CONTINUUM ELECTROMECHANICS GROUP

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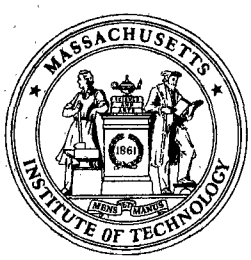
CHARGED DROPLET TECHNOLOGY FOR REMOVAL
OF PARTICULATES FROM INDUSTRIAL GASES

by

J. R. Melcher and K. S. Sachar

Final Report under Task No. 8
Contract #68-002-0018

August 1, 1971



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CHARGED DROPLET TECHNOLOGY FOR REMOVAL OF PARTICULATES FROM INDUSTRIAL GASES

by

J. R. Melcher and K. S. Sachar

I. Introduction

A. Background

Nature gives an example of how drops can scavenge particulate from the atmosphere. It is well documented that falling raindrops effectively reduce the number density of particulate in the range of 1 - 50 μm[1]. Further, the existence of atmospheric electric fields has encouraged investigation of possible influences of particle and drop charging on the collection of particulate by the drops [2]. Hence, it is not surprising that the idea of using electric fields to enhance collection of particulate on drops is often suggested as an approach to controlling particle emissions in industrial gases.

We are concerned with a class of devices that is hybrid between scrubbers and electrostatic precipitators. The scrubber makes use of drops usually moving relative to the particle-laden gas, while the precipitator depends on electrical forces caused by charging the particles and subjecting them to an electric field. The contrast between these conventional devices correctly suggests that there are a number of basically different mechanisms for collecting the particles on the drops.

B. Outline

The first objective is a review of what has been reported through patents and in the formal literature relating to the use of drops and

electric fields in particulate control. Devices range from "electrically augmented inertial impact scrubbers" to "electrostatic precipitators with pretreated particles". Somewhere between is a class of devices which take unique advantage of the combination of fields and drops. It is the intent of the survey given in §II not only to cite the relevant reported work, but also to classify the devices according to the type of interaction between drops and particles, and organize the literature within that framework.

There are many combinations of scrubbers and precipitators that offer no new mechanism for removing particles, albeit possible improvements in a technological sense. Our approach to the classification emphasizes what is unique to the use of drops and electric fields in collecting particles: the interaction between fields-drops and particles.

A second objective, in §III, is a sketch of fundamental models appropriate to each class of interaction. The simple models given in §III serve at least two purposes: they further clarify the basic mechanisms used for classification in §II, and by showing parameter dependences, they make possible comparison of systems in §V.

The third objective is to place the basic interaction mechanisms in the context of systems. Important factors beyond the basic drop-particle interaction are the means by which drops are formed, charged, and injected, and means for their removal. Electric fields have been used in devices for producing charged drops, hence the discussion of drop generators is taken up in a separate section, §IV. Then comparison between systems is discussed in § V.

The fourth objective is an assessment of needed research and an identification of promising types of devices; this is given in §VI.

II. Survey and Classification of Devices

A. Process Functions

A schematic view of the processes called for in an electrostatic-droplet type of device is shown in Fig. II-1.

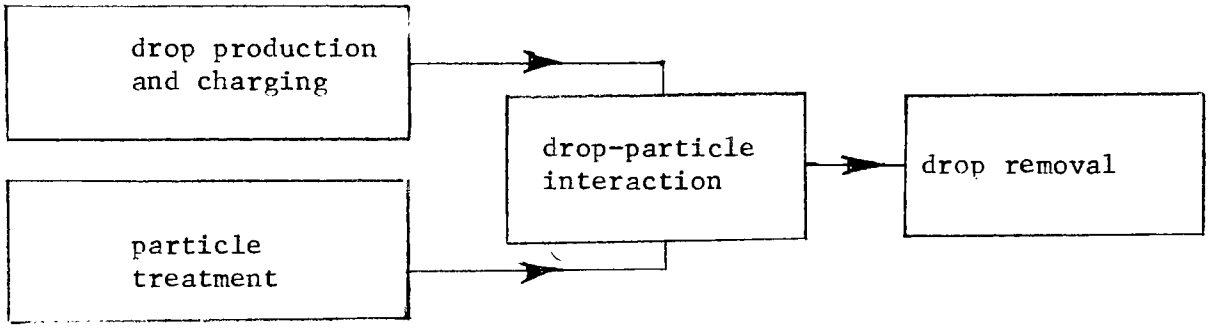

Fig. II-1 Schematic of processes that must occur in complete particle removal system

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The practicality of a device, of course, depends on the means for producing and charging the droplets and for charging the gas-entrained particulate prior to collection of particles on drops. Also, once collected on the drops, particles are removed by removing the drops and the provision for drop collection is also essential. But the heart of the drop-particle approach is in the mechanism by which the particles are collected on the drops. In reviewing and classifying devices, we first focus on the interaction mechanism. Distinction between devices in a given class is then based on methods for prior or subsequent processing of the drops and particles.

B. Classification of Drop-Particle Interactions

In the zone of drop-particle interaction, there can be many combinations of drop-charge, particle-charge, and electric field configuration.

The classification given by Table II-1 makes a compromise between the proliferation of categories that results from a refined view of the interaction mechanism, and the need for simplification which is the objective of a classification in the first place. A given class is specified by the electrical state of the drops and of the particles, and by whether or not there is a significant ambient or macroscopic electric field playing a dominant role. Before discussing the classes of interaction, comments pertaining to terminology are appropriate.

Ambient Electric Fields: By an "ambient electric field intensity", $\bar{E}$, we mean one which should be distinguished from the component of electric field that fluctuates spatially over distances on the order of the interparticle or interdrop spacing. The sketch of Fig. II-2a shows a region in the interaction zone occupied by charged drops and particles. In general, these can each be of both polarities. Gauss' law requires that the total flux of electric field normal to the surface, S, of a volume V enclosing some of the drops and particles be equal to the net charge contained within that volume.

$$\oint_S \varepsilon_o \bar{E} \cdot \bar{n} da = \int_V \rho_f dV \quad . \tag{II.1}$$

Here, $\varepsilon_o = 8.85 \times 10^{-12}$, $\bar{n}$ is the unit normal to S, and ρ_f the net charge density. Made small enough, V can contain one particle or drop. As it increases in size so as to include more charges, it is evident that a part of the associated electric field varies rapidly over a characteristic length typified by the distance between particles. But, as the volume begins to include many particles, there is a contribution to the electric field that is steady, and that increases in proportion to the net charge

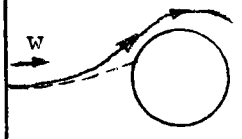
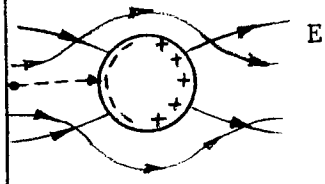
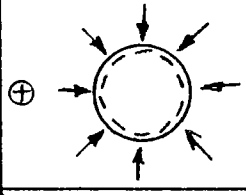
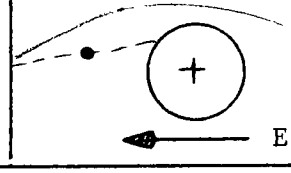
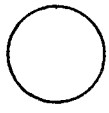
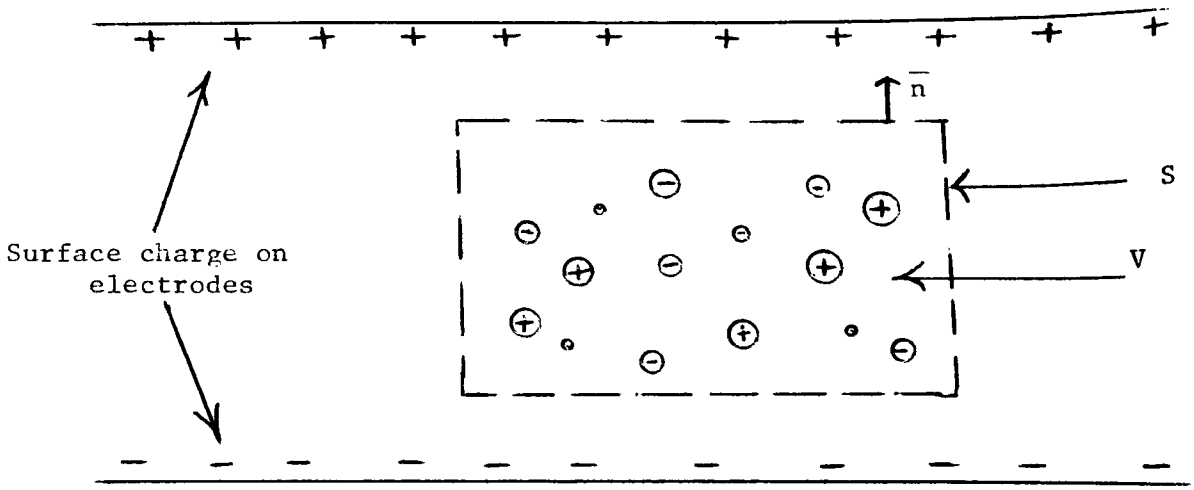
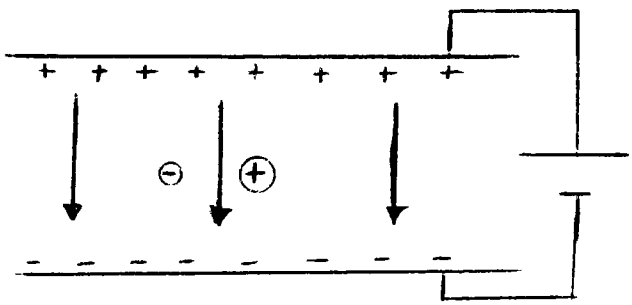
Drop \ Particle	Uncharged	Charged; No Ambient E	Charged, Ambient E
Uncharged	I. Mechanical scrubbers; inertial impact from drop injection. 		IV. Collection on half-surface of polarized drop having no net charge. 
Charged; no ambient E		III. Drops having net charge collect particles having net opposite charge. 	
Charged, and ambient E	II. Electrical scrubbing; inertial impact through electrical propulsion of drops. 		V. Combination of II, III and IV 

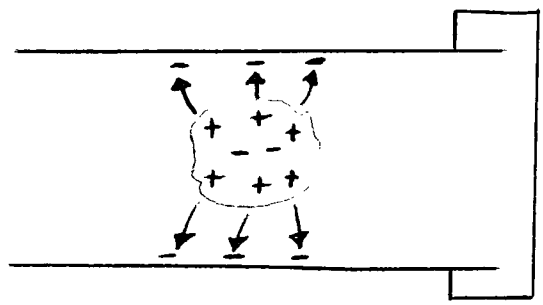
Table II-1 Classification of Particle-Drop Interaction Mechanisms



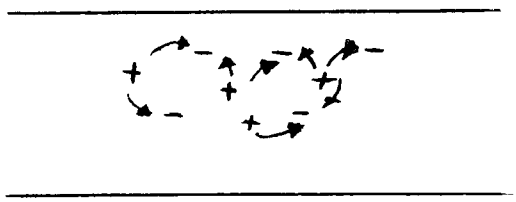
(a)



(b)



(c)



(d)

Fig. II-2 (a) Flux of electric field out of S enclosing V, is proportional to net charge enclosed: (b) Field due almost entirely to surface charges on electrodes; (c) Field induced by "space charge" caused by net effect of drop and particle charging; (d) No ambient field, but microfield due to particles.

enclosed. It is this average component of $\bar{E}$ that we call the "ambient" $\bar{E}$.

There are two ways of producing an ambient $\bar{E}$. Usually, as in a conventional electrostatic precipitator, a voltage is applied to electrodes, as sketched in Fig. II-2b. Charged particles and drops in the intervening volume contribute to the field, but often to a negligible degree simply because they are outnumbered by charges on the walls. The ambient field is then termed an "imposed field". If, on the other hand, there is enough net charge in the volume, even with the electrodes grounded, an ambient $\bar{E}$ can be generated by virtue of net charge carried by the combination of drops and particles in the volume. Such an ambient $\bar{E}$ will be called a "space charge field", since it arises from the space-average net charge density in the volume making an appreciable contribution.

The microelectric fields are illustrated by the extreme depicted in Fig. II-2d, where there are as many charges of one sign as of the other and no external excitations to induce charges on electrodes. Then, the only electric fields are at the micro level, because by Gauss' law, any volume enclosing many particles includes no net charge.

The electrical force on a particle having charge q is $q\bar{E}$. Hence, the micro fields represent forces of attraction or repulsion between neighboring particles, while the macro, or ambient, fields represent forces tending to carry particles from one macroscopic region of net charge to another. In Fig. II-2b, the ambient field carries the particles toward one or the other electrode, while in Fig. II-2c, the ambient field tends to make the positive charge carriers "explode" toward the image charges on the electrodes.

It is, of course, not possible to have even one charge in the interaction region without having it make a contribution to the electric field. But for practical purposes, we distinguish between interactions that are dominated by the ambient $\overline{E}$ or by a micro-$\overline{E}$. We will further distinguish between ambient electric fields generated by external sources applied to electrodes and those generated by space charge from the drops and particles. These ambient electric fields are respectively termed "imposed" and "space charge" electric fields.

Attachment: The classification pertains to practical mechanisms by which particles are made to collide with drops. The implicit assumption is that the collision is tantamount to attachment. Although somewhat conflicting, what studies there are of attachment between typical "dust" particles and water drops tend to support this view. In a review of wet scrubbers, Weber makes the statement [3]: "If dust particles and water drops collide, the former will invariably attach themselves to the latter". He goes on to qualify this statement by distinguishing between the manner in which wetting and nonwetting particles adhere to the drop. The wetted particles are ingested into the drop volume, while the nonwetting particles tend to be retained at the surface. In either case, the particles adhere to the drops. Weber's emphatic prediction that industrial particulates are attached upon collision is at some variance with McCully's observations [1] in which experiments are cited that show nonwettable beads can bounce off water drops, and hence that some collisions with nonwetting particles are not tantamount to attachment. With additives in the water, it appears that a wide range of particulate readily attach on collision, at least in the absence of an electrical charging.

But a further question is the possible influence of the electric charging on attachment. Here, it is important to define what is meant by "charging" and the region occupied by the associated electric field. When we refer to "charging a drop", we mean that there is a layer of net charges per unit area at the gas-liquid interface responsible for the discontinuity between electric field just outside the water, and the zero field region inside. This field is on the order of 10^6 v/m, and is limited by electrical breakdown or electrohydrodynamic instability, as discussed in § IV. It should be distinguished from fields associated with double layers of charge in a zone on the order of 100 Å at the drop-gas interface. It is this double-layer charge that is closely tied up with the theory of wetting. Fields in the double layer can be far greater than those possible in the gas phase region outside the drop.

The surface tension is known to depend on charging of the double layer, and if it were such a charging that was of interest here, then we would expect that the attachment would depend on charging. By definition, charging of the double layer, which can be thought of as a thin capacitor with spherical electrodes in the neighborhood of the drop interface, leaves the drop with no net charge. The type of charging referred to here leaves the drop with a net charge, and results in an electric field in the region exterior to the drop. This exterior field is far weaker than that in the double layer, and hence too small to influence the surface tension. It has been experimentally verified that even water-air interfaces which support monomolecular films have surface tensions that are independent of electric field up to strengths sufficient to produce electrical breakdown [4].

Because we would expect attachment to be closely related to surface tension, this finding tends to support the view that there should be little effect of the field on the mechanisms of attachment present in the absence of the fields.

One mechanism by which the electric field can influence attachment is through charge exchange upon collision. The drop is relatively highly conducting, and depending on their electrical properties, the particles can acquire the same sign of charge as the drop on collision, and then be repelled by electrostatic forces. Some work has been done on the competition between these electrostatic forces and forces of adhesion on solid surfaces [5]. Also, there are studies of electric field influence on drop-drop 'adhesion" [65]. The evidence is that attachment may be a consideration, but that the relative merits of using electrical forces for initiating particle-drop collision can over-ride the attachment question. That attachment takes place with a wide range of particle types is supported by experiments operated under practical conditions [7].

The classes outlined in Fig. II-1 are as follows:

Class I: Drops and Particles Uncharged, no Ambient $\bar{E}$.

Here, there is purely mechanical scrubbing with no electrical interaction. This class includes venturi and cyclone scrubbers. Particles are collected on drops mainly by inertial impact. Advantage is taken of the tendency of the particles, by virtue of their inertia, to leave the gas stream and be collected on the drops. Also included are such mechanisms as entrainment in the wake of drops moving relative to the gas. Especially for extremely small particles, further collection

mechanisms are molecular and turbulent diffusion of particles through boundary layers adjacent to the drop.

In an inertial scrubber, it is the inertial force on a particle which prevents it from "turning the corner" and following the gas stream around the drop. Typically, the particle of radius a, having relative velocity w and mass density ρ_a, suffers an acceleration on the order w^2/R in the neighborhood of a drop, where R is the drop radius. Hence, if the gas mass density relative to that of the particle is ignored, the particle experiences an inertial force $4/3\ \rho_a \pi a^3 (w^2/R)$. That force is retarded by a viscous drag force which, if represented by Stokes' law, is of the order $6\pi\mu aw$, where μ is the gas viscosity. The inertial impact parameter [8] is the ratio of these two forces and must be appreciable compared to unity for the collection process to be effective.

$$\text{inertial impact parameter} = \frac{\text{inertial force}}{\text{viscous drag}} = \frac{2}{9}\frac{\rho_a a^2 w}{R\mu} \qquad (1)$$

Class II: Drops Charged, Particles Uncharged, Ambient $\bar{E}$

In a mechanical scrubber, relative motion between drops and particle-laden gas is often obtained by injecting the drops at a velocity differing from that of the gas. This relative velocity can also be achieved by charging the drops and subjecting them to an ambient electric field. The resulting electric force gives a drop motion, relative to the gas, that can be used to scrub uncharged particles from the gas. The advantage of the field-induced scrubbing is that the relative velocity does not decay to zero from the point of injection, but rather reaches a steady value determined by the hydrodynamic

drag, particle charge and ambient electric field. Thus, the device scaling is altered because the penetration distance associated with the mechanical scrubber is no longer a limitation.

The inertial impact parameter of Eq. (1) remains appropriate for the class II interactions, with the relative velocity w determined by the balance of electric and viscous forces on the drop. If we let the drops be charged to $Q = C\pi a^2 \varepsilon_o E_*$, where R is the drop radius, E_* is a charging field, and C is a geometry factor varying between about three and 12 depending on the charging method, and again assume Stokes' drag, in an electric field E, the drop reaches a steady velocity $w = [(C\pi R^2 \varepsilon_o E_*)/6\pi\mu R]E$. Thus, the appropriate inertial impact parameter follows from Eq. (1) as

$$\text{electrically induced inertial impact parameter} = \frac{\rho_a a^2 C \varepsilon_o E_* E}{27\,\mu^2} \quad . \tag{2}$$

Class III: Drops Charged, Particles Charged, No Ambient $\bar{E}$

By contrast with Classes I and II, there is negligible relative velocity between gas and drops in Class III interactions. Particles are collected on a charged drop surface by electrical attraction of opposite charge. Hence the drop surfaces attract particles much as do the precipitating electrodes of a conventional electrostatic precipitator. Fields responsible for collection of particles are largely "micro" fields. Clearly, Class III collection dominates if the charge density of small particles is equal and opposite to that of the drops. Then, there is no net space charge, hence no ambient field. Other possibilities are mixtures of both positive and negative particles along with

both positive and negative drops, in proportions such that there is negligible field due to net space charge. But, from a practical point of view, the charge density of the drops could far outweigh that of the particles and still result in an interaction of essentially the Class III type.

The fundamental collection mechanism is placed in the perspective of conventional electrostatic precipitators by introducing the following commonly used simplified model of a conventional precipitator [9]. Suppose that a section of a precipitator having length ℓ_{prec} collects an appreciable fraction of particles on electrode collecting surfaces used to impose an ambient E normal to their surfaces. The field causes a particle velocity normal to the electrodes w, and the electrodes are characterized geometrically by a circumference S, and cross-sectional area perpendicular to the flow, A. The number of particles passing a given cross section per second is then UAn, and that must be on the order of the number per second $\ell_{prec.}$ $Sw_{prec}n$ collected at the walls. Hence,

$$\ell_{prec}\, Sw_{prec} \approx UA \tag{3}$$

For efficient precipitation, the device length, L, should be large compared to ℓ_{prec}.

$$\frac{L}{\ell_{prec}} = \left(\frac{LS\, w_{prec}}{UA}\right) > 1 \quad . \tag{4}$$

Now, consider a length ℓ_{III} in which a large fraction of the particles is collected on drops. In Eq. (3), the particle collection surface is $\ell_{prec}S$, and with N-per-unit volume of drops, each of which

has surface area $4\pi R^2$, this collection area is replaced by $4\pi R^2 NA\ell_{III}$. We define α_R as the number of drop radii by which the drops are separated on the average, so that $N = 1/(\alpha_R R)^3$, and it follows that the equivalent of Eq. (3) for the drop precipitator is

$$\frac{4\pi R^2 A\ell_{III}}{\alpha_R{}^3 R^3} w_{III} \gtrsim UA \quad . \tag{5}$$

Thus, for efficient collection of particles on drops, we must have $L/\ell_{II} > 1$, where

$$\frac{L}{\ell_{III}} = \frac{L4\pi w_{III}}{U\alpha_R{}^3 R} \quad . \tag{6}$$

The necessity for having close packing of the drops, or low α_R's if they are to compete with the conventional precipitator, is emphasized by taking the ratio of characteristic collection lengths

$$\frac{\ell_{III}}{\ell_{prec}} = \frac{\alpha_R{}^3 RS}{4\ \pi A} \frac{w_{prec}}{w_{III}} \quad . \tag{7}$$

For the drop system to compete favorably, Eq. (7) should give a ratio large compared to unity. Note that the particle velocities w_{prec} and w_{III} at the collecting surfaces are functions of the particle charge and electric field intensity at the respective surfaces.

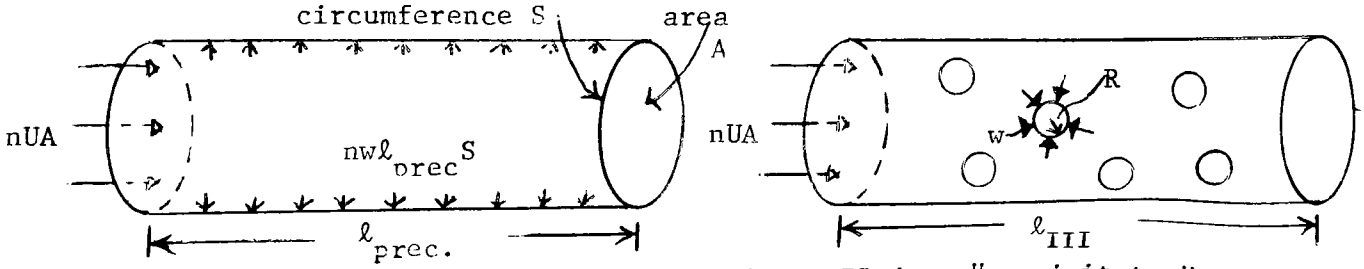


Fig. II-3 Configurations for comparing conventional precipitator and Class III interactions

Class IV:Drops Uncharged, Particles Charged, Ambient E

Neutral drops introduced into the flowing gas and subjected to an ambient electric field are polarized. The flux of electric field lines over an area three times that of the drop cross section is intercepted by the drop. Thus, charged particles migrating toward the drop along these lines are intercepted by the drop. If particles are charged to only one sign, then they are initially collected over half of the drop surface. If particles are charged to both polarities, then collection takes place over the whole drop surface.

In the unipolar particle case, the drop collects particles at the expense of acquiring a net charge, and the field in the drop's vicinity is altered so as to limit particle collection. Because of its resultant net charge and the imposed E, drops drift relative to the gas. Hence, Class II interactions can come into play as the process proceeds. Also, the net charge on a drop can be the basis for subsequent drop collection using conventional precipitator configurations or space charge precipitators.

In the case of bipolar charging of the particulate, it is possible to have the neutral drops collect particles without significant alteration of the charge neutrality. In that case, contribution to the ambient E from particle charging is also minimized. Assuming unipolar charging, the effective drop-collecting surface is initially $3\pi R^2$, and hence the collecting area in a collection length ℓ_{IV} is $3\pi R^2 NA\ell_{IV}$. Following arguments similar to those accompanying Eqs. (5)-(7), effective particle collection on the drops requires that $L/\ell_{IV} > 1$, where

$$\frac{L}{\ell_{IV}} = \frac{L3\pi w_{IV}}{U\alpha_R{}^3 R} \tag{8}$$

Class V: Hybrid Interactions with Drops and Particles Charged and An Ambient Field

With the combination of ambient electric field and charges of opposite sign on drops and particles, it is possible to collect particles on the drops through combinations of the mechanisms included under Classes II, III, and IV. In this class, the drops can be charged to differing signs, as can also the particles. The ambient electric field is imposed by means of external electrodes, or caused by space charge. For example, suppose that drops of one sign are mixed with the dusty gas charged to the opposite sign, and the mixture subjected to an ambient electric field. Then, droplets are driven at a relative velocity with respect to the gas, so that the inertial impact mechanisms of Class II come into play. The particles also collect on drops through the micro field interaction of Class III. Once neutralized by the collection of oppositely charged particles, the drops are the sites of further particle collection through the mechanisms of Class IV. Thus, depending on the relative magnitudes of charge and electric field, on the size of the particles and drops, and on the interaction time between the particles and drops, all three mechanisms come into play to one extent or another.

Further Types of Devices Not Involving Basically New Phenomena

The classification of drop-particle collection mechanisms emphasizes the interaction, rather than differences in systems. It is through these mechanisms that there is promise for making a new departure in particle

collection technology. Devices have been proposed that combine drops and fields for particle collection, but that really do not exploit new mechanisms which might therefore hold promise for developing a technology for handling sub-micron particles.

An example is the combination of a scrubber and electrostatic precipitator into a single volume, where each functions in an essentially conventional manner. Reference is made in the following review of the literature to "miscellaneous" devices that fall into this category. They are not regarded as exploiting what is basically new about the interaction of drops and particles in an electric field.

C. Classification of Devices Disclosed in the Patent Literature

In the summary of patents directly relating to the use of electric fields and drops for collecting particulate, primary emphasis is given to the drop-particle interaction. Also included is a categorization of the method of: (a) charging particulate, (b) charging and producing drops, (c) removing drops, and (d) method of creating ambient fields, if any. Class I interactions encompass conventional mechanical scrubbing, and are included in the classification only for purposes of comparison. Hence, they are not surveyed in the following. Table II-2 lists patents described in this section.

Table II-2

Patents Relating Directly to Electrically-Induced Collection of Particles on Drops

Class	Inventor	Patent No.	Title	Date
II	Wintermute	1,959,752	Liquid Flushing for Discharge Electrodes	1934
II	Schmid	2,962,115	Apparatus for Separating Solid and Liquid Particles for Gases and Vapours	1960

Class	Inventor	Patent No.	Title	Date
II	Marks	3,503,704	Method and Apparatus for Suppressing Fumes w/Charged Aerosols	1970
II	Marks	3,520,662	Smokestack Aerosol Gas Purifier	1970
III	Wagner	1,940,198	Apparatus for Cleaning Gas	1933
III	Penney	2,357,354	Electrified Liquid Spray Dust Precipitators	1944
III	Penney	2,357,355	Electrical Dust Precipitator for Utilizing Liquid Sprays	1944
III	Gilman	2,523,618	Electrostatic Apparatus	1950
III	Gilman	2,525,347	Electrostatic Apparatus	1950
III	Peterson	2,949,168	Electrical Precipitator Apparatus of the Liquid Spray Type	1960
III	Peterson	3,331,192	Electrical Precipitator Apparatus of the Liquid Spray Type	1967
III	Romell	3,440,799	Gas Scrubber	1967
V	Hodson and Klemperer	2,615,530	Liquid Cleaned Precipitator	1952
V	Ransburg	2,788,081	Electrostatic Gas-Treating Apparatus	1957
V	DeGraaf, Haas van Dorsser and Zaalberg	2,864,458	Liquid-Electrostatic Precipitation	1958
V	Ziems and Bönicke	3,384,446	Apparatus for Disinfecting Gases	1968

Class	Inventor	Patent No.	Title	Date
Miscell.	Buff	1,905,993	Treatment of Gases	1933
Miscell.	Vicard	2,983,332	Process and Apparatus for the Purification of Gases	1956
Miscell.	Wiemer	3,221,475	Wet Electrostatic Precipitator	1963
Miscell.	Vicard	3,363,403	Electrostatic Filtering Apparatus	1968
Miscell.	Ebert	3,492,790	Gas Cleaning Apparatus and More Particularly to an Improved Electrical Precipitator	1970
Miscell.	Humbert	3,523,407	Method for Electrostatic Precipitation of Dust Particles	1970
III*	Prentiss	2,758,666	Carbon Black Separation	1956
III*	Johnstone	2,924,294	Apparatus for Cleaning Gases with Electrostatically Charged Particles	1960
III*	Silverman	2,992,700	Electrostatic Air Cleaning Device and Method	1961
III**	Allemann, Moore & Upson	3,218,781	Electrostatic Apparatus for Removal of Dust Particles from a Gas Stream	1965
IV*	Cole	2,990,912	Electrical Precipitator and Charged Particle Collection Structure Therefor	1961

* Solid particles used instead of drops

** Drops replaced by bubbles

Class II: Electrically Augmented Impact Scrubbing

Wintermute: 1,959,752

The invention relates to the addition of liquids to a conventional electrostatic precipitator. In the configuration described, a single-stage precipitator of tubular construction is fitted with a water injection system so that the discharge wire (passing down the center of the cylindrical collecting electrode, which in this case is at high voltage) is surrounded by a liquid stream. Because of the electric stress, this stream breaks up into drops. As described, the device operates much like a conventional precipitator with the water electrically made to strike the outer electrode, and hence clean it. No interaction mechanism between drops and particles is described. However, we know from related work on corona discharges from wet surfaces [10] that the wire must be the source of both ions and drops having the same polarity. Thus, the annulus is a region filled by radially expanding ions and drops.

Particles are introduced to the tube structure as in a conventional precipitator. Upon entering the field region, the particles assume whatever sign of charge is carried by the flux of ions. Hence, particles and drops have the same sign, and there is at best a limited tendency of a particle to be attracted to a drop because of the polarization of the drops in the ambient field. (That collection mechanism would cut off if a drop were charged on formation to the saturation charge, as discussed in § III.A). Thus, the only remaining mechanism for electrical augmentation of the particle removal process by collection on the drops is via inertial impact caused by the relative velocity between drops and particles traveling radially outward in the imposed

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particle charging - ion impact, but not consequential to drop-particle interaction
drop charging - induction and/or ion impact
drop removal - electric precipitation
ambient field - imposed

electric field.

Schmid: 2,962,115

A problem with centrifugal separators is that, once the particulate has been forced to the outside wall, it must still be removed from the device by inertial (gravitational) settling. To hasten this process, the size of the particulate could be increased, possibly by agglomeration. The dirty air from one cyclone is passed to another. Inside the latter are a pair of parallel plates, with a high voltage placed between them. Charged water spray is then introduced between them; this is accomplished by inducing electrodes placed a slight distance away from the nozzles. The drops oscillate between the plates, and collect the uncharged particulate by inertial impaction. These drops are heavy enough to fall to the bottom of the chamber where they can be drained away.

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particle charging	-	none
drop charging	-	by induction
drop removal	-	settling
ambient field	-	imposed.

Marks: 3,520,662

To clean the flow from a smokestack of particulate and fumes, the gas is deflected through an annular region containing an array of small nozzles. Downstream of this is a perforated charging electrode. The fluid is drawn out of the nozzles by the electric field in the form of very fine drops. Past the charger is a mixing region, where the droplets mix with the flow and pick up the dust and fumes. Since the walls are grounded, the mist is space-charge deposited there. No provision is made to charge the dust. Another way to form the charged drops is to draw fluid through a grounded porous plug surrounded by a screen set at high voltage. The drops produced

are claimed to be about the same size as the holes, 1 - 100 μ. The inertial impact interaction is inferred from the patent because the drop-particle interaction is not actually described.

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particle charging - none
drop charging - by induction
drop removal - electric precipitation
ambient field - space charge

Marks: 3,503,704

A method to cleanse a gas flow of noxious fumes is offered. Inside the channel, an array of capillary tubes is placed across the flow and connected to ground. Downstream of this is positioned a grid connected to high voltage. This arrangement allows the droplets to be inductively charged and moved relatively rapidly through the gas flow. In the process, the air which each drop contacts will be cleansed by the oxidant dissolved in the liquid. Next, the drops encounter a grounded matrix of needles. A discharge produced at the needles by the high voltage source is of such a polarity as to discharge the incoming drops; this allows them to coalesce and be inertially removed from the flow. Another method to effect coalescence would be to combine two streams carrying droplets of opposite polarities. Still another means would be to allow the charged drops to be precipitated on an inner wall set to some voltage. No attempt is made to charge the dust. The inertial impact interaction is inferred because the patent does not specify how the fume is "entrained" on the drops.

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particle charging - none
drop charging - by induction
drop removal - inertial or electrostatic precipitation
ambient field - space charge

Class III: Oppositely Charged Drops and Particles with Essentially No Ambient E

Wagner: 1,940,198

Dust, presumably charged during combustion, is passed through a region saturated with water droplets. The latter are frictionally charged either in passing through the nozzle or through the air. Collection of the particulate by the droplets then occurs in this mixing region, promoted to a large extent by the turbulence established by the injection of the spray. The flow is next directed through low-temperature regions which cause the droplets to coalesce. These larger drops are then removed from the gas flow by inertial means.

particle charging	-	combustion
drop charging	-	frictional
drop removal	-	inertial
ambient field	-	negligible

Penney: 2,357,354

Dust-laden gas first passes through a tubular electrostatic ionizing section to charge the particulate by ion impact. Just downstream from this is an array of nozzles which issue a spray of inductively charged or ion-impact-charged drops. In experiments reported, drops are in the size range of 500 μm. Penney emphasizes that drops must be of sufficient size that they can be easily removed from the gas. Drops and particulate have opposite polarity. During passage down the channel, the drops scavenge the particulate, until finally they are removed. Suggested means of removal are inertial impact and other conventional means.

This patent not only makes a primary disclosure, but is an informative and useful guide to the design of the general class of devices. Penney clearly recognizes the competition between the space-charge-generated ambient field and the micro fields responsible for the collection of particles on drops. To alleviate the adverse effect of the space-charge ambient field, particularly

in electrical breakdown, a modification of his device is described in which an array of grounded parallel plates are placed just after the high voltage charge, inducing electrodes of the sprayer. Their purpose is to keep the droplet space charge from growing to discharge levels. In § III-C, we discuss the role of particulate space charge, which Penney chose to make negligible compared to that of the drops.

Limiting the volume occupied by the space charge is necessary if the ambient field is to be limited. Equation (II.1) shows that, for a given net charge per unit volume, the greater the volume enclosed, the greater the average electric field intensity at the surface of the volume. The space charge fields act to promote collection of the drops, but this is tantamount to removal of the dust only if the drops have had time to collect the dust. Space-charge ambient fields also bring into possible play Class II interactions.

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particle charging	- ion impact
drop charging	- ion impact or induction
drop removal	- inertial impact, precipitator
ambient field	- present due to space charge, but not basic to the interaction

Penney: 2,357,355

The "principles and teachings" of Penney's Patent 2,357,354 are basic to what is disclosed here. Further arrangements of electrodes, gas stream and drop orifices are described.

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particle charging	- ion impact
drop charging	- ion impact or induction
drop removal	- inertial impact, etc.
ambient field	- inadvertently due to space charge

Gilman: 2,523,618

One problem arising in Penney's devices is that water collects on the high voltage electrode opposite the source of water spray. The field at this inducing electrode is sufficient to cause discharge of such a sign that the drops leaving the nozzle will be discharged before encountering the charged dust. A solution offered is to use two parallel rods as the high-voltage-inducing electrodes, with the row of nozzles situated between and slightly above them. These electrodes would be slightly tilted, so that any liquid collecting on them would run off. Drops would then form at the lower end, where the field strength would be the weakest. These take the place of Penney's ring inducing electrodes. In addition, a pneumatic type of sprayer is described that produces much more highly charged drops than the other non-pneumatic types.

See Penney - 2,357,354

Gilman: 2,525,347

As in his patent 2,523,618, an improvement is disclosed on Penney's devices. The problem is that, in a spray device with induced charging, water collects on the high voltage end and discharges. This results in a loss of charge on the escaping drops. To prevent this, another grounded ring electrode is added beneath the high voltage one, which tends to pull the drops and their discharge away from the main dust flow.

See Penney - 2,357,354

Peterson: 2,949,168

This device is similar to Penney's. The dust and spray are charged to opposite polarities by passing the respective flows through corona discharge regions. Although the spray is injected transverse to the dust flow, a fan

at the output of the system pulls both the particulate and the drops downstream together. There is no imposed ambient electric field in this region, since the outer walls of the device are constructed of insulating material. This will be true as long as wetting and subsequent fouling of the walls by the drops can be prevented. Collection of the drops is accomplished by inertial impaction at baffle plates placed downstream. The patent pertains to arrangements of apparatus leading to reliable operation without the necessity for cleaning.

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particle charging	-	ion impact
drop charging	-	ion impact
drop removal	-	inertial impact
ambient field	-	negligible

Allemann, Moore and Upson: 3,218,781

Strictly, the disclosed device does not involve drops, but rather bubbles. In a sense, it is the Class III type of interaction "turned inside out". Ion impact charged particles are entrained in a gas forced through holes in an insulating plate. A conducting liquid is on the other side of the plate, and the gas passes through in the form of bubbles. Hence, the charged particles find themselves within a bubble and are precipitated on the bubble walls. Thus, they are transferred to the water, and removed.

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particle charging	-	ion impact
drop charging	-	induction
"drop removal"	-	(bubbles removed by "settling")
ambient field	-	none

Peterson: 3,331,192

Apparatus improvements relating directly to Petersons patent 2,949,168 are disclosed. See also Peterson's patent 3,098,890 for device capable of ducting a conducting liquid between two points that must remain electrically insulating.

Romell: 3,440,799

The main innovation in this device is the use of Kelvin's influence machine [13] for both charging the drops and providing the high voltage for the corona source. The basic interactions seem most similar to Penney's. Considering the significant emphasis given to problems of fouling and maintenance of electrical insulation in the region of dirty gas and drops, it is extremely doubtful that the proposed technique of converting hydrostatic head into the required high voltage electricity is a practical or appreciable innovation.

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particle charging	-	ion impact
drop charging	-	induction
drop removal	-	settling
ambient field	-	negligible

Class IV: Ambient E with Particles Charged but Drops Initially Uncharged

There do not appear to be patents that clearly fall in this class. See Class V patents for devices that probably involve the Class IV collection under some conditions. A disclosure making use of solid particles instead of drops is:

Cole: 2,990,912

A two-stage electrostatic precipitator is made with a conventional corona source for charging particles by ion impact. The collection section of the conventional precipitator is replaced by a packed bed of spherical semi-insulating large particles. The bed is subjected to an imposed electric field so that the spherical large particles are polarized with positive and negative surface charges, respectively, over half-surfaces. Hence, the solid large particles play a role similar to that of the drops in a Class IV

type of interaction. By contrast with the drops, the particles of Cole's invention are fixed in position and packed between conducting grids to the point of sustaining a conduction current.

Class V: Hybrid Interactions with Both Drops and Particles Charged in an Ambient Field

Hodson & Klemperer: 2,615,530

After the dust flow is mixed with steam it passes into the ionization section, which is cooled by a water jacket. This promotes condensation of the vapor, with the particulate as nuclei. This effectively increases their size and allows a larger amount of charge to be placed on them. In the collector, a rod instead of a wire (as in the ionization section) is placed along the axis. The outer wall is again water cooled. The "thermal head" and the electrostatic force act to drive the charged particulate to the wall. Water drops that were not charged in the ionization section experience a dipole force produced by the nonuniform field tending to pull them toward the center conductor. The relative motion produced will enable these uncharged drops to collect, by means of impaction, other particulate which may have escaped charging in the ionization section.

Although not apparently recognized by the inventors, those drops arriving in the collector section with less than the saturation charge will collect particles through the Class IV mechanism, thus giving rise to a drop charging that results both in particle collection and the net charge on the drop necessary for conventional precipitation.

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particle charging	-	ion impact
drop charging	-	condensation and ion impact claimed
drop removal	-	electrostatic precipitation
ambient field	-	imposed

Ransburg: 2,788,081

This device is very similar to Penney's. The charging of the dust is accomplished with a cone-shaped device having a sharp base edge to promote corona discharge, which presents much less resistance to the flow than an array of bars. The droplets are produced and charged by placing a spinning disk atomizer at high voltage with respect to the grounded wall. A relatively non-conducting fluid is used, so that the pump can be placed at ground. The electric field present causes the drops to disperse and migrate to the walls. Provision is made to allow collected liquid to drain away to prevent the drops there from becoming a source of back ionization.

The transverse motion of the drops across the dust flow tends to remove the particulate. In this case, there is no mixing along the flow, where the dust and drops are made to flow together downstream. The basic collection process is not specified, but it can be inferred that Class II and III interactions are intended. The spinning disk used to produce the charged drops is the main innovation. This has been highly successful in the electrostatic paint spraying application [11,12].

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particle charging	-	ion impact
drop charging	-	induction
drop removal	-	inertial impact and electrostatic precipitation
ambient field	-	imposed

DeGraaf et al: 2,864,458

The device described consists of a grounded metal pipe with a discharge electrode set at high voltage running down its axis. Beginning just above the wire, and spaced between it and the outer wall, is a curtain of water emanating from grounded nozzles. One reason for this arrangement is to prevent discharge from the nozzles. The closest ground to the wire is the curtain. A short distance downstream, the curtain breaks up into individual

drops. These are inductively charged to a polarity opposite from that of the dust, as a result of the corona. The electric field causes the dust and drops to move in opposite directions - one toward the wire, and the other toward the wall. This encourages mixing and further agglomeration. The performance of this scrubber was compared to that of a wet Cottrell and Penney's, and fared much better, especially at high gas velocities. Note that the particles are in competition with the ions in discharging the drops (Class III), and charging the drops (Class IV). Also, the drops are accelerated relative to the gas, so as to encourage inertial impact of particles.

particle charging	-	ion impact
drop charging	-	induction
drop removal	-	inertial impact and electrostatic precipitation
ambient field	-	imposed.

Ziems & Bonicke: 3,384,446

A mechanism for disinfecting a room is described. Disinfectant is sprayed from nozzles placed far above the floor and connected to a high voltage source. The charged drops passing into the air will tend to collect dus, which apparently is naturally charged. Through the action of both gravity and the electric field between the tower and the ground plane, the dirty drops will be collected on the floor. Provision is made to allow the drops to be either positively or negatively charged, depending on whether the dust is predominantly one sign or the other. The device might operate in any or all of the classes, depending on the degree of drop charging, the strength of the imposed field, and the charge of the particulate. The inventors give few clues as to the basic mechanism.

particle charging	-	random "natural"
drop charging	-	induction
drop removal	-	gravitational settling and electrostatic precipitation
ambient field	-	imposed

Class: Miscellaneous:Patents Combining Scrubbers and Electrostatic Precipitation Without Exploiting New Interactions

Buff: 1,905,993

A combination scrubber and wet-wall electrostatic precipitator is disclosed. The electric field is used largely to remove the water drops not removed by inertial impact.

Vicard: 2,983,332

The innovation consists of the combination of conventional venturi scrubber and electrostatic precipitator to make a single device. The air just upstream of the venturi is saturated with water vapor. A discharge zone occupies some of the space in front of the throat, hence succeeds in charging a portion of the drops passing through. When the flow passes through the converging section, the submicron-size particles are accelerated to a much higher velocity than the droplets. This relative motion results in collisions and, effectively, in the charging of the dust. The micron , ionic water droplets can then be electrically precipitated much further downstream at the end of the diverging section. Additional efficiency can be obtained by having a pair of parallel, oppositely charged plates further downstream; these induce dipoles on the uncharged drops passing between them. The latter collect the remaining charged particulate.

Wiemer: 3,221,475

The major innovation in this patent is the use of thermal effects to augment the formation of water droplets on dust particles. Incorporated into the device is a conventional electrostatic precipitation system.

Ebert: 3,492,790

The main innovation is a system for rotating the corona charging electrodes so as to charge more completely the particulate. The device is essentially a conventional scrubber placed in series with a wet-wall precipitator.

Humbert: 3,523,407

Use of liquid additives to precondition particulate before conventional electrostatic precipitation is described as a means of alleviating problems with high resistitivity particles.

Vicard: 3,363,403

In this version, relating to Patent 2,983,332, vanes are added to the venturi throat to impart rotational motion to the flow composed of condensed droplets and particulate. This forces the drops to the outside and prevents the system from arcing, and aids in the later electrostatic precipitation of the wet particulate, since the outer wall is the collecting electrode.

Patents Disclosing Use of Solid Particles Instead of Drops

Prentiss: 2,758,666

Particles are carbon black, charged by ion impact to differing signs in separate regions, or by means of an ac electric field. Agglomeration of small particles on large ones may be considered as equivalent to the Class III interaction with one of the carbon black families playing the role of the drops.

Johnstone: 2,924,294

Solid pellets are charged by frictional electrification and used as collection sites for fine particles through electrostatic attraction. Pellets are essentially insulating and collected in a cyclone-type filter. With the pellets playing the role of charged drops of insulating liquid, the interaction would be Class III.

Silverman: 2,992,700

A fluidized bed of solid insulating particles is charged "by triboelectrification". The dirty gas is apparently naturally charged, and upon being filtered through the bed, leaves particulate on the insulating particles of the bed. The collection process is of type III, if that explanation is accepted.

D. Basic Work Generally Relevant to Drop-Particle Interactions

Once it is recognized that charge and field effects on particle interactions are of interest in such widely separated areas as meteorology, colloid chemistry, and industrial process control, it is not surprising that the literature of the basic area is extensive, in some respects highly developed, and somewhat fragmented. There are a large number of investigations reported on charge effects in the stability and aging of aerosols [14,15,16,17,18,19]. Other reports relate to cloud-drop interactions with ions, particulate, and droplets. An excellent overview of the electrical behavior of aerosols is given by Whitby and Liu[20]. The spectrum of phenomena is, of course, relevant to the use of drop-particle interactions in cleaning industrial gases.

However, we can avoid becoming overwhelmed by the large number of parameters and process that could be brought into play by recognizing at the outset certain limitations on the systems of interest here. First, the drops are probably in the range of 10 μ. Penney states that there is little point in introducing the drops unless they themselves are easily removed [7], which tends to place a lower bound on the size of useful drops. He gives an example of drops in the size range of 0.1 - 0.5 mm, with 10 μm as a lower bound. Typical devices such as inertial impact scrubbers, cyclone scrubbers and the like, as well as electrostatic precipitators, begin to have limited capabilities as the particle size is reduced much below 10 μ. Hence, 1 μ is certainly a lower limit on the range of drop size of practical interest, and 10μ is likely to be typical.

Second, for industrial applications we are concerned with processes that occur in seconds, rather than aging processes that take minutes or more to produce a significant effective change in particle size. This is a major reason for considering interactions between particles and relatively large drops. A single collision results in a drastic change in effective size. In view of the relatively large size of the drops and the submicron size of particles of interest, we can think of a "sea" of small particles and relatively widely dispersed much larger drops. Figure II-4 shows the drops, with radius R, having average spacing $\alpha_R R$ and the particles with radius a having average spacing $\alpha_a a$. In general, the particles can be both positively and negatively charged.

Under the assumption that $\alpha_R R >> \alpha_a a$, a particle can be considered as interacting with an isolated drop. Considerable literature exists for studies of interactions between isolated "drops" and various types of charged particles. The manner in which a particle acquires charges from a sea of ions in an ambient electric field is fundamental to the theory of electrostatic corona charging as used in conventional precipitators. With a primary motivation from problems in atmospheric electricity, many authors have studied the interplay between drop charging in an ambient field and the relative motion of the gas in which both drops and ions are entrained [21,22 23,24].

As long as the particle inertia is not an important factor in the particle collection (Classes III-IV), the charged particles can be viewed as heavy ions, and much of the theory developed in a meteorological context is directly applicable to the particle collection process. Since results of this work are needed in the next chapters, the impact charging mechanisms are reviewed here. The derivations summarized in the Appendix follow most closely those of Whipple and Chalmers [22].

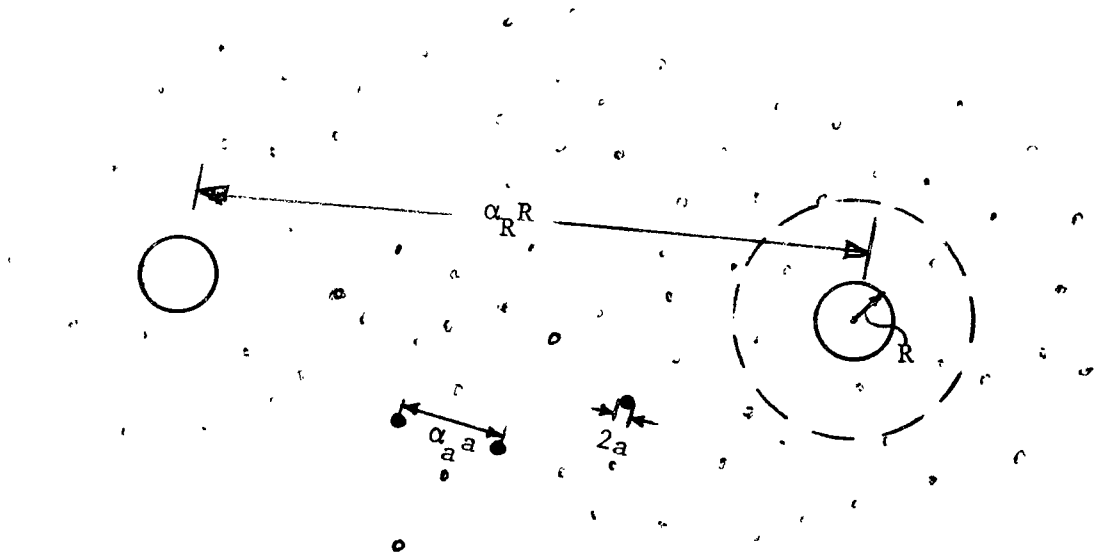


Fig. II-4 Relatively large and widely spaced drops in a "sea" of particles

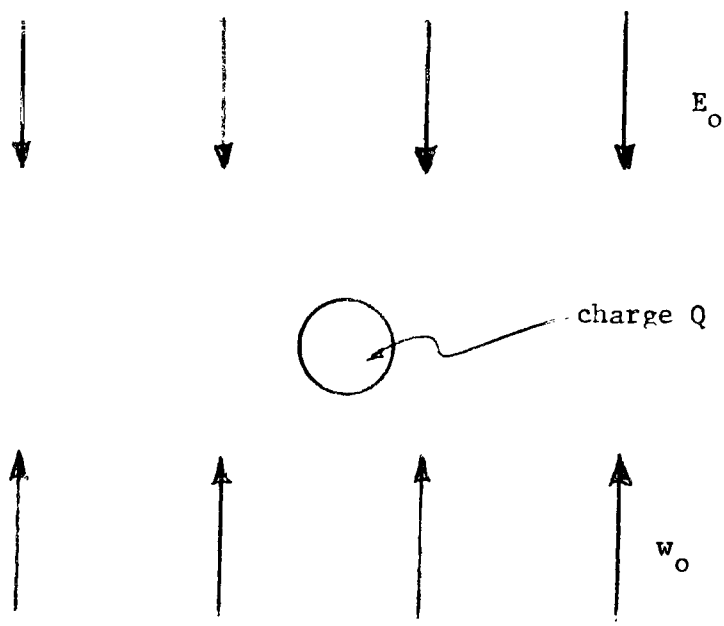


Fig. II-5 Drop generally having net charge Q in an ambient field E_o, and having velocity w_o relative to the gas.

For convenience, consider the drop as fixed with the gas having a relative velocity w_o and an ambient field E_o far from the drop. The definitions of positive flow and field are shown in Fig. II-5. The drop is taken as perfectly conducting, since the relaxation time of the water is far shorter than other dynamical times of interest (see Appendix B for a discussion of this point). Note that the relative velocity w_o could be caused by the electrical force on the drop due to its charge, Q, and the ambient field E_o. Then the relative velocity w_o and net drop charge Q would not be independent quantities. For the present, think of the state of the drop in terms of its net charge Q and the ambient field multiplied by the particle mobility, b, and the relative velocity w_o. Specification of the first two of these quantities gives a point in the (Q,bE_o) plane shown in Fig. II-6. The relative velocity w_o then specifies the location of the vertical line at $bE_o = w_o$. The figure summarizes the manner in which particles charge or discharge, given an initial state. The critical charge, Q_c, is proportional to E_o

$$Q_c = 12\pi\varepsilon_o b^2 E_o$$

while currents i_1 and i_2 are proportional to b and the particle charge density, and depend on Q itself [see Eqs. (A.2), (A.22) and (A.26)].

Remember that an electrical current is equivalent to the deposition of particles on the drop. In the Fig. II-6, the particles are assumed positive. Hence, anywhere on the surface of the drop that the electric field is directed inward, it can collect particles. The "window" through which particles can reach the drop is sketched to typify each regime. As the charge changes, so also does the "window". For example, suppose that $bE_o > w_o$ and a drop initially has a sufficiently negative net charge that

it is in regime (ℓ) of the figure. Then, it has at first a "window" that is the whole surface and will collect particles with a current i_2 until it has discharged to the critical charge $-Q_c$. As it continues into regime (i) where it discharges with current i_1, the "window" begins to close and finally it charges to $+Q_c$ by passing through regime (f). The drop is then saturated and incapable of collecting more particles. It has acquired a sufficient positive charge to insure that there are no lines of electric field intensity directed inward on its surface. Note that the saturation charge Q_c is familiar from field charging theory for conventional precipitators (see, for example, page 133 of Ref. [9]). However, here the charging is caused by the charged particles, and is tantamount to particle collection.

The relationship between particle collection and drop charge is further complicated if the relative velocity w_o is caused by the electric field itself, because then the vertical line $bE_o = w_o$ demarking regimes in Fig. II-6 shifts according to the drop charge.

That the drop tends to reach a saturation charge, after which it is no longer useful as a collecting site, suggests that the particles might be charged to more than one sign. Then, we would expect that the charging effect of one group of particles would be canceled by that of the other. To summarize the combined effects of both positive and negative particles, first consider the charging diagram analogous to Fig. II-6, but for negative particles. That is given in Fig. II-7. Then, with both positive and negative particles, the drop acquires particles following the charging trajectories shown in Fig. II-8. Whereas an electrical current in the unipolar particle cases is equivalent to a particle current, the net electrical current to the drop in the bipolar particle case of Fig. II-8 is not. For

example, a drop having an initial charge sufficiently positive to place it in regime (d) will discharge until it reaches an equilibrium somewhere in regimes (h) or (ℓ). If the degree of particle charging is symmetric, then the equilibrium charge will be zero. That the net electrical current vanishes as the particle reaches this final state of zero charge does not mean that the particle current approaches zero; on the contrary, the individual particle currents from each family of particles remain finite, and the drop continues to collect particles after it reaches its equilibrium charge. This state of zero charge collection is one possible basis for Class IV interactions. Positive particles impact the drop over half of its surface, while negative ones impact over the other half. If the particle charging is symmetric, the net current is zero, and hence the drop remains in a state of zero net charge, even though it continues to collect particles.

One of the main values of the Whipple and Chalmers analysis is that it places the role of gas convection in perspective. The theory is based on the assumption that a high Reynolds number laminar flow exists around the drop. But, because particle conduction to the drop at the interface is controlled only by the electric field there, the details of the gas flow have little to do with the collection process. Only the switching from one collection regime to another is determined by the flow, and that is a matter of the relative values of bE_o, the particle velocity relative to the gas, and w_o far from the drop.

With the objective of refining the theory for the collection of extremely small particles, Zebel [2] has extended the Whipple and Chalmers analysis to include a diffusion boundary layer around the drop. His analysis appears to be of value mainly because of the light it sheds on the processes by which the particles are actually collected. The addition of

diffusion to the model allows a prediction of the diffusion boundary layer thickness. But for relatively strong electric forces, the Whipple and Chalmers picture adequately predicts the collection rate.

By using ions, Cott [23] has verified the Whipple and Chalmers model of electrical charging. Kraemer and Johnstone [25] consider the interactions between a charged aerosol and a single charged or potential-constrained spherical particle. Their work also relates largely to an inertialess representation of the particle collisions with the spherical collector, but it includes some numerical representations of the effects of particle inertia. They make the comment that, in general, the effects of inertial impact and other distinguishable collection mechanisms can be represented by superimposing collection efficiencies computed for the mechanisms considered separately. This is an especially good approach if one of the mechanisms is dominant. In the overview taken here, that viewpoint is implicitly taken. We ignore the effects of inertia and compare the electrical collection process with that caused by inertia acting in the absence of an electrical augmentation.

Kraemer and Johnstone also consider effects of particle space charge on the collection process, but do not include an ambient field. One of their interaction mechanisms which involves the mutual attraction of charged drop and particle charges of opposite sign is identical to the Class III mechanism and model. The "$-4K_E$" collection efficiency is consistent with Whipple and Chalmers in the limit of no ambient E. Hence, their experimental observations on the collection efficiency of dioctyl phthalate aerosol particles on a spherical collector are in agreement with that model, and hence lend support to its use. Their spherical collector,

which played the role of the drop and was fixed to a stinger in the flowing gas, was metal and in the diameter range 1/4 - 7/16 inch.

Complications of self-consistent field and particle charge are addressed in the theoretical work of Smirnov and Deryagin [26]. These complications, also included in the theoretical work of Kraemer and Johnstone, represent a refinement beyond our present needs. The effects of finite particle size and inertia are brought into the picture in various numerical studies [27,28,29] aimed at understanding cloud drop-droplet interactions. Again, these meteorologically motivated studies are relevant, but bring in complications not of immediate interest here.

E. Charged Drop Devices Described in the Literature

Hanson [30,31] and Marks [32,33] suggest the use of charged drops for implementing a space-charge type of precipitator. The drops are either charged by induction during their formation at a nozzle, or by ion impact after being injected into the flow of dirty gas. In either case, their self-fields are responsible for the collection of the drops and/or particles on conducting walls. The use of space charge to replace one of the electrodes in a conventional precipitator is an application of charged drop technology that uses the conventional collection mechanism. The objective is a simplification of systems, or perhaps an improvement in capacity to handle low-conductivity particles [34]. However, the space-charge type of drop devices is of interest here, because whether designed to do so or not, it does have the inherent possibility of giving an electrically augmented collection of particles by the drops.

Marks describes devices in which the entering particles are not intentionally charged. The mechanism by which they are collected by the drops before the drops are space-charge-precipitated is not specified.

As pointed out in §II.C, one possible mechanism is a Class II type of scrubbing interaction, with the drops moving under the influence of the space-charge field and collecting particles by inertial impact. Such a mechanism of collection is possible in the devices described by Hanson as well, although there the particles have the same sign of charge as the drops, hence the coulombe contribution to the particle-drop interaction tends to obstruct collection.

Eyraud [35,36] describes the use of charged drops for collecting submicron biological particles. His interaction is one in which the drops are introduced and charged in the immediate vicinity of a corona wire. The arrangement is otherwise similar to that of a single-stage tube-type precipitator. Again, the interaction mechanism is not specified. The author cites the ease with which the drops are removed by the precipitator, but alludesonly to the mechanism by which the drops pick up the particles (before being themselves removed) with the statement: "Furthermore, each drop plays the role of a high-voltage electrode for a very limited region of the gas to be cleaned". This irplies the interaction is of Class III type. However, particles and drops seem to have the same charge, so it is difficult to see how such a mechanism can be effective. But certainly the field-induced radial velocity of the drops caused by their charging and the imposed electric field can lead to a Class II type of induced inertial impact scrubbing.

There seems to be remarkably little work reported in the formal literature which can serve as a guide to making efficient use of drops in collecting particles. The patent literature of § II.C is a considerably better indication of what has been accomplished in this area than is the formal literature. Even though considerable progress has been made toward understanding the basic

particle-drop interaction (see § II.d), the relationship of these processes to practical systems such as those disclosed in the patent literature is undeveloped. In many respects, the particle-drop interaction can be regarded as a type of agglomeration akin to that studied recently [37] between large and small solid particles.

To discuss properly the feasibility of using drops to collect particulate, attention must be paid to the charging dynamics itself. Because the volume of water introduced is critical to the success of such methods, complete utilization of the collecting capabilities of the drop is highly desirable. These systems aspects of using charged drops are discussed in the next section. Results from the work [25] with electrically-induced agglomeration between solids will be directly applicable to the Class III interactions and applicable with some modification to the Class IV interactions.

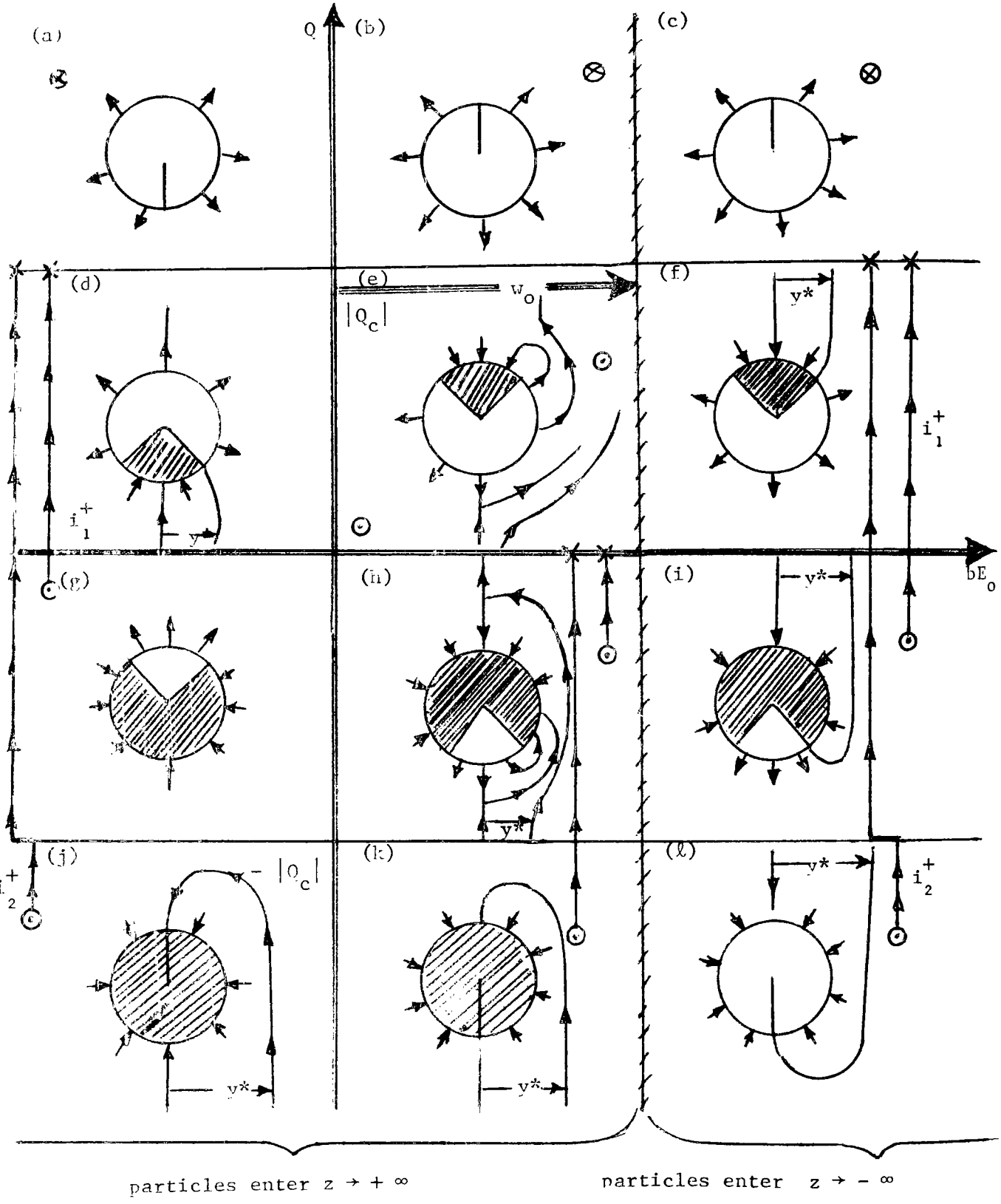


Fig. II-6 Positive particle charging diagram. Charging regimes depicted in the plane of drop charge Q and mobility-field product. With increasing fluid velocity, the vertical line of demarcation indicated by w_o moves to the right. Initial charges, indicated by ⊙, follow the trajectories shown until they reach a final value given by X. If there is no charging, the final and initial charges are identical, and are indicated by ⊗. The inserted diagrams show the force lines $\bar{v} \pm b\bar{E}$.

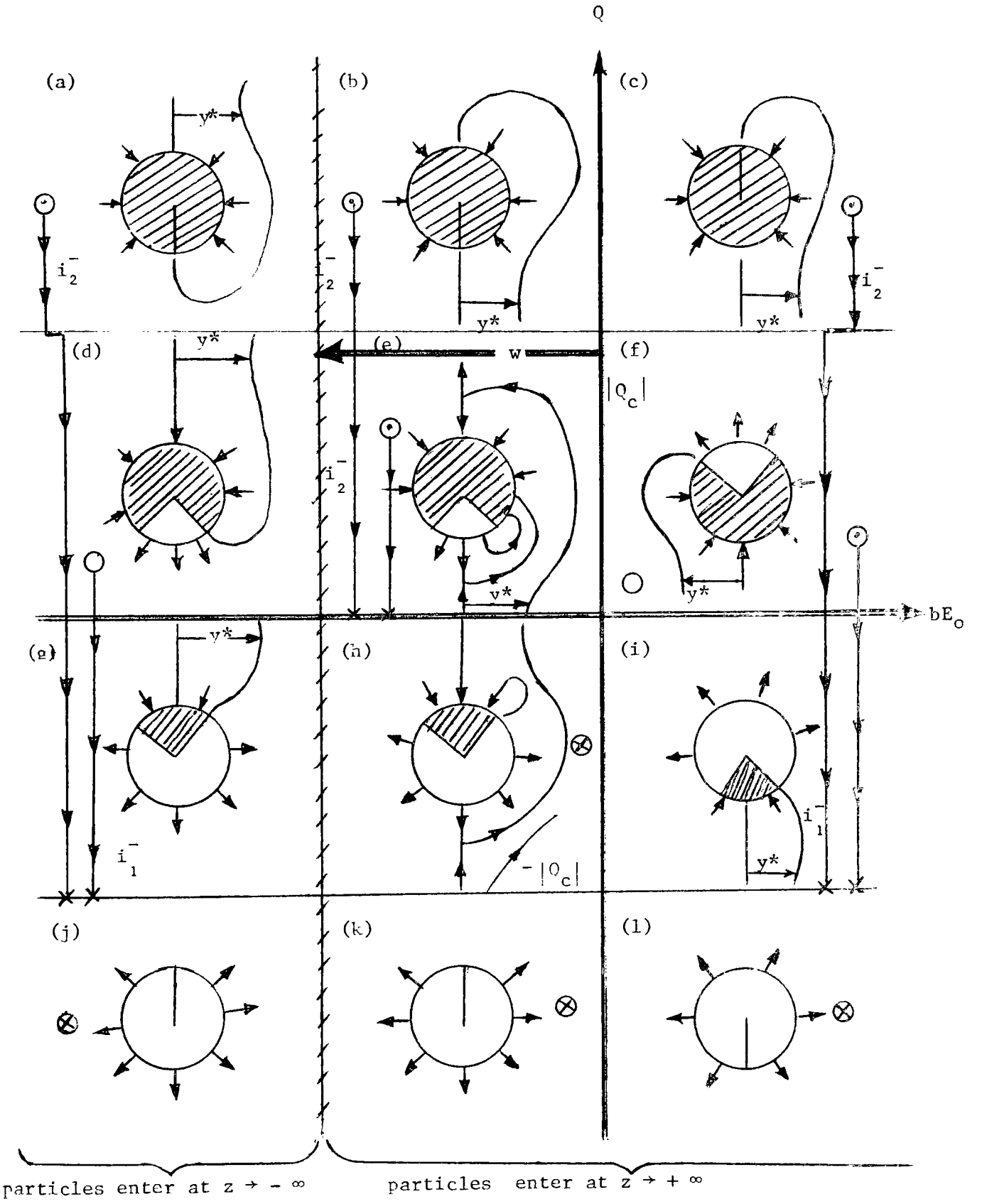


Fig. II-7 Negative particle charging diagram. Drop charge vs mobility-field product. Conventions are as in the previous figure. With increasing fluid velocity, the line of demarcation indicated by w_o is moved to the left.

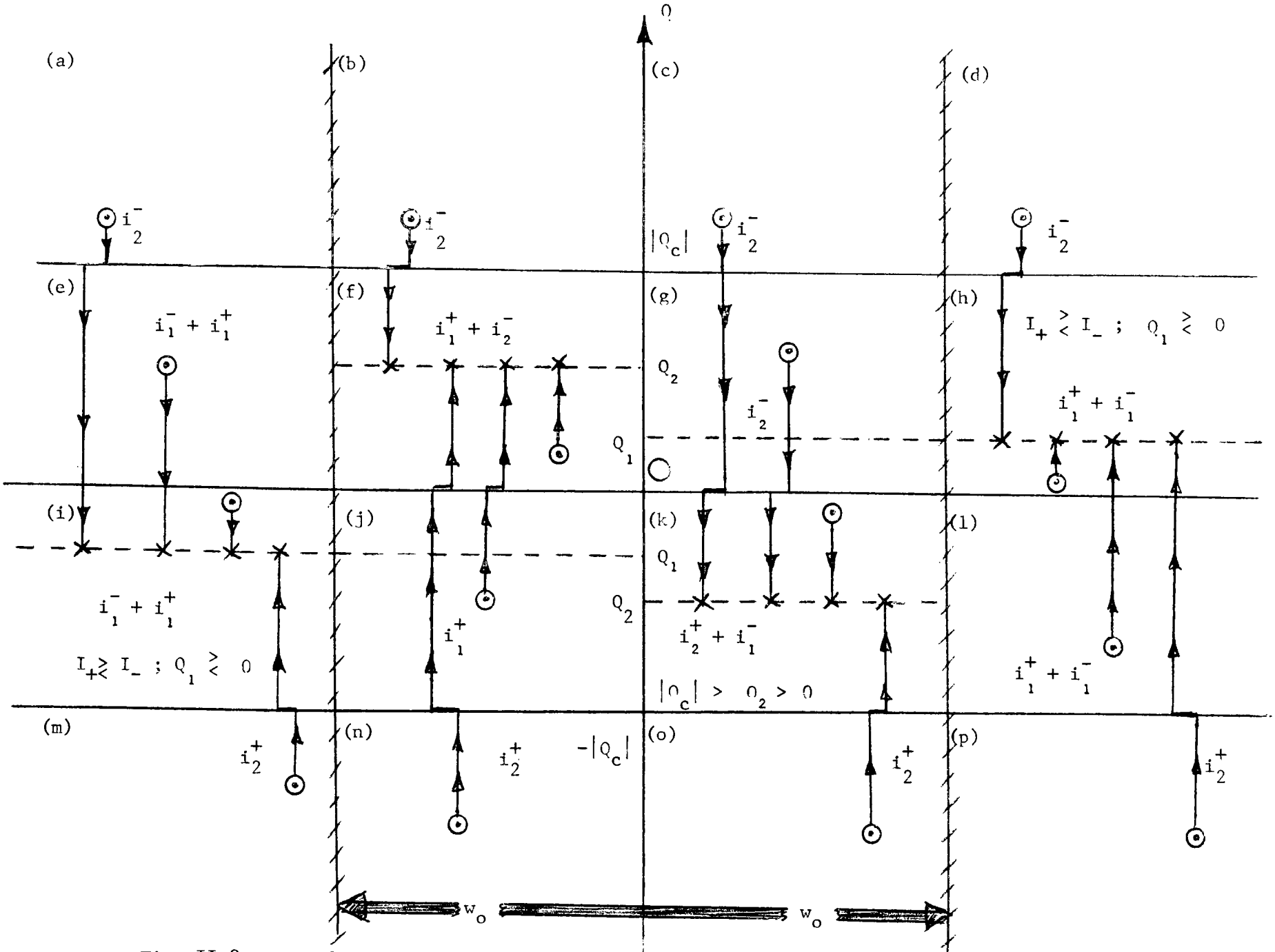


Fig. II-8 Drop-charge-imposed field trajectories for impact charging in combined flux of positive and negative particles

III. Simple Models for Classes of Drop-Particle Interaction

Before an assessment can be made of the relative merits of a scheme using electric fields and drops, the scaling of the interaction to a practical system must be delineated. It is the purpose of this section to highlight the dependence of the various collection mechanisms on systems design factors. In the subsections to follow, only preliminary estimates are made. Obviously, any one of the device classes could involve many pages of theoretical development and still give only a theory capable of defining trends and orders of magnitude. Our major effort is given to systems factors with Class III interactions, since they appear to hold the most promise for a new approach.

A. Class I: Inertial Impact Scrubbing

Weber [3] gives a comprehensive review of wet scrubbers. The scrubbing can be implemented by a number of different mechanisms. He classifies the processes by which drops and particles combine as

a) Direct inertial impact collision
b) Condensation
c) Diffusion through a boundary layer
d) Sedimentation effects

If a condensation mechanism is used, particles can be first charged and then serve as nuclei for the drops, or the drops can be first condensed on particles and then charged. Then the drops can be precipitated by using an electric field. In either case, the resulting device is a condensation type scrubber in series with a conventional precipitator. Any virtues of the system must be argued by comparing the condensation scrubber to conventional scrubbers. Devices which use the condensation mechanism are not regarded as uniquely involving the drops and fields to provide a collection

mechanism. Weber indicates that, within the context of scrubbers, little is known about the condensation type of devices and there is little to suggest that they are competitive.

Of the other mechanisms listed, the dominant is usually impact scrubbing. As a particle entrained in the gas approaches a drop with the relative velocity w , it undergoes an acceleration in trying to follow the gas streamline. The resulting inertial force can overcome the viscous drag and cause collision with the drop. The impaction parameter K defined with Eq. (II.1) determines the effective cross section of the drop for collision. In representing the systems aspect of scrubbers. Calvert [38] uses the empirical formula

$$\left(\frac{y^*}{R}\right)^2 = \frac{1}{\left(1 + \frac{0.7}{K}\right)^2} \quad ; \quad K \equiv \frac{2}{9} \frac{\rho_a a^2 w}{R \mu_c} \tag{1}$$

where y* is the radius of the effective collection cross section. If particles are submicronic, μ_c is the air viscosity diminished by the Cunningham factor. The drop is introduced into the flow, perhaps by an atomization process, and is effective as a collector only so long as it has a relative velocity with respect to the gas. Because relative motion is damped out by the gas, there is a limited distance over which the drops are effective and that distance is a function of the drop size. A simplified version of the systems analysis given by Calvert [8] is given here with the objective of identifying the essential parameter values.

Most suspect in our simple model is the use of Stokes' drag for the drops, since in many scrubbers the relative velocity can be large. But, with the understanding that the predictions are restricted, the drops

initiated into the flow with a relative velocity w obey the equation of motion

$$\frac{4}{3}\pi R^3 \rho_R \frac{dw}{dt} + 6\pi\mu R w = 0 \quad . \tag{2}$$

Thus, the relative velocity decays with a time constant

$$\tau_i = \frac{2\rho_R R^2}{9\mu} \tag{3}$$

and we can think of that time constant as the effective lifetime for particle collection of the drop.

The distance traveled by the drop relative to the gas during the lifetime given by Eq. (3) is $w\tau_i$, and hence we can use Eq. (1) to write the volume of the gas "cutout" by the drop, and hence cleaned of particles as

$$\text{volume of gas cleaned/drop} = \frac{\pi R^2}{\left(1 + \frac{0.7}{K}\right)^2} \frac{2\rho_R R^2}{9\mu} w \tag{4}$$

By identifying the mass of the drop, $\frac{4}{3}\pi R^3 \rho_R$, in Eq. (4), and dividing it out, we obtain an expression for the volume of gas cleaned per unit mass of water required.

$$\frac{\text{Vol. cleaned}}{\text{mass of water}} = \frac{1}{6\mu} \frac{Rw}{\left(1 + \frac{0.7}{K}\right)^2} \tag{5}$$

Again, with the understanding that the domain of validity of our model is limited, we can recognize from Eq. (5) that there is an optimum value of the drop radius, R, given by

$$R_{opt} = \frac{2\rho_a a^2 w}{6.3\ \mu_c} \tag{6}$$

and if that optimum values is used, then Eq. (5) becomes

$$\frac{\text{Vol. cleaned}}{\text{mass of water}} \frac{m^3}{kg} = \frac{w^2 \rho_a a^2}{75\ \mu\mu_c} \tag{7}$$

In obtaining Eq. (4), we have ignored the continuous change in relative velocity by approximating the drop motion as being constant throughout a finite "lifetime". Calvert's more refined development leads to an expression which is essentially consistent with Eq. (5), provided the device efficiency is low.

Note that the performance of the scrubber, according to the model developed here, does not depend on the particulate loading of the gas. The volume of gas cleaned per unit mass of water used in no way reflects the amount of particulate removed. By contrast, the Class III type of interactions are characterized in terms of a mass of particulate removed per unit volume of water used. The dramatic dependence of the scrubber efficiency on drop relative velocity (usually on the order of gas velocity relative to duct) w and, more important, on particle size, a, is illustrated by expressing Eq. (7) as

$$\frac{\text{vol. cleaned - ft}^3}{\text{weight of water - lbs.}} = 4.95 \times 10^{-2} \; w^2 a^2 \left[1 + \frac{0.086}{a}\right] \left\{ \begin{matrix} a \text{ in microns} \\ w \text{ ft/sec.} \end{matrix} \right\} \tag{8}$$

Typical values of the volume of air cleaned per unit water used are shown in Table III-1

Table III-1

Volume of Air Cleaned per Unit Weight of Water in $ft^3/lb.$, for Various Particle Radii and Drop Relative Velocities

w-ft/sec \ a microns	0.1	0.5	1	5
1	9.2×10^{-4}	1.45×10^{-2}	5.37×10^{-2}	1.26
10	9.2×10^{-2}	1.45	5.37	126
100	9.2	145	537	12,600

If the drops' relative velocity does not tend to zero, then what limits the amount of gas that can be cleaned by a single drop? The answer to this question comes from practical limitations on potential differences that can be used to pull the drop through the gas. To achieve a given E within the gas volume, the voltage must be raised in proportion to the distance between electrodes. Operating voltages are limited, probably to the range of 100 kv (in view of the wet environment of the electrodes). Hence, the electrode spacing is taken here as a limit on the effective path length of the drops as they pass through the gas. Thus, instead of Eq. (4), the gas volume cleaned by a single drop is the collision cross section multiplied by the path length, ℓ. Using Eq. (1),

$$\frac{\text{vol. cleaned}}{\text{drop}} = \pi(y^*)^2\ell = \frac{\pi R^2 \ell}{\left(1 + \frac{0.7}{K_e}\right)^2} \tag{10}$$

Here, the impact parameter K_e is based on the electrically induced drop velocity given by Eq. (9):

$$K_e = \frac{2}{9} \frac{\rho_a a^2 QE}{R^2 \mu_c 6\pi\mu} \tag{11}$$

By dividing (10) by the drop mass, we obtain the volume of air cleaned per unit mass of water required:

$$\frac{\text{vol. cleaned}}{\text{mass of water}} = \frac{3\ell}{4\rho_R R\left(1 + \frac{0.7}{K_e}\right)^2} \tag{12}$$

To determine if the electrically augmented scrubbing should be given practical consideration, first consider the electrical impact parameters that can be achieved in practice. Whether created by corona charging or by influence charging, the drop charge is likely to be on the order of

$$Q = 12\,\pi\varepsilon_o R^2 E^* \tag{13}$$

Using a relative velocity w = 100 ft/sec., a plant operating at 10^5 cfm would require $(10^5/9.2) \approx 10^4$ lb/min. of water to clean 0.1-micron particles and $10^5/12{,}600 \approx 8$ lb/min. of water to clean 5-micron particles.

B. Class II: Electrically Augmented Impact Scrubbing

In this class of device, the particles are not charged. Drops are given charge, Q, and there is an ambient field, E. Under the assumption that dipole moments induced in the particles by the imposed electric field do not produce a significant interaction with the nonuniform electric field from the drops, the collection mechanism is mechanical and is the same as for the conventional scrubber. The difference is that the drops are charged and made to move relative to the gas by an electrical force. As pointed out in § II.E, space-charge fields can also be used to provide the ambient field. For present considerations, it will be assumed, though, that E is caused by a voltage, V, applied to external electrodes having the spacing ℓ. It does not appear that scaling laws will differ appreciably if E is developed by space charge.

In the conventional scrubber, there is a limitation on the useful life of a drop because of the finite penetration distance. Drops injected at a velocity differing from that of the gas tend to the gas velocity. In the electrically augmented device, each drop has a charge Q which, because the particles are not charged and the other drops have the same charge, can be regarded as constant. In an electric field E, the drop experiences a constant driving force QE, hence tends toward a constant velocity relative to the gas. Under the assumption that the QE force is equilibrated in the steady state by Stokes' drag, we have the drift velocity

$$w = \frac{QE}{6\pi\mu R} \quad . \tag{9}$$

where E* is a charging fields. If we take both the imposed field and the charging field as being E = E* = 5 $\times 10^5$ v/m, then Eq. (11) becomes

$$K_e = \frac{4}{9} \frac{\rho_a a^2 \varepsilon_o E^* E}{\mu_c \mu} = 2.46\ a^2 [1 + \frac{0.086}{a}]\ (a - \text{micron}), \quad (14)$$

Typical values of the electrical impact parameter are given in Table III-2 as a function of particle radius.

Note that K_e does not depend on R. Hence, according to Eq. (12), the cleaning efficiency is inversely proportional to the drop size, R. Of course, there is a lower limit on useful drop sizes imposed by the requirement that the drops be removed from the gas volume before leaving the device. For example, suppose that the drops are injected across the flow. They must traverse the distance ℓ between electrodes before being carried by the gas out of the field region. This requirement is similar to that for particle collection in a conventional electrostatic precipitator, and places a lower limit on R in the range of one micron.

Table III-2

Electrical Inpact Parameter K_e as Function of Particle Radius. Charging and Imposed Fields are Assumed to be 5 kv/cm.

a- micron	K_e
0.1	0.046
0.5	0.722
1	2.68
5	61.5
10	246

A typical device, using an electrode voltage of 100 kv and an ambient E = 5 kv/cm, has electrode spacing ℓ = 0.2 m. Evaluation of Eq. (12)

then gives

$$\frac{\text{gas vol. cleaned}}{\text{water weight}} \quad \frac{ft^3}{lb} = \frac{2.4 \times 10^{-3}}{R\left(1 + \frac{0.7}{K_e}\right)^2} \quad (R \text{ in meters}) \tag{15}$$

Typical values from this expression are tabulated in Table III-3, where use is made of K_e from Table III-2.

Table III-3

Gas Volume Cleaned per Water Weight Required in ft^3/lb as Functions of Particle Radius a and Drop Radius R.

R meters \ a micr.	0.1	0.5	1	5
10^{-6}	9.1	-	-	-
10^{-5}	0.91	62	150	-
10^{-4}	0.091	6.2	15	24

C. Class III: Oppositely Charged Drops and Particles with No Ambient E

In terms of the Whipple and Chalmers model of §II.D, the Class III interactions involve particle collection with $bE_o = 0$, and hence $Q_c = 0$; therefore, interactions fall in domains (j), (k) and (ℓ) of Fig. II-6. The drops are formed and charged in one volume, while the particles are oppositely charged in another. As depicted schematically by Fig. II-1, the drops and particles are then mixed in a separate interaction region. Note that am ambient field is probably used to charge both the drops and the particles. But, in the much larger interaction region, the ambient field is negligible. Also, there can be more than one family of particles and of drops. For example, both positive and negative particles can be used. In the following, we consider only one family of particles and one of drops. Our objective is to obtain relationships between the efficiency of removing particles and residence time, given systems parameters such as drop and particle size, water volume loading, etc.

The theoretical model used here is similar to one developed for the agglomeration between large and small solid particles [37]. It is aimed at determining the residence time, τ, required to remove a given fraction of particulate. As the drops and particles interact, there is a simultaneous decay of the number density, n, of the fine particles and of the charge per drop, Q. Design of a system for making best use of the charged drops requires that account be taken of the play-off between a short residence time and the requirement for large amounts of water. In fact, a certain minimum loading of water is required if an arbitrary fraction of particles is to be removed in a single-stage device, because the collected particles neutralize the drops, and hence nullify the collection mechanism. By contrast with the Class I and II interactions, the scaling laws now depend on the particle loading.

Consider the collection process from a frame of reference moving with the drops. Because the ambient electric field is negligible, we can assume that the relative velocity w between drops and gas is negligible, and hence that the residence time of the drops is equivalent to the effective length of the device divided by the gas velocity.

In regimes (j), (k) and (ℓ) of Fig. II-6, the electrical current to the drop caused by the particle collection is simply

$$i_2 = - nqbE(4\pi R^2) = \frac{dQ}{dt} \tag{16}$$

where b is the particle mobility and Q and q are both positive. Moreover, the electric field, E, at the surface of the drop is merely that due to its own charge, and hence Eq. (16) becomes

$$\frac{dQ}{dt} = - \frac{nqbQ}{\varepsilon_o} \quad . \tag{17}$$

In this expression, the dependent variables are (Q,n).

To obtain a second expression for the decay of the particle density n(t), observe that in a unit volume there are N drops, each collecting one particle for each charge q collected. Hence, the rate of decrease of the particle density is simply N/q multiplied times i_2 from Eq. (16). Thus, the particle density equation is

$$\frac{dn}{dt} = - \frac{nbQN}{\varepsilon_o} \quad . \tag{18}$$

These last two coupled equations determine the decay of the charge and particle density, and hence the residence time required to achieve a given efficiency.

For design purposes, it is convenient to define the following parameters:

$$\zeta \equiv \left[\frac{Q(0)N}{q\ n(0)} - 1\right] \quad - \quad \text{Collection capacity} \tag{19}$$

$$\tau^* \equiv \frac{\varepsilon_o}{bq\ n(0)} \quad - \quad \text{Particle relaxation time} \tag{20}$$

$$\tau_d \equiv \frac{\varepsilon_o}{bQ(0)N} \quad - \quad \text{Drop relaxation time} \tag{21}$$

where $Q(0)$ and $n(0)$ denote the initial drop charge and particle density. Then, in terms of these parameters, solutions to Eqs. (17) and (18) are:

$$\frac{n}{n(0)} = \left[-\frac{1}{\zeta} + \left(1 + \frac{1}{\zeta}\right) e^{\zeta t/\tau^*}\right]^{-1} \tag{22}$$

$$\frac{Q}{Q(0)} = e^{\zeta t/\tau^*}\left[-\frac{1}{\zeta} + \left(1 + \frac{1}{\zeta}\right) e^{\zeta t/\tau^*}\right]^{-1} \tag{23}$$

Also, it follows from Eqs. (17) and (18) that

$$\frac{1}{N}\frac{dn}{dt} = \frac{1}{Q}\frac{dQ}{dt} \tag{24}$$

so that Q and n are simply related by

$$\frac{n(t)}{n(0)} = (\zeta + 1)\frac{Q(t)}{Q(0)} - \zeta \quad . \tag{25}$$

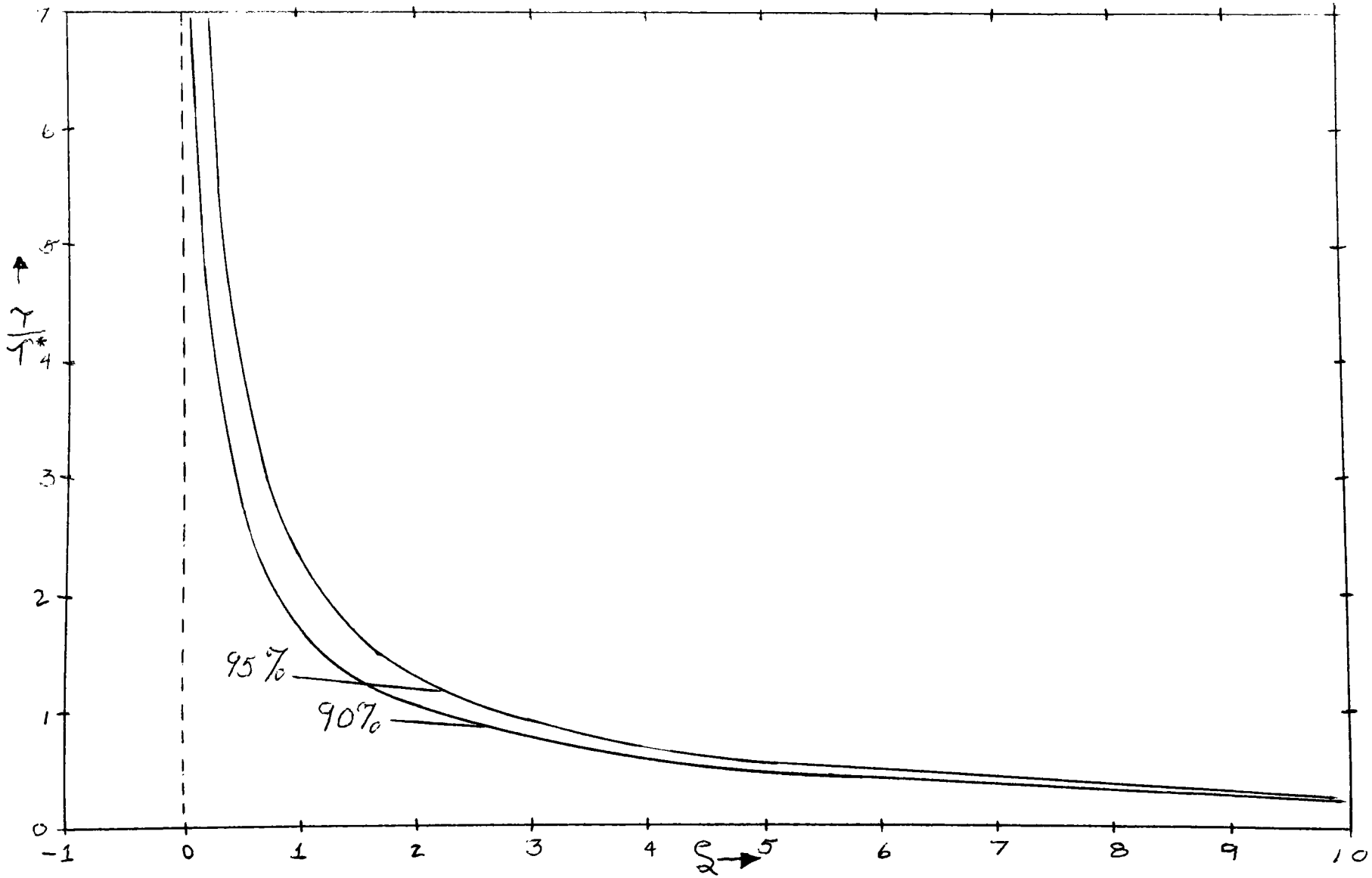
If $\zeta = 0$, then n and Q decay proportionately. If $\zeta < 0$, then Q decays to zero before the particle density is reduced to zero, and the system of drops does not have the capacity to collect all of the particles. By contrast, if $\zeta > 0$, then all of the particles can be collected before the drop charge decays to zero, and there is more than sufficent capacity to collect all of the particles in a single stage. The only excuse for making $\zeta > 0$ is to achieve a given efficiency with a shorter residence time.

If we define the efficiency of single-stage particle removal as

$$\text{Eff} = \frac{n(0) - n}{n(0)} \tag{26}$$

then it follows from Eq. (22) that the residence time τ required for

Fig. III-1 Residence time, τ, required to achieve particle removal efficiences of 90% and 95% as a function of collecting capacity parameter.



achieving that efficiency is

$$\frac{\tau}{\tau^*} = \frac{1}{\zeta} \ln \left[\frac{\frac{\zeta}{(1 - \text{Eff})} + 1}{\zeta + 1} \right] . \qquad (27)$$

The residence time has been normalized to the particle relaxation time τ^* and is plotted as a function of the collection capacity ζ for two efficiencies in Fig. III-1. To make use of Eq. (27) or Fig. III-1, specify the required efficiency of particle removal and the loading capacity ζ. Then, the required residence time is given in units of τ^*.

Particle charge and mobility in the submicron range are typified by values given in Table III-4, where a charging field of 3.6 kv/cm is used. If we specify the particle mass loading, m, in grains/ft^3 (here the particles are assumed to be spherical and to have the mass density of water), and the particle radius is given in microns, then the particle density is

$$n(\#/m^3) = 5.46 \times 10^{11} \frac{m}{a^3} \qquad \begin{matrix} (\text{grains/ft}^3) \\ (\text{microns}) \end{matrix} \qquad (28)$$

Table III-4

Submicron Particle Charge and Mobility and m* (Particles Charged in a Corona Discharge with Field E = 3.6 kv/cm; see White [9], 146-147.)

a-μ	q (coulombs)	b (m^2/v sec)	m* (grain/ft^3)
0.1	1.3×10^{-18}	1×10^{-7}	0.125
0.2	4.0×10^{-18}	1.1×10^{-7}	0.295
0.3	9.5×10^{-18}	1.3×10^{-7}	0.354
0.4	1.7×10^{-17}	1.6×10^{-7}	0.381
0.5	2.6×10^{-17}	1.9×10^{-7}	0.41
0.6	3.7×10^{-17}	2.2×10^{-7}	0.43

Table III-5

M* as a Function of Drop and Particle Radius; mobilities are those of Table III-4.

R - m \ M* lbs/ft^3	a = 0.1	a = 0.3	a = 0.6
10^{6}	1.39×10^{-4}	1.07×10^{-4}	6.3×10^{-5}
10^{-5}	1.39×10^{-3}	1.07×10^{-3}	6.3×10^{-4}
10^{-4}	1.39×10^{-2}	1.07×10^{-2}	6.3×10^{-3}
10^{-3}	1.39×10^{-1}	1.07×10^{-1}	6.3×10^{-2}

It follows from Eq. (20) that

$$\tau^* = \frac{m^*}{n} ; \qquad m^* = \frac{\varepsilon_o a^3}{bq(5.46 \times 10^{11})} \quad . \tag{29}$$

Values of m* are given in Table III-4 for the 0.1 - 0.6 micron radius range of particles (remember, charged in a 3.6 kv/cm field). Thus, with a mass loading of one grain/ft^3 of one micron diameter particles (a = 0.5), the particle relaxation time is given from Eq. (29) with m* from Table III-4 as 0.41 seconds.

The drop relaxation time is similarly written in terms of engineering quantities, by assuming that the drops are charged in a charging field E* to the value

$$Q = 12 \pi \varepsilon_o R^2 E^* \quad . \tag{30}$$

This would be the case if the drops were charged to saturation in a corona field. Also, essentially the same type of relationship between applied field and charge would exist with induction charging of the drops. The only difference would be in a geometric constant, which can be thought of here as absorbed into the effective charging field. For purposes of

establishing the magnitudes of typical quantities, consider a charging field of 5×10^5 v/m. Then, if M is the drop mass loading in lb/ft^3, we can write Eq. (21) as

$$\tau_d = \frac{M^*}{M} : \quad M^* = \frac{R}{(7.2 \times 10^4)b} \ (lb/ft^3). \ (R \text{ in meters}) \tag{31}$$

Values of M* are tabulated in Table III-5, where the particle mobilities have been used from Table III-4. Note that the collecting capacity is

$$\zeta = [\frac{\tau^*}{\tau_d} - 1] = (\frac{M}{M^*})(\frac{m^*}{m}) - 1 \quad . \tag{32}$$

For very small particle loadings, such that $\zeta >> 1$, the drop relaxation time typifies the residence time. This can be seen by taking the limit of Eqs. (27) and (32) where $\zeta >> 1 \Rightarrow \zeta \rightarrow \tau^*/\tau$

$$\frac{\tau}{\tau_d} = \ln[\frac{1}{1 - Eff}] \quad . \tag{33}$$

The tabulated results give a picture of the critical design parameters for a wide range of systems parameters. To illustrate how they can be used, consider the removal of particles characterized by:

particle loading = 0.1 grains/ft^3

2a = 0.2 micron

charging field = 3.6 kv/cm

From Table III-4, it follows that m* = 0.124 grains/ft^3. Hence, using Eq. (29),

$$\tau^* = \frac{m^*}{m} = \frac{0.124}{0.1} = 1.24 \text{ seconds} \quad . \tag{34}$$

To achieve 95% efficiency with a residence time of 2.5 seconds, Fig. III-4 with $\tau/\tau^* \simeq 2$ shows that

$$\zeta = 1.25 \quad . \tag{35}$$

It follows from Eq. (32) that

$$\frac{M}{M^*} = 1.82 \quad . \tag{36}$$

From Table III-5, the water mass loading required is then

$$\begin{aligned} M &= 2.5 \times 10^{-4} \text{ lb/ft}^3 \text{ for } R = 10^{-6} \text{ m} \\ M &= 2.5 \times 10^{-3} \text{ lb/ft}^3 \text{ for } R = 10^{-5} \text{ m} \quad . \end{aligned} \tag{37}$$

This means that, in a plant having a flow rate of 10^5 cfm, 25 lb/min. of water injected in the form of 2-μ-diameter drops would be required.

As a rule of thumb in determining effects of collecting capacity, the charging is proportional to radius, and under the assumption that both particle and drop charging is accomplished in fields of the same typical intensity, we have $Q/q \propto R^2/a^2$. Also, $N/n \propto \alpha_a^3/\alpha_R^3$ and hence we can, under these assumptions, write the loading capacity as

$$\zeta = \left[\frac{QN}{qn} - 1\right] = \left[\frac{a}{R}\frac{R^3}{a^3}\frac{\alpha_a^3}{\alpha_R^3} - 1\right] = \left[\frac{V_R}{V_a}\frac{a}{R} - 1\right] \quad . \tag{38}$$

where V_R and V_a are the volume loading of drops and particles, respectively. That is, in order to achieve a loading capacity $\zeta = 0$, the volume ratio of water to particles must be equal to the ratio of the drop radii to the particle radii. To achieve an arbitrary efficiency in a single stage, we must have $\zeta > 0$. Thus, to obtain a large fractional removal of particles of 0.1 micron radius using drops of 10 micron radius, the volume of water must exceed that of the particulate by a factor of approximately 100. This is essentially what has been found with Eq. (37), since 2.5×10^{-3} lb/ft^3 = 17.5 grains/ft^3 of water to clean 0.1 grains/ft^3 of particles,

and we have taken particles and water drops as having the same mass densities ρ_a and ρ_R.

As a further rule of thumb in determining residence times, τ_d is a characteristic time if the collection capacity is high, and τ^* is a typical time if it is in the range 2 - 10 (see Fig. III-1).

D. Class IV: Ambient E, Particles Charged and Drops Initially Uncharged

The interaction region for this class of devices is one in which the previously charged particles are mixed with uncharged drops under the influence of an ambient electric field. In terms of the drop charging diagrams (summarized for positive, for negative, and for both positive and negative particles by Figs. II-6, II-7 and II-8 respectively), the drop starts out on the horizontal axis, where $Q = 0$. Because there is initially no net charge on a drop, the electric field does not give rise to a relative velocity w, and hence the drop state is on the horizontal axis between either regions (f) and (i) or (d) and (g) in Figs. II-6 and II-7, or in the case of particles charged to two different signs, between regions (h) and (ℓ) or (e) and (i) in Fig. II-8.

The drop charging subsequent to mixing with the particles has many possibilities, depending on the relative numbers of positive and negative particles. Probably the most practical and certainly the simplest is the one in which the number of particles charged positively just equals that charged negatively. Then the currents charging the drop, depicted in Fig. II-8, and given by Eqs. (A.29) and (A.2), are:

$$i_1^+ = -i_1^- = 3\pi R^2 bnqE \quad . \tag{39}$$

Thus, the drop charge remains zero as it collects particles. The number of positively charged particles collected per unit volume is $(N/q)i_1^+$ and an

equal number of negatively charged particles per unit volume are also collected by the drops. Thus, the particle density can be written as

$$\frac{dn}{dt} = - \frac{n}{\tau_{IV}} \tag{40}$$

where the time constant for collection is

$$\tau_{IV} = \frac{1}{6\pi R^2 bNE} \quad . \tag{41}$$

The solution to Eq. (4) is simply

$$n = n(0)e^{- t/\tau_{IV}} \tag{42}$$

and hence the efficiency of particle removal for a system having the residence time τ can be written as

$$Eff = 1 - e^{- \tau/\tau_{IV}} \quad . \tag{43}$$

To determine the dependence of the time constant τ_{IV} on drop loading, note that it can be written as

$$\tau_{IV} = \frac{2R\rho_p}{9MbE} = 2.7 \times 10^{-5} \left(\frac{R}{Mb}\right) \quad \begin{matrix} R - \text{meters} \\ M - lb/ft^3 \\ b - m^2/sec\text{-}V \end{matrix} \quad . \tag{44}$$

Typical values of τ_{IV} are given as functions of drop radius R and lb/ft^3 of water required for removal of two sizes of particles in Table III-6. Note that it is desirable to have as small a drop radius as possible. Just how small R can be made, practically, is a matter of the means used to remove the drops, once they have been used to collect the particles. Because conventional electrostatic precipitation techniques would probably be used to remove the drops, a drop radius of one micron is a reasonable minimum.

If one of the particle families dominates the other, the collection of particles on drops and removal of drops to the electrodes could be combined. Then, in the process of collecting particles, the drops would become charged. Because of the ambient electric field, there would result a precipitating force tending to collect the drops on the electrodes. Also, space-charge fields could be used to precipitate the drops, once charged by the particle collection. But note that, as the drops pick up speed relative to the gas, the relative velocity w in the charging diagrams becomes finite. In fact, if the charging field for the particles is on the order of the ambient field in the interaction region, the drops have mobilities larger than those of the particles, by a factor R^2/a^2. Hence, the drops cannot reach the critical charge Q_c without first being limited by the expansion of the regime (e), in which the drop velocity relative to the gas exceeds that of the particle.

Table III-6

Collection τ_{IV} in Seconds for Class IV Interactions with Bipolar Particles: Ambient E = 5×10^5 v/m : Particle Mobilities from Table III-4.

M(lb/ft^3)	R = 1μ , a = 0.1 μ	R = 5 μ, a = 0.5 μ
10^{-5}	27.0	71.0
10^{-4}	2.70	7.10
10^{-3}	0.27	0.71

E. Class V: Hybrid Interactions

In hybrid interactions, both the effects of a drop charge and of an ambient field are significant. The result is a combination of the Class III and IV interactions. Practical systems considerations motivate the use of a hybrid interaction. For example, once drops have exhausted their collection capacity in a Class III interaction, it might be desirable to subject them to an ambient field. With unipolar charging of the particles, they would then recharge while continuing to collect particles. Also, they could then be removed by means of the imposed electric field.

One of our reasons for introducing the charging diagrams of Figs. II-6-8 is to emphasize the large number of ways in which a drop can attract a current of charged particles. No attempt will be made to give performance equations for hybrid systems. Partly, this is due to the diversity of possible combinations of field, drop charge, and relative drop velocity. But also, we can expect that, basically, no new collection phenomena are brought into play.

IV. Production of Charged Drops

A. Classification of Particle Production Techniques

It is convenient to classify methods of making charged drops according to:

a) the manner in which the drops are formed in a mechanical sense. Mechanisms are of two types: i) Mechanical atomization, in which a liquid bulk is broken up into drops typically in stages by first forming liquid sheets or jets or jets that reduce to drops which in turn subdivide to the required size. ii) Condensation; a saturated phase is condensed on nuclei leading to droplets that can be made to grow to the proper size.

b) the means used to charge the drops. Again, methods fall into one of two categories: i) Bulk charging; the drops are charged after they have been established essentially as mechanical entities. Charging by subjecting drops to the combination of an electric field and an ion flux from a corona discharge is an example. ii) Charging at birth. Condensation on ions is a charging mechanism in this category. Another is influence or induction charging, which occurs as drops are in contact with a reservoir of charge and under the influence of a charging field... the "splashing" charging of Lenard and Zeleny is strictly in this category, with the charging taking place essentially by induction as drops encounter a metallic or insulating surface.

In the following § B and C, we first comment on the alternative means of mechanically forming drops. Then, § D through F relate to the charging techniques.

The formation of charged drops usually amounts to adding one of the electrical charging methods to a conventional mechanical process. Although the electric field can be used to augment the instability of sheets, jets, and drops in an atomization process, the main source of energy for making the drops is usually mechanical or thermodynamic. In the case of influence charging, no electrical energy is required in principle, and with corona charging, the process is typically in a region removed from that where the particles are formed. Hence, the electric field energy does not contribute in an essential way to the energy supplied to the mechanical drop formation.

The electrohydrodynamic spraying technique for producing charged drops is discussed in § G, and does make use of the electric field to form the drops in a mechanical sense. Fluid pumping, atomization, and charging are accomplished in a single process. That process can still be categorized as indicated above. The drops are formed by atomization and primarily charged by induction, but the electric field is intimately involved with both charging and atomization.

The main objective of the following discussion of drop production techniques is to determine the feasibility of using schemes outlined in the previous sections. Does the particle production problem make the use of fields and drops impractical? However, the electrohydrodynamic spraying introduces a broader issue. Is there an electrically driven alternative to existing mechanical techniques for atomization? Even if the drop charging is superfluous, an alternative to the extremely inefficient devices used for atomization would be highly significant. Our discussions therefore bear on what is known about the efficiency with which current-driven jets produce drops.

B. Mechanical Atomization

A general review of conventional means for atomizing liquids into a gas is given by Perry [38]. It is not appropriate to repeat his review here. Relevant, however, are the observations that in conventional devices (whether they make use of liquid ejected under pressure, of rotating members, or of liquid and gas ejected under pressure), fundamentally the energy required to form drops of radius R is simply that necessary to make new surface. To make a single drop, the energy required is

$$W_\gamma = 4\pi R^2 \gamma \tag{1}$$

where γ is the surface tension. Thus, the energy required per unit volume of the water is

$$W_\gamma = \frac{3\gamma}{R} = \frac{2.16 \times 10^{-3}}{R} \frac{\text{joules}}{\text{m}^3 \text{H}_2\text{O}} \tag{2}$$

where R is in meters.

In practice, dominant power losses result from making drops at a finite rate. These are attributable to: i) residual kinetic energy resulting from dynamic breakup as, for example, that caused by mixing with high velocity air stream, (ii) viscous losses as the water passes from the bulk state to a system of drops, and iii) pump-type losses associated with the production of hydrodynamic head. Of these, the greatest factor is the second. In making 1μ drops, it has been estimated to be 10^5 to 10^7 times greater than that theoretically needed to form the new surface in one case study [39].

Calculation of power requirements for atomization can at best be crude. Atomization devices are diverse in design, and even the laboratory model designed to simplify correlations with theory shows that practical atomization processes are extremely complex. Perry states of the losses in atomization: "They are generally incalculable, and the power requirement of an atomizing system must be predicted from experience".

Here, two examples based on data for commercial sprayer nozzles are used to establish order-of-magnitude estimates of energy requirements in producing drops. Perry [38] cites data for a hollow-cone pressure nozzle producing drops with a median radius of 50 μ. Drops are produced by a single unit at 0.2 gal/min using a pressure of 250 lb/in^2. Thus, the power requirement for the single unit is 20 watts. If we assume a spray system comprised of a plurality of such units, then the power requirement is 100 watts/(gal/min).

It is usual to give power requirements in watts/cfm of gas. To this end, recognize that the gal/min of water required to process 1000 cfm of air using M lb. H_2O/ft^3 gas is

$$\frac{\text{Gal-}H_2O/\text{min}}{1000\ \text{cfm}} = 120\ M \quad . \tag{3}$$

A typical water loading for drops in this 50μ radius range [see Eq. (37)] is:

$$M \cong 10^{-2}\ lb/ft^3 \quad \text{or, from Eq. (3), } 1.2\ (\text{Gal/min})/1000\ \text{cfm}.$$

Note that relatively large drops have been used so as to be consistent with the sprayer characteristics. This rather large water requirement is the penalty paid for using such large drops. The power requirement is (1.2)(100) = 120 watts/1000 cfm. Power requirements for typical electrostatic precipitators are in the range of 30 - 130 watts/1000 cfm (see Ref. [9], page 204). Note that the conventional sprayer considered here produces a wide range of drop sizes, hence is inherently wasteful of the given water supply.

Drop sizes an order of magnitude smaller than produced by the spray nozzle just considered are most likely required. A given mass of these is certainly produced at considerably greater expense. Pneumatic nozzles are more adaptable than pressure types to drop sizes in the micron range. An

example is a sprayer producing 5μ median-radius drops [40]. (Sprayer Setup #1A supplied by Spray Systems Co., 3201 Randolph Street, Bellwood, Illinois). This device produces 0.5 gal/hr of drops with the major portion of the energy supplied by compressed gas; 50 psi at 1.13 standard cubic ft/min. Estimated power supplied based on an isothermal expansion of the air [38] is 7×10^4 joules/lb H_2O. Again, using the case study for Class III devices, Eq. (37), a typical water loading using R = 5μ is 10^{-3} lb H_2O/ft^3 gas, or 0.0166 lb H_2O/sec/1000 cfm. Hence, the required power is $(7 \times 10^4)(0.0166) \approx$ 1 kw/1000 cfm ! To accomplish this, approximately 12 of the sprayer units would be required per 1000 cfm of gas processed.

The two examples presented here only establish the relative significance of the atomization problem. Certainly, no suggestion is made that the power requirement in this last example is so high as 1 kw/1000 cfm. For one thing, the sprayer systems considered are not designed for the scale of the application. More important, note that the particle loading of 0.1 grain/ft^3 of 0.2 micron diameter particles is extremely large. But it should be clear that atomization of drops is a major consideration in making the use of drops as particle collectors feasible. In the Class III type of interaction, the drops play the role of electrodes, and in a sense the capital investment in an electrode system is offset by the operating cost of making the drops.

Means of atomizing liquid mechanically are extremely diverse. Spinning disks and cylinders are often used to induce breakup of the liquid bulk [11,46]. Most of these methods can be used with the charging methods discussed in § E and F to produce charged drops. In a sense, the hydrodynamic pumping is supplied in part by the atomizer itself in such devices. As with the pressure and pneumatic type nozzles, hydrodynamic losses are the dominant power sink.

c. Condensation

Clouds give an example of droplets formed by condensation of a supersaturated vapor. In fact, the water loading commonly found in clouds of 0.2 - 1 g/m^3 approaches the 10^{-4} lb/ft^3 found in the Class III interactions as giving efficient collection of 0.1μ particles. Scientifically, a distinction is made between the condensation from a vapor phase to a system of droplets, according to whether nuclei such as ions or microscopic particles augment the formation (heterogeneous nucleation), or if the process requires a relatively high supersaturation because of the absence of nuclei (homogeneous nucleation). In practice, and most certainly in the environment of a drop collection system, the heterogeneous condensation is dominant. Fletcher gives a summary of the theory involved (Ref. [41], Chapter 3) from a cloud physics point of view.

There are two issues to be raised in connection with condensation as a mechanism for forming drops. First, simply as an alternative method for making drops, is a condensation method attractive? If ions are used as nuclei, there is the possibility of simultaneously providing for not only the mechanical formation, but the charging as well.

Second, the condensation nuclei can be the particles to be precipitated. Then the question is broadened to the viability of a collection system, rather than a drop-producing mechanism. A collection device might consist of two stages. In the first stage, particles are used as condensation sites. In the second, the drop-entrained particles are electrically removed, much as in a conventional precipitator. If the condensation sites are not charged prior to the drop formation, then charging can be achieved by means of a corona discharge after the condensation is complete. The device described by Vicard and classified in § III as "Miscellaneous" is in this category.

As pointed out, Vicard's type of device really does not reveal an innovation in the use of the electric fields and drops. The essence of the process is in the condensation on the particles. Once enveloped by water, the particles are collected in a conventional manner, and the mechanism of bringing particle and drop together is only indirectly related to the electric field. (Although there is a connection between the charge on nuclei and, for example, critical condensation radii, particles can act as nuclei, whether charged or not.)

There are at least two approaches to forming drops by condensation. The first involves processing a major fraction of the stack gas by humidifying (if necessary) and then cooling below the dew point. The second makes use of nozzles for the processing of a relatively small volume, perhaps of steam available from a thermal cycle, with this volume of vapor-entrained drops injected into the dirty gas [42]. A determination of the feasibility of using these methods, and particularly the latter, which appears to be the more practical of the two approaches, requires a knowledge of the incidence of condensation as well as the growth dynamics, so that drop sizes can be accounted for. Condensing methods generally come into their own for producing extremely fine drops; 0.1μ or less. Here, we are interested in relatively large drops, 1μ or greater. No attempt will be made here to calculate the energy requirements of condensing methods of drop formation.

D. Limits on Electrical Charging

In all but the Class IV interactions, charged drops are required. The two main charging mechanisms are discussed in the following two sections. Both are limited by how much charge can be placed on a drop without inducing electromechanical instability or fissioning of the drop. The maximum charge that can be placed on a drop without an ambient electric field is called "Rayleigh's limit" [43], and is given in MKS units by

$$Q_{Ray} = 8\pi \sqrt{\varepsilon \gamma R^3} \tag{4}$$

where γ is the surface tension. This is one upper bound on the drop charge used in the Class II and III interactions. For water, $\gamma = 7.2 \times 10^{-2}$ newt/m and Eq. (4) is conveniently written as

$$Q_{Ray}(\text{coulombs}) = (2 \times 10^{-5}) R^{3/2} \tag{5}$$

where R is in meters.

Similar electromechanical considerations place an upper limit on the electric stress that can be applied to an uncharged drop in an initially uniform electric field without producing rupture. Here, Class IV interactions are of interest. In this case, instability results in two or more drops which, because of the initial drop polarization, are likely to be charged. The critical electric field is [44]:

$$E_{Tay} = 0.458 \sqrt{\frac{\gamma}{\varepsilon_o}} R^{-1/2} \tag{6}$$

For water drops in air, Eq. (3) becomes

$$E_{Tay}(\text{v/m}) = \frac{4.12 \times 10^4}{\sqrt{R}} \tag{7}$$

Table IV-1

Rayleigh's Limiting Charge Q_{Ray}, Taylor's Limiting Electric Field Intensity E_{Tay} and the Saturation Charge as a Function of Drop Radius

R - m	Q_{Ray} - coulombs	E_{Tay}	Q_c at $E^* = 10^6$ v/m
10^{-6}	2×10^{-14}	4.12×10^7	3.34×10^{-16}
10^{-5}	6.32×10^{-13}	1.3×10^7	3.34×10^{-14}
10^{-4}	2×10^{-11}	4.12×10^6	3.34×10^{-12}
10^{-3}	6.32×10^{-10}	1.3×10^6	3.34×10^{-10}

In § III, for estimating drop charges, we made use of the saturation charge on a drop charged in a field E*:

$$Q_c = 12\,\pi\varepsilon_o R^2 E^* = 3.34 \times 10^{-10} R^2 E^* \left\{ \begin{matrix} R \text{ - meters} \\ E^* \text{ - v/m} \end{matrix} \right\} \qquad (8)$$

Table IV-1 shows the dependence on drop radius of Rayleigh's limiting charge, of Taylor's limiting electric field intensity, and the saturation charge given by Eq. (8). Note first that, even under the assumption of a charging field E* twice as large as that used in typical calculations in § III, the saturation charge is less than Rayleigh's limit. Hence, there is some leeway on the drop charge that can be used insofar as Rayleigh's limit is concerned. Second, only for the extremely large drops are electric field intensities required to produce rupture of an uncharged drop within a range where they might be encountered in a practical device. (The breakdown strength of dry air between uniform electrodes is about 3×10^6 v/m.) Thus, under practical conditions, either in the Class III or IV interactions, there does not appear to be any limit on the drop charging brought about by the electrohy-

drodynamic instability of the drop.

E. Corona Charging; Charging in the Bulk

The conventionally used theory of "field charging" [9] is a limiting case of the Whipple and Chalmers [22] drop charging picture introduced in § II.D. As a drop or particle passes through the region of a coron discharge, it experiences simultaneously an ambient electric field and an ion flux. Hence, the role of the "particles" in the Whipple and Chalmers theory is played by ions which typically have mobilities much greater than those of the particles... on the order of 10^{-4} m^2/v-sec. Hence, under field conditions practical in a device, the ions move at hundreds of meters per second, and the relative velocity of the gas can be neglected. In the charging diagrams, this means that $w \to 0$.

As shown by the charging diagrams of Figs. II-6, drops or particles entering with no charge Q are charged to a limiting "critical" or "saturation" charge given by Eq. (8). (This charging process caused by particles or drops collecting ions should be distinguished from the one in which drops collect particles.) The charging equation for a drop or particle is

$$\frac{d(Q/Q_c)}{dt} = \frac{bn_i q_i}{4\varepsilon_o}(1 - Q/Q_c)^2 \qquad (9)$$

where b, n_i, and q_i are the ion mobility particle density and the charge, respectively. Thus, the charging rate is governed by the time constant $4\varepsilon_o/bn_i q_i$. Because this quantity does not depend on the nature of the drop or particle being charged, we can conclude that it is typified by charging time constants in conventionally used corona chargers. It is well known that the charging time in a conventional precipitator can be made quite short compared to typical residence times [9]. Because the well developed theories

of particle and drop charging for conventional precipitators are equally applicable here, further discussion of the corona charging mechanism is not required.

F. Influence Charging

A powerful mechanism for charging highly conducting drops is influence or induction charging. By "highly conducting", we mean drops which have electrical relaxation times $\varepsilon_o(2 + \varepsilon/\varepsilon_o)/\sigma$ short compared to typical times involved in the drop formation. As discussed in Appendix B, this condition is easily met in the use of water, unless the formation time is extremely short.

Influence, or induction, charging is characterized by the configuration of Fig. IV-1.

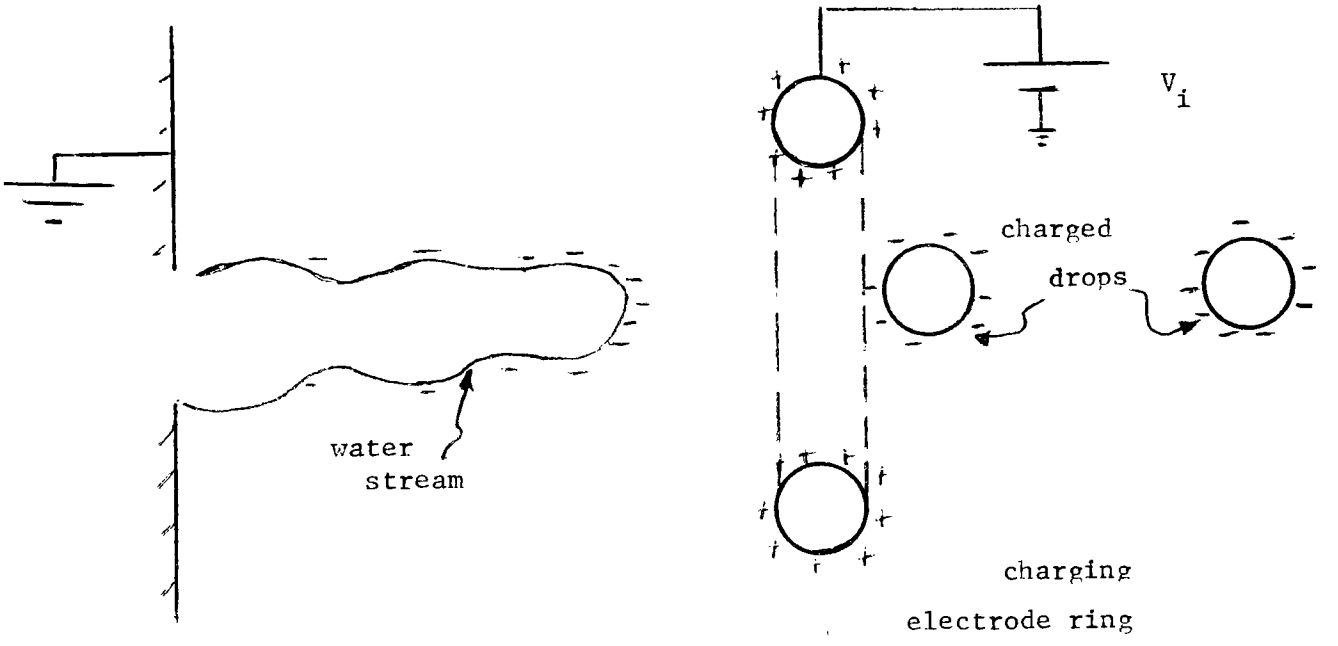


Fig. IV-1 Induction charging of drops

Drops are formed in a nozzle so that they break away either from the conducting nozzle, or from a short jet, under the influence of an electric

field. This field is applied by means of a charging electrode having a potential V_c relative to that of the water; thus, as it forms, the drop is one electrode of a capacitor having an applied voltage V_c. Maximum charge per drop, Q, is obtained by making the drop break away from the nozzle or jet as it accrues the maximum charge.

Estimates of the charge/drop attempt to account for the geometry of the water interface as the drop breaks away. As a rule of thumb, the charge is on the order of the cross-sectional area of the drop, πR^2 multiplied by $\varepsilon_o E^*$, where E^* is a typical charging electric field intensity. For example, if at the instant of breaking loose, the drop can be represented as a sphere on a conducting plane, then the charge is [45]:

$$Q = 6.56\ \pi R^2 \varepsilon_o E^* \quad . \tag{10}$$

Here, E^* is the uniform field intensity in the absence of the drops, so the field concentration around the drop is accounted for. Note that, within a factor of 2, this expression for the drop charge caused by induction is the same as that given by Eq. (8) for corona charging.

By contrast with the corona method of charging, the induction charger in principle, requires no electrical power input. To see this, observe that over the period of drop charging, the amount of energy required from the source V_c is

$$\int V_c\, \delta Q_e \tag{11}$$

where Q_e is the charge on the charging electrode. Because V_c is constant, the integration of Eq. (11) amounts to taking the difference between the Q_e's at the end and beginning the charging cycle. Because the drop charging is

periodic, the electrode charge at beginning and end of the cycle are the same, and hence no electrical energy is supplied.

Even though the electrical energy required for the idealized charging is zero, the losses connected with the influence charger are a major consideration. As discussed in §II, much of the patent literature pertains to means for avoiding the fouling of the charging electrode by the drops. From Fig. IV-1, it is clear that a charged drop will tend toward the charging electrode. Invariably, the charging ring intercepts at least a fraction of the drops and becomes the seat of electrical activity. Drops re-emitted from the charging electrode have a polarity opposite to that from the nozzle. In a scheme which depends on having only one sign of drops, this is a nuisance. Also, drops formed from the charging electrode by electro hydrodynamic spraying consume power from the source V_i, and so the ring capture brings into play electrical losses.

The powerful charging mechanism provided by influence charging makes it an attractive basis for drop charging. If the configuration can be properly designed to avoid fouling of electrodes by drops and particles, the power requirements for drop formation are essentially the same as for a wet scrubber. The drops are formed mechanically, and charged with a negligible electrical loss.

In actual devices, it is not always clear whether the charging process is mainly inductive, or involves corona charging caused by ions generated in the vicinity of the orifice. But the use of induction charging with various configurations of atomizing devices is well established [11, 47, 48, 51]. There is little doubt but what the physical principles dominating this method of producing charged drops are well understood [49].

Induction charging is responsible for what is sometimes called "Lenard" charging [50] of drops, caused as they form from larger drops splashing on a solid surface.

G. Electrohydrodynamic Spraying

Over the past several decades, with various applications in mind, inventors have recognized the advantages of using an electric field to atomize liquids [47, 48,63]. Although investigated qualitatively by a number of researchers [52,53], it is only recently that work has begun to give a rational description. Motivation comes not only from the hope for a more efficient technique from the standpoint of power requirements, but,at least for some applications, from the tendency of the drops to be highly charged.

Typical electrohydrodynamic sprayers are shown in Fig. IV-1. In (a), liquid is forced through an orifice much as in an ordinary pressure nozzle, while in (b) liquid is injected into a mixing region where it is entrained in the fashion of a pneumatic nozzle. In both cases, the drops form in the region of an electric field, acting either to charge the interface inductively, or if fields are sufficient to produce local electrical breakdown, to provide for corona charging through the action of an ion flux in the drop formation region. As thus far described, each of these devices is simply a conventional nozzle fitted with a charger as described in previous sections. However, in electrohydrodynamic spraying, the field has a radical effect on the drop formation.

It is helpful to distinguish between two mechanisms by which the electric stress can contribute to liquid atomization:

i) The electric field contributes to the stability of the interface. Just as surface tension is responsible for the breakup of a laminar jet into drops, the electric field can be the agent

of increasing, or the sole agent of causing, the instability of an interfacial configuration. Rayleigh's limit on the charge that can be placed on an isolated drop (see § D) is caused by the incipience of such an electromechanical instability.

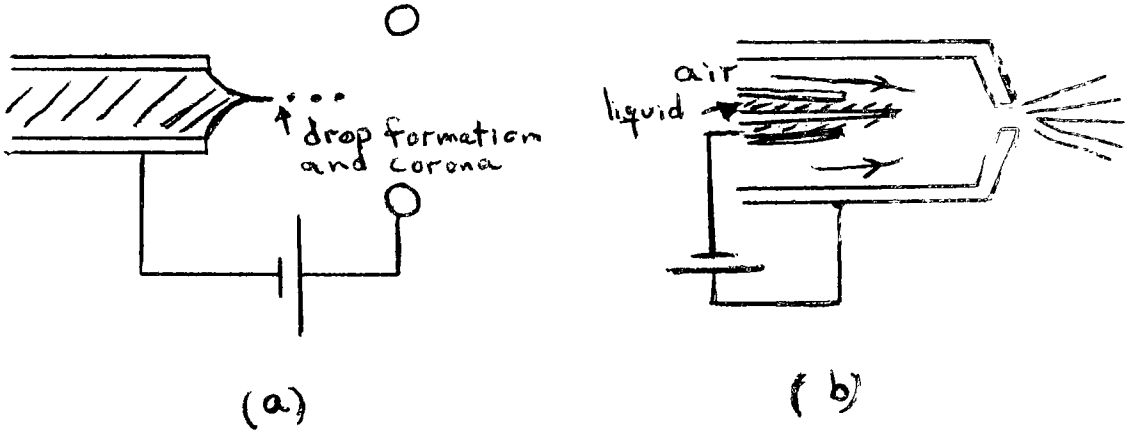


Figure IV-1 Nozzle type electrohydrodynamic spray configurations

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Even a flat interface will "buckle" under a sufficient electric stress. Typically, the electric field acts normal to the interface, as sketched in Fig. IV-2. A perturbance on the interface is accompanied by a local field concentration and an accumulation of surface charge. The result is such an increase in the local surface traction that the surface deflection is further increased. Liquid jets in a radial electric field not only exhibit an enhanced rate of growth for sausage instabilities associated with the surface tension, but are also unstable in kinking modes [60].

ii) If a mechanism is available to force a current flow through the liquid, an interface can be subjected to a tangential electric field as sketched in Fig. IV-3. With the interface also charged, there results an electrical shear stress on the interface which is capable of accelerating and effectively pumping the fluid [61].

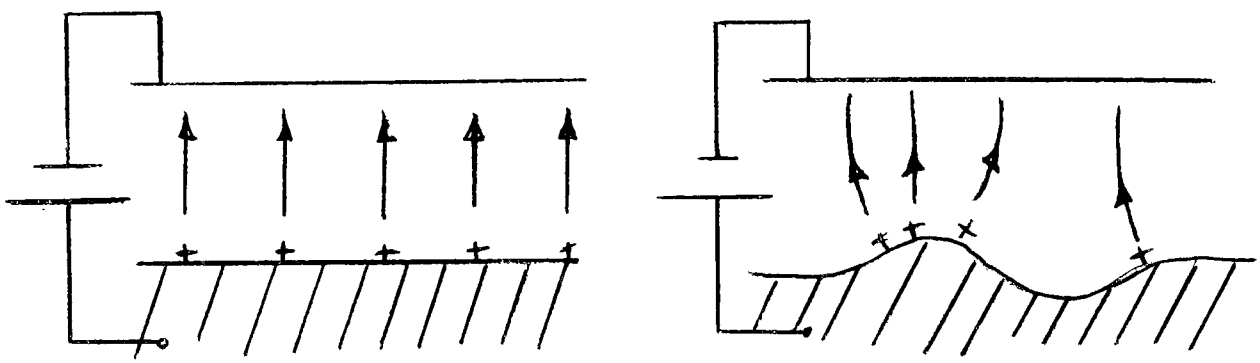

Figure IV-2 Local interface subject to normal stress tends to buckle and exhibit instability.

The destabilizing effect of (i) alone is enough to have an appreciable effect on drop production. However, under practically realizable electric stresses at atmospheric pressure, the tendency of the field to produce instability is not sufficient to create single-step processes that change typical dimensions by orders of magnitude. For example, even with the help of a field perpendicular to its surface, under realizable conditions a jet breaks up under the action of surface tension into drops that are within an order of magnitude in diameter of the jet. Similarly, the direct consequences

of instability attributable to normal stresses alone on drop breakup are fission fragments within an order of magnitude of the same size.

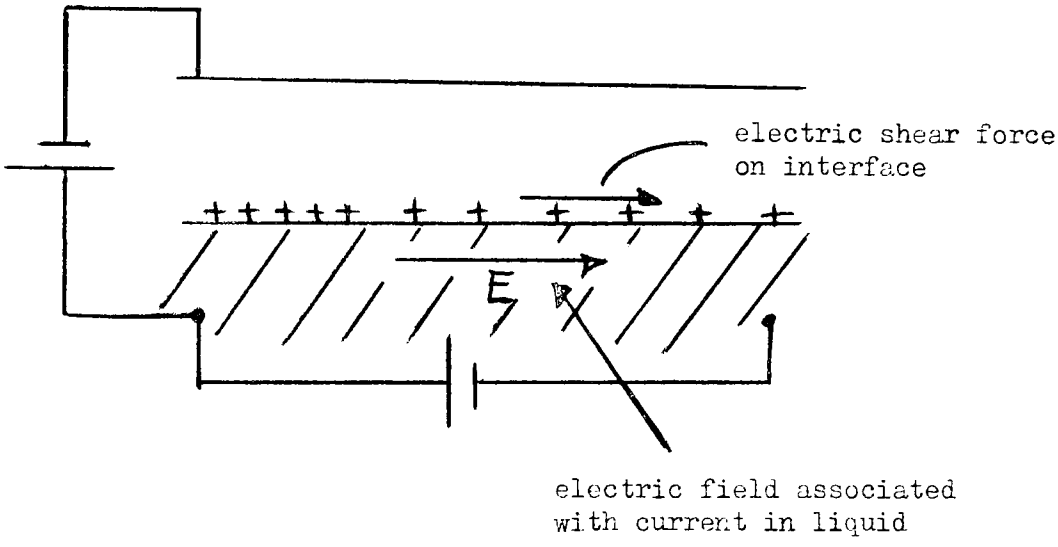

Figure IV-3 Combination of current in liquid and surface charge results in electric shear force on interface

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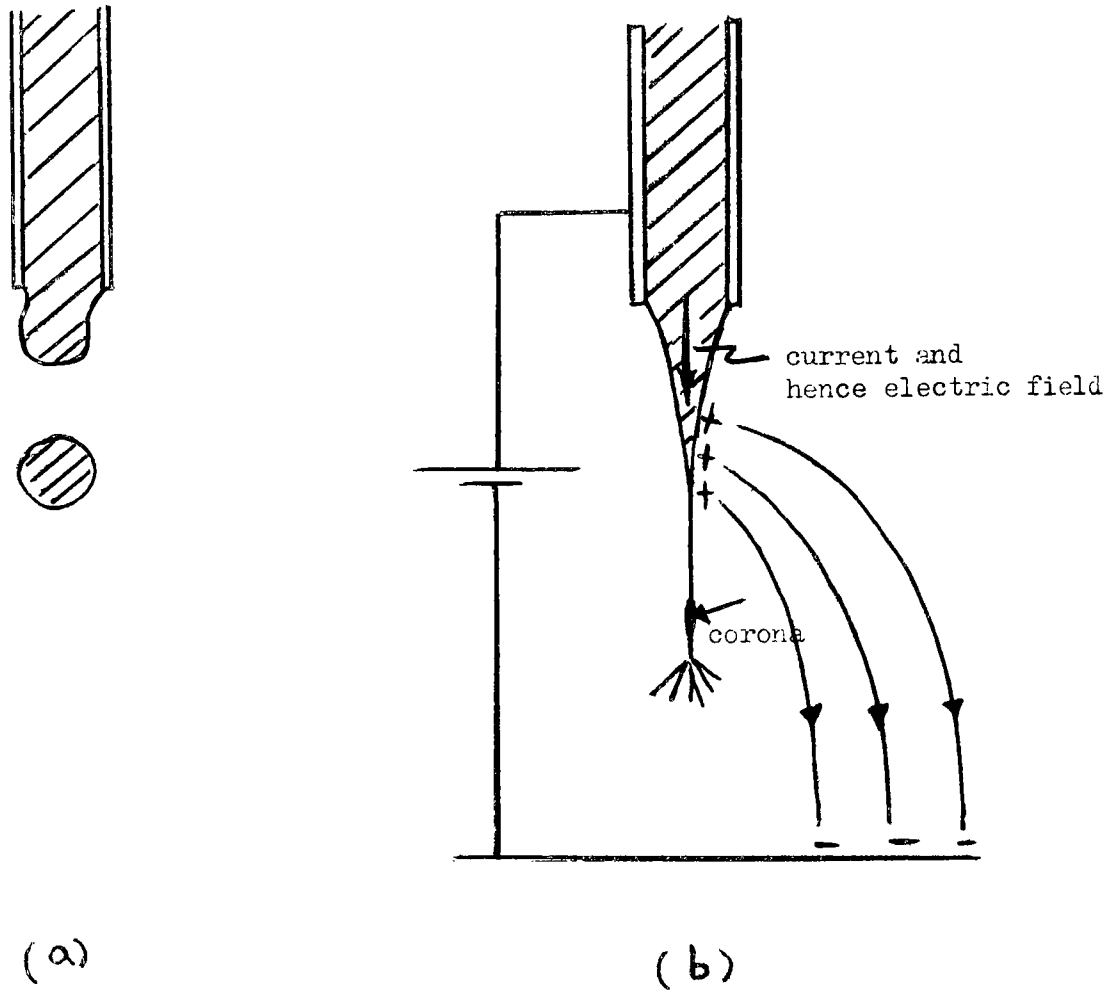
To account for the drastic changes in size that can be obtained using electric fields, proper credit must be given to the role played by the electric shear stresses. Consider how a detailed investigation has shown the current-driven jets to produce first, a fine thread of liquid, and then a spray of charged drops [62,10].

Consider the orifice shown in Fig. IV-4, with a potential difference applied between the liquid and an electrode structure which might consist of a ring-shaped electrode, so as to allow for passage of drops (like the inducer rings in Fig. IV-1). Without a field, the fluid is supplied with sufficient rate to produce slow dripping in the range of 1 mm drop radii.



Fi . IV-4 (a) In the absence of an electric field, liquid drips from orifice in l rge drops. (b) With field above threshold, liquid forms jet which accelerates into fine thread breaking up into spray and corona.

As the voltage is raised, with the volume rate of flow kept constant, the drop rate increases and hence the size of the drops decreases. This trend continues until a transition is achieved to a regime in which the dripping is replaced by aperiodic spitting. Drops are now accompanied by corona discharge and a spectrum of sizes is produced.

Although the trend with increasing field is still to produce smaller drops, these are only about a factor of 10 smaller than those that form in the absence of the field. As the field is raised still further, a dramatic switch in the flow configuration is obtained, with the dripping replaced by a steady stream terminated in a fuzzy region of spray, as sketched in Fig. IV-4(b). The liquid narrows to a fine jet which is steady from orifice to point of breakup. This stream has been observed to be as fine as a micron in diameter [64].

To understand the current-driven jet, the combined effects of the shear stress, corona discharge, polarization forces, and of the normal stress-induced instability must be taken into account. Observation of jets in the dark shows that just above the region of spraying, there is a corona discharge. Hence, current is carried by the jet from the orifice to the position at which the electrical breakdown occurs. This current insures that the jet interface experiences a tangential electric field. The interface is also charged. In the point-to-"plane" configuration, positive charges generally are distributed along the interface, as sketched. Hence, there is an electrical shear stress tending to accelerate the stream and draw it out into the fine thread. Further, as long as the jet carries the current, it can be shown that the field, through the agent of polarization forces, tends to stabilize the stream. Thus, the natural tendency

of surface tension to cause jet breakup is offset by the field and the stream tends to be stable, again as observed. However, once the stream narrows to a sufficiently fine stream that electrical breakdown occurs, the corona discharge carries away the stabilizing current. From that point on,the electric field is normal to the interface and tends to cause instability. In fact, the conditions for instability are greatly exceeded and the jet breaks into drops violently. Because the breakup occurs in a region occupied by both field and ions from the corona discharge, these drops are highly charged.

In a current-driven jet, the electrical shear stresses act on the interface and reduce the size of the stream by orders of magnitude before processes of instability subdivide the stream into drops. Because these stresses act on the interface itself, it can be expected that the atomization process is a more efficient one than that obtained, say, by pushing the liquid through a rigid orifice.

In recent years, studies have been made of electrohydrodynamic spraying for its application to drop production in space propulsion [54, 55, 56, 57, 58, 59]. Unfortunately, the interest of that work is in drop formation in vacuum. As the above description suggests, electrical breakdown is closely related to the spraying obtained at atmospheric pressure. In fact, although notgenerally given much attention, it is difficult, if not impossible, to obtain electrohydrodynamic spraying under standard conditions without producing large numbers of ions through attendent corona discharge. Electrical breakdown may occur prior to the drop formation, but generally extends throughout the region where the drops break away.

It is essential to recognize that current-driven jets are the source of both ions and charged drops. That the electrical source must supply

power for the ions means that there is an inherent inefficiency. Further, particles passing through the field region of the jet will be subject to an ion flux which can result in corona charging to the same sign as the drops.

Probably the most attractive attribute of the electrical spraying is the opportunity it gives to avoid use of orifices in forming the drops. The jets can be formed from a continuous surface, such as a film. The generation of audible noise on high voltage transmission lines under foul-weather conditions is a reminder of this [10]. A film of liquid on a high voltage cylindrical electrode is unstable in an electromechanical sense and forms perturbances on its surface which are the sites of current-driven jets ... of corona discharge and drops. If the production of ions does not present an unreasonable penalty, the wetted electrode has the advantage of being relatively invulnerable to clogging and deterioration. In any case, the orifice used with an electrical sprayer can be made much larger than is typical of a nozzle for producing the same size of drops.

Reported investigations of electrohydrodynamic spraying give scant information about electrical power and flow relationships. Hoburg's work [10] simulating the spraying from a wetted transmission line is probably of most direct use in determining the merits of electrical sprayers for drop formation. He studied spraying through a 1/8" hole in the wall of a cylindrical conductor approximately one inch in diameter. The conductor was grounded and placed coaxial with a high voltage cylindrical cage. Unfortunately, although Hoburg's work gives a thorough representation of the relations between spraying regimes and flow rate, voltage, and current,

it does not report the drop size under the spraying conditions. His drops were relatively large, probably between 10 and 100 μ. With the simple arrangement of coaxial conductors, voltages up to 50 kv were used. Only a small fraction of the inter-electrode spacing represented the potential drop across the liquid stream, hence a large part of the power input was unnecessarily dissipated in dragging ions across most of the gap. Even so, a typical mass of water converted to drops by a single source was 15 cm^3/min, with an electrical excitation of 40 kv and 2×10^{-5} amps. This converts to about 4×10^{-2} (lb/min)/watt. With a water loading of 2.5×10^{-3} lb/ft^3 [Class III loading case study, see Eq. (III.37)], this amounts to a power requirement of 62.5 watts/1000 cfm.

Again, we must emphasize that the calculation is at best a rough indication. The numbers used are for an apparatus not designed for the application at hand. The drops produced were too large and no advantage was taken of a moving gas to sweep drops from the volume subject to electric stress. Flashover caused by the drops in this region places a limit on how small an electrode-jet spacing can be used, and hence on how much voltage is required to achieve the spraying. We can make the following observations:

i) The role of corona discharge in electrohydrodynamic spraying is essential at atmospheric pressure, and requires that work done at elevated pressures or in vacuum be viewed with circumspection, especially as regards the efficiency of the atomizing process.

ii) The electrical stresses do offer an alternative to producing charged drops in the 1- 50μ range with improved efficiency and simplicity in the apparatus. Presently, inadequate information is available to make an engineering evaluation, but such a study would be relatively straightforward.

H. Condensation Charging

Condensation methods both of forming and charging drops can be divided into the two categories for condensation methods in general, suggested in § C...(low velocity-high volume, and high velocity-low volume). Methods of charging extremely small drops by condensation on ions are a standard part of aerosol technology [66]. More recent work has been directed toward condensation charging in nozzles, usually in the vicinity of a shock [63,70,71,72.73]. Processes have been demonstrated experimentally in which drops are so efficient in condensing on ions created by a corona source that virtually all of the corona current is carried away by the charged drops [68]. Generally, these condensation devices involve high concentrations of energy and generate submicron drops. Primary interest here is in the generation of relatively large drops; hence, no attempt is made to assess the viability of these techniques in transforming large amounts of water into the form of drops.

V. Comparison of Systems

Classes I and II

Generally, devices using drops and fields are in competition with scrubbers and electrostatic precipitators of conventional design. But, because the Class I and II type of interactions are closely related, a comparison between these systems is particularly meaningful. Here, the question is: in a system already involving the use of injected water for collecting particles, is the additional technology introduced by the requirement for particle charging and creating an ambient E offset by the improvement in performance? One way to measure performance is in terms of water usage.

TablesIII-1 and III-3 indicate the volume of gas cleaned per pound of water required in devices which utilize conventional scrubbing and electrically induced scrubbing, respectively. It appears that the electrical scrubber competes favorably with mechanical scrubbers operating at injection velocities on the order of 10 ft/sec or less. Clearly, the electrical process is at the greatest advantage in collecting the smallest particles. For example, it seems that, by using one-micron particles, the electrical device can do as well as the mechanical scrubber using an injection velocity of 100 ft/sec.

We can conclude that, in the submicron collection, the Class II interactions are viable, but not overwhelmingly so. Further comparison of the two systems must be based on energy requirements. Mechanical losses in the high-velocity conventional scrubber are a major consideration. But on the other hand, the Class II interactions require an ambient field. Hence, the interaction volume must be filled with high voltage electrodes fitted for collecting and injecting the drops. Power requirements are the

major drawback of the mechanical scrubber in the collection of submicron sized particles. Reliability and systems complexity resulting from having not only the water injection and removal equipment, but also the high-voltage electrodes in a hostile environment, is the principal disadvantage of the electrical Class II scrubber.

Classes III and IV

A comparison of performance in removing 0.2 micron diameter particles by Class III and IV interactions can be made using results summarized by Eqs. (III.34 - III.37) and Table (III-6). With water loading in the neighborhood of 10^{-4} lb/ft^3, both interactions give reasonable efficiencies for removing 0.2 micron particles with residence times of a few seconds. The Class III and IV systems will generally have performance characteristics in the same range, if the charge induced on the poles of the drop surface by the imposed field in the Class IV interaction is on the order of that caused by the net charge in the Class III interaction.

For the Class III interactions:

(a) The interaction volume requires no ambient E, hence is simply a mixing region devoid of electrodes and not involved with electrical insulation problems in an environment filled by drops and particles.

(b) However, the loading of drops is critical if high efficiency is to be achieved in a single stage. If the loading capacity defined by Eq. (III-19) does not exceed zero, high efficiency is impossible, no matter what the residence time.

For the Class IV interactions:

(a) No drop charging is required. Further, with bipolar charging of the particles, the effective lifetime of a drop is not limited by the charge collection.

(b) However, the interaction region must be filled by an ambient electric field, although the electrodes used to impose that field need not be used to inject or collect the drops, hence need not be fitted with hydraulic equipment.

Classes III and IV Compared to Conventional Precipitator

The models used in § III indicate that the water requirement for the electrical collection processes is not unreasonable; orders of magnitude more water are required in a conventional scrubber. Hence, the question of viability for the devices based on Class III and IV interactions is asked by making comparisons with conventional electrostatic precipitators. A fair comparison gives the conventional precipitator the option of a wet wall, since the use of drops implies wet walls somewhere, in any case. Hence, care must be taken in arguing that the drop devices are at an advantage in solving the re-entrainment problem [see Ref. 34, p. 311].

The residence time for high efficiency in a conventional precipitator is of the order

$$\tau_{prec} = \frac{3A}{Sw_{prec}} = \frac{3r}{2bE} \tag{1}$$

where we assume tubular electrodes of radius r. Here, we have made L/U from Eq. (II-4) three times A/Sw_{prec} to give approximately 95% efficiency with an exponential decay law. Also, $w_{prec} = bE$ in going from Eq. (II-4) to Eq. (1). For 0.2 micron-diameter particles, we use b from Table III-4 r = 20 cm. and E = 5×10^5 kv/cm. to obtain a residence time from Eq. (1) of six seconds. This is comparable with 2.5 seconds for the case study of Eqs. (III-34 - 37). This latter time constant is achieved with a relatively modest water loading.

There is a major difference between the Class III type of interaction and that conventionally used in precipitators. Performance of the drop Class III devices is strongly influenced by particle loading. Such devices would be compromised in fine particle efficiency by the existence of a significant population of large particles. All particles tend to neutralize the drops and limit their effective collection time.

If a spectrum of particles is present, it may be best to have a "topping" precipitator to eliminate the largest particles first. The advantage of the Class III interaction is that, in achieving the residence time required for removal of extremely fine particles, the interaction region is simply free space. Hence, much greater residence times can be contemplated in systems that can practically remove particles in sizes well under 0.1 micron.

The Class IV interactions require systems of electrodes. However, because the drops do not need to be charged, it appears that such interactions should be considered wherever wet -wall electrostatic precipitators appear attrative.

VI. Conclusions and Recommendations

It is clearly evident from information available in the literature, from patents and from calculations presented here, that the practical use of drops and electric fields for the collection of submicron particles is feasible. It is also clear that fields and drops do not represent a panacea. Calculations, and what few quantitative results are available do not suggest orders-of-magnitude improvement in device performance.

Drop-field devices involve a superposition of the technologies of wet scrubbers and electrostatic precipitators. With the complications come additional degrees of flexibility. For example, drop loading becomes an alternative to device length in improving performance. However, improvements tend to be offset by a considerable additional device complexity. This report cannot answer the final question of whether or not the complications are worth the improvement; that will depend intimately on the specific application and the particulars of the configuration.

It is important to recognize that one capability of the drop-type devices is not shared by the conventional precipitator. The drop devices, like their scrubber relatives, can remove gases as well as particulate. (Marks' [**] primary intent is to provide removal of gaseous materials, not particles.) Each of the electrical interactions described here has the capability of simultaneously acting as a gas and particle scrubber. Because of differences between Class II and Classes III and IV interactions relative to the gas, these can be expected to have differing performance characteristics in the removal of gases. Because the drop-type devices can compete, performancewise, with conventional precipitators, it is clear

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[**] see Table II-2

that further preliminary work is called for in determining the characteristics of such devices acting as combine gas and particle scrubbers.

As illustrated by § III, performance is determined by the collective interactions of drops and particles. It is well known that "real" hydrodynamic effects are a dominating influence in conventional precipitators. There, the distance between electrodes is great enough that we can think of the particles as entrained in a turbulent gas core, and migrating to the electrodes across boundary layers. Although we are certainly justified in thinking of the drop surface as a conventional collecting electrode, there is no analogy to the relative gas flow. According to whether the drops are in Class II or Classes III and IV, there may or may not be a velocity relative to the gas. In either case, the scale of the turbulence relative to the typical collecting surface dimension has changed completely. There is a lack of experimental work aimed at giving support to quantitative descriptions of the collective particle interactions with entrained drops. A next logical step toward device design with confidence is experiments using drops in the Class II, III and IV interactions. Of these, the most important seems to be the Class III interactions.

A necessary next step in refining comparisons between field-drop devices and conventional ones is the identification of specific applications that provide a context for a complete systems design. Only in this way is it possible to compare alternatives meaningfully. So far as we can tell, drop-field type devices have not been investigated for large-scale applications. Yet it is to large systems, perhaps with a need for collecting gaseous effluent also, that the Class III type interaction would seem to have the most to offer.

Questions that must next be answered are these:

a) What confidence can we have in the models given in § III for particle collection on drops through the agency of an electric field? Here, the need is for experiments that place the particle and drop populations and charges under control and allow for measurements resolved in space and time. The need is for the collection performance measurements of the system of drops and particles under realistic flow conditions. The models have already been given considerable support for collection on isolated drops.

b) What are the implications of thermodynamic effects in the face of evaporation? What is the lifetime of drops under practical industrial conditions? What are the relative merits of using a thermal source of energy to make drops by one of the condensation mechanisms cited in § IV C?

c) Penney emphasizes the necessity of recognizing the implications of space charge effects for device scaling. Included in the Class III interactions are collection on bipolar charged drops, or systems of heterogeneously charged drops. The use of such techniques for eliminating space charge effects as the devices are scaled up in size requires careful investigation of the implications for the collection properties of the system, under (a) above. A specific example of heterogeneous charging, used to prevent electrical breakdown limitations as the scale is increased, is shown in the hybrid system of Fig. VI-1. Space charge problems are avoided in the Class III interaction using drops charged to only one sign by making the collection capacity ζ on the order of zero. This generally leads to operation with a relatively low water requirement loading and long residence times. For example, for 95% efficiency, the residence time at $\zeta = 0$ is about $20\,\tau_d$.

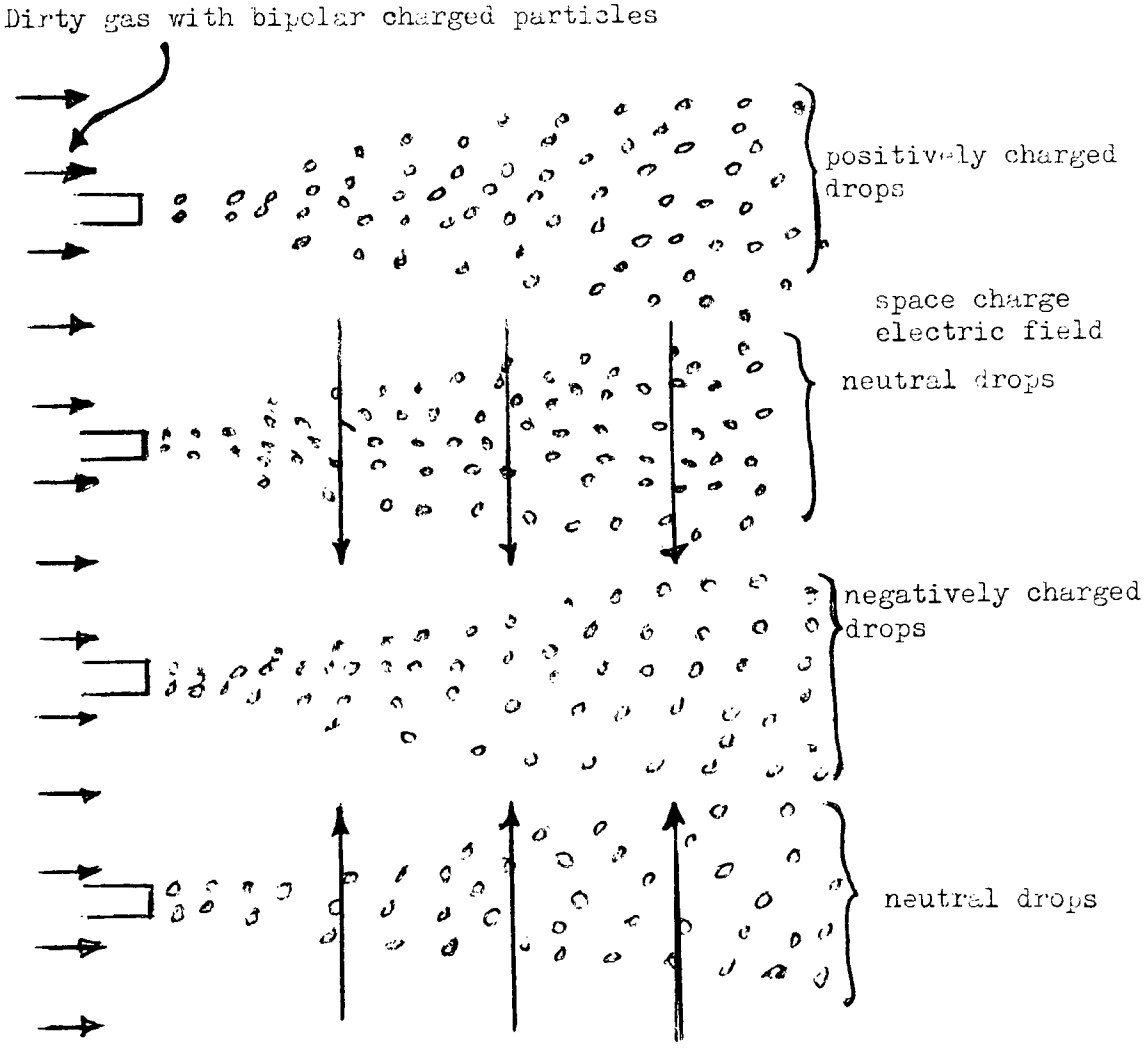

Fig. VI-1 Drops are charged to alternate polarities in neighboring regions with a buffer region between of uncharged drops. The charged regions collect particles by the Class III mechanism, while the uncharged drops are in the ambient field generated by the space charge from the charged drops, and therefore collect particles by the Class IV mechanism.

d) The necessity for having a viable large-scale scheme for making drops has been shown to be an important consideration. Further studies should be made of current-driven jets as the basis for electrohydrodynamic spraying to determine the power requirements and particle production capabilities,using apparatus simple and rugged enough in design to be used in a large-scale system.

e) Further consideration should be given to designs exploiting the dual capability of the drop systems to remove both particulate and noxious gases. As shown in this study, there is a considerable disparity between the liquid loading associated with the scrubber-like devices and the Class III and IV precipitator-like devices. Is there a range of loadings in which a device operates efficiently in both modes?

Bibliography

1. McCully, C. R. et al., "Scavenging Action of Rain on Air-borne Particulate Matter", Indus. and Eng. Chem. 48, #9 (1956), pp. 1512-1516.

2. Zebel, G., "Capture of Small Particles by Drops Falling in Electric Fields", J. Colloid & Interface Science, 27, #2 (1968), pp. 294-304.

3. Weber, E., "Present State and Aim of Basic Research in Web Scrubbers", Staub-Reinhalt Luft, 29, #7 (1969), pp. 12-18.

4. Zelazo, R. E., "Dynamic Interactions of Monomolecular Films with Imposed Electric Fields", Ph.D. Thesis in preparation, Department of Electrical Engineering, M.I.T., 1971.

5. Niedra, J. M., and Penney, G. W., "Orientation and Adhesion of Particles", IEEE Trans. on Industrial Elec., IECI-12, #2 (1965).

6. Lindblad, N. R. "Effects of Relative Humidity and Electric Charge on the Coalescence of Curved Water Surfaces", J. Colloid Sci., 19, (1964), pp. 729-743.

7. Penney, G. W., "Electrified Liquid Spray Dust Precipitator", U. S. Patent 2,357,354 (1944).

8. Calvert, S., "Venturi and Other Atomizing Scrubbers' Efficiency and Pressure Drop", AIChE. J., 16, #3 (1970), pp. 392-396.

9. White, H. J., Industrial Electrostatic Precipitation, Addison-Wesley Pub. Co., Inc., Reading, Mass., (1963), p. 165.

10. Hoburg, J. F., "Modes of Behavior of Water on High Voltage Trans-Mission Lines", M.S. Thesis, Dept. of Electrical Engineering, M.I.T., January 1971.

11. Ransburg, E. M., "Electrostatic Coating Method and Apparatus", U.S. Patent 2,658,009 (1953).

12. Miller, E. P., and Spiller, L. L., Paint and Varnish Production, June-July (1964).

13. Woodson, H. H., and Melcher, J. R., Electromechanical Dynamics, Part II: Fields, Forces and Motion, John Wiley & Sons, Inc., N. Y., (1968), pp. 388-392.

14. Fuchs, N. A., The Mechanics of Aerosols, Pergamon Press and MacMillan Company, N. Y. (1964).

15. Devir, S. E., "On the Coagulation of Aerosols, III: Effect of Weak Electric Charge on Rate", J. of Colloid & Interfacial Sci., 21, (1967), pp. 80-89.

16. Gillespie, T., "The Effect of the Electric Charge Distribution on the Aging of an Aerosol", Proc. of the Roy. Soc., A 216 (1953).

17. Gillespie, T., "Electric Charge Effects in Aerosol Particle Collision Phenomena", Aerodynamic Capture of Particles, Pergamon Press, N.Y. (1960), pp. 44-49.

18. Kunkel, W. B., "Charge Distribution in Coarse Aerosols as a Function of Time", J. App. Phys., 21, (1950), pp. 833-837.

19. Müller, H., "Zur Theorie der Elektrischer Ladung und der Koagulation der Kolloide", Kolloidchemische Beihefte, 26 (1928), pp. 257-311.

20. Whitby, K. T., and Liu, B. Y. H., "The Electrical Behavior of Aerosols", Chap III. in Aerosol Science, ed. by Davies, C. N., Academic Press, N. Y., (1966).

21. Pauthenier, M. M., and Moreau-Hanot, M., "La Charge des Particules Spheriques dans un Champ Ionisé", J. de Physique et le Radium, 3, (1932), p. 590- 613.

22. Whipple, F. J. W., and Chalmers, J. A., "On Wilson's Theory of the Collection of Charge by Falling Drops", Quart. J. of the Roy. Met. Soc., 70, (1944), pp. 103-119.

23. Gott, J. P., "On the Electric Charge Collected by Water Drops Falling Through Ionized Air in a Vertical Electric Field", Proc. Roy. Soc. A 142 (1933), p. 248.

24. Gunn, R., "The Electrification of Precipitation and Thunderstorms", Proc. of I.R.E., October (1957), pp. 1331-1358.

25. Kraemer, H. F., and Johnstone, H. F., "Collection of Aerosol Particles in Presence of Electrostatic Fields", Indus. and Eng. Chem., 47, (1955), pp. 2426-2434.

26. Smirnov, L. P., and Deryagin, B. V., "Inertialess Electrostatic Deposition of Aerosol Particles on a Sphere Surrounded by a Viscous Stream", Kolloidnyi Zhurnal, 29, #3 (1967), pp. 400-412.

27. Paluch, I. R., "Theoretical Collision Efficiencies of Charged Cloud Droplets", J. Geophys. Res. 75, #9 (1970), pp. 1633-1640.

28. Sartor, J. D., "Some Electrostatic Cloud-Droplet Collision Efficiencies", J. Geophys. Res., 65, #7 (1960), pp. 1953-1957.

29. Semonin, R. G., and Plumlee, H. R., "Collision Efficiency of Charged Cloud Droplets in Electric Fields", J. Geophys. Res., 71, #18, (1966), pp. 4271-4278.

30. Faith, L. E., Bustany, S. N., Hanson, D. N., and Wilke, C. R., "Particle Precipitation by Space Charge in Tubular Flow", I. & EC Fundamentals, 6, #4 (1967), pp. 519-526.

31. Hanson, D. N., and Wilke, C. R., "Electrostatic Precipitator Analysis", I & EC Process Design and Development, 8, #3 (1969), pp. 357-364.

32. Anonymous, "More Help for Solid Waste Disposability", Modern Plastics, August (1970), pp. 67-68.

33. Marks, A. M., "Charged Aerosols for Air Purification and Other Uses", paper presented at Symposium on Useful Electrical Field-Induced Phenomena in Physical and Chemical Operations, 69th National AIChE Meeting, May 16-19, 1971, Cincinnati, Ohio.

34. Oglesby, S. J., et al., "A Manual of Electrostatic Precipitator Technology, Part I - Fundamentals", Report PB 196380, National Air Pollution Control Admn., August (1970), p. 317.

35. Eyraud, C., Joubert, J., Henry, C., and Morel, R., "Étude D'un Nouveau Type de Dépoussiereur à Pulvérisation Electrostatique D'Eau", C. R. Acad. Sci., Paris, t262 (1966), pp. 1224-1226.

36. Joubert, J., Henry, C., and Eyraud, C., "Étude des Trajectoires des Particules Sub-microniques dans les Champs Ionisés", Le J. de Phys. Phys. Appliqués, Supplément An. #3, Tome 25 (1964), pp. 67A-72A.

37. Melcher, J. R., and Sachar, K. S., "Electrical Induction of Particulate Agglomeration," Final Report to Environmental Protection Agency, Task 7, Contract 68-002-0018 (1971).

38. Perry, J. H., Ed., Chemical Engineers Handbook, McGraw-Hill Book Co., New York (1963), pp. 18-59 - 18-68.

39. Monk, G. W., "Viscous Energy Dissipated During the Atomization of a Liquid", J. App. Phys., 23 (1952), p. 288.

40. McElroy, A. J., "Flashover Mechanisms of Insulators with Contaminated Surfaces", Ph.D. Thesis, Dept. of Electrical Engineering, MIT, June, 1969.

41. Fletcher, N. H., The Physics of Rainclouds, Cambridge University Press, Cambridge, England (1962), p. 8.

42. Hill, P. G., "Condensation of Water Vapor during Supersonic Expansion in Nozzles", J. Fl. Mech. 25 (1966), pp. 593-620.

43. Rayleigh, Lord, "On the Equilibrium of Liquid Conducting Masses Charged with Electricity", Phil. Magazine, 14 (1882), pp. 184-186.

44. Taylor, G. I., "Disintegration of Water Drops in an Electric Field", Proc. Roy. Soc. London , A 280, (1964), pp. 383-397.

45. Lebedev, N. N., and Skal'skaya, I. P., "Force Acting on a Conducting Sphere in the Field of a Parallel Plate Condenser", Zhurnal Tekhnicheskoi Fiziki, 32, #3, (1962), pp. 375-378.

46. Bowman, R. G., U.S. Patent 2,182,533 "Spray Tower" (1939).

47. Darrah, W. A., U.S. Patent 1,958,406, "Electrical Spraying Device" (1934).

48. Campbell, J. A., U.S. Patent 2,302,185, "Electrified Spray Apparatus", (1942).

49, Centanni, F. A., "An Electrohydrodynamic Synchronous Generator", M.S. Thesis, Dept. of Electrical Engineering, MIT, (1969).

50. Gill, E. W. B., and Alfrey, G. F., "Electrification of Liquid Drops", Proc. Phys. Soc., London 65B (1952), pp. 546-551.

51. Herold, Lance, "Traveling-Wave Charged Particle Generation", M.S. Thesis, Dept. of Electrical Engineering, MIT, June 1967.

52. Vonnegut, B., and Neubauer, R. L., J. Colloid Sci., 7, (1952), p. 616.

53. Drozin, V. G., "The Electrical Dispersion of Liquids as Aerosols", J. Colloid Sci., 10 (1955), pp. 158-164.

54. English, W. N., "Corona from a Water Drop", Physical Review, 74, No. 2 (July 15, 1948), pp. 179-189.

55. Carson, R. S., and Hendricks, C. D., "Natural Pulsations in Electrical Spraying of Liquids", AIAA Paper 64-675, AIAA Fourth Electric Propulsion Conference, Aug. 31-Sept. 2, 1964, Philadelphia, Pa.

56. Pfeifer, R. J., and Hendricks, C. D., "Parametric Studies of Electrohydrodynamic Spraying", AIAA Journal, 6 (1968), pp. 496-502.

57. Pfeifer, R. J., and Hendricks, C. D., "Charge-to-Mass Relationships for Electrohydrodynamically Sprayed Liquid Droplets", Phys. Fluids, 10, #10 (1967), pp. 2149-2154.

58. Krohn, V. E., "Research on the Generation and Acceleration of Submicron-Size Particles", Report ARL 8937-6005-CU-000, (1962). (Work done at Physical Electronics Laboratory, Space Technology Laboratories, Inc., Canoga Park, California).

59. Hogan, J. J., Carson, R. S., Schneider, J. M., and Hendricks, C. D., "Factors Influencing Electrically Sprayed Liquids", AIAA Journal, 2, #8, (1964), pp. 1460-1461.

60. Melcher, J. R., Field-Coupled Surface Waves, MIT Press, Cambridge, Mass. (1963), pp. 123-135.

61. Melcher, J. R., and Taylor, G. I., "Electrohydrodynamics: A Review of the Role of Interfacial Shear Stresses", Annual Review of Fluid Mechanics, Vol. 1 (1969), Annual Reviews, Inc., Palo Alto, Cal., pp. 111-146.

62. Melcher, J. R., and Warren, E. P., "Electrohydrodynamics of a Current-carrying Semi-insulating Jet", J. Fluid Mech., 47, Part 1, (1971), pp. 127-143.

63. Marks, A. M., "Method and Apparatus for Producing Charged Aerosols" , U.S. Patent 3,411,025 (1968).

64. Taylor, G. I., "Electrically Driven Jets", Proc. Roy. Soc. London A313 (1969), pp. 4530375.

65. Montgomery, D. N., and Dawson, G. A., "Collisional Charging of Water Drops", J. of Geophys. Res., 74, #4 (1969), pp. 962-972.

66. LaMer, V. K., and Wilson, I. B., "The Methods of Forming, Detecting and Measuring the Size and Concentration of Liquid Aerosols in the Size Range 0.01 to 0.25 Microns Diameter", J. Colloid Sci., 5 (1950), pp. 471-496.

67. Cox, A. L., "Particle Growth in the Ionic Nucleation Condensation Colloidal Ion Source", Tech. Note R-139, March 1965, Research Laboratories, Brown Engineering, Co., Inc., Huntsville, Ala., (available via Defense Documentation Center, Alexandria, Va., No. AD 614111).

68. Barreto, E., and Mulcahy, M. J., "Production and Neutralization of a Charged Aerosol by Corona Fields", J. Geophys. Res., 70, (1965), pp. 1303-1309.

69. Barreto, E., "Electrical Discharges From and Between Clouds of Charged Aerosols", J. of Geophys. Res., 74, #28 (1969), pp. 6911-6925.

70. Decaire, J. A., "Effects of Partial Condensation Around Ions in Fluid-Dynamic Energy Conversion Processes", M.S. Thesis, 1964, Air Force Inst. of Technology, Wright-Patterson AFB, Ohio, (available via Defense Documentation Center, Alexandria, Va., Access. No. N64-33832).

71. Decaire, J. A., and Lawson, M. O., "Electrofluiddynamic Power Generation Trends and Expectations", Conference Paper, Symposium on Engineering Aspects of Magnetohydrodynamics, Stanford University, March 1967, Stanford, Calif.

72. Brandmaier, H. E., and Dimmock, T. H., "Factors Influecing Electrofluid Dynamic Power Generation", J. of Spacecraft & Rockets, 4 No. 8 (August 1967), pp. 961-966.

73. Solbes, A., "Research on Charged Alkali Colloids", December, 1970, USAF Aerospace Research Laboratories, Wright-Patterson AFB, Ohio (available via Defense Documentation Center, Alexandria, Va., # ARL 70-0344).

Appendix A : Charged Particle Collection on Charged Drop in Ambient Flow and Field (Adapted from book, Electrohydrodynamics, by J. R. Melcher, to be published by M.I.T. Press).

Whipple and Chalmers [22] use an imposed field and flow model to predict the charging of water drops as they fall through positive, negative, and both positive and negative ions. Their theories cover the results given earlier by Pauthenier and Mme. Moreault-Hanot [21]. This latter work did not include the hydrodynamic effect of the streaming neutral gas. Because the particle inertia is ignored, the model developed here for collection of charged particles on a drop is essentially the same as these ion-impact models.

A schematic view of the physical situation is depicted by Fig. A-1 , wherein the particle is viewed as fixed in the frame of reference, and hence the neutral gas streams relative to the drop with a velocity w_o. The imposed field, like the imposed flow, is uniform at infinity with the amplitude E_o, also shown in Fig. A-1 . In the following, E_o will be considered positive or negative, with the positive directions as defined by the figure.

Objectives in the following derivations are to determine the rate of charging of the particle, given its initial charge, and to find the final charge established after the drop has been falling through the particle flux for a long period of time. To look ahead, the desired information is summarized in a plot like that given by Fig. II-6 . Coordinates are the instantaneous drop charge, Q, and the field-induced velocity, E_o. Given that the charge on the drop and field are represented by some point in the plane, the questions we ask are: what, then, is the rate of charging of the particle? And what is the trajectory followed in this plane, ending at a final equilibrium charge?

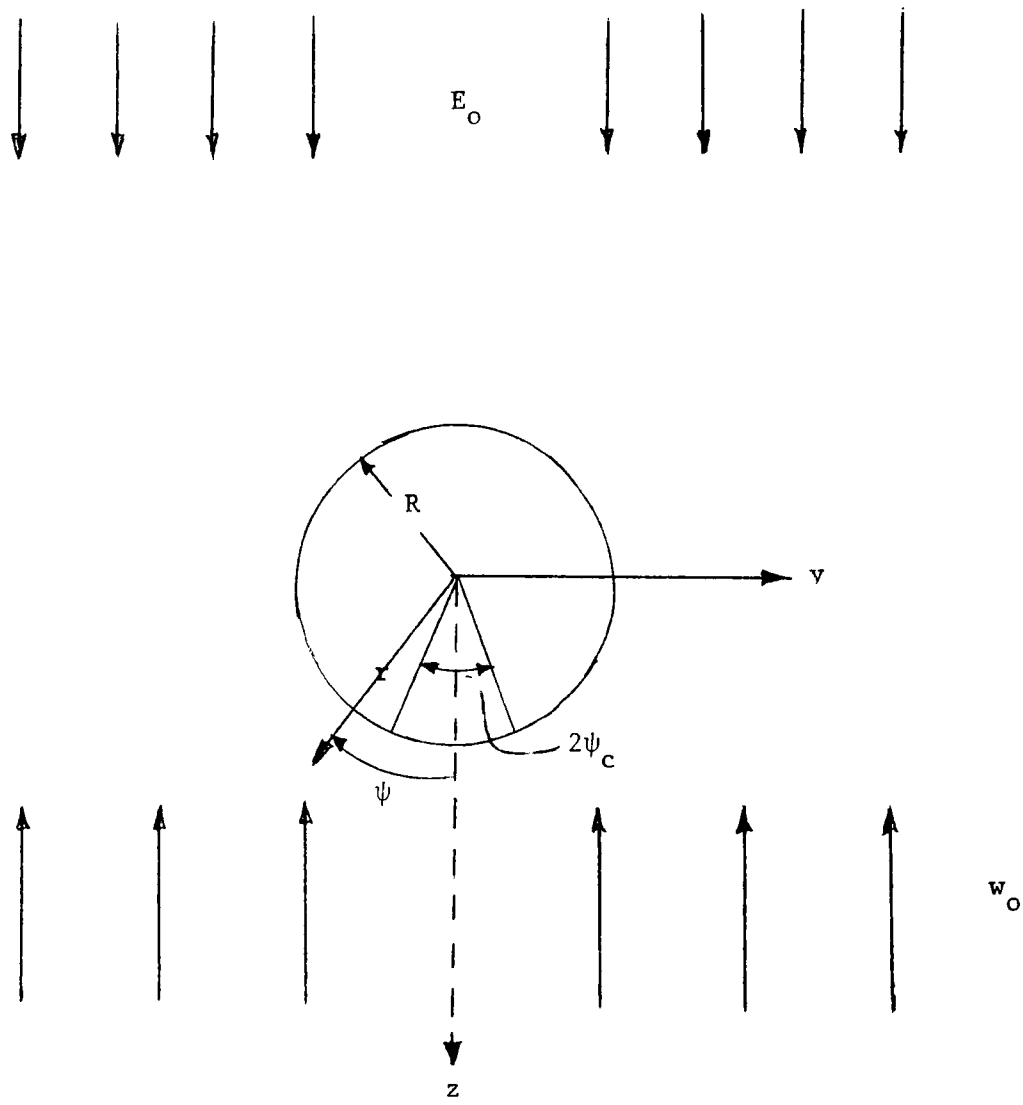

Fig. A-1 Spherical conducting drop in imposed electric field and flow that are uniform at infinity. E_o and w_o are positive if directed as shown; in general, the electric field intensity E_o can be either positive or negative.

At the outset, we draw attention to parameters which will be found useful throughout. Regimes of charging are demarked by the critical charge

$$Q_c = 12\ \pi\varepsilon_o R^2 E_o \tag{A.1}$$

which can be positive or negative, depending on the sign of E_o. Rates of charging will be characterized by the currents

$$I_{\pm} = \pi R^2 b_{\pm}\ \rho_{\pm}\ E_o \tag{A.2}$$

which are also determined in sign by E_o. The magnitudes of the positive and negative particle charge densities are $\rho_{\pm}$ respectively, at infinity.

That the charging rate is to be calculated infers that the particle motions are not in the steady state. We assume that transit times through several drop radii R are short compared to charging times of interest, and hence, at any instant take the drop charge, Q, as a known constant, which then makes a contribution to the instantaneous electric field intensity imposed throughout. The particle is taken as perfectly conducting, hence the potential and electric field intensity follow from well known solutions. The imposed electric field intensity is therefore

$$\bar{E} = -\nabla\phi = \{ E_o\left(\frac{2R^3}{r^3} + 1\right) \cos\psi + \frac{Q}{4\pi\varepsilon_o r^2} \}\, \bar{i}_r + \{E_o\left(\frac{R^3}{r^3} - 1\right) \sin\psi\}\bar{i}_{\psi} \ . \tag{A.3}$$

A stream function for the electric field intensity can be defined as

$$\Sigma = -E_o R^2 \left[\frac{R}{r} + \frac{1}{2}\left(\frac{r}{R}\right)^2\right] \sin^2\psi + \frac{Q\cos\psi}{4\pi\ \varepsilon_o} \tag{A.4}$$

where $\bar{E} = -\nabla x[\bar{i}_{\theta}\ \frac{1}{r\sin\psi}\ \Sigma]$.

It will be evident shortly that the particular details of the velocity

distribution are surprisingly unimportant. We could use potential flow, but here we follow Whipple and Chalmers [22] and use low Reynolds number flow. Thus, it is possible to make both components of velocity at the spherical surface vanish. In terms of the stream function, the boundary condition at infinity is $\Psi = \frac{1}{2} w_o (r \sin \psi)^2$. In fact, the stream function is readily available in the literature .

$$\Psi = \frac{w_o R^2}{2} \left[\left(\frac{r}{R}\right)^2 - \left(\frac{R}{r}\right) \right] \sin^2 \psi \tag{A.5}$$

where $$\bar{v} = - \nabla \times \left[\bar{i}_\theta \frac{1}{r \sin \psi} \Psi \right] \tag{A.6}$$

The mobility, b_i, is taken as constant. Hence we are dealing with a system of charged particles with the charge-per-particle a constant.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \bar{J}^i = 0 ; \qquad \rho = \pm \rho_i \quad . \tag{A.7}$$

The neutral fluid can be taken as having a known velocity; in addition, however, we require that it be incompressible, so that $\nabla \cdot \bar{v} = 0$. If space charge effects were important, Gauss' law would also relate the local net charge density to the electric field intensity. But, in our approximation, the electric field intensity, like the velocity of the neutral fluid, is solenoidal: $\nabla \cdot \bar{E} = 0$. With these assumptions, substitution of Eq. (A.6) into (A.7) gives a simple expression for the transport of each charge density

$$\frac{\partial \rho_i}{\partial t} + (\bar{v} \pm b_i \bar{E}) \cdot \nabla \rho_i = 0 \quad . \tag{A.8}$$

Thus, the only coupling between the particles occurs because of alterations in the imposed fields due to the charging of boundaries.

Equation (A.8) can be integrated with complete generality using the method of characteristics. The equations reduce to

$$d\rho_i = 0 \quad \text{on} \quad \frac{d\bar{r}}{dt} = \bar{v} \pm b_i\bar{E} \quad . \tag{A.9}$$

That is, the charge density of each species of particle is constant along given force lines in $(\bar{r},t)$ space. If we are interested in determining the charge distribution within a volume enclosed by the surface S, then it is appropriate to imposed boundary conditions on the i'th species, wherever

$$\bar{n}\cdot(\bar{v} \pm b_i\bar{E}) < 0 \tag{A.10}$$

where $\bar{n}$ is taken as positive if directed out of the volume of interest. At points on the surface other than those that satisfy Eq. (A.10), a boundary condition cannot be imposed. These observations, although seemingly obvious in the transient problem, are crucial to making sense out of quasi-steady motions.

Motions in the face of a given flow can be described with considerable generality for those problems in which stream functions can be defined for both the electric field intensity and the velocity of the fluid, so as to insure that both $\bar{E}$ and $\bar{v}$ are solenoidal. These functions are given by Eqs. (A.4) and (A.5). The functions Σ and Ψ have the simple physical interpretation of being the flux of the electric field intensity of fluid velocity through an open surface, S.

With the stream functions given, the lines of constant charge density given by Eq. (A.9) take the form

$$\frac{dr}{dt} = -\frac{-1}{r^2 \sin\psi}\frac{\partial}{\partial\psi}(\pm b\Sigma + \Psi) \tag{A.11a}$$

$$r\frac{d\psi}{dt} = \frac{1}{r\sin\psi}\frac{\partial}{\partial r}(\pm b\Sigma + \Psi) \tag{A.11b}$$

A complete solution can now be obtained by combining this pair of equations. Multiplication of (A.11b) by $dr/d\psi$ and equating that expression to Eq. (A.11a) multiplied by r gives the exact differential:

$$d(\pm b\Sigma + \Psi) = 0 \tag{A.12}$$

provided Σ and Ψ are independent of time. Here, our fundamental assumption of quasi-steady particle motions must be invoked. We will be interested in non-steady phenomena. To integrate particle equations of motion, as we have in writing Eq. (A.12), we require that the particles have essentially the same field and flow distribution throughout their motions in the volume of interest. In that sense, the motions are steady. But the particle transit times are likely to be brief compared to a dynamical time of interest (perhaps that required for a surface upon which the particles impinge to change, and hence change the electric field intensity significantly). Thus, over a longer time scale, the flow and field distribution, hence the stream functions, may be functions of time. In summary, Eq. (A.12) shows that the charge density of a given species is constant along constant stream function lines

$$\rho_i = \text{constant on } \pm b_i\Sigma + \Psi = \text{constant} \quad . \tag{A.13}$$

Given Eqs. (A.4) and (A.5), the characteristic lines are determined by substituting into (A.11) to get

$$\mp \left[\frac{R}{r} + \frac{1}{2}\left(\frac{r}{R}\right)^2\right] \sin^2\psi \pm \frac{3Q}{Q_c} \cos\psi + \frac{1}{2}\frac{w_o}{b_\pm E_o}\left[\left(\frac{r}{R}\right)^2 + \frac{1}{2}\left(\frac{R}{r}\right) - \frac{3}{2}\left(\frac{r}{R}\right)\right] \sin^2\psi = C \quad . \tag{A.14}$$

Lines of force $\overline{v} \pm b_{\pm}\overline{E}$ originating on the spherical surface carry zero charge density. At those points on the surface where the force lines are into the drop, we have the possibility of particle migration. At the drop surface, the velocity normal to the surface is zero, hence the force lines degenerate to $\pm b_{\pm}\overline{E}$. This greatly simplifies the charging process, because we can now use the electric field intensity given by Eq. (A.3) to decide whether or not a given point on the particle surface can accept charge. Evaluation of (A.3) shows that force lines are directed into the particle surface wherever

$$E_o \gtrless 0 \; ; \quad \psi_c < \psi < \pi \tag{A.17}$$

$$E_o \lessgtr 0 \; ; \quad 0 < \psi < \psi_c \tag{A.17b}$$

with the upper and lower inequalities indicating positive and negative particles. The critical angle ψ_c demarking regions of inward and outward force lines follows from Eq. (A.3) as satisfying

$$\cos \psi_c = -\frac{Q}{Q_c} \tag{A.18}$$

where Q_c is given by Eq. (A.1).

A graphical representation of what has been determined is given by the direction of incident force lines on the particle surfaces sketched in Fig. A-2. Where directed inward, these force lines indicate a possible particle current. Whether or not the current is finite depends on whether the given force line originates on the boundary at infinity. In any case, if the force line is directed outward, we can be assured that there is no charging current to the particle, and so without further derivations, we know that regimes (a), (b) and (c) for the positive particles and (j),(k) and

The upper and lower signs, respectively, refer to positive and negative particles and C is a constant which determines the particular characteristic line. Just what constant charge density should be associated with each of these lines is determined by a single boundary condition imposed wherever the line "enters" the volume of interest at a point satisfying the condition

$$\bar{n}\cdot[\bar{v} \pm b_{\pm}\bar{E}] < 0 \quad . \tag{A.15}$$

Here, $\bar{n}$ is taken as pointing out of the volume of interest; that is, outward at infinity and inward on the spherical surface of the drop.

To reiterate our objectives in terms of parameters now introduced, we wish to obtain the net instantaneous current of particles to the drop $i(Q,E_o,w_o,\rho_+,\rho_-)$. With the imposed field, velocity and charge densities held fixed, this expression then serves to give the rate of drop charging as

$$\frac{dQ}{dt} = i(Q) \quad . \tag{A.16}$$

Boundary Conditions: Permutations and combinations of flow velocity, imposed field, instantaneous particle charge, and sign of the incident particles are extremely large, so an orderly approach is required to sort out the possible charging regimes. A large step in this direction is taken by first recognizing the surfaces which satisfy the condition of Eq. (A.15), and hence imposed boundary conditions.

For positive particles (upper sign) the distribution of charge density for particles entering at $z \to -\infty$ is required if $bE_o > w_o$. Otherwise, the charge density is imposed as $z \to +\infty$ because the positive particles enter from below. Thus, the charge-imposed field plane divides into two regimes, as shown by the notations at the bottom of Fig. II-6.

(1) for the negative particles give no charging current; from Eq. (A.16) the particle charge remains at whatever its initial value within the regime was.

Regimes (f) and (i) for Positive Particles: (b) and (g) for Negative Particles

We now characterize each regime shown in Figs. II-6 and II-7. Upper and lower signs respectively will be used to refer to the positive and negative ion cases.

The characteristic line terminating at the critical angle on the particle surface reaches the $z \to -\infty$ surface at the radius y* shown in the respective regimes in Figs. II-6 and II-7 . Particles entering within that radius strike the surface of the drop within the range of angles wherein the drop can accept particles. Hence, to compute the instantaneous drop charging current, we can simply find this radius y* and compute the total current passing within that radius at $z \to -\infty$. The particular line is defined by Eq. (A.14) evaluated at the critical angle, and on the particle surface: $\psi = \psi_c$, $r = R$. Thus, the constant is evaluated to be

$$C = \pm \frac{3}{2}\left[1 + \left(\frac{Q}{Q_c}\right)^2\right] \qquad (A.19)$$

To find y*, we now take the limit of Eq. (A.14) using the constant of (A.19) to determine that

$$(y^*)^2\left(1 \pm \frac{w_o}{b_\pm E_o}\right) = 3R^2\left[1 - \frac{Q}{Q_c}\right]^2 . \qquad (A.20)$$

Our problem is particularly simple because the particle flux is constant, so the current passing through the surface with radius y* is simply the product of the current density and the circumscribed area

$$i_1^\pm = \pm \rho_\pm(\pm b_\pm E_o - w_o)\pi(y^*)^2 . \qquad (A.21)$$

Now, if we combine Eqs. (A.20) and (A.21), we arrive at

$$i_1^{\pm} = 3I_{\pm}\left(1 - \frac{Q}{Q_c}\right)^2 = \pm 3|I_{\pm}| \left(1 \mp \frac{Q}{|Q_c|}\right)^2 . \qquad (A.22)$$

The second equality is written by recognizing the sign of E_o in the respective regimes.

We conclude that in the positive particle regimes (f) and (i), the charging current is positive, tending to increase the particle charge until it reaches the limiting value $Q = |Q_c|$. Charging trajectories are shown in the figures, with i_1 the rate of charging, whether the initial particle charge is within the respective regimes or the charge passes from another regime into one of these regimes, and then passes on to its final value, $|Q_c|$. For example, in the case of the positive particle charging, we will soon find that a particle charges at one rate in regime (1) and then, on reaching regime (i), assumes the charging rate given by Eq. (A.22), which it obeys until the charge reaches a final value on the boundary between regimes (f) and (c).

Also summarized in the charge field, plots of Figs.II-6 and II-7 are the force line patterns, and the critical angles defining those portions of the drop over which conduction can occur. As a drop charges and then passes from regime (i) to (f), and finally to the boundary between regimes (f) and (c) in the positive particle case, we see that the angle over which the drop can accept charge decreases from a maximum of 2π to π at $q = 0$, and finally to zero when $Q = |Q_c|$. It is the closing of this "window" through which charge can be accepted to the particle surface which is the essence of the charging process.

Regimes (d) and (g) for Positive Particles; (f) and (i) for Negative Particles

These regimes are analogous to the four just discussed except that the ions enter at $z \to \infty$, rather than at $z \to -\infty$. The derivation is therefore as just described except that the limiting form of (A.14) is taken as $\psi \to 0$, with C again given by Eq. (A.19) to obtain

$$(y^*)^2\left(1 \mp \frac{w_o}{b_\pm E_o}\right) = 3R^2\left(1 + \frac{Q}{Q_c}\right)^2 . \tag{A.23}$$

Then, the particle currents can be evaluated as

$$i_1^\pm = -3I_\pm \left(1 + \frac{Q}{Q_c}\right)^2 = \pm 3|I_\pm|\left(1 \mp \frac{Q}{|Q_c|}\right)^2 . \tag{A.24}$$

As would be expected on physical grounds, the positive particle case gives charging currents and final drop charges in regimes (d) and (g), which are the same as those in (f) and (i). Similar remarks apply in the negative particle charging case. Certainly, if the fluid velocity is zero, the charging conditions must be the same, whether the electric field is positive or negative; we must have symmetry about the Q axis.

Regimes (j) and (k) for Positive Particles; (b) and (c) for Negative Particles:

For these regimes, the total surface of the drop can accept particles. The radius for the circular cross section of particles reaching the surface of the drop from $z \to \infty$ is determined by the line intersecting the drop surface at $\psi = \pi$. This line is defined by evaluating Eq. (A.14) at $r = R$, $\psi = \pi$ to obtain

$$C = \mp \frac{3Q}{Q_c} . \tag{A.25}$$

Then, if the limit is taken $r \to \infty$, $\psi \to 0$ of Eq. (A.14), v^* is obtained and the current can be evaluated as

$$i_1^{\pm} = \pm \rho_{\pm}(\pm b_{\pm}E_o + w_o)\pi(y^*)^2 = - \frac{12|\ I_{\pm}|}{|Q_c|} Q \quad . \tag{A.26}$$

Note that in the positive particle regimes, Q is negative, so our result indicates that the particle charges at this rate until it leaves the respective regimes when the charge $Q = - |Q_c|$.

Regime (1) for Positive Particles; (a) for Negative Particles: The situation here is similar to that for the previous cases, except that particles enter at $z \to - \infty$, so the appropriate constant for the critical characteristic lines given by Eq. (A.14) evaluated at $r = R, \psi = \pi$, is the negative of Eq. (A.25). The limit of that equation given at $r \to \infty$, $\psi \to \pi$ gives y* and evaluation of the current gives a value identical to that found with Eq. (A.26). In terms of Fig. II-6 for positive particles, we have found that in the regime (1), where the initial charge is negative, the charging current is positive, and tends to reduce the magnitude of the particle charge until it enters regime (i), where its rate of charging shifts to i_1 and it continues to acquire positive charge until it reaches the final value $|Q_c|$ indicated on the diagram.

Regime (e), Positive Particles; Regime (h), Negative Particles: In regimes (e) and (h) for either sign of particles, the window through which the drop can accept a particle flux is on the opposite side from the incident particles. This gives the opportunity for force lines terminating within the window through which the drop can accept particles to originate on the drop itself. In that case, the charge density on the characteristic line is zero, since the drop surface is incapable of providing particles.

To determine the particle charge that just prevents force lines originating at $z \to \infty$ from terminating on the particle surface, follow a lines from the z- axis where the drops enter at infinity back to the drop surface. That line has a constant determined by evaluating Eq. (A.14) with $\psi = 0$

$$C = \pm \frac{3Q}{Q_c} \tag{A.27}$$

Now, if we evaluate (A.14) using the constant of Eq. (A.27) and r = R, we achieve an expression for the angular position at which that characteristic line meets the drop surface

$$\frac{3}{2} \sin \psi = \frac{3Q}{Q_c} (\cos \psi - 1) \quad . \tag{A.28}$$

Note that the quantity on the right is always negative if Q/Q_c is positive, as it is in regimes (e) for the positive particles and (h) for the negative. [Remember that Q_c can be positive or negative, according to the sign of E_o, as expressed by Eq. (A.1).] We conclude that in regime (e) for the positive particles and (h) for the negative, the rate of charging vanishes; the drop remains at its initial charge.

Regime (h) for Positive Particles; (e) for Negative Particles: In these regimes, Q/Q_c is negative and Eq. (A.28) gives an angle at which the characteristic line along the z- axis meets the particle surface. To compute the rate of charging, we do not require the solution to this equation, because a circular area of incidence for particles at $z \to \infty$ is then determined by the characteristic line reaching the drop at $\psi = \pi$. Actually, no new calculation is necessary because that radius is the same as that found for regime (k) for the positive particles and (b) for the negative, so we can conclude immediately that the charging current is $i_2^{\pm}$, as given by Eq. (A.26). Drops in these regimes discharge until they reach the charge zero. Moreover, we can now see that, if the initial drop charges place the drop in regimes (k) for the positive particles or (b) for the negative particles, the rate of discharge follows the same law through regimes (h) for the positive particles and (e) for the negative until the drop reaches zero charge.

Positive and Negative Particles Simultaneously: Implicit to our imposed fields and flow approximation is the non-interaction of particle species except through charges on the drop surface, If both positive and negative particles are present simultaneously, the drop charging is characterized by simply superimposing the results summarized with Figs. II-6 and II-7 . The diagrams are especially helpful in this regard, in that the charging current for any given imposed field and drop charge is obtained as the superposition of the respective charging currents. Practically, the diagrams are superimposed with their origins (marked 0) coincident. A given point in either plane then specifies the charge and field experienced by both families of charges. This justifies merely superimposing the respective currents at the given point to find the total charging current.

Charging trajectories are summarized for the combined positive and negative particle configuration by Fig. II-8 . Each of the 16 regimes shown is just the superposition of two regimes from Figs. II-6 and II-7. For example, regime (c) in Fig. II-8 has the charging current which is the sum of those for regime (b) of Fig. II-6 and regime (c) of Fig. II-7 . Note that this particular regime characterizes all of those in the top and bottom rows in Fig. II-8 in that only one of the particle species is conducted to the particle, while the other is totally repelled.

In regimes (h), (l), (e) and (i), the rate of charging is

$$i = i_1^+ + i_1^- = 3|I_+| \left(1 - \frac{Q}{|Q_c|}\right)^2 - 3 |I_-|\left(1 + \frac{Q}{|Q_c|}\right)^2 \quad . \qquad (A.29)$$

The particle charges toward that value of Q that makes the current given by Eq. (A.29) vanish. So the final charge, which we call here Q_1 , is the solution to the quadratic expression of (A.29) set equal to zero:

$$Q = |Q_c| \left\{ \frac{\left(\frac{|I_+|}{|I_-|} + 1 \right)}{\left(\frac{|I_+|}{|I_-|} - 1 \right)} - \left[\frac{\left(\frac{|I_+|}{|I_-|} + 1 \right)^2}{\left(\frac{|I_+|}{|I_-|} - 1 \right)^2} - 1 \right]^{\frac{1}{2}} \right\} \qquad \text{(A.30)}$$

Here, one root is extraneous because it corresponds to a value of $Q_1 > |Q_c|$. As summarized in the figure, the drop charge is positive if the positive particle current exceeds the negative particle current, and vice versa. It is clearer from Eq. (A.29) than from (A.30) that, if the positive and negative particle current densities are equal, the final particle charge vanishes.

Appendix B: Drop Electrical Conductivity

Many of the theoretical discussions of mechanisms by which "drops" collect particles with the help of an electric field represent the "drop" as having a dielectric constant ε and being electrically insulating. For almost all applications, the drop should be regarded as perfectly conducting, especially if it is water. Fortunately, ε is sufficiently high in water that little error is committed in using the field distribution predicted, assuming the conductivity is zero.

To see how the electrical conductivity σ of the collecting particles comes into play, consider the typical situation of Fig. B-1 in which an uncharged drop arrives in a region where the electric field in initially uniform. In the frame of the drop, the field is essentially "turned on" when $t = 0$.

Figure B-1

With the application of the electric field, there is at first a field in the interior of the drop, hence a current density $\overline{J}$ must exist there given by $\overline{J} = \sigma\overline{E}$. As a result, positive surface charges accumulate on half of the drop equal in number to the negative charges that accumulate over the other half. These charges build up until the electric field is excluded from the volume of the drop: that is, until the drop achieves the field

configuration of a perfect conductor. The process of charge relaxation is exponential, with time constant

$$\tau = \frac{\varepsilon_o(2 + \frac{\varepsilon}{\varepsilon_o})}{\sigma}$$

In tap water, where σ is typically 1 mho/m and $\varepsilon \simeq 80\ \varepsilon_o$, this time constant is approximately 10^{-9} secs. Most processes of interest take longer than this, hence we are justified in regarding the drop as perfectly conducting.