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ORP/CSD-76-1

A STATISTICAL ANALYSIS OF THE PROJECTED PERFORMANCE OF MULTI-UNIT REACTOR SITES

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U.S. ENVIRONMENTAL PROTECTION AGENCY
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WASHINGTON, D.C. 20460

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Byron M. Bunger
Mary K. Barrick

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PREFACE

The emphasis when performing this research was the application of the formal procedures of statistics to the multiple unit reactor siting problem. Expertise in the technology of light water reactors was necessary to the correct application of the statistics used here. James M. Gruhlke and James W. Phillips of the Technical Assessment Division very helpfully provided this expertise. They also devoted a great amount of time and effort to the collection and assessment of the data used here and to the criticism of earlier drafts of this report.

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SECTION 1
INTRODUCTION AND SUMMARY

The objective of this investigation is the statistical analysis of the additivity of doses from multiple reactor sites. The problem is hypothetical, but is of interest because it bears directly on the question of how many reactors can be located at a single site under provisions of the Environmental Protection Agency's proposed uranium fuel cycle standard.[1,2] The specific problem is to estimate the potential dose that would result from a multi-reactor site where each reactor individually just meets the provisions of Appendix I to 10 CFR Part 50.[3]

A basic premise underlying this analysis is that the maximum possible dose to hypothetical receptor under the provisions of Appendix I is allowed to exceed the 5 mrem per year design objective whole body dose only under temporary and unusual circumstances.[3] It is assumed that one may assign a reasonable probability that this dose is exceeded for any single reactor. For example, it is assumed that the 0.25% fuel failure assumption commonly used, until recently, as a design basis for pressurized water reactors is not exceeded, on the basis of current operating history, more than 5% of the time.[4] The implication of this assumption is that current operating data is representitive of reactor operation that would exceed the Appendix I limit of 5 mrem per year no more than 5% of the time. A conservative approach is used for the purpose of this analysis; it is assumed that the effluent control system is designed so that 5 mrem per year is exceeded exactly 5% of the time.

Additionally it is assumed that when two or more reactors occupy a site, each reactor operates independently. This means that the operation or non-operation of individual reactors and the resultant dose is independent of (i.e., is not correlated with) the operation of other reactors on the site. A worst case set of assumptions is used for modeling the multiple reactor cases in order that the resultant doses be conservatively estimated. These assumptions are that all reactors on a site occupy the same point rather than being areally dispersed over the site, that the size of the site does not increase as the number of reactors occupying it increases, that no economies in received dose are to be realized from shared control measures, and that those reactors exhibiting best performance are not operated in preference for the other reactors at the site.

The data used in the analysis is taken from operating reports of single unit reactors which are in commercial operation. Separate analyses have been done for the pressurized water reactors (PWRs) and the boiling water reactors (BWRs). The data for the PWRs is monthly readings on the concentration of I-131 in the primary coolant and for BWRs is monthly readings on noble gas releases. Both sets of data are considered to be lognormally distributed. Section 2 contains an evaluation of the data and a discussion of the assumption of lognormality.

In order to use monthly emissions data from single unit reactors in an analysis of annual doses from multiple reactor sites, a method of transformation and statistical aggregation has been developed and is described in Section 3. Briefly, it is assumed that monthly readings on I-131 concentrations (PWRs) and noble gas releases (BWRs) are randomly selected from lognormal populations of monthly readings whose parameters are estimated by statistical procedures. Monthly readings are averaged to determine annual averages; then are assumed to be linearly transformed to annual dose levels for single reactors. These annual doses are then aggregated to determine the annual doses from 4, 5 and 6 reactors on a site. An important part of the solution is the premise concerning the distributions of annual doses from single units and multi-units on a site. The premise is that the true distributions are unknown, but that the Central Limit Theorem gives assurance that the true distribution is somewhere between that for the lognormal and normal distributions. Reactor down time has also been treated in two ways, with the intention that calculated estimates bracket the correct value.

An example solution is outlined in Section 4. This section also contains the complete results of the analyses. These results are summarized here.

Table 1
DOSE LEVELS (MREM/YR) THAT WILL BE
STATISFIED 95% OF THE TIME

	4 Units	5 Units	6 Units
PWR	14	17	20
BWR	15	18	21

SECTION 2
EVALUATION OF DATA

Data for Pressurized Water Reactors

The data set provides monthly readings on the concentration of I-131 in the primary coolant of seventeen pressurized water reactors (PWRs). Data is taken from the semiannual operating reports for the respective reactors. There have been over 500 months of combined operation of these seventeen reactors. For a variety of reasons, the observations for many of these months either are not available or are of limited or no use to this analysis. No observations are available for over 150 months. The first year's operation is excluded because it is not considered to be representative of typical reactor operation. Additional information is lost because, on occasion, the average of the readings for several months is provided instead of the monthly readings themselves (since averaging over several months results in the loss of information on the variation of monthly observations, these averages were not made a part of the data set analyized). In addition, the data for Ginna was excluded because it is not considered to be typical of the other reactors. The result is that the data set used for this analysis represents the concentration levels for 132 months of operation and 23 months of "down time", when the reactors are not in operation. The entire data set is listed in Table 2.

Considerable effort was devoted to the inspection of the data to determine how it could be best analyzed. A question of particular concern is whether the variation in successive monthly readings on the same fuel loading may be highly correlated. Another concern is the relative variance of observations within a fuel loading as compared to the variation between different fuel loadings. Questions of this sort can be treated by techniques of regression analysis and analysis of variance, but data limitations prevented these approaches. The reason is that there are only five reasonably complete sets of monthly readings for fuel loadings, and four of these are from the same reactor, Haddam Neck. This effectively prevents any elaborate analysis of the separate fuel loadings. A simplistic approach to displaying the information on separate fuel loadings is provided in Figure 1, which shows the mean and variance of each of the 22 separate fuel loadings. Observations for the first year's operation for each reactor are excluded. The five fuel loadings represented by only the averages of monthly readings are shown on the right hand side of the figure. Variance of monthly readings cannot be calculated for these five fuel loadings. The other 17 fuel loadings are ordered on the number of observations and plotted from left to right in the figure. Many of these fuel loadings are represented by six or fewer observations. The most

Table 2
PWR PRIMARY COOLANT I-131 CONCENTRATIONS (nCi/l)

Location	Year	January	February	March	April	May	June	July	August	September	October	November	December
Haddam Neck (7/24/67)	1967							NR	NR	NR	NR	3.9(2)	5.2(2)
	68	2.47(3)	2.6(2)	2.9(2)	4.7(2)	1.5(2)	1.8(2)	*9.0(1)	3.68(3)	3.9(2)	5.1(2)	1.3(2)	2.7(2)
	69	2.3(2)	1.0(2)	1.3(2)	1.8(2)	1.1(4)	2.9(2)	2.6(2)	2.6(2)	4.0(2)	1.31(3)	1.54(3)	9.10(3)
	70	3.78(3)	2.98(3)	1.48(3)	1.34(2)	0	0	4.65(3)	5.86(3)	1.38(4)	5.6(4)	2.79(4)	8.5(4)
	71	3.4(4)	4.53(4)	2.95(4)	1.9(4)	2.54(4)	1.74(4)	1.58(4)	1.65(4)	1.56(4)	2.13(4)	1.75(4)	1.76(4)
	72	0	2.5(4)	2.7(4)	0	0	0	6.29(3)	6.83(3)	6.24(3)	6.99(3)	6.51(3)	7.24(3)
	73	5.5(3)	6.55(3)	5.50(3)	5.19(3)	0	0	0	0	0	0	0	5.14(3)
	74	2.99(3)	5.89(3)	3.78(3)	4.30(3)	6.45(3)	3.93(3)	4.1(3)	2.69(4)	4.22(3)	4.33(4)	4.72(3)	4.03(3)
	75	4.36(3)	3.45(3)	4.69(3)	3.92(3)	0							
Ginna (11/9/69)	1969											NR	NR
	70	NR	NR	NR	1.08(6)	5.31(5)	4.7(5)	9.9(5)	7.5(5)	NR	NR	*NR	NR
	71	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR
	72	NR	NR	NR	NR	NR	NR	NR	6.1(4)	2.6(4)	8.2(4)	NR	NR
	73	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR
	74	0	0	0	1.2(4)	3.1(4)	3.8(4)	4.0(4)	3.6(4)	5.3(4)	6.3(4)	4.8(4)	8.3(4)
H.B. Robinson (9/20/70)	1970									NR	NR	NR	NR
	71	NR	NR	NR	NR	NR	NR	NR	NR	* NR	NR	NR	NR
	72	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR
	73	NR	NR	0	NR	NR	NR	NR	NR	NR	NR	NR	NR
	74	9.68(3)	6.07(3)	1.30(4)	2.48(4)	0	0	1.02(4)	1.12(4)	3.08(3)	4.75(3)	1.04(4)	2.37(3)
Point Beach 1 (11/2/70)	1970											NR	NR
	71	NR	NR	NR	NR	NR	NR	→		1.41(4)		*	←
	72	→	1.40(5)				←	→	2.16(5)	←	0	0	0
	73	NR	NR	NR	NR	NR	NR	→		6.2(4)			←
	74	→	1.05(5)		←	0	3.36(5)	→		3.26(5)			←

Table 2 (cont.)
PWR PRIMARY COOLANT I-131 CONCENTRATIONS (nCi/l)

Location	Year	January	February	March	April	May	June	July	August	September	October	November	December
Palisades	1971					NR	NR	NR	NR	NR	NR	NR	NR
(5/24/71)	72	NR	NR	NR	NR	*NR	NR	→		2.9(3)			←
	73	→		3.4(4)			←	0	0	0	0	0	0
	74	0	0	0	0	0	0	0	0	0	0	0	0
Point Beach 2	1972					NR	NR	NR	→		2.33(2)		←
(5/30/72)	73	NR	NR	NR	NR	*NR	NR	→		6.73(3)			←
	74	→		1.10(4)			←	→	7.8(3)		←	0	3.4(3)
Surry 1	1972							NR	NR	7.77(1)	0	0	6.75(2)
(7/1/72)	73	1.13(3)	1.54(3)	3.65(3)	2.31(3)	3.74(3)	2.76(3)	*3.90(3)	4.24(3)	3.05(3)	1.05(3)	2.53(3)	0
	74	0	0	7.0(2)	3.17(3)	2.06(4)	2.30(4)	1.29(4)	1.97(4)	3.68(4)	7.13(3)	0	0
	75	0	1.25(5)	7.06(4)	1.43(5)	1.11(5)	8.78(4)						
Maine Yankee	1972										NR	NR	NR
(10/23/72)	73	NR	NR	NR	3.37(2)	1.74(2)	2.3(1)	NR	NR	NR	*NR	NR	NR
	74	NR	NR	NR	NR	NR	NR	0	0	0	4.96(2)	1.27(3)	2.82(5)
Surry 2	1973			4.7(1)	5.16(2)	5.94(3)	2.94(3)	3.85(3)	3.99(3)	7.48(3)	2.16(3)	3.86(3)	4.91(3)
(3/7/73)	74	3.95(3)	3.04(3)	*1.57(3)	2.35(3)	0	0	6.31(3)	4.41(3)	4.19(3)	0	0	0
	75	2.87(3)	6.02(3)	4.87(3)	6.72(3)	0	0						
Oconee 1	1973				NR	NR	NR	NR	NR	NR	NR	NR	NR
(4/19/73)	74	NR	NR	NR	*NR	NR	NR	8.2(4)	1.1(5)	5.75(4)	8.2(4)	0	0
Indian Point 2	1973					1.7(0)	5.5(0)	1.1(2)	6.2(2)	6.8(3)	2.0(3)	0	0
(5/22/73)	74	0	0	8.4(2)	2.8(4)	*1.5(4)	6.0(4)	6.9(4)	7.6(4)	5.1(4)	5.4(4)	5.4(4)	5.1(4)
Fort Calhoun	1973								NR	NR	NR	NR	NR
(8/5/73)	74	NR	NR	NR	NR	NR	NR	1.83(2)	*3.65(3)	5.71(2)	3.41(2)	3.20(3)	1.88(2)

Table 2 (cont.)
PWR PRIMARY COOLANT I-131 CONCENTRATION (nCi/1)

Location	Year	January	February	March	April	May	June	July	August	September	October	November	December
Oconee 2	1973											NR	NR
(11/11/73)	74	NR	NR	NR	NR	NR	NR	1.7(3)	7.1(2)	9.2(3)	6.4(3)	*3.4(4)	5.4(4)
Kewaunee	1974			NR	<1.0(1)	3.1(1)	7.3(1)	3.46(2)	4.17(4)	9.92(4)	5.32(3)	1.87(4)	1.27(4)
(3/7/74)	75	9.18(3)	2.97(4)	*3.30(4)	2.16(4)	1.02(4)	8.73(3)						
Three Mile Island(6/5/1974)	1974						2.5(0)	→			1.54(4)		←
Oconee 3 (9/5/74	1973									<1.0(1)	3.0(1)	<1.0(1)	1.3(2)
Rancho Seco (9/16/74)	1974									<MDA	<MDA	<MDA	<MDA

NR = Not Reported

MDA = Minmum Detectable Activity

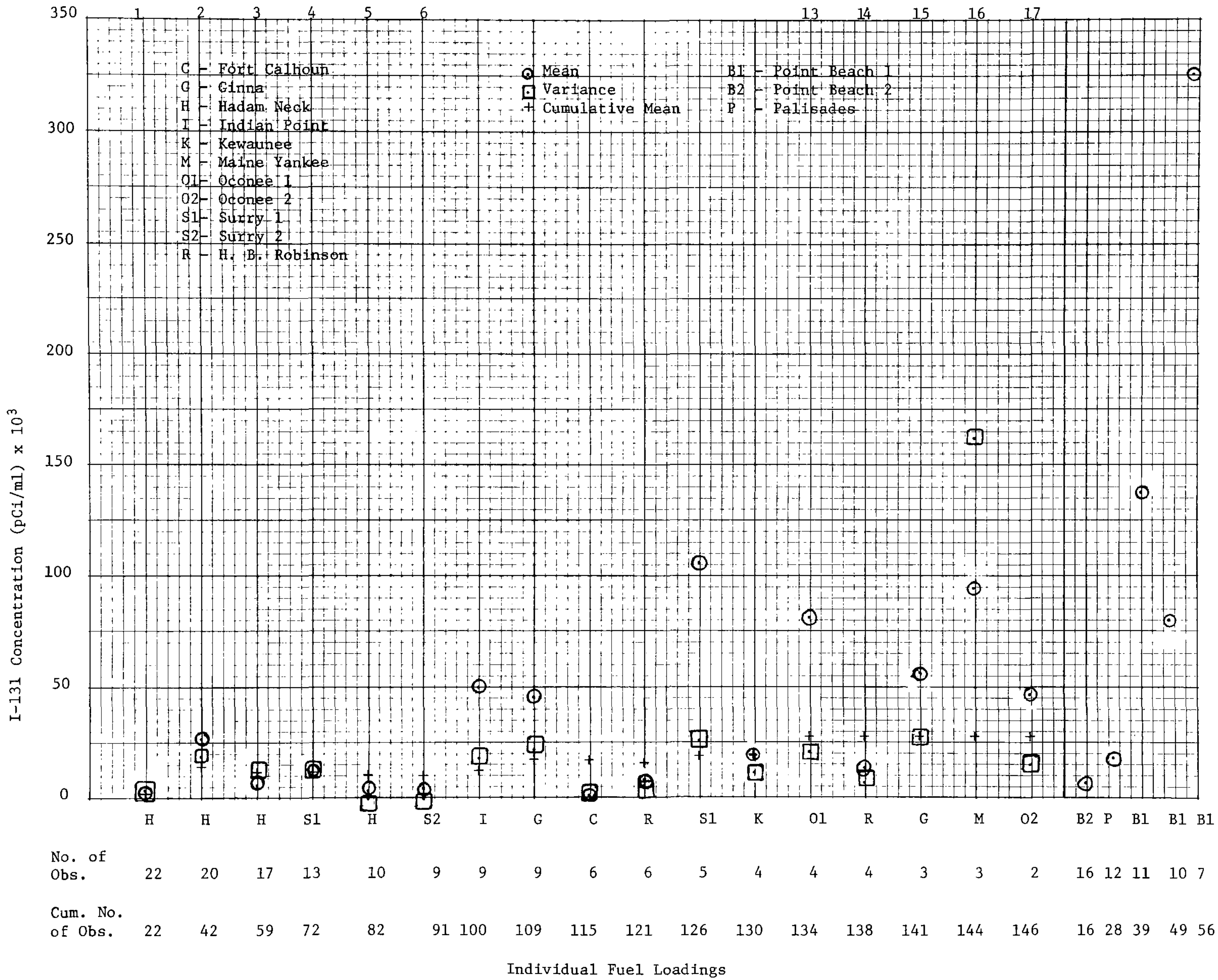
0 = Reactor not in Operation - Down Time

Numbers in parentheses represent exponents, for example: 3.9(2) = 3.9×10^2

* = Start of Second Year of Commercial Operation

→ ← = Average Concentration for Indicated Period

Figure 1
PWR PRIMARY COOLANT I-131 CONCENTRATIONS FOR INDIVIDUAL FUEL LOADINGS



conspicuous characteristic of the data apparent from this figure is that the fuel loadings with the most information have the smallest mean and variance. No completely satisfactory reason for this is readily apparent. It might be proposed that the reactors with the longest history have had time to work the "bugs" out but, this explanation is not borne out by the data. The early history of the reactors with the longest records do not show behavior similar to that seen in the reactors with relatively short histories. Differences in size of reactors does not appear to be an explanation.

A characteristic of the data that is important, but difficult to evaluate, is that Haddam Neck has the longest history of any of the reactors. Most of its data is good, so that its behavior threatens to dominate the analysis. Four of the five longest fuel loading records are from Haddam Neck.

The mean and variance of groupings of three, six and twelve consecutive monthly observations were calculated. The variance declined with increases in group size, but data was lost with each increase in group size, because some reactor histories had less than three or six observations. Therefore, the results are difficult to interpret.

The tentative conclusion drawn from the results of the grouping on three, six and twelve months as well as from the grouping on separate fuel loadings is that individual fuel loadings probably exhibit different variances. This proposition could be more fully explored if there were more complete records for the newer reactors. As things now stand, however, there is insufficient information to proceed with the analyses of separate fuel loadings.

Another approach is to consider each observation to be a member of a "grand" population of monthly observations. Although this approach may be overly simplified, it is tractable. It is the approach used in this analysis. The implications of the assumptions underlying this approach will be discussed in Section 3.

One necessary step when using this approach to the analysis is the determination of the form of distribution of the observations.

Professional opinion is that the monthly observations are lognormally distributed. Therefore the data was tested for normality and lognormality by chi-square goodness of fit. The 132 observations representing operating date were tested and found to fail the test for normality and lognormality at the 5% level of significance. However the data was found to more closely fit the lognormal than the normal distribution.

The operating data for Haddam Neck was then tested separately for lognormality. The chi-square goodness of fit test verified that the concentration levels for Haddam Neck are adequately described by a lognormal distribution (test not significant at the 5% level). This provides some evidence that the reactors with the longer operating experience do display a lognormal distribution.

Analysis was continued on the assumption that the whole data set of 132 observations is lognormally distributed. The justification is: 1) professional opinion that monthly concentrations are lognormally distributed and 2) the test showing that the Haddam Neck data is lognormally distributed. Haddam Neck is also the best data available. The assumption that the distribution is lognormal implies the expectation that a more complete data set for the other reactors would determine a lognormal distribution for the entire set of data.

<u>Sample Estimates of Mean and Variance for PWRs</u>

The analysis to be performed here will require that the data be expressed in its original form and also in its log form. The mean and standard deviation for the 132 months of operating data, in original form, expressed in nanocuries (10^{-9} Ci) per liter are:

$$\begin{aligned} \bar{x} &= 2.108 \times 10^{4} \\ s_x &= 3.602 \times 10^{4} \end{aligned} \qquad (1)$$

The mean and standard deviation of the 132 months of operating data expressed in log form are:

$$\begin{aligned} \bar{y} &= 8.778 \\ s_y &= 1.772 \end{aligned} \qquad (2)$$

where $\bar{y} = \frac{1}{n} \sum y_i - \frac{1}{n} \sum \ln x_i$ and $s_y = \frac{1}{n-1} \sum (y_i \;\; \bar{y})^2 = \frac{1}{n-1} \sum (\ln x_i - \bar{y})^2$

<u>Data for Boiling Water Reactors</u>

Monthly releases of noble gases from single-unit Boiling Water Reactors (BWRs) form the basis for this analysis. The release data is taken from the semiannual operating reports for each reactor. The noble gases being considered are those normally reported by the utilities; namely Kr-85m, Kr-87, Kr-88, Xe-133, Xe-135 and Xe-138. Information has been obtained for the 6 reactors with at least 12 months of commercial operating experience as of December 31, 1974. The first 12 months operation have been discarded as atypical of future performance. The data set thus defined comprises 184 months of operating experience including 27 months of "down time." As with the previous analysis, these monthly releases are assumed to be representative of random monthly releases from a typical single-unit reactor currently in operation.

Although the data (stack releases reported by the utilities) may be assumed to represent current operations, there are at least two reasons why they are not as yet in the proper form for this analysis. Therefore the data requires further modifications.

The first problem involves the off-gas treatment system. The present data come from reactors with various systems for treating releases from the steam jet air ejector. In modeling releases the normal procedure is to use a source term based on a 30-minute hold-up system regardless of the system actually in use at a particular reactor. Therefore the data should be recalculated using only a 30-minute delay system.

A difficulty encountered in using the above modification is determining the fraction of a reported stack release which has passed through the hold-up system and therefore should be adjusted. To do this, the estimated gland seal system release for each isotope (taken from AEC figures[4]) has been subtracted from the stack release for that isotope and the remainder, if any, has been recalculated based on 30 minutes delay. The sum of the adjusted releases and the gland seal releases for all six isotopes constitutes the estimated stack release adjusted to 30 minutes delay.

The second problem involves the size and operating level of future reactors. These facilities are assumed to have a generating capacity of 1000 MWe (3400 MW(th)) and to operate on the average at 80% of that capacity. Current reactors are smaller and on most occasions operate at a lower percent capacity. It is also known that noble gas release levels are related to power generation. In order to use the available data, a linear relationship between releases and power generation has been assumed. With such an assumption a monthly release can be scaled easily to the equivalent release expected from any given size reactor for any specified percent capacity.

To scale the data, a monthly release (normalized to 30 minute hold-up) is first divided by the power generated in that month and then multiplied by constants to reflect a 3400 MW (th) plant operating at 80% capacity. Since the constants apply to all the data and therfore will have no bearing on the underlying distribution, they have been omitted from the scaling process. The releases will thus be in units of Curies /MWd(th). The major factor in the analysis is the relationship between the mean and variance of the underlying distribution. This relationship will be the same with or without the constant multiplier. It should be noted, however, that by not employing the constant multiplier, the transformed data will not reflect the expected absolute amount of releases and therefore should not be used in other analyses where such values are required.

The modified data set used in the analysis is shown in Table 3.

With the data set redefined so that it fits the problem to be investigated here, the first step in the analysis is to determine if the 157 data points representing actual operation are lognormally distributed. The precedure involved is a chi-quare goodness of fit test. Using this test, it can be shown that the operating data fit a lognormal distribution (test not significant at the 5% level).

<u>Sample Estimates of Mean and Variance for BWRs</u>

The mean and standard deviation for the 157 months of operating data, in the original form, expressed in curies per megawatt day thermal are:

$$\bar{x} = 2.196$$
$$s_x = 2.647 \tag{3}$$

The mean and standard deviation for the 157 months of operating data, transfered to log form, are:

$$\bar{y} = 0.140$$
$$s_y = 1.268 \tag{4}$$

Table 3

BWR NOBLE GAS RELEASES ADJUSTED FOR GLAND SEAL AND 30 Min. DELAY (Ci/MWd(th))

Location	Year	January	February	March	April	May	June	July	August	September	October	November	December
Oyster Creek	1969									→		3.81(-1)	←
	70	→		6.21(-1)			←	→		*	7.37(-1)		←
	71	→		1.65			←	1.84	3.27	3.63	0	1.18	3.81
	72	4.52	4.55	5.45	7.19	9.15	1.21	9.57(-1)	9.75(-1)	1.23	1.48	2.07	3.41
	73	2.89	6.07	1.02(1)	1.02(1)	0	1.20	1.88	2.14	1.78	1.98	1.54	1.92
	74	1.88	2.19	2.74	2.92	0	0	9.43(-1)	9.31(-1)	7.82(-1)	1.39	1.95	2.51
Nine Mile Point	1969											→5.63(-3)	←
	70	→	5.69(-3)	←	→				5.76(-2)			*	←
	71	6.02(-2)	1.16(-1)	5.27(-1)	2.83	1.51(-1)	4.03(-1)	4.42(-1)	5.56(-1)	1.98	5.95(-1)	1.33	1.47
	72	8.55(-1)	9.40(-1)	1.85	3.28	0	2.15(-1)	7.75(-1)	1.39	9.87(-1)	5.80(-1)	2.01	2.82
	73	3.03	3.24	5.86	7.34	0	4.33(-1)	6.59(-1)	6.94(-1)	6.96(-1)	8.86(-1)	1.44	1.55
	74	→	1.61	←	→	0	←	→	2.05	←	→	1.82	←
Millstone	1970											→3.41(-2)	←
	71	8.17(-2)	1.82(-1)	1.82(-1)	1.64(-1)	4.30(-1)	3.74(-1)	2.74(-1)	4.96(-1)	1.20	1.45	*1.14	1.06
	72	5.36(-1)	3.83(-1)	6.56(-1)	4.86(-1)	1.40	3.97	3.53	5.40	0	0	0	0
	73	0	0	1.93(-1)	4.70(-1)	0	0	7.59(-2)	2.02(-1)	3,52(-1)	3.79(-1)	3.46(-1)	3.55(-1)
	74	6.49(-1)	3.91(-1)	5.50(-1)	5.15(-1)	8.27(-1)	7.29(-1)	2.73	8.30	0	0	1.98	4.15

Table 3 (cont.)
BWR NOBLE GAS RELEASES ADJUSTED FOR GLAND SEAL AND 30 Min. DELAY (Ci/MWd(th))

Location	Year	January	February	March	April	May	June	July	August	September	October	November	December
Pilgrim I	1972							7.05(-3)	0	1.12(-2)	1.92(-2)	1.56(-1)	2.38(-1)
	73	2.45(-1)	2.12(-1)	2.94(-1)	4.80(-1)	5.27(-1)	3.58(-1)	*3.38(-1)	5.06(-1)	1.22	3.66(-1)	2.59(-1)	2.23(-1)
	74	0	0	0	0	0	0	3.17(-1)	4.43(-1)	2.12	2.99	6.51	1.66
Monticello	1971			→	3.18(-2)		←	3.78(-2)	4.39(-1)	5.01(-1)	4.75(-1)	5.47(-1)	0
	72	4.71(2)	5.69(-1)	*8.17(-1)	7.15(-1)	1.35	1.16	1.19	2.34	2.68	2.37	3.36	3.18
	73	2.53	1.93	0	0	3.29(-1)	5.27(-1)	8.03(-1)	1.25	1.38	2.51	5.20	4.57
	74	7.89	8.27	7.29	0	3.23	4.37	6.47	6.05	2.02(1)	7.19	2.44	8.40(-1)
Vermont Yankee	1972									7.25(-3)	2.84(-1)	8.63(-1)	1.62
	73	5.56	0	6.40(-1)	7.17(-1)	6.80(-1)	5.47(-1)	4.08(-1)	4.50(-1)	*9.85(-1)	0	1.60(-1)	2.31(-2)
	74	1.85(-2)	2.31(-1)	1.30(-1)	3.52(-1)	2.91(-1)	7.99(-2)	3.94(-1)	8.96(-2)	1.16(-1)	7.23(-1)	0	4.74(-2)

0 = Reactor Not in Operation - Down Time

Numbers in parentheses represent exponents, for example: 3.81(-1) = 3.81×10^{-1}

* = Start of Second Year of Commercial Operation

→ ← = Average Release for Indicated Period

SECTION 3
METHOD OF ANALYSIS

Analytical Solution Based on Randomly Selected Monthly Observations

The objective of this analysis is to estimate the exposure from a multi-reactor site under the restriction that each reactor, if tested individually, would just meet the Appendix I operating criteria (five millirem per year). Since this limit and the uranium fuel cycle standard are both expressed in millirem per year, the distribution of exposures must also be expressed in annual terms (i.e., millirem per year).

Based on the assumption that the 132 observations on I-131 concentrations and the 157 observations on noble gas releases are lognormally distributed, there are at least two conceivable ways to proceed to an estimate of the mean and variance of the annual exposures. The first is to use simulation techniques to determine the distribution of the annual mean concentration, then transform to annual exposure and to again simulate to determine the distribution of the mean level of exposure from multiple reactors on a site. This method is not pursued in this analysis. The second possible method, the one adopted here, is to use an analytical technique for estimating as closely as possible the distribution of the mean annual concentration levels and, after transformation to exposures, to again use an analytical technique to approximate the distribution of the exposures from a number of reactors on a site. The technique for performing this analysis is discussed in detail later in this section.

The method used here, as well as the simulation method discussed above, relies on a concept that needs special attention. Both methods assume that the annual average concentration (PWRs) and noble gas released (BWRs) are represented by the mean of individual monthly observations selected randomly from their respective populations, independently of reactor and fuel loadings. Since a number of different fuel loadings are represented in the data, this assumption is quite different from that underlying an analysis based on the separate fuel loadings, as discussed in Section 2.

Aside from the argument that the method used here is justified because it is manageable, it can also be argued that when the analysis is complete, with a model for four or more reactors on a site, the annual mean emissions level is based on the sum of 48 or more monthly observations from at least four fuel loadings. At this level of aggregation, it may make little difference whether the 48 individual observations were randomly selected or not, so long as they are representative of typical reactor behavior.

The Transformation to Exposure Levels

The Appendix I limitation and the proposed uranium fuel cycle standard are expressed in terms of radiation exposures. Therefore this analysis must assume some form of transformation from coolant concentrations (PWRs) and stack releases (BWRs) to exposure levels. Linear transformations will be assumed. For example, the distribution of radiation emissions from PWRs that cause exposures will have the same distribution as that assumed for the concentration levels (i.e. if the coolant concentration is assumed to be lognormally distributed, radiation exposures at a given location will also be lognormally distributed). The linear transformation will also transform zero observations for coolant concentration to zero levels of radiation exposures.

Reactor "Down Time"

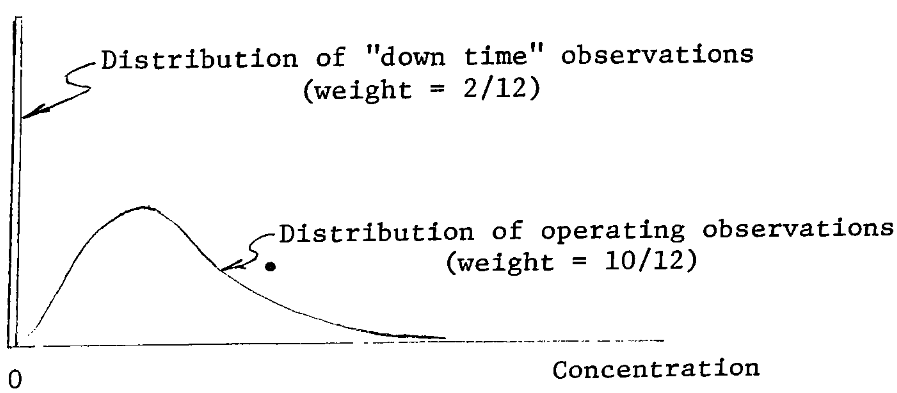
Reactor "down time" is a problem that must be given special attention. When shut down for refueling or repairs, past practice has been that I-131 concentrations in the primary coolant for PWRs have not been reported. Stack releases for BWRs are not measured during shutdown. Therefore readings of zero have been assumed for months when the reactors are out of operation. Since there are numerous months of down time in the data, and since it is to be expected that even the best future experience with reactors will still include frequent down-time periods (if for nothing more than refueling), the zero level readings should be included in the data set. The "zero" readings account for about 1/6 of each of the data sets. They occur frequently enough that they cannot be included in the data set for operating months, since the lognormal distribution does not provide for any observations at zero and provides for only a limited number at values close to zero. Therefore operating months must be handled seperately from down time. The following is a discussion of the two possible approaches to the treatment of these zero values.

The months when there is no data representing coolant concentration levels (PWRs), because of refueling or repairs, may be handled in at least two ways. The first is to assume that a zero level of exposure (after transformation from a zero level concentration in the primary coolant in PWRs, for example) is a valid representation of the true exposure level. This, of course, implies that the major source of radiation exposure is leakage from primary coolant and little is contributed by such things as wastes stored at the reactor site, or from other plant operations releasing radioactivity more or less independently of power production, such as containment pipe purge or waste gas decay tank releases. The second method is to assume that the emissions level remains relatively high during periods of refueling and repair, so that exposures are best represented by assuming that the monthly concentration levels, after transformation to exposures, stay at the same levels. This method would assume that exposures are just as high during periods when the reactor is down as when it is operating.

It is probable that neither of these methods is completely satisfactory. The first may underestimate the true exposures, especially for PWRs. The second may overestimate exposures for the BWRs, but may closely approximate them for the PWRs because it has been observed that radioactivity releases for PWRs remain quite high during shutdown periods, especially noble gas releases. Both methods will be used in this analysis, to see what impacts the different methodologies have. The first analysis will be performed assuming that exposures are zero when reactors are down for refueling or repairs. Later it will be assumed that down time is represented by the same exposure level as operating time. Each year is, therefore, assumed to be represented by twelve months of operating data. Each of these techniques will be more fully elaborated upon in Section 4, where, the calculations are discussed. With appropriate modifications, similar arguments apply to BWRs.

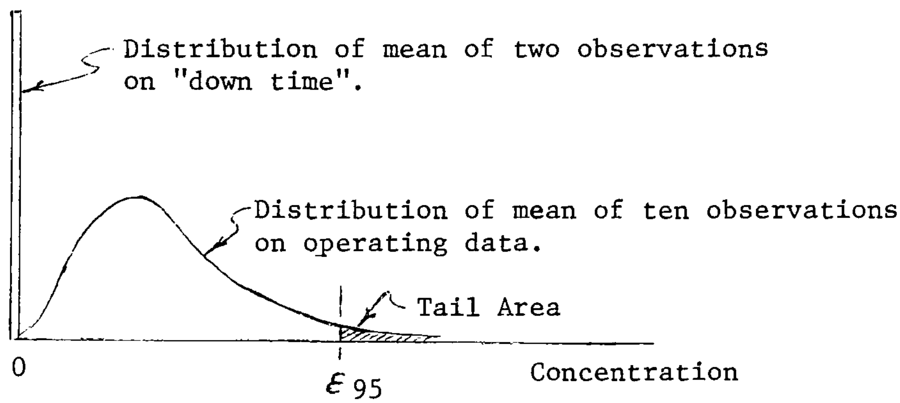
The data on which the analysis of the PWR is based has 132 months of operating data and 23 months of time of no operation because of refueling or repairs. The total months of operation are 132 + 23= 155. The 23 months of down time represent 23/155 = 14.8% of the time, which is close to 1/6 of the time, or 2 months out of each year. Therefore, it will first be assumed that there are 10 months of operation each year and 2 months of down time.

It will be assumed that the observations on operating months and on "down time" represent independent distributions, and that independent monthly observations are drawn from each of them. For each year's data set, ten observations are drawn from the operating data and two from the "down time" data. It is assumed that this holds true for every year's operation. This is, of course, a restrictive assumption, but facilitates the solution. To combine the two distributions into one creates a population that is not easily treated, for simulation techniques would be needed to estimate the distribution of a sum of observations drawn from this distribution.



The areas represented on the sketch above should be in proportion to the weight given them in drawing samples. The distribution of "down time" is a narrow vertical strip approaching zero width, but whose area is 1/6 that of the distribution of operating months. The variance of these "down time" observations is equal to zero.

Since the "down time" observations have zero variance and since the proportion of sample observations from the two distributions is constant, the variation in the operating data accounts for the entire variation observed in the yearly performance. Yearly performance is characterized by the weighted average of the mean of ten observations drawn from the distribution of operating data and the mean of two zero observations from the distribution of "down time". The following sketch represents the distributions of these two means.



Let $\mathcal{E}_{95}$ determine a tail area that represents 5% of the total area for the distribution on the mean of ten months of operation. Averages over ten months of operation that exceed $\mathcal{E}_{95}$ can be expected 5% of the time. Since the only source of variation in a years performance is in the operating data, the mean value for a years operation will be directly proportional to the average for the ten operating months in each year. Therefore, the tail area represents the probability (5%) that the annual average concentration will exceed $(10/12)\mathcal{E}_{95}$. The $(10/12)\mathcal{E}_{95}$ represents the weighted average of ten months of operation and two months of zero readings observed when the reactor is not in operation.

For the second case to be investigated, it will be assumed that reactor down time contributes as much to exposure as does operating time. This case is handled by assuming that all 12 monthly observations are drawn from the distribution of operating data. Therefore, no weighting of operating and down time months is necessary.

The analysis of the BWR case is similar. The data set represents 157 months of operation and 28 months of down time for refueling and repairs. Down time represents 15.2% of the total months of operation (27/185), approximately 1/6 of the time. Therefore, the same two cases used to investigate the PWRs are used for the BWRs, one assuming two months down time with emissions equal zero and the other assuming the two months down time having emission levels equal to those experienced when in operation.

The Basic Premise Underlying the Methodology

This analysis is based upon the use of the Central Limit Theorem, one of the fundamental theorems of statistics. Loosely expressed, this theorem states that the distribution of the sum of randomly selected observations from any population will approach normal as the number of summed observations increases. The great strength of this theorem is that there are no restrictions on the distribution from which the observations are selected, except that it have a finite variance. Based on this theorem, the sample average, which is a sum of randomly selected observations divided by the sample size, will approach a normal distribution as the sample size is increased. It is generally accepted that a sample size greater than thirty is sufficiently large to assume that the sample average is normally distributed.[5]

The data for both types of reactors (PWRs and BWRs) are monthly, and the standard against which exposures (after transformation) are to be compared is expressed in annual terms. The Central Limit Theorem cannot be exercised to claim that the annual average concentration is normally distributed, because the annual average is based on only twelve monthly readings. However, the Central Limit Theorem is still of use, for it is necessary to the method of analysis developed here. This methodology will now be described.

A sample of size one selected from a parent population will have the same distribution as the parent distribution. In most cases the mean of samples sized two and larger will take a form different from that of the parent population, and this difference will increase as the sample size increases. According to the Central Limit Theorem, the distribution of the mean will approach a normal as the sample size increases.[6] The basic premise upon which this solution is based is that the distribution of the mean of a sample of 10 to 12 observations will describe a distribution that is somewhere between that of the parent population and the normal. Even though the precise distribution the mean will follow is unknown, it must be somewhere between that of the parent distribution and that of the normal.

Specifically, the area in the right tail of the distribution of the mean is of interest. The probability that the true value of the mean will exceed some specific value in the right tail decreases as the sample size increases. For any selected sample size, this probability will be somewhere between that which would be measured if the sample mean were distributed as the parent distribution and that which would be measured if it were normally distributed.

The problem of interest here is not the determination of the area enclosed in the right hand tail, but is, instead, the closely related problem of determining the value that would enclose an area in the right tail that is equal to a selected percentage of the area under the entire distribution (e.g. 5%). The premise upon which this solution is based can, therefore, be restated: it is that the value that cuts off 5% of the area under the right tail of the true distribution of the sample mean lies somewhere between the values that would enclose 5% tails of the parent distribution and the normal distribution, where the parameters specifying the parent and normal distributions are based on estimators calculated from the sample data. In order to apply this methodology, the distribution of the parent population must be known or assumed. As discussed in Section 2, the parent population is assumed to be lognormally distributed.

Section 4
EXAMPLE SOLUTIONS AND RESULTS

Estimation of the Parameters of the Lognormal Distribution

The methodology used here requires estimates of the mean μ and variance σ^2 of the lognormal distribution of concentration levels (for PWRs) and noble gas releases (for BWRs). The properties of different estimation techniques are discussed in Chapter 5 of Aitchison and Brown.[7] A variety of techniques are available. Each can be expected to provide slightly different estimates of the parameters μ and σ^2. Aitchison and Brown's discussion shows that the method of maximum liklihood is preferred for estimating the mean and variance of the lognormal distribution.*

The discussion in Aitchison and Brown is based on an article by Finney, who derives the maximum likelihood estimators.[8] His works shows that the estimators incorporate an infinite series that converges, but only slowly. Finny went on to develop an approximation method for estimating the value of the infinite series. This approximation is sufficiently accurate for most work, but holds only for data having relatively small sample variance (the variance of the transformed data must be less than approximately 0.69), and sample size greater than 50. Neither data set used here has a variance this small (compare to equations (2) and (4)). Therefore, Finney's method of approximation is not usable. However, computational methods have greatly advanced since Finney's article was written, so that rapid convergence is no longer of such importance. A computer program was developed to test the feasibility of attaining a solution to the infinite series, based on Finney's equation (10), p.157. The series was found to converge sufficiently rapidly that this method of solution could be used. Derived estimates of μ and σ^2 are represented by a and b^2 respectively. This method of solution requires sample estimates of the mean and standard deviation of logarithms of the original data. These estimates are provided in equation (2) and (4).

For the PWRs:

$$a = 3.029 \times 10^4$$
$$b^2 = 1.565 \times 10^{10} \tag{5}$$

For the BWRs:

$$a = 2.547$$
$$b^2 = 24.060 \tag{6}$$

*Preliminary estimates, based on the method of quantiles, were made before the computer solution of the maximum likelihood estimates had been derived. These estimates were the basis of the conclusions presented in: Environmental Radiation Protection for Nuclear Power Operations, Supplementary Information, U.S. Environmental Protection Agency, January 1976. These estimates do not differ significantly from the results calculated here.

Development of an Expression for the Percentile Point for the Lognormal Distribution

Before working through an example solution, it will be useful to develop the expression for the value of L_v, which cuts off an area in the right hand tail of the lognormal density equal to (1-v). From Aitchesin and Brown, p. 9, the expression for L_v is:

$$L_v = exp\,(\mu + z_v \sigma)$$

where z_v is the value that cuts off an area in the right hand tail of the standard normal density equal to (1-v). This expression is needed because no table of areas under the lognormal density are available, so corresponding areas under the normal density are used. In order to use this formulation for L_v, the values μ and σ are required. Since the methodology developed here is not directed towards the estimation of μ and σ, but is instead, directed towards the estimation of α and β, the paramenters of the lognormal density, an expression for L_v in terms of α and β will be needed. From Aitchison and Brown, p 8, α and β are expressed in terms of μ and σ.

$$\alpha = exp\,(\mu + \tfrac{1}{2}\sigma^2)$$

$$\beta \quad [exp\,(2\mu + \sigma^2)](exp\,\sigma^2 - 1)$$

These equations can be solved for μ and σ, with the following results:

$$\mu = ln\,\alpha - \tfrac{1}{2}\,ln\left(\frac{\beta^2}{\alpha^2} + 1\right)$$

$$\sigma^2 = ln\left(\frac{\beta^2}{\alpha^2} + 1\right)$$

Therefore: $$L_v = exp\left\{ln\,\alpha - \tfrac{1}{2}\,ln\left(\frac{\beta^2}{\alpha^2} + 1\right) + z_v\left[ln\left(\frac{\beta^2}{\alpha^2} + 1\right)\right]^{\frac{1}{2}}\right\}$$

$$= \alpha\; exp\left[-\tfrac{1}{2}w + z_v w^{\frac{1}{2}}\right]$$

Where: $$w = ln\left(\frac{\beta^2}{\alpha^2} + 1\right) \qquad (7)$$

Solution for PWR Assuming Ten Operating Months are Representitive of Annual Operation

The mean for 10 months operation for PWRs will be assumed to be distributed as; first, a lognormal deviate and; secondly, as a normal deviate, on the premise that the true distribution is somewhere between the two. It will first be assumed that the ten month mean is lognormally distributed. Estimates of the paramenters of the lognormal distribution were given in equations (5). Let g_m and h_m represent estimates of the sample mean and standard deviation respectively. Therefore:

$$g_m = a = 3.029 \times 10^4$$
$$h_m = b = (1.565 \times 10^{10})^{\frac{1}{2}} = 1.251 \times 10^5 \qquad (8)$$

These are estimates of the parameters of the distribution of monthly operation, from which a sample of 10 months operating data are drawn. The sample of 10 monthly concentration levels will have a mean g_A and standard deviation h_A.

$$g_A = g_m = 3.029 \times 10^4$$
$$h_A = h_m/\sqrt{10} = 3.956 \times 10^4 \qquad (9)$$

These paramenters can be used to determine the value of $\mathcal{E}_{95}$, relying on the derivation developed in equation (7).

$$\mathcal{E}_{95} = g_A \exp\left(-\tfrac{1}{2}w + z_{95} w^{\frac{1}{2}}\right)$$

Where: $$w = \ln\left(g_A^2/h_A^2 + 1\right) = \ln\left[\left(\frac{3.029 \times 10^4}{3.956 \times 10^4}\right)^2 + 1\right]$$
$$= 0.995$$
$$\mathcal{E}_{95} = 3.029 \times 10^4 \exp\left[-\tfrac{1}{2}(0.995) + 1.645(0.995)^{\frac{1}{2}}\right]$$
$$= 9.502 \times 10^4$$

The transformation to exposures can now be performed. In performing this transformation, no attempt is made to develop a pathway model. Instead, it is assumed that the concentration level represented by $\mathcal{E}_{95}$ corresponds to the operating criteria established in Appendix I, which determines a 5 millirem annual limit for each reactor. The correspondence between $\mathcal{E}_{95}$ and 5 millirem per year is assumed to be represented by a linear transformation. In making this transformation, it has been assumed that the 95% point for concentration ($\mathcal{E}_{95}$) can be linearly transformed to the 95% point for exposure. Equating the 95% point for exposure to 5 mrem imposes an interpretation on the Appendix I limit; that the 5 mrem limit is not to be exceeded more than 5% of the time under normal operation. A worst case has been assumed in this solution; that 5 mrem is exceeded exactly 5% of the time. This same linear transformation can be used to transform the mean annual concentration to mean annual exposure and also the standard deviation of annual concentrations to that for annual exposures. There is, however, one conceptual clarification necessary before performing the transformation. The primary interest is the transformation of the paramenters of the distribution of operating data, since it is reasonable that a zero concentration level for "down time" will transform to zero exposure. Since the operating data represents only ten months operation, the concentration level will be assumed to transform to exposure levels that produces 5 millirem during a whole years operation, rather than 5 millirems for ten months operation. Therefore the parameters of the distribution for operating data is weighted by the factor 10/12. The following calculations determine the transformation factor.

$$\tfrac{10}{12}\, \mathcal{E}_{95}\, T = 5 \text{ mrem}$$

$$\begin{aligned} T &= 5/(\tfrac{10}{12}\mathcal{E}_{95}) \\ &= 5/(\tfrac{10}{12}\, 9.502 \times 10^{4}) \\ &= 6.314 \times 10^{-5} \end{aligned}$$

The mean g_E and standard deviation h_E of the ten month level of exposures can then be calculated.

$$\begin{aligned} g_E &= T\, g_A = (6.314 \times 10^{-5})(3.029 \times 10^{4}) \\ &= 1.903 \\ h_E &= T\, h_A = (6.314 \times 10^{-5})(1.251 \times 10^{5}) \\ &= 2.498 \end{aligned}$$

Note that g_E does not represent the mean annual exposure level, but represents the mean exposure rate for the ten months of operating time. The mean for a years operation is represented by $(10/12)g_E$.

The aggregation of r reactors on a site can now be considered. Since exposures from individual reactors are summed to determine the exposures from r reactors, the mean and standard deviation of the aggregation are represented by the following, where g_r and h_r represent the mean and standard deviation of the exposures from the aggregation. Each reactor is assumed to operate only ten months each year.

$$\begin{aligned} g_r &= r\, g_E \\ h_r &= \sqrt{r}\, h_E \end{aligned}$$

It should be noted that these equations do not necessairly imply that all the r reactors operate the same ten months out of each year. The down time for different reactors could be spaced at different times throughout the year.

Either of two assumptions can be made concerning the distribution of the exposures from the aggregation. One assumption is that the aggregation continues to be lognormally distributed and the other is that it is normally distributed. These two alternatives come from the basic premise upon which this solution is based, that these two distributions will "bracket" the true distribution, as was discussed above in the section on methodology.

First, it will be assumed that the aggregation is lognormally distributed. The value of r will be set to equal six for demonstrating the technique.

$$\begin{aligned} g_6 &= 6\, g_E \\ &= 6\,(1.913) \\ &= 11.478 \\ h_6 &= \sqrt{6}\, h_E \\ &= \sqrt{6}\,(2.498) \\ &= 6.119 \end{aligned} \tag{10}$$

The exposure level that "cuts off" a 5% tail is determined by the formula introduced earlier, in equation (7).

$$\nu_{95} = g_6 \exp\left(-\tfrac{1}{2} w + z_{95}\, w^{\frac{1}{2}}\right)$$

In this case: $w = \ln\left[\left(\frac{6.119}{11.478}\right)^2 + 1\right]$

$$= 0.250$$

Therefore: $\nu_{95} = 23.059$

These parameters represent the distribution of exposures from six reactors, for ten months of operations. The exposure level for the other two months operation for each reactor is equal zero, so, taking the weighted average to determine the 95% level will give:

$$\begin{aligned} R_{95} &= \frac{10}{12}\, \nu_{95} \\ &= 19.215 \end{aligned}$$

This completes the demonstration for the case assuming annual concentration levels and annual exposure levels are both lognormally distributed. It will now be assumed that annual concentration levels continue to be lognormally distributed, but that the exposure level for the aggregation of six reactors will be normally distributed. The same mean and standard deviation estimates calculated in equation (10) still apply, but the equation for ν_{95} will now be:

$$\begin{aligned}\nu_{95} &= g_6 + z_{95}\, h_6 \\ &= 11.478 + 1.645(6.119) \\ &= 21.544\end{aligned}$$

Therefore: $$R_{95} = \tfrac{10}{12}\, 21.544 = 17.953$$

This completes the analysis based on the assumption that the annual concentration is lognormally distributed. It will now be assumed that annual concentration is normally distributed as was discussed in Section 3. The sample estimates based on the data in its original form serve as the starting point. Let the mean and standard deviation of the distribution of the annual average under this assumption be g'_A and h'_A. From equations (1):

$$\begin{aligned}g'_A &= a = 2.108 \times 10^4 \\ h'_A &= b/\sqrt{10} = (3.602 \times 10^4)/\sqrt{10} \\ &= 1.139 \times 10^4\end{aligned}$$

The 95th percentile point for the normal distribution is represented by ε'_{95}.

$$\begin{aligned}\varepsilon'_{95} &= g'_A + z_{95}\, h'_A \\ &= 2.108 \times 10^4 + 1.645\,(1.139 \times 10^4) \\ &= 3.982 \times 10^4\end{aligned}$$

The transformation to exposures will give:

$$\begin{aligned}\tfrac{10}{12}\, \varepsilon_{95}\, T &= 5 \\ T &= 1.507 \times 10^{-4}\end{aligned}$$

The mean g_E and standard deviation h_E of the ten month exposure level is found to be:

$$g_E' = T g_A' = 1.507 \times 10^{-4} (2.108 \times 10^4)$$
$$= 3.177$$
$$h_E' = T h_A' = 1.507 \times 10^{-4} (1.139 \times 10^4)$$
$$= 1.716$$

Note again that these parameters represent the distribution for ten months of operation only. They must be weighted by the two months of down time to attain the annual values.

Aggregation to 6 reactors on a site gives:

$$g_6' = 6 g_E' = 6(3.177)$$
$$= 19.062$$
$$h_6' = \sqrt{6}\, h_E' = \sqrt{6}(1.716)$$
$$= 4.203$$

Since the distribution of the mean for ten months operation has been assumed to be normal, the aggregation to six reactors will also be normal.

$$v_{95}' = 19.062 + 1.645 (4.203)$$
$$= 25.976$$

Weighting this by the two months of zero emissions for each reactor gives

$$R_{95}' = \frac{10}{12}(25.976)$$
$$= 21.647$$

Summarizing these results, it is found that when assuming that each reactor operates ten months every year and is down for repairs or refueling two months every year, the technique of analysis provides three estimates of exposures from multiple reactors at a site. For six reactors, the 95% points are:

Method 1 (Lognormal at one reactor, lognormal at six reactors): 19.2 mrem/year

Method 2 (Lognormal at one reactor, normal at six reactors): 18.0 mrem/year

Method 3 (Normal at one reactor, normal at six reactors: 21.6 mrem/year

<u>Solution for PWR Assuming Twelve Operating Months are Representative of Annual Operation</u>

As discussed above, it might be argued that the exposure level observed from a reactor is best assumed to be the same when down for repairs as when in operation. Therefore, it will now be assumed that twelve monthly observations are representative of a years operation. The solution is similar to that used when assuming ten operating months represents annual operation. Using equations (8):

$$g_A = g_m = 3.029 \times 10^4$$
$$h_A = h_m/\sqrt{12} = 1.251 \times 10^5/\sqrt{12} = 3.611 \times 10^4 \qquad (11)$$

A lognormal distribution will now be fitted to these parameters, for the purpose of determining the 95% point (equation 7).

$$\mathcal{E}_{95} = g_A \exp\left(-\tfrac{1}{2} w + Z_{95} w^{\frac{1}{2}}\right)$$
$$w = \ln\left[\frac{h_A^2}{g_A^2} + 1\right] = \ln\left[\left(\frac{3.611 \times 10^4}{3.029 \times 10^4}\right)^2 + 1\right]$$
$$= 0.884$$
$$\mathcal{E}_{95} = 3.029 \times 10^4 \exp\left[-\tfrac{1}{2}(0.884) + 1.645(0.884)^{\frac{1}{2}}\right]$$
$$= 9.142 \times 10^4$$

The transformation to exposures will give the transformation factor (T).

$$\mathcal{E}_{95}\, T = 5$$
$$T = 5/9.142 \times 10^4 = 5.469 \times 10^{-5}$$

The mean and standard deviation of the exposure level for one year is:

$$g_E = 5.469 \times 10^{-5}\,(3.029 \times 10^4)$$
$$= 1.657$$

$$h_E = 5.469 \times 10^{-5}\,(3.611 \times 10^4$$
$$= 1.975$$

Aggregation to six reactors will give

$$g_6 = 6\,(1.657)$$
$$= 9.942$$
$$h_6 = \sqrt{6}\,(1.975)$$
$$= 4.838$$

Assuming that the aggregated mean annual exposure level follows a lognormal distribution, it is found that:

$$\vartheta_{95} = g_6 \exp\left(-\tfrac{1}{2} w + z_{95} w^{\frac{1}{2}}\right)$$
$$w = \ln\left[\frac{h_6^2}{g_6^2} + 1\right] = \ln\left[\left(\frac{4.838}{9.942}\right)^2 + 1\right]$$
$$\doteq 0.213$$
$$\vartheta_{95} = 9.942 \exp\left[-\tfrac{1}{2}(0.213) + 1.645\,(0.213)^{\frac{1}{2}}\right]$$
$$= 19.099$$

Since no adjustment for "down time" is necessary, the value for R_{95} is:

$$R_{95} = \vartheta_{95} = 19.099$$

This completes the solution, assuming the aggregated mean for six reactors is lognormally distributed. Now, assuming that the aggregated annual exposure level follows a normal distribution, it is found that:

$$R_{95} = \vartheta_{95} = g_6 + z_{95}\, h_6 = 9.942 + 1.645\,(4.838)$$
$$= 17.901$$

This completes the analysis for the cases assuming the annual concentration level is lognormally distributed. The case assuming annual concentration to be normally distributed will now be demonstrated. As for the ten months case, sample estimates of the data in its original form must be used. From equations (1):

$$g_A = a = 2.108 \times 10^4$$
$$h_A' = b/\sqrt{12} = (3.602 \times 10^4)/\sqrt{12}$$
$$= 1.040 \times 10^4$$
$$\varepsilon_{95}' = 2.108 \times 10^4 + \sqrt{12}\,(1.040 \times 10^4)$$
$$= 3.819 \times 10^4$$

Performing the transformation to exposures:

$$\mathcal{E}'_{95} \; T = 5$$

$$T = 5/3.819 \times 10^{4}$$

$$= 1.309 \times 10^{-4}$$

The parameters for annual exposures are:

$$g'_A = 1.309 \times 10^{-4}(2.108 \times 10^{4})$$

$$= 2.759$$

$$h'_A = 1.309 \times 10^{-4}(1.139 \times 10^{4})$$

$$= 1.361$$

Aggregation to six reactors on a site gives:

$$g'_6 = 6\ (2.756)$$

$$= 16.557$$

$$h'_6 = \sqrt{6}\ (1.361)$$

$$= 3.334$$

The aggregation to six reactors must be normally distributed since the distribution of one reactor is normal. Therefore:

$$R'_{95} = \nu'_{95} = 16.557 + 1.645(3.334)$$

$$= 22.038$$

These results can now be summarized. Assuming the exposures are best represented by assuming the monthly operating level emissions continue when the reactor is down for repairs or refueling, the 95% levels are determined to be:

Method 1 (Lognormal at one reactor, lognormal at six reactors): 19.1 mrem/year

Method 2 (Lognormal at one reactor, normal at six reactors): 17.9 mrem/year

Method 3 (Normal at one reactor, normal at six reactors): 22.0 mrem/year

These results, as well as those for the analysis assuming zero contribution to exposure when the reactors are not in operation, are summarized in Table 4. This table also summarizes the results of the analysis for aggregates of 4 and 5 reactors on a site. Solutions for the cases of 4 and 5 reactors on a site have not been demonstrated here, but are similar to those for the 6 reactors cases.

The averages of the six solutions investigated for the 4, 5 and 6 reactors on a site cases are shown. The summary results provided in Section 1 are based on these averages. The two solutions assuming lognormal distributions at the annual level and normal distributions at the aggregated level (methods LN-N) appear to be the most realistic, considering the implications of the Central Limit Therom. Since the solutions based on this method have the lowest estimated exposure levels, the use of the average of all methods of solution for the estimate of the exposure levels is probably conservative.

Table 4
ESTIMATED EXPOSURE LEVELS FOR PWRs

Method[1]	Reactor Operation Months per year	Number of Reactors on a Site 4	5	6
LN-LN	10	14.2[2]	16.8	19.2
LN-N	10	13.2	15.6	18.0
N-N	10	15.3	18.5	21.6
LN-LN	12	14.1	16.6	19.1
LN-N	12	13.1	15.5	17.9
N-N	12	15.5	18.8	22.0
Average		14.2	17.0	19.6

[1] Method refers to the distribution assumed in the solutions. LN means lognormal and N means normal. They are in the order of their incorporation into the solutions.

[2] units are mrem/year

Results of Analysis of BWR

Solutions for the BWR are completely analogous to those for the PWR. The only difference is that the starting point for the BWR analysis is stack release data rather than the coolant concentration data used for the PWR. Sample estimates of mean and standard deviation for the data in original form and after transformation were provided in equations (3) and (4). Maximum likelihood estimates of μ and σ^2 were given in equations (6). The stack releases are assumed to be linearly transformed to exposure levels in a manner similar to that for the PWR. After transformation to exposure levels, the solutions are exactly the same. They will not be demonstrated.

Table 5 summarizes the results of the analysis of the BWRs. The averages shown are summarized in Section 1. The discussion for Table 4 is appropriate for this table as well. As for the PWR cases, the solutions assuming lognormal distributions at the annual level and normal distributions at the aggregated level appear to be the most realistic. Therefore, the use of averages for the BWR cases also appears to be conservative.

Table 5
ESTIMATED EXPOSURE LEVELS FOR BWRs

Method[1]	Reactor Operation Months per year	Number of Reactors on a Site 4	5	6
LN-LN	10	14.5[2]	17.4	20.2
LN-N	10	14.0	16.8	19.6
N-N	10	16.1	19.7	23.2
LN-LN	12	14.7	17.7	20.6
LN-N	12	14.2	17.2	20.1
N-N	12	16.4	20.0	23.5
Average		15.0	18.1	21.2

[1] Method refers to the distribution assumed in the solutions. LN means lognormal and N means normal. They are in the order of their incorporations into the solution.

[2] Units are mrem/year

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