	707
50	c-e	161	2	20892	8	30992	8	701
51	c-e	163	3	10992	12	20892	12	707
52	c-e	164	4	20892	13	30992	13	701

Virginia Power Co., Chesapeake Energy Center, Unit #4
Alternative Monitoring System Study --- Subsets Summary

OBS	LABEL	Setno (in dataset)	Subset (in tables)	Start Date	Start Time	End Date	End Time	n (before lagging)
53	c-e	166	5	10992	17	20892	17	707
54	c-e	167	6	20892	18	30992	18	701
55	c-e	169	7	10992	22	20892	22	707
56	c-e	170	8	20892	23	30992	23	701
57	c-e	172	9	11092	3	20992	3	707
58	c-e	173	10	20992	4	31092	4	701
59	c-e	175	11	11092	8	20992	8	707
60	c-e	176	12	20992	9	31092	9	701
61	d-e	180	1	10992	7	20892	7	647
62	d-e	181	2	20892	8	30992	8	658
63	d-e	183	3	10992	12	20892	12	647
64	d-e	184	4	20892	13	30992	13	658
65	d-e	186	5	10992	17	20892	17	647
66	d-e	187	6	20892	18	30992	18	658
67	d-e	189	7	10992	22	20892	22	647
68	d-e	190	8	20892	23	30992	23	658
69	d-e	192	9	11092	3	20992	3	647
70	d-e	193	10	20992	4	31092	4	658
71	d-e	195	11	11092	8	20992	8	647
72	d-e	196	12	20992	9	31092	9	658

Pennsylvania Electric Co., Homer City Unit #1
Alternative Monitoring System Study --- Subsets Summary

OBS	Unit	Setno	Start Date	End Date	n (before lagging)
1	1	6	53185	62985	30
2	1	10	92885	102785	28
3	1	21	82486	92286	28
4	1	24	112286	122186	30

Pennsylvania Electric Co., Homer City Unit #3
Alternative Monitoring System Study --- Subsets Summary

OBS	Unit	Setno	Start Date	End Date	n (before lagging)
1	3	3	30285	33185	28
2	3	5	50185	53085	30
3	3	7	63085	72985	28
4	3	8	73085	82885	28
5	3	16	32786	42586	30
6	3	17	42686	52586	30
7	3	20	72586	82386	29
8	3	21	82486	92286	29

Northern States Power Co., Sherburne County Unit #3
Alternative Monitoring System Study --- Subsets Summary

OBS	Setno	Start Date	Start Time	End Date	End Time	n (before lagging)
1	1	10189	1	13089	24	715
2	4	40189	1	43089	24	672
3	6	53189	1	62989	24	720
4	7	63089	1	72989	24	720
5	8	73089	1	82889	24	680
6	9	82989	1	92789	24	720
7	10	92889	1	102789	24	720
8	11	102889	1	112689	24	696
9	12	112789	1	122689	24	720
10	13	122789	1	12590	24	720
11	14	12690	1	22490	24	720
12	15	22590	1	32690	24	720
13	16	32790	1	42590	24	698
14	18	52690	1	62490	24	720
15	21	82490	1	92290	24	704
16	22	92390	1	102290	24	720
17	23	102390	1	112190	24	720

Niagara Mohawk, Oswego Unit #6
Alternative Monitoring System Study --- Subsets Summary

OBS	Setno	Start Date	Start Time	End Date	End Time	n (before lagging)
1	2	13190	0	30190	23	690
2	5	50190	0	53090	23	671