United States
Environmental Protection
Agency

Industrial Environmental Research
Laboratory
Research Triangle Park NC 27711

EPA-600/7-80-107
May 1980

EPA

Effect of Physical Coal Cleaning on Sulfur Content and Variability

Interagency Energy/Environment R&D Program Report

RESEARCH REPORTING SERIES

Research reports of the Office of Research and Development, U.S. Environmental Protection Agency, have been grouped into nine series. These nine broad categories were established to facilitate further development and application of environmental technology. Elimination of traditional grouping was consciously planned to foster technology transfer and a maximum interface in related fields. The nine series are:

1. Environmental Health Effects Research
2. Environmental Protection Technology
3. Ecological Research
4. Environmental Monitoring
5. Socioeconomic Environmental Studies
6. Scientific and Technical Assessment Reports (STAR)
7. Interagency Energy-Environment Research and Development
8. "Special" Reports
9. Miscellaneous Reports

This report has been assigned to the INTERAGENCY ENERGY-ENVIRONMENT RESEARCH AND DEVELOPMENT series. Reports in this series result from the effort funded under the 17-agency Federal Energy/Environment Research and Development Program. These studies relate to EPA's mission to protect the public health and welfare from adverse effects of pollutants associated with energy systems. The goal of the Program is to assure the rapid development of domestic energy supplies in an environmentally-compatible manner by providing the necessary environmental data and control technology. Investigations include analyses of the transport of energy-related pollutants and their health and ecological effects; assessments of, and development of, control technologies for energy systems; and integrated assessments of a wide range of energy-related environmental issues.

EPA REVIEW NOTICE

EPA-600/7-80-107
May 1980

Effect of Physical Coal Cleaning on Sulfur Content and Variability

by

D.H. Sargent, B.A. Woodcock,
J.R. Vaill, and J.B. Strauss

Versar, Inc.
6621 Electronic Drive
Springfield, Virginia 22151

Contract No. 68-02-2136
Task No. 300
Program Element No. EHE623A

EPA Project Officer: James D. Kilgroe

Industrial Environmental Research Laboratory
Office of Environmental Engineering and Technology
Research Triangle Park, NC 27711

Prepared for

U.S. ENVIRONMENTAL PROTECTION AGENCY
Office of Research and Development
Washington, DC 20460

ABSTRACT

Sulfur content and heating value data for 53 different coal source - cleaning plant combinations were statistically analyzed, to document the effectiveness of commercial operating coal cleaning plants in reducing sulfur and enhancing heating value, and to investigate the effect of physical coal cleaning upon sulfur variability.

Cleaning plants for which matched pairs of feed and product coal data were available exhibited reductions (from feed to product) in the mean lbs SO_2/MM Btu of 24 to 50 percent. These empirical data fall within the range (8 to 81 percent reduction) of the calculated design performance of coal cleaning plants. The wide ranges reflect the sensitivity of performance to coal washability and to plant design.

These matched pairs of empirical feed and product coal data exhibited a reduction in sulfur variability averaging 55 percent and ranging from 9 to 90 percent. An indirect analysis of a larger data base, where matched pairs of data were not available, resulted in similar reductions in sulfur variability attributable to physical coal cleaning.

Much of the coal data exhibited autocorrelation, verifying expectations based upon geology and engineering rationale. Analysis of the data resulted in estimates of the long-term (geostatistical) component of variability, in the short-term component of variability, and in the component of variability attributable to coal sampling and analysis. By removing the long-term component (which includes autocorrelation), an inverse relationship between relative standard deviation and lot size was empirically demonstrated.

CONTENTS

CONTENTS (Continued)

Appendices

FIGURES

TABLES

TABLES (Continued)

ACKNOWLEDGEMENTS

This study was made possible by the cooperation of the National Coal Association and several of its member companies in supplying the extensive data required for statistical analysis. The authors also thank Mr. Carl Nelson of PEDCo Environmental, Inc., for his help in supplying the original detailed data used in his prior study of sulfur variability, much of which has been incorporated into the data base for this study.

Two mandates for this study had to be carefully balanced: the need for a useful product and the requirement for meticulous statistical treatment. Fortunately, the authors were guided in this effort by EPA Project Officers and advisors with the required sensitivity for this balance: Mr. James D. Kilgroe, Mr. David Kirchgessner, Mr. Robert Lagemann and Dr. Kenneth E. Rowe. Our special gratitude is extended to Dr. Constancio F. Miranda, for his guidance, for his careful and constructive reviews of the drafts leading to this final document, and for his active participation in the detailed analytical work.

SECTION 1

INTRODUCTION

..1 OBJECTIVES OF THE STUDY

The Office of Research and Development (ORD), Industrial Environmental Research Laboratory of the U.S. Environmental Protection Agency (EPA) at Research Triangle Park is studying coal cleaning technology to determine its potential for controlling the emissions of sulfur dioxide from coal-fired boilers. Two mechanisms are straightforward: lowering of the mean sulfur content in coal by removal of pyritic sulfur, and lowering of the mean pounds of sulfur (or equivalent SO_2) per million Btu by enhancing the heating value. The original purpose of this study was to document the performance of U.S. coal preparation plants by gathering and analyzing existing data. This documentation of sulfur reduction and of heating value enhancement varifies the effectiveness of existing and operating coal cleaning technology as a sulfur dioxide control for boilers.

The ability of boiler operators to comply with existing or proposed emission regulations, and the costs associated with such compliance, also depend upon the variabilities of coal sulfur content and heating value. An emission limitation, expressed as a maximum heat-specific SO_2 value (lbs SO_2/MM Btu), has the effect of requiring combustion of a coal with a mean lbs SO_2/MM Btu value lower than the emission limit, to prevent exceeding the limit when the coal sulfur excursions about the mean are positive. Two factors determine how much lower the mean heat-specific SO_2 value must be than the emission limit: the fractional time that the regulations permit a boiler to exceed the nominal limit (confidence level), and the characteristic variability in the coal feed (standard deviation or relative standard deviation).

Quantification of this second factor, i.e., the characteristic variability of heat-specific sulfur content in coal, is a second prime objective of this study. A previous EPA-sponsored study[1] served as a beginning in quantifying this factor. This present study extends the data base to include higher sulfur coals. The primary objective of this study (which the previous study did not address) is to quantify the reduction of sulfur variability achieved by physical coal cleaning. Secondary objectives are to better understand the fundamental variability of sulfur and heating value in coal, and to add to the understanding of how lot size affects variability.

1.2 BACKGROUND ON COAL CLEANING EFFECTIVENESS AS A SULFUR DIOXIDE CONTROL TECHNOLOGY

The available information on the effectiveness of physical coal cleaning in reducing the lbs SO_2/MM Btu values of coal is generally limited to calculated performance, for specific coals and for specific plant designs, using washability data as the basis. An objective of this study is to provide actual performance data from operating plants to supplement the published calculated performance values.

The calculated data may be classified into two types. In the first type, the theoretical potential for reduction of lbs SO_2/MM Btu was calculated using coal washability data and some arbitrary degree of crushing and operating specific gravity. The U.S. Bureau of Mines, in RI 8118, published washability data for 455 coal samples, representing mines which currently provide more than 70 percent of the utility coal. Table 1 lists the raw coal characteristics composited by region, the calculated theoretical product characteristics (upon crushing to 3/8-inch top size and separating, with no misplaced material, at 1.60 specific gravity), and the resulting percent reduction in lbs SO_2/MM Btu.[7] Under the conditions stated, the average potential reduction in lbs SO_2/MM Btu ranged (by region) from 16 to 43 percent.

For the second type of published effectiveness data, a coal cleaning plant of a desired level of complexity was tailored for each coal considered, and the performance of that plant was calculated, taking into consideration the inefficiencies of separation on a commercial scale. Each cleaning level includes one or more of the major unit operations. Although the levels may oversimplify a complex technology, they illustrate and identify the basic coal preparation principles.

TABLE 1. AVERAGE CALCULATED POTENTIAL FOR DESULFURIZATION BY PHYSICAL COAL CLEANING*

(Crushing to 3/8-inch Top Size, Separation at 1.60 S.G.)

Coal region		Northern Appalachia	Southern Appalachia	Alabama	Eastern Midwest	Western Midwest	Western	Total U.S.
Number of samples		227	35	10	95	44	44	455
Raw Coal Characteristics	Pyritic S, %	2.01	0.37	0.69	2.29	3.58	0.23	1.89
	Total S, %	3.01	1.03	1.33	3.92	5.25	0.68	3.00
	Ash, %	15.1	11.0	9.5	14.2	16.2	8.9	14.0
	Btu/lb	12,693	13,314	13,696	12,189	12,072	12,437	12,573
	lbs $SO_2/10^6$Btu	4.74	1.55	1.94	6.43	8.69	1.09	4.77
Cleaned Coal Characteristics	Pyritic S, %	0.85	0.19	0.49	1.03	1.80	0.10	0.85
	Total S, %	1.86	0.91	1.16	2.74	3.59	0.56	2.00
	Ash, %	8.0	5.1	5.8	7.5	8.3	6.3	7.5
	Btu/lb	13,766	14,197	14,264	13,138	13,209	12,779	13,530
	lbs $SO_2/10^6$Btu	2.70	1.28	1.63	4.17	5.43	0.88	2.95
Btu Recovery, %		92.5	96.1	96.4	94.9	91.7	97.6.	93.8
Weight Recovery, %		85.3	90.1	92.6	88.0	83.8	95.0	87.2
Increase in Btu/lb, %		8.5	6.6	4.1	7.8	9.4	2.7	7.6
Decrease in Pyritic S, %		58	49	29	55	50	57	55
Decrease in Total S, %		38	12	13	30	32	18	33
Decrease in $\frac{\text{lbs } SO_2}{10^6\text{Btu}}$, %		43	17	16	35	38	19	38

*Source: U.S. Bureau of Mines [7]

Level 1 - Breaker for top size control and for the removal of coarse refuse.

Level 2 - Coarse beneficiation - where larger fractions of coal (plus 3/8 inch) are treated. The separated and untreated minus 3/8 inch portion of the coal is combined with the cleaned coarse coal for shipment.

Level 3 - Fine and coarse size beneficiation - where all the feed is wetted. Plus 28M is beneficiated; 28M x 0 material is dewatered and either shipped with clean coal or discarded as refuse.

Level 4 - Very fine beneficiation - where all the feed is wetted and washed. Thermal drying of 1/4" x 0 fraction generally is often desired to limit moisture content.

Level 5 - Full beneficiation resulting in multiproducts - where the raw coal is crushed to much finer sizes, resulting in multistage cleaning and multiproduct operation. A plant optimized to remove both pyritic sulfur and ash from amenable coals would most likely be of this type.

Tables 2A, 2B, and 2C summarize the performance calculations conducted by Bechtel Corporation for the Department of Energy[8] and by Versar, Inc., for the Environmental Protection Agency.[9,10] The calculated reductions in lbs SO_2/MM Btu ranged from 8 to 81 percent. The data in Tables 2A and 2C clearly show that the effectiveness of coal cleaning as an SO_2 control technology depends to a large extent upon the cleanability potential of the raw coal, which varies widely from coal to coal. The data in Table 2B, all for the same raw coal, clearly show that the effectiveness is highly sensitive to the complexity of the cleaning plant design. These data demonstrate that no valid "typical" effectiveness can be quoted for physical coal cleaning technology.

Because the existing published measures of coal cleaning effectiveness are based upon design calculations for hypothetical plants, a verification was needed. Acquiring actual data from operating plants to fulfill this need was one of the original objectives of this study.

TABLE 2A*.

CALCULATED DESIGN PERFORMANCE OF CLEANING PLANTS

Cleaning level		II	III	IV	V - Multiproduct plant		
Coal source		West Va. Cedar Grove	Colorado Montrose	Pennsylvania Lower Kittaning	Pennsylvania Upper Freeport		
Raw coal character-istics	Pyritic S, %	0.11	0.25	2.19	2.79		
	Total S, %	0.81	0.80	2.77	3.40		
	Ash, %	22.7	19.4	13.0	23.4		
	Btu/lb	11,810	11,790	12,830	11,486		
	lbs $SO_2/10^6$ Btu	1.37	1.36	4.32	5.92		
Cleaned coal character-istics					Clean Pdt	Middlings	Composite
	Pyritic S, %	0.06	0.19	0.22	0.22	1.56	0.71
	Total S, %	0.75	0.73	0.80	0.83	2.17	1.32
	Ash, %	17.0	10.5	4.7	3.3	17.5	8.5
	Btu/lb	12,655	13,120	14,250	14,608	12,342	13,774
	lbs/$SO_2/10^6$ Btu	1.19	1.11	1.12	1.14	3.52	1.92
Btu Recovery, %		97.4	94.3	76.2	61.2	30.1	91.3
Weight Recovery, %		90.9	84.7	68.6	48.1	28.0	76.1
Increase in Btu/lb, %		7.2	11.3	11.1	27.2	7.5	19.9
Decrease in Pyritic S, %		45	24	90	92	44	75
Decrease in Total S, %		7	9	71	76	36	61
Decrease in lbs $SO_2/10^6$ Btu, %		13	18	74	81	41	68

*Source: Bechtel Corporation[8]

TABLE 2B*.

CALCULATED DESIGN PERFORMANCE OF CLEANING PLANTS
RAW COAL: UPPER FREEPORT "E" SEAM, BUTLER, PENNSYLVANIA**

Cleaning level		II	III	IV	V - Multiproduct plant		
					Clean Pdt	Middlings	Composite
Product	Total S, %	3.00	1.89	1.30	1.08	1.69	1.53
	Ash, %	20.0	11.5	7.6	5.80	11.31	8.66
	Btu/lb	12,260	13,551	14,000	14,426	13,612	14,004
	lbs $SO_2/10^6$ Btu	4.89	2.79	1.85	1.50	2.48	2.01
Btu recovery, %		97.0	84.0	87.5	43.4	44.0	87.4
Weight recovery, %		92.8	73.2	70.0	35.3	38.0	73.3
Increase in Btu/lb, %		6.5	17.7	21.6	25.3	18.3	21.7
Decrease in total S, %		13	45	62	69	51	56
Decrease in $\frac{\text{lbs } SO_2}{10^6 \text{ Btu}}$, %		18	53	69	75	59	66

*Source: Versar Inc.(9)
**Upper Freeport Coal:
Pyritic S, 2.51%
Total S, 3.45%
Ash, 23.90%
Btu/lb 11,510
lbs $SO_2/10^6$ Btu, 5.99

TABLE 2C.* CALCULATED DESIGN PERFORMANCE OF CLEANING PLANTS

Cleaning level	IV	IV	III	IV	IV	III	IV	IV	II	III	IV	IV	IV
Coal source	Upper Freeport, PA.	Lower Kitt., PA	Upper Freeport, PA	Bakerstown, WV	Clintwood, VA	Upper Cliff, AL	Splash Dam, VA	Eagle, VA	Hagy, VA	No. 6 IL	No. 5 IL	No. 5 IL	Bevier, MO
Raw coal													
Pyritic S, %	2.51	1.34	2.51	--	--	0.41	--	--	--	3.13	2.01	2.01	--
Total S, %	3.45	1.86	3.45	0.92	0.87	0.85	1.56	1.18	2.95	4.35	2.88	2.88	5.22
Ash, %	23.9	12.8	23.9	28.7	11.2	10.9	25.9	10.4	23.1	29.9	16.4	16.4	27.2
Btu/lb	11,510	13,508	11,510	10,750	13,891	13,845	11,275	13,622	11,688	9,782	12,120	12,120	8,011
lbs $SO_2/10^6$ Btu	5.99	2.75	5.99	1.71	1.25	1.23	2.77	1.73	5.04	8.89	4.75	4.75	13.04
Product coal													
Pyritic S, %	0.95	0.67	1.34	--	--	0.34	--	--	--	1.80	1.06	0.91	--
Total S, %	1.57	1.22	2.21	0.82	0.83	0.75	1.24	0.94	2.31	3.30	1.99	1.85	4.05
Ash, %	9.7	8.7	14.4	19.9	8.1	9.5	9.7	4.7	15.5	11.4	7.5	7.1	8.0
Btu/lb	13,704	14,139	12,971	12,072	14,382	14,094	13,960	14,487	13,002	12,370	13,407	13,464	10,123
lbs $SO_2/10^6$ Btu	2.30	1.72	3.41	1.35	1.15	1.06	1.77	1.30	3.55	5.34	2.97	2.75	7.99
Btu recovery, %	88.7	96.9	94.2	90.3	93.0	98.0	92.1	89.3	93.8	91.6	92.9	88.6	83.1
Weight recovery, %	74.5	92.6	83.6	80.5	89.9	96.3	74.4	84.0	84.3	72.4	84.0	79.8	65.8
Increase in Btu/lb, %	19.1	4.7	12.7	12.3	3.5	1.8	23.8	6.4	11.2	26.5	10.6	11.1	26.4
Dec. in Pyritic S, %	62	50	47	--	--	17	--	--	--	42	47	55	--
Dec. in Total S, %	54	34	36	11	5	12	21	20	22	24	31	36	22
Dec. in $\frac{\text{lbs } SO_2}{10^6 \text{ Btu}}$, %	62	37	43	22	8	14	36	25	30	40	37	42	39

*Source: Versar, Inc.[10]

1.3 IMPORTANCE OF COAL SULFUR VARIABILITY

The variability of coal has a pronounced effect upon the ability and costs for boiler operators to comply with existing or proposed emission regulations. An emission limitation, expressed as a maximum value in lbs SO_2/MM Btu, to be exceeded only for a specified percentage of the time, has the effect of requiring a coal with a mean lbs SO_2/MM Btu value lower than the emission limit.

The relationship between μ, the mean coal value for lbs SO_2/MM Btu and E, the emission limitation value or maximum permissible emission in lbs SO_2/MM Btu, may be described by the equation:[3]

$$E = \beta(\mu + t_\alpha S) = \beta\mu(1 + t_\alpha RSD) \tag{1}$$

where β = the fraction of sulfur in the coal which is emitted (less losses to bottom ash and fly ash). Although β has a distribution of its own, its variability is comparatively small, and β is generally assumed constant at 0.95.

RSD = the relative standard deviation for lbs SO_2/MM Btu, defined as the standard deviation divided by the mean.

S = the absolute standard deviation for lbs SO_2/MM Btu.

t_α = the one-tailed students' "t" value assuring a fractional compliance time of α. For the purpose of illustrating the importance of coal sulfur variability in this section, it is assumed that the data follow a quasinormal or "t" distribution. The validity of this assumption is discussed in Section 5.5.

Values of t_α for large numbers (>100) of observations are[2]:

α, Fractional compliance time	0.80	0.90	0.95	0.99	0.9995
t_α	0.85	1.29	1.66	2.36	3.37

A prior EPA study[1] of sulfur variability in coal resulted in RSD values ranging from 5 to 34 percent (excluding core samples which resulted in higher RSD values).

Table 3, calculated from Equation 1, lists the ratio of μ (the required mean coal value for lbs SO_2/MM Btu) to E (the emission limitation value). It is apparent that this ratio is much less than unity for typical RSD values, and is also quite sensitive to RSD. Hence, the variability of sulfur in coal assumes importance in formulating new emission standards and in complying with existing or new standards. This study addresses the variability of sulfur and of heating value in coal with the objectives of better understanding the sources of variability, of better quantification of variability for a given coal, and ultimately of suggesting means for attenuating this variability.

1.4 EXPECTED ATTENUATION OF VARIABILITY BY COAL CLEANING PROCESSES

Three reasons exist for expecting washed coals to exhibit a lower sulfur variability than raw coals. First, is the reduction of the variance in coal sampling and sample preparation, as a result of coal cleaning. Most cleaning plants include a size reduction step, ameliorating the influence of large integral impurity particles upon each sample. The cleaning process removes much of these integral impurity particles, thereby making the product more uniform. The increased precision of sampling washed coals (as opposed to raw coals) has been experimentally documented.[5,6] Keller has proposed a revision to the ASTM coal sample preparation method in which half as much of the reduced sample quantity was required for cleaned coals as for raw coals.[5]

The second reason is that the coal preparation process contains many opportunities for blending. Coal unloading, loading, and storage operations, size reduction operations, and some separation operations feature mixing of the coal, which should reduce short-term variability. A third reason is that specific gravity separation processes should dampen the feed variations. If the instantaneous pyritic sulfur and/or ash content is greater than nominal, the instantaneous refuse stream should be larger, thereby resulting in a product stream of greater uniformity.

These reasons for expecting attenuation of variability, coupled with the importance of sulfur variability, led to the primary objective for this study - to quantitatively determine, by analyzing data from operating coal cleaning plants, the attenuation of sulfur variability achieved by physical coal cleaning.

TABLE 3. CALCULATED VALUES OF MEAN COAL-TO-EMISSION LIMIT RATIO (μ/E)

RSD of coal	Fractional compliance time			
	0.90	0.95	0.99	0.9995
0.05	0.99	0.97	0.94	0.90
0.10	0.93	0.90	0.85	0.79
0.15	0.88	0.84	0.78	0.70
0.20	0.84	0.79	0.72	0.63
0.30	0.76	0.70	0.62	0.52
0.40	0.69	0.63	0.54	0.45

SECTION 2

SOURCES OF SULFUR VARIABILITY

2.1 CHARACTERIZATION OF VARIABILITY IN COAL AND IN COAL DATA

One approach to characterizing variability in coal is to relate measured coal properties to the geologic history and current geologic characteristics of the deposit. Included in this approach is the documentation of the precise mining location for each lot of coal sampled and analyzed. Also included in this approach is the correlation of sulfur and heating value data to petrographic data and to the mineralogical and chemical composition of the impurities in coal. These subjects are currently under study by the U.S. Geological Survey.

An alternative approach, taken in this study, is to characterize sulfur variability from existing coal analysis data itself. This approach is empirical, compared to the more fundamental approach described above. The ultimate objective, however, is the same: to be able to predict with some precision the variability of each coal resource, and further, to find ways of attenuating or accomodating this variability.

To implement this empirical approach, the observed variance for each coal source was divided into three basic categories:

1. Long-Term Component. For each coal source, the coal characteristics change as mining progresses, and mining and processing methods may change as well. From a utilization standpoint (although the rationale is based upon geology and technology), the long-term component of variance is defined in this study as the month-to-month variation.

2. Short-Term Component. This is defined, for practicality, as lot-to-lot variability. The lot is the smallest unit for which coal characteristics data are reported and maintained for each source. The lot size may be a daily quantity, a unit-train quantity, or some other discrete value. Rationale for day-to-day variance may include local coal variations within the mine and short-term variations in processing efficien-

cies. This component of variance may depend upon the quantity of coal in a lot for each source.

3. Sampling/Sample Preparation/Analysis Component. This component of the variance depends upon whether ASTM methods are employed. It also depends upon whether or not the coal has been washed.[5,6]

In summary, the model adopted for examining sulfur variability has the following generalized components of variability:

$$V_{Total} = V_{\substack{Long\\Term}} + V_{\substack{Short\\Term}} + V_{\substack{Sampling\\\&\ Analysis}}$$

This model is applicable to data from a specific coal source. There are additional components of variance, when considering multiple coal sources, that are geographical and source-specific in nature. The sulfur level and its variability are unique according to the region and the seam of the deposit, and to the particular mine and mining method. Components of variance may be defined as region-to-region, seam-to-seam, and (mining or processing) source-to-source. Rationale for the first two and for part of the third is geology; part of the source-to-source variance may be attributable to mining methods and coal preparation methods employed.

2.2 PERMISSIBLE VALUES FOR THE SAMPLING COMPONENT OF VARIANCE

As defined above, the "sampling" component of variance includes sample preparation (both field and laboratory steps) and laboratory analysis, as well as sampling itself. The American Society for Testing and Materials (ASTM) Standards D-492 and D-2234 widely adopted in the commercial sampling of coal, are intended to provide an accuracy of $\pm$ 10 percent (of the ash content) in 95 out of 100 test results.[4] It may be implied that the sampling accuracy intended for ash content applies as well to sulfur content and to heating value. At a confidence level of 95 percent, approximately equivalent (for the two-tailed "t" distribution) to two standard deviations, the ASTM-intended relative standard deviation is 0.05.

Other ASTM standards address laboratory analysis precision. The permissible variance for ash analysis is 20 percent of the total sampling and analysis variance. The implied permissible relative standard deviation of ash analysis is therefore $0.05\sqrt{0.2} = 0.0224$. The permissible sampling relative standard deviation (which is also valid for sulfur content and for heating value) is $0.05\sqrt{0.8} = 0.0447$.

For sulfur determinations, the following precision limits are specified by ASTM:

Coal containing less than 2 percent sulfur, 0.1 percent

Coal containing 2 percent sulfur or more, 0.2 percent.

Translating into relative standard deviation (assuming a 95 percent confidence level for a two-tailed "t" distribution is implied),

Mean sulfur content %	Permissible analysis std. dev. %	Permissible analytical RSD	Permissible total RSD for sulfur*
0.5	0.05	0.100	0.110
1.0	0.05	0.050	0.067
1.5	0.05	0.033	0.056
2.0	0.10	0.050	0.067
2.5	0.10	0.040	0.060
3.0	0.10	0.033	0.056
4.0	0.10	0.025	0.051
5.0	0.10	0.020	0.049
6.0	0.10	0.017	0.048

*The last column is derived (since sampling and analytical RSDs are independent) from: Permissible Total RSD = $[0.0447^2 + (\text{Permissible Analytical RSD})^2]^{1/2}$

The ASTM precision limits for the laboratory determination of heating value is 100 Btu/lb. At an assumed 95 percent confidence level, the corresponding relative standard deviations are:

Mean heating value , Btu/lb	Permissible for RSD	Permissible total RSD for heating value*
9,000	0.0056	0.045
10,000	0.0050	0.045
11,000	0.0045	0.045
12,000	0.0042	0.045
13,000	0.0038	0.045
14,000	0.0036	0.045

*Including permissible sampling variance.

Assuming the total RSD for sulfur value is independent from the total value for heating value (see Section 5.3), the permissible RSD for lbs SO_2/MM Btu is:

$$\left[(\text{RSD for S})^2 + (\text{RSD for Heating Value})^2\right]^{1/2}$$

The derived values are listed below:

Mean sulfur content, %	Permissible total RSD for lbs. SO_2/MM Btu
0.5	0.119
1.0	0.081
1.5	0.072
2.0	0.081
2.5	0.075
3.0	0.072
4.0	0.068
5.0	0.067
6.0	0.066

These values, for the permissible RSDs in lbs SO_2/MM Btu, based upon ASTM sampling and analysis protocols, are 7 or 8 percent of the mean (for mean sulfur contents greater than one percent).

2.3 EXPERIMENTAL VALUES FOR THE SAMPLING COMPONENT OF VARIANCE

Keller[5] and Aresco and Orning[6] experimentally determined for ash the components of the coal sampling, sample preparation, and analysis variance. In their tests, field sampling, field sample preparation, laboratory sample preparation, and laboratory analyses were conducted according to ASTM standards. Their results, in terms of the components of variance in ash content, are listed in Table 4. Also listed in Table 4 are the calculated relative standard deviations for each component of variance. These were pooled according to:

$$\text{Pooled RSD} = \left[\frac{\Sigma N(\text{RSD})^2}{\Sigma N}\right]^{1/2}$$

where N = number of tests in each data set. Since RSDs have been arithmetically averaged in prior work,[1] average RSDs were also calculated. The results are:

	Raw coals			Washed coals		
	d.f.	$(RSD)_p$	$\overline{RSD}$	d.f.	$(RSD)_p$	$\overline{RSD}$
RSD_s for Sampling	146	0.0291	0.0275	54	0.0160	0.0150
RSD_p for Sample Preparation	206	0.0149	0.0148	156	0.0126	0.0122
RSD_A for Lab Analysis	206	0.0080	0.0071	156	0.0114	0.0073

TABLE 4. COMPONENTS OF ASH VARIANCE IN SAMPLING AND ANALYSIS

Data source*	Source ID	Coal condition	No. tests	Ash content, percent								
				Mean	V_S	V_P	V_A	V_T	RSD_S	RSD_P	RSD_A	RSD_T
K	A	Raw	10	12	--	0.0407	0.0961	--	--	0.0168	0.0258	--
K	B	Raw	20	12.5	--	0.0360	0.0054	--	--	0.0152	0.0059	--
K	C	Raw	10	29	--	0.2935	0.0052	--	--	0.0187	0.0025	--
K	K	Raw	10	22	--	0.0329	0.1471	--	--	0.0082	0.0174	--
K	L	Raw	10	16	--	0.2948	0.0036	--	--	0.0339	0.0038	--
AO	1	Raw	22	9.05	0.0644	0.0090	0.0012	0.0746	0.0280	0.0105	0.0038	0.0302
AO	2	Raw	34	9.22	0.0627	0.0153	0.0015	0.0795	0.0272	0.0134	0.0042	0.0306
AO	3	Raw	17	10.95	0.1472	0.0162	0.0017	0.1651	0.0350	0.0116	0.0038	0.0371
AO	4	Raw	12	5.11	0.0234	0.0063	0.0012	0.0309	0.0299	0.0155	0.0068	0.0344
AO	5	Raw	28	8.21	0.0442	0.0112	0.0011	0.0565	0.0256	0.0129	0.0040	0.0290
AO	6	Raw	29	11.96	0.1498	0.0175	0.0018	0.1691	0.0324	0.0111	0.0035	0.0344
AO	7	Raw	4	15.14	0.0471	0.0245	0.0028	0.0744	0.0143	0.0103	0.0035	0.0180
K	D	Washed	10	7.0	--	0.0036	0.0037	--	--	0.0086	0.0087	--
K	F	Washed	42	5.5	--	0.0045	0.0098	--	--	0.0122	0.0180	--
K	I	Washed	10	4.5	--	0.0090	0.0050	--	--	0.0211	0.0157	--
K	M	Washed	20	4.2	--	0.0030	0.0015	--	--	0.0130	0.0092	--
K	N	Washed	10	6.5	--	0.0064	0.0009	--	--	0.0123	0.0046	--
K	P	Washed	10	7.0	--	0.0027	0.0037	--	--	0.0074	0.0087	--
AO	8	Washed	18	8.25	0.0195	0.0100	0.0012	0.0307	0.0169	0.0121	0.0042	0.0212
AO	9	Washed	6	7.02	0.0062	0.0064	0.0004	0.0130	0.0112	0.0114	0.0028	0.0162
AO	10	Washed	3	8.54	0.0315	0.0070	0.0006	0.0391	0.0208	0.0098	0.0029	0.0232
AO	11	Washed	6	5.11	0.0026	0.0083	0.0008	0.0117	0.0100	0.0178	0.0055	0.0212
AO	12	Washed	16	8.34	0.0223	0.0086	0.0010	0.0329	0.0179	0.0111	0.0038	0.0217
AO	13	Washed	5	10.44	0.0187	0.0103	0.0009	0.0299	0.0131	0.0097	0.0029	0.0166

*K = Keller(5), AO = Aresco and Orning(6)

V = Variance
RSD = Relative Standard Deviation
Subscripts = S = Sampling
P = Sample Preparation
A = Laboratory Analysis
T = Total

$(RSD)_p$ refers to the pooled RSD, and $\overline{RSD}$ to the average RSD. Although $\overline{RSD}$ is always less than $(RSD)_p$, the differences are slight.

The total RSD, calculated from

$$(RSD_T)_p = [(RSD_s)_p^2 + (RSD_p)_p^2 + (RSD_A)_p^2]^{1/2}$$

is: $(RSD_T)_p = 0.0337$ for raw coals,

$(RSD_T)_p = 0.0233$ for washed coals.

These values are smaller than the permissible maximum relative standard deviation for ash analysis, derived from the ASTM standard, of 0.05.

The combined RSDs for sample preparation and laboratory analysis are:

$(RSD_{P,A})_p = [0.0149^2 + 0.0080^2]^{1/2} = 0.0169$ for raw coals,

$(RSD_{P,A})_p = [0.0126^2 + 0.0114^2]^{1/2} = 0.0170$ for washed coals.

Combined RSD values are virtually equal for raw and washed coals, leading to the conclusion that the larger total RSD for raw coals (compared to washed coals) results from coal sampling differences and not from laboratory differences. This conclusion is rational, since washed coal is expected to have a smaller particle size and to be more uniform in composition, leading to lower sampling-induced variability. The empirical sampling RSD_s, which should apply to sulfur content and heating value as well as to ash content, are:

$(RSD_s)_p = 0.029$ for raw coals,

$(RSD_s)_p = 0.016$ for washed coals.

Keller[5] also reported the variances for total sulfur in coal, and for sample preparation and analysis. His data, listed in Table 5, did not include the variance for sampling.

The pooled values for RSD (laboratory) for sulfur are:

0.019 for raw coals,

0.006 for washed coals.

TABLE 5. LABORATORY (SAMPLE PREPARATION AND ANALYSIS) VARIANCE FOR TOTAL SULFUR

SOURCE: KELLER[5]

Coal ID	Coal condition	Number of tests	Mean sulfur, percent	Variance (laboratory)	RSD (laboratory)
A	Raw	10	3.30	0.0040	0.019
B	Raw	50	2.38	0.0014	0.016
C	Raw	10	1.20	0.0003	0.014
K	Raw	10	0.72	0.0004	0.028
L	Raw	10	1.30	0.0013	0.028
F	Washed	30	0.60	0.0000	0.000
M	Washed	10	0.65	0.0000	0.000
N	Washed	10	1.09	0.0001	0.009
O	Washed	10	1.70	0.0004	0.012
P	Washed	10	1.52	0.0001	0.007

It appears that the sample preparation and analysis component of variance for raw coals is considerably greater than for washed coals. This observation is different from that reached for ash analysis, where the RSDs were equivalent at 0.017.

Since the sampling procedures for coal are the same (and in practice the identical samples are utilized) for the ash determination and for the sulfur and the heating value determinations, it is reasonable to adopt the experimental sampling RSDs for ash both to the sulfur and heating value. The total RSDs for sulfur are:

$$[(0.029)^2 + (0.019)^2]^{1/2} = 0.035 \text{ for raw coals,}$$

$$[(0.016)^2 + (0.006)^2]^{1/2} = 0.017 \text{ for washed coals.}$$

No experimental data were found for the laboratory heating value determination. Adopting the permissible laboratory RSD of 0.004 and the experimental (for ash) sampling RSD, the total sampling and analysis RSDs for heating value are:

$$[(0.029)^2 + (0.004)^2]^{1/2} = 0.029 \text{ for raw coals,}$$

$$[(0.016)^2 + (0.004)^2]^{1/2} = 0.016 \text{ for washed coals.}$$

Finally, the sampling and analysis RSDs for lbs SO_2/MM Btu, based primarily upon experimental data, would be:

$$[(0.035)^2 + (0.029)^2]^{1/2} = 0.045 \text{ for raw coals,}$$

$$[(0.017)^2 + (0.016)^2]^{1/2} = 0.023 \text{ for washed coals.}$$

These values represent the best current estimates for the actual sampling and analysis components of variance, consistent with ASTM standards for sampling and analysis. They are considerably less than the maximum permissible values.

SECTION 3

DESCRIPTION OF THE DATA BASE

3.1 DATA SETS

The data base for this study consists of 53 individual data sets, with a total of 3,204 data points. Each data set represents an identifiable and unique coal stream, either raw coal or cleaned coal, from a particular cleaning plant or loading facility, with the source of the coal (seam and county) and cleaning level specified. Tables 6 and 7 list the data sets for unwashed coals and for washed coals (respectively) together with the characteristics of the data sets. Unwashed coals were defined as either run-of-mine (ROM) coals, where no preparation at all was conducted, or coals cleaned at Level I (crushing and sizing), with removal of large pieces of rock and overburden). Washed coals were defined as coals cleaned at Levels II and above, where specific gravity separation is conducted on one or more size fractions.

The data sets were obtained in three different ways. Forty-one data sets resulted from a request by Versar to coal preparation plant owners through the National Coal Association (NCA). Nine data sets published in a previous EPA study [1] were abstracted for use in this study. Three data sets from the Homer City, Pennsylvania, preparation plant were obtained from EPA.

The NCA data request was aimed at obtaining matching pairs of plant feed (unwashed coal) and product (washed coal) data sets so that direct comparisons could be used to determine the effectiveness of coal cleaning plants. The NCA contacted a number of companies which, taken together, operate 111 preparation plants (over 25 percent of the national total) in different coal regions. Since most such plants are in the East and the Midwest, these are the primary areas covered by the study — although data was also requested from one (non-NCA) company which operates four coal preparation plants in the Alabama region. Eight separate coal companies responded to the data request; eight coal preparation plants provided data sets for both feed and product coal; and approximately

TABLE 6.

DATA SET IDENTIFICATION - UNWASHED COALS

Data set	State	County	Seams	Cleaning level	Number of data points
101	Ky.	Ohio	9,14	I (F)	6
102	Ky.	Ohio	9	I (F)	6
103	Ky.	Ohio	9,11	ROM	25
104	Ky.	Ohio	9	I	25
105	Ky.	Ohio	9,11,13	I	25
106	Ky.	Muhlenberg	11,12	I (F)	4
107	Ky.	Muhlenberg	9	I	25
108	Ky.	Muhlenberg	9	I	25
109	Ky.	Muhlenberg	11	I	25
110	Ky.	Muhlenberg	12	I	25
111	Ky.	Muhlenberg	9,12	I	25
112	Il.	Randolph	6	I	26
113	Il.	Christian	6	ROM (F)	6
114	Il.	Randolph	6	ROM (F)	12
115	Oh.	Perry	6	I (F)	6
116	Pa.	Indiana	Upper Free.	ROM	44
117	Pa.	Indiana	Upper Free.	ROM	44
118	Va.	-	SW	ROM (F)	5
119	WV.	Hancock	Low Kitt.	ROM (F)	5
120	Mt.	Rosebud	Rosebud	I	25
121	Ok.	Craig, Nowata	Ft. Scott	I	24
C-1	WV.	-	-	ROM	704
C-8	Oh.	-	-	ROM	275
U-4	Ky.	-	-	ROM	162
U-5	Ky.	-	-	ROM	250
U-11	Pa.	-	-	ROM	250
U-12	Pa.	-	-	I	250
U-13	Pa.	-	-	I	250

TABLE 7.

DATA SET IDENTIFICATION - WASHED COALS

Data set	State	County	Seams	Cleaning level	Number of data points
201	Ky.	Ohio	9,14	V	6
202	Ky.	Ohio	9,14	V	24
203	Ky.	Ohio	9	V	6
204	Ky.	Ohio	9	V	25
205	Ky.	Muhlenberg	11,12	V	6
206	Il.	Randolph	6	V	24
207	Il.	Christian	6	VI	6
208	Il.	St. Clair	6	VI	26
209	Il.	Gallatin	6	V	25
210	Il.	Gallatin	5	VI	25
211	Il.	Randolph	6	III	12
212	Oh.	Perry	6	V	6
213	Oh.	Perry	6	V	24
214	In.	Greene, Knox	5, 6	V	25
215	In.	Warrick	5, 6	VI	25
216	In.	Warrick	5, 6	V	25
217	In.	Greene, Sullivan	3, 4, 5, 6, 7	V	25
218	In.	Vermillion	6	V	25
219	Pa.	Indiana	Upper Free.	VI	46
220	Mo.	Henry	Beviar Lit. Tabo	VI	6
221	Mo.	Henry	Beviar	VI	18
222	Va.	-	SW	V	5
223	WV.	Hancock	Low.Kitt.	III	5
C-2	Ky.	-	-	II	115
C-3	Ky.	-	-	V	115

40 others submitted only single values for feed and product measurements. The remaining plants provided product data without the corresponding feed values.

The matched pairs of feed and product data sets for individual coal preparation plants are as follows:

Plant ID	Feed data set ID	Product data set ID
1	101	201
2	114	211
3	113	207
4	102	203
5	106	205
6	115	212
7	119	223
8	118	222

3.2 DATA POINTS

Each of the data points within each data set represents a "lot" of coal from the coal stream. A "lot" may be a day's or a month's contiguous quantity from the coal stream, or it may be a car load, a unit train load, or a barge load. The lots (data points) are ordered chronologically, from earliest to latest, within each data set. Associated with each lot (data point) is the lot size (in tons), and single reported values for measured total sulfur content (percent, dry basis) and for calculated equivalent pounds SO_2 per million Btu (lbs SO_2/MM Btu).

Table 8 illustrates the contents of a representative data set. Appendix A is a listing of all of the data acquired in this study.

3.3 SAMPLING PROCEDURES UTILIZED

Little specific information was provided with each data set regarding the procedures used (sampling, analyzing, compositing, or mathematical averaging) to generate the single reported values of percent sulfur and heating value for each data point. The coal companies were asked to describe their sampling procedure, and the replies were that specific procedures differ from plant to plant relative to how the sample is taken, sampling frequency, the method of

TABLE 8.

CONTENTS OF DATA SET 208

6" x 100M Cleaned Coal (Level VI), Illinois, St. Clair County

Data point (lot) no.	Lot size tons	Total S, %	Heating value, Btu/lb	lbs SO_2 / MM Btu
1	2,806	4.29	12,215	7.02
2	2,626	4.20	12,171	6.90
3	1,023	4.15	12,115	6.84
4	4,201	4.13	12,300	6.71
5	6,870	3.96	12,162	6.51
6	8,361	4.04	12,085	6.68
7	5,463	4.03	12,099	6.66
8	2,790	4.31	11,934	7.22
9	7,983	4.08	12,134	6.72
10	5,652	4.04	11,805	6.84
11	6,837	4.12	12,284	6.70
12	4,060	4.10	12,001	6.83
13	2,621	4.00	12,105	6.60
14	5,598	4.08	12,192	6.69
15	7,015	3.87	12,109	6.39
16	1,388	3.99	12,232	6.52
17	7,961	4.15	12,279	6.75
18	2,847	4.09	12,271	6.66
19	5,231	4.24	12,421	6.82
20	1,353	4.20	11,991	7.00
21	9,864	3.83	12,016	6.37
22	2,844	3.98	11,896	6.69
23	2,798	4.25	11,671	7.28
24	5,675	4.28	12,074	7.08
25	1,426	3.55	11,831	6.00
26	1,395	3.44	12,622	5.45

producing a composite sample, and sampling locations for feed and product coals.

For feed coal, infrequent, manual sampling is the norm. The terms "occasionally," "weekly," "only when we have problems," and "periodically" were used to describe feed coal sample frequency. In a majority of the cases the feed coal belt is stopped and an American Society for Testing and Materials (ASTM) belt sample is taken. The sample provides a dependable representation of the input coal at that time; however, it should not be considered to reliably represent actual coal properties in the long- and medium-term. This was an overriding factor that led some of the coal companies to send monthly and yearly average values, rather than the daily, weekly, or lot shipment information requested. The values provided to Versar were generally weighted averages of feed coal belt sample analyses.

In contrast the product coal is extensively sampled and analyzed. The coal companies typically take a one or two hour composite sample of the product, if the plant has an automatic sampler, or manually sample unit train carloads or barges according to ASTM sampling procedures. The automatically sampled composites consist of individual samples taken at 5-15 minute intervals. The manual samples are usually taken off a conveyor discharge as the railroad car or barge is loaded.

The frequency of product sampling is determined in part by the origin of the coal feed to the preparation plant. For a mine mouth coal cleaning plant, only one composite sample per day may be analyzed; at the other extreme, where specifications are tight and contract coal is blended and cleaned, the composite samples may be taken and analyzed every 30 minutes. Where possible, Versar has specified data which were received from plants with automatic product samplers.

In Table 6, most of the data sets represent product coal (despite the lack or low level of cleaning). Those data sets in Table 6, which represent feed coal to a preparation plant and are not subjected to the more extensive product coal sampling requirements, are signified by an (F) in the cleaning level column.

SECTION 4

ANALYSIS OF PAIRED COAL FEED AND PRODUCT DATA SETS FROM INDIVIDUAL PLANTS

Table 9 lists the statistics calculated for the eight data sets from the individual coal preparation plants which provided multiple data samples for both feed and product coal. For each plant (and separately for both feed and product samples) the mean ($\overline{Y}$), the standard deviation (Sy), and the relative standard deviation (RSD = Sy/$\overline{Y}$) were calculated for:

Y_1 = Total sulfur content (percent)

Y_2 = Heating value (Btu/lb)

Y_3 = Heat-specific SO_2 content (lbs SO_2/MM Btu)

4.1 EFFECT OF COAL CLEANING UPON SULFUR CONTENT, HEATING VALUE, AND HEAT-SPECIFIC SO_2 CONTENT

Table 10 lists, for each plant, the changes (between the feed coal and the product coal) in the mean values of Y_1, Y_2, and Y_3. The percentage changes are also listed. It is observed that (except for Plant No. 8), the coal cleaning process resulted in reductions in sulfur content (Y_1) of 14 to 45 percent, in increases in heating value (Y_2) of 6 to 23 percent, and in reductions in heat-specific SO_2 content (Y_3) of 24 to 50 percent. Coal cleaning plant No. 8 operated upon a very high ash feed, and the heating value was increased by 45 percent, but in the process, the sulfur content was also increased.

To test whether the observed changes in the mean values are statistically significant, the "t" statistics were calculated for each plant:

$$t = |\overline{Y}_{product} - \overline{Y}_{feed}|/s,$$

where s is the pooled standard deviation for both feed and product coal for that plant.

TABLE 9. STATISTICS FOR PREPARATION PLANT FEED AND PRODUCT

Cleaning plant ID			1	2	3	4	5	6	7	8
Cleaning level			V	III	VI	V	V	V	III	V
State			KY	IL	IL	KY	KY	OH	WV	VA
County			Ohio	Randolph	Christian	Ohio	Muhlenberg	Perry	Hancock	-
Seams			9 & 14	6	6	9	11 & 12	6	Lower Kitt.	SW
	Data set no.		101	114	113	102	106	115	119	118
	No. points		6	12	6	6	4	6	5	5
Feed coal statistics	Y_1=	$\bar{Y}$	4.16	4.75	5.12	4.29	4.26	3.94	1.94	0.93
	Percent	SY	0.249	0.461	0.199	0.477	0.550	0.339	0.588	0.233
	Tot.S.	SY/$\bar{Y}$	0.060	0.097	0.039	0.111	0.129	0.086	0.303	0.250
	Y_2=	$\bar{Y}$	11,320	10,762	10,650	12,230	10.270	11,070	12,950	9,980
	Btu/	SY	581	248	242	371	500	331	624	636
	lb	SY/$\bar{Y}$	0.051	0.023	0.023	0.030	0.049	0.030	0.048	0.064
	Y_3=	$\bar{Y}$	7.35	8.82	9.60	7.01	8.33	7.13	3.00	1.85
	lbs SO_2	SY	0.427	0.938	0.301	0.786	1.405	0.730	0.864	0.389
	MM Btu	SY/$\bar{Y}$	0.058	0.106	0.031	0.112	0.169	0.102	0.288	0.210
	Data set no.		201	211	207	203	205	212	223	222
	No. points		6	12	6	6	6	6	5	5
Product coal statistics	Y_1=	$\bar{Y}$	3.19	4.10	4.39	3.37	3.22	3.01	1.07	1.17
	Percent	SY	0.046	0.316	0.065	0.038	0.148	0.077	0.166	0.230
	Tot.S.	SY/$\bar{Y}$	0.015	0.077	0.015	0.011	0.046	0.026	0.155	0.197
	Y_2=	$\bar{Y}$	13,000	12,177	12,300	12,970	12,650	12,460	14,370	14,470
	Btu/	SY	55	136	37	48	119	22	182	145
	lb	SY/$\bar{Y}$	0.0043	0.0112	0.0030	0.0037	0.0094	0.0018	0.0127	0.0101
	Y_3=	$\bar{Y}$	4.90	6.73	7.13	5.18	5.09	4.83	1.49	1.61
	lbs SO_2	SY	0.123	0.447	0.097	0.066	0.280	0.124	0.240	0.307
	MM Btu	SY/$\bar{Y}$	0.025	0.067	0.014	0.013	0.055	0.026	0.161	0.191

TABLE 10. CHANGES IN MEAN VALUES FROM COAL CLEANING: DIRECT COMPARISON OF FEED AND PRODUCT FOR INDIVIDUAL PLANTS

Plant number	Change in Y			% change in Y			"t" statistics			d.f.	t.95
	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
1	-0.97	+1680	-2.45	-23	+15	-33	10.7	7.5	14.1	5	2.02
2	-0.65	+1415	-2.09	-14	+12	-24	9.1	9.3	11.7	11	1.80
3	-0.73	+1650	-2.47	-14	+15	-26	8.6	18.5	19.9	5	2.02
4	-0.92	+ 740	-1.83	-21	+ 6	-26	4.8	4.8	5.7	5	2.02
5	-1.04	+2380	-3.24	-24	+23	-39	3.5	9.4	4.4	3	2.35
6	-0.93	+1390	-2.30	-24	+13	-32	7.4	10.5	5.0	5	2.02
7	-0.87	+1420	-1.51	-45	+11	-50	3.8	6.5	4.5	4	2.13
8	+0.24	+4490	-0.24	+26	+45	-13	2.7	16.7	1.5	4	2.13

These values of t are listed in Table 10. It is apparent from Table 9 that for each plant, the standard deviations for feed coal are not equal to the standard deviations for product coal. The values for s in the above equation were therefore determined by the method of Crow, Davis, and Maxfield,[2]

$$s = \sqrt{\frac{Q}{n_1^2 (n_1-1)}}$$

Where

$$Q = n_1 \sum_{i=1}^{n_1} U_i^2 - \left(\sum_{i=1}^{n_1} U_i \right)^2$$

$$U_i = Y_{1i} - Y_{2i}\sqrt{\frac{n_1}{n_2}} \qquad (i = 1,2,\ldots, n_1)$$

Y_{1i} and Y_{2i} are paired individual observations such that $n_1 \leq n_2$.

It may be observed that for plant Nos. 1 through 7, the reductions in sulfur content, the increases in heating value, and the reductions in heat-specified SO_2 content are all significant at the 95 percent confidence level. For plant No. 8, only the decrease in lbs SO_2/MM Btu was not significant at this confidence level.

The table on the following page summarizes the ranges of effectiveness in physical coal cleaning, comparing the calculated average potential by coal producing region, the calculated design performance of hypothetical plants, and the measured performance of operating plants. Effectiveness is measured by the decrease in total sulfur content, the increase in heating value, and the decrease in lbs SO_2/MM Btu.

	Decrease in total sulfur percent	Increase in heating value percent	Decrease in lbs SO_2/MM Btu, percent
Calculated Average Potential (Table 1)	12-38	4-9	16-43
Calculated Performance of Hypothetical Plants (Tables 2A, 2B, 2C)	5-76	5-27	8-81
Measured Performance of Operating Plants (Table 10, plants 1-7)	14-45	6-23	24-50

These ranges of effectiveness are wide, reflecting the sensitivity of coal cleaning efficiency to the washability characteristics of specific coals and to the complexity and sophistication of the plant design. The ranges of measured performance, however, fall within the ranges of calculated performance. The effectiveness as measured in operating coal cleaning plants does in fact confirm the effectiveness calculated for many hypothetical plants, although the very highest removal efficiencies (greater than 50 percent decrease in lbs SO_2/MM Btu) were not empirically validated by the available data.

4.2 EFFECT OF COAL CLEANING UPON SULFUR VARIABILITY

In each of the eight plants, both the absolute standard deviation and the relative standard deviation were reduced (for all three coal characteristics) by the coal preparation process. The percent reductions are summarized in Table 11. The average reductions in relative standard deviation attributable to cleaning are 57 percent (for Y_1, percent sulfur), 81 percent (for Y_2, Btu/lb), and 54 percent (for Y_3, lbs SO_2/MM Btu).

Although data from only eight plants were available for this direct analysis of individual cleaning plant variability reduction, the results appear to be relatively consistent from plant to plant. Both percent sulfur and lbs SO_2/MM Btu variabilities (as measured by relative standard deviation) were reduced by an average of 55 percent, while heating value variability was reduced by an average of 80 percent.

TABLE 11. REDUCTIONS IN VARIABILITY FROM COAL CLEANING: DIRECT COMPARISON OF FEED AND PRODUCT FOR INDIVIDUAL PLANTS

Plant number	Percent reductions in SY			Percent reductions in $SY/\bar{Y}$		
	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3
1	72	91	71	75	92	57
2	31	45	52	21	51	37
3	45	85	68	62	87	55
4	92	87	92	90	88	88
5	73	76	80	64	81	67
6	77	93	83	70	94	75
7	72	71	72	49	74	44
8	3	77	21	21	84	9
Averages				57	81	54

SECTION 5

ANALYSIS OF UNPAIRED DATA SETS

5.1 APPROACH AND LIMITATIONS

The analysis of matched pairs of feed and product data for eight coal cleaning plants, presented in Section 4, was a straightforward comparison of the means and variabilities before and after the coal cleaning process. This analysis followed the approach that was intended at the time of the data request.

As explained in Section 3, however, the preponderance of the responses did not provide matched pairs of feed and product data. Twenty coal data sets in Table 6, out of a total of 28, were for raw coals or for coals cleaned at Level I, with no matching data sets for corresponding washed product coals. Similarly, 17 coal data sets in Table 7, out of a total of 25, were for washed product coals, with no matching data sets for the corresponding feed coals. A second statistical analysis, distinct from the direct comparison of Section 4, was conducted to exploit the entire available data base. This second analysis was an indirect comparison between feed and product variabilities for the data sets of all washed coals relative to the data sets of all unwashed coals.

This approach is not nearly as satisfying as the direct approach of analyzing matched pairs of data sets, but it does allow consideration of more of the available data. A major difficulty in the indirect approach is that neither the group of unwashed coal data sets nor the group of washed coal data sets forms a logically consistent or homogeneous population susceptible to rigorous statistical analysis. Indeed, much of the reported data is mixed with respect to mining method, cleaning method, sampling frequency and procedure, methods of compositing or averaging, definition of a "lot" of coal and the nominal lot size, and the analytical laboratory precision. Each of the two groups of data sets (unwashed and washed coals) is comprised of coals from different regions, seams, and mines, with inherent geological and engineering differences. Furthermore, the data sets are not necessarily representative of their respective regions or seams.

Because of these inherent comparability problems, the results of this second statistical analysis are not as definitive as those of the first analysis. The comparison of mixed data sets can lead to only a rough picture of the effects of coal cleaning on sulfur variability.

5.2 STATISTICS FOR THE DATA SETS

When the data sets were assembled, those with large lot sizes ($>10^5$ tons) were deleted because they were unlikely to represent single analyses of single coal sample composites.

For each data set, the mean, the standard deviation, and the coefficient of variation (relative standard deviation) were calculated for:

Y_1 = total sulfur content (%)

Y_2 = heating value (Btu/lb)

Y_3 = heat-specific SO_2 content (lbs SO_2/MM Btu)

These statistics are listed in Table 12 (for unwashed coals) and in Table 13 (for washed coals).

5.3 VARIABILITY OF HEAT-SPECIFIC SULFUR CONTENT CALCULATED FROM VARIABILITIES OF SULFUR AND OF HEATING VALUE

In the previous EPA Study[1], a contingency table analysis showed that the percent total sulfur data was independent from the heating value data. There are potential mechanisms for explaining either the existence or non-existence of a correlation between the two measurements. Since total sulfur is measured in the laboratory by different tests from heating value, measurement errors should be independent. Fundamentally, the sample-to-sample variation of heating value should be similar to that of ash content,

TABLE 12. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S UNWASHED COAL

Batch ID.	N= No. of lots	$\bar{X}$ = Mean lot size tons	Y_1 = sulfur,(%)			Y_2 = Btu/lb			$Y_3 = \frac{\text{LBS } SO_2}{\text{MM Btu}}$		
			$\bar{Y}_1$	SY_1	$SY_1/\bar{Y}_1$	$\bar{Y}_2$	SY_2	$SY_2/\bar{Y}_2$	$\bar{Y}_3$	SY_3	$SY_3/\bar{Y}_3$
103	25	2,800	4.44	0.45	0.10	11,840	333	0.0281	7.51	0.81	0.11
104	25	10,900	4.11	0.16	0.039	12,030	172	0.0143	6.82	0.30	0.044
105	25	11,200	4.85	0.58	0.12	12,000	340	0.0283	8.11	1.22	0.15
107	25	5,300	5.04	0.47	0.093	12,310	255	0.0207	8.20	0.88	0.11
108	25	6,800	4.16	0.23	0.055	11,900	169	0.0142	6.99	0.44	0.063
109	25	7,100	5.21	0.47	0.090	11,520	461	0.0400	9.04	0.89	0.098
110	25	1,100	4.80	0.49	0.101	10,350	371	0.0358	9.28	1.01	0.11
111	25	6,700	4.66	0.35	0.075	11,610	216	0.0186	8.03	0.67	0.083
112	26	3,800	4.22	0.24	0.057	11,510	418	0.0363	7.35	0.62	0.084
116	44	7,600	2.00	0.28	0.14	11,440	322	0.0281	3.50	0.55	0.16
117	44	3,400	3.09	0.24	0.078	11,090	263	0.0237	5.59	0.48	0.086
120	25	10,300	1.49	0.44	0.30	11,400	250	0.0219	2.63	0.79	0.30
121	24	17,300	3.78	0.19	0.050	12,550	226	0.0180	6.00	0.29	0.048
C-1	704	12,000	2.79	0.23	0.082	13,052	231	0.0177	4.27	0.36	0.084
C-8	275	10,000	2.60	0.13	0.050	12,481	703	0.0563	4.17	0.27	0.065
U-4	164	12,000	1.04	0.16	0.15	12,000	310	0.026	1.73	0.28	0.16
U-5	250	13,000	0.92	0.15	0.16	11,856	299	0.025	1.55	0.25	0.16
U-11	250	———	3.13	0.44	0.14	11,522	378	0.0328	5.44	0.85	0.16
U-12	250	———	2.34	0.26	0.11	12,046	257	0.0213	3.88	0.46	0.12
U-13	250	———	2.31	0.22	0.095	12,135	273	0.0225	3.81	0.35	0.092

TABLE 13. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S WASHED COAL

Batch ID.	N= No. of lots	$\bar{X}$= Mean lot size tons	$\bar{Y}_1$= Sulfur (%)			$\bar{Y}_2$= Btu/lb			$\bar{Y}_3$= lbs SO_2 / MM Btu		
			$\bar{Y}_1$	SY_1	$SY_1/\bar{Y}_1$	$\bar{Y}_2$	SY_2	$SY_2/\bar{Y}_2$	$\bar{Y}_3$	SY_3	$SY_3/\bar{Y}_3$
202	24	6,500	3.18	0.15	0.047	13,000	76	0.0058	4.88	0.25	0.051
203	6	100,000	3.37	0.04	0.012	12,970	48	0.0037	5.18	0.07	0.014
204	25	18,300	3.36	0.12	0.036	12,990	80	0.0062	5.18	0.20	0.039
206	24	3,900	3.85	0.20	0.052	12,060	210	0.0174	6.38	0.38	0.060
208	26	4,500	4.05	0.21	0.052	12,120	200	0.0165	6.69	0.37	0.055
209	25	6,300	2.77	0.20	0.072	12,760	172	0.0135	4.33	0.32	0.074
210	25	4,200	3.34	0.13	0.039	12,990	107	0.0082	5.14	0.22	0.043
213	24	10,300	2.97	0.15	0.050	12,440	68	0.0055	4.78	0.25	0.052
214	25	16,900	2.58	0.24	0.093	12,670	93	0.0073	4.07	0.38	0.093
215	25	2,100	3.47	0.16	0.046	12,740	144	0.0113	5.44	0.26	0.048
216	25	5,400	3.75	0.21	0.056	12,620	121	0.0096	5.94	0.34	0.057
217	25	4,700	2.92	0.28	0.10	12,510	205	0.0164	4.66	0.46	0.099
218	25	51,500	2.59	0.13	0.050	12,250	119	0.0097	4.21	0.22	0.052
219	46	7,600	1.66	0.18	0.11	12,350	428	0.0346	2.69	0.33	0.12
220	6	4,200	4.14	0.55	0.13	12,290	265	0.0216	6.72	0.95	0.14
221	18	5,400	4.46	0.44	0.10	12,780	194	0.0152	6.97	0.70	0.10
222	5	2,700	1.17	0.23	0.20	14,470	145	0.0100	1.61	0.31	0.19
223	5	600	1.07	0.17	0.16	14,370	182	0.0127	1.49	0.24	0.16
C-2	115	5,600	0.66	0.03	0.045	13,240	162	0.0122	0.99	0.05	0.051
C-3	115	2,500	0.78	0.08	0.103	12,179	373	0.0306	1.28	0.12	0.094

since Btu per pound for a given coal is almost a monotonic decreasing function of ash content. Conversely, the percent total sulfur is, to a considerable extent, sensitive to the organic sulfur content, which is not at all linked to ash-forming inorganic impurity concentrations. However, two factors may cause total sulfur variability measurements to be correlated to heating value variability measurements: first, whatever contribution the inadequacies of coal sampling make to the observed variabilities should be the same for total sulfur as for heating value; second, the reduction of variability by coal preparation processes (either by blending operations or by attenuation in specific-gravity separation processes) should be effective for both percent sulfur and for heating value.

Accepting the conclusion of independence established in the previous EPA study[1] means that Y_1 varies about its mean in a random fashion relative to the variations of Y_2 about its mean. In this case, the relative standard deviation of Y_3 may be predicted by:

$$\left(\frac{SY_3}{\bar{Y}_3}\right) = \left[\left(\frac{SY_1}{\bar{Y}_1}\right)^2 + \left(\frac{SY_2}{\bar{Y}_2}\right)^2\right]^{1/2} \qquad (2)$$

The statistics in Tables 12 and 13 may be used to verify the above equation. Figure 1 is a plot of the actual values of the relative standard deviation of Y_3 vs. the value as calculated from equation (2). It can be seen that the points scatter above, but generally parallel to, the 45-degree line. This would confirm that, in fact, the variation in total sulfur content (in both cleaned and uncleaned coals) is essentially independent of the variation in heating value; and the utility of such a conclusion is that the relative standard deviation of heat-specific SO_2 content can be estimated via equation (2) using the relative standard deviations of percent sulfur and heating value.

5.4 AUTOCORRELATION OF DATA POINTS

Elementary statistical tests and confidence intervals are based on the assumptions that the individual data points are samples of a single

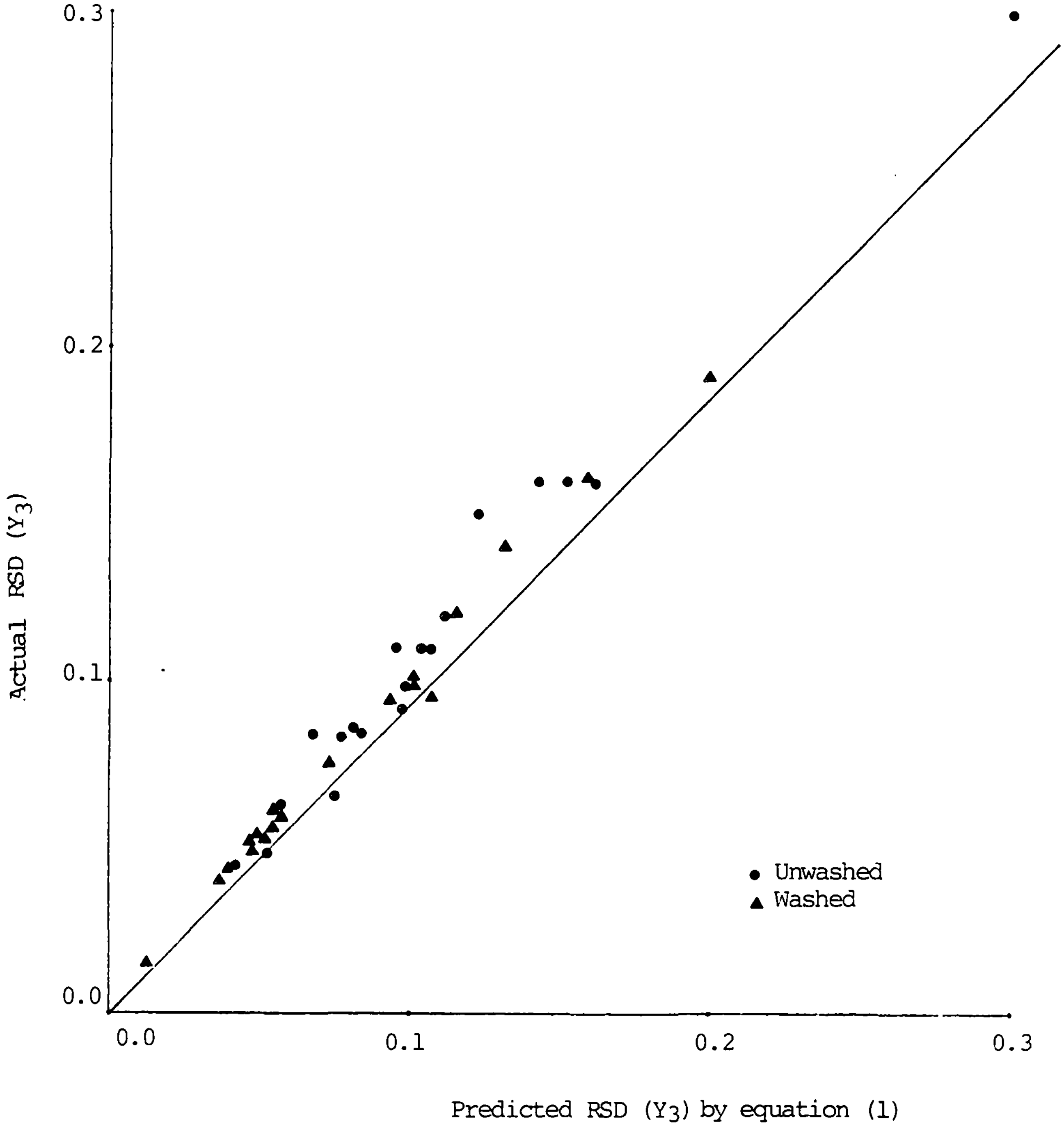

Figure 1. Relative standard deviation of lbs SO_2/MM Btu: actual vs. predicted.

population and that they are independent of each other - that they are randomly distributed about a mean.

As Thomas[11] pointed out, however, there is good reason why coal data points are neither samples of a single population nor independent. Even within a given mine, geological factors are responsible for the inhomogeneity of the deposit. The deposit likely consists of multiple populations, separated by distance along the seam (or by time of extraction). The coals mined on successive days are potentially highly correlated because they are geologically related.

The science of geostatistics is being applied to coal data to measure and account for autocorrelation. The total variance in a set of data points is separated into two components: the "nugget" component Co associated with samples taken close together and the long-range components C_i. Co is conceptually associated with samples taken close together in the mine, and thus represents the imprecision of replicate samples due to local discontinuities or inhomogeneities in the coal and to sampling and analytical uncertainties. C is conceptually associated with samples taken far apart, so that it represents geologic differences in the deposit[11]. There may be several distinctly different mineralized zones within a deposit with different values of C_i. The values of Co and C_i are determined empirically from a data set (in which the data points are spatially or chronologically ordered*) by generating a variogram: a plot of $\gamma(k)$ vs k, where:

$$\gamma(k) = \frac{1}{2(n-k)} \sum_{i=1}^{n-k} [X_{i+k} - X_i]^2 \quad , \quad k = 1,2,\ldots,n/3$$

For small values of k, as k approaches zero, $\gamma(k)$ approaches Co, the "nugget" variance. At large values of k, $\gamma(k)$ typically becomes constant with k and is equivalent to the combined long-range variance $Co + C = S^2$, where S^2 is the classical variance computed by:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} [X_i - \overline{X}]^2 = \frac{1}{n-1} \left[\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}\right]$$

*One-dimensional spatial and time-correlated variances are analogous.

For the special case of k = 1, where $\gamma(k)$ should approach Co,

$$\gamma(1) = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} [X_{i+1}-X_i]^2$$

An alternative method for detecting a trend in data (an autocorrelation) is given by Crow[2]. The mean square successive difference δ^2 is calculated by:

$$\delta^2 = \frac{1}{n-1} \sum_{i=1}^{n-1} [X_{i+1}-X_i]^2$$

An estimate of the component of variance that reduces the trend effect is $\delta^2/2$, which is the same as $\gamma(1)$ -an estimate of the "nugget" variance- in the geostatistical approach. Crow's test for autocorrelation consists of computing δ^2/S^2 (= 2 $\gamma(1)/S^2$). If the data are not serially correlated, a value of δ^2/S^2 near 2 is expected. Values of δ^2/S^2 less than 2 occur with autocorrelation, and values greater than 2 occur with serial oscillation greater than random variation.[2]

The computed value of δ^2/S^2 is compared with the appropriate critical value from a table given by Crow[2] and abstracted below for illustrative purposes:

n	Autocorrelation			Rapid oscillation		
	P=0.95	P=0.99	P=0.999	P=0.95	P=0.99	P=0.999
4	0.7805	0.6256	0.5898	3.2195	3.3744	3.4102
10	1.0623	0.7518	0.4816	2.9378	3.2482	3.5184
20	1.2996	1.0406	0.7852	2.7004	2.9593	3.2148
40	1.4921	1.2934	1.0850	2.5079	2.7066	2.9151
60	1.5814	1.4144	1.2349	2.4186	2.5856	2.7651

The lbs SO_2/MM Btu values for the data sets listed in Tables 6 and 7 were tested. Tables 14 and 15 list (respectively), for unwashed coal data sets and for washed coal data sets, values of $\gamma(1) = \delta^2/2$, of S^2, and of the ratio δ^2/S^2. The appropriate critical values (at the 95 percent confidence level) are also listed, and those data sets meeting the test for autocorrelation or for rapid oscillation are noted by an asterisk. A double asterisk signifies that the test was positive at the 99 percent confidence level, and a triple asterisk refers to a positive test result at 99.9 percent confidence.

Only one data set (No. 103) out of 48 in Tables 14 and 15 had a positive rapid-oscillation test at the 95 percent confidence level, and the test was not positive at the 99 percent level. This result is discounted; one of 20 data sets with true random variation may be expected to erroneously give positive test results.

The autocorrelation test, however, gave positive results in 6 of 23 unwashed coal data sets and in 10 of 25 washed coal data sets, at the 95 percent confidence level. Of these 16 data sets giving positive results, 9 were positive at the 99 percent level, and 4 were positive at the 99.9 percent confidence level. There is little doubt that much of these coal data are serially correlated, verifying the expectations based upon geology and engineering rationale.

Several reasons may be proposed to explain why autocorrelation was not evident in all the data sets:

- The measurement (sampling and analysis) imprecision in some data sets may be so large as to overshadow serial correlations.
- The local coal inhomogeneities and discontinuities in some data sets may be of such magnitude as to overshadow serial correlations.
- There may be significant blending of coal in some data sets which overshadows serial correlation. This blending might be the result of simultaneous mining from multiple mine faces (or

TABLE 14 .

AUTOCORRELATION TEST, UNWASHED COAL DATA SETS

PARAMETER TESTED: LBS SO_2/MM BTU

Data set	n	$\gamma(1) = \delta^2/2$	S^2	δ^2/S^2	(δ^2/S^2) 0.95 Autocorr.	Oscill.	Auto-correl.	Rapid oscill.
101	6	0.265	0.182	2.91	0.89	3.11		
102	6	0.789	0.618	2.55	0.89	3.11		
103	25	0.907	0.656	2.77	1.37	2.63		*
104	25	0.059	0.090	1.31	1.37	2.63	*	
105	25	1.324	1.488	1.78	1.37	2.63		
106	4	2.235	1.974	2.26	0.78	3.22		
107	25	0.893	0.774	2.31	1.37	2.63		
108	25	0.248	0.194	2.56	1.37	2.63		
109	25	0.991	0.792	2.50	1.37	2.63		
110	25	1.037	1.020	2.03	1.37	2.63		
111	25	0.544	0.449	2.42	1.37	2.63		
112	26	0.253	0.384	1.32	1.38	2.62	*	
113	6	0.138	0.091	3.03	0.89	3.11		
114	12	0.255	0.880	0.58	1.13	2.87	**	
115	6	0.486	0.533	1.82	0.89	3.11		
116	44	0.261	0.303	1.72	1.52	2.48		
117	44	0.223	0.230	1.94	1.52	2.48		
118	5	0.146	0.151	1.93	0.82	3.18		
119	5	0.159	0.740	0.43	0.82	3.18	**	
120	25	0.492	0.624	1.58	1.37	2.63		
121	24	0.065	0.084	1.55	1.36	2.64		
C-1	704	0.093	0.130	1.43	1.87	2.13	***	
C-8	275	0.034	0.073	0.92	1.80	2.20	***	

TABLE 15.

AUTOCORRELATION TEST, WASHED COAL DATA SETS

PARAMETER TESTED: LBS SO_2/MM BTU

Data set	n	$\gamma(1) = \delta^2/2$	S^2	δ^2/S^2	(δ^2/S^2) 0.95 Autocorr.	Oscill.	Auto-correl.	Rapid oscill.
201	6	0.0024	0.0151	0.32	0.89	3.11	***	
202	24	0.0708	0.0625	2.27	1.36	2.64		
203	6	0.0033	0.0044	1.50	0.89	3.11		
204	25	0.0395	0.0400	1.98	1.37	2.63		
205	6	0.0219	0.0784	0.56	0.89	3.11	**	
206	24	0.1648	0.1444	2.28	1.36	2.64		
207	6	0.0078	0.0094	1.66	0.89	3.11		
208	26	0.0672	0.1369	0.98	1.38	2.62	**	
209	25	0.0633	0.1024	1.24	1.37	2.63	*	
210	25	0.0292	0.0485	1.20	1.37	2.63	*	
211	12	0.0637	0.1998	0.64	1.13	2.87	**	
212	6	0.0192	0.0154	2.49	0.89	3.11		
213	24	0.0574	0.0625	1.84	1.36	2.64		
214	25	0.0934	0.1444	1.29	1.37	2.63	*	
215	25	0.0773	0.0676	2.29	1.37	2.63		
216	25	0.1101	0.1156	1.90	1.37	2.63		
217	25	0.1922	0.2116	1.82	1.37	2.63		
218	25	0.0379	0.0484	1.57	1.37	2.63		
219	46	0.0773	0.1089	1.42	1.53	2.47	*	
220	6	0.7194	0.9025	1.59	0.89	3.11		
221	18	0.4571	0.4900	1.87	1.27	2.73		
222	5	0.0514	0.0942	1.09	0.82	3.22		
223	5	0.0521	0.0576	1.81	0.82	3.22		
C-2	115	0.0017	0.0078	0.44	1.69	2.31	***	
C-3	115	0.0109	0.0136	1.60	1.69	2.31	*	

longwall mining with the same effect), feed coal to a preparation plant being composed of shipments from multiple mines, or blending in the feed coal or product coal storage, handling, or shipping operations.

- The coal seam may not exhibit large inhomogeneities at distances equivalent to a "lot" quantity for some data sets.
- An equivalent effect to the previous hypothesis would be achieved if, for some data sets, the "lot" quantity is very large compared to the zones of influence in the coal seam.

5.5 DISTRIBUTION OF THE DATA POINTS

Many statistical tests and confidence intervals are based upon the assumption that the data populations have normal distributions. The data sets in Tables 12 and 13 were examined for normality. In addition, two variance-stabilizing transformations were applied in an attempt to obtain some approximation of normality. For each data set, the individual data points of total sulfur content (Y_1), heating value (Y_2), and lbs SO_2/MM Btu (Y_3) were transformed into natural logarithms:

$$Z_1 = \ln (Y_1)$$
$$Z_2 = \ln (Y_2)$$
$$Z_3 = \ln (Y_3)$$

The resulting statistics for the transformed data sets are listed in Table 16 (for uncleaned coals) and in Table 17 (for cleaned coals).

Similarly, the original data points were transformed into radical form:

$$W_1 = \sqrt{Y_1}$$
$$W_2 = \sqrt{Y_2}$$
$$W_3 = \sqrt{Y_3}$$

Table 18 (for uncleaned coals) and Table 19 (for cleaned coals) list the resulting statistics.

TABLE 16. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S UNCLEANED COAL (Logarithmic Transformation)

Batch ID.	N= No. of lots	$\bar{X}$= Mean lot size, tons	$Z_1=\ln(\text{Sulfur}(\%))$			$Z_2=\ln(\text{Btu/lb})$			$Z_3=\ln\left(\frac{\text{lbs } SO_2}{\text{MM Btu}}\right)$		
			$\bar{Z}_1$	SZ_1	$SZ_1/\bar{Z}_1$	$\bar{Z}_2$	SZ_2	$SZ_2/\bar{Z}_2$	$\bar{Z}_3$	SZ_3	$SZ_3/\bar{Z}_3$
103	25	2,800	1.49	0.10	0.067	9.38	0.028	0.0030	2.01	0.11	0.055
104	25	10,900	1.41	0.038	0.027	9.39	0.014	0.0015	1.92	0.043	0.022
105	25	11,200	1.57	0.11	0.070	9.39	0.029	0.0031	2.08	0.14	0.067
107	25	5,300	1.61	0.090	0.056	9.42	0.021	0.0022	2.10	0.10	0.048
108	25	6,800	1.42	0.056	0.039	9.38	0.014	0.0015	1.94	0.063	0.032
109	25	7,100	1.65	0.089	0.054	9.35	0.041	0.0044	2.20	0.15	0.068
110	25	1,100	1.56	0.101	0.065	9.24	0.036	0.0039	2.22	0.11	0.050
111	25	6,700	1.54	0.075	0.049	9.36	0.019	0.0020	2.08	0.083	0.040
112	26	3,800	1.44	0.056	0.039	9.35	0.036	0.0038	1.99	0.082	0.041
116	44	7,600	0.68	0.17	0.25	9.34	0.028	0.0030	1.24	0.18	0.14
117	44	3,400	1.13	0.078	0.069	9.31	0.024	0.0026	1.72	0.085	0.049
120	25	10,300	0.36	0.29	0.80	9.34	0.022	0.0024	0.92	0.31	0.34
121	24	17,300	1.33	0.050	0.038	9.44	0.018	0.0019	1.79	0.049	0.027
C-1	704	12,000	1.02	0.08	0.078	9.477	0.018	0.0019	1.45	0.09	0.062
C-8	275	10,000	0.96	0.05	0.052	9.430	0.058	0.0062	1.43	0.06	0.042
U-4	164	12,000	0.02	0.15	7.50	9.392	0.026	0.0028	0.53	0.15	0.28
U-5	250	13,000	-0.09	0.13	-1.44	9.380	0.025	0.0027	0.43	0.13	0.30
U-11	250	-	1.13	0.15	0.13	9.352	0.032	0.0034	1.68	0.17	0.10
U-12	250	-	0.85	0.10	0.12	9.396	0.021	0.0022	1.35	0.11	0.081
U-13	250	-	0.83	0.09	0.11	9.404	0.023	0.0024	1.33	0.09	0.068

TABLE 17. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S-CLEANED COAL (Logarithmic Transformation)

Batch ID.	N= No. of lots	$\bar{X}$= Mean lot size, tons	$Z_1=\ln(\text{Sulfur}(\%))$			$Z_2=\ln(\text{Btu/lb})$			$Z_3=\ln\left(\frac{\text{lbs } SO_2}{\text{MM Btu}}\right)$		
			$\bar{Z}_1$	SZ_1	$SZ_1/\bar{Z}_1$	$\bar{Z}_2$	SZ_2	$SZ_2/\bar{Z}_2$	$\bar{Z}_3$	SZ_3	$SZ_3/\bar{Z}_3$
202	24	6,500	1.15	0.049	0.043	9.473	0.0059	0.00062	1.58	0.051	0.032
203	6	100,000	1.21	0.011	0.0091	9.471	0.0037	0.00039	1.64	0.013	0.0079
204	25	18,300	1.21	0.035	0.029	9.471	0.0062	0.00065	1.64	0.038	0.023
206	24	3,900	1.35	0.052	0.038	9.40	0.018	0.0019	1.86	0.058	0.031
208	26	4,500	1.40	0.053	0.038	9.40	0.016	0.0017	1.90	0.058	0.030
209	25	6,300	1.015	0.070	0.069	9.45	0.013	0.0014	1.463	0.074	0.050
210	25	4,200	1.206	0.040	0.033	9.47	0.0080	0.00084	1.637	0.044	0.027
213	24	10,300	1.09	0.050	0.046	9.43	0.0054	0.00057	1.56	0.053	0.034
214	25	16,900	0.945	0.096	0.10	9.45	0.0073	0.00077	1.40	0.095	0.068
215	25	2,100	1.24	0.046	0.037	9.45	0.011	0.0012	1.69	0.048	0.028
216	25	5,400	1.32	0.056	0.042	9.44	0.0096	0.0010	1.78	0.057	0.032
217	25	4,700	1.065	0.098	0.092	9.43	0.016	0.0017	1.53	0.103	0.067
218	25	51,500	0.95	0.047	0.049	9.41	0.010	0.0011	1.44	0.050	0.035
219	46	7,600	0.50	0.107	0.21	9.42	0.035	0.0037	0.98	0.12	0.12
220	6	4,200	1.41	0.14	0.099	9.42	0.021	0.0022	1.90	0.14	0.074
221	18	5,400	1.49	0.097	0.065	9.46	0.015	0.0016	1.94	0.10	0.052
222	5	2,700	0.14	0.20	1.43	9.58	0.010	0.0010	0.463	0.19	0.41
223	5	600	0.055	0.164	2.98	9.57	0.013	0.0014	0.38	0.17	0.45
C-2	115	5,600	-0.42	0.05	-0.12	9.491	0.012	0.0013	-0.01	0.05	-5.00
C-3	115	2,500	-0.24	0.10	-0.40	9.407	0.032	0.0034	0.25	0.09	0.36

TABLE 18. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S UNCLEANED COAL (Radical Transformation)

Batch ID.	N= No. of lots	$\bar{X}$= Mean lot size, tons	$W_1 = \sqrt{\text{Sulfur (\%)}}$			$W_2 = \sqrt{\text{Btu/lb}}$			$W_3 = \sqrt{\frac{\text{lbs } SO_2}{\text{MM Btu}}}$		
			$\bar{W}_1$	SW_1	$SW_1/\bar{W}_1$	$\bar{W}_2$	SW_2	$SW_2/\bar{W}_2$	$\bar{W}_3$	SW_3	$SW_3/\bar{W}_3$
103	25	2,800	2.11	0.11	0.052	108.8	1.53	0.014	2.74	0.15	0.055
104	25	10,900	2.03	0.039	0.019	109.7	0.78	0.0071	2.61	0.056	0.021
105	25	11,200	2.20	0.13	0.059	109.5	1.57	0.014	2.84	0.20	0.070
107	25	5,300	2.24	0.10	0.045	110.9	1.15	0.010	2.86	0.15	0.052
108	25	6,800	2.04	0.057	0.028	109.1	0.77	0.0071	2.64	0.083	0.031
109	25	7,100	2.28	0.10	0.044	107.3	2.17	0.020	3.00	0.097	0.032
110	25	1,100	2.19	0.11	0.050	101.7	1.83	0.018	3.04	0.16	0.053
111	25	6,700	2.16	0.081	0.038	107.7	1.01	0.0094	2.83	0.12	0.042
112	26	3,800	2.05	0.058	0.028	107.3	1.95	0.018	2.71	0.11	0.041
116	44	7,600	1.41	0.11	0.078	106.9	1.49	0.014	1.86	0.15	0.081
117	44	3,400	1.76	0.069	0.039	105.3	1.26	0.012	2.36	0.101	0.043
120	25	10,300	1.21	0.17	0.14	106.8	1.17	0.011	1.60	0.24	0.15
121	24	17,300	1.94	0.048	0.025	112.2	1.01	0.0090	2.45	0.059	0.024
C-1	704	12,000	1.67	0.07	0.60	114.2	1.0	0.0088	2.07	0.09	0.043
C-8	275	10,000	1.61	0.04	0.025	111.7	3.2	0.029	2.04	0.07	0.034
U-4	164	12,000	1.02	0.08	0.078	109.5	1.4	0.013	1.31	0.10	0.076
U-5	250	13,000	0.96	0.07	0.073	108.9	1.4	0.013	1.24	0.09	0.073
U-11	250	-	1.76	0.13	0.074	107.3	1.7	0.016	2.32	0.19	0.082
U-12	250	-	1.53	0.08	0.052	109.7	1.2	0.011	1.97	0.11	0.056
U-13	250	-	1.52	0.07	0.046	110.2	1.2	0.011	1.95	0.09	0.046

TABLE 19. STATISTICS FOR VARIABILITY OF SULFUR, HEATING VALUE, AND LBS SO_2 PER MILLION BTU'S-CLEANED COAL (Radical Transformation)

Batch ID.	N= No. of lots	$\bar{X}$= Mean lot size, tons	$W_1 = \sqrt{\text{Sulfur}(\%)}$			$W_2 = \sqrt{\text{Btu/lb}}$			$W_3 = \sqrt{\frac{\text{lbs } SO_2}{\text{MM Btu}}}$		
			$\bar{W}_1$	SW_1	$SW_1/\bar{W}_1$	$\bar{W}_2$	SW_2	$SW_2/\bar{W}_2$	$\bar{W}_3$	SW_3	$SW_3/\bar{W}_3$
202	24	6,500	1.78	0.043	0.024	114.0	0.33	0.0029	2.21	0.056	0.025
203	6	100,000	1.83	0.010	0.0055	113.9	0.21	0.0018	2.28	0.015	0.0066
204	25	18,300	1.83	0.03	0.016	114.0	0.35	0.0031	2.27	0.043	0.019
206	24	3,900	1.96	0.051	0.026	109.8	0.0961	0.00088	2.53	0.074	0.029
208	26	4,500	2.01	0.052	0.026	110.1	0.91	0.0083	2.58	0.073	0.028
209	25	6,300	1.68	0.059	0.035	112.9	0.76	0.0067	2.08	0.077	0.037
210	25	4,200	1.83	0.037	0.020	114.0	0.468	0.0041	2.27	0.049	0.021
213	24	10,300	1.72	0.043	0.025	111.5	0.30	0.0027	2.18	0.058	0.027
214	25	16,900	1.61	0.077	0.048	112.6	0.412	0.0037	2.02	0.095	0.047
215	25	2,100	1.86	0.043	0.023	112.9	0.64	0.0057	2.33	0.056	0.024
216	25	5,400	1.94	0.054	0.028	112.3	0.54	0.0048	2.44	0.069	0.028
217	25	4,700	1.705	0.083	0.049	111.8	0.92	0.0082	2.16	0.11	0.051
218	25	51,500	1.61	0.038	0.024	110.7	0.54	0.0049	2.05	0.052	0.025
219	46	7,600	1.28	0.069	0.054	111.1	1.94	0.017	1.64	0.10	0.061
220	6	4,200	2.03	0.15	0.074	110.8	1.19	0.011	2.59	0.18	0.069
221	18	5,400	2.11	0.103	0.049	113.0	0.86	0.0076	2.64	0.13	0.049
222	5	2,700	1.08	0.106	0.098	120.3	0.605	0.0050	1.26	0.12	0.095
223	5	600	1.031	0.082	0.080	119.9	0.761	0.0063	1.22	0.100	0.082
C-2	115	5,600	0.81	0.02	0.025	115.1	0.7	0.0061	1.00	0.03	0.030
C-3	115	2,500	0.88	0.04	0.045	110.3	1.7	0.015	1.13	0.05	0.044

Now, to test the hypothesis that a (random) sample comes from a population having a normal distribution, a normal curve can be fitted to the data and a chi-square test (for goodness of fit) applied to determine whether the hypothesis is justified. In such a case, the mean and standard deviation for the fitted normal curve is usually estimated from grouped sample data because the chi-square test requires the presence of several ($\geq$ 5) observations in each class if the Stirling-approximation (used in the classical derivation of the test) is to be valid. Inspection of Tables 12 and 13 shows that sufficient lot numbers exist (for known mean lot-sizes) for seven batches of uncleaned coals (C1, C8, U4, U5, U11, U12, and U13) and two batches of cleaned coals (C2 and C3). Taking the corresponding sample means and standard deviations as estimates for the population means and standard deviations*, chi-square tests at the 5 percent level of significance were made to determine whether the (transformed and untransformed) data could be assumed to have come from a normal distribution. Detailed chi-quare analyses for three data sets (C-8, U-11, and C-3) in their three forms (original, logarithmic transformation, and radical transformation) are included in Appendix B. Table 20 presents the summarized results of these calculations for all data sets, with italicized entries designating acceptance of the hypothesis at the 5 percent level of significance.

As can be seen, the general tendency is that, if the actual data satisfies the test**, the transformed data does also. The exceptions in U-12 and U-13 can be seen to have failed the test** by a relatively small margin. Observe, however, that <u>all</u> of the data for both cleaned coal batches satisfy the acceptance criteria.** Recalling that uncleaned coals are generally subject to far less stringent sampling procedures than are

---

* Using the sample standard deviation as an estimate of the population standard deviation is admittedly tenuous, for those samples exhibiting autocorrelation (Section 5.4).

** The chi-square test is designed to refute an hypothesis of normality, not to validate it. Therefore, "satisfying the test" means an absence of evidence (at the stated level of significance) to reject the hypothesis, and "failing the test" means rejection of the hypothesis of normality.

TABLE 20. COMPUTED CHI-SQUARE VALUES

Data set	No. of data points	Degrees of freedom	S	ln(S)	$\sqrt{S}$	Btu	ln(Btu)	$\sqrt{Btu}$	SO_2	$\ln(SO_2)$	$\sqrt{SO_2}$	chi-square (0.05)
Unwashed												
C1	704	17	31.8	31.8	29.0	31.7	37.3	35.9	*19.4*	*24.9*	*20.2*	27.59
C8	275	13	23.5	35.7	33.7	292.0	328.0	314.2	*22.2*	*14.0*	*16.8*	22.36
U4	164	5	80.6	91.3	91.3	*5.2*	*5.2*	*5.2*	55.2	31.7	47.6	11.07
U5	250	7	186.7	197.5	190.6	*6.9*	*4.6*	*4.6*	78.1	28.0	34.5	14.07
U11	250	13	44.2	54.1	50.0	74.8	74.4	73.6	53.1	74.4	72.6	22.36
U12	250	13	32.0	36.0	32.3	*8.7*	*8.2*	*8.6*	*21.6*	23.1	*17.5*	22.36
U13	250	13	37.3	29.1	32.0	26.4	*22.0*	26.9	*13.8*	*13.9*	*15.1*	22.36
Washed												
C2	115	9	*10.2*	*7.7*	*7.8*	*13.3*	*12.5*	*12.5*	*14.7*	*12.1*	*13.3*	16.92
C3	115	9	*6.5*	*15.1*	*7.1*	*13.7*	*16.6*	*12.9*	*7.1*	*7.9*	*6.8*	16.92

Italicized numbers indicate acceptance of the hypothesis that the samples were obtained from populations that are normal at the 5% level of significance.

the cleaned (product) coals, no satisfactory conclusions can be drawn from assumptive distribution testing before the sampling and analysis protocols which produce the data are sufficiently defined and standardized. Indeed, the results of the chi-square evaluation tend to indicate the absence of sensible evidence for preferring any one distribution over the others, and Figures 2, 3, and 4 (where graphs of the fitted normal curve are plotted on the same scale as the histogram of the grouped data) illustrate this characteristic for three of the data sets of Table 20. It should also be noted that there exists an inherent difficulty in judging "by eye" how much departure from the normal pattern should be expected in such figures. In fact, the plots for a sample of 1,000 data points drawn from a population known to be normal often exhibit substantial irregularity -- which is why the chi-square test is generally taken to be far superior to graphical methods for testing the fit of a distribution.*

5.6 COMPARISON OF VARIABILITIES: UNCLEANED VS. CLEANED COALS

The conclusion reached in Section 5.5 was that no sensible evidence exists for preferring any one distribution (original data points, logarithmic transformation, or radical transformation) over the others. In general, if the original data satisfied the chi-square test for normality, then the transformed data does also. The converse is also generally true. For reasons of simplicity, then, the following comparison of variabilities was conducted using the original (untransformed) data.

The following comparison of variabilities was conducted with due respect to the limitations and caveats expressed about this analysis in Section 5.1, and especially with respect to the evidence of autocorrelation in Section 5.4. It would be a mistake to construe the results of this comparison beyond the point of being, perhaps, roughly indicative of the effects of the coal cleaning process.

The statistics for individual (untransformed) data sets are listed in Tables 12 and 13. The variances ($S^2_{Y_1}$, $S^2_{Y_2}$, and $S^2_{Y_3}$) for all of the uncleaned coal data sets in Table 12 were pooled according to:

---

* More rigorous tests of normality should also include tests for skewness and kurtosis.

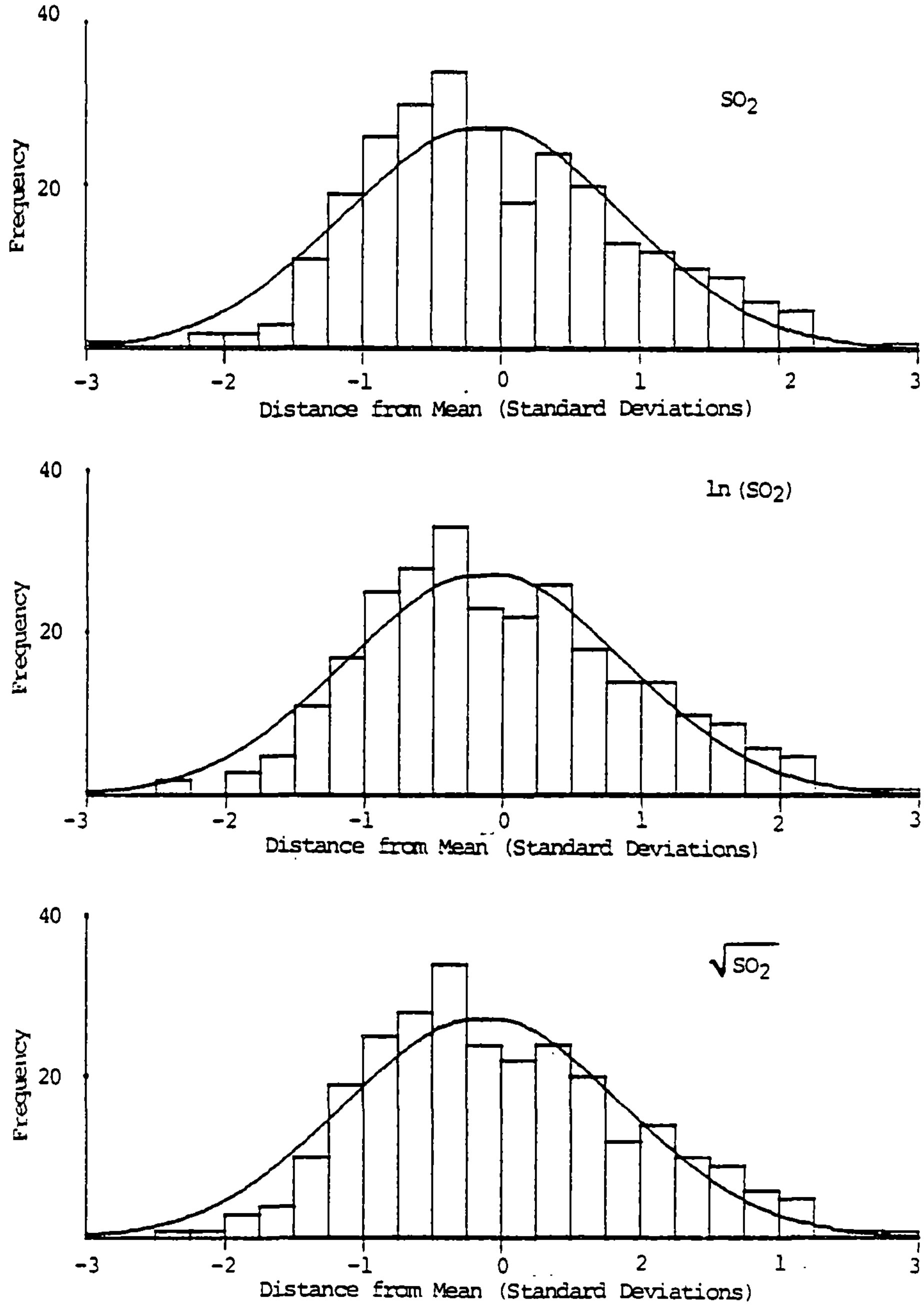

Figure 2. Fitted Normal Curve and Histogram of Grouped SO_2 Data for Batch C-8

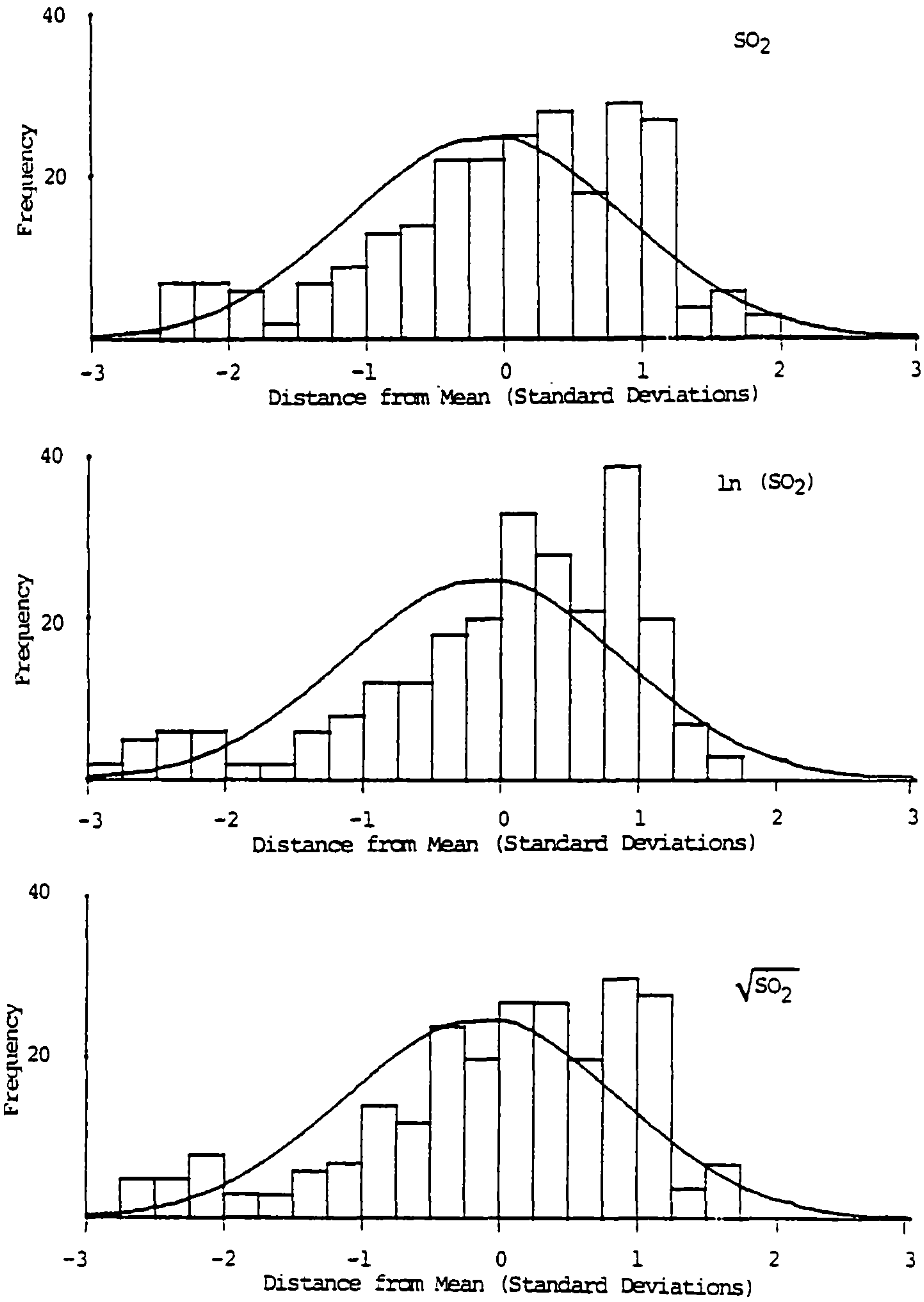

Figure 3. Fitted Normal Curve and Histogram of Grouped SO_2 Data for Batch U-11

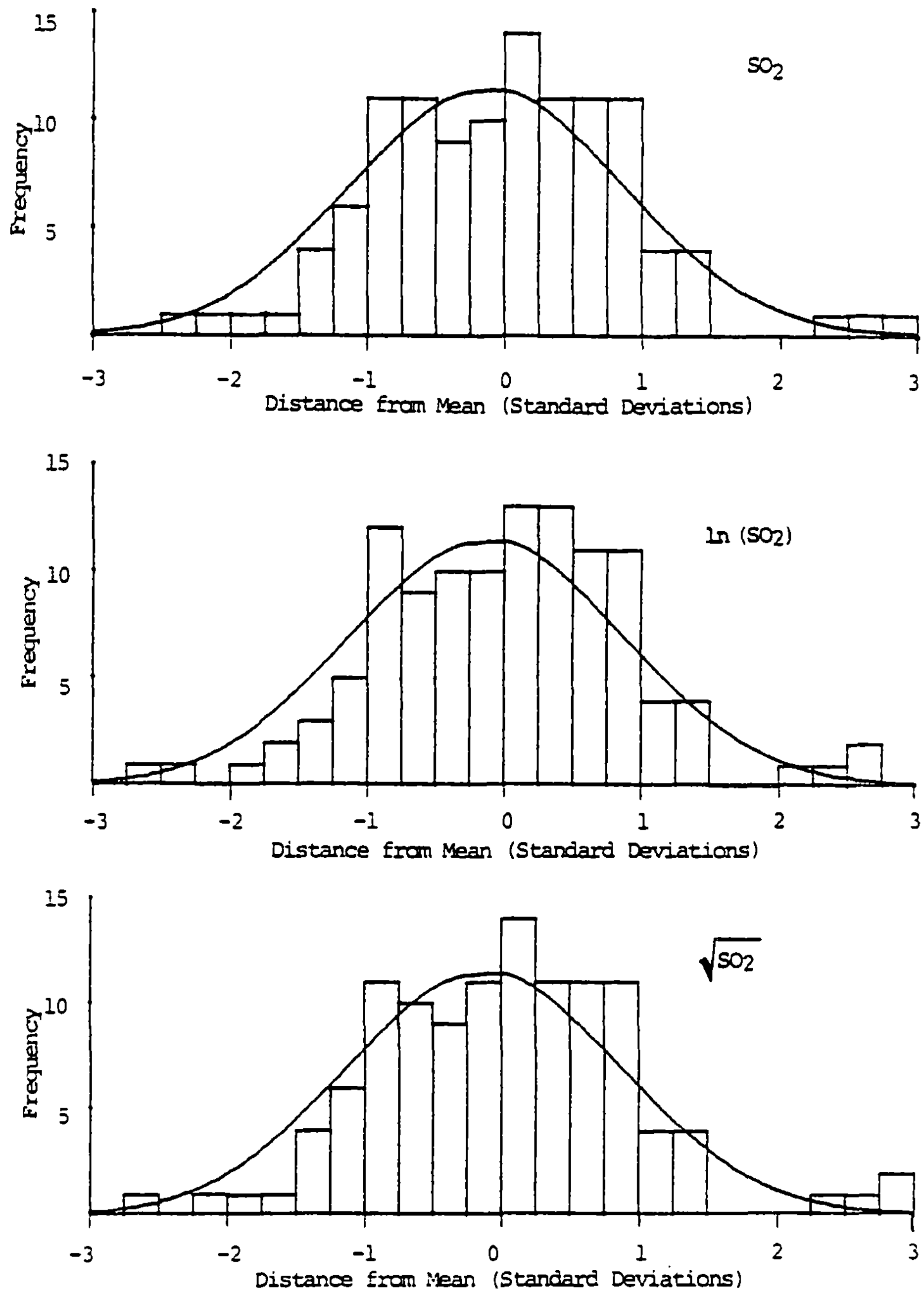

Figure 4. Fitted Normal Curve and Histogram of Grouped SO_2 Data for Batch C-3

$$(S^2_{yi})_{pooled} = \frac{1}{\Sigma N_j} \sum_{j=1}^{J} N_j \, (S^2_{yi})_j \quad ; \quad i = 1,2,3$$

where N_j = number of data points (lots) in data set j

J = the number of data sets in Table 12.

The same calculation was performed for the cleaned coal data sets in Table 13.

Similarly, the squares of the relative standard deviations were pooled according to:

$$\left(\frac{Sy_i}{\overline{Y}_i}\right)^2_{pooled} = \frac{1}{\Sigma N_j} \sum_{j=1}^{J} N_j \left(\frac{Sy_i}{\overline{Y}_i}\right)^2_j \quad , i = 1,2,3$$

$$\text{and } (RSD_i) \text{ pooled} = \left(\left[\left(\frac{Sy_i}{\overline{Y}_i}\right)^2_{pooled} \right] \right)^{1/2}$$

Finally, because we wish to evaluate average RSDs,

$$(RSD_i)_{average} = \frac{1}{J} \sum_{j=1}^{J} \left(\frac{Sy_i}{\overline{Y}_i}\right)_j \quad , i = 1,2,3 \ .$$

The results of these calculations are listed in Table 21. The reductions in variability, depending upon how variability is measured, range from 25 to 64 percent. Despite the limitations of the statistical treatment, these results surely suggest that the variability is reduced by the coal cleaning process. Moreover, these results are consistent with the percent reductions derived (in Section 4.2) from the eight paired coal feed and product data sets.

TABLE 21.

COMPARISON OF VARIABILITIES

		Y_1 % Total sulfur	Y_2 Btu/lb	Y_3 lb SO_2/MM Btu
Pooled variances	Uncleaned coals	0.0710	126,100	0.236
$\left(Sy_1{}^2\right)$ pooled	Cleaned coals	0.0303	56,800	0.084
	Percent reduction	57	55	64
Pooled RSDs	Uncleaned coals	0.111	0.0291	0.118
$\left[\left(\frac{Sy_1}{y_1}\right)^2_{pooled}\right]^{1/2}$	Cleaned coals	0.078	0.0192	0.078
	Percent reductions	30	34	34
Average	Uncleaned coals	0.104	0.0265	0.114
RSDs	Cleaned coals	0.078	0.0134	0.080
	Percent reductions	25	49	30

SECTION 6

COMPONENTS OF VARIANCE

As outlined in Section 2.1, the model adopted for examining sulfur variability has the following generalized components of variance:

$$V_{Total} = V_{\substack{Long\\Term}} + V_{\substack{Short\\Term}} + V_{\substack{Sampling\ \&\\Analysis}}$$

V_{Total} represents the variability of the data points within each data set about the mean for that data set. No attempt has been made in this study to evaluate the additional geographical and source components of variance which apply among data sets. Hence, V_{Total} for each coal source (e.g., each data set) will be influenced by characteristics of the coal region, the coal seam, the particular mine, the mining methods employed, and the coal preparation methods employed.

For example, the coal shipped from an eastern underground mine may originate from several quite-different coal faces, i.e., the sulfur content at one face may have a mean of one percent, and may be three percent at another face. A large value for V_{Total} and for $V_{Long-Term}$ would result. In contrast, another mine might have a single face in operation at any one time, and the deposit may be relatively homogeneous, resulting in a smaller value for V_{Total}.

The model for examining sulfur variability is a temporal model. Further, the definition of long-term as the month-to-month variation is rather arbitrary, and is influenced by the definition of short-term as considerably less than one-month (e.g., daily variation). There is no fundamental reason, such as matching "long-term" in a temporal model with "long-range" in a corresponding geostatistical spatial model, for choosing a month as the long-term/short-term boundary.

6.1 ANALYSIS OF AUTOCORRELATED DATA SETS

In Section 5.4, tests for autocorrelation of the heat-specific sulfur content yielded positive results (at the 95 percent confidence level) for 16 data sets. These "select" data sets differ from the broader group of data sets specifically because the long-term component of variance was discernible above the noise level of lot-to-lot (short-term) variance. An opportunity therefore exists to use the select group of data sets to estimate a generalized long-term component of variance.

Table 22 presents these selected data sets. The values for the total (source-specific) variance, S^2, and the estimates of the "nugget" variance, $\gamma(1)$, are taken from Tables 14 and 15. For the purposes of this analysis, $\gamma(1)$ is used as a conveniently calculated surrogate for the true nugget variance, $\gamma(0)$, with due appreciation of possible added uncertainty. This "nugget" variance includes both the short-term variance and the sampling-and-analysis variance. The difference, $S^2 - \gamma(1)$, is listed in Table 22 as representing the long-term component of variance. The transformation of variances to relative standard deviations (also shown in Table 22) serves to normalize the variability data for the "geographical" component which influences the mean for each data set:

$$\text{Total RSD} = (S^2)^{1/2}/\bar{Y}_3$$

$$\text{(Short-Term + S/A) RSD} = (\gamma(1))^{1/2}/\bar{Y}_3$$

$$\text{Long-Term RSD} = (S^2-\gamma(1))^{1/2}/\bar{Y}_3$$

The RSDs of Table 22 were then pooled according to:

$$RSD_{pooled} = \left[\frac{1}{\Sigma n} \quad \Sigma n(RSD)_i^2 \right]^{1/2} ,$$

resulting in:

	Pooled RSD		
	Total	Short + S/A	Long
Uncleaned coals	0.0815	0.0649	0.0493
Cleaned coals	0.0882	0.0665	0.0580
All coals	0.0834	0.0653	0.0519

TABLE 22.

ANALYSIS OF AUTOCORRELATED DATA SETS

PARAMETER: LBS SO_2/MM BTU

Data set	n	Average lot size, tons	Mean, $\bar{Y}_3$	Variances			RSD		
				S^2	$\gamma(1)$	$S^2-\gamma(1)$	Total	Short + S/A	Long
104	25	10,900	6.82	0.090	0.059	0.031	0.0440	0.0356	0.0258
112	26	3,800	7.35	0.384	0.253	0.131	0.0843	0.0684	0.0492
114	12	319,700	8.82	0.880	0.255	0.625	0.1064	0.0573	0.0896
119	5	640	3.00	0.740	0.159	0.581	0.2867	0.1329	0.2541
C-1	704	12,000	4.27	0.130	0.093	0.037	0.0844	0.0714	0.0450
C-8	275	10,000	4.17	0.073	0.034	0.039	0.0648	0.0442	0.0474
201	6	214,500	4.90	0.0151	0.0024	0.0127	0.0251	0.0100	0.0230
205	6	228,300	5.09	0.0784	0.0219	0.0565	0.0550	0.0291	0.0467
208	26	4,500	6.69	0.1369	0.0672	0.0697	0.0553	0.0387	0.0395
209	25	6,300	4.33	0.1024	0.0633	0.0391	0.0739	0.0581	0.0457
210	25	4,200	5.14	0.0485	0.0292	0.0193	0.0428	0.0332	0.0270
211	12	251,900	6.73	0.1998	0.0637	0.1361	0.0664	0.0375	0.0548
214	25	16,900	4.07	0.1444	0.0934	0.0510	0.0934	0.0751	0.0555
219	46	7,600	2.69	0.1089	0.0773	0.0316	0.1227	0.1034	0.0661
C-2	115	5,600	0.99	0.0078	0.0017	0.0061	0.0892	0.0416	0.0789
C-3	115	2,500	1.28	0.0136	0.0109	0.0027	0.0911	0.0816	0.0406

The total RSDs above for the select group of autocorrelated data sets may be compared with the total RSDs for all data sets (from Table 21):

Uncleaned Coals, 0.118

Cleaned Coals, 0.078

The results of the analysis of the select group of data sets are then useful in estimating a generalized value for the long-term RSD. This long-term RSD, defined as a month-to-month component of variance, is 0.052, which is the best estimate applicable to all of the coals in the data base. No difference should exist for this long-term component between uncleaned coals and cleaned coals.

6.2 GENERALIZED ESTIMATES FOR COMPONENTS OF VARIANCE

In Section 2.3, experimentally based estimates were derived for the actual sampling and analysis components of variability of lbs SO^2/MM Btu:

(RSD) S&A for Uncleaned Coals = 0.045

(RSD) S&A for Cleaned Coals = 0.023

In Section 5.6 (Table 21), estimates were derived for the total RSD (of each coal source about the mean for that source) for lbs SO_2/MM Btu:

(RSD) Total, Each Source, for Uncleaned Coals = 0.118

(RSD) Total, Each Source, for Cleaned Coals = 0.078

In Section 6.1, an estimate was derived for the long-term component of variability, applicable to all coal data sets:

(RSD) Long-Term = 0.052

The determination, by difference, of the short-term component of variability is therefore possible:

(RSD) Short-Term for Uncleaned Coals

$$= \left[(0.118)^2 - (0.052)^2 - (0.045)^2\right]^{1/2} = 0.096$$

(RSD) Short-Term for Cleaned Coals

$$= \left[(0.078)^2 - (0.052)^2 - (0.023)^2\right]^{1/2} = 0.053$$

The table below summarizes these estimates for the components of variability:

	Uncleaned coals	Cleaned coals
RSD for long-term	0.052	0.052
RSD for short-term	0.096	0.053
RSD for S&A	0.045	0.023
(RSD) total for each source	0.118	0.078

It must be emphasized that these are generalized estimates, representing aggregated data sets. In no way may these values be utilized to characterize any one particular coal. Actual variabilities of individual data sets, as evidenced by Tables 12 and 13, may be quite different from the generalized values shown above.

SECTION 7

EFFECT OF LOT SIZE UPON VARIABILITY

7.1 RESULTS OF PREVIOUS STUDY

In a previous study of sulfur variability sponsored by EPA[1], it was pointed out on both theoretical and empirical grounds that the sulfur variability of small lots of coal should be greater than that of large lots. A qualitative explanation was the application of the Central Limit Theorem to the component of variance which corresponds to the averaging process. However, the evidence in the present study (Section 5.4) for serial correlation of coal sulfur variability data places doubt upon the validity of applying the Central Limit Theorem to such data.

The empirical rationale in this previous study was based upon 12 data sets for coals with less than one percent sulfur, where each data point represented a unit train (approximately 10,000 tons). Despite averaging across heterogeneous populations (different regions, both unwashed and washed coals, etc.), the average RSD among data points (unit trains) within each month was 0.143, while the average RSD from month-to-month (perhaps 16 unit trains per month) was 0.059. This inverse relationship between RSD and lot size was not, however, demonstrated with consistency in the previous study.

7.2 ANALYSIS OF TOTAL VARIABILITY, ALL DATA SETS

The data of Tables 12 and 13 are plotted on Figure 5. Shown are the relative standard deviation of percent sulfur as the ordinate, and the lot size (on a logarithmic scale) as the abcissa. As observed, the data in Figure 5 are highly scattered, but it may be argued that the general inverse relationship (higher RSD's at lower lot sizes) does exist.

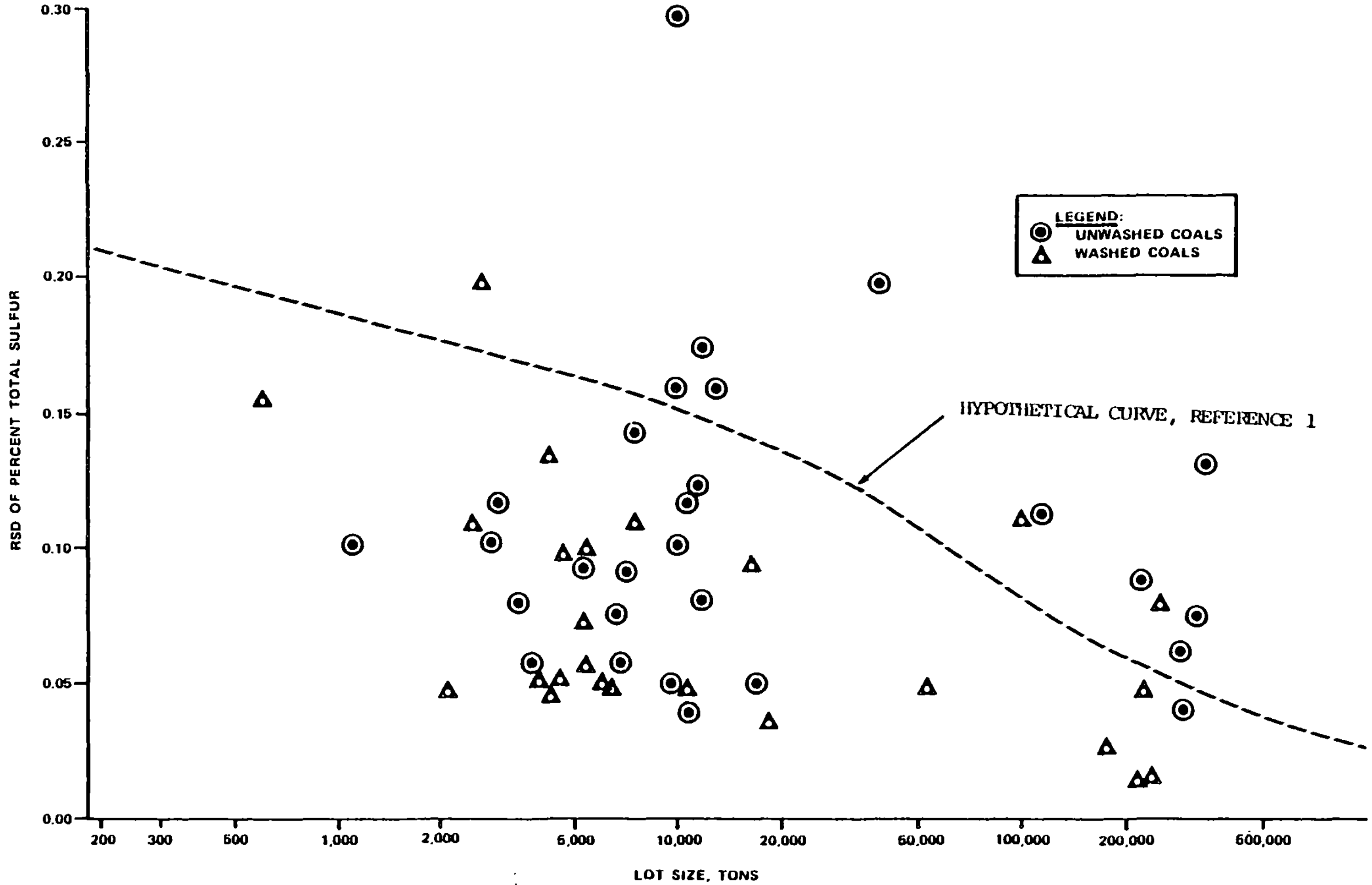

FIGURE 5. EFFECT OF LOT SIZE UPON PERCENT SULFUR RELATIVE STANDARD DEVIATION

Also shown (for reference) in Figure 5 is a curve developed in the previous EPA study [1] for a "hypothetical" coal. The origin of this curve has both theoretical and empirical rationale, but the prior study pointed out that a different curve would result for any specific coal deposit (data set) or for different groups of data sets. The RSD data points (from the database in this study) are generally lower than the hypothetical curve.

7.3 ANALYSIS OF SELECT DATA SETS

It is possible that an observed difference between a relative standard deviation for data points within months, and a relative standard deviation for month-to-month aggregated data, may be explained either by a lot size difference (the month-to-month aggregated data would, of course, be associated with large lots) or by the difference between the short-term and the long-term components of variability.

The 16 select data sets of Table 22, where autocorrelation was demonstrated, provide an opportunity for discerning lot-size effects from long-term/short-term effects, because the long-term and short-term components of variability were independently estimated. Table 22 also lists the average lot sizes for these select data sets. Figure 6 is a plot of the long-term component of the relative standard deviation (for lbs SO_2/MM Btu) vs. lot size. Except for the one data set (No. 119) with an average lot size of 640 tons, the long-term variability data of Figure 6 does not exhibit a dependence of RSD upon lot size.

Figure 7 is a plot of the short-term (including sampling and analysis) component of the RSD vs. lot size. Although these data are scattered to a large extent, the inverse relationship is clearly observable. This result is expected, since the short-term component of variability has been separated from the component which is associated with autocorrelation effects. A least-squares straight line (shown in Figure 7) through the data points of Figure 7 is:

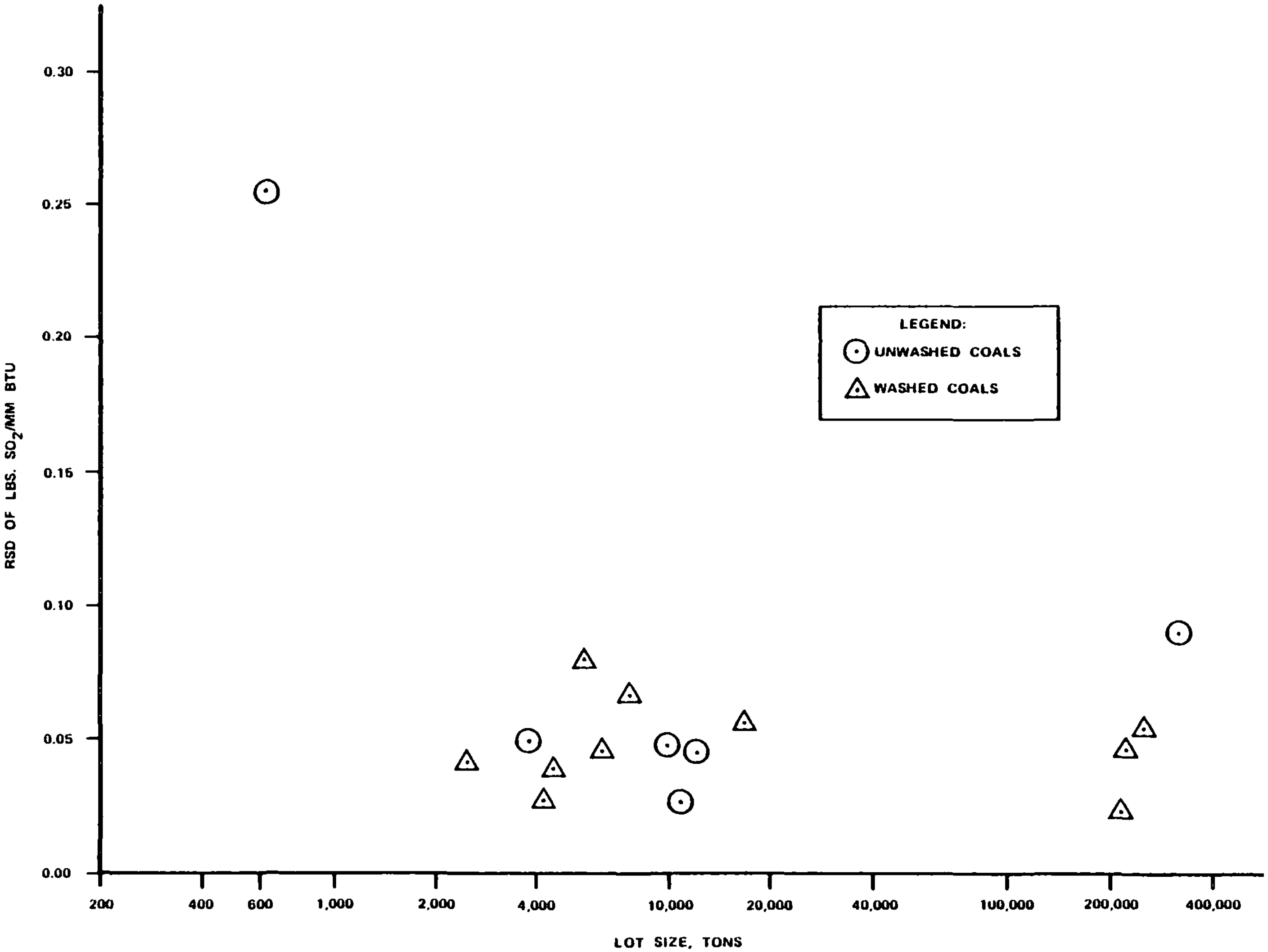

FIGURE 6. EFFECT OF LOT SIZE UPON LONG TERM RSD SELECT DATA SETS.

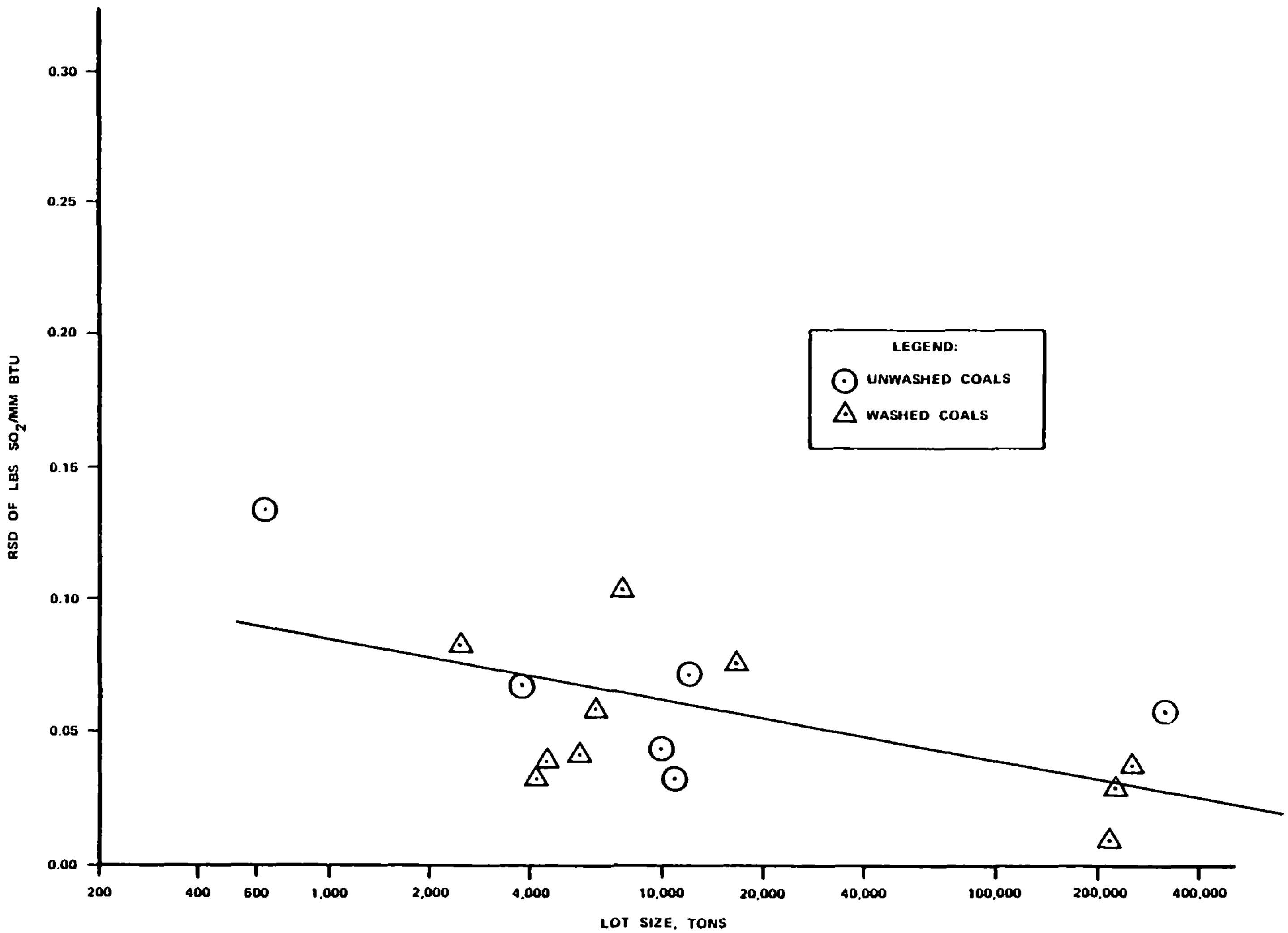

FIGURE 7. EFFECT OF LOT SIZE UPON SHORT-TERM RSD SELECT DATA SETS

$$(RSD)\ \text{Short-Term} = 0.150 - 0.0223 \log_{10} (\text{lot size, tons})$$

The correlation coefficient of this least-square line is 0.59. Hence, 34 percent of the total variance of short-term RSD values was accounted for by the regression on (the log of) lot size. Considering that the 16 data sets are from non-homogeneous populations - different coal regions, seams, and mines, both unwashed and washed coals, and non-uniform sampling and analytical procedures - the remaining variance is explainable.

SECTION 8

CONCLUSIONS

The analysis of the collected data in this report supports the following conclusions:

- In seven of eight coal cleaning plants, for which matched pairs of feed and product coal data were available, the coal cleaning process resulted in significant changes in coal properties. The mean total sulfur content was reduced by 14 to 45 percent, the mean heating value was increased by 6 to 23 percent, and the mean lbs SO_2/MM Btu was reduced by 24 to 50 percent. The range of effectiveness of physical coal cleaning as a sulfur dioxide control technology is demonstrated by these data from operating commercial preparation plants. These empirical data fall within the range of calculated physical coal cleaning performance of hypothetical plants. The ranges of both demonstrated and calculated effectiveness are wide, reflecting the sensitivity of coal cleaning efficiency to the washability characteristics of specific coals and to the complexity of the plant design. These actual and calculated data demonstrate that no valid "typical" effectiveness can be quoted for physical coal cleaning technology.

- In each of the eight plants for which matched pairs of feed and product data were available, both the absolute standard deviation and the relative standard deviation for all three coal characteristics were reduced by the coal preparation process. The reductions in both percent sulfur variability and lbs SO_2/MM Btu variability averaged approximately 55 percent and ranged from 9 to 90 percent, while the heating value variability reduction averaged approximately 80 percent and ranged from 51 to 94 percent.

- Data from 20 sets of unwashed coal data and from 17 sets of washed coal data did not permit direct comparison of feed and product pairs. A second statistical analysis, conducted to exploit the entire available data base, compared the data sets of all unwashed coals to the data sets of all washed coals. This indirect approach is hampered because the two groups of data sets do not form logically consistent or homogeneous populations sufficient for rigorous statistical analysis. Because of these inherent compatibility problems, the results of this second statistical analysis should not be regarded as definitive as those of the first analysis. Despite the limitations of the statistical treatment, the comparison of variabilities of the two groups of data sets surely suggest that the variability is reduced by the coal cleaning process. The reductions, from unwashed coals to washed coals, range from 25 to 64 percent depending upon how variability is measured. These results are consistent with the percent reductions in variability derived from the paired feed/product data sets.

- Nine data sets (which accounted for 2,373 data points) were examined in three ways: without transformation, with a logarithmic transformation, and with a radical transformation. The distributions of the untransformed and transformed data were tested for normality. Six of the nine batches satisfied the chi-square test (for lbs SO_2/MM BTU) for normality, with either the untransformed data or the transformed data. The three batches failing the test failed regardless of whether the data were transformed or not. These results indicate the absence of sensible evidence for preferring any one distribution over the others.

- Tests for autocorrelation of the data points within data sets gave positive results in 16 of 48 data sets (at the 95 percent confidence level). There is little doubt, therefore, that much of these coal data are serially correlated, verifying the expectations based upon geology and engineering rationale.

- For each of 16 data sets which exhibited autocorrelation, the total variance (of lbs SO_2/MM Btu) was resolved into the long-term component, associated with the serial correlation according to geostatistical concepts, and the residual short-term (including sampling and analysis) component. An estimate of a generalized long-term component of relative standard deviation was 0.052, applicable to both unwashed coals and washed coals.

- From previously published data representing actual commercial practice, the component of relative standard deviation attributable to ASTM coal sampling, sample preparation, and laboratory analysis (in terms of lbs SO_2/MM Btu) was 0.045 for unwashed coals and 0.023 for washed coals. These values are smaller than the 0.07 to 0.08 maximum permitted by the ASTM protocols.

- Estimates of the components of variability are:

	Uncleaned coals	Cleaned coals
RSD for long-term	0.052	0.052
RSD for short-term	0.096	0.053
RSD for S&A	0.045	0.023
(RSD) total for each source	0.118	0.078

It must be emphasized that these are generalized estimates, representing aggregated data sets. In no way may these values be utilized to characterize any one particular coal. Actual variabilities of individual data sets may be quite different from the generalized values shown above.

- A prior study concluded that the relative standard deviation should be inversely related to lot size. By removing the long-term component of variability (which includes autocorrelation) from data in this study, an inverse relationship between the short-term component of RSD and lot size was discerned. A least-squares line had a correlation coefficient of 0.6, indicating a much clearer inverse relationship than was previously demonstrated.

SECTION 9

REFERENCES

1 Preliminary Evaluation of Sulfur Variability in Low-Sulfur Coals from Selected Mines. EPA 450/3-77-044, November 1977.

2 Crow, E. L., F. A. Davis, and M. W. Maxfield. Statistics Manual. Dover Publications Inc., New York. (S599).

3 Kilgroe, J. D. Coal Sulfur Variability Estimates for Industrial Boiler Study. EPA/IERL/RTP Memo. (October 20, 1978).

4 Leonard, J. W., and D. R. Mitchell (ed.). Coal Preparation. 3rd edition. The American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc., New York, 1968.

5 Keller, G. E. Determination of Quantities Needed in Coal Sample Preparation and Analysis. Transactions Society of Mining Engineers, September 1965. pp. 218-26.

6 Aresco, S. J., and A. A. Orning. A Study of the Precision of Coal Sampling. Sample Preparation and Analysis. Transactions Society of Mining Engineers, September 1965. pp. 258-64.

7 Cavallaro, J. A., M. T. Johnson, and A. W. Deurbrouck. Sulfur Reduction Potential of the Coals of the United States. Report of Investigations 8118. U.S. Department of the Interior, Bureau of Mines, 1976.

8 Bechtel Corporation. Environmental Control Implications of Generating Electric Power from Coal. Appendix A (Part 1). Argonne National Laboratory, U.S. Department of Energy, December 1977.

9
Technology Assessment Report for Industrial Boiler Applications: Coal Cleaning and Low Sulfur Coal. EPA-600/7-79-178c. EPA Contract No. 68-02-2199, Task No. 12. December 1979.

10
Versar, Inc. Coal Cleaning Sensitivity Analysis. (Draft) for U.S. Environmental Protection Agency. July 1979.

11 Thomas, R. E. Interpreting Statistical Variability. Battelle-Columbus Laboratories. In Proceedings: Symposium on Coal Cleaning to Achieve Energy and Environmental Goals. September 1978. Hollywood, Florida. Volume 1. EPA-600/7-79-098a, April 1979. pp. 126-147.

APPENDIX A

LISTING OF THE DATA BASE

TABLE A-1. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
101	271,402	4.17	11,035	7.55
	238,303	4.64	11,827	7.84
	275,873	4.08	12,009	6.79
	270,487	3.96	10,529	7.51
	394,101	3.98	11,611	6.85
	271,355	4.13	10,914	7.56
102	143,854	4.72	12,447	7.58
	134,799	4.07	12,385	6.57
	118,004	3.99	12,019	6.63
	106,709	3.96	11,664	6.78
	104,390	5.05	12,154	8.30
	88,544	3.93	12,728	6.17
103	4,120	4.61	11,813	7.80
	2,871	4.22	11,498	7.33
	1,358	5.39	11,834	9.10
	1,404	3.94	12,027	6.55
	4,276	4.54	12,189	7.44
	1,530	3.99	12,342	6.46
	2,825	4.83	12,010	8.04
	4,339	3.91	12,063	6.48
	2,908	4.25	12,185	6.97
	4,181	5.41	11,939	9.05
	5,642	4.00	12,110	6.60
	4,201	4.44	11,905	7.45
	5,392	4.57	11,974	7.63
	1,245	3.40	12,088	5.62
	1,465	4.99	11,707	8.52
	1,421	4.49	11,338	7.91
	1,409	4.13	11,558	7.14
	1,343	4.46	12,435	7.17
	1,438	4.57	11,472	7.96
	1,427	4.28	11,812	7.24
	10,100	4.63	11,902	7.77
	1,383	4.74	11,515	8.22
	1,432	4.48	11,636	7.69
	1,383	4.15	11,032	7.52
	1,436	4.64	11,479	8.08

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
104	13,129	3.99	12,360	6.45
	8,983	3.86	12,151	6.35
	11,787	4.30	12,218	7.03
	15,092	4.14	12,007	6.89
	11,662	3.94	11,942	6.59
	16,837	4.07	11,912	6.83
	13,618	4.13	12,021	6.86
	7,604	4.43	11,890	7.44
	5,825	4.16	12,128	6.85
	12,055	4.46	12,126	7.35
	12,828	4.10	12,124	6.76
	11,982	3.95	12,210	6.46
	11,175	3.93	12,042	6.52
	5,992	4.05	12,314	6.57
	7,579	4.01	12,146	6.60
	16,131	3.89	12,040	6.46
	4,412	4.08	12,000	6.79
	10,376	4.05	11,640	6.95
	14,163	4.23	11,896	7.10
	13,533	4.11	11,965	6.86
	10,389	4.25	11,673	7.27
	11,501	4.26	11,902	7.15
	8,360	3.99	11,907	6.70
	8,612	3.98	12,028	6.61
	9,903	4.27	12,177	7.01
105	10,974	5.14	11,766	8.73
	11,111	5.88	11,119	10.57
	11,237	4.53	12,055	7.51
	11,009	4.36	12,169	7.16
	11,170	4.48	12,264	7.30
	10,957	4.83	12,175	7.93
	11,912	4.56	11,940	7.63
	11,499	5.15	12,063	8.53
	9,668	4.41	12,178	7.24
	10,910	4.30	12,087	7.11
	9,632	4.59	12,215	7.51
	11,073	5.20	12,244	8.49
	11,185	4.35	12,051	7.21
	11,912	5.20	11,666	8.91
	10,862	5.04	11,908	8.46
	12,417	6.61	11,100	11.90
	11,834	4.49	12,227	7.34
	11,313	4.20	12,253	6.85
	11,338	4.48	12,123	7.38
	11,549	5.33	12,081	8.82
	11,911	5.71	11,472	9.95
	11,488	4.70	11,866	7.91
	11,304	5.04	12,404	8.12
	11,737	4.42	12,350	7.15

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
106	399,192	3.99	10,803	7.38
	344,441	4.25	9,821	8.65
	371,490	3.77	10,590	7.11
	272,336	5.03	9,268	10.18
107	8,613	4.7	12,360	7.60
	8,495	4.9	12,240	8.00
	3,629	5.1	12,600	8.09
	7,786	5.3	12,280	8.02
	3,384	5.1	12,440	8.19
	4,613	4.6	12,340	7.45
	4,693	4.7	12,380	7.59
	5,382	5.5	11,900	9.23
	5,125	4.9	11,840	8.27
	5,047	6.5	11,980	10.84
	8,821	4.7	12,320	7.62
	5,618	5.4	12,340	8.74
	5,670	4.5	12,100	7.43
	9,079	4.9	12,200	8.03
	4,233	4.6	12,600	7.29
	4,407	5.3	12,060	8.78
	3,597	4.8	12,580	7.62
	4,817	4.3	12,780	6.72
	4,311	5.4	12,400	8.70
	3,486	4.7	12,620	7.44
	4,727	5.6	12,240	9.14
	3,734	5.5	12,100	9.08
	3,350	4.7	12,640	7.43
	5,272	5.0	12,460	8.02
	3,614	5.4	11,900	9.07

TABLE A-2. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
108	6,852	4.29	11,744	7.30
	6,927	4.03	11,760	6.85
	6,727	3.92	11,742	6.67
	7,010	4.14	11,664	7.09
	7,142	4.39	11,774	7.45
	7,083	4.23	11,898	7.10
	7,058	3.86	11,852	6.51
	6,872	4.03	12,009	6.67
	6,746	4.58	11,853	7.72
	7,081	4.15	12,126	6.84
	7,055	3.95	11,975	6.59
	6,809	4.32	11,948	7.22
	7,017	4.01	12,032	6.66
	6,780	4.29	11,643	7.36
	6,900	3.91	11,981	6.52
	6,793	4.23	11,641	7.26
	7,030	3.85	12,202	6.30
	6,826	4.47	11,771	7.59
	7,008	3.80	12,035	6.31
	5,456	3.96	11,917	6.64
	7,165	4.39	11,818	7.42
	6,743	4.33	12,029	7.19
	6,672	4.04	11,978	6.74
	6,608	4.15	12,262	6.76
	6,351	4.66	11,803	7.89
109	8,807	4.9	11,800	8.30
	9,135	5.7	11,780	9.67
	8,233	4.8	12,080	7.94
	4,709	5.2	11,320	9.18
	8,171	4.5	10,240	8.78
	7,976	5.4	11,700	9.22
	4,004	6.2	11,800	10.50
	7,256	5.2	11,920	8.72
	4,499	5.0	11,220	8.90
	10,350	5.3	11,820	8.96
	2,932	4.8	11,620	8.25
	10,317	6.0	11,560	10.37
	6,575	4.8	12,060	7.95
	9,193	4.7	11,740	8.00
	16,731	5.0	11,640	8.58
	4,095	5.9	11,720	10.06
	9,041	4.9	11,420	8.57
	5,253	5.9	11,100	10.62
	3,555	5.5	10,600	10.37
	4,912	4.5	11,260	7.99
	6,166	5.2	11,200	9.28
	5,365	5.3	11,960	8.85
	5,635	4.8	11,969	8.02
	10,129	5.6	10,840	10.32
	5,406	5.1	11,740	8.68

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
110	2,320	4.7	10,100	9.30
	1,465	4.4	9,240	9.51
	1,434	5.2	9,980	10.41
	755	4.1	11,140	7.35
	978	4.8	10,440	9.19
	1,155	4.6	10,360	8.87
	1,200	4.8	10,660	9.00
	1,037	4.6	10,600	8.67
	1,066	4.7	10,680	8.79
	761	5.3	10,280	10.30
	666	4.5	10,940	8.22
	887	4.7	10,680	8.79
	2,043	5.0	10,540	9.48
	858	3.9	10,240	7.61
	866	4.9	10,280	9.52
	1,126	4.9	10,600	9.24
	890	5.2	10,560	9.84
	1,014	4.6	10,240	8.98
	800	5.4	10,100	10.68
	955	4.1	10,260	7.98
	881	4.2	10,060	8.34
	1,170	5.0	10,140	9.85
	970	5.6	10,120	11.06
	726	4.8	10,100	9.50
	751	6.0	10,500	11.42
111	6,227	5.0	11,620	8.60
	10,570	4.5	11,300	7.96
	7,484	4.7	12,000	7.83
	9,403	4.2	11,580	7.25
	5,247	4.5	11,440	7.86
	9,885	4.4	11,820	7.44
	5,692	5.1	11,060	9.21
	3,068	4.8	11,420	8.40
	9,477	5.0	11,600	8.61
	3,133	4.4	11,640	7.55
	8,877	4.6	11,680	7.87
	9,600	5.4	11,340	9.51
	7,969	4.2	11,640	7.21
	7,782	4.6	11,620	7.91
	7,452	4.1	11,760	6.97
	6,101	4.9	11,740	8.34
	3,426	4.4	11,320	7.77
	5,454	4.6	11,780	7.80
	5,979	4.3	11,920	7.21
	14,585	4.8	11,800	8.13
	4,824	5.1	11,760	8.67
	5,843	4.7	11,680	8.04
	3,537	5.2	11,580	8.97
	3,674	4.2	11,740	7.15
	3,122	4.9	11,380	8.60

Data Set No.	Lot Size Tons	Total Sulfur %	Btu/lb	lbs SO_2/10^6 Btu
112	1,443	4.27	12,106	7.05
	2,842	4.35	11,172	7.78
	2,851	4.79	11,103	8.62
	1,424	4.42	11,779	7.50
	2,890	4.24	11,397	7.43
	2,829	4.34	11,953	7.25
	2,851	4.26	11,175	7.62
	2,842	4.10	10,683	7.67
	2,826	4.18	11,409	7.32
	8,489	4.06	11,197	7.25
	5,585	4.35	11,226	7.74
	2,833	4.75	10,717	8.86
	2,803	4.33	11,145	7.76
	5,750	4.10	11,482	7.13
	2,869	4.34	11,350	7.64
	7,089	4.06	11,832	6.83
	2,804	4.02	12,035	6.67
	2,873	3.91	11,595	6.74
	8,333	4.04	12,007	6.72
	1,407	4.11	11,880	6.91
	2,800	3.85	11,760	6.54
	5,744	4.63	11,033	8.39
	4,250	4.18	11,792	7.08
	5,647	4.20	11,443	7.33
	4,260	4.05	11,935	6.78
	1,426	3.87	12,109	6.39
113	299,805	4.87	10,304	9.44
	300,584	5.16	10,707	9.63
	318,967	5.05	10,956	9.21
	280,974	5.44	10,741	10.12
	303,269	4.98	10,417	9.55
	244,479	5.20	10,765	9.65
114	-	3.98	11,113	7.16
	-	4.27	10,780	7.91
	-	4.74	10,929	8.67
	-	4.72	10,991	8.58
	-	4.10	10,940	7.49
	-	4.45	10,499	8.47
	-	4.87	10,304	9.44
	-	5.16	10,707	9.63
	-	5.05	10,956	9.21
	-	5.44	10,741	10.12
	-	4.98	10,417	9.55
	-	5.20	10,765	9.65
115	212,173	4.07	11,054	7.36
	196,633	3.73	11,665	6.39
	222,090	3.98	11,179	7.11
	203,919	4.46	10,742	8.30
	262,582	3.96	10,836	7.30
	226,344	3.45	10,929	6.31

TABLE A-3. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
116	2,108	2.15	11,562	3.72
	2,280	2.21	11,192	3.95
	4,088	2.15	11,181	3.85
	5,554	1.97	11,161	3.53
	6,213	2.11	11,356	3.72
	6,357	2.05	11,171	3.67
	6,394	2.09	11,030	3.79
	6,494	2.26	10,951	4.13
	6,762	2.15	10,859	3.96
	6,952	2.00	11,329	3.53
	7,143	1.79	11,538	3.10
	7,344	1.94	11,607	3.34
	7,350	1.64	11,253	2.91
	7,354	2.36	11,247	4.20
	7,465	2.07	11,432	3.62
	7,634	2.03	11,565	3.51
	7,645	1.96	11,467	3.42
	7,649	2.07	11,523	3.59
	7.672	1.88	11,466	3.28
	7,674	1.93	11,732	3.29
	7,702	2.18	12,833	3.40
	7,704	1.82	11,328	3.21
	7,741	2.01	11,540	3.48
	7,851	2.25	11,326	3.97
	7,914	2.11	11,589	3.64
	7,974	1.82	11,769	3.09
	7,981	2.01	11,317	3.55
	8,053	2.93	10,833	5.41
	8,119	1.89	11,616	3.25
	8,169	1.76	11,708	3.01
	8,176	2.11	11,272	3.74
	8,310	2.27	11,279	4.03
	8,413	2.10	11,609	3.62
	8,488	1.93	11,324	3.41
	8,584	1.91	11,692	3.27
	8,627	1.99	11,386	3.50
	8,766	1.97	11,481	3.43
	8,896	1.98	11,329	3.50
	8,942	2.05	11,117	3.69
	9,134	0.81	11,782	1.37
	9,310	1.89	11,757	3.22
	9,824	1.55	11,643	2.66
	9,923	1.82	11,615	3.13
	10,782	1.82	11,493	3.17
117	1,500	3.09	11,445	5.40
	1,526	3.01	11,191	5.38
	1,576	3.14	10,889	5.77
	1,895	3.42	10,976	6.23
	2,009	2.83	11,200	5.05
	2,235	3.38	11,345	5.96
	2,273	3.06	11,034	5.55
	2,401	2.97	11,162	5.32
	2,478	2.92	11,574	5.05
	2,501	3.07	11,132	5.52
	2,506	3.53	10,398	6.79
	2,754	2.82	10,849	5.20
	2,770	3.09	11,137	5.55
	2,808	3.11	11,206	6.09
	2,813	3.04	11,106	5.47
	2,828	3.10	11,420	5.43
	2,936	3.16	11,149	5.67
	3,072	2.95	10,972	5.38
	3,171	2.64	11,105	4.75
	3,310	2.91	11,308	5.15
	3,384	3.02	11,312	5.34
	3,406	3.00	11,375	5.27
	3,422	3.01	11,490	5.24
	3,450	3.00	10,118	5.93
	3,518	2.87	10,898	5.27
	3,542	3.52	11,259	6.25
	3,635	2.89	11,274	5.13
	3,732	2.67	11,257	4.74
	3,800	3.36	11,000	6.11
	3,800	3.24	10,656	6.08
	3,891	3.51	11,182	6.28
	3,953	2.86	10,938	5.23
	4,007	2.72	11,199	4.86
	4,016	2.94	10,903	5.39
	4,084	3.60	10,976	6.56
	4,374	3.25	11,106	5.85
	4,432	3.01	11,011	5.47
	4,742	2.75	11,055	4.98
	4,770	3.12	11,098	5.62
	4,801	3.31	11,268	5.88
	4,998	3.39	11,081	6.12
	5,126	2.99	11,078	5.40
	5,414	3.27	10,908	6.00
	5,590	3.29	11,013	5.97

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
118	-	1.20	10,834	2.21
	-	0.92	10,377	1.77
	-	0.82	9,771	1.68
	-	0.61	9,182	1.33
	-	1.10	9,752	2.25
119	-	2.80	13,485	4.15
	-	2.24	12,879	3.48
	-	1.84	12,102	3.04
	-	1.46	12,657	2.30
	-	1.38	13,629	2.02

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
120	11,269	0.90	11,287	1.59
	10,160	1.20	11,489	2.09
	10,137	1.51	11,006	2.74
	10,380	1.74	11,126	3.12
	10,285	1.98	11,053	3.58
	10,437	1.76	10,987	3.20
	10,769	2.34	11,300	4.14
	10,284	1.29	11,592	2.22
	10,254	1.08	11,594	1.86
	10,112	1.89	11,370	3.32
	10,202	1.22	11,386	2.14
	10,177	1.41	11,407	2.47
	10,704	0.98	11,629	1.68
	10,480	1.62	11,585	2.79
	9,627	1.95	11,432	3.39
	9,065	1.18	11,657	2.02
	9,145	1.38	11,345	2.43
	10,016	0.98	11,663	1.68
	10,526	1.39	11,709	2.37
	10,292	1.78	11,120	3.20
	10,463	1.16	11,581	2.00
	10,297	1.52	11,548	2.63
	10,143	0.83	11,848	1.40
	10,740	1.92	11,019	3.48
	10,471	2.33	11,150	4.18

TABLE A-4. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	$\frac{Btu}{lb}$	$\frac{lbs\ SO_2}{10^6 Btu}$
121	18,889	3.79	12,364	6.12
	30,903	3.74	12,507	5.98
	17,362	3.94	12,629	6.23
	13,442	3.99	12,740	6.26
	23,145	3.88	12,831	6.04
	10,574	3.66	12,974	5.64
	18,169	3.71	12,978	5.71
	10,921	3.78	12,545	6.02
	6,004	3.69	12,727	5.79
	16,925	3.74	12,610	5.93
	29,162	3.90	12,630	6.17
	24,219	3.40	12,400	5.48
	14,619	3.69	12,477	5.91
	5,777	3.34	12,330	5.41
	14,691	3.69	12,477	5.91
	20,468	3.59	12,435	5.77
	6,007	3.80	12,247	6.20
	22,728	3.72	12,273	6.06
	31,152	3.76	12,616	5.96
	17,662	4.01	12,597	6.36
	8,100	4.15	12,692	6.53
	7,998	3.85	13,092	5.88
	22,065	3.72	12,510	5.94
	18,100	4.07	12,314	6.60

TABLE A-5. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
201	203,873	3.21	13,052	4.91
	179,374	3.23	13,063	4.94
	209,280	3.24	13,030	4.97
	201,994	3.14	12,927	4.85
	293,460	3.13	12,977	4.82
	198,878	3.18	12,956	4.90
202	6,810	3.20	13,026	4.91
	6,548	3.22	13,062	4.93
	5,582	3.39	13,071	5.18
	6,683	2.89	13,145	4.39
	6,404	3.30	13,051	5.05
	6,613	3.16	13,118	4.81
	6,120	3.15	13,052	4.82
	6,841	3.19	13,095	4.87
	6,761	3.34	13,015	5.13
	6,556	3.11	13,084	4.75
	6,517	2.95	13,044	4.52
	6,129	3.02	12,883	4.68
	6,784	3.30	12,927	5.10
	6,418	3.12	12,959	4.81
	5,993	3.38	12,978	5.20
	6,459	3.30	12,899	5.11
	6,687	3.27	12,947	5.05
	6,575	2.89	13,039	4.43
	6,607	3.10	12,944	4.79
	6,762	3.28	12,936	5.07
	6,714	3.04	12,956	4.69
	6,271	3.24	12,887	5.02
	6,259	3.38	12,924	5.23
	6,407	2.97	13,047	4.55
203	124,662	3.40	13,021	5.22
	116,037	3.40	12,887	5.27
	101,978	3.36	12,995	5.17
	91,848	3.30	12,993	5.07
	90,102	3.35	12,954	5.17
	75,501	3.38	12,988	5.20

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs S/10^6 B
204	18,498	3.32	13,075	5.07
	19,656	3.38	13,159	5.13
	22,867	3.32	12,996	5.10
	17,399	3.54	12,944	5.46
	26,023	3.30	12,930	5.10
	24,458	3.45	12,898	5.34
	21,620	3.39	12,924	5.24
	4,177	3.26	13,133	4.96
	12,558	3.35	13,065	5.12
	18,030	3.51	12,894	5.44
	23,770	3.29	12,891	5.10
	8,209	3.25	13,001	4.99
	24,577	3.24	13,026	4.97
	15,878	3.42	12,901	5.30
	9,006	3.18	13,107	4.85
	30,906	3.29	12,989	5.06
	26,560	3.21	12,969	4.95
	13,407	3.32	12,935	5.13
	17,816	3.67	12,919	5.68
	21,954	3.41	12,930	5.27
	10,340	3.55	13,057	5.43
	26,005	3.24	12,980	4.99
	13,840	3.34	12,954	5.15
	18,059	3.49	12,924	5.40
	10,960	3.39	13,091	5.17
205	124,662	3.31	12,552	5.27
	116,037	3.39	12,495	5.42
	101,978	3.29	12,633	5.20
	91,848	3.20	12,689	5.04
	90,102	3.15	12,692	4.96
	75,501	2.97	12,832	4.62

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
206	2,912	3.99	11,937	6.68
	5,721	3.79	12,044	6.29
	5,694	4.05	12,074	6.70
	5,696	3.91	12,107	6.45
	5,554	3.57	12,239	5.83
	1,378	4.27	11,552	7.39
	4,285	3.67	12,218	6.00
	2,800	3.42	12,037	5.68
	2,868	3.84	12,022	6.38
	2,838	3.80	12,397	6.12
	1,434	3.77	11,963	6.30
	1,374	3.87	11,527	6.71
	2,851	3.99	11,937	6.68
	2,925	4.32	12,403	6.96
	8,734	3.70	12,309	6.01
	2,787	3.69	12,103	6.09
	2,860	3.72	12,078	6.15
	2,834	3.96	12,174	6.50
	1,460	3.78	11,979	6.31
	1,428	3.71	12,042	6.16
	4,263	3.89	11,949	6.51
	9,959	3.89	12,226	6.34
	5,703	3.76	12,051	6.23
	4,720	4.05	12,017	6.73
207	237,056	4.40	12,297	7.15
	244,514	4.34	12,278	7.06
	251,592	4.44	12,370	7.17
	226,517	4.46	12,320	7.23
	242,190	4.42	12,267	7.20
	197,683	4.29	12,292	6.97

TABLE A-6. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
208	2,806	4.29	12,215	7.02
	2,626	4.20	12,171	6.90
	1,023	4.15	12,115	6.84
	4,201	4.13	12,300	6.71
	6,870	3.96	12,162	6.51
	8,361	4.04	12,085	6.68
	5,463	4.03	12,099	6.66
	2,790	4.31	11,934	7.22
	7,983	4.08	12,134	6.72
	5,652	4.04	11,805	6.84
	6,837	4.12	12,284	6.70
	4,060	4.10	12,001	6.83
	2,621	4.00	12,105	6.60
	5,598	4.08	12,192	6.69
	7,015	3.87	12,109	6.39
	1,388	3.99	12,232	6.52
	7,961	4.15	12,279	6.75
	2,847	4.09	12,271	6.66
	5,231	4.24	12,421	6.82
	1,353	4.20	11,991	7.00
	9,864	3.83	12,016	6.37
	2,844	3.98	11,896	6.69
	2,798	4.25	11,671	7.28
	5,675	4.28	12,074	7.08
	1,426	3.55	11,831	6.00
	1,395	3.44	12,622	5.45

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
209	7,150	2.45	12,938	3.78
	6,230	2.79	12,844	4.34
	6,023	2.56	12,902	3.96
	5,444	2.76	12,873	4.28
	5,972	2.69	12,880	4.17
	6,255	2.46	12,712	3.87
	5,906	2.68	12,715	4.21
	6,556	2.84	13,119	4.33
	5,720	2.54	12,471	4.07
	5,817	2.96	12,652	4.67
	5,964	2.76	12,802	4.31
	6,597	2.84	12,895	4.40
	6,260	2.73	12,851	4.24
	6,099	3.04	12,754	4.76
	5,974	3.27	12,742	5.13
	8,067	2.95	12,439	4.74
	7,498	2.81	12,674	4.43
	6,570	2.96	12,752	4.64
	6,196	2.97	12,631	4.70
	6,224	2.67	12,707	4.20
	5,972	2.75	12,376	4.44
	6,258	2.68	12,596	4.25
	6,143	2.69	12,808	4.20
	5,881	2.45	12,981	3.77
	6,252	2.84	12,849	4.42
210	4,174	3.12	12,997	4.80
	4,269	3.35	13,011	5.14
	4,356	3.20	13,131	4.87
	4,219	3.20	13,094	4.88
	4,464	3.45	12,835	5.37
	4,402	3.13	12,980	4.82
	4,444	3.36	13,020	5.16
	4,387	3.29	13,035	5.04
	4,383	3.51	13,011	5.39
	4,367	3.51	13,063	5.37
	4,593	3.65	12,968	5.62
	4,129	3.05	13,157	4.63
	4,018	3.44	13,034	5.27
	2,755	3.35	13,070	5.12
	4,147	3.34	12,970	5.15
	4,139	3.43	13,061	5.25
	4,291	3.28	13,187	4.97
	4,395	3.35	13,059	5.16
	4,290	3.29	12,982	5.06
	4,345	3.34	12,823	5.20
	4,109	3.42	12,934	5.28
	4,285	3.32	12,934	5.13
	4,351	3.35	12,800	5.23
	4,280	3.43	12,895	5.31
	4,375	3.41	17,779	5.33

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
211	150,234	3.64	12,073	6.02
	315,719	3.93	12,073	6.50
	295,914	3.83	12,011	6.37
	320,532	3.94	12,047	6.53
	294,042	3.83	12,059	6.35
	246,346	3.71	12,032	6.46
	237,056	4.40	12,297	7.15
	244,514	4.34	12,278	7.06
	251,592	4.44	12,370	7.18
	226,517	4.46	12,320	7.23
	242,190	4.42	12,267	7.20
	197,683	4.29	12,292	6.97
212	170,413	3.03	12,494	4.85
	152,329	2.86	12,464	4.58
	178,680	3.06	12,462	4.91
	159,949	3.05	12,443	4.90
	208,650	3.06	12,465	4.91
	179,994	2.99	12,428	4.81
213	9,829	2.87	12,459	4.60
	10,558	3.27	12,439	5.25
	10,402	3.02	12,425	4.86
	10,072	2.89	12,536	4.61
	9,731	2.84	12,478	4.55
	10,521	3.01	12,503	4.81
	10,292	2.96	12,390	4.77
	10,565	2.66	12,539	4.24
	10,467	3.01	12,467	4.82
	9,990	2.99	12,453	4.80
	10,002	2.79	12,588	4.43
	10,610	3.12	12,392	5.03
	10,508	3.05	12,412	4.91
	10,336	3.15	12,354	5.09
	10,000	2.88	12,430	4.63
	10,532	3.20	12,365	5.17
	10,534	3.09	12,281	5.03
	9,842	3.10	12,430	4.98
	10,069	3.02	12,365	4.88
	10,447	3.02	12,370	4.88
	9,799	2.85	12,443	4.57
	10,205	2.96	12,495	4.73
	10,078	2.76	12,474	4.42
	10,686	2.83	12,438	4.55

TABLE A-7. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu	Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu	Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
214	13,115	2.87	12,932	4.43	216	900	4.28	12,652	6.76	218	29,086	2.90	12,250	4.73
	14,897	2.20	12,716	3.46		5,700	3.38	12,549	5.38		31,875	2.43	12,200	3.98
	15,428	2.40	12,757	3.76		9,667	3.80	12,509	6.07		45,584	2.54	12,060	4.21
	24,217	2.56	12,660	4.04		2,866	4.08	12,380	6.59		54,055	2.62	12,171	4.30
	9,580	2.22	12,723	3.49		3,825	3.49	12,611	5.53		65,572	2.55	12,086	4.22
	18,826	2.77	12,801	4.32		1,875	3.76	12,738	5.90		51,656	2.58	12,078	4.27
	28,960	2.77	12,702	4.36		5,925	3.69	12,688	5.81		65,654	2.65	12,120	4.37
	17,473	2.82	12,638	4.46		5,400	3.75	12,599	5.95		54,053	2.56	12,146	4.21
	11,551	2.80	12,683	4.41		5,775	3.68	12,766	5.76		63,352	2.58	12,169	4.24
	32,901	2.82	12,714	4.43		6,450	3.90	12,782	6.10		54,144	2.60	12,138	4.38
	15,247	2.25	12,781	3.52		7,650	3.84	12,735	6.02		31,839	2.56	12,192	4.20
	15,815	2.24	12,568	3.56		5,625	4.00	12,803	6.24		8,861	2.72	12,409	4.38
	19,800	2.47	12,638	3.91		5,925	3.72	12,415	5.99		55,367	2.54	12,376	4.10
	24,806	2.83	12,733	4.44		3,450	3.95	12,395	6.37		48,727	2.70	12,301	4.39
	10,042	2.78	12,598	4.41		6,450	3.85	12,652	6.08		61,182	2.53	12,397	4.08
	7,169	2.78	12,554	4.42		4,575	3.67	12,707	5.77		44,064	2.69	12,308	4.37
	13,873	2.78	12,751	4.36		4,575	3.86	12,693	6.08		49,681	2.57	12,453	4.12
	8,512	2.34	12,636	3.70		6,450	4.01	12,665	6.33		55,536	2.63	12,403	4.24
	4,312	2.24	12,569	3.56		5,850	3.62	12,543	5.77		47,273	2.69	12,359	4.35
	17,981	2.28	12,625	3.61		6,600	3.51	12,662	5.54		73,512	2.55	12,324	4.13
	28,434	2.58	12,565	4.10		5,100	3.49	12,513	5.57		78,538	2.39	12,374	3.86
	25,009	2.77	12,541	4.41		5,100	3.63	12,636	5.74		53,775	2.44	12,362	3.94
	15,132	2.51	12,653	3.96		4,800	3.71	12,569	5.90		41,351	2.41	12,267	3.93
	15,248	2.74	12,722	4.30		8,475	3.53	12,778	5.52		43,733	2.42	12,228	3.95
	10,906	2.79	12,580	4.43		5,925	3.57	12,538	5.09		79,239	2.85	12,146	4.69
215	2,570	3.13	12,917	4.84	217	2,221	3.34	12,650	5.28					
	589	3.84	12,731	6.03		5,759	2.34	12,461	3.75					
	2,373	3.54	12,906	5.48		5,712	2.99	12,373	4.83					
	4,079	3.23	12,862	5.02		2,025	2.91	12,336	4.71					
	1,587	3.44	12,724	5.40		1,664	3.18	12,142	5.23					
	2,277	3.60	12,776	5.63		4,003	2.97	12,473	4.76					
	868	3.57	12,792	5.58		3,873	2.36	12,603	3.74					
	2,405	3.65	12,853	5.67		4,342	2.55	12,667	4.02					
	2,729	3.42	12,536	5.45		5,800	2.75	12,484	4.40					
	1,522	3.53	12,576	5.61		4,466	2.86	12,606	4.53					
	2,576	3.40	12,573	5.40		5,455	2.55	12,729	4.00					
	481	3.49	12,825	5.44		2,820	2.66	12,700	4.19					
	2,576	3.42	12,897	5.30		2,207	3.16	12,923	4.88					
	4,962	3.36	12,839	5.23		1,294	2.97	12,741	4.66					
	1,547	3.33	12,925	5.15		5,213	3.27	12,567	5.20					
	1,303	3.50	12,413	5.63		5,835	2.67	12,706	4.20					
	2,055	3.18	12,762	4.98		7,504	3.30	12,627	5.22					
	1,566	3.65	12,694	5.75		4,859	2.97	12,183	4.87					
	1,984	3.65	12,738	5.73		7,732	3.14	12,584	4.99					
	615	3.41	12,620	5.40		4,344	3.00	12,593	4.76					
	4,996	3.39	12,474	5.43		4,483	3.26	12,314	5.29					
	2,350	3.39	12,635	5.36		6,033	2.81	12,441	4.51					
	1,374	3.55	12,719	5.58		5,674	2.95	12,058	4.89					
	1,472	3.54	12,906	5.48		6,614	2.90	12,440	4.60					
	2,733	3.52	12,757	5.51		6,420	3.01	12,353	4.87					

TABLE A-8. DATA SETS

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
219	45	1.57	12,818	2.45
	1,737	1.44	12,641	2.28
	1,916	1.75	13,040	2.68
	3,102	1.92	11,518	3.33
	3,161	1.59	11,745	2.71
	3,373	1.88	12,419	3.03
	4,471	1.48	12,624	2.34
	4,473	1.36	12,196	2.23
	4,673	1.81	12,860	2.81
	5,240	1.44	12,726	2.26
	5,505	1.57	12,918	2.43
	5,616	1.58	12,065	2.62
	5,831	1.63	12,528	2.60
	6,292	1.60	12,233	2.62
	6,807	1.64	12,178	2.69
	6,982	1.63	12,439	2.62
	7,016	1.45	12,909	2.25
	7,165	1.73	12,446	2.78
	7,467	1.5.	12,589	2.40
	7,517	1.65	12,211	2.70
	7,658	1.44	12,972	2.22
	7,749	1.49	12,441	2.40
	8,109	1.58	12,686	2.49
	8,173	1.85	12,316	3.00
	8,473	1.77	12,661	2.80
	8,586	1.45	12,555	2.31
	8,602	1.93	12,340	3.13
	8,611	1.72	12,109	2.84
	8,733	1.38	12,235	2.26
	8,862	1.45	11,783	2.46
	9,030	1.50	12,003	2.50
	9,045	1.74	10,977	3.17
	9,131	1.64	12,262	2.67
	9,177	1.54	12,643	2.44
	9,914	1.70	12,686	2.68
	9,961	1.57	12,675	2.48
	9,984	1.47	12,577	2.34
	9,989	1.90	12,315	3.09
	10,040	1.70	12,706	2.68
	10,198	1.80	12,152	2.96
	10,206	2.01	12,400	3.24
	10,444	2.05	12,010	3.41
	11,180	2.01	12,204	3.29
	11,283	1.81	11,827	3.06
	12,100	1.80	11,912	3.02
	13,814	1.62	11,432	2.33

Data Set No.	Lot Size, Tons	Total Sulfur, %	Btu/lb	lbs SO_2/10^6 Btu
220	5,760	3.79	12,455	6.02
	3,120	3.31	12,170	5.43
	3,920	4.16	12,276	6.77
	3,920	4.95	12,126	8.16
	6,400	3.94	11,972	6.58
	2,320	4.66	12,715	7.32
221	1,050	4.35	12,486	6.96
	4,720	4.80	12,575	7.63
	4,560	4.57	12,932	7.06
	6,880	4.45	12,497	7.11
	4,240	4.97	12,983	7.65
	4,320	4.14	13,093	6.32
	4,640	3.97	12,903	6.15
	5,360	4.48	13,039	6.87
	7,200	4.72	12,747	7.40
	4,560	4.37	12,626	6.92
	3,791	4.46	12,739	7.00
	4,260	5.14	12,824	8.01
	9,108	4.14	12,931	6.40
	2,874	3.78	12,913	5.85
	5,340	4.73	12,577	7.51
	8,240	3.85	12,546	6.13
	6,800	4.01	12,906	6.21
	8,889	5.31	12,710	8.35
222	2,700	1.48	14,591	2.03
	2,700	1.31	14,449	1.81
	2,700	0.89	14,260	1.25
	2,700	1.06	14,428	1.47
	2,700	1.10	14,624	1.50
223	640	1.11	14,622	1.52
	640	1.20	14,249	1.68
	640	1.22	14,146	1.72
	640	0.82	14,392	1.14
	640	0.99	14,435	1.37

APPENDIX B

DETAILED CHI-SQUARE ANALYSIS

$$\chi^2 = \sum_{i=1}^{r} \frac{(N_i - E_i)^2}{E_i}$$

N_i = Observed frequency in group i

e_i = Theoretical (normal distribution) frequency in group i

r = Number of Groups

f = Degrees of freedom = r-3, when both $\bar{Y}$ and S_Y are estimated from the data.

Table B.1. DATA SET C-8, 275 DATA POINTS.

$\frac{Y-\bar{Y}}{Sy}$	Group i	Theoretical Frequency, e_i	Observed Frequency, N_i								
			Y_1	log Y_1	$\sqrt{Y_1}$	Y_2	log Y_2	$\sqrt{Y_2}$	Y_3	log Y_3	$\sqrt{Y_3}$
− ∞	1	11.0275	12	15	12	15	15	15	5	6	6
-1.75	2	7.3425	7	5	8	15	15	15	3	5	4
-1.50	3	10.6700	10	9	9	19	18	18	11	11	10
-1.25	4	14.6025	11	11	11	18	19	19	19	17	19
-1.00	5	18.6725	11	11	11	15	14	15	26	25	25
-0.75	6	22.5225	23	23	23	10	11	10	30	28	28
-0.50	7	25.5200	33	21	21	1	1	1	34	33	34
-0.25	8	27.1425	27	39	39	3	2	2	27	23	23
0.00	9	27.1425	24	24	24	5	6	6	18	22	22
0.25	10	25.5200	37	37	37	38	35	38	24	26	24
-.50	11	22.5200	27	27	27	54	61	56	20	18	20
0.75	12	18.6725	14	14	14	67	70	70	13	14	12
1.00	13	14.6025	11	11	11	15	8	10	12	14	14
1.25	14	10.6700	15	20	20	0	0	0	10	10	10
1.50	15	7.3425	10	5	5	0	0	0	9	9	9
1.75	16	11.0275	3	3	3	0	0	0	14	14	14
+ ∞											

Table B.2. DATA SET U-11 , 250 DATA POINTS.

$\frac{Y-\bar{Y}}{Sy}$	Group i	Theoretical Frequency, e_i	Observed Frequency, Ni Y_1	$\log Y_1$	$\sqrt{Y_1}$	Y_2	$\log Y_2$	$\sqrt{Y_2}$	Y_3	$\log Y_3$	$\sqrt{Y_3}$
$-\infty$	1	10.025	20	20	20	2	3	3	21	21	21
-1.75	2	6.675	4	5	4	4	3	3	2	2	3
-1.50	3	9.700	9	7	8	1	1	1	7	6	6
-1.25	4	13.275	10	7	8	17	17	17	9	8	7
-1.00	5	16.975	11	9	13	41	41	41	13	12	14
-0.75	6	20.475	17	17	16	25	25	25	14	12	12
-0.50	7	23.200	24	20	21	32	31	31	22	18	24
-0.25	8	24.675	18	24	21	25	26	26	22	20	20
0.00	9	24.675	17	19	19	24	24	24	25	33	27
0.25	10	23.200	29	30	29	15	14	15	28	28	27
0.50	11	20.475	23	26	23	16	16	16	18	21	20
0.75	12	16.975	25	32	28	14	15	14	29	39	30
1.00	13	13.275	27	22	27	7	6	7	27	20	28
1.25	14	9.700	7	8	8	6	7	6	4	7	4
1.50	15	6.675	6	4	4	2	2	2	6	3	7
1.75	16	10.025	3	0	1	19	19	19	3	0	0
$+\infty$											

Table B.3. DATA SET C-3, 115 DATA POINTS.

$\frac{Y-\bar{Y}}{Sy}$	Group i	Theoretical Frequency, e_i	Observed Frequency, N_i								
			Y_1	$\log Y_1$	$\sqrt{Y_1}$	Y_2	$\log Y_2$	$\sqrt{Y_2}$	Y_3	$\log Y_3$	$\sqrt{Y_3}$
$-\infty$											
	1	12.1440	8	11	8	8	7	8	9	9	9
-1.27											
	2	6.1065	6	3	6	6	6	5	6	5	6
-1.00											
	3	7.8085	10	10	10	4	4	5	11	12	11
-0.75											
	4	9.4185	12	12	12	12	10	11	11	9	10
-0.50											
	5	10.6720	10	10	10	16	17	16	9	10	9
-0.25											
	6	11.3505	14	14	14	7	9	8	10	10	11
0.00											
	7	11.3505	12	7	12	11	11	11	14	13	14
0.25											
	8	10.6720	9	12	9	15	16	15	11	13	11
0.50											
	9	9.4185	11	7	5	12	13	14	11	11	11
0.75											
	10	7.8085	4	15	10	11	11	9	11	11	11
1.00											
	11	6.1065	8	7	8	5	4	6	4	4	4
1.25											
	12	12.1440	11	7	11	8	7	7	8	8	8
$+\infty$											

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO. EPA-600/7-80-107	2.	3. RECIPIENT'S ACCESSION NO.
4. TITLE AND SUBTITLE Effect of Physical Coal Cleaning on Sulfur Content and Variability		5. REPORT DATE May 1980
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) D.H. Sargent, B.A. Woodcock, J.R. Vaill, and J.B. Strauss		8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Versar, Inc. 6621 Electronic Drive Springfield, Virginia 22151		10. PROGRAM ELEMENT NO. EHE623A
		11. CONTRACT/GRANT NO. 68-02-2136, Task 300
12. SPONSORING AGENCY NAME AND ADDRESS EPA, Office of Research and Development Industrial Environmental Research Laboratory Research Triangle Park, NC 27711		13. TYPE OF REPORT AND PERIOD COVERED Task Final; 6/78-4/80
		14. SPONSORING AGENCY CODE EPA/600/13

15. SUPPLEMENTARY NOTES IERL-RTP project officer is James D. Kilgroe, Mail Drop 61, 919/541-2851.

16. ABSTRACT The report gives results of a statistical analysis of the sulfur content and heating value data for 53 different coal-source/cleaning-plant combinations, both to document the operational effectiveness of commercial coal cleaning plants in reducing sulfur and enhancing heating value, and to define the effect of physical coal cleaning on sulfur variability. Cleaning plants, for which matched pairs of feed and product coal data were available, showed 24-50% reductions (from feed to product) in the mean lb SO2/million Btu. These empirical data are consistent with the calculated performance of hypothetical coal cleaning plants. The wide ranges reflect the sensitivity of performance to both coal washability and plant design. These matched pairs of data showed a 60% reduction in sulfur variability. An indirect analysis of a larger data base, where matched pairs were not available, showed similar sulfur variability reductions, attributable to physical coal cleaning. Much of the coal data showed serial autocorrelation, verifying expectations based on geology and engineering rationale. Data analysis resulted in estimates of the long-term (geostatistical) and short-term components of variability, and in the component of variability attributable to coal sampling and analysis. Removing the long-term component empirically showed an inverse relationship between relative standard deviation and lot size.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
Pollution	Pollution Control	13B
Coal Preparation	Stationary Sources	08I
Sulfur	Physical Coal Cleaning	07B
Desulfurization	Sulfur Content	07A,07D
Variability		14G
Calorific Value		20M

18. DISTRIBUTION STATEMENT Release to Public	19. SECURITY CLASS (This Report) Unclassified	21. NO. OF PAGES 93
	20. SECURITY CLASS (This page) Unclassified	22. PRICE

EPA Form 2220-1 (9-73)