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Stream Models for Calculating Pollutional Effects of Stormwater Runoff

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STREAM MODELS FOR CALCULATING POLLUTIONAL EFFECTS OF STORMWATER RUNOFF

by

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FOREWORD

The Environmental Protection Agency was created because of increasing public and government concern about the dangers of pollution to the health and welfare of the American people. Noxious air, foul water, and spoiled land are tragic testimonies to the deterioration of our natural environment. The complexity of that environment and the interplay between its components require a concentrated and integrated attack on the problem.

Research and development is that necessary first step in problem solution, and it involves defining the problem, measuring its impact, and searching for solutions. The Municipal Environmental Research Laboratory develops new and improved technology and systems to prevent, treat, and manage wastewater and solid and hazardous waste pollutant discharges from municipal and community sources, and to minimize the adverse economic, social, health, and aesthetic effects of pollution. This publication is one of the products of that research--a most vital communications link between the research and the user community.

The information presented here describes the use of computer models to simulate the biological, physical, and chemical reactions that occur in a flowing stream. Hydraulic aspects are also considered. These models make it possible to estimate the pollutional effect of stormwater runoff on receiving streams. As such, this study fulfills the need for continuing technology assessment in emerging areas.

Francis T. Mayo
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ABSTRACT

Three related studies are described that provide the means to quantify the pollutional and hydraulic effects on flowing streams caused by stormwater runoff. Mathematical stream models were developed to simulate the biological, physical, chemical, and hydraulic reactions that occur in a stream. Relationships take the form of differential equations with the two independent variables of time and distance. The differential equations can be solved directly by means of calculus or by digital computer using numerical methods. The solution would be the concentration of species of pollutional interest, such as BOD and dissolved oxygen, within the stream as a function of distance and time. The solution can be steady-state or transient. There is sufficient information presently available for finding steady-state solutions. However, when the pollution loads and/or the initial conditions for the flowing stream vary with time, the problem becomes much more difficult, and the technology for handling the transient situation has not been adequately developed. The purpose of this report is to show how the solution can be found for the case where the pollution loading is a transient, especially as it applies to stormwater overflow.

The work described in this report was done inhouse by the Systems and Economic Analysis Section of EPA and covers a period from April 1977 to November 1977 and the work was completed as of December 1977.

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METRIC CONVERSION TABLE

Customary Units	Multiplier	Metric Units	Reciprocal
cubic foot, cu ft	.02832	cubic meter, m^3	35.31
cubic foot, cu ft	28.32	liter, l	.03531
cubic feet per minute, cfm	.04719	liters per second, l/sec	2.119
cubic feet per second, cfs	.02832	cubic meters per second, m^3/sec	35.31
foot, ft	.3048	meter, m	3.281
gallon, gal	3.785	liter, l	.2642
mile, mi	1.609	kilometer, km	.6215
miles per hour, mi/hr	1.609	kilometers per hour, km/hr	.6215
million gallons, mil gal	3785.0	cubic meters, m^3	.0002642
million gallons per day, mgd	43.81	liters per second, l/sec	.02282
million gallons per day, mgd	.04381	cubic meters per second, m^3/sec	22.82
pound, lb	.4536	kilogram, kg	2.205
square miles, sq mi	2.590	square kilometers, km^2	.3861

SECTION 1

INTRODUCTION

This report consists of separate descriptions of the following three related studies that were performed inhouse by the U.S. Environmental Protection Agency:

1. Stormwater Overflow Pollution Stream Model (SWOPS)

2. Stormwater Overflow Hydraulic Stream Model (SWOHS)

3. Effect of Stormwater on Stream Dissolved Oxygen

These studies were made in order to develop better methods for quantifying the pollutional effects in streams caused by stormwater overflows.

Mathematical stream models were developed to simulate the biological, physical, and chemical reactions that occur in a flowing stream. Hydraulic aspects are also taken into account. The relationships take the form of differential or difference equations with two independent variables: time and distance along the stream. The differential equations can be solved with analog or digital computers or, in some cases, calculus can be used to find analytical (closed-form) solutions. The solution of the differential equations is the concentration of all species of pollutional interest, such as BOD and dissolved oxygen, within the stream as a function of distance and time resulting from the assumed pollution load on the stream.

The solution is divided into the transient regime and the steady-state regime. The steady-state solution is simply the solution that will apply when time becomes infinitely large, and therefore the steady-state solution is a function of distance only. The transient solution is the concentration of all pollution species as a function of distance and time beginning at time zero and ending when the transient solution equals the steady-state solution.

The pollution loads on the stream can also be specified as steady-state or transient. A steady-state load on the stream is one where the rate of pollution entering the stream is constant. A transient load on the stream is one where the rate of pollution entering varies with time. It should be pointed out that the solution is required only for a specified segment of the stream, and the range of interest for time is from zero to the time at which the solution reaches the steady-state regime.

When the initial conditions for the river at distance zero are constant for all time and the pollution loads on the stream are steady-state, the solution of interest is the steady-state solution. This set of assumptions is made in most stream modeling, and many analytical expressions and computer programs are available for finding the steady-state solution. However, when the pollution loads and/or the initial conditions for the river at distance zero vary with time, the problem is much more difficult and the technology for treatment of this transient situation is poorly developed. The purpose of these reports is to show how the solution can be found for the case where the pollutional load is a transient, especially as it applies to storm overflow events.

SECTION 2

SUMMARY AND CONCLUSIONS

Since stormwater overflows from urban areas served by combined sewers contribute a mass of pollutants to the stream roughly equivalent to the sanitary sewage treated to a secondary level, environmental planners must estimate the effect of stormwater on the dissolved oxygen level in the receiving stream. Because of the transient nature of the problem, computational methods available were complex and time consuming. Although the Streeter-Phelps model neglects the very important dispersion term, it is often used for estimating the impact of stormwater overflows because of the simplicity of the solution.

In an effort to improve the technology for assessing the importance of stormwater overflows a digital computer stream model (SWOPS) was developed for computing the dissolved oxygen (DO) deficit in the stream as a function of time and distance along the stream caused by any specified overflow. The accuracy of the computation was judged by comparsion with the known closed-form solution for the BOD concentration in the stream resulting from an impulsive load. The superposition principle was used to find the BOD response to any overflow from the closed-form solution for the impulsive load. It was discovered that the computational accuracy was greatly improved by the use of the Lagrange coordinate system instead of the commonly used Euler coordinate system. A second digital computer program (SWOHS) was developed to study the hydraulic effect on the stream of large volume overflows. Since the reaeration coefficient is known to be related to the stream velocity and water depth the hydraulic aspects of the problem can affect the reaeration coefficient and thus the dissolved oxygen levels in the stream. Computations made with SWOHS show that the hydraulic effect is likely to be minimal in all except the most extreme cases.

It was found that a time interval of 15 minutes gave sufficient accuracy for computing the DO deficit profiles with SWOPS. Dissolved oxygen deficit profiles were computed with SWOPS using a wide range for all variables and a method for presenting the results in a succinct form was sought. It was found that the ratio of the peak DO deficit in the stream to the peak DO deficit computed with the Streeter-Phelps model could be presented as a single-valued function of a non-dimensional grouping. It was also found that if the time-to-the-peak was expressed as a non-dimensional grouping this grouping could also be presented as a single-valued function of the first non-dimensional grouping. Thus, a method was found for summarizing the results of the computations in an easily used manner.

In performing the computations with SWOPS it was noted that as the width of the square pollution pulse was decreased the form of the DO deficit profiles approached the shape of the normal probability density function. Using this fact as a clue, a closed-form solution for the DO deficit response to an impulsive BOD load was then discovered. This closed-form solution can now be used with the superposition principle to find the DO response in the stream to any stormwater pollution event. A short digital computer program (SWOCFS) was then developed to apply the closed-form solution for the impulsive load with the superposition principle although the computations can be made by hand if necessary. Thus, the technology for estimating the impact of any stormwater overflow on the dissolved oxygen resources of the stream has been advanced to a point where making the computations is feasible even by hand computation.

SECTION 3

RECOMMENDATIONS

The effect of storm and combined sewer overflows on the quality of the receiving stream is poorly understood. One reason for this is that the effect of time varying or transient concentrations of dissolved oxygen deficit or toxicants on aquatic life is difficult and expensive to investigate. Another reason is the difficulty of calibrating and validating a time dependent stream model. The prevailing opinion at this time is that in flowing streams of reasonably large volume the impact of periodic dissolved oxygen depressions caused by storm and combined sewer overflows is probably not critical to aquatic life. It is also understood that in lakes or even estuaries the impact of storm overflows can be critical. There is some evidence that storm overflow solids can settle out on the stream bottom and build up benthic deposits which can have a significant oxygen demand in dry weather or low flow periods which may seriously deplete the oxygen resources of the stream. These benthic deposits can also become resuspended in high flow periods to cause significant BOD loads in the stream. This aspect of the storm water pollution problem needs to be investigated first by locating and investigating sites where this mechanism occurs and second by devising a stream model capable of predicting the magnitude of the effect based on the physical parameters of the stream and the pollution sources.

SECTION 4

STORMWATER OVERFLOW POLLUTION STREAM MODEL (SWOPS)

BACKGROUND

The Stormwater Overflow Pollution Stream Model (SWOPS) is a mathematical model for a natural flowing river or stream developed primarily to study the effect of dispersion within the stream on transient changes in water quality caused by storm and combined sewer overflow events. The stream is assumed to be prismatic, that is, the cross-sectional area and velocity are fixed for all distances along the stream.

The length of the river to be studied, the solution reach, is divided into equal intervals of distance (ΔX) along the stream. Processes such as advection, dispersion, depletion of oxygen reserves by BOD degradation, and reaeration from atmospheric air are expressed as difference equations with time (t) and distance (X) as the independent variables. These difference equations can be expressed as partial differential equations when the increments of time and distance are allowed to approach zero.

The point along the stream where the storm overflow occurs is specified and the characteristics of the overflow are expressed as a volume flow (cfs)* and a concentration of pollutant (BOD) for each time increment beginning at time equals zero. The solution to the differential equations over the domain of time and distance along the stream is found by means of numerical integration using the Crank-Nicolson method. Lagrange coordinates, using a distance reference point moving downstream at the stream velocity, are used instead of the more conventional Euler coordinates, using a distance reference point fixed on the stream bank, to improve the accuracy of the numerical solution. The accuracy of the numerical integration scheme is checked against a closed form solution for BOD concentration in the stream caused by a square pulse of BOD pollution entering the stream. Finally, the computed results are presented as functions of non-dimensional groupings to allow the analyst to estimate the effect of dispersion without use of the digital computer program.

The solution is roughly divided into two regimes; the transient regime and the steady-state regime. The steady-state solution is simply the solution

*For simplicity, customary units of measurement are being used in this report instead of the required metric units, and a metric conversion table is given on page ix.

which will apply when time becomes infinitely large and, therefore, the steady state solution is a function of distance only. The transient solution is the concentration of all pollutional species as a function of distance and time beginning at time zero and ending when the transient solution equals the steady-state solution.

The pollutional loads on the stream can be specified as steady-state or transient. For example, a steady-state load on the stream is one where the rate of pollution entering the stream (lb/day) is constant. A transient load on the stream is one where the rate of pollution entering varies with time. An example of a transient load is a storm overflow in the form of a square pulse of four hours duration with a fixed concentration of BOD.

It should also be pointed out that the solution is required only for a specified segment of the river, the solution reach corresponding to the distance from X=0 to X=XMAX. Similarly, the range of interest for time is restricted from t=0 to the time at which the solution reaches the steady-state regime.

When the initial conditions for the river upstream of the pollution sources are constant for all time and the pollutional loads on the stream are steady-state the solution of interest is the steady-state solution. This set of assumptions is made in most river modeling and many analytical expressions and digital computer programs are available for finding the steady-state solution. However, when the pollutional loads and/or the initial conditions for the river vary with time the problem is more difficult and the technology for treatment of this transient problem is not well documented. The purpose of this report is to show how the solution can be found for the case where the pollutional load is a transient especially as it applies to storm overflow events.

The solution for storm overflow events will be found by means of numerical integration of the basic differential equations. However, errors of various kinds are indigenous to numerical integration and it is, therefore, necessary to compare the numerical integration solution to a solution known to be mathematically accurate in order to develop a knowledge of the limitations of the numerical solution. Analytical solutions (closed-form) for a number of simple stream modeling problems are available. First of these is the Streeter-Phelps (1) steady-state solution for BOD and dissolved oxygen profiles in a prismatic stream assuming that no dispersion exists. Analytical expressions are also available (2) for steady-state BOD and dissolved oxygen profiles in a prismatic stream including dispersion. However, it was shown by Dobbins (2) that for the range of dispersion coefficients normally experienced in natural streams the Streeter-Phelps solution is a good approximation to the steady state solution including dispersion. A closed-form expression is available (3) for calculating the concentration of BOD in a stream resulting from an impulsive load of BOD entering the stream as a function of time and distance from the dump point. By an impulsive load we mean a load which enters the stream instantaneously. The Streeter-Phelps solution and the solution for an impulsive load were used to test the accuracy of the numerical integration scheme used in SWOPS. These two closed form solutions will be derived in the following section.

CLOSED-FORM SOLUTIONS

The pioneering work of H. W. Streeter and Earle B. Phelps of the U. S. Public Health Service is presented in Public Health Bulletin No. 146 (1) dated February, 1925. They visualized the Ohio River as many segments divided by planes perpendicular to the flow of the river which move in the direction of flow with no mixing across the dividing planes. Biological activity, using the BOD pollutional load, depletes the dissolved oxygen reserves of the river and additional oxygen enters each segment from the atmosphere at a rate directly proportional to the dissolved oxygen deficit. The Streeter-Phelps model is described by the following two differential equations.

$$dL/dt = -K_r L \tag{1}$$

$$dD/dt = K_d L - K_a D \tag{2}$$

where

L = 5-day BOD concentration, mg/l

t = time of travel, days

K_r = rate constant for deoxygenation and sedimentation, day^{-1}

D = dissolved oxygen deficit, mg/l

K_d = deoxygenation rate constant, day^{-1}

K_a = reaeration rate constant, day^{-1}

L_o = BOD concentration at X=0 for all time, mg/l

Equation (1) can be integrated simply as follows:

$$L = L_o e^{-K_r t} \tag{3}$$

If the volume flow of the pollutional load is small compared to the volume flow of the river L_o is computed simply as follows:

$$L_o = Q_i \; L_i / Q \tag{4}$$

Q_i = volume flow of pollutional stream, cfs

Q = volume flow of river, cfs

L_i = 5-day BOD concentration in pollutional stream, mg/l

Solving equation (2) is slightly more involved than solving equation (1). Equation (2) can be written in the standard form for a linear first order differential equation as follows:

$$dD/dt + K_a D = K_d L \tag{5}$$

If we multiply both sides of equation (5) by the integrating factor, $e^{K_a t}$, it can be seen that the left-hand side is an exact derivative of $De^{K_a t}$. Therefore, equation (5) can be written as follows:

$$\left[D\ e^{K_a t}\right]_0^t = K_d\ L_o \int_0^t e^{(K_a - K_r)t}\ dt \tag{6}$$

This equation is readily integrated to give the following result:

$$D = K_d\ L_o/(K_a - K_r)\ \left[e^{-K_r t} - e^{-K_a t}\right] + D_o e^{-K_a t} \tag{7}$$

In equation (7), D_o is the initial dissolved oxygen deficit in the stream as it enters the solution reach. Equations (3) and (7) can be used to check the numerical integration solution in certain cases. If it is assumed that the cross-sectional area of the stream and the stream velocity are constant with distance, time and distance are related by X = time x velocity and equations (3) and (7) can be used to find the steady-state solution.

It will be shown later that as the length of the pollution pulse approaches 24 hr in length the effect of dispersion is minimized and the Streeter-Phelps solution gives a good approximation to the true transient solution. For shorter pulses, the solution can only be found by means of numerical integration. A partial check on the accuracy of the numerical solution can be had by comparison with the closed-form solution for BOD concentration in the stream resulting from an impulsive BOD load.

The following expression for the BOD concentration in the stream caused by an impulsive or spike load of BOD entering the stream can be derived by Laplace transforms as shown by Thomann (3).

$$L(x,t) = C\ \exp(-\tfrac{1}{2}((x-vt)/s)^2)\ \exp(-K_r t) \tag{8}$$

L = BOD concentration in stream, $(\text{cu mi})^{-1}$

x = distance from spike input, miles

t = time from spike input, days

v = stream velocity, miles/day

K_r = rate constant for BOD removal, $days^{-1}$

$C = 1/(As \sqrt{2\Pi})$

$s = (2Et)^{\frac{1}{2}}$

E = dispersion coefficient, sq miles/day

A = cross-sectional area of stream, sq miles

Equation (8) expresses the concentration of BOD in the stream as lb/cu mi per pound of BOD introduced in the spike input. Pounds per cubic mile can be converted to mg/l by the factor 1.008×10^{-7}. The stream cross-sectional area (A) equals Q/v where Q is the stream volume flow in cubic miles/day and v is the stream velocity in miles/day. Cubic feet per second (cfs) for stream flow can be converted to cu mi/day by the factor 5.87×10^{-7}. Thus, equation (8) can be expressed as mg/l of BOD per pound of BOD introduced in the spike as follows:

$$L(x,t) = (0.07395/(sQ/v)) \exp(-\tfrac{1}{2}((x-vt)/s)^2) \exp(-K_r t) \qquad (9)$$

To find the expression for BOD in the stream (L) as a function of time and distance equation (9) must be multiplied by the mass of pollutant introduced. If the combined sewer overflow is a square pulse with a volume flow of QIN (cfs) and a BOD concentration of LOIN (mg/l) and a duration of TAU (days) the amount of pollutant is found as follows:

$$\text{lb 5-day BOD} = QIN*0.646*LOIN*TAU*8.33 \qquad (10)$$

Multiplying equation (9) by equation (10) finally gives an expression for BOD in the stream as a function of time and distance.

$$L, \text{mg/l} = (0.3979/(Qs/v))*QIN*LOIN*TAU \exp(-\tfrac{1}{2}((x-vt)/s)^2)\exp(-K_r t) \qquad (11)$$

It can be seen from equation (11) that the BOD profile in the stream at any time (t) has the form of the normal probability density function with a mean of stream velocity times time from the spike and a standard deviation of $(2Et)^{\frac{1}{2}}$.

Equation (11) cannot be used directly to check the results of the numerical integration because it applies only to a mass of pollutant introduced instantaneously. However, since the differential equations for the processes occurring within the stream are linear any square pulse can be divided up into sub-pulses and the responses to the sub-pulses can be summed to find the response to the square pulse. A simple computer program named STREAM-4 (4) has been devised to perform this type of computation. The results from STREAM-4 can then be used to check the accuracy of the numerical integration scheme.

SOLUTION BY NUMERICAL INTEGRATION

The processes of advection, dispersion, deoxygenation caused by the presence of BOD, and reaeration at the water surface can all be expressed as difference equations. The corresponding partial differential equations can then be found by letting the increments of time and distance approach zero.

The difference equations are solved simultaneously by means of a numerical integration scheme to find the solution which consists of computed values for each of the two dependent variables, BOD (L) and dissolved oxygen deficit (D) at each of the lattice points DX apart in the X direction and DT apart in the time direction. This is shown diagrammatically in Figure 1c. Values for the dependent variables must be supplied for all points along the time=0 axis and for all points along the X=0 axis which represents the upstream boundry of the solution reach. At the downstream boundry of the solution reach, shown in Figure 1c, the values for L and D are assumed to be zero. In addition, the characteristics of the pollutional pulse and the point along the X axis at which it enters must be specified. The pollutional pulse is assumed to begin at time=0.

The first step is to divide the solution reach into equal increments (ΔX) by planes perpendicular to the stream velocity vector. The volumes between the dividing planes are sometimes called control volumes. This is shown in Figure 1a and three adjacent control volumes are shown in Figure 1b. If the volume of each increment is V and the volumetric flow rate for the stream is Q, advection of BOD (L) and dissolved oxygen deficit (D) into and out of control volume (2) is expressed by the following two difference equations:

$$V \, \Delta L_2/\Delta t = Q \, (L_1 - L_2) \tag{12}$$

$$V \, \Delta D_2/\Delta t = Q \, (D_1 - D_2) \tag{13}$$

These two expressions apply only when the Euler coordinate system is used. The Euler coordinate system assumes that the incremental volumes are fixed with respect to the shore and the stream flows through each volume.

When the Lagrange coordinate system is used the X=0 point is fixed with respect to the contents of one of the control volumes at time=0. Therefore, in the Lagrange system no advection will occur in the tubular stream, and the relationships for advection shown in equations (12) and (13) can be dropped from the set of equations to be solved.

When the Euler coordinate system is used the pollutional pulse will enter a single control volume and will cause a positive rate of change of BOD in that control volume. For example, assume that an outfall is located at control volume (2). When the hydrograph for the outfall is known, such as the unit hydrograph shown in Figure 2, the hydrograph is divided into increments of Δt and the flow assigned to each increment is the average of the

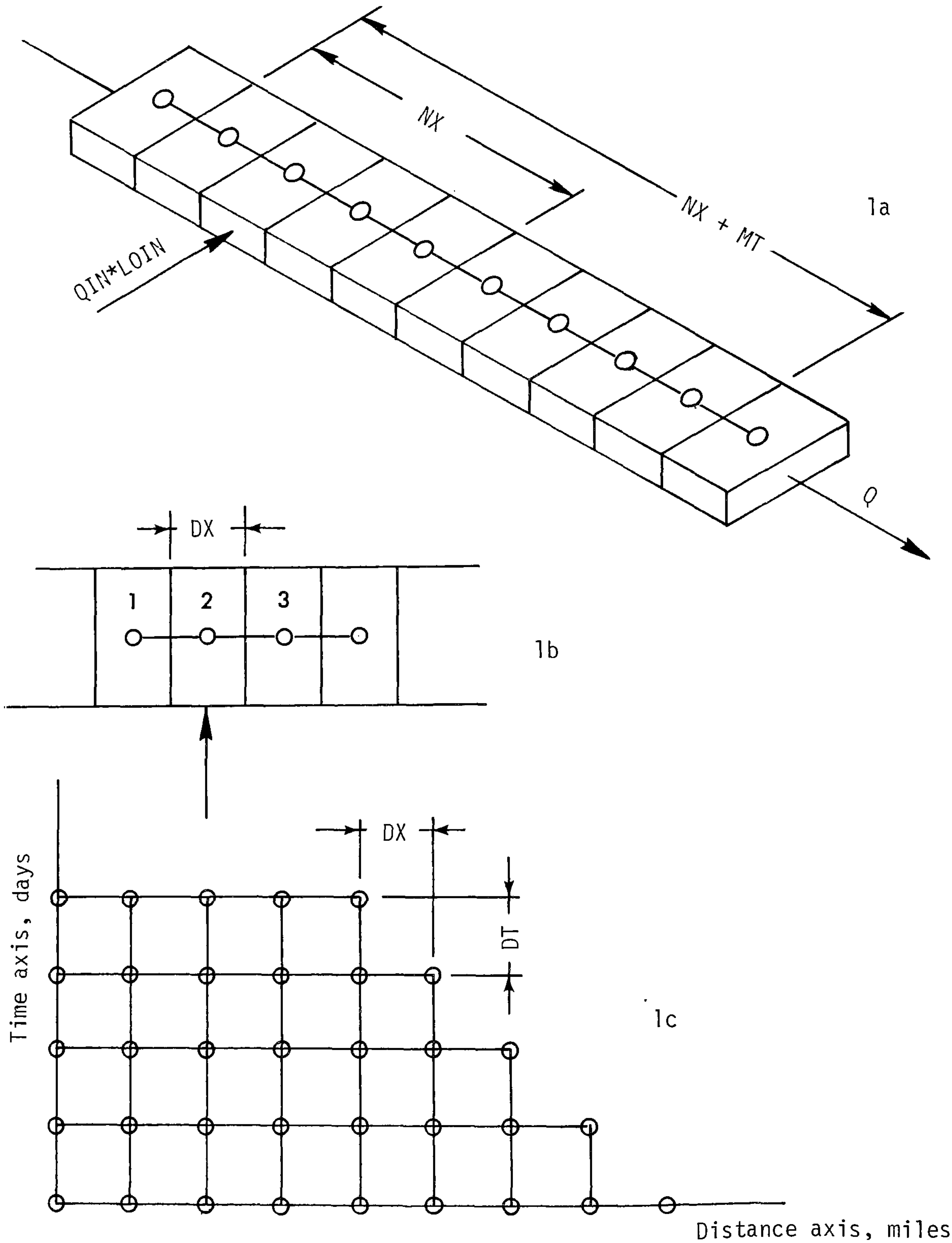

Figure 1. Diagrams for explanation of numerical integration used in stream model for study of storm overflow events.

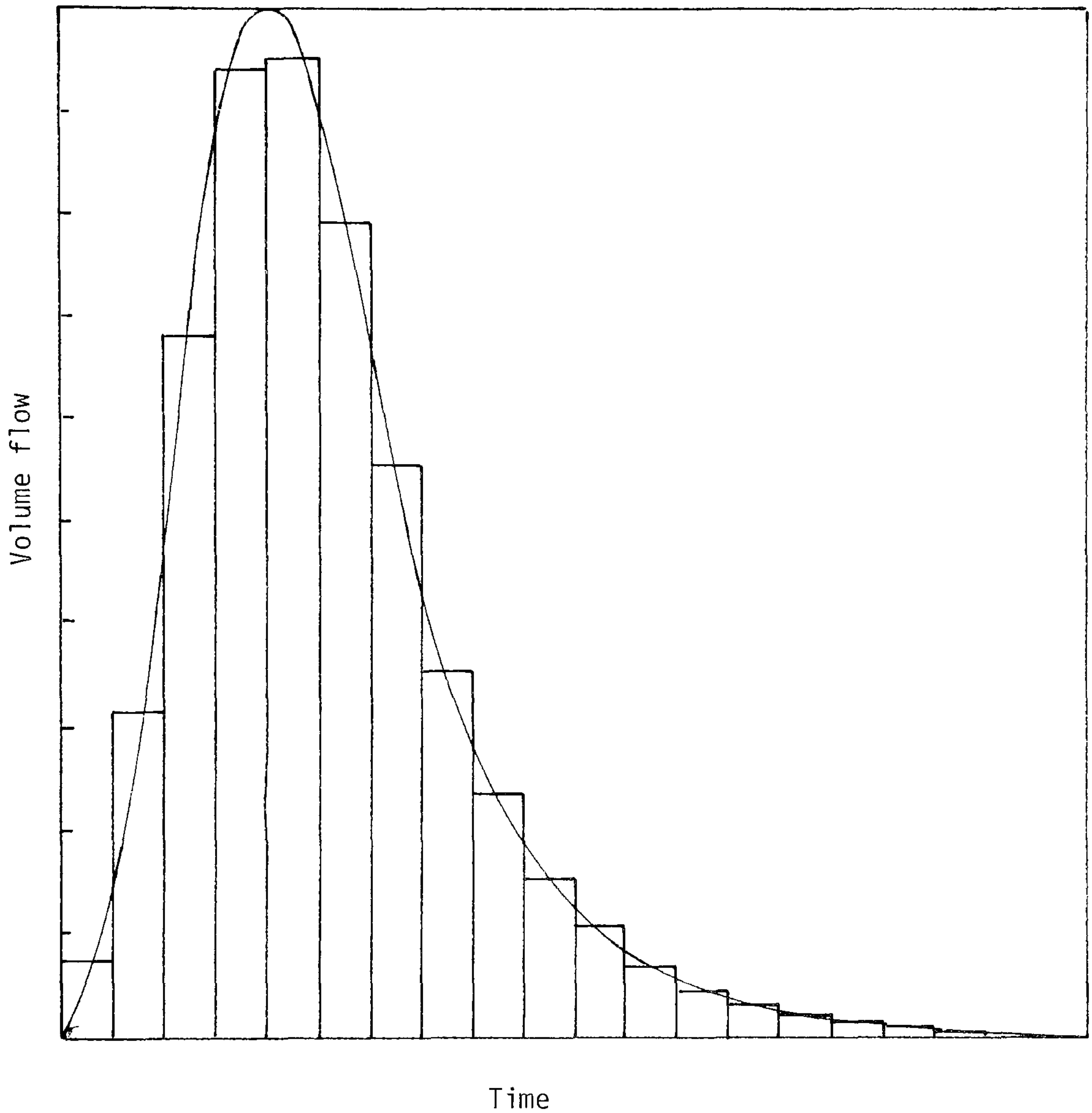

Figure 2. Diagram illustrating how a storm overflow hydrograph can be divided into discrete flow rates for use in the stream model.

flow at the boundries of Δt. Thus, the hydrograph is replaced with a bar-graph as shown in Figure 2. When a plot of BOD concentration versus time is known, this is also divided into Δt increments in a similar way. The average volume flow from the hydrograph for each Δt is called Q_{in}. Similarly, the average BOD concentration over each Δt is called L_{in}. Using the Euler coordinates, all input of pollution will occur at segment (2) and the rate of change of BOD can be written as follows where Q_{in} and L_{in} are functions of Δt.

$$V\ \Delta L_2/\Delta t = Q_{in}\ L_{in} \qquad (14)$$

When Lagrange coordinates are used, the ratio of ΔX to Δt must always equal the stream velocity. When this is true, it can be seen that pollution associated with the first Δt interval of the hydrograph will enter increment (N), pollution associated with the second Δt will enter increment (N-1), etc. where increment (N-1) is adjacent to and upstream of increment (N).

Equations for simulating the loss of BOD and the loss and replenishment of dissolved oxygen in the stream are the same for both sets of coordinates. For example, the loss of BOD in the stream in written simply as follows:

$$V\ \Delta L_2/\Delta t = -K_r L_2\ V \qquad (15)$$

Here, K_r is the rate constant for the loss of BOD by sedimentation and biological activity combined. The rate constant for loss of BOD by biological activity alone is K_d.

The use of dissolved oxygen by biological activity is expressed as follows:

$$V\ \Delta D_2/\Delta t = K_d\ L_2\ V \qquad (16)$$

Reaeration of the stream through the water surface is expressed as follows:

$$V\ \Delta D_2/\Delta t = -K_a\ D_2\ V \qquad (17)$$

Similar terms are included in the stream model for the oxygen consumption of nitrogenous compounds and the benthic demand of sludge deposits on the stream bottom but these are omitted here to simplify the discussion.

The final term to be considered is dispersion of BOD and dissolved oxygen across the planes which separate the incremental stream volumes. Although dispersion is known to consist of both eddy mixing and molecular diffusion, the form of the equation used to simulate dispersion has the same form as Fick's law for molecular diffusion. Fick's law states that the rate of diffusion across the cross-sectional area (A) of the stream is equal to a constant (E, sq miles/day) times the product of stream cross-sectional area and the gradient of the concentration of the diffusion material along the stream.

Again, referring to the second segment, the rate of diffusion of BOD upstream from the segment is written as:

$$\text{rate of upstream diffusion, lb/day} = A\ E\ (L_2 - L_1)/\Delta X \tag{18}$$

Similarly, the rate of diffusion downstream is:

$$\text{rate of diffusion downstream, lb/day} = A\ E\ (L_2 - L_3)/\Delta X \tag{19}$$

Since the Area A, (sq miles) can be expressed as $V/\Delta X$ the net rate of diffusion into segment 2 is:

$$\text{Net rate of diffusion into segment 2} = EV(L_1 - L_2)/\Delta X^2 + EV(L_3 - L_2)/\Delta X^2 \tag{20}$$

$$V\ \Delta L/\Delta t = EV(L_1 - 2L_2 + L_3)/\Delta X^2 \tag{21}$$

The following analogous expression can be used to describe the diffusion of dissolved oxygen deficit.

$$V\ \Delta D/\Delta t = EV(D_1 - 2D_2 + D_3)/\Delta X^2 \tag{22}$$

If the diffusion coefficient (E) is different for BOD and dissolved oxygen deficit we can use (E_c) for BOD and (E_d) for dissolved oxygen deficit. By summing the right-hand sides of equations (12), (14), (15), and (21) the complete difference equation for the rate of change of BOD in increment (2) is written as follows:

$$\Delta L_2/\Delta t = Q(L_1 - L_2)/V + Q_{in}\ L_{in}/V - K_r L_2 + E_c(L_1 - 2L_2 + L_3)/\Delta X^2 \tag{23}$$

Similarly, the right-hand sides of equations (13), (16), (17), and (22) can be summed to find the rate of change of dissolved oxygen deficit in increment (2).

$$\Delta D_2/\Delta t = Q(D_1 - D_2)/V + K_d L_2 \quad K_a D_2 + E_d(D_1 - 2D_2 + D_3)/\Delta X^2 \tag{24}$$

These equations (23) and (24) are written in the Euler form. To arrange them into the Lagrange form we need only drop the first term on the right-hand side of both equations and apply the input term for BOD as described previously.

Although equations (23) and (24) refer specifically to segment (2), they apply equally well to any segment with the exception that if the segment is not located at an outfall the term ($Q_{in}\ L_{in}/V$) is zero.

A crude solution of equations (23) and (24) can be found by taking the values for L and D as those at the I'th time row to estimate values for L and D at the (I+1)'th time row. Since the solution for L does not involve D the solution for L should be computed first. If this approach is taken, it is known from experience that the time and distance intervals must be very small to achieve any accuracy and this results in extreme amounts of computer time as well as magnification of computational error. A more efficient approach is to use the sum of values at the I'th and the (I+1)'th time points divided by two as the values to be used in equations (23) and (24). To implement this type of solution, called the Crank-Nicolson method by Smith (5), a linear equation must be written for each point along the X axis within the solution reach and these equations must be solved to advance the solution one time interval. Fortunately, the linear equations contain only three unknowns except for the first and last which contain two unknowns. Thus, a square tridiagonal matrix with a dimension equal to the number of distance intervals in the solution reach must be solved for each time point. It will be shown that the solution to the set of simultaneous linear equations is very simply accomplished with the digital computer.

To facilitate the following discussion, equations will be written in the FORTRAN language. For example, the value for BOD concentration (mg/l) at the I'th distance point from the upstream boundry of the solution reach and J'th time point from time=0 will be written as L(I,J). Also, the asterisk (*) will indicate multiplication and the slash (/) will indicate division. Since in equations (23) and (24) the dispersion coefficient always appears divided by delta X squared we will redefine $E_c/\Delta X^2$ and $E_d/\Delta X^2$ as EC and ED.

The time increment (days) will be called DT and DT/2 will be called DT2. Similarly, the ratio of increments on the left-hand side of equation (23) will be called DLDT(I,J) and the left-hand side of equation (24) will be called DDDT(I,J). Since the solution is built up one row at a time progressing towards greater values of time, the following two equations will apply:

$$L(I,J+1) = L(I,J) + ((DLDT(I,J) + DLDT(I, J+1))/2)*DT \tag{25}$$

$$D(I,J+1) = D(I,J) + ((DDDT(I,J) + DDDT(I, J+1))/2)*DT \tag{26}$$

Combining equations (23) and (25) it can be seen that a linear equation of the following form will result for each point within the solution reach:

$$A*L(I-1, J+1) + B*L(I, J+1) + C*L(I+1, J+1) = D \tag{27}$$

The constants A, B, C, and D can all be found from known values of L (I,J) along the J'th time line, from the characteristics of the incoming pollutional pulse, or from the assumed coefficients. L(1, J) is the value of BOD along the vertical left-hand boundry of the solution plane and is supplied as input to the program.

It can be seen from Figure 1c that as the solution progresses from one time row to the next the solution contains one less valid point each time a new line is computed. Therefore, if the solution is required for a solution reach containing NX distance intervals for all times out to MT time increments the number of distance points must be MT + NX + 1 initially. The number of distance points must then be reduced by one for each time line computed to insure that only valid points on the (J-1)'th line are used to compute points on the J'th line. The number of distance points which enter into the solution is NINC+1 which has an initial value of MT+NX+1. The values for L and D at the NINC+2 interval are assumed to be zero. This is shown diagrammatically in Figure 1c.

The index (I) in equation (27) ranges from (2) to NINC+1 as shown in Figure 1. Values for the constants A-D in equation (27) are calculated as follows:

$$A = -Q*DT2/V - EC*DT2 \tag{28}$$

$$B = Q*DT2/V + 1 + EC*DT + KR*DT2$$

$$C = -EC*DT2$$

$$\begin{aligned} D(I,J) = {} & Q*DT2*(L(I-1,J)-L(I,J))/V \\ & + DT2*(QIN(I,J)*LIN(I,J) + QIN(I,J+1)*LIN(I,J+1))/V \\ & + EC*DT2(L(I-1,J) - 2*L(I,J) + L(I+1,J)) \\ & - KR*DT2*L(I,J) \\ & + L(I,J) \end{aligned}$$

When (I) has a value of (2) an additional term of -A*L(1,J+1) should be added to the expression for D(I,J) if the initial concentration of BOD in the stream upstream of the pollutional pulse is not zero. The equations for A, B, C, and D as given apply to the Euler coordinate system. Corresponding equations for the Lagrange system are obtained by simply deleting the first term in the expressions for A, B, and D.

Equations (24) and (26) can similarly be combined to form one linear equation for each point along the X axis all with the same form as equation (27) except that the D(I,J) is substituted for L(I,J). Relationships for A, B, and D which apply to dissolved oxygen deficit (D) are given below:

$$A = -Q*DT2/V - ED*DT2$$

$$B = Q*DT2/V + 1 + 2*ED*DT2 + KA*DT2$$

$$C = -ED*DT2$$

$$D(I,J) = Q*DT2*(D(I-1,J) - D(I,J)/V$$
$$+ ED*DT2*(D(I-1,J) - 2*D(I,J) + D(I+1,J)$$
$$+ D(I,J)*(1 - KA*DT2)$$
$$+ KD*DT2*(L(I,J) + L(I,J+1))$$

If the dissolved oxygen deficit (D) is not zero along the left-hand vertical boundry of the X-T plane, an additional term should be added to the expression for D(I,J) when I=2. This term is -A*D(1,J+1).

The coefficients as they appear above apply to the Euler coordinate system. The first term of the equations for A, B, and D should be deleted when the Lagrange coordinate system is used.

The number of simultaneous linear equations to be solved to advance the solution one time increment is NINC. Each equation has three terms centered along the diagonal of the matrix except the first and last which have two. This tridiagonal of the matrix can be solved by Gauss Elimination but a more direct and convenient method is given by Bunce and Hetling (6). To apply this method, first, compute two vectors R(N) and S(N) as follows:

$$S(2) = B \qquad (30)$$

$$S(N) = B - C*A/S(N-1) \qquad (31)$$

In equation (31) the index ranges from 3 to NINC+1. The vector R(N) is computed as follows:

$$R(2) = L(2,J) \qquad (32)$$

$$R(N) = L(2,N) - R(N-1)*A/S(N-1) \qquad (33)$$

In equation (33) the index (N) ranges from 3 to NINC+1. Notice that the vector S(N) must be computed first in order that it can be used in generating vector R(N). The values for L on the (J+1)'th row are then computed as follows beginning with a value of NINC for N and working backward down the numerical scale until N=2.

$$L(N,J+1) = (R(N) - C*L(N+1,J+1))/S(N) \qquad (34)$$

Using equation (34) values for BOD on the J+1 row are calculated between an index of 2 and an index of NINC. The BOD value in the last (NINC+1) distance point is computed as follows:

$$L(NINC+1, J+1) = R(NINC+1)/S(NINC+1) \qquad (35)$$

To solve for the values of dissolved oxygen deficit on the (J+1)'th row the vectors S(N) are computed in a completely analogous way. The value of D(NINC+1,J+1) is found as follows:

$$D(NINC+1,J+1) = R(NINC+1)/S(NINC+1) \qquad (36)$$

Letting the index (N) take on all values between NINC and 2 working backward down the numerical scale, the remaining values for dissolved oxygen deficit are computed as follows:

$$D(N,J+1) = (R(N)-C*D(N+1,J+1))/S(N) \qquad (37)$$

This method of solution is also discussed on page 136 of the book by R. V. Thomann (3). The report (6) by R. E. Bunch and Leo J. Hetling entitled "A Steady State Segmented Estuary Model" gives a particularly clear presentation of the method. A diagram of the stream divided into segments is shown in Figure 1a. The solution reach is between segment 2 and segment NINC+1. Initial conditions in segment (1) are assumed to be fixed for all time and both the BOD and dissolved oxygen deficit are assumed to be zero in segment NINC+2. The pollutional pulse is shown entering segment (3). This segment number is called POINT in the computer program and it must be far enough downstream so that the dispersion will not carry the pollutant upstream beyond the second segment.

A number of computer programs have been developed based on the process relationships and the computational scheme described here. Listings of these programs and instructions for their use are given in references 7-11.

ACCURACY OF NUMERICAL INTEGRATION SOLUTION

The computational accuracy of these programs was compared to the Streeter-Phelps solution and to the closed-form solution for an impulsive load. It was found that the use of Lagrange coordinates allowed much greater accuracy of computation than that possible with Euler coordinates. When the Lagrange coordinates are used the distance interval must equal the time interval times the stream velocity. Using the Lagrange coordinates, it was found that a 15 minute time interval gave sufficient accuracy when the duration of the square pollutional pulse was one hour. Similarly, it was found that when the duration of the pollutional pulse was 24 hours or greater the solution approximated the Streeter-Phelps solution. A one-hour time interval gives sufficient accuracy when the duration of the pulse is 24 hours.

For example, Figure (3) shows computed points connected by a dashed line for a 24 hour pulse entering a stream with a velocity of 13 miles per day. The corresponding Streeter-Phelps solution is shown by the solid line. It can be seen that the accuracy is adequate. The corresponding values for dissolved oxygen deficit are shown in Figure (4) which also shows that the solution for a 24 hour pulse agrees well with the Streeter-Phelps solution.

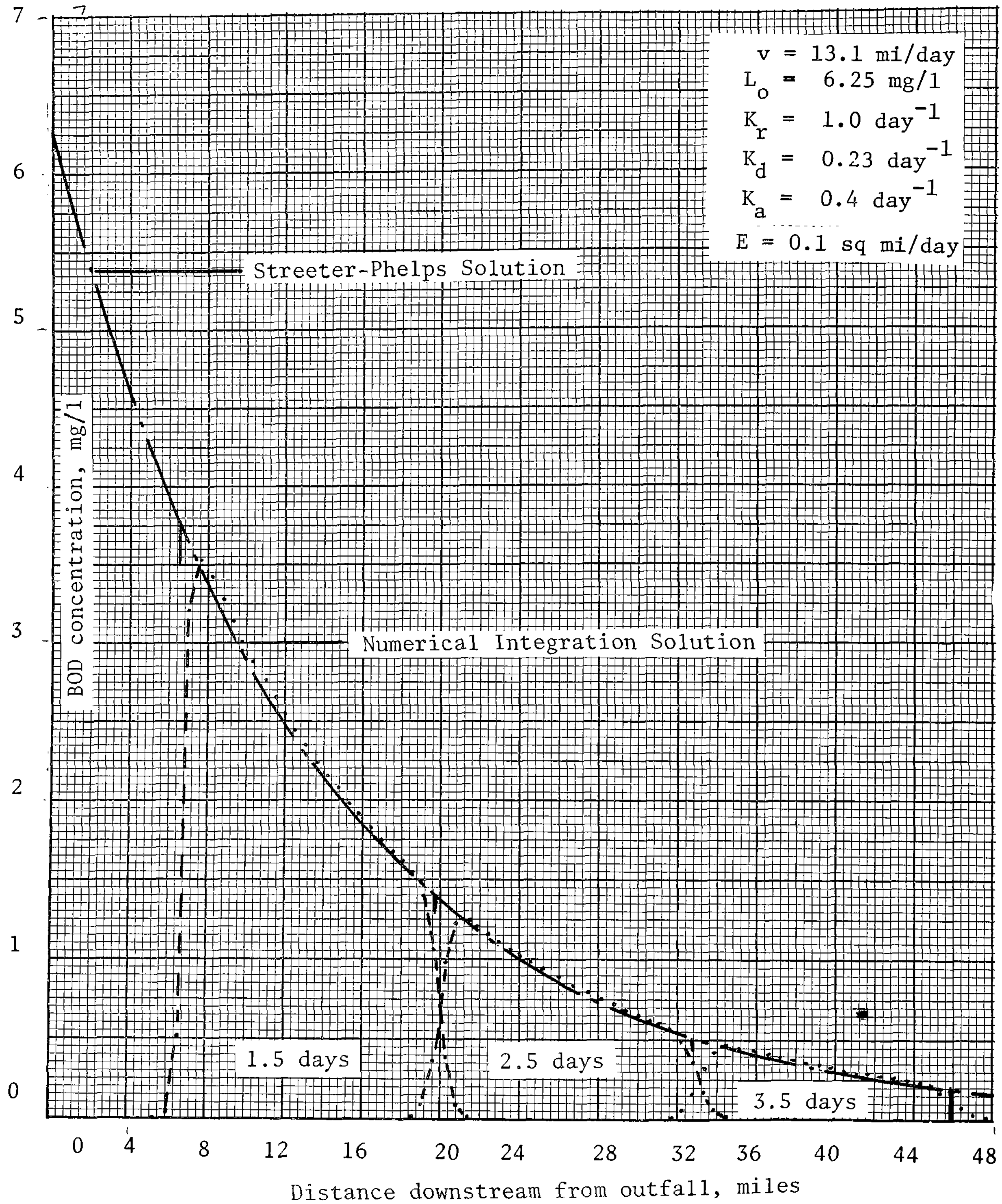

Figure 3. Computed BOD profiles in the stream caused by 24 hour storm overflow event

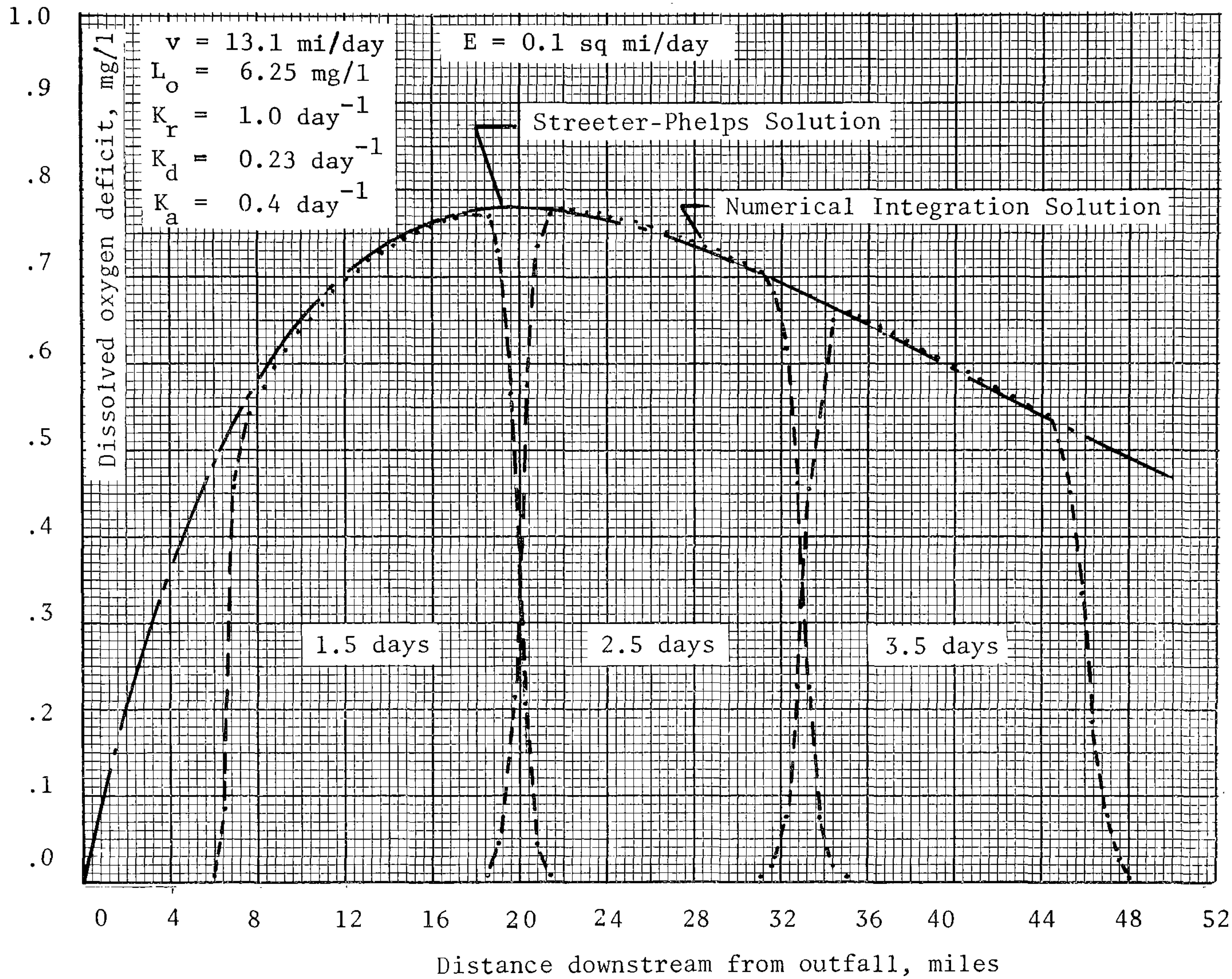

Figure 4. Dissolved oxygen deficit profiles in the stream caused by 24 hour storm overflow event

The computer program described in reference (4) was used to find BOD profiles in the stream caused by a square pollutional (BOD) pulse of one hour duration. The one hour duration was broken up into ten increments and each increment was taken as an impulsive load for which a closed-form solution is available. The solution for the one hour pulse was then taken as the sum of the solutions for the ten increments. BOD profiles in the stream caused by the one hour BOD pulse are shown by the solid lines in Figure (5) which have the shape of the normal probability density function. The computed points are circled in Figure (5). The corresponding Streeter-Phelps solutions are shown by the vertical bars with dashed lines. It can be seen that the computed points fit the closed-form solution reasonably well. The time interval used was 15 minutes. The corresponding profiles for dissolved oxygen deficit are shown in Figure (6). The solid lines represent the numerical integration solution and the bars enclosed by dashed lines represent the corresponding Streeter-Phelps solution. It can be seen that dispersion can have a significant effect on the shape of the BOD and dissolved oxygen deficit profiles caused by storm overflows.

SUMMARIZING THE IMPACT OF SQUARE POLLUTION PULSES

The digital computer program described in this report can be used to precisely compute the BOD and dissolved oxygen deficit (DO) profiles in the receiving stream caused by any defined storm overflow event.

Because of other uncertainties associated with the storm overflow problem it is seldom of interest to compute the stream impact with great precision. Therefore, it is clearly desirable to present the computed results in terms of graphical relationships which can be easily used to find a rough solution to any problem.

In general, when a square pollutant pulse is introduced into the stream a short segment of the stream is polluted and this segment is carried downstream at the velocity of the tubular stream. The initial length of the polluted segment is simply the duration of the square pollutant pulse times the stream velocity. When dispersion is assumed to be zero the length of the polluted segment is fixed and the Streeter-Phelps solution can be used. If dispersion is introduced into the problem the length of the polluted segment will increase with time and the shape of the BOD and DO profiles (at any time) will have a shape which approaches the normal error function as time becomes great. The shorter the width of the pollution pulse and the greater the value of the dispersion coefficient the less time is required for the shape of the DO profile to reach the normal curve shape. The standard deviation of both the BOD and DO profiles will equal $(2Et)^{1/2}$ where t is the time from entry of the initial portion of the pulse. The standard deviations of the profiles from a one hour square pulse with values of dispersion coefficient (E) of 0.1, 0.2, and 1.0 sq mi/day are shown in Figure (7). It can be seen that the profiles are normal after about 150 15 minute time intervals or 1.56 days. Since the maximum peak occurred at 1-1.2 days it can be seen that the profiles did not become normal until after the maximum peak. The effect of dispersion is minimized when the initial

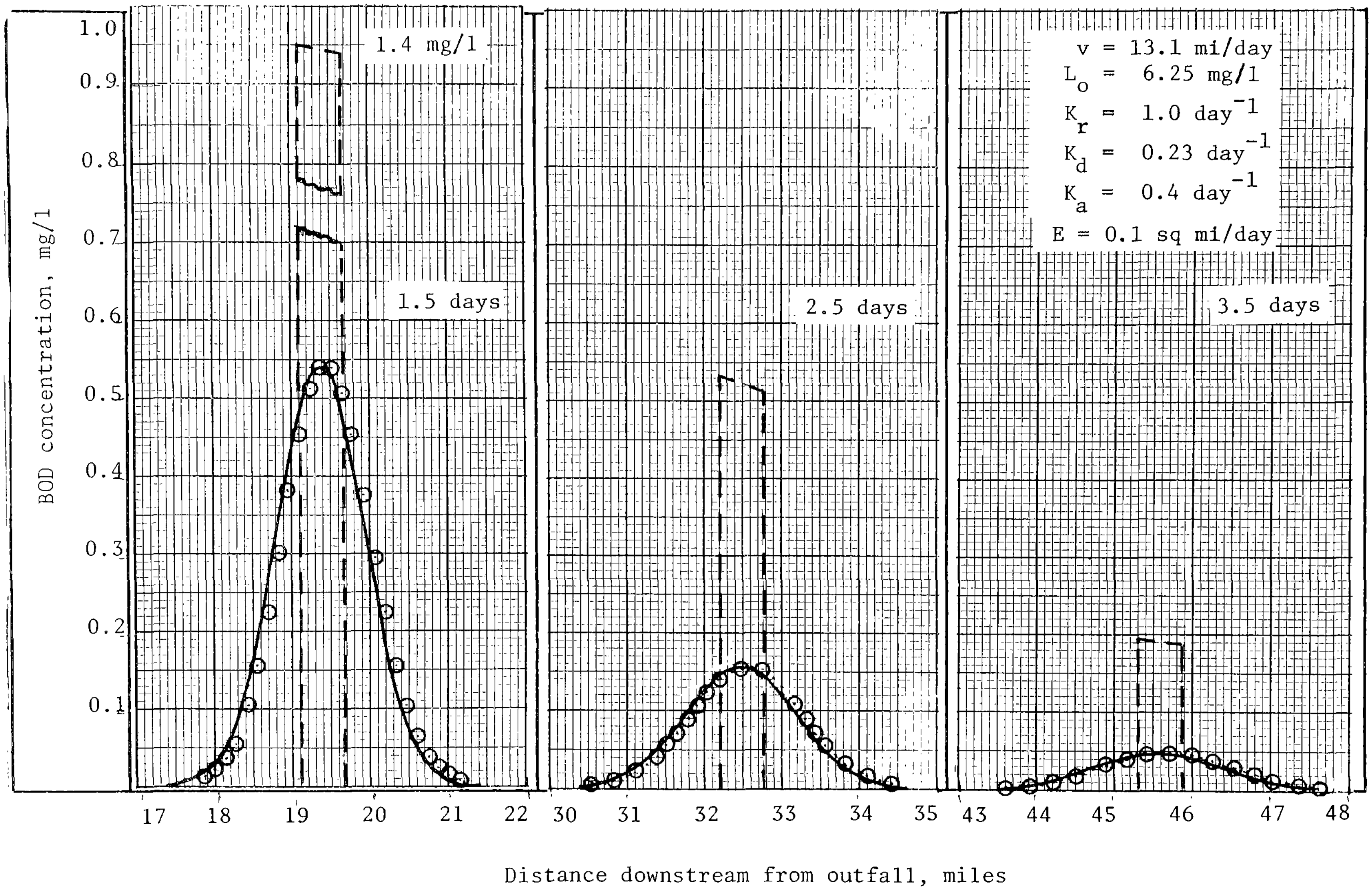

Figure 5. Computed BOD profiles in the stream caused by one hour storm overflow event. Solid line = closed-form solution, dashed line = Streeter-Phelps solution, circled points = numerical solution

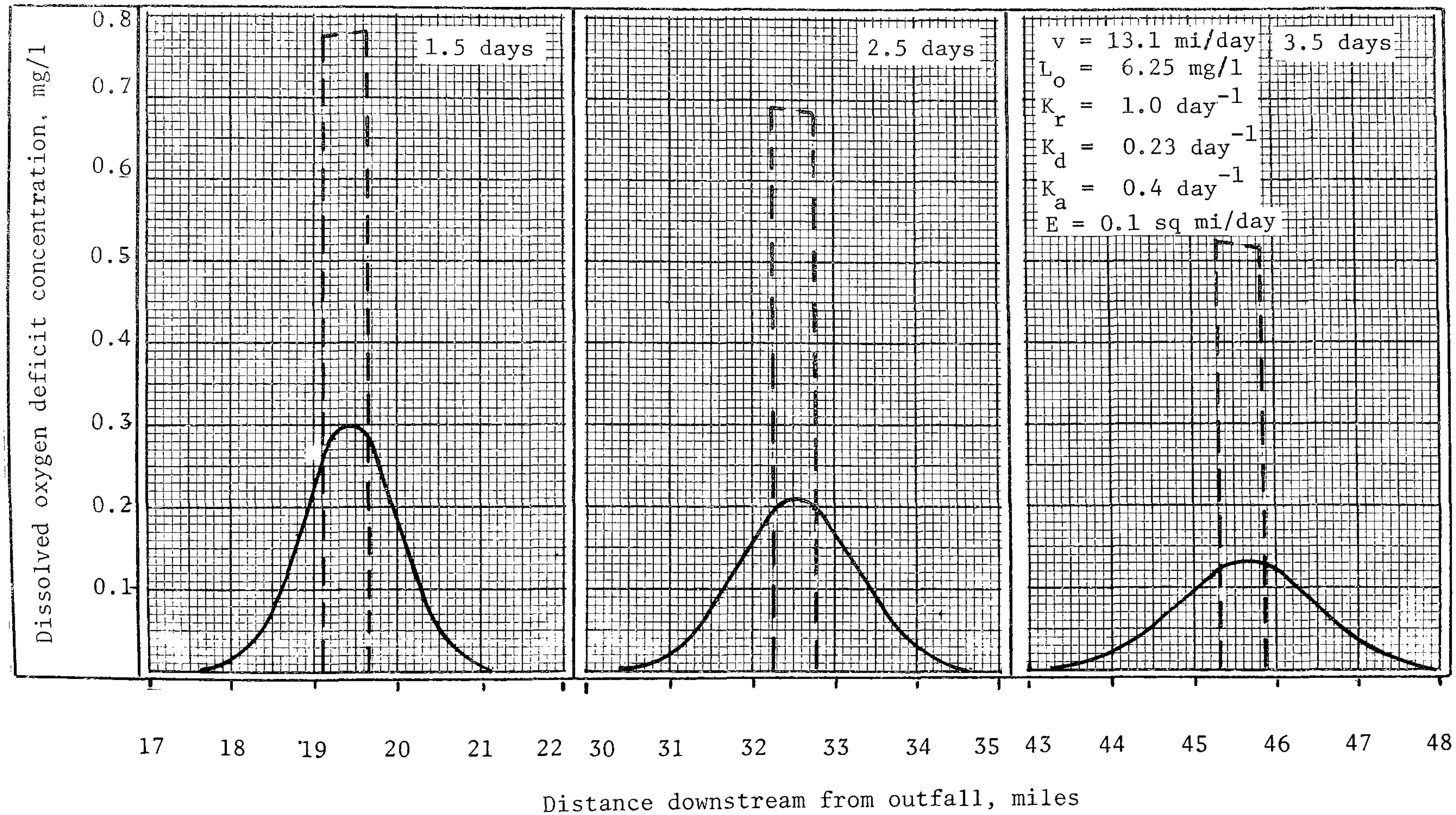

Figure 6. Dissolved oxygen deficit profiles in the stream caused by one hour storm overflow event.
Solid line = numerical solution dashed line = Streeter-Phelps solution

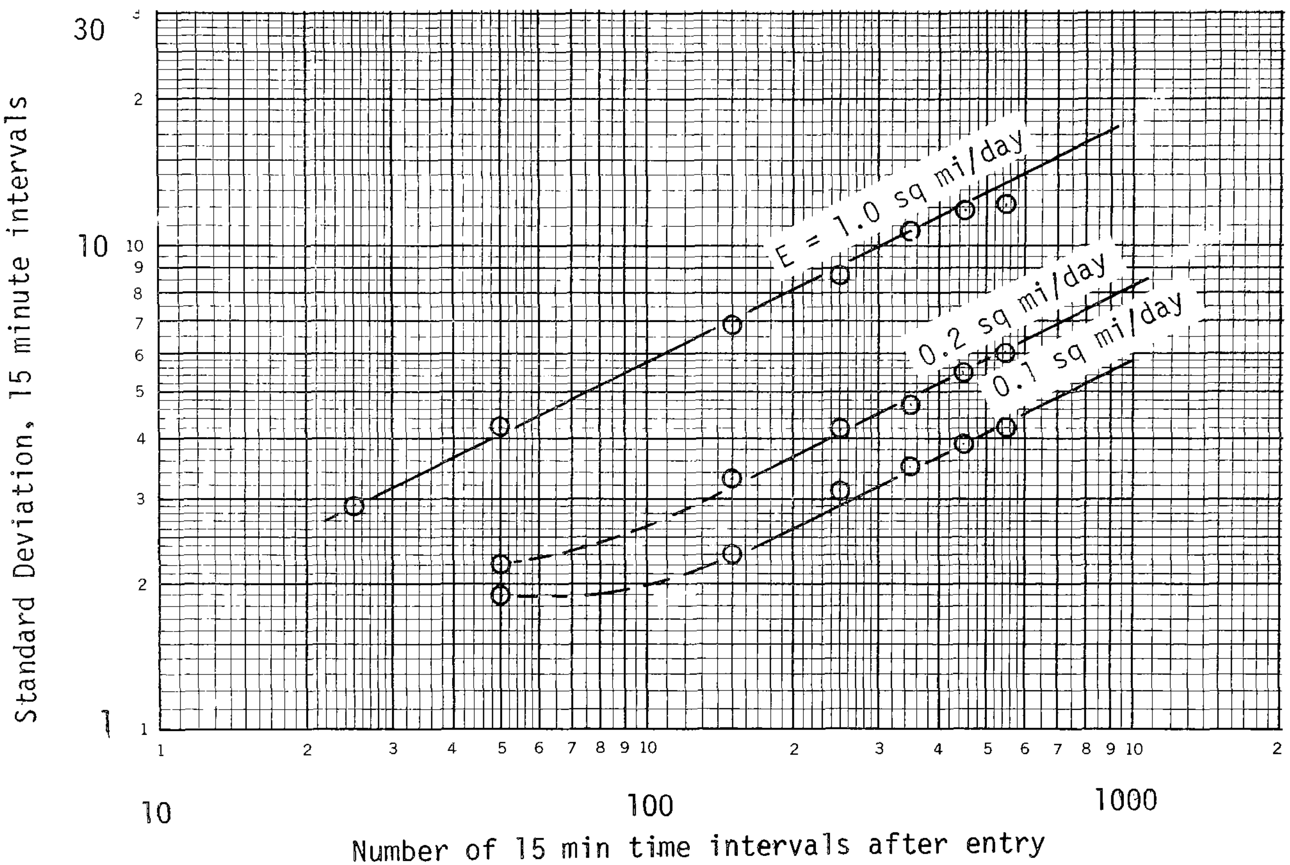

Figure 7. Computed standard deviations for dissolved oxygen deficit profiles at time from entry of leading edge of pulse for dispersion coefficient values of 0.1, 0.2, and 1.0 sq mi/day.

length of the polluted segment is large or the dispersion coefficient is small. The BOD profile will be square initially and the DO profile will be zero initially.

The peak DO values within the polluted segment will increase from zero to a maximum (DOMAX) and then descrease to zero. It was found that the ratio of the maximum peak to the Streeter-Phelps peak DO value could be plotted as a single valued function of the following non-dimensional grouping:

$$E^{\frac{1}{2}}/(W^*(KA^*KD)^{\frac{1}{4}}))$$

E = dispersion coefficient, sq miles/day

W = pulse width in the stream (duration x stream velocity), miles

KA = reaeration rate coefficient, days^{-1}

KD = deoxygenation rate coefficient, days^{-1}

The relationship between this non-dimensional grouping and the ratio of DOMAX to the Streeter-Phelps DOMAX is shown in Figure (8).

The time from the introduction of the initial part of the square pollutant pulse to the point where DOMAX occurs is also of interest. It was found that if this time (t) in days is expressed in terms of the following non-dimensional grouping it will plot as a single valued function of the first non-dimensional grouping:

$$Et/W2$$

E = dispersion coefficient, sq miles/day

t = time from initial introduction of pollutant to time at which DOMAX occurs, days

W = initial pulse width, miles

A plot of this second non-dimensional grouping as a function of the first is shown in Figure (9). Computed results used to plot Figures 8 and 9 are shown in Table 1.

In summary, it has been shown that by the use of non-dimensional relationships a reasonably complete picture of the impact on the dissolved oxygen resources of the stream caused by a storm overflow can be estimated. However, if a closed-form expression for the time-distance profiles of dissolved oxygen deficit caused by an impulsive or spike load could be found a much more straight-forward and satisfactory method for estimating the impact of storm overflows would be possible.

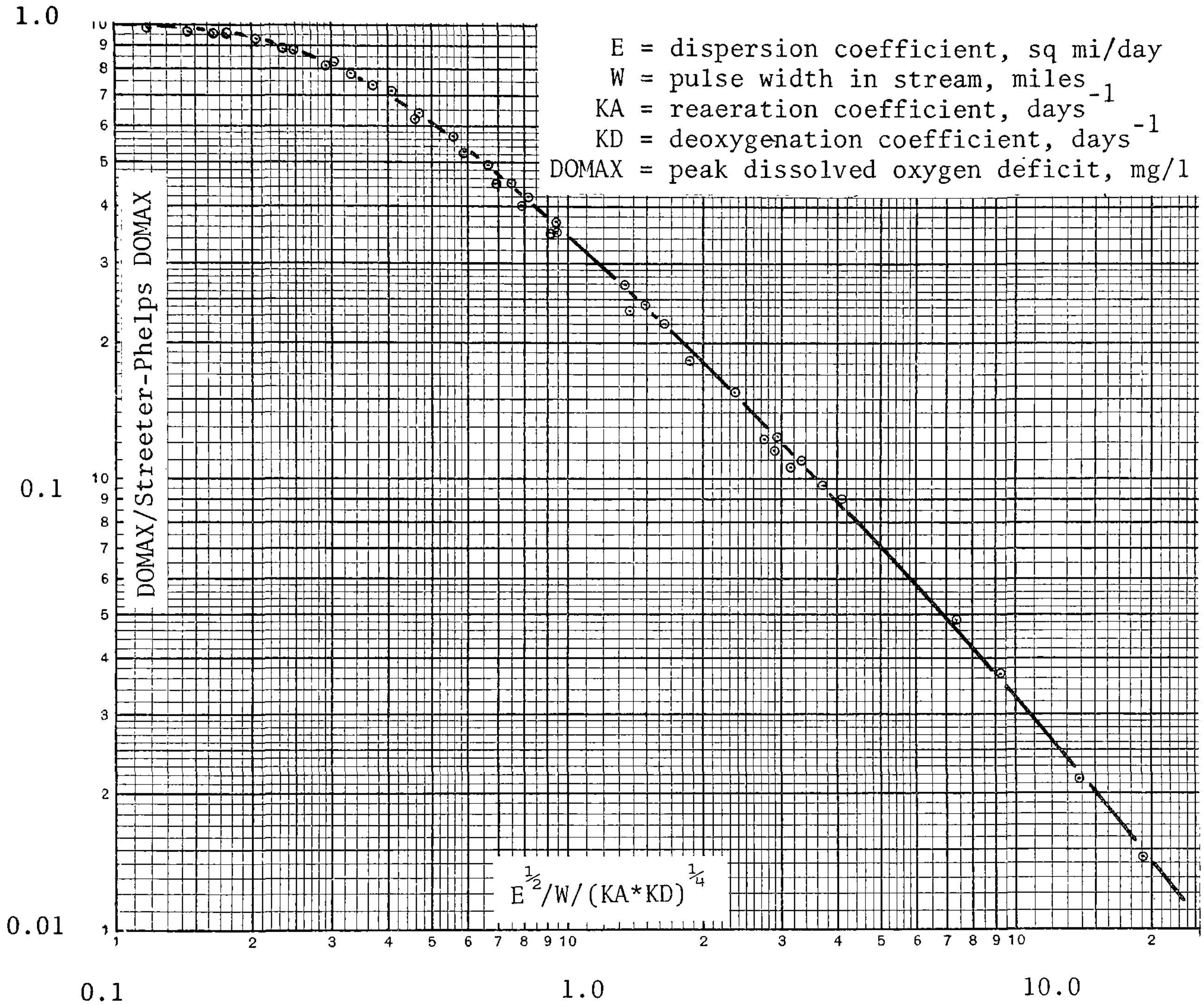

Figure 8. Correction factor for peak dissolved oxygen deficit to be applied to the value computed with the Streeter-Phelps model

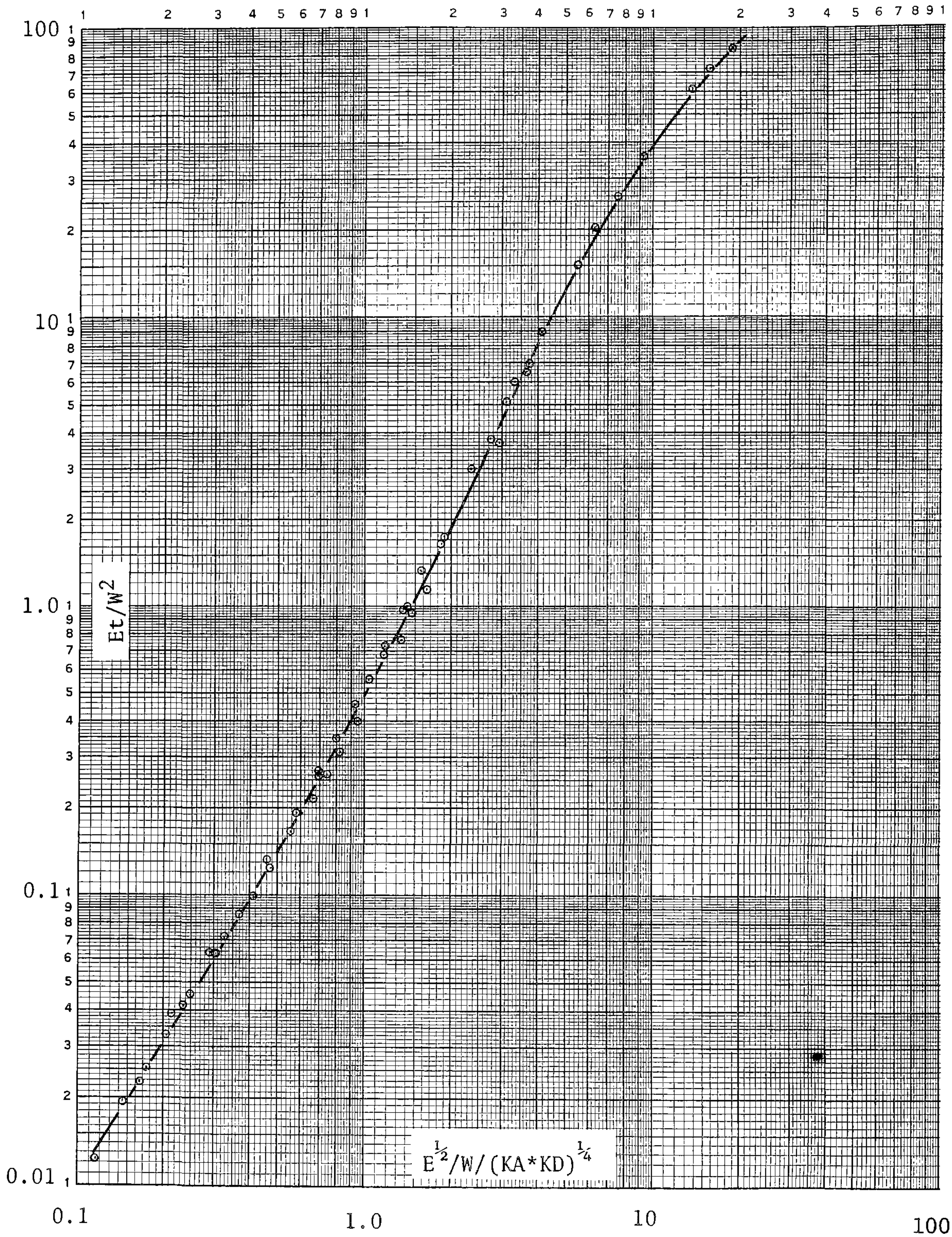

Figure 9. Time (t, days) between entry of leading edge of square stormwater pulse and time of peak DO deficit in terms of non-dimensional groupings where: E=dispersion coef, sq mi/day W=pulse width in stream, miles KA=reaeration coef, 1/day KD=deoxygenation coef, 1/day.

Table 1

Computed Results

Case	Parameter	Value	Quantity						
	Velocity, mi/day			2.4	6.0	12.0	24.0	32.15	48.0
	Pulse width (W), mi			0.1	0.25	0.5	1.0	1.34	2.0
I	K	4.68							
	K_r K_d	0.7	t* =	0.229	.250	.312	.417	.458	.490
	E =	0.1	$DOMAX/L_o$ =	0.01658	.03965	.06911	.09602	.10201	.10569
	SP-t* =	0.4774	N	2.351	.940	.470	.235	.175	.118
	$SP\text{-}DOMAX/L_o$	0.1071	T*	2.99	.400	.125	.042	.026	.012
II	K_a	4.68							
	$K_r = K_d$	0.7	t*	0.229	.240	.271	.354	.406	.458
	E	0.2	$DOMAX/L_o$	0.01178	.02878	.05339	.08435	.09452	.10276
	SP-t*	0.4774	N	3.324	1.330	.665	.332	.248	.166
	$SP\text{-}DOMAX/L_o$	0.1071	T*	5.98	.768	.217	.071	.045	.023
III	K_a	4.68							
	K_r K_d	0.7	t*	0.229	.240	.260	.323	.375	.437
	E	0.3	$DOMAX/L_o$	0.00963	.02371	.04503	.07569	.08781	.09940
	SP-t*	0.4774	N	4.071	1.628	.814	.407	.304	.204
	$SP\text{-}DOMAX/L_o$	0.1071	T*	8.97	1.152	.312	.097	.063	.033
IV	K_a -	4.68							
	$K_r = K_d$	0.7	t*	0.229	.229	.240	.260	.292	.344
	E	1.0	$DOMAX/L_o$	0.00528	.01316	.02588	.04868	.06146	.07970
	SP-t*	0.4774	N	7.433	2.973	1.487	.743	.555	.372
	$SP\text{-}DOMAX/L_o$	0.1071	T*	29.9	3.664	.960	.260	.163	.086
V	K_a	2.0							
	$K_r = K_d$	0.7	t*	0.406	.427	.490	.635	.708	.781
	E	0.1	$DOMAX/L_o$	0.02316	.05647	.10405	.16146	.17908	.19247
	SP-t*	0.8076	N	2.907	1.163	.581	.291	.217	.145
	$SP\text{-}DOMAX/L_o$	0.1989	T*	4.06	.683	.196	.635	.039	.020
VI	K_a	2.0							
	$K_r = K_d$	0.7	t*	0.354	.406	.417	.448	.469	.531
	E	1.0	$DOMAX/L_o$	0.00722	.01835	.03636	.07021	.09081	.12407
	SP-t* -	0.8076	N	9.193	3.677	1.839	.919	.686	.460
	$SP\text{-}DOMAX/L_o$	0.1989	T*	35.40	6.469	1.668	.448	.261	.133
VII	K_a	0.4							
	$K_r = K_d$	0.2	t*	0.844	1.635	1.729	1.760	1.792	1.865
	E	1.0	$DOMAX/L_o$	0.00344	.01104	.02213	.04389	.05827	.08496
	SP-t*	3.466	N	18.803	7.521	3.761	1.880	1.403	.940
	$SP\text{-}DOMAX/L_o$	0.25	T*	84.40	26.160	6.916	1.760	.998	.466
VIII	K_a	0.4							
	K_r K_d	0.4	t*	0.719	1.210	1.281	1.312	1.333	1.406
	E	1.0	$DOMAX/L_o$	0.00637	.01910	.03811	.07536	.09976	.14428
	SP-t*	2.5	N	15.811	6.324	3.162	1.581	1.180	.791
	$SP\text{-}DOMAX/L_o$	0.368	T*	71.90	20.160	5.124	1.312	.742	.352
IX	K_a	0.4							
	$K_r = K_d$	0.7	t*	0.604	.937	.948	.979	1.010	1.083
	E	1.0	$DOMAX/L_o$	0.01014	.02860	.05698	.11222	.14796	.21182
	SP-t* -	1.865	N	13.747	5.499	2.749	1.375	1.026	.687
	$SP\text{-}DOMAX/L_o$	0.474	T*	60.40	14.992	3.792	.979	.563	.271

The FORTRAN listing, input and output definitions, and sample printouts for the SWOPS computer program are given in the Appendix

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8. Eilers, Richard G. "Stream Model (STREAM2): Closed-Form Solution for Stream Simulation Using Euler Coordinates", Internal Memo, May 17, 1977.

9. Eilers, Richard G. "Stream Model (STREAM3): Crank-Nicolson Implicit Method Solution Using Euler Coordinates", Internal Memo, May 17, 1977.

10. Eilers, Richard G. "Stream Model (STREAM5): Crank-Nicolson Implicit Method Solution Using Lagrange Coordinates with a Single Influent Point", Internal Memo, May 17, 1977.

11. Eilers, Richard G. "Stream Model (STREAM6): Crank-Nicolson Implicit Method Solution Using Lagrange Coordinates with Multiple Influent Points", Internal Memo, May 17, 1977.

APPENDIX FOR SECTION 4

Table 2

Definitions of FORTRAN Symbols Used in SWOPS

Input Variables

NCASE	number of data cases to be calculated by the program.
LIST	alpha-numeric identification for each data case.
KR	rate constant for removal of BOD by deoxygenation and sedimentation, $days^{-1}$.
KN	rate constant for removal of ammonia nitrogen by deoxygenation, $days^{-1}$.
KA	rate constant for reaeration from the atmosphere, $days^{-1}$.
KD	rate constant for removal of BOD by deoxygenation, $days^{-1}$.
DX	distance interval for the numerical integration procedure, miles.
DT	time interval for the numerical integration procedure, days
XN	number of distance increments in the solution reach.
TM	number of time increments to be calculated.
EC	dispersion coefficient for BOD, sq mi/day.
EN	dispersion coefficient for ammonia nitrogen, sq mi/day.
ED	dispersion coefficient for dissolved oxygen, sq mi/day.
VEL	velocity of the stream, miles/day.
BEN	benthic oxygen demand, mg/l/day.

Table 2 (continued)

POINT	program control: distance point (number of increments from the first upstream point of the solution reach) at which the storm overflow enters the stream.
Q	total stream flow, cfs.
ZM	program control: number of time increments in the storm hydrograph.
DAY	program control: time point at which the program begins printing output, days.
Y1-Y5	set concentrations for the BOD or DO increment calculations, mg/l.
DL	program control: 0 = BOD calculation, 1 = DO calculation.
PRNT	program control: 0 = increment calculation printout, 1 = selected concentrations printout.
X1	distance increment at which program printout begins.
X2	distance increment at which program printout stops (note that X2-X1 must equal 20).
QNT(M)	volume of the storm overflow stream at time increment M, cfs
LOT(M)	BOD concentration of the storm overflow stream, mg/l.
LNT(M)	ammonia nitrogen concentration of the storm overflow stream, mg/l.

Output Parameters

N	distance increment along the stream.
DIST	distance along the stream at increment N, miles.
TT	time of travel downstream, days.
LO(N)	carbonaceous oxygen demand (5-day BOD) at N, mg/l.
LN(N)	nitrogen ultimate oxygen demand (exclusive of LO) at N, mg/l.
DO(N)	dissolved oxygen deficit at N, mg/l.

Table 2 (continued)

M	time increment of calculation.
IT	time interval at which the maximum BOD or DO occurs.
TTRAV	time of travel (I*DT), days.
DOMAX	maximum DO, mg/l.
LOMAX	maximum LO, mg/l.
SUM1-SUM5	sum of DO or BOD at time increment M based on Y1-Y5 inputs.
IM1-IM5	number of increments exceeding the Y1-Y5 inputs at time increment M.
DOT(I)	if DL=0 - DO deficit in the stream at distance interval I/initial BOD concentration in the stream.
	if DL=1 - BOD concentration in the stream at distance interval I/initial BOD concentration in the stream.

Table 3

FORTRAN Source Listing for the SWOPS Program

```
C     (SWOPS)
C     CRANK-NICOLSON IMPLICIT METHOD SOLUTION (LAGRANGE COORDINATES)
C     SINGLE INFLUENT POINT
C     BOD OR DO SUMS ARE CALCULATED
C     R. G. EILERS      SEPTEMBER 1977
      REAL KR,KN,KA,KD,LO(700),LN(700),LOS(700),LOT(30),LNT(30),
     . LOTA,LOMAX
      DIMENSION LIST(40),DO(700),QNT(30),DE(700),R(700),S(700),
     . DOT(700),NA(21)
      OPEN(UNIT=1,NAME='SWOPS.DAT',TYPE='OLD',FORM='FORMATTED',
     . READONLY)
      IN=1
      IO=6
      READ(IN,10) NCASE
   10 FORMAT(I2)
      DO 1000 III=1,NCASE
      DO 20 N=1,700
      LO(N)=0.
      LN(N)=0.
      DO(N)=0.
      LOS(N)=0.
      DE(N)=0.
      R(N)=0.
      S(N)=0.
      DOT(N)=0.
   20 CONTINUE
      DO 30 M=1,30
      QNT(M)=0.
      LOT(M)=0.
      LNT(M)=0.
   30 CONTINUE
      READ(IN,40) LIST
   40 FORMAT(40A2)
      READ(IN,50) KR,KN,KA,KD,DX,DT,XN,TM
      READ(IN,50) EC,EN,ED,VFL,BFN,POINT,Q,ZM
      READ(IN,50) DAY,Y1,Y2,Y3,Y4,Y5,DL,PRNT
      READ(IN,50) X1,X2
   50 FORMAT(8F10.0)
      NX=XN
      MT=TM
      IP=POINT+1.
      MZ=ZM
      N1=X1
      N2=X2
      NT=N1-1
```

```
      DO 55 N=1,21
      NA(N)=NT+N
   55 CONTINUE
      READ(IN,60) (QNT(M),M=1,30)
      READ(IN,60) (LOT(M),M=1,30)
      READ(IN,60) (LNT(M),M=1,30)
   60 FORMAT(10F8.0)
      WRITE(IO,70) LIST
   70 FORMAT(1H1,/////,20X,40A2,//)
      WRITE(IO,80) KR,KN,KA,KD,DX,DT,XN,TM
   80 FORMAT(8X,'KR',10X,'KN',10X,'KA',10X,'KD',10X,'DX',10X,'DT',
     . 10X,'XN',10X,'TM',/,8F12.4,/)
      WRITE(IO,90) EC,EN,ED,VEL,BEN,POINT,Q,ZM
   90 FORMAT(8X,'EC',10X,'EN',10X,'ED',9X,'VEL',9X,'BEN',7X,'POINT',
     . 11X,'Q',10X,'ZM',/,8F12.4,/)
      WRITE(IO,95) DAY,Y1,Y2,Y3,Y4,Y5,DL,PRNT
   95 FORMAT(7X,'DAY',10X,'Y1',10X,'Y2',10X,'Y3',10X,'Y4',10X,'Y5',
     . 10X,'DL',8X,'PRNT',/,8F12.4,/)
      WRITE(IO,97) X1,X2
   97 FORMAT(8X,'X1',10X,'X2',/,2F12.4,/)
      WRITE(IO,100) (QNT(M),M=1,30)
  100 FORMAT(7X,'QNT',/,10F12.4,/,10F12.4,/,10F12.4,/)
      WRITE(IO,110) (LOT(M),M=1,30)
  110 FORMAT(7X,'LOT',/,10F12.4,/,10F12.4,/,10F12.4,/)
      WRITE(IO,120) (LNT(M),M=1,30)
  120 FORMAT(7X,'LNT',/,10F12.4,/,10F12.4,/,10F12.4,////)
      IF (PRNT) 128,121,128
  121 IF (DL) 124,122,124
  122 WRITE(IO,123)
  123 FORMAT(8X,'M',9X,'I',5X,'TTRAV',5X,'LOMAX',6X,'SUM1',6X,'SUM2',
     . 6X,'SUM3',6X,'SUM4',6X,'SUM5',9X,'IM1',3X,'IM2',3X,'IM3',3X,
     . 'IM4',3X,'IM5',/)
      GO TO 128
  124 WRITE(IO,125)
  125 FORMAT(8X,'M',9X,'I',5X,'TTRAV',5X,'DOMAX',6X,'SUM1',6X,'SUM2',
     . 6X,'SUM3',6X,'SUM4',6X,'SUM5',9X,'IM1',3X,'IM2',3X,'IM3',3X
     . 'IM4',3X,'IM5',/)
      GO TO 137
  128 IF (DL) 134,130,134
  130 WRITE(IO,132) (NA(II),II=1,21)
  132 FORMAT(12X,'BOD IN STREAM / INITIAL BOD IN STREAM',//,
     . 4X,'M',21(2X,I3,1X),/)
      GO TO 137
  134 WRITE(IO,136) (NA(II),II=1,21)
  136 FORMAT(12X,'DO DEFICIT IN STREAM * 100 / INITIAL BOD ',
     . 'IN STREAM',//,4X,'M',21(2X,I3,1X),/)
  137 DT=DT/24.
      DX=VEL*DT
```

Table 3 (continued)

```
      IDAY=DAY/DT+1.
      EC=EC/DX**2
      EN=EN/DX**2
      ED=ED/DX**2
      NINC=MT+NX+2
      DT2=DT/2.
      I1=0
      I2=0
      I3=0
      I4=0
      I5=0
      SUM1=0.
      SUM2=0.
      SUM3=0.
      SUM4=0.
      SUM5=0.
      BOD=LOT(1)*QNT(1)/(QNT(1)+Q)
      DO 500 M=1,MT
      DOMAX=0.
      LOMAX=0.
      TTRAV=0.
      IT=0
      TRAV1=0.
      IT1=0
      TRAV2=0.
      IT2=0
      NINC=NINC-1
      A=-EC*DT2
      B=1.+EC*DT2*2.+KR*DT2
      C=-EC*DT2
      LO(1)=0.
      IP=IP-1
      K=NINC+1
      DO 150 N=2,K
      LOS(N)=LO(N)
      IF (N-IP) 140,138,140
  138 IF (M-MZ) 139,139,140
  139 QNTA=QNT(M)
      LOTA=LOT(M)
      GO TO 145
  140 QNTA=0.
      LOTA=0.
  145 DE(N)=QNTA*LOTA/(Q+QNTA)+EC*(LO(N-1)-2.*LO(N)+LO(N+1))*DT2+
     . LO(N)*(1.-KR*DT2)
  150 CONTINUE
      R(2)=DE(2)
      S(2)=B
      DO 160 N=3,K
```

Table 3 (continued)

```
      S(N)=B-C*A/S(N-1)
  160 CONTINUE
      DO 170 N=3,K
      R(N)=DE(N)-R(N-1)*A/S(N-1)
  170 CONTINUE
      LO(NINC+1)=R(NINC+1)/S(NINC+1)
      K=NINC-1
      DO 180 N=1,K
      I=NINC+1-N
      J=I+1
      LO(I)=(R(I)-C*LO(J))/S(I)
      IF (LOMAX-LO(I)) 175,175,180
  175 LOMAX=LO(I)
      TRAV1=I*DT
      IT1=I
  180 CONTINUE
      DO(1)=0.
      DO(NINC+2)=0.
      A=-ED*DT2
      B=1.+2.*ED*DT2+KA*DT2
      C=-ED*DT2
      K=NINC+1
      DO 190 N=2,K
      DE(N)=(DO(N-1)-2.*DO(N)+DO(N+1))*ED*DT2+DO(N)*(1.-KA*DT2)+
     . (LO(N)+LOS(N))*KD*DT2+BFN
  190 CONTINUE
      R(2)=DE(2)
      S(2)=B
      DO 200 N=3,K
      S(N)=B-C*A/S(N-1)
  200 CONTINUE
      DO 210 N=3,K
      R(N)=DE(N)-R(N-1)*A/S(N-1)
  210 CONTINUE
      DO(NINC+1)=R(NINC+1)/S(NINC+1)
      K=NINC-1
      DO 220 N=1,K
      I=NINC-N+1
      J=I+1
      DO(I)=(R(I)-C*DO(J))/S(I)
      IF (DOMAX-DO(I)) 215,215,220
  215 DOMAX=DO(I)
      TRAV2=I*DT
      IT2=I
  220 CONTINUE
      IF (DL) 224,222,224
  222 TTRAV=TRAV1
      IT=IT1
```

Table 3 (continued)

```
      GO TO 226
224   TTRAV=TRAV2
      IT=IT2
226   IM1=0
      IM2=0
      IM3=0
      IM4=0
      IM5=0
      DO 320 N=1,NINC
      IF (DL) 230,228,230
228   VAR=LO(N)
      GO TO 232
230   VAR=DO(N)
232   IF (VAR-Y1) 240,235,235
235   IM1=IM1+1
240   IF (VAR-Y2) 260,250,250
250   IM2=IM2+1
260   IF (VAR-Y3) 280,270,270
270   IM3=IM3+1
280   IF (VAR-Y4) 300,290,290
290   IM4=IM4+1
300   IF (VAR-Y5) 320,310,310
310   IM5=IM5+1
320   CONTINUE
      I1=I1+IM1
      I2=I2+IM2
      I3=I3+IM3
      I4=I4+IM4
      I5=I5+IM5
      SUM1=I1*DX*DT
      SUM2=I2*DX*DT
      SUM3=I3*DX*DT
      SUM4=I4*DX*DT
      SUM5=I5*DX*DT
      IF (PRNT) 340,370,340
340   DO 350 N=N1,N2
      IF (DL) 345,342,345
342   DOT(N)=LO(N)/BOD
      GO TO 350
345   DOT(N)=DO(N)/BOD*100.
350   CONTINUE
      WRITE(IO,360) M,(DOT(II),II=N1,N2)
360   FORMAT(I5,21F6.3)
      GO TO 500
370   IF (DL) 390,380,390
380   WRITE(IO,400) M,IT,TTRAV,LOMAX,SUM1,SUM2,SUM3,SUM4,SUM5,
     . IM1,IM2,IM3,IM4,IM5
      GO TO 405
```

Table 3 (continued)

```
 390 WRITE(IO,400) M,IT,TTRAV,DOMAX,SUM1,SUM2,SUM3,SUM4,SUM5,
    , IM1,IM2,IM3,IM4,IM5
 400 FORMAT(5X,I5,5X,I5,7F10.5,5X,5I6)
 405 IF (M-IDAY) 500,410,500
 410 WRITE(IO,420) M,DAY
 420 FORMAT(///,5X,'M =',I5,5X,'DAY =',F7.2,///)
     WRITE(IO,430)
 430 FORMAT(9X,'N',8X,'DIST',10X,'TT',10X,'LO',10X,'LN',10X,'DO',/)
     DO 450 N=1,NX
     DIST=(N-POINT)*DX+VEL*DAY
     TT=(N-POINT)*DT+DAY
     WRITE(IO,440) N,DIST,TT,LO(N),LN(N),DO(N)
 440 FORMAT(6X,I6,5F12.5)
 450 CONTINUE
 500 CONTINUE
     IF (PRNT) 1000,505,1000
 505 WRITE(IO,510) I1,I2,I3,I4,I5
 510 FORMAT(///,95X,5I6)
1000 CONTINUE
     CLOSE(UNIT=1)
     END
```

Table 4

Sample Printout from SWOPS (DL = 0, PRNT = 1)

BIOCHEMICAL OXYGEN DEMAND - DISTANCE INCREMENTS 40 TO 60   9-21-77

KR	KN	KA	KD	DX	DT	XN	TM
0.2500	0.0000	0.9800	0.2500	0.0000	0.2500	100.0000	50.0000

EC	EN	ED	VEL	BEN	POINT	Q	ZM
0.1000	0.1000	0.1000	24.0000	0.0000	50.0000	950.0000	4.0000

DAY	Y1	Y2	Y3	Y4	Y5	DL	PRNT
28.0000	6.0000	4.0000	2.0000	1.0000	0.5000	0.0000	1.0000

X1	X2
40.0000	60.0000

QNT

1294.0000	1294.0000	1294.0000	1294.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

LOT

110.0000	110.0000	110.0000	110.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

LNT

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

BOD IN STREAM / INITIAL BOD IN STREAM

M	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.982	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	1.006	0.956	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	1.007	0.994	0.939	0.038	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.008	1.007	0.996	0.991	0.923	0.052	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.988	0.993	0.987	0.908	0.065	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.001	0.040	0.970	0.990	0.983	0.893	0.078	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.002	0.054	0.953	0.987	0.979	0.879	0.090	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.003	0.068	0.936	0.984	0.975	0.866	0.101	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.004	0.080	0.920	0.980	0.971	0.853	0.112	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.005	0.093	0.905	0.976	0.966	0.840	0.122	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.000	0.006	0.104	0.891	0.972	0.962	0.829	0.131	0.011	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.000	0.008	0.115	0.877	0.968	0.957	0.817	0.140	0.013	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.001	0.010	0.125	0.864	0.964	0.953	0.806	0.149	0.015	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.001	0.011	0.135	0.851	0.960	0.948	0.796	0.157	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.001	0.013	0.144	0.839	0.955	0.943	0.786	0.165	0.019	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 5

Sample Printout from SWOPS (DL = 1, PRNT = 1)

BIOCHEMICAL OXYGEN DEMAND - DISTANCE INCREMENTS 40 TO 60   9-21-77

KR	KN	KA	KD	DX	DT	XN	TM
0.2500	0.0000	0.9800	0.2500	0.0000	0.2500	100.0000	50.0000
FC	**FN**	**FD**	**VEL**	**BEN**	**POINT**	**Q**	**ZM**
0.1000	0.1000	0.1000	24.0000	0.0000	50.0000	950.0000	4.0000
DAY	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**DL**	**PRNT**
28.0000	6.0000	4.0000	2.0000	1.0000	0.5000	1.0000	1.0000
X1	**X2**						
40.0000	60.0000						

QNT

1294.0000	1294.0000	1294.0000	1294.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

LOT

110.0000	110.0000	110.0000	110.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

LNT

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

DO DEFICIT IN STREAM * 100 / INITIAL BOD IN STREAM

M	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.125	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.135	0.368	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.136	0.393	0.599	0.025	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.136	0.395	0.646	0.820	0.047	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.393	0.648	0.894	1.032	0.076	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.001	0.027	0.639	0.899	1.138	1.235	0.109	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.002	0.050	0.874	1.145	1.378	1.431	0.148	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.003	0.079	1.099	1.388	1.614	1.620	0.191	0.012	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.000	0.000	0.000	0.000	0.000	0.005	0.115	1.315	1.627	1.845	1.801	0.238	0.017	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.008	0.155	1.522	1.862	2.073	1.976	0.289	0.022	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.001	0.012	0.200	1.721	2.093	2.296	2.146	0.342	0.029	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.001	0.017	0.249	1.913	2.320	2.515	2.309	0.399	0.037	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.001	0.023	0.302	2.097	2.543	2.730	2.468	0.459	0.046	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.002	0.030	0.359	2.275	2.762	2.941	2.621	0.521	0.056	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.002	0.039	0.418	2.447	2.976	3.147	2.769	0.584	0.067	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000

SECTION 5

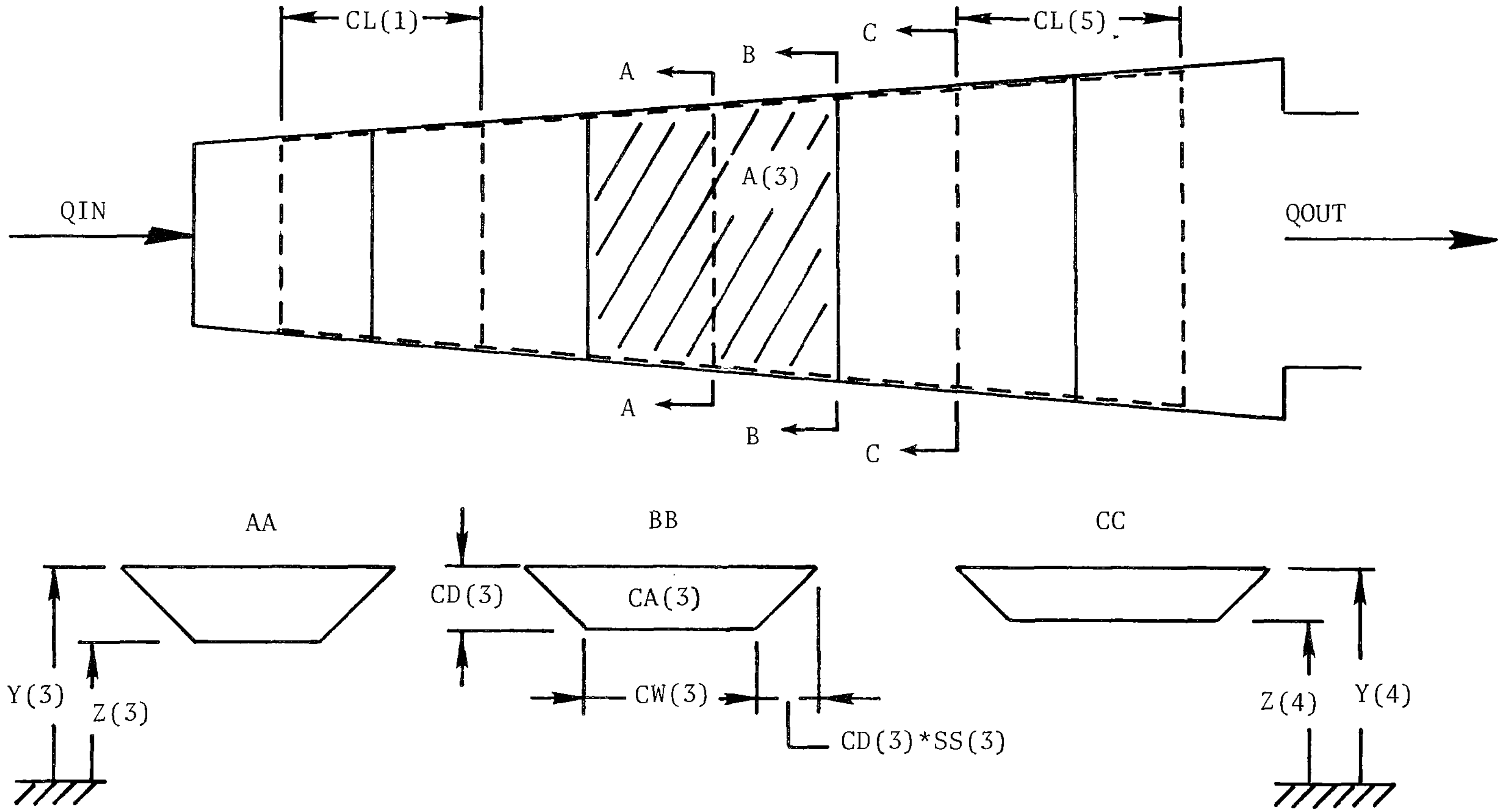
STORMWATER OVERFLOW HYDRAULIC STREAM MODEL (SWOHS)

BACKGROUND

A digital computer program (SWOHS) has been developed to study the dynamic (time-dependent) hydraulic response of the receiving stream to large stormwater overflow hydrographs entering the stream. This problem can be important in stream pollution studies because the reaeration coefficient is known to depend on both stream velocity and stream depth. The two physical laws used in the program are conservation of mass and conservation of momentum. When these two laws are expressed as partial differential equations they are sometimes (1) referred to as Saint-Venant's equations after an early (1848) hydrologist who expressed them for the first time. The program numerically integrates the St. Venant equations.

STREAM CONCEPTUALIZATION

In order to perform the integration, the stream is first divided into a number of increments by planes perpendicular to the direction of flow. A stream diagram with trapezoidal cross-sectional areas is shown divided by the solid lines in Figure 10. Terms used in Figure 10 are defined in Table 6. It is convenient to visualize the stream as a series of nodes (or junctions) connected by channels. The nodes are visualized as having the property of mass storage and the channels are visualized as having the properties associated with flow of water. For example, properties such as elevation (above sea level) of the stream bottom, elevation of the water surface, and stream surface area are associated with the nodes. Channels, on the other hand, are visualized as having the properties of length, width, velocity, roughness, and side-slope if the trapezoidal shape is assumed. A rectangular channel can be assumed by setting the value for the side-slope equal to zero. In Figure 10, stream increments bounded by the solid lines are represented by nodes and the increments bounded by the dashed lines are represented by channels. The node is visualized as being located at the center of the increment bounded by the the solid lines and the length of the channel which connects two adjacent nodes is, therefore, one-half the length of the two adjacent increments bounded by solid lines. Elevation of the stream bottom (Z) and elevation of the water surface (Y) are associated with the node. The depth of the node is simply Y-Z. The surface area (A) is also associated with the node. The depth of the channel connecting two nodes is taken as the average depth of the two nodes as shown by the first equation in Figure 10. The inter-connection between nodes and channels is described by the




```
1.  CD(I) = (Y(I) + Y(I+1) - Z(I) - Z(I+1))/2
2.  CA(I) = CD(I)*CW(I) + CD(I)**2*SS(I)
3.  WP(I) = CW(I) + 2*CD(I)*(1 + SS(I)**2)**0.5
4.  HR(I) = CA(I)/WP(I)
5.  A(I)  = (CW(I-1) + 2*CD(I-1)*SS(I-1))*CL(I-1)/2 + (CW(I) + 2*CD(I)*SS(I))*CL(I)/2
6.  A(1)  = CW(1) + 2*CD(1)*SS(1))*CL(1)
7.  A(5)  = (CW(5) + 2*CD(5)*SS(5))*CL(5)
```

Figure 10. Diagram of solution reach used in SWOHS showing cross-sections and basic equations used for depth, cross-sectional area, wetted perimeter, hydraulic radius, and surface area.

Table 6

DEFINITIONS FOR VARIABLES USED IN THE SWOHS PROGRAM

NCASE	number of cases to be computed by the program
LIST	alpha-numeric title of the data case
XI = NI	number of nodes
TN = NT	number of time increments to be calculated
DT	length of time increment, seconds
XNCH = NCH	number of channels
PRIN = NPRIN	time increment at which the program printout is to begin
TIDE = NTIDE	identifying number of the tidally forced node
FMC1	a constant having a value of 7.285 when English units (cfs and sec) are used and 4.9 when metric units are used
FMC2	a constant having a value of 16.1 when English units (cfs and sec) are used and 4.9 when metric units are used
T1, T2, T3, T4, T5, T6, T7	coefficients in sinusoidal expression for elevation of tidally forced node as a function of time
TP	tidal period, hours
STR = JSTRT	time increment at which the storm input begins
STO = JSTOP	time increment at which the storm input ends
XSTRM = NSTRM	number of node where storm flow enters
QSTRM(I)	volume of storm flow at time increment I during the duration of the storm, cfs (cu m/sec)

Table 6 (continued)

Z(I)	elevation (above some reference point) of the stream bed at the I'th node, feet (m)
Y2(I)	elevation of the water surface at the I'th node at time point T+DT, feet (m)
QSS(I)	volume flow of the steady stream into the I'th node, cfs (cu m/sec)
CV2(I)	velocity of the water in the I'th channel at time point T+DT, feet/second (m/sec)
CL(I)	length of the I'th channel, feet (m)
CW(I)	width of the flat bottom of the stream bed for the I'th channel, feet (m)
RN(I)	Manning roughness coefficient for the I'th channel
SS(I)	side-slope of the I'th channel
NDAM(I)	vector to indicate the location of the dam/weir controlling the flow from the I'th node - if the integer I appears in the I'th row of NDAM a dam/weir controls the flow out of the I'th node
W1(I)	first weir coefficient for the I'th node
W2(I)	second weir coefficient for the I'th node, ft (m)
W3(I)	third weir coefficient for the I'th node
NCN(I,1)	number of the upstream node for the I'th channel
NCN(I,2)	number of the downstream node for the I'th channel
NCN(I,3)	set equal to one if the I'th channel enters the downstream channel laterally
CV1(I)	velocity of the water in the I'th channel at time point T, feet/second
Y1(I)	elevation of the water surface at the I'th node at time point T, feet (m)
CD1(J)	depth of J'th node at time T, feet (m)

Table 6 (continued)

CD2(J)	depth of J'th node at time T+DT, feet (m)
A1	one-half surface area of the channel at time T, sq ft (sq m)
A2	one-half surface area of channel at time T+DT, sq ft (sq m)
CA1	cross-sectional area of channel at time T, sq ft (sq m)
CA2	cross-sectional area of channel at time T+DT, sq ft (sq m)
QO1	flow leaving a node through all downstream channels at time T, cfs (cu m/sec)
QO2	flow leaving a node through all downstream channels at time T+DT, cfs (cu m/sec)
QI1	flow entering a node from all upstream channels at time T, cfs (cu m/sec)
QI2	flow entering a node from all upstream channels at time T+DT, cfs (cu m/sec)
QIN(I)	steady flow entering the I'th node over and above the flow entering through numbered channels and is equal to QSS(I) + QSTRM(I), cfs (cu m/sec)
NO	number of upstream node
NI	number of downstream node
WP1	wetted perimeter of channel cross-sectional area at time T, feet (m)
WP2	wetted perimeter of channel cross-sectional area at time T+DT, feet (m)
HR1	hydraulic radius of channel equal to the cross-sectional area/wetted perimeter at time T, feet (m)
HR2	hydraulic radius of channel equal to the cross-sectional area/wetted perimeter at time T+DT, feet (m)

matrix NCN(NCH,3) with one row for each channel. The first column of NCN contains the number of the upstream node (IN) and the second column contains the number of the downstream node (NO). The third column refers to the direction at which side streams enter the main stream and this will be discussed later in this report.

PHYSICAL LAWS USED

The two independent variables used in the problem are time and distance along the stream. The Euler coordinate system which uses a point attached to the river bank as the zero distance point is used for the computation because stream properties such as roughness coefficients and shape of the trapezoidal cross-section are associated with points along the stream bed. As mentioned earlier, the purpose of the program is to apply the principle of conservation-of-mass to each node and the principle of conservation-of momentum to each channel. For example, each node can be thought of as a control volume and if the net flow entering the node (flow entering - flow leaving) is positive the water depth in the node will increase. Alternatively, if the net flow into the node is negative the water level in the node will decrease. The principle of conservation-of-momentum applied to each channel requires that if the resultant of the force system acting on the water within the channel acts upstream the flow in the channel will be decelerated and if the net force acts downstream the water will be accelerated. The forces acting on the mass within the channel consist of the gravity force, the pressure acting across the vertical dividing planes, the frictional force exerted by the stream bed and the force equivalent to the net momentum flux entering and leaving the control volume. For clarification, these concepts will be discussed with respect to the stream diagram shown in Figure 10.

SYSTEMS OF UNITS USED

Definitions given in the following table are expressed in the British gravitational system which used pounds as the unit of force, feet as the unit of length, and seconds as the unit of time. When the British system is used the value of 7.285 for FMC1 and a value of 16.1 for FMC2 are supplied as input. When the metric system is used, the values for FMC1 and FMC2 are both 4.9. The metric gravitational system uses kilograms as the unit of force, meters as the unit of length and seconds as the unit of time. Corresponding units for other variables used in the program are listed below:

	British	Metric
1. Force	lbf	kgf
2. Length	ft	m
3. Time	sec	sec
4. Acceleration of Gravity, g	$32.2\ ft/sec^2$	$9.8\ m/sec^2$

5.	Mass (force/g)	slug	metric slug
6.	Volume	cu ft	cu m
7.	Velocity	ft/sec	m/sec
8.	Volume Flow	cfs	cu m/sec
9.	Area	sq ft	sq m

CONSERVATION-OF-MASS

To apply the conservation-of-mass principle to each node the surface area of the node must first be computed. The nodal surface area is taken as one-half the surface area of the two adjacent channels except for terminal nodes with only one adjacent channel. For terminal nodes the nodal area is taken as equal to the area of the one adjacent channel. This is demonstrated by equations 5-7 in Figure 10. The net flow into each node is then computed and divided by the nodal area to find the rate of change of the elevation of the water surface in the node. When this rate is multiplied by the time increment (DT) an incremental change in water surface elevation is found.

When the flow out of a node is controlled by a dam or a weir the weir equation must be used to compute the flow out of the node. No distinction between dams and weirs is made in the program because the flow is computed from the following relationship in both cases.

QOUT = W1*(Y(5)-W2)**W3 (2)

In this relationship QOUT is the flow over the weir (cfs), Y(5) is the elevation of the water surface in the 5'th node and W1, W2, and W3 are weir constants supplied as input to the program.

CONSERVATION-OF-MOMENTUM

To apply the principle of conservation-of-momentum to each channel it is first necessary to compute the average channel depth, the channel cross-sectional area, the wetted perimeter, and the hydraulic radius as shown by equation 1-4 in Figure 10. Having computed these variables the mass of water in the channel is known and the principle of conservation-of-momentum can be applied by dividing the net force acting on the water by the mass to find the time rate of change of velocity within the channel. For example, the gravity force on the I'th channel can be computed as follows:

FG = 62.4*CA(I)*(Z(NI)-Z(NO)) (3)

In this equation NI is the number of the upstream node and NO is the number of the downstream node.

The force caused by pressure acting across the upstream and downstream boundry planes can be written as follows:

$$FP = 62.4*CA(I)*((Y(NI)-Z(NI))-(Y(NO)-Z(NO))) \qquad (4)$$

It can be seen that the gravity and pressure forces can be added to give the following simplier result.

$$FPG = FG+FP = 62.4*CA(I)*(Y(NI)-Y(NO)) \qquad (5)$$

The frictional force which acts in a direction opposite to the direction of flow can be written as follows:

$$FF = 62.4*CA(I)*CL(I)*CV(I)**2*RN(I)**2/2.21/HR(I)**(4/3) \qquad (6)$$

Finally, a momentum force must be computed if the momentum flux entering the channel differs from the momentum flux leaving the channel. For the simple case where momentum flux enters the I'th channel only from the upstream (I-1)'th channel the momentum force (FM) can be computed as follows:

$$FM = 62.4/32.2*(CA(I-1)*CV(I-1)**2 - CA(I)*CV(I)**2) \qquad (7)$$

The incremental change in velocity in the I'th channnel (DV) can now be found from the following relationship. The mass of water (M) in the channel is computed simply as 62.4*CA(I)*CL(I)/32.2.

$$M*(DV/DT) = FPG + FM - FF \qquad (8)$$

A number of complications are involved and these will be discussed later on, but the equations presented here show in a rudimentary way the physical laws used in the program.

NUMERICAL INTEGRATION SCHEME USED

Because of the quadratic terms which appear in equation (6) for the frictional force (FF) an implicit numerical integration scheme such as the Crank-Nicolson method could not be used for this problem. Instead, an iterative scheme was used which is based on making the derivatives used for advancing the solution one time increment (DT) equal to the mean of their values at T and T+DT. Also, the conservation-of-mass equations and the conservation-of-momentum equations must be solved simultaneously. Therefore, the value of each variable at T and T+DT must be separately identified. For example, Y1(I) is the water elevation in the I'th node at time = T and Y2(I) is the water elevation in the I'th node at time = T+DT. All necessary variables are similarly marked.

Values for all variables at time = 0 must, of course, be supplied as input to the program. To advance the solution by one time increment (DT) the value of all variables at time = T+D are set equal to their values at time = T. This constitutes the first guess at the value of the variables at time = T+DT.

All of the channels are then considered one-at-a-time. The upstream node of the I'th channel (NI) is identified and value for Y2(NI) at the later time point (T+DT) is found from equations for conservation-of-mass using the most up-to-date estimates for all variables at the T+TD time points. The downstream node (NO) for the I'th channel is identified and a new value for Y2(NO) is found in the same way. The difference between the new computed values for Y2(NI) and Y2(NO) and the previous values is noted. If they differ by more than a set tolerance a flag (NTEST) is set equal to one.

Continuing our consideration of the I'th channel, all forces acting on the water within the channel are evaluated using the mean of values at time = T and the most up-to-date values at time = T+DT. The mean mass of water in the channel between T and T+DT is computed and equation (8) is used to compute a new value of CV2(I) for T+DT. The newly computed value is compared to the previous value and if they differ by more than some set tolerance the flag (NTEST) is set equal to one. This procedure is performed for each channel in turn.

After all channels have been considered and new values for Y2(J) and CV2(I) have been found the flag NTEST is examined and if it has been set equal to one the whole procedure is repeated using values for Y2(J) and CV2(I) computed during the previous iteration. This iteration is repeated until NTEST no longer equals one indicating that the computation has converged. The final values for Y2(J) and CV2(I) are then assumed to be the correct values and are printed. The numerical integration scheme is then advanced one time increment and the whole procedure described above is repeated until the iteration converges. These values are printed and the solution advances to the next time point until the specified number (NT) of time increments have been computed.

DESCRIPTION OF INPUT REQUIRED

The time increment to be used (DT) is read in as seconds and the number of time increments over which a solution is to be found is read in as NT. The number of channels in the stream system is read as NCH. The placement of nodes and channels is described by a matrix (NCN) with NCN rows and three columns. The I'th row of NCN, corresponding to the I'th channel, contains the identifying number of the upstream node (NI) in the first column, the number of the downstream node (NO) in the second column, and a one in the third column if the I'th channel enters the main stream laterally. If a zero appears in the third column the direction of flow in the I'th channel is assumed to be in the same direction as the flow in the downstream channel. This distinction is made to properly handle the momentum flux term.

Two constants, FMC1 and FMC2, are read-in to provide the flexibility of changing the system of units. When the British system (lbf, ft, sec) is used the values for FMC1 is 7.285 (32.2/2/2.21)and the value for FMC2 is 16.1 (32.2/2). When the metric system (kgf, meter, sec) is used the value for both FMC1 and FMC2 is 4.9 (9.8 m/sec^2 divided by 2).

Before the descriptive parameters for the stream are read in, the analyst should assure himself that the variables used to describe the stream are feasible. That is, they must be capable of corresponding to some steady-state solution. Otherwise, the numerical integration procedure can drive Y(J) below Z(J) corresponding physically to the node running dry. A program HYSS described in the Appendix has been developed to check the input to guard against this problem. When a feasible steady-state solution is known the elevation of the stream bed, Z(J), and the elevation of the water surface, Y2(J) is read into the program for each node at time = 0. For each channel the length, CL(I), the width of the trapezoidal stream bottom, CW(I), the side-slope, SS(I), the Manning roughness coefficient, RN(I), and the stream velocity, CV2(I) are read into the program.

A steady flow into each node is provided for by the input variables QSS(J) where J is the node number. Normally, QSS(J) is provided only for the terminal nodes although this restriction is unnecessary. The hydrograph of the stormwater overflow is divided into time intervals (DT) of the same length as the interval used in the integration scheme. The volume flow (cfs) read from the hydrograph at the mid-point of each interval is read into the program as the vector QSTRM which provides for a maximum of 50 values. The number of the node to which QSTRM is applied is input as NSTRM. The variable L indexes time in the program. When L equals JSTRT the first value of QSTRM is applied to node NSTRM. Successive values of QSTRM are applied for all L values greater than JSTRT until L exceeds JSTOP. Thus, the first and last values of L over the storm duration are JSTRT and JSTOP.

The number of the node which is tidally controlled is input as NTIDE. The elevation of the node numbered NTIDE is computed as follows:

Y(NTIDE = T1 + T2*SIN(WT) + T3*SIN(2*WT) + T4*SIN(3*WT) + T5*COS(WT)

+ T6*COS(2*WT) + T7*COS(3*WT) (9)

WT = 6.283185*L*DT/TP/3600 (10)

Thus, the constants T1-T7 must be supplied as input and the period in hours (TP) must also be supplied as input. In this expression for the elevation of the NTIDE node; L is the number of time increments since time = 0.

When flow into or out of a node is controlled by a weir or a dam (hydraulically they are both treated the same) this is specified by the input vector NDAM with J rows; one for each node. When the flow out of the J'th node is controlled by a weir/dam this is indicated by placing the number J in the J'th row of NDAM. If flow into the J'th node is controlled by a weir/dam the number of the node (KD) which controls the flow into the J'th node is placed in the J'th row of NDAM. The three weir coefficients are provided by the vector W1(J), W2(J), and W3(J) where J is the node number of the node controlled by the weir/dam. The flow out of the J'th node is computed as follows:

$$QO = W1(J)*(Y(J) - W2(J))**W3(J) \tag{11}$$

Flow into the J'th node controlled by the KD'th node is computed as follows:

$$QI = W1(KD)*(Y(KD) - W2(KD))**W3(KD) \tag{12}$$

If a dam/weir is located in a non-terminal node no channel is assigned downstream of the node. The logic of the program is arranged to assume that the momentum flux of the channel upstream of the dam/weir is dissipated by the dam/weir and no incoming momentum flux is assigned to the channel downstream of the node on the downstream side of the dam/weir.

LOGICAL STRUCTURE OF THE PROGRAM

When input to the program is completed, the first step in advancing the solution from time = 0 to time = DT is to set the values of node water surface elevation and channel velocity at time = DT equal to their values at time = 0. This constitutes the first guess at the solution at time = DT. At any time other than zero this procedure involves setting Y1(J) equal to Y2(J) and CV1(I) equal to CV2(I). Thus, in order to use the same logic in the program, initial values for Y and CV are input as Y2(J) and CV2(I).

The iterative numerical integration procedure for advancing the solution by one time increment continues by considering each channel (I) one-at-a-time and first identifying the node (K) downstream of the I'th channel. The first column of NCN is scanned to identify channels which flow out of node K and the second column of NCN is scanned to identify channels which flow into node K. When a channel which flows into or out of node K is identified the other terminal node of the channel is identified from NCN and called M. The depth, cross-sectional area, and surface area of any channel flowing into or out of node K is then computed from the properties of nodes K and M. The variable NSEG is given the value of the total number of channels which flow into or out of node K.

A test is made to determine if a dam/weir controls flow into or out of node K. If NDAM(K) equals K the flow out of node K over a weir is computed from the inputted weir coefficients and the elevation of water surface in node K. If NDAM(K) is not zero, and not K, it must equal KD and the flow into node K is controlled by the water level in node KD. The flow from node KD into node K is computed from the inputted weir coefficients W1(KD), W2(KD), and W3(KD). If NSEG is greater than one, the surface area of node K is taken as one-half the sum of the surface areas of all incoming or outgoing channels. If NSEG is less than two, the surface area of the one channel entering or leaving node K is taken as the surface area of node K.

The net flow into node K is then computed. Flow into node K includes the sum of the flow in all channels entering node K, the steady flow QSS(K), the flow entering over a weir from an upstream node, and stormwater overflow. If K equals NSTRM and the time index (L) falls between JSTRT and JSTOP inclusive, the volume flow of stormwater entering node K is given by the vector QSTRM. The flow out of node K is found by summing the flow in all channels leaving node K with flow over a weir, if one exists at node K. Subtracting the flow out of node K from the flow into node K gives the net in-flow which can then be divided by the surface area of node K to find the change in water surface elevation at node K. The new value for Y2(K) is then compared to the previously computed value YTEMP(K) and if they differ (in absolute value) by more than a specified tolerance (0.01 ft is used in the program) the flag NTEST is set equal to one. In either case YTEMP(K) is set equal to the newly computed value for Y2(K).

If NTIDE equals K the whole procedure outlined above is bypassed and the level of water in node K is found from the tidal equation (9). The same procedure is then repeated for the node upstream of the I'th channel unless that node is tidally controlled.

Conservation of momentum in the I'th channel is considered next. To consider conservation of momentum in channel I, the depth, cross-sectional area, wetted perimeter, and hydraulic radius of channel I are computed from the most up-to-date values of variables involved. The mass of water in the I'th channel is computed as follows:

$$M = CA(I)*CL(I)*62.4/32.2 \tag{13}$$

The derivative of velocity over the time interval DT is expressed as follows:

$$DVDT = (CV2(I) - CV1(I))/DT \tag{14}$$

The principle of conservation of momentum as applied to the I'th channel is expressed as follows:

$$M*MVDT = FPG + FM - FF \tag{15}$$

Where FPG is the sum of the gravity and pressure forces acting on the mass M, FF is the frictional force acting on M, and FM is the equivalent momentum force caused by the net momentum flux entering and leaving the I'th channel.

To assure good accuracy in the numerical integration, all variables must be averaged over the interval DT. That is, variables used in applying the principle of conservation of momentum must be the mean of values existing at T and T+DT. In terms of the average values of all variables, the three forces acting on the I'th channel are expressed as follows:

$$FPG = 62.4*CA(I)*(Y(NI) - Y(NO)) \tag{16}$$

$$FF = 62.4*CA(I)*CL(I)*ABS(CV(I))*CV(I)*RN(I)**2/2.21/HR(I)**(4/3) \tag{17}$$

$$FM = 62.4/32.2*(CA(J)*ABS(CV(J))*CV(J) - CA(I)*ABS(CV(I))*CV(I)) \tag{18}$$

The absolute values are used in the expressions for FF amd FM to make sure that the FF force acts in a direction opposite to the direction of flow and that the momentum force acts downstream when the momentum flux entering from upstream exceeds the momentum flux leaving the I'th channel. Also, in the expression for FM the first term in the brackets must be summed for all channels (J) upstream of the I'th channel.

When equations 13, 14, 16, 17, and 18 are substituted into equation 15 it can be seen that the resulting equation is quadratic in terms of the unknown variable CV2(I). This can be readily solved if care is taken to select the proper root. If we let X be equal to CV2(I) the quadratic equation can be put into the following form:

$$AX^2 + BX + C = 0 \tag{19}$$

The values for the constants in this equation can be shown to be as follows:

```
A    =  1/DT

B    =  7.285*RN(I)**2/HR2(I)**(4/3) + 0.5/CL(I)

C    =  CP1 + CP2

CP1  =  7.285*ABS(CV1(I))*CV1(I)*RN**2/HR1**(4/3)          (20)

     -  16.1*(Y1(NI)+Y2(NI)-Y1(NO)-Y2(NO))/CL(I)

     -  CV1(I)/DT

CP2  =  (ABS(CV1(I)) - FMI1 - FMI2)*CV1(I)/CL(I)/2

FMI1 =  CA1(J)*ABS(CV1(J))*CV1(J)/CA1(I)

FMI2 =  CA2(J)*ABS(CV2(J))*CV2(J)/CA2(I)
```

The index (J) which appears in the expression for FMI1 and FMI2 must be summed over all channels which flow into the I'th channel. If the number which appears in the third column of NCN of the row corresponding to the J'th channel is one, the direction of flow in the J'th channel is perpendicular to the flow in the receiving channel and the momentum flux (FMI1 or FMI2) is set equal to zero. If the value FMI1 is found to have a value of zero the I'th channel is assumed to be a terminal channel and the values of FMI1 and FMI2 are computed as follows: The upstream node of the I'th channel is K.

```
FMI1 = QSS(K)*CV1(I)/CA1(I)          (21)

FMI2 = QSS(K)*CV2(I)/CA2(I)          (22)
```

FMI1 will also be zero if the I'th channel is downstream of a dam/weir but since QSS(K) will be zero the value for FMI1 and FMI2 will remain zero. Care should be taken to make QSS(K) zero at any node downstream of a dam/weir. This type of calculation is made for each of the NCH channels and if the newly computed value for CV2(I) differs from the previously computed value VTEMP(I), by more than the tolerance (0.001 ft/sec) the flag NTEST is set equal to one and a second iteration is necessary. If NTEST is found to be zero, indicating that the iteration has converged, Y1(I) is set equal to Y2(I) and CV1(I) is set equal to CV2(I) and the numerical integration is advanced one time increment. This procedure is continued until the solution has been found for NT time increments. The results of the computation are printed after each time step.

SOLUTION FOR HYPOTHETICAL PROBLEMS

The first problem selected for testing SWOHS involved a prismatic stream flowing at a constant velocity into which a square hydrograph of 15 minutes duration and a volume flow roughly equal to the stream flow was introduced. The trapezoidal cross-section of the stream had a width of 30 ft and a side-slope of four. The solution reach was divided into 20 lengths of 4000 ft. each. The Manning roughness coefficient was set at 0.023 and the bed slope at 0.0005 ft/ft. Before entry of the square hydrograph, the steady stream flow was 598.2 cfs, the velocity of flow was 3.036 ft/sec and the depth in each node was 4.207 ft. The length of the time interval used in the numerical integration was 300 seconds.

To simulate the effect of a large stormwater overflow a square hydrograph with a duration of 15 minutes and a volume flow of 600 cfs was introduced into the 5'th node. The computed elevations of the water surface in each of the 20 nodes above the steady-state elevation is shown for one-hour intervals after entry in Figure 11. From Figure 11 it can be seen that the maximum increase in water surface of 1.2 ft occurred at the time corresponding to the end of the square hydrograph. One hour later the pulse has spread significantly and the amplitude has reduced to about 0.35 ft. In general, the square hydrograph produced a wave which moved downstream at a velocity of about 4 ft/sec and attenuated rapidly.

The computed flow over the weir in node 20 minus the steady-state flow of 598.2 cfs is shown plotted in Figure 12. Thus, in about 11.4 miles of stream the peak of 600 cfs introduced at node 5 has been reduced to about 46 cfs. The volume of flow introduced (540,000 cu ft) equals the area under the hydrograph at node 20 as shown by the cumulative curve in Figure 12. The interval over which the 600 cfs overflow was introduced is also shown in Figure 12 between the 10'th and 12'th time intervals. Thus, about 5 hours elapsed between the time the overflow was introduced and the time at which the flow over the weir began to increase. The fact that the integrated flow over the weir equaled the integrated flow into the reach shows that the conservation-of-mass principle is being satisfied. A simple closed-form expression for the computed result is not available to check the numerical integration scheme.

One of the reasons for simulating the hydraulic response of the stream to large stormwater hydrographs was to consider the effect on the reaeration coefficient. Many relationships are available for estimating the reaeration

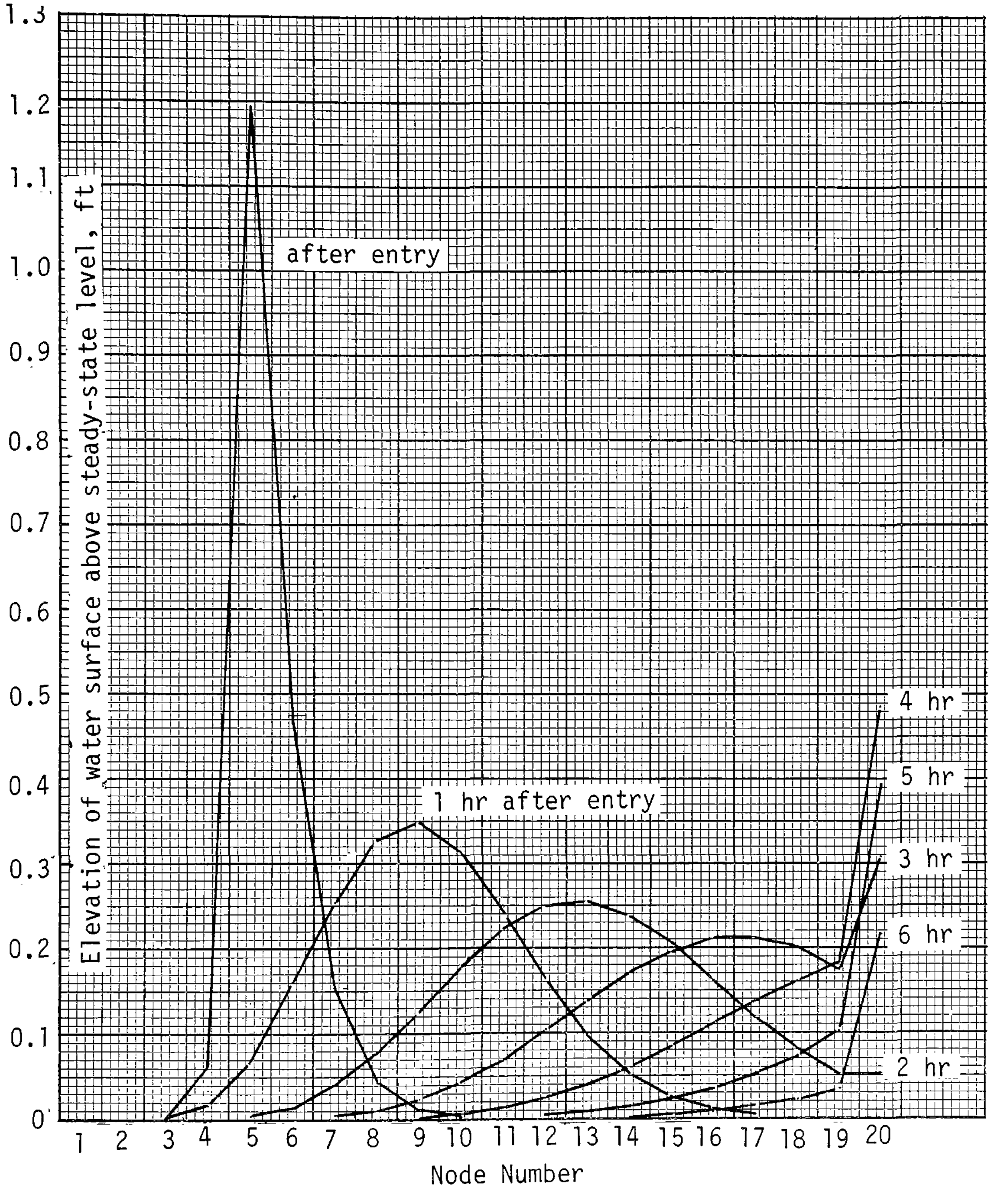


Figure 11. Computed wave profiles caused by a square hydrograph (600 cfs-15 min duration) entering a steady-state stream with 598.2 cfs volume flow.

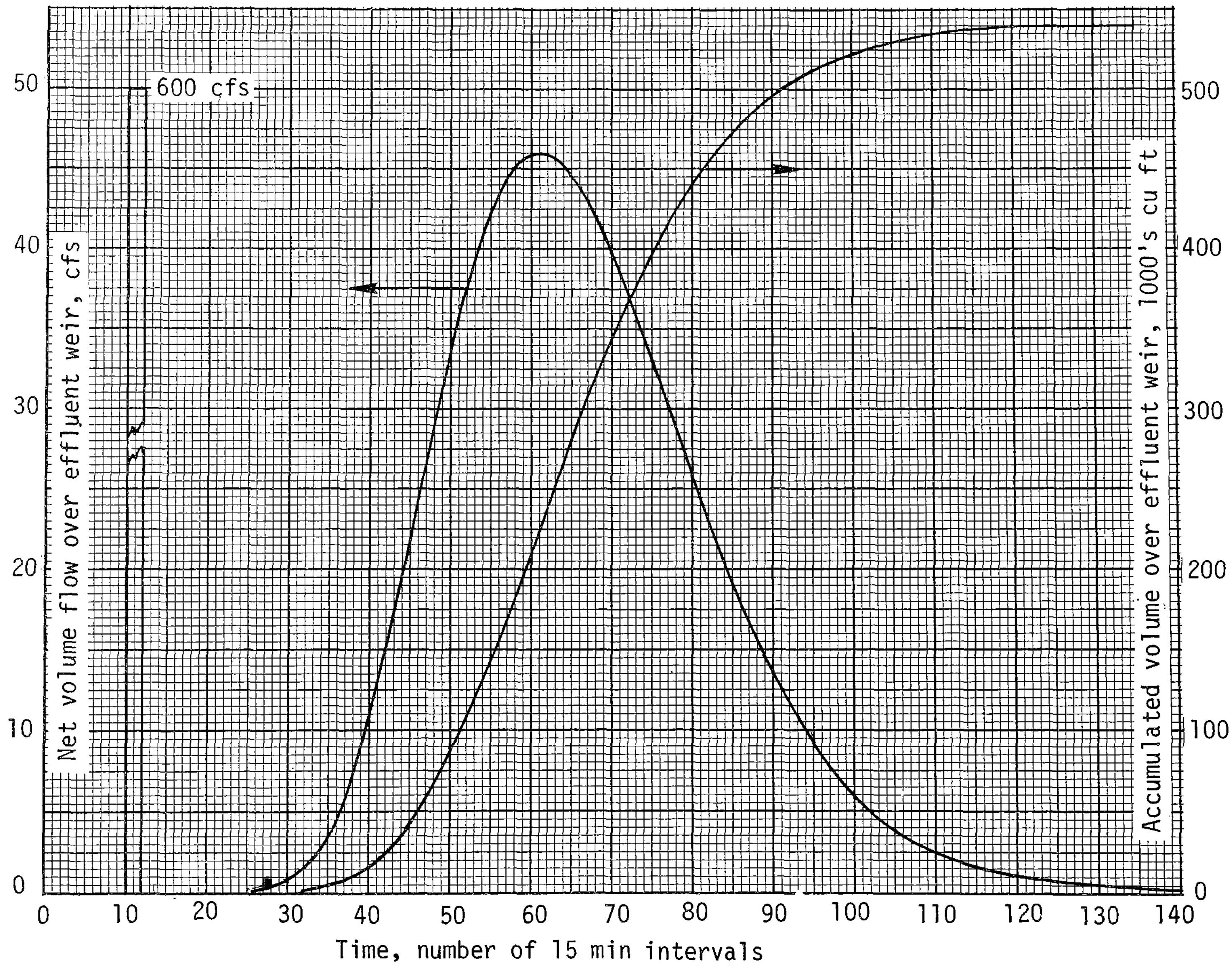


Figure 12. Computed flow over terminal weir minus steady-state flow resulting from a square hydrograph (600 cfs-15 min duration) introduced 11.4 miles upstream of the weir.

coefficient as a function of the depth and velocity of the stream. The work of O'Connor and Dobbins (2) is widely used for this purpose. The two relationships developed by O'Connor and Dobbins, one for deep and one for shallow streams are shown by equations (23) and (24) below:

$$K_a = 1440\ (DU)^{\frac{1}{2}}/H^{1.5} \qquad \text{(H greater than 5 ft)} \tag{23}$$

$$K_a = 1016.4\ D^{\frac{1}{2}}S^{\frac{1}{4}}/H^{1.25} \qquad \text{(H less than 5 ft)} \tag{24}$$

K_a = reaeration coefficient, days^{-1}

D = diffusivity, sq ft/hr (80 x 10^{-6} sq ft/hr typical value)

H = water depth, ft

S = slope of stream channel, ft/ft

U = stream velocity, ft/sec

If the typical value of 80 x 10^{-6} sq ft/hr for diffusivity is substituted into equations (23) and (24) the following two equations result:

$$K_a = 12.9\ U^{\frac{1}{2}}/H^{1.5} \tag{25}$$

$$K_a = 48.5\ S^{\frac{1}{4}}/H^{1.25} \tag{26}$$

In the first example the steady state parameters for the stream were 3.036 ft/sec for velocity, 4.207 ft for depth, and 0.0005 for slope. Substituting these values into equations (25) and (26) yields an estimate of 2.6 days^{-1} from equation (25) and 1.2 days^{-1} from equation (26). Since the depth of the stream is less than 5 ft equation (26) should be used according to O'Connor and Dobbins.

The greatest change in the stream characteristics occurred just after the 600 cfs event in node 5 where the depth increased by 1.3 ft and the velocity increased by 0.73 ft/sec. Substituting these into equation (25) gives an estimate of K_a of 1.94 instead of 2.6 days^{-1}. From equation (26) the estimate for K_a is 0.86 days^{-1} instead of 1.2 days^{-1}. Thus, the change is about 25% in both cases and this is the peak variation which lasts a very short period of time. Since the correct value of the reaeration coefficient is probably not known within 25% it can be concluded that for the problem selected the effect of the hydraulic transient on the value of K_a is negligible.

The second hypothetical problem was taken from the report (3) on the RECEIVE-II model written by the Raytheon Company. This problem which is described in Section V of the Raytheon report involves one tidally controlled terminal node and one dam/weir located in the center of the solution reach. A diagram for the problem is given in Figure 13. The solution reach is divided into ten 2000 meter lengths. The dam/weir is located in the 5'th node and the 10'th node is tidally controlled. The steady flow into the upstream terminal node is 5.66 cu m/sec and a second steady flow of 0.566 cu m/sec enters the 3'rd node. The solution showed that the elevation of the water surface in nodes 1-5 and the stream velocity in channels 1-4 approach a steady-state condition. For nodes 6-10 the water surface elevation as a function of time beginning at 750 minutes and ending at 1500 minutes from time = 0 is shown in Figure 14. The water surface elevation in node 6 is essentially constant and the water surface elevation in node 7 varies only slightly. The water surface elevation in nodes 8 and 9 varies significantly in phase with the variation in the tidally controlled node number 10. These results are in substantial agreement with the results of the RECEIV-II model. One significant difference between the two models is that the surface areas are supplied as input in the RECEIV-II simulation. The input for the two problems and a sample output at one time point are shown in Tables (7) and (8).

PROGRAM LISTING

The FORTRAN listing for the program SWOHS is shown in Table 9 and the definitions for variables used in the program is given in Table 6. Examples of input and output are shown in Tables 7 and 8.

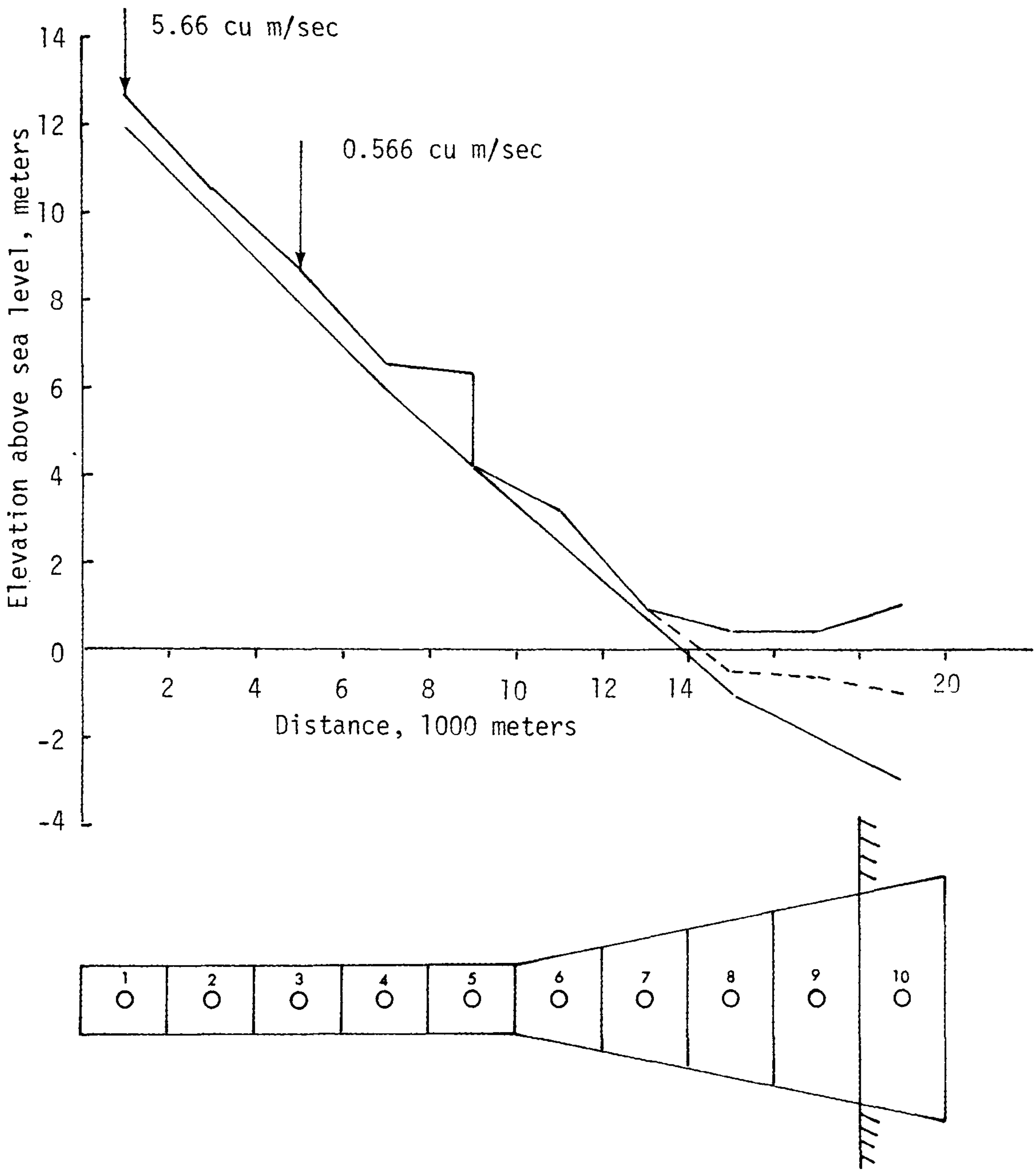


Figure 13. Diagram of second hypothetical problem solved with the SWOHS program.

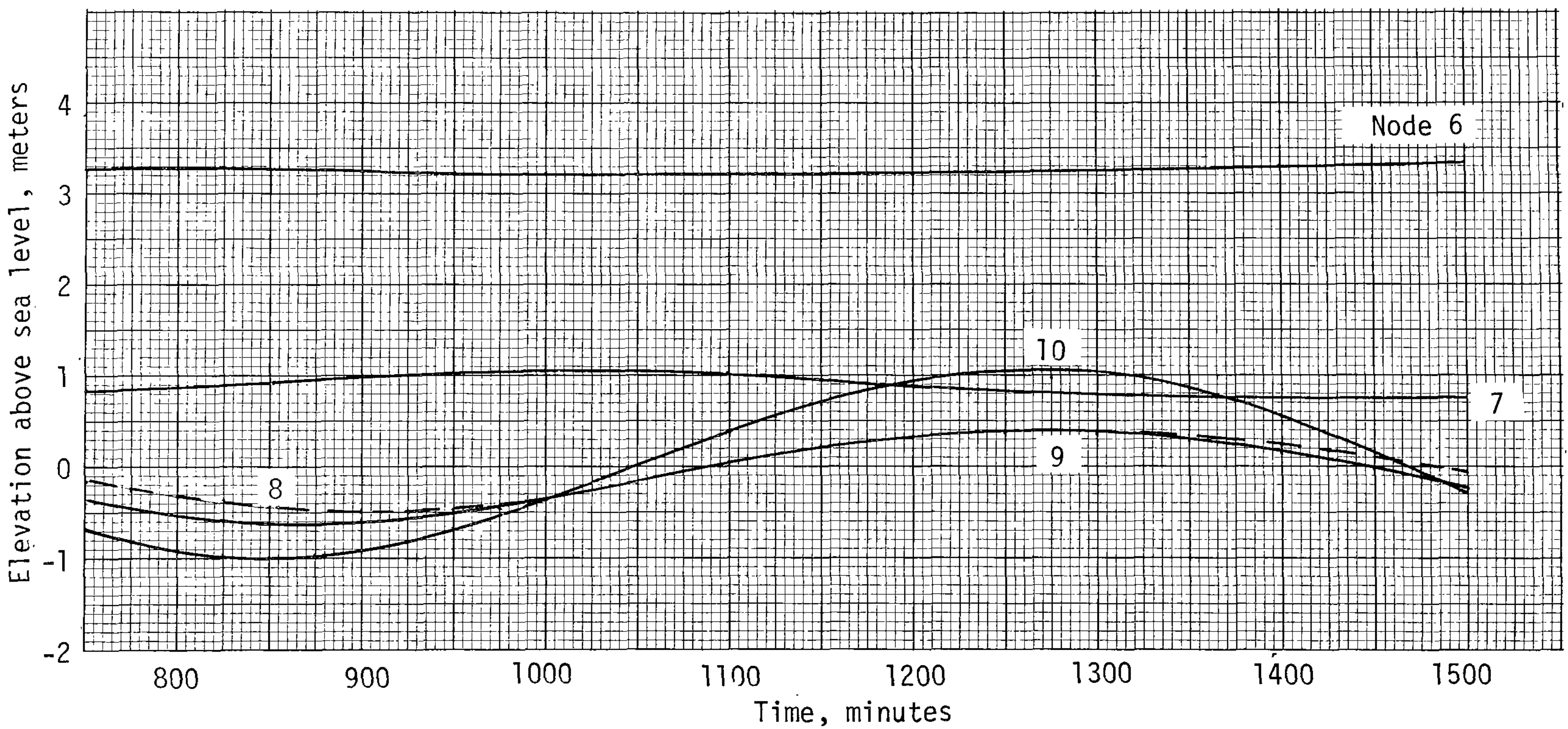


Figure 14. Steady-state variation of water surface elevation of nodes 6-10 in the second hypothetical problem solved with the SWOHS program.

Table 7

Printout for First Hypothetical Problem Solved with SWOHS

STREAM MODEL TEST CASE - DYNAMIC/HYDRAULIC (1) (SWOHS) 9-12-1977

XI	NT	DT	XNCH	PRIN	TIDE	FMC1	FMC2
20.000	25.000	300.000	19.000	0.000	0.000	7.285	16.100

T1	T2	T3	T4	T5	T6	T7	TP
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

STR	STO	XSTRM
10.000	12.000	5.000

QSTRM

600.0000	600.0000	600.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

I	Z(I)	Y2(I)	QSS(I)	CV2(I)	CL(I)	CW(I)	RN(I)	SS(I)	NDAM(I)
1	800.0000	804.2070	598.2000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
2	798.0000	802.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
3	796.0000	800.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
4	794.0000	798.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
5	792.0000	796.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
6	790.0000	794.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
7	788.0000	792.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
8	786.0000	790.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
9	784.0000	788.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
10	782.0000	786.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
11	780.0000	784.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
12	778.0000	782.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
13	776.0000	780.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
14	774.0000	778.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
15	772.0000	776.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
16	770.0000	774.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
17	768.0000	772.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
18	766.0000	770.2070	0.0000	3.0360	4000.0000	30.0000	0.0230	4.0000	0
19	764.0000	768.2070	0.0000	3.1220	4000.0000	30.0000	0.0230	4.0000	0
20	762.0000	766.0360	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	20

I	W1(I)	W2(I)	W3(I)	NCN-1	NCN-2	NCN-3
1	0.0000	0.0000	0.0000	1	2	0
2	0.0000	0.0000	0.0000	2	3	0
3	0.0000	0.0000	0.0000	3	4	0
4	0.0000	0.0000	0.0000	4	5	0
5	0.0000	0.0000	0.0000	5	6	0
6	0.0000	0.0000	0.0000	6	7	0
7	0.0000	0.0000	0.0000	7	8	0
8	0.0000	0.0000	0.0000	8	9	0
9	0.0000	0.0000	0.0000	9	10	0
10	0.0000	0.0000	0.0000	10	11	0
11	0.0000	0.0000	0.0000	11	12	0
12	0.0000	0.0000	0.0000	12	13	0
13	0.0000	0.0000	0.0000	13	14	0
14	0.0000	0.0000	0.0000	14	15	0
15	0.0000	0.0000	0.0000	15	16	0
16	0.0000	0.0000	0.0000	16	17	0
17	0.0000	0.0000	0.0000	17	18	0
18	0.0000	0.0000	0.0000	18	19	0
19	0.0000	0.0000	0.0000	19	20	2
20	20.0000	756.4000	1.5000	0	0	0

Table 7 (continued)

L = 10 JT= 50 MIN

I	CV2(I)	Y2(I)	QIN(I)	CQ	CA2(I)
1	3.0365	804.2070	598.2000	598.2054	197.0035
2	3.0365	802.2070	0.0000	598.2047	197.0074
3	3.0385	800.2070	0.0000	598.0182	196.8132
4	2.8044	798.2009	0.0000	608.6623	217.0403
5	3.3330	796.8307	600.0000	731.1630	219.3700
6	3.1076	794.2712	0.0000	619.6157	199.3869
7	3.0487	792.2174	0.0000	601.7906	197.3922
8	3.0386	790.2087	0.0000	598.8104	197.0696
9	3.0369	788.2073	0.0000	598.3096	197.0152
10	3.0366	786.2070	0.0000	598.2323	197.0074
11	3.0365	784.2070	0.0000	598.2027	197.0035
12	3.0365	782.2070	0.0000	598.2008	197.0035
13	3.0365	780.2070	0.0000	598.2006	197.0035
14	3.0365	778.2070	0.0000	598.2006	197.0035
15	3.0365	776.2070	0.0000	598.2006	197.0035
16	3.0365	774.2070	0.0000	598.2006	197.0035
17	3.0365	772.2070	0.0000	598.2006	197.0035
18	3.0365	770.2070	0.0000	598.2006	197.0035
19	3.1223	768.2070	0.0000	598.1823	191.5818
20	0.0000	766.0357	0.0000	0.0000	0.0000

L = 11 JT= 55 MIN

I	CV2(I)	Y2(I)	QIN(I)	CQ	CA2(I)
1	3.0365	804.2070	598.2000	598.1995	197.0035
2	3.0366	802.2070	0.0000	598.2045	196.9996
3	3.0354	800.2068	0.0000	598.6759	197.2289
4	2.4361	798.2142	0.0000	564.4816	231.7133
5	3.6837	797.2552	600.0000	883.2436	239.7731
6	3.2849	794.4501	0.0000	678.7817	206.6395
7	3.0980	792.2638	0.0000	617.1514	199.2073
8	3.0499	790.2192	0.0000	602.2786	197.4738
9	3.0393	788.2095	0.0000	599.0403	197.1006
10	3.0371	786.2075	0.0000	598.3716	197.0229
11	3.0366	784.2071	0.0000	598.2379	197.0074
12	3.0365	782.2070	0.0000	598.2034	197.0035
13	3.0365	780.2070	0.0000	598.2009	197.0035
14	3.0365	778.2070	0.0000	598.2007	197.0035
15	3.0365	776.2070	0.0000	598.2007	197.0035
16	3.0365	774.2070	0.0000	598.2007	197.0035
17	3.0365	772.2070	0.0000	598.2007	197.0035
18	3.0365	770.2070	0.0000	598.2007	197.0035
19	3.1224	768.2070	0.0000	598.1871	191.5818
20	0.0000	766.0356	0.0000	0.0000	0.0000

L = 12 JT= 60 MIN

I	CV2(I)	Y2(I)	QIN(I)	CQ	CA2(I)
1	3.0365	804.2070	598.2000	598.2005	197.0035
2	3.0369	802.2070	0.0000	598.1810	196.9685
3	3.0127	800.2057	0.0000	599.3221	198.9303
4	2.2226	798.2685	0.0000	537.8660	242.0036
5	3.7682	797.5016	600.0000	966.3295	256.4422
6	3.4653	794.6815	0.0000	753.2759	217.3754
7	3.1883	792.3601	0.0000	648.0488	203.2580
8	3.0797	790.2491	0.0000	611.8713	198.6808
9	3.0474	788.2174	0.0000	601.6057	197.4155
10	3.0391	786.2094	0.0000	599.0009	197.1006
11	3.0371	784.2075	0.0000	598.3951	197.0268
12	3.0366	782.2071	0.0000	598.2395	197.0074
13	3.0365	780.2070	0.0000	598.2034	197.0035
14	3.0365	778.2070	0.0000	598.2009	197.0035
15	3.0365	776.2070	0.0000	598.2006	197.0035
16	3.0365	774.2070	0.0000	598.2006	197.0035
17	3.0365	772.2070	0.0000	598.2006	197.0035
18	3.0365	770.2070	0.0000	598.2006	197.0035
19	3.1224	768.2070	0.0000	598.1906	191.5818
20	0.0000	766.0356	0.0000	0.0000	0.0000

Table 8

STREAM MODEL TEST CASE - DYNAMIC/HYDRAULIC (2)   (SWQHS)   9-12-1977

XT	NT	DT	XNCH	PRIN	TIDE	FMC1	FMC2
10.000	25.000	300.000	8.000	0.000	10.000	4.900	4.900

T1	T2	T3	T4	T5	T6	T7	TP
0.001	-0.116	-0.878	-0.047	-0.090	-0.475	0.103	25.000

STP	STD	XSTRM
0.000	0.000	0.000

QSTRM

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

I	Z(I)	Y2(I)	QSS(I)	CV2(I)	CL(I)	CW(I)	RN(I)	SS(I)	NDAM(I)
1	12.0000	13.0000	5.6600	0.1000	2000.0000	40.0000	0.1000	0.0000	0
2	10.0000	11.0000	0.0000	0.1000	2000.0000	40.0000	0.1000	0.0000	0
3	8.0000	9.0000	0.5660	0.1000	2000.0000	40.0000	0.1000	0.0000	0
4	6.0000	7.0000	0.0000	0.1000	2000.0000	40.0000	0.1000	0.0000	0
5	4.2000	6.2000	0.0000	0.1000	2000.0000	60.0000	0.1000	0.0000	5
6	2.4666	4.0000	0.0000	0.1000	2000.0000	80.0000	0.1000	0.0000	5
7	0.7333	2.0000	0.0000	0.1000	2000.0000	100.0000	0.1000	0.0000	0
8	-1.0000	1.0000	0.0000	0.1000	2000.0000	120.0000	0.1000	0.0000	0
9	-2.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
10	-3.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0

I	W1(I)	W2(I)	W3(I)	NCN=1	NCN=2	NCN=3
1	0.0000	0.0000	0.0000	1	2	0
2	0.0000	0.0000	0.0000	2	3	0
3	0.0000	0.0000	0.0000	3	4	1
4	0.0000	0.0000	0.0000	4	5	0
5	73.5000	6.2000	1.5000	6	7	0
6	0.0000	0.0000	0.0000	7	8	0
7	0.0000	0.0000	0.0000	8	9	0
8	0.0000	0.0000	0.0000	9	10	0
9	0.0000	0.0000	0.0000	0	0	0
10	0.0000	0.0000	0.0000	0	0	0

L = 1   JT= 5 MIN

I	CV2(I)	Y2(I)	QIN(I)	CQ	CA2(I)
1	0.3948	12.9844	5.6600	15.6629	39.6769
2	0.3980	10.9995	0.0000	15.9353	40.0348
3	0.3979	9.0022	0.5660	15.8684	39.8820
4	0.3000	6.9919	0.0000	18.2136	60.7195
5	0.4829	6.2441	0.0000	39.5048	81.8022
6	0.3695	3.9410	0.0000	48.2124	130.4684
7	0.1024	1.9857	0.0000	25.0308	244.5321
8	0.5692	1.0093	0.0000	189.8422	333.4993
9	0.0000	0.8813	0.0000	0.0000	0.0000
10	0.0000	-0.3230	0.0000	0.0000	0.0000

Table 9

FORTRAN Listing for the SWOHS Program

```
 (SWOHS)      STREAM MODEL --- DYNAMIC/HYDRAULIC
 R. G. EILERS          SEPTEMBER 1977
      DIMENSION LIST(40),Z(30),Y1(30),Y2(30),QIN(30),CD1(30),CD2(30),
     . CV1(30),CV2(30),CL(30),CW(30),RN(30),SS(30),YTEMP(30),VTEMP(30),
     . CA1(30),CA2(30),QSS(30),W1(30),W2(30),W3(30),NCN(30,3),
     . NDAM(30),QSTRM(50)
      OPEN(UNIT=1,NAME='SWOHS.DAT',TYPE='OLD',FORM='FORMATTED',
     . READONLY)
      IN=1
      IO=6
      READ(IN,10) NCASE
   10 FORMAT(I2)
      DO 1000 III=1,NCASE
      DO 20 I=1,30
      Z(I)=0.
      Y1(I)=0.
      Y2(I)=0.
      QIN(I)=0.
      CD1(I)=0.
      CD2(I)=0.
      CV1(I)=0.
      CV2(I)=0.
      CL(I)=0.
      CW(I)=0.
      RN(I)=0.
      SS(I)=0.
      YTEMP(I)=0.
      VTEMP(I)=0.
      CA1(I)=0.
      CA2(I)=0.
      QSS(I)=0.
      W1(I)=0.
      W2(I)=0.
      W3(I)=0.
      NCN(I,1)=0
      NCN(I,2)=0
      NCN(I,3)=0
   20 CONTINUE
      DO 25 I=1,50
      QSTRM(I)=0.
   25 CONTINUE
      READ(IN,40) LIST
   40 FORMAT(40A2)
      READ(IN,50) XI,TN,DT,XNCH,PRIN,TIDE,FMC1,FMC2
      READ(IN,50) T1,T2,T3,T4,T5,T6,T7,TP
```

```
      READ(IN,50) STR,STO,XSTRM
 50   FORMAT(8F10.0)
      NXI=XI
      NT=TN
      NPRIN=PRIN
      NCH=XNCH
      NTIDE=TIDE
      QSUM=25.
      JSTRT=STR
      JSTOP=STO
      NSTRM=XSTRM
      READ(IN,60) (QSTRM(I),I=1,50)
      READ(IN,60) (Z(I),I=1,NXI)
      READ(IN,60) (Y2(I),I=1,NXI)
      READ(IN,60) (QSS(I),I=1,NXI)
      READ(IN,60) (CV2(I),I=1,NXI)
      READ(IN,60) (CL(I),I=1,NXI)
      READ(IN,60) (CW(I),I=1,NXI)
      READ(IN,60) (RN(I),I=1,NXI)
      READ(IN,60) (SS(I),I=1,NXI)
      READ(IN,70) (NDAM(I),I=1,NXI)
      READ(IN,60) (W1(I),I=1,NXI)
      READ(IN,60) (W2(I),I=1,NXI)
      READ(IN,60) (W3(I),I=1,NXI)
 60   FORMAT(10F8.0)
      READ(IN,70) (NCN(I,1),I=1,NXI)
      READ(IN,70) (NCN(I,2),I=1,NXI)
      READ(IN,70) (NCN(I,3),I=1,NXI)
 70   FORMAT(10I8)
      WRITE(IO,100) LIST
100   FORMAT(1H1,/////,20X,40A2,//)
      WRITE(IO,110) XI,TN,DT,XNCH,PRIN,TIDE,FMC1,FMC2
110   FORMAT(8X,'XI',10X,'NT',10X,'DT',8X,'XNCH',8X,
     . 'PRIN',8X,'TIDE',8X,'FMC1',8X,'FMC2',/,8F12.3,/)
      WRITE(IO,115) T1,T2,T3,T4,T5,T6,T7,TP
115   FORMAT(8X,'T1',10X,'T2',10X,'T3',10X,'T4',10X,'T5',10X,'T6',
     . 10X,'T7',10X,'TP',/,8F12.3,/)
      WRITE(IO,116) STR,STO,XSTRM
116   FORMAT(7X,'STR',9X,'STO',7X,'XSTRM',/,3F12.3,//,5X,'QSTRM')
      WRITE(IO,118) (QSTRM(I),I=1,50)
118   FORMAT(10F12.4)
      WRITE(IO,120)
120   FORMAT(//,11X,'I',6X,'Z(I)',7X,'Y2(I)',6X,'QSS(I)',6X,'CV2(I)',7X,
     . 'CL(I)',7X,'CW(I)',7X,'RN(I)',7X,'SS(I)',8X,'NDAM(I)')
      DO 140 I=1,NXI
      WRITE(IO,130) I,Z(I),Y2(I),QSS(I),CV2(I),CL(I),CW(I),RN(I),SS(I),
     . NDAM(I)
130   FORMAT(8X,I4,8F12.4,I12)
```

Table 9 (continued)

```
140 CONTINUE
    WRITE(IO,150)
150 FORMAT(//,11X,'I',5X,'W1(I)',7X,'W2(I)',7X,'W3(I)',
   . 10X,'NCN=1',7X,'NCN=2',7X,'NCN=3')
    DO 170 I=1,NXI
    WRITE(IO,160) I,W1(I),W2(I),W3(I),NCN(I,1),NCN(I,2),NCN(I,3)
160 FORMAT(8X,I4,3F12.4,3I12)
170 CONTINUE
    LL=0
    KOUT=0
    DO 600 L=1,NT
    IF (L-JSTRT) 200,180,180
180 IF (L-JSTOP) 190,190,200
190 LL=LL+1
200 ICNT=0
    JT=L*DT/60.
    DO 210 I=1,NXI
    Y1(I)=Y2(I)
    CV1(I)=CV2(I)
    YTEMP(I)=0.
    VTEMP(I)=0.
210 CONTINUE
    IF (NTIDE) 220,220,215
215 Y1(NTIDE)=T1+T5+T6+T7
    Y2(NTIDE)=Y1(NTIDE)
220 NTEST=0
    ICNT=ICNT+1
    DO 470 I=1,NCH
    NI=NCN(I,1)
    NO=NCN(I,2)
    K=NO
230 A1=0.
    A2=0.
    QI1=0.
    QI2=0.
    QO1=0.
    QO2=0.
    NSEG=0.
    DO 360 J=1,NCH
    IF (NDAM(K)-K) 239,250,239
239 IF (NCN(J,1)-K) 270,240,270
240 NSEG=NSEG+1
    M=NCN(J,2)
    CD1(J)=(Y1(M)+Y1(K)-Z(M)-Z(K))/2.
    CD2(J)=(Y2(M)+Y2(K)-Z(M)-Z(K))/2.
    A1=A1+(CW(J)+2.*CD1(J)*SS(J))*CL(J)/2.
    A2=A2+(CW(J)+2.*CD2(J)*SS(J))*CL(J)/2.
    CA1(J)=CD1(J)*CW(J)+CD1(J)**2*SS(J)
```

```
      CA2(J)=CD2(J)*CW(J)+CD2(J)**2*SS(J)
      QO1=QO1+CA1(J)*CV1(J)
      QO2=QO2+CA2(J)*CV2(J)
      GO TO 270
  250 QO1=W1(K)*(Y1(K)-W2(K))**W3(K)
      QO2=W1(K)*(Y2(K)-W2(K))**W3(K)
  270 IF (NDAM(K)) 275,275,271
  271 IF (NDAM(K)-K) 272,275,272
  272 KD=NDAM(K)
      GO TO 285
  275 IF (NCN(J,2)-K) 290,280,290
  280 NSEG=NSEG+1
      M=NCN(J,1)
      CD1(J)=(Y1(M)+Y1(K)-Z(M)-Z(K))/2.
      CD2(J)=(Y2(M)+Y2(K)-Z(M)-Z(K))/2.
      A1=A1+(CW(J)+2.*CD1(J)*SS(J))*CL(J)/2.
      A2=A2+(CW(J)+2.*CD2(J)*SS(J))*CL(J)/2.
      CA1(J)=CD1(J)*CW(J)+CD1(J)**2*SS(J)
      CA2(J)=CD2(J)*CW(J)+CD2(J)**2*SS(J)
      QI1=QI1+CA1(J)*CV1(J)
      QI2=QI2+CA2(J)*CV2(J)
      GO TO 290
  285 QI1=W1(KD)*(Y1(KD)-W2(KD))**W3(KD)
      QI2=W1(KD)*(Y2(KD)-W2(KD))**W3(KD)
  290 IF (L-JSTRT) 350,330,330
  330 IF (L-JSTOP) 340,340,350
  340 IF (K-NSTRM) 350,345,350
  345 QIN(K)=QSS(K)+QSTRM(LL)
      GO TO 360
  350 QIN(K)=QSS(K)
  360 CONTINUE
      IF (NSEG-1) 366,365,366
  365 A1=2.*A1
      A2=2.*A2
  366 IF (K-NTIDE) 368,367,368
  367 WT=6.283185*L*DT/TP/3600.
      Y2(K)=T1+T2*SIN(WT)+T3*SIN(2.*WT)+T4*SIN(3.*WT)+T5*COS(WT)+
     . T6*COS(2.*WT)+T7*COS(3.*WT)
      GO TO 370
  368 Y2(K)=Y1(K)+((QIN(K)+QI1-QO1)/A1/2.+(QIN(K)+QI2-QO2)/A2/2.)*DT
  370 IF (Y2(K)-Z(K)) 372,374,374
  372 WRITE(IO,373) K
  373 FORMAT(///,5X,'STOP PROGRAM - NODE HAS RUN DRY AT I =',I3)
      KOUT=1
  374 IF (ABS(YTEMP(K)-Y2(K))-.01) 380,380,375
  375 NTEST=1
  380 YTEMP(K)=Y2(K)
      IF (NI-K) 400,410,400
```

Table 9 (continued)

```
400 K=NI
    GO TO 230
410 NI=NCN(I,1)
    NO=NCN(I,2)
    CD1(I)=(Y1(NI)+Y1(NO)-Z(NI)-Z(NO))/2.
    CD2(I)=(Y2(NI)+Y2(NO)-Z(NI)-Z(NO))/2.
    CA1(I)=CD1(I)*CW(I)+CD1(I)**2*SS(I)
    CA2(I)=CD2(I)*CW(I)+CD2(I)**2*SS(I)
    WP1=CW(I)+2.*CD1(I)*(1.+SS(I)**2)**.5
    WP2=CW(I)+2.*CD2(I)*(1.+SS(I)**2)**.5
    HR1=CA1(I)/WP1
    HR2=CA2(I)/WP2
    A=FMC1*RN(I)**2/HR2**(4./3.)+.5/CL(I)
    B=1./DT
    CP1=FMC1*RN(I)**2*CV1(I)*ABS(CV1(I))/HR1**(4./3.)-FMC2*
   . (Y1(NI)+Y2(NI)-Y1(NO)-Y2(NO))/CL(I)-CV1(I)/DT
    FMI1=0.
    FMI2=0.
    K=NCN(I,1)
    DO 440 J=1,NCH
    IF (NCN(J,2)-K) 440,420,440
420 IF (NCN(J,3)-1) 430,440,430
430 FMI1=FMI1+CA1(J)*CV1(J)*ABS(CV1(J))/CA1(I)
    FMI2=FMI2+CA2(J)*CV2(J)*ABS(CV2(J))/CA2(I)
440 CONTINUE
    IF (FMI1) 444,442,444
442 FMI1=QSS(K)*CV1(I)/CA1(I)
    FMI2=QSS(K)*CV2(I)/CA2(I)
444 CP2=(FMI1+FMI2-CV1(I)*ABS(CV1(I)))/CL(I)/2.
    C=CP1-CP2
    IF (C) 446,448,448
446 A=ABS(A)
    GO TO 449
448 A=-ABS(A)
449 CV2(I)=((B**2-4.*A*C)**.5-B)/2./A
    IF (ABS(VTEMP(I)-CV2(I))-.001) 460,460,450
450 NTEST=1
460 VTEMP(I)=CV2(I)
470 CONTINUE
    IF (ICNT-25) 480,500,500
480 IF (NTEST-1) 500,220,220
500 IF (L-NPRIN) 600,510,510
510 WRITE(IO,520) L,JT
520 FORMAT(/,3X,'L =',I4,3X,'JT=',I4,1X,'MIN',6X,'I',5X,'CV2(I)',
   . 7X,'Y2(I)',6X,'QIN(I)',10X,'CQ',6X,'CA2(I)')
    KKK=NI+1
    DO 540 I=1,KKK
    CQ=CV2(I)*CA2(I)
```

Table 9 (continued)

```
      WRITE(IO,530) I,CV2(I),Y2(I),QIN(I),CQ,CA2(I)
  530 FORMAT(27X,I6,5F12.4)
  540 CONTINUE
      IF (KOUT) 1000,600,1000
  600 CONTINUE
 1000 CONTINUE
      CLOSE(UNIT=1)
      END
```

REFERENCES FOR SECTION 5

1. Chow, V. T., "Open-Channel Hydraulics, McGraw-Hill Book Company, 1959.

2. O'Connor, D. J. and Dobbins, W. E., "Mechanism of Reaeration in Natural Streams" ASCE Transactions, Vol. 123, 1958, pp. 641-684.

3. Raytheon Company, Oceanographic and Environmental Services, Portsmouth, Rhode Island, New England River Basins Modeling Project, Volume III, Part 1, RECEIV-II Water Quantity and Quality Model, December 1974, EPA Contract No. 68-01-1890.

APPENDIX FOR SECTION 5

DESCRIPTION OF HYDRAULIC STEADY-STATE (HYSS) PROGRAM

To avoid computational problems in the use of SWOHS initial values for water surface elevation, Y(J), at all nodes and the velocity, CV(I) in all channels should correspond to a feasible steady-state solution. The program HYSS has been developed to find an initial feasible steady-state solution. The variable names used in HYSS correspond, where possible, to variable names used in SWOHS. Variables read into HYSS as input include QOUT (cfs) which is the volume flow out of the downstream terminal node. The downstream terminal node is assumed to be controlled by a dam/weir and the three weir coefficients, W1, W2, and W3 must be read in as input. A maximum nodal depth, CDMAX, in feet must be supplied as input. The number of channels in the stream is supplied as NCH. Two constants, VAR1 = 32.2 and VAR2 = 14.57 corresponding to th English system of unit are read in as input. The steady flow into each node, QIN(J), is read in as input. The following additional variables whose names correspond to input for SWOHS must be supplied for each channel; CL(I), CW(I), RN(I), and SS(I).

The first thing to be computed is the water surface elevation in the downstream terminal node, Y(KI), from the value for QOUT and the weir/dam coefficients. The flow in all channels is found next. This is done by taking each channel (I) in turn, identifying the upstream node (NI) and then setting the flow in the I'th channel equal to the sum of QIN(NI) plus the flow in all channels whose downstream node is NI. Care should be taken to make the number of any channel which flows into the I'th channel less than the number of the I'th channel.

Initial values for Y(J) and CV(I) are set equal to zero. Each channel is then considered one-at-a-time beginning with the channel upstream of the terminal node (KI) and working backward through the list of channel numbers. The upstream node (NI) and the downstream node (NO) are identified for each channel. The elevation of the water surface in the downstream node is known. The elevation of the water surface in the upstream node is known to be between Z(NI) and Z(NI)+CDMAX. A continuously halving or Bolzano iteration is then performed to find the elevation of the water surface Y(NI) in the upstream node. This iteration is performed for each channel in turn finding the channel velocity and the elevation of the water surface for each node. As a check, DVDT, the rate of change of velocity with respect to time is computed and this should always be zero or a very small number.

If the variables input to the program are incompatible with a feasible steady state solution, this will be detected by noting that the elevation of the water surface equals Z(J) (within the tolerance) or equals Z(J) plus CDMAX. If this happens new values for RN, CW, or SS must be input and the program rerun until a feasible solution is found. The definitions for the variables used in the HYSS program are given in Table 10. The listing for HYSS is shown in Table 11 and the output for the first and second hypothetical problem is shown in Table 12.

Table 10

Definitions of FORTRAN Symbols Used in HYSS

NCASE	number of cases to be executed by the program
LIST	alpha-numeric title of the data case
QOUT	sum total of all flows entering the system, cfs
W1	first weir coefficient
W2	second weir coefficient
W3	third weir coefficient
CDMAX	maximum feasible depth of channel, ft (M)
XNCH = NCH	number of channels
VAR1	a constant having a value of 32.2 when English units (cfs and sec) are used (9.8 metric units)
VAR2	a constant having a value of 14.57 when English units (cfs and sec) are used (9.8 metric units)
QIN(I)	steady flow entering the I'th node, cfs (M/Sec)
CL(I)	length of the I'th channel, ft (M)
CW(I)	width of the flat bottom of the stream bed for the I'th channel, ft (M)
RN(I)	Manning roughness coefficient for the I'th channel
SS(I)	side-slope of the I'th channel
Z(I)	elevation (above some reference point) of the stream bed at the I'th node, ft (M)
NCN(I,1)	number of the upstream node for the I'th channel

Table 10 (continued)

NCN(I,2)	number of the downstream node for the I'th channel
NCN(I,3)	set equal to one if the I'th channel enters the downstream channel laterally

Table 11

FORTRAN Source Listing for HYSS

```
 (HYSS)      PROGRAM FOR PREPARING INPUT TO SWOHS
 R.G. EILERS      SEPTEMBER 1976
   DIMENSION LIST(40),Z(30),QIN(30),CL(30),CW(30),RN(30),SS(30),
  . Y(30),CV(30),DVDT(30),CD(30),CA(30),VTEMP(30),NCN(30,3),Q(30)
   OPEN(UNIT=1,NAME='HYSS.DAT',TYPE='OLD',FORM='FORMATTED',
  . READONLY)
   IN=1
   IO=6
   READ(IN,10) NCASE
10 FORMAT(I2)
   DO 1000 III=1,NCASE
   DO 20 I=1,30
   Z(I)=0.
   QIN(I)=0.
   CL(I)=0.
   CW(I)=0.
   RN(I)=0.
   SS(I)=0.
   Y(I)=0.
   CV(I)=0.
   DVDT(I)=0.
   CD(I)=0.
   NCN(I,1)=0
   NCN(I,2)=0
   NCN(I,3)=0
   Q(I)=0.
20 CONTINUE
   READ(IN,30) LIST
30 FORMAT(40A2)
   READ(IN,50) QOUT,W1,W2,W3,CDMAX,XNCH,VAR1,VAR2
50 FORMAT(8F10.0)
   NCH=XNCH
   KI=NCH+1
   READ(IN,60) (QIN(I),I=1,KI)
   READ(IN,60) (CL(I),I=1,KI)
   READ(IN,60) (CW(I),I=1,KI)
   READ(IN,60) (RN(I),I=1,KI)
   READ(IN,60) (SS(I),I=1,KI)
   READ(IN,60) (Z(I),I=1,KI)
   READ(IN,55) (NCN(I,1),I=1,KI)
   READ(IN,55) (NCN(I,2),I=1,KI)
   READ(IN,55) (NCN(I,3),I=1,KI)
55 FORMAT(10I8)
60 FORMAT(10F8.0)
```

```
      Y(KI)=W2+(QOUT/W1)**(1./W3)
      DO 75 J=1,NCH
      NI=NCN(J,1)
      Q(J)=QIN(NI)
      DO 70 K=1,NCH
      IF (NCN(K,2)-NI) 70,65,70
   65 Q(J)=Q(J)+Q(K)
   70 CONTINUE
   75 CONTINUE
   78 DO 80 I=1,NCH
      VTEMP(I)=CV(I)
   80 CONTINUE
      DO 200 II=1,NCH
      I=KI-II
      NI=NCN(I,1)
      NO=NCN(I,2)
      YMAX=Z(NI)+CDMAX
      YMIN=Z(NI)
      Y(NI)=(YMAX+YMIN)/2.
  100 YTEMP=Y(NI)
      CD(I)=(Y(NI)+Y(NO)-Z(NI)-Z(NO))/2.
      CA(I)=CD(I)*CW(I)+CD(I)**2*SS(I)
      WP=CW(I)+2.*CD(I)*(1.+SS(I)**2)**.5
      HR=CA(I)/WP
      CV(I)=Q(I)/CA(I)
      FMO=CA(I)*CV(I)**2
      CONST=VAR1*(Y(NI)-Y(NO))/CL(I)-VAR2*RN(I)**2*CV(I)**2/HR**(4./3.)
      FMI=0.
      K=NCN(I,1)
      DO 120 J=1,NCH
      IF (NCN(J,2)-K) 120,115,120
  115 IF (NCN(J,3)-1) 117,120,117
  117 FMI=FMI+CA(J)*CV(J)**2.
  120 CONTINUE
      IF (FMI) 126,125,126
  125 FMI=QIN(K)*CV(I)
  126 DVDT(I)=CONST+(FMI-FMO)/CA(I)/CL(I)
      IF (DVDT(I)) 140,140,130
  130 YMAX=Y(NI)
      GO TO 150
  140 YMIN=Y(NI)
  150 Y(NI)=(YMAX+YMIN)/2.
      IF (ABS(YTEMP-Y(NI))-.00001) 200,200,100
  200 CONTINUE
      NTEST=0
      DO 220 I=1,NCH
      IF (ABS(VTEMP(I)-CV(I))-.001) 220,220,210
  210 NTEST=1
```

Table 11 (continued)

```
 220 CONTINUE
     IF (NTEST) 290,290,78
 290 WRITE(IO,300) LIST
 300 FORMAT(1H1,/////,20X,40A2,//)
     WRITE(IO,310)
 310 FORMAT(15X,'QOUT',10X,'W1',10X,'W2',10X,'W3',7X,
    . 'CDMAX',8X,'XNCH',8X,'VAR1',8X,'VAR2')
     WRITE(IO,320) QOUT,W1,W2,W3,CDMAX,XNCH,VAR1,VAR2
 320 FORMAT(9X,8F12.4,///)
     WRITE(IO,330)
 330 FORMAT(8X,'I',5X,'QIN',8X,'CL',8X,'CW',8X,'RN',8X,'SS',9X,'Z',
    . 3X,'NCN=1',1X,'NCN=2',1X,'NCN=3',7X,'Y',8X,'CD',8X,'CV',
    . 6X,'DVDT',/)
     DO 350 I=1,KT
     WRITE(IO,340) I,QIN(I),CL(I),CW(I),RN(I),SS(I),Z(I),
    . NCN(I,1),NCN(I,2),NCN(I,3),Y(I),CD(I),CV(I),DVDT(I)
 340 FORMAT(3X,I6,6F10.3,3I6,4F10.3)
 350 CONTINUE
1000 CONTINUE
     CLOSE(UNIT=1)
     END
```

Table 12

Printout for the First and Second Hypothetical Problems Using HYSS

FIRST HYPOTHETICAL PROBLEM (HYSS) 9-12-1977

QOUT	W1	W2	W3	CDMAX	XNCH	VAR1	VAR2
598.2396	20.0000	756.4000	1.5000	10.0000	19.0000	32.2000	14.5700

I	QIN	CL	CW	RN	SS	Z	NCN-1	NCN-2	NCN-3	Y	CD	CV	DVDT
1	598.240	4000.000	30.000	0.023	4.000	800.000	1	2	0	804.207	4.207	3.037	0.000
2	0.000	4000.000	30.000	0.023	4.000	798.000	2	3	0	802.207	4.207	3.037	0.000
3	0.000	4000.000	30.000	0.023	4.000	796.000	3	4	0	800.207	4.207	3.037	0.000
4	0.000	4000.000	30.000	0.023	4.000	794.000	4	5	0	798.207	4.207	3.037	0.000
5	0.000	4000.000	30.000	0.023	4.000	792.000	5	6	0	796.207	4.207	3.037	0.000
6	0.000	4000.000	30.000	0.023	4.000	790.000	6	7	0	794.207	4.207	3.037	0.000
7	0.000	4000.000	30.000	0.023	4.000	788.000	7	8	0	792.207	4.207	3.037	0.000
8	0.000	4000.000	30.000	0.023	4.000	786.000	8	9	0	790.207	4.207	3.037	0.000
9	0.000	4000.000	30.000	0.023	4.000	784.000	9	10	0	788.207	4.207	3.037	0.000
10	0.000	4000.000	30.000	0.023	4.000	782.000	10	11	0	786.207	4.207	3.037	0.000
11	0.000	4000.000	30.000	0.023	4.000	780.000	11	12	0	784.207	4.207	3.037	0.000
12	0.000	4000.000	30.000	0.023	4.000	778.000	12	13	0	782.207	4.207	3.037	0.000
13	0.000	4000.000	30.000	0.023	4.000	776.000	13	14	0	780.207	4.207	3.037	0.000
14	0.000	4000.000	30.000	0.023	4.000	774.000	14	15	0	778.207	4.207	3.037	0.000
15	0.000	4000.000	30.000	0.023	4.000	772.000	15	16	0	776.207	4.207	3.037	0.000
16	0.000	4000.000	30.000	0.023	4.000	770.000	16	17	0	774.207	4.207	3.037	0.000
17	0.000	4000.000	30.000	0.023	4.000	768.000	17	18	0	772.207	4.207	3.037	0.000
18	0.000	4000.000	30.000	0.023	4.000	766.000	18	19	0	770.207	4.207	3.036	0.000
19	0.000	4000.000	30.000	0.023	4.000	764.000	19	20	2	768.207	4.122	3.122	0.000
20	0.000	0.000	0.000	0.000	0.000	762.000	0	0	0	766.036	0.000	0.000	0.000

SECOND HYPOTHETICAL PROBLEM (HYSS) 9-12-1977

QOUT	W1	W2	W3	CDMAX	XNCH	VAR1	VAR2
23.0000	20.0000	488.9000	1.5000	2.0000	7.0000	32.2000	14.5700

I	QIN	CL	CW	RN	SS	Z	NCN-1	NCN-2	NCN-3	Y	CD	CV	DVDT
1	16.000	12700.000	23.000	0.080	6.000	575.000	1	2	0	575.781	0.998	0.553	0.000
2	0.000	12500.000	27.000	0.080	2.000	560.000	2	3	0	561.215	0.822	0.679	0.000
3	0.000	10200.000	43.000	0.060	3.000	537.000	3	4	0	537.429	0.548	0.655	0.000
4	0.000	14100.000	41.000	0.080	1.000	520.000	5	4	0	520.666	0.333	0.508	0.241
5	7.000	32500.000	31.000	0.080	1.000	673.000	4	6	0	673.000	1.268	0.562	0.000
6	0.000	3750.000	30.000	0.050	4.000	495.000	6	7	0	496.869	0.935	0.729	0.000
7	0.000	3600.000	30.000	0.050	4.000	494.000	7	8	2	494.000	0.999	0.677	0.016
8	0.000	0.000	0.000	0.000	0.000	488.000	0	0	0	489.998	0.000	0.000	0.000

SECTION 6

EFFECT OF STORMWATER ON STREAM DISSOLVED OXYGEN

BACKGROUND

From an analysis of stormwater overflow measurements made in seven cities, Heaney et al (5) concluded that the BOD load originating from separate storm sewers in an urban area depends primarily on the population density and ranges from 25-38 lb BOD/acre-yr (28-43 kg/ha-yr). The corresponding load from combined sewers was found to be 3-4 times this amount. Using the load estimating relationships developed by Heaney et al, the pollution caused by combined sewer overflows in a community of 100,000 can be estimated at about 100 lb BOD/acre-yr (112 kg/ha-yr). If the community treats the municipal sewage to the secondary level (30 mg/l BOD) the load from the steady dry weather flow can be estimated at about 92 lb BOD/acre-yr (103 kg/ha-yr) assuming a population density of 10 persons per acre. Thus, the amount of pollution from combined sewer overflows is roughly equivalent to the load from the dry weather secondary treatment plant. This type of reasoning has generated an interest in estimating the impact of stormwater overflow events on the dissolved oxygen resources of the receiving stream.

If dispersion in the receiving stream is neglected, the Streeter-Phelps model (7) can be used to find the peak dissolved oxygen deficit caused by any overflow event with minimal computational effort. However, if the analyst wishes to include the effect of dispersion, no model of equivalent simplicity has been available.

If the stormwater overflow event is divided into increments as shown in Figure 15 and the stream is prismatic, each incremental load can be approximated as an impulsive load for which a closed form solution is available (1). Since the partial differential equations governing the deoxygenation and reaeration process are linear the superposition principle applies and the response of the incremental loads can be summed to find the response of the entire stormwater overflow event. An alternative approach, which must be used when the stream cannot be assumed to be prismatic is to numerically integrate the governing partial differential equations. This approach is tedious and time-consuming but when the pollutional pulse is square and the stream is prismatic numerical integration results can be conveniently summarized by means of non-dimensional groupings providing preliminary estimates of the peak dissolved oxygen deficit occurring in the stream as well as the time after entry of the pollutional pulse at which the peak occurs.

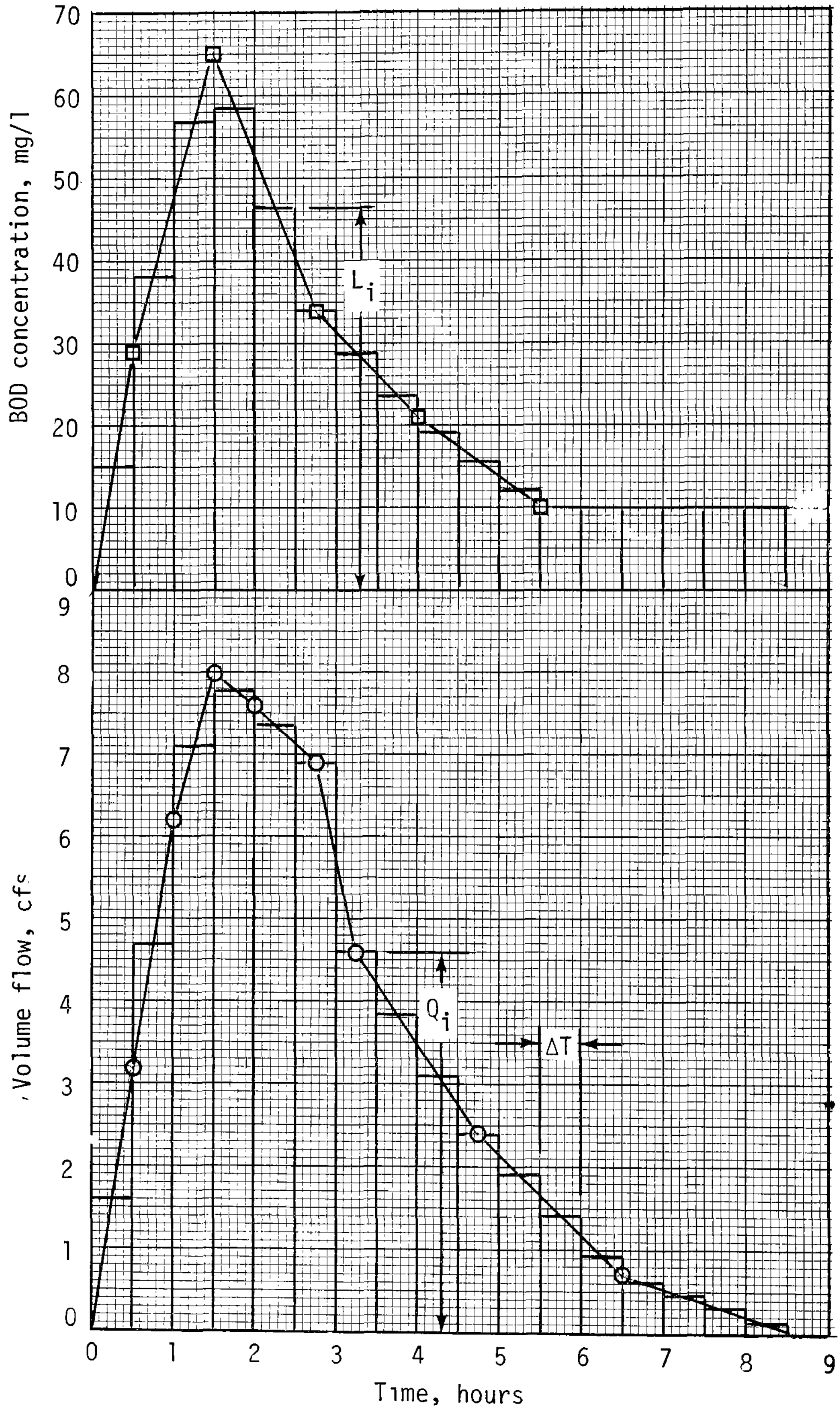


Figure 15. Typical stormwater overflow event divided into half hour sub-loads

PROBLEM OF DEOXYGENATION AND REAERATION IN TUBULAR STREAMS

When the Euler coordinate system, which measures distance along the stream (X) from a point on the stream bank is used, the following two partial differential equations describe the concentration of BOD (L) and the concentration of dissolved oxygen deficit (D) as functions of distance and time.

$$\partial L/\partial t = -v(\partial L/\partial X) + E(\partial^2 L/\partial X^2) - K_r L \qquad (1)$$

$$\partial D/\partial t = -v(\partial D/\partial X) + E(\partial^2 D/\partial X^2) + K_d L - K_a D \qquad (2)$$

These equations can be simplified by converting to the Lagrange coordinate system which measures distance (x) from a point which moves downstream with the velocity of the stream. When the Lagrange coordinate system is used, the first term on the right-hand side of both equations becomes zero and the Lagrangian equations take the following simplier form:

$$\partial L/\partial t = E(\partial^2 L/\partial x^2) - K_r L \qquad (3)$$

$$\partial D/\partial t = E(\partial^2 D/\partial x^2) + K_d L - K_a D \qquad (4)$$

If dispersion (E) in the stream is assumed to be zero, distance is no longer an independent variable and equations (3) and (4) reduce to the following two equation which govern the Streeter-Phelps model:

$$dL/dt = -K_r L \qquad (5)$$

$$dD/dt = K_d L \quad K_a D \qquad (6)$$

Equation (5) can be integrated simply as follows:

$$L/L_o = e^{-K_r t} \qquad (7)$$

If the hydrograph of the storm overflow entering the stream is square with a flow rate of Q_i and a BOD concentration of L_i, the initial BOD concentration (L_o) in the stream is calculated by the following relationship:

$$L_o = Q_i L_i / Q \qquad (8)$$

The Streeter-Phelps model was developed for steady state analysis of deoxygenation and reaeration in streams but can be applied equally well to a plane in the stream perpendicular to the direction of flow in which the initial BOD concentration is given by equation (8). Equation (7) shows that the BOD concentration in that plane falls off exponentially as the plane moves downstream at the velocity of the stream. The DO deficit in the plane will increase from the initial value (D_0) to some maximum value and then approach zero as the time form entry becomes great. The relations describing the behavior of DO deficit can be found by substituting equation (7) into equation (6) and integrating to find the following result:

$$D/L_0 = K_d/(K_a-K_d) \left[e^{-K_r t} - e^{-K_a t}\right] + D_0/L_0 \, e^{-K_a t} \qquad (9)$$

When disperison is assumed to exist in the stream values for L and D can no longer be associated with planes and their values are functions of both time and distance along the stream.

STREAM RESPONSE WITH DISPERSION

When BOD is introduced into the stream by a storm overflow event, the BOD and DO deficit concentrations produced in the stream are functions of time and distance from the point in the stream at which the load is introduced and are referred to as the BOD and DO deficit responses in the stream to the storm overflow load. The form of the BOD and DO deficit responses must satisfy equations (3) and (4) and will also depend on the nature of the load introduced.

Since equations (3) and (4) are linear partial differential equations, the principle of superposition applies. The storm overflow load can be divided into a number of sub-loads, the response of each sub-load solved for, and the response of the total load found as the sum of the responses of the sub-loads. Therefore, if the solution for a sub-load is known, the solution for any overflow event can be found by superposition. The most convenient kind of sub-load is the impulsive load. An impulsive load can be visualized physically as a load which enters the stream instantaneously. A following closed-form expression for the BOD response to an impulsive load is given by Thomann (8).

$$L' = \frac{M/A}{(2Et)^{\frac{1}{2}} (2\Pi)^{\frac{1}{2}}} \quad e^{-\frac{1}{2}(x^2/(2Et))} \quad e^{-K_r t} \qquad (10)$$

This equation, which is a solution to equation (3) for an impulsive load, shows that the shape of the BOD concentration profile at any time has the shape of the normal probability density function with a standard deviation equal to $(2Et)^{\frac{1}{2}}$. The peak value of the profile is infinite initially and falls off quickly with time. Since equation (3) is analogous to the classical one-dimensional heat conduction equation, methods for deriving the solution given by equation (10) can be found in textbooks such as Operational Mathematics by Churchill (3).

To use equation (10) the storm overflow hydrograph and the corresponding plot of BOD concentration versus time are both divided into a number of equal time increments. The value of M for each time increment is then found by multiplying the average flow (Q_i) over the interval by the average BOD concentration (L_i) and multiplying the product by ΔT. This procedure is shown diagrammatically in Figure 15.

The cross-sectional area of the stream (A) is found by dividing the stream flow (Q) after introduction of the stormwater by the stream velocity after introduction of the stormwater. Adjusting the units gives the following modification of equation (10) applied to a single impulsive load.

$$L/L_o = \left[\frac{1}{2\Pi}\right] (v\ \Delta T/s)\ e^{-\frac{1}{2}(x/s)^2}\ e^{-K_r t} \qquad (11)$$

In equation (11) <u>s</u> is defined as $(2Et)^{\frac{1}{2}}$ and L_o is computed using equation (8).

A stormwater overflow pollution stream model (SWOPS) was developed as described in ref. 6 using conventional numerical integration methods as described by Bunce and Hetling (2). The DO deficit profiles in the stream resulting from square stormwater overflow events were computed using a wide range for the various variables involved. It was noted from the computed results that, as the width of the square pollution pulse was decreased, the shape of the DO deficit profiles at any time approached the shape of the normal probability density function. Assuming that the shape of the DO deficit response at any time has the shape of the normal probability density function it follows that the second partial derivative of D with respect to distance could be expressed as follows:

$$\partial^2 D/\partial x^2 = (D/2Et)\ \left((x^2/2Et) - 1\right) \qquad (12)$$

Substituting equations (11) and (12) into the governing partial differential equation (4) then produced the following expression for the DO deficit response in the stream to an impulsive load of BOD when D_o is zero.

$$D/L_o = \left[\frac{1}{2\Pi}\right] (v\ \Delta T/s)\ \left(K_d/(K_a - K_r)\right) \left[e^{-K_r t} - e^{-K_a t}\right] e^{-\frac{1}{2}(x/s)^2} \qquad (13)$$

The second term on the right side of equation (9) should be added to equation (13) if D_o is positive.

A rigorous derivation of equation (13) is contained in the referenced (1) paper by Bennett. A recurrence formula derived by Di Toro (4) which relates the concentrations of sequentially reacting substances to the concentration of a single first order reacting substance can be used to derive equation (13) from equation (11).

To compare the accuracy of the two methods of solution the DO deficit response to a square pollution pulse was computed with the digital computer stream model SWOPS (6) and with equation (13) using the superposition principle. A listing of the program SWOCFS developed to apply equation (13) using the superposition principle is shown in Table 13. A sample printout is shown in Table 14.

A comparison of values computed by the two methods is shown in Figure 16. The values plotted are the peak values for D/L_0 at 15-minute intervals after the leading edge of the square wave enters the stream. Values computed with SWOPS are shown by the solid line and values computed using equation (13) are shown by the circled points. In this analysis, the hypothetical stream had a volume flow of 950 cfs (26.9 m^3/sec) and a velocity of one mile/hr (1.61 km/hr). The values used for rate constants were 0.25 $days^{-1}$ for both K_r and K_d and 0.98 $days^{-1}$ for K_a. The width of the square pulse was taken as one hour and the time increment used was 15 minutes. The volume flow of the pulse was 1294 cfs (36.6 m^3/sec) and the BOD concentration was 110 mg/l. The difference between the two solutions can be attributed to computational error.

NON-DIMENSIONAL GROUPINGS USED TO SUMMARIZE COMPUTED RESULTS

Although equation (13) can be used with the superposition principle to find the complete DO deficit response to any known stormwater overflow with a minimum of computation effort, many problems require only an estimate of the peak DO deficit concentration in the stream resulting from a square pollution pulse. However, the peak DO deficit concentration might be required for hundreds or even thousands of individual square pollution pulses. Therefore, a method which would allow estimation of the peak DO deficit concentration simply by reading a graph is needed for many problems. To fill this need the program SWOPS was used to compute the peak DO deficit concentration from square pollution pulses over a wide range of all variables involved. These computed results are shown in Table 15. It can be seen from equations (9) and (13) that the term $K_d/(K_a-K_r)$ appears in both expressions for D and will, therefore, not appear in the ratio. Thus, only K_r and K_a need be specified in computing the ratio of the two dissolved oxygen deficits.

By analyzing the computed results it was found that, if the computed peak DO deficit concentration was divided by the corresponding DO deficit concentration computed from the Streeter-Phelps model, this ratio will plot as a single-valued function of the following non-dimensional grouping.

Table 13

Listing for the Program SWOCFS which Applies the Superposition Principle to the Closed-Form Solution for an Impulsive BOD Load

```
C     ( SWOCFS )          CLOSED FORM SOLUTION FOR STREAM SIMULATION
C     R. G. FILERS        SEPTEMBER 1977
      REAL KR,KA,KD,LO(200),DO(200),LOIN(30),MDT,LOMAX
      DIMENSION LIST (40),QIN(30)
      OPEN(UNIT=1,NAME='SWOCFS.DAT',TYPE='OLD',FORM='FORMATTED',
     . READONLY)
      IN=1
      IO=6
      READ(IN,10) NCASE
   10 FORMAT(I2)
      DO 1000 III=1,NCASE
      DO 15 I=1,200
      LO(I)=0.
      DO(I)=0.
   15 CONTINUE
      DO 16 I=1,30
      QIN(I)=0.
      LOIN(I)=0.
   16 CONTINUE
      READ(IN,20) LIST
   20 FORMAT(40A2)
      READ(IN,30) KR,EC,VEL,QSS,KD,KA
      READ(IN,30) DT,DN,TM,FORK
   30 FORMAT(8F10.0)
      READ(IN,35) (QIN(I),I=1,30)
      READ(IN,35) (LOIN(I),I=1,30)
   35 FORMAT(10F8.0)
      ND=DN
      MT=TM
      WRITE(IO,40) LIST
   40 FORMAT(1H1,/////,20X,40A2,//)
      WRITE(IO,50) KR,EC,VEL,QSS,KD,KA
   50 FORMAT(8X,'KR',10X,'EC',9X,'VEL',
     . 9X,'QSS',10X,'KD',10X,'KA',/,6F12.4,/)
      WRITE(IO,60) DT,DN,TM,FORK
   60 FORMAT(8X,'DT',10X,'DN',10X,'TM',8X,'FORK',/,
     . 4F12.4,//)
      WRITE(IO,65) (QIN(I),I=1,30)
   65 FORMAT(7X,'QIN',/,10F12.4,/,10F12.4,/,10F12.4,/)
      WRITE(IO,66) (LOIN(I),I=1,30)
   66 FORMAT(6X,'LOIN',/,10F12.4,/,10F12.4,/,10F12.4,/)
      DT=DT/24.
```

```
      DX=VEL*DT
      T=0.
      DO 200 M=1,MT
      T=T+DT
      NX=M
      IF (M-ND) 80,80,70
   70 NX=ND
   80 DO 85 I=1,21
      LO(I)=0.
      DO(I)=0.
   85 CONTINUE
      MDT=T
      DO 150 K=1,NX
      Q=QIN(K)+QSS
      LOMAX=.2814*QIN(K)*LOIN(K)*DT/Q*VEL/EC**.5
      DOMAX=LOMAX*KD/(KA-KD)
      DO 110 I=1,21
      X=(I-11)*DX+(K-1)*DX
      VAR=X**2/4./EC/MDT
      IF (VAR-18.) 90,110,110
   90 IF (FORK) 96,95,96
   95 LO(I)=LO(I)+LOMAX/MDT**.5/EXP(X**2/4./EC/MDT)/
     . EXP(KR*MDT)
      GO TO 110
   96 DO(I)=DO(I)+DOMAX/MDT**.5/EXP(X**2/4./EC/MDT)*
     . (EXP(-KD*MDT)-EXP(-KA*MDT))
  110 CONTINUE
      MDT=MDT-DT
  150 CONTINUE
      DO 155 I=1,21
      LO(I)=LO(I)*100.
      DO(I)=DO(I)*100.
  155 CONTINUE
      IF (FORK) 165,160,165
  160 WRITE(IO,170) M,(LO(I),I=1,21)
      GO TO 200
  165 WRITE(IO,170) M,(DO(I),I=1,21)
  170 FORMAT(I5,21F6.2)
  200 CONTINUE
 1000 CONTINUE
      CLOSE(UNIT=1)
      END
```

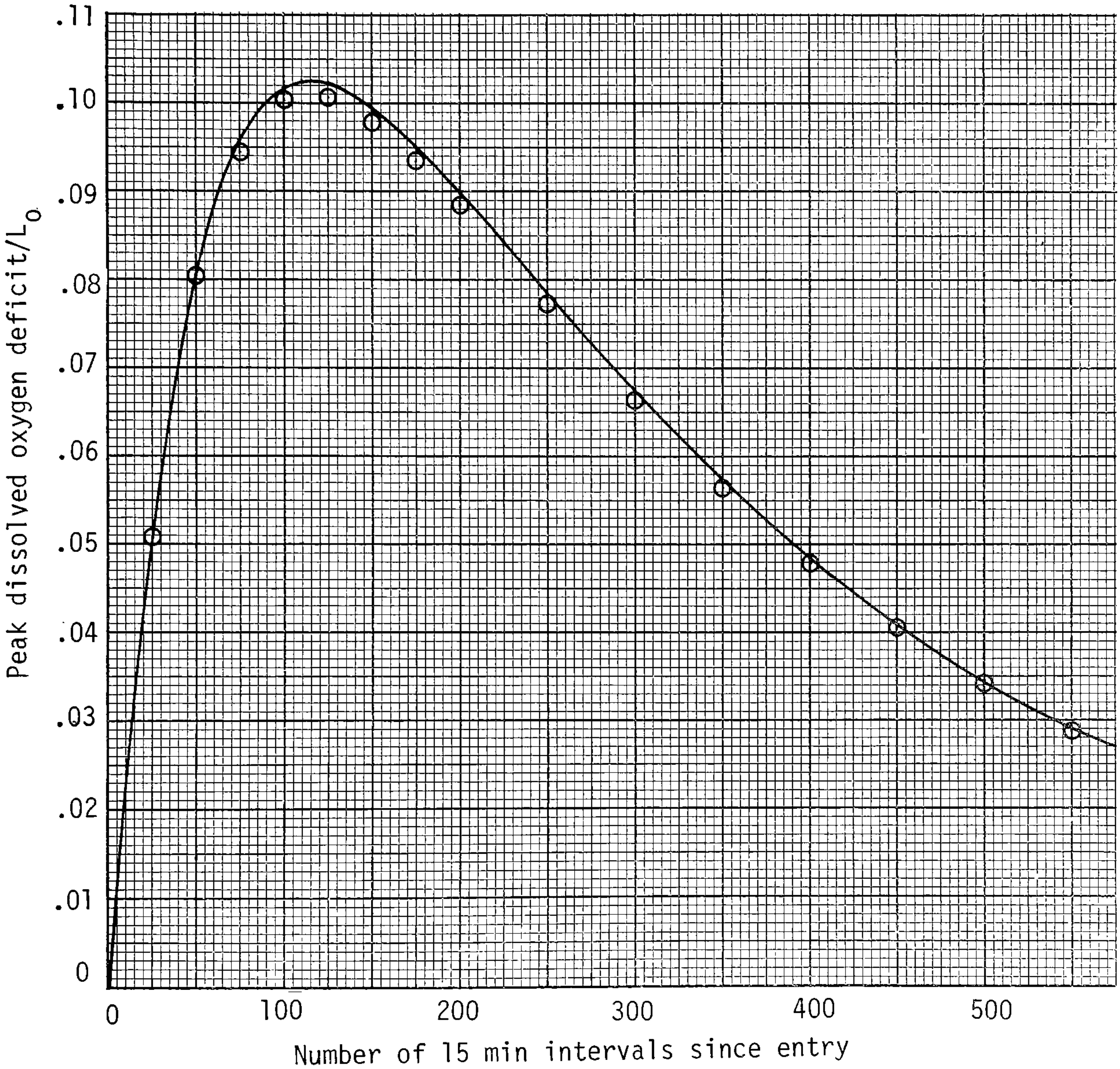


Figure 16. Comparison between computed dissolved oxygen deficit peak values using the program SWOPS shown by the solid line and using the closed form solution shown by the circled points. Time is measured as time intervals (15 min) from the entry of the first increment of the one-hour square pollution pulse.

Table 14

Sample Printout from SWOCFS

SAMPLE TEST CASE FOR THE CLOSED FORM SOLUTION --- 7-12-1977

KR	EC	VEL	QSS	KD	KA
0.2500	0.1000	24.0000	510.0000	0.2500	0.9800

DT	DN	TM	FORK
0.2500	16.0000	600.0000	1.0000

QIN

3.0000	3.8000	3.8000	3.8000	5.8000	5.8000	5.8000	5.8000	4.2000	4.2000
4.2000	4.2000	9.4000	9.4000	9.4000	9.4000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

LQIN

7.7000	8.3000	8.3000	8.3000	5.2000	5.2000	5.2000	5.2000	5.5000	5.5000
5.5000	5.5000	5.5000	5.5000	5.5000	5.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.07	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.07	0.08	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.07	0.08	0.09	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.03	0.05	0.06	0.07	0.08	0.10	0.11	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.03	0.05	0.06	0.07	0.08	0.10	0.11	0.12	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.03	0.04	0.06	0.07	0.08	0.09	0.11	0.12	0.13	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.03	0.04	0.04	0.07	0.08	0.09	0.10	0.12	0.14	0.14	0.10	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	0.04	0.04	0.05	0.08	0.09	0.10	0.12	0.14	0.15	0.15	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	0.04	0.05	0.06	0.09	0.10	0.12	0.13	0.15	0.16	0.17	0.12	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	0.05	0.06	0.07	0.10	0.12	0.13	0.14	0.16	0.18	0.18	0.12	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	0.06	0.07	0.08	0.11	0.13	0.14	0.16	0.18	0.19	0.19	0.13	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	0.07	0.08	0.09	0.13	0.14	0.16	0.17	0.19	0.20	0.20	0.14	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.08	0.09	0.11	0.14	0.16	0.17	0.18	0.20	0.22	0.21	0.15	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18	0.09	0.10	0.12	0.15	0.17	0.18	0.20	0.22	0.23	0.22	0.15	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	0.10	0.11	0.13	0.16	0.18	0.20	0.21	0.23	0.24	0.23	0.16	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	0.11	0.12	0.14	0.18	0.19	0.21	0.22	0.24	0.25	0.24	0.17	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	0.12	0.13	0.15	0.19	0.21	0.22	0.24	0.25	0.27	0.25	0.17	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.13	0.14	0.16	0.20	0.22	0.23	0.25	0.27	0.28	0.26	0.18	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
23	0.14	0.15	0.17	0.21	0.23	0.24	0.26	0.28	0.29	0.27	0.19	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
24	0.15	0.16	0.18	0.22	0.24	0.26	0.27	0.29	0.30	0.28	0.19	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.16	0.17	0.19	0.23	0.26	0.27	0.28	0.30	0.31	0.29	0.20	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
26	0.17	0.18	0.20	0.24	0.27	0.28	0.30	0.31	0.32	0.30	0.20	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
27	0.18	0.19	0.21	0.25	0.28	0.29	0.31	0.32	0.33	0.31	0.21	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	0.19	0.20	0.22	0.26	0.29	0.30	0.32	0.34	0.34	0.32	0.22	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	0.20	0.21	0.23	0.27	0.30	0.31	0.33	0.35	0.36	0.33	0.22	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	0.21	0.22	0.24	0.28	0.31	0.33	0.34	0.36	0.37	0.33	0.23	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 15

Computed Values for Peak Dissolved Oxygen Deficit and Time of Occurrence for Square Pollutographs

	Parameter	Value	Velocity, mi/day	2.4	6.0	12.0	24.0	32.15	48.0
			Pulse width (W), mi	0.1	0.25	0.5	1.0	1.34	2.0
I	K	4.68							
	K_r K_d	0.7	t*	0.229	.250	.312	417	.458	.490
	E	0.1	$DOMAX/L_o$	0.01658	.03965	.06911	.09602	.10201	.10569
	SP-t*	0.4774	N	2.351	.940	470	.235	.175	.118
	$SP\text{-}DOMAX/L_o$	0.1071	T*	2.99	.400	.125	.042	.026	.012
II	K_a	4.68							
	K_r - K_d	0.7	t*	0.229	.240	.271	.354	.406	.458
	E	0.2	$DOMAX/L_o$	0.01178	.02878	.05339	.08435	.09452	.10276
	SP-t*	0.4774	N	3.324	1.330	.665	.332	.248	.166
	$SP\text{-}DOMAX/L_o$	0.1071	T*	5.98	.768	.217	.071	.045	.023
III	K_a	4.68							
	K_r K_d	0.7	t*	0.229	.240	.260	.323	.375	.437
	E	0.3	$DOMAX/L_o$	0.00963	.02371	.04503	.07569	.08781	.09940
	SP-t*	0.4774	N	4.071	1.628	.814	.407	.304	.204
	$SP\text{-}DOMAX/L_o$	0.1071	T*	8.97	1.152	.312	.097	.063	.033
IV	K_a	4.68							
	K_r K_d	0.7	t*	0.229	.229	.240	.260	.292	.344
	E	1.0	$DOMAX/L_o$	0.00528	.01316	.02588	.04868	.06146	.07970
	SP-t*	0.4774	N	7.433	2.973	1.487	.743	.555	.372
	$SP\text{-}DOMAX/L_o$	0.1071	T*	29.9	3.664	.960	.260	.163	.086
V	K_a	2.0							
	K_r K_d	0.7	t*	0.406	.427	.490	.635	.708	.781
	E	0.1	$DOMAX/L_o$	0.02316	.05647	.10405	.16146	.17908	.19247
	SP-t*	0.8076	N	2.907	1.163	.581	.291	.217	.145
	$SP\text{-}DOMAX/L_o$	0.1989	T*	4.06	.683	.196	.635	.039	.020
VI	K_a	2.0							
	K_r K_d	0.7	t*	0.354	.406	.417	.448	.469	.531
	E	1.0	$DOMAX/L_o$	0.00722	.01835	.03636	.07021	.09081	.12407
	SP-t*	0.8076	N	9.193	3.677	1.839	.919	.686	.460
	$SP\text{-}DOMAX/L_o$	0.1989	T*	35.40	6.469	1.668	.448	.261	.133
VII	K_a	0.4							
	K_r K_d	0.2	t*	0.844	1.635	1.729	1.760	1.792	1.865
	E	1.0	$DOMAX/L_o$	0.00344	.01104	.02213	.04389	.05827	.08496
	SP-t*	3.466	N	18.803	7.521	3.761	1.880	1.403	.940
	$SP\text{-}DOMAX/L_o$	0.25	T*	84.40	26.160	6.916	1.760	.998	466
VIII	K_a	0.4							
	K_r K_d	0.4	t*	0.719	1.210	1.281	1.312	1.333	1.406
	E	1.0	$DOMAX/L_o$	0.00637	.01910	.03811	.07536	.09976	.14428
	SP-t*	2.5	N	15.811	6.324	3.162	1.581	1.180	.791
	$SP\text{-}DOMAX/L_o$	0.368	T*	71.90	20.160	5.124	1.312	.742	.352
IX	K_a	0.4							
	K_r K_d	0.7	t*	0.604	.937	.948	.979	1.010	1.083
	E	1.0	$DOMAX/L_o$	0.01014	.02860	.05698	.11222	.14796	.21182
	SP-t*	1.865	N	13.747	5.499	2.749	1.375	1.026	.687
	$SP\text{-}DOMAX/L_o$	0.474	T*	60.40	14.992	3.792	.979	.563	.271

$$N = \frac{E^{\frac{1}{2}}}{W\ (K_a K_r)^{\frac{1}{4}}} \tag{14}$$

In this relationship 'W' is the pulse width in the stream (miles) defined as the duration of the square pulse (days) times the stream velocity (miles per day). The single-valued relationship between the ratio and N is shown in Figure 17.

The time between entry of the leading edge of the square pollution pulse and the time at which the peak DO deficit concentration occurs (t^*) is also of interest in some problems. By analysis of the same computed data, shown in Table 14, it was found that the non-dimensional time (T^*) between entry and occurrence of the DO deficit peak when expressed as follows also plotted as a single-valued function of the non-dimensional group N.

$$T^* = Et^*/W^2 \tag{15}$$

The single valued relationship between T^* and N is shown in Figure 18.

It can be seen from Figure 17 that the effect of dispersion in the stream is to reduce the peak DO deficit concentration significantly. The time at which the peak DO deficit occurs is shown by Figure 18 to be much shorter than the corresponding time to the peak DO deficit as computed with the Streeter-Phelps model.

SUMMARY AND CONCLUSIONS

A closed-form expression for the dissolved oxygen deficit response in the stream to an impulsive BOD load can be used with the superposition principle to estimate the response to any known stormwater overflow event. The closed-form solution agrees well with corresponding numerical integration solution. Non-dimensional groupings have been developed to summarize computed values for peak dissolved oxygen deficit and time of occurrence for any square stormwater pollution pulse.

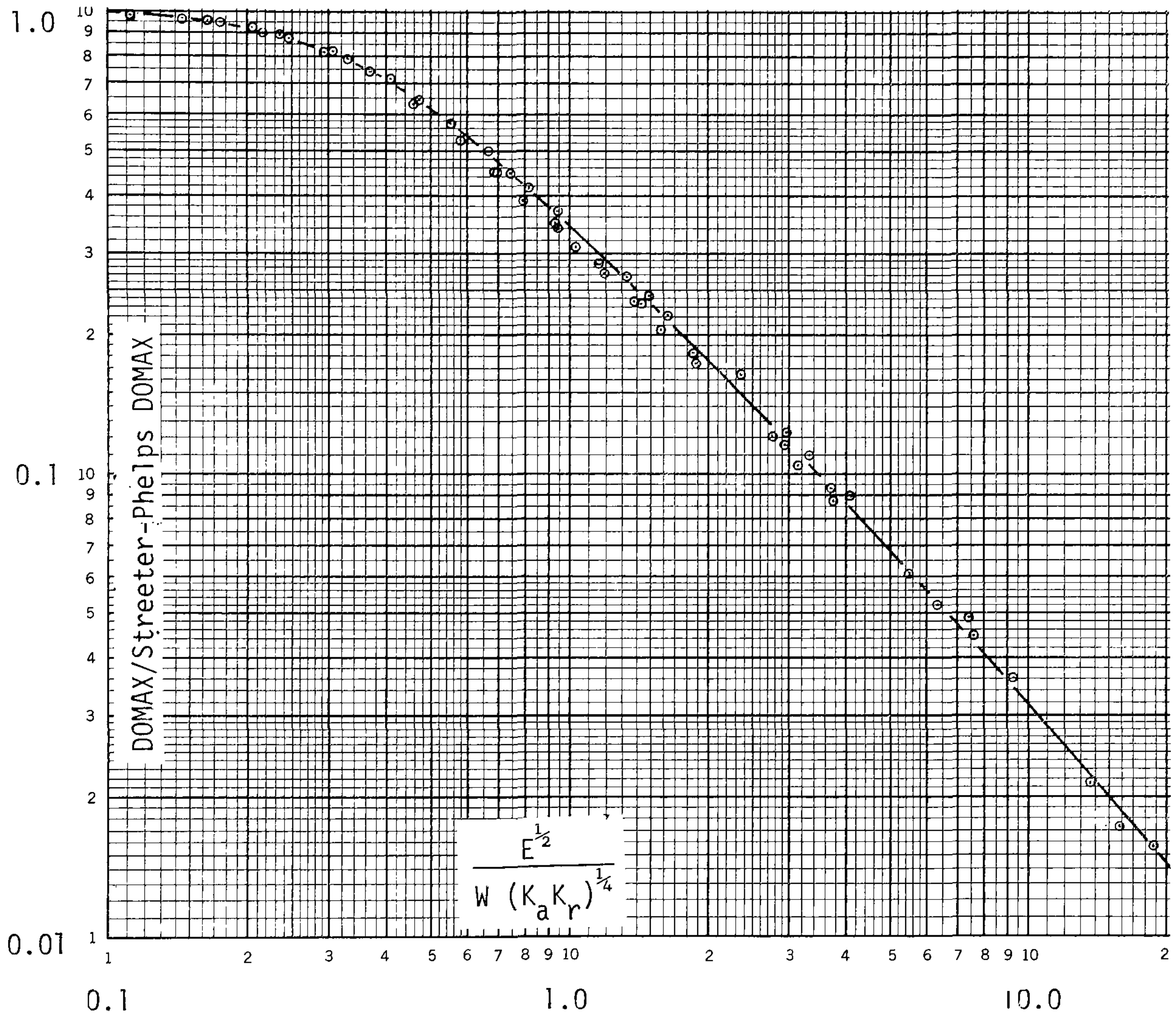


Figure 17. Correction factor for peak dissolved oxygen deficit to be applied to the value computed with the Streeter-Phelps model.

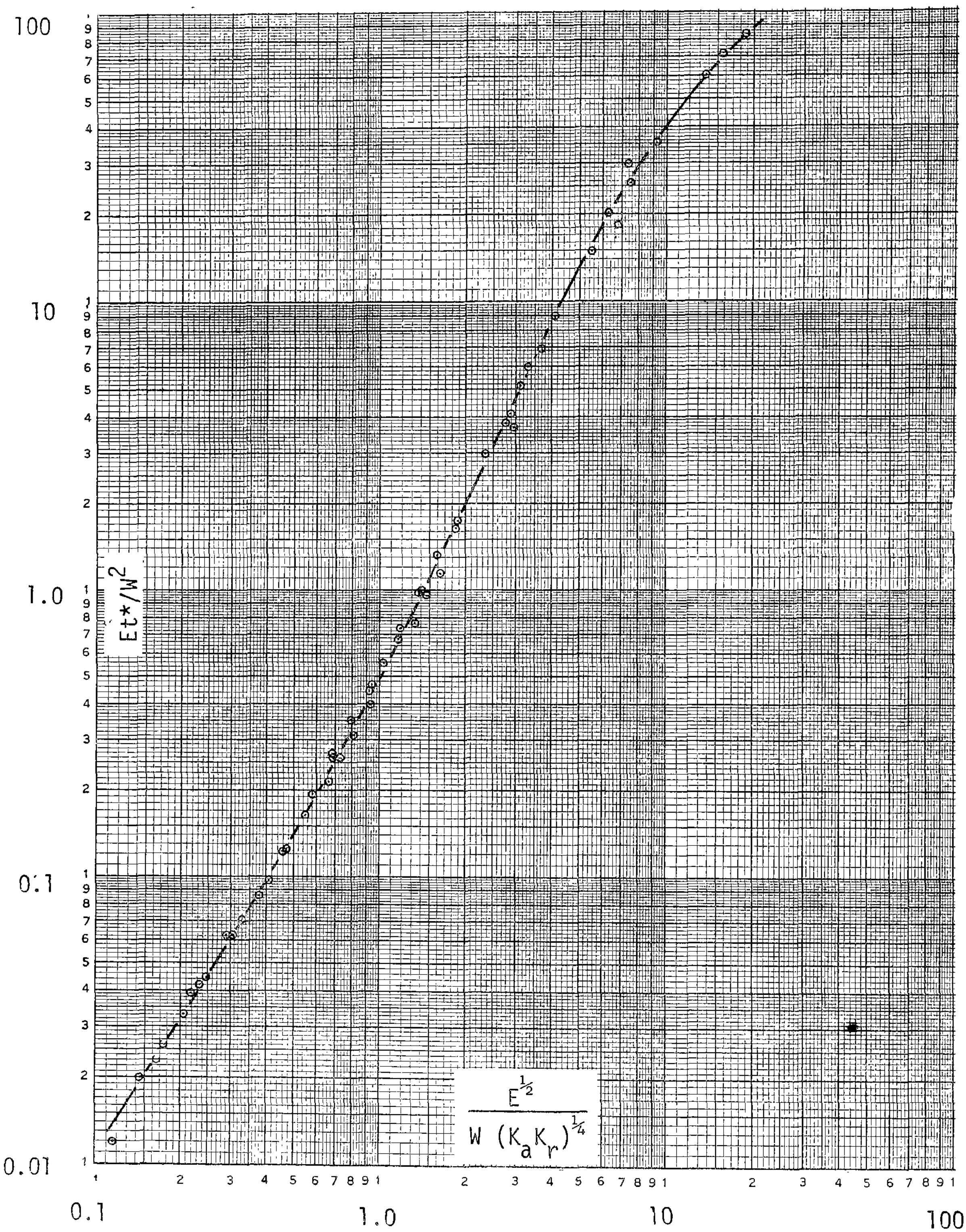


Figure 18. Relationship for finding the time (t^*) to the peak dissolved oxygen deficit from the non-dimensional grouping N.

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APPENDIX FOR SECTION 6

The following symbols are used in this section:

t = time, days

ΔT = incremental time for segmented storm overflow load, days

v = stream velocity, miles/day (km/day)

A = stream cross-sectional area, sq miles (km^2)

X = distance measured downstream from a point on the stream bank, miles (km)

x = distance measured downstream from a point in the stream, miles (km)

M = mass of impulsive BOD load lbs (kg)

L = concentration of BOD in the stream, mg/l

L_o = initial BOD concentration in the stream, mg/l

L' = concentration of BOD in the stream, lb/cu mile (kg/km^3)

L_i = concentration of BOD in the I'th segment of the storm overflow, mg/l

Q_i = volume flow of I'th segment of the storm overflow, cfs (m^3/sec)

Q = volume flow of receiving stream downstream of the storm overflow, cfs (m^3/sec)

D = concentration of dissolved oxygen deficit in the stream, mg/l

D_o = initial concentration of dissolved oxygen deficit in the stream, mg/l

E = dispersion coefficient, sq miles/day (km^2/day)

K_r = rate of BOD removal by oxidation and sedimentation, $days^{-1}$

K_d = rate of deoxygenation in the stream, $days^{-1}$

K_a = rate of stream reaeration, $days^{-1}$

Π = the constant 3.1416

APPENDIX (continued)

The following symbols are used in the program SWOCFS. Symbols corresponding to those used in the paper are KR = K_r, KA = K_a, EC = E, VEL = v, Q = Q and DT = ΔT in hours. Symbols not used in the paper are defined as follows:

DN = number of time increments in storm overflow

TM = number of time increments over which solution is required

QIN(I) = volume flow of storm overflow at center of I'th time increment, cfs (m^3/sec)

LOIN(I) = BOD concentration in overflow at center of I'th time increment, mg/l

DO(J) = computed dissolved oxygen deficit at J'th stream segment, mg/l

LO(J) = computed BOD concentration at J'th stream segment, mg/l

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15. SUPPLEMENTARY NOTES

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16. ABSTRACT

Three related studies are described which provide the means to quantify the pollutional and hydraulic effects on flowing streams caused by stormwater runoff. Mathematical stream models were developed to simulate the biological, physical, chemical, and hydraulic reactions which occur in a stream. Relationships take the form of differential equations with the two independent variables of time and distance. The differential equations can be solved directly by means of calculus or by digital computer using numerical methods. The solution would be the concentration of species of pollutional interest, such as BOD and dissolved oxygen, within the stream as a function of distance and time. The solution can be steady-state or transient. There is a sufficient amount of information presently available for finding steady-state solutions. However, when the pollutional loads and/or the initial conditions for the flowing stream vary with time, the problem becomes much more difficult and the technology for handling the transient situation has not been adequately developed. The purpose of this report is to show how the solution can be found for the case where the pollution loading is a transient, especially as it applies to the stormwater overflow.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
*Surface water runoff *Mathematical models *Stream pollution *Stream flow *Water pollution	Stormwater overflow Streeter-Phelps Dissolved oxygen deficit Stormwater hydrograph	13 B

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