

BACKGROUND DOCUMENT

GROUND WATER SCREENING PROCEDURE

OFFICE OF SOLID WASTE

U.S. ENVIRONMENTAL PROTECTION AGENCY

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1 INTRODUCTION

The Resource Conservation and Recovery Act (RCRA) as amended in 1984 requires EPA to promulgate regulations which restrict land disposal of hazardous wastes unless it is determined by EPA that a prohibition is not required to protect human health and the environment. The restrictions apply for as long as the waste remains hazardous.

In promulgating such regulations, Congress has directed EPA to consider:

- A. The long-term uncertainties associated with land disposal;
- B. The goal of managing hazardous wastes in an appropriate manner; and
- C. The persistence, toxicity, mobility, and propensity of hazardous wastes and their constituents to bioaccumulate.

EPA's Office of Solid Waste (OSW) reviewed a number of approaches which could serve to evaluate the need to ban or further restrict hazardous wastes from land disposal. The general approaches examined included decision criteria schemes, numerical schemes, modeling schemes, and combinations of all three. The approach presented in this paper, developed by OSW represents one option considered by EPA. It combines a mathematical model for solute transport with health based concentration limits and a leach test.

EPA's approach provides a systematic, consistent, and non-capricious method for determining the need to restrict hazardous wastes from land disposal. EPA's approach is designed to establish acceptable concentrations for specific chemical constituents in waste extracts by working backward from a point of potential human exposure (through air, groundwater, and surface water) to the land disposal unit. Health assessment levels for each constituent will be appropriately partitioned into the three media.

This paper presents the groundwater screening component of EPA's approach. It is important to note that in this approach decisions on restricting land disposal of hazardous wastes will be made on a national basis. The RCRA reauthorization legislation establishes a petition process for consideration of site specific factors.

The groundwater screening procedure presented in this paper involves application of an analytic solute transport equation in conjunction with values for hydrogeologic parameters, such as groundwater velocity, soil porosity, net infiltration, etc., that are applied in a Monte Carlo routine to derive a distribution of outputs.

The EPA approach is based on the following general assumptions:

- (1) Decisions to restrict the land disposal of hazardous wastes will be made on a national basis.
- (2) Restrictions decisions will be linked to health assessment levels, i.e., the reference dose.
- (3) The procedure developed by EPA to restrict the land disposal of hazardous wastes are to accomodate the variations in environmental settings, and the uncertainties in specific chemical properties.

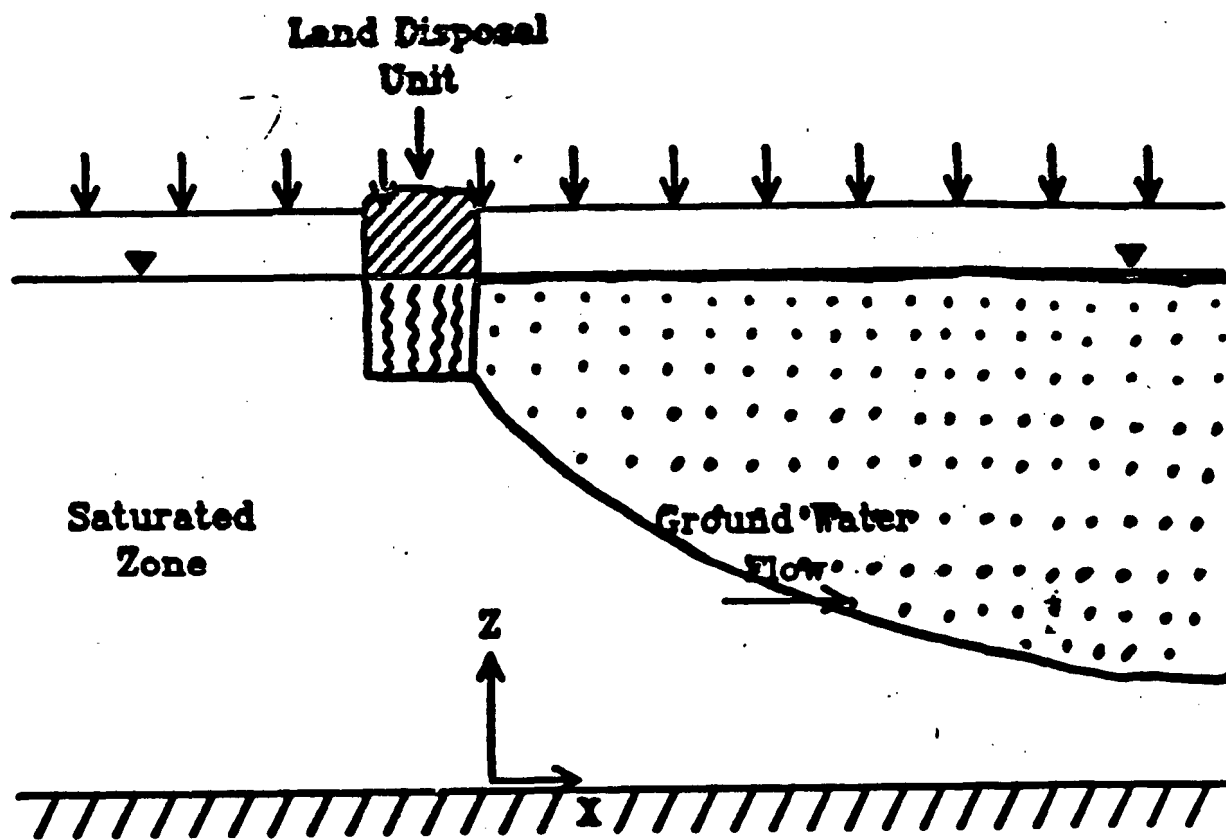
The scenario to be applied in conjunction with the ground water screening procedure is a land disposal unit with a fixed point of measurement established at distance x_1 down gradient. Figure 1 illustrates a cross section and a plan view of this system.

The groundwater screening procedure can be described qualitatively as follows.

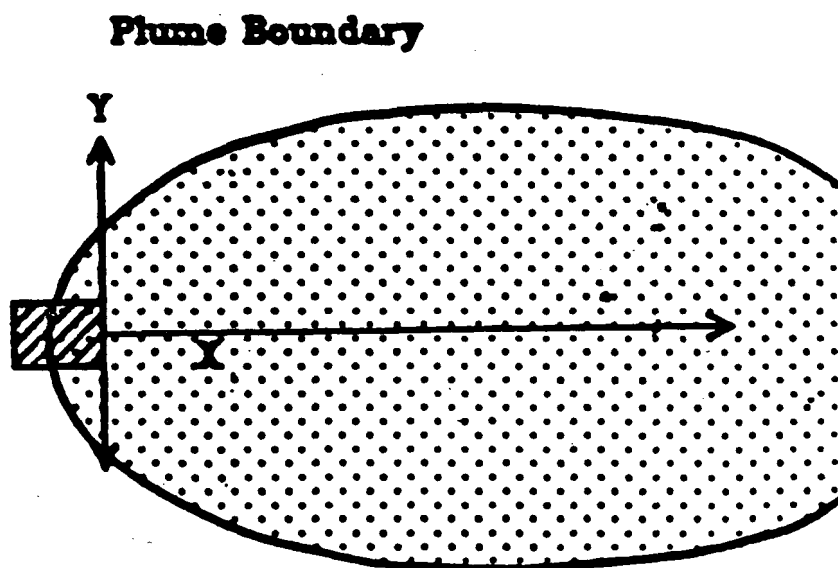
- (1) A waste will be determined acceptatble for land disposal if specific chemical constituents are projected to migrate to the fixed measurement point at concentrations equal to or less than the reference dose for those chemicals.
- (2) If constituents in hazardous wastes migrate to the fixed measurement point at concentrations above the reference dose, then the waste will be restricted from land disposal.
- (4) Land disposal will be permitted, however, if it is possible to pretreat the waste such that the constituent concentration in the groundwater at the measurement point no longer exceeds the reference dose.

In order to apply the groundwater screening procedure, one must be able to measure, estimate, or predict:

- (1) leachate concentrations of specific chemical constituents from a waste; and
- (2) the migration of the chemical constituents in the leachate to the measurement point.



Cross Section



Plan View

Figure 1

2 MATHEMATICAL MODEL

A mathematical model is used to provide an instrument for computing concentration distributions in the generic groundwater system (depicted in Figure 1). The model is explained below in general terms, and sample calculations are provided. A detailed explanation of the mathematical techniques employed in the model is presented in Appendix A at the end of the paper. The mathematical techniques employed in the model are, however, based upon analytical solution procedures well established in the scientific literature.

The ground water back-calculation model properly accounts for most of the major physical and chemical processes known to influence movement and transformations of chemicals in simple, homogeneous and isotropic porous media under steady flow conditions (constant velocity). The important mechanisms considered include advection, hydrodynamic dispersion in the longitudinal, lateral and vertical dimensions, adsorption, and chemical degradation. The model assumes first-order chemical reactions, and linear equilibrium sorption isotherms.

The three-dimensional solute transport equation upon which the ground water screening model is based is presented below as Equation (1), written in the form (Bear, 1979):

$$D_{xx} \frac{\partial^2 c}{\partial x^2} + D_{yy} \frac{\partial^2 c}{\partial y^2} + D_{zz} \frac{\partial^2 c}{\partial z^2} - \frac{v}{R_f} \frac{\partial c}{\partial x} = \frac{\partial c}{\partial t} + \theta c + I \quad (1)$$

where:

- x, y, z = spatial coordinates in the longitudinal, lateral and vertical directions, respectively, (m)
- c = dissolved concentration of chemical, (g/ml)
- D_{xx}, D_{yy}, D_{zz} = retarded dispersion coefficients in the x, y and z directions, respectively, (m^2/yr)
- v = groundwater seepage velocity, assumed to be in the x-direction, (m/yr)
- R_f = retardation factor, (dimensionless)
- t = elapsed time, (yr)
- θ = effective first-order decay constant, (yr^{-1})
- θ = volumetric water content of the porous medium, (cm^3/cm^3)
- I = rate of ground water recharge from precipitation (yr^{-1})

The retardation factor, R , and the effective decay constant, λ , are defined as follows:

$$R_f = 1 + \frac{\rho_b K_d}{\theta} \quad (2)$$

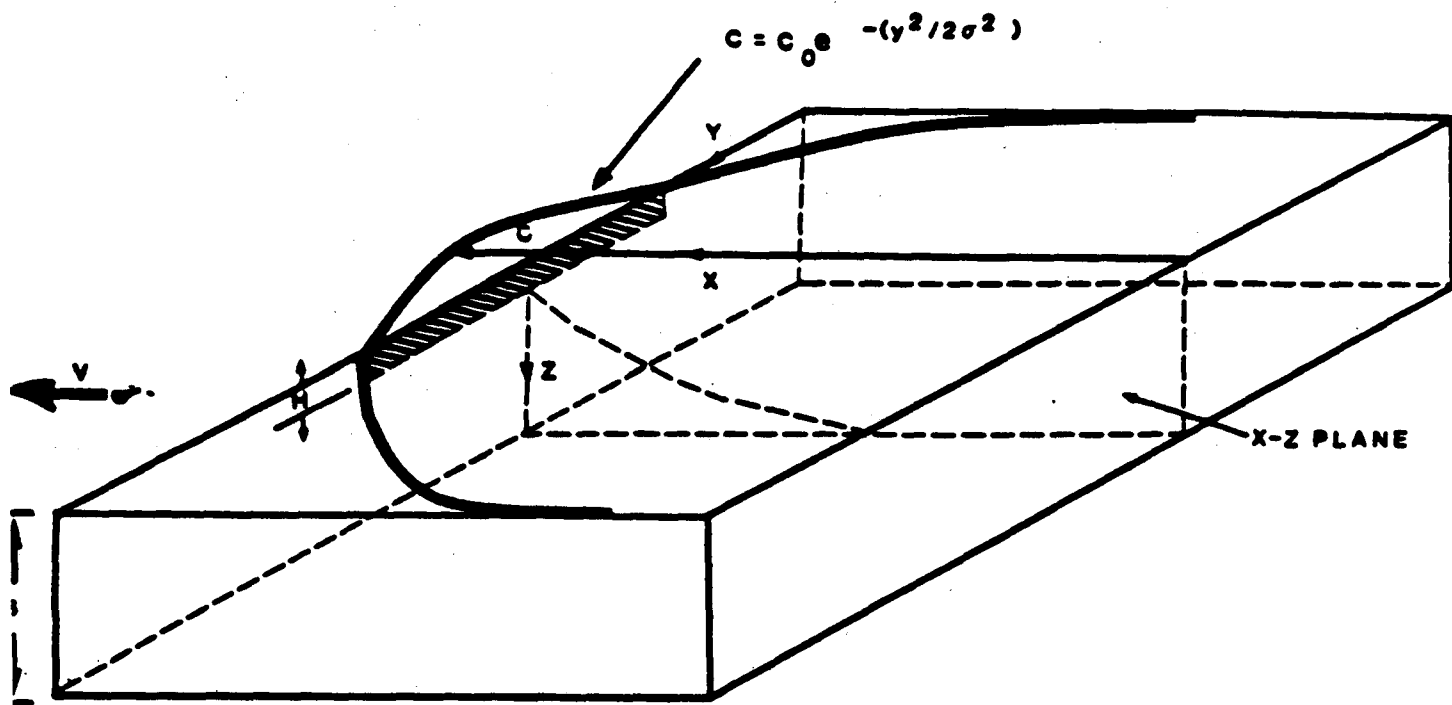
and,
$$\lambda = \frac{\lambda_1 \theta + \lambda_2 \rho_b K_d}{\theta + \rho_b K_d} \quad (3)$$

where:

- ρ_b = bulk density of the porous medium, (g/cm³)
- K_d = distribution coefficient, (cm³/g)
- θ = volumetric water content, (cm³/cm³)
- λ_1 = decay constant for dissolved phase, (yr⁻¹)
- λ_2 = decay constant for sorbed phase, (yr⁻¹)

A schematic description of the three-dimensional region considered is presented in Figure 2. The region is regarded as semi-infinite in the x-direction ($0 \leq x < \infty$), infinite in the y-direction ($-\infty < y < \infty$), and finite in the z-direction ($0 \leq z \leq B$). Initially, the aquifer is assumed to be free of contamination

The solution treats the source concentration (i.e., the contaminant concentration in the leachate directly below the land disposal unit) as a Gaussian distribution in the lateral direction (along the y-axis corresponding to the leading, down gradient edge of the unit), and a uniform distribution over the vertical mixing or penetration depth, H . The maximum dissolved concentration of the contaminant, c_0 , occurs at the center of the Gaussian distribution (i.e., at $y = 0$; under the midpoint of the disposal unit). The Gaussian distribution of the contaminant concentration is then defined by its standard deviation, σ , as illustrated in Figure 2. The standard deviation of the distribution, σ , is measured in terms of distance (meters), and is related but not equal to the width of the disposal unit.



PERSPECTIVE PLOT

FIGURE: 2 Schematic Description of Three-Dimensional Region For The Analytical Solution.

Based on the stated assumptions, the initial and boundary conditions employed may be written as follows:

$$c(x, y, z, 0) = 0 \quad (4)$$

$$c(0, y, z, t) = c_0 e^{-y^2/2\sigma^2} \quad (5)$$

$$c(x, \infty, z, t) = 0 \quad (6)$$

$$c(x, -\infty, z, t) = 0 \quad (7)$$

$$c(\infty, y, z, t) = 0 \quad (8)$$

$$\frac{\partial c}{\partial z}(x, y, 0, t) = 0 \quad (9)$$

$$\frac{\partial c}{\partial z}(x, y, B, t) = 0 \quad (10)$$

where B is the aquifer thickness.

The solution of equation (1) subjected to equations (4)-(10) is derived in Appendix A. This three-dimensional analytical solution can be written in a simple functional form as follows:

$$c_p(x, y, z, t) = \frac{H}{B} c_f(x, y, t) + \Delta c_p(x, y, z, t) \quad (11)$$

where H/B is the penetration ratio and $c_f(x, y, t)$ and $\Delta c_p(x, y, z, t)$ are functions defined in Appendix A.

Note that the function $c_f(x, y, t)$ turns out to be identical to the two-dimensional analytical solution of a corresponding case where full penetration of the source (or complete vertical mixing) over the entire aquifer thickness is assumed. According to equation (11), the solution for the general partial penetration case, $c_p(x, y, z, t)$, consists of two terms. The first term is the product of the penetration ratio and the solution of the corresponding full-penetration case. It may be interpreted as the concentration that would be predicted if vertical spreading of the contaminant plume

is not allowed. The second term, $c_p(x, y, z, t)$, is a correction term necessary to allow for the partial penetration effect which leads to vertical spreading or vertical dispersion. Vertical dispersion will cause the contaminant plume to grow in the vertical direction with increasing longitudinal distance from the source. The vertical extent of the plume is, however, limited by the available aquifer thickness.

3 GENERAL DIMENSIONLESS RELATIONSHIPS

To allow simple use of the analytical model, dimensionless relationships are introduced. First, equation (11) is rewritten as

$$c_{pD} = \frac{H}{B} c_{fD} + \Delta c_{pD} \quad (12)$$

where c_{pD} , c_{fD} , and Δc_{pD} are dimensionless functions obtained by normalizing c_p , c_f and Δc_p with respect to the maximum source concentration, c_0 .

Second, another dimensionless function, referred to as a dilution factor for partial penetration, γ , is introduced as

$$\gamma = \frac{c_{pD}}{c_{fD}} = \frac{c_p}{c_f} \quad (13)$$

Thus it follows that γ is given by

$$\gamma = \frac{H}{B} c_{fD} + c_{pD} \quad (14)$$

The dilution factor will be used as a vehicle for evaluating concentration values for the partial penetration case using concentration values for the corresponding fully-penetration case. The normalized concentrations, c_{pD} and c_{fD} , and the dilution factor γ , can be expressed as functions of several dimensionless variables as shown in Appendix A. For convenience, the important dimensionless variables are categorized as follows:

(1) Dimensionless coordinates: x_D , y_D and z_D , defined as

$$x_D = \frac{V_s x}{2D_{xx}} = \frac{x}{2\alpha_L} \quad (15a)$$

$$y_D = \frac{V_s y}{2D_{yy}} = \frac{y}{2\alpha_T} \quad (15b)$$

$$z_D = \frac{V_s z}{2D_{zz}} = \frac{z}{2\alpha_z} \quad (15c)$$

where the retarded dispersion coefficients, D_{xx} , D_{yy} and D_{zz} are computed from:

$$D_{xx} = \alpha_L V_s, \quad D_{yy} = \alpha_T V_s, \quad \text{and} \quad D_{zz} = \alpha_z V_s$$

with α_L , α_T and α_z representing the longitudinal, transverse and vertical dispersivity coefficients, respectively, and V_s representing the solute velocity.

(2) Dimensionless time: t_D , defined as

$$t_D = \frac{V_s t}{x} = \frac{t}{T} \quad (16)$$

where t denotes the elapsed time, and T denotes the transit time of the solute. The transit time is the time taken by a solute particle to travel from the source to the observation point $(x, 0, 0)$ on the x -axis.

(3) Dimensionless decay constant: λ_D , defined as

$$\lambda_D = \lambda_T = \frac{V_s}{x} \quad (17)$$

(4) Dimensionless standard deviation: σ_D , defined as

$$\sigma_D = \frac{\sigma}{2(\alpha_L \alpha_T)^{1/2}} \quad (18)$$

(5) Penetration ratio of contaminant source: H/B .

(6) Aquifer thickness to vertical dispersivity ratio: B/α_z .

In general, the relative concentration for the partial penetration case can be expressed as

$$c_{pD} = f c_{fD} = c_{pD}(x_D, y_D, z_D, t_D, \lambda_D, \sigma_D, \frac{H}{B}, \frac{B}{\alpha_z}) \quad (19)$$

where c_{fD} and f are dimensionless functions of the form

$$c_{fD} = c_{fD}(x_D, y_D, t_D, \lambda_D, \sigma_D) \quad (20)$$

$$f = (x_D, y_D, z_D, t_D, \lambda_D, \sigma_D, \frac{H}{B}, \frac{B}{\alpha_z}) \quad (21)$$

4 SIMPLIFIED DIMENSIONLESS RELATIONSHIPS FOR STEADY-STATE CONCENTRATION DISTRIBUTIONS ALONG THE x-AXIS

In a case where one is concerned with maximum attainable or steady-state concentration values at measurement points along the x-axis ($y = z = 0$), simplified dimensionless relationships can be derived.

These relationships may be written as

$$c^*_{pD} = f^* c^*_{fD} = c^*_{pD}(x_D, \lambda_D, \sigma_D, \frac{H}{B}, \frac{B}{\alpha_z}) \quad (22)$$

where c^*_{pD} and c^*_{fD} denote the dimensionless steady-state concentration distributions for the partial-penetration and the corresponding fully penetration cases, respectively, and f^* denotes the steady-state dilution factor for partial penetration. c^*_{fD} and f^* can be written in the form

$$c^*_{fD} = c^*_{fD}(x_D, \lambda_D, \sigma_D) \quad (23)$$

$$f^* = f^*(x_D, \lambda_D, \sigma_D, \frac{H}{B}, \frac{B}{\alpha_z}) \quad (24)$$

It is seen that c^*_{fD} is a function of three dimensionless variables, whereas $\mathcal{J}^*$ is a function of five dimensionless variables. Physical considerations suggest, however, some of the variables in equation (24) may not have significant influences on the value of the dilution factor $\mathcal{J}^*$. Via sensitivity analysis, it is concluded in Appendix A that an approximate functional expression for $\mathcal{J}^*$ may be written as

$$\mathcal{J}^* = \mathcal{J}^* \left(\frac{x}{B}, \frac{B}{\alpha_z}, \frac{H}{B} \right) \quad (25)$$

A comparison of equations (24) and (25) shows that the dimensionless decay and standard deviation parameters would have secondary effects on the value of $\mathcal{J}^*$ for specified values of x/B , B/α_z and H/B .

5 EVALUATION OF STEADY-STATE MAXIMUM CONCENTRATION VALUES

The steady-state, three-dimensional analytical solution developed in Appendix A has been implemented into a FORTRAN computer code, EPASMOD-P. This code can be used to evaluate concentration values at any specified location and time. It can also be used to obtain dimensionless type curves (or tables of values) of the dimensionless concentration and the dilution factor. Once they have been produced, the type curves can be used as an alternative tool for providing a quick estimate or prediction of concentration values at measurement points. The case where the contaminant concentration will be maximum corresponds to the situation in which the point of measurement depicted in Figure 1 is located on the x-axis along the top of the aquifer. The maximum concentration values attainable at this point corresponds to the steady-state concentration values at the distance x from the source.

According to the analysis given in the previous section, steady-state concentration distributions along the x-axis can be represented by equation (22), which expresses c^*_{pD} for the partially-penetrating case as the product of the steady-state dilution factor, $\mathcal{J}^*$, and the steady-state dimensionless concentration for the corresponding fully-penetrating case, c^*_{fD} . To provide a simple means for evaluating concentration values at the point of measurement, the functions c^*_{fD} and $\mathcal{J}^*$ are evaluated for realistic ranges of physical parameters.

Figure 3 shows two graphs of dimensionless concentration function c^*_{fD} . Each graph is prepared for a specified value of σ_D , and contains one family of type curves of c^*_{fD} versus x_D , for specified values of λ_D . The effect of λ_D and σ_D on values of c^*_{fD} may be noted.

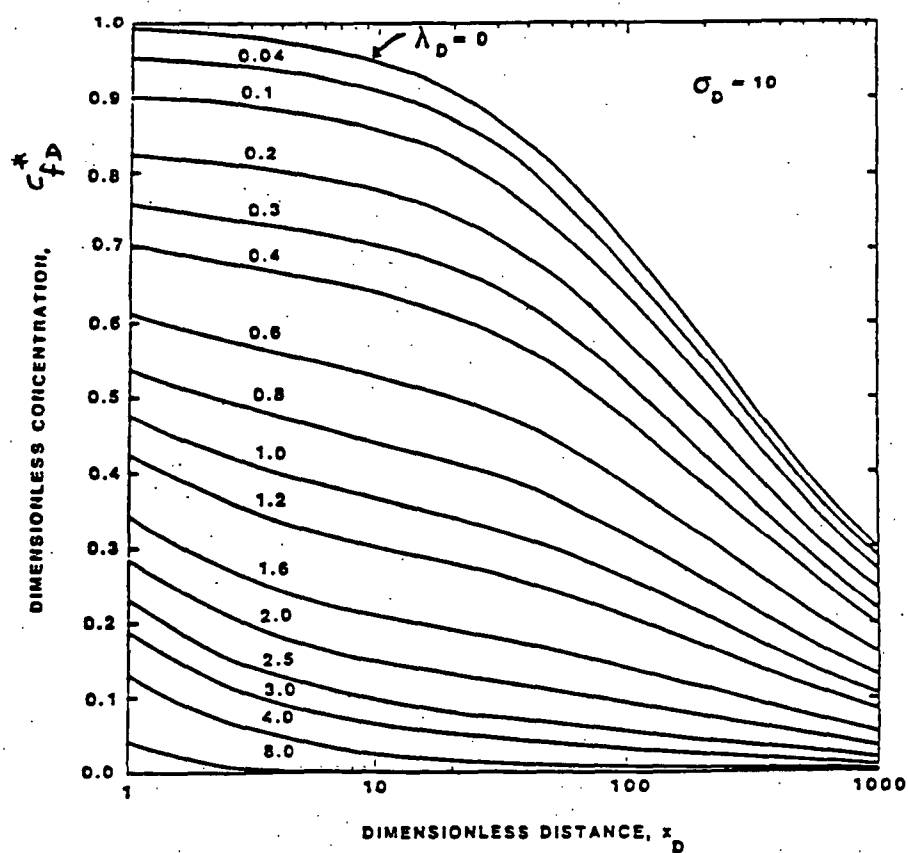
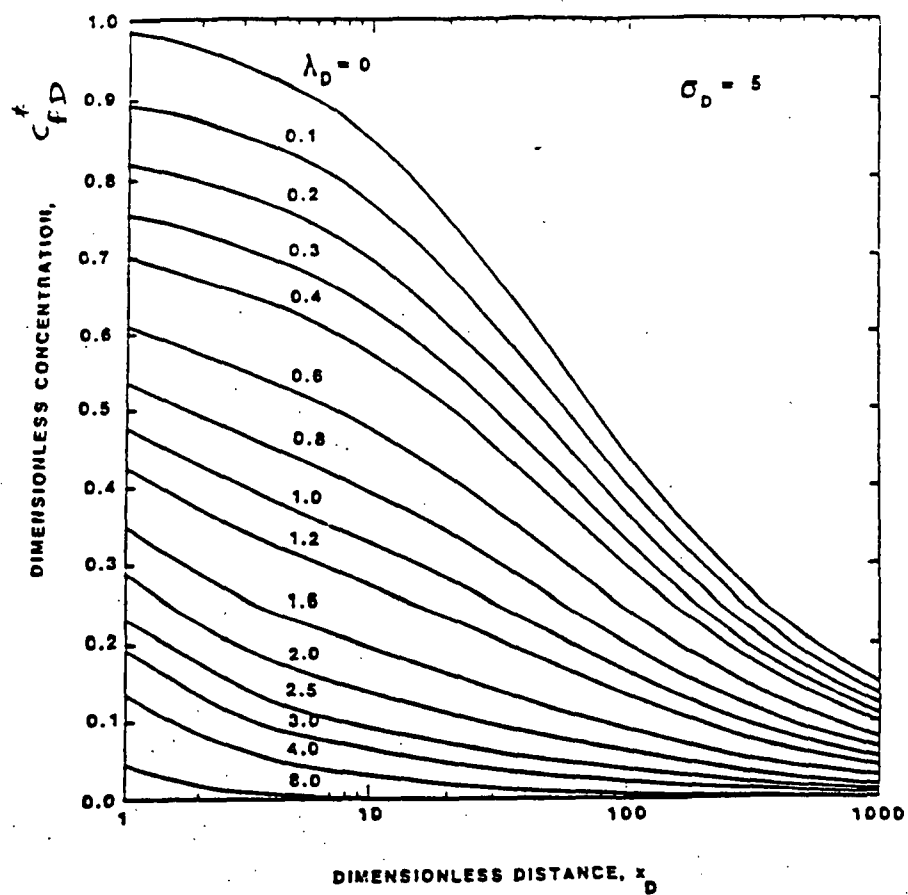


Figure 3. Type curves of dimensionless concentration, c_{fD}^* versus dimensionless distance, x_D .

As expected, λ_D has a significant influence on the type curves whereas σ_D has only a slight influence. The effect of σ_D is almost negligible for values of $x_D \leq 10$.

Figure 4 shows two graphs of dilution factor ζ^* . The first graph is prepared for a specified value of penetration ratio, $H/B = 0.25$, and contains type curves corresponding to eight selected values of B/α_z . The second graph is prepared for a specified value of ratio of aquifer thickness to vertical dispersivity, $B/\alpha_z = 25$. It contains type curves corresponding to eight selected values of H/B .

From Figure 4, the following observations of the behavior of the partial-penetration dilution factor, ζ^* , can be made.

1. The dimensionless parameters that have significant influences on ζ^* are x/B , H/B and B/α_z , which denote distance to aquifer thickness ratio, penetration ratio, and aquifer thickness to vertical dispersivity ratio, respectively.
2. For specified values of B/α_z and H/B , the ζ^* -value is unity at the source, where $x/B = 0$. As x/B increases, ζ^* decreases. The value of ζ^* approaches a lower limit as x/B reaches a value corresponding to the distance where vertical mixing is virtually completed and the concentration becomes uniform with depth. The lower limit of the ζ^* -value is equal to the specified value of penetration ratio H/B . A critical distance, x_m , where the difference between ζ^* and H/B values equal to approximately 0.02 will be referred to as the "mixing length" for a given transport condition.
3. There is a relationship between the ratio of mixing length to aquifer thickness x_m/B , and the ratio of aquifer thickness to vertical dispersivity, B/α_z . This relationship is approximately linear and is defined by the following equation

$$\frac{x_m}{B} = k \cdot \frac{B}{\alpha_z} \quad (26)$$

where k is a constant and is approximately equal to one half.

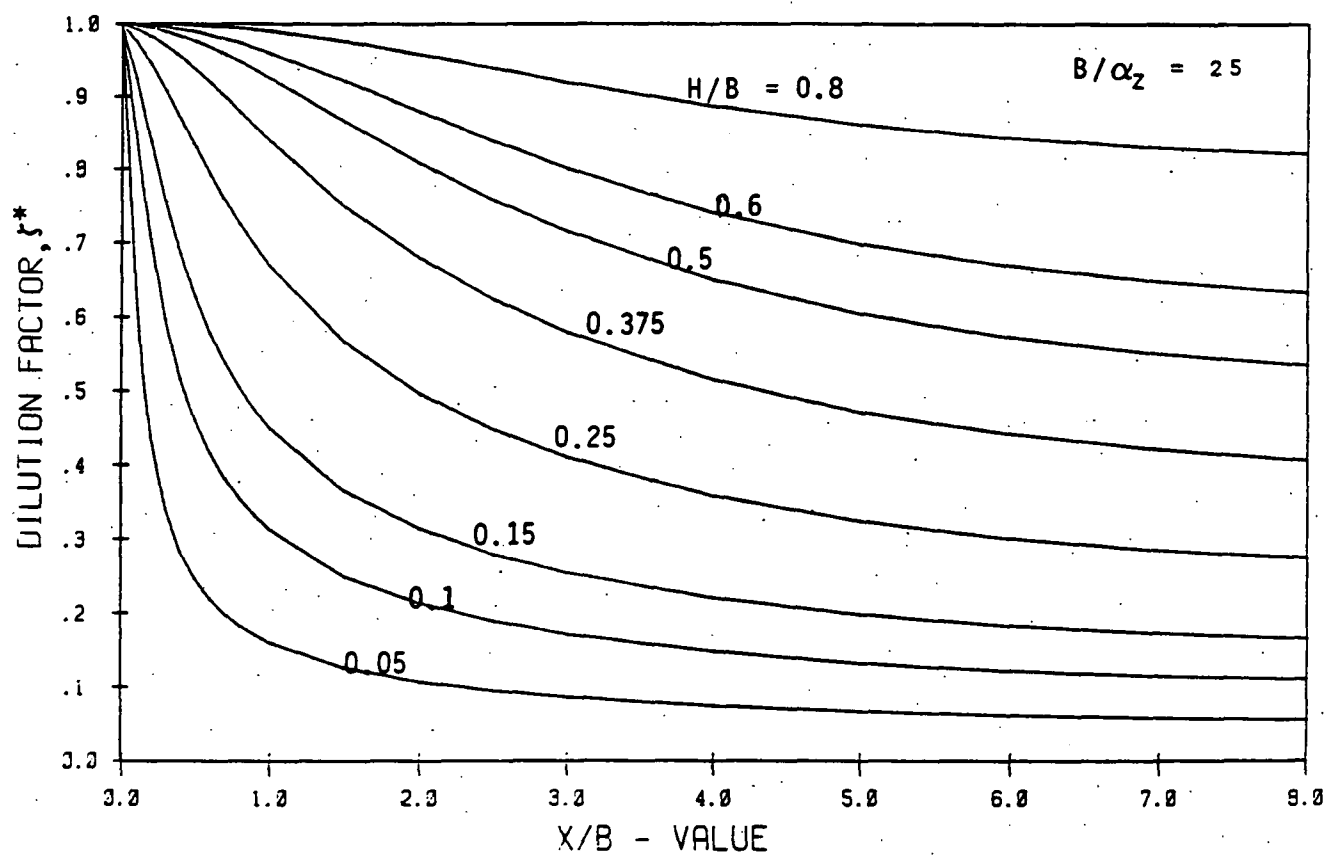
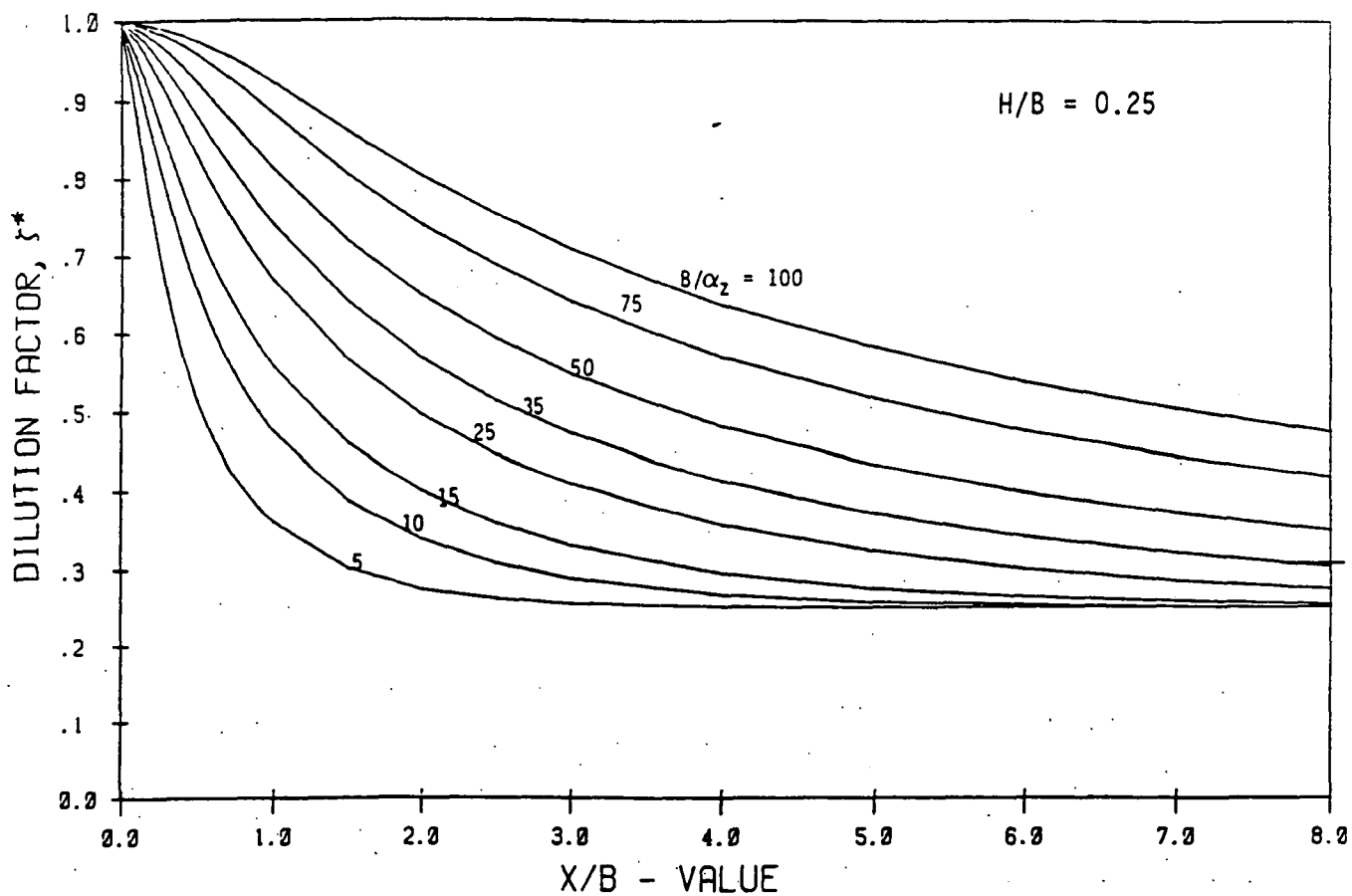


Figure 4. Type curves of dilution factor ζ^* versus ratio of distance to aquifer thickness, x/B . ($\lambda_D = 0$, $\sigma_D = 4.2$).

6 EXAMPLE CALCULATIONS: DOWN-GRADIENT CONCENTRATION

For the sake of convenience in the computation of actual concentration, tabulated values of c^*_{fD} versus x_D and ζ^* versus x/B are given for fairly wide and realistic ranges of the relevant dimensionless variables. The tables of c^*_{fD} and ζ^* values are given in Appendix B. (Note that the tables should be used in preference to the graphs since the graphs may not provide sufficient accuracy in the interpolation of functional values.)

The values of steady-state concentration along the x-axis can be readily computed using either the type curves or the tables of c^*_{fD} and ζ^* values. To show the utility of the type curves and the tables, the following sample calculations are provided.

Example 1. Consider the migration of Chemical A from a land disposal unit. Let the average groundwater velocity in the flow region, V , be taken as 30 m/yr. The saturated water content and the bulk density of the aquifer material are taken as 0.35 and 1.70 g/ml, respectively. The longitudinal, transverse and vertical dispersivities, α_L , α_T and α_z are taken as 15.4 m, 1.54 and 1.54 m, respectively. The standard deviation of the Gaussian source, σ , is taken as 97.3 m. The chemical properties of Chemical A are:

$$\begin{aligned}\text{Solubility, } S &= 3.8 \times 10^{-3} \text{ mg/l} \\ \text{Distribution coefficient, } K_d &= 5.5 \times 10^{-3} \text{ ml/g} \\ \text{Effective decay constant, } \lambda &= 3.65 \times 10^{-7} \text{ yr}^{-1}\end{aligned}$$

We wish to calculate the concentration, c , at a measurement point located at $x = 308$ m from the landfill. We will assume that the initial source concentration, c_0 , equals the solubility, S . The penetration thickness of the contaminant source and the aquifer thickness are assumed to be 10 and 40 m, respectively.

The calculation procedure consists of three steps. The first and second steps involve the determination of values of c^*_{fD} and ζ^* from the type curves using the given values of various physical parameters to compute values of the relevant dimensionless variables. The third step involves the determination of the required concentration value from known values of c^*_{fD} , ζ^* and c_0 .

Step 1: Determination of c^*_{fD} -value

To use the type curves, we need to compute the values of the dimensionless parameters, x_D , σ_D and λ_D . Using the given data, one obtains:

$$x_D = \frac{x}{2\alpha_L} = \frac{308}{2(15.4)} = 10 ; \quad D = \frac{\lambda x}{v_s}$$

$$\sigma_D = \frac{\sigma}{2(\alpha_L \alpha_T)^{1/2}} = \frac{97.3}{2(15.4 \times 1.54)^{1/2}} = 10$$

Since, $\lambda = 3.65 \times 10^{-7} \text{ yr}^{-1}$

$$\begin{aligned} \text{and, } v_s &= \frac{V}{R_f} = \frac{V}{1 + K_d \rho_b / \theta} = \frac{30 \text{ m/yr}}{1 + 5500 \text{ ml/g} \times 1.7 \text{ g/ml} / 0.35} \\ &= 30 \text{ m/yr} / 26715 = 1.123 \times 10^{-3} \text{ m/yr} \end{aligned}$$

$$\text{thus, } \lambda_D = \frac{3.65 \times 10^{-7} \text{ yr}^{-1} \times 308 \text{ m}}{1.123 \times 10^{-3} \text{ m/yr}} = 0.10$$

Next, we use the computed value of σ_D to select the most appropriate family of type curves. The values of λ_D and x_D are then entered into the graph to determine the corresponding value of c^*_{fD} . The procedure for locating c^*_{fD} -value is illustrated in Figure 5. It is seen that $c^*_{fD} = 0.863$.

Step 2: Determination of value of dilution factor γ^*

We elect to use the type curves of γ^* versus x/B and select the family that corresponds to the given penetration ratio H/B of 0.25. Using the given data, we obtain:

$$\frac{x}{B} = \frac{308}{40} = 7.7$$

and

$$\frac{B}{\alpha_z} = \frac{40}{1.54} = 25.97$$

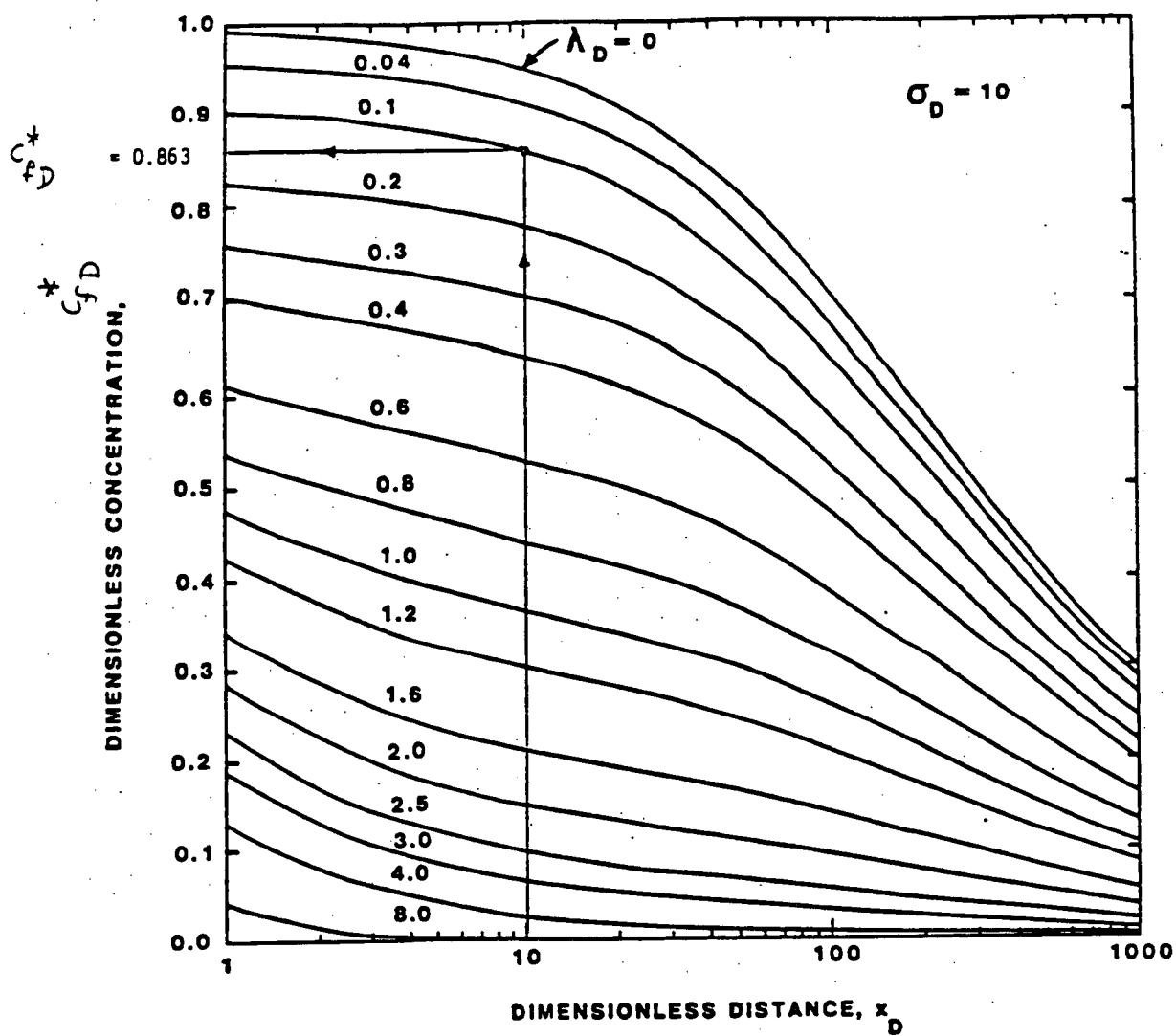


Figure 5. Determination of c_{fD}^* -value using type curves of c_{fD}^* versus x_D .

We then use the computed value of B/α_z to select the most appropriate curve, and enter the value x/B to determine the corresponding value of ζ^* as illustrated in Figure 6. From the graph, it is seen that $\zeta^* = 0.28$.

Step 3: Determination of concentration value at the measurement point

The required value of maximum attainable or steady-state concentration c_p^* at $x = 308$ m is obtained as follows:

$$c_p^* = c_{pD}^* c_0$$

$$c_{pD}^* = \zeta^* c_{fD}^* = 0.28 \times 0.863 = 0.242$$

$$\begin{aligned} \text{Hence } c_p^* &= 0.242 \times 3.8 \times 10^{-3} \\ &= 0.92 \times 10^{-3} \quad \text{mg/l} \end{aligned}$$

Example 2. In this example, we consider the migration of Chemical B from a land disposal unit. The chemical properties of Chemical B are

Solubility, S	0.18 mg/l
Distribution coefficient, K_d	96 ml/g
Effective decay constant,	$7.3 \times 10^{-3} \text{ yr}^{-1}$

Suppose we wish to know the steady-state concentration at $x = 154$ m, given that the groundwater velocity at the site is now 300 m/yr (ten times the velocity used in Example 1). The remaining transport parameters and aquifer and contaminant source characteristics are the same as before.

Step 1: Determination of c_{fD}^* -value

From the given data, we obtain:

$$x_D = 5$$

$$\sigma_D = 10 \quad (\text{as in Example 1})$$

$$\lambda_D = \frac{x}{v_s} = \frac{x}{v/(1 + K_d \rho_b/\theta)}$$

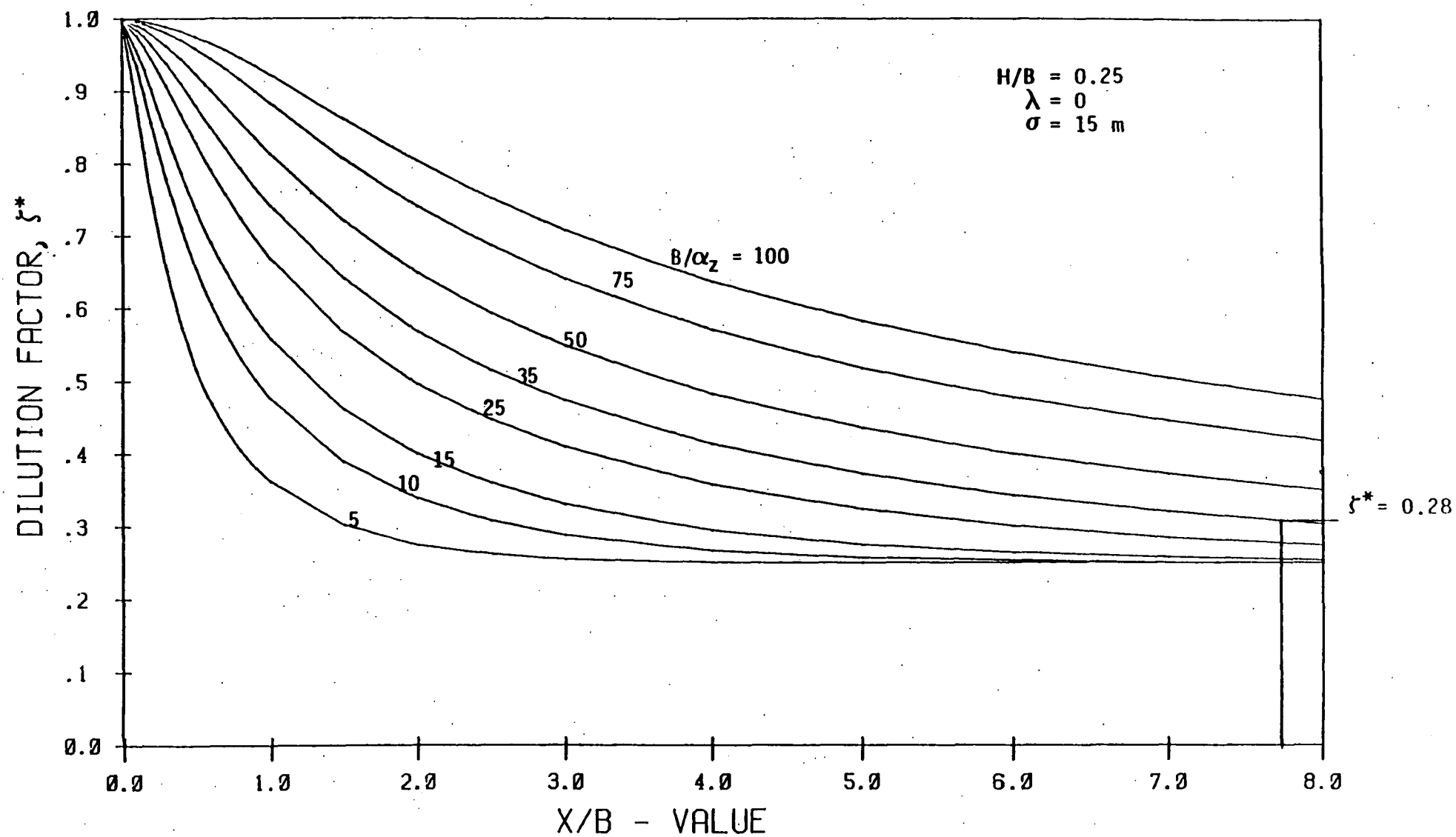


Figure 6. Determination of ζ^* -value using type curves of ζ^* versus x/B .

$$\begin{aligned}
 &= \frac{7.3 \times 10^{-3} \text{yr}^{-1} \times 154 \text{m}}{300 \text{m/yr} / (1 + 96 \text{ml/g} \times 1.76 \text{g/ml} / 0.35)} \\
 &= 1.75
 \end{aligned}$$

We then use the determined value of σ_D and λ_D to locate the appropriate table in Appendix B. In this case, the most appropriate table to use is Table 7. Thus, the values of λ_D and x_D are entered into this table. The next step is to determine c^*_{fD} via linear interpolation shown below.

Given $\lambda_D = 1.75$, and using the given value of $x_D = 5$, one obtains from Table 7 in Appendix C: $c^*_{fD} = 0.241$ for $\lambda_D = 1.6$ and $c^*_{fD} = 0.207$ for $\lambda_D = 1.8$. The desired value of c^*_{fD} is that which corresponds to $\lambda_D = 1.75$, and is given by

$$\begin{aligned}
 c^*_{fD} &= 1.75 = 0.241 - \left(\frac{0.241 - 0.207}{1.6 - 1.8} \right) (1.6 - 1.75) \\
 &= 0.216
 \end{aligned}$$

Step 2: Determination of value of dilution factor β^*

From the given data, we obtain:

$$\begin{aligned}
 \frac{x}{B} &= \frac{154}{40} = 3.85 \\
 \frac{B}{z} &= \frac{40}{1.54} = 25.97
 \end{aligned}$$

We use the value of H/B to locate the appropriate table in Appendix C. In this case, the most appropriate table to use is Table 3. The values of B/σ_z and x/B are then entered into this table to determine the corresponding value of the dilution factor β^* . It follows that $\beta^* = 0.36$.

Step 3: Determination of concentration value at the measurement point

The required value of steady-state concentration c^*_p at $x = 154 \text{ m}$ is obtained as follows:

$$c^*_p = c^*_{pD} c_o$$

$$c^*_{pD} = \beta^* c^*_{fD} = 0.36 \times 0.216 = 0.0778$$

Hence

$$\begin{aligned}c^*_p &= 0.0778 \times 0.18 \\ &= 0.014 \text{ mg/l}\end{aligned}$$

7 BACK-CALCULATION OF MAXIMUM LEACHATE CONCENTRATION

It has been shown that for any combination of parameter values, the steady-state, down-gradient concentration, $c = c_p$, at the fixed point x can be readily determined using the type curves or tables of function values. One can also determine the maximum concentration at the source, (c_0) that corresponds to a prescribed or set value of the downstream concentration, c . (We call this the back-calculation process.) Thus, if c is assumed to be a concentration value sufficient to meet a health assessment level such as the reference dose for a specific chemical, then the computed c_0 will be the leachate quality required to yield the reference dose at the fixed distance. This "back calculated" leachate concentration can then be compared to expected values from a leach test or by a comparison to a maximum solubility value.

The above examples illustrate the calculation of steady-state concentration distributions along the x -axis for a given set of input parameters using the steady-state solution. However, rather than specify one "reasonable worst case" land disposal scenario, EPA has developed a procedure that accommodates the possible variation in environmental settings, uncertainties in chemical specific properties, and uncertainties in the range of impact of engineered system releases from land disposal units. This procedure termed Monte Carlo simulation is most suitable for investigating the land disposal restrictions process.

8 LOGIC FLOW CHART FOR LAND DISPOSAL SCREENING

A logic flow chart for the screening procedure is presented in Figure 7. The flow chart combines the steps EPA must take in developing and implementing the procedure with the steps generators and owner/operators must take in determining whether or not their waste can be land disposed.

Using the information available at each critical step in the flow chart (diamond shaped boxes), generators and owner/operators must answer the following questions:

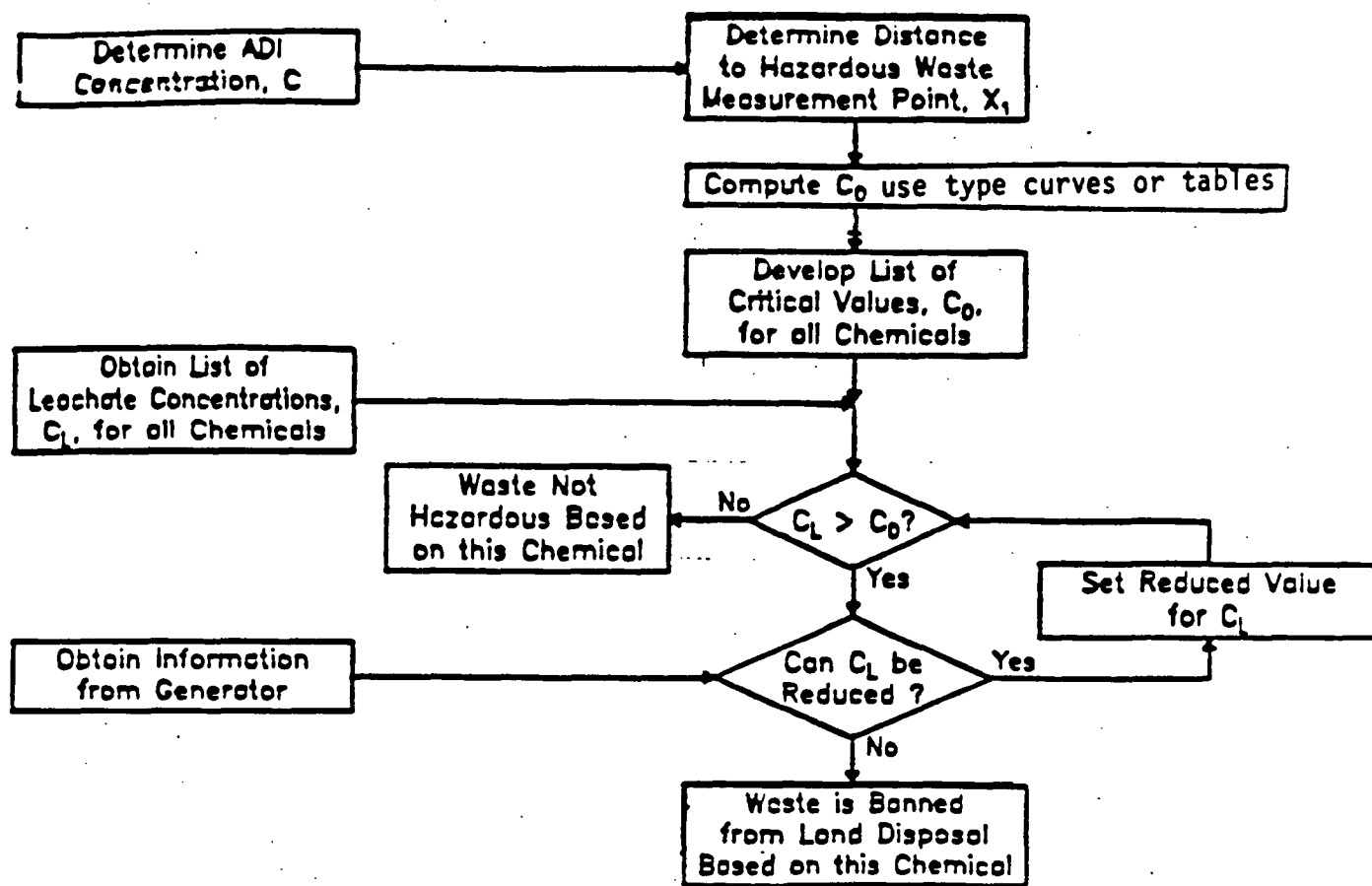


FIGURE 7
SCREENING PROCEDURE FLOW CHART

1. Does the leachate quality for each constituent in the hazardous waste indicate that it is acceptable for land disposal?
2. If the leachate quality indicates that the hazardous waste is not acceptable for land disposal, can it be pretreated or treated such that the leachate quality indicates that the hazardous waste is acceptable for land disposal?

The answers to these questions lead to yes or no decisions and dictate the action to be taken. Further, question two provides a point where treatment of wastes can be taken into account prior to making a final decision. A detailed, step-by-step description of the screening procedure is given below. The parentheses indicate who is responsible for each step, EPA or the generator/owner/operator (Gen;0/0).

PART A: Restriction of Hazardous Waste From Land Disposal

Step 1 (EPA).. Determine "Reference Dose Concentration

For each individual constituent in the waste, there must be an appropriate toxicologically based limit expressed in concentration units. This value, once determined, is set to be the steady-state down gradient concentration, c , at the fixed measurement point (x).

The steady-state value downgradient, c , will be pre-determined by the levels evaluated in a risk assessment of each constituent in the waste. The maximum concentration of each constituent at the point of measurement cannot exceed either reference dose for non-carcinogens or the a dose at a 10^{-6} risk level for carcinogens.

Step 2 (EPA): Determine Distance to Measurement Point

The lateral, downgradient distance from the land disposal facility to the measurement point, x , must be specified. At this point, the presence of a constituent at concentrations exceeding a health assessment level will result in a waste being banned from land disposal. In this approach, we are setting x equal to 500ft.

Step 3 (EPA).. Compute c_0

This step can be achieved using the type curves or tables of dimensionless functions, c^*_{FD} and J^* .

For the determination of c^*_{FD} -value, input values of the dimensionless parameters x_D , λ_D , and σ_D are required. By definition, these dimensionless parameters depend on the distance, x , the longitudinal and transverse dispersivities, α_L and α_T , the standard deviation of the Gaussian distribution at the source, σ , the chemical decay constant, λ , and the transit time T ($T = x/V_s$).

For the determination of J^* -value, input values of x/B , H/B , and B/α_z are required. By definition, these dimensionless parameters depend on the distance, x , the aquifer thickness, B , the penetration thickness of the source, H , and the vertical dispersivity, α_z .

Values of the longitudinal, the transverse, and the vertical dispersivities can be determined from published data. In the field, the longitudinal dispersivity may vary from 5 to 100 m. The transverse dispersivity value is usually taken to be the same as the vertical dispersivity. The value of lateral dispersivity is a fraction of the value of longitudinal dispersivity. The ratio of α_L to α_T may vary from 2 to 20.

The standard deviation has a similar range of variation to α_L and α_T . For a conservative prediction of downstream concentration, the value of σ should be selected to be at least the same or twice the longitudinal dispersivity. The effective decay constant, λ , is chemical specific. Assuming simple hydrolysis, λ , can be determined using the procedures described in detail in Section 9.

Step 4 (EPA): Develop and Promulgate List of Screening levels, c_0 , for Land Disposal Restrictions Determinations for All Chemicals

Through the use of the computer model, EPASMOD, EPA will develop a list of c_0 values for all chemical constituents. The c_0 values can also be computed using the dimensionless functions and tabulated values. The c_0 values can be viewed as maximum acceptable extract concentrations for each waste constituent if the waste is to be considered acceptable for land disposal.

Step 5 (Gen;0/0).. Determine The Leachate Concentration, c_L

The concentration in the leachate c_L will be determined by a test similar to the EPA extraction procedure (EP), developed specifically to address organic as well as inorganic constituents. (This test, is termed the Toxicity Characteristic Leaching Procedure.) c_L must be determined for each chemical constituent

in the waste for which the Agency has established values for c_0 . This step is the responsibility of the waste generator.

Step 6 (Gen;0/0).. Is $c_L > c_0$?

Once a generator determines the actual leachate concentration, c_L , of each chemical constituent in a waste, a simple comparison is made with the list of maximum acceptable leach concentrations, c_0 , developed by EPA in Step 4. If, for each chemical constituent in the waste, the leachate concentration, c_L , is not greater than the listed maximum acceptable concentration, c_0 , for that constituent, then the waste can be land disposed. If, the leachate concentration, c_L , for any constituent in the waste exceeds the listed maximum acceptable concentration, c_0 , for that constituent, the waste is banned from land disposal.

Step 7 (Gen;0/0): Can c_L be Reduced?

If it is possible for the generator to reduce the leachate concentration via pretreatment or treatment such that the resulting leachate concentration, c_L , for each chemical constituent in the waste is no longer greater than the listed maximum acceptable concentration, c_0 , then the waste (following pretreatment or treatment) can be land disposed as a hazardous waste. If the leachate concentration, c_L , for any chemical constituent in the waste can not be reduced via pretreatment or treatment to a level equal to or less than its listed maximum acceptable concentration, c_0 , the waste is banned from land disposal.

9 CALCULATION OF EFFECTIVE DECAY CONSTANT

The effective decay constant, λ , is chemical specific. Assuming simple hydrolysis, λ , can be determined from

$$\lambda = \left(\frac{\lambda_1 \theta + \lambda_2 K_d \rho_b}{\theta + K_d \rho_b} \right) \quad (27)$$

where λ = lumped first-order degradation rate constant, yr^{-1}
 λ_1 = first-order hydrolysis rate constant for the dissolved constituent, yr^{-1}
 λ_2 = first-order hydrolysis rate constant for the sorbed constituent, yr^{-1}
 K_d = distribution coefficient, $\text{cm}^3 \text{g}^{-1}$
 θ = volumetric water content of soil, cm^3/cm^3
 ρ_b = soil bulk density, g/cm^3

$$\text{and } R_f = 1 + \frac{k_d \rho_b}{\theta}$$

From process theory (Mill et al., 1981) we know that for hydrolysis, the decay constant for the i phase can be determined from:

$$\lambda_i = K_a[H^+] + K_n + K_b[OH^-], \quad i = 1, 2 \quad (28)$$

where K_a = second order rate constant for acid catalyzed hydrolysis, $M^{-1} \text{ yr}^{-1}$

K_n = neutral hydrolysis rate constant, yr^{-1}

K_b = second order rate constant for base catalyzed hydrolysis, $M^{-1} \text{ yr}^{-1}$

$[H^+]$ = hydrogen ion concentration, M

$[OH^-]$ = hydroxyl ion concentration, M

Because both the sorbed and dissolved states exist we must develop expressions for λ_1 and λ_2 . The dissolved case is simply as depicted above. The pH of the groundwater must be known. For the sorbed case, an "effective" pH of the sorbent must be known, i.e., what is the pH of the adsorbing surface? This can vary depending upon the soil type but a conservative estimate is one pH unit less than the bulk fluid. Evidence also exists to suggest alkaline hydrolysis does not operate for the sorbed state (Wolfe, 1984). Using this approximation we have:

$$\lambda_2 = 10K_a [H^+] + K_n \quad (29)$$

We can now write

$$\lambda = \frac{(K_a[H^+] + K_n + K_b[OH^-])\theta + (10K_a[H^+] + K_n) K_d \rho_b}{\theta + K_d \rho_b} \quad (30)$$

From this analysis it is clear that two things are required to compute λ . First the second-order rate constants must be known, measured, or estimated and the pH of the groundwater must be known.

Other parameter values required are: the bulk density, ρ_b , saturated moisture content, θ , the solute velocity, V_s , and the distribution coefficient, K_d . The bulk density, ρ_b and the saturated moisture content, θ , vary over a relatively narrow range and the flow velocity, V , may be quite variable. The development of these data are described in the discussion of Monte Carlo Uncertainty Analysis.

The problem remains to estimate the distribution coefficient, K_d , needed for both the overall rate expression and the retardation coefficient, R_f . In general, this term is dependent upon both the chemical properties and the sorbent properties. For organic, hydrophobic chemicals K_d is commonly correlated to the organic carbon fraction of the sorbent (Karickhoff, 1979; Lyman, 1982). Thus, if one knows the normalized distribution coefficient for organic carbon, K_{oc} , for a specific chemical and the fraction organic carbon, f_{oc} for the soil then,

$$K_d = f_{oc}K_{oc} \quad (31)$$

Values for K_{oc} are often available for chemicals and in many cases can be estimated from K_{ow} (octanol-water partition coefficient) (Karickhoff, 1979). K_{ow} values can be either measured or calculated from well-known and usually reliable relationships (Lyman et al., 1982; Mill et al., 1984). The problem remains to specify the organic carbon content, f_{oc} , of the saturated zone. This value is expected to be quite low and can be determined from surveys, published data, or from engineering judgement. Caution must be exercised in assigning a very low value for f_{oc} . At such values (e.g., $f_{oc} < .001 - .005$) other sorbent characteristics contribute more to sorption. The soil mineral content, for example, will influence sorption. In any case, our assumptions are conservative in that estimated K_d values will be smaller than if other mechanisms were included. Sample calculations of λ and R are given below.

Example Calculations for λ and R_f

Given: groundwater pH at 6.5

$$\begin{aligned} \theta &= 0.35 \\ f_{oc} &= 0.001 \\ \rho_b &= 1.20 \text{ gm cm}^{-3} \end{aligned}$$

chemical is X

chemical properties

water solubility, 700 mg/l

$$\begin{aligned} K_a &= 5.04 \times 10^{-4} \text{ M}^{-1} \text{ hr}^{-1} \\ K_n &= 2.5 \times 10^{-3} \text{ hr}^{-1} \\ K_b &= 1638 \text{ M}^{-1} \text{ hr}^{-1} \\ \text{Log } K_{ow} &= 2.55 \end{aligned}$$

$$\text{Find } \lambda_1 = K_a[H^+] + K_n + K_b[OH]$$

$$\lambda_2 = 10 K_a[H^+] + K_n$$

$$\begin{aligned} \text{at pH} = 6.5 \quad [H^+] &= 3.16 \times 10^{-7} \text{ M} \\ [OH^-] &= 3.16 \times 10^{-8} \text{ M} \end{aligned}$$

Converting K_a , K_n , K_b to units of yr^{-1} we have

$$\begin{aligned} K_a &= 5.04 \cdot 10^{-4} \text{M}^{-1} \text{hr}^{-1} * 8.76 \text{ hr yr}^{-1} * 10^3 = 4.415 \text{ M}^{-1} \text{yr}^{-1} \\ K_n &= 2.5 \cdot 10^{-3} \text{ hr}^{-1} * 8,760 \text{ hr yr}^{-1} * 10^3 = 21.9 \text{ yr}^{-1} \\ K_b &= 1638 \text{M}^{-1} * 8.76 \cdot 10^3 \text{ hr yr}^{-1} = 14.35 \cdot 10^6 \text{ M}^{-1} \text{yr}^{-1} \end{aligned}$$

$$\begin{aligned} \text{Now for } \lambda_1 &= 4.415 * 3.16 \cdot 10^{-7} + 21.9 + 14.35 \cdot 10^6 * 3.16 \cdot 10^{-8} \\ \lambda_1 &= 1.4 \cdot 10^{-6} + 21.9 + .453 \\ \lambda_1 &= 22.4 \text{ yr}^{-1} \\ \lambda_2 &= 10 * 4.415 [10^{-7}] + 21.9 \\ \lambda_2 &= 21.9 \text{ yr}^{-1} \end{aligned}$$

Find: K_d
 From Karickhoff (1979) we have
 $\log K_{oc} = 1.00 \log K_{ow} - 0.21$
 so $\log K_{oc} = 1.00 * 2.55 - 0.21$
 and $K_{oc} = 219$

$$K_d = f_{oc} * K_{oc} = .001 * 219 = 0.219$$

$$\text{Finally, } \lambda = \frac{(22.4 * 0.35 + 21.9 * 0.219 * 1.2)}{0.35 + 0.219 * 1.2}$$

$$\lambda = 22.19 \text{ yr}^{-1}$$

$$\text{and } R = \left(1 + \frac{.219 * 1.2}{0.35}\right) = 1.75$$

10 SUMMARY AND OBSERVATIONS

The proposed approach is conservative but not unrealistic, since it accommodates the opportunity for degradation (due to hydrolysis), retardation, and dispersion in the longitudinal, lateral and vertical directions. The proposed procedure does not require considerable effort to execute. A careful analysis of the data required to run the model is presented in the following section on Monte Carlo simulation techniques.

REFERENCES

- Carslaw, H.S., and J.C. Jaeger, Conduction of Heat in Solids, 510 pp, 1959.
- Churchill, R.V., Operational Mathematics, McGraw-Hill, 481 pp, 1972.
- Dominico, P.A., and V.V. Palciaukus, 1985, Alternative methods for the prediction of leachate plume migration, Ground Water Monitoring Review, 5(2), pp 46-59.
- Gureghian, A.B., D.S. Ward, and R.W. Cleary, 1980, A finite element model for the migration of leachate from a sanitary landfill in Long Island, New York--Part I: Theory, Water Resources Bulletin, 16(5), pp 900-096.
- Gureghian, A.B., D.S. Ward, and R.W. Cleary, 1981, A finite element model for the migration of leachate from a sanitary landfill in Long Island, New York--Part II: Application, Water Resources Bulletin, 17(1), pp 62-66.
- Kent, D.C., N.A. Pettyjohn, and T.A. Prickett, 1985, Analytical methods for the prediction of leachate plume migration, Ground Water Monitoring Review, 5(2), pp 46-59.
- Prakash, A., 1984, Ground-water contamination due to transient sources of pollution, Amer. Soc. Civ. Engrs., J. Hydraulics Div., 110(HYII), pp 1642-1658.
- Sagar, B., 1982, Dispersion in three dimensions: Approximate analytic solutions, Amer. Soc. Civ. Engrs., J. Hydraulics Div., 108(HYI), pp 47-62.
- Shen, H.T., 1976, Transient dispersion in uniform porous media flow, Amer. Soc. Civ. Engrs., J. Hydraulics Div., 102(HY6), pp 707-718.
- Wilson, J.L., and P.J. Miller, 1978, Two-dimensional plume in uniform ground water flow, Amer. Soc. Civ. Engrs., J. Hydraulics Div., 104(HY4), pp 503-514.

APPENDIX A
DEVELOPMENT OF THREE-DIMENSIONAL
ANALYTICAL SOLUTIONS FOR CONTAMINANT TRANSPORT

INTRODUCTION

The model described in the main body of this report is based on three-dimensional, transient and steady-state, analytical solutions for solute transport from a distributed source in a homogeneous and isotropic aquifer through which there is uniform groundwater flow. These solutions are derived herein.

GOVERNING EQUATIONS

Consider the region with the distributed source shown in Figure 1.1. The advective-dispersive equation for the transport of a nonconservative contaminant in an adsorbing homogeneous and isotropic porous medium with fully-saturated uniform flow can be written as

$$V_s \frac{\partial c}{\partial x} - D_x \frac{\partial^2 c}{\partial x^2} - D_y \frac{\partial^2 c}{\partial y^2} - D_z \frac{\partial^2 c}{\partial z^2} + \lambda_c c + \frac{\partial c}{\partial t} = 0 \quad (1)$$

$$0 \leq x \leq \infty \quad -\infty \leq y \leq \infty \quad 0 \leq z \leq B$$

where D_x , D_y and D_z are retarded dispersion coefficients in the longitudinal, x , transverse, y , and vertical, z , directions, respectively ($D_x \equiv D_{xx}$, $D_y \equiv D_{yy}$, and $D_z \equiv D_{zz}$), V_s is the retarded solute velocity, λ is the first-order decay coefficient, c is the solute concentration, and t is the elapsed time.

The initial and boundary conditions of the problem may be expressed as follows:

$$c(x, y, z, 0) = 0 \quad (2a)$$

$$c(\infty, y, z, t) = 0 \quad (2b)$$

$$c(x, \infty, z, t) = 0 \quad (2c)$$

$$c(x, -\infty, z, t) = 0 \quad (2d)$$

$$\frac{\partial c}{\partial z}(x, y, 0, t) = 0 \quad (2e)$$

$$\frac{\partial c}{\partial z}(x, y, B, t) = 0 \quad (2f)$$

$$c(0, y, z, t) = c_0 \exp\left(\frac{-y^2}{2\sigma^2}\right) U(z) \quad (2g)$$

where c_0 is the peak concentration at the source, σ is the standard deviation of the Gaussian distribution centered at $x = y = 0$, and $U(z)$ is the unit step function defined as

$$\begin{aligned} U(z) &= 1 & , H_1 \leq z \leq H_2 \\ U(z) &= 0 & , z > H_1 \text{ or } z > H_2. \end{aligned}$$

DERIVATION OF TRANSIENT ANALYTICAL SOLUTION

To derive the transient analytical solution for a Gaussian distributed source in y with the concentration at any point y for $x = 0$ being uniform between $z = H_1$ and $z = H_2$, we will first derive a fundamental solution for an infinitely thin Gaussian source located at depth $z = z'$. We will then integrate this solution with respect to z' between the depth limits H_1 and H_2 .

The fundamental solution must satisfy equation (1) and the initial and boundary conditions (2a)-(2f), as well as the following source boundary condition:

$$c(0, y, z, t) = (H_2 - H_1) c_0 \exp\left(\frac{-y^2}{2\sigma^2}\right) \delta(z - z') \quad (2h)$$

where c_0 and σ are the peak concentration and the standard deviation of the Gaussian distribution, respectively, and $\delta(z - z')$ is the Dirac delta function.

We introduce the following exponential Fourier transform in the y -space:

$$\begin{aligned} \bar{c}(x, \alpha, z, t) &= \int_{-\infty}^{\infty} c(x, y, z, t) \exp(-i \alpha y) dy \\ &= F_e [c(x, y, z, t)] \end{aligned} \quad (3)$$

Applying the above transform to equation (1) and making use of boundary conditions (2c) and (2d), one obtains:

$$\frac{\partial \bar{c}}{\partial t} + v_s \frac{\partial \bar{c}}{\partial x} - D_x \frac{\partial^2 \bar{c}}{\partial x^2} + D_y \alpha^2 \bar{c} - D_z \frac{\partial^2 \bar{c}}{\partial z^2} + \lambda \bar{c} + I_c = 0$$

By means of the exponential transform, the remaining initial and boundary conditions (2a), (2b), (2e), (2f), and (2h) can be converted to

$$\bar{c}(x, \alpha, z, 0) = 0 \quad (5a); \quad \bar{c}(\infty, \alpha, z, t) = 0 \quad (5b)$$

$$\frac{\partial \bar{c}}{\partial z}(x, \alpha, B, t) = 0 \quad (5c); \quad \frac{\partial \bar{c}}{\partial z}(x, \alpha, 0, t) = 0 \quad (5d)$$

$$\bar{c}(0, \alpha, z, t) = (2\pi)^{\frac{1}{2}} m \sigma \exp\left(\frac{-\sigma^2 \alpha^2}{2}\right) \delta(z-z') \quad (5e)$$

where, for the sake of convenience, we let $m = (H_2 - H_1)c_0$. Note that equation (5e) was obtained using the relation:

$$F_e \left[\exp\left(-\frac{y^2}{4c}\right) \right] = 2(\pi c)^{\frac{1}{2}} \exp(-c \alpha^2) \quad (6)$$

Next, we introduce the finite Fourier cosine transform in the z-space:

$$\bar{\bar{c}}(x, \alpha, n, t) = \int_0^B \bar{c}(x, \alpha, z, t) \cos\left(\frac{n\pi z}{B}\right) dz \quad (7)$$

Using this transform, equation (4), with (5c) and (5d), and the remaining initial and boundary conditions (5a), (5b), and (5e) can be converted to

$$\frac{\partial \bar{\bar{c}}}{\partial t} + v_s \frac{\partial \bar{\bar{c}}}{\partial x} - D_x \frac{\partial^2 \bar{\bar{c}}}{\partial x^2} + D_y \alpha^2 \bar{\bar{c}} + \frac{n^2 \pi^2}{B^2} D_z \bar{\bar{c}} + \lambda \bar{\bar{c}} + I_c = 0 \quad (8)$$

$$\text{and} \quad \bar{\bar{c}}(\infty, \alpha, z, t) = 0 \quad (9a)$$

$$\bar{\bar{c}}(x, \alpha, n, 0) = 0 \quad (9b)$$

$$\bar{\bar{c}}(0, \alpha, n, t) = (2\pi)^{\frac{1}{2}} m \sigma \exp\left(-\frac{\sigma^2 \alpha^2}{2}\right) \cos\left(\frac{n\pi z'}{B}\right) \quad (9c)$$

Finally, we introduce the Laplace transform in the time domain:

$$\bar{\bar{\bar{c}}}(x, \alpha, n, p) = \int_0^\infty \exp(-pt) \bar{\bar{c}}(x, \alpha, n, t) dt \quad (10)$$

Using the Laplace transform, equation (8) with (9a), and the remaining boundary conditions (9b) and (9c) can be converted to

$$p\bar{\bar{c}} + V_s \frac{d\bar{\bar{c}}}{dx} - D_x \frac{d^2\bar{\bar{c}}}{dx^2} + D_y \alpha^2 \bar{\bar{c}} - \frac{n^2 \pi^2}{B^2} D_z \bar{\bar{c}} + \lambda \bar{\bar{c}} + I_c = 0 \quad (11)$$

and

$$\bar{\bar{c}}(\infty, \alpha, n, p) = 0 \quad (12a)$$

$$\bar{\bar{c}}(0, \alpha, n, p) = \frac{(2\pi)^{\frac{1}{2}}}{p} m \sigma \exp\left(\frac{-\sigma^2 \alpha^2}{2}\right) \cos\left(\frac{n\pi z'}{B}\right) \quad (12b)$$

Equation (11) can be rearranged to yield:

$$\frac{d^2\bar{\bar{c}}}{dx^2} - \frac{V_s}{D_x} \frac{d\bar{\bar{c}}}{dx} - \frac{1}{D_x} (p + \lambda + I + D_y \alpha^2 + \frac{n^2 \pi^2}{B^2} D_z) \bar{\bar{c}} = 0 \quad (13)$$

The general solution to the second-order, ordinary, homogeneous differential equation (13) is given by

$$\bar{\bar{c}} = A_{\exp}(r_+) + B_{\exp}(r_-) \quad (14)$$

where

$$r_+ = \frac{V}{2D_x} + \frac{1}{2} \left[\frac{V_s^2}{D_x^2} + \frac{4}{D_x} (p + \lambda + I + D_y \alpha^2 + \hat{n} D_z) \right]^{1/2}$$

$$r_- = \frac{V}{2D_x} + \frac{1}{2} \left[\frac{V_s^2}{D_x^2} + \frac{4}{D_x} (p + \lambda + I + D_y \alpha^2 + \hat{n} D_z) \right]^{1/2}$$

and

$$\hat{n} = \frac{n^2 \pi^2}{B^2}$$

Because the solution must be bounded as $x \rightarrow D$ according to (12a), the constant of integration $A = 0$. Boundary condition gives:

$$B = \frac{(2\pi)^{1/2} m}{p} \sigma \exp\left(-\frac{\sigma^2 \alpha^2}{2}\right) \cos\left(\frac{n\pi z'}{B}\right) \quad (15)$$

Substitution of $A = 0$ and B given by (15) into (14) gives, after slight algebra:

$$\bar{\bar{c}} = (2\pi)^{1/2} m \sigma \exp\left(-\frac{\sigma^2 \alpha^2}{2}\right) \cos\left(\frac{n\pi z'}{B}\right) \exp\left(\frac{V_s x}{2D_x}\right) \quad (16)$$

$$\cdot \frac{1}{p} \exp \left[- \left\{ \frac{p + \lambda + I + \frac{V_s^2}{4D_x} + \alpha^2 D_y + \hat{n} D_z}{D_x} \right\}^{1/2} x \right]$$

the subsidiary solution (15) must now be inverted. We will apply the inverse Laplace transform first. Although the function

$$\frac{1}{p} \exp \left[-\frac{1}{2} \left\{ \cdot \right\}^{1/2} x \right]$$

is easily inverted using tabulated results (see Carslaw and Jaeger, 1959, p. 495), the resulting function is complicated and will therefore make application of the inverse Exponential Fourier transform difficult.

Therefore, we will make use of the following relations:

$$L^{-1} \left[\exp \left(-a(p + b)^{1/2} \right) \right] = \frac{a}{2\pi^{1/2} t^{3/2}} \exp \left(-bt - \frac{a^2}{4t} \right) \quad (17a)$$

and

$$L^{-1} \left[\frac{1}{p} f(p) \right] = \int_0^t f(\tau) d\tau \quad (17b)$$

where the shift theorem was made use of in the first inverse. Using the results, the inverse Laplace transform of (16) is

$$\begin{aligned} \bar{c}(x, \alpha, n, t) = & \frac{\max}{(2D_x)^{1/2}} \exp \left(-\frac{\sigma^2 \alpha^2}{2} \right) \cos \left(\frac{n\pi z'}{B} \right) \exp \left(\frac{V_s x}{2D_x} \right) \\ & \cdot \int_0^t \frac{1}{\tau^{3/2}} \exp \left[-\frac{V_s^2}{4D_x} + \alpha^2 D_y + \hat{n} D_z + \lambda + I \right) \tau - \frac{x^2}{4D_x \tau} \right] d\tau \end{aligned} \quad (18)$$

Although the above integral can be evaluated analytically, the resulting expression will make inversion with respect to α extremely difficult.

Thus, the time integral will be retained.

The inverse Exponential Fourier transform will be applied next. To facilitate inversion, (18) can be written in the form:

$$\begin{aligned} \bar{c}(x, \alpha, n, t) = & \frac{\max}{(2D_x)^{1/2}} \exp \left(\frac{V_s x}{2D_x} \right) \cos \left(\frac{n\pi z'}{B} \right) \\ & \cdot \int_0^t \frac{1}{\tau^{3/2}} \exp \left\{ -\alpha^2 \left(D_y \tau + \frac{\sigma^2}{2} \right) - \left(\frac{V_s^2}{4D_x} + \hat{n} D_z + \lambda + I \right) \tau - \frac{x^2}{4D_x \tau} \right\} d\tau \end{aligned} \quad (19)$$

We need the following inverse Exponential Fourier transform
(Churchill, 1979, p. 472)

$$F_e^{-1} \left[\exp \left\{ -\alpha^2 \left(D_y \tau + \frac{\sigma^2}{2} \right) \right\} \right] = \frac{\exp \left[-y^2 / 4 \left(D_y \tau + \frac{\sigma^2}{2} \right) \right]}{2 \left[\pi \left(D_y \tau + \frac{\sigma^2}{2} \right) \right]^{1/2}} \quad (20)$$

Using (20) the inverse of (19) with respect to α is:

$$\begin{aligned} \bar{c}(x, y, n, t) &= \frac{m\sigma x}{2(2\pi D_x)^{1/2}} \exp \left(\frac{V_s x}{2D_x} \right) \cos \left(\frac{n\pi z'}{B} \right) \\ &\cdot \int_0^t \frac{1}{\tau^{3/2} (D_y \tau + \frac{\sigma^2}{2})^{1/2}} \exp \left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4(D_y \tau + \frac{\sigma^2}{2})} - \left(\frac{V_s^2}{4D_x} + \hat{n} D_z + \lambda + I \right) \tau \right) d\tau \end{aligned} \quad (21)$$

Finally, our fundamental solution will be completed after making use of the following inversion formula for the Finite Fourier Cosine transform:

$$c(x, y, z, t) = \bar{c} \left(\frac{x, y, n=0, t}{B} \right) + \frac{2}{B} \sum_{n=1}^{\infty} \bar{c}(x, y, n, t) \cos \left(\frac{n\pi x}{B} \right) \quad (22)$$

Substitution of (21) into (22) gives

$$\begin{aligned} c(x, y, z, t) &= \frac{m\sigma x}{2B(2\pi D_x)^{1/2}} \exp \left(\frac{V_s x}{2D_x} \right) \left[\int_0^t \frac{1}{\tau^{3/2} (D_y \tau + \frac{\sigma^2}{2})^{1/2}} \right. \\ &\exp \left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4(D_y \tau + \frac{\sigma^2}{2})} - \frac{V_s^2 \tau}{4D_x} - \lambda \tau - I \tau \right) d\tau + 2 \sum_{n=1}^{\infty} \cos \left(\frac{n\pi z}{B} \right) \cos \left(\frac{n\pi z'}{B} \right) \\ &\cdot \int_0^t \frac{1}{\tau^{3/2} (D_y \tau + \frac{\sigma^2}{2})^{1/2}} \exp \left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4(D_y \tau + \frac{\sigma^2}{2})} - \frac{V_s^2 \tau}{4D_x} - \frac{n^2 \pi^2 D_z \tau}{B^2} - \lambda \tau - I \tau \right) d\tau \left. \right] \end{aligned} \quad (23)$$

Having obtained the fundamental solution, the remaining step is to use it to obtain the final solution by converting the infinitely thin Gaussian source to the finite Gaussian source of equal strength and extending from $z = H_1$ to $z = H_2$. This is achieved by integrating (23) with respect to z' between the limits H_1 and H_2 and dividing by the source thickness, $H_2 - H_1$. The result is given by

$$\begin{aligned}
c(x, y, z, t) = & \frac{c_0 \sigma x}{2(2\pi D_x)^{1/2}} \exp\left(\frac{V_s x}{2D_x}\right) \left[\frac{H_2 - H_1}{B} \int_0^t \frac{1}{\tau^{3/2} (D_y \tau + \frac{\sigma^2}{2})^{1/2}} \right. \\
& \cdot \exp\left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4(D_y \tau + \frac{\sigma^2}{2})} - \frac{V_s^2 \tau}{4D_x} - \lambda \tau - I \tau\right) d\tau \\
& + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos\left(\frac{n\pi z}{B}\right) \left[\sin\left(\frac{n\pi H_2}{B}\right) - \sin\left(\frac{n\pi H_1}{B}\right) \right] \int_0^t \frac{1}{\tau^{3/2} (D_y \tau + \frac{\sigma^2}{2})^{1/2}} \\
& \cdot \exp\left\{-\frac{x^2}{4D_x \tau} - \frac{y^2}{4(D_y \tau + \frac{\sigma^2}{2})} - \left(\frac{V_s^2}{4D_x} + \frac{n^2 \pi^2 D_z}{B^2} + \lambda + I\right) \tau\right\} d\tau \Bigg]
\end{aligned} \quad (24)$$

Equation (24) may be expressed in a simple form as

$$c_p(x, y, z, t) = \frac{H}{B} c_f(x, y, t) + \Delta c_p(x, y, z, t) \quad (25)$$

where we have assumed that $H_1 = 0$ and $H_2 = H$ (i.e., the source extends from the top of the aquifer through the thickness H), $c_p(x, y, z, t)$ now replaces $c(x, y, z, t)$ and $c_f(x, y, t)$ and $\Delta c_p(x, y, z, t)$ are defined as

$$\begin{aligned}
c_f(x, y, t) = & \xi \int_0^t \frac{1}{\tau^{3/2} (2\sigma^2 + 4D_y \tau)^{1/2}} \\
& \cdot \exp\left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4D_y \tau + 2\sigma^2} - \eta \tau\right) d\tau
\end{aligned} \quad (26)$$

$$\Delta c_p(x, y, z, t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos\left(\frac{n\pi z}{B}\right) \sin\left(\frac{n\pi H}{B}\right) \quad (27)$$

$$\cdot \xi \int_0^t \frac{1}{\tau^{3/2} (2\sigma^2 + 4D_y \tau)^{1/2}} \cdot \exp\left(-\frac{x^2}{4D_x \tau} - \frac{y^2}{4D_y \tau + 2\sigma^2} - \beta_n \tau\right) d\tau$$

in which

$$\xi = \frac{c_0 \sigma x}{(2\pi D_x)^{1/2}} \exp\left(\frac{V_s x}{2D_x}\right) \quad (28)$$

$$\eta = \frac{V_s^2}{4D_x} + \lambda + I \quad (29)$$

$$\beta_n = n + \frac{n^2 \pi^2 D_z}{B^2} \quad (30)$$

It should be noted that $c_f(x, y, t)$ is identical to the two-dimensional analytical solution for the case where the Gaussian source fully penetrates the entire aquifer thickness.

According to equation (25), the analytical solution for the case of a partially-penetrating Gaussian source consists of two terms. The first term is the product of penetration ratio H/B , and the solution for the corresponding fully-penetrating case, c_f . The second term is Δc_p , which may be interpreted as a correction term that accounts for the effect of partial penetration.

STEADY-STATE ANALYTICAL SOLUTION

A steady-state condition may be approached by letting t approach infinity. Under this condition, the analytical solution given in (25) may be expressed as

$$c_p^*(x, y, z) = \frac{H}{B} c_f^*(x, y) + \Delta c_p^*(x, y, z) \quad (31)$$

where the asterisk superscript is used to denote the steady-state condition, and $c_f^*(x, y)$ and $\Delta c_p^*(x, y, z)$ are obtained from (26) and (27) with the upper limit of the time integral set to infinity.

An alternative steady-state analytical solution can also be obtained by direct solution of the steady-state advective-dispersive transport equation (equation (1) without the time-derivative term), subjected to the boundary conditions given by (2b)-(2g). This solution can be expressed in a concise form using equation (31) but with the function $c_f^*(x, y)$ and $\Delta c_p^*(x, y, z)$ defined as

$$c_f^*(x, y) = \xi' \int_{-\infty}^{\infty} \frac{\exp(-y'^2/2\sigma^2)}{\left\{ x^2 + (y'-y)^2 D_x/D_y \right\}^{\frac{1}{2}}} K_1 \left\{ \left[\frac{nx^2}{D_x} + \frac{n(y'-y)^2}{D_y} \right]^{\frac{1}{2}} \right\} dy' \quad (32)$$

$$\Delta c_p^*(x, y, z) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi H}{B}\right) \cos\left(\frac{n\pi z}{B}\right) \cdot \xi' \int_0^{\infty} \frac{\exp(-y'/2\sigma^2)}{\left\{x^2 + (y'-y)^2 D_x/D_y\right\}^{\frac{1}{2}}} \cdot K_1 \left\{ \left[\frac{\beta_n x^2}{D_x} + \frac{\beta_n (y'-y)^2}{D_y} \right]^{\frac{1}{2}} \right\} dy' \quad (33)$$

in which y' is a dummy variable of integration, ξ' is a constant defined as

$$\xi' = \frac{x c_0}{\pi} \sqrt{\frac{\beta_n}{D_y}} \exp\left(\frac{V_s x}{2D_x}\right) \quad (34)$$

n and β_n are given by equations (29) and (30), and $K_1(\cdot)$ is the modified Bessel function of the second kind and of first order.

In a special case where one is interested in obtaining a steady-state concentration distribution along the x -axis, equation (31) becomes

$$c_p^*(x, 0, 0) = \frac{H}{B} c_f^*(x, 0) + \Delta c_p^*(x, 0, 0) \quad (35)$$

where $c_f^*(x, 0)$ and $\Delta c_p^*(x, 0, 0)$ are obtained by reducing equations (32) and (33) to the following:

$$c_f^*(x, 0) = \xi^* \int_0^{\infty} \exp \left\{ - \left[\frac{\sigma^2 \xi^2}{2} + x \sqrt{\frac{\xi^2 D_y}{D_x} + \frac{n}{D_x}} \right] \right\} d\xi \quad (36)$$

$$\Delta c_p^*(x, 0, 0) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi H}{B}\right) \cos\left(\frac{n\pi z}{B}\right)$$

$$\cdot \xi^* \int_0^{\infty} \exp \left\{ - \left[\frac{\sigma^2 \xi^2}{2} + x \sqrt{\frac{\xi^2 D_y}{D_x} + \frac{\beta_n}{D_x}} \right] \right\} d\xi \quad (37)$$

with ξ^* defined as

$$\xi^* = \frac{2c_0\sigma}{\sqrt{2}\pi} \exp \left\{ \frac{V_s x}{2D_x} \right\} \quad (38)$$

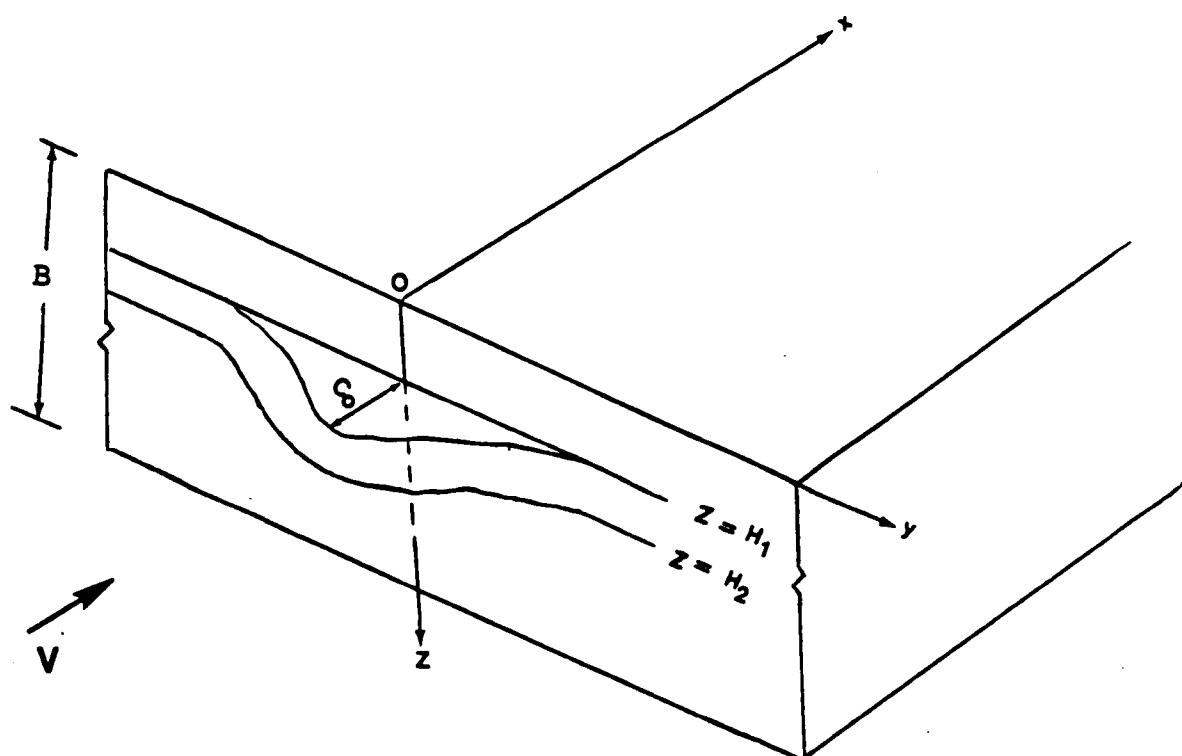


Figure 1.1. Schematic picture of the solution region with a contaminant source having concentration distribution that is Gaussian in the lateral direction and uniform between $H_1 \leq z \leq H_2$ in the vertical direction.

APPENDIX B
TABLES OF DIMENSIONLESS CONCENTRATION c_{fD}^* VERSUS x_D

TABLE 1: XD VERSUS CD FOR SIGD = 0.500E 01

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
0.100E 01	0.991E 00	0.944E 00	0.893E 00	0.819E 00	0.756E 00
0.150E 01	0.972E 00	0.935E 00	0.884E 00	0.809E 00	0.741E 00
0.200E 01	0.963E 00	0.926E 00	0.875E 00	0.798E 00	0.731E 00
0.300E 01	0.946E 00	0.910E 00	0.859E 00	0.782E 00	0.714E 00
0.400E 01	0.930E 00	0.894E 00	0.844E 00	0.767E 00	0.699E 00
0.500E 01	0.915E 00	0.879E 00	0.830E 00	0.754E 00	0.686E 00
0.600E 01	0.900E 00	0.865E 00	0.816E 00	0.741E 00	0.674E 00
0.700E 01	0.886E 00	0.852E 00	0.803E 00	0.730E 00	0.663E 00
0.800E 01	0.873E 00	0.839E 00	0.791E 00	0.718E 00	0.653E 00
0.900E 01	0.860E 00	0.827E 00	0.780E 00	0.707E 00	0.643E 00
0.100E 02	0.848E 00	0.815E 00	0.768E 00	0.697E 00	0.633E 00
0.150E 02	0.793E 00	0.763E 00	0.719E 00	0.652E 00	0.591E 00
0.200E 02	0.748E 00	0.719E 00	0.678E 00	0.614E 00	0.557E 00
0.250E 02	0.710E 00	0.682E 00	0.643E 00	0.583E 00	0.528E 00
0.300E 02	0.677E 00	0.650E 00	0.613E 00	0.555E 00	0.503E 00
0.400E 02	0.622E 00	0.598E 00	0.564E 00	0.511E 00	0.463E 00
0.500E 02	0.579E 00	0.557E 00	0.525E 00	0.475E 00	0.430E 00
0.600E 02	0.544E 00	0.523E 00	0.493E 00	0.446E 00	0.404E 00
0.700E 02	0.514E 00	0.494E 00	0.466E 00	0.422E 00	0.392E 00
0.800E 02	0.489E 00	0.470E 00	0.443E 00	0.401E 00	0.363E 00
0.900E 02	0.467E 00	0.449E 00	0.423E 00	0.383E 00	0.347E 00
0.100E 03	0.449E 00	0.431E 00	0.406E 00	0.367E 00	0.333E 00
0.150E 03	0.379E 00	0.364E 00	0.343E 00	0.310E 00	0.281E 00
0.200E 03	0.334E 00	0.321E 00	0.302E 00	0.273E 00	0.248E 00
0.300E 03	0.278E 00	0.267E 00	0.251E 00	0.227E 00	0.206E 00
0.400E 03	0.243E 00	0.233E 00	0.220E 00	0.199E 00	0.180E 00
0.500E 03	0.218E 00	0.210E 00	0.198E 00	0.179E 00	0.162E 00
0.600E 03	0.200E 00	0.192E 00	0.181E 00	0.164E 00	0.148E 00
0.700E 03	0.186E 00	0.179E 00	0.168E 00	0.152E 00	0.138E 00
0.800E 03	0.174E 00	0.167E 00	0.158E 00	0.143E 00	0.129E 00
0.900E 03	0.164E 00	0.158E 00	0.149E 00	0.135E 00	0.122E 00
0.100E 04	0.156E 00	0.150E 00	0.141E 00	0.129E 00	0.116E 00

TABLE 2: XD VERSUS CD FOR SIGD = 0.500E 01

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
J.100E 01	0.700E 00	0.609E 00	0.535E 00	0.476E 00	0.425E 00
0.150E 01	0.683E 00	0.586E 00	0.508E 00	0.445E 00	0.392E 00
J.20JE 01	0.671E 00	0.571E 00	0.491E 00	0.425E 00	0.371E 00
0.300E 01	0.653E 00	0.550E 00	0.468E 00	0.400E 00	0.344E 00
J.400E 01	0.638E 00	0.535E 00	0.451E 00	0.383E 00	0.327E 00
0.500E 01	0.626E 00	0.523E 00	0.439E 00	0.370E 00	0.314E 00
J.60JE 01	0.614E 00	0.512E 00	0.428E 00	0.360E 00	0.304E 00
J.70JE 01	0.604E 00	0.502E 00	0.419E 00	0.351E 00	0.296E 00
J.800E 01	0.594E 00	0.493E 00	0.411E 00	0.344E 00	0.286E 00
J.90JE 01	0.594E 00	0.485E 00	0.403E 00	0.337E 00	0.282E 00
0.100E 02	0.576E 00	0.477E 00	0.396E 00	0.330E 00	0.276E 00
0.150E 02	0.537E 00	0.443E 00	0.367E 00	0.304E 00	0.253E 00
0.200E 02	0.506E 00	0.417E 00	0.344E 00	0.285E 00	0.236E 00
0.250E 02	0.479E 00	0.394E 00	0.325E 00	0.269E 00	0.222E 00
0.300E 02	0.456E 00	0.375E 00	0.309E 00	0.255E 00	0.211E 00
0.400E 02	0.419E 00	0.345E 00	0.284E 00	0.233E 00	0.192E 00
0.500E 02	0.390E 00	0.320E 00	0.263E 00	0.217E 00	0.178E 00
0.600E 02	0.366E 00	0.300E 00	0.247E 00	0.203E 00	0.167E 00
J.700E 02	0.346E 00	0.294E 00	0.233E 00	0.192E 00	0.157E 00
J.800E 02	0.329E 00	0.270E 00	0.222E 00	0.182E 00	0.150E 00
J.900E 02	0.314E 00	0.258E 00	0.211E 00	0.174E 00	0.143E 00
J.100E 03	0.301E 00	0.247E 00	0.203E 00	0.166E 00	0.137E 00
J.150E 03	0.254E 00	0.208E 00	0.171E 00	0.140E 00	0.115E 00
J.20JE 03	0.224E 00	0.184E 00	0.151E 00	0.123E 00	0.101E 00
J.30JE 03	0.186E 00	0.153E 00	0.125E 00	0.102E 00	0.840E-01
J.40JE 03	0.163E 00	0.133E 00	0.109E 00	0.895E-01	0.733E-01
0.50JE 03	0.146E 00	0.120E 00	0.983E-01	0.803E-01	0.659E-01
J.600E 03	0.134E 00	0.110E 00	0.900E-01	0.737E-01	0.604E-01
0.70JE 03	0.123E 00	0.102E 00	0.836E-01	0.684E-01	0.561E-01
0.800E 03	0.117E 00	0.956E-01	0.783E-01	0.641E-01	0.525E-01
0.900E 03	0.110E 00	0.903E-01	0.740E-01	0.606E-01	0.496E-01
0.100E 04	0.105E 00	0.858E-01	0.702E-01	0.575E-01	0.471E-01

TABLE 3: XD VERSUS CD FOR SIGD = 0.500E 01

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
0.100E 01	0.383E 00	0.347E 00	0.315E 00	0.288E 00	0.233E 00
0.150E 01	0.348E 00	0.310E 00	0.278E 00	0.250E 00	0.195E 00
0.200E 01	0.325E 00	0.287E 00	0.254E 00	0.226E 00	0.172E 00
0.300E 01	0.298E 00	0.259E 00	0.226E 00	0.198E 00	0.145E 00
0.400E 01	0.280E 00	0.241E 00	0.208E 00	0.181E 00	0.129E 00
0.500E 01	0.268E 00	0.229E 00	0.196E 00	0.169E 00	0.118E 00
0.600E 01	0.258E 00	0.219E 00	0.187E 00	0.160E 00	0.110E 00
0.700E 01	0.250E 00	0.212E 00	0.180E 00	0.153E 00	0.104E 00
0.800E 01	0.243E 00	0.205E 00	0.174E 00	0.148E 00	0.994E-01
0.900E 01	0.237E 00	0.200E 00	0.169E 00	0.143E 00	0.954E-01
0.100E 02	0.232E 00	0.195E 00	0.164E 00	0.139E 00	0.919E-01
0.150E 02	0.211E 00	0.176E 00	0.147E 00	0.123E 00	0.799E-01
0.200E 02	0.196E 00	0.163E 00	0.136E 00	0.113E 00	0.722E-01
0.250E 02	0.184E 00	0.153E 00	0.127E 00	0.105E 00	0.667E-01
0.300E 02	0.174E 00	0.144E 00	0.119E 00	0.991E-01	0.624E-01
0.400E 02	0.159E 00	0.131E 00	0.108E 00	0.996E-01	0.560E-01
0.500E 02	0.147E 00	0.121E 00	0.999E-01	0.925E-01	0.513E-01
0.600E 02	0.137E 00	0.113E 00	0.932E-01	0.769E-01	0.476E-01
0.700E 02	0.130E 00	0.107E 00	0.878E-01	0.723E-01	0.446E-01
0.800E 02	0.123E 00	0.101E 00	0.832E-01	0.685E-01	0.422E-01
0.900E 02	0.117E 00	0.964E-01	0.792E-01	0.652E-01	0.401E-01
0.100E 03	0.112E 00	0.922E-01	0.758E-01	0.624E-01	0.383E-01
0.150E 03	0.944E-01	0.774E-01	0.636E-01	0.522E-01	0.319E-01
0.200E 03	0.830E-01	0.681E-01	0.558E-01	0.458E-01	0.280E-01
0.300E 03	0.688E-01	0.564E-01	0.463E-01	0.379E-01	0.231E-01
0.400E 03	0.601E-01	0.493E-01	0.404E-01	0.331E-01	0.201E-01
0.500E 03	0.540E-01	0.443E-01	0.363E-01	0.297E-01	0.181E-01
0.600E 03	0.495E-01	0.405E-01	0.332E-01	0.272E-01	0.165E-01
0.700E 03	0.459E-01	0.376E-01	0.308E-01	0.253E-01	0.153E-01
0.800E 03	0.430E-01	0.353E-01	0.289E-01	0.237E-01	0.144E-01
0.900E 03	0.406E-01	0.333E-01	0.273E-01	0.223E-01	0.136E-01
0.100E 04	0.386E-01	0.316E-01	0.259E-01	0.212E-01	0.129E-01

TABLE 4: XD VERSUS CD FOR SIGD = 0.500E 01

XD-VALUE	LAMD= 0.300E 01 CD-VALUE	LAMD= 0.400E 01 CD-VALUE	LAMD= 0.600E 01 CD-VALUE	LAMD= 0.800E 01 CD-VALUE	LAMD= 0.100E 02 CD-VALUE
0.100E 01	0.191E 00	0.134E 00	0.735E-01	0.438E-01	0.277E-01
0.150E 01	0.155E 00	0.102E 00	0.493E-01	0.265E-01	0.152E-01
0.200E 01	0.133E 00	0.829E-01	0.367E-01	0.181E-01	0.961E-02
0.300E 01	0.108E 00	0.624E-01	0.239E-01	0.103E-01	0.485E-02
0.400E 01	0.933E-01	0.512E-01	0.176E-01	0.688E-02	0.294E-02
0.500E 01	0.838E-01	0.441E-01	0.140E-01	0.503E-02	0.198E-02
0.600E 01	0.771E-01	0.393E-01	0.116E-01	0.390E-02	0.144E-02
0.700E 01	0.720E-01	0.357E-01	0.995E-02	0.315E-02	0.110E-02
0.800E 01	0.679E-01	0.329E-01	0.873E-02	0.263E-02	0.872E-03
0.900E 01	0.645E-01	0.307E-01	0.781E-02	0.225E-02	0.713E-03
0.100E 02	0.617E-01	0.289E-01	0.708E-02	0.196E-02	0.597E-03
0.150E 02	0.523E-01	0.231E-01	0.495E-02	0.119E-02	0.311E-03
0.200E 02	0.466E-01	0.198E-01	0.392E-02	0.855E-03	0.202E-03
0.250E 02	0.426E-01	0.177E-01	0.331E-02	0.674E-03	0.148E-03
0.300E 02	0.396E-01	0.162E-01	0.290E-02	0.562E-03	0.117E-03
0.400E 02	0.351E-01	0.141E-01	0.238E-02	0.429E-03	0.823E-04
0.500E 02	0.320E-01	0.126E-01	0.205E-02	0.354E-03	0.642E-04
0.600E 02	0.296E-01	0.115E-01	0.183E-02	0.305E-03	0.532E-04
0.700E 02	0.277E-01	0.107E-01	0.166E-02	0.270E-03	0.458E-04
0.800E 02	0.261E-01	0.100E-01	0.154E-02	0.244E-03	0.404E-04
0.900E 02	0.247E-01	0.948E-02	0.143E-02	0.224E-03	0.364E-04
0.100E 03	0.236E-01	0.900E-02	0.135E-02	0.208E-03	0.333E-04
0.150E 03	0.196E-01	0.739E-02	0.107E-02	0.159E-03	0.241E-04
0.200E 03	0.171E-01	0.641E-02	0.915E-03	0.133E-03	0.196E-04
0.300E 03	0.141E-01	0.525E-02	0.737E-03	0.105E-03	0.150E-04
0.400E 03	0.123E-01	0.456E-02	0.633E-03	0.889E-04	0.126E-04
0.500E 03	0.110E-01	0.408E-02	0.564E-03	0.786E-04	0.110E-04
0.600E 03	0.101E-01	0.373E-02	0.513E-03	0.712E-04	0.994E-05
0.700E 03	0.933E-02	0.345E-02	0.474E-03	0.656E-04	0.911E-05
0.800E 03	0.874E-02	0.323E-02	0.443E-03	0.611E-04	0.846E-05
0.900E 03	0.824E-02	0.305E-02	0.417E-03	0.574E-04	0.793E-05
0.100E 04	0.782E-02	0.289E-02	0.395E-03	0.543E-04	0.749E-05

TABLE 5: XD VERSUS CD FOR SIGD = 0.100E 02

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
J.100E 01	0.995E 00	0.957E 00	0.905E 00	0.829E 00	0.764E 00
U.150E 01	0.993E 00	0.954E 00	0.901E 00	0.823E 00	0.755E 00
U.200E 01	0.990E 00	0.952E 00	0.899E 00	0.819E 00	0.749E 00
U.300E 01	0.985E 00	0.947E 00	0.893E 00	0.813E 00	0.741E 00
U.400E 01	0.981E 00	0.943E 00	0.889E 00	0.807E 00	0.735E 00
U.500E 01	0.976E 00	0.938E 00	0.884E 00	0.803E 00	0.730E 00
U.600E 01	0.971E 00	0.934E 00	0.880E 00	0.799E 00	0.726E 00
U.700E 01	0.967E 00	0.929E 00	0.876E 00	0.795E 00	0.722E 00
J.900E 01	0.962E 00	0.925E 00	0.872E 00	0.791E 00	0.718E 00
U.700E 01	0.958E 00	0.921E 00	0.868E 00	0.787E 00	0.714E 00
J.100E 02	0.954E 00	0.917E 00	0.864E 00	0.783E 00	0.711E 00
J.150E 02	0.933E 00	0.897E 00	0.845E 00	0.765E 00	0.694E 00
U.200E 02	0.913E 00	0.878E 00	0.827E 00	0.749E 00	0.679E 00
U.250E 02	0.895E 00	0.860E 00	0.810E 00	0.734E 00	0.665E 00
U.300E 02	0.878E 00	0.843E 00	0.795E 00	0.720E 00	0.652E 00
U.400E 02	0.846E 00	0.813E 00	0.766E 00	0.693E 00	0.624E 00
U.500E 02	0.817E 00	0.785E 00	0.740E 00	0.670E 00	0.607E 00
J.600E 02	0.791E 00	0.760E 00	0.716E 00	0.648E 00	0.587E 00
U.700E 02	0.768E 00	0.738E 00	0.695E 00	0.629E 00	0.570E 00
U.800E 02	0.746E 00	0.717E 00	0.675E 00	0.611E 00	0.553E 00
J.900E 02	0.726E 00	0.698E 00	0.657E 00	0.595E 00	0.539E 00
U.100E 03	0.708E 00	0.680E 00	0.641E 00	0.580E 00	0.525E 00
U.150E 03	0.683E 00	0.608E 00	0.573E 00	0.519E 00	0.469E 00
U.200E 03	0.578E 00	0.555E 00	0.523E 00	0.473E 00	0.428E 00
U.300E 03	0.500E 00	0.481E 00	0.453E 00	0.410E 00	0.371E 00
U.400E 03	0.447E 00	0.430E 00	0.405E 00	0.366E 00	0.332E 00
U.500E 03	0.408E 00	0.392E 00	0.370E 00	0.334E 00	0.303E 00
U.500E 03	0.378E 00	0.363E 00	0.342E 00	0.310E 00	0.290E 00
J.700E 03	0.354E 00	0.340E 00	0.320E 00	0.290E 00	0.262E 00
U.800E 03	0.335E 00	0.320E 00	0.302E 00	0.273E 00	0.247E 00
J.900E 03	0.316E 00	0.304E 00	0.286E 00	0.259E 00	0.234E 00
J.100E 04	0.302E 00	0.290E 00	0.273E 00	0.247E 00	0.223E 00

TABLE 6: XD VERSUS CD FOR SIGD = 0.100E 02

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
U.100E 01	0.708E 00	0.615E 00	0.540E 00	0.480E 00	0.429E 00
U.150E 01	0.695E 00	0.596E 00	0.516E 00	0.451E 00	0.397E 00
J.20JE 01	0.687E 00	0.584E 00	0.501E 00	0.434E 00	0.378E 00
U.300E 01	0.677E 00	0.570E 00	0.485E 00	0.413E 00	0.355E 00
J.400E 01	0.671E 00	0.561E 00	0.473E 00	0.401E 00	0.341E 00
U.500E 01	0.665E 00	0.555E 00	0.465E 00	0.392E 00	0.332E 00
U.600E 01	0.661E 00	0.549E 00	0.459E 00	0.385E 00	0.325E 00
0.700E 01	0.656E 00	0.545E 00	0.454E 00	0.380E 00	0.320E 00
0.800E 01	0.652E 00	0.541E 00	0.450E 00	0.376E 00	0.315E 00
J.90JE 01	0.649E 00	0.537E 00	0.446E 00	0.372E 00	0.311E 00
U.100E 02	0.645E 00	0.534E 00	0.443E 00	0.369E 00	0.308E 00
J.150E 02	0.630E 00	0.519E 00	0.429E 00	0.355E 00	0.295E 00
U.200E 02	0.616E 00	0.507E 00	0.418E 00	0.345E 00	0.286E 00
J.250E 02	0.603E 00	0.496E 00	0.408E 00	0.337E 00	0.278E 00
0.300E 02	0.591E 00	0.486E 00	0.400E 00	0.329E 00	0.272E 00
J.400E 02	0.569E 00	0.467E 00	0.384E 00	0.316E 00	0.260E 00
J.500E 02	0.549E 00	0.451E 00	0.370E 00	0.305E 00	0.251E 00
0.500E 02	0.532E 00	0.436E 00	0.358E 00	0.294E 00	0.242E 00
J.700E 02	0.516E 00	0.423E 00	0.347E 00	0.285E 00	0.234E 00
0.80JE 02	0.501E 00	0.411E 00	0.337E 00	0.277E 00	0.227E 00
J.900E 02	0.488E 00	0.400E 00	0.328E 00	0.269E 00	0.221E 00
0.100E 03	0.475E 00	0.390E 00	0.320E 00	0.262E 00	0.215E 00
0.150E 03	0.425E 00	0.348E 00	0.285E 00	0.234E 00	0.192E 00
J.20JE 03	0.388E 00	0.318E 00	0.260E 00	0.213E 00	0.175E 00
0.30JE 03	0.336E 00	0.275E 00	0.225E 00	0.185E 00	0.151E 00
J.400E 03	0.300E 00	0.246E 00	0.201E 00	0.165E 00	0.135E 00
0.50JE 03	0.274E 00	0.224E 00	0.194E 00	0.151E 00	0.123E 00
J.50JE 03	0.254E 00	0.208E 00	0.170E 00	0.139E 00	0.114E 00
U.70JE 03	0.237E 00	0.194E 00	0.159E 00	0.130E 00	0.107E 00
U.80JE 03	0.224E 00	0.183E 00	0.150E 00	0.123E 00	0.101E 00
J.90JE 03	0.212E 00	0.174E 00	0.142E 00	0.116E 00	0.954E-01
0.100E 04	0.202E 00	0.166E 00	0.136E 00	0.111E 00	0.910E-01

TABLE 7: XD VERSUS CD FOR SIGD = 0.100E 02

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
0.100E 01	0.386E 00	0.349E 00	0.318E 00	0.290E 00	0.234E 00
0.150E 01	0.352E 00	0.314E 00	0.281E 00	0.253E 00	0.197E 00
0.200E 01	0.331E 00	0.292E 00	0.259E 00	0.230E 00	0.174E 00
0.300E 01	0.307E 00	0.266E 00	0.232E 00	0.203E 00	0.148E 00
0.400E 01	0.292E 00	0.251E 00	0.217E 00	0.183E 00	0.134E 00
0.500E 01	0.282E 00	0.241E 00	0.207E 00	0.178E 00	0.124E 00
0.600E 01	0.275E 00	0.234E 00	0.199E 00	0.171E 00	0.117E 00
0.700E 01	0.270E 00	0.229E 00	0.194E 00	0.163E 00	0.112E 00
0.800E 01	0.263E 00	0.224E 00	0.189E 00	0.161E 00	0.108E 00
0.900E 01	0.261E 00	0.220E 00	0.185E 00	0.157E 00	0.104E 00
0.100E 02	0.258E 00	0.216E 00	0.182E 00	0.154E 00	0.102E 00
0.150E 02	0.246E 00	0.205E 00	0.171E 00	0.143E 00	0.924E-01
0.200E 02	0.237E 00	0.197E 00	0.164E 00	0.136E 00	0.869E-01
0.250E 02	0.230E 00	0.191E 00	0.158E 00	0.131E 00	0.830E-01
0.300E 02	0.224E 00	0.186E 00	0.154E 00	0.127E 00	0.800E-01
0.400E 02	0.215E 00	0.177E 00	0.146E 00	0.121E 00	0.753E-01
0.500E 02	0.206E 00	0.170E 00	0.140E 00	0.116E 00	0.717E-01
0.600E 02	0.199E 00	0.164E 00	0.135E 00	0.111E 00	0.688E-01
0.700E 02	0.193E 00	0.158E 00	0.130E 00	0.107E 00	0.662E-01
0.800E 02	0.187E 00	0.154E 00	0.126E 00	0.104E 00	0.640E-01
0.900E 02	0.182E 00	0.149E 00	0.123E 00	0.101E 00	0.620E-01
0.100E 03	0.177E 00	0.145E 00	0.119E 00	0.982E-01	0.603E-01
0.150E 03	0.158E 00	0.129E 00	0.106E 00	0.871E-01	0.533E-01
0.200E 03	0.144E 00	0.118E 00	0.966E-01	0.792E-01	0.484E-01
0.300E 03	0.124E 00	0.102E 00	0.833E-01	0.683E-01	0.416E-01
0.400E 03	0.111E 00	0.908E-01	0.744E-01	0.610E-01	0.371E-01
0.500E 03	0.101E 00	0.829E-01	0.678E-01	0.556E-01	0.338E-01
0.600E 03	0.935E-01	0.766E-01	0.628E-01	0.514E-01	0.313E-01
0.700E 03	0.874E-01	0.716E-01	0.587E-01	0.481E-01	0.292E-01
0.800E 03	0.824E-01	0.675E-01	0.553E-01	0.453E-01	0.275E-01
0.900E 03	0.781E-01	0.640E-01	0.524E-01	0.429E-01	0.251E-01
0.100E 04	0.745E-01	0.610E-01	0.500E-01	0.409E-01	0.249E-01

TABLE 8: XD VERSUS CD FOR SIGD = 0.100E 02

XD-VALUE	LAMD= 0.300E 01 CD-VALUE	LAMD= 0.400E 01 CD-VALUE	LAMD= 0.600E 01 CD-VALUE	LAMD= 0.800E 01 CD-VALUE	LAMD= 0.100E 02 CD-VALUE
0.100E 01	0.193E 00	0.135E 00	0.738E-01	0.440E-01	U.278E-01
0.150E 01	0.156E 00	0.103E 00	0.497E-01	0.266E-01	U.153E-01
0.200E 01	0.135E 00	0.940E-01	0.371E-01	0.183E-01	U.959E-02
0.300E 01	0.110E 00	0.638E-01	0.244E-01	0.105E-01	U.493E-02
0.400E 01	0.966E-01	0.529E-01	0.181E-01	0.706E-02	U.301E-02
0.500E 01	0.878E-01	0.461E-01	0.145E-01	0.520E-02	0.205E-02
0.600E 01	0.816E-01	0.414E-01	0.122E-01	0.407E-02	U.150E-02
0.700E 01	0.770E-01	0.380E-01	0.105E-01	0.332E-02	U.115E-02
0.800E 01	0.734E-01	0.354E-01	0.934E-02	0.280E-02	U.923E-03
0.900E 01	0.705E-01	0.333E-01	0.843E-02	0.241E-02	U.762E-03
0.100E 02	0.681E-01	0.317E-01	0.771E-02	0.212E-02	U.644E-03
0.150E 02	0.603E-01	0.265E-01	0.564E-02	0.134E-02	U.349E-03
0.200E 02	0.559E-01	0.237E-01	0.465E-02	0.101E-02	U.236E-03
0.250E 02	0.529E-01	0.219E-01	0.406E-02	0.921E-03	U.179E-03
0.300E 02	0.506E-01	0.206E-01	0.366E-02	0.70E-03	U.146E-03
0.400E 02	0.472E-01	0.185E-01	0.316E-02	0.568E-03	U.108E-03
0.500E 02	0.447E-01	0.176E-01	0.294E-02	0.498E-03	U.981E-04
0.600E 02	0.427E-01	0.166E-01	0.262E-02	0.435E-03	U.756E-04
0.700E 02	0.410E-01	0.158E-01	0.245E-02	0.397E-03	U.669E-04
0.800E 02	0.395E-01	0.152E-01	0.232E-02	0.367E-03	U.606E-04
0.900E 02	0.382E-01	0.146E-01	0.220E-02	0.344E-03	U.557E-04
0.100E 03	0.371E-01	0.141E-01	0.211E-02	0.325E-03	U.518E-04
0.150E 03	0.326E-01	0.123E-01	0.178E-02	0.264E-03	U.400E-04
0.200E 03	0.296E-01	0.111E-01	0.158E-02	0.229E-03	U.338E-04
0.300E 03	0.254E-01	0.945E-02	0.133E-02	0.189E-03	U.270E-04
0.400E 03	0.226E-01	0.839E-02	0.117E-02	0.164E-03	U.232E-04
0.500E 03	0.206E-01	0.763E-02	0.105E-02	0.147E-03	U.206E-04
0.600E 03	0.190E-01	0.704E-02	0.970E-03	0.134E-03	U.198E-04
0.700E 03	0.178E-01	0.657E-02	0.903E-03	0.125E-03	U.173E-04
0.800E 03	0.167E-01	0.618E-02	0.848E-03	0.117E-03	U.162E-04
0.900E 03	0.159E-01	0.586E-02	0.802E-03	0.110E-03	U.152E-04
0.100E 04	0.151E-01	0.558E-02	0.763E-03	0.105E-03	U.145E-04

TABLE 9: XD VERSUS CD FOR SIGD = 0.150E 02

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
0.100E 01	0.998E 00	0.959E 00	0.907E 00	0.831E 00	0.766E 00
0.150E 01	0.997E 00	0.958E 00	0.905E 00	0.826E 00	0.758E 00
0.200E 01	0.996E 00	0.957E 00	0.903E 00	0.823E 00	0.753E 00
0.300E 01	0.993E 00	0.955E 00	0.901E 00	0.819E 00	0.746E 00
0.400E 01	0.991E 00	0.953E 00	0.898E 00	0.816E 00	0.743E 00
0.500E 01	0.989E 00	0.951E 00	0.896E 00	0.813E 00	0.739E 00
0.600E 01	0.987E 00	0.948E 00	0.894E 00	0.811E 00	0.737E 00
0.700E 01	0.985E 00	0.946E 00	0.892E 00	0.809E 00	0.735E 00
0.800E 01	0.983E 00	0.944E 00	0.890E 00	0.807E 00	0.732E 00
0.900E 01	0.981E 00	0.942E 00	0.888E 00	0.805E 00	0.730E 00
0.100E 02	0.979E 00	0.940E 00	0.886E 00	0.803E 00	0.729E 00
0.150E 02	0.968E 00	0.930E 00	0.877E 00	0.794E 00	0.720E 00
0.200E 02	0.958E 00	0.921E 00	0.868E 00	0.786E 00	0.712E 00
0.250E 02	0.949E 00	0.912E 00	0.859E 00	0.778E 00	0.705E 00
0.300E 02	0.939E 00	0.903E 00	0.850E 00	0.770E 00	0.697E 00
0.400E 02	0.922E 00	0.886E 00	0.834E 00	0.755E 00	0.684E 00
0.500E 02	0.905E 00	0.869E 00	0.819E 00	0.741E 00	0.671E 00
0.600E 02	0.889E 00	0.854E 00	0.804E 00	0.728E 00	0.659E 00
0.700E 02	0.874E 00	0.839E 00	0.791E 00	0.716E 00	0.648E 00
0.800E 02	0.859E 00	0.826E 00	0.778E 00	0.704E 00	0.637E 00
0.900E 02	0.845E 00	0.812E 00	0.765E 00	0.693E 00	0.627E 00
0.100E 03	0.832E 00	0.800E 00	0.753E 00	0.682E 00	0.617E 00
0.150E 03	0.775E 00	0.745E 00	0.701E 00	0.635E 00	0.574E 00
0.200E 03	0.728E 00	0.699E 00	0.659E 00	0.596E 00	0.540E 00
0.300E 03	0.655E 00	0.629E 00	0.593E 00	0.536E 00	0.485E 00
0.400E 03	0.600E 00	0.577E 00	0.543E 00	0.492E 00	0.445E 00
0.500E 03	0.557E 00	0.535E 00	0.504E 00	0.456E 00	0.413E 00
0.600E 03	0.522E 00	0.502E 00	0.473E 00	0.429E 00	0.387E 00
0.700E 03	0.493E 00	0.474E 00	0.446E 00	0.404E 00	0.366E 00
0.800E 03	0.469E 00	0.450E 00	0.424E 00	0.384E 00	0.347E 00
0.900E 03	0.447E 00	0.430E 00	0.405E 00	0.366E 00	0.331E 00
0.100E 04	0.429E 00	0.412E 00	0.388E 00	0.351E 00	0.318E 00

TABLE 10: XD VERSUS CD FOR SIGD = 0.150E 02

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
J.100E 01	0.709E 00	0.616E 00	0.541E 00	0.480E 00	0.430E 00
J.150E 01	0.698E 00	0.598E 00	0.518E 00	0.452E 00	0.398E 00
J.200E 01	0.691E 00	0.587E 00	0.503E 00	0.435E 00	0.379E 00
J.300E 01	0.682E 00	0.574E 00	0.487E 00	0.416E 00	0.357E 00
J.400E 01	0.677E 00	0.566E 00	0.477E 00	0.404E 00	0.344E 00
J.500E 01	0.673E 00	0.561E 00	0.470E 00	0.396E 00	0.336E 00
U.500E 01	0.670E 00	0.557E 00	0.466E 00	0.391E 00	0.329E 00
U.700E 01	0.668E 00	0.554E 00	0.462E 00	0.386E 00	0.325E 00
0.900E 01	0.666E 00	0.551E 00	0.459E 00	0.383E 00	0.321E 00
J.700E 01	0.663E 00	0.549E 00	0.456E 00	0.380E 00	0.318E 00
0.100E 02	0.662E 00	0.547E 00	0.454E 00	0.377E 00	0.315E 00
J.150E 02	0.653E 00	0.538E 00	0.445E 00	0.369E 00	0.306E 00
U.200E 02	0.645E 00	0.531E 00	0.438E 00	0.362E 00	0.299E 00
U.250E 02	0.639E 00	0.525E 00	0.432E 00	0.357E 00	0.294E 00
0.300E 02	0.632E 00	0.519E 00	0.427E 00	0.352E 00	0.290E 00
J.400E 02	0.619E 00	0.509E 00	0.418E 00	0.344E 00	0.283E 00
0.500E 02	0.608E 00	0.499E 00	0.410E 00	0.337E 00	0.277E 00
U.500E 02	0.597E 00	0.490E 00	0.402E 00	0.330E 00	0.271E 00
J.700E 02	0.587E 00	0.481E 00	0.395E 00	0.324E 00	0.266E 00
U.800E 02	0.577E 00	0.473E 00	0.388E 00	0.319E 00	0.262E 00
J.700E 02	0.568E 00	0.465E 00	0.382E 00	0.313E 00	0.257E 00
J.100E 03	0.559E 00	0.458E 00	0.376E 00	0.309E 00	0.253E 00
J.150E 03	0.520E 00	0.426E 00	0.349E 00	0.286E 00	0.235E 00
J.200E 03	0.488E 00	0.400E 00	0.328E 00	0.269E 00	0.220E 00
U.300E 03	0.439E 00	0.360E 00	0.295E 00	0.242E 00	0.198E 00
U.400E 03	0.403E 00	0.330E 00	0.270E 00	0.221E 00	0.181E 00
0.500E 03	0.374E 00	0.306E 00	0.251E 00	0.205E 00	0.168E 00
J.500E 03	0.350E 00	0.287E 00	0.235E 00	0.192E 00	0.158E 00
U.700E 03	0.331E 00	0.271E 00	0.222E 00	0.182E 00	0.149E 00
J.900E 03	0.314E 00	0.257E 00	0.211E 00	0.173E 00	0.141E 00
U.900E 03	0.300E 00	0.246E 00	0.201E 00	0.165E 00	0.135E 00
J.100E 04	0.287E 00	0.235E 00	0.193E 00	0.158E 00	0.129E 00

TABLE 11: XD VERSUS CD FOR SIGD = 0.150E 02

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
0.100E 01	0.387E 00	0.350E 00	0.318E 00	0.290E 00	0.234E 00
0.150E 01	0.353E 00	0.314E 00	0.281E 00	0.253E 00	0.197E 00
0.200E 01	0.332E 00	0.293E 00	0.259E 00	0.231E 00	0.175E 00
0.300E 01	0.308E 00	0.268E 00	0.234E 00	0.205E 00	0.149E 00
0.400E 01	0.295E 00	0.253E 00	0.219E 00	0.190E 00	0.135E 00
0.500E 01	0.285E 00	0.244E 00	0.209E 00	0.180E 00	0.125E 00
0.600E 01	0.279E 00	0.237E 00	0.202E 00	0.173E 00	0.118E 00
0.700E 01	0.274E 00	0.232E 00	0.197E 00	0.167E 00	0.113E 00
0.800E 01	0.270E 00	0.228E 00	0.192E 00	0.163E 00	0.110E 00
0.900E 01	0.266E 00	0.224E 00	0.189E 00	0.160E 00	0.106E 00
0.100E 02	0.264E 00	0.221E 00	0.186E 00	0.157E 00	0.104E 00
0.150E 02	0.254E 00	0.212E 00	0.177E 00	0.148E 00	0.955E-01
0.200E 02	0.248E 00	0.206E 00	0.171E 00	0.143E 00	0.908E-01
0.250E 02	0.243E 00	0.202E 00	0.167E 00	0.139E 00	0.877E-01
0.300E 02	0.240E 00	0.198E 00	0.164E 00	0.136E 00	0.853E-01
0.400E 02	0.233E 00	0.192E 00	0.159E 00	0.131E 00	0.818E-01
0.500E 02	0.228E 00	0.188E 00	0.155E 00	0.128E 00	0.792E-01
0.600E 02	0.223E 00	0.184E 00	0.151E 00	0.125E 00	0.770E-01
0.700E 02	0.219E 00	0.180E 00	0.148E 00	0.122E 00	0.752E-01
0.800E 02	0.215E 00	0.177E 00	0.145E 00	0.120E 00	0.735E-01
0.900E 02	0.211E 00	0.174E 00	0.143E 00	0.117E 00	0.721E-01
0.100E 03	0.208E 00	0.171E 00	0.140E 00	0.115E 00	0.707E-01
0.150E 03	0.193E 00	0.158E 00	0.130E 00	0.107E 00	0.651E-01
0.200E 03	0.181E 00	0.148E 00	0.122E 00	0.997E-01	0.609E-01
0.300E 03	0.162E 00	0.133E 00	0.109E 00	0.894E-01	0.544E-01
0.400E 03	0.149E 00	0.122E 00	0.998E-01	0.819E-01	0.498E-01
0.500E 03	0.138E 00	0.113E 00	0.925E-01	0.758E-01	0.461E-01
0.600E 03	0.129E 00	0.106E 00	0.867E-01	0.710E-01	0.432E-01
0.700E 03	0.122E 00	0.999E-01	0.818E-01	0.670E-01	0.407E-01
0.800E 03	0.115E 00	0.948E-01	0.777E-01	0.636E-01	0.387E-01
0.900E 03	0.110E 00	0.905E-01	0.741E-01	0.607E-01	0.369E-01
0.100E 04	0.106E 00	0.867E-01	0.710E-01	0.582E-01	0.353E-01

TABLE 12: XD VERSUS CD FOR SIGD = 0.150E 02

XD-VALUE	LAMD= 0.300E 01 CD-VALUE	LAMD= 0.400E 01 CD-VALUE	LAMD= 0.600E 01 CD-VALUE	LAMD= 0.800E 01 CD-VALUE	LAMD= 0.100E 02 CD-VALUE
0.100E 01	0.193E 00	0.135E 00	0.738E-01	0.440E-01	0.278E-01
0.150E 01	0.156E 00	0.103E 00	0.497E-01	0.267E-01	0.153E-01
0.200E 01	0.135E 00	0.342E-01	0.371E-01	0.183E-01	0.971E-02
0.300E 01	0.111E 00	0.641E-01	0.244E-01	0.105E-01	0.495E-02
0.400E 01	0.973E-01	0.532E-01	0.182E-01	0.710E-02	0.302E-02
0.500E 01	0.886E-01	0.465E-01	0.146E-01	0.524E-02	0.206E-02
0.600E 01	0.825E-01	0.418E-01	0.123E-01	0.410E-02	0.151E-02
0.700E 01	0.780E-01	0.385E-01	0.107E-01	0.336E-02	0.116E-02
0.800E 01	0.746E-01	0.359E-01	0.946E-02	0.283E-02	0.934E-03
0.900E 01	0.718E-01	0.339E-01	0.856E-02	0.245E-02	0.772E-03
0.100E 02	0.695E-01	0.323E-01	0.785E-02	0.216E-02	0.654E-03
0.150E 02	0.623E-01	0.273E-01	0.581E-02	0.135E-02	0.358E-03
0.200E 02	0.586E-01	0.247E-01	0.483E-02	0.104E-02	0.245E-03
0.250E 02	0.558E-01	0.231E-01	0.426E-02	0.861E-03	0.158E-03
0.300E 02	0.539E-01	0.219E-01	0.389E-02	0.747E-03	0.154E-03
0.400E 02	0.512E-01	0.204E-01	0.342E-02	0.612E-03	0.117E-03
0.500E 02	0.493E-01	0.193E-01	0.312E-02	0.535E-03	0.964E-04
0.600E 02	0.478E-01	0.186E-01	0.292E-02	0.484E-03	0.839E-04
0.700E 02	0.465E-01	0.179E-01	0.277E-02	0.447E-03	0.753E-04
0.800E 02	0.454E-01	0.174E-01	0.265E-02	0.420E-03	0.691E-04
0.900E 02	0.444E-01	0.170E-01	0.255E-02	0.398E-03	0.642E-04
0.100E 03	0.435E-01	0.166E-01	0.247E-02	0.380E-03	0.604E-04
0.150E 03	0.397E-01	0.150E-01	0.217E-02	0.322E-03	0.497E-04
0.200E 03	0.372E-01	0.139E-01	0.198E-02	0.287E-03	0.424E-04
0.300E 03	0.332E-01	0.124E-01	0.173E-02	0.246E-03	0.353E-04
0.400E 03	0.303E-01	0.112E-01	0.156E-02	0.219E-03	0.310E-04
0.500E 03	0.281E-01	0.104E-01	0.144E-02	0.200E-03	0.291E-04
0.600E 03	0.263E-01	0.972E-02	0.134E-02	0.186E-03	0.259E-04
0.700E 03	0.248E-01	0.916E-02	0.126E-02	0.174E-03	0.242E-04
0.800E 03	0.235E-01	0.869E-02	0.119E-02	0.164E-03	0.227E-04
0.900E 03	0.224E-01	0.828E-02	0.113E-02	0.156E-03	0.216E-04
0.100E 04	0.215E-01	0.793E-02	0.108E-02	0.149E-03	0.205E-04

TABLE 13: XD VERSUS CD FOR SIGD = 0.200E 02

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
J.100E 01	0.999E 00	0.960E 00	0.908E 00	0.832E 00	0.767E 00
J.150E 01	0.998E 00	0.960E 00	0.906E 00	0.827E 00	0.759E 00
J.200E 01	0.998E 00	0.959E 00	0.905E 00	0.824E 00	0.754E 00
0.300E 01	0.996E 00	0.958E 00	0.903E 00	0.821E 00	0.748E 00
J.400E 01	0.995E 00	0.956E 00	0.902E 00	0.819E 00	0.745E 00
0.500E 01	0.994E 00	0.955E 00	0.900E 00	0.817E 00	0.743E 00
0.600E 01	0.993E 00	0.954E 00	0.899E 00	0.815E 00	0.741E 00
0.700E 01	0.991E 00	0.953E 00	0.898E 00	0.814E 00	0.739E 00
0.800E 01	0.990E 00	0.951E 00	0.897E 00	0.813E 00	0.738E 00
0.900E 01	0.989E 00	0.950E 00	0.895E 00	0.812E 00	0.736E 00
J.100E 02	0.988E 00	0.949E 00	0.894E 00	0.810E 00	0.735E 00
0.150E 02	0.982E 00	0.943E 00	0.889E 00	0.805E 00	0.730E 00
J.200E 02	0.976E 00	0.938E 00	0.883E 00	0.800E 00	0.725E 00
J.250E 02	0.970E 00	0.932E 00	0.878E 00	0.795E 00	0.720E 00
0.300E 02	0.965E 00	0.927E 00	0.873E 00	0.790E 00	0.716E 00
0.400E 02	0.954E 00	0.916E 00	0.863E 00	0.781E 00	0.707E 00
0.500E 02	0.943E 00	0.906E 00	0.853E 00	0.772E 00	0.699E 00
0.600E 02	0.933E 00	0.896E 00	0.844E 00	0.764E 00	0.692E 00
0.700E 02	0.923E 00	0.887E 00	0.835E 00	0.756E 00	0.684E 00
0.800E 02	0.913E 00	0.877E 00	0.826E 00	0.748E 00	0.677E 00
J.900E 02	0.904E 00	0.869E 00	0.818E 00	0.740E 00	0.670E 00
0.100E 03	0.895E 00	0.860E 00	0.810E 00	0.733E 00	0.663E 00
0.150E 03	0.885E 00	0.820E 00	0.772E 00	0.699E 00	0.632E 00
0.200E 03	0.877E 00	0.785E 00	0.739E 00	0.669E 00	0.605E 00
J.300E 03	0.756E 00	0.726E 00	0.694E 00	0.619E 00	0.560E 00
0.400E 03	0.707E 00	0.680E 00	0.640E 00	0.579E 00	0.524E 00
J.500E 03	0.667E 00	0.641E 00	0.603E 00	0.546E 00	0.494E 00
0.600E 03	0.633E 00	0.608E 00	0.572E 00	0.518E 00	0.469E 00
0.700E 03	0.603E 00	0.580E 00	0.546E 00	0.494E 00	0.447E 00
0.800E 03	0.577E 00	0.555E 00	0.523E 00	0.475E 00	0.428E 00
J.900E 03	0.555E 00	0.533E 00	0.502E 00	0.454E 00	0.411E 00
0.100E 04	0.535E 00	0.514E 00	0.494E 00	0.438E 00	0.396E 00

TABLE 14: XD VERSUS CD FOR SIGD = 0.200E 02

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
0.10JE 01	0.710E 00	0.616E 00	0.542E 00	0.481E 00	0.430E 00
0.15JE 01	0.698E 00	0.598E 00	0.518E 00	0.453E 00	0.399E 00
J.20JE 01	0.692E 00	0.588E 00	0.504E 00	0.436E 00	0.380E 00
0.30JE 01	0.686E 00	0.575E 00	0.488E 00	0.416E 00	0.358E 00
0.400E 01	0.590E 00	0.568E 00	0.479E 00	0.405E 00	0.345E 00
0.50JE 01	0.676E 00	0.564E 00	0.472E 00	0.398E 00	0.337E 00
0.60UE 01	0.674E 00	0.560E 00	0.468E 00	0.393E 00	0.331E 00
0.70UE 01	0.672E 00	0.558E 00	0.465E 00	0.389E 00	0.327E 00
0.800E 01	0.670E 00	0.555E 00	0.462E 00	0.386E 00	0.323E 00
J.900E 01	0.669E 00	0.553E 00	0.460E 00	0.383E 00	0.320E 00
J.10UE 02	0.668E 00	0.552E 00	0.458E 00	0.381E 00	0.318E 00
0.15UE 02	0.652E 00	0.545E 00	0.451E 00	0.373E 00	0.310E 00
J.20UE 02	0.657E 00	0.541E 00	0.446E 00	0.366E 00	0.305E 00
0.25JE 02	0.653E 00	0.537E 00	0.442E 00	0.364E 00	0.301E 00
J.30UE 02	0.649E 00	0.533E 00	0.438E 00	0.361E 00	0.295E 00
0.40UE 02	0.641E 00	0.526E 00	0.432E 00	0.355E 00	0.293E 00
0.50JE 02	0.633E 00	0.520E 00	0.427E 00	0.351E 00	0.298E 00
0.60JE 02	0.626E 00	0.514E 00	0.422E 00	0.346E 00	0.285E 00
0.70JE 02	0.619E 00	0.508E 00	0.417E 00	0.342E 00	0.291E 00
0.90UE 02	0.613E 00	0.502E 00	0.412E 00	0.338E 00	0.278E 00
J.70JE 02	0.607E 00	0.497E 00	0.408E 00	0.335E 00	0.275E 00
J.100E 03	0.600E 00	0.492E 00	0.404E 00	0.331E 00	0.272E 00
J.150E 03	0.572E 00	0.469E 00	0.384E 00	0.315E 00	0.258E 00
0.20UE 03	0.548E 00	0.449E 00	0.368E 00	0.301E 00	0.247E 00
0.30JE 03	0.507E 00	0.415E 00	0.340E 00	0.279E 00	0.228E 00
J.40JE 03	0.474E 00	0.388E 00	0.318E 00	0.261E 00	0.214E 00
J.500E 03	0.447E 00	0.366E 00	0.300E 00	0.246E 00	0.201E 00
0.500E 03	0.424E 00	0.347E 00	0.285E 00	0.233E 00	0.191E 00
0.70UE 03	0.404E 00	0.331E 00	0.271E 00	0.222E 00	0.182E 00
0.90JE 03	0.387E 00	0.317E 00	0.260E 00	0.213E 00	0.174E 00
J.70JE 03	0.372E 00	0.305E 00	0.249E 00	0.204E 00	0.167E 00
J.10UE 04	0.358E 00	0.294E 00	0.240E 00	0.197E 00	0.161E 00

TABLE 15: XD VERSUS CD FOR SIGD = 0.200E 02

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
0.100E 01	0.387E 00	0.350E 00	0.318E 00	0.290E 00	0.235E 00
0.150E 01	0.353E 00	0.315E 00	0.282E 00	0.253E 00	0.197E 00
0.200E 01	0.333E 00	0.293E 00	0.260E 00	0.231E 00	0.175E 00
0.300E 01	0.309E 00	0.268E 00	0.234E 00	0.205E 00	0.149E 00
0.400E 01	0.295E 00	0.254E 00	0.219E 00	0.190E 00	0.135E 00
0.500E 01	0.287E 00	0.245E 00	0.210E 00	0.180E 00	0.125E 00
0.600E 01	0.280E 00	0.238E 00	0.203E 00	0.173E 00	0.119E 00
0.700E 01	0.275E 00	0.233E 00	0.198E 00	0.168E 00	0.114E 00
0.800E 01	0.272E 00	0.229E 00	0.194E 00	0.164E 00	0.110E 00
0.900E 01	0.268E 00	0.226E 00	0.190E 00	0.161E 00	0.107E 00
0.100E 02	0.266E 00	0.223E 00	0.188E 00	0.158E 00	0.105E 00
0.150E 02	0.257E 00	0.215E 00	0.179E 00	0.150E 00	0.967E-01
0.200E 02	0.252E 00	0.209E 00	0.174E 00	0.145E 00	0.923E-01
0.250E 02	0.249E 00	0.206E 00	0.171E 00	0.142E 00	0.895E-01
0.300E 02	0.246E 00	0.203E 00	0.168E 00	0.139E 00	0.874E-01
0.400E 02	0.241E 00	0.199E 00	0.164E 00	0.136E 00	0.845E-01
0.500E 02	0.237E 00	0.195E 00	0.161E 00	0.133E 00	0.824E-01
0.600E 02	0.234E 00	0.193E 00	0.159E 00	0.131E 00	0.807E-01
0.700E 02	0.231E 00	0.190E 00	0.156E 00	0.129E 00	0.793E-01
0.800E 02	0.228E 00	0.188E 00	0.154E 00	0.127E 00	0.780E-01
0.900E 02	0.226E 00	0.185E 00	0.152E 00	0.125E 00	0.769E-01
0.100E 03	0.223E 00	0.183E 00	0.151E 00	0.124E 00	0.759E-01
0.150E 03	0.212E 00	0.174E 00	0.143E 00	0.117E 00	0.716E-01
0.200E 03	0.203E 00	0.166E 00	0.136E 00	0.112E 00	0.682E-01
0.300E 03	0.187E 00	0.153E 00	0.126E 00	0.103E 00	0.628E-01
0.400E 03	0.175E 00	0.143E 00	0.118E 00	0.963E-01	0.586E-01
0.500E 03	0.165E 00	0.135E 00	0.111E 00	0.907E-01	0.552E-01
0.600E 03	0.156E 00	0.128E 00	0.105E 00	0.860E-01	0.523E-01
0.700E 03	0.149E 00	0.122E 00	0.100E 00	0.819E-01	0.498E-01
0.800E 03	0.143E 00	0.117E 00	0.957E-01	0.784E-01	0.476E-01
0.900E 03	0.137E 00	0.112E 00	0.919E-01	0.753E-01	0.457E-01
0.100E 04	0.132E 00	0.108E 00	0.885E-01	0.725E-01	0.441E-01

TABLE 16: XD VERSUS CD FOR SIGD = 0.200E 02

XD-VALUE	LAMD= 0.300E 01 CD-VALUE	LAMD= 0.400E 01 CD-VALUE	LAMD= 0.600E 01 CD-VALUE	LAMD= 0.800E 01 CD-VALUE	LAMD= 0.100E 02 CD-VALUE
0.100E 01	0.193E 00	0.135E 00	0.738E-01	0.440E-01	0.278E-01
0.150E 01	0.156E 00	0.103E 00	0.498E-01	0.267E-01	0.153E-01
0.200E 01	0.135E 00	0.943E-01	0.372E-01	0.183E-01	0.972E-02
0.300E 01	0.111E 00	0.642E-01	0.245E-01	0.106E-01	0.495E-02
0.400E 01	0.975E-01	0.533E-01	0.183E-01	0.711E-02	0.303E-02
0.500E 01	0.989E-01	0.466E-01	0.147E-01	0.525E-02	0.206E-02
0.600E 01	0.829E-01	0.420E-01	0.123E-01	0.412E-02	0.151E-02
0.700E 01	0.794E-01	0.387E-01	0.107E-01	0.337E-02	0.117E-02
0.800E 01	0.750E-01	0.361E-01	0.951E-02	0.285E-02	0.938E-03
0.900E 01	0.723E-01	0.341E-01	0.961E-02	0.246E-02	0.776E-03
0.100E 02	0.700E-01	0.325E-01	0.790E-02	0.217E-02	0.657E-03
0.150E 02	0.631E-01	0.276E-01	0.587E-02	0.139E-02	0.362E-03
0.200E 02	0.593E-01	0.251E-01	0.490E-02	0.106E-02	0.248E-03
0.250E 02	0.569E-01	0.235E-01	0.434E-02	0.977E-03	0.191E-03
0.300E 02	0.552E-01	0.225E-01	0.398E-02	0.763E-03	0.157E-03
0.400E 02	0.529E-01	0.210E-01	0.352E-02	0.630E-03	0.120E-03
0.500E 02	0.513E-01	0.201E-01	0.324E-02	0.555E-03	0.999E-04
0.600E 02	0.500E-01	0.194E-01	0.306E-02	0.505E-03	0.876E-04
0.700E 02	0.490E-01	0.189E-01	0.292E-02	0.470E-03	0.791E-04
0.800E 02	0.481E-01	0.185E-01	0.281E-02	0.444E-03	0.730E-04
0.900E 02	0.474E-01	0.181E-01	0.272E-02	0.423E-03	0.693E-04
0.100E 03	0.467E-01	0.178E-01	0.264E-02	0.407E-03	0.646E-04
0.150E 03	0.439E-01	0.165E-01	0.239E-02	0.353E-03	0.534E-04
0.200E 03	0.417E-01	0.156E-01	0.222E-02	0.322E-03	0.474E-04
0.300E 03	0.383E-01	0.143E-01	0.200E-02	0.293E-03	0.406E-04
0.400E 03	0.357E-01	0.132E-01	0.184E-02	0.258E-03	0.365E-04
0.500E 03	0.336E-01	0.124E-01	0.172E-02	0.239E-03	0.336E-04
0.600E 03	0.318E-01	0.118E-01	0.162E-02	0.225E-03	0.313E-04
0.700E 03	0.303E-01	0.112E-01	0.154E-02	0.212E-03	0.295E-04
0.800E 03	0.289E-01	0.107E-01	0.147E-02	0.202E-03	0.280E-04
0.900E 03	0.278E-01	0.103E-01	0.141E-02	0.193E-03	0.267E-04
0.100E 04	0.268E-01	0.988E-02	0.135E-02	0.186E-03	0.256E-04

TABLE 17: XD VERSUS CD FOR SIGD = 0.250E 02

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
0.100E 01	0.999E 00	0.961E 00	0.908E 00	0.832E 00	0.767E 00
0.150E 01	0.999E 00	0.960E 00	0.907E 00	0.828E 00	0.759E 00
0.200E 01	0.998E 00	0.960E 00	0.906E 00	0.825E 00	0.754E 00
0.300E 01	0.998E 00	0.959E 00	0.904E 00	0.822E 00	0.749E 00
0.400E 01	0.997E 00	0.958E 00	0.903E 00	0.820E 00	0.746E 00
0.500E 01	0.996E 00	0.957E 00	0.902E 00	0.819E 00	0.744E 00
0.600E 01	0.995E 00	0.956E 00	0.901E 00	0.818E 00	0.743E 00
0.700E 01	0.994E 00	0.956E 00	0.901E 00	0.817E 00	0.741E 00
0.800E 01	0.994E 00	0.955E 00	0.900E 00	0.816E 00	0.740E 00
0.900E 01	0.993E 00	0.954E 00	0.899E 00	0.815E 00	0.739E 00
0.100E 02	0.992E 00	0.953E 00	0.898E 00	0.814E 00	0.738E 00
0.150E 02	0.988E 00	0.950E 00	0.895E 00	0.810E 00	0.734E 00
0.200E 02	0.984E 00	0.946E 00	0.891E 00	0.807E 00	0.731E 00
0.250E 02	0.981E 00	0.942E 00	0.888E 00	0.804E 00	0.728E 00
0.300E 02	0.977E 00	0.939E 00	0.884E 00	0.800E 00	0.725E 00
0.400E 02	0.969E 00	0.932E 00	0.877E 00	0.794E 00	0.719E 00
0.500E 02	0.962E 00	0.925E 00	0.871E 00	0.788E 00	0.714E 00
0.600E 02	0.955E 00	0.918E 00	0.864E 00	0.782E 00	0.708E 00
0.700E 02	0.948E 00	0.911E 00	0.858E 00	0.777E 00	0.703E 00
0.800E 02	0.942E 00	0.905E 00	0.852E 00	0.771E 00	0.698E 00
0.900E 02	0.935E 00	0.898E 00	0.846E 00	0.766E 00	0.693E 00
0.100E 03	0.929E 00	0.892E 00	0.840E 00	0.760E 00	0.688E 00
0.150E 03	0.898E 00	0.863E 00	0.813E 00	0.736E 00	0.666E 00
0.200E 03	0.870E 00	0.836E 00	0.788E 00	0.713E 00	0.645E 00
0.300E 03	0.822E 00	0.790E 00	0.744E 00	0.673E 00	0.609E 00
0.400E 03	0.781E 00	0.750E 00	0.707E 00	0.640E 00	0.579E 00
0.500E 03	0.745E 00	0.716E 00	0.675E 00	0.610E 00	0.552E 00
0.600E 03	0.714E 00	0.686E 00	0.646E 00	0.585E 00	0.529E 00
0.700E 03	0.687E 00	0.660E 00	0.622E 00	0.562E 00	0.509E 00
0.800E 03	0.662E 00	0.636E 00	0.599E 00	0.542E 00	0.491E 00
0.900E 03	0.640E 00	0.615E 00	0.579E 00	0.524E 00	0.474E 00
0.100E 04	0.620E 00	0.596E 00	0.561E 00	0.508E 00	0.450E 00

TABLE 15: XD VERSUS CD FOR SIGD = 0.250E 02

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
0.100E 01	0.710E 00	0.616E 00	0.542E 00	0.481E 00	0.430E 00
0.150E 01	0.699E 00	0.598E 00	0.518E 00	0.453E 00	0.399E 00
0.200E 01	0.692E 00	0.588E 00	0.504E 00	0.436E 00	0.380E 00
0.300E 01	0.685E 00	0.576E 00	0.488E 00	0.417E 00	0.358E 00
0.400E 01	0.681E 00	0.569E 00	0.479E 00	0.406E 00	0.346E 00
0.500E 01	0.678E 00	0.565E 00	0.473E 00	0.399E 00	0.338E 00
0.600E 01	0.676E 00	0.562E 00	0.469E 00	0.394E 00	0.332E 00
0.700E 01	0.674E 00	0.559E 00	0.466E 00	0.390E 00	0.327E 00
0.800E 01	0.673E 00	0.557E 00	0.463E 00	0.387E 00	0.324E 00
0.900E 01	0.671E 00	0.555E 00	0.461E 00	0.384E 00	0.321E 00
0.100E 02	0.670E 00	0.554E 00	0.459E 00	0.382E 00	0.319E 00
0.150E 02	0.666E 00	0.549E 00	0.453E 00	0.375E 00	0.311E 00
0.200E 02	0.663E 00	0.545E 00	0.449E 00	0.371E 00	0.307E 00
0.250E 02	0.660E 00	0.542E 00	0.446E 00	0.368E 00	0.304E 00
0.300E 02	0.657E 00	0.540E 00	0.444E 00	0.365E 00	0.301E 00
0.400E 02	0.651E 00	0.535E 00	0.439E 00	0.361E 00	0.297E 00
0.500E 02	0.646E 00	0.530E 00	0.435E 00	0.358E 00	0.294E 00
0.600E 02	0.641E 00	0.526E 00	0.432E 00	0.355E 00	0.291E 00
0.700E 02	0.637E 00	0.522E 00	0.428E 00	0.352E 00	0.289E 00
0.800E 02	0.632E 00	0.518E 00	0.425E 00	0.349E 00	0.286E 00
0.900E 02	0.627E 00	0.514E 00	0.422E 00	0.346E 00	0.284E 00
0.100E 03	0.623E 00	0.511E 00	0.419E 00	0.344E 00	0.282E 00
0.150E 03	0.602E 00	0.494E 00	0.405E 00	0.332E 00	0.272E 00
0.200E 03	0.584E 00	0.478E 00	0.392E 00	0.321E 00	0.263E 00
0.300E 03	0.551E 00	0.452E 00	0.370E 00	0.303E 00	0.248E 00
0.400E 03	0.524E 00	0.429E 00	0.351E 00	0.288E 00	0.236E 00
0.500E 03	0.500E 00	0.409E 00	0.335E 00	0.275E 00	0.225E 00
0.600E 03	0.479E 00	0.392E 00	0.321E 00	0.263E 00	0.216E 00
0.700E 03	0.461E 00	0.377E 00	0.309E 00	0.253E 00	0.207E 00
0.800E 03	0.444E 00	0.364E 00	0.298E 00	0.244E 00	0.200E 00
0.900E 03	0.429E 00	0.352E 00	0.288E 00	0.236E 00	0.193E 00
0.100E 04	0.416E 00	0.341E 00	0.279E 00	0.228E 00	0.187E 00

TABLE 19: XD VERSUS CD FOR SIGD = 0.250E 02

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
0.100E 01	0.387E 00	0.350E 00	0.318E 00	0.290E 00	0.235E 00
0.150E 01	0.353E 00	0.315E 00	0.282E 00	0.253E 00	0.197E 00
0.200E 01	0.333E 00	0.293E 00	0.260E 00	0.231E 00	0.175E 00
0.300E 01	0.309E 00	0.269E 00	0.234E 00	0.205E 00	0.150E 00
0.400E 01	0.296E 00	0.254E 00	0.220E 00	0.190E 00	0.135E 00
0.500E 01	0.287E 00	0.245E 00	0.210E 00	0.181E 00	0.126E 00
0.600E 01	0.281E 00	0.238E 00	0.203E 00	0.174E 00	0.119E 00
0.700E 01	0.276E 00	0.234E 00	0.198E 00	0.169E 00	0.114E 00
0.800E 01	0.272E 00	0.230E 00	0.194E 00	0.165E 00	0.110E 00
0.900E 01	0.269E 00	0.227E 00	0.191E 00	0.162E 00	0.107E 00
0.100E 02	0.267E 00	0.224E 00	0.188E 00	0.159E 00	0.105E 00
0.150E 02	0.259E 00	0.216E 00	0.180E 00	0.151E 00	0.972E-01
0.200E 02	0.254E 00	0.211E 00	0.176E 00	0.146E 00	0.930E-01
0.250E 02	0.251E 00	0.208E 00	0.172E 00	0.143E 00	0.904E-01
0.300E 02	0.249E 00	0.206E 00	0.170E 00	0.141E 00	0.884E-01
0.400E 02	0.245E 00	0.202E 00	0.167E 00	0.138E 00	0.858E-01
0.500E 02	0.242E 00	0.199E 00	0.164E 00	0.136E 00	0.840E-01
0.600E 02	0.240E 00	0.197E 00	0.162E 00	0.134E 00	0.826E-01
0.700E 02	0.237E 00	0.195E 00	0.161E 00	0.132E 00	0.814E-01
0.800E 02	0.235E 00	0.193E 00	0.159E 00	0.131E 00	0.804E-01
0.900E 02	0.233E 00	0.192E 00	0.157E 00	0.129E 00	0.795E-01
0.100E 03	0.231E 00	0.190E 00	0.156E 00	0.128E 00	0.787E-01
0.150E 03	0.223E 00	0.183E 00	0.150E 00	0.123E 00	0.754E-01
0.200E 03	0.216E 00	0.177E 00	0.145E 00	0.119E 00	0.727E-01
0.300E 03	0.204E 00	0.167E 00	0.137E 00	0.112E 00	0.683E-01
0.400E 03	0.193E 00	0.158E 00	0.130E 00	0.106E 00	0.647E-01
0.500E 03	0.184E 00	0.151E 00	0.124E 00	0.101E 00	0.616E-01
0.600E 03	0.177E 00	0.145E 00	0.118E 00	0.971E-01	0.590E-01
0.700E 03	0.170E 00	0.139E 00	0.114E 00	0.933E-01	0.567E-01
0.800E 03	0.164E 00	0.134E 00	0.110E 00	0.899E-01	0.546E-01
0.900E 03	0.158E 00	0.130E 00	0.106E 00	0.864E-01	0.528E-01
0.100E 04	0.153E 00	0.125E 00	0.103E 00	0.842E-01	0.511E-01

TABLE 20: XD VERSUS CD FOR SIGD = 0.250E 02

XD-VALUE	LAMD= 0.300E 01 CD-VALUE	LAMD= 0.400E 01 CD-VALUE	LAMD= 0.600E 01 CD-VALUE	LAMD= 0.800E 01 CD-VALUE	LAMD= 0.100E 02 CD-VALUE
J.100E 01	0.193E 00	0.135E 00	0.738E-01	0.440E-01	U.278E-01
J.150E 01	0.157E 00	0.103E 00	U.498E-01	0.267E-01	U.153E-01
J.200E 01	0.135E 00	0.843E-01	U.372E-01	0.193E-01	U.972E-02
0.300E 01	0.111E 00	0.642E-01	U.245E-01	0.106E-01	U.495E-02
J.403E 01	0.976E-01	0.534E-01	0.193E-01	0.711E-02	U.303E-02
J.500E 01	0.890E-01	0.467E-01	U.147E-01	0.525E-02	U.207E-02
0.603E 01	0.830E-01	0.421E-01	U.123E-01	U.412E-02	U.152E-02
0.700E 01	0.786E-01	0.387E-01	U.107E-01	0.337E-02	U.117E-02
J.803E 01	0.752E-01	0.362E-01	U.953E-02	0.285E-02	U.740E-03
0.900E 01	0.725E-01	0.342E-01	U.863E-02	0.247E-02	U.777E-03
J.103E 02	0.703E-01	0.326E-01	0.792E-02	0.218E-02	U.659E-03
0.150E 02	0.634E-01	0.278E-01	0.590E-02	0.140E-02	U.363E-03
J.203E 02	0.598E-01	0.253E-01	0.494E-02	0.107E-02	U.250E-03
0.250E 02	0.575E-01	0.238E-01	0.438E-02	0.884E-03	U.192E-03
J.303E 02	0.559E-01	0.227E-01	0.402E-02	0.771E-03	U.159E-03
0.400E 02	0.537E-01	0.214E-01	U.357E-02	0.639E-03	U.122E-03
J.500E 02	0.523E-01	0.205E-01	0.331E-02	0.565E-03	U.102E-03
0.600E 02	0.512E-01	0.199E-01	U.312E-02	0.516E-03	0.894E-04
J.700E 02	0.503E-01	0.194E-01	U.299E-02	0.482E-03	U.511E-04
0.903E 02	0.496E-01	0.190E-01	U.289E-02	0.457E-03	U.751E-04
J.900E 02	0.490E-01	0.187E-01	U.291E-02	0.437E-03	U.705E-04
0.100E 03	0.484E-01	0.184E-01	U.274E-02	0.421E-03	U.669E-04
J.150E 03	0.461E-01	0.174E-01	U.251E-02	0.371E-03	U.561E-04
J.200E 03	0.444E-01	0.166E-01	U.236E-02	0.342E-03	U.504E-04
0.300E 03	0.416E-01	0.155E-01	U.217E-02	0.307E-03	U.441E-04
0.400E 03	0.394E-01	0.146E-01	0.203E-02	0.284E-03	U.402E-04
J.500E 03	0.375E-01	0.139E-01	U.192E-02	0.267E-03	U.375E-04
0.603E 03	0.359E-01	0.133E-01	U.183E-02	0.253E-03	U.353E-04
0.700E 03	0.345E-01	0.127E-01	U.175E-02	0.242E-03	U.336E-04
0.803E 03	0.332E-01	0.123E-01	U.168E-02	0.232E-03	U.321E-04
J.900E 03	0.321E-01	0.118E-01	U.162E-02	0.223E-03	U.308E-04
J.103E 04	0.310E-01	0.115E-01	U.157E-02	0.215E-03	U.297E-04

TABLE 21: XD VERSUS CD FOR SIGD = 0.300E 02

XD-VALUE	LAMD= 0.000E 00 CD-VALUE	LAMD= 0.400E-01 CD-VALUE	LAMD= 0.100E 00 CD-VALUE	LAMD= 0.200E 00 CD-VALUE	LAMD= 0.300E 00 CD-VALUE
0.100E 01	0.999E 00	0.961E 00	0.909E 00	0.832E 00	0.767E 00
0.150E 01	0.999E 00	0.961E 00	0.907E 00	0.828E 00	0.759E 00
0.200E 01	0.999E 00	0.960E 00	0.906E 00	0.825E 00	0.755E 00
0.300E 01	0.998E 00	0.959E 00	0.905E 00	0.823E 00	0.750E 00
0.400E 01	0.998E 00	0.959E 00	0.904E 00	0.821E 00	0.747E 00
0.500E 01	0.997E 00	0.958E 00	0.903E 00	0.820E 00	0.745E 00
0.600E 01	0.997E 00	0.958E 00	0.903E 00	0.819E 00	0.744E 00
0.700E 01	0.996E 00	0.957E 00	0.902E 00	0.818E 00	0.743E 00
0.800E 01	0.996E 00	0.957E 00	0.901E 00	0.817E 00	0.742E 00
0.900E 01	0.995E 00	0.956E 00	0.901E 00	0.817E 00	0.741E 00
0.100E 02	0.994E 00	0.956E 00	0.900E 00	0.816E 00	0.740E 00
0.150E 02	0.992E 00	0.953E 00	0.898E 00	0.813E 00	0.737E 00
0.200E 02	0.989E 00	0.950E 00	0.895E 00	0.811E 00	0.734E 00
0.250E 02	0.986E 00	0.948E 00	0.893E 00	0.808E 00	0.732E 00
0.300E 02	0.984E 00	0.945E 00	0.890E 00	0.806E 00	0.730E 00
0.400E 02	0.979E 00	0.940E 00	0.886E 00	0.802E 00	0.726E 00
0.500E 02	0.973E 00	0.935E 00	0.881E 00	0.797E 00	0.722E 00
0.600E 02	0.968E 00	0.930E 00	0.876E 00	0.793E 00	0.718E 00
0.700E 02	0.963E 00	0.926E 00	0.872E 00	0.789E 00	0.714E 00
0.800E 02	0.958E 00	0.921E 00	0.867E 00	0.785E 00	0.710E 00
0.900E 02	0.953E 00	0.916E 00	0.863E 00	0.781E 00	0.707E 00
0.100E 03	0.949E 00	0.912E 00	0.859E 00	0.777E 00	0.703E 00
0.150E 03	0.926E 00	0.890E 00	0.838E 00	0.758E 00	0.686E 00
0.200E 03	0.905E 00	0.869E 00	0.819E 00	0.741E 00	0.670E 00
0.300E 03	0.865E 00	0.832E 00	0.784E 00	0.709E 00	0.642E 00
0.400E 03	0.832E 00	0.800E 00	0.753E 00	0.681E 00	0.617E 00
0.500E 03	0.802E 00	0.770E 00	0.726E 00	0.657E 00	0.594E 00
0.600E 03	0.775E 00	0.744E 00	0.701E 00	0.634E 00	0.574E 00
0.700E 03	0.750E 00	0.721E 00	0.679E 00	0.614E 00	0.556E 00
0.800E 03	0.728E 00	0.699E 00	0.658E 00	0.596E 00	0.539E 00
0.900E 03	0.707E 00	0.679E 00	0.640E 00	0.579E 00	0.524E 00
0.100E 04	0.688E 00	0.661E 00	0.623E 00	0.564E 00	0.510E 00

TABLE 22: XD VERSUS CD FOR SIGD = 0.300E 02

XD-VALUE	LAMD= 0.400E 00 CD-VALUE	LAMD= 0.600E 00 CD-VALUE	LAMD= 0.800E 00 CD-VALUE	LAMD= 0.100E 01 CD-VALUE	LAMD= 0.120E 01 CD-VALUE
J.100E 01	0.710E 00	0.617E 00	0.542E 00	0.481E 00	0.430E 00
J.150E 01	0.699E 00	0.599E 00	0.518E 00	0.453E 00	0.399E 00
0.200E 01	0.693E 00	0.588E 00	0.505E 00	0.436E 00	0.380E 00
0.300E 01	0.685E 00	0.576E 00	0.489E 00	0.417E 00	0.356E 00
0.400E 01	0.681E 00	0.570E 00	0.480E 00	0.406E 00	0.346E 00
J.500E 01	0.679E 00	0.565E 00	0.474E 00	0.399E 00	0.338E 00
0.600E 01	0.677E 00	0.562E 00	0.470E 00	0.394E 00	0.332E 00
J.700E 01	0.675E 00	0.560E 00	0.467E 00	0.390E 00	0.328E 00
0.800E 01	0.674E 00	0.558E 00	0.464E 00	0.387E 00	0.325E 00
J.900E 01	0.673E 00	0.557E 00	0.462E 00	0.385E 00	0.322E 00
0.100E 02	0.672E 00	0.555E 00	0.460E 00	0.383E 00	0.320E 00
J.150E 02	0.668E 00	0.551E 00	0.455E 00	0.377E 00	0.312E 00
0.200E 02	0.666E 00	0.548E 00	0.451E 00	0.373E 00	0.308E 00
0.250E 02	0.663E 00	0.545E 00	0.449E 00	0.370E 00	0.306E 00
0.300E 02	0.661E 00	0.543E 00	0.447E 00	0.368E 00	0.303E 00
0.400E 02	0.657E 00	0.540E 00	0.443E 00	0.365E 00	0.300E 00
0.500E 02	0.654E 00	0.536E 00	0.440E 00	0.362E 00	0.298E 00
0.600E 02	0.650E 00	0.533E 00	0.438E 00	0.359E 00	0.295E 00
0.700E 02	0.647E 00	0.530E 00	0.435E 00	0.357E 00	0.293E 00
0.800E 02	0.643E 00	0.527E 00	0.432E 00	0.355E 00	0.291E 00
0.900E 02	0.640E 00	0.524E 00	0.430E 00	0.353E 00	0.290E 00
0.100E 03	0.637E 00	0.522E 00	0.428E 00	0.351E 00	0.288E 00
0.150E 03	0.621E 00	0.509E 00	0.417E 00	0.342E 00	0.280E 00
0.200E 03	0.607E 00	0.497E 00	0.407E 00	0.334E 00	0.274E 00
0.300E 03	0.591E 00	0.476E 00	0.390E 00	0.319E 00	0.262E 00
0.400E 03	0.558E 00	0.457E 00	0.374E 00	0.307E 00	0.251E 00
0.500E 03	0.538E 00	0.440E 00	0.361E 00	0.295E 00	0.242E 00
0.600E 03	0.519E 00	0.425E 00	0.348E 00	0.285E 00	0.234E 00
0.700E 03	0.503E 00	0.412E 00	0.337E 00	0.276E 00	0.226E 00
0.800E 03	0.498E 00	0.400E 00	0.327E 00	0.268E 00	0.219E 00
J.900E 03	0.474E 00	0.388E 00	0.318E 00	0.260E 00	0.213E 00
J.100E 04	0.461E 00	0.378E 00	0.309E 00	0.253E 00	0.208E 00

TABLE 23: XD VERSUS CD FOR SIGD = 0.300E 02

XD-VALUE	LAMD= 0.140E 01 CD-VALUE	LAMD= 0.160E 01 CD-VALUE	LAMD= 0.180E 01 CD-VALUE	LAMD= 0.200E 01 CD-VALUE	LAMD= 0.250E 01 CD-VALUE
J.100E 01	0.387E 00	0.350E 00	0.318E 00	0.290E 00	0.235E 00
0.150E 01	0.355E 00	0.315E 00	0.282E 00	0.253E 00	0.197E 00
0.200E 01	0.333E 00	0.294E 00	0.260E 00	0.231E 00	0.175E 00
J.300E 01	0.310E 00	0.269E 00	0.234E 00	0.205E 00	0.150E 00
J.400E 01	0.290E 00	0.255E 00	0.220E 00	0.190E 00	0.135E 00
0.500E 01	J.287E 00	0.245E 00	0.210E 00	0.181E 00	0.126E 00
J.600E 01	0.281E 00	0.239E 00	0.203E 00	0.174E 00	0.119E 00
J.700E 01	0.270E 00	0.234E 00	0.198E 00	0.169E 00	0.114E 00
0.800E 01	0.273E 00	0.230E 00	0.195E 00	0.165E 00	0.111E 00
J.900E 01	0.270E 00	0.227E 00	0.191E 00	0.162E 00	0.108E 00
J.100E 02	0.260E 00	0.224E 00	0.189E 00	0.159E 00	0.105E 00
J.150E 02	0.260E 00	0.216E 00	0.181E 00	0.151E 00	0.975E-01
0.200E 02	0.256E 00	0.212E 00	0.176E 00	0.147E 00	0.934E-01
J.250E 02	0.253E 00	0.209E 00	0.173E 00	0.144E 00	0.908E-01
0.300E 02	0.250E 00	0.207E 00	0.171E 00	0.142E 00	0.890E-01
J.400E 02	0.247E 00	0.204E 00	0.168E 00	0.139E 00	0.866E-01
0.500E 02	0.245E 00	0.202E 00	0.166E 00	0.137E 00	0.849E-01
J.600E 02	0.243E 00	0.200E 00	0.164E 00	0.135E 00	0.837E-01
0.700E 02	0.241E 00	0.198E 00	0.163E 00	0.134E 00	0.827E-01
J.800E 02	0.239E 00	0.197E 00	0.162E 00	0.133E 00	0.818E-01
J.900E 02	0.230E 00	0.195E 00	0.161E 00	0.132E 00	0.811E-01
0.100E 03	0.230E 00	0.194E 00	0.159E 00	0.131E 00	0.804E-01
0.150E 03	0.230E 00	0.189E 00	0.155E 00	0.127E 00	0.777E-01
J.200E 03	0.224E 00	0.184E 00	0.151E 00	0.124E 00	0.755E-01
0.300E 03	0.214E 00	0.176E 00	0.144E 00	0.118E 00	0.719E-01
J.400E 03	0.206E 00	0.169E 00	0.138E 00	0.113E 00	0.689E-01
0.500E 03	0.198E 00	0.162E 00	0.133E 00	0.109E 00	0.663E-01
J.600E 03	0.191E 00	0.157E 00	0.128E 00	0.105E 00	0.640E-01
J.700E 03	0.185E 00	0.152E 00	0.124E 00	0.102E 00	0.619E-01
J.800E 03	0.180E 00	0.147E 00	0.121E 00	0.988E-01	0.600E-01
0.900E 03	0.175E 00	0.143E 00	0.117E 00	0.960E-01	0.583E-01
0.100E 04	0.170E 00	0.139E 00	0.114E 00	0.934E-01	0.567E-01

APPENDIX C
TABLES OF DILUTION FACTOR ζ^* VERSUS x/B

TABLE 1: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.500E-01

X/B-Value	B/ALFA _z = 0.500E 01 Zeta-Value	B/ALFA _z = 0.100E 02 Zeta-Value	B/ALFA _z = 0.150E 02 Zeta-Value	B/ALFA _z = 0.250E 02 Zeta-Value
0.10000E 00	0.35931E 00	0.47179E 00	0.54392E 00	0.63613E 00
0.15000E 00	0.26821E 00	0.36268E 00	0.42835E 00	0.51969E 00
0.20000E 00	0.21738E 00	0.29757E 00	0.35546E 00	0.43971E 00
0.30000E 00	0.16260E 00	0.22462E 00	0.27087E 00	0.34104E 00
0.40000E 00	0.13330E 00	0.18469E 00	0.22353E 00	0.28349E 00
0.50000E 00	0.11487E 00	0.15927E 00	0.19311E 00	0.24581E 00
0.60000E 00	0.10212E 00	0.14152E 00	0.17175E 00	0.21910E 00
0.70000E 00	0.92725E-01	0.12832E 00	0.15582E 00	0.19905E 00
0.80000E 00	0.85516E-01	0.11808E 00	0.14342E 00	0.18338E 00
0.90000E 00	0.79812E-01	0.10986E 00	0.13345E 00	0.17074E 00
0.10000E 01	0.75200E-01	0.10309E 00	0.12522E 00	0.16029E 00
0.15000E 01	0.61483E-01	0.81532E-01	0.98720E-01	0.12646E 00
0.20000E 01	0.55408E-01	0.69965E-01	0.84021E-01	0.10748E 00
0.25000E 01	0.52562E-01	0.62952E-01	0.74579E-01	0.95013E-01
0.30000E 01	0.51213E-01	0.58483E-01	0.68054E-01	0.86090E-01
0.40000E 01	0.50271E-01	0.53669E-01	0.59951E-01	0.74081E-01
0.50000E 01	0.50060E-01	0.51589E-01	0.55534E-01	0.66450E-01
0.60000E 01	0.50013E-01	0.50687E-01	0.53084E-01	0.61344E-01
0.70000E 01	0.50003E-01	0.50297E-01	0.51718E-01	0.57852E-01
0.80000E 01	0.50001E-01	0.50128E-01	0.50957E-01	0.55442E-01
0.90000E 01	0.50000E-01	0.50055E-01	0.50533E-01	0.53773E-01
0.10000E 02	0.50000E-01	0.50024E-01	0.50296E-01	0.52617E-01
0.15000E 02	0.50000E-01	0.50000E-01	0.50016E-01	0.50419E-01
0.20000E 02	0.50000E-01	0.50000E-01	0.50001E-01	0.50067E-01
0.30000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50002E-01
0.40000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.50000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.60000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.70000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.80000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.90000E 02	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01
0.10000E 03	0.50000E-01	0.50000E-01	0.50000E-01	0.50000E-01

TABLE 2: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.500E-01

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.69468E 00	0.75253E 00	0.81109E 00	0.84725E 00
0.15000E 00	0.58286E 00	0.64986E 00	0.72285E 00	0.77076E 00
0.20000E 00	0.50102E 00	0.56926E 00	0.64796E 00	0.70235E 00
0.30000E 00	0.39475E 00	0.45777E 00	0.53563E 00	0.59338E 00
0.40000E 00	0.33043E 00	0.38690E 00	0.45920E 00	0.51498E 00
0.50000E 00	0.28755E 00	0.33845E 00	0.40495E 00	0.45747E 00
0.60000E 00	0.25685E 00	0.30326E 00	0.36464E 00	0.41386E 00
0.70000E 00	0.23367E 00	0.27646E 00	0.33352E 00	0.37971E 00
0.80000E 00	0.21548E 00	0.25530E 00	0.30870E 00	0.35223E 00
0.90000E 00	0.20077E 00	0.23811E 00	0.28839E 00	0.32960E 00
0.10000E 01	0.18857E 00	0.22382E 00	0.27143E 00	0.31060E 00
0.15000E 01	0.14899E 00	0.17721E 00	0.21564E 00	0.24758E 00
0.20000E 01	0.12669E 00	0.15082E 00	0.18380E 00	0.21134E 00
0.25000E 01	0.11200E 00	0.13340E 00	0.16271E 00	0.18723E 00
0.30000E 01	0.10142E 00	0.12083E 00	0.14746E 00	0.16977E 00
0.40000E 01	0.86975E-01	0.10359E 00	0.12649E 00	0.14572E 00
0.50000E 01	0.77452E-01	0.92091E-01	0.11246E 00	0.12961E 00
0.60000E 01	0.70704E-01	0.83761E-01	0.10225E 00	0.11786E 00
0.70000E 01	0.65738E-01	0.77416E-01	0.94390E-01	0.10881E 00
0.80000E 01	0.62011E-01	0.72430E-01	0.88113E-01	0.10157E 00
0.90000E 01	0.59184E-01	0.68434E-01	0.82964E-01	0.95599E-01
0.10000E 02	0.57029E-01	0.65190E-01	0.78655E-01	0.90578E-01
0.15000E 02	0.51851E-01	0.55856E-01	0.64758E-01	0.73862E-01
0.20000E 02	0.50487E-01	0.52268E-01	0.57751E-01	0.64538E-01
0.30000E 02	0.50034E-01	0.50340E-01	0.52152E-01	0.55509E-01
0.40000E 02	0.50002E-01	0.50051E-01	0.50598E-01	0.52094E-01
0.50000E 02	0.50000E-01	0.50008E-01	0.50166E-01	0.50796E-01
0.60000E 02	0.50000E-01	0.50001E-01	0.50046E-01	0.50303E-01
0.70000E 02	0.50000E-01	0.50000E-01	0.50013E-01	0.50115E-01
0.80000E 02	0.50000E-01	0.50000E-01	0.50004E-01	0.50044E-01
0.90000E 02	0.50000E-01	0.50000E-01	0.50001E-01	0.50017E-01
0.10000E 03	0.50000E-01	0.50000E-01	0.50000E-01	0.50006E-01

TABLE 3: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.100E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.60207E 00	0.72059E 00	0.78246E 00	0.84875E 00
0.15000E 00	0.48521E 00	0.61228E 00	0.68640E 00	0.77272E 00
0.20000E 00	0.40771E 00	0.53067E 00	0.60808E 00	0.70459E 00
0.30000E 00	0.31449E 00	0.42193E 00	0.49555E 00	0.59585E 00
0.40000E 00	0.26105E 00	0.35478E 00	0.42173E 00	0.51748E 00
0.50000E 00	0.22637E 00	0.30953E 00	0.37038E 00	0.45991E 00
0.60000E 00	0.20197E 00	0.27693E 00	0.33268E 00	0.41621E 00
0.70000E 00	0.18383E 00	0.25223E 00	0.30378E 00	0.38196E 00
0.80000E 00	0.16981E 00	0.23281E 00	0.28085E 00	0.35438E 00
0.90000E 00	0.15868E 00	0.21708E 00	0.26216E 00	0.33166E 00
0.10000E 01	0.14965E 00	0.20405E 00	0.24659E 00	0.31257E 00
0.15000E 01	0.12267E 00	0.16211E 00	0.19568E 00	0.24921E 00
0.20000E 01	0.11068E 00	0.13939E 00	0.16701E 00	0.21275E 00
0.25000E 01	0.10506E 00	0.12557E 00	0.14848E 00	0.18854E 00
0.30000E 01	0.10240E 00	0.11675E 00	0.13564E 00	0.17110E 00
0.40000E 01	0.10054E 00	0.10725E 00	0.11965E 00	0.14752E 00
0.50000E 01	0.10012E 00	0.10314E 00	0.11093E 00	0.13248E 00
0.60000E 01	0.10003E 00	0.10136E 00	0.10609E 00	0.12240E 00
0.70000E 01	0.10001E 00	0.10059E 00	0.10339E 00	0.11551E 00
0.80000E 01	0.10000E 00	0.10025E 00	0.10189E 00	0.11075E 00
0.90000E 01	0.10000E 00	0.10011E 00	0.10105E 00	0.10745E 00
0.10000E 02	0.10000E 00	0.10005E 00	0.10059E 00	0.10517E 00
0.15000E 02	0.10000E 00	0.10000E 00	0.10003E 00	0.10083E 00
0.20000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10013E 00
0.30000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.40000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.50000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.60000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.70000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.80000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.90000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00
0.10000E 03	0.10000E 00	0.10000E 00	0.10000E 00	0.10000E 00

TABLE 4: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.100E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.88461E 00	0.91587E 00	0.94358E 00	0.95872E 00
0.15000E 00	0.82261E 00	0.86804E 00	0.90987E 00	0.93338E 00
0.20000E 00	0.76389E 00	0.82038E 00	0.87463E 00	0.90611E 00
0.30000E 00	0.66333E 00	0.73265E 00	0.80472E 00	0.84945E 00
0.40000E 00	0.58555E 00	0.65925E 00	0.74080E 00	0.79440E 00
0.50000E 00	0.52576E 00	0.59961E 00	0.68509E 00	0.74390E 00
0.60000E 00	0.47901E 00	0.55116E 00	0.63743E 00	0.69888E 00
0.70000E 00	0.44165E 00	0.51137E 00	0.59676E 00	0.65922E 00
0.80000E 00	0.41113E 00	0.47825E 00	0.56192E 00	0.62438E 00
0.90000E 00	0.38573E 00	0.45027E 00	0.53184E 00	0.59371E 00
0.10000E 01	0.36423E 00	0.42633E 00	0.50566E 00	0.56660E 00
0.15000E 01	0.29192E 00	0.34430E 00	0.41325E 00	0.46818E 00
0.20000E 01	0.24976E 00	0.29558E 00	0.35671E 00	0.40624E 00
0.25000E 01	0.22155E 00	0.26269E 00	0.31799E 00	0.36321E 00
0.30000E 01	0.20106E 00	0.23865E 00	0.28944E 00	0.33125E 00
0.40000E 01	0.17286E 00	0.20532E 00	0.24960E 00	0.28629E 00
0.50000E 01	0.15416E 00	0.18289E 00	0.22258E 00	0.25563E 00
0.60000E 01	0.14087E 00	0.16657E 00	0.20275E 00	0.23304E 00
0.70000E 01	0.13108E 00	0.15410E 00	0.18741E 00	0.21552E 00
0.80000E 01	0.12372E 00	0.14428E 00	0.17512E 00	0.20143E 00
0.90000E 01	0.11814E 00	0.13640E 00	0.16501E 00	0.18979E 00
0.10000E 02	0.11388E 00	0.13000E 00	0.15654E 00	0.17996E 00
0.15000E 02	0.10366E 00	0.11157E 00	0.12915E 00	0.14711E 00
0.20000E 02	0.10096E 00	0.10448E 00	0.11531E 00	0.12871E 00
0.30000E 02	0.10007E 00	0.10067E 00	0.10425E 00	0.11088E 00
0.40000E 02	0.10000E 00	0.10010E 00	0.10118E 00	0.10414E 00
0.50000E 02	0.10000E 00	0.10002E 00	0.10033E 00	0.10157E 00
0.60000E 02	0.10000E 00	0.10000E 00	0.10009E 00	0.10060E 00
0.70000E 02	0.10000E 00	0.10000E 00	0.10003E 00	0.10023E 00
0.80000E 02	0.10000E 00	0.10000E 00	0.10001E 00	0.10009E 00
0.90000E 02	0.10000E 00	0.10000E 00	0.10000E 00	0.10003E 00
0.10000E 03	0.10000E 00	0.10000E 00	0.10000E 00	0.10001E 00

TABLE 5: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.150E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.74390E 00	0.83871E 00	0.88263E 00	0.92559E 00
0.15000E 00	0.63944E 00	0.75900E 00	0.81967E 00	0.88248E 00
0.20000E 00	0.55858E 00	0.68868E 00	0.76023E 00	0.83878E 00
0.30000E 00	0.44818E 00	0.57857E 00	0.65895E 00	0.75635E 00
0.40000E 00	0.37872E 00	0.50062E 00	0.58104E 00	0.68544E 00
0.50000E 00	0.33155E 00	0.44397E 00	0.52135E 00	0.62658E 00
0.60000E 00	0.29751E 00	0.40125E 00	0.47479E 00	0.57803E 00
0.70000E 00	0.27181E 00	0.36794E 00	0.43762E 00	0.53771E 00
0.80000E 00	0.25175E 00	0.34122E 00	0.40730E 00	0.50387E 00
0.90000E 00	0.23570E 00	0.31928E 00	0.38208E 00	0.47511E 00
0.10000E 01	0.22262E 00	0.30092E 00	0.36075E 00	0.45037E 00
0.15000E 01	0.18327E 00	0.24081E 00	0.28923E 00	0.36492E 00
0.20000E 01	0.16569E 00	0.20776E 00	0.24799E 00	0.31377E 00
0.25000E 01	0.15743E 00	0.18754E 00	0.22106E 00	0.27919E 00
0.30000E 01	0.15352E 00	0.17460E 00	0.20230E 00	0.25401E 00
0.40000E 01	0.15079E 00	0.16065E 00	0.17886E 00	0.21969E 00
0.50000E 01	0.15018E 00	0.15461E 00	0.16606E 00	0.19769E 00
0.60000E 01	0.15004E 00	0.15199E 00	0.15895E 00	0.18291E 00
0.70000E 01	0.15001E 00	0.15086E 00	0.15499E 00	0.17278E 00
0.80000E 01	0.15000E 00	0.15037E 00	0.15278E 00	0.16579E 00
0.90000E 01	0.15000E 00	0.15016E 00	0.15155E 00	0.16095E 00
0.10000E 02	0.15000E 00	0.15007E 00	0.15086E 00	0.15759E 00
0.15000E 02	0.15000E 00	0.15000E 00	0.15005E 00	0.15122E 00
0.20000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15019E 00
0.30000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.40000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.50000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.60000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.70000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.80000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.90000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00
0.10000E 03	0.15000E 00	0.15000E 00	0.15000E 00	0.15000E 00

TABLE 6: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.150E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.94693E 00	0.96427E 00	0.97841E 00	0.98548E 00
0.15000E 00	0.91499E 00	0.94208E 00	0.96461E 00	0.97607E 00
0.20000E 00	0.88137E 00	0.91793E 00	0.94916E 00	0.96534E 00
0.30000E 00	0.81403E 00	0.86680E 00	0.91465E 00	0.94062E 00
0.40000E 00	0.75171E 00	0.81587E 00	0.87762E 00	0.91283E 00
0.50000E 00	0.69686E 00	0.76812E 00	0.84034E 00	0.88352E 00
0.60000E 00	0.64956E 00	0.72475E 00	0.80434E 00	0.85394E 00
0.70000E 00	0.60898E 00	0.68598E 00	0.77044E 00	0.82498E 00
0.80000E 00	0.57404E 00	0.65152E 00	0.73897E 00	0.79718E 00
0.90000E 00	0.54378E 00	0.62090E 00	0.71000E 00	0.77085E 00
0.10000E 01	0.51736E 00	0.59362E 00	0.68344E 00	0.74610E 00
0.15000E 01	0.42363E 00	0.49320E 00	0.58005E 00	0.64487E 00
0.20000E 01	0.36599E 00	0.42906E 00	0.51004E 00	0.57258E 00
0.25000E 01	0.32641E 00	0.38418E 00	0.45956E 00	0.51894E 00
0.30000E 01	0.29724E 00	0.35070E 00	0.42122E 00	0.47749E 00
0.40000E 01	0.25660E 00	0.30344E 00	0.36624E 00	0.41713E 00
0.50000E 01	0.22940E 00	0.27118E 00	0.32815E 00	0.37479E 00
0.60000E 01	0.20998E 00	0.24751E 00	0.29984E 00	0.34306E 00
0.70000E 01	0.19563E 00	0.22934E 00	0.27776E 00	0.31817E 00
0.80000E 01	0.18484E 00	0.21498E 00	0.25997E 00	0.29800E 00
0.90000E 01	0.17665E 00	0.20344E 00	0.24527E 00	0.28122E 00
0.10000E 02	0.17040E 00	0.19405E 00	0.23292E 00	0.26701E 00
0.15000E 02	0.15537E 00	0.16699E 00	0.19281E 00	0.21914E 00
0.20000E 02	0.15141E 00	0.15658E 00	0.17249E 00	0.19217E 00
0.30000E 02	0.15010E 00	0.15099E 00	0.15625E 00	0.16599E 00
0.40000E 02	0.15001E 00	0.15015E 00	0.15173E 00	0.15608E 00
0.50000E 02	0.15000E 00	0.15002E 00	0.15048E 00	0.15231E 00
0.60000E 02	0.15000E 00	0.15000E 00	0.15013E 00	0.15088E 00
0.70000E 02	0.15000E 00	0.15000E 00	0.15004E 00	0.15033E 00
0.80000E 02	0.15000E 00	0.15000E 00	0.15001E 00	0.15013E 00
0.90000E 02	0.15000E 00	0.15000E 00	0.15000E 00	0.15005E 00
0.10000E 03	0.15000E 00	0.15000E 00	0.15000E 00	0.15002E 00

TABLE 7: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.250E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.87966E 00	0.93491E 00	0.95723E 00	0.97666E 00
0.15000E 00	0.81504E 00	0.89635E 00	0.93088E 00	0.96176E 00
0.20000E 00	0.75437E 00	0.85658E 00	0.90257E 00	0.94511E 00
0.30000E 00	0.65200E 00	0.77971E 00	0.84401E 00	0.90815E 00
0.40000E 00	0.57416E 00	0.71168E 00	0.78754E 00	0.86889E 00
0.50000E 00	0.51514E 00	0.65400E 00	0.73612E 00	0.82980E 00
0.60000E 00	0.46956E 00	0.60567E 00	0.69056E 00	0.79242E 00
0.70000E 00	0.43362E 00	0.56511E 00	0.65063E 00	0.75751E 00
0.80000E 00	0.40473E 00	0.53080E 00	0.61570E 00	0.72535E 00
0.90000E 00	0.38114E 00	0.50150E 00	0.58506E 00	0.69593E 00
0.10000E 01	0.36162E 00	0.47625E 00	0.55804E 00	0.66909E 00
0.15000E 01	0.30165E 00	0.38939E 00	0.46070E 00	0.56567E 00
0.20000E 01	0.27441E 00	0.33943E 00	0.40047E 00	0.49647E 00
0.25000E 01	0.26158E 00	0.30833E 00	0.35988E 00	0.44702E 00
0.30000E 01	0.25548E 00	0.28828E 00	0.33115E 00	0.40987E 00
0.40000E 01	0.25123E 00	0.26658E 00	0.29491E 00	0.35796E 00
0.50000E 01	0.25027E 00	0.25718E 00	0.27501E 00	0.32411E 00
0.60000E 01	0.25006E 00	0.25311E 00	0.26394E 00	0.30121E 00
0.70000E 01	0.25001E 00	0.25134E 00	0.25777E 00	0.28547E 00
0.80000E 01	0.25000E 00	0.25058E 00	0.25433E 00	0.27459E 00
0.90000E 01	0.25000E 00	0.25025E 00	0.25241E 00	0.26705E 00
0.10000E 02	0.25000E 00	0.25011E 00	0.25134E 00	0.26183E 00
0.15000E 02	0.25000E 00	0.25000E 00	0.25007E 00	0.25189E 00
0.20000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25030E 00
0.30000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25001E 00
0.40000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.50000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.60000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.70000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.80000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.90000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00
0.10000E 03	0.25000E 00	0.25000E 00	0.25000E 00	0.25000E 00

TABLE 8: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.250E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.98518E 00	0.99134E 00	0.99568E 00	0.99752E 00
0.15000E 00	0.97555E 00	0.98564E 00	0.99279E 00	0.99586E 00
0.20000E 00	0.96457E 00	0.97903E 00	0.98940E 00	0.99388E 00
0.30000E 00	0.93930E 00	0.96329E 00	0.98108E 00	0.98895E 00
0.40000E 00	0.91093E 00	0.94474E 00	0.97081E 00	0.98270E 00
0.50000E 00	0.88109E 00	0.92416E 00	0.95881E 00	0.97515E 00
0.60000E 00	0.85105E 00	0.90234E 00	0.94539E 00	0.96642E 00
0.70000E 00	0.82171E 00	0.87998E 00	0.93088E 00	0.95667E 00
0.80000E 00	0.79362E 00	0.85762E 00	0.91564E 00	0.94606E 00
0.90000E 00	0.76705E 00	0.83564E 00	0.89995E 00	0.93478E 00
0.10000E 01	0.74213E 00	0.81431E 00	0.88408E 00	0.92301E 00
0.15000E 01	0.64054E 00	0.72090E 00	0.80745E 00	0.86165E 00
0.20000E 01	0.56830E 00	0.64891E 00	0.74135E 00	0.80352E 00
0.25000E 01	0.51482E 00	0.59314E 00	0.68654E 00	0.75232E 00
0.30000E 01	0.47360E 00	0.54888E 00	0.64110E 00	0.70812E 00
0.40000E 01	0.41398E 00	0.48295E 00	0.57062E 00	0.63693E 00
0.50000E 01	0.37290E 00	0.43591E 00	0.51848E 00	0.58252E 00
0.60000E 01	0.34313E 00	0.40049E 00	0.47815E 00	0.53957E 00
0.70000E 01	0.32096E 00	0.37289E 00	0.44588E 00	0.50468E 00
0.80000E 01	0.30422E 00	0.35087E 00	0.41940E 00	0.47568E 00
0.90000E 01	0.29149E 00	0.33306E 00	0.39724E 00	0.45113E 00
0.10000E 02	0.28176E 00	0.31853E 00	0.37845E 00	0.43004E 00
0.15000E 02	0.25837E 00	0.27646E 00	0.31661E 00	0.35734E 00
0.20000E 02	0.25220E 00	0.26025E 00	0.28503E 00	0.31563E 00
0.30000E 02	0.25015E 00	0.25154E 00	0.25973E 00	0.27490E 00
0.40000E 02	0.25001E 00	0.25023E 00	0.25270E 00	0.25947E 00
0.50000E 02	0.25000E 00	0.25003E 00	0.25075E 00	0.25360E 00
0.60000E 02	0.25000E 00	0.25001E 00	0.25021E 00	0.25137E 00
0.70000E 02	0.25000E 00	0.25000E 00	0.25006E 00	0.25052E 00
0.80000E 02	0.25000E 00	0.25000E 00	0.25002E 00	0.25020E 00
0.90000E 02	0.25000E 00	0.25000E 00	0.25000E 00	0.25008E 00
0.10000E 03	0.25000E 00	0.25000E 00	0.25000E 00	0.25003E 00

TABLE 9: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.375E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.94506E 00	0.97443E 00	0.98494E 00	0.99306E 00
0.15000E 00	0.91164E 00	0.95806E 00	0.97511E 00	0.98845E 00
0.20000E 00	0.87650E 00	0.93983E 00	0.96390E 00	0.98307E 00
0.30000E 00	0.80670E 00	0.89966E 00	0.93804E 00	0.97011E 00
0.40000E 00	0.74306E 00	0.85756E 00	0.90906E 00	0.95455E 00
0.50000E 00	0.68801E 00	0.81623E 00	0.87863E 00	0.93696E 00
0.60000E 00	0.64139E 00	0.77725E 00	0.84810E 00	0.91797E 00
0.70000E 00	0.60214E 00	0.74132E 00	0.81836E 00	0.89816E 00
0.80000E 00	0.56910E 00	0.70860E 00	0.78997E 00	0.87803E 00
0.90000E 00	0.54119E 00	0.67900E 00	0.76320E 00	0.85796E 00
0.10000E 01	0.51753E 00	0.65228E 00	0.73816E 00	0.83823E 00
0.15000E 01	0.44209E 00	0.55258E 00	0.63705E 00	0.74929E 00
0.20000E 01	0.40685E 00	0.49064E 00	0.56679E 00	0.67862E 00
0.25000E 01	0.39012E 00	0.45087E 00	0.51674E 00	0.62315E 00
0.30000E 01	0.38217E 00	0.42493E 00	0.48033E 00	0.57909E 00
0.40000E 01	0.37660E 00	0.39665E 00	0.43357E 00	0.51473E 00
0.50000E 01	0.37536E 00	0.38438E 00	0.40766E 00	0.47145E 00
0.60000E 01	0.37508E 00	0.37906E 00	0.39321E 00	0.44180E 00
0.70000E 01	0.37502E 00	0.37675E 00	0.38515E 00	0.42131E 00
0.80000E 01	0.37500E 00	0.37576E 00	0.38065E 00	0.40712E 00
0.90000E 01	0.37500E 00	0.37533E 00	0.37815E 00	0.39728E 00
0.10000E 02	0.37500E 00	0.37514E 00	0.37675E 00	0.39045E 00
0.15000E 02	0.37500E 00	0.37500E 00	0.37509E 00	0.37747E 00
0.20000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37540E 00
0.30000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37501E 00
0.40000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.50000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.60000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.70000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.80000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.90000E 02	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00
0.10000E 03	0.37500E 00	0.37500E 00	0.37500E 00	0.37500E 00

TABLE 10: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.375E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.99615E 00	0.99810E 00	0.99925E 00	0.99964E 00
0.15000E 00	0.99357E 00	0.99682E 00	0.99873E 00	0.99940E 00
0.20000E 00	0.99053E 00	0.99529E 00	0.99812E 00	0.99911E 00
0.30000E 00	0.98302E 00	0.99145E 00	0.99655E 00	0.99835E 00
0.40000E 00	0.97368E 00	0.98652E 00	0.99446E 00	0.99733E 00
0.50000E 00	0.96269E 00	0.98049E 00	0.99183E 00	0.99601E 00
0.60000E 00	0.95029E 00	0.97342E 00	0.98862E 00	0.99437E 00
0.70000E 00	0.93679E 00	0.96540E 00	0.98483E 00	0.99238E 00
0.80000E 00	0.92251E 00	0.95655E 00	0.98046E 00	0.99002E 00
0.90000E 00	0.90771E 00	0.94701E 00	0.97555E 00	0.98730E 00
0.10000E 01	0.89263E 00	0.93692E 00	0.97013E 00	0.98421E 00
0.15000E 01	0.81879E 00	0.88243E 00	0.93726E 00	0.96385E 00
0.20000E 01	0.75398E 00	0.82852E 00	0.89939E 00	0.93755E 00
0.25000E 01	0.69971E 00	0.77962E 00	0.86108E 00	0.90846E 00
0.30000E 01	0.65452E 00	0.73658E 00	0.82459E 00	0.87882E 00
0.40000E 01	0.58459E 00	0.66602E 00	0.75990E 00	0.82245E 00
0.50000E 01	0.53379E 00	0.61149E 00	0.70614E 00	0.77253E 00
0.60000E 01	0.49598E 00	0.56848E 00	0.66143E 00	0.72922E 00
0.70000E 01	0.46744E 00	0.53400E 00	0.62389E 00	0.69171E 00
0.80000E 01	0.44573E 00	0.50602E 00	0.59203E 00	0.65905E 00
0.90000E 01	0.42916E 00	0.48314E 00	0.56474E 00	0.63043E 00
0.10000E 02	0.41648E 00	0.46435E 00	0.54120E 00	0.60519E 00
0.15000E 02	0.38593E 00	0.40957E 00	0.46188E 00	0.51446E 00
0.20000E 02	0.37788E 00	0.38839E 00	0.42075E 00	0.46063E 00
0.30000E 02	0.37520E 00	0.37701E 00	0.38771E 00	0.40753E 00
0.40000E 02	0.37501E 00	0.37530E 00	0.37853E 00	0.38737E 00
0.50000E 02	0.37500E 00	0.37504E 00	0.37598E 00	0.37970E 00
0.60000E 02	0.37500E 00	0.37501E 00	0.37527E 00	0.37679E 00
0.70000E 02	0.37500E 00	0.37500E 00	0.37508E 00	0.37568E 00
0.80000E 02	0.37500E 00	0.37500E 00	0.37502E 00	0.37526E 00
0.90000E 02	0.37500E 00	0.37500E 00	0.37501E 00	0.37510E 00
0.10000E 03	0.37500E 00	0.37500E 00	0.37500E 00	0.37504E 00

TABLE 11: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.500E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.97232E 00	0.98872E 00	0.99400E 00	0.99765E 00
0.15000E 00	0.95455E 00	0.98126E 00	0.98999E 00	0.99606E 00
0.20000E 00	0.93480E 00	0.97264E 00	0.98528E 00	0.99417E 00
0.30000E 00	0.89178E 00	0.95237E 00	0.97385E 00	0.98944E 00
0.40000E 00	0.84760E 00	0.92899E 00	0.95998E 00	0.98340E 00
0.50000E 00	0.80526E 00	0.90374E 00	0.94412E 00	0.97608E 00
0.60000E 00	0.76636E 00	0.87772E 00	0.92681E 00	0.96759E 00
0.70000E 00	0.73150E 00	0.85181E 00	0.90859E 00	0.95807E 00
0.80000E 00	0.70071E 00	0.82661E 00	0.88990E 00	0.94769E 00
0.90000E 00	0.67374E 00	0.80250E 00	0.87114E 00	0.93664E 00
0.10000E 01	0.65025E 00	0.77970E 00	0.85258E 00	0.92508E 00
0.15000E 01	0.57213E 00	0.68667E 00	0.76824E 00	0.86464E 00
0.20000E 01	0.53441E 00	0.62364E 00	0.70169E 00	0.80741E 00
0.25000E 01	0.51636E 00	0.58171E 00	0.65114E 00	0.75741E 00
0.30000E 01	0.50775E 00	0.55393E 00	0.61313E 00	0.71499E 00
0.40000E 01	0.50173E 00	0.52343E 00	0.56326E 00	0.64956E 00
0.50000E 01	0.50039E 00	0.51015E 00	0.53533E 00	0.60392E 00
0.60000E 01	0.50009E 00	0.50439E 00	0.51971E 00	0.57216E 00
0.70000E 01	0.50002E 00	0.50190E 00	0.51098E 00	0.55009E 00
0.80000E 01	0.50000E 00	0.50082E 00	0.50612E 00	0.53476E 00
0.90000E 01	0.50000E 00	0.50035E 00	0.50341E 00	0.52411E 00
0.10000E 02	0.50000E 00	0.50015E 00	0.50190E 00	0.51672E 00
0.15000E 02	0.50000E 00	0.50000E 00	0.50010E 00	0.50268E 00
0.20000E 02	0.50000E 00	0.50000E 00	0.50001E 00	0.50043E 00
0.30000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50001E 00
0.40000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.50000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.60000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.70000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.80000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.90000E 02	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00
0.10000E 03	0.50000E 00	0.50000E 00	0.50000E 00	0.50000E 00

TABLE 12: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.500E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.99886E 00	0.99952E 00	0.99985E 00	0.99994E 00
0.15000E 00	0.99808E 00	0.99920E 00	0.99975E 00	0.99990E 00
0.20000E 00	0.99715E 00	0.99880E 00	0.99962E 00	0.99985E 00
0.30000E 00	0.99478E 00	0.99779E 00	0.99929E 00	0.99972E 00
0.40000E 00	0.99168E 00	0.99643E 00	0.99884E 00	0.99954E 00
0.50000E 00	0.98781E 00	0.99469E 00	0.99825E 00	0.99930E 00
0.60000E 00	0.98316E 00	0.99253E 00	0.99750E 00	0.99899E 00
0.70000E 00	0.97776E 00	0.98994E 00	0.99657E 00	0.99860E 00
0.80000E 00	0.97165E 00	0.98690E 00	0.99545E 00	0.99811E 00
0.90000E 00	0.96490E 00	0.98343E 00	0.99411E 00	0.99752E 00
0.10000E 01	0.95759E 00	0.97953E 00	0.99255E 00	0.99682E 00
0.15000E 01	0.91548E 00	0.95464E 00	0.98139E 00	0.99140E 00
0.20000E 01	0.87028E 00	0.92389E 00	0.96517E 00	0.98255E 00
0.25000E 01	0.82696E 00	0.89110E 00	0.94536E 00	0.97058E 00
0.30000E 01	0.78746E 00	0.85865E 00	0.92354E 00	0.95621E 00
0.40000E 01	0.72107E 00	0.79895E 00	0.87837E 00	0.92337E 00
0.50000E 01	0.66964E 00	0.74803E 00	0.83517E 00	0.88881E 00
0.60000E 01	0.63009E 00	0.70547E 00	0.79581E 00	0.85512E 00
0.70000E 01	0.59973E 00	0.67012E 00	0.76066E 00	0.82344E 00
0.80000E 01	0.57644E 00	0.64082E 00	0.72953E 00	0.79415E 00
0.90000E 01	0.55857E 00	0.61655E 00	0.70205E 00	0.76732E 00
0.10000E 02	0.54488E 00	0.59646E 00	0.67784E 00	0.74283E 00
0.15000E 02	0.51183E 00	0.53741E 00	0.59386E 00	0.64994E 00
0.20000E 02	0.50311E 00	0.51450E 00	0.54950E 00	0.59252E 00
0.30000E 02	0.50022E 00	0.50217E 00	0.51376E 00	0.53520E 00
0.40000E 02	0.50001E 00	0.50033E 00	0.50382E 00	0.51339E 00
0.50000E 02	0.50000E 00	0.50005E 00	0.50106E 00	0.50509E 00
0.60000E 02	0.50000E 00	0.50001E 00	0.50029E 00	0.50193E 00
0.70000E 02	0.50000E 00	0.50000E 00	0.50008E 00	0.50073E 00
0.80000E 02	0.50000E 00	0.50000E 00	0.50002E 00	0.50028E 00
0.90000E 02	0.50000E 00	0.50000E 00	0.50001E 00	0.50011E 00
0.10000E 03	0.50000E 00	0.50000E 00	0.50000E 00	0.50004E 00

TABLE 13: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.600E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.1000E 00	0.98345E 00	0.99387E 00	0.99698E 00	0.99896E 00
0.1500E 00	0.97256E 00	0.98976E 00	0.99494E 00	0.99825E 00
0.2000E 00	0.96015E 00	0.98493E 00	0.99252E 00	0.99739E 00
0.3000E 00	0.93186E 00	0.97321E 00	0.98648E 00	0.99522E 00
0.4000E 00	0.90089E 00	0.95903E 00	0.97887E 00	0.99236E 00
0.5000E 00	0.86938E 00	0.94290E 00	0.96977E 00	0.98877E 00
0.6000E 00	0.83888E 00	0.92542E 00	0.95937E 00	0.98445E 00
0.7000E 00	0.81035E 00	0.90716E 00	0.94792E 00	0.97941E 00
0.8000E 00	0.78428E 00	0.88861E 00	0.93565E 00	0.97369E 00
0.9000E 00	0.76082E 00	0.87018E 00	0.92281E 00	0.96735E 00
0.1000E 01	0.73996E 00	0.85217E 00	0.90963E 00	0.96046E 00
0.1500E 01	0.66823E 00	0.77347E 00	0.84427E 00	0.92054E 00
0.2000E 01	0.63268E 00	0.71644E 00	0.78746E 00	0.87756E 00
0.2500E 01	0.61555E 00	0.67739E 00	0.74203E 00	0.83668E 00
0.3000E 01	0.60737E 00	0.65120E 00	0.70692E 00	0.80005E 00
0.4000E 01	0.60165E 00	0.62228E 00	0.66006E 00	0.74096E 00
0.5000E 01	0.60037E 00	0.60966E 00	0.63358E 00	0.69846E 00
0.6000E 01	0.60008E 00	0.60418E 00	0.61874E 00	0.66853E 00
0.7000E 01	0.60002E 00	0.60181E 00	0.61044E 00	0.64761E 00
0.8000E 01	0.60000E 00	0.60078E 00	0.60582E 00	0.63305E 00
0.9000E 01	0.60000E 00	0.60034E 00	0.60324E 00	0.62293E 00
0.1000E 02	0.60000E 00	0.60014E 00	0.60180E 00	0.61590E 00
0.1500E 02	0.60000E 00	0.60000E 00	0.60010E 00	0.60255E 00
0.2000E 02	0.60000E 00	0.60000E 00	0.60001E 00	0.60041E 00
0.3000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60001E 00
0.4000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.5000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.6000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.7000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.8000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.9000E 02	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00
0.1000E 03	0.60000E 00	0.60000E 00	0.60000E 00	0.60000E 00

TABLE 14: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.600E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.1000E 00	0.99954E 00	0.99983E 00	0.99996E 00	0.99999E 00
0.1500E 00	0.99923E 00	0.99972E 00	0.99992E 00	0.99997E 00
0.2000E 00	0.99885E 00	0.99958E 00	0.99989E 00	0.99996E 00
0.3000E 00	0.99787E 00	0.99921E 00	0.99979E 00	0.99993E 00
0.4000E 00	0.99657E 00	0.99871E 00	0.99965E 00	0.99988E 00
0.5000E 00	0.99489E 00	0.99806E 00	0.99947E 00	0.99982E 00
0.6000E 00	0.99280E 00	0.99723E 00	0.99923E 00	0.99974E 00
0.7000E 00	0.99029E 00	0.99620E 00	0.99893E 00	0.99963E 00
0.8000E 00	0.98735E 00	0.99495E 00	0.99856E 00	0.99949E 00
0.9000E 00	0.98397E 00	0.99348E 00	0.99810E 00	0.99932E 00
0.1000E 01	0.98018E 00	0.99177E 00	0.99755E 00	0.99911E 00
0.1500E 01	0.95589E 00	0.97965E 00	0.99322E 00	0.99736E 00
0.2000E 01	0.92585E 00	0.96229E 00	0.98592E 00	0.99406E 00
0.2500E 01	0.89389E 00	0.94140E 00	0.97575E 00	0.98896E 00
0.3000E 01	0.86250E 00	0.91872E 00	0.96325E 00	0.98207E 00
0.4000E 01	0.80592E 00	0.87269E 00	0.93386E 00	0.96383E 00
0.5000E 01	0.75963E 00	0.82996E 00	0.90216E 00	0.94168E 00
0.6000E 01	0.72306E 00	0.79242E 00	0.87088E 00	0.91777E 00
0.7000E 01	0.69459E 00	0.76029E 00	0.84136E 00	0.89355E 00
0.8000E 01	0.67259E 00	0.73317E 00	0.81422E 00	0.86992E 00
0.9000E 01	0.65567E 00	0.71047E 00	0.78963E 00	0.84737E 00
0.1000E 02	0.64267E 00	0.69155E 00	0.76756E 00	0.82617E 00
0.1500E 02	0.61125E 00	0.63558E 00	0.68912E 00	0.74184E 00
0.2000E 02	0.60296E 00	0.61379E 00	0.64707E 00	0.68787E 00
0.3000E 02	0.60020E 00	0.60207E 00	0.61308E 00	0.63348E 00
0.4000E 02	0.60001E 00	0.60031E 00	0.60363E 00	0.61273E 00
0.5000E 02	0.60000E 00	0.60005E 00	0.60101E 00	0.60484E 00
0.6000E 02	0.60000E 00	0.60001E 00	0.60028E 00	0.60184E 00
0.7000E 02	0.60000E 00	0.60000E 00	0.60008E 00	0.60070E 00
0.8000E 02	0.60000E 00	0.60000E 00	0.60002E 00	0.60027E 00
0.9000E 02	0.60000E 00	0.60000E 00	0.60001E 00	0.60010E 00
0.1000E 03	0.60000E 00	0.60000E 00	0.60000E 00	0.60004E 00

TABLE 15: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.800E 00

X/B-Value	B/ALFAz = 0.500E 01 Zeta-Value	B/ALFAz = 0.100E 02 Zeta-Value	B/ALFAz = 0.150E 02 Zeta-Value	B/ALFAz = 0.250E 02 Zeta-Value
0.10000E 00	0.99440E 00	0.99822E 00	0.99923E 00	0.99979E 00
0.15000E 00	0.99063E 00	0.99701E 00	0.99870E 00	0.99964E 00
0.20000E 00	0.98621E 00	0.99556E 00	0.99807E 00	0.99946E 00
0.30000E 00	0.97562E 00	0.99191E 00	0.99644E 00	0.99899E 00
0.40000E 00	0.96316E 00	0.98726E 00	0.99429E 00	0.99836E 00
0.50000E 00	0.94951E 00	0.98164E 00	0.99159E 00	0.99754E 00
0.60000E 00	0.93537E 00	0.97515E 00	0.98834E 00	0.99650E 00
0.70000E 00	0.92135E 00	0.96795E 00	0.98453E 00	0.99522E 00
0.80000E 00	0.90788E 00	0.96020E 00	0.98022E 00	0.99370E 00
0.90000E 00	0.89528E 00	0.95208E 00	0.97546E 00	0.99193E 00
0.10000E 01	0.88371E 00	0.94375E 00	0.97031E 00	0.98989E 00
0.15000E 01	0.84180E 00	0.90336E 00	0.94115E 00	0.97620E 00
0.20000E 01	0.82015E 00	0.87084E 00	0.91170E 00	0.95825E 00
0.25000E 01	0.80961E 00	0.84751E 00	0.88609E 00	0.93864E 00
0.30000E 01	0.80456E 00	0.83155E 00	0.86541E 00	0.91938E 00
0.40000E 01	0.80102E 00	0.81376E 00	0.83702E 00	0.88586E 00
0.50000E 01	0.80023E 00	0.80597E 00	0.82074E 00	0.86049E 00
0.60000E 01	0.80005E 00	0.80258E 00	0.81158E 00	0.84225E 00
0.70000E 01	0.80001E 00	0.80112E 00	0.80645E 00	0.82939E 00
0.80000E 01	0.80000E 00	0.80048E 00	0.80360E 00	0.82042E 00
0.90000E 01	0.80000E 00	0.80021E 00	0.80200E 00	0.81417E 00
0.10000E 02	0.80000E 00	0.80009E 00	0.80111E 00	0.80983E 00
0.15000E 02	0.80000E 00	0.80000E 00	0.80006E 00	0.80157E 00
0.20000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80025E 00
0.30000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80001E 00
0.40000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.50000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.60000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.70000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.80000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.90000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00
0.10000E 03	0.80000E 00	0.80000E 00	0.80000E 00	0.80000E 00

TABLE 16: Dilution Factor(ZETA) versus X/B for Penetration Ratio(H/B) = 0.800E 00

X/B-Value	B/ALFAz = 0.350E 02 Zeta-Value	B/ALFAz = 0.500E 02 Zeta-Value	B/ALFAz = 0.750E 02 Zeta-Value	B/ALFAz = 0.100E 03 Zeta-Value
0.10000E 00	0.99992E 00	0.99998E 00	0.10000E 01	0.10000E 01
0.15000E 00	0.99987E 00	0.99996E 00	0.99999E 00	0.10000E 01
0.20000E 00	0.99980E 00	0.99994E 00	0.99999E 00	0.10000E 01
0.30000E 00	0.99963E 00	0.99989E 00	0.99998E 00	0.10000E 01
0.40000E 00	0.99939E 00	0.99982E 00	0.99997E 00	0.99999E 00
0.50000E 00	0.99908E 00	0.99973E 00	0.99995E 00	0.99999E 00
0.60000E 00	0.99867E 00	0.99961E 00	0.99992E 00	0.99998E 00
0.70000E 00	0.99816E 00	0.99945E 00	0.99989E 00	0.99997E 00
0.80000E 00	0.99754E 00	0.99925E 00	0.99985E 00	0.99996E 00
0.90000E 00	0.99679E 00	0.99900E 00	0.99980E 00	0.99995E 00
0.10000E 01	0.99590E 00	0.99870E 00	0.99973E 00	0.99993E 00
0.15000E 01	0.98929E 00	0.99625E 00	0.99915E 00	0.99976E 00
0.20000E 01	0.97915E 00	0.99187E 00	0.99794E 00	0.99938E 00
0.25000E 01	0.96636E 00	0.98548E 00	0.99587E 00	0.99865E 00
0.30000E 01	0.95210E 00	0.97734E 00	0.99280E 00	0.99745E 00
0.40000E 01	0.92305E 00	0.95766E 00	0.98372E 00	0.99334E 00
0.50000E 01	0.89698E 00	0.93639E 00	0.97151E 00	0.98683E 00
0.60000E 01	0.87539E 00	0.91597E 00	0.95749E 00	0.97827E 00
0.70000E 01	0.85820E 00	0.89757E 00	0.94283E 00	0.96826E 00
0.80000E 01	0.84476E 00	0.88155E 00	0.92840E 00	0.95740E 00
0.90000E 01	0.83437E 00	0.86789E 00	0.91469E 00	0.94622E 00
0.10000E 02	0.82636E 00	0.85639E 00	0.90200E 00	0.93510E 00
0.15000E 02	0.80696E 00	0.82198E 00	0.85493E 00	0.88689E 00
0.20000E 02	0.80183E 00	0.80852E 00	0.82908E 00	0.85418E 00
0.30000E 02	0.80013E 00	0.80128E 00	0.80809E 00	0.82069E 00
0.40000E 02	0.80001E 00	0.80019E 00	0.80225E 00	0.80787E 00
0.50000E 02	0.80000E 00	0.80003E 00	0.80062E 00	0.80299E 00
0.60000E 02	0.80000E 00	0.80000E 00	0.80017E 00	0.80114E 00
0.70000E 02	0.80000E 00	0.80000E 00	0.80005E 00	0.80043E 00
0.80000E 02	0.80000E 00	0.80000E 00	0.80001E 00	0.80016E 00
0.90000E 02	0.80000E 00	0.80000E 00	0.80000E 00	0.80006E 00
0.10000E 03	0.80000E 00	0.80000E 00	0.80000E 00	0.80002E 00

**Development of Land Disposal Banning Decisions
Under Uncertainty**

**Environmental Research Laboratory
U.S. Environmental Protection Agency
Athens, Georgia**

APPENDIX C

Development of Land Disposal Banning Decisions Under Uncertainty

1.0 Introduction

1.1 Background and Relevant History

Increasingly, the Agency is required to regulate the use or disposal of toxic chemicals including hazardous wastes before such materials become part of a specific, permitted system for use or disposal. The regulatory approach developed in response to such legislative mandates is necessarily generic in nature and must be largely based upon inherent chemical properties. In general, however, the behavior of specific chemicals in the environment and the subsequent health risks posed to populations are highly dependent on both the environmental setting and the chemical properties.

An obvious dilemma arises from the requirement to regulate (based on risks posed to human health) on a national basis when the exposure is determined by a combination of site- and chemical-specific factors. The errors and uncertainties inherent in site-specific analyses are further complicated by uncertainty about how to properly specify a national environment. The land disposal restrictions requirements of the RCRA amendments pose just such a dilemma. Specifically, how can the Agency determine the national acceptability (or specify required treatment levels) of the land disposal of hazardous wastes based on an evaluation of risk to human health?

As initially conceived by the OSW, "screening levels were to be developed to specify maximum acceptable contaminant concentrations in waste extracts. Concentration-based fate and transport models for groundwater, air, and surface waters were to be developed for

a "worst case" or "reasonable worst case" land disposal scenario to allow back calculation of acceptable leachate contaminations commencing from a reference dose (or a q_1^* concentration for carcinogens) at a point of potential human exposure at a specified distance directly down gradient (e.g. water supply well, surface water intake, or downwind exposure point). As this approach was further developed two major issues emerged: (1) how can the reasonable worst case be defined to avoid unrealistic situations and permit some site-specificity and; (2) how can the considerable uncertainties inherent in any analysis of this nature be factored into the process.

1.2 Objectives

The objectives of the material presented in the following sections are to develop and demonstrate a procedure for applying the initially-developed concept that accommodates the possible variation in environmental settings, the uncertainties in specific chemical properties, and the range of impact of engineered system releases from disposal facilities. The developed approach is intended to present decision-makers, for each chemical, the level of treatment (or restriction) required to achieve any desired level of "protection" expressed as the percentage of all possible land disposal scenarios that are more protective than the level chosen. In this manner the scenario selected can be evaluated for its probability of occurrence. At one level one may say, for example, that 90% of all possible sites are more protective than the health based threshold while 10% of all possible sites are less protective. In this case the decision-maker can be assured that the level of treatment selected will ensure that downgradient concentrations will not exceed the specified target concentration in more than 10% of all possible environmental settings.

Meeting this overall objective requires that the following subobjectives be met:

- o Identify and characterize the environmental and chemical properties contributing to variation in the release, fate, and transport of chemicals.
- o Identify and characterize the errors in estimating or measuring relevant chemical properties.
- o Develop a technique to synthesize input data for use in investigating model variations and uncertainties.

2.0 The Concept of Uncertainty

2.1 Definitions

Uncertainty is generally defined as the absence of information about the subject under consideration. In this context the absence or lack of information usually refers to information about past, present, or future events, values, or conditions. Rowe (1977) describes two basic types of uncertainty: (1) descriptive uncertainty and (2) measurement uncertainty. Descriptive uncertainty is a measure of the lack of information about the identity of variables that define the system under study. Measurement uncertainty is a measure of the lack of information about how to quantify those variables. Uncertainty analyses are typically completed for measurement uncertainty with the assistance of a system model that transforms inputs to outputs via some mathematical description of the "real world" system of interest. The objective is to quantify the model output uncertainties given a knowledge of the input uncertainties. Several formal methods exist to conduct such analyses as summarized by Cox and Baybutt (1981). Usually, such analyses do not address descriptive uncertainty. That is, the model itself is assumed to adequately describe the "real

world" system being simulated. Uncertainty (descriptive) about the system model will almost always exist and in some cases this uncertainty is acknowledged but ignored because of policy objectives. In other cases, descriptive uncertainty is unknown (and unknowable) and is simply ignored.

2.2 Application to the Land Disposal Banning Process

The application of uncertainty analyses to the procedures developed for setting limits on land disposal is highly desirable. Indeed, the RCRA amendments directed that such regulations be developed with proper accounting for the long-term uncertainties of land disposal technologies. It follows that any provisions granted for continued use of land disposal should properly account for the uncertainties in the technical basis for such provisions. Following the previous discussion of uncertainty, two problems must be addressed: descriptive uncertainty and measurement uncertainty.

The choice of the models used in the procedure and the conceptualization of the physical system acknowledges the descriptive uncertainty in the process. In this sense, assumptions made about land disposal performance, location of fixed, downgradient exposure points, and the use of unit risk numbers (e.g., Reference Dose Values) are ways to include descriptive uncertainty. For example, it is generally conceded that landfills have and could in the future be designed such that their lower boundary is near or actually in the saturated zone. The uncertainty about this descriptive variable is simply handled by assuming all facilities are located such that any leakage will be directly to the saturated zone. Additional, specific assumptions and decisions imbedded in the procedure that fall into the descriptive uncertainty category are:

- (1) all engineered systems fail given sufficient time
- (2) the advective-dispersion equation with reaction and sorption is an accurate description of chemical fate and transport in groundwater, surface water, and air
- (3) all processes that can degrade organics in groundwater other than hydrolysis are ignored
- (4) for metals and organics, equilibrium, reversible speciation and sorption are appropriate
- (5) the leach test (EP III) is an accurate measure of the leachate concentration resulting when failure of the facility occurs
- (6) the contaminant source is sufficiently large in mass to enable an assumption of an infinite source, i.e., down gradient contamination once reached will be maintained
- (7) transient behavior is unimportant and the overall response can be represented by steady-state estimates

Most of the assumptions listed above eliminate consideration of possible effects or variables that can conceivably lead to differences between the real and modeled systems. These expected differences resulting from omission of such effects represent uncertainty that is quantified only in one sense--their exclusion will result in a more conservative analysis. Conservative in this case simply means that a more restrictive outcome for land disposal is generated. To the extent that this is desired, the assumptions per se represent an accommodation of descriptive uncertainty. The modeling approach and its assumptions with their descriptive uncertainty in land-related hazardous waste management are consistent with the strong legislative presumption against land disposal.

The problem remains to evaluate measurement uncertainty. In this case, measurement is taken to mean assignment of specific values to each of the model variables and parameters. Once a model is chosen to represent the conceptualized system, specific values must be chosen to further describe the behavior of the modeled system. The groundwater model relies on two general types of data:

- data on the physical, chemical, and toxicological properties of the hazardous constituents (e.g., K_{OC} , K_{OW} , acid, neutral, and base-catalyzed hydrolysis rates, and Reference Dose)
- data on the environmental properties of land disposal settings that impact the release, fate, and transport of each hazardous constituent (e.g., groundwater velocity, thickness of the saturated zone, bulk density, groundwater temperature and pH, dispersivities)

Measurement uncertainty applied to the first data type is taken to mean the errors resulting from laboratory or theoretical analyses used to estimate their numerical values. In addition to experimental precision and accuracy, errors may arise in extrapolation from measurement conditions to environmental conditions (e.g., hydrolysis experiments conducted at elevated temperatures extrapolated to groundwater temperatures). For some parameters, most notably partition coefficients, semi-empirical methods are used to estimate values and in such cases the errors in the empiricisms contribute to uncertainty.

Measurement uncertainty applied to the second data type, characterization of the relevant environmental properties, is taken to mean the variation expected to occur across possible land disposal settings. In the strictest sense analyses of known variation are more properly referred

to as sensitivity analyses. Within the context of defining the range of possible regulatory outcomes, however, we elect to describe the characterization of variation as representing uncertainty. An important distinction is made here between variation (and measurement errors in characterizing that variation) at a given site and variation from site to site. Variation within a site is an important phenomenon in site-specific issues like permit evaluations, designs, and remedial action alternatives. The uncertainty in the performance of a given system will in large part reflect the uncertainty about the characterization of the given site. For example, the location of down-gradient monitoring wells to detect future leaks is chosen based on measurements that define the direction of groundwater flow. If uncertainty exists in the determination of the flow direction, then the performance of the detection system is also made uncertain. This "within-site" variation is not the most important source of uncertainty, however, for the land disposal banning process because we are concerned with a presumed national (generic) site. The major concern for this case is uncertainty in specifying the range of possible sites.

The generic nature of the present regulatory process leads to a major uncertainty in characterization of the presumed national site. Assuming it is possible to accurately measure site properties (e.g., groundwater velocities and pH) the problem reduces to one of properly specifying the many possible combinations of conditions that can exist in possible sites for land disposal. Uncertainty in this sense, is simply an explicit representation of the variation in site conditions that can exist. From such information the levels chosen for regulating land disposal restrictions can be easily referenced to the full range of possible outcomes or restrictions.

3.0 Technical Approach

3.1 Monte Carlo Analyses

The recent review by Cox and Baybutt (1981) noted five different methods for conducting uncertainty analyses. The choice of any one method is largely determined by the structure of the system model and the objective of the analyses. In some cases the methods are quite limited because of the requirements imposed for their use. For example, analytic techniques require a mathematically tractable system model that permits direct calculation of output uncertainties given the input distributions. In some cases, knowledge about the source of the output uncertainty given a multitude of possible input uncertainties is needed in order to minimize or reduce uncertainty; for these cases, techniques that randomly combine individual inputs to produce a single output are unacceptable because it is not possible to know the source of the uncertainty. However, the monte carlo method remains the most widely-used procedure and is suitable for investigating the land disposal restrictions process.

Before further elaboration on the monte carlo method it is useful to review the needs for the land disposal restrictions process:

- (1) portray in a concise manner the field of information describing an estimate of all possible exposures
- (2) demonstrate that all reasonable steps were taken to reflect the wide range of possible site conditions for land disposal systems
- (3) investigate the range and distribution of possible restrictions needed to insure specified levels of protection

- (4) properly reflect the uncertainty introduced by laboratory-measured model parameters for hydrolysis and sorption

The emphasis here is on the output of the modeling process. The exact cause of output variation is less important at this stage of developing the restrictions. For these reasons, the monte carlo approach is appropriate, relatively easy to implement, and computationally efficient for the system model used.

Figure 1 illustrates the monte carlo procedure used in this analysis. If $G(X)$ represents the desired output distribution for a given variable, i.e., C_L , the acceptable leachate concentration, then the process is represented by:

$$G(X) = f(F_1, F_2, \dots F_n) \quad (1)$$

where $f(\quad) =$ the system model

$F_1, F_2 \dots F_n =$ input distributions for
parameters or variables.

$G(X)$ is defined from the distribution produced from a large number of simulations, i.e., 5000 - 10,000.

Note from Figure 1 that two kinds of inputs are defined - parameters and forcing function values. Model parameters are representations of physical or chemical properties (e.g. soil bulk density, hydrolysis rate constants). Forcing function values correspond to variables or constants that specify conditions or states of the system. In some groundwater transport models such inputs are internally estimated rather than externally derived as shown here.

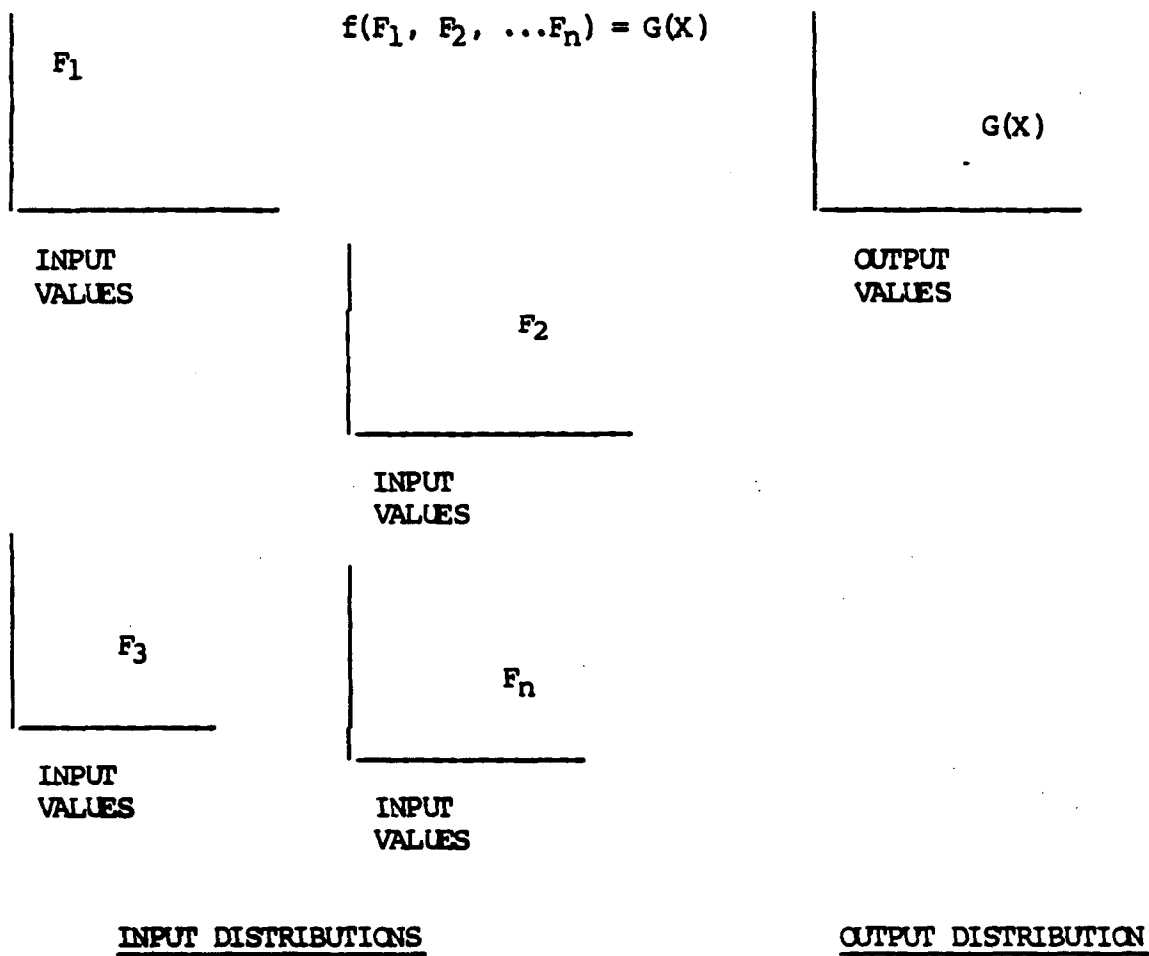
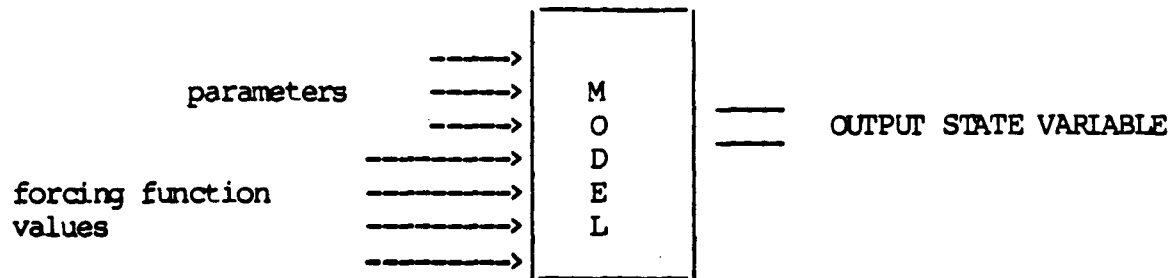


Figure 1. The Monte Carlo Process for Developing Distributional (Cumulative Frequencies) Results from Models

The monte carlo process proceeds as follows:

- o values from each input distribution are selected at random.
- o a value for the desired output variable, G , is computed for each randomly selected set of inputs
- o the input selection and computation steps are repeated a large number of times, e.g., 5,000
- o the output values are analyzed for presentation as a distribution.

An important point to remember in completing the monte carlo process is the notion that the derived output distribution, $G(X)$, is greatly influenced by the input distributions, $F_1, F_2 \dots F_n$. This is true even if the system model is known to be a perfectly correct representation of the physical system (i.e., no descriptive uncertainty).

3.2 Model Parameters and Input Data

The groundwater model parameters and input data requirements are given in detail in the mathematical description of the model. The major question for their use in monte carlo analyses is their relationship one to the other. In the simplest case where all input values are independent, the monte carlo process is straightforward. Once each input distribution is developed the process is executed as described. Where inputs are correlated, and therefore dependent, the proper specification of each distribution becomes much more difficult. For example, if F_2 is dependent upon F_1 then F_1 and F_2 have a joint distribution. That is, any value of F_2 is the probability of F_2 given a corresponding value of F_1 (and vice versa). For other situations where dependence exists among more than two variables the problem is further compounded with requirements for joint probability functions. Before developing input data distributions,

it is first necessary to determine the existence of any dependence among the data.

Table 1 illustrates the expected dependence among the groundwater model's parameters and input data. This table was constructed from a combination of documented observations and engineering judgement. In some cases very weak dependence may exist but an assumption of independence is acceptable in light of the model's sensitivity to the assumption. The data pairs denoted by D are thought to be sufficiently dependent to require generation of correlated input sequences. All pairs denoted by I are considered independent. The downgradient distance, X will be fixed and thus it can be deleted because it will take on a single, known value.

3.3 Data and Information Sources

The single values or distributions of values for the various input parameters and variables must be developed from data, theoretical constructs, expert judgement or policy considerations. A number of different sources can be used in developing this information including the following:

- o the scientific and engineering literature
- o laboratory experiments and measurements for specific chemical reactions
- o EPA directed surveys
- o chemical structure-property correlations and structure-activity relationships
- o mathematical modeling

Table 1. Summary of Relationships Among Model Parameters and Input Values

	V	θ	ρ	α_L	α_T	α_V	X	σ	H	B	f_{OC}	pH	T	k_a	k_n	k_b	k_D
V		D	D	I	I	I	I	D	I	I	I	I	I	I	I	I	I
θ	D		D	D	D	D	I	D	I	I	I	I	I	I	I	I	I
ρ	D	D		D	D	D	I	I	I	I	I	I	I	I	I	I	I
α_L	I	D	D		D	D	D	I	I	I	I	I	I	I	I	I	I
α_T	I	D	D	D		D	D	I	I	I	I	I	I	I	I	I	I
α_V	I	D	D	D	D		D	I	I	I	I	I	I	I	I	I	I
X	I	I	I	D	D	D		I	I	I	I	I	I	I	I	I	I
σ	D	D	I	I	I	I	I		D	I	I	I	I	I	I	I	I
H	I	I	I	I	I	I	I	D		D	I	D	I	I	I	I	I
B	I	I	I	I	I	I	I	I	I		I	I	I	I	I	I	I
f_{OC}	I	I	I	I	I	I	I	I	I	I		I	I	I	I	I	D
pH	I	I	I	I	I	I	I	I	I	I	I		I	D	I	D	D
T	I	I	I	I	I	I	I	I	I	I	I	I		D	D	D	I
k_a	I	I	I	I	I	I	I	I	I	I	I	D	D		I	I	I
k_n	I	I	I	I	I	I	I	I	I	I	I	I	D	I		I	I
k_b	I	I	I	I	I	I	I	I	I	I	I	D	D	I	I		I
k_D	I	I	I	I	I	I	I	I	I	I	D	D	I	I	I	I	

D denotes dependence

I denotes weak or no dependence

The required data can be further characterized according to type. That is, data that describe pollutant transport, data that characterize the environment, and data that characterize the individual chemicals. The major data influencing transport are flow velocity, V , and dispersivities α_L , α_T , and α_V . Flow velocities for various sites have been reported in the literature and were included in the EPA survey of Part B permit applications. In many cases velocities can be estimated using a relationship incorporating Darcy's law when the gradient, hydraulic conductivity, and porosity are known. Similarly, dispersivities have been reported for a number of case studies and recent research has defined both empirical and theoretical methods for relating dispersivities to other system properties.

Environmental characterization data can be developed from national data bases. For example, many groundwater pH and temperature measurements are recorded as entries to the EPA STORET data base. Another source is the EPA survey of Part B permit applications for bulk densities (ρ), porosities (θ), and facility geometries (related to σ). The literature reports individual values and ranges of values for these and other parameters including organic carbon content, foc. The parameter most closely related to the performance of the engineered components of land disposal is sigma, σ . In this case almost no data from case studies are available but expert judgement enhanced by modeling can be used to generate appropriate values.

Chemical-specific data can be derived from the literature, generated by specific laboratory experiments, or estimated from theoretical or empirical relationships. In some cases useful qualitative information can be generated by expert judgement. Most notable is judgement on the existence of hydrolyzable functional groups for each compound. Expert

judgement can determine for many chemicals if hydrolysis will degrade the chemical. The assignment of a zero rate constant for non-hydrolyzable chemicals allows the analysis to proceed without the need for experimental data. In other cases structure--activity relationships can be used to estimate needed parameters. For example, the octanol-water partition coefficient can be reliably estimated from knowledge of molecular structure. The proper interpretation of data generated from the above methods. This can be addressed by convening expert panels to review reported data, specify limits of uncertainty, and delineate appropriate ranges of application.

3.4 Data Generation Procedures

The input data and parameters listed in Table 1 must be generated before monte carlo analyses can proceed. It is convenient to separate this activity into two tasks: generating independent data sets; and generating correlated or dependant data sets. Independent data sets can be developed as empirical distributions of observed data, as theoretical distributions from a "best-fit" analyses of observed data, or as assumed distributions. Dependent data sets can be developed as empirical, joint, or multivariate distributions, theoretical distributions, or from functional dependencies among the variables and parameters.

3.4.1 Independent Parameters and Variables

The summary shown in Table 1 suggests that none of the variables or parameters is totally independent from all other variables or parameters. In some cases, however, an independent "seed" distribution can be generated to which other variables are correlated. For example, the temperature (T) will influence the hydrolysis rate constants (k_a , k_n , k_p) but the

reverse is not true nor for the system studied does anything else influence temperature. Thus temperature distributions can be developed as an independent variable. Following this rationale, the parameters and variables to be generated independently are as follows:

- B - the thickness of the saturated zone
- foc - the fractional organic carbon content of the soils
- pH - the groundwater hydrogen ion activity
- T - the groundwater temperature
- H - the leachate penetration depth into the saturated zone

The thickness of the saturated zone, B, influences the opportunity for vertical dispersive mixing as the plume moves down gradient. Literature values taken from measurements and surveys were used to derive a distribution for this parameter.

The fractional organic carbon content, foc, is used to determine the distribution coefficient, K_D . Recall from the groundwater model description that

$$K_D = \text{foc } K_{oc} \quad (2)$$

where K_{oc} = distribution coefficient normalized to organic carbon. It is clear from this relationship that the variation in foc leads directly to variation in the K_D and hence retardation of the solute in groundwater. Unfortunately, few if any comprehensive subsurface characterizations of organic carbon content exist. In general the values are known to be very low, typically less than .01. In the absence of evidence to the contrary the approach taken was to assume a low range of foc. A distribution shape for this range was determined by the distribution of measured dissolved organic carbon recorded as entries to STORET. The assumption is that dissolved organic carbon reflects the existence (and hence distri-

bution) of organic carbon in the subsurface environment being considered.

The model assumes that the groundwater is sufficiently buffered to insure that the pH is not influenced by input of contaminants or changes in temperature. This permits a pH distribution to be derived independently. STORET data were analyzed, a distribution developed and summary statistics generated for pH.

Assumptions about the independence of groundwater temperature are essentially the same as for pH. Temperature influences hydrolysis reactions but the reverse is ignored. Actually, temperature can also influence sorption but such effects are ignored in this analysis.

The depth, H, to which the leachate flow penetrates the saturated zone is probably related to the relative differences in the leachate velocity and the groundwater velocity. Because disposal of free liquids is not permitted, density gradients or stratification of "floaters" or "sinkers" are not likely to occur. Lacking any meaningful data, a simple, independent, uniform distribution ranging from a fixed minimum to a fixed maximum was used.

A summary of the procedures used to generate the independent input data sets is given in Table 2. The fitted distribution (FD) method refers to the development of a mathematically-defined frequency distribution function by "fitting" various possible distributions (e.g., normal, log normal, exponential) or mixtures of distributions to the "observed" data and selecting the "best fit" distribution for use in the monte carlo process. The distributions selected for each variable or parameter are given in section 4.0 of this report. In all cases we recognize the possibility that the data in STORET may represent a biased sample. However, lacking a better alternative the data were accepted as representative of groundwater and subsurface conditions.

Table 2. Summary of Procedures Used to Generate Independent Input Data Sets

Input Data	Method of Generation	Source of Data and/or Reference
B	FD*	Various literature
F _{OC}	FD	STORET, assumptions
pH	FD	STORET
T	FD	STORET
H	AD**	—

*fitted distribution to empirical data

**assumed distribution

3.4.2 Dependent Parameters and Variables

The remaining input parameters and variables are dependent and cannot be generated without properly "matching" each value with other related values. Recall that our objective is to provide a consistent set of data that when viewed as descriptions of site-specific scenarios will produce situations that occur in nature. The main purpose of building in dependencies is to avoid unrealistic or impossible sets of data. For example, a uniform soil having high porosity because of high clay content will rarely if ever have high groundwater velocities because of the low hydraulic conductivities. Failure to exclude such possibilities however by assuming that porosity and velocity are independent will lead to unrealistic if not incorrect modeling results.

The "consistency" criterion intended here needs further elaboration. In general, precise functional relationships among all the dependent

variables or parameters do not exist. Similarly, observed data for all values taken in sets do not exist or are inadequate in number to permit a statistical representation of the dependencies. Fortunately, however, equations do exist in the engineering and scientific literature to permit generation of sets of "possible" combinations of input data. Generation of consistent sets of input data is much easier to accomplish than the more rigorous but related task of predicting a precise, site-specific set of values given only one or two measurements at that site. The parameters and variables to be generated as dependent values are as follows:

- $\alpha_L, \alpha_T, \alpha_V$ = dispersivities in the longitudinal, lateral, and vertical directions assumed to be largely dependent on distance, x
- θ = porosity of the soil or porous media is assumed to be largely dependent on soil properties including particle sizes textural classes, bulk densities, and parent material
- ρ = bulk density of the soil or porous media is assumed to be largely dependent on soil properties including porosity
- V = groundwater flow velocity is assumed to be largely dependent on soil properties including hydraulic conductivities, porosity, bulk density and on hydraulic gradient (groundwater slope)
- σ = the standard deviation of the gaussian distribution representation of the source concentration is related via mass balance principles to leachate volumes, groundwater velocity, porosity, and depth of leachate penetration into the saturated zone

- k_a, k_n, k_b = hydrolysis rate constants are assumed to be largely dependent on groundwater pH and temperature and on specific chemical properties
- k_D = effective distribution coefficient for each specific chemical is assumed to be largely dependent on the organic carbon content of the soil or porous media and in some cases on the pH as well as on specific chemical properties of the pollutant

3.4.2.1 Dispersivity

The "spreading" of solutes transported by groundwater is usually described as a combination of molecular diffusion and mechanical mixing. The relative magnitudes of each are such that molecular diffusion can be ignored. The property of the soil or porous medium that is commonly used to define the magnitude and direction of dispersion is included in the dispersivity parameters. A generalized theory to describe dispersivity has not yet been developed but recent work has noted a strong dependence on scale (Sudicky et al., 1983; Anderson, 1979; Pickens and Grisak, 1981; Pickens and Grisak, 1981; Molz et al., 1983; Gelhar et al., 1985). Some investigators (Pickens and Grisak, 1981) have reported simple, linear dependencies for longitudinal dispersivity, α_L , as

$$\alpha_L = 0.1 X \quad (3)$$

where X = mean travel distance

More recently Guven et al. (1984) completed a detailed theoretical analysis and suggested an expression of similar form

$$\alpha_L = 0.093 X + 0.007 \text{ (X in meters)} \quad (4)$$

Many experimental data suggest that equation 3 is a reasonable approximation for α_L given our limited objective of "consistency" in the input data set.

Transverse dispersivity, α_T , has been studied to a lesser degree but its magnitude is known to be less than longitudinal dispersivity while maintaining scale dependency (Santy, 1980; Anderson, 1979; and Sudicky et al., 1983). Typically, α_T is related to α_L by a simple ratio leading to the following expression

$$\alpha_L / \alpha_T = \text{LTR} \quad (5)$$

where LTR = longitudinal-transverse dispersivity ratio

A range of LTR's has been reported and appears to center around a value of 3.0. Again, to insure consistency in input data while maintaining a straightforward approach, an LTR of 3.0 was selected for the monte carlo analysis.

For unidirectional flow in the longitudinal direction the vertical dispersivity, α_v , is quite low. Using the ratio, α_L / α_v , to describe vertical dispersivity, Gelhar et al., 1985, reported a range of 30-1860 with an average of about 400. Because of the uncertainty surrounding proper specification of values for vertical dispersivity, it is varied uniformly from 40 to 400 m in the monte carlo routine.

The data generation approach for dispersivity can be summarized by the following equations:

$$\alpha_L = 0.1 \times \quad (6)$$

$$\alpha_T = 0.0333 \times \quad (7)$$

$$\alpha_v = 0.0025(x) - 0.01(x) \quad (8)$$

where x is the downgradient exposure point distance selected for implementation of the decision rule.

3.4.2.2 Porosity

The porosity, θ , of a uniform porous media is largely a function of particle size. For small particle sizes like clay, porosity increases to a maximum of around 50%. Porosities of coarser media like gravel decrease to a minimum of around 30%. These measured ranges of porosities suggest a strong correlation with mean particle diameter, d . Data reported by Davis (1969) were used to develop a regression equation relating porosity to d as follows

$$\theta = 0.261 - 0.0385 \ln d \quad (9)$$

Because porosities are generally known for a wide range of soils and porous media another approach to generating input values is to determine a distribution for θ from observed data. It will be shown later, however, that θ is linked to velocity, V , through the mean particle size, d . Thus, to preserve this relationship, the distribution for θ is generated from a "seed" distribution for d via equation 9. The mean particle diameter as the single correlated property to porosity ignores the influence of particle sorting within porous media and hence may not be the most accurate approach in developing the dependence. Unfortunately, at the present time the distribution of sorting and mean particle size in materials in the saturated zones is not well enough known to be used in the methodology.

The distribution of particle sizes selected for this study should reflect the distribution of subsurface characteristics in all areas subject to potential use for land disposal. While specific case studies exist, apparently no general characterization is available. One approach is to assume a distribution bounded by reported ranges. In the absence

of data both a uniform and a log-uniform distribution were investigated. As will be shown later, the log-uniform distribution was selected because it more heavily weights the influence of smaller particle sizes and because the related, derived velocity distribution is more consistent with observed data.

3.4.2.3 Bulk Density

The soil's bulk density, ρ , is defined as the mass of dry soil divided by its total (or bulk) volume. Bulk density directly influences the retardation of solutes and is related to soil structure. In general, as soils become more compact their bulk density increases. This relationship produces a dependency between porosity, θ , and ρ . Freeze and Cherry (1979) note that

$$\theta = 1 - \frac{\rho}{\rho_p} \quad (10)$$

where ρ_p = particle density, g/cm^3

By assuming $\rho_p = 2.65$, equation 10 can be rearranged to yield an expression for estimating ρ given θ as follows

$$\rho = 2.65 (1 - \theta) \quad (11)$$

The particle density of soil materials varies over a very narrow range and can be fixed at a value of 2.65 gm/cm^3 . Equation 11 can be used to derive a frequency distribution for ρ given the previously generated distribution for θ .

3.4.2.4 Velocity

The velocity of groundwater is a major determinant of the transport of solutes in subsurface systems. In uniform porous media it is the

dominant factor and must be properly specified in the monte carlo process. Dependencies among the input data (θ and ρ) must be preserved while generating realistic values of velocity.

Groundwater flow velocities vary widely. Mackay et al. (1985) report that velocities typically range between 1-100 m/yr. These ranges apply to typical "natural gradient" conditions and higher velocities can exist under both man-induced (e.g., well-field drawdown) and extreme natural situations. For example, velocities in excess of 9000 m/yr have been reported (Guvén et al., 1984) for a glacial outwash material. Such data sources could be used to develop an empirical frequency distribution for velocity but the requirement to maintain dependencies with soil properties is not easily met using this approach. The EPA survey of Part B permit applications for velocity could be used to generate empirical distribution, but in addition to the dependency problem just described, this limited sample is likely to be biased.

Velocities are related to soil properties and other site-specific factors through Darcy's law. Using Darcy's law and assumptions of steady flow in uniform, saturated media yields the following expression for average pore velocity, V

$$V = \frac{K_s S}{\theta} \quad (12)$$

where K_s = saturated hydraulic conductivity, cm/sec

S = hydraulic gradient

Because an expression for porosity, θ , has already been developed, equation 12 properly relates V and θ . The saturated hydraulic conductivity, K_s , reflects the "ease" with which water is transported through porous media

and for any given fluid K_s is a function of porous medium properties such as particle size, grain shape, connectivity, and tortuosity. To the extent that K_s is related to such properties if functional relationships exist for K_s then dependencies among V , K_s and θ can also be represented.

Individual, site-specific measurements for K_s are usually difficult to make and the spatial variability of "point" measurements is the subject of much current research. Also, site-specific variations in K_s values introduce considerable uncertainty in modeling groundwater flow when point estimates or averaged point values are used as model inputs. Recall that the objective here is to insure consistency in results while representing the wide variations expected from site to site. Given this objective (that is much less demanding than an attempt to predict an accurate K_s for any given site), it is reasonable to use an approximate functional relationship. The most notable among these is the Karmen-Cozeny equation (Bear, 1979),

$$K_s = 478 \left[\frac{\theta^3}{(1-\theta)^2} d^2 \right] \quad (13)$$

where θ = porosity

d = mean particle diameter

Note that equation 13 relates K_s to θ and d . Furthermore, θ is derived from d that is generated from a "seed" distribution.

The remaining factor in equation 12 for velocity is the gradient, S . In general the gradient is a function of the local topography, groundwater recharge volumes and locations, and the influence of withdrawals (e.g., well fields). It is also likely to be indirectly related to porous media properties. Rarely are large gradients associated with very high conductivities. No functional relationships exist, however, to express this

association. Thus, another independent "seed" distribution is required. The potential problem with the independence assumption is in "extreme" values. Data sets having large values for both K_s and S will also have very large values for velocities resulting in unrealistic conditions. Such condition can be prevented by bounding the velocities such that a fixed maximum is not exceeded. The observed value of 9250 m/yr was selected for this purpose.

The distribution for the gradient can be assumed or derived from observations. Gradient data were included in the EPA survey of Part B permit applications and were analyzed to develop a frequency distribution. Results are given in later sections of this report.

3.4.2.5 Standard Deviation of the Gaussian Distribution for the Source Concentration

The parameter defines the nature of the leachate after it has mixed with the underlying saturated zone. Because sigma reflects the nature and extent of the leachate interaction with the groundwater beneath the facility, it and hence also reflects the failure of the engineered controls on the facility (e.g., liners, caps). From mass balance principles is related to the environmental setting by

$$\sigma = q A_w C_L / \sqrt{2\pi} V \theta H C_0 \quad (14)$$

where q = unit areal flux of leachate through the land disposal facility, $m \text{ yr}^{-1}$

A_w = area of disposal facility, m^2

V = groundwater velocity, $m \text{ yr}^{-1}$

θ = saturated zone porosity

H = leachate penetration into the saturated zone, m

C_L = contaminant concentration in the leachate

C_0 = contaminant concentration in the mixing zone
beneath the facility

If we assume that the leachate concentration, C_L , is the same as the maximum concentration C_0 (at $y = 0$) of the gaussian concentration distribution, then equation 14 enables direct calculation of σ given the other, known variables. Setting $C_L = C_0$ in equation 14 simply means that the leachate displaces the groundwater and dilution occurs after advective transport is initiated; a reasonable assumption given the low velocities for leachate fluxes.

Values for q vary depending on the location of facilities, their vertical configuration (e.g., liners, caps), and their performance over time. The procedure used to produce a distribution for q is described in the E. C. Jordan (1985) report. A distribution was generated from the EPA survey of Part B permit applications for the area term, A_w . Velocity, V , and porosity, θ , and penetration depth, H , are generated as previously discussed. For mathematical reasons (boundary effects) the constraint that the ratio, H/B , where B is the saturated zone thickness, be less than 0.5 must also be made. The minimum saturated thickness is 3 meters.

3.4.2.6 Hydrolysis Rates

Hydrolysis rate constants are unique to each chemical and will be determined from the literature or from laboratory experiments. All rates (acid-catalyzed (k_a), neutral (k_n), base-catalyzed (k_b)) are influenced by groundwater temperature. k_a and k_b are also influenced by pH. The pH dependency is included directly in the groundwater model and the rates will be adjusted accordingly via the independently derived distribution for pH. The temperature dependency requires further elaboration.

Rate constants are often adjusted for temperature effects by the Arrhenius equation. Because the rate constant values are given for a specific temperature, the need exists to adjust these values to account for different temperatures in the groundwater. Using the generic activation energy recommended by Wolfe (1985) of approximately 20 k cal/mole, the temperature correction factor can be written as

$$\frac{k_{a,n,b}^T}{k_{a,n,b}^{Tr}} = \exp \left[10^4 \left(\frac{1}{T_r} - \frac{1}{T} \right) \right] \quad (15)$$

where $k_{a,n,b}^T$ = second-order hydrolysis rate constants for acid, neutral, or base conditions at temperature T
 $k_{a,n,b}^{Tr}$ = second-order hydrolysis rate constants for acid, neutral, or base conditions at reference temperature, T_r
 T, T_r = temperature, °Kelvin

The temperature can also influence the base-catalysed hydrolysis rate through influence on autoprotolysis of water. Porous medium properties and groundwaters are sufficiently buffered, however, to minimize this effect. Temperature corrections to pH are not made.

The measurement and extrapolation errors for hydrolysis measurements are not yet fully developed. Once the experimental program to develop such values is fully implemented the nature and magnitude of these errors will be included.

3.4.2.7 Distribution Coefficient

The relationships most suited for relating the chemical distribution coefficient, K_p , to soil or porous medium properties are discussed in detail by Karickhoff (1985). In cases where reliable relationships do not exist, measurements are required. For many cases hydrophobic binding

dominates the sorption process and it is possible to relate the distribution coefficient directly to soil organic carbon. For these cases the dependency is given by

$$K_D = K_{OC} f_{OC} \quad (16)$$

where K_{OC} = normalized distribution coefficient for organic carbon

f_{OC} = fraction organic carbon

Recall that f_{OC} values will be generated as an independent parameter as previously described. Equation 16 will be used to preserve dependency between porous media properties and chemical sorptive properties. For other binding mechanisms described by Karickhoff (1985) including those for polar, ionizable compounds adjustments will be made on a case-by-case basis as appropriate.

4.0 Data Generation Results

The combination of data sources and approaches described in Sections 3.3 and 3.4 were used to generate input frequency distributions for each of the parameters and forcing function variables. In some cases intermediate or precursor variables were also generated to enable representation of appropriate dependencies among the variables and parameters. Results for these variables are also included in this section. Table 3 gives a summary of the distribution types and parameters for each model parameter or variable. For derived distributions, only the mean and range of the synthesized data are given--it's not necessary to approximate these results by a mathematical distribution function. In some cases only single, fixed values were selected largely based upon their nature (e.g., a chemical-specific rate constant) or upon policy directions (e.g., the downgradient distance, x).

Table 3. Summary of Results for Input Data Generation

Parameter or Variable	Distribution Type	Distribution Parameters		Range min-max
		mean	std. dev.	
Temperature, °C	Normal	14.4	5.29	(0.0 - 30.0)
pH	Normal	6.2	1.28	(0.3 - 14.0)
Dissolved Organic-C, mg/l	Lognormal	1.99	1.09	(0.01 - 6.89)
f_{oc}	Lognormal	-5.76	3.17	(.001 - .01)
d, cm	Log ₁₀ uniform	.0063	—	(.0004 - 0.10)
θ , cm ³ m ⁻³	derived from d			(.30 - .56)
ρ , gm cm ⁻³	derived from θ			(1.16 - 1.8)
K_s , cm sec ⁻¹	derived from θ , d			(.0001 - .48)
S	exponential	.0309	—	(.00001- 0.10)
V, myr ⁻¹	derived from S, K_s , θ			(.01 - 9250)
B, m	exponential	78.6	—	(3.0 - 560)
A_w	mixed;exponential uniform			(23 - 930,000)
H, m	derived from uniform	6.0	—	(2.0 - 10.0)
q, m	modeling			(0.0 - .3)
σ , m	derived from q, A_w , V, H, θ			(.001 - 60,000)
α_L , m	single values			
α_T , m	single values			
α_V , m	single values			
X, m	single values			
k_a MM ⁻¹ yr ⁻¹	chemical specific value			
k_n yr ⁻¹	chemical specific value			
k_b MM ⁻¹ yr ⁻¹	chemical specific value			
k_D	chemical specific value			

4.1 Fraction Organic Carbon Content, f_{OC}

Recall from Section 3.4.1 that f_{OC} was generated by assuming a fixed, low range of .001 - .01 and using the observed distribution of dissolved organic carbon, DOC, to "shape" the f_{OC} distribution. This was done by assuming both variables had identical distributions and scaling the f_{OC} by the coefficient of variation (CV) for the DOC. That is, the mean value for f_{OC} (based upon a lognormal distribution with a stated range) was multiplied by the DOC coefficient of variation to yield a standard deviation for the f_{OC} . This fully specified the distribution.

4.2 Mean Particle Size, d

The distribution of d should correspond to the distribution of soil types in the saturated zones across the country. As previously discussed such a characterization does not exist and the mean values alone do not describe sorted grain sites. Following the rationale developed in Section 3, the $\log_{10}$ uniform distribution was assumed. This distribution, shown in Figure 2, is bounded over the range from .0004 cm, corresponding to a clay material, to 0.1 cm, corresponding to a gravelly sand. The mean is .0063 cm corresponding to a soil somewhat finer than a silty fine sand but coarser than a clayey sandy silt. This assumed distribution is a key component of the input because it influences θ and V . As will be shown the $\log_{10}$ uniform distribution produces subsequent distributions for V that are more consistent with observed data than alternative assumed distributions investigated. Clearly, however, this is an area that should be more fully investigated.

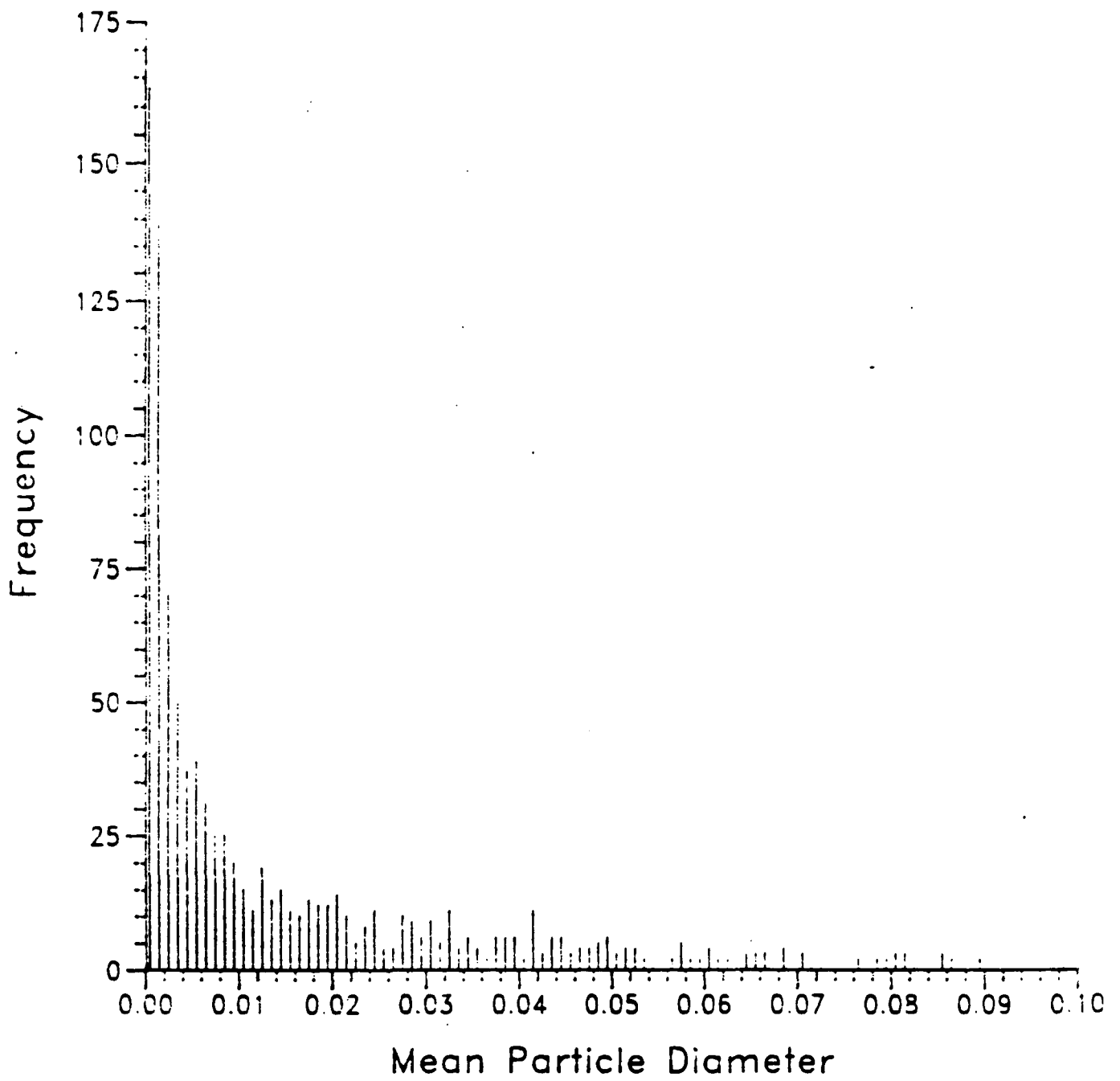


FIGURE 2

4.3 Porosity (θ), Bulk Density (ρ), Hydraulic Conductivity (K_s) and Gradient (S)

Graphical representations of frequency distributions for each of these parameters are given in Figures 3-6. Only one distribution, S , required a mathematical description since each data point for all others in this group was derived through functional relationships with mean particle size, d .

4.4 Saturated Zone Depth (B), and Facility Area (A_w)

The graphical representations for these fitted distributions are shown in Figures 7 and 8. Because A_w is used in developing for "engineered" facilities, Figure 8 applies only to landfills. The rationale for this approach is the requirement that impoundments must be "closed" as if the system were a landfill.

4.5 Percolation Flux (q) and Initial Concentration Standard Deviation (σ)

The distribution for q is described in the E. C. Jordan report (1985). Because a relatively few data points make up this distribution a direct interpretation from the data expressed as a cumulative frequency table was made. That is, linear interpolation for intermediate data points on the Figure 9 developed in the E. C. Jordan report (1985) was employed. Subsequent generation of the distributions for A_w , H , V , and σ produced Figures 10-12.

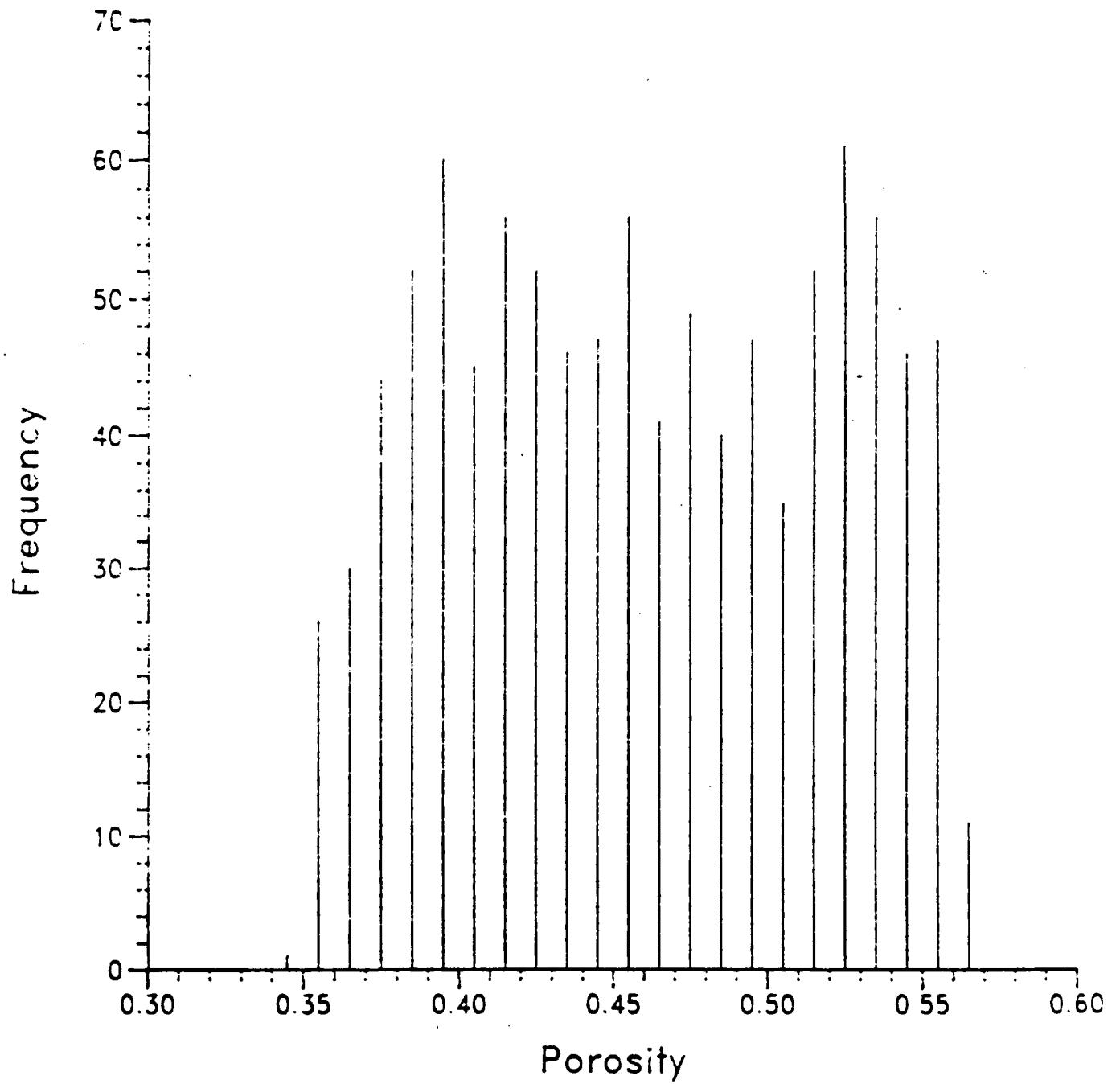


FIGURE 3

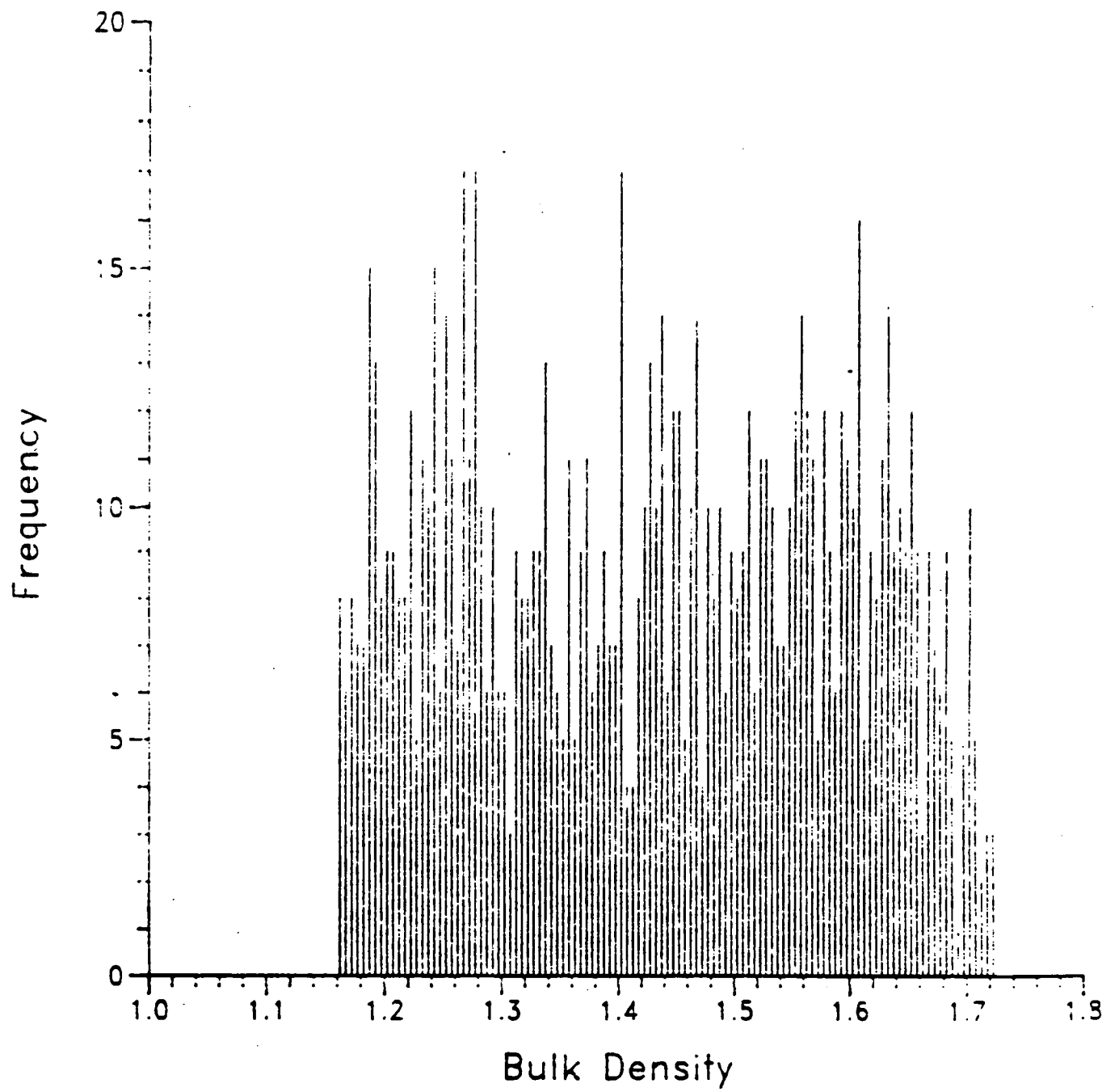


FIGURE 4

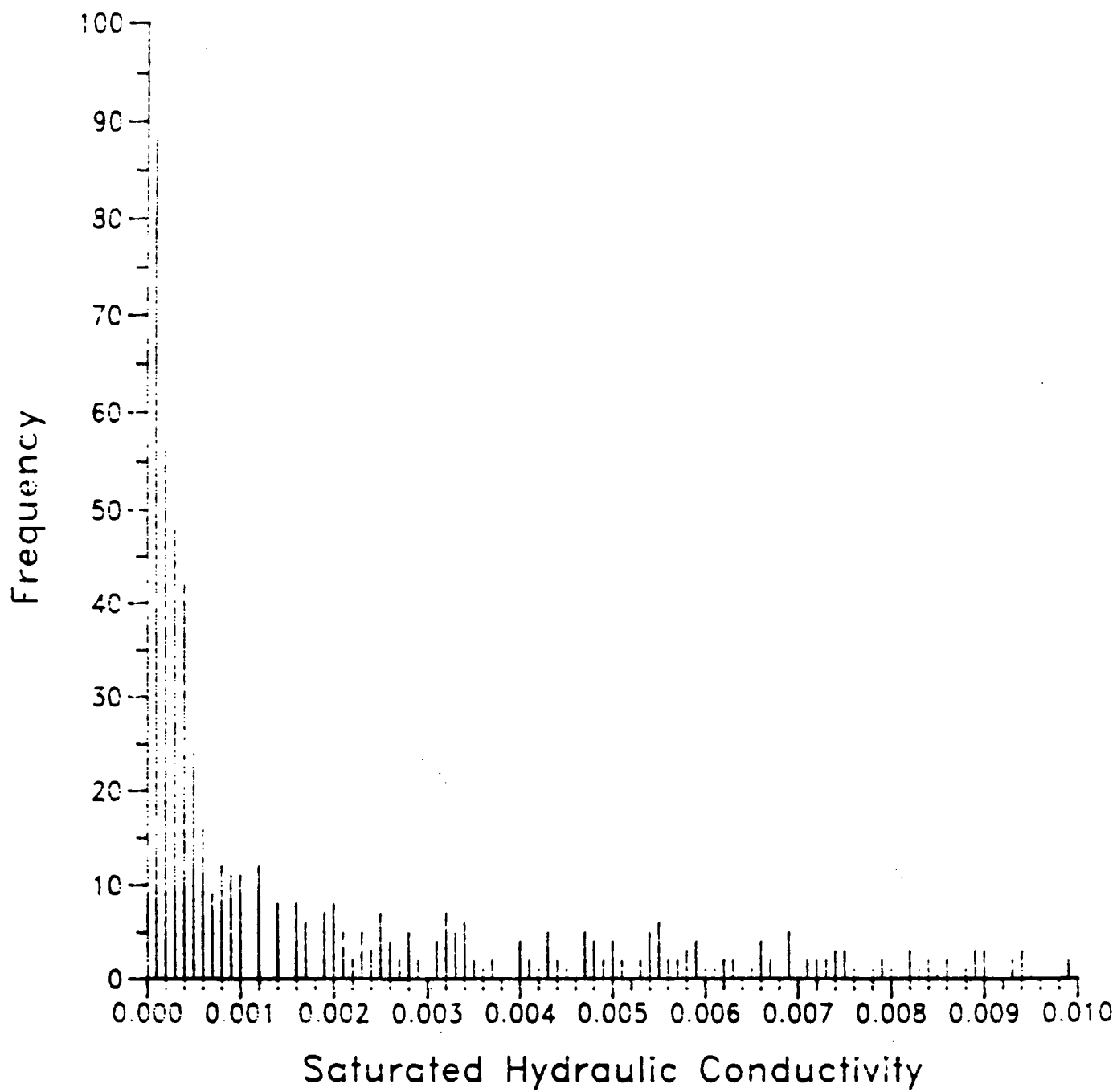


FIGURE 5

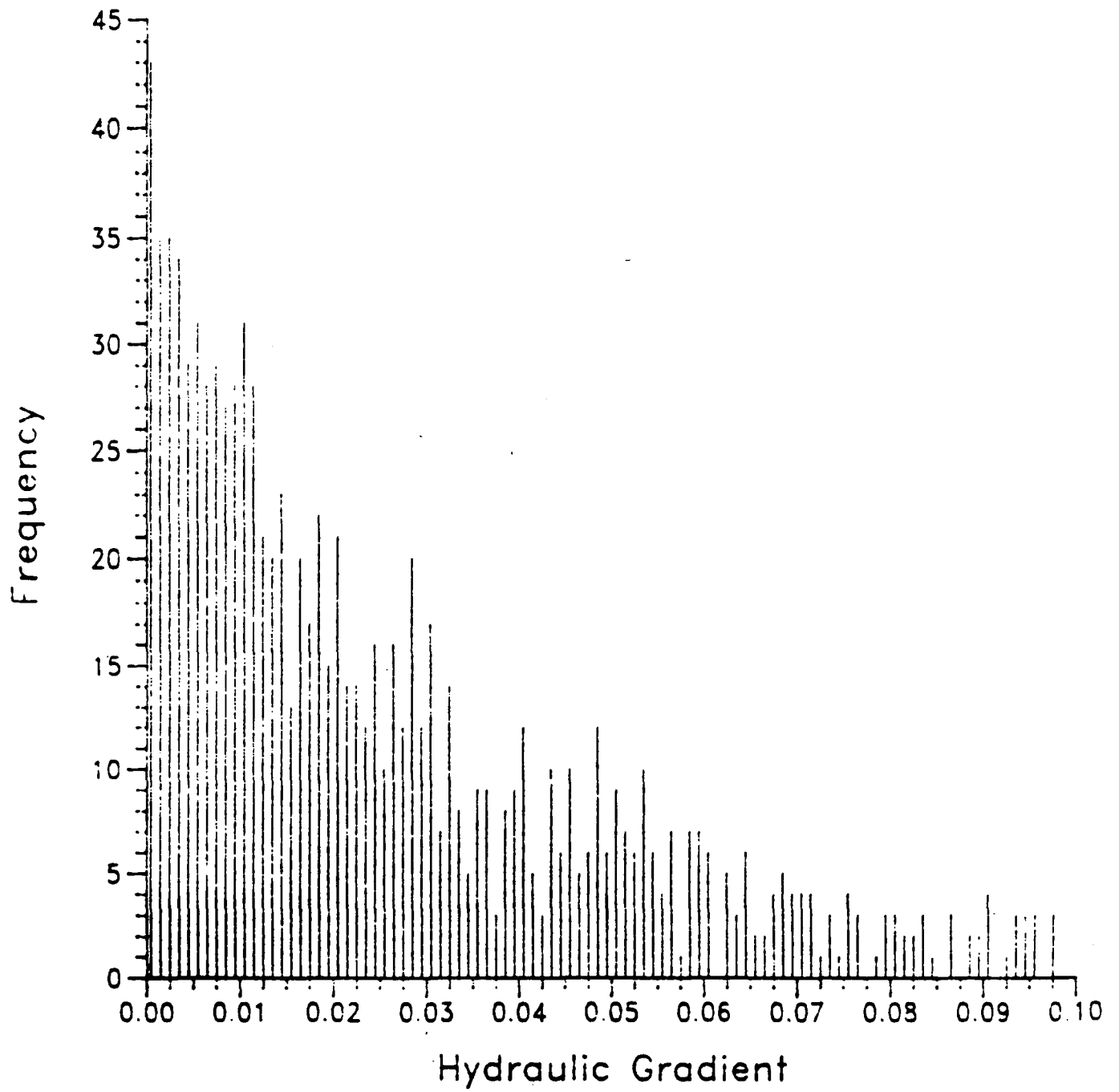


FIGURE 6

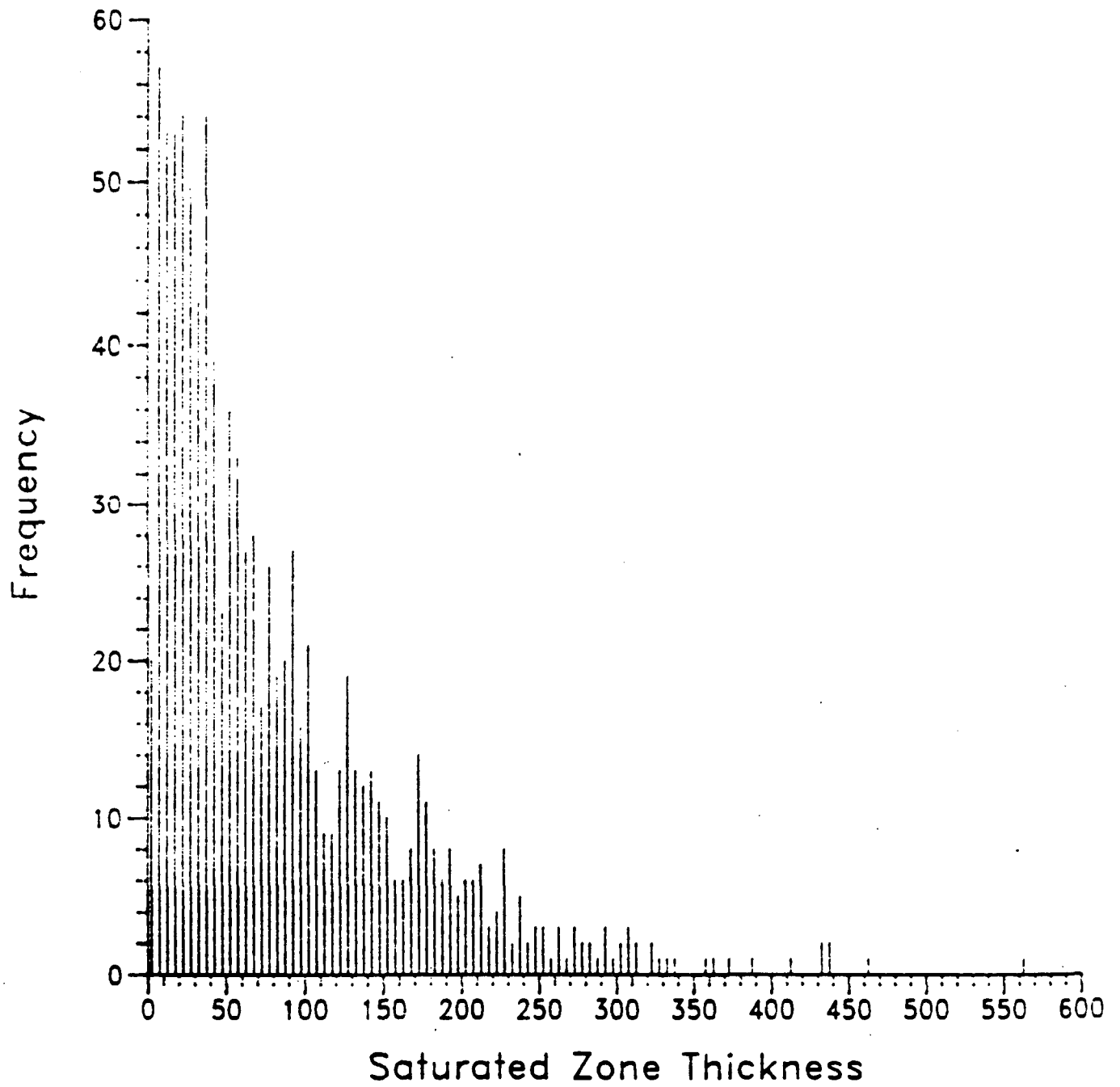


FIGURE 7

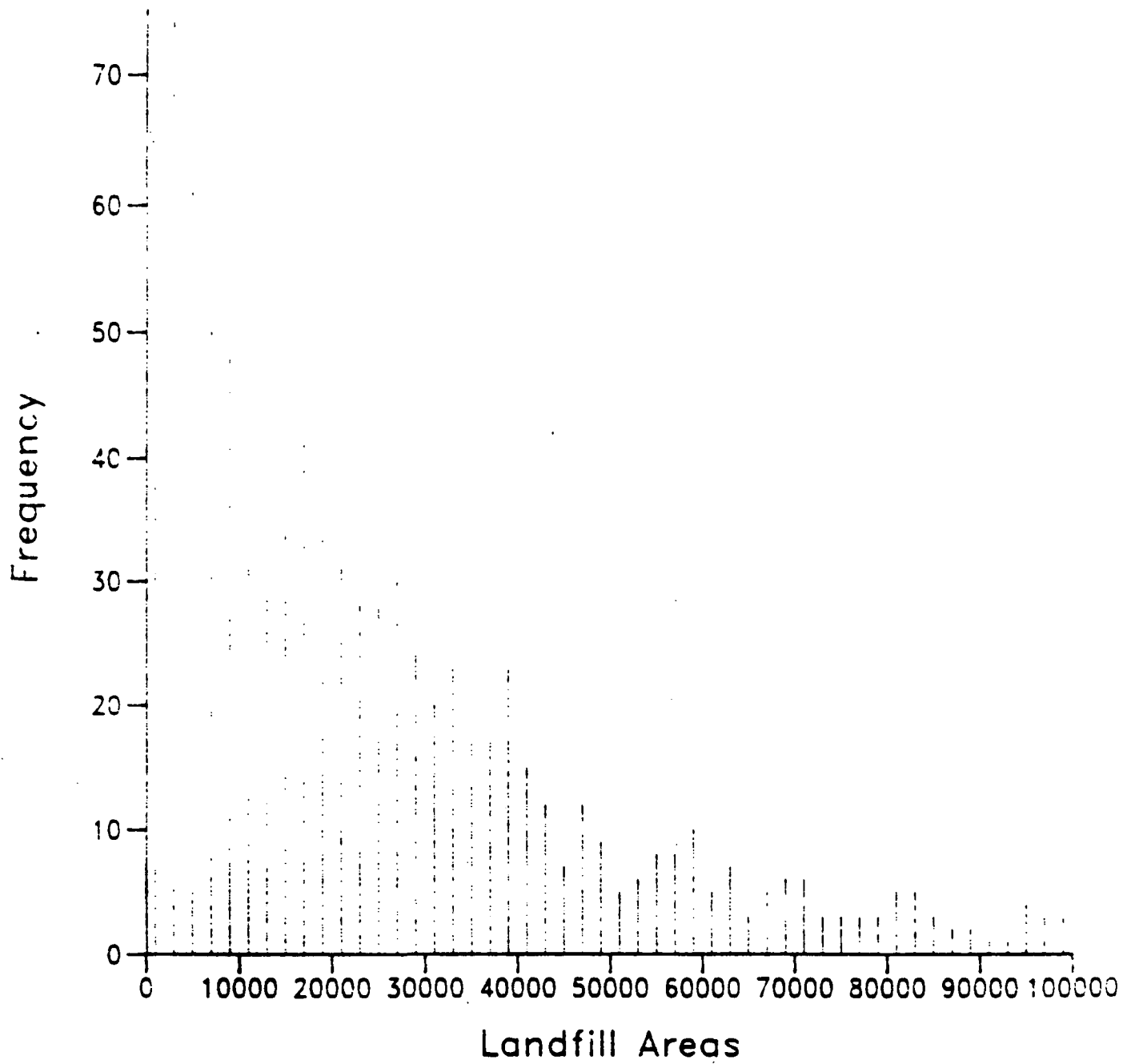


FIGURE 8

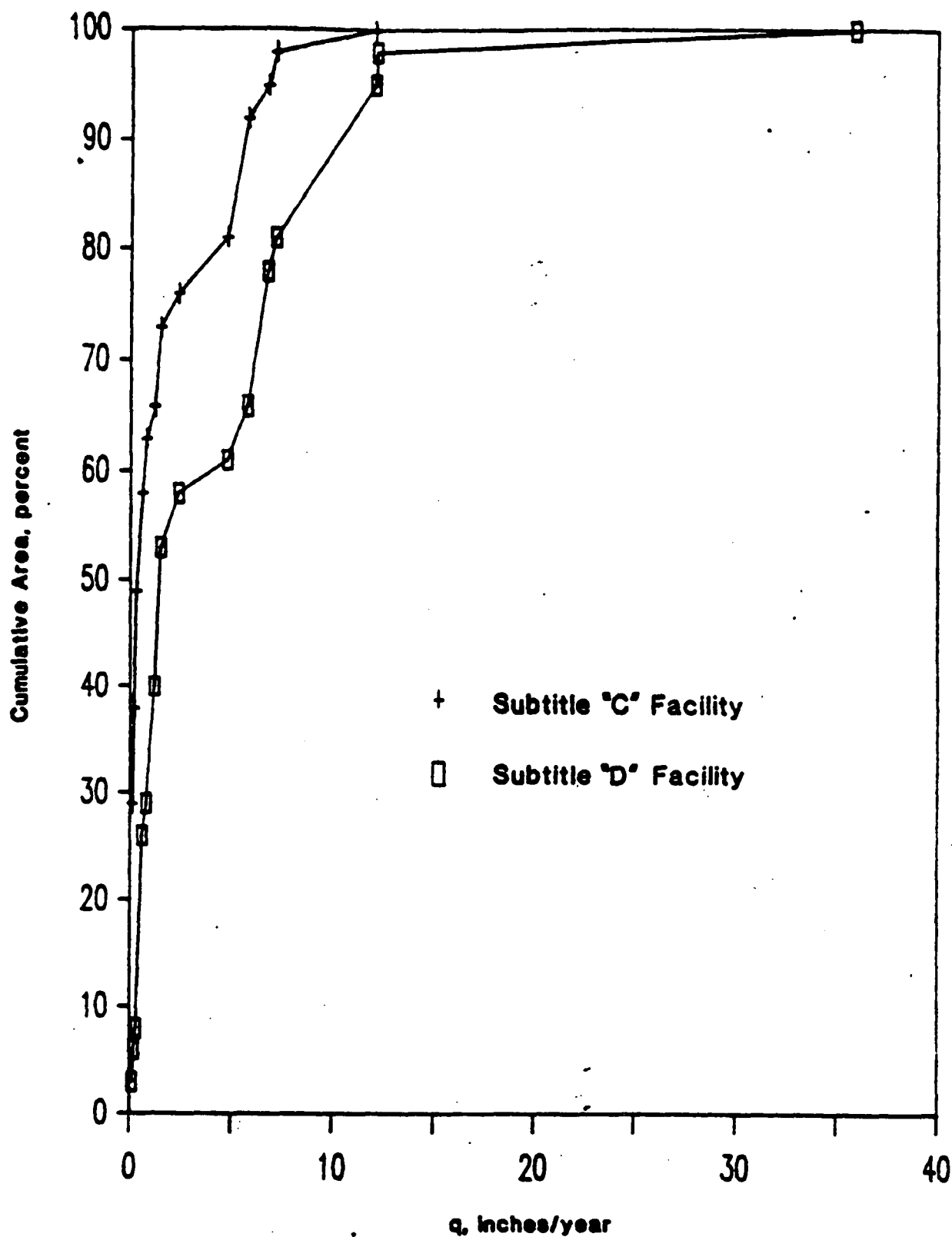


FIGURE 3-1
ESTIMATED DISTRIBUTION OF UNIT AREA LEACHING
RATES FOR SUBTITLE C AND D FACILITIES

ECJORDANCO

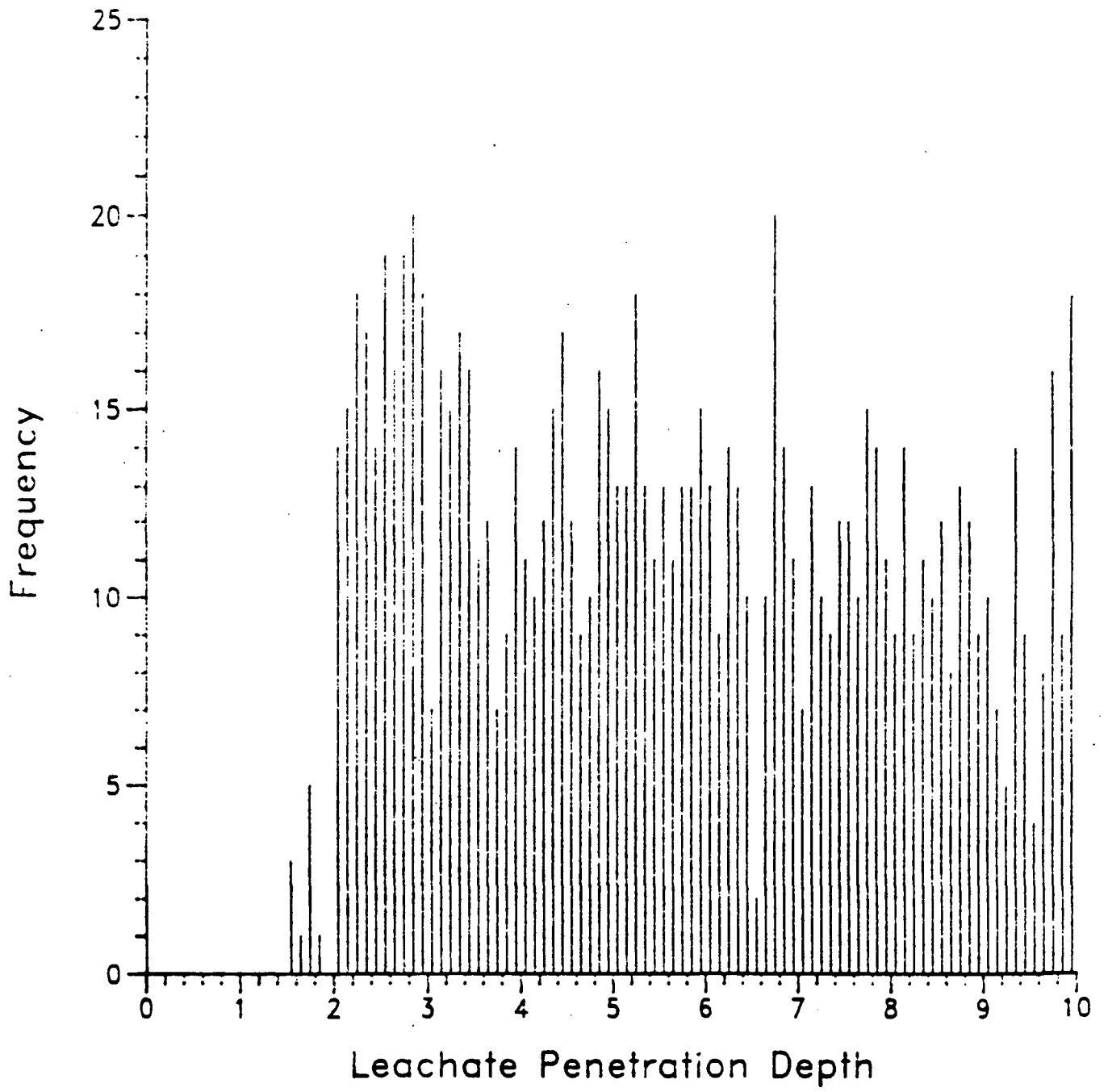


FIGURE 10

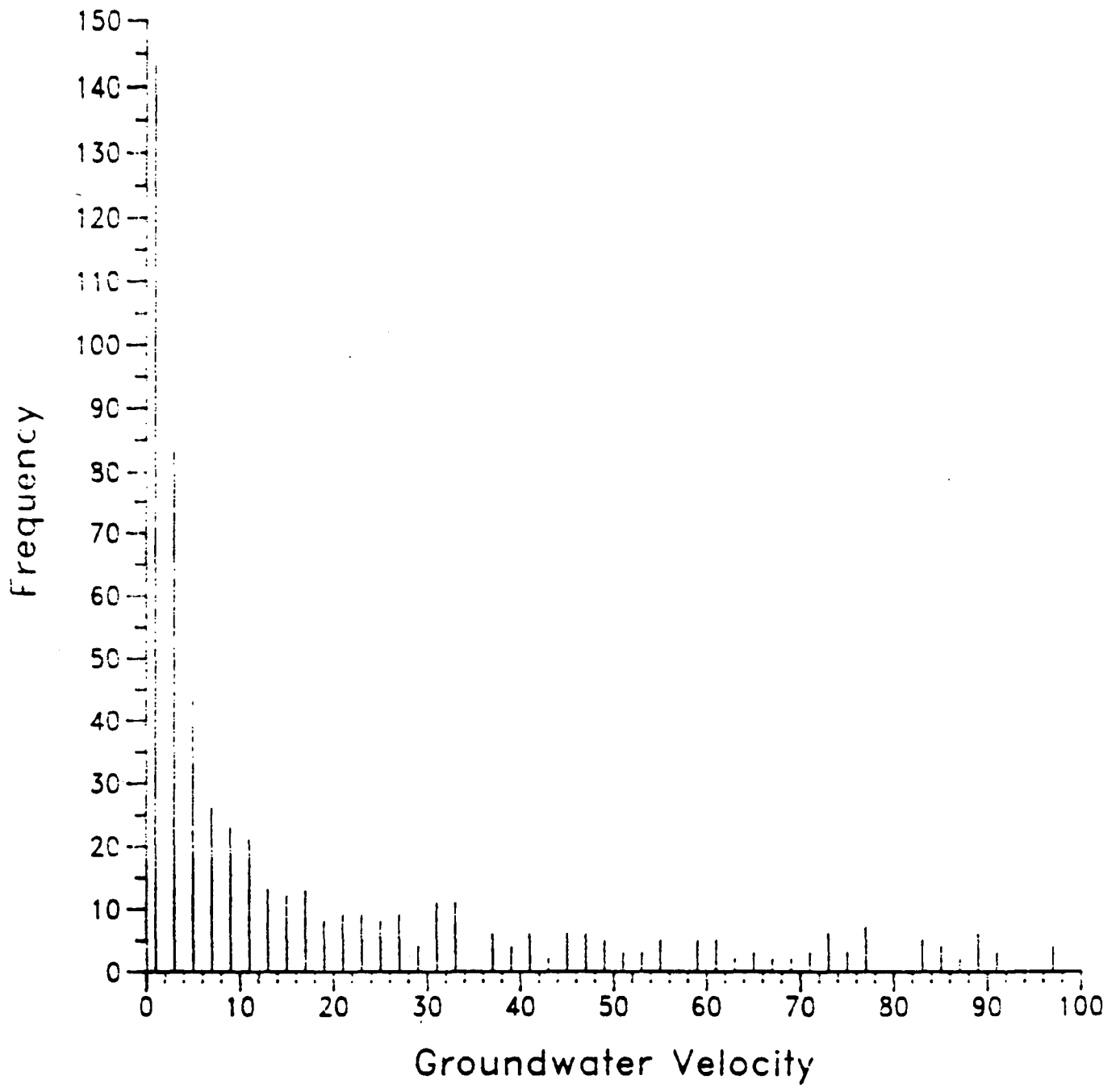


FIGURE 11

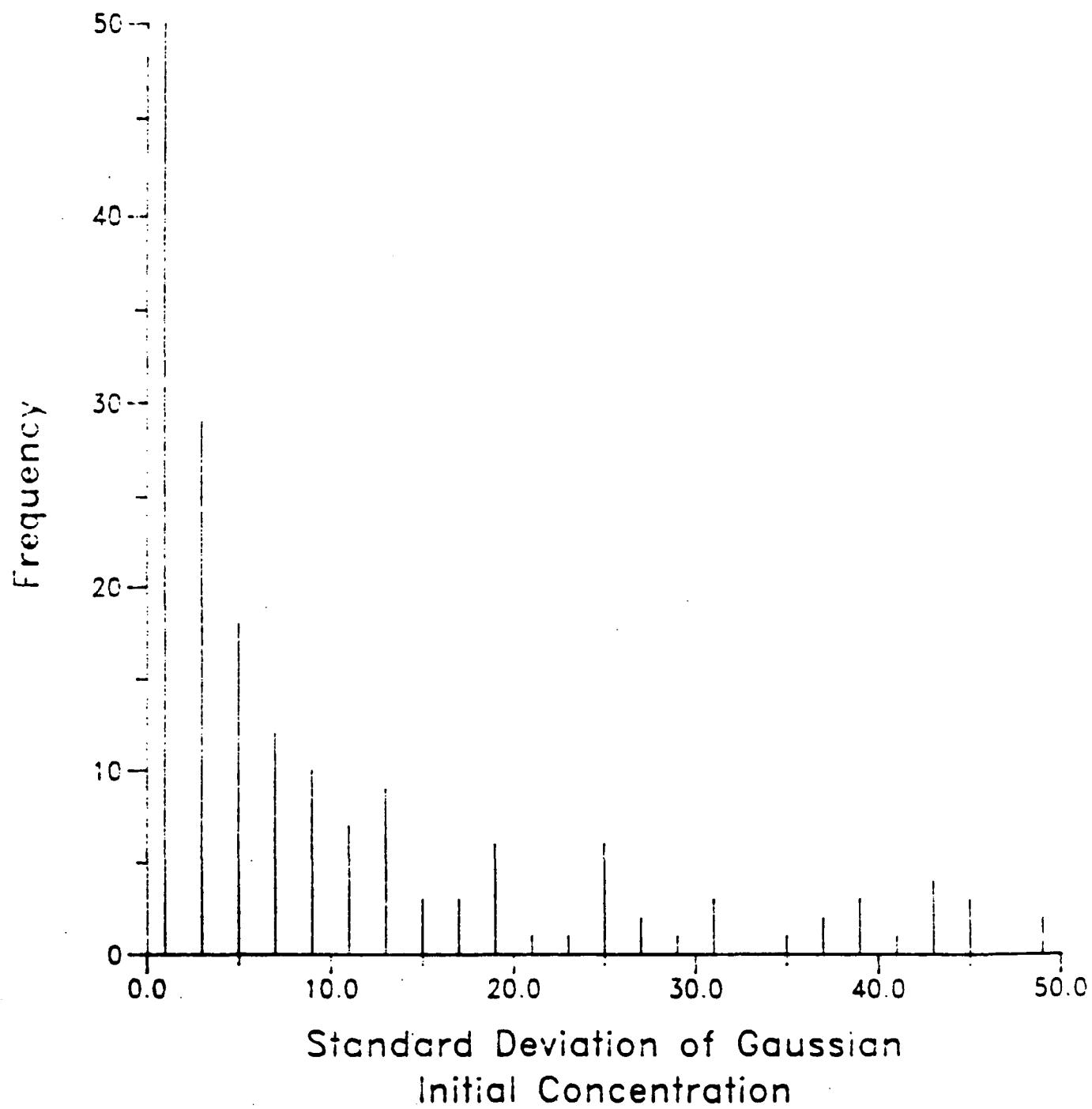


FIGURE 12

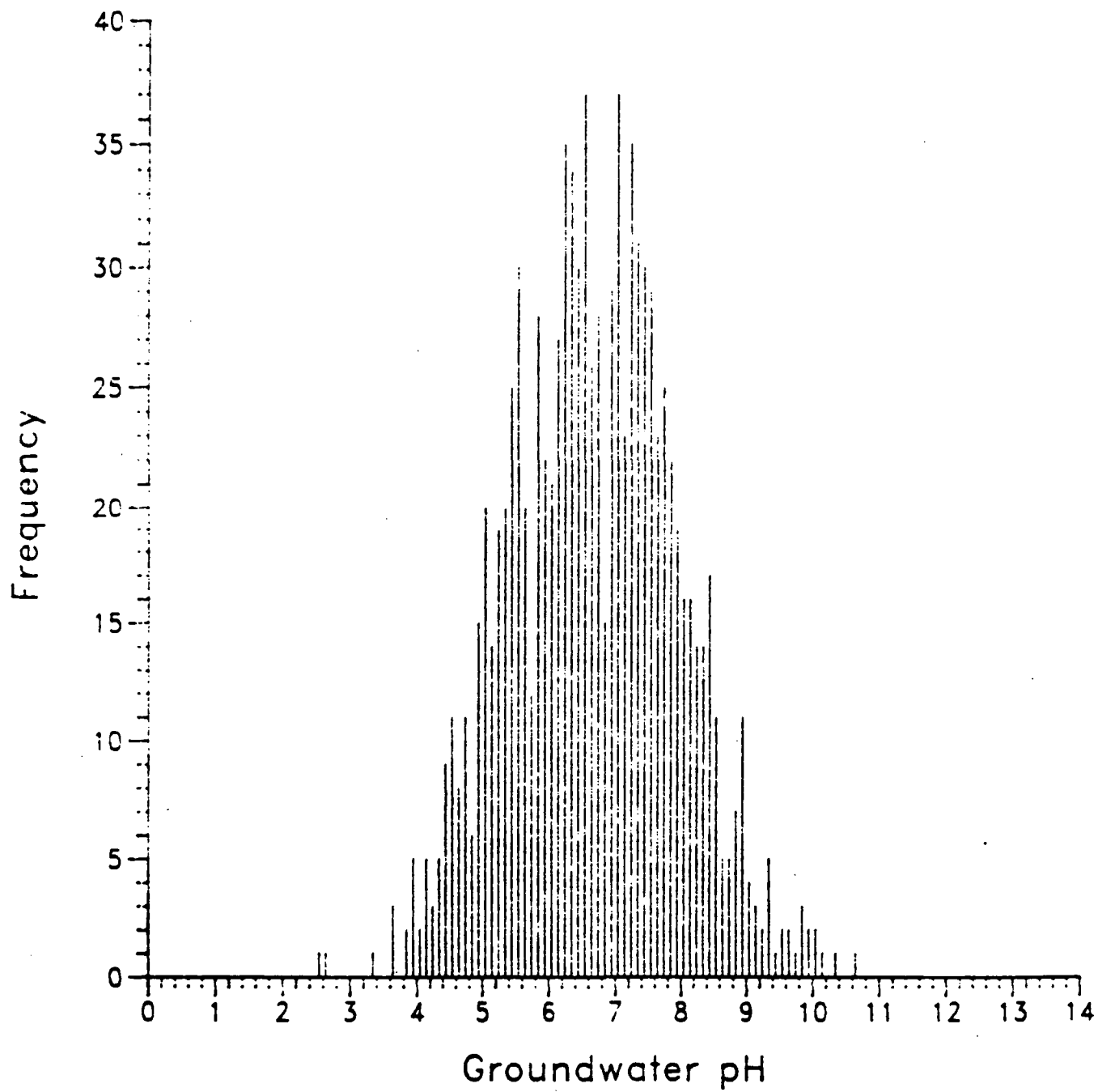


FIGURE 13

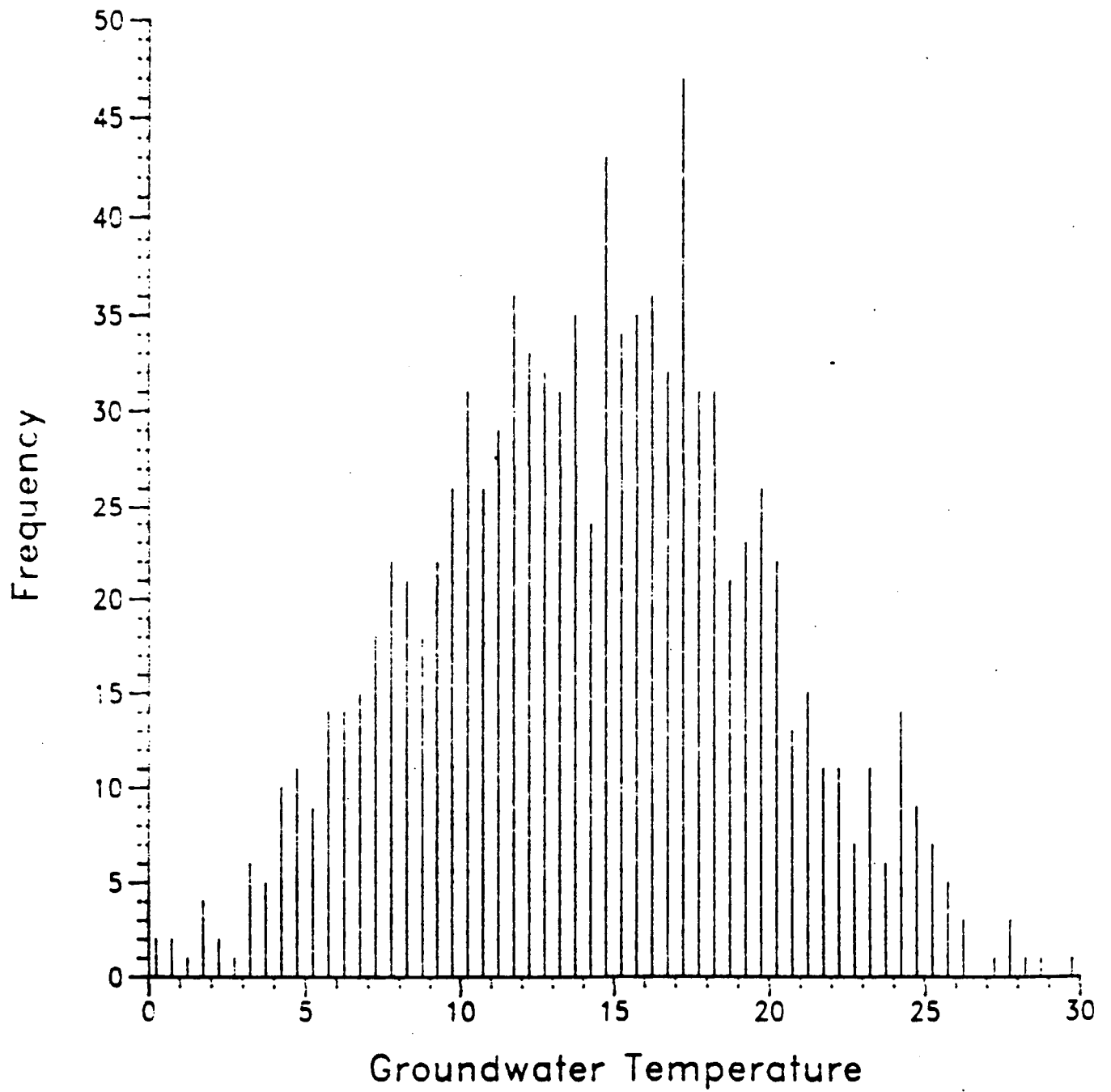


FIGURE 14

5.0 Modeling Results

The groundwater model was implemented with the input data generated by the data, assumptions, or modeling reported in the previous sections. Two general cases are of interest: modeling results for non-degrading chemicals, and for degrading chemicals.

5.1 Results for Nondegrading Chemicals

The behavior of all non-degrading organic chemicals will be identical because sorption does not (as implemented in the model) influence dissolved concentrations. Thus, it is possible to produce a single cumulative frequency distribution for all such compounds. The resulting distribution is given in Figure 15. In addition to the graphical results, tabulated values for the frequency distribution are given in Table 4. Also given in the table are results from simulations using a neutral hydrolysis rate corresponding to a one-year half-life with no sorption.

Table 4. Model Simulation Results for Non-Degrading Chemicals and Chemicals with a One Year Halflife

Cumulative Percentile	*CD Value	
	Non-Degrading Case	One Year Halflife
0.25	$<0.0 \times 10^{-7}$	1.0×10^{-6}
0.50	1.8×10^{-5}	2.0×10^{-5}
0.75	0.0044	0.0046
0.90	0.043	0.043
0.95	0.09	0.10

$$*CD = C_{ADI}/C_L$$

where C_{ADI} = health-based threshold

C_L = leachate concentration

365 DAYS (12 MONTHS)

SUBTITLE C DKN9 = 0.0000811

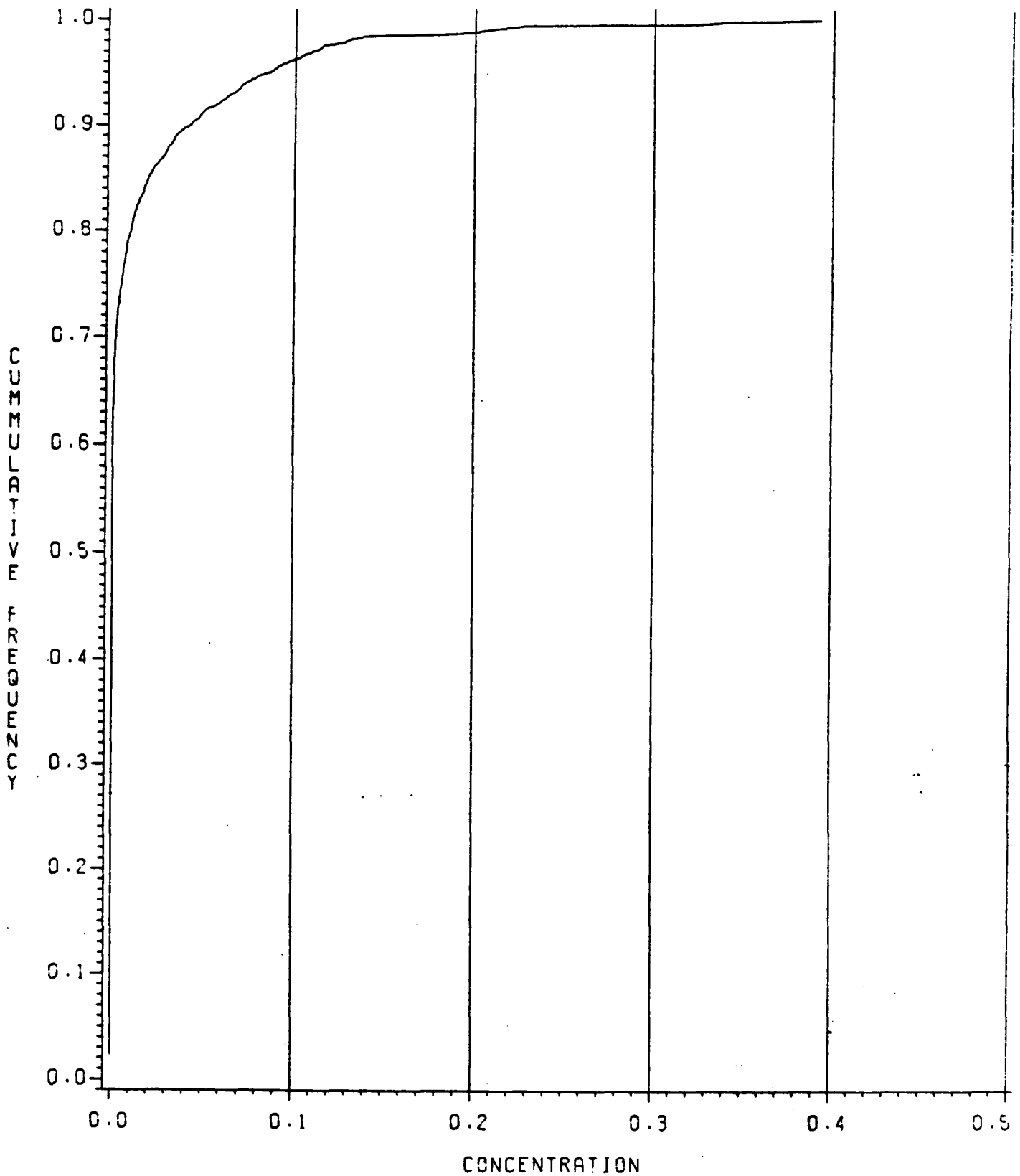


FIGURE 15

5.2 Results for Degrading Chemicals

All degrading chemicals will respond to differences in both rate constants (k_a , k_n , k_b) and partition coefficients (K_D). For this reason no general result can be developed, rather, a unique cumulative frequency curve exists for each individual chemical. To illustrate general differences between behavior of chemicals that do not degrade and those that degrade a series of simulations shown in Figures 16 - 21 were produced representing chemicals that do not sorb but have varying effective half-lives. These results compared to those of Figure 13 illustrate the dramatic impact that degradation can have at "fast" rates but also show a surprisingly small effect at "slow" rates. Table 5 summarizes the results for different rate constants.

Table 5. Model Simulation Results for Degrading Chemicals That Do Not Sorb

Cumulative Percentile	CD Value for Each Noted Halflife					
	30 days	60 days	90 days	8 months	16 months	2 years
0.25	8.0×10^{-6}	9.0×10^{-6}	9.2×10^{-6}	9.7×10^{-6}	1.0×10^{-5}	1.0×10^{-5}
0.50	1.7×10^{-5}	1.8×10^{-5}	1.8×10^{-5}	1.8×10^{-5}	1.8×10^{-5}	1.84×10^{-5}
0.75	3.6×10^{-3}	4.0×10^{-3}	4.4×10^{-3}	4.5×10^{-3}	4.5×10^{-3}	4.5×10^{-3}
0.90	3.5×10^{-2}	3.8×10^{-2}	3.9×10^{-2}	4.2×10^{-2}	4.3×10^{-2}	4.3×10^{-2}
0.95	7.5×10^{-2}	8.2×10^{-2}	8.5×10^{-2}	8.6×10^{-2}	8.7×10^{-2}	8.8×10^{-2}

The graph plots Percent Conversion (Y-axis, 0.0 to 1.0) against Concentration (X-axis, 0.0 to 0.5). The curve starts at (0,0) and rises steeply, reaching approximately 95% conversion at a concentration of 0.1. Beyond this point, the conversion increases very slowly, approaching 100% as the concentration reaches 0.5.

Concentration	Percent Conversion
0.0	0.0
0.02	0.65
0.04	0.80
0.06	0.88
0.08	0.92
0.10	0.95
0.15	0.98
0.20	0.99
0.30	1.00
0.40	1.00
0.50	1.00

FIGURE 16



SUBTITLE C DRNO = 0.0001203



90 DAYS

SUBTITLE C DKN9 = 0.0003209

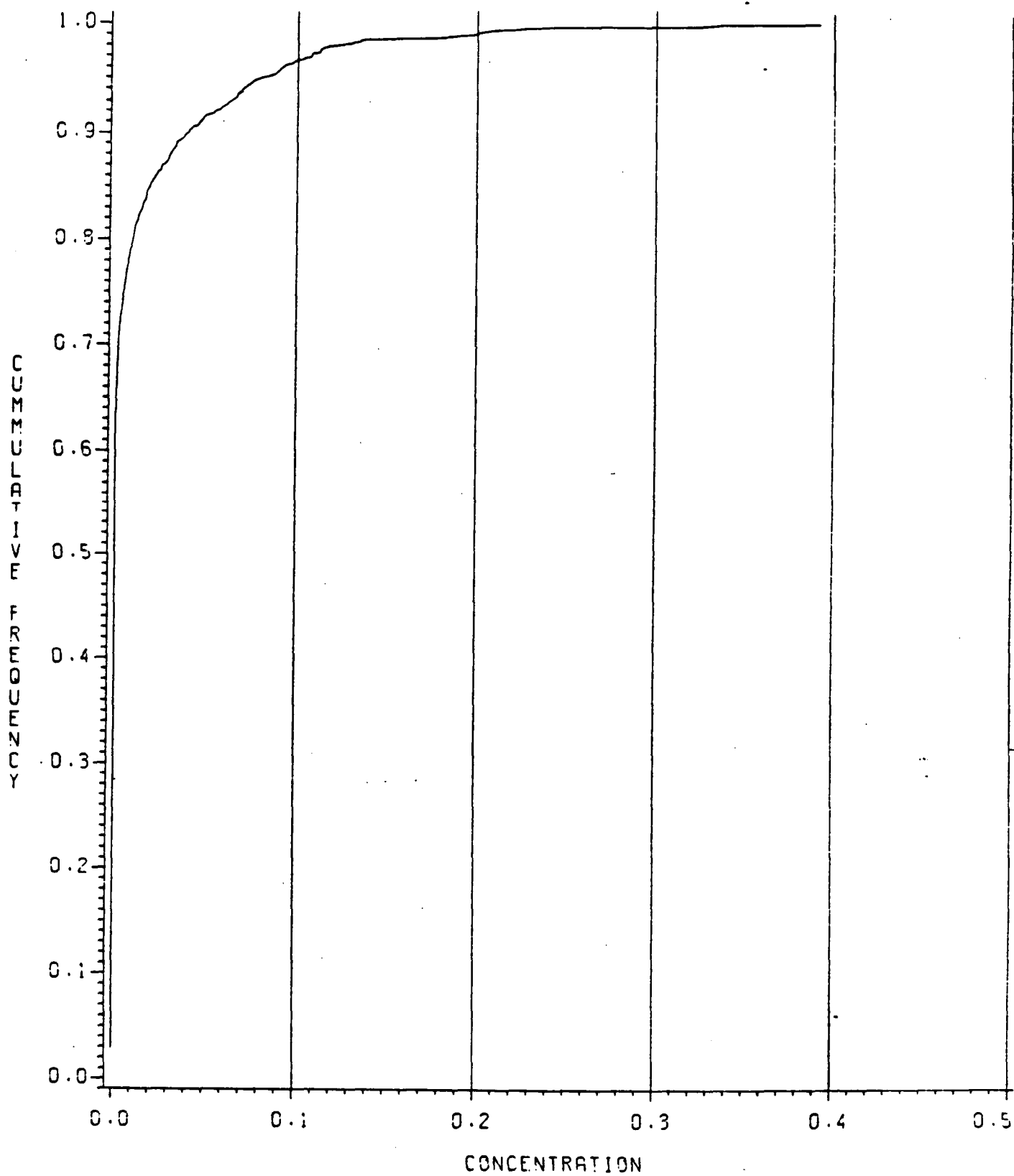


FIGURE 19

60 DAYS

SUBTITLE C DKN0 = 0.0004813

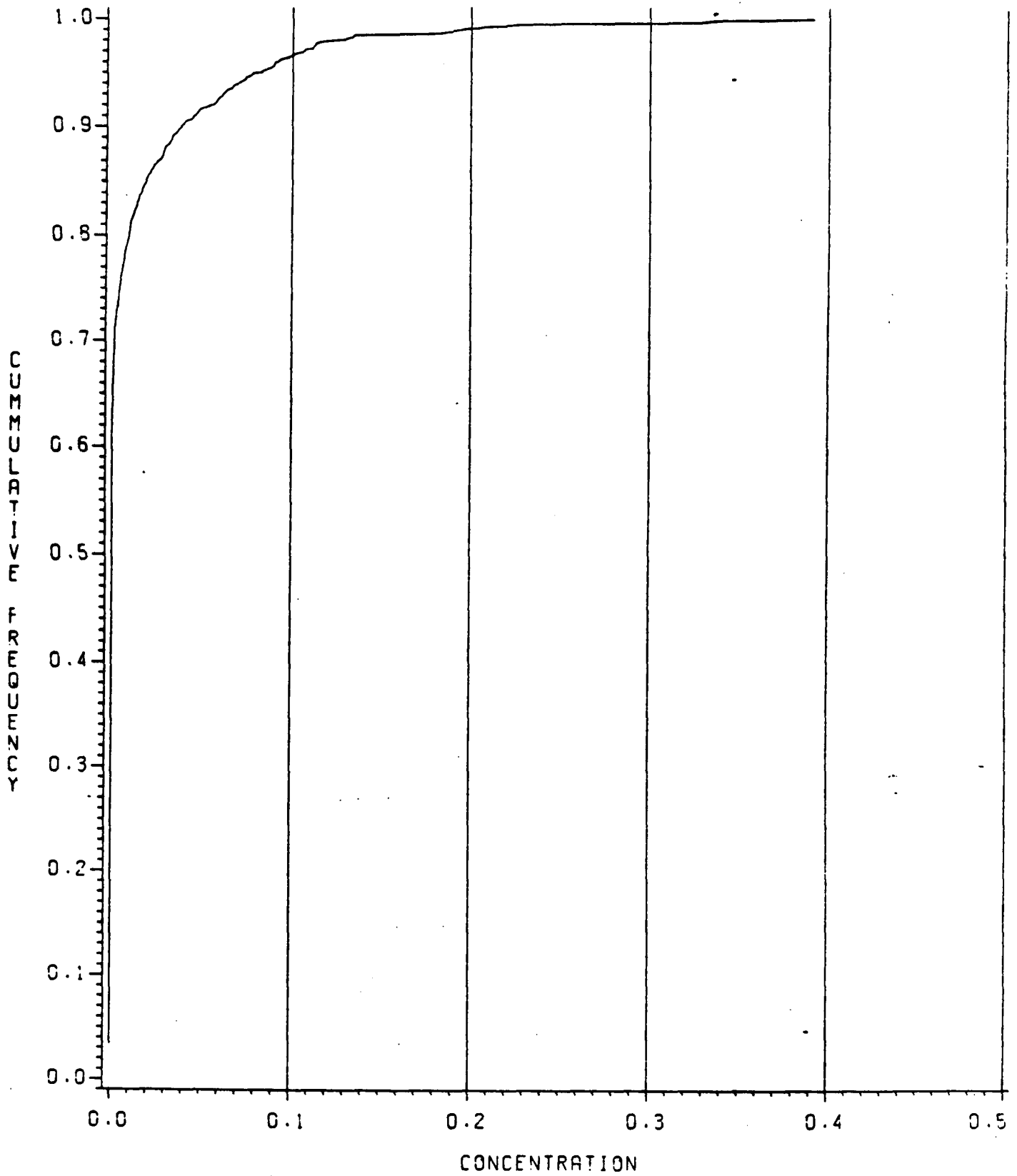
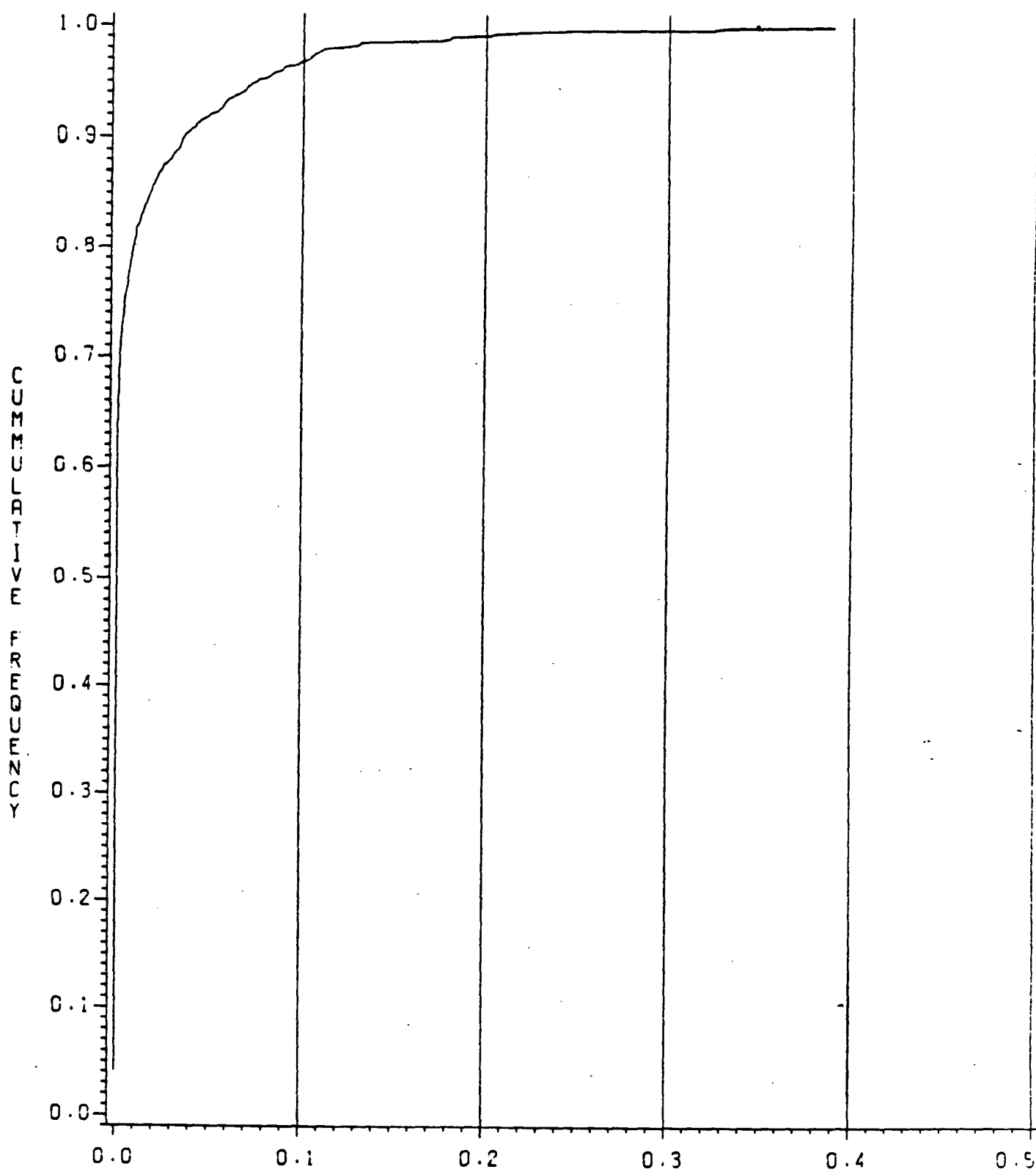


FIGURE 20

30 DAYS

SUBTITLE C DKNO = 0.0009625



CONCENTRATION

FIGURE 21

6.0 Limitations in the Approach and Issues

The results obtained from the monte carlo simulations are intended to represent the performance of land disposal systems as they can exist over the entire country. Attempts have been made as presented in this report to accurately represent the range of possible outcomes of this process. Given the required assumptions, however (including in some cases assumptions of independence when weak dependence is known to exist), it is recommended that extreme values of the derived distributions be viewed as suspect. For this reason, values in excess of the 95th percentile level and below the 5th percentile level should not be viewed as having the "accuracy" of those values intermediate to these extremes.

Recall from the statement of objectives the intent to include an explicit representation of the measurement errors associated with hydrolysis and sorption experiments. This has not yet been done but is under development. For chemicals that do not degrade this is not a requirement. Limited sensitivity analysis to date for degrading compounds suggests that for certain ranges of halflives the outcomes are relatively insensitive, especially near distribution extremes.

Increased confidence in the modeling results can be achieved by completing the following activities:

- (1) developing better distributions for subsurface properties, particularly mean particle sizes and related phenomena based on a more comprehensive review of subsurface data
- (2) expanding the spatial and temporal extent of the HELP model simulations (as described in the E. C. Jordan report, 1985) for leachate volumes

- (3) developing a correlated distribution generation approach for the leachate penetration depths, H , q , and V
- (4) relating dispersivities to porous medium properties (probably hydraulic conductivities) as well as scale
- (5) developing analytic expressions for the sensitivity functions of the model
- (6) conducting exhaustive sensitivity analyses to determine exact sources of the major model sensitivity

REFERENCES

- Anderson, M. P. 1979. Using Models to Simulate the Movement of Contaminants Through Groundwater Flow Systems. CRC Critical Reviews in Environmental Control. Vol. 9, Issue 2, pp 97-156.
- Bear, Jacob. 1979. Hydraulics of Groundwater. McGraw-Hill. 569 p.
- Cox, P. C. and P. Bayhutt. 1981. Methods for Uncertainty Analysis: A Comparative Survey. Risk Analysis 1, 251-258.
- Davis, S. N. 1969. Porosity and Permeability of Natural Materials. In: Flow Through Porous Media. Ed. Roger J. M. DeWiest. Academic Press, 1969. 530 p.
- E. C. Jordan Co. 1985. Analysis of Engineered Controls of Subtitle C Facilities for Land Disposal Restrictions Determinations. Report Submitted to EPA, Contract 68-01-7075.

Freeze, R. A. and J. A. Cherry. 1979. Groundwater. Prentice-Hall Inc. 604p.

Gelhar, L. W., A. Mantoglou, C. Welty, and K. R. Rehfeldt. 1985. A Review of Field Scale Subsurface Solute Transport Processes Under Saturated and Unsaturated Conditions. Electric Power Research Institute, Groundwater Studies EPRI EA-CCCC. Palo Alto, Calif. (Draft Report) 107 p.

Guen, O., F. J. Molz, and J. G. Melville. 1984. An Analysis of Dispersion in a Stratified Aquifer. Water Resources Research. Vol. 20, No. 10, pp 1337-1354.

Huyakorn, P. S., M. J. Unga, E. D. Sudicky, L. A. Mulkey, and T. D. Wadsworth. 1985. RCRA Hazardous Waste Identification and Land Disposal Restrictions Groundwater Screening Procedure. U.S. EPA, Washington, DC.

Karickhoff, S. W. 1985. Sorption Protocol Evaluation for OSW Chemicals. U.S. EPA, Athens Environmental Research Laboratory, Athens, GA.

Mackay, D. M., P. V. Roberts, and J. A. Cherry. 1985. Transport of Organic Contaminants in Groundwater. Environ. Sci. Technol., Vol. 19, No. 5, pp 384-392.

Molz, F. J., O. Guen, and J. G. Melville. 1983. An Examination of Scale-Dependent Dispersion Coefficients. Groundwater. Vol. 21, No. 6, pp 715-725.

Pickens, J. F. and G. E. Grisah. 1981. Scale-Dependent Dispersion in a Stratified Granular Aquifer. J. Water Resources Research. Vol. 17, No. 4, pp 1191-1211.

Pickens, J. F. and G. E. Grisak. 1981. Modeling of Scale-Dependent Dispersion in Hydrogeologic Systems. Water Resources Research. Vol. 17, No. 6, pp 1701-1711.

Rowe, William D. 1977. An Anatomy of Risk. John Wiley and Sons. 488 p.

Sauty, Jean-Pierre. 1980. An Analysis of Hydrodispersive Transfer in Aquifers. Water Resources Research. Vol. 16, No. 1, pp 145-158.

Wolfe, N. L. 1985. Screening of Hydrolytic Reactivity of OSW Chemicals. U.S. EPA, Athens Environmental Research Laboratory, Athens, GA.

Sudicky, E. A., J. A. Cherry, and E. O. Frind. 1983. Migration of Contaminants in Groundwater at a Landfill: A Case Study. Journal of Hydrology, 63, pp 81-108.

ANALYSIS OF ENGINEERED
CONTROLS OF SUBTITLE C
FACILITIES FOR LAND DISPOSAL
RESTRICTIONS DETERMINATIONS

APPENDIX D

ANALYSIS OF ENGINEERED CONTROLS OF SUBTITLE C
FACILITIES FOR LAND DISPOSAL RESTRICTIONS DETERMINATIONS

FINAL REPORT

Contract No. 68-01-7075
Work Assignment No. 24
RTI Project No. 3224
ECJ Project No. 4756-00

Contractor: Research Triangle Institute
Subcontractor: E.C. Jordan Co.
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Washington, D.C.

June, 1985

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1.0 INTRODUCTION

In accordance with recent federal legislation, land disposal of certain hazardous wastes is to be banned. Short-term hazards posed by these wastes can be controlled by specially engineered and constructed facilities. The banning of wastes is intended to prevent hazards posed by certain wastes as the long-term performance of the engineered facilities change. Analysis of these changes in long-term performance provides information needed to determine which wastes to ban.

1.1 Background

In accordance with the Resource Conservation and Recovery act of 1976 (RCRA) and the Solid Waste Amendments of 1984 (Amendments), the U.S. Environmental Protection Agency (EPA) must review all listed hazardous wastes and hazardous constituents and determine which wastes should be banned from hazardous waste land disposal facilities. The Amendments require waste to be banned for as long as the waste is hazardous if the wastes are highly mobile, highly toxic, persistent or bioaccumulate and pose a significant threat to human health and the environment. Further, the EPA is developing a complementary test, EP-III, for the toxicity characteristic of hazardous wastes. This test, similar to the existing test (EP-I) which screens wastes for the presence of selected mobile heavy metals and organic compounds (40 CFR 261.24), will screen wastes for the presence of other mobile organic constituents. Both tests are intended to be

used by generators of solid wastes to determine if a waste is hazardous. While specific criteria were established (i.e., 100 times National Interim Primary Drinking Water Regulations) for the concentration of chemicals in the extract of EP-I, new criteria need to be identified for chemicals that exceed these criteria to determine if they should be banned from land disposal facilities. Criteria will also be needed for waste classification and banning for chemicals detected in the extract of EP-III.

The EPA, as part of a comprehensive regulatory development program to protect human health and the environment from mobile and toxic wastes has identified the following needs:

- o identify wastes that should be classified hazardous and managed in Subtitle C (hazardous waste) facilities; and
- o identify those wastes that should be banned from Subtitle C land disposal facilities.

Wastes that are not classified as hazardous may be managed in Subtitle D facilities such as sanitary landfills, municipal incinerators and resource recovery facilities.

EPA's Office of Solid Waste (OSW) is currently developing a framework for a regulatory program to address these needs. This proposed framework is intended to provide a generic screening procedure for all wastes generated in the United

States. Under OSW's proposed framework, EPA will establish screening levels for chemical constituents of wastes.

The screening levels will be transformed as maximum acceptable concentrations for chemicals in air emissions, surface runoff and leachate emanating from land disposal facilities. These transformed criteria will be established through a back-calculation procedure. This procedure uses fate and transport models that start from a point of potential chemical exposure at a concentration protective of public health and the environment and estimates the chemical concentration in leachate that will not cause the criterion to be exceeded. Wastes that yield a chemical concentration in the EP extract that exceeds the transformed criterion will be classified as hazardous.

The transformed criteria will be further adjusted to account for the additional control over hazardous wastes provided by engineered structures at Subtitle C land disposal facilities. If chemical concentrations in the EP extract exceed these adjusted criteria, then the waste would be banned from Subtitle C land disposal facilities.

Adjustments may be based on one or more of the following:

- o leaching environment and chemical flux to air, surface water or groundwater;
- o containment facilities (e.g., cover system, liner system); and

- o management controls (e.g., groundwater monitoring, operating and post-closure care period procedures).

Leaching environments and chemical flux are highly site specific and waste specific. Further, the interaction of wastes and mobile constituents of wastes may significantly alter the chemical flux from a facility that occurs as precipitation (e.g., rainfall) percolates through the waste deposit. Increases in the chemical flux may occur by enhancing the solubility of a constituent by the presence of a mobile chemical from another waste in percolating precipitation. For example, hydrophobic chemicals may dissolve to a greater extent if the percolating water contains sufficient organic solvents to enhance the solubility of these chemicals. Decreases in chemical flux may occur due to waste-waste interactions or other physicochemical or biological activity that may occur during facility operation and containment of the waste after closure. Examples of processes that may reduce chemical flux to the environment include:

- o collection of mobile chemicals in leachate during operations and post-closure care period (PCP);
- o hydrolysis of organic wastes;
- o waste/waste interactions;
- o chemical, physical and/or biological transformation or degradation of hazardous constituents; and

o volatilization.

Management of land disposal facilities is typically more structured and controlled at Subtitle C facilities than at Subtitle D facilities. During operations, wastes are screened, analyzed, and, if appropriate, rejected to protect the performance of engineered containment systems and public health and the environment. Undesirable waste-waste interactions can also be avoided. Groundwater monitoring, inspection, maintenance and reporting enhance the intended performance of the facility. Contingency plans and a regulatory program for corrective action exist through the post-closure care period to prevent and control chemical flux from the facility.

A greater degree of containment is typically provided at Subtitle C facilities than at Subtitle D facilities. Installation and operation of leachate collection systems is specifically regulated. Cover and liner system components and performance are also specifically regulated. Engineered controls are specified in 40 CFR Parts 264 and 265 for Subtitle C and in 40 CFR 257 for Subtitle D land disposal facilities.

1.2 Objective

The objective of this report is to identify how these engineered controls can be incorporated into the back-calculation procedure. Engineered controls will be analyzed and evaluated to factor these controls into OSW's approach for restricting land disposal of hazardous waste.

The analysis takes into account the time importance of low permeable barrier layers in cover and liner systems (e.g., polymeric geomembranes, clay), leachate collection and removal systems, groundwater monitoring and corrective action requirements of 40 CFR Parts 264 and 265. Particular emphasis is placed on long-term events and performance of these control systems.

OSW's proposed regulatory framework is intended to provide a generic screening procedure, independent of site specific factors and protective of human health and the environment. The analysis and evaluation was conducted on a national scale to incorporate engineered controls into OSW's proposed regulatory framework.

1.3 General Approach

A procedure to incorporate engineering controls into OSW's proposed regulatory framework needs to consider the following factors:

- o chemical flux depends on the presence of a transport medium and the leachability of specific chemicals in that medium; and
- o performance of engineered controls depends on duration of exposure to natural weathering processes and chemicals contained.

Chemical flux to groundwater from a land disposal facility occurs as a result of chemicals dissolving into liquids migrating through the waste. Since bulk and containerized liquid wastes are currently banned from land disposal facili-

ties, chemical flux is the result of precipitation that percolates through the waste.

Performance of engineered control structures in the short-term depends on appropriate design, material selection and specification, construction, waste screening, inspection, maintenance and monitoring. Over time, performance characteristics of cover or liner system components can be expected to change. Some components, such as polymeric geomembranes, may undergo dramatic change in performance while other components, such as clay layers, may undergo more limited change in performance. The effect these changes have on component and system performance will control the leaching rate and thus the mass and concentration of chemical leached.

Duration of containment is important for evaluating the threat posed by hazardous wastes if the processes identified in Section 1.1 can be reasonably expected to significantly reduce mobility, toxicity or tendency to bioaccumulate during the containment period. Currently, there is insufficient information available to accurately account for such significant reductions in waste characteristics in a generic screening procedure. Where information is available, then the generator may seek a variance through a site-specific petition procedure.

The back-calculation procedure proposed by OSW incorporates several important characteristics:

- o the procedure should not be dependent on a limited mass of waste (the procedure does not limit hazards due to the size of a facility or the quantity of waste deposited in a facility); and
- o the procedure is independent of long-term (on the order of thousands of years) events or changes in facility performance.

These characteristics indicate that short-term performance of engineered control, such as during operations or the typical 30 year PCP, are not important in transforming the screening levels to classification and banning criteria. The most important characteristic is the leachate rate caused by precipitation percolating through the waste.

Land disposal facilities typically include:

- o waste piles;
- o surface impoundments;
- o landfill;
- o land treatment; and
- o subsurface injection well facilities.

Requirements of 40 CFR Parts 264 and 265 state that closure of waste piles and subsurface impoundments must remove all hazardous waste or close in compliance with all landfill regulations. Due to the long-term characteristic of the screening procedure, only closed landfills are considered in assessing engineered controls. Land treatment and subsurface injection well facilities represent special cases and are not considered in this analysis.

Analysis and evaluation of engineered controls at land disposal facilities were incorporated into the proposed regulatory framework by the following approach:

- o identify significant mechanisms and extent of changes in leaching rates from Subtitle C and D facilities;
- o determine long-term leaching rates representative of climatic and site conditions within the contiguous 48 states for Subtitle C and D facilities;
- o develop an area weighted distribution of leaching rates; and
- o conduct a sensitivity analysis of the parameters used to estimate the leaching rates.

This approach was selected because the primary transport mechanism to the groundwater is percolating precipitation, long-term performance will yield the greatest chemical flux to the groundwater, and the procedure should be generic.

2.0 ANALYTICAL APPROACH

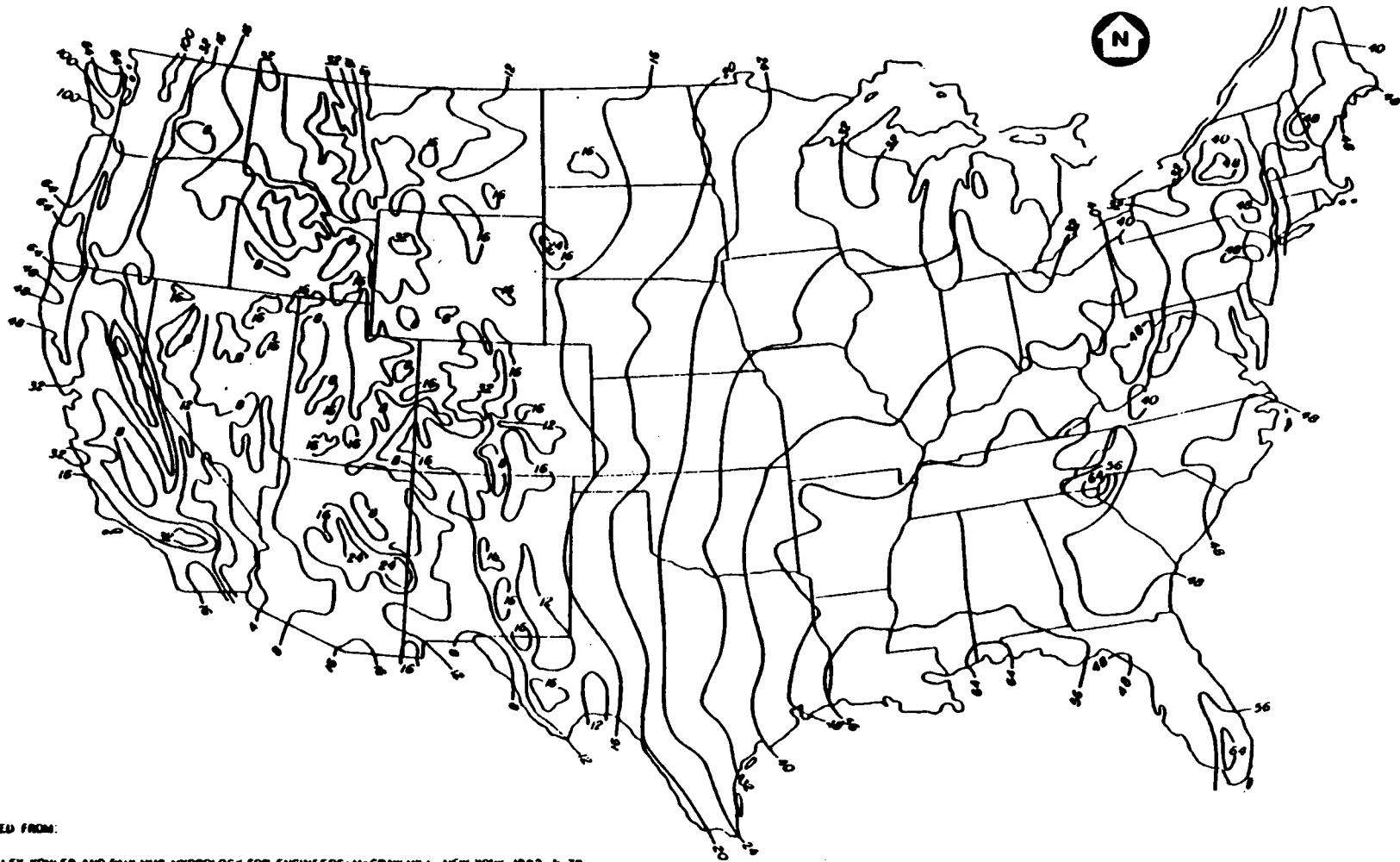
The analytical approach used to incorporate engineered controls into land disposal restrictions is based on the leaching process and anticipated long-term conditions at the disposal facility. A standard set of conditions is used to develop national leaching rate distributions for Subtitle C and D facilities. Sensitivity analysis of the standard conditions describes the potential variation for each distribution.

2.1 Leaching Process

Long-term chemical flux at land disposal sites is caused by precipitation percolating through the waste. The leaching process depends on regional climatic conditions and site specific facility conditions. Quantitative estimates of long-term leaching rates can be made using historic climatic and anticipated facility conditions.

2.1.1 Climatic Conditions

Distribution of precipitation in the contiguous 48 states is shown in Figure 2-1. Precipitation will vary from year to year, but typically will vary less than 20 percent of the average precipitation. Precipitation in the form of rainfall will initially enter or infiltrate the cover soil of a disposal facility until the precipitation rate exceeds the infiltration rate and will then drain from the facility as surface runoff. Water that infiltrates the sur-



ADAPTED FROM:

LINSLEY, KOHLER, AND PAULMUS, HYDROLOGY FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1962, P. 79

FIGURE 2-1
MEAN ANNUAL PRECIPITATION (IN INCHES)

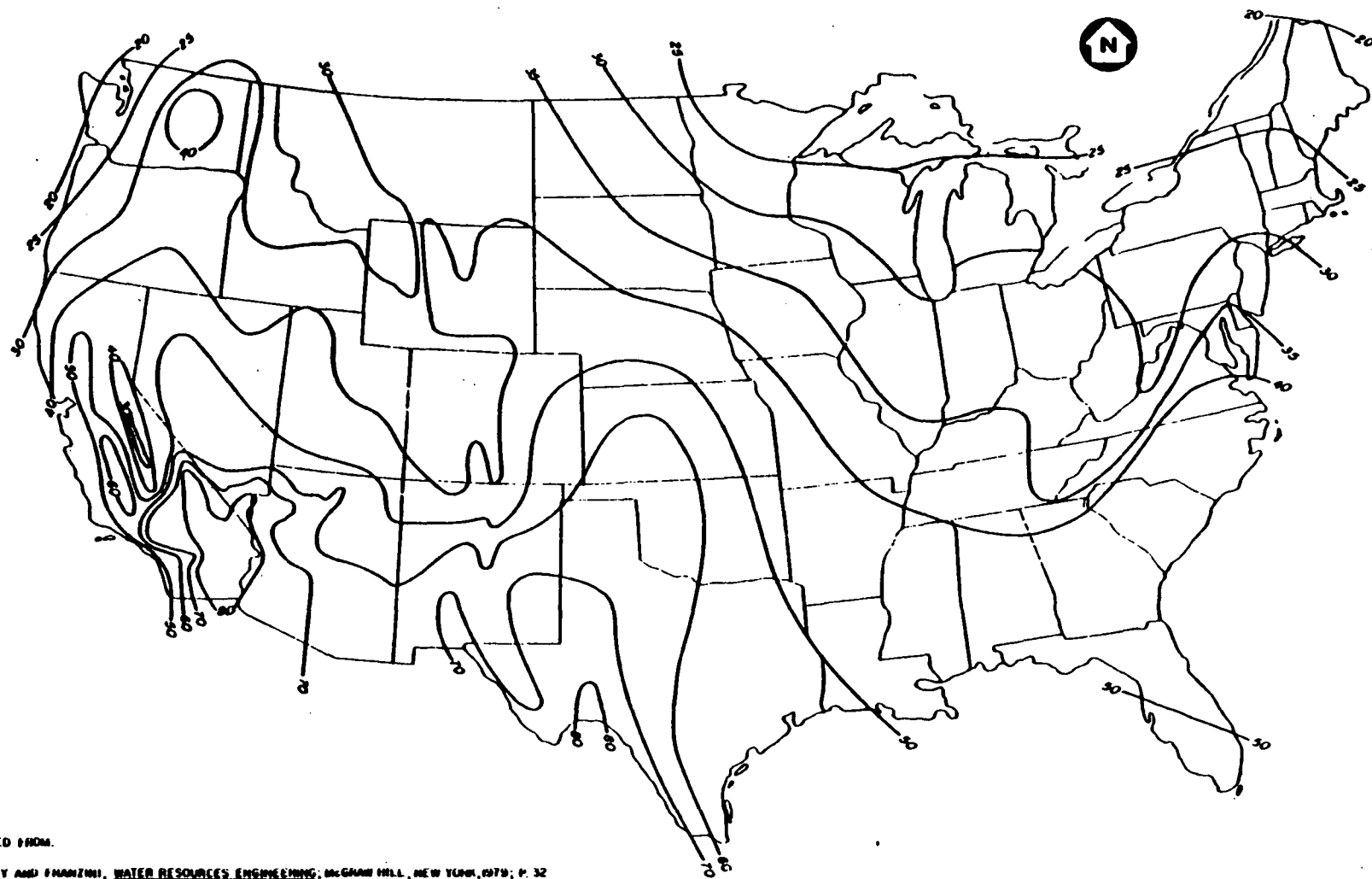
E.C. JORDANCO

face soil is stored in the soil and transmitted to lower, less moist soils. Infiltrating water is removed from the soil through evaporation and transpiration. Average evaporative losses are shown in Figure 2-2. Evaporative losses from pans or lakes are used because of the difficulty in directly measuring evaporation and transpiration losses from soil. Areas with high evaporative losses are also found to have high evaporative-transpiration (E-T) losses. Actual E-T losses will depend on such factors as solar radiation, wind speed, humidity, plant type, (including leaf area and root zone) texture of surface soil, and availability of water.

Precipitation in excess of losses due to surface runoff and E-T will result in net infiltration or deep percolation to soils below the root zone. This net infiltration is considered comparable to the long-term leaching rate at land disposal facilities. Site specific facility conditions such as low permeable layers or drainage layers may cause the leaching rate to be less than net infiltration since these layers may divert percolating water from underlying waste.

2.1.2 Analysis Method

Quantitative estimates of leaching rates are typically calculated using the Hydrologic Evaluation of Landfill Performance (HELP) model or the Water Budget (WATBUG) model. The HELP model relies on the Penman technique to estimate net infiltration and was developed by EPA in 1980 (Perrier and Gibson, 1980) and most recently revised in 1984 (Schroeder, et al., 1984) specifically for hazardous waste landfills. The WATBUG model relies on the Thornthwaite method



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LINSLEY AND MANZINI, WATER RESOURCES ENGINEERING, MCGRAW HILL, NEW YORK, 1979, P. 32

FIGURE 2-2
AVERAGE ANNUAL LAKE EVAPORATION (IN INCHES)

—E.C. DIAMANTO—

(Thornthwaite and Mather, 1957) of estimating net infiltration and has been used for landfill applications in automated form since 1980 (Mather, et al., 1980) and manual form since 1975 (Fenn, et al., 1975). The HELP model conducts the water balance computations on a daily time step, but relies on average monthly temperatures; and the WATBUG model relies on a monthly time step.

The HELP model was adapted from similar models developed by the US Department of Agriculture and incorporates analytical modules that account for drainage barrier layers that can be installed in cover or liner systems. The WATBUG, in the form presented by Mather, does not account for such drainage layers.

The HELP model is readily available through a National Technical Information Service (NTIS) time-share computer and includes the most recent 5 years of climatic data at more than 100 reporting stations. The WATBUG uses a 25-year average of climatic conditions and requires that precipitation data be entered into the computer for each station.

The HELP model was chosen to conduct this analysis of the leaching rate for the following reasons:

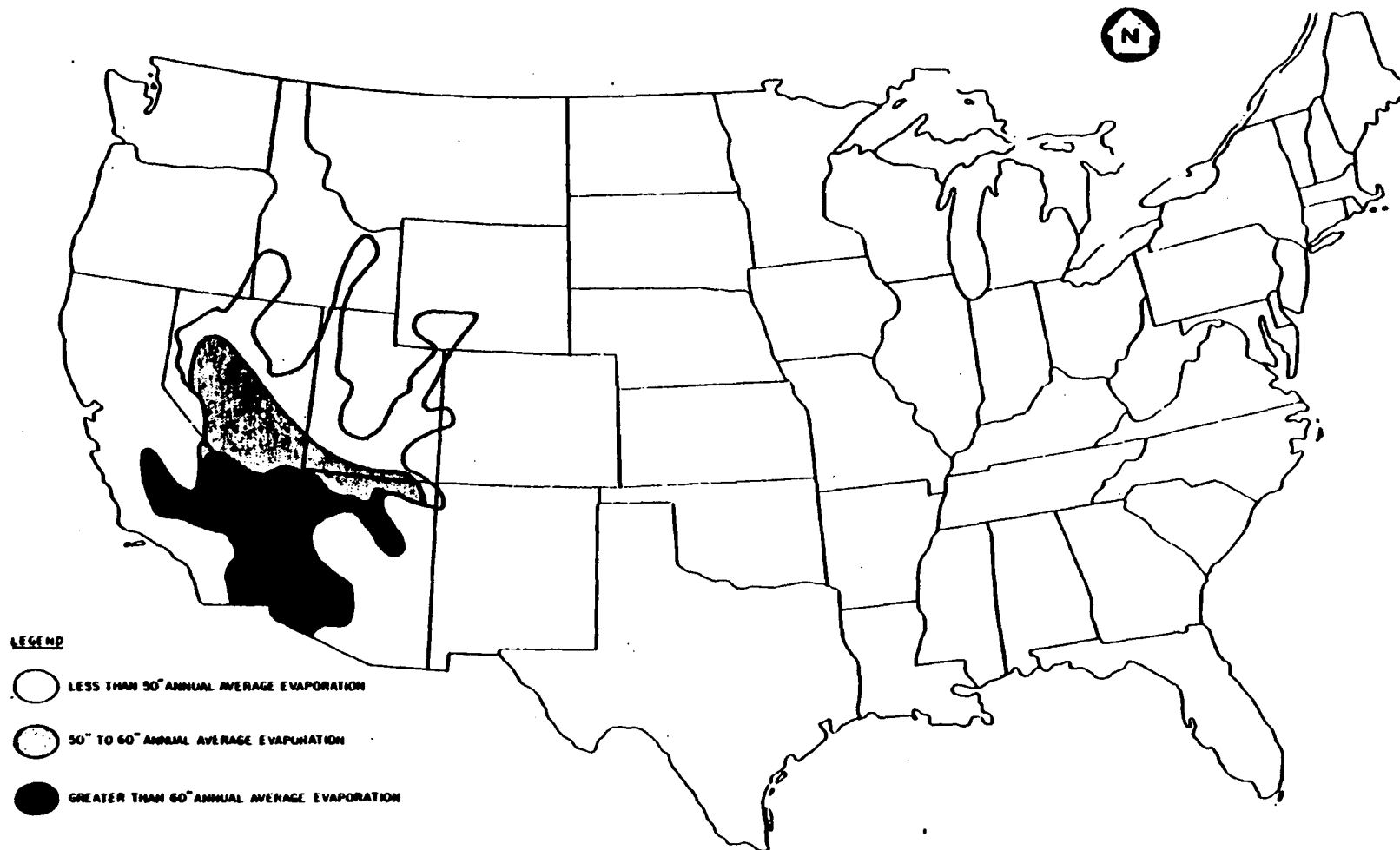
- o the model is currently being validated and documented by the EPA for land disposal sites; whereas a comparable validation and documentation is not available for the WATBUG model;
- o ease of access to the large number of reporting stations compared to manual entry for WATBUG;

- o 5 years of data was not considered to be an overly restrictive data base compared to 25 years (see Appendix A); and
- o barrier and drainage layer performance is already incorporated into the HELP model.

2.2 Climatic Data Base

The precipitation data, Figure 2-1, and evaporation data, Figure 2-2, were used to identify ranges of conditions that may be encountered in the contiguous 48 states. These conditions are summarized in Figures 2-3 through 2-8. Six precipitation ranges were selected as representative of the U.S. and three E-T conditions were selected. A total of 18 climatic conditions were identified for developing the national leaching rate distributions. Greater definition of the distribution is possible by selecting a larger number of precipitation and/or evaporation conditions. If greater definition is desired, emphasis should be placed in areas of the country where precipitation is greater than E-T because these areas yield the larger leaching rates.

A reporting station of climatic data was selected in each of the 18 areas shown in Figures 2-3 through 2-8. Stations were selected from the more than 100 cities included in the HELP model data base. Cities selected are listed in Table 2-1. Since some areas selected are larger than others, it may be desirable to relate the estimated leaching rate to the percent of the total area of the contiguous 48 states.



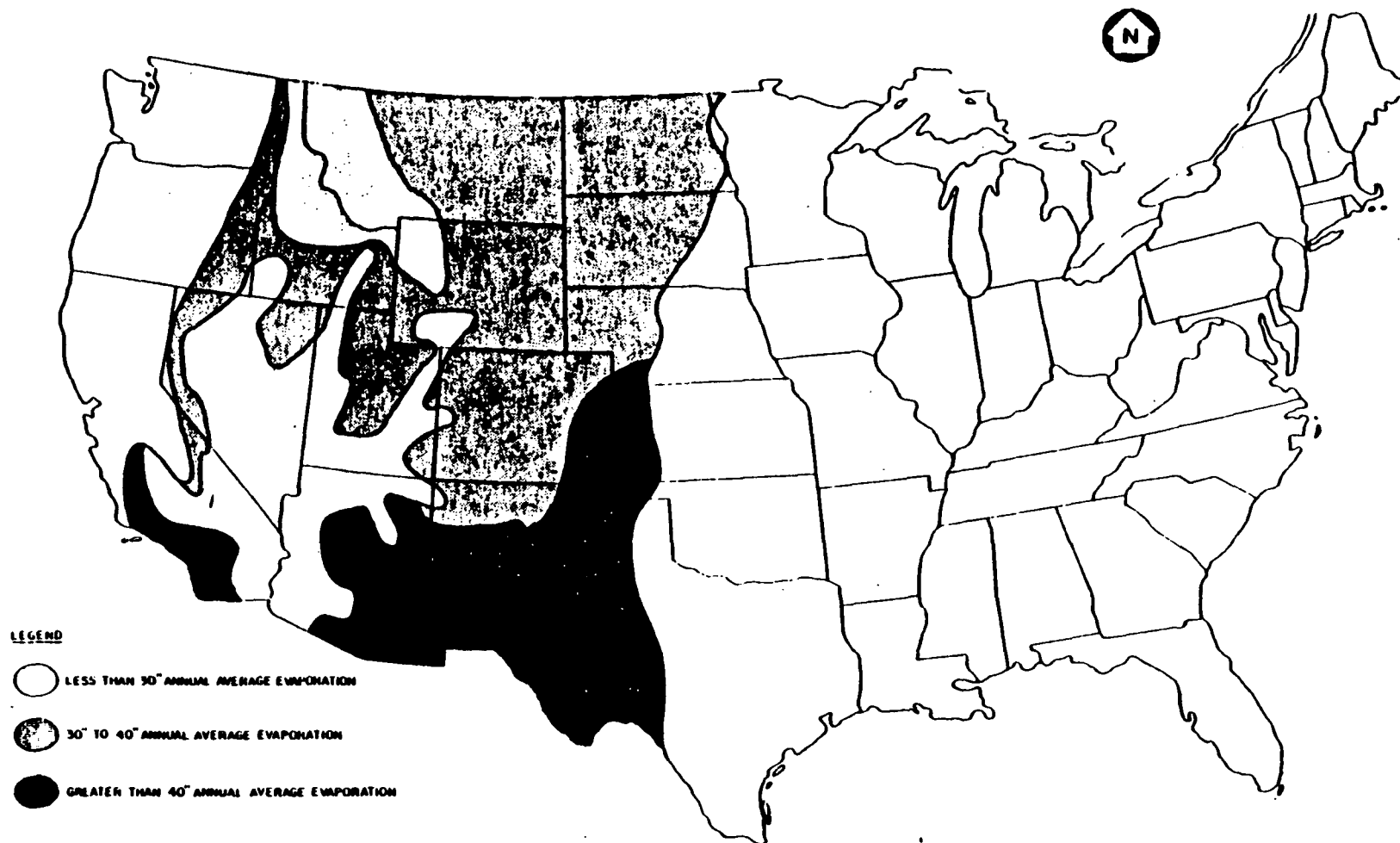
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PRECIPITATION - LINSLEY, KOHLER, AND PAULMUS, HYDROLOGY FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1982, P. 70

FIGURE 2-3
LESS THAN 10" ANNUAL AVERAGE PRECIPITATION

— EC JORDANCO —



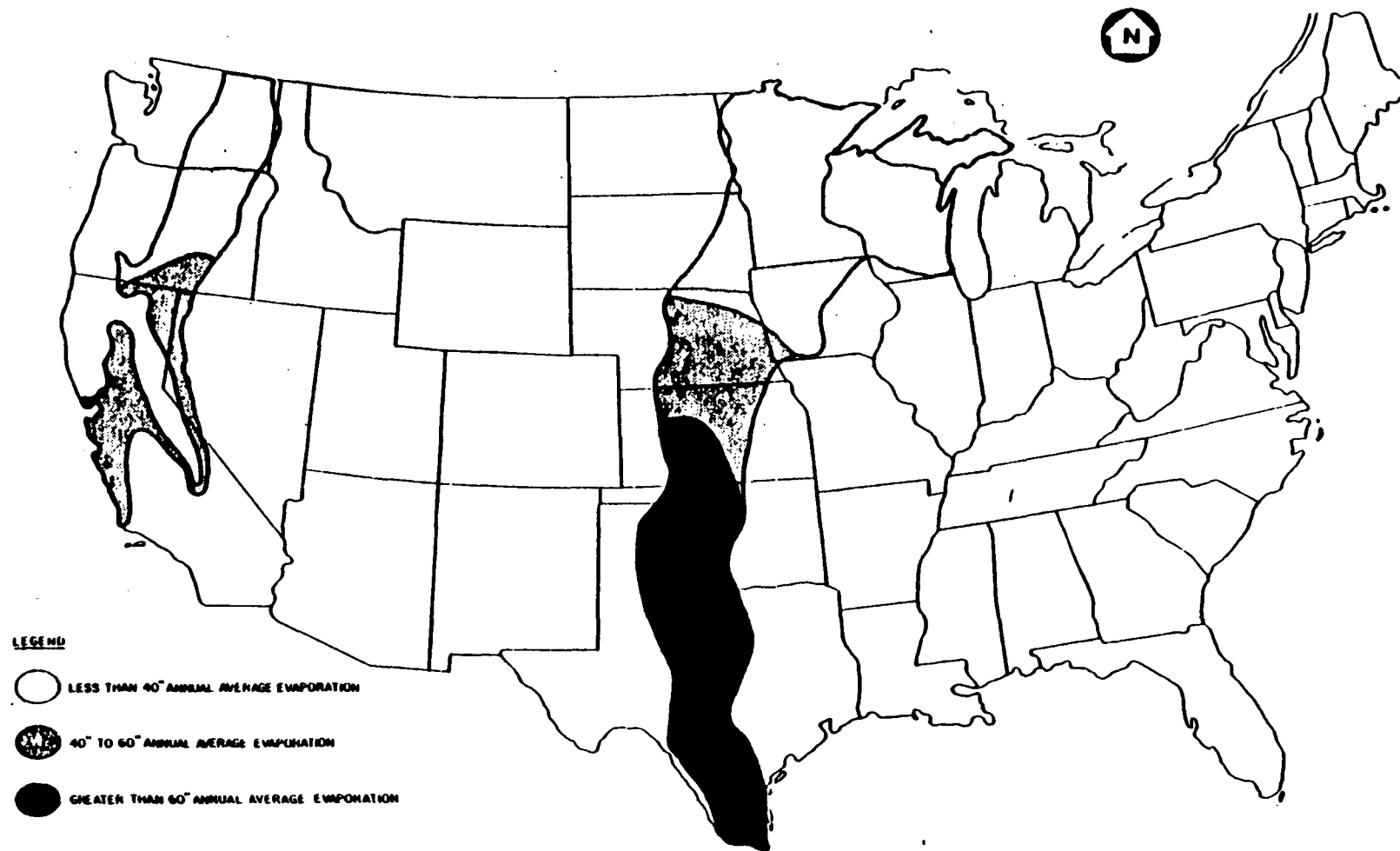
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PRECIPITATION - LINSLEY, KOHLER, AND PAULHUS, HYDROLOGY FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1982, P. 78

FIGURE 2-4
10" TO 20" ANNUAL AVERAGE PRECIPITATION

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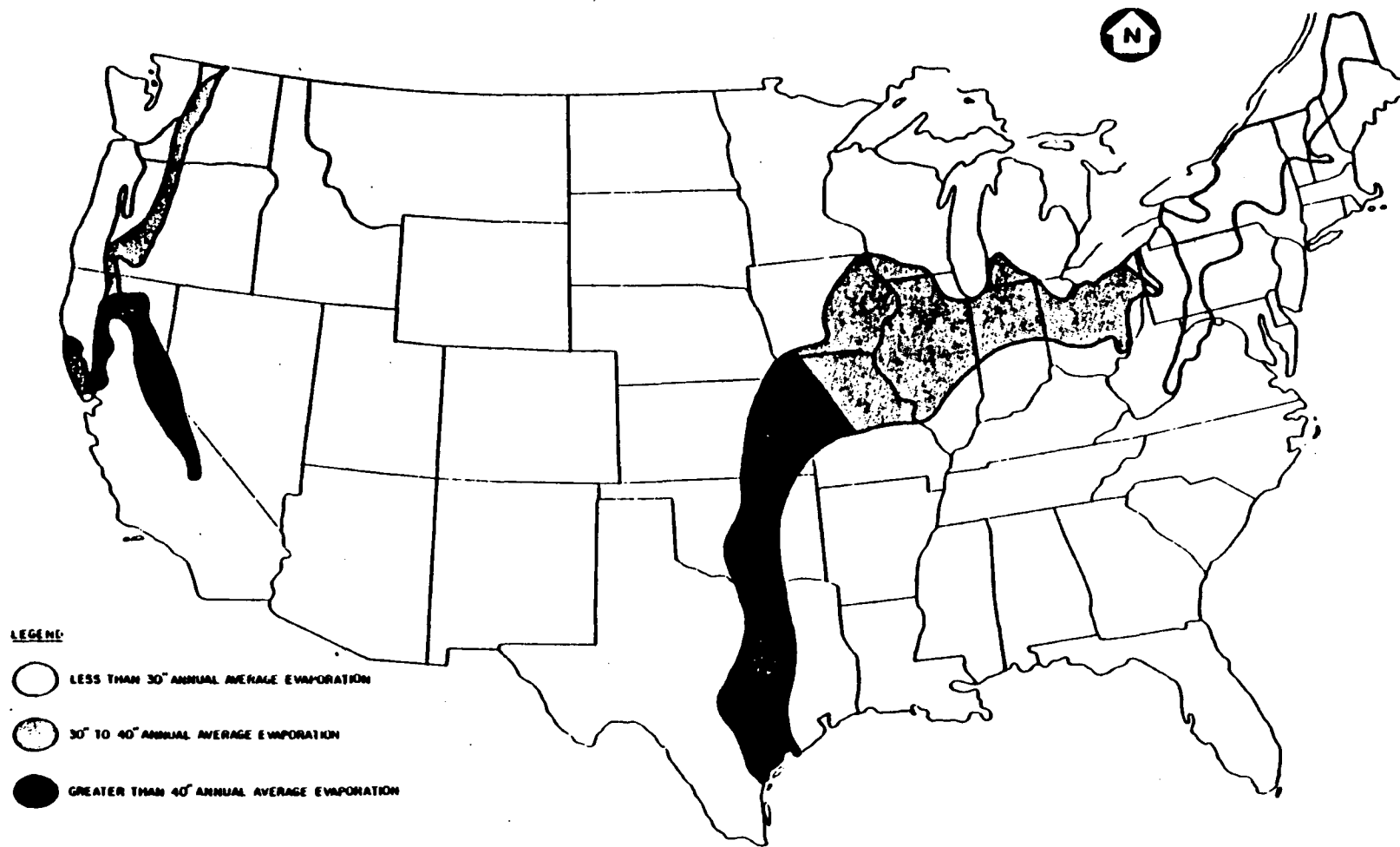
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PRECIPITATION - LINSLEY, KOHLER, AND PAUL HUS, HYDROLOGY FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1962, P. 78

FIGURE 2-5
20" TO 32" ANNUAL AVERAGE PRECIPITATION

E.C. JORDANCO

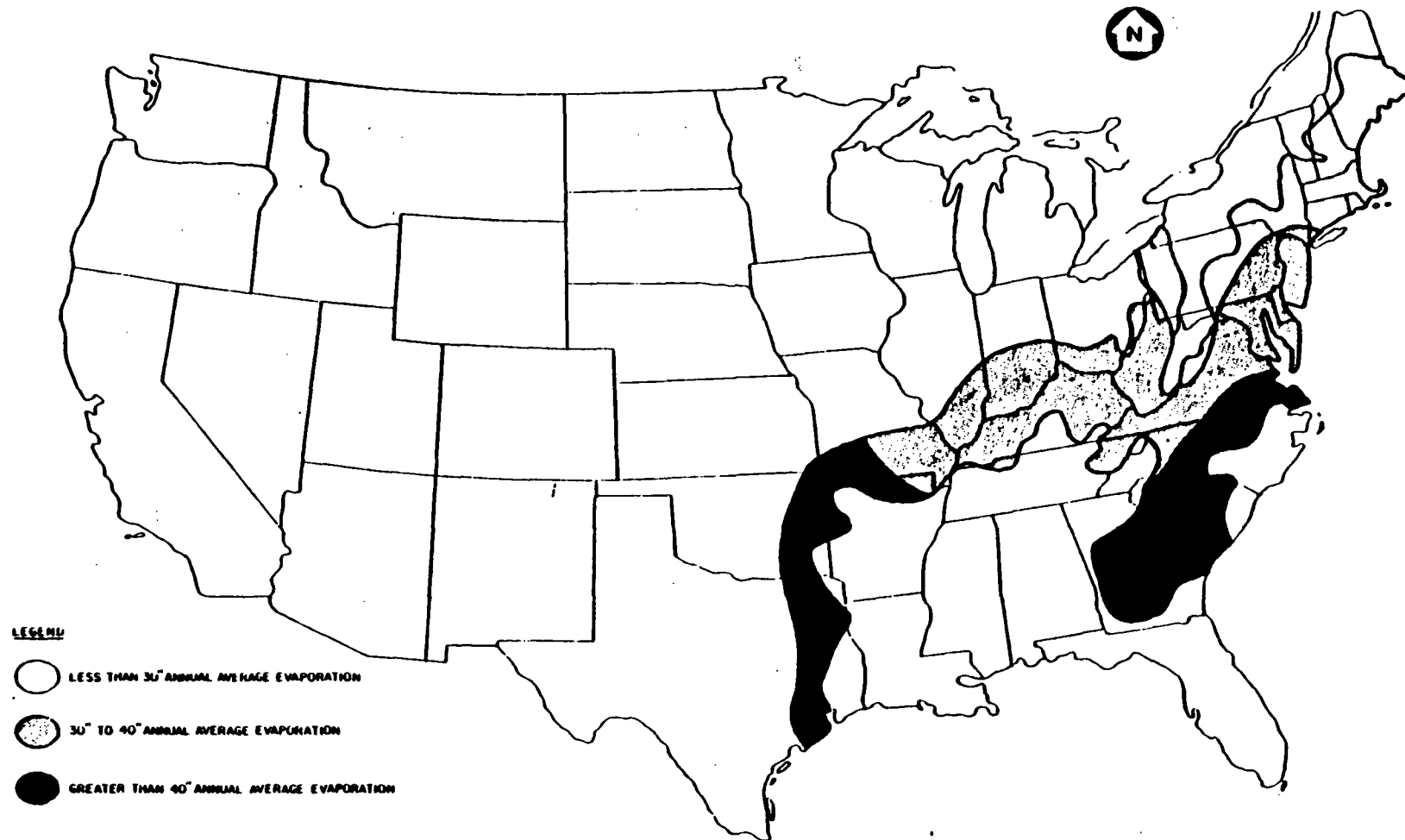


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 PRECIPITATION - LINSLEY, RUMEN, AND PALAHUS, HYDROLOGY FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1962, P. 78

FIGURE 2-6
32" TO 40" ANNUAL AVERAGE PRECIPITATION

- E.C. JORDANCO -



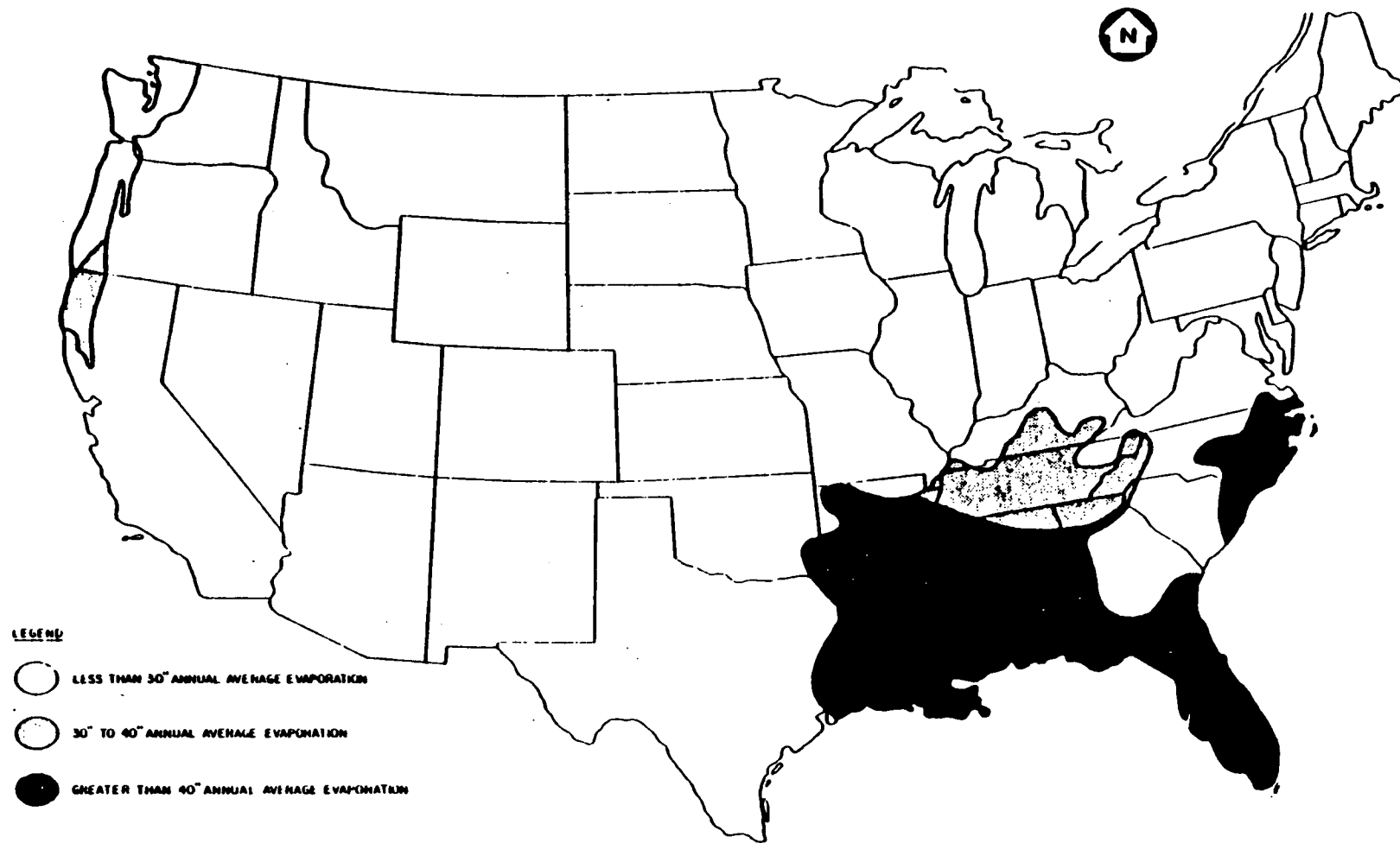
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PRECIPITATION - LINSLEY, KUMLEN, AND PAULIUS, HYDROLOGIST FOR ENGINEERS, MCGRAW HILL, NEW YORK, 1982, P. 78

FIGURE 2-7
40" TO 48" ANNUAL AVERAGE PRECIPITATION

EC JORDANCO



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LAKE EVAPORATION—LINSLEY AND FRANZONI, WATER RESOURCES ENGINEERING, MCGRAW HILL, NEW YORK, 1979, P. 32

PRECIPITATION—LINSLEY, KUMLEN, AND PAULMUS, HIGHLY U.S. FIRM ENGINEERS, MCGRAW HILL, NEW YORK, 1982, P. 78

FIGURE 2-8
GREATER THAN 48" ANNUAL AVERAGE PRECIPITATION

—E.C. JORDAN CO.

TABLE 2-1

CITIES SELECTED FOR CLIMATIC DATA^a

PRECIPITATION RANGE ^b	EVAPORATION POTENTIAL	CLIMATOLOGICAL CENTER	AVERAGE 5-YEAR PRECIPITATION ^b	PERCENT OF TOTAL AREA ^c
<10	Low	Pocatello, ID	9.8	3
	Medium	Cedar City, UT	9.8	2
	High	Las Vegas, NV	5.3	3
10-20	Low	Great Falls, MT	17.9	3
	Medium	Rapid City, SD	15.4	18
	High	Midland, TX	16.3	11
20-32	Low	St. Cloud, MN	25.8	9
	Medium	Grand Island, NE	22.2	4
	High	Oklahoma City, OK	30.4	5
32-40	Low	Montpelier, VT	34.5	3
	Medium	Cleveland, OH	36.7	5
	High	Columbia, MS	37.1	3
40-48	Low	Boston, MA	40.9	3
	Medium	Indianapolis, IN	40.3	7
	High	Charleston, SC	48.3	5
>48	Low	Astoria, OR	65.6	2
	Medium	Knoxville, TN	48.1	3
	High	Tallahassee, FL	68.4	11

^a Based on USEPA HELP model.^b Inches per year (assumes unit area).^c Percent of total area of contiguous 48 states.

2.3 Facility Conditions

The design conditions used in predicting the leaching rate are based on current state and federal regulations, and guidance criteria from EPA documents. The major design conditions considered in analyzing engineered controls include: cover and liner configuration (e.g., slope, soil types, barrier layer materials), type of vegetative cover and depth of root (evaporative) zone. The standard set conditions used in the analysis of Subtitle D and Subtitle C facilities are discussed below.

2.3.1 Subtitle D Facility. Subtitle D facilities are typically municipal solid waste landfills. These types of landfills are regulated by state agencies. These regulations include specific design requirements as well as site location, operating and other requirements. In addition, federal guidelines for Subtitle D facilities are found at 40 CFR 257. A review of state regulations concerning cover and liner requirements found the following:

1. 90 percent of the 50 states require a minimum of 2-feet of cover soil; and
2. 90 percent of the 50 states have no specific requirements for liner systems beneath Subtitle D facilities.

The type of soil selected for use in the cover must be able to support vegetation and to minimize infiltration. Soil textures, as classified by the U.S. Department of Agriculture (USDA), commonly used to meet this requirement

include loamy sand, loam and clayey loam. A 24-inch thick loam soil placed on a slope of 2 to 5 percent will comply with the regulations found in over 90 percent of the 50 states and was selected for the standard conditions of this analysis. The type and condition of vegetative cover at the Subtitle D landfill influences the leaching rate by controlling infiltration rates, evapotranspiration and surface runoff. Landfill covers are usually seeded with a mixture of grass to control erosion. Because the cover soils required are expected to be conducive to grass growth, complete coverage or a good grass stand would be expected shortly (several years) after seeding the cover. The root zone of grass species can extend to 36 inches and more. However, the root zone is not expected to penetrate the waste layer, which is at a depth of 24 inches, due to unsuitable growing conditions in the waste layer. Waste material in sanitary landfills will decompose (e.g., production of acids, methane and carbon dioxide), creating an unsuitable growing condition which would inhibit the roots from penetrating below the cover soil layer.

Vegetative cover may progress to other species in the long-term, depending on the climatic conditions, indigenous species in the vicinity of the landfill, and growth conditions prevalent in the landfill cover. This long-term vegetative species progression was not considered since such a long-term vegetative species progression is highly sensitive to site-specific conditions, it may also be expected at Subtitle C facilities, and the greatest chemical flux at sanitary landfills occurs in the first several years after waste placement.

Based on the above information, the conditions used in analyzing long-term Subtitle D facility performance using the HELP model included:

- o 2-feet of cover soil consisting of a loam texture and a good grass stand on a 2 to 5 percent slope;
- o root zone of 24 inches; and
- o no liner system beneath the site.

These conditions are depicted in Figure 2-9. A sensitivity analysis of soil texture and root zone depth is discussed in Section 3.0.

2.3.2 Subtitle C Facility. Subtitle C facilities are regulated by federal regulations for the disposal of hazardous wastes. Specifically, the regulations cited in the 40 CFR Parts 264 and 265 and requirements of the Amendments provide the framework for engineered controls. In addition, the EPA has issued guidance to owners and operators of land disposal facilities (Lutton, 1980; EPA, 1982; EPA, 1983; EPA, 1984; EPA, 1985). These engineered controls for disposal facilities include multi-layered cover/liner systems consisting of drainage layers, geomembrane barriers and soil barriers. Several cover/liner system configurations that will meet the regulations are depicted in Figure 2-10. The configurations include one cover system and three liner systems. The cover system consists of a 2-foot cover soil layer above a 1-foot drainage layer, a geomembrane liner bedded with filter fabric, followed by a 3-foot clay layer.

The liner systems shown on Figure 2-10 all include leachate collection and detection layers. The difference between the liner systems is the combination

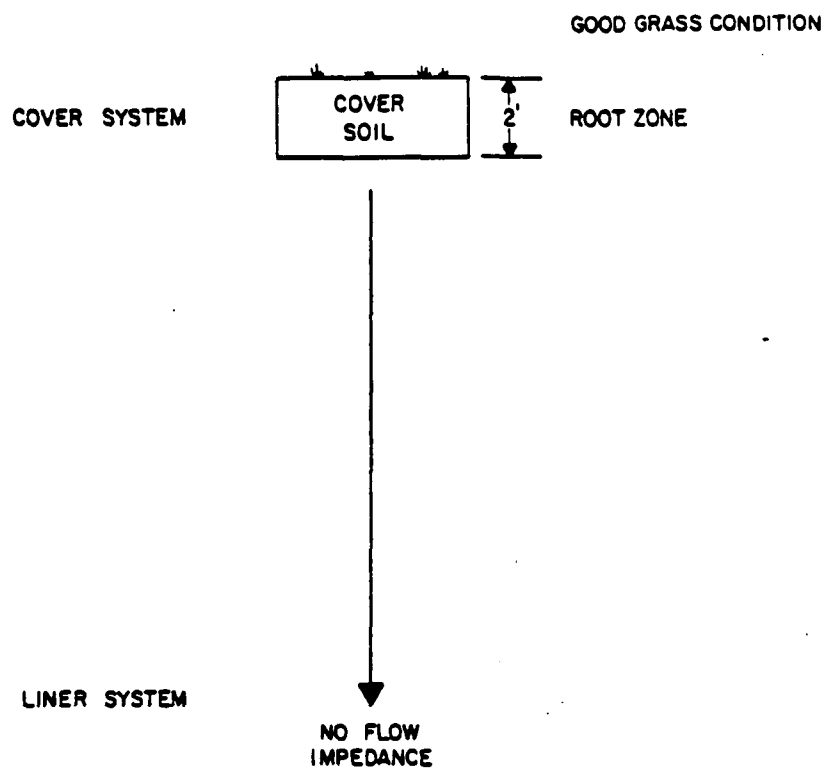


FIGURE 2-9
SUBTITLE "D"
FACILITY CONDITIONS USED IN
ESTIMATING LEACHING RATES

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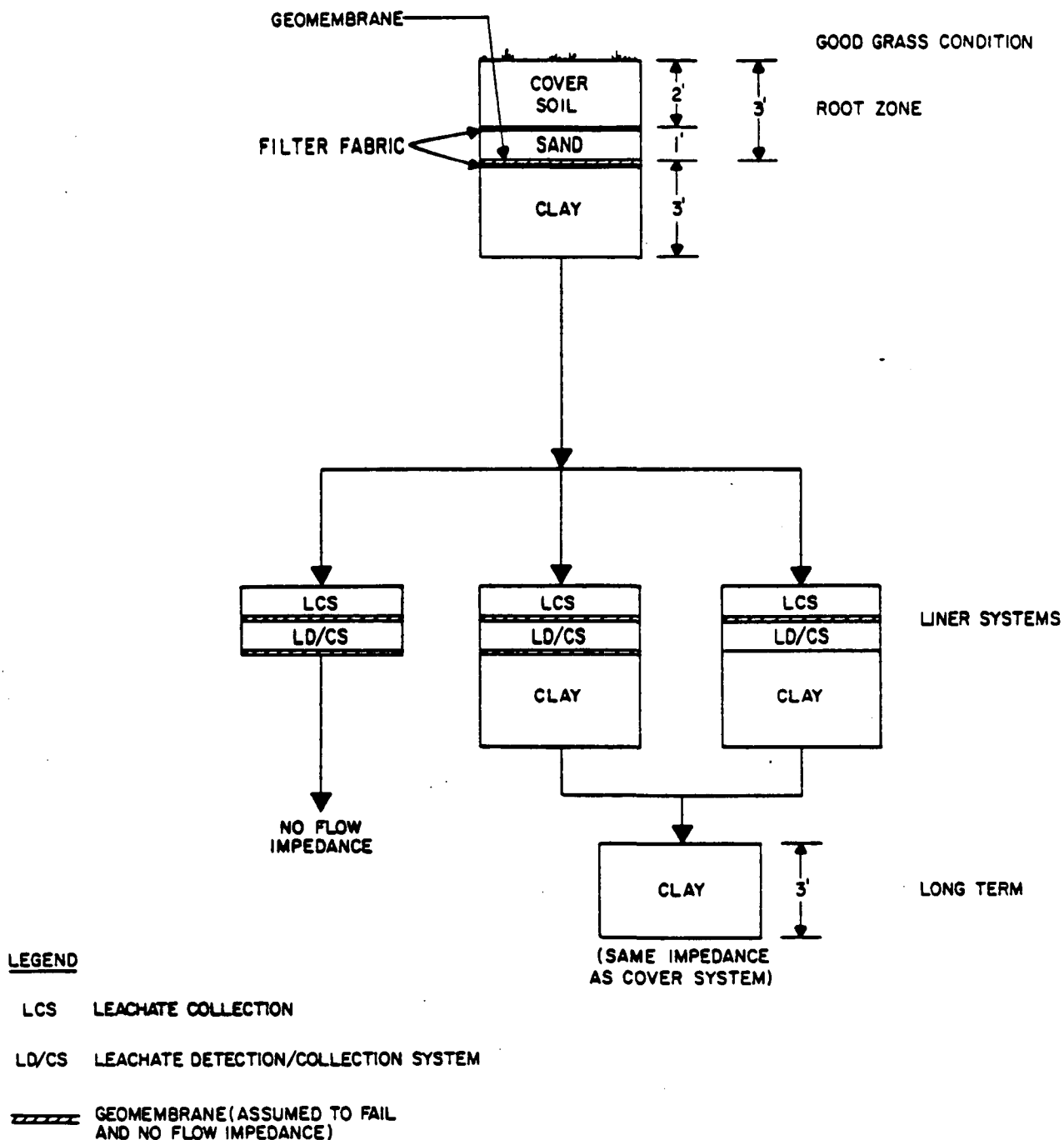


FIGURE 2-10
SUBTITLE 'C'
FACILITY CONDITIONS USED IN
ESTIMATING LEACHING RATES

of barrier layers used. The combinations include the use of two geomembranes separated by a drain layer, a geomembrane separated from a clay layer, and a geomembrane separated from a geomembrane-clay composite layer.

Since the analysis of engineered controls is intended to evaluate long-term performance (e.g., several centuries), the leaching rate of configurations described above is expected to change.

The principal change affecting leaching rate is expected to be the degradation of the geomembranes to a point where they are no longer effective in controlling water movement. In addition, the leachate collection/detection systems that remove leachate from the facility are not expected to be operating over this period. The clay liners are expected to have greater hydraulic conductivity as a result of geologic change (e.g., weathering) and exposure to chemicals.

Practical experience with the performance of polymeric geomembranes as barriers to water flow is limited to a few decades. Use of geomembranes to contain mobile wastes in landfill environments is limited to less than 20 years. An assessment was made of the service life of geomembranes used to contain hazardous wastes at land disposal facilities (Lyman, et al., 1983) this assessment indicated there is limited data available on which to project long-term service life but containment may be expected over several decades and could possibly extend over 100 years. Since the long-term analysis of engineering controls considers performance over several centuries and the service life of geomembranes is estimated to be a fraction of this period, the effectiveness of

geomembranes barriers to control leaching rates is expected to be equal to or less than clay barriers.

Clays are residual soils created through physical and chemical weathering of rocks and minerals such as feldspars in granite and pegmatites. The silicon-oxygen sheet structures of these minerals combine with aluminum, sodium, calcium, potassium and magnesium cations, some iron oxides and hydroxyl molecules from water to form hydrous silicates -- mainly kaolinite, illite and montmorillinite clays. Clays are relatively stable, although they exchange cations and dehydrate and hydrate to variable extents.

Clays are regionally distributed in the U.S. and are susceptible to specific geologic weathering processes. In general, the most important long-term weathering process of clay barriers at waste disposal facilities is the development of soil structure (fabric). Soil fabric may result from wetting and drying in response to percolating precipitation. Clay particles tend to flocculate and clump together in aggregates, or peds, when the clay dehydrates. The formation of peds is a slow gradual process that transforms the clay barrier into blocks of soil. These blocks of soil cause an increase in the hydraulic conductivity of the clay barrier. The extent of this increase depends on the extent of dehydration. Clay barriers in cover and liner systems are not as susceptible to formation of peds as similar clays at the ground surface since they are less susceptible to large fluctuations in moisture content caused by evapotranspiration of percolating water.

Naturally occurring clay soils near the ground surface can be considered representative of the extent of geologic weathering to which clay barriers in Subtitle C facilities will be exposed. A Soil Conservation Service survey of clay soils for states across the U.S. indicates hydraulic conductivity ranged from 1.4×10^{-4} to 4.3×10^{-5} cm/sec for soils at depth of 30 to 80 inches.

It is anticipated that natural clays with an in-situ hydraulic conductivity on the order of 10^{-5} cm/sec can be compacted to achieve the 10^{-7} cm/sec required by the Amendments. Therefore, the slower hydraulic gradient (4.3×10^{-5} cm/sec) is probably representative of the upper limit for hydraulic conductivity that may result from geologic weathering of clay barriers in engineered controls at Subtitle C facilities. An even slower hydraulic conductivity may be more typical of the long-term clay performance of clay barriers since some weathering processes, such as transport of fines by percolating water from the soils overlying the clay barrier tends to reduce hydraulic conductivity by plugging pores of the clay soil.

The use of clay soils as barrier layers in cover or liner systems is subject to both geologic weathering and alteration of the clay soil structure on exposure to chemicals. These alterations can lead to increased hydraulic conductivity.

Alteration of the clay soil structure may be caused by dissolving constituents of the clay mineral, including aluminum, iron and silica (Brown and Anderson, 1983). Changes in the colloidal characteristics of the clay mineral may cause flocculation and increased hydraulic conductivity. The extent of alteration is controlled by such factors as type of clay mineral, waste constituent, mixture of chemicals and concentration of chemicals. As these alterations progress increased flow of percolating liquids may cause piping. These alternations may

cause the hydraulic conductivity of some clay soils to increase by a factor of 100 (Anderson, 1982). While much of the current knowledge of chemical interactions with clay is based on laboratory studies with concentrated chemicals, there is a potential for increased hydraulic conductivity to occur under field conditions. Additional information is needed to determine the increase in hydraulic conductivity caused by aqueous solutions of chemicals under field conditions. The increase, however, is not expected to be greater than the 100 fold increase observed for concentrated chemicals under laboratory conditions. A hydraulic conductivity range of 5×10^{-6} to 5×10^{-7} cm/sec appears to be reasonable to estimate long-term performance of clay barriers exposed to chemicals.

Based on the above information, the conditions used in analyzing long-term Subtitle C facility performance using the HELP model included:

- o 2 feet of cover soil consisting of a loam texture and good grass stand;
- o 1-foot drainage layer at a two percent slope to free drain at toe of slope;
- o root zone of 36 inches; and
- o 3-foot clay layer with a hydraulic conductivity of 1×10^{-6} cm/sec.

A bottom liner system was not included in the design because the clay layer in the bottom liner is subject to similar conditions as the clay layer in the cover system and is also exposed to chemicals leaching from the waste. Therefore, the hydraulic conductivity of the bottom clay liner is expected to be equal to or greater than the clay layer in the cover and is not expected to control the leaching rate.

A sensitivity analysis was conducted to determine the variation in estimated leaching rates by varying the cover soil texture, clay hydraulic conductivity and root zone depth. The results of this analysis is presented in Section 3.2.

3.0 CONCLUSIONS AND RECOMMENDATIONS

3.1 Leaching Rate Distributions

Leaching rates for Subtitle C (hazardous) and Subtitle D landfill facilities were estimated with the HELP model for conditions considered representative of long-term performance. Leaching rates were estimated for 18 climatic conditions encountered in the contiguous 48 states. A summary of leaching rates is provided in Table 3-1. This information is expressed as percent of area in the contiguous 48 states where leaching rates may be expected to equal or be less than the rate shown.

Estimates of leaching rates suggest the rates are not randomly distributed in the contiguous 48 states, but are highly dependent on the annual precipitation. Leachate was estimated for all chosen locations of Subtitle D facilities. These leaching rates ranged approximately 2 orders-of-magnitude (100 fold) from Pocatello, ID to Astoria, OR (35.9 inches per year), while precipitation ranged over one order-of-magnitude. Long-term leaching was not estimated to occur at Subtitle C facilities receiving less than 20 inches per year of precipitation (with one exception). Where leaching was estimated to occur at Subtitle C facilities, the rate was generally 2 to 3 times less than for Subtitle D facilities.

The distributions of leaching rates shown in Figure 3-1 indicate Subtitle C facilities are expected to leach at a rate less than for Subtitle D facilities.

TABLE 3-1
LEACHING RATES SUMMARY^a

PRECIPITATION RANGE ^b	EVAPORATION POTENTIAL	LEACHING RATE ^b		CLIMATOLOGICAL CENTER	AVERAGE 5-YEAR ^b PRECIPITATION ^b	PERCENT OF TOTAL AREA ^c
"C"	"D"					
<10	Low	0.0	0.1	Pocatello, ID	9.8	3
	Medium	0.0	0.3	Cedar City, UT	9.8	2
	High	0.0	0.2	Las Vegas, NV	5.3	3
10-20	Low	0.0	0.8	Great Falls, MT	17.9	3
	Medium	0.0	0.6	Rapid City, SD	15.4	18
	High	0.4	1.2	Midland, TX	16.3	11
20-32	Low	0.3	1.5	St. Cloud, MN	25.8	9
	Medium	0.5	1.5	Grand Island, NE	22.2	4
	High	0.5	2.4	Oklahoma City, OK	30.4	5
32-40	Low	3.6	7.2	Montpelier, VT	34.5	3
	Medium	3.7	6.8	Cleveland, OH	36.7	5
	High	2.9	4.8	Columbia, MS	37.1	3
40-48	Low	7.0	12.1	Boston, MA	40.9	3
	Medium	3.3	6.8	Indianapolis, IN	40.3	7
	High	2.5	5.8	Charleston, SC	48.3	5
>48	Low	11.9	35.9	Astoria, OR	65.6	2
	Medium	7.1	12.2	Knoxville, TN	48.1	3
	High	6.9	12.1	Tallahassee, FL	68.4	11

- ^a Based on USEPA HELP model estimates. Estimates for Subtitle C facilities assume an evapotranspiration zone of 36 inches and 24 inches for Subtitle D facilities.
- ^b Inches per year (assumes unit area).
- ^c Percent of total area of contiguous 48 states.

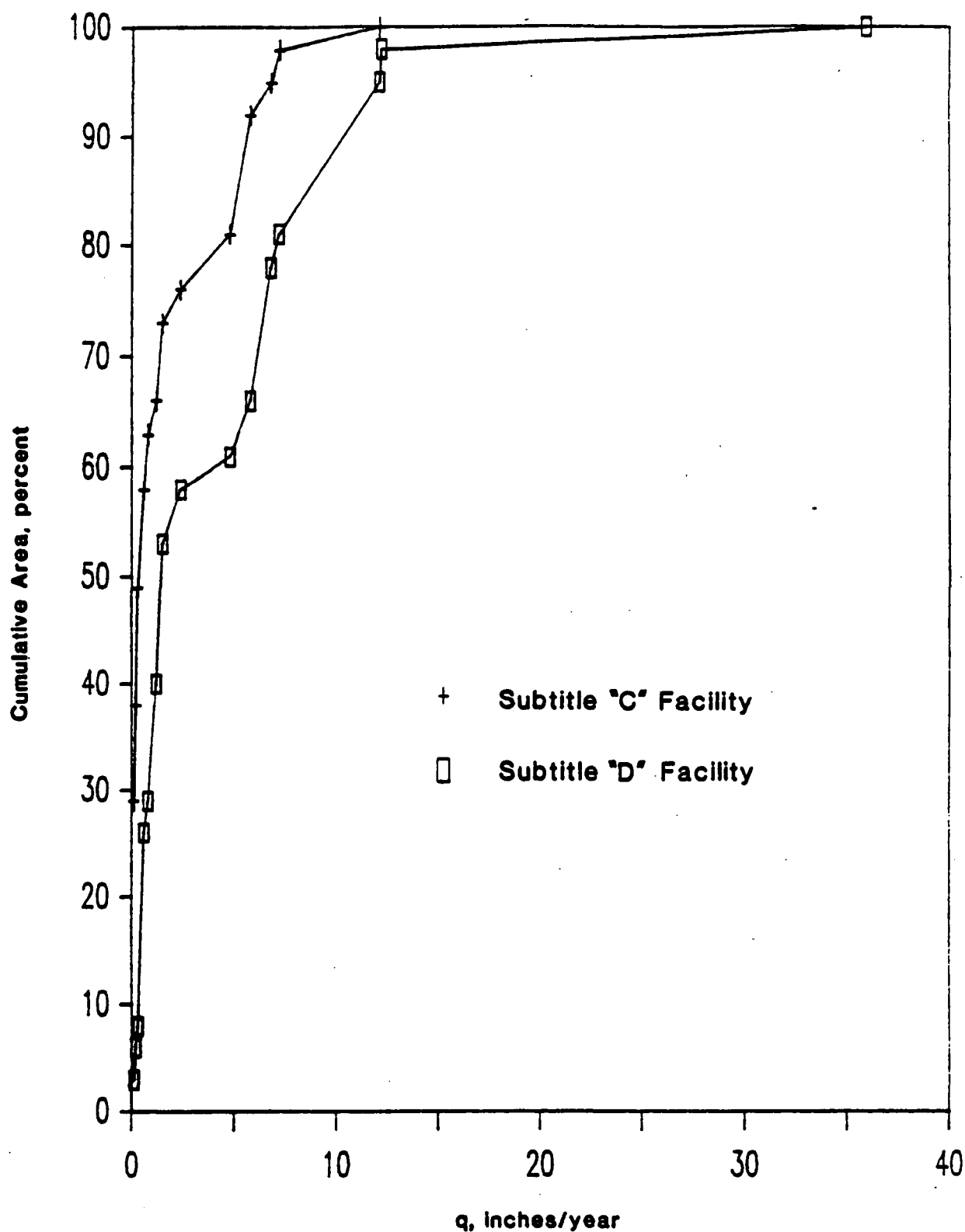


FIGURE 3-1
ESTIMATED DISTRIBUTION OF UNIT AREA LEACHING
RATES FOR SUBTITLE C AND D FACILITIES

The distributions may be better defined if more climatological centers are evaluated (up to 100 reporting cities are included in the HELP data base), but the basic trends and differences are not expected to change dramatically.

3.2 Sensitivity Analysis

Since the leaching rates shown in Table 3-1 and Figure 3-1 were estimated for a standard set of conditions, an analysis was conducted to determine the effect on leaching rate over a range of conditions that may be encountered. This analysis was conducted using Indianapolis, IN. Other climatological centers would be expected to respond in a similar manner. Conditions that were varied included cover soil texture, hydraulic conductivity of the barrier layer and evapotranspiration zone.

Cover soil texture effect on estimated leaching rates was analyzed for loamy sand and clayey loam. These soils are considered reasonable substitutes for the loam soil that was used in the standard conditions. As may be expected, use of more permeable loamy sand in the cover to support vegetation was found to yield more leachate than loam. The leaching rate was almost doubled for the Subtitle C facilities while it was tripled for Subtitle D facilities. The leaching rate was less affected by use of clayey loam instead of loam (<10 percent at Subtitle C facilities and <20 percent at Subtitle D facilities). This suggests that the use of cover soils less permeable than loam will have a minor effect on the estimated leaching rate whereas the use of more permeable cover soils will increase leaching rates more at Subtitle D facilities than at Subtitle C facilities.

TABLE 3-2
SENSITIVITY ANALYSIS SUMMARY

CONDITION	LEACHING RATE ^a	
	q _C	q _D
Standard Conditions ^b	3.3	6.8
Soil Texture		
Loamy Sand	6.11	19.71
Clayey Loam	2.97	5.22
Hydraulic Conductivity, k		
5 x 10 ⁻⁶ cm/sec	4.3	-
5 x 10 ⁻⁷ cm/sec	2.2	-
Evapotranspiration (E-T) Zone		
Depth = half of standard condition	7.34	11.6

^a inches per year (assumes unit area) for Indianapolis, IN
location: q_C = Subtitle C facility
 q_D = Subtitle D facility

^b Standard Conditions included:
cover soil texture
barrier layer hydraulic conductivity 10⁻⁶ loam cm/sec
evapotranspiration zone 36 inches for Subtitle C facility
 24 inches for Subtitle D facility

3 Hydraulic conductivity in barrier layers in cover or liner systems was varied from 5×10^{-6} to 5×10^{-7} cm/sec to determine the effect on leaching rate from the standard condition of 10^{-6} cm/sec. Since Subtitle D facility cover systems were not assumed to provide such a barrier layer, the analysis was limited to Subtitle C facilities. The analysis indicated changes in leaching rates on the order of approximately 50 percent.

Reducing the depth of the zone where evapotranspiration may occur increased the Subtitle C leaching rate by a factor of almost 2.5, while the Subtitle D leaching rate increased by a factor of 1.7. This difference in effect on leaching rates caused by reducing the evapotranspiration zone suggests that the greater depth used for the Subtitle C facility (36 inches) provides more control over leaching rates than the shallower depth (24 inches) used for Subtitle D facilities.

Comparison of the relative importance of the conditions analyzed indicated the evapotranspiration depth is the most important condition for estimating Subtitle C leaching rates followed by use of more permeable soils for barriers and cover systems. The same two conditions, but in reverse order, were the most important in affecting Subtitle D leaching rates. In all cases, Subtitle D leaching rates were greater than for Subtitle C.

The sensitivity analysis was conducted over a reasonable range of long-term conditions. The estimated leaching rates over these ranges therefore provide a means of estimating the range of leaching rates that may be expected for these facilities under climatic conditions represented by Indianapolis, IN. Sub-

title C facilities in Indianapolis may leach at a rate ranging from 2.2 to 7.34 inches per year, whereas, a Subtitle D leaching rate may range from 5.22 to 19.71 inches per year. The range of leaching rates may be greater if more than one standard condition is varied.

3.3 Application of Distributions

3.3.1 Use. Leaching rates and their frequency of occurrence in the contiguous 48 states were estimated for use in developing a generic procedure to classify wastes as hazardous and determine hazardous wastes to be banned from Subtitle C land disposal facilities. The leaching rates provide a means of estimating the chemical flux (mass rate of chemical release) from both Subtitle C and D facilities. The volumetric flux (volumetric rate of leachate production) identifies the transport rate which is leaching mobile chemicals out of the facility. When leaching rate is multiplied by concentration of chemicals in the leachate, then the chemical flux is described on a unit area basis. It is appropriate to express this flux on a unit area to maintain the generic application of the procedure.

The generic procedure is to be used to select chemical concentration criteria for use with a simple extraction procedure (e.g., Extraction Procedure III - organics). The EPA must ultimately select a specific leaching rate to determine the chemical concentration criterion that is protective of human health and the environment. Maximum protection is provided by selecting the greatest leaching rate.

Selection of an appropriate chemical concentration may be a complex task. An upper concentration limit may be identified as the solubility of the chemical in an aqueous solution. This assumes that wastes placed in the facility are not, or do not, contain free liquids and must dissolve into precipitation that percolates through the wastes. This approach to identifying a chemical concentration has several limitations:

- o not all chemicals have solubility limits in water (i.e., some chemicals are miscible in water);
- o solubility concentrations are rarely attained because of the inherent kinetics of the leaching process and chemical equilibria; and
- o may far exceed actual leachate concentrations.

Alternatively, an appropriate concentration can be estimated by an iterative process to determine the concentration that, when released to an underlying groundwater system and transported downgradient to a receptor, does not cause adverse human health or environmental effects. This iterative process can be conducted after a specific leaching rate is selected or can be conducted on a mass flux basis (e.g., product of leaching rate and chemical concentration). The appropriate leaching rate and concentration that meets the chemical flux criterion can then be selected.

3.3.2 Limitations

Limitations of the leaching rate distributions and their use may be described as:

- o analytical assumptions; and
- o geographical distribution of leaching rates.

The analytical approach has several limitations that were described in Section 2. These limitations relate to the data base used to project long-term performance. Use of the HELP model is based on 5 consecutive years of climatic data. Although an assessment was made that suggested limited differences in average precipitation for the 5-year and 25-year periods, the significance of several years of abnormally high precipitation or permanent shift in climatic conditions has not been evaluated.

The conditions projected to prevail in the long-term are not based on an extensive long-term data base. Reasonable assumptions were made to identify a range of conditions that may prevail in the future. A particularly important assumption was that performance is controlled by natural activities and not by human disturbance of the low permeable barrier.

Geographical distribution of the estimated leaching rates presents a limitation to its use in the generic procedure. Low leaching rates prevail in certain areas of the nation and high leaching rates in other areas. If a leaching rate

is selected in establishing criteria for specific chemicals, then two undesirable effects may occur:

- o over-regulation of wastes in areas with low leaching rates; and
- o under-regulation of wastes if the leaching rate is larger than the rate selected for the criterion.

Under-regulation of wastes can be avoided by using the maximum estimated leaching rate for selecting chemical concentration criteria. Both limitations can be avoided by establishing land disposal banning criteria expressed as a mass flux (e.g., the product of leaching rate and concentration). The owner/operator of a specific facility could readily identify if a waste with appropriate testing results (i.e., EP-III) is banned. This approach allows higher concentrations in areas with low leaching rates and requires low concentrations in areas with high rates. Using the mass flux rate approach, the rule or classification procedure becomes national in scope, avoids over- or under-regulation, protects human and the environment, and allows climatic conditions and costs to control where wastes are deposited in a protective manner.

3.3.3 Summary

The analysis of engineered controls for land disposal of wastes yielded distributions of leaching rates (Figure 3-1) for the contiguous 48 states. The Subtitle C distribution yielded estimated leaching rates that were always less (generally 1/2 to 1/3) than the Subtitle D leaching rates. The depth of the

evaporative zone was found to be the most sensitive long-term site condition used to estimate leaching rates at Subtitle C facilities. Soil texture was found to be the most sensitive long-term site condition for Subtitle D facilities.

The distributions are useful in developing a generic procedure for classifying hazardous wastes and banning wastes from land disposal facilities. The EPA may use the distributions to develop chemical criteria in one of the following approaches:

- o select a leaching rate and then determine an appropriate chemical concentration criterion; or
- o determine an appropriate chemical mass flux rate from a facility by back-calculation from a health/environmental based criterion; and establish a mass flux criterion with regional maps of leaching rates for use by owners/operators of disposal facilities; or
- o establish a concentration criterion using the maximum leaching rate and over-regulate wastes in low leaching rate areas; or
- o establish a concentration criterion using a leaching rate that will under-regulate higher leaching rate areas and over-regulate lower leaching rate areas.

REFERENCES

- Anderson, D., 1982. "Does Landfill Leachate Make Clay Liners More Permeable?" Civil Engineering, September, pp 66-69.
- Brown, K.W. and D.C. Anderson, 1983. "Effects of Organic Solvents on the Permeability of Clay Soils." EPA-600/S2-83-016, April.
- Environmental Protection Agency, 1982. "Draft RCRA Guidance Document: Landfill Design, Liner Systems and Final Cover." Washington, D.C.
- Environmental Protection Agency, 1983. "Lining of Waste Impoundment and Disposal Facilities." SW-870. Revised Edition. Washington, D.C.
- Environmental Protection Agency, 1984. "Permit Applicants' Guidance Manual for Hazardous Waste Land Treatment, Storage, and Disposal Facilities Final Draft." EPA 530 SW-84-004. Washington, D.C.
- Environmental Protection Agency, 1985. "Minimum Technology Guidance for Double Liner Systems." Washington, D.C.
- Environment Reporter, State Solid Waste-Land Use Bureau of National Affairs.
- Perrier, E.R., and A.C. Gibson, 1980. "Hydrologic Simulation on Solid Waste Disposal Sites (HSSWDS)." SW-868, U.S. Environmental Protection Agency, Cincinnati, OH.
- Fenn, D.G., K.J. Hanley and T.V. DeGeare, 1975. "Use of the Water Balance Method for Predicting Leachate Generation from Solid Waste Disposal Sites". EPA/630/SW-168, U.S. Environmental Protection Agency, Cincinnati, OH.
- Lutton, R.J., 1980. "Evaluating Cover Systems for Solid and Hazardous Waste." SW-867. U.S. Environmental Protection Agency. Washington, D.C.
- Lyman, W.J., et al., 1983. "Expected Life of Synthetic Liners and Caps." Draft Final Report. Contract 68-01-6160. Arthur D. Little, Inc. for U.S. Environmental Protection Agency, Washington, D.C.
- Mather, J.R., and P.A. Rodriguez, 1980. "The Use of the Water Budget in Evaluating Leaching Through Solid Waste Landfills." PB 80-180888, National Technical Information Service, Springfield, VA.
- NOAA 1980. Climates of the States. National Oceanic and Atmospheric Administration, Second Edition.
- Schroeder, P.R., et al., 1984. "The Hydrologic Evaluation of Landfill Performance (HELP) Model: Volume I - User's Guide for Version I and Volume II - Documentation for Version I." EPA/530-SW-84-009 and -010, U.S. Environmental Protection Agency, Washington, D.C.

Thornthwaite, C.W., and J.R. Mather, 1957. "Instructions and Tables for Computing Potential Evapotranspiration and the Water Balance" in Publications in Climatology, Volume X, No. 3. Laboratory of Climatology, Drexel Institute of Technology, Elmer, New Jersey.

APPENDIX A
COMPARISON OF 5 AND 30 YEAR PRECIPITATION DATA

APPENDIX A

COMPARISON OF 5-YEAR AND 30-YEAR

PRECIPITATION DATA

Analysis of the long-term performance of engineered controls at waste disposal facilities relied on the 5-year precipitation data base used in the HELP model. The appropriateness of using a 5-year data base was assessed by comparing the 5 year average annual precipitation to an average from a longer period.

Average annual precipitation collected over a 23- to 30-year period for the climatological centers used in the analysis of engineering controls compiled from "Climates of the States" (NOAA, 1980). These data are presented in Table A-1 along with the 5-year annual average precipitation computed for the HELP model data base. Comparison of the two averages for each center showed differences in averages from less than 1 percent to 27 percent. The larger differences tend to occur in the low precipitation ranges. Two-thirds of the centers exhibited average annual precipitation differences of less than 5 percent. The 5-year HELP averages for those centers in the higher precipitation ranges (>32 inches per year) were generally greater than for the 30 year average. This general trend was reversed for the lower precipitation ranges. In addition, the larger differences (>5 percent) were generally because the HELP data base provided a larger average annual precipitation than the 30-year average.

Based on the above assessment of average annual precipitation, the following observations are made:

TABLE A-1

COMPARISON OF 5 AND 30 YEAR
AVERAGE PRECIPITATION

PRECIPITATION ^a RANGE	EVAPORATION POTENTIAL	CLIMATOLOGICAL CENTER	AVERAGE PRECIPITATION ^a		PERCENT DIFFERENCE
			5-YEAR ^b	30-YEAR ^c	
<10	Low	Pocatello, ID	9.8	10.8	9
	Medium	Cedar City, UT	9.8	9.97 ^d	1
	High	Las Vegas, NV	5.3	4.16	27
10-20	Low	Great Falls, MT	17.9	14.99	19
	Medium	Rapid City, SD	15.4	17.55	12
	High	Midland, TX	16.3	13.31	22
20-32	Low	St. Cloud, MN	25.8	26.84	3
	Medium	Grand Island, NE	22.2	23.41	5
	High	Oklahoma City, OK	30.4	31.73	4
32-40	Low	Montpelier, VT	34.5	33.23 ^e	3
	Medium	Cleveland, OH	36.7	34.93	5
	High	Columbia, MS	37.1	37.39	0.7
40-48	Low	Boston, MA	40.9	41.50	1
	Medium	Indianapolis, IN	40.3	39.90	0.8
	High	Charleston, SC	48.3	48.73	0.8
>48	Low	Astoria, OR	65.6	66.34	1
	Medium	Knoxville, TN	48.1	46.18	4
	High	Tallahassee, FL	68.4	61.58	11

a Inches per year (assumes unit area)

b Based on HELP model, May, 1985.

c Based on "Climates of the States".

d 24-year average.

e 23-year average.

- o distribution of leaching rates estimated using the HELP model tend to over-estimate the long-term performance of land disposal facilities;
- o the larger differences between the two precipitation averages are associated with the lower precipitation ranges (<20 inches per year) which were estimated to yield small leaching rates.

APPENDIX E

ANALYSIS OF ENGINEERED
CONTROLS OF SUBTITLE C
FACILITIES FOR LAND DISPOSAL
RESTRICTIONS DETERMINATIONS

REVISED DISTRIBUTION OF LEACHING RATES

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REFERENCES

1.0 INTRODUCTION

As part of recent federal legislation, the U.S. Environmental Protection Agency (EPA) must review all listed hazardous wastes and determine which wastes should be banned from land disposal facilities. The general approach for identifying such wastes was developed by EPA's Office of Solid Waste (OSW). OSW's analytical procedure includes credits for engineering controls that are designed into Subtitle C facilities. These engineering controls are used to determine leaching rates from Subtitle C facilities based on long-term performance (e.g., beyond anticipated design life). An analysis of leaching rates from Subtitle C facilities and Subtitle D facilities was performed and reported in June and August of 1985 (Jordan, 1985). Jordan's analysis evaluated 18 climatic centers throughout the contiguous 48 states. Recommendations for future studies included 1) an increase in the number of cities evaluated; and 2) comparing leaching rates based on the 5-year HELP¹ model data base to a long-term (i.e., 20-years) precipitation data base. This report presents the results of the additional analyses conducted in accordance with those recommendations.

¹ Hydrologic Evaluation of Landfill Performance (Schroeder, et. al., 1984)

2.0 REVISED LEACHING RATE DISTRIBUTIONS

2.1 Analytical Approach

The analytical approach used for this evaluation is identical to that used in the June 1985 study in which a standard set of design and long-term conditions was used to estimate the leaching rates for Subtitle C and D facilities. In both studies these standard sets, along with climatological data, were entered into the HELP model resulting in a calculated leaching rate.

2.2 Climatic Data Base

In the June 1985 study a total of 18 climatic centers were identified for developing the national leaching rate distribution. As fifty percent of these locations yielded leaching rates of less than one inch for Subtitle C facilities, few data points were provided to establish the upper end of the leaching rate. To obtain more data for the higher leaching rates, 12 additional climatic centers with precipitation averages of greater than 20 inches were evaluated in the September 1985 study. The combined list of cities selected for evaluation is presented in Table 2-1. Included in the group are cities with an extended data base of up to 20 years that were also used for a sensitivity analysis (Section 3.0) of the leaching rate based on a comparison of the 5-year and 20-year climatic data.

TABLE 2-1
LEACHING RATES SUMMARY^a

PRECIPITATION RANGE ^b	EVAPORATION POTENTIAL	LEACHING RATE ^b		CLIMATOLOGICAL CENTER	AVERAGE 5-YEAR ^d PRECIPITATION ^d (20-YEAR)	PERCENT OF TOTAL AREA ^c
<10	Low	0.0	0.1	Pocatello, ID	9.8	3
	Medium	0.0	0.3	Cedar City, UT	9.8	2
	High	0.0	0.2	Las Vegas, NV	5.3	3
10-20	Low	0.0	0.8	Great Falls, MT	17.9	3
	Medium	0.0	0.6	Rapid City, SD	15.4	13
	Medium	0.02(0.04)	0.04	Denver, CO	13.0 (14.9)	5
	High	0.4	1.2	Midland, TX	16.3	11
20-32	Low	0.3	1.5	St. Cloud, MN	25.8	7.8
	Low	3.1 (3.4)	3.4	Milwaukee, WI	34.1 (33.1) ^d	2.2
	Medium	3.4	3.4	San Francisco, CA	(19.7)	2
	Medium	0.5	1.5	Grand Island, NE	22.2	2
	High	0.5	2.4	Oklahoma City, OK	30.4	5
32-40	Low	3.6	7.2	Montpelier, VT	34.5	3
	Medium	3.7	6.8	Cleveland, OH	36.7	2
	Medium	2.7	5.4	E.St.Louis, IL	36.2	3
	High	2.9	4.8	Columbia, MS	37.1	3
40-48	Low	7.0	12.1	Boston, MA	40.9	0.4
	Low	6.6	12.8	Providence, RI	46.7	0.1
	Low	4.6 (5.3)	10.0	Hartford, CT	41.1 (44.1)	0.2
	Low	5.5	14.2	Worcester, MA	46.5	1.6
	Low	5.7	13.0	Portland, ME	45.5	0.8
	Medium	(5.2)		Newark, NJ	(40.4)	0.3
	Medium	6.6	10.6	Philadelphia, PA	43.7	1.4
	Medium	3.3	6.8	Indianapolis, IN	40.3	5.2
	High	2.5	5.8	Charleston, SC	48.3	4.6
	High	6.4	11.4	Norfolk, VA	44.4	0.4
>48	Low	11.9	35.9	Astoria, OR	65.6	2
	Medium	7.1	12.2	Knoxville, TN	48.1	3
	High	6.9	12.1	Tallahassee, FL	68.4	5
	High	9.8(5.9)	16.7	New Orleans, LA	70.1(56.7)	5

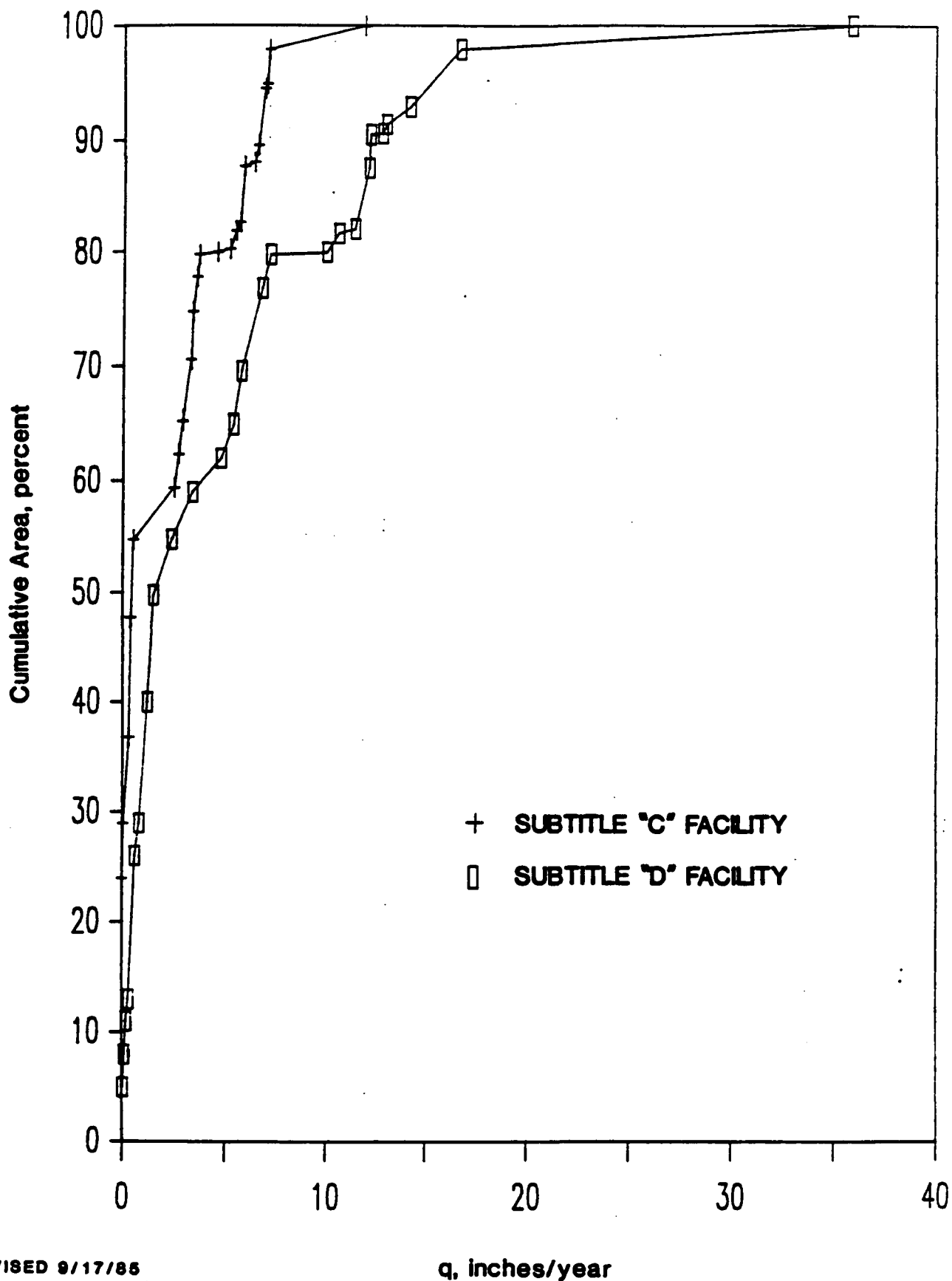
- ^a Based on USEPA HELP model estimates. Estimates for Subtitle C facilities assume an evapotranspiration zone of 36 inches and 24 inches for Subtitle D facilities.
- ^b Inches per year (assumes unit area); number in parenthesis indicates average based on 20 year database.
- ^c Percent of total area of contiguous 48 states.
- ^d 11 years of precipitation data.

2.3 Leaching Rate Distribution

In both studies leaching rates for Subtitle C and Subtitle D landfill facilities were estimated with the HELP model. Leaching rates were estimated in the September 1985 study for 30 climatic centers in the contiguous 48 states. A summary of the leaching rates is provided in Table 2-1. This information is expressed as percent of area in the contiguous 48 states where leaching rates may be expected to equal or be less than the rate shown.

The distributions of the leaching rates shown on Figure 2-1 do not indicate a significant change from the June 1985 distribution. The maximum leaching rates for Subtitle C and D facilities repeated the pattern of the initial study in that the Subtitle C leaching rates were generally 50 percent less than those for Subtitle D facilities. The distribution represents the estimated leaching rates based on the 5-year precipitation data base with the exception of New Orleans (Louisiana), San Francisco (California) and Newark (New Jersey) which were based on a more extensive data base. The 5-year precipitation data for New Orleans is substantially greater than the 20-year average (70.1 inches versus 56.7 inches); therefore, the 20-year leaching rate was selected to estimate long-term conditions. San Francisco and Newark were not available in the 5-year data base, but estimated leaching rates for both cities are presented using the 20-year data base.

With the inclusion of the 12 additional climatic centers, the greatest change in the leaching distributions was observed for Subtitle D facilities with estimated leaching rates greater than 7 inches. The leaching distribution from



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FIGURE 2-1
ESTIMATED DISTRIBUTION OF UNIT AREA LEACHING
RATES FOR SUBTITLE C AND D FACILITIES

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that point is shifted to the right an average of 35 percent compared to the previous distribution submitted in August 1985. While the Subtitle C distribution did not change to any extent, the additional data more clearly defines the 4 to 7 inches/year rate range in estimated leaching rates.

3.0 COMPARISON OF 5-YEAR AND 20-YEAR PRECIPITATION DATA

A 5-year precipitation data base is typically used in the HELP model. Jordan's initial comparison provided a sensitivity to the duration of precipitation used in the HELP model based on a comparison of average annual precipitation for 5 years and 30-years of record. The assessment concluded that the HELP model would tend to over-estimate the long-term performance of land disposal facilities by an unknown percentage.

In the September 1985 study, to analyze sensitivity of leaching rates to duration of precipitation, a 20-year data base consisting of eight cities was loaded into the HELP model. The data base was obtained from the Waterways Experiment Station, U.S. Army Corps of Engineers. Half of the cities loaded were not in the original 5-year HELP data base, therefore the nearest climatic center was used to allow a comparison of leaching rates. However, for three cities the nearest climatic center was not appropriate and only five cities were finally used in the analysis. The cities are listed in Table 3-1.

Comparison of the two annual precipitation averages (5-year and 20-year) for each city showed differences ranging from 1 percent to 19 percent (Table 3-1). In contrast to the June 1985 findings, the largest difference occurred in the highest precipitation range, greater than 48 inches. In this case the 5-year average was considerably greater than the 20-year average. The remaining differences were not significant in relation to the range of precipitation.

TABLE 3-1
COMPARISON OF 5 AND 20-YEAR
CLIMATOLOGICAL DATA BASE

PRECIPITATION RANGE	EVAPORATION POTENTIAL	CLIMATOLOGICAL CENTER	AVERAGE PRECIPITATION ¹		PERCENT DIFFERENCE	LEACHING RATE "C" ¹		PERCENT DIFFERENCE
			5 YR. ²	20 YR.		5 YR.	20 YR	
10-20	Medium	Denver, CO	13.0	14.9	15	0.02	0.04	100
20-32	Low	Milwaukee, WI	34.1 ³	33.1 ⁴	3	3.1	3.4	9
40-48	Low	Hartford, CT	41.1	44.1	7	4.6	5.3	15
>48	Low	Seattle, WA	35.6	35.8	1	8.1	8.7	11
	High	New Orleans, LA	70.1	56.7	19	9.8	5.9	40

Notes: 1. Inches per year (assumes unit area).
2. Based on HELP model, May 1985.
3. Nearest climatic center Chicago.
4. 11 years of precipitation data.

A HELP model was run for the 5-year and 20-year data bases utilizing standard conditions for Subtitle C facilities. The results of the 5-year and 20-year leaching rates, shown on Table 3-1, showed differences ranging from 9 percent to 100 percent. However, the difference of 100 percent occurred in the lowest precipitation range and is not considered a significant increase. The next largest difference, 40 percent for New Orleans, is considered significant as it occurred in the highest precipitation range. In contrast to the initial findings, the 5-year leaching estimate for New Orleans is greater than the long-term average. Generally, the remaining data agree with our original findings that the 5-year data base will tend to over-estimate the long-term performance of land disposal facilities.

The results of the additional study point to an apparent relationship between the percent differences of the precipitation and leaching rates in that the percent difference of the leaching rate appears to be approximately twice the percent difference of the precipitation data for moderate to high precipitation areas.

4.0 CONCLUSIONS

The evaluation of 12 additional climatic centers yielded distributions of leaching rates (Figure 2-1) very similar to the original distribution presented in the June 1985 study. In both studies the Subtitle C distribution yielded leaching rates that were always less, generally 50 percent, than the Subtitle D leaching rates. The inclusion in the September 1985 study of additional climatic centers did provide better definition of the leaching curves for the moderate precipitation ranges.

Comparison of 5-year and 20-year climatological data bases for the moderate to high rainfall ranges indicated a general correlation between the precipitation and leaching rate differences: the percent difference of the 5-year and 20-year leaching rate is approximately twice the percent difference of the 5-year and 20-year precipitation averages. Using this assumption, a review of the percent differences of the 5-year and 30-year average precipitation data presented in Table A-1 of the June 1985 report shows that the percent differences for the rainfall ranges greater than 20 inches are very small. This would indicate that the 18 climatic centers chosen for the initial study are appropriate for use in determining the national leaching distribution rates for Subtitle C facilities.

REFERENCES

E.C. Jordan Co., June 1985. "Analysis of Engineered Controls of Subtitle C Facilities for Land Disposal Restrictions Determinations." EPA Contract No. 68-01-7075. Revised August 1985.

Schroeder, P.R., et al., 1984. "The Hydrologic Evaluation of Landfill Performance (HELP) Model: Volume I - User's Guide for Version I and Volume II - Documentation for Version I." EPA/530-SW-84-009 and -010, U.S. Environmental Protection Agency, Washington, D.C.

DRAFT

APPENDIX F

**Analysis of the "Infinite Source" Assumption
Used in the Groundwater Model for Land
Disposal Banning Evaluation**

by

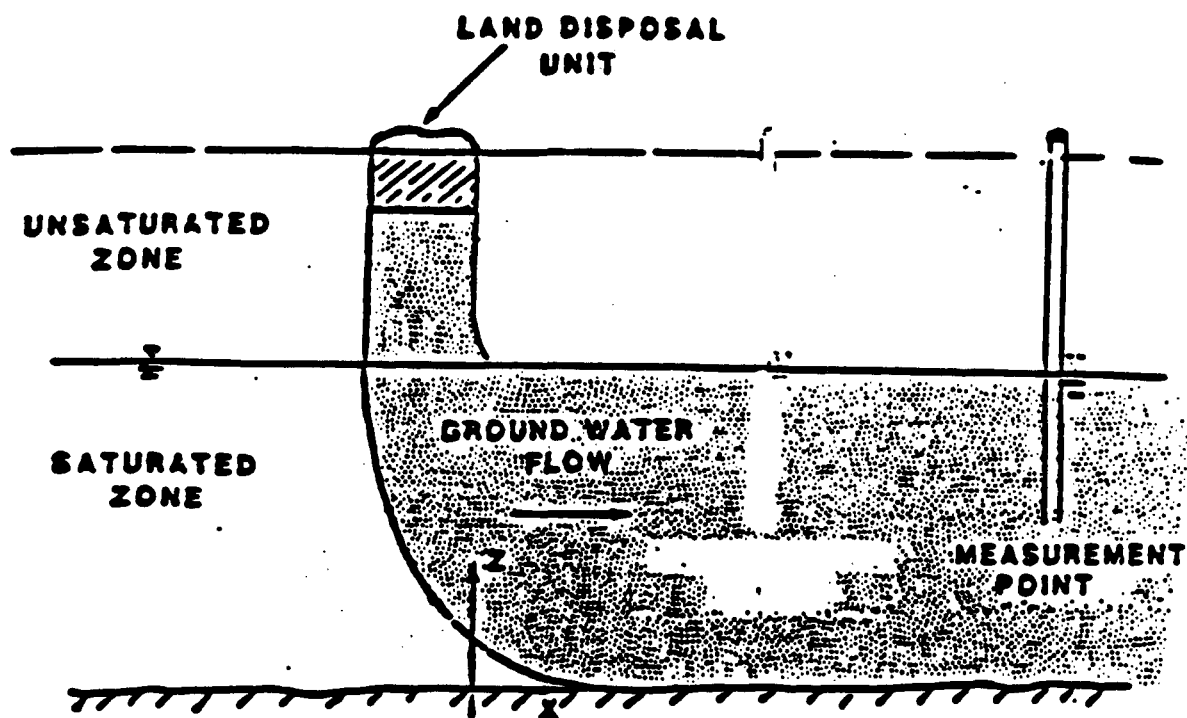
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A key assumption of the ground water fate and transport model is that the mass of each individual Chemical constituent deposited in the facility is sufficiently large to justify the mathematical assumption of an infinite source. In addition to the mathematical convenience resulting from such an assumption, the desire to regulate on leachate concentration values rather than total waste characteristics (e.g., mass loading to the facility) makes the assumption appealing. The purpose of the following analysis is to evaluate the physical significance of the "infinite source" assumption and its impact on the outcomes of the ground water model. The results demonstrate that the "infinite source assumption" of the model is a physically realistic part of the approach and not overly conservative.

The groundwater fate and transport equations are solved for the steady-state case which requires an evaluation of the solution at infinite time. Obviously, the source term under these conditions must also be infinite. The resulting solution allows the plume to fully develop in the groundwater system and the desired maximum concentration at the plume centerline along the x-axis (see Figure 1) is achieved. A useful graphical depiction as C_{LEACH} is the assumed, constant, and infinite source term that produces the down-gradient exposure concentration, C_{ADI} . The time period, ψ_t , is the travel time to the downgradient exposure point and is the approximate time required for the plume shown in Figure 1 to fully develop. The result for the infinite source of leachate is represented by Figure 2. The difference between C_{ADI} and C_{LEACH} results from the combined effects of dispersion, dilution from recharge, and degradation.

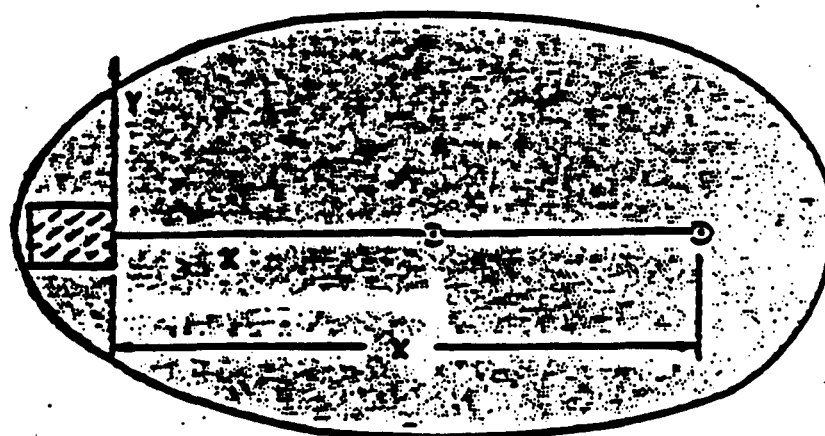
The corresponding result for a finite source of the leachate is represented by Figure 3. This results in a pulse input moving through the groundwater system. Depending on the duration of the pulse, the downgradient exposure concentration value (set to the fix value of C_{ADI} in the rule) may be different than that given by the solution in Figure 2. In any case, the duration of the exposure value, ψ_e , is largely determined by the duration of the pulse input, ψ_i . The differences between exposure and C_{LEACH} are also related to dispersion, dilution and degradation, but their relative effects may be different than that from the infinite source case. If the input duration is short (i.e., ψ_i is small), corresponding to a relatively small amount of mass available for transport, then the down-gradient concentration may not reach the levels obtained for the infinite source case. If the input duration is sufficiently long, corresponding to a relatively large amount of mass available for transport, then the down-gradient value will reach the same level as in the infinite source case albeit for a finite period of time, ψ_e , and the assumption of an infinite source is most likely to be met. The exact mathematical solution for the pulse-load case and its sensitivity to the pulse input time, ψ_i , is not now implemented for the decision rule. We can, however, approximate the impact of the pulse duration via the steady-state solution by noting that this will yield the maximum values for the down-gradient concentration in the pulse load case.

Another useful characteristic shown in Figure 3 is the exposure time, ψ_e . Because the reference dose masses for carcinogens are based upon a seventy-year life span it follows that a minimum value for ψ_e is 70 years. Simply stated, the idea is that the leachate must be emitted at a minimum for a time sufficient to produce the 70-year exposure period. Full development of the plume may or may not actually be achieved depending on the value for ψ_t . If $\psi_e \gg \psi_t$ then the plume is fully developed, the requirements for the steady-state solution are met, and a pulse



CROSS SECTION

PLUME BOUNDARY



PLAN VIEW

Figure 1. Schematic description of contaminated groundwater system.

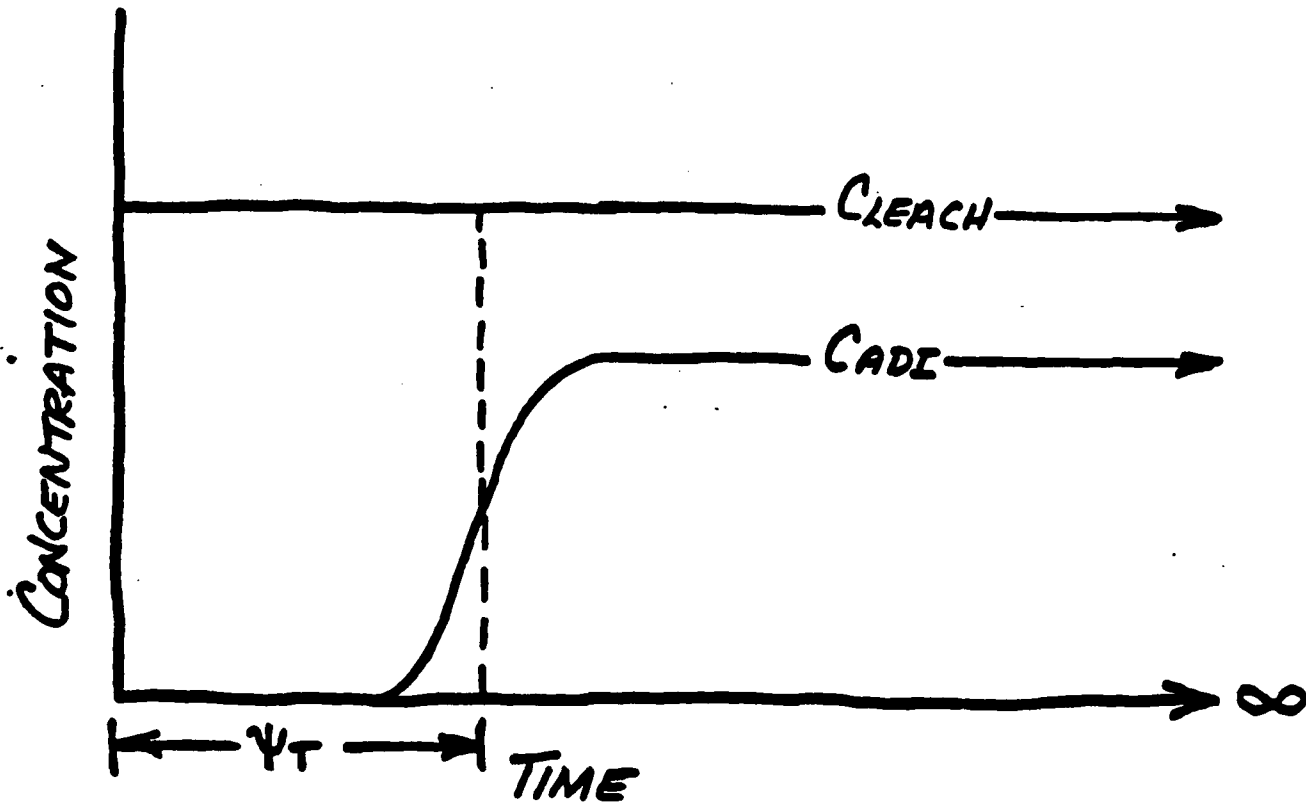


FIGURE 2 TIME HISTORY OF CONCENTRATIONS
WITH AN INFINITE SOURCE

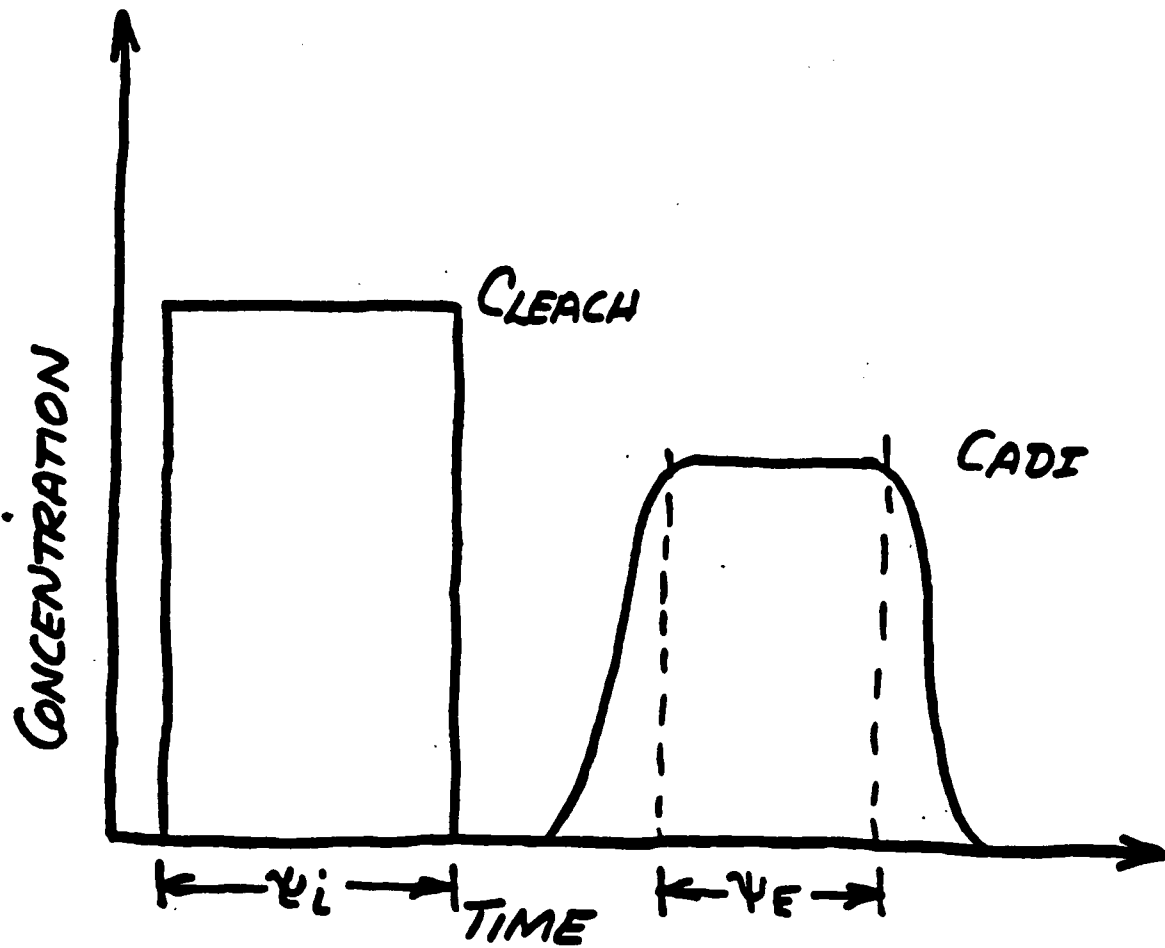


FIGURE 3 TIME HISTORY OF CONCENTRATIONS
WITH A FINITE SOURCE

load of duration, V_e , can be accurately evaluated via the current model. If $V_e \leq V_t$ (or perhaps V_e somewhat greater than V_t) then the pulse or wave concentration observed at the downgradient point may be somewhat less than that predicted by the current solution. It remains useful, however, to use the steady-state equations to yield the maximum values for the down-gradient concentrations.

The above discussion suggests that for purposes of estimating the source mass required for an accurate implementation of the decision rule, two leaching durations are of interest. The time to fully develop the plume, V_t , is useful because this determines the absolute minimum time required for the solutions to the equations to be exact. The exposure time, V_e , is also important because if insufficient mass is available to produce the C_{ADI} value for this time period then the toxicological assumptions of the health-based thresholds are violated. Thus, for any specific set of conditions in a monte carlo run of the model, we can calculate a mass corresponding to both these time durations.

The mass leaving the facility during time period, , is calculated as,

$$SM = Q * A_w * C_L * V * 10^{-3} \quad (1)$$

where Q = leachate volume flux, m/yr
 A_w = facility area, m^2
 C_L = leachate concentration, mg/l
 V = time duration of leak or emission, yr
 SM_V = available mass of constituent, kg

Recall from the solution of the groundwater model that we can express C_L as

$$C_L = C_{ADI}/CD \quad (2)$$

where C_{ADI} = concentration corresponding to the health-based threshold, mg/l

CD = dimensionless "back-calculated" solution to the groundwater equation

Substitution of (2) into (1) yields

$$SM_V = 10^{-3} * Q * A_w * V * C_{ADI}/CD \quad (3)$$

Equation (3) can now be used to calculate the total mass of each specific chemical constituent required to be present in the waste material for any time, Ψ , of interest.

A more useful measure of the mass corresponding to the time periods depicted in Figure 2 and 3 is concentration in the waste. That is, the concentration of each constituent of the waste that if made totally available for leaching would produce a sufficient mass of material at the leachate concentration, C_{LEACH} , to yield the exposure values, C_{ADI} . Consider the mass of the waste, M_W , in the facility. By combining this total waste mass with the constituent mass expression from equation (3), we obtain

$$\frac{SM}{MW} = \frac{Q * A_W * \Psi * C_{ADI}}{CD * DW * \rho_W * A_W}$$

$$\text{or } LWC = Q * \Psi * C_{ADI} / CD * DW * \rho_W \quad (4)$$

where LWC_{Ψ} = leachate concentration of the constituent in the waste material for leach time, Ψ , mg/kg

DW = depth of the waste material in the facility, m

ρ_W = density of the waste material, gm/cm³

Equation (4) requires two new parameters not heretofore needed. Fortunately, both will vary over relatively narrow ranges and can be either fixed at mean values or can be allowed to vary according to some distribution. Interestingly, the facility area (which varies widely) cancels in the equation and does not influence the result except through the values for CD taken from the groundwater model.

Equation (4) enables us to calculate for any desired value of CD (say that corresponding to the 90th percentile level) the required concentration level in the waste (assuming all can be leached) for any given duration, Ψ . Recall that at the minimum Ψ must be 70 years. Thus, a minimum waste concentration for any constituent is given by

$$LWC_{min} = 70 * Q * C_{ADI} / CD * DW * \rho_W \quad (5)$$

The value obtained from equation (5) can be viewed as the minimum constituent concentration in the waste (assuming it is all leachable) required to insure that the conditions represented by the ground water model are physically possible. If the actual concentration is greater than LWC_{min} , then the outcome of the ground water model is also physically possible. If the actual concentration is less than LWC_{min} , then the ground water model overestimates the downgradient concentration because the assumed source term is greater than the waste can provide.

Another useful waste characteristic to consider is the concentration of constituent required to fully develop the plume corresponding to the travel time, t_t . This value can be determined from

$$t_t = X (1 + K_D \rho_B / \theta) / V \quad (6)$$

where X = distance downgradient to the exposure point, m

K_D = partition coefficient, cm^3/gm

ρ_B = soil bulk density, gm/cm^3

θ = porosity, cm^3/cm^3

V = groundwater flow velocity, m/yr

By combining equation (6) and (4) we obtain

$$LWC_T = C_{ADI} * Q * X(1 + K_D \rho_B / \theta) / V * CD * DW * P_w \quad (7)$$

where LWC_T can be interpreted as the minimum constituent concentration in the waste required to insure that the decision rule outcome is derived from a fully developed plume. If the actual waste concentration is greater than LWC_T , then the assumptions become more nearly those of an infinite source and the rule outcomes are accordingly correct. If the actual waste concentration is less than LWC_T , then the plume is not fully developed and the source is exhausted before the downgradient concentration, C_{ADI} , is reached. Note also that if

$$X(1 + K_D \rho_B / \theta) / V < 70 \quad (8)$$

then the minimum exposure time of 70 years will not be achieved by exposure to the mass resulting from the fully developed plume. Thus, LWC_T only has meaning if the travel time is given by equation (6) is greater than 70 years.

Consideration of equations (1)-(8) leads to a useful additional concept in regulating land disposal via leachate concentrations. We can now specify for any given allowable leachate level the associated minimum applicable constituent concentration given as

$$\text{Max } [LWC_{\min}, LWC_T]$$

This value can be interpreted as the constituent waste concentration below which land disposal is acceptable regardless of the results from the leach test. These values can be viewed as bench-mark measures of the reasonableness of the infinite source assumption embedded in the decision rule.

The problem remains to calculate the LWC_{min} and LWC_T values for any percentile level chosen from the frequency distribution generated by the monte carlo runs. Note from equations 5 and 7 that each monte carlo run (i.e., single values for each variable and parameter in the model) will have a unique set of values for all defined variables except the two new waste parameters, DW and P_w . Thus, if these are known we can calculate LWC_{min} and LWC_T values for each data point on the curve and hence the one ultimately chosen for regulation. Example calculations for a non-degrading, sorbing constituent ($KOC = 1000.0$) with an assumed facility depth of 5 meters, a waste density of 1.0 and C_{ADI} of 1.0 are given in the attached figures. Subtitle C facility results are presented in Figure 4-7 and Subtitle D facility results are presented in Figures 8-11.

The figures demonstrate several important outcomes. First, the magnitudes of both concentration levels ($LWOMIN$, $LWCTOT$) and the required masses (S_{MIN} , S_{TOT}) are quite reasonable and thus the "infinite source" assumption of the model remains a physically realistic part of the approach. For example, at the 90th percentile level for the Subtitle C facility, $LWOMIN$ ranges from 7-90 mg/kg and S_{MIN} ranges from 500-50,000 kg if the reference dose is 1.0 mg/l.

Second, pursuing further the ideal of specifying an additional "critical value," the figures suggest that if the 90th percentile is chosen for regulation one can further state that if the constituent concentration in the waste is less than 7 mg/kg then land disposal is safe regardless of the leachate test values.

Mass Balance Analysis of the Infinite Source Assumption

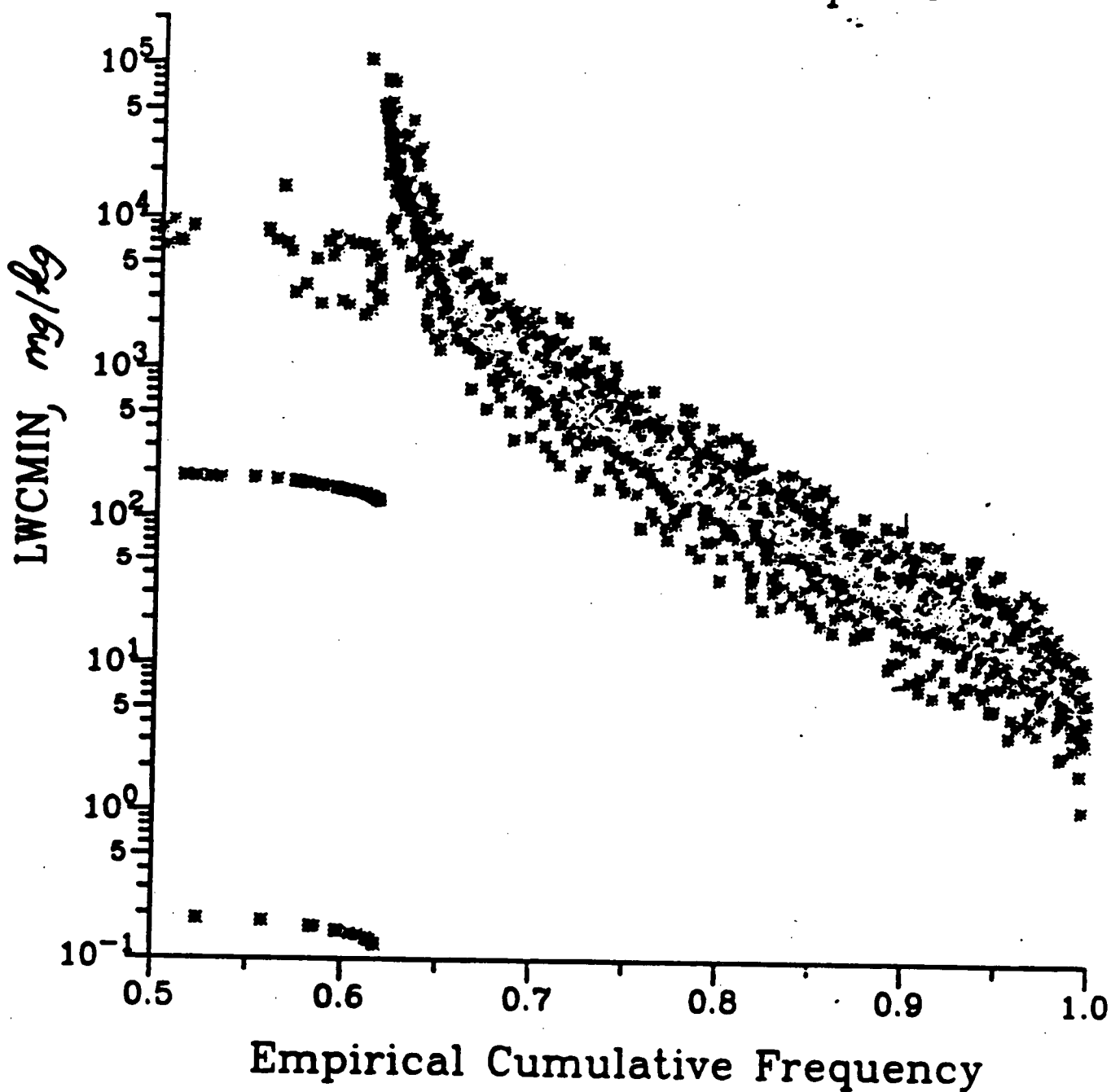


Figure 4. Minimum Applicable Concentration of
Constituent in Waste for an Exposure
Duration of 70 years

Mass Balance Analysis of the Infinite Source Assumption

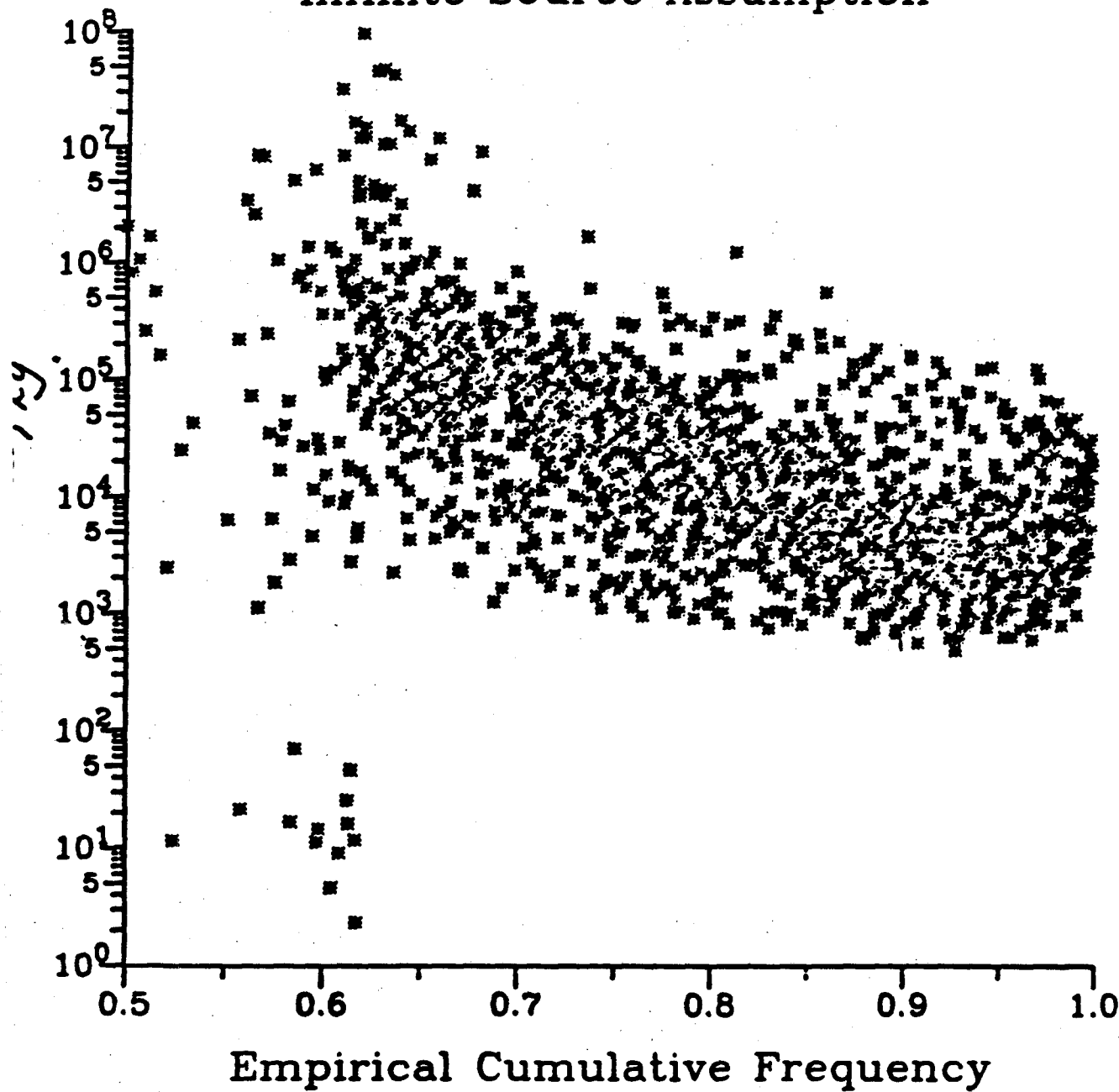
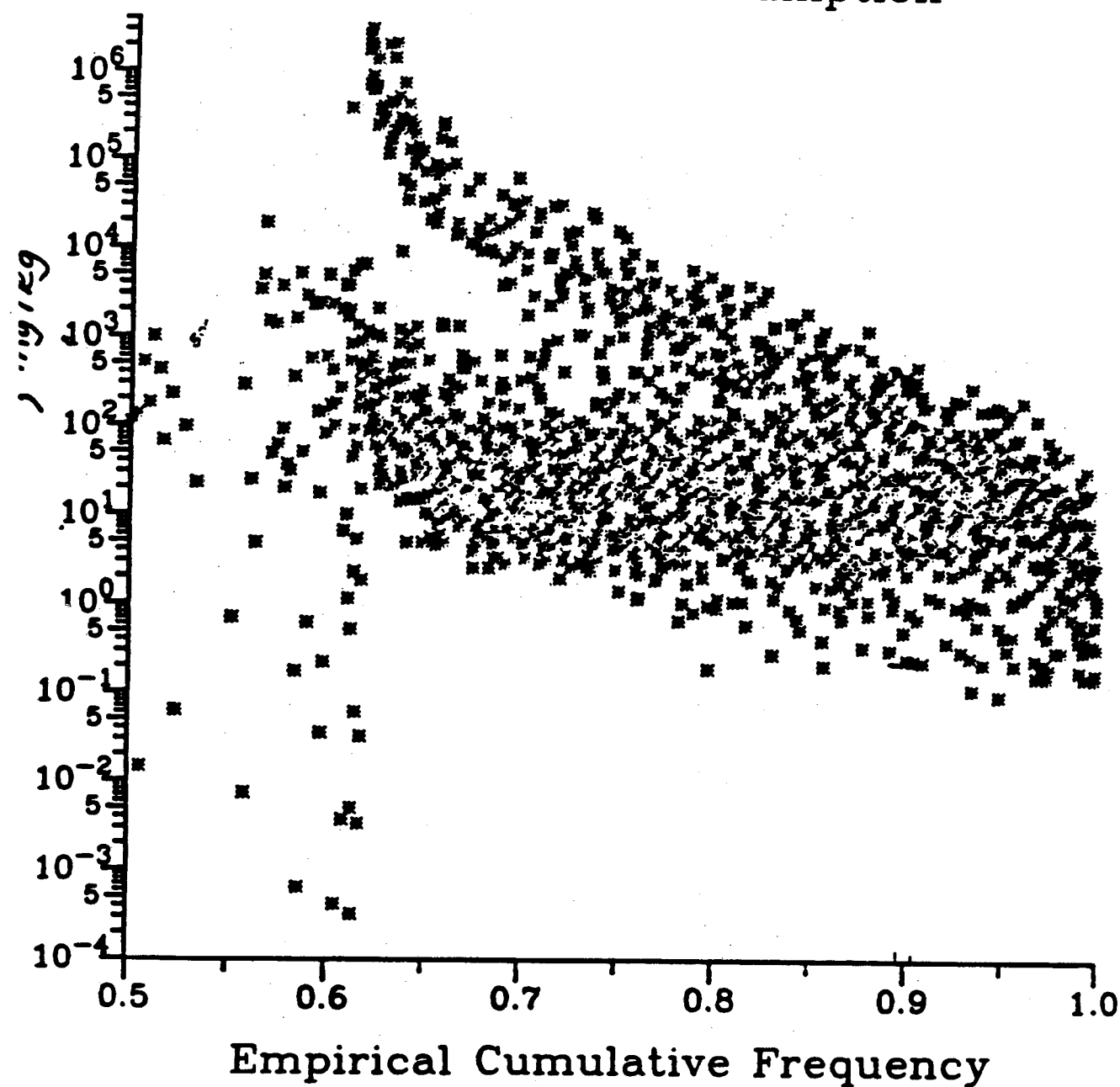


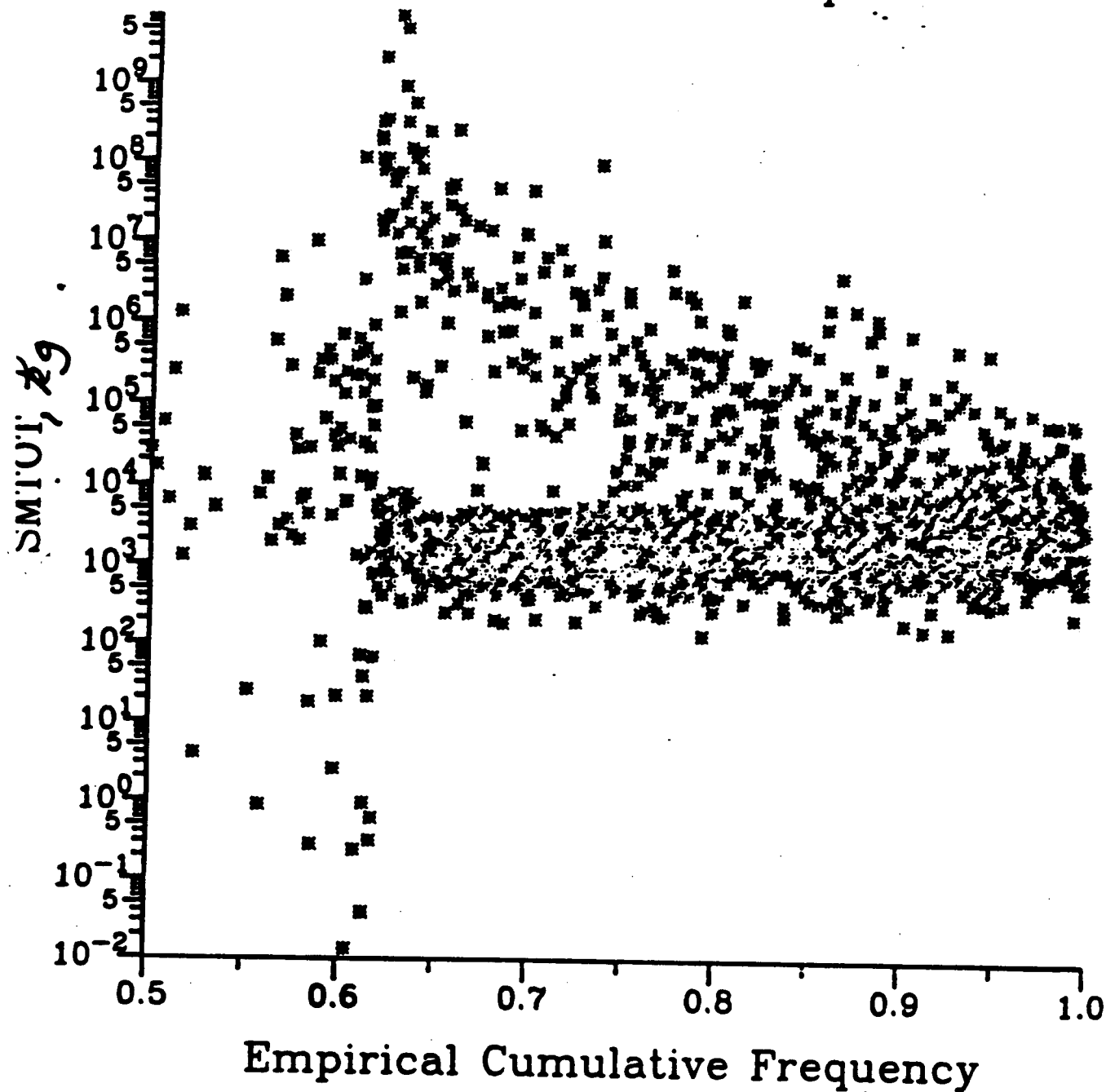
Figure 5. Minimum Mass of Constituent Required in Facility to Produce 70-year Exposure

Mass Balance Analysis of the Infinite Source Assumption



*Figure 6. Minimum Concentration of Constituent
in Waste Required to Produce Full
Development of Groundwater Plume*

Mass Balance Analysis of the Infinite Source Assumption



*Figure 7. Minimum Mass of Constituent Required
in Facility to Produce Fully Developed
Groundwater Plume*

Mass Balance Analysis of the Infinite Source Assumption

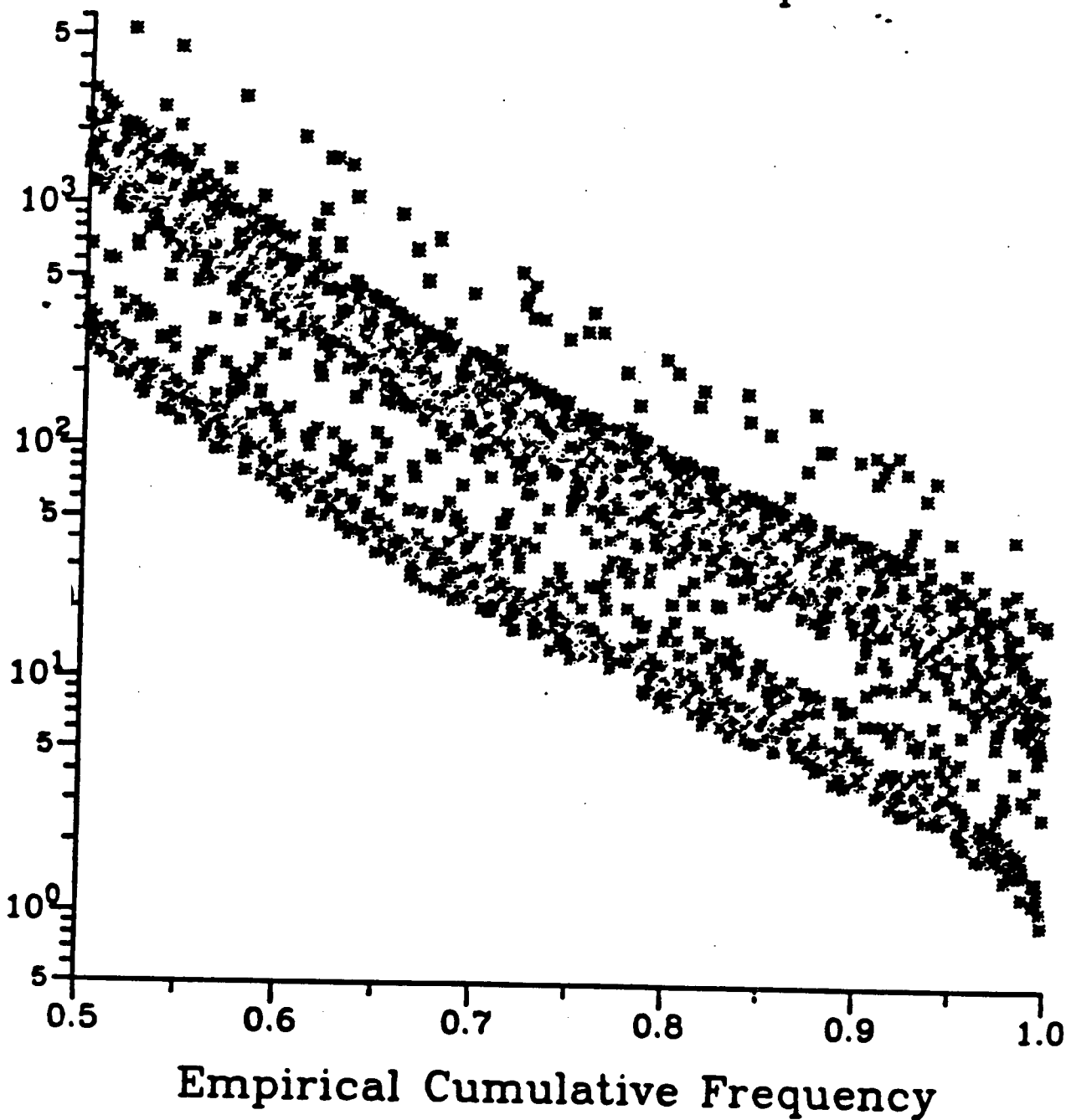
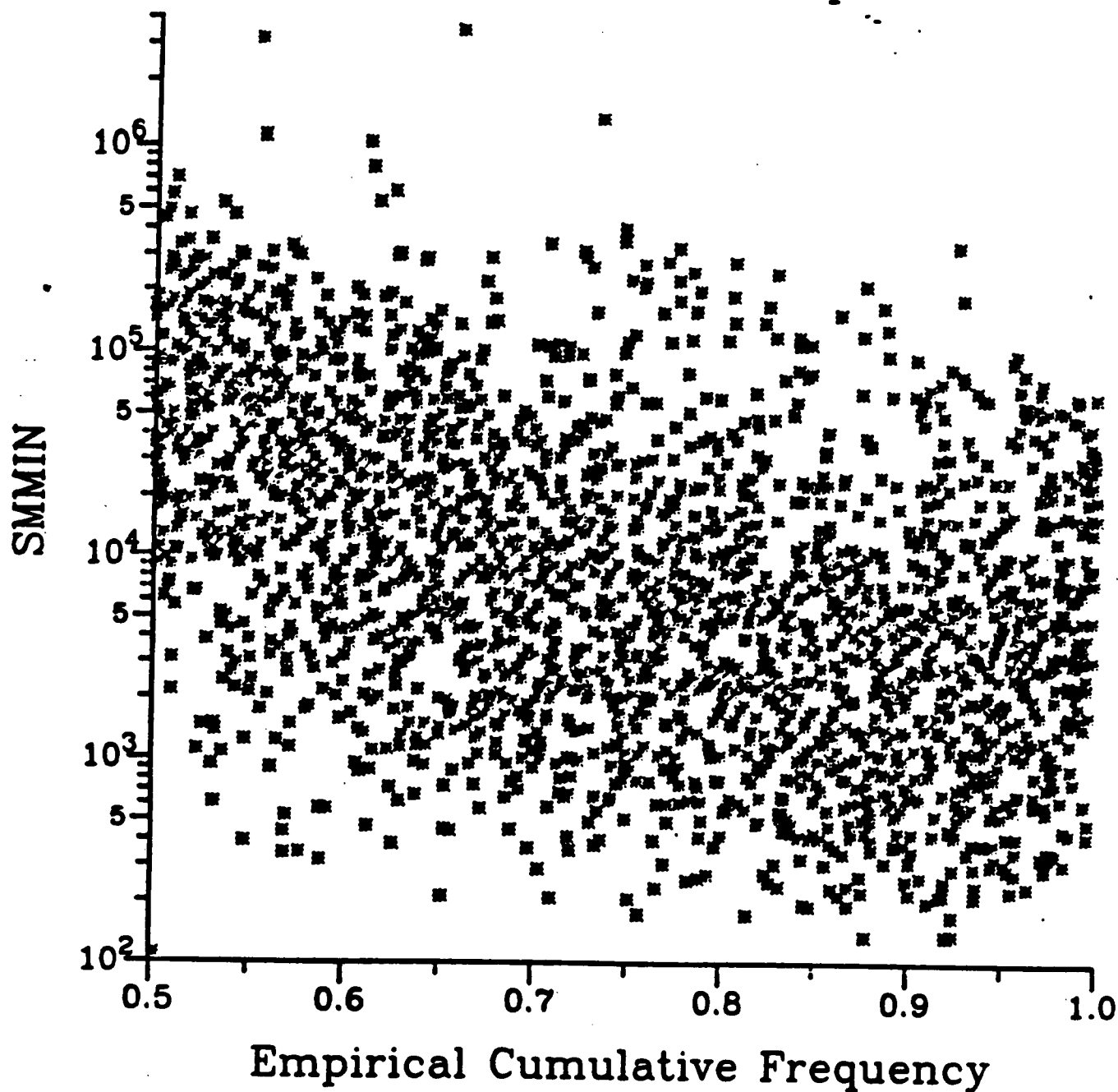


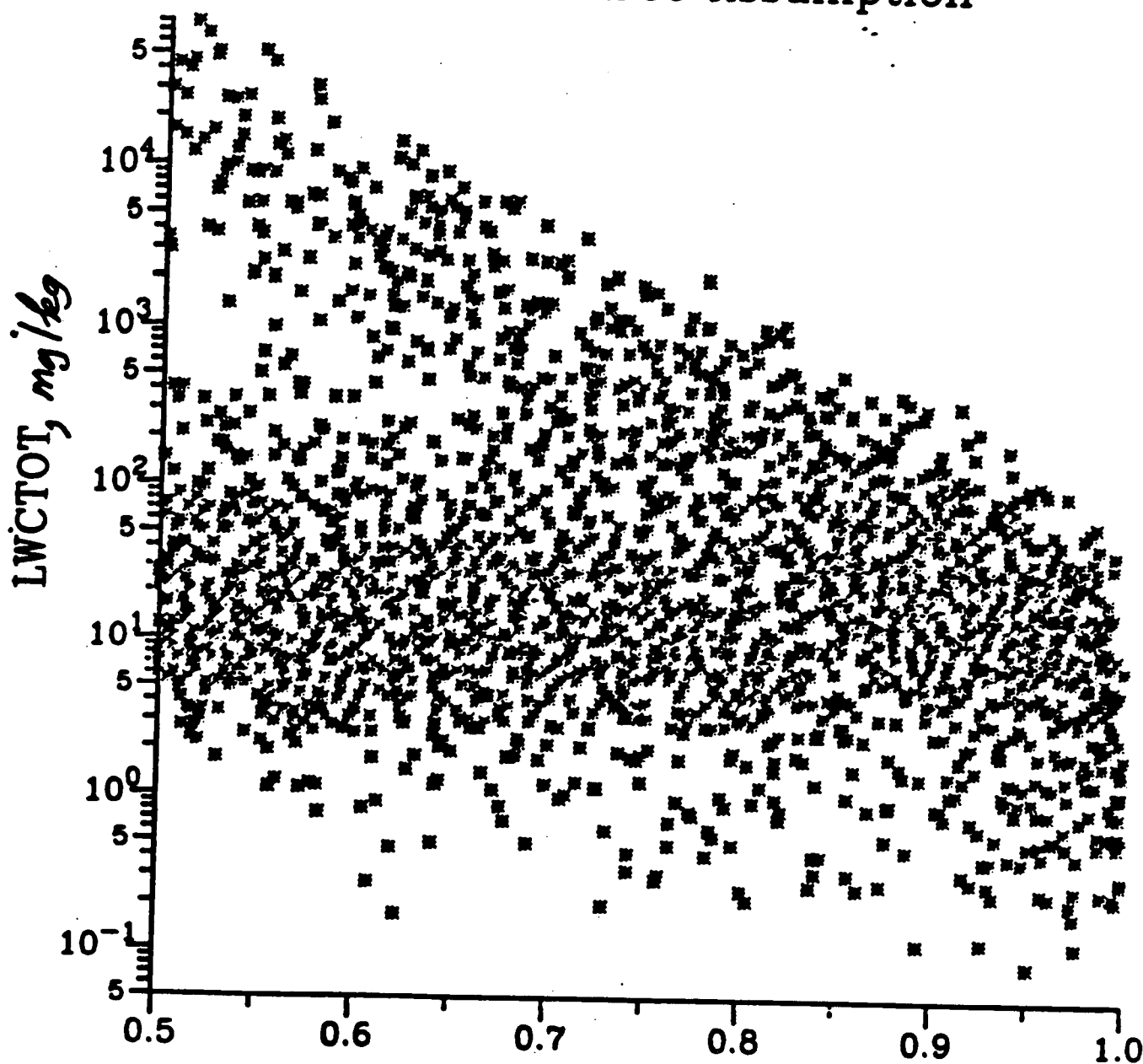
Figure 8. Minimum Constituent Concentration in
Waste Required to Produce Exposure Duration
of 70 years.

Mass Balance Analysis of the Infinite Source Assumption



*Figure 9. Minimum Mass of Constituent
Required in Facility to Produce
Exposure Duration of 70 years*

Mass Balance Analysis of the
Infinite Source Assumption



Empirical Cumulative Frequency

*Figure 10. Minimum Concentration of Constituent
in Waste to Produce Fully Developed
Groundwater Plume*

Mass Balance Analysis of the Infinite Source Assumption

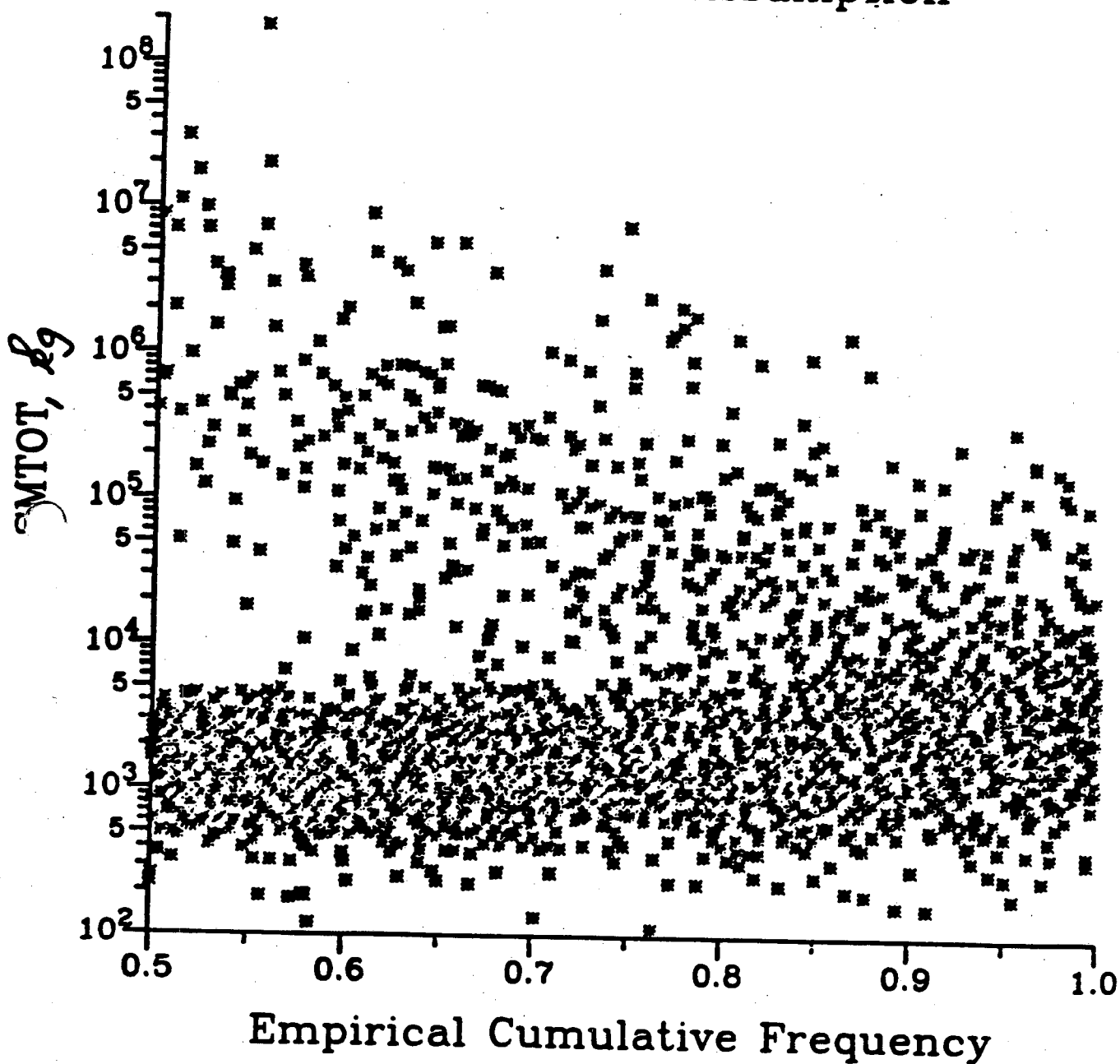


Figure 11. Minimum Mass of Constituent Required
in Facility to Produce Fully Developed
Groundwater Plume.

APPENDIX G

FORTRAN LISTING

GROUND WATER SCREENING PROCEDURE

```

C-----00000010
C   This version was modified to run on an IBM system by DPRA, Inc. 00000020
C-----00000030
C   00000040
C   00000050
C   00000060
C   MCARLO.FTH          DD2:[205,202]          MCARLO.FTH 00000070
C   00000080
C   This version was created 10/15/85 from the QSTAR9 version 00000090
C   created 10/12/85 except now print CD, LMCHIN 00000100
C   00000110
C   IMPLICIT REAL*8      (A-H,O-Z) 00000120
C   REAL*8               LMCHIN,LMCTOT 00000130
C   INTEGER*2 NPROB      00000140
C   INTEGER*2 NMAXCK(9),IUNIT1,IUNIT2,IUNIT3,IPSPEC, 00000150
C   x                   NSIGMA,NLAMDA,NX,NVM,IPCAL,IRCHSE,NMAX,CHOICE, 00000160
C   x                   IX,IY,ILEACH 00000170
C   CHARACTER MENU,ANS,TITLE*60,TERM,VTANSI(4),LEACH(2) 00000180
C   DOUBLE PRECISION SEED1,SEED2,SEED3,SEED4 00000190
C   COMMON/SEEDS/ SEED1,SEED2,SEED3,SEED4 00000200
C   COMMON/CLEAR/ TERM 00000210
C   COMMON/INTEG/ IPSPEC,NSIGMA,NLAMDA,NX,NVM,IPCAL,IRCHSE,NMAX 00000220
C   COMMON/REALS/ SIGMA(1),RHOB,THETA,TCONV,DKA,DKB,DKH,PH,DKDC,FOC, 00000230
C   x                   ADI,XX(1),ALFAL,ALFAT,VM(1),RCHSE(1),DKS, 00000240
C   x                   HSOURC,BQFR,AM,ALFAZ 00000250
C   COMMON/IO/ IUNIT1,IUNIT2,IUNIT3 00000260
C   COMMON/BLOCKA/CD(1,1,1),XD(1),GLAND(1),SD(1), 00000270
C   1                   ZZ(1),GLAM(1) 00000280
C   COMMON/BLOCKP/CPF(1,1,1),DLFAC(1,1,1) 00000290
C   COMMON/NEWMOUT/SPMIN,SMTOT,LMCHIN,LMCTOT,DF,RHOM,Q 00000300
C   Declare all data statements. 00000310
C   00000320
C   DATA VTANSI / 24, ' ', 27, '<' / 00000330
C   DATA NPROB, TITLE / 1000, ' ' / 00000340
C   DATA NMAXCK / 4, 5, 6, 10, 15, 20, 60, 104, 236 / 00000350
C   DATA THOPI / 6.283185307179600 / 00000360
C   DATA ILEACH, LEACH / 1, 'C', 'D' / 00000370
C   DATA DKA0, DKB0, DKH0 / 0.0, 0.0, 0.0 / 00000380
C   DATA DLOW, DRIGH / 40.0, 330.0 / 00000390
C   00000400

```

```

C 45 WRITE (6,50) 00000410
C 50 FORMAT (' Enter (1) for VT100 or (2) for VT52 terminal > ') 00000420
C READ (5,100) TERM 00000430
C IF ( TERM .NE. '1' .AND. TERM .NE. '2' ) GO TO 45 00000440
C IF (TERM .EQ. '1') WRITE (6,100) VTANSI 00000450
100 FORMAT (4A1) 00000460
C 00000470
C Display the Menu. 00000480
C 00000490
200 CALL CLSCR 00000500
WRITE (6,305) ADI,DKA0,DKB0,DKN0,DKOC,NMAX,NPROB, 00000510
TCONV,TR,XC(1),LEACH(ILEACH),DF,RHOM 00000520
305 FORMAT (3(/), 00000530
X 5X,'A - ADI (Value =',G11.5,')',/, 00000540
X 5X,'B - DKA0 (Value =',G11.5,')',/, 00000550
X 5X,'C - DKB0 (Value =',G11.5,')',/, 00000560
X 5X,'D - DKN0 (Value =',G11.5,')',/, 00000570
X 5X,'E - DKOC (Value =',G11.5,')',/, 00000580
X 5X,'F - NMAX (Value =',I11,') # of Gauss points',/, 00000590
X 5X,'G - NPROB (Value =',I11,') # of plot points',/, 00000600
X 5X,'H - TCONV (Value =',G11.5,')', 00000610
X ' Units are per year for rate constants',/, 00000620
X 5X,'I - T(ref) (Value =',G11.5,')',/, 00000630
X 5X,'J - XC (Value =',G11.5,')',/, 00000640
X 5X,'K - ( Subtitle ',A1,' Used )', 00000650
X ' Leaching Rate Distribution',/, 00000660
X 5X,'L - DF (Value =',G11.5,')',/, 00000670
X 5X,'M - RHOM (Value =',G11.5,')',/, 00000680
X 5X,'S - Start the Calculations',/, 00000690
X 5X,'X - Exit Program',/, 00000700
X 5X,'Select a letter -> ') 00000710
READ (5,315,END=2300,ERR=200) MENU 00000720
315 FORMAT (A1) 00000730
355 FORMAT (F12.0) 00000740
365 FORMAT (I6) 00000750
C CHOICE = INDEX('ABCDEFGHIJKLMNabdefghijklmn',MENU) 00000760
IF (CHOICE .EQ. 0) GO TO 200 00000770
IF (CHOICE .GT. 15) CHOICE = CHOICE - 15 00000780
C GO TO 200 00000790
380 CALL CLSCR 00000800
GO TO (400,440,480,520,560,600,640,680,720,760, 00000810
X 800,840,880,920,2300), 00000820
X CHOICE 00000830
GO TO 200 00000840
C 400 WRITE(6,405) 00000850
405 FORMAT(' Enter ADI > ') 00000860
READ (5,355,ERR=400,END=2300) ADI 00000870
IF (ADI .LE. 0.000) GO TO 380 00000880
GO TO 200 00000890
C 440 WRITE(6,445) 00000900
445 FORMAT(' Enter DKA0 > ') 00000910
READ (5,355,ERR=440,END=2300) DKA0 00000920
IF (DKA0 .LT. 0.000) GO TO 380 00000930
GO TO 200 00000940
C 480 WRITE(6,485) 00000950
485 FORMAT(' Enter DKB0 > ') 00000960
READ (5,355,ERR=480,END=2300) DKB0 00000970
00000980
00000990
00001000
00001010

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```

      IF (DKBO .LT. 0.000) GO TO 300
      GO TO 200
C
520 WRITE(6,525)
525 FORMAT(' Enter DKN0 > ')
      READ (5,355,ERR=520,END=2300) DKN0
      IF (DKN0 .LT. 0.000) GO TO 300
      GO TO 200
C
560 WRITE(6,565)
565 FORMAT(' Enter DKOC > ')
      READ (5,355,ERR=560,END=2300) DKOC
      IF (DKOC .LT. 0.000) GO TO 300
      GO TO 200
C
600 WRITE(6,605)
605 FORMAT(////,' Enter NMAX (number of Gauss points) > ')
      READ (5,365,ERR=600,END=2300) NMAX
      DO 610 I = 1,9
610 IF ( NMAX .EQ. NMAXCK(I) ) GO TO 200
      WRITE (6,615) NMAX
615 FORMAT(//,' You entered ',I9,'. Please reenter',/,
x      ' 4, 5, 6, 10, 15, 20, 60, 104, 256',//,
x      ' Hit Carriage Return to Continue')
      READ (5,315,ERR=300,END=2300) MENU
      GO TO 300
C
640 WRITE(6,645)
645 FORMAT(////,' Enter NPROB (number of Plot points) > ')
      READ (5,365,ERR=640,END=2300) NPROB
      IF (NPROB .LT. 500) GO TO 300
      GO TO 200
C
680 WRITE(6,685)
685 FORMAT(' Enter TCONV > ')
      READ (5,355,ERR=680,END=2300) TCONV
      IF (TCONV .LE. 0.000) GO TO 300
      GO TO 200
C
720 WRITE(6,725)
725 FORMAT(' Enter T<ref> ')
      READ (5,355,ERR=720,END=2300) TR
      IF (TR .LE. 0.000 .OR. TR .GT. 50.000) GO TO 300
      GO TO 200
C
760 WRITE(6,765)
765 FORMAT(' Enter XX > ')
      READ (5,355,ERR=760,END=2300) XX(1)
      IF (XX(1) .LE. 0.000) GO TO 300
      GO TO 200
C
800 ILEACH = 3 - ILEACH
      GO TO 200
C
810 WRITE(6,811)
811 FORMAT(' Enter DF > ')
      READ (5,355,ERR=810,END=2300) DF
      IF (DF .LE. 0.000) GO TO 300
      GO TO 200
C
820 WRITE(6,821)

```

```

00001020
00001030
00001040
00001050
00001060
00001070
00001080
00001090
00001100
00001110
00001120
00001130
00001140
00001150
00001160
00001170
00001180
00001190
00001200
00001210
00001220
00001230
00001240
00001250
00001260
00001270
00001280
00001290
00001300
00001310
00001320
00001330
00001340
00001350
00001360
00001370
00001380
00001390
00001400
00001410
00001420
00001430
00001440
00001450
00001460
00001470
00001480
00001490
00001500
00001510
00001520
00001530
00001540
00001550
00001560
00001570
00001580
00001590
00001600
00001610
00001620

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```

821 FORMAT(' Enter RHOM > ')
READ (5,355,ERR=820,END=2300) RHOM
IF (RHOM .LE. 0.000) GO TO 380
GO TO 200

C
C Perform calculations
C
840 CALL CLSCR
C OPEN(UNIT=IUNIT1,FILE='MCARLO.DAT',STATUS='NEW',
C x CARRIAGECONTROL='LIST')
C OPEN(UNIT=IUNIT2,FILE='MCARLO.INF',STATUS='NEW')
C
C Write information
C
WRITE (2,2150) ADI,DKA0,DKB0,DKN0,DKOC,NMAX,NPROB,
x TCONV,TR,XX(1),LEACH(ILEACH),DF,RHOM
2150 FORMAT (/ ,2X,'A - ADI (Value =',012.6,')',/,
x 2X,'B - DKA0 (Value =',012.6,')',/,
x 2X,'C - DKB0 (Value =',012.6,')',/,
x 2X,'D - DKN0 (Value =',012.6,')',/,
x 2X,'E - DKOC (Value =',012.6,')',/,
x 2X,'F - NMAX (Value =',I12,') # of Gauss points',/,
x 2X,'G - NPROB (Value =',I12,') # of plot points',/,
x 2X,'H - TCONV (Value =',012.6,')',
x ' Units are per year for rate constants',/,
x 2X,'I - T(REF) (Value =',012.6,')',/,
x 2X,'J - XX (Value =',012.6,')',/,
x 2X,'K - ( Subtitle ',A1,' Used )',
x ' Leaching Rate Distribution',/,
x 2X,'L - DF (Value =',011.5,')',/,
x 2X,'M - RHOM (Value =',011.5,')',/)
CALL CLSCR
WRITE(6,2160)
2160 FORMAT (10(/),15X,'Calculations are being Performed',/)
SEED1 = 123457.000
SEED2 = 123457.000
SEED3 = 123457.000
SEED4 = 123457.000
IX = 1235
IY = 8765
DO 2200 I = 1, NPROB

C
C New code
C
2170 DDLMIN = DLOG10(4.00-4)
DDLMAX = DLOG10(1.00-1)
CALL RANDOM(DDLMIN,DDLMAX,DD)
DD = 10.000*DD
THETA = 0.261 - 0.0385 * ( DLOG(DD) )
RHOB = 2.65 * ( 1.000 - THETA )
DKS = 470.0 * (THETA**3.000 + DD**2.000
x / (1.00 - THETA)**2.000)
CCCCC CALL RANDOM(1.00-5,0.100,SS)
2180 CALL EXRAND(0.0309,1,SS)
IF ( SS .LT. 1.00-5 .OR. SS .GT. 1.00-1 ) GO TO 2180
VM(1) = (3.183605 * DKS * SS) / THETA
IF ( VM(1) .GT. 9250.000 ) GO TO 2170

C
C Code added 06/05/85
C
TRK = TR + 273.000

```

```

00001630
00001640
00001650
00001660
00001670
00001680
00001690
00001700
00001710
00001720
00001730
00001740
00001750
00001760
00001770
00001780
00001790
00001800
00001810
00001820
00001830
00001840
00001850
00001860
00001870
00001880
00001890
00001900
00001910
00001920
00001930
00001940
00001950
00001960
00001970
00001980
00001990
00002000
00002010
00002020
00002030
00002040
00002050
00002060
00002070
00002080
00002090
00002100
00002110
00002120
00002130
00002140
00002150
00002160
00002170
00002180
00002190
00002200
00002210
00002220
00002230

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```

2190      CALL RANROR ( 14.400, 27.9500, T )
      IF ( T .LT. 0.000 .OR. T .GT. 30.000 ) GO TO 2190
      T = T + 273.000
C
      DKA = DKA0 * ( DEXP( 1.004 * ( 1.000 / TRK - 1.000 / T )) )
      DKN = DKN0 * ( DEXP( 1.004 * ( 1.000 / TRK - 1.000 / T )) )
      DKB = DKB0 * ( DEXP( 1.004 * ( 1.000 / TRK - 1.000 / T )) )
C
2195      CALL RANROR ( 6.6200, 1.6500, PH )
      IF ( PH .LT. 0.300 .OR. PH .GT. 14.000 ) GO TO 2195
C
C CODE ADDED 8/26/85 BY JEROME COLEMAN
C QC IS THE Q VALUE FOR SUBTITLE C
C QD IS THE Q VALUE FOR SUBTITLE D
C
C      IF(ILEACH.EQ.1)THEN
C          Call Rudy's subroutines
C          CALL TBLC(QC,QD,IX,IY)
C          IF ( QC .LE. 0.0100 ) QC = 0.0000100
C          QC = QC * 0.025400
C          IF ( QD .LE. 0.0100 ) QD = 0.0000100
C          QD = QD * 0.025400
C          Q=QC
C      ELSE
C          Call Rudy's subroutines
C          CALL TBLD(QC,QD,IX,IY)
C          IF ( QC .LE. 0.0100 ) QC = 0.0000100
C          QC = QC * 0.025400
C          IF ( QD .LE. 0.0100 ) QD = 0.0000100
C          QD = QD * 0.025400
C          Q=QD
C      ENDIF
C
      CALL EXRAND(70.600,1,BAQFR)
      BAQFR = BAQFR + 3.000
C      WRITE(2,111)' BAQFR= ',BAQFR
C111  FORMAT(IX,A10,F15.7)
C
C      B2 added coded on 6/10/85.
C
      CALL RANDUM (2.00,10.00,HINIT)
      IF ( HINIT / BAQFR .GT. 0.500 ) THEN
      HSOURC = 0.500 * BAQFR
      ELSE
      HSOURC = HINIT
      END IF
C
CC
CC ADDITION 06/19/85 BY JEROME COLEMAN
CC
      RCHSE(1)=QD/HSOURC
C
C
2199      CALL RANDUM(0.000,1.000,UI)
      IF ( UI .LE. 0.910/100 ) THEN

```

```

00002240
00002250
00002260
00002270
00002280
00002290
00002300
00002310
00002320
00002330
00002340
00002350
00002360
00002370
00002380
00002390
00002400
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00002500
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00002600
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00002660
00002670
00002680
00002690
00002700
00002710
00002720
00002730
00002740
00002750
00002760
00002770
00002780
00002790
00002800
00002810
00002820
00002830
00002840

```



```

x          TS , '      XX = ',G12.6,/,          00003460
x          TS , '      DF = ',G12.6,T30, '  RHOM = ',G12.6,/  00003470
C          CLOSE(UNIT=IUNIT1)          00003480
C          CLOSE(UNIT=IUNIT2)          00003490
C                                          00003500
2220      CALL CLSCR                    00003510
          WRITE(6,2225)                00003520
2225      FORMAT(/////////,            00003530
x          ' Do you want to make another run (Y/N) > ' )      00003540
          READ(5,315,END=2300,ERR=2220)MENU 00003550
          IF ( INDEX ( 'YyNn',MENU ) .EQ. 0 ) THEN 00003560
              CALL CLSCR                00003570
              WRITE (6,2235) MENU        00003580
2235      FORMAT ( ' You entered <',A1,'>. Please reenter' ) 00003590
              GO TO 2220                00003600
          ENDIF                          00003610
          IF ( MENU .EQ. 'Y' .OR. MENU .EQ. 'y' ) GO TO 200 00003620
2300      CONTINUE                      00003630
          STOP                          00003640
          END                            00003650
                                          00003660
          BLOCK DATA                   00003670
          IMPLICIT REAL*8 (A-H,O-Z)     00003680
          REAL*8 LNCMIN,LNCTOT          00003690
          INTEGER*2 IUNIT1,IUNIT2,IUNITS,IPSPEC, 00003700
x          NSIGMA,NLAMDA,NX,NVW,IPCAL,IRCHGE,NMAX,CHOICE 00003710
          CHARACTER TERM                00003720
          COMMON/CLEAR/ TERM            00003730
          COMMON/INTGS/ IPSPEC,NSIGMA,NLAMDA,NX,NVW,IPCAL,IRCHGE,NMAX 00003740
          COMMON/REALS/ SIGMA(1),RHOB,THETA,TCONV,DKA,DKB,DKN,PH,DKOC,FOC, 00003750
x          ADI,XX(1),ALFAL,ALFAT,VW(1),RCHGE(1),DKS, 00003760
x          NSOURC,BAGFR,AM,ALFAZ        00003770
          COMMON/IO/ IUNIT1,IUNIT2,IUNITS 00003780
          COMMON/BLOCKA/CD(1,1,1),XD(1),GLAND(1),SD(1),ZZ(1),GLAM(1) 00003790
          COMMON/BLOCKP/CPF(1,1,1),DLFAC(1,1,1) 00003800
          COMMON/NEOUT/SPMIN,SHTOT,LNCMIN,LNCTOT,DF,RHOM,Q 00003810
C          Declare all data statements. 00003820
          DATA IUNIT1,IUNIT2,IUNITS,TERM / 1, 2, 5, '1' / 00003830
          DATA IPSPEC,NSIGMA,NLAMDA,NX,NVW,IPCAL,IRCHGE / 00003840
x          1, 1, 1, 1, 1, 1, 1 /      00003850
          DATA NMAX,SIGMA,RHOB,THETA,TCONV / 00003860
x          15, 15.0, 1.6, 0.35, 1.0 / 00003870
          DATA PH, DKOC, FOC, ADI, XX, ALFAL,ALFAT, VW, RCHGE / 00003880
x          7.0, 100.0, 0.01, 1.0, 150.0, 10.0, 1.0, 0.0, 0.0 / 00003890
          DATA DF,RHOM / 5.0, 1.0 /    00003900
          END                            00003910
                                          00003920
          SUBROUTINE CLSCR               00003930
C          CHARACTER CLR1(10),CLR2(10),TERM 00003940
C          INTEGER*2 IUNIT1,IUNIT2,IUNITS 00003950
C          COMMON/IO/ IUNIT1,IUNIT2,IUNITS 00003960
C          COMMON/CLEAR/ TERM            00003970
C          DATA CLR1 / ' ',27,'I','1',' ','1',' ','27','I','J' / 00003980
C          DATA CLR2 /6*' ',27,'H',27,'J' / 00003990
C          IF ( TERM .EQ. '1' ) WRITE(6,10)CLR1 00004000
C          IF ( TERM .EQ. '2' ) WRITE(6,10)CLR2 00004010
C 10      FORMAT (10A1)                 00004020
          DO 10 I=1,24                  00004030
              WRITE(6,11)                00004040
              CONTINUE                   00004050
          11
          CONTINUE

```

```

      RETURN
      END
C=====
C  TBLC.FTN - 5/31/85
C
C  ROUTINE TO PRODUCE A RANDOM NUMBER ON THE BASIS OF A
C  TABLE-SPECIFIED EMPIRICAL DISTRIBUTION
C
C=====
C
C      SUBROUTINE TBLC (RESC,RESO,IX,IY)
C
C      IMPLICIT REAL*8  (A-H,O-Z)
C      INTEGER*2 IX,IY
C      DIMENSION TABLEC(13,2)
C      DIMENSION TABLED(15,2)
C
C      DATA TABLEC/.01,.3,.4,.5,2.5,2.9,3.3,3.6,3.7,6.9,7.0,8.0,11.9,
C      * .29,.30,.49,.50,.63,.64,.73,.76,.81,.92,.95,.98,1.0/
C
C      DATA TABLED/.1,.2,.3,.6,.8,1.2,1.5,2.4,4.0,5.0,6.0,7.2,12.1,12.2,
C      * 35.9,.03,.06,.08,.26,.29,.40,.53,.50,.61,.64,.70,.81,.95,.98,
C      * 1.0/
C
C      CALL RANDU(IX,IY,YFL)
C
C  CHECK VS TABLEC
C      IF(YFL.GT.TABLEC(1,2))GO TO 100
C
C      RESC=TABLEC(1,1)
C      GO TO 102
C
C  DO TABLE LOOKUP, INTERPOLATION
C
C100  DO 101 I=2,13
C      IF(YFL.GT.TABLEC(I,2))GO TO 101
C      SLOPE = (TABLEC(I,2)-TABLEC(I-1,2))/(TABLEC(I,1)-TABLEC(I-1,1))
C      RESC = TABLEC(I-1,1) + (YFL-TABLEC(I-1,2))/SLOPE
C      GO TO 102
C101  CONTINUE
C
C      RESC = TABLEC(13,1)
C102  CONTINUE
C
C  now for D
C
C  CHECK VS TABLE
C
C      IF(YFL.GT.TABLED(1,2))GO TO 200
C
C      RESO=TABLED(1,1)
C      GO TO 299
C
C  DO TABLE LOOKUP, INTERPOLATION
C
C200  DO 201 I=2,15
C      IF(YFL.GT.TABLED(I,2))GO TO 201
C      SLOPE = (TABLED(I,2)-TABLED(I-1,2))/(TABLED(I,1)-TABLED(I-1,1))
C      RESO = TABLED(I-1,1) + (YFL-TABLED(I-1,2))/SLOPE
C      GO TO 299

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201  CONTINUE
C
C      RESD = TABLED(15,1)
C
299  RETURN
END
C=====
C  TBLD.FTH - 5/31/85
C
C  ROUTINE TO PRODUCE A RANDOM NUMBER ON THE BASIS OF A
C  TABLE-SPECIFIED EMPIRICAL DISTRIBUTION
C
C=====
C
C      SUBROUTINE TBLD (RESC,RESD,IX,IY)
C
C      IMPLICIT REAL*8  (A-H,O-Z)
C      INTEGER*2 IX,IY
C      DIMENSION TABLEC(13,2)
C      DIMENSION TABLED(15,2)
C
C      DATA TABLEC/.01,.3,.4,.5,2.5,2.9,3.3,3.6,3.7,6.9,7.0,8.0,11.9,
C      * .29,.30,.49,.50,.63,.64,.73,.76,.81,.92,.95,.98,1.0/
C
C      DATA TABLED/.1,.2,.3,.6,.8,1.2,1.3,2.4,4.0,5.0,6.0,7.2,12.1,12.2,
C      * 35.9, .03,.06,.08,.26,.29,.40,.53,.58,.61,.64,.70,.81,.95,.98,
C      * 1.0/
C
C  GENERATE A UNIFORM (0,1)
C
C      CALL RANDU(IX,IY,YFL)
C
C  now for D
C
C  CHECK VS TABLE
C
C      IF(YFL.GT.TABLED(1,2))GO TO 200
C
C      RESD=TABLED(1,1)
C      GO TO 299
C
C  DO TABLE LOOKUP, INTERPOLATION
C
200  DO 201 I=2,15
C      IF(YFL.GT.TABLED(I,2))GO TO 201
C      SLOPE = (TABLED(I,2)-TABLED(I-1,2))/(TABLED(I,1)-TABLED(I-1,1))
C      RESD = TABLED(I-1,1) + (YFL-TABLED(I-1,2))/SLOPE
C      GO TO 299
201  CONTINUE
C
C      RESD = TABLED(15,1)
C
299  CONTINUE
C
C  now for C
C
C  CHECK VS TABLEC
C      IF(YFL.GT.TABLEC(1,2))GO TO 100
C
C      RESC=TABLEC(1,1)

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      GO TO 102
C
C DO TABLE LOOKUP, INTERPOLATION
C
100 DO 101 I=2,13
      IF(YFL.GT.TABLEC(I,2))GO TO 101
      SLOPE = (TABLEC(I,2)-TABLEC(I-1,2))/(TABLEC(I,1)-TABLEC(I-1,1))
      RESC = TABLEC(I-1,1) + (YFL-TABLEC(I-1,2))/SLOPE
      GO TO 102
101 CONTINUE
C
      RESC = TABLEC(13,1)
102 RETURN
      END
C=====
C TBLINTRP.FTN - 8/31/85
C
C ROUTINE TO PRODUCE A RANDOM NUMBER ON THE BASIS OF A
C TABLE-SPECIFIED EMPIRICAL DISTRIBUTION
C
C=====
C
      SUBROUTINE TBLINT (RESULT,IX,IY)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER*2 IX,IY
      DIMENSION TABLE(13,2)
C
      DATA TABLE/.01,.3,.4,.5,2.5,2.9,3.3,3.6,3.7,6.9,7.0,8.0,11.9,
      * .29,.30,.49,.50,.63,.66,.73,.76,.81,.92,.95,.98,1.0/
C
      DO 200 I=1,13
C
C GENERATE A UNIFORM (0,1)
C
      CALL RANDU(IX,IY,YFL)
C
C CHECK VS TABLE
      IF(YFL.GT.TABLE(1,2))GO TO 100
C
      RESULT=TABLE(1,1)
      GO TO 99
C
C DO TABLE LOOKUP, INTERPOLATION
C
100 DO 1 I=2,13
      IF(YFL.GT.TABLE(I,2))GO TO 1
      SLOPE = (TABLE(I,2)-TABLE(I-1,2))/(TABLE(I,1)-TABLE(I-1,1))
      RESULT = TABLE(I-1,1) + (YFL-TABLE(I-1,2))/SLOPE
      GO TO 99
      1 CONTINUE
C
      RESULT = TABLE(13,1)
C
      99 RETURN
      END
C=====
C TBLINTRPD.FTN - 07/10/85
C
      SUBROUTINE TBLIN2 (RESULT,IX,IY)
C

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      IMPLICIT REAL*8  (A-H,O-Z)
      INTEGER*2 IX,IY
      DIMENSION TABLE(15,2)
C
      DATA TABLE/.1,.2,.3,.6,.8,1.2,1.5,2.4,4.8,8.8,6.8,7.2,12.1,12.2,
* 35.9,.03,.06,.08,.26,.29,.40,.53,.58,.61,.66,.78,.81,.95,.98,
* 1.0/
C
C  GENERATE A UNIFORM (0,1)
C
      CALL RANDU(IX,IY,YFL)
C
C  CHECK VS TABLE
      IF(YFL.GT.TABLE(1,2))GO TO 100
C
      RESULT=TABLE(1,1)
      GO TO 99
C
C  DO TABLE LOOKUP, INTERPOLATION
C
100  DO 1 I=2,15
      IF(YFL.GT.TABLE(I,2))GO TO 1
      SLOPE = (TABLE(I,2)-TABLE(I-1,2))/(TABLE(I,1)-TABLE(I-1,1))
      RESULT = TABLE(I-1,1) + (YFL-TABLE(I-1,2))/SLOPE
      GO TO 99
1  CONTINUE
C
      RESULT = TABLE(15,1)
C
99  RETURN
      END
C=====
      SUBROUTINE RANDU(IX,IY,YFL)
C
      IMPLICIT REAL*8  (A-H,O-Z)
      INTEGER*2 IX,IY
      DATA IA,IB,IC/259,32767,1/
C
      IY=IX*IA
      IF(IY)5,6,6
5  IY=(IY+IB)*IC
6  YFL=IY
      YFL=YFL*.30518509E-4
      IX=IY
      RETURN
      END
C
      LNDGEN.FTH
C
      SUBROUTINE LNDGEN (MEAN,VARI,VALUE)
      IMPLICIT REAL*8  (A-H,O-Z)
      DOUBLE PRECISION  VALUE,SEED1,SEED2,SEED3,SEED4
      COMMON/SEEDS/ SEED1,SEED2,SEED3,SEED4
      REAL  MEAN, VARI, VALUES(2)
10  CALL CGNLS (SEED1, 1, MEAN, VARI, VALUES)
      VALUE = VALUES(1)
      IF (VALUE .LT. 1.00D-3 .OR. VALUE .GT. 1.00D-2) GO TO 10
      RETURN
      END
C
      RANDNR.FTH

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C
SUBROUTINE RANNOR(MEAN,VARI,VALUEO)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION VALUEO,SEED1,SEED2,SEED3,SEED4
COMMON/SEEDS/ SEED1,SEED2,SEED3,SEED4
REAL MEAN, VARI, VALUE(1)
CALL GGNML ( SEED2, 1, VALUE(1) )
VALUEO = MEAN + SQRT(VARI) * VALUE(1)
RETURN
END

C
RANDOM.FTN

C
SUBROUTINE RANDUM(LOWER,UPPER,VALUEO)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION VALUEO,SEED1,SEED2,SEED3,SEED4
COMMON/SEEDS/ SEED1,SEED2,SEED3,SEED4
REAL LOWER, UPPER, VALUE
VALUE = GSUBFS(SEED3)
VALUEO = LOWER + VALUE * ( UPPER - LOWER)
RETURN
END

C
EXRAND.FTN

C
SUBROUTINE EXRAND(XM,NR,VALUEO)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION VALUEO,SEED1,SEED2,SEED3,SEED4
COMMON/SEEDS/ SEED1,SEED2,SEED3,SEED4
REAL XM, VALUE
CALL GSEXN ( SEED4, XM, NR, VALUE )
VALUEO = VALUE
RETURN
END

SUBROUTINE CAD
C..... EPASHOD-1: COMPUTES SUDICKY'S STEADY-STATE ANALYTICAL SOLUTION
C DISTRIBUTION OF CONCENTRATION AT X = 0. THE CODE
C DETERMINES ONLY CONCENTRATION VALUES ALONG THE X-AXIS
C INSERT SYSCOM>ERRD.F
C INSERT SYSCOM>KEYS.F
IMPLICIT REAL*8(A-H,O-Z)
INTEGER*2 IUNIT1,IUNIT2,IUNIT3,
X IPSPEC,NSIGMA,NLANDA,NX,NVM,IPCAL,IRCHSE,NMAX
CHARACTER MENU,ANS,TITLE*60,TERM
REAL*8 KRD,KRDSTO,KRDN,LNCHIN,LNCTOT
DIMENSION RETARD(1)
COMMON/INTESS/ IPSPEC,NSIGMA,NLANDA,NX,NVM,IPCAL,IRCHSE,NMAX
COMMON/IO/ IUNIT1,IUNIT2,IUNIT3
COMMON/REALS/ SIGMA(1),RHOB,THETA,TCONV,DKA,DKB,DKN,PH,DKQC,FOC,
X ADI,XX(1),ALFAL,ALFAT,VM(1),RCHSE(1),DKB,
X HSOURC,BAQFR,AW,ALFAZ
COMMON/BLOCKA/CD(1,1,1),XD(1),GLAND(1),SD(1),
1 ZZ(1),GLAM(1)
COMMON/BLOCKP/CPF(1,1,1),DLFAC(1,1,1)
COMMON/NEWOUT/SHMIN,SHTOT,LNCHIN,LNCTOT,DF,RHOB,Q
C-----
CALL SRCH00(KOREAD,'IPEPMS',7,1,ITT,ICOD)
CALL SRCH00(KSHRT,'OPEPMS',7,2,ITT,ICOD)
CALL SRCH00(KORDR,'OTEPMS',7,3,ITT,ICOD)

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C..... CARD TYPE 00007120
      IF(NMAX.EQ.0) NMAX=104 00007130
C..... READ PARAMETER VALUES 00007140
C..... CARD T00007150
C 00007160
      VS = VM(1) 00007170
C 00007180
      ISPEC=IPSPEC 00007190
      IF(IPSPEC.EQ.1) GO TO 10 00007200
C..... FOR IPSPEC=0, READ IN DIMENSIONLESS VALUES OF 00007210
C STANDARD DEVIATION, DECAY CONSTANT AND DISTANCE 00007220
C----- 00007230
C..... CARD 00007240
      ALFAL=0.5 00007250
      ALFAT=0.5 00007260
      VS=1. 00007270
      DO 12 J=1,NSIGMA 00007280
12 SIGMA(J)=SD(J) 00007290
      DO 14 J=1,NX 00007300
14 XX(J)=XD(J) 00007310
      GO TO 20 00007320
10 CONTINUE 00007330
C..... FOR IPSPEC=1, READ IN REAL VALUES OF 00007340
C STANDARD DEVIATION, DECAY CONSTANT AND DISTANCE 00007350
C----- CARD TY00007360
      IF(IPCAL.EQ.0) GO TO 129 00007370
C..... "D TY 00007380
      IF(DABS(TCONV).LE.1.D-20) TCONV=1.D0 00007390
      DO 149 J=1,NLMDA 00007400
      DKD = DKOC * FOC 00007410
C..... CARD TY00007420
      POH=14.-PH 00007430
      PH=-PH 00007440
      POH=-POH 00007450
      GLAM1=DKA*(10.**PH)+DKN+DKB*(10.**POH) 00007460
      GLAM2=10.*DKA*(10.**PH)+DKN 00007470
      GLAM1=GLAM1*TCONV 00007480
      GLAM2=GLAM2*TCONV 00007490
      GLAM(J)=(GLAM1*THETA+GLAM2*DKD*RHOB)/(THETA+DKD*RHOB) 00007500
      RETARD(J)=1.+RHOB*DKD/THETA 00007510
149 CONTINUE 00007520
129 CONTINUE 00007530
22 FORMAT(//10X,'BULK DENSITY =',E12.4,6X,'WATER CONTENT =',E12.4, 00007540
1 6X,'TIME CONVERSION FACTOR =',E12.4//10X, 00007550
2 'LIST OF CHEMICAL PROPERTIES OF SOLUTE SPECIES'// 00007560
3 5X,'SPECIES$',7X,'CONSTANT KA ',5X,'CONSTANT KB ',5X, 00007570
4 'CONSTANT KN',5X,' PH-VALUE ',3X,'CONST. GLAM1', 00007580
5 3X,'CONST. GLAM2'// 00007590
27 FORMAT(1X,15,6X,6(E14.4,3X)) 00007600
34 FORMAT(//10X,'LIST OF RETARDATION VALUES'//) 00007610
C----- 00007620
      IF(NZ.EQ.0) NZ=1 00007630
C..... 00007640
C..... FOR IPSPEC = 1, READ IN VALUES OF LONGITUDINAL AND TRANSVERSE 00007650
C DISPERSIVITIES, WATER VELOCITY, AND RECHARGE PARAMETER INDEX 00007660
C----- CARD TY00007670
C 00007680
      IF(ALFAZ.LE.1.D-10) ALFAZ=ALFAT 00007690
      IF(BAQR.LE.1.D-10) BAQR=1. 00007700
      IF(NSOURC.LE.1.D-10) NSOURC=1. 00007710
      RHOB=NSOURC/BAQR 00007720

```



```

DO 1 J=1,NSIGMA
101 SD(J)=SIGMA(J)
DO 102 J=1,NLAMD
102 GLAMD(J)=GLAM(J)
DO 104 J=1,NX
104 XD(J)=XX(J)
IF(IRCHGE.EQ.0) GO TO 49
48 CONTINUE
C.....
DO 69 J=1,NLAMD
RFAC=1./(THETA*RETARD(J))
69 GLAM(J)=GLAM(J)*RFAC
49 CONTINUE
1032 FORMAT(/10X,'SOLUTE VELOCITY'='E12.3)
1034 FORMAT(/10X,'GROUNDWATER SEEPAGE VELOCITY'='E12.3)
103 FORMAT(/10X,' LONGITUDINAL DISPERSIVITY'='E12.3//
1 10X,' TRANSVERSE DISPERSIVITY'='E12.3//
1 10X,' VERTICAL DISPERSIVITY'='E12.3//
1 10X,' AQUIFER THICKNESS'='E12.3//
1 10X,' SOURCE THICKNESS'='E12.3//
1 10X,' PENETRATION RATIO'='E12.3/)
20 CONTINUE
C.....
C-----
IPRCHK=1
C-----
DO 40 ISIG=1,NSIGMA
S=SIGMA(ISIG)
C
DO 36 ILAM=1,NLAMD
C
J=ILAM
IF(ISPEC.EQ.0) RETARD(J)=1.
IF(ISPEC.EQ.1) KRD=GLAM(ILAM)
VX=VS/RETARD(J)
DX=ALFAL*VX
DY=ALFAT*VX
DZ=ALFAZ*VX
DO 36 IX=1,NX
XP=XX(IX)
IF(ISPEC.NE.1) KRD=GLAMD(ILAM)/XP
KRDSTO=KRD
C..... FULL PENETRATION EVALUATION
DELCCP=0.
CALL GWP2D0(XP,S,DX,DY,VX,KRD,CCF)
IF(DABS(CCF).LT.1.D-20) CCF=1.D-20
C..... PARTIAL PENETRATION EFFECT
DO 254 IZ=1,NZ
ZP=ZZ(IZ)
IF(XP.LE.0.01) GO TO 409
IF(HONB.LT.0.9900) CALL GWP3D0(XP,ZP,S,DX,DY,DZ,VX,KRDSTO,
1 BARFR,HONB,DELCCP,NKEEP)
CD(IX,ILAM,ISIG)=CCF*HONB+DELCCP
DLFAC(IX,ILAM,ISIG)=HONB+DELCCP/CCF
CPF(IX,ILAM,ISIG)=CCF
GO TO 45
409 CONTINUE
NKEEP=0
CCF=1.0
CD(IX,ILAM,ISIG)=1.
DLFAC(IX,ILAM,ISIG)=1.

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CARD

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CPF(IX,ILAM,ISIG)=1.
IF(ZP.LT.HSOURC) GO TO 45
DLFAC(IX,ILAM,ISIG)=0.
CPF(IX,ILAM,ISIG)=0.
45 CONTINUE
C
C code added 10/07/85 by R. Moon
C
SMIN = (Q * AM * 70.000 * ADI * 10.00-04) / CD(IX,ILAM,ISIG)
TAU = (XX(1) * ( 1.000 + ( DKOC * FOC * RHOB ) / THETA ))
      / VM(1)
SMTOT = (Q * AM * TAU * ADI * 10.00-04) / CD(IX,ILAM,ISIG)
LMCHIN = (Q * 70.000 * ADI) / (CD(IX,ILAM,ISIG) * DF * RHOM)
LMCTOT = (Q * TAU * ADI) / (CD(IX,ILAM,ISIG) * DF * RHOM)
C
IF(IPRCHK.NE.0) WRITE(1,4993)CD(IX,ILAM,ISIG),
      LMCHIN
4993 FORMAT (F10.7,1X,G11.5)
983  FORMAT(//10X,'PRINT CHECK:XP-ZP-DX-DY-DZ-KRD-CD-CPF-DLFAC',
      1 '- NKEEP'/)
1093 FORMAT(1X,114(' '))
993 FORMAT(9E12.4,I7)
254 CONTINUE
36 CONTINUE
38 CONTINUE
40 CONTINUE
CALL OUTPUT
100 CONTINUE
C----- FORMATS
1 FORMAT(16I5)
11 FORMAT(20A4)
29 FORMAT(3E10.3,15,3E10.3)
3 FORMAT(//10X,'NUMBER OF PROBLEMS TO BE SOLVED =',IS//)
13 FORMAT(//10X,'PROBLEM NUMBER =',IS//)
1 10X,'NUMBER OF GAUSS POINTS FOR NUMERICAL INTEGRATION =',IS/)
23 FORMAT(/10X,'PARAMETER SPECIFICATION INDEX =',IS//10X,
1 'NUMBER OF STANDARD DEVIATION VALUES =',IS//10X,
2 'NUMBER OF DECAY COEFFICIENT VALUES =',IS//10X,
3 'NUMBER OF X-COORDINATE VALUES =',IS//10X,
4 'NUMBER OF Z-COORDINATE VALUES =',IS)
21 FORMAT(6E10.3)
33 FORMAT(//10X,'LIST OF STANDARD DEVIATION PARAMETER VALUES'/)
43 FORMAT(6E12.3)
53 FORMAT(//10X,'LIST OF DECAY PARAMETER VALUES'/)
63 FORMAT(//10X,'LIST OF DISTANCE PARAMETER VALUES'/)
C-----
RETURN
END

SUBROUTINE OUTPUT
IMPLICIT REAL*8(A-H,O-Z)
INTEGER*2 IUNIT1,IUNIT2,IUNIT3,
      X IPSPEC,NSIGMA,NLAMD,NX,NVW,IPCAL,IRCHSE,NMAX
CHARACTER MENU,ANS,TITLE*60,TERM
REAL*8 KRD,KRSTO,KRDN,LMCHIN,LMCTOT
DIMENSION RETARD(1)
COMMON/INTEG/ IPSPEC,NSIGMA,NLAMD,NX,NVW,IPCAL,IRCHSE,NMAX
COMMON/IO/ IUNIT1,IUNIT2,IUNIT3
COMMON/REALS/ SIGMA(1),RHOB,THETA,TCOM,OKA,OKB,OKN,PH,DKOC,FOC,
      X ADI(1),ALFA(1),ALFA2(1),XN(1),RCHSE(1),OKS,
      X HSOURC,BAGN,AM,ALFAZ

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COMMON/BLOCKA/CD(1,1,1),XD(1),GLAND(1),SD(1),
1      ZZ(1),GLAM(1)
COMMON/BLOCKP/CPF(1,1,1),DLFAC(1,1,1)
COMMON/NEWOUT/SHMIN,SHTOT,LNCHIN,LNCTOT,DF,RHOW,Q
NCOL=5
ITABL=0
DO 10 ISIG=1,NSIGMA
SIGD=SD(ISIG)
ILST=1
ILEND=ILST+NCOL-1
IF(ILEND.GT.NLAMDA) ILEND=NLAMDA
14 CONTINUE
ITABL=ITABL+1
IF(IPSPEC.EQ.0) GO TO 19
133 FORMAT(/12X,5('RCNGE=',E10.3,1X))
GO TO 69
19 CONTINUE
69 CONTINUE
DO 16 IX=1,NX
IF(CD(IX,ILST,ISIG).LT.1.E-7) GO TO 16
16 CONTINUE
123 FORMAT(2X,93('-')//)
39 FORMAT(///2X,'TABLE',I3,':',2X,'X VERSUS CD FOR SIGMA =',
1 E11.3//2X,93('-'))
49 FORMAT(/12X,5('LAMD=',E10.3,1X))
59 FORMAT(3X,'X-VALUE',6X,'CD-VALUE',
1 4(6X,'CD-VALUE')//2X,93('-')//)
ILST=ILEND+1
ILEND=ILST+NCOL-1
IF(ILST.GT.NLAMDA) GO TO 18
IF(ILEND.GT.NLAMDA) ILEND=NLAMDA
GO TO 14
18 CONTINUE
10 CONTINUE
C----- FORMATS
3 FORMAT(///2X,'TABLE',I3,':',2X,'XD VERSUS CD FOR SIGD =',
1 E11.3//2X,93('-'))
13 FORMAT(/12X,5('LAMD=',E10.3,2X))
113 FORMAT(3X,'XD-VALUE',6X,'CD-VALUE',4(6X,'CD-VALUE')//2X,93('-')//)
23 FORMAT(1X,E10.3,5(4X,E11.3))
RETURN
END

SUBROUTINE GMP2D0(X,S,DX,DY,VX,KRD,CCP)
IMPLICIT REAL*8(A-H,O-Z)
INTEGER*2 IUNIT1,IUNIT2,IUNIT3,
* IPSPEC,NSIGMA,NLAMDA,NX,NVH,IPCAL,IRCHGE,NMAX
REAL*8 KRD,KRDSTO,KRDN
COMMON/ID/ IUNIT1,IUNIT2,IUNIT3
COMMON/INTEG3/ IPSPEC,NSIGMA,NLAMDA,NX,NVH,IPCAL,IRCHGE,NMAX
DIMENSION Z(256),M(256)
TLRNC=1.D-06
20 FORMAT(///50X,'ERROR IN SUBROUTINE GMP2D0'//20X,
1 'THE PARAMETERS DX,DY,S MUST ALL BE GREATER THAN ZERO BUT YOU',
2 ' SET'///40X,' DX =',E12.4,20X,' DY =',E12.4/
3 21X,' S = ',E12.4//)
IF(DX.LE.0..OR.DY.LE.0..OR.S.LE.0.) STOP
PI=3.1415926535897900
CCP=1.D0
IF(X.LE.TLRNC) RETURN
ALPHA=VX*VX/(4.D0*DX)*KRD
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A=0.00
B=1.0+20
CALL DGAUSS(A,B,Z,M)
SUM=0.00
EX0=VX=X/(2.00*DX)
DO 100 I=1,NMAX
  Z2=Z(I)*Z(I)
C ACCELERATE THE INTEGRATION BY LETTING Y=Z*Z*DSQRT(2)/S
  Z4=Z2*Z2
  EX=EX0-Z4-X*DSQRT(2.00*DY*Z4/(DX*S*S)+ALPHA/DX)
  TERM=0.00
  IF(EX.GT.-72.) TERM = Z(I)*M(I)*DEXP(EX)
100 SUM=SUM+TERM
  CCP=4.00*SUM/DSQRT(PI)
RETURN
END

```

```

SUBROUTINE GWP3DP(XP,ZP,S,DX,DY,DZ,VX,KRSTO,BAQFR,
1 HONB,DELCCP,NKEEP)
IMPLICIT REAL*8(A-H,O-Z)
INTEGER*2 IUNIT1,IUNIT2,IUNIT3,
+ IPSPEC,NSIGNA,NLAMD,NO,NVM,IPCAL,IRCNSE,NMAX
REAL*8 KRD,KRSTO,KRDN
COMMON/IO/ IUNIT1,IUNIT2,IUNIT3
COMMON/INTEG/ IPSPEC,NSIGNA,NLAMD,NO,NVM,IPCAL,IRCNSE,NMAX
PI=3.1415926535897900
NTERM=200
ERROR=1.0-4
CONST=2./PI
DZCONS=DZ*PI*PI/(BAQFR*BAQFR)
DELCCP=0.
DO 100 N=1,NTERM
  EN=N
  ARG=EN*PI*HONB
  ZPARG=EN*PI*ZP/BAQFR
  ATERM=DSIN(ARG)*DCOS(ZPARG)/EN
  KRDN=KRSTO+EN*EN*DZCONS
  CALL GWP2D0(XP,S,DX,DY,VX,KRDN,DCTERM)
  DELCCP=DELCCP+DCTERM*ATERM
  CTOL=ERROR*DABS(DELCCP)
  IF(DABS(DCTERM).LT.CTOL) GO TO 200
100 CONTINUE
200 CONTINUE
  NKEEP=N
  DELCCP=DELCCP*CONST
RETURN
END

```

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SUBROUTINE DGAUSS(A,B,Z,M)
C COMPUTE THE FIRST NMAX ROOTS AND WEIGHT FACTORS TO THE
C GAUSS-LEGENDRE QUADRATURE INTEGRATION SCHEME

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FOR THE SOLUTION OF INTEGRALS OF THE FORM

INTEGRAL OF $F(Z) \cdot dZ = \text{SUM OF } W(I) \cdot F(Z(I))$
AS $I = 1, \dots, NMAX$
INTEGRATED FROM A TO B

IMPLICIT REAL*8(A-H,O-Z)
INTEGER*2 IUNIT1,IUNIT2,IUNIT3,
* IPSPEC,NSIGMA,NLAMD,NX,NYN,IPCAL,IRCHGE,NMAX
DIMENSION W(256),Z(256)
COMMON/IO/ IUNIT1,IUNIT2,IUNIT3
COMMON/INTSS/ IPSPEC,NSIGMA,NLAMD,NX,NYN,IPCAL,IRCHGE,NMAX

A LOWER LIMIT OF INTEGRATION

B UPPER LIMIT OF INTEGRATION

Z ROOTS OF THE LEGENDRE POLYNOMIALS $P(N+1)(Z)$

W WEIGHT FACTORS FOR THE GAUSS-LEGENDRE QUADRATURE

NMAX NUMBER OF INTEGRATION POINTS
NMAX CAN ONLY HAVE THE VALUE OF EITHER 4,5,6,10,15,20,60,104,256

THE ROOTS AND WEIGHT FACTORS FOR THE NORMALIZED INTEGRAL ARE TAKEN
FROM :

APPLIED NUMERICAL METHODS

BY : B. CARNAHAN, H.A. LUTHER AND J.O. MILKES
JOHN WILEY AND SONS, INC. , 1969

AND IN ADDITION :

GAUSSIAN QUADRATURE FORMULAS

BY : A.H. STROUD AND DON SECREST
PRINTICE-HALL, INC. 1966

PROGRAMME WAS WRITTEN BY : MICHAEL J. UNOS JULY, 1976

CHECK IF A > B

IF(B.LT.A) WRITE(2,1) A,B,NMAX

1 FORMAT(////////,50X,'***** ERROR IN SUBROUTINE DGAUSS *****',//,00010630
2 ,40X,'THE FOLLOWING VALUES WERE SPECIFIED',//,30X,'LOWER LIMIT 00010640
3F INTEGRATION A = ',D20.12,/,30X,'UPPER LIMIT OF INTEGRATION 00010650
4 = ',D20.12,/,17X,'NUMBER OF GAUSSIAN INTEGRATION POINTS NMAX = '00010660
5, 15,/,35X,'YOU VIOLATED THE CONDITION THAT LIMIT B MUST BE GREAT00010670
6ER OR EQUAL THEN LIMIT A',//,30X,'PROGRAM SHALL STOP AFTER FORMAT00010680
7 1 IN SUBROUTINE DGAUSS')
IF(B.LT.A) STOP

NORMALIZED ROOTS AND WEIGHTS

IF(NMAX.NE.4) GO TO 50
Z(1)=.339981043584656200
W(1)=.652145154862246100
Z(2)=.731781136475910660
W(2)=.347854845137753900

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      M(3)=.347854845137453800
      GO TO 10000
50    IF(NMAX.NE.5) GO TO 60
      Z(1)=0.
      Z(2)=.538469310105683000
      Z(4)=.906179845938663900
      M(1)=.568888888888888800
      M(2)=.478628670499366400
      M(4)=.236926885056189000
      GO TO 10000
60    IF(NMAX.NE.6) GO TO 100
      Z(1)=.238619186083196900
      Z(3)=.661209386466264500
      Z(5)=.932469514203152000
      M(1)=.467913934572691000
      M(3)=.360761573048138600
      M(5)=.171324492379170300
      GO TO 10000
100   IF(NMAX.NE.10) GO TO 150
      Z(1)=.14887433898163100
      Z(3)=.43339539412924700
      Z(5)=.67940956829902400
      Z(7)=.86306336668898500
      Z(9)=.97390652851717200
      M(1)=.29552422471475300
      M(3)=.26926671930999600
      M(5)=.21908636251598200
      M(7)=.14945134915058100
      M(9)=.06667134438868800
      GO TO 10000
150   IF(NMAX.NE.15) GO TO 200
      Z(1)=0.
      Z(2)=.20119409399743500
      Z(4)=.39415134707756300
      Z(6)=.57097217268853900
      Z(8)=.72441773136017000
      Z(10)=.84820658341042700
      Z(12)=.93727339240070600
      Z(14)=.98799251802048500
      M(1)=.20257824192556100
      M(2)=.19843148532711100
      M(4)=.18616100011556200
      M(6)=.16626920581699400
      M(8)=.13957067792615400
      M(10)=.10715922046717200
      M(12)=.07036604748810800
      M(14)=.03075324199611700
      GO TO 10000
200   IF(NMAX.NE.20) GO TO 600
      Z(1)=.76526521133497330-01
      Z(3)=.227785851141645000
      Z(5)=.373706088715419500
      Z(7)=.510867001950827000
      Z(9)=.636053680726515000
      Z(11)=.746331906460150700
      Z(13)=.839116971822218800
      Z(15)=.912234428251325900
      Z(17)=.963971927277913700
      Z(19)=.993128599105099900
      M(1)=.122783387138728800
      M(3)=.149172986472683700

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600

M(5)=.142096109318382000
M(7)=.131688638449176600
M(9)=.118194531961518400
M(11)=.101930119817240400
M(13)=.83276741576704740-01
M(15)=.62672048334109060-01
M(17)=.40601429800386940-01
M(19)=.17614007139182110-01
GO TO 10000
IF(NMAX.NE.60) GO TO 1040
Z(1)=.85959772301247790-01
Z(3)=.77809333949536560-01
Z(5)=.129449135396945000
Z(7)=.180739964873425400
Z(9)=.231543551376029300
Z(11)=.281722937423261600
Z(13)=.331142848268448100
Z(15)=.379670056576797900
Z(17)=.427173741583078300
Z(19)=.473525841761707100
Z(21)=.518601400058569700
Z(23)=.562278900753944500
Z(25)=.604440597048510300
Z(27)=.644972828489477800
Z(29)=.683766327381355400
Z(31)=.720716813355730300
Z(33)=.755723775306885600
Z(35)=.788693739932264800
Z(37)=.819537526162145700
Z(39)=.848171984785929600
Z(41)=.874519922644898300
Z(43)=.898510310810045900
Z(45)=.920078476177627800
Z(47)=.939166276116423200
Z(49)=.955722255839996100
Z(51)=.969701788765052700
Z(53)=.981067201752598100
Z(55)=.98978789522221700
Z(57)=.995840525118838100
Z(59)=.999210123227436000
M(1)=.51907877631220630-01
M(3)=.51767943174910180-01
M(5)=.51488451500988930-01
M(7)=.51070156069855620-01
M(9)=.50514184532509370-01
M(11)=.49822035690550160-01
M(13)=.48995875455756830-01
M(15)=.48037031819971180-01
M(17)=.46948988848912200-01
M(19)=.45734379716114480-01
M(21)=.44396478795787110-01
M(23)=.42938892835935640-01
M(25)=.41365551235584780-01
M(27)=.39680695452380790-01
M(29)=.37888867869243440-01
M(31)=.35994898051084800-01
M(33)=.34003892724946420-01
M(35)=.31921219019296320-01
M(37)=.29752491500708940-01
M(39)=.27583384728892740-01
M(41)=.25188477621821240-01

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W(43)=.2278951694399781D-01
W(45)=.2033712072945728D-01
W(47)=.1782990101420772D-01
W(49)=.1527461859678479D-01
W(51)=.1267816647681596D-01
W(53)=.1004755718220798D-01
W(55)=.7389931163345455D-02
W(57)=.4712729926953568D-02
W(59)=.2026811968873758D-02
GO TO 10800

1040 IF(NMAX.NE.104) GO TO 2560

Z(1)=.1503080570428580D-01
Z(3)=.4507883345537786D-01
Z(5)=.7508612251067031D-01
Z(7)=.1058255346478664D0
Z(9)=.1348700729684854D0
Z(11)=.1645927063496751D0
Z(13)=.1941665938185881D0
Z(15)=.22356800888272125D0
Z(17)=.2527613834087209D0
Z(19)=.2817293321725080D0
Z(21)=.3104426761792209D0
Z(23)=.3388754665692306D0
Z(25)=.3670020080281650D0
Z(27)=.3947968820053119D0
Z(29)=.4222349696849036D0
Z(31)=.4492914746865266D0
Z(33)=.4759419454741398D0
Z(35)=.5021622974534582D0
Z(37)=.5279288347376772D0
Z(39)=.5532182718620344D0
Z(41)=.5780077533275774D0
Z(43)=.6022748772554004D0
Z(45)=.6259977126325182D0
Z(47)=.6491548206311185D0
Z(49)=.6717252736833362D0
Z(51)=.6936886743939371D0
Z(53)=.7150251739739257D0
Z(55)=.7357154981783885D0
Z(57)=.7557489247321753D0
Z(59)=.7750833802283033D0
Z(61)=.7937253764827685D0
Z(63)=.8116500663320458D0
Z(65)=.8288412508583829D0
Z(67)=.8458283394829362D0
Z(69)=.8609616367385050D0
Z(71)=.8758618102342710D0
Z(73)=.8899784489254110D0
Z(75)=.9032748025512220D0
Z(77)=.9157628477059157D0
Z(79)=.9274232987070844D0
Z(81)=.9382456177991042D0
Z(83)=.9482200246834838D0
Z(85)=.9573375053700733D0
Z(87)=.9655898203461570D0
Z(89)=.9729695120673583D0
Z(91)=.9794699117905919D0
Z(93)=.9850851458144620D0
Z(95)=.9898101513367139D0
Z(97)=.9936731714868389D0

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00012590

Z(101)=-.998605162651977300
 Z(103)=-.999735221876088200
 M(1) = .3005934726091462D-01
 M(3) = .3003218199259360D-01
 M(5) = .2997787600578057D-01
 M(7) = .2989647837794739D-01
 M(9) = .2978806266985445D-01
 M(11) = .2965272685988227D-01
 M(13) = .2949059325146727D-01
 M(15) = .2930180837059142D-01
 M(17) = .2908654282535595D-01
 M(19) = .2884499115580049D-01
 M(21) = .2857737165729427D-01
 M(23) = .2828392618325631D-01
 M(25) = .2796491992658968D-01
 M(27) = .2762064118802044D-01
 M(29) = .2725140107556215D-01
 M(31) = .2685753330333987D-01
 M(33) = .2643939381802810D-01
 M(35) = .2599736847717519D-01
 M(37) = .2553183277970497D-01
 M(39) = .2504323142490426D-01
 M(41) = .2453199797222264D-01
 M(43) = .2399859443422828D-01
 M(45) = .2344350285908004D-01
 M(47) = .2286722489489493D-01
 M(49) = .2227028133640237D-01
 M(51) = .2165321165429792D-01
 M(53) = .2101657358772073D-01
 M(55) = .2036894224029680D-01
 M(57) = .1968691036020432D-01
 M(59) = .1899808780473277D-01
 M(61) = .1828489738982289D-01
 M(63) = .1756858224588301D-01
 M(65) = .1681919723481482D-01
 M(67) = .1606261236555251D-01
 M(69) = .1529151138869267D-01
 M(71) = .1450659114276789D-01
 M(73) = .1378856108396590D-01
 M(75) = .1289814216552733D-01
 M(77) = .1207686702671645D-01
 M(79) = .1124307852416543D-01
 M(81) = .1039992946259468D-01
 M(83) = .9547381838326911D-02
 M(85) = .8686206157923711D-02
 M(87) = .7817180756425343D-02
 M(89) = .6941091125180975D-02
 M(91) = .6058729274750651D-02
 M(93) = .5170893207327752D-02
 M(95) = .4278386752472274D-02
 M(97) = .3382020818687030D-02
 M(99) = .2482621802176784D-02
 M(101) = .1581095291194842D-02
 M(103) = .6794761824845529D-03

GO TO 10000

2560 IF(NMAX.NE.256) GO TO 3000

Z(1)=-.6123912375189529D-02
 Z(3)=-.1837081847881366D-01
 Z(5)=-.3061497697799728D-01
 Z(7)=-.4285245887383788D-01
 Z(9)=-.5588768884946338D-01

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Z(13)=.67312521165716400-01
 Z(13)=.79527289100232960-01
 Z(15)=.91730127163519550-01
 Z(17)=.103919204810509400
 Z(19)=.116092693560332800
 Z(21)=.128248767270607000
 Z(23)=.140305602411375800
 Z(25)=.152501378338656300
 Z(27)=.164894277547553800
 Z(29)=.176662486044901900
 Z(31)=.188704193421308000
 Z(33)=.200717593323126600
 Z(35)=.212700883622625900
 Z(37)=.224652266709131900
 Z(39)=.236569949750204000
 Z(41)=.2484521458001056600
 Z(43)=.260297069991942500
 Z(45)=.272102947076336600
 Z(47)=.283868007657081700
 Z(49)=.295590484460138600
 Z(51)=.307268619799319000
 Z(53)=.318900661840106200
 Z(55)=.330484865662416900
 Z(57)=.342019493522371600
 Z(59)=.353502815112969900
 Z(61)=.364933107823654800
 Z(63)=.376388656998716300
 Z(65)=.387627756194518500
 Z(67)=.398888707435459100
 Z(69)=.410089821468716500
 Z(71)=.421229418017623800
 Z(73)=.432305826833741300
 Z(75)=.443317383947527300
 Z(77)=.454262439917589900
 Z(79)=.465139352078479300
 Z(81)=.475946488786983300
 Z(83)=.486682228866898300
 Z(85)=.497344961852181400
 Z(87)=.507933088228616800
 Z(89)=.518445019673674400
 Z(91)=.528879179294822200
 Z(93)=.539234801866059100
 Z(95)=.549507934862718500
 Z(97)=.559699434694481100
 Z(99)=.569806974936568700
 Z(101)=.579829038559082900
 Z(103)=.589764122184454300
 Z(105)=.599610735362968300
 Z(107)=.609367401096333900
 Z(109)=.619032655759261200
 Z(111)=.628605049469014900
 Z(113)=.638083146272911300
 Z(115)=.647465524363724800
 Z(117)=.656750776292973200
 Z(119)=.665937509182848500
 Z(121)=.675024344931162700
 Z(123)=.684009920426075900
 Z(125)=.692892887742576900
 Z(127)=.7016711915349695100
 Z(129)=.7103412893483971400

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Z(133)=.727372259649652100
Z(135)=.735722512885917800
Z(137)=.743962400549111500
Z(139)=.752090686575492000
Z(141)=.760106151642655400
Z(143)=.768007593352445600
Z(145)=.775793826411325700
Z(147)=.783463682808183800
Z(149)=.791016011989545900
Z(151)=.798449681032170700
Z(153)=.805763574012998600
Z(155)=.812956596176431800
Z(157)=.820027666098917800
Z(159)=.826975723850812500
Z(161)=.833799727155504800
Z(163)=.840498652345762700
Z(165)=.847071494517296200
Z(167)=.853517267679502900
Z(169)=.859835004903376300
Z(171)=.866023758466554500
Z(173)=.872082599995488200
Z(175)=.878010620604706500
Z(177)=.883806931033158200
Z(179)=.889470661777610800
Z(181)=.895000963223084500
Z(183)=.900397005770303500
Z(185)=.905657979960144600
Z(187)=.910783096595065800
Z(189)=.915771586857490300
Z(191)=.920622702425146400
Z(193)=.925335715583316200
Z(195)=.929909919334085400
Z(197)=.934344627502003000
Z(199)=.938639174837814800
Z(201)=.942792917117462400
Z(203)=.946805231239127400
Z(205)=.950675515316628200
Z(207)=.954403188769716200
Z(209)=.957987692411178100
Z(211)=.961428488530732100
Z(213)=.964725060975706400
Z(215)=.967876915228489400
Z(217)=.970883578480743000
Z(219)=.973744599704378400
Z(221)=.976459549719234100
Z(223)=.979028021257622000
Z(225)=.981449629025464400
Z(227)=.983724089760315400
Z(229)=.985880822286125900
Z(231)=.987829747564860600
Z(233)=.989660488745065200
Z(235)=.991342771207583000
Z(237)=.992876342608822100
Z(239)=.994260972922409600
Z(241)=.995496454481096300
Z(243)=.996582602023381500
Z(245)=.997519252756720800
Z(247)=.998306266473006400
Z(249)=.998943525047400800
Z(251)=.999338515242721500
Z(253)=.999786437489283100

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Z(255)=.9999560500189922D0
 M(1)=.1224767164028975D-01
 M(3)=.1224583436974792D-01
 M(5)=.1224216010427280D-01
 M(7)=.1223664939504015D-01
 M(9)=.1222930306871027D-01
 M(11)=.1222012222730396D-01
 M(13)=.1220910824803724D-01
 M(15)=.1219626278311471D-01
 M(17)=.1218158775948177D-01
 M(19)=.1216508537853580D-01
 M(21)=.1214675811579448D-01
 M(23)=.1212660872052732D-01
 M(25)=.1210464021534046D-01
 M(27)=.1208085589572454D-01
 M(29)=.1205525932956014D-01
 M(31)=.1202785435658257D-01
 M(33)=.1199844508780581D-01
 M(35)=.1196763598490589D-01
 M(37)=.1193483148956386D-01
 M(39)=.1190023667276648D-01
 M(41)=.1186385673407107D-01
 M(43)=.1182369710082397D-01
 M(45)=.1178576349734342D-01
 M(47)=.1174486191406055D-01
 M(49)=.1170059860662074D-01
 M(51)=.1165538009494524D-01
 M(53)=.1160841316225310D-01
 M(55)=.1155970485404363D-01
 M(57)=.1150926247703949D-01
 M(59)=.1145709359809063D-01
 M(61)=.1140320604303918D-01
 M(63)=.1134768789534549D-01
 M(65)=.1129030749587550D-01
 M(67)=.1123131343964964D-01
 M(69)=.1117063457655344D-01
 M(71)=.1110828000900984D-01
 M(73)=.1104425909081390D-01
 M(75)=.1097858142572957D-01
 M(77)=.1091125686604983D-01
 M(79)=.1084229551111479D-01
 M(81)=.1077170770580462D-01
 M(83)=.1069950403897978D-01
 M(85)=.1062569534189656D-01
 M(87)=.1055029268658148D-01
 M(89)=.1047330738417048D-01
 M(91)=.1039475098321172D-01
 M(93)=.1031463526793401D-01
 M(95)=.1023297225647821D-01
 M(97)=.1014977419909486D-01
 M(99)=.1006505357630638D-01
 M(101)=.9978823097034910D-02
 M(103)=.9891095696695828D-02
 M(105)=.9801884535257327D-02
 M(107)=.9711202995266279D-02
 M(109)=.9619064679840727D-02
 M(111)=.9525483410629284D-02
 M(113)=.9430473225737752D-02
 M(115)=.9335049377623587D-02
 M(117)=.9240229377623587D-02
 M(119)=.9145912728488888D-02

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W(121)=.90364315486628730-02
W(123)=.89344947837582070-02
W(125)=.88312177572487500-02
W(127)=.87266159616988070-02
W(129)=.86207050884010140-02
W(131)=.85135010250224900-02
W(133)=.84050198532215350-02
W(135)=.82952778462352250-02
W(137)=.81842914664382690-02
W(139)=.80720773628734990-02
W(141)=.79586523687843480-02
W(143)=.78440334989397110-02
W(145)=.77282379473815550-02
W(147)=.76112830845454590-02
W(149)=.74931864548058830-02
W(151)=.73739657738123440-02
W(153)=.72536389258339130-02
W(155)=.71322239610753980-02
W(157)=.70097390929698220-02
W(159)=.68862826954463280-02
W(161)=.67616333001737980-02
W(163)=.66360495937810650-02
W(165)=.65094784150536600-02
W(167)=.63819147521078880-02
W(169)=.62534017395424010-02
W(171)=.61239506555679320-02
W(173)=.59935809191153380-02
W(175)=.58623120869226530-02
W(177)=.57301638506814370-02
W(179)=.55971560336829180-02
W(181)=.54633085806443100-02
W(183)=.53286415939159300-02
W(185)=.51931752508492880-02
W(187)=.50569298807848420-02
W(189)=.49199259218138450-02
W(191)=.47821839258926910-02
W(193)=.46437245556800600-02
W(195)=.45045685814478970-02
W(197)=.43647348779680540-02
W(199)=.42242584213815360-02
W(201)=.40831302860526480-02
W(203)=.39413976414088330-02
W(205)=.37990737487662570-02
W(207)=.36561799581425020-02
W(209)=.35127377058563070-02
W(211)=.33687685873155510-02
W(213)=.32242939617941980-02
W(215)=.30793387411993370-02
W(217)=.29339155908297160-02
W(219)=.27880553253277060-02
W(221)=.26417768254274900-02
W(223)=.24951020347037060-02
W(225)=.23480529363273120-02
W(227)=.22006516498399100-02
W(229)=.20529202279661430-02
W(231)=.19048808534997180-02
W(233)=.17565557363307290-02
W(235)=.16079671307493270-02
W(237)=.14591373333107330-02
W(239)=.13108886813015940-02
W(241)=.11608418578677240-02

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W(243)=.1011424393208440D-02      00015660
W(245)=.0610537014200890D-03      00015670
W(247)=.7121541634733206D-03      00015680
W(249)=.5623409540314090D-03      00015690
W(251)=.4124632544261763D-03      00015700
W(253)=.2625349442964459D-03      00015710
W(255)=.1127890178222721D-03      00015720
GO TO 10000                          00015730
3000  CONTINUE                        00015740
      WRITE(2,4000)NMAX               00015750
4000  FORMAT(////,40X,'***** ERROR FLAG FROM SUBROUTINE DGAUSS *****',00015760
      2//,20X,'SUBROUTINE DGAUSS DOES NOT CONTAIN THE NMAX= ',I3, 00015770
      0' ROOTS AND WEIGHT FACTORS', 00015780
      3      /, 20X,'FOR THE GAUSS-LEGENDRE QUADRATURE INTEG00015790
      4RATION SCHEME, THEY MUST BE ADDED ',/, 30X,'PROGRAM SHALL END AFTE00015800
      5R FORMAT 4000 IN SUBROUTINE DGAUSS')
      STOP                             00015810
10000 CONTINUE                        00015820
C                                     00015830
C      CONSTRUCT THE REMAINING COEFFICIENTS FROM THE GIVEN ONES 00015840
C                                     00015850
      N0=0                             00015860
      NF=NMAX/2                         00015870
      IF(Z(1).EQ.0.) N0=1              00015880
      NJ=N0                             00015890
      DO 10100 J=1,NF                  00015900
      NJ=NJ+2                           00015910
      Z(NJ)=-Z(NJ-1)                   00015920
10100  W(NJ)=W(NJ-1)                   00015930
C                                     00015940
C      CHECK IF LOWER AND UPPER INTEGRATION LIMITS ARE FINITE 00015950
C      OR INFINITE                    00015960
C      IF(A.LE.-1.D+10) GO TO 30000   00015970
C      IF(B.GE.1.D+10) GO TO 50000   00015980
C                                     00015990
C      ADJUST THE COEFFICIENTS TO ACCOUNT FOR THE NON-NORMALIZED 00016000
C      LIMITS OF INTEGRATION          00016010
C      WHERE                          00016020
C      W(I)      W(I)=(B-A)/2          00016030
C      Z(I)      (Z(I)=(B-A)+B+A)/2    00016040
C                                     00016050
C      WHERE W(I) AND Z(I) ARE THE NORMALIZED 00016060
C      GAUSSIAN HEIGHTS AND ROOTS        00016070
C      SEE PAGE 104 AND EQ 2.98 IN THE " APPLIED NUMERICAL METHODS" 00016080
C                                     00016090
C      DO 20000 I=1,NMAX               00016100
C      W(I)=W(I)*(B-A)/2.D0             00016110
C      Z(I)=(Z(I)*(B-A)+B+A)/2.D0       00016120
20000  Z(I)=(Z(I)*(B-A)+B+A)/2.D0       00016130
      RETURN                            00016140
30000  CONTINUE                        00016150
C                                     00016160
C      SPECIAL CASE WHEN A --> - INFINITY 00016170
C                                     00016180
C      DO 40000 I=1,NMAX               00016190
C      W(I)=W(I)/(Z(I)+1.D0)            00016200
40000  Z(I)=B+DLOG((Z(I)+1.D0)/2.D0)    00016210
C      CHECK IF THIS INTEGRAL HAS INFINITE UPPER AND LOWER LIMITS 00016220
C      OF INTEGRATION                  00016230
C      IF(B.GE.1.D+10) GO TO 80000     00016240
      RETURN                            00016250

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50000	CONTINUE	00016270
C		00016280
C	SPECIAL CASE WHEN B--> INFINITY	00016290
C		00016300
	DO 60000 I=1,NMAX	00016310
	W(I)=W(I)/(Z(I)+1.00)	00016320
60000	Z(I)=A+DLOG(2.00/(Z(I)+1.00))	00016330
C	CHECK IF ONE IS TRYING TO SOLVE THE INFINITE INTEGRAL	00016340
	IF(A.LE.-1.0+10) GO TO 80000	00016350
	RETURN	00016360
80000	CONTINUE	00016370
	II=0	00016380
C		00016390
C	IF WE WANT TO SOLVE A PROBLEM WITH A--> - INFINITY AND	00016400
C	B --> + INFINITY, THEN BREAK THE INTEGRATION INTO TWO	00016410
C	SEMI-INFINITE INTEGRALS	00016420
C	THUS : $F(X)DX (-INF,+INF) = F(X)DX (-INF,0) + F(X)DX (0,+INF)$	00016430
C		00016440
	WRITE(2, 90000) A,B,NMAX	00016450
90000	FORMAT(//////////,30X,'***** ERROR IN SUBROUTINE DGAUSS *****')	00016460
	2',//, 30X,'DGAUSS HAS CALLED WITH A = ',D20.12,/, 54X,'B = ',	00016470
	3020.12,/, 51X,'NMAX= ', I3,///,10X,'THIS SUBROUTINE CAN NOT HANDLE	00016480
	4E AN INTEGRAL WHERE BOTH THE UPPER AND LOWER LIMITS OF INTEGRATION	00016490
	5 ARE INFINITE', ///, 20X,'WHAT TO DO ...DEFINE A NEW PROBLEM SUCH	00016500
	6THAT',//,20X, ' $F(X)DX (-INF,+INF) = F(X)DX (-INF,0) + F(X)$	00016510
	7DX (0,+INF) WHERE INF = 1.0+20 ',//,20X,'PROGRAM SHALL END AFTE	00016520
	8R FORMAT 90000 IN SUBROUTINE DGAUSS')	00016530
	IF(II.EQ.0) STOP	00016540
	RETURN	00016550
	END	00016560

