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# Hypolimnetic Flow Regimes in Lakes and Impoundments

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# HYPOLIMNETIC FLOW REGIMES

# IN LAKES AND IMPOUNDMENTS

By

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## ABSTRACT

The "hypolimnetic flow" is a two-layered flow with the upper layer stagnant. This report presents the possibility of different flow regimes for the hypolimnetic flow which may be determined from the parameters of slope of channel bottom, flow depth, flowrate, density difference of water in the two layers, and channel roughness. The analysis is limited to the steady-state case of the hypolimnetic flow and the "upper layer analysis" in which the lower layer is stagnant. Interfacial profile equations which predict possible existence of ten different flow regimes for the hypolimnetic flow and two regimes for the upper layer analysis were obtained form the equations of continuity and momentum for two-layered flow.

Experimental apparatus, consisting of a large scale open-channel tilting flume and water supply and control systems capable of circulating water of two different temperatures was designed and constructed. The experiment employed observations of a dyed flow layer and measurements of vertical temperature distributions in the flume. It was found that eight different flow regimes could be generated in the flume for the hypolimnetic flow on the positive and horizontal slopes.

To use the governing differential equation of the interfacial shapes for the prediction of the flow regimes, it is recommended to investigate the non-established portion of the flow, the definition of the critical depth, and the interfacial friction factor.



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# SECTION I

## CONCLUSIONS

The interfacial profile equations were developed for steady hypolimnetic flow, including the upper layer analysis, of a unit width. These can be understood with a slight extension of the knowledge of open-channel hydraulics and predict the existence of twelve different flow regimes.

A large scale experimental flume which is capable of handling the requirements to produce the hypolimnetic flow by temperature difference was constructed.

The existence of eight different flow regimes on the positive and horizontal slopes was verified in the experimental flume.

## .SECTION II

## RECOMMENDATIONS

This investigation was limited to the laboratory study of the established hypolimnetic flow. It was not within the scope of the investigation to develop the method of the accurate prediction of the interfacial shapes for the different flow regimes. It is recommended that the following items should be investigated to fulfill the above purpose.

1. The location and size of the frontal zone which exists at the point where the flowing layer goes under the stagnant upper layer.
2. The location and depth of the vena contracta occurring after the gate.
3. The method to obtain the value of the interfacial friction factor.
4. The definition of the stratified critical depth.
5. The effect of the internal circulation in the layer of no net flow-rate, although it did have little effect in this study.
6. The magnitude of the momentum coefficient in the flowing layer.

## SECTION III

## INTRODUCTION

This report presents the verification of the existence of the different flow regimes of the hypolimnetic flow by the experiment using a large scale experimental apparatus.

The hydraulic analysis of the hypolimnetic flow was reported earlier. However, the analysis is presented again in this report, since further analysis has been done, and to provide the references for the equations used in the data analysis.

Section 4 deals with the derivation of the equations of continuity and motion for one-dimensional, unsteady, non-uniform two-layered flow, which are the base for the interfacial profile equations derived in section 5.

In section 5, interfacial profile equations for the hypolimnetic flow of a unit width are derived for the steady state case. These interfacial profile equations suggest many possible flow regimes. The term "upper layer analysis" is used for the case with stagnant lower layer.

The uniformity of velocity distribution is assumed in both layers in the derivation. Then, the change in the interfacial profile equations is studied with the assumption of non-uniform velocity distribution in the flowing layer. For convenience of computation and to obtain a more universal expression of results, non-dimensionalized forms of the interfacial and free surface equations are included. In addition, the effect of the following factors on the form of the interfacial profile equations is examined: a circulation of water in the layer with no net flowrate and the wall of the experimental flume.

The free surface profile equation for open-channel flow on a horizontal slope is derived and is used as a basis for estimating the roughness of the flume section.

Section 6 is concerned with the design and construction of the experimental system. The conditions required for the system are listed.

Section 7 deals with the experiment and the data. After the reason for the change of the original plan of the experiment is explained, the experimental procedure is described in detail. Inconveniences and deficiencies of the experimental system found during the experiment and crude discussions on the accuracy of parameters are included.

The verification of the existence of the predicted flow regimes is done by visual observations, photographs, and temperature distribution curves.

Some discrepancies between the stratified critical depth determined by its definition using the data and the stratified critical depth expected from the interfacial profile were found. Further, reliable values for bottom friction factor and a good method to estimate interfacial friction factor could not be obtained. Therefore, a method to estimate the bottom friction factor was developed. An attempt was made to correct the stratified critical depth and estimate the interfacial friction factor by using the interfacial profile equation, the interface and data obtained, and the estimated bottom friction factor. The results of the experiments and this attempt are shown in Sec. 7.3. Observations obtained throughout the experiment are described in Sec. 7.1. Observations for each profile designation are in Sec. 7.3.

# SECTION IV

## HYDRODYNAMIC THEORY OF HYPOLIMNETIC FLOW

Schijf and Schönfeld[1] presented the equations of continuity and motion for one-dimensional unsteady non-uniform two-layered flow of a unit width which are the base for this investigation. The equations may be derived from the most general expressions of conservation of mass and linear momentum and described in the following sections.

The concept of continuum field theories is premised for the derivation.

### 4.1 EQUATIONS OF CONTINUITY

If there are no sources or sinks in the material volume of fluid under consideration, and mass transfer due to diffusion is ignored, then conservation of mass can be written as

$$\frac{D}{Dt}\int_{V_m(t)} \rho \, dV = 0 \tag{1}$$

where

| | | |
|---|---|---|
| $D/Dt$ | = | material derivative |
| $V_m(t)$ | = | material volume |
| $\rho = \rho(\underline{x},t)$ | = | density of the homogeneous fluid at $\underline{x}$, spatial coordinates and t, time, the underbar indicates vector |
| $V$ | = | volume |

The Reynolds transport theorem[2] can be stated as

$$\frac{D}{Dt}\int_{V_m(t)}\Phi dV=\int_{V_m(t)}\frac{\partial\Phi}{\partial t}dV+\int_{A_m(t)}\Phi\underline{v}\cdot\underline{n}dA \tag{2}$$

where $\Phi$ = arbitrary scalor or vector

$\partial/\partial t$ = partial derivative

$A_m(t)$ = surface bounding $V_m(t)$

$\underline{v} = \underline{v}(\underline{x},t)$ = velocity of the fluid at $\underline{x}$ and t

$\underline{n}$ = unit normal vector

A = area

If $\rho$ is substituted for $\Phi$ in Eq. 2 and Eq. 1 is used,

$$\int_{V_m(t)}\frac{\partial\rho}{\partial t}dV+\int_{A_m(t)}\rho\underline{v}\cdot\underline{n}dA=0 \tag{3}$$

If use is made of the divergence theorem,

$$\int_V\nabla\cdot\Psi dV=\int_A\Psi\cdot\underline{n}dA \tag{4}$$

where $\Psi$ = arbitrary vector or tensor

$\nabla$ = gradient vector operator,

Eq. 3 can be written, by letting $\Psi = \rho\underline{v}$, as

$$\int_{V_m(t)}\left[\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\underline{v})\right]dV=0 \tag{5}$$

Since $V_m(t)$ is arbitrary,

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\underline{v})=0 \tag{6}$$

Eq. 6 can also be written as

$$\frac{D\rho}{Dt}+\rho\nabla\cdot\underline{v}=0 \tag{7}$$

Assume an incompressible flow, which is usually done for water; then, $D\rho/Dt=0$. This states that the density of a fluid particle does not change along its particle path. Since the density of water is mainly determined by its temperature and substances dissolved (or perhaps suspended) in it, this assumption is equivalent to the neglect of the molecular transport of energy and of the dissolved substances across the boundaries of fluid particles. Then

$$\nabla\cdot\underline{v}=0 \tag{8}$$

In order to take into consideration the turbulence usually existing in open-channel flow, instantaneous values of flow properties are written in terms of mean values and fluctuating components. Substituting $\underline{v}=\bar{\underline{v}}+\underline{v}'$ (in which "overbar" indicates mean value and "prime" denotes fluctuating components) into Eq. 8 and taking the average yields

$$\nabla\cdot\bar{\underline{v}}=0 \tag{9}$$

as the mass conservation equation for an incompressible flow. [For reference, the condition of incompressibility expressed in the average can be obtained by taking the average of Eq. 6 and substituting Eq. 9:

$$\frac{\partial \bar{\rho}}{\partial t}+\bar{\underline{v}}\cdot\nabla\bar{\rho}+\nabla\cdot\overline{\rho'\underline{v}'}=0 \tag{10}$$

The integration of Eq. 9 over $V_a(t)$, which is a control volume moving through the space in an arbitrary manner, is carried out in order that the equation can be used for a specific problem:

$$\int_{V_a(t)} \nabla \cdot \underline{v} \, dV = 0 \tag{11}$$

From Eq. 11 on, the overbar indicating mean value on $\underline{v}$ will be understood.

If the divergence theorem with $\underline{v}$ in place of $\Psi$ in Eq. 4 and the general transport theorem,[2]

$$\frac{d}{dt}\int_{V_a(t)} \Phi \, dV = \int_{V_a(t)} \left(\frac{\partial \Phi}{\partial t}\right) dV + \int_{A_a(t)} \Phi (\underline{w} \cdot \underline{n}) \, dA \tag{12}$$

where $d/dt$ = total derivative

$A_a(t)$ = surface bounding $V_a(t)$

$\underline{w}$ = mean velocity of points on the surface of $V_a(t)$

with substituting 1 for $\Phi$, are used, Eq. 11 may be written as

$$\frac{d}{dt}\int_{V_a(t)} dV + \int_{A_a(t)} (\underline{v} - \underline{w}) \cdot \underline{n} \, dA = 0 \tag{13}$$

Assume, in a channel, the existence of the two distinct flow layers of different densities separated by an interface across which little mixing occurs. It is also assumed that the free surface and the interface are horizontal in the direction of the width of the channel. A control volume can be described as in Fig. 1.

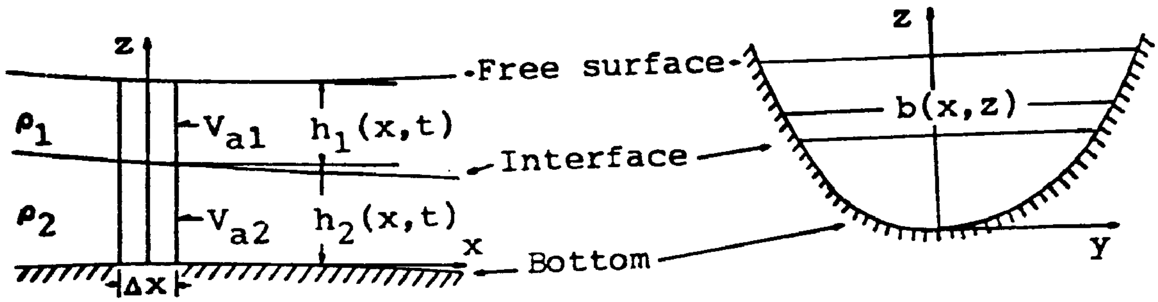



Fig. 1. Definition sketch for two-layered flow.

In Fig. 1, x = longitudinal distance (in the direction of the flow

y = transversal distance (in the direction of the width of the channel)

z = vertical distance (in the direction normal to the x direction)

h = h(x,t) = depth of a layer

b = b(x,z) = width of a channel

subscripts 1 and 2 indicate upper and lower layer, respectively.

$A_a(t)$ can be divided into the following three areas:

(1) $A_e(t)$ = area of entrances and exits through which the flow enters or leaves the control volume (cross section);

(2) $A_s(t)$ = area of moving boundary (free surface and interface);

(3) $A_s$ = area of fixed boundary (bottom and other wetted perimeter).

If there is no appreciable precipitation on or evaporation from the free surface, and no mixing occurs at the interface, the velocity $\underline{w}=\underline{v}$ can be assumed on $A_s(t)$. If there is no inflow or outflow, including groundwater flow at the wetted perimeter, $\underline{w}=\underline{v}=0$ on $A_s$ may be used. Let $A_e(t)$ be fixed ($\underline{w}=0$), and Eq. 13 reduces to

$$\frac{d}{dt}\int_{V_a(t)} dV+\int_{A_e(t)} \underline{v}\cdot\underline{n}dA=0 \tag{14}$$

For the upper layer, it is

$$\frac{d}{dt}\int_{V_{a1}(t)} dV+\int_{A_{e1}(t)} \underline{v}\cdot\underline{n}dA=0 \tag{15}$$

Leibnitz's rule is used to calculate the total derivative of integrals. The first of Eq. 15:

$$\frac{d}{dt}\int_{V_{a1}(t)} dV=(\text{in the limit})dx\frac{d}{dt}\int_{h_2}^{h_1+h_2} bdz$$

$$=dx\left[b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial t}-b(h_2)\frac{\partial h_2}{\partial t}\right]$$

The second term of Eq. 15:

$$\int_{A_{e1}(t)} \underline{v}\cdot\underline{n}dA=(\text{in the limit})dx\frac{d}{dx}\int_{h_2}^{h_1+h_2} \langle v_x\rangle bdz$$

$$=dx\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\left[\langle v_x\rangle b\right]dz$$

$$+dx\langle v_x(h_1+h_2)\rangle b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}$$

$$-dx\langle v_x(h_2)\rangle b(h_2)\frac{\partial h_2}{\partial x}$$

where $\langle v_x \rangle = \langle v_x(x,z,t) \rangle$ = average of the velocity in the direction of the flow integrated over a horizontal lamina.

Assume $\partial b(x,z)/\partial x = \partial b(x,z)/\partial z = 0$, or $b(x,z)$=constant and cancel $bdx$. Then,

$$\frac{\partial h_1}{\partial t}+\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x \rangle dz+\langle v_x(h_1+h_2)\rangle\frac{\partial(h_1+h_2)}{\partial x}$$

$$-\langle v_x(h_2)\rangle\frac{\partial h_2}{\partial x}=0 \quad (16)$$

If it is assumed that

$$\langle v_x(h_1+h_2)\rangle=\langle v_x(h_2)\rangle=U_1(x,t) \quad (17)$$

and

$$\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x \rangle dz=h_1\frac{\partial U_1}{\partial x} \quad (18)$$

where $U_1$ = average velocity in the upper layer, Eq. 16 can be written in the following form.

$$\frac{\partial h_1}{\partial t}+h_1\frac{\partial U_1}{\partial x}+U_1\frac{\partial h_1}{\partial x}=0 \quad (19)$$

It can also be written as

$$\frac{\partial h_1}{\partial t}+\frac{\partial}{\partial x}(h_1U_1)=0 \quad (20)$$

Similarly, for the lower layer (integrating from 0 to $h_2$), if the assumptions,

$$\langle v_x(h_2)\rangle=U_2(x,t) \quad (21)$$

and

$$\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x \rangle dz=h_2\frac{\partial U_2}{\partial x} \quad (22)$$

are made,

$$\frac{\partial h_2}{\partial t}+h_2\frac{\partial U_2}{\partial x}+U_2\frac{\partial h_2}{\partial x}=0 \tag{23}$$

and also

$$\frac{\partial h_2}{\partial t}+\frac{\partial}{\partial x}(h_2U_2)=0 \tag{24}$$

## 4.2 EQUATIONS OF MOTION

The conservation of linear momentum can be stated as

$$\frac{D}{Dt}\int_{V_m(t)}\rho\underline{v}dV=\int_{V_m(t)}\rho\underline{F}dV+\int_{A_m(t)}\underline{\underline{T}}\cdot\underline{n}dA, \tag{25}$$

where $\underline{F}$= body force per mass and $\underline{T}$= stress tenor.
If the gravitational force is considered as the only body force, $\underline{F}=\underline{g}$, which is the gravitational acceleration vector. Using the Reynolds transport theorem ($\phi=\rho\underline{v}$ in Eq. 2) and the divergence theorem ($\psi=\rho\underline{v}\underline{v}$ and $=\underline{\underline{T}}$ in Eq. 4) and also the relation

$$\underline{\underline{T}}=-P\underline{\underline{I}}+\underline{\underline{\sigma}}$$

where P = pressure

$\underline{\underline{I}}$ = unit tensor

$\underline{\underline{\sigma}}$ = shear stress tenor

yields

$$\int_{V_m(t)}\left[\frac{\partial(\rho\underline{v})}{\partial t}+\nabla\cdot(\rho\underline{v}\underline{v}+P\underline{\underline{I}}-\underline{\underline{\sigma}})-\rho\underline{g}\right]dV=0 \tag{26}$$

Since $V_m(t)$ is arbitrary,

$$\frac{\partial(\rho\underline{v})}{\partial t}+\nabla\cdot(\rho\underline{v}\underline{v}+P\underline{\underline{I}}-\underline{\underline{\sigma}})-\rho\underline{g}=0 \tag{27}$$

Before accounting for the turbulence, assume the incompressible flow of a constant viscosity, Newtonian fluid. Then,

$$\nabla\cdot\underline{\underline{\sigma}}=\mu\nabla^2\underline{v} \tag{28}$$

where $\mu$ = viscosity

Substituting $\underline{v}=\bar{\underline{v}}+\underline{v}'$, $P=\bar{P}+P'$, $\rho=\bar{\rho}+\rho'$ , and $\underline{\underline{\sigma}}=\bar{\underline{\underline{\sigma}}}+\underline{\underline{\sigma}}'$ [and therefore $\nabla\cdot(\bar{\underline{\underline{\sigma}}}+\underline{\underline{\sigma}}')=\mu\nabla^2(\bar{\underline{v}}+\underline{v}')$] into Eq. 27 and taking the average yeilds

$$\frac{\partial(\bar{\rho}\bar{\underline{v}}+\overline{\rho'\underline{v}'})}{\partial t}+\nabla\cdot(\bar{\rho}\bar{\underline{v}}\bar{\underline{v}}+\bar{\rho}\overline{\underline{v}'\underline{v}'}+\overline{\rho'\bar{\underline{v}}\underline{v}'}+\overline{\rho'\underline{v}'}\bar{\underline{v}}+\overline{\rho'\underline{v}'\underline{v}'}$$

$$+\bar{P}\underline{\underline{I}}-\bar{\underline{\underline{\sigma}}})-\bar{\rho}\underline{g}=0 \tag{29}$$

If $\partial(\overline{\rho'\underline{v}'})/\partial t$ is ignored and let $\bar{\underline{\underline{\tau}}}=\bar{\underline{\underline{\sigma}}}-\bar{\rho}\overline{\underline{v}'\underline{v}'}-\overline{\rho'\bar{\underline{v}}\underline{v}'}-\overline{\rho'\underline{v}'}\bar{\underline{v}}-\overline{\rho'\underline{v}'\underline{v}'}$ so that $\tau$ may be called the stress considering the turbulence, Eq. 29 becomes

$$\frac{\partial}{\partial t}(\rho\underline{v})+\nabla\cdot(\rho\underline{v}\underline{v}+P\underline{\underline{I}}-\underline{\underline{\tau}})-\rho\underline{g}=0 \tag{30}$$

where overbars for the mean are understood.

By integrating Eq. 30 over $V_a(t)$ and using the divergence theorem ($\underline{\underline{\psi}}=\rho\underline{v}\underline{v}+P\underline{\underline{I}}-\underline{\underline{\tau}}$ in Eq. 4) and the general transport theorem ($\underline{\phi}=\rho\underline{v}$ in Eq. 12),

$$\frac{d}{dt}\int_{V_a(t)}\rho \underline{v}dV+\int_{A_a(t)}\rho \underline{v}(\underline{v}-\underline{w})\cdot \underline{n}dA+\int_{A_a(t)}P\underline{\underline{I}}\cdot \underline{n}dA-\int_{A_a(t)}\underline{\underline{\tau}}\cdot \underline{n}dA$$

$$-\int_{V_a(t)}\rho \underline{g}dV=0 \qquad (31)$$

Again, assume the existence of the two-layered flow stated in Sec. 4.1 and a control volume shown in Fig. 1, and take the same division of $A_a(t)$ and boundary conditions; then,

$$\frac{d}{dt}\int_{V_a(t)}\rho \underline{v}dV+\int_{A_e(t)}\rho \underline{v}\underline{v}\cdot \underline{n}dA+\int_{A_a(t)}P\underline{n}dA-\int_{A_a(t)}\underline{\underline{\tau}}\cdot \underline{n}dA$$

$$-\int_{V_a(t)}\rho \underline{g}dV=0 \qquad (32)$$

The change in variables in the transverse section of the channel is not considered hence forth.

In order to consider the z component of Eq. 32, dot $\underline{k}$, which is a unit vector in the z direction, with Eq. 32 to obtain

$$\frac{d}{dt}\int_{V_a(t)}\rho v_z dV+\int_{A_e(t)}\rho v_z \underline{v}\cdot \underline{n}dA+\int_{A_a(t)}P\underline{k}\cdot \underline{n}dA-\int_{A_a(t)}\underline{k}\cdot(\underline{\underline{\tau}}\cdot\underline{n})dA$$

$$-\int_{V_a(t)}\rho g_z dV=0 \qquad (33)$$

where subscript z indicates the z component of a vector.

Assume that

(1) the channel bottom is practically straight in the direction of the flow;

(2) the mean particle paths of the flow are nearly parallel to each other and to the channel bottom; and

(3) the slope of the channel bottom is less than 1/10.

Since we can define

$$\underline{g}=-g\underline{p}$$

where $\underline{p}$ = unit vector in the perpendicular direction

$g$ = acceleration of gravity,

$$g_z=\underline{g}\cdot\underline{k}=-g\underline{p}\cdot\underline{k}=-g\cos\theta\approx-g$$

where $\theta$ = angle between the x axis and the horizontal

Eq. 33 is applied to a differential volume dxdydz.
Cancelling dxdydx yields

$$\frac{\partial}{\partial t}(\rho v_z)+\frac{\partial}{\partial x}(\rho v_z v_x)+\frac{\partial P}{\partial z}-\frac{\partial \tau_{xz}}{\partial x}-\frac{\partial \tau_{yz}}{\partial y}-\frac{\partial \tau_{zz}}{\partial z}+\rho g=0 \tag{34}$$

where the first subscript for $\tau$ indicates the plane on which the stress acts and the second one denotes the direction in which the stress acts. It is assumed that

$$\frac{\partial}{\partial t}(\rho v_z)=0,\ \frac{\partial}{\partial x}(\rho v_z v_x)=0,\ \frac{\partial \tau_{xz}}{\partial x}=0,\ \text{and}\ \frac{\partial \tau_{zz}}{\partial z}=0$$

(Of course, $\partial\tau_{yz}/\partial y=0$, since the change in the y direction is not considered.)

Further, for the upper layer, the density of the water is assumed to be $\rho_1$ and constant in the z direction. In addition, the surface and interfacial tension can be ignored. The, what is left is

$$\frac{\partial P}{\partial z}+\rho g=0 \tag{35}$$

Eq. 35 can be solved with a boundary condition, $P=P_0$= atmospheric pressure at $z=h_1+h_2$, to yield

$$P=P_0+\rho_1 g(h_1+h_2-z) \tag{36}$$

For the lower layer, it is assumed that $\rho_2$ is constant in the z direction, and the interfacial tension can be ignored. In a similar way,

$$P=P_0+\rho_1 g h_1+\rho_2 g(h_2-z) \tag{37}$$

Take the x component of Eq. 32.

$$\frac{d}{dt}\int_{V_a(t)}\rho v_x dV+\int_{A_e(t)}\rho v_x \underline{v}\cdot\underline{n}dA+\int_{A_a(t)}P\underline{i}\cdot\underline{n}dA-\int_{A_a(t)}\underline{i}\cdot(\underline{T}\cdot\underline{n})dA$$

$$+\int_{V_a(t)}\rho g_x dV=0 \tag{38}$$

where $\underline{i}$= unit vector in the x direction,the subscript x indicates the x component of a vector.

Each term of Eq. 38 for the upper layer is examined. They are expressed in the limit.

$$\bullet\frac{d}{dt}\int_{V_{a1}(t)}\rho_1 v_x dV=dx\frac{d}{dt}\int_{h_2}^{h_1+h_2}\rho_1\langle v_x\rangle b dz$$

$$=dx\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial t}(\rho_1\langle v_x\rangle b)dz$$

$$+dx\rho_1(h_1+h_2)\langle v_x(h_1+h_2)\rangle b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial t}$$

$$-dx\rho_1(h_2)\langle v_x(h_2)\rangle b(h_2)\frac{\partial h_2}{\partial t}$$

$$\bullet\int_{A_{e1}(t)}\rho_1 v_x\underline{v}\cdot\underline{n}dA=dx\frac{d}{dx}\int_{h_2}^{h_1+h_2}\rho_1\langle v_x^2\rangle b dz$$

$$=dx\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}(\rho_1\langle v_x^2\rangle b)dz$$

$$+dx\rho_1(h_1+h_2)\langle v_x^2(h_1+h_2)\rangle b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}$$

$$-dx\rho_1(h_2)\langle v_x^2(h_2)\rangle b(h_2)\frac{\partial h_2}{\partial x}$$

$$\cdot\int_{A_{a1}(t)} P\underline{i}\cdot\underline{n}dA=\int_{A_{a1}(t)}[P_0+\rho_1 g(h_1+h_2-z)]\underline{i}\cdot\underline{n}dA \text{ (Eq. 36 is used.)}$$

At the cross sections $[A_{e1}(t)]$:

$$\int_{A_{e1}(t)}[P_0+\rho_1 g(h_1+h_2-z)]\underline{i}\cdot\underline{n}dA$$

$$=dx\frac{d}{dx}\int_{h_2}^{h_1+h_2}[P_0+\rho_1 g(h_1+h_2-z)]bdz$$

$$=dx\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\{[P_0+\rho_1 g(h_1+h_2-z)]b\}dz$$

$$+dxP_0 b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}-dx(P_0+\rho_1 gh_1)b(h_2)\frac{\partial h_2}{\partial x}$$

At the free surface $[A_s(t)]$:
See Fig. 2. Since $\delta$ is small according to the assumptions that the mean particle paths of the flow are nearly parallel to each other and to the channel bottom,

$$\underline{i}\cdot\underline{n}=\cos(90^{\circ}+\delta)=-\sin\delta\approx-\tan\delta=-\frac{\partial(h_1+h_2)}{\partial x}.$$

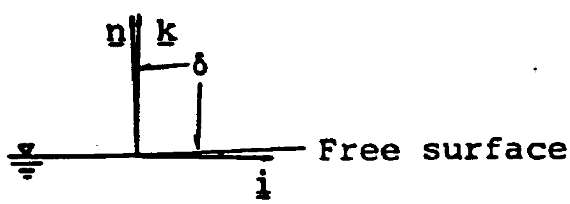



Fig. 2. Definition sketch for the free surface.

Therefore,

$$\int_{A_s(t)}-P_0\frac{\partial(h_1+h_2)}{\partial x}dA=-dxP_0 b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}$$

At the interface [$A_i(t)$]:

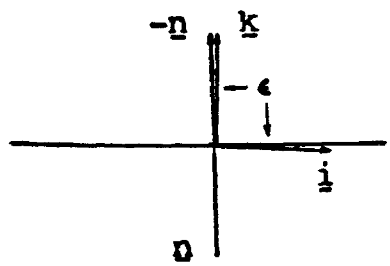



Fig. 3. Definition sketch for the interface.

$$\mathbf{i}\cdot\mathbf{n}=\cos(90^{\circ}-\epsilon)=\sin\epsilon\approx\tan\epsilon=\partial h_2/\partial x$$

$$\int_{A_i(t)}(P_0+\rho_1 g h_1)\frac{\partial h_2}{\partial x}dA=dx(P_0+\rho_1 g h_1)b(h_2)\frac{\partial h_2}{\partial x}$$

At the side walls [$A_{w1}(t)$]:
It is assumed that the change in width, both in the flow and z directions, is gradual.

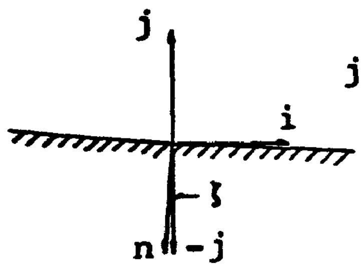



Fig. 4. Definition sketch for the side wall.

$$\mathbf{i}\cdot\mathbf{n}+\cos(90^{\circ}+\zeta)=-\sin\zeta\approx-\tan\zeta=-\frac{\partial b(x,z)}{\partial x}$$

$$\int_{A_{w1}(t)}-[P_0+\rho_1 g(h_1+h_2-z)]\frac{\partial b}{\partial x}dA$$

$$=-dx\int_{h_2}^{h_1+h_2}[P_0+\rho_1 g(h_1+h_2-z)]\frac{\partial b}{\partial x}dz$$

$\cdot\int_{A_{a1}(t)} \mathbf{i}\cdot(\boldsymbol{\tau}\cdot\mathbf{n})\,dA$

At $A_{e1}(t)$:

$$\int_{A_{e1}(t)} \tau_{xx}\,dA = dx\frac{d}{dx}\int_{h_2}^{h_1+h_2} \tau_{xx}\,b\,dz$$

$$= dx\int_{h_2}^{h_1+h_2} \frac{\partial}{\partial x}(\tau_{xx}b)\,dz$$

$$+dx\,\tau_{xx}(h_1+h_2)\,b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}$$

$$-dx\,\tau_{xx}(h_2)\,b(h_2)\frac{\partial h_2}{\partial x}$$

At $A_s(t)$:

$$\int_{A_s(t)} \left(\tau_{zx}-\frac{\partial(h_1+h_2)}{\partial x}\tau_{xx}\right)dA$$

$$= dx\,\tau_{zx}(h_1+h_2)\,b(h_1+h_2) - dx\,\tau_{xx}(h_1+h_2)\,b(h_1+h_2)\frac{\partial(h_1+h_2)}{\partial x}$$

At $A_i(t)$:

$$\int_{A_i(t)} \left(-\tau_{zx}+\frac{\partial h_2}{\partial x}\ {}_{xx}\right)dA = -dx\,\tau_{zx}(h_2)\,b(h_2) + dx\,\tau_{xx}(h_2)\,b(h_2)\frac{\partial h_2}{\partial x}$$

At $A_{w1}(t)$:

$$\int_{A_{w1}(t)} \left[-\frac{\partial b(x,z)}{\partial x}\tau_{xx} - \tau_{yx} - \frac{\partial b(x,z)}{\partial z}\tau_{zx}\right]dA$$

$$= dx\int_{h_2}^{h_1+h_2} \left(-\frac{\partial b}{\partial x}\tau_{xx} - \tau_{yx} - \frac{\partial b}{\partial z}\tau_{zx}\right)dz$$

(We define the sign of $\tau_{yx}$ in this manner.)

$$\cdot\int_{V_a(t)}\rho_1 g_x dV$$

$g_x=g\cdot i=-gp\cdot i=-g\sin\theta\approx-g\tan\theta=gS_b$, in which $S_b=-\tan\theta$=slope of the channel bottom.

$$\int_{V_a(t)}-\rho_1 gS_b dV=-gS_b dx\int_{h_2}^{h_1+h_2}\rho_1 b dz$$

It is assumed that

(1) $\partial\rho_1/\partial t=\partial\rho_1/\partial x=0$ ($\partial\rho_1/\partial z$ is already assumed. For reference, $\nabla\cdot\overline{\rho'\underline{v}'}=0$ in Eq. A1.2-10, since $\partial\rho_1/\partial y=0$.);

(2) $\partial b(x,z)/\partial x=\partial b(x,z)/\partial z=0$, or $b(x,z)$=constant;

(3) $\partial P_0/\partial x=0$; and

(4) $\partial\tau_{xx}/\partial x=0$.

Use the following definitions:

$$\tau_{zx}(h_1+h_2)=\tau_s,\quad \tau_{zx}(h_2)=\tau_i,\quad \text{and}\quad \int_{h_2}^{h_1+h_2}\tau_{yx}dz=\tau_w W_1(x,t),$$

where $\tau_s$ = surface shear stress

$\tau_i$ = interfacial shear stress

$\tau_w$ = wall shear stress

$W_1$ = wetted perimeter corresponding to the upper layer

Then, Eq. 38 takes the following form for the upper layer, by cancelling bdx.

$$\rho_1\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial t}\langle v_x\rangle dz+\rho_1\langle v_x(h_1+h_2)\rangle\frac{\partial(h_1+h_2)}{\partial t}-\rho_1\langle v_x(h_2)\rangle\frac{\partial h_2}{\partial t}$$

$$+\rho_1\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x^2\rangle dz+\rho_1\langle v_x^2(h_1+h_2)\rangle\frac{\partial(h_1+h_2)}{\partial x}-\rho_1\langle v_x^2(h_2)\rangle\frac{\partial h_2}{\partial x}$$

$$+\rho_1 g\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}(h_1+h_2-z)dz-\tau_s+\tau_i+\tau_w\frac{W_1}{b}-\rho_1 gS_b h_1=0 \tag{39}$$

If it is assumed that

$$\int_{h_2}^{h_1+h_2} \frac{\partial}{\partial t} \langle v_x \rangle dz = h_1 \frac{\partial U_1}{\partial t} \tag{40}$$

$$\langle v_x^2 \rangle = \langle v_x \rangle^2 \tag{41}$$

$$\int_{h_2}^{h_1+h_2} \frac{\partial}{\partial x} \langle v_x^2 \rangle dz = h_1 \frac{\partial U_1^2}{\partial x} \quad \text{and} \tag{42}$$

$$\langle v_x(h_1+h_2) \rangle = \langle v_x(h_2) \rangle = U_1 \tag{43}$$

then, Eq. 39 can be written as

$$\rho_1 h_1 \frac{\partial U_1}{\partial t} + \rho_1 U_1 \frac{\partial h_1}{\partial t} + \rho_1 h_1 \frac{\partial U_1^2}{\partial x} + \rho_1 U_1^2 \frac{\partial h_1}{\partial x} + \rho_1 g h_1 \frac{\partial h_1}{\partial x} + \rho_1 g h_1 \frac{\partial h_2}{\partial x}$$

$$- \tau_s + \tau_i + \tau_w \frac{W_1}{b} - \rho_1 g h_1 S_b = 0 \tag{44}$$

It can also be written as

$$\rho_1 \frac{\partial (U_1 h_1)}{\partial t} + \rho_1 \frac{\partial (U_1^2 h_1)}{\partial x} + \rho_1 g h_1 \frac{\partial h_1}{\partial x} + \rho_1 g h_1 \frac{\partial h_2}{\partial x}$$

$$- \tau_s + \tau_i + \tau_w \frac{W_1}{b} - \rho_1 g h_1 S_b = 0 \tag{45}$$

Subtracting the equation of continuity (Eq. 19) multiplied by $\rho_1 U_1$ from Eq. 44 yields

$$\rho_1 h_1 \frac{\partial U_1}{\partial t} + \rho_1 h_1 U_1 \frac{\partial U_1}{\partial x} + \rho_1 g h_1 \frac{\partial h_1}{\partial x} + \rho_1 g h_1 \frac{\partial h_2}{\partial x}$$

$$- \tau_s + \tau_i + \tau_w \frac{W_1}{b} - \rho_1 g h_1 S_b = 0 \tag{46}$$

If Eq. 46 is divided by $\rho_1 g h_1$, applied to a unit width of the flow ($\tau_w=0$), and the definition

$$S_{f1}=\frac{\tau_i-\tau_s}{\rho_1 g h_1} \tag{47}$$

is used,

the equation of motion for the upper layer is obtained as

$$\frac{1}{g}\frac{\partial U_1}{\partial t}+\frac{U_1}{g}\frac{\partial U_1}{\partial x}+\frac{\partial h_1}{\partial x}+\frac{\partial h_2}{\partial x}+S_{f1}-S_b=0 \tag{48}$$

For the lower layer, similar operation can be used. Some of terms are listed below.

$\cdot \int_{A_{a2}(t)} P\underline{i}\cdot\underline{n}dA=\int_{A_{a2}(t)}[P_0+\rho_1 g h_1+\rho_2 g(h_2-z)]\underline{i}\cdot\underline{n}dA$ (Eq. 37 is used.)

At the cross sections $[A_{e2}(t)]$:

$$dx\frac{d}{dx}\int_0^{h_2}[P_0+\rho_1 g h_1+\rho_2 g(h_2-z)]bdz$$

$$=dx\int_0^{h_2}\frac{\partial}{\partial x}\{[P_0+\rho_1 g h_1+\rho_2 g(h_2-z)]b\}dz+dx(P_0+\rho_1 g h_1)b(h_2)\frac{\partial h_2}{\partial x}$$

At the interface $[A_i(t)]$:

$$\underline{i}\cdot\underline{n}=\cos(90^{\circ}+\epsilon)=-\sin\epsilon\approx-\tan\epsilon=-\partial h_2/\partial x$$

$$\int_{A_i(t)}-(P_0+\rho_1 g h_1)\frac{\partial h_2}{\partial x}dA=-dx(P_0+\rho_1 g h_1)b(h_2)\frac{\partial h_2}{\partial x}$$

At the bottom $[A_b]$: $\underline{n}=-\underline{k}$, so that $\underline{i}\cdot\underline{n}=0$.

At the side walls $[A_{w2}(t)]$: The same as for the upper layer, except for integrating from 0 to $h_2$.

$$\cdot\int_{A_{a2}(t)}\underline{i}\cdot(\underline{\underline{\tau}}\cdot\underline{n})dA$$

At $A_{e2}(t)$:

$$dx\int_0^{h_2}\frac{\partial}{\partial x}(\tau_{xx}b)dz+dx\tau_{xx}(h_2)b(h_2)\frac{\partial h_2}{\partial x}$$

At $A_i(t)$:

$$\int_{A_i(t)}(\tau_{zx}-\frac{\partial h_2}{\partial x}\tau_{xx})dA=dx\tau_{zx}(h_2)b(h_2)-dx\tau_{xx}(h_2)b(h_2)\frac{\partial h_2}{\partial x}$$

At $A_b$:

$$\int_{A_b}-\tau_{zx}dA=-dx\tau_{zx}(0)b(0)$$

At $A_{w2}(t)$: The same as for the upper layer.

Assumptiopns and definitions in addition to the ones for the upper layer are

$$\partial\rho_2/\partial t=\partial\rho_2/\partial x=0,\ \tau_{zx}(0)=\tau_b=\text{ bottom shear stress,}$$

and

$$\int_0^{h_2}\tau_{yx}dz=\tau_w W_2(x,t),$$

where $\overline{W}_2$= wetted perimeter corresponding to the lower layer except for the bottom.

Eq. 38 becomes, for the lower layer

$$\rho_2\int_0^{h_2}\frac{\partial}{\partial t}<v_x>dz+\rho_2<v_x(h_2)>\frac{\partial h_2}{\partial t}+\rho_2\int_0^{h_2}\frac{\partial}{\partial x}<v_x{}^2>dz+\rho_2<v_x{}^2(h_2)>\frac{\partial h_2}{\partial x}$$

$$+\rho_1 g\int_0^{h_2}\frac{\partial h_1}{\partial x}dz+\rho_2 g\int_0^{h_2}\frac{\partial h_2}{\partial x}dz-\tau_i+\tau_b+\tau_w\frac{W_2}{b}-\rho_2 gS_b h_2=0 \tag{49}$$

If it is assumed that, in addition to Eq. 41,

$$\int_0^{h_2}\frac{\partial}{\partial t}<v_x>dz=h_2\frac{\partial U_2}{\partial t} \tag{50}$$

$$\int_0^{h_2}\frac{\partial}{\partial x}<v_x{}^2>dz=h_2\frac{\partial U_2{}^2}{\partial x} \tag{51}$$

and $$\langle v_x(h_2)\rangle=U_2 \tag{52}$$

Eq. 49 can be written as

$$\rho_2 h_2\frac{\partial U_2}{\partial t}+\rho_2 U_2\frac{\partial h_2}{\partial t}+\rho_2 h_2\frac{\partial U_2{}^2}{\partial x}+\rho_2 U_2{}^2\frac{\partial h_2}{\partial x}+\rho_1 g h_2\frac{\partial h_1}{\partial x}+\rho_2 g h_2\frac{\partial h_2}{\partial x}$$

$$-\tau_i+\tau_b+\tau_w\frac{W_2}{b}-\rho_2 g h_2 S_b=0 \tag{53}$$

or $$\rho_2\frac{\partial(U_2 h_2)}{\partial t}+\rho_2\frac{\partial(U_2{}^2 h_2)}{\partial x}+\rho_1 g h_2\frac{\partial h_1}{\partial x}+\rho_2 g h_2\frac{\partial h_2}{\partial x}$$

$$-\tau_i+\tau_b+\tau_w\frac{W_2}{b}-\rho_2 g h_2 S_b=0 \tag{54}$$

If the equation of continuity (Eq. 23) multiplied by $\rho_2 U_2$ is subtracted from Eq. 53,

$$\rho_2 h_2\frac{\partial U_2}{\partial t}+\rho_2 h_2 U_2\frac{\partial U_2}{\partial x}+\rho_1 g h_2\frac{\partial h_1}{\partial x}+\rho_2 g h_2\frac{\partial h_2}{\partial x}$$

$$-\tau_i+\tau_b+\tau_w\frac{W_2}{b}-\rho_2 g h_2 S_b=0 \tag{55}$$

Dividing by $\rho_2 g h_2$, using the definition

$$S_{f2}=\frac{\tau_b-\tau_i}{\rho_2 g h_2} \tag{56}$$

and applying this to a unit width of the flow yields

$$\frac{1}{g}\frac{\partial U_2}{\partial t}+\frac{U_2}{g}\frac{\partial U_2}{\partial x}+\frac{\rho_1}{\rho_2}\frac{\partial h_1}{\partial x}+\frac{\partial h_2}{\partial x}+S_{f2}-S_b=0 \tag{57}$$

The term of density is examined.

$$\frac{\rho_1}{\rho_2}=\frac{\rho_2-(\rho_2-\rho_1)}{\rho_2}=1-\frac{\Delta\rho}{\rho_2}, \text{ in which } \Delta\rho=\rho_2-\rho_1.$$

If it is assumed that

$$\rho=(\rho_1+\rho_2)/2\approx\rho_1\approx\rho_2, \tag{58}$$

$\rho_1/\rho_2=1-(\Delta\rho/\rho)$ can be used. [For example, the density difference is approximately 0.001 gm/cm$^3$ between water of temperation 20$^o$C (68$^o$F) and 25$^o$c (77$^o$F) and 0.026 gm/cm$^3$ between fresh water and sea water at 15$^o$C (59$^o$F)].

Now the equation of motion for the lower layer is

$$\frac{1}{g}\frac{\partial U_2}{\partial t}+\frac{U_2}{g}\frac{\partial U_2}{\partial x}+(1-\frac{\Delta\rho}{\rho})\frac{\partial h_1}{\partial x}+\frac{\partial h_2}{\partial x}+S_{f2}-S_b=0 \tag{59}$$

As a consequence, $\rho$ in Eq. 58 can be used in $S_{f1}$ (Eq. 47) and $S_{f2}$ (Eq. 56).

## 4.3 SUMMARY OF THE ASSUMPTIONS USED IN SEC. 4.1 AND 4.2

1. Continuum field theories are used.
2. The flow of water is the incompressible flow of a constant viscosity, Newtonian fluid.
3. Only gravity is considered as body force.
4. The surface and interfacial tensions are ignored.
5. Only the mean values of fluid properties are used to consider the turbulence in the flow. (Related to this, $\partial(\overline{\rho'\underline{v}'})/\partial t$ and $\nabla\cdot\overline{\rho'\underline{v}'}$ are ignored.)
6. There exist, in a channel, two distinct flow layers of different densities separated by an interface across which little mixing occurs.
7. The free surface and the interface are horizontal in the direction of width of the channel.
8. There is no appreciable precipitation on or evaporation from the free surface, and no inflow or outflow, including groundwater at the wetted perimeter.
9. The variation in the transverse direction of the channel is not considered.

10. The channel bottom is practically straight in the direction of the flow.
11. The mean particle paths of the flow are nearly parallel to each other and to the channel bottom.
12. The slope of the channel bottom is less than 1/10.
13. Only a unit width of the flow is considered.
14. The density of water is constant in a layer.
15. The density difference between the layers is small. $[\rho=(\rho_1+\rho_2)/2\approx\rho_1\approx\rho_2]$
16. $\partial v_z/\partial t=0$, and $\partial(v_z v_x)/\partial x=0$.
17. $\partial P_0/\partial x=0$.
18. $\partial\tau_{xx}/\partial x=0$, $\partial\tau_{xz}/\partial x=0$, and $\partial\tau_{zz}/\partial z=0$.
19. $\langle v_x(h_1+h_2)\rangle=\langle v_x(h_2)\rangle=U_1(x,t)$ for the upper layer.
    $\langle v_x(h_2)\rangle=U_2(x,t)$ for the lower layer.
20. $\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz=h_1\frac{\partial U_1}{\partial x}$ and $\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz=h_2\frac{\partial U_2}{\partial x}$.
21. $\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial t}\langle v_x\rangle dz=h_1\frac{\partial U_1}{\partial t}$ and $\int_0^{h_2}\frac{\partial}{\partial t}\langle v_x\rangle dz=h_2\frac{\partial U_2}{\partial t}$.
22. $\langle v_x^2\rangle=\langle v_x\rangle^2$.
23. $\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x^2\rangle dz=h_1\frac{\partial U_1^2}{\partial x}$ and $\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x^2\rangle dz=h_2\frac{\partial U_2^2}{\partial x}$.

The assumptions 6 and 15 can be considered to differ from the assumptions generally used for the gradually varied open-channel flow.

### 4.4 NON-UNIFORM VELOCITY DISTRIBUTION CONSIDERED IN A LAYER

In the derivation of the equation of motion shown in Sec 4.2, it is assumed that the velocity is distributed uniformly in both layers. If non-uniform velocity distribution is assumed in the flowing layer, the assumption $\langle v_x^2\rangle=\langle v_x\rangle^2$ made in Sec. 4.2 becomes approximate and it is necessary to have a correction factor, $\beta$, in the assumption:

$$\langle v_x^2\rangle=\beta\langle v_x\rangle^2 \tag{60}$$

$\beta$ may be called the momentum coefficient.[3] The range of $\beta$ is $1<\beta<1.2$, although the value of the upper limit is not definite.[2,3] The effect of the introduction of $\beta$ on the form of the equation of motion is now examined. This examination is done only for the steady-state case, since the derivation is not easy for the unsteady-state case.

Then, Eq. 42 for the upper layer becomes

$$\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz=\beta h_1\frac{dU_1^2}{dx} \tag{61}$$

Therefore, Eq. 44 can be written as

$$\beta\rho_1 h_1\frac{dU_1^2}{dx}+\beta\rho_1 U_1^2\frac{dh_1}{dx}+\rho_1 g h_1\frac{dh_1}{dx}+\rho_1 g h_1\frac{dh_2}{dx}$$
$$-\tau_s+\tau_i+\tau_w\frac{W_1}{b}-\rho_1 g h_1 S_b=0 \tag{62}$$

The equation of continuity for the steady-state case is reduced to (from Eq. 19)

$$h_1\frac{dU_1}{dx}+U_1\frac{dh_1}{dx}=0 \tag{63}$$

If Eq. 63 multiplied by $\beta\rho_1 U_1$ is subtracted from Eq. 62,

$$\beta\rho_1 h_1 U_1\frac{dU_1}{dx}+\rho_1 g h_1\frac{dh_1}{dx}+\rho_1 g h_1\frac{dh_2}{dx}$$
$$-\tau_s+\tau_i+\tau_w\frac{W_1}{b}-\rho_1 g h_1 S_b=0 \tag{64}$$

If it is applied only to a unit width of the flow with the definition of $S_{f1}$ (Eq. 47), the equation of motion for the upper layer is obtained to be

$$\frac{\beta U_1}{g}\frac{dU_1}{dx}+\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1}-S_b=0 \tag{65}$$

Similarly, the equation of motion for the lower layer becomes

$$\frac{\beta U_2}{g}\frac{dU_2}{dx}+(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f2}-S_b=0 \tag{66}$$

## 4.5 EQUATIONS OF CONTINUITY AND MOTION FOR ONE-DIMENSIONAL UNSTEADY NON-UNIFORM OPEN-CHANNEL FLOW

The method of derivation used for two-layered flow can naturally be used for open-channel flow. Using the assumptions mentioned for the two-layered flow, for an open-channel flow with a constant width,
equation of continuity:

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(hU)=0 \tag{67}$$

and equation of motion:

$$\frac{1}{g}\frac{\partial U}{\partial t}+\frac{U}{g}\frac{\partial U}{\partial x}+\frac{\partial h}{\partial x}+\frac{\tau_b-\tau_s+\tau_w(W/b)}{\rho g h}-S_b=0 \tag{68}$$

For a unit width of the flow, Eq. 68 can be written as

$$\frac{1}{g}\frac{\partial U}{\partial t}+\frac{U}{g}\frac{\partial U}{\partial x}+\frac{\partial h}{\partial x}+S_f-S_b=0 \tag{69}$$

with a definition

$$S_f=\frac{\tau_b-\tau_s}{\rho g h} \tag{70}$$

If the discussion on the momentum coefficient is applied, $\beta$, mentioned in Sec. 4.4, Eq. 68 becomes, for the steady-state case,

$$\frac{\beta U dU}{g\ dx}+\frac{dh}{dx}+\frac{\tau_b-\tau_s+\tau_w(W/b)}{\rho g h}-S_b=0 \tag{71}$$

For a unit width of the flow, it reduces to

$$\frac{\beta U dU}{g\ dx}+\frac{dh}{dx}+S_f-S_b=0 \tag{72}$$

# SECTION V

# INTERFACIAL PROFILE EQUATIONS

## 5.1 INTERFACIAL PROFILE EQUATIONS FOR HYPOLIMNETIC FLOW OF A UNIT WIDTH

One-dimensional equations of continuity and motion for the steady two-layered flow can be written from the equations for the unsteady-state case (eqs. 20, 24, 48, and 59). These equations are listed below with the definition sketch.

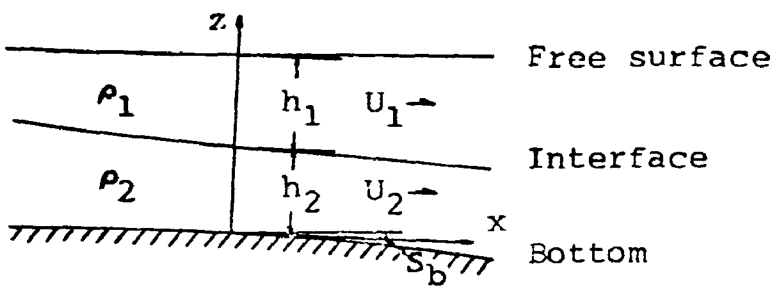



Fig. 5. Definition sketch for steady two-layered flow of a unit width.

Equation of continuity for the upper layer:

$$h_1U_1=q_1=\text{constant} \tag{73}$$

Equation of continuity for the lower layer:

$$h_2U_2=q_2=\text{constant} \tag{74}$$

Equation of motion for the upper layer:

$$\frac{U_1}{g}\frac{dU_1}{dx}+\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1}-S_b=0 \tag{75}$$

Equation of motion for the lower layer:

$$\frac{U_2}{g}\frac{dU_2}{dx}+(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f2}-S_b=0 \tag{76}$$

where $\rho = (\rho_1+\rho_2)/2$

$\Delta\rho = \rho_1-\rho_2$

$q$ = flowrate per unit width

$S_f$ is the friction slope, which is defined by the relations

$$S_{f1}=\frac{\tau_i-\tau_s}{gh_1} \tag{77}$$

and

$$S_{f2}=\frac{\tau_b-\tau_i}{gh_2} \tag{78}$$

$\tau_b$ and $\tau_i$ may be given empirically as

$$\tau_b=\frac{\rho f_b}{8}|U_2|U_2 \tag{79}$$

$$\tau_i=\frac{\rho f_i}{8}|U_1-U_2|(U_1-U_2) \tag{80}$$

where $f_b$ = friction factor at the channel bottom, which is called "bottom friction factor"

$f_i$ = friction factor at the interface, which is called "interfacial friction factor

$\tau_s$ can be ignored for the case in which the wind effect on the free surface is small.

For the hypolimnetic flow, the upper layer is assumed to have no motion, i.e., $U_1 = 0$. (See Sec. 5.4 for a discussion on the case in which the internal circulation of water is allowed in a layer having no net flow rate.) Then, Eqs. 75 and 76 can be written by substituting $U_2 = q_2/h_2$ from Eq. 74, as follows.

$$\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1L}-S_b=0 \tag{81}$$

$$(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx}+(1-\frac{q_2^2}{gh_2^3})\frac{dh_2}{dx}+S_{f2L}-S_b=0 \tag{82}$$

where subscript L indicates the hypolimnetic flow.

$\tau_b$ and $\tau_i$ may be written, from Eqs. 79 and 80, as

$$\tau_b=\frac{\rho f_b}{8}U_2^2 \tag{83}$$

and

$$\tau_i=-\frac{\rho f_i}{8}U_2^2 \tag{84}$$

Eqs. 77 and 78 can now be written as

$$S_{f1L}=-\frac{f_i q_2^2}{8gh_1h_2^2} \tag{85}$$

and

$$S_{f2L}=\frac{q_2^2(f_b+f_i)}{8gh_2^3} \tag{86}$$

Substituting $dh_1/dx$ from Eq. 81, and Eqs. 85 and 86 into Eq. 82 and re-arranging it yields

$$[1-\frac{q_2^2}{(\Delta\rho/\rho)gh_2^3}]\frac{dh_2}{dx}$$

$$=S_b-\frac{q_2^2}{8(\Delta\rho/\rho)gh_2^3}\frac{(f_b+f_i)H-f_bh_2-(\Delta\rho/\rho)f_ih_2}{H-h_2} \tag{87}$$

where $H = h_1+h_2$ = total depth.

Since $\Delta\rho/\rho$ is expected to be at most 0.03,* the term $(\Delta\rho/\rho)\ f_ih_2$

* This is for a case of fresh and sea water. For water with temperature difference of 16.7°C (30°F), $\Delta\rho/\rho$ is at most 0.005.

may be ignored in Eq. 87, provided that $f_i$ is not very large compared to $f_b$. Then Eq. 87 becomes

$$[1-\frac{q_2^2}{(\Delta\rho/\rho)gh_2^3}]\frac{dh_2}{dx}=S_b-\frac{q_2^2 f_b}{8(\Delta\rho/\rho)gh_2^3}\frac{H+(f_i/f_b)H-h_2}{H-h_2} \tag{88}$$

Positive Slope Flow

When $S_b$ is positive, Eq. 88 can be rewritten as

$$[1-\frac{q_2^2}{(\Delta\rho/\rho)gh_2^3}]\frac{dh_2}{dx}$$

$$=S_b\{1-\frac{q_2^2 f_b}{8(\Delta\rho/\rho)gS_b h_2^3}\frac{[1+(f_i/f_b)]H-h_2}{H-h_2}\} \tag{89}$$

The stratified critical depth for the lower layer, $h_{c2}$ is defined by the relation

$$h_{c2}^3=\frac{q_2^2}{(\Delta\rho/\rho)g} \tag{90}$$

The stratified normal depth for the lower layer, $h_{n2}$, may be defined by the relation

$$h_{n2}^3=\frac{q_2^2 f_b}{8(\Delta\rho/\rho)gS_b}\frac{[1+(f_i/f_b)]H-h_{n2}}{H-h_{n2}}$$

$$=h_{c2}^3\frac{f_b}{8S_b}[1+\frac{f_i/f_b}{1-(h_{n2}/H)}] \tag{91}$$

A factor related to the stratified normal depth for the lower layer, G, is defined by the relation

$$G^3=\frac{(1+f_i/f_b)H-h_2}{(1+f_i/f_b)H-h_{n2}}\frac{H-h_{n2}}{H-h_2}$$

$$=\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right]/\left[1+\frac{f_i/f_b}{1-(h_{n2}/H)}\right] \tag{92}$$

If the above definitions are used, an equation of interfacial profiles for the hypolimnetic flow can be obtained as

$$\frac{dh_2}{dx}=S_b\frac{1-(Gh_{n2}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{93}$$

It is almost the same in the form as the equation presented by Chow[3] for the free surface profiles of gradually varied open-channel flow.

The term $Gh_{n2}$ is now examined. From Eqs. 91 and 92,

$$Gh_{n2}=h_{c2}\left\{\frac{f_b}{8S_b}\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right]\right\}^{1/3} \tag{94}$$

If $q_2$, $S_b$, $f_i$ and $f_b$ are assumed to be constant, change in $Gh_{n2}$ depends only on $\{1+(f_i/f_b)/[1-(h_2/H)\}^{1/3}$, which is later called $G_H$. Examples of the values of $G_H$ are given in Fig. 6. $G_H$ is fairly constant when $h_2/H < 0.8$ especially for smaller $f_i/f_b$. The term $Gh_{n2}$ may be called "virtual stratified normal depth."

General characteristics of interfacial profiles for the positive slope flow can be found from Eq. 93.

As $Gh_{n2}/h_2$ approaches 1, change in the lower layer depth, $dh_2/dx$, goes to zero. This means that the interface becomes parallel to the channel bottom if the lower layer depth equals the virtual stratified normal depth. When $h_{c2}/h_2$ approaches 1, $dh_2/dx$ goes to infinity. This indicates that the interface will intersect the stratified critical depth sharply and disturbances can be expected at the interface.

(Where the interface is too steep, the method used in this study is not applicable, since streamlines are assumed to be nearly parallel to each other and to the channel bottom.) If $h_2$ goes to zero, it is easily deduced that $dh_2/dx = (f_b+f_i)/8$, which is a small angle to the channel bottom. If $h_2$ becomes much larger than $Gh_{n2}$ and $h_{c2}$, $dh_2/dx$ approaches

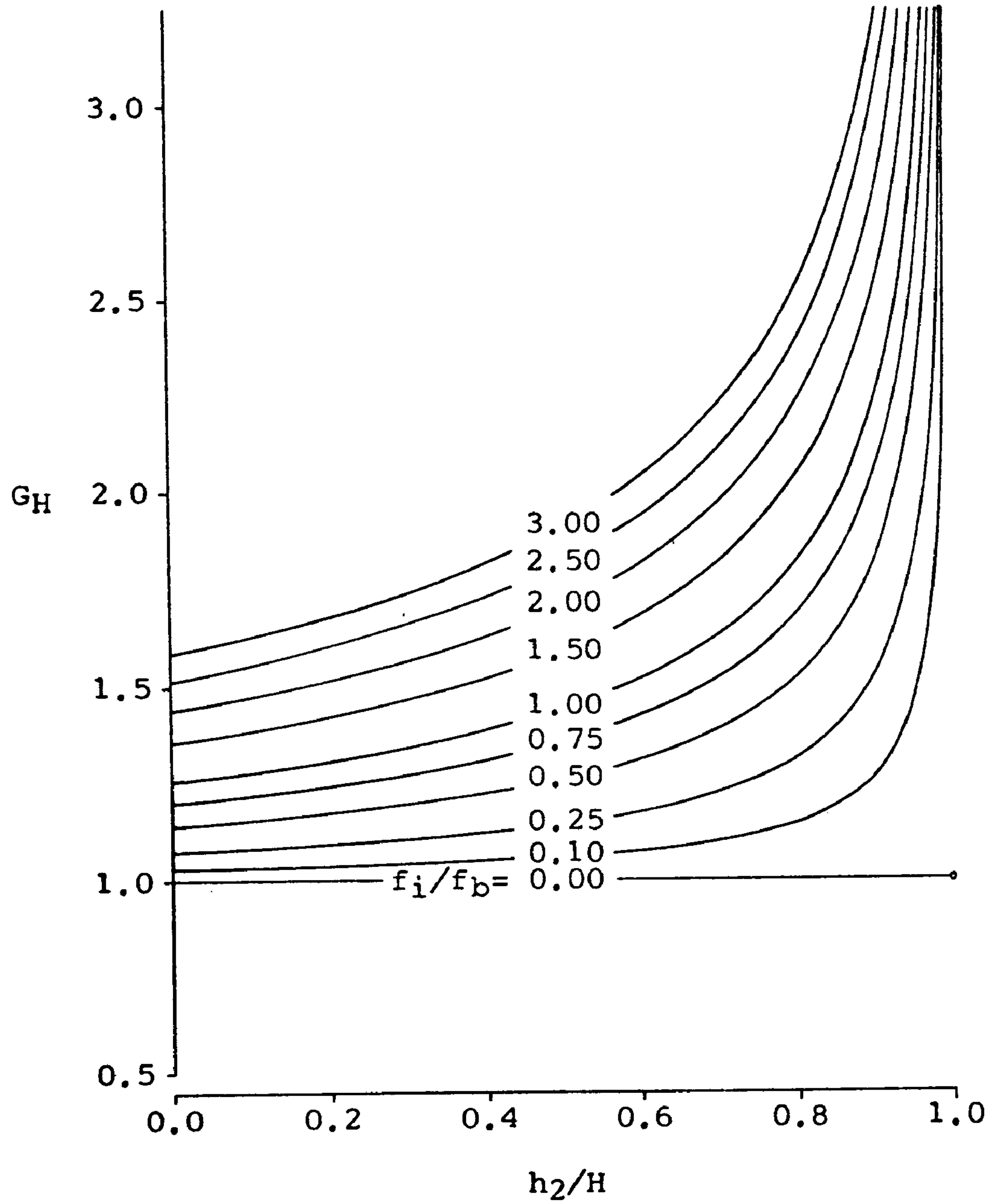



Fig. 6 Values of $G_H$.

$S_b$. This means that the interface becomes close to the horizontal in cases in which the lower layer can extend toward the downstream with its depth increasing. In addition, as can be observed from the values of $G_H$, if $h_2$ approaches H, $dh_2/dx$ may go to infinity. This indicates that the interface may intersect the free surface sharply. Therefore disturbance can be expected at the frontal zone of the upper layer where the two layers start to establish themselves.

The position of the interface relative to the stratified critical depth and the virtual stratified normal depth will dictate whether $dh_2/dx$ assumes a positive or negative value, i.e., whether the lower layer depth increases or decreases in the downstream direction. Equation 94 shows that it depends on the magnitude of $G_H[f_b/(8S_b)]^{1/3}$ whether $Gh_{n2}$ is greater or less than $h_{c2}$, that is, whether the flow is on a mild or steep slope.

From these discussions, it can be expected that there would exist the following interfacial profiles, which are illustrated in Fig. 7.

1. Mild slope flow (denoted by "M"), when $Gh_{n2}>h_{c2}$:

   M1 profile, when $h_2>Gh_{n2}>h_{c2}$;

This profile could represent a density flow in an impoundment created by a dam downstream on a mild slope. Note that the downstream end of the interface approaches the horizontal. Mixing between the two layers would not occur quickly.

M2 profile, when $Gh_{n2}>h_2>h_{c2}$;

This could be a case of the discharge of lighter water into running denser water on a mild slope. The lighter water forms a pool. Mixing between the two layers can be expected downstream.

M3 profile when $Gh_{n2}>h_{c2}>h_2$;

This could be the discharge of denser water into the lower part of the lighter water pool from an opening lower than the stratified critical depth on a mild slope. An internal jump may occur downstream and mixing between the layers may result.

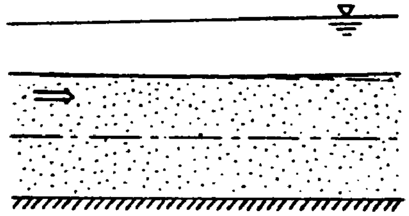
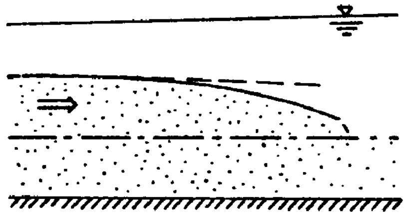
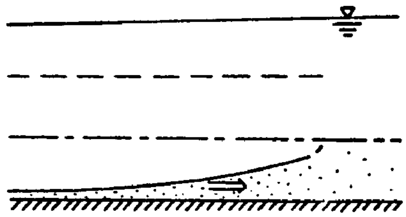
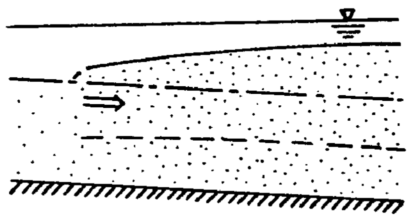
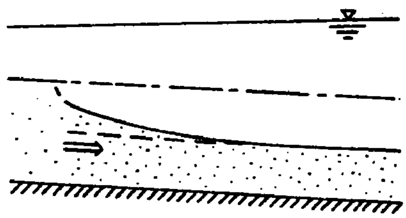
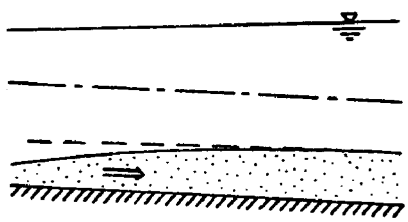
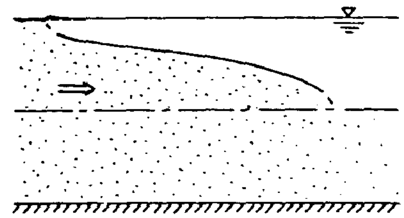
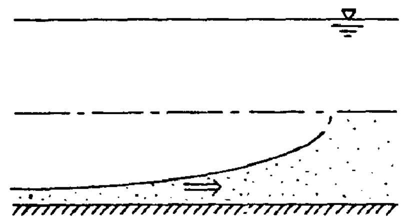
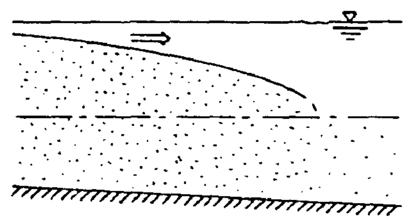
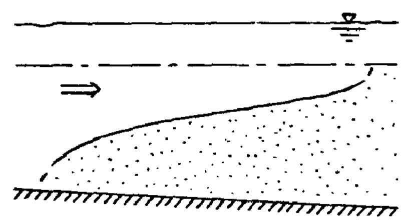
| Name of flow | Designation | Relation of $h_2$ to $h_{c2}$ and $Gh_{n2}$ | Sketch of profile |
|---|---|---|---|
| Mild slope | M1 | $h_2 > Gh_{n2} > h_{c2}$ |  |
| | M2 | $Gh_{n2} > h_2 > h_{c2}$ |  |
| | M3 | $Gh_{n2} > h_{c2} > h_2$ |  |
| Steep slope | S1 | $h_2 > h_{c2} > Gh_{n2}$ |  |
| | S2 | $h_{c2} > h_2 > Gh_{n2}$ |  |
| | S3 | $h_{c2} > Gh_{n2} > h_2$ |  |

Fig. 7 Possible flow regimes.

| Name of flow | Desig-nation | Relation of h to $h_c$ | Sketch of profile |
| --- | --- | --- | --- |
| Horizontal slope | H1 | $h_2>h_{c2}$ |  |
| | H2 | $h_{c2}>h_2$ |  |
| Adverse slope | A1 and A2 are essentially the same as H flow above. | | |
| Upper layer analysis | U1 | $h_{c1}>h_1$ |  |
| | U2 | $h_1>h_{c1}$ |  |

Note: Depth in U flow is measured from the free surface.

Notation: h= depth of a layer; $h_c$= stratified critical depth; $Gh_n2$= virtual stratified normal depth; subscripts: 1= upper layer; 2= lower layer.

Legend: = free surface; = interface;
= bottom; ⇒ = flow;
--- = virtual stratified normal depth line; —·— = stratified critical depth line.

Fig. 7 Continued.

2. Steep slope flow (indicated by "S"), when $h_{c2}>Gh_{n2}$:

S1 profile, when $h_2>h_{c2}>Gh_{n2}$;

This case might represent the flow of denser water into an impoundment created by a dam in a steep slope when the thickness of lower layer is larger than the stratified critical depth.

S2 profile, when $h_{c2}>h_2>Gh_{n2}$;

This might be the case of the density flow often observed in lakes and impoundments, if denser water happens to start out at a height between the stratified critical and virtual stratified normal depths. It may also represent the discharge of lighter water into running denser water on a steep slope. In either case, the two layers would not mix with each other quickly.

S3 profile, when $h_{c2}>Gh_{n2}>h_2$;

This might be the discharge of denser water into the lower portion of the lighter water pool from an opening lower than the virtual stratified normal depth on a steep slope. The two layers would not mix with ease.

Horizontal Slope Flow (denoted by "H")

When the flow is on a horizontal channel, the interfacial profile equation, Eq. 93, can be rewritten as follows.

$$\left[1-\frac{q_2^{\,2}}{(\Delta\rho/\rho)gh_2^{\,3}}\right]\frac{dh_2}{dx}=-\frac{q_2^{\,2}f_b}{8(\Delta\rho/\rho)gh_2^{\,3}}\,\frac{H+(f_i/f_b)H-h_2}{H-h_2} \qquad (95)$$

If a definition

$$G_H^{\,3}=\frac{H+(f_i/f_b)H-h_2}{H-h_2}=1+\frac{f_i/f_b}{1-(h_2/H)} \qquad (96)$$

is introduced*, then Eq. 93 becomes, for the H profile,

---

* $G_H$ is the G for the H profile. $G_H^{\,3}$ can be obtained by letting $h_{n2}=\infty$ in Eq. 92.

$$\frac{dh_2}{dx}=-\frac{f_b(G_H h_{c2}/h_2)^3}{8\ 1-(h_{c2}/h_2)^3} \tag{97}$$

with the definition of $h_{c2}$, Eq. 90.

It is easily observed that only the stratified critical depth is important to the horizontal slope flow. Applying arguments similar to the ones for the positive slope flow, the following two possible profiles were found.

H1 profile, when $h_2>h_{c2}$;
This could represent the discharge of lighter water into the running denser water. It is a profile similar to the M2 profile. Chow[3] would prefer the term "H2" profile; however, the term "H1" profile was chosen to show that there exist only two different profiles for the H profile. Oil slicks show a similar profile.[4]

H2 profile, when $h_{c2}>h_2$;
This profile is similar to the M3 profile.

<u>Adverse Slope Flow</u> (denoted by "A")

Adverse slope means the flow on a slope which has a negative value. The same Eq. 93 can be used for this flow; however, the positive value of the slope is used for convenience.

$$S_b=-|S_b| \tag{98}$$

Substitute this into Eqs. 93 and 94 and

$$\frac{dh_2}{dx}=-|S_b|\frac{1+(Gh_{n2}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{99}$$

where

$$Gh_{n2}=h_{c2}\ \frac{f_b}{8|S_b|}\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right] \tag{100}$$

Subsequently, only the stratified critical depth is an important factor for the A profile.

A1 profile, when $h_2 > h_{c2}$;
This is the same as the H1 profile, except that it is on the adverse slope.

A2 profile, when $h_{c2} > h_2$;
This profile is similar to the H2 profile.

Free Surface Profile Equation for the Hypolimnetic Flow

The equation for the total depth profile can be found by using Eqs. 81 and 85

$$\frac{dH}{dx} = S_b + \frac{f_i}{8}\frac{\Delta\rho}{\rho}\frac{{h_{c2}}^3}{{h_2}^2(H-h_2)} \tag{101}$$

This indicates that the free surface is approximately horizontal and shows a very slight increase toward the downstream direction, except when $h_2$ approaches either zero or the total depth.

Upper Layer Analysis (Designated by "U")

For this analysis, it is assumed that $U_2 = 0$. Then, Eqs. 75 and 76 can be written in the following form by substituting $U_1 = q_1/h_1$ from Eq. 73,

$$-\frac{q_1}{g{h_1}^3}\frac{dh_1}{dx} + \frac{dh_1}{dx} + \frac{dh_2}{dx} + S_{f1U} - S_b = 0 \tag{102}$$

$$(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx} + \frac{dh_2}{dx} + S_{f2U} - S_b = 0 \tag{103}$$

where subscript U indicates the upper layer analysis.

If $\tau_s$ is neglected again, Eqs. 77 and 78 can be written as

$$S_{f1U} = \frac{f_i {q_1}^2}{8g{h_1}^3} \tag{104}$$

and

$$S_{f2U}=-\frac{f_i q_1^2}{8gh_1^2 h_2} \tag{105}$$

Eliminating $dh_2/dx$ from Eqs. 102 and 103, substituting Eqs. 104 and 105, and rearranging them yields

$$\left[1-\frac{q_1^3}{(\Delta\rho/\rho)gh_1^3}\right]\frac{dh_1}{dx}=-\frac{f_i q_1^2 H}{8(\Delta\rho/\rho)gh_1^3 h_2} \tag{106}$$

If the stratified critical depth for the upper layer, $h_{c1}$, is defined by the relation

$$h_{c1}^3=\frac{q_1^2}{(\Delta\rho/\rho)g} \tag{107}$$

an equation of the interfacial profiles for the upper layer analysis can be obtained, in terms of $h_1$, as

$$\frac{dh_1}{dx}=-\frac{f_i}{8}\frac{(h_{c1}/h_1)^3/[1-(h_1/H)]}{1-(h_{c1}/h_1)^3} \tag{108}$$

As in the cases of the horizontal and adverse slope flow profiles, only the stratified critical depth is important for this upper layer analysis. Thus, two profiles appear possible. The interface becomes vertical at the points where $h_1$ approaches either $h_{c1}$ or H. Eq. 108 can be applied on any sloped bottom, since $S_b$ is not included in it.

U1 profile, when $h_{c1}>h_1$;

This could represent the release of less dense water on a pool of denser water. Mixing of the two layers may be expected downstream.

U2 profile, when $h_1>h_{c1}$;

This case could represent the familiar shape of the arrested salt wedge found in estuaries. As pointed out above, the shape of the wedge could be a sinuous curve even on a sloped bottom although it is well known that the wedge takes the sinuous form on the horizontal slope.

## Free Surface Profile Equation for the Upper Layer Analysis

The equation of total depth change can be obtained by using Eqs. 102, 104 and 108.

$$\frac{dH}{dx}=S_b-\frac{f_i}{8}\frac{\Delta\rho}{\rho}\frac{(h_{c1}/h_1)^3\{1+h_{c1}{}^3/[h_1{}^2(H-h_1)]\}}{1-(h_{c1}/h_1)^3} \qquad (109)$$

This indicates that the free surface would remain approximately horizontal and show a very slight increase towards the downstream direction for the U1 profile and a very slight decrease for the U2 profile, except when $h_1 = h_{c1}$, $h_1 = H$, and $h_1 = 0$.

### 5.2 NON-UNIFORM VELOCITY DISTRIBUTION CONSIDERED IN THE FLOWING LAYER

When non-uniform velocity distribution is assumed in a layer, the momentum coefficient, $\beta$, may be included as discussed in Sec. 4.4. Then, the equations of motion can be written as follows.

The equation of motion for the upper layer:

$$\frac{\beta U_1}{g}\frac{dU_1}{dx}+\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1}-S_b=0 \qquad (110)$$

The equation of motion for the lower layer:

$$\frac{\beta U_2}{g}\frac{dU_2}{dx}+(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f2}-S_b=0 \qquad (111)$$

Then, it is easy to derive interfacial profile equations by following the method of Sec. 5.1.

Positive Slope Flow

If the same definitions are used for the stratified normal depth, critical depth, and G, the interfacial profile equation for the positive slope flow can be written as

$$\frac{dh_2}{dx}=S_b\frac{1-(Gh_{n2}/h_2)^3}{1-\beta(h_{c2}/h_2)^3} \tag{112}$$

If a definition

$$h_{c2M}=\beta^{1/3}h_{c2} \tag{113}$$

is used, Eq. 112 becomes

$$\frac{dh_2}{dx}=S_b\frac{1-(Gh_{n2M}/h_2)^3}{1-(h_{c2M}/h_2)^3} \tag{114}$$

where

$$Gh_{n2M}=h_{c2M}\left\{\frac{f_b}{8\beta S_b}\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right]\right\}^{1/3} \tag{115}$$

Note that $Gh_{n2M} = Gh_{n2}$.

Horizontal Slope Flow

Similarly, the interfacial profile equation for the horizontal slope flow can be

$$\frac{dh_2}{dx}=-\frac{f_b}{8}\frac{(G_H h_{c2}/h_2)^3}{1-\beta(h_{c2}/h_2)^3} \tag{116}$$

With the definition $h_{c2M} = \beta^{1/3}h_{c2}$,

$$\frac{dh_2}{dx}=-\frac{f_b}{8\beta}\frac{(G_H h_{c2M}/h_2)^3}{1-(h_{c2M}/h_2)^3} \tag{117}$$

Adverse Slope Flow

$$\frac{dh_2}{dx}=-|S_b|\frac{1+(Gh_{n2}/h_2)^3}{1-\beta(h_{c2}/h_2)^3} \tag{118}$$

$$=-|S_b|\frac{1+(Gh_{n2M}/h_2)^3}{1-(h_{c2M}/h_2)^3} \tag{119}$$

where

$$Gh_{n2M}=h_{c2M}\left\{\frac{f_b}{8\beta|S_b|}\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right]\right\}^{1/3} \tag{120}$$

Free Surface Profile Equation for the Hypolimnetic Flow

The free surface profile equation for the hypolimnetic flow is not influenced by $\beta$, but will change its appearance slightly.

$$\frac{dH}{dx}=S_b+\frac{f_i}{8}\frac{\Delta\rho}{\rho}\frac{1}{\beta}\frac{h_{c2M}{}^3}{h_2{}^2(H-h_2)} \tag{121}$$

Upper Layer Analysis

A similar derivation results in the following interfacial profile equation for the upper layer analysis.

$$\frac{dh_1}{dx}=-\frac{f_i}{8}\frac{(h_{c1}/h_1)^3/[1-(h_1/H)]}{1-\beta(h_{c1}/h_1)^3} \tag{122}$$

If a definition

$$h_{c1M}=\beta^{1/3}h_{c1} \tag{123}$$

is used, Eq. 122 becomes

$$\frac{dh_1}{dx}=-\frac{f_i}{8\beta}\frac{(h_{c1M}/h_1)\ /[1-(h_1/H)]}{1-(h_{c1M}/h_1)^3} \tag{124}$$

Free Surface Profile Equation for the Upper Layer Analysis

The free surface profile equation for the upper layer analysis can be written as

$$\frac{dH}{dx}=S_b-\frac{f_i}{8}\frac{\Delta\rho}{\rho}\frac{1}{\beta}\frac{(h_{c1M}/h_1)^3\quad 1+h_{c1M}{}^3/[\beta h_1{}^2(H-h_1)]}{1-(h_{c1M}/h_1)^3} \qquad (125)$$

This indicates that the free surface would remain approximately horizontal.

## 5.3 NON-DIMENSIONAL INTERFACIAL AND FREE SURFACE PROFILE EQUATIONS

For convenience of computation and to obtain a more universal expression of results, the interfacial and free surface profile equations obtained in the previous sections can be non-dimensionalized.

Non-dimensional depth of a running layer, total depth, virtual stratified normal depth, and distance along a channel are defined as functions of the stratified critical depth as follows.

$$h_N=h/h_c \qquad (126)$$

$$H_N=H/h_c \qquad (127)$$

$$Gh_{n2N}=Gh_{n2}/h_c \text{ for positive slope flow} \qquad (128)$$

$$x_N= S_b\ x/h_c \text{ for positive and adverse slope flow} \qquad (129)$$

and

$$x_N=x/h_c \text{ for horizontal slope flow and upper layer analysis} \qquad (130)$$

Also, if the momentum coefficient, $\beta$, is included in the equations,

$$h_{cM}=\beta^{1/3}h_c \qquad (131)$$

$$h_{NM}=h/h_{cM} \qquad (132)$$

$$H_{NM}=H/h_{cM} \qquad (133)$$

$$Gh_{n2MN}=Gh_{n2M}/h_{c2M} \text{ for positive slope flow} \qquad (134)$$

$$x_{NM}= S_b\ x/h_{cM} \text{ for positive and adverse slope flow} \qquad (135)$$

and $x_{NM}=x/h_{cM}$ for horizontal slope flow and upper layer analysis (136)

<u>For Positive Slope Flow</u>

Interfacial profile equation:

$$\frac{dh_{2N}}{dx_N}=\frac{h_{2N}^3-G_{hn2N}^3}{h_{2N}^3-1} \tag{137}$$

where $$G_{hn2N}^3=\frac{f_b}{8S_b}\left[1+\frac{f_i/f_b}{1-(h_{2N}/H_N)}\right] \tag{138}$$

If the momentum coefficient, β, is included,

$$\frac{dh_{2NM}}{dx_{NM}}=\frac{h_{2NM}^3-G_{hn2MN}^3}{h_{2NM}^3-1} \tag{139}$$

where $$G_{hn2MN}^3=\frac{f_b}{8\beta S_b}\left[1+\frac{f_i/f_b}{1-(h_{2NM}/H_{NM})}\right] \tag{140}$$

Free surface profile equation:

$$\frac{dH_N}{dx_N}=1+\frac{(f_i/8)(\Delta\rho/\rho)/S_b}{h_{2N}^2(H_N-h_{2N})} \tag{141}$$

With β,

$$\frac{dH_{NM}}{dx_{NM}}=1+\frac{(f_i/8)(\Delta\rho/\rho)/(S_b\beta)}{h_{2NM}^2(H_{NM}-h_{2NM})} \tag{142}$$

<u>For Horizontal Slope Flow</u>

Interfacial profile equation:

$$\frac{dh_{2N}}{dx_N}=\frac{(f_b/8)G_{HN}^3}{1-h_{2N}^3} \tag{143}$$

where $$G_{HN}^3=1+\frac{f_i/f_b}{1-(h_{2N}/H_N)} \tag{144}$$

With $\beta$,

$$\frac{dh_{2NM}}{dx_{NM}}=\frac{[f_b/(8\beta)]G_{HNM}^3}{1-h_{2NM}^3} \tag{145}$$

where $$G_{HNM}^3=1+\frac{f_i/f_b}{1-(h_{2NM}/H_{NM})} \tag{146}$$

Free surface profile equation:

$$\frac{dH_N}{dx_N}=\frac{(f_i/8)(\Delta\rho/\rho)}{h_{2N}^2(H_N-h_{2N})} \tag{147}$$

With $\beta$,

$$\frac{dH_{NM}}{dx_{NM}}=\frac{(f_i/8)(\Delta\rho/\rho)/\beta}{h_{2NM}^2(H_{NM}-h_{2NM})} \tag{148}$$

<u>For Adverse Slope Flow</u>

Interfacial profile equation:

$$\frac{dh_{2N}}{dx_N}=\frac{h_{2N}^3+Gh_{n2N}^3}{1-h_{2N}^3} \tag{149}$$

With $\beta$,

$$\frac{dh_{2NM}}{dx_{NM}}=\frac{h_{2NM}^3+Gh_{n2MN}^3}{1-h_{2NM}^3} \tag{150}$$

Free surface profile equation:

$$\frac{dH_N}{dx_N}=-1+\frac{(f_i/8)(\Delta\rho/\rho)/|S_b|}{h_{2N}^2(H_N-h_{2N})} \tag{151}$$

With $\beta$,

$$\frac{dH_{NM}}{dx_{NM}}=-1+\frac{(f_i/8)(\Delta\rho/\rho)/(|S_b|\beta)}{h_{2NM}^2(H_{NM}-h_{2NM})} \tag{152}$$

For Upper Layer Analysis

Interfacial profile equation:

$$\frac{dh_{1N}}{dx_N}=\frac{(f_i/8)/[1-(h_{1N}/H_N)]}{1-h_{1N}^3} \tag{153}$$

With $\beta$,

$$\frac{dh_{1NM}}{dx_{NM}}=\frac{[f_i/(8\beta)]/[1-(h_{1NM}/H_{NM})]}{1-h_{1NM}^3} \tag{154}$$

Free surface profile equation:

$$\frac{dH_N}{dx_N}=S_b+\frac{(f_i/8)(\Delta\rho/\rho)\{1+1/[h_{1N}^2(H_N-h_{1N})]\}}{1-h_{1N}^3} \tag{155}$$

With $\beta$,

$$\frac{dH_{NM}}{dx_{NM}}=S_b+\frac{[f_i/(8\beta)](\Delta\rho/\rho)\{1+1/[\beta h_{1NM}^2(H_{NM}-h_{1NM})]\}}{1-h_{1NM}^3} \tag{156}$$

## 5.4 INTERFACIAL PROFILE EQUATIONS FOR HYPOLIMNETIC FLOW OF A UNIT WIDTH, WHEN INTERNAL CIRCULATION IS CONSIDERED WITHIN A LAYER OF NO NET FLOWRATE

In the previous section, one of the layers is assumed to have no motion at all. It is, however, learned [Ippen[5], Harleman[6], and Polk et al[7]] that circulation of water within the layer having no flow rate can be generated by the moving water of the other layer and its average flow rate in the layer is very close to zero. Therefore, the layer can be said to have no net flow rate.

However, in this case, the assumptions related to the homogenuity in the layer of no net flow rate (assumptions 19, 20 and 23 listed in Sec. 4.3) cannot be made. The changes in the forms of the interfacial profile equations derived in the previous section are studied below.

If the assumptions mentioned above are not used for the upper layer and homogenuity of velocity is still assumed for the lower layer, equations of continuity and motion for this analysis can be written as follows (see Eqs. 16 and 39). Equation of continuity for the upper layer:

$$\int_{h_2}^{h_1+h_2} \frac{\partial}{\partial x} \langle v_x \rangle dz + \langle v_x(h_1+h_2) \rangle \frac{d(h_1+h_2)}{dx} - \langle v_x(h_2) \rangle \frac{dh_2}{dx} = 0 \qquad (157)$$

Equation of continuity for the lower layer:

$$h_2 U_2 = q_2 = \text{constant} \qquad (158)$$

Equation of motion for the upper layer:

$$\frac{1}{gh_1} \int_{h_2}^{h_1+h_2} \frac{\partial}{\partial x} \langle v_x \rangle^2 dz + \frac{\langle v_x(h_1+h_2) \rangle^2}{gh_1} \frac{d(h_1+h_2)}{dx} - \frac{\langle v_x(h_2) \rangle^2}{gh_1} \frac{dh_2}{dx} + \frac{dh_1}{dx} + \frac{dh_2}{dx} + S_{f1rL} - S_b = 0 \qquad (159)$$

Equation of motion for the lower layer:

$$\frac{U_2}{g} \frac{dU_2}{dx} + \left(1 - \frac{\Delta \rho}{\rho}\right) \frac{dh_1}{dx} + \frac{dh_2}{dx} + S_{f2rL} - S_b = 0 \qquad (160)$$

where $\langle v_x(h) \rangle$ = average velocity in the x direction integrated over lamina at depth h,

$$S_{f1rL} = -\frac{f_{ir} q_2^2}{8 g h_1 h_2^2} \qquad (161)$$

and

$$S_{f2rL} = \frac{q_2^2 (f_b + f_{ir})}{8 g h_2^3} \qquad (162)$$

Subscript r in $f_{ir}$ ans $S_{frL}$ indicates that $f_i$ is influenced by circulation within a layer: it is assumed that the definitions of $\tau_b$ and $\tau_i$, Eqs. 79 and 80, can still be used for this case and it may be considered that, after stable circulation is established in a layer, the internal friction would be different from that for the case of no circulation due to the motion of the lower part of the upper layer.

An assumption of no net flow rate can be expressed as

$$\int_{h_2}^{h_1+h_2}\langle v_x\rangle dz=0 \tag{163}$$

Naturally, this assumption satisifies Eq. 157, since it says

$$\frac{d}{dx}\int_{h_2}^{h_1+h_2}\langle v_x\rangle dz=0$$

Subtracting [Eq. 157 x $(1/gh_1)\langle v_x(h_2)\rangle$ ] from Eq. 159 yields

$$\frac{1}{gh_1}\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz-\frac{1}{gh_1}\langle v_x(h_2)\rangle\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz$$

$$+\frac{1}{gh_1}[\langle v_x(h_1+h_2)\rangle^2-\langle v_x(h_1+h_2)\rangle\langle v_x(h_2)\rangle]\frac{d(h_1+h_2)}{dx}$$

$$+\frac{d(h_1+h_2)}{dx}+S_{f1rL}-S_b=0 \tag{164}$$

If following definitions are used,

$$B=\frac{1}{gh_1}\langle v_x(h_1+h_2)\rangle[\langle v_x(h_1+h_2)\rangle-\langle v_x(h_2)\rangle] \tag{165}$$

and $$S_{r1}=\frac{1}{gh_1}\left(\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz\right.$$

$$\left.-\langle v_x(h_2)\rangle\int_{h_2}^{h_1+h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz\right) \qquad (166)$$

where $S_r$ is used for the friction slope caused by the circulation, Eq. 164 is now

$$(1+B)\frac{d(h_1+h_2)}{dx}+S_{f1rL}-S_b+S_{r1}=0 \qquad (167)$$

From Eqs. 158 and 160,

$$(1-\frac{\Delta\rho}{\rho})\frac{dh_1}{dx}+(1-\frac{q_2^2}{gh_2^3})\frac{dh_2}{dx}+S_{f2rL}-S_b=0 \qquad (168)$$

The elimination of $dh_1/dx$ from Eqs. 167 and 168 with the use of Eqs. 161 and 162 gives

$$[1-\frac{q_2^2}{(\Delta\rho/\rho)gh_2^3}]\frac{dh_2}{dx}=\frac{(\Delta\rho/\rho)+B}{(\Delta\rho/\rho)(1+B)}S_b$$

$$x\{1-\frac{1+B}{(\Delta\rho/\rho)+B}\frac{q_2^2}{8gS_bh_2^3}\frac{(f_b+f_{ir})h_1+f_{ir}h_2[1-(\Delta\rho/\rho)]/(1+B)}{h_1}$$

$$+\frac{1-(\Delta\rho/\rho)}{B+(\Delta\rho/\rho)}\frac{S_{r1}}{S_b}\} \qquad (169)$$

The definition of the stratified critical depth for the lower layer, $h_{c2}$, is used, since the left hand side of the above equation has not changed from Eq. 88. Then, the interfacial profile equation for hypolimnetic flow considering the effect of internal circulation can be expressed as

$$\frac{dh_2}{dx}=\frac{1}{1-(h_{c2}/h_2)^3}\frac{(\Delta\rho/\rho)+B}{(\Delta\rho/\rho)(1+B)}S_b(1-D_p+E_p) \qquad (170)$$

where $$D_P=\frac{(\Delta\rho/\rho)(1+B)}{(\Delta\rho/\rho)+B}\frac{f_b}{8S_b}\left(\frac{h_{c2}}{h_2}\right)^3$$

$$\times\left\{1+\frac{f_{ir}/f_b}{1-(h\ /H)}\frac{[1-(h_2/H)](1+B)-[1-(\Delta\rho/\rho)]}{1+B}\right\} \tag{171}$$

and $$E_P=\frac{1-(\Delta\rho/\rho)S_{r1}}{B+(\Delta\rho/\rho)\ S_b} \tag{172}$$

It was not attempted to define the stratified normal depth for this equation.

For the horizontal slope flow, the derivation of the interfacial profile equation is done as follows. $S_b$ = 0 is used in Eqs. 167 and 168 and the same procedure as used for the sloped flow is followed.

$$\frac{dh_2}{dx}=-\frac{f_b}{8}\frac{(h_{c2}/h_2)^3}{1-(h_{c2}/h_2)^3}$$

$$\times\left\{1+\frac{f_{ir}/f_b}{1-(h_2/H)}\frac{[1-(h_2/H)](1+B)-[1-(\Delta\rho/\rho)]}{1+B}-E_H\right\} \tag{173}$$

where $$E_H=\frac{S_{r1}}{(f_b/8)(h_{c2}/h_2)^3}\frac{1-(\Delta\rho/\rho)}{(\Delta\rho/\rho)(1+B)} \tag{174}$$

For the adverse slope flow, Eq. 170 for the positive slope flow can be used.

The free surface profile equation can be obtained from Eqs. 161 and 167.

$$\frac{dH}{dx}=\frac{1}{1+B}\left[S_b+\frac{f_{ir}}{8}\frac{\Delta\rho}{\rho}\frac{h_{c2}{}^3}{h_2{}^2(H-h_2)}-S_{r1}\right] \tag{175}$$

If the free surface is found to remain horizontal in experiments, this equation can indicate that the effect of B and $S_{r1}$ may be ignored. Both B and $S_{r1}$ are expected to assume positive value and, if so, dH/dx should be smaller than that for the case without considering the internal circulation.

Upper Layer Analysis

For this analysis, the equations of continuity and motion are as follows.
Equation of continuity for the upper layer:

$$h_1U_1=q_1=\text{constant} \tag{176}$$

Equation of continuity for the lower layer:

$$\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz+\langle v_x(h_2)\rangle\frac{dh_2}{dx}=0 \tag{177}$$

Equation of motion for the upper layer:

$$\frac{U_1}{g}\frac{dU_1}{dx}+\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1rU}-S_b=0 \tag{178}$$

Equation of motion for the lower layer:

$$\frac{1}{gh_2}\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz+\frac{\langle v_x(h_2)\rangle^2}{gh_2}\frac{dh_2}{dx}$$

$$+\left(1-\frac{\Delta\rho}{\rho}\right)\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f2rU}-S_b=0 \tag{179}$$

where

$$S_{f1rU}=\frac{f_{ir}q_1^2}{8gh_1^3} \tag{180}$$

and

$$S_{f2rU}=\frac{(f_{br}-f_{ir})q_1^2}{8gh_1^2h_2} \tag{181}$$

Again, it is assumed that $\tau_b$ can be expressed in terms of $f_{br}$, $q_1$, $h_1$, and $h_2$. The subscript r in $f_{br}$ indicates the effect of the circulation as before. Subtracting Eqs. 177 multiplied by $\langle v_x(h_2)\rangle/(gh_2)$ from Eq. 179 gives

$$\frac{1}{gh_2}\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz-\frac{\langle v_x(h_2)\rangle}{gh_2}\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz$$

$$+\left(1-\frac{\Delta\rho}{\rho}\right)\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f2rU}-S_b=0 \tag{182}$$

If $S_{r2}$ is defined by

$$S_{r2}=\frac{1}{gh_2}\left(\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle^2 dz-\langle v_x(h_2)\rangle\int_0^{h_2}\frac{\partial}{\partial x}\langle v_x\rangle dz\right) \tag{183}$$

Eq. 182 is

$$\left(1-\frac{\Delta\rho}{\rho}\right)\frac{dh_1}{dx}+\frac{dh_2}{dx}-S_b+S_{f2rU}+S_{r2}=0 \tag{184}$$

Also, from Eqs. 176 and 178,

$$-\frac{q_1{}^2}{gh_1{}^3}\frac{dh_1}{dx}+\frac{dh_1}{dx}+\frac{dh_2}{dx}+S_{f1rU}-S_b=0 \tag{185}$$

The interfacial profile equation can be found, from Eqs. 180, 181, 184 and 185 and the definition of the stratified critical depth for the upper layer, as

$$\frac{dh_1}{dx}=-\frac{(f_{ir}/8)(h_{c1}/h_1)^3[1-(h_1/H)(f_{br}/f_{ir})]}{[1-(h_{c1}/h_1)^3][1-(h_1/H)]}+E_U \tag{186}$$

where
$$E_U=\frac{S_{r2}}{(\Delta\rho/\rho)[1-(h_{c1}/h_1)^3]} \tag{187}$$

For the free surface profile equation, Eqs. 176 and 186 are used to find

$$\frac{dH}{dx}=S_b-\frac{f_{ir}}{8}\frac{\Delta\rho}{\rho}\left(\frac{h_{c1}}{h_1}\right)^3\frac{1-(h_1/H)+(h_{c1}/h_1)^3(h_1/H)(1-f_{br}/f_{ir})}{[1-(h_{c1}/h_1)^3][1-(h_1/H)]}$$

$$+S_{r2}\frac{(h_{c1}/h_1)^3}{1-(h_{c1}/h_1)^3} \tag{188}$$

As can be seen, the forms of the interfacial and free surface profile equations are not as simple as the previous ones because of the terms arising from the internal circulation. However, the general forms of the equations remain unchanged. Furthermore, it is learned where the effect of the circulation appears in the equations.

## 5.5 INTERFACIAL PROFILE EQUATIONS FOR HYPOLIMNETIC FLOW IN A RECTANGULAR EXPERIMENTAL FLUME

In this section, the change in the form of the interfacial profile equation is studied when the effect of the side walls of a rectangular flume is considered. The width of the flume is designated by "b" and assumed to be constant. The change does not occur in the equations of continuity and motion, Eqs. 73, 74, 75 and 76, but in the definitions of the friction slope, which are designated by $S_{fW}$.

$$S_{f1w}=\frac{\tau_i-\tau_s+\tau_w(W_1/b)}{\rho g h_1} \tag{189}$$

and

$$S_{f2w}=\frac{\tau_b-\tau_i+\tau_w(W_2/b)}{\rho g h_2} \tag{190}$$

where $\tau_w$ = wall shear stress

$W_1$ = wetted perimeter corresponding to the upper layer

$W_2$ = wetted perimeter corresponding to the lower layer, except for the bottom.

If it is assumed that

$$\tau_w=\frac{\rho f_w}{8}U_2{}^2 \tag{191}$$

the friction slopes, $S_f$, can be written as

$$S_{f1L}=-\frac{f_i q_2^2}{8gh_1h_2^2} \tag{192}$$

and

$$S_{f2wL}=-\frac{q_2^2[f_b+f_i+2(h_2/b)f_w]}{8gh_2^3} \tag{193}$$

since $W_2=2h_2$ and $\tau_w=0$ can be assumed for the upper layer. By a similar derivation shown in Sec. 5.1,

$$[1-\frac{q_2^2}{(\Delta\rho/\rho)gh_2^3}]\frac{dh_2}{dx}=S_b[1-\frac{q_2^2 f_b}{8(\Delta\rho/\rho)gS_bh_2^3}$$

$$\times\frac{H+(f_i/f_b)H-h_2+2h_2(H-h_2)f_w/(f_bb)}{H-h_2}] \tag{194}$$

The stratified normal depth for the lower layer including the wall friction, $h_{n2w}$, is defined by

$$h_{n2w}^3=h_{c2}^3\frac{f_b}{8S_b}[1+\frac{f_i/f_b}{1-(h_{n2w}/H)}+\frac{2h_{n2w}f_w}{f_bb}] \tag{195}$$

Note that the stratified critical depth has not changed. Also

$$G_w^3=[1+\frac{f_i/f_b}{1-(h_2/H)}+\frac{2h_2f_w}{f_bb}]/[1+\frac{f_i/f_b}{1-(h_{n2w}/H)}+\frac{2h_{n2w}f_w}{f_bb}] \tag{196}$$

Then,

$$G_wh_{n2w}=h_{c2}[\frac{f_b}{8S_b}(1+\frac{f_i/f_b}{1-(h_2/H)}+\frac{2h_2f_w}{f_bb})]^{1/3} \tag{197}$$

Now the equation of the interfacial profile for the hypolimnetic flow can be written in the form

$$\frac{dh_2}{dx}=S_b\frac{1-(G_w h_{n2w}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{198}$$

If this equation is compared with Eq. 93, it is easily found that the difference between the two is the term $2h_2f_w/(f_bb)$.

For the horizontal slope flow, $G_H$, now becomes $G_{Hw}$ defined by

$$G_{Hw}^3=1+\frac{f_i/f_b}{1-(h_2/H)}+\frac{2h_2f_w}{f_bb} \tag{199}$$

And

$$\frac{dh_2}{dx}=-\frac{f_b}{8}\frac{(G_{Hw}h_{c2}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{200}$$

On the adverse slope,

$$\frac{dh_2}{dx}=-|S_b|\frac{1+(G_w h_{n2w}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{201}$$

where $$G_w h_{n2w}=h_{c2}\left[\frac{f_b}{8|S_b|}\left(1+\frac{f_i/f_b}{1-(h_2/H)}+\frac{2h_2f_w}{f_bb}\right)\right]^{1/3} \tag{202}$$

The free surface profile equation remains the same regardless of the inclusion of the wall friction.

Upper Layer Analysis

For this analysis, the friction slope may be stated as

$$S_{f1wU}=\frac{q_1^2[f_i+2(h_1/b)f_w]}{8gh_1^3} \tag{203}$$

and $$S_{f2U}=-\frac{f_iq_1^2}{8gh_1^2h_2} \tag{204}$$

Then,

$$\left[1-\frac{q_1^2}{(\Delta\rho/\rho)gh_1^3}\right]\frac{dh_1}{dx}=-\frac{q_1^2 f_i H+[2q_1^2 h_1(H-h_1)f_w/b]}{8(\Delta\rho/\rho)gh_1^3 h_2} \quad (205)$$

With the stratified critical depth (Eq. 107)

$$\frac{dh_1}{dx}=-\frac{(f_i/8)(h_{c1}/h_1)^3}{[1-(h_{c1}/h_1)^3]}\left[\frac{1}{1-(h_1/H)}+\frac{2h_1 f_w}{f_i b}\right] \quad (206)$$

The free surface profile equation may be written as

$$\frac{dH}{dx}=S_b-\frac{f_i}{8}\frac{\Delta\rho}{\rho}\frac{(h_{c1}/h_1)^3}{1-(h_{c1}/h_1)^3}\left[1+\frac{h_{c1}^3}{h_1^2(H-h_1)}+\frac{2f_w h_1}{f_i b}\right] \quad (207)$$

## 5.6 FREE SURFACE PROFILE EQUATION FOR OPEN-CHANNEL FLOW ON A HORIZONTAL SLOPE IN A RECTANGULAR EXPERIMENTAL FLUME

### Free Surface Profile Equation

For a channel with constant width, equations of continuity and motion for steady state open-channel flow are as follows. (See Sec. 4.5).
Equation of continuity:

$$hU=q=\text{constant} \quad (208)$$

Equation of motion:

$$\frac{U}{g}\frac{dU}{dx}+\frac{dh}{dx}+S_f-S_b=0 \quad (209)$$

where

$$S_f=\frac{\tau_b-\tau_s+(\tau_w W/b)}{\rho g h} \quad (210)$$

If it is assumed that

$$\tau_b=\frac{\rho}{8}f_b U^2 \quad (211)$$

$$\tau_w=\frac{\rho}{8}f_w U^2 \quad (212)$$

and $$\tau_s=0,$$
Eq. 210

$$S_f=\frac{q^2[f_b+(2hf_w/b)]}{8gh^3} \tag{213}$$

where b = width of the flume and h is used for W.

If an equivalent friction factor for the bottom and the wall, $f_q$, can be defined by

$$f_q(1+\frac{2h}{b})=f_b+\frac{2hf_w}{b} \tag{214}$$

$S_f$ becomes

$$S_f=\frac{f_q[1+(2h/b)]q^2}{8gh^3} \tag{215}$$

The free surface profile equation can be derived easily from Eqs. 208 and 209 by eliminating U.

$$\frac{dh}{dx}=S_b\frac{1-(S_f/S_b)}{1-(q^2/gh^3)} \tag{216}$$

For the horizontal slope flow,

$$\frac{dh}{dx}=-\frac{S_f}{1-(q^2/gh^3)} \tag{217}$$

The critical depth is defined by

$$h_c{}^3=q^2/g \tag{218}$$

Then, the free surface flow profile for the horizontal slope flow can be expressed as

$$\frac{dh}{dx}=-\frac{S_f}{1-(h_c/h)^3} \tag{219}$$

where
$$S_f=\frac{f_q}{8}\left(1+\frac{2h}{b}\right)\left(\frac{h_c}{h}\right)^3 \tag{220}$$

If the momentum coefficient in this discussion is included, Eq. 219 can be written as
$$\frac{dh}{dx}=-\frac{S_f}{1-\beta(h_c/h)^3} \tag{221}$$

with the same definition of $S_f$.

Relation Between Hydraulic Radius and Friction Factor

Recalling the assumption that, in open-channel flow, the uniform flow forumula can be used for the gradually varied flow, some relations between hydraulic radius and friction factor are derived for later use.

For uniform flow, in Eq. 209
$$S_f = S_b \tag{222}$$

From the assumption in Eq. 215,
$$U^2=\frac{q^2}{h^2}=\frac{8g}{f_q}S_b\frac{bh}{b+2h} \tag{223}$$

Since hydraulic radius, R, is
$$R=\frac{bh}{b+2h},$$

$$U^2=\frac{8g}{f_q}S_bR \tag{224}$$

If it is assumed that this uniform formula can be applied to the hydraulic radius corresponding to the bottom, $R_b$, and to the wall, $R_w$,[8] relations
$$U^2=\frac{8g}{f_b}S_bR_b \tag{225}$$

and

$$U^2=\frac{8g}{f_w}S_bR_w \tag{226}$$

can be obtained.

Then, a relation

$$\frac{R}{f_q}=\frac{bh}{f_q(b+2h)}=\frac{R_b}{f_b}=\frac{R_w}{f_w} \tag{227}$$

may be obtained.
Furthermore, a relation

$$bh=(b+2h)R=bR_b+2hR_w \tag{228}$$

may be found if the definition of the hydraulic radius is considered.

## SECTION VI

## EXPERIMENTAL SYSTEM

Prior to the design of the experimental system, all requirements and restructions were considered in order to construct it in an organized manner.

### 6.1 CONSIDERATION OF PARAMETERS INVOLVED

As shown in Sec. 5.1, the positions of the interface relative to the stratified critical and virtual stratified normal depths produce the various interfacial profiles. The interfacial profile equations show the parameters to be included in the design and construct an experimental system for the verification of the proposed interfacial profiles.

The definitions of the stratified critical and virtual stratified normal depths in the lower layer analysis and the stratified critical depth for the upper layer analysis are listed below.

$$h_{c2}=\left[\frac{q_2^{\,2}}{(\Delta\rho/\rho)g}\right]^{1/3} \tag{229}$$

$$Gh_{n2}=h_{c2}\left(\frac{f_b}{8S_b}\right)^{1/3}\left[1+\frac{f_i/f_b}{1-(h_2/H)}\right]^{1/3} \tag{230}$$

$$h_{c1}=\left[\frac{q_1^{\,2}}{(\Delta\rho/\rho)g}\right]^{1/3} \tag{231}$$

The stratified critical depths (Eqs. 229 and 231) can be determined when the flow rate of the flowing layer and density of the two layers are known.

The parameters needed for the virtual stratified normal depth (Eq. 230) are: the slope of the channel bottom, the total depth of the flow, the depth of one of the layers, the friction factors at the bottom and at the interface.

The experimental system should be able to provide various flow rates, densities of the water, and slopes of the channel bottom. Further, artificial roughness is necessary to provide bottom friction and to offer somewhat easier determination of its magnitude.

## 6.2 REQUIREMENTS AND RESTRICTIONS FOR THE EXPERIMENTAL SYSTEM

As indicated in the previous section, the experimental system is designed to handle the following parameters: slope of channel bottom, density of water, depth of flow, flow rate, roughness of a channel, and end conditions.

As pointed out in the introduction, most of the studies which have been published on the two-layered flow were limited to the case of uniform total depth in the flow, i.e., to the horizontal slope flow. In this study, the flume can be tilted to give the desired slope to the flume bottom.

In addition, the previous studies have used chemicals, such as salt, to obtain density differences in the flow. This experiment uses the difference in density produced by temperature differences.

The depth of one of the layers, or the position of the interface is determined by measuring the vertical temperature distribution in the flow.

The space available for all structures, piping and equipment was approximately 19.2 meters (63 feet) long, 3.05 meters (10 feet) wide and 3.36 meters (11 feet) high, including unusuable space which is round stairwells for two spiral staircases of approximately 1.83 meter (6 foot) diameter. The system includes a flume with a tilting device, a system to supply water of two different temperatures (or of different chemical concentrations) at different flowrates, and equipment to measure the temperature of water (or chemical concentration).

Preliminary studies proved that the flume needed "inlet-outlet boxes" at both ends where water enters and leaves the flume and a "supporting frame" to support a load -- that is, water, the flume, and the boxes. Therefore, the flume and its accompanying structure is called the "flume structure." The system to supply water is called the "water supply system," The equipment to measure the temperature is called the "temperature measuring equipment."

Detailed requirements and restrictions for the experimental system are listed in the following sections.

Flume Structure

1. The maximum gross space available for the flume structure is approximately 12.2 meters (40 feet) long, 1.22 meters (4 feet) wide, and 3.36 meters (11 feet) high. This height includes the clearance required when one end of the flume is tilted up.
2. The walls of the flume are transparent so that the state of flow can be observed from the sides of the flume.
3. The inside of the flume is as smooth as possible.
4. The flume is made in sections to provide control weirs, side inlet and outlet devices, and changes of geometry, when necessary.
5. Two inlet-outlet boxes with the depth deeper and the width wider than the flume are placed at both ends of the flume to give the water supply system a chance to stabilize before flowing into the flume, and to provide a drain.
6. The deflection of the supporting frame is as small as possible. The deflections of the frame at the center and both ends are approximately equal.
7. The supporting frame is made in sections for ease of construction and transport.
8. The supporting frame is able to support the flume with a width of approximately 0.914 meters (3 feet).
9. A "tilting device" is able to give a slope of approximately 1/10 to the flume bottom.

10. Bottom roughness provides friction much larger than that found in nature since the flow rate obtainable in the flume is limited. The bottom roughness is removable from the flume.
11. A gate can be installed at several points along the flume, and its opening is adjustable.

Temperature Measuring Equipment

1. The temperature of the water in the system supplying the flume and the air temperature is recorded in order to determine if the flow in the flume has reached a nearly-steady state.
2. The temperature of the water in the flume is measurable vertically and longitudinally to find the temperature distribution profile of the flow.

## 6.3 DESIGN AND CONSTRUCTION OF THE EXPERIMENTAL SYSTEM

The experimental system is designed and constructed to fulfill the requirements and restrictions are stated in Sec. 6.2.

A detailed sketch of the flume is shown in Figure 8. A schematic diagram of the water supply system is shown in Figure 9. Further details of the system are given in Yanagida.[8]

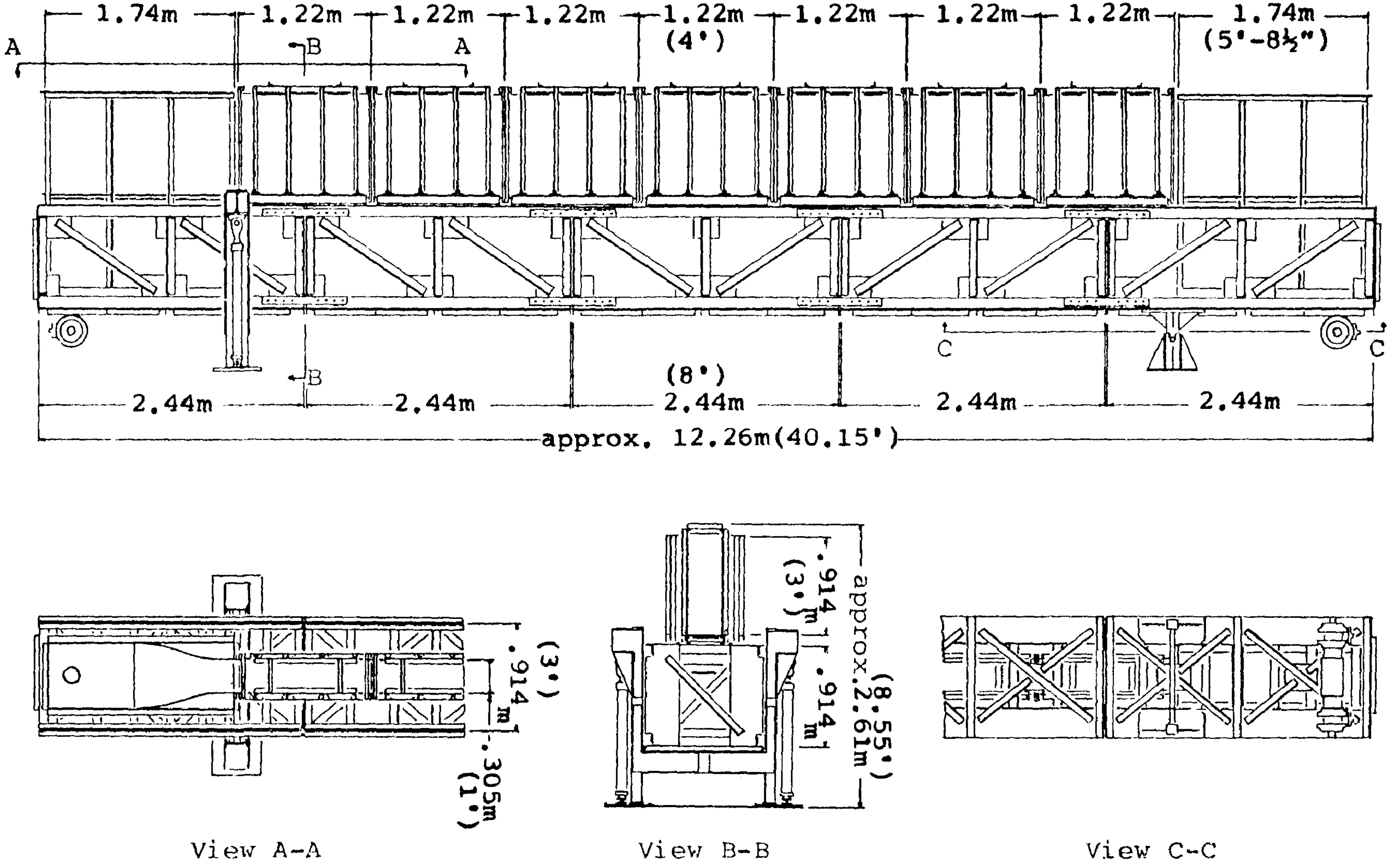
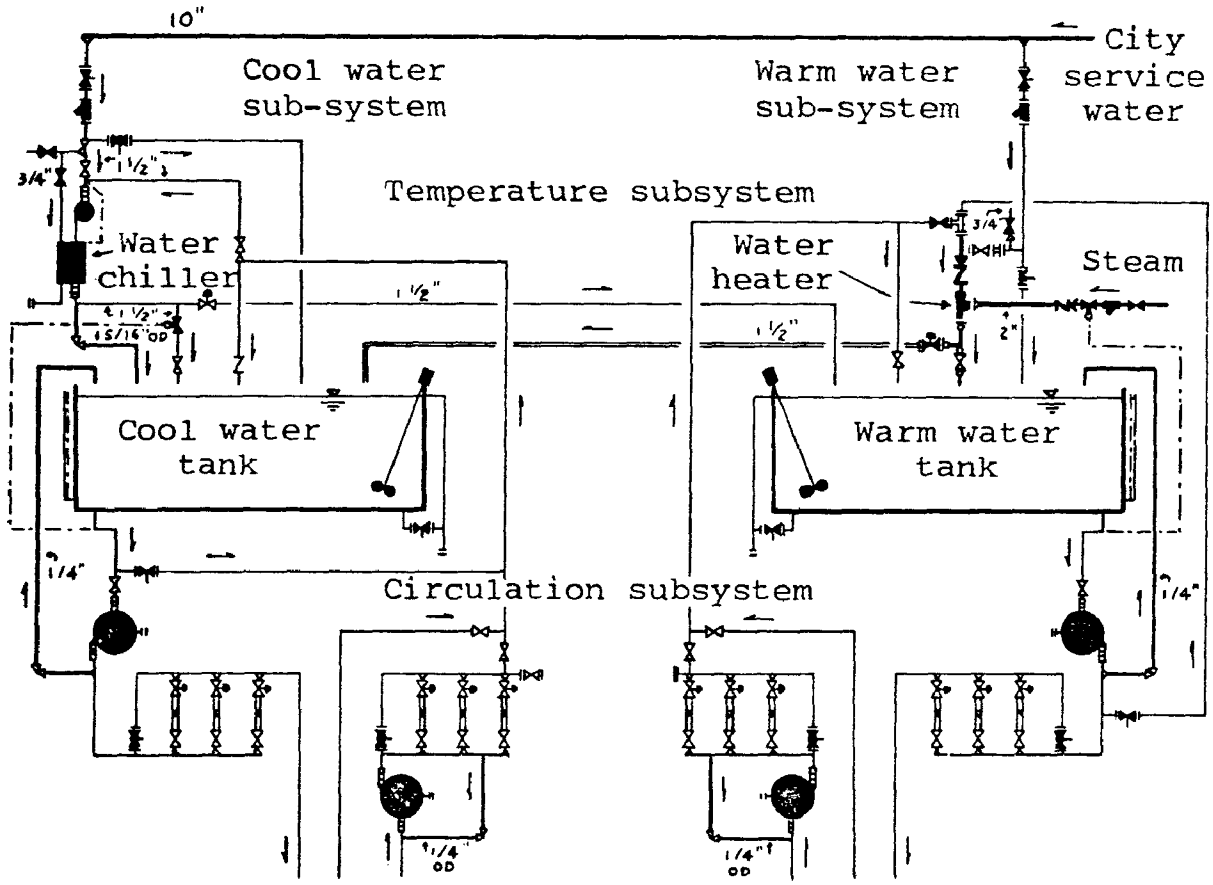



Fig. 8 Flume structure.




Legend

Material:

- PVC
- CPVC
- copper or other metal
- reinforced rubber hose

Pipes are 3", unless otherwise stated.

- ½"O.D. clear plastic glass tube

Valves have screwed joints, except for

- flanged
- solder-jointed

Fittings:

- ball valve
- diaphragm valve
- butterfly valve step control
- butterfly valve infinite control
- automatic control valve
- check valve
- angle valve
- strainer
- reducer
- 1½"rotometer
- pump
- mixer
- to drain

Fig. 9 Water supply system flow diagram.

## SECTION VII

## EXPERIMENT AND DATA

### 7.1 EXPERIMENT

#### Experimental Plan

The principal object of this experiment is to generate as many of the predicted interfacial profiles as possible in the experimental flume.

Prior to the experiment, an experimental plan was formulated out as follows, assuming that each profile could be generated if the proper values were given to the parameters included in the interfacial profile equations.

For each profile, a set of parameters was found to satisfy the requirement that it could be observed within the length of the flume. The parameters are the slope of the flume bottom, the flowrate of moving layers, the temperature of the two layers to determine their density, the total depth of the flow, the depth of one of the layers, and the interfacial and bottom friction factors.

The selection of the parameters was done through trial and error by numerically integrating the proposed interfacial profile equations with the free surface profile equations.* The temperature of the two layers was chosen so that the difference in the average temperature of the two layers was at least 3.3$^{o}$C (6$^{o}$F).[6] The bottom friction factor was estimated by the formula suggested by Ghosh and Roy.[10] For the interfacial friction factor, the value calculated from the friction factor for the smooth channel of open-channel flow was substituted after Polk et al.[7]

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* The computation used the 4-th order Milne predictor-corrector method. Since the same program is used in Sec. 7.2, see Appendix C.

Care was taken to give sufficient flowrate to the moving layer so that its flow would be in the turbulent region if it was a non-stratified flow. The effect of side walls was ignored at that time.

However, the following observations were obtained from the preliminary experiment for the lower layer analysis.

1. An attempt to determine the depth of the lower layer with an underflow gate resulted in having the effect of changing the S1 or S2 profiles into the S3 profile. The gate seemed to be helpful to produce the S3, M3, and H2 profiles.
2. The S1, S2, M1, M2, and H1 profiles could be generated without the gate mainly by changing the flowrate and the slope.

Thereafter, the experiment was conducted by trial and error, using the obtained parameters as a starting point. The experiment was conducted only for the important cases of the lower layer analysis.

## Experimental Procedure

After testing several different methods of introducing the water into the flume, most of the experiments were conducted by the following procedure. The term "cool water" is used for the lower layer water; "warm water" is used for the upper layer.

Prior to the experiment, the connection between the flume and the water supply system was established by means of reinforced hoses. For the S, M, and H profiles, the discharging line of the cool water system was connected to the upstream end of the flume. Its return line started from the downstream end of the flume. The ends of both the discharging and the return lines of the warm water supply are placed in the upper part of the downstream inlet-outlet box. They are equipped with baffle boards to prevent the force of the flow from mixing the two layers.

### Preparation -

The flume is tilted to the desired slope. The flume must be raised to an elevation slightly higher than the elevation desired in the experiment, since the weight of water lowers the flume slightly.

The ends of the pipes of the warm water circulation subsystem are placed at the desired position in the inlet-outlet box.

The desired total depth is marked on the side wall of the flume.

The main switches of the electricity for the pumps are on; the thermister thermometers are warming up. Each valve should be examined if it is open or closed correctly before each specific operation.

The main valve of the service water is opened.

Making the Cool Water -

The cool water tank is filled with the service water. To measure the temperature of this water, it is circulated in the temperature sub-system through the short cut line by the pump of the water chiller. To do this, the temperature control of the chiller is set so high that it does not chill the water, while the setting of the cool water control valve is adjusted low to open the valve fully. Desired temperature is a few degrees [1.1 to 1.7°C (2 to 3°F)] lower than that to be used in the experiment. If its temperature is close to the desired one, the circulation is stopped and the water in the tank is fed into the flume through the by-pass line in the rotometer manifold. Initially, the discharge pump has to be run with the valve in the by-pass line being half-closed for a very short period to expel the air in the line from the air bleeder.

After that, the water can be fed to the flume by gravity, if the water level in the tank is maintained higher than the level in the flume by supplying the service water into the tank. Of course, the drain valves have to be closed. However, both valves were kept open until some water was discarded from the drains to clean the flume.

If the temperature of the service water is higher than that desired, the water chillder has to be used. If it does not require too much cooling, the water can be fed continuously to the flume while cooling it by the circulation through the short cut line. However, when considerable cooling is necessary, a batch method has to be used; that is, one tankful of water is cooled down through circulation and then fed to the flume. This procedure is repeated until the water reaches the desired level in the flume. The desired level is a little higher than the total depth to be used in the experiment, since it is easier to adjust the water level by reducing the water level than by increasing it.

If the service water temperature is lower than desired, the warm water temperature subsystem must warm the water. In a similar manner, the service water put into the warm water tank is warmed by steam while circulating through the short cut line. If the desired temperature is reached, this warmed water has to be sent to the cool water tank before being introduced to the flume. This inconvenience arises because the end of the warm water discharge line is in the air with the baffle board which splashes water until it is submerged in the water. Both batch and continuous feeding to the flume can be used depending on the degree of warming.

Making the Warm Water -

The water for the upper layer should have a temperature 1.7 to 2.2$^{o}$C (3 to 4 degrees F) higher than that we desire to have in the flume during the experiment. As described previously, the service water is warmed by steam injected at the in-line type heater, while circulated by the discharge pump through the short cut line. After the desired temperature is obtained, the value of one of the rotometers is opened to introduce the water from the end of the discharge pipe submerged in the cool water. This introduction of the warm water into the flume is done very slowly. At the same time, the drain valve at the upstream end of the flume is slightly opened to discard the cool water already in the flume. The cool water is removed at about the same flow rate as the introduction of the warm water to maintain a static water level in the flume. This process has to be repeated if the flume is not filled to the desired level by one tankful of water.

The preparation of water for the two layers takes 1-1/2 to 2 hours.

Before Running the Water -

The warm water, with the temperature a little higher [0.6 to 1.1$^{o}$C (1 to 2$^{o}$F)] than we desire to have in the flume, is made in the warm water tank. All the valves in the circulation subsystems are open and all air bleeder cocks are open to expel the air. Then the water level in the flume is brought down to the marked total depth by discarding the cool water from the bottom drain of the flume.

Thermister thermometers have to be calibrated, if necessary. The camera should be ready. The slope of the flume is recorded.

Running the Water -

The mixer in the warm water is started. The main steam valve is opened. The two pumps in the warm water subsystem are started. First, both flow rates are brought to 0.000631 to 0.000758 m$^3$/sec (10 to 12 GPM) for a short while; then, they are reduced to around 0.000316 m$^3$/sec (5 GPM). After these flow rates become relatively constant, the two pumps in the cool water subsystem come into action. The mixer in the cool water tank is on and the water chiller must start to work with the appropriate setting of the cool water control valve. The flow rate of the cool water should be brought gradually to the desired rate. The temperature probe bar is submerged in the upper layer to measure the temperature at a certain point of the layer. This probe and two probes in the warm and cool water supply are connected to the auto scan type thermometer.

The flow rate of the 4 pumps usually fluctuates and changes considerably in the beginning of their operation. These have to be adjusted by watching the level of the water in the flume and in both tanks. Adjustment of the flow rate has to be made until these water levels can be considered to be nearly constant.

Also, the temperature of the discharging water has to be adjusted. If the warm water tends to lose heat, steam has to be injected. If the warm water temperature rises, the cool water has to be brought from the cool water tank. In this case, approximately the same amount of the warm water has to be returned to the cool water tank to maintain the water level. The cool water temperature was fairly well maintained by the cool water control valve.

If the temperature at the three points becomes almost constant, then dye (methyl green) is put into the cool water tank. After dye spread through the flume and reaches the return line (dye can be seen through the glass of the rotometer), photographs are taken.

The air temperature probe is connected to the scan type thermometer in place of the probe which has been measuring the temperature at a point of the upper layer. Now, the probes in the probe bar are connected to the thermometer with a rotary switch. It took about 1 to 1-1/2 hours to achieve a nearly steady state.

Measuring the Temperature of the Flow in the Flume -

The vertical distribution of the temperature in the flume was measured at the middle of each section of the flume during most of the experiment.

The probe bar is fixed to the holder so that the top probe in the bar is slightly below the free surface. After several minutes, the probes reach the temperature of their surrounding, and the recorder is set to recording position and the rotary switch is turned at specific intervals to profuce the record of the vertical temperature distribution. Since only 11 probes are in the bar, the probe bar has to be lowered to measure the lower part of the flow by using a guide mark put on the bar. If the probe bar is lowered, the temperature is recorded after the transient period. This procedure is repeated, if necessary. Meanwhile, the visual observations of the state of the flow are made.

The flow rate and water level have to be checked and adjusted, if necessary, before going to the next measuring point. These measurements take about 1 hour.

Stopping the Run -

The main steam valve is closed. The mixer in the warm water tank is stopped. The two pumps in the warm water subsystem are stopped. The water chiller is shut. The mixer in the cool water tank is stopped. The main valve for the service water is closed. The flow rate of the two pumps in the cool water subsystem is lowered gradually to a small rate and the pumps are shut. The water in the flume is drained slowly from the two drains. The water in both tanks is drained. The electric source is shut.

Deficiencies and Inconveniences of the System

Although the experimental system works well in general, a few deficiencies and inconveniences were noticed during the experiment. They are listed below.

1. The supporting frame is slightly twisted toward the upstream end to produce a slope of approximately 3.18 to 6.35 mm (1/8" to 1/4") per 0.305 meter (1 foot) in the direction of the width of the flume. This was found by observing that only one of the two jacks started to rise about 6.4 mm (1/4") and then both jacks went up with the same speed when the frame was lifted from the horizontal position. This situation may be due to inaccuracy in the construction of the yoke-shaped upstream end's support or unexpected force exerted by aluminum welding. A few attempts were made to correct the problem, but they were of little help. The fact that the bottom of the flume at the upstream end is not horizontal in the direction of the width is an inconvenience when reading the depth and determining the slope of the flume. However, the slope is not so large that it does not influence the results of the experiment.
2. The automatic temperature control valve for the warm water (steam regulating valve) is not reliable. This seems to be due to overdesign for small flow rate of warm water circulation occurring at the experiment corresponding to the lower layer analysis. One correctional measure was taken, in which the flow rate through the heater is increased by returning a portion of discharging water through the heater into the warm water tank, and it did help a little. However, manual control of the steam was used for most of the experiment.
3. The flowmeters, which must be checked quite often during an experiment, are located far apart between the warm and cool water subsystem because the limitations of space and the requirements of the piping.

4. The roughness plates made of plastic glass sheet warped in the water. They had to be attached to the flume bottom with adhesive to prevent them from disturbing the flow.
5. When the water in one subsystem was sent to the other subsystem through the 1-1/2" pipe to maintain the temperature of the water in the tank, the adjustment of its flow rate was quite difficult and took time to obtain constancy. This is because these lines are not provided with flow rate measuring devices.
6. The gate can be installed only at limited points of the flume because the gate hits lighting fixtures or a concrete girder on the ceiling when the flume is tilted.

Accuracy of Parameters

This is a rather crude discussion of the accuracy of the parameters measured in the experiment.

1. Slope

The slope is measured by finding the elevation from the horizontal of a point on the flume located at a distance of 7.63 mm (25 feet) from the center of the pivot toward the upstream end.

The point on the flume is 1.22 m (4 feet) above the level of the center of the pivot. This is because of the height of a leveling instrument.

These relations are shown schematically in Fig. 10.

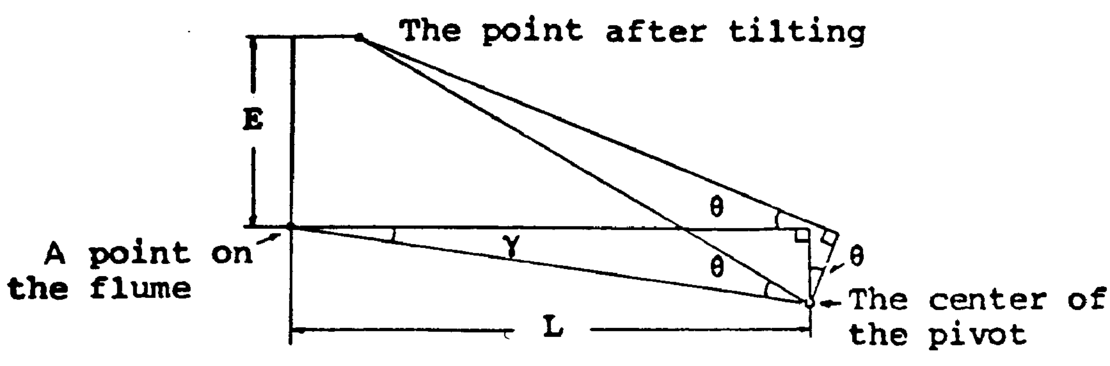



Fig. 10. Definition sketch for the tilted frame.

The slope is obtained by E/L [L = 7.63 mm (25 feet)]. If trigonometry is used, we easily find $E/L = \sin\theta - (1-\cos\theta)\tan\gamma$. $\tan\theta$ is at most 1/20 and $\tan\gamma = 4/25$ in this experiment. Since the slope is equal to $\tan\theta$ by its definition, the slope obtained by E/L generates an error of -0.0003 at the maximum.

Other errors come from the measurement of L and the reading of E. Also, as mentioned in the deficiencies of the system, the bottom of the flume, where the E is measured, is not horizontal in the direction of the width. Although the averaged bottom elevation is adjusted to the horizontal, it certainly introduced more error. These errors could amount to ±0.5%.

2. Flow Rate

The flow rate measured by a rotometer was examined by leading the water to a weighing tank after going through the rotometer while measuring the time. Errors were found to be at most ±1% of the flow rate for flow rates greater than 0.00378 $m^3$/sec (60 GPM). For 0.00252 $m^3$/sec, it is about ±2.5% of the flow rate. For 0.00158 $m^3$/sec (25 GPM), it could be approximately ±4.5% of the flow rate.

3. Temperature

The manufacturer of the thermister probes states that ±1% of 50°C is the error for this temperature reading. The recorder used for the temperature in the flume has ±0.5% error. Therefore, the accuracy of the temperature measured by the instrument is ±1.5% of 50°C. Furthermore, water temperature in the flume could not be maintained constant during an experiment. Although data to be used in later analysis were chosen by the standard of less variation in the temperature in the flume, the temperature averaged in the lower layer may have an error of ±0.2°C around 18.0°C (64.4°F). Therefore, the error in obtaining temperature could amount to ±1°C at the most. This could cause a maximum error of ±0.027% in density and of ±55% in the value of $\Delta\rho/\rho$.

If the errors of the flow rate and density difference are used, the error in the calculated stratified critical depth can be estimated. These errors are included in the tables shown in Sec. 7.3.

4. Depth

The fluctuation of the total depth during an experiment was maintained within ±0.79mm (1/32 inch). However, the total depth was measured at two points close to the ends of the flume after the free surface was found to be practically horizontal; the total depth at the other points and the lower layer depth were obtained from the temperature distribution curve on which the total depth obtained at the two points were plotted. This may introduce ±2.54 mm (0.1 inch) of error. Therefore, it is reasonable to consider that the depth used in data analysis has approximately ±3.18 mm (1/8") of error.

5. Width

The width of the flume could be 304.8 ± 1.59 millimeters (12 ± 1/16 inches). It is difficult to examine its width close to the bottom since its depth is 0.914 meters (3 feet). If the above estimate is used, the error may be ±0.52%.

General Observations

The observations made through the entire experiment are listed in the following. Specific observations on each interfacial profile are described in Sec. 7.3.

1. The two layers of different temperatures can be generated fairly easily in the experimental flume. The average value of $\Delta\rho/\rho$ was approximately 0.001 (ranging from 0.0007 to 0.0015). Even when the disturbance was seen at the head of the warm water layer, the two layers did not disappear rapidly.
2. The free surface remained practically horizontal through the experiment, as far as the length of this experimental flume is concerned. This indicates that we may ignore the effect of the internal circulation in the upper layer as stated in Sec. 5.4.
3. The interface shown by dye coloration in the water and the interface obtained by connecting the inflection points of the temperature distribution curves agree reasonably well in most of the experiments.
4. If the warm water is not maintained by supplying the warm water of constant temperature with a small flow rate, the upper layer dis-

appears rather quickly.

5. The change in the flow rate of the warm water does not change the position of the interface if the flow rate remains within approximately 1/5 of the flow rate of the lower layer. The flow rate of about 0.000316 $m^3$/sec (5 GPM) was used for the upper layer circulation in most of the experiments.
6. The position of the ends of the discharging and return lines of the warm water supply does not influence the position of the interface unless they are located close to the interface or in the lower layer.
7. Changes in the total depth of the flow and in the temperature of the water influence the position of the interface.
8. The interface is almost straight or slightly dipped at the center in the direction of the width of the flow in the experimental flume.
9. When the gate is used, it introduces more disturbance just behind the gate. However, the temperature of the layers are maintained better than in the cases without the gate.
10. When the head of the warm water layer is seen in the flume, it is very difficult to determine where the stable two layers start.

## 7.2 DATA ANALYSIS

Data used in the analysis were selected mainly by examining how well the temperature of the water in both tanks was maintained during the experiment. Then, the location of the interface was determined by connecting inflection points on vertical temperature distribution curves. The temperature of the two layers was calculated from the area between the temperature distribution curve and the vertical line through the inflection point. Water density in the two layers was found from the table of density of water;[11] then, $\Delta\rho/\rho$ was calculated. The stratified critical depth was calculated from the flow rate and $\Delta\rho/\rho$.

However, a few difficulties stopped continuation of the analysis at this point.

1. Although an open-channel flow experiment was conducted to determine the bottom friction factor for this experimental flume, only the value for very shallow depth could be obtained, since only a fairly crude method was available. Therefore, a method to estimate the values of bottom friction factor for deeper depth must be found.
2. A good method to estimate the value of the interfacial friction factor is not available. Thus, it must be estimated.
3. It seemed that most of the stratified critical depth calculated from its definition (Eq. 30) was lower than the data indicated.

Some methods to cope with these problems were implemented and are described later; however, the outline of methods is given in the following, since these problems are related to each other.

Eqs. 198 and 200 are used as the interfacial profile equations and rewritten for the experimental flume (b=1) as follows.

For the positive slope flow:

$$\frac{dh_2}{dx}=S_b\frac{1-(G_w h_{n2w}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{232}$$

where

$$G_w h_{n2w}=h_{c2}\{\frac{f_b}{8S_b}[1+\frac{f_i/f_b}{1-(h_2/H)}+\frac{2h_2f_w}{f_b}]\}^{1/3}$$

$$=h_{c2}\{\frac{1}{8S_b}[f_b+\frac{f_i}{1-(h_2/H)}+2h_2f_w]\}^{1/3}$$

$$=h_{c2}\{\frac{f_b+2h_2f_w}{8S_b}[1+\frac{f_i/(f_b+2h_2f_w)}{1-(h_2/H)}]\}^{1/3}$$

$$=h_{c2}\{\frac{f_q(1+2h_2)}{8S_b}[1+\frac{f_i/f_q/(1+2h_2)}{1-(h_2/H)}]\}^{1/3} \tag{233}$$

where $f_q$ = equivalent friction factor.

For the horizontal slope flow:

$$\frac{dh_2}{dx}=-\frac{f_b}{8}\frac{(G_{Hw}h_{c2}/h_2)^3}{1-(h_{c2}/h_2)^3} \tag{234}$$

where $$f_bG_{Hw}{}^3=f_b+\frac{f_i}{1-(h_2/H)}+2h_2f_w$$

$$=(f_b+2h_2f_w)[1+\frac{f_i/(f_b+2h_2f_w)}{1-(h_2/H)}]$$

$$=f_q(1+2h_2)[1+\frac{f_i/f_q/(1+2h_2)}{1-(h_2/H)}] \tag{235}$$

If the relation between the equivalent friction factor, bottom friction factor and wall friction factor obtained for open-channel flow can be used for the two-layer flow, and the term $f_b$ is used for $f_q(1+2h_2)$, then Eqs. 232 and 234 become the same forms as Eqs. 93 and 97.

Therefore, if the value of $f_q$ from the open-channel experiment can be obtained, the estimated value of the stratified critical depth and the interfacial friction factor can be found by the following trial and error method.

1. Integrate numerically the interfacial profile equations (eqs. 93 and 97) with the free surface profile equations (Eq. 101), using a set of the values of the stratified critical depth and the interfacial friction factor.
2. Compare the interface thus obtained with the interface obtained by the experiment.
3. If the two interfaces are quite different, choose another set of the values of the stratified critical depth and the interfacial friction factor and integrate.
4. This is repeated until the two interfaces reasonable match.

## Bottom Friction Factor

An attempt was made to find the value of the bottom friction factor for the experimental flume using Eq. 219 with Eq. 220, that is,

$$\frac{dh}{dx}=-\frac{f_q(1+2h)}{8}\frac{(h_c/h)^3}{1-(h_c/h)^3} \tag{236}$$

When open-channel flow was run on the horizontal slope in the experimental flume, it was observed that the downstream depth was slightly lower than the upstream depth. Therefore, this can be considered as an H2 profile according to Chow.[3] (This is similar to an H1 profile in this study).

Unfortunately, the flow rate designed for the two-layered flow is very small for open-channel flow in the experimental flume and better equipment to measure the depth is not available. Fairly reliable data was obtained only from the measurement at approximately 102 millimeters (4 inches) of depth for three different flow rates. After compensating for the irregularity of the channel bottom, the free surface profile was obtained for this flow. Numerically integrating Eq. 236 with different equivalent friction factors gave the calculated free surface profile. (The Milne predictor-corrector method is used to calculate the profile. See Appendix A.) Comparison between this and the profile obtained from the experiment allows to estimate the equivalent friction factor for a 102 millimeter (4 inch) depth of flow as approximately 0.136. Furthermore, the effect of the momentum coefficient on this result was examined. (See Eq. 221). Whe $\beta$=1.2 was assumed, it did not change the above value. $\beta$=1.2 could be the largest value occurring in the straight experimental flume.

However, equivalent friction factors are necessary for greater depths of the flow. Therefore, the following method was developed to estimate the equivalent friction factors.

According to Ghosh and Roy,[10] the friction factor for roughwalled channel can be expressed as

$$\frac{1}{\sqrt{f}}-\frac{14\alpha}{\sqrt{8g}}=2.031\log\frac{R}{r\frac{rs}{lm}}+C \tag{237}$$

where f = friction factor for roughwalled channel

R = hydraulic radius

r = height of roughness element

s = width of roughness element

l = longitudinal spacing of roughness element

m = transverse spacing of roughness element

C = a constant to be determined

Also, $\alpha$ is defined for a rectangular channel as

$$\alpha=\ln(1+\frac{2h}{b})-\frac{h}{b},\ \text{if}\ h\leqq\frac{b}{2} \tag{238}$$

$$=\ln(1+\frac{b}{2h})-\frac{1}{2},\ \text{if}\ h>\frac{b}{2} \tag{239}$$ [12]

where h = depth of flow

b = width of the channel

For the experimental flume, Eq. 2 can be written as

$$\frac{1}{\sqrt{f_b}}-\frac{14\alpha}{\sqrt{8g}}=2.031\log R_b+K \tag{240}$$

where $f_b$ = bottom friction factor

$R_b$ = bottom hydraulic radius

K = C-2.03log [r (rs/lm)].

Since b=1,

$$\alpha=\ln(1+2h)-h,\ \text{if}\ h\leqq 0.5 \tag{241}$$

$$=\ln[1+(0.5/h)]-0.5,\ \text{if}\ h>0.5 \tag{242}$$

Now, if the relations derived in Sec. 5.6 are used (Eqs. 214, 227, and 228),

$$R_b=h(1-2R_w) \tag{243}$$

$$f_b=f_q(1+2h)-2hf_w \tag{244}$$

$$f_w=f_q(1+2h)R_w/h \tag{245}$$

and therefore $$f_b=f_q(1+2h)(1-2R_w) \tag{246}$$

It is assumed that the Blasius equation defined with the hydraulic radius instead of the pipe diameter can be used to estimate the wall friction factor (Chow[3])

$$f_w=0.223/Re_w^{0.25} \tag{247}$$

where $Re_w$ = Reynolds number related to the wall = $UR_w/\nu$

$U$ = average velocity = $q/h$

$R_w$ = wall hydraulic radius

$\nu$ = kinematic viscosity of water

Note that the Blasius equation is valid only for $750<re_w<25{,}000$. $f_w$ therefore, can be written as,

$$f_w=0.223/[(q/\nu h)R_w]^{0.25} \tag{248}$$

Equating Eqs. 245 and 248 gives

$$R_w=0.223^{0.8}\ \nu^{0.2}h/\{q^{0.2}[f_q(1+2h)]^{0.8}\} \tag{249}$$

Therefore, $f_b$ and $R_b$ are expressed in terms of $f_q(1+2h)$, h, $\nu$, and q. When $(1/\sqrt{f_b})-(14\alpha/\sqrt{8g})$ and $R_b$ were calculated and plotted in a semi-logarithmic paper, K=3.1 was obtained.

Therefore,

$$\frac{1}{\sqrt{f_b}}-\frac{14\alpha}{\sqrt{8g}}=2.03\log R_b+3.1 \tag{250}$$

This equation can be solved by an approximate method* to obtain $f_q(1+2h)$ as a function of q, $\nu$, and h for each experiment. At the same time, $Re_w$ can be calculated and examined if it is between 750 and 25,000. q and $\nu$ are considered to be constant for an experiment.+ The average $h_2$ is used for h. One value of $f_q(1+2h)$ was chosen for each experiment by examining the average depth for that experiment. The values of $f_q(1+2h)$ ($=f_b+2hf_w$) used in the data analysis together with the values of the interfacial friction factor used in the analysis, are included in the tables comparing data with the methods estimating the stratified critical depth shown in Sec. 7.3.

Interfacial Friction Factor

Few studies have been conducted on interfacial friction factor and no one has offered a definite method to estimate its value.[13, 14, 15, 16, 17]

Therefore, an attempt was made to estimate the magnitude of the interfacial friction factor from the obtained data by numerically computing the interfacial profiles with different values of the interfacial friction factor and comparing these calculated profiles with the data. As a basis for estimating value of the interfacial friction factor, the value calculated from the Blasius equation with the Reynolds number defined by the hydraulic radius, 0.5/(1+h), is used,‡ namely,

$$f_i=0.223/Re_i^{0.25} \tag{251}$$

*An iterative method to solve this equation, though somewhat primitive, was developed by the author. The computer program is shown in the Appendix B.

+The values of $\nu$ are obtained from the values of viscosity found in Reference 11.

‡In this hydraulic radius, 2(1+h) is used for the wetted perimeter to reflect the effect of the lower layer running under the upper layer surrounded by the walls and the bottom.

where $$Re_i = \frac{q}{\nu h}\frac{h}{2(1+h)} = \frac{q}{2\nu(1+h)} \tag{252}$$

Unfortunately, as stated in the beginning of Sec. 7.2, the stratified critical depth has to be estimated as well. Therefore, an accurate estimation of the interfacial friction factor cannot be expected.

As shown in the next section, two methods to estimate the stratified critical depth and thus two different values of the interfacial friction factor can be obtained.

Since the calculation of the interfacial friction factor by Eq. 251 uses the values of $q/\nu$ as the equivalent friction factor, the calculation of the interfacial friction factor is included in the computer program calculating the equivalent friction factor. (See the Appendix B.)

Stratified Critical Depth

As stated at the beginning of Sec. 7.2, the indication was that the stratified critical depth calculated by its definition (Eq. 90) is smaller than the data suggests for most of the experiments conducted. The variables governing this critical depth are the flow rate, the density difference, and the momentum coefficient, if it is included in the critical depth (Eq. 113). Errors in the flow rate and the density difference are mentioned in Sec. 7.1. If the stratified critical depth indicated by the data exceeds the value including the error, then the definition of the interface and the method of obtaining the temperature of the two layers may have to be reconsidered.

When the word interface appears in the derivation of the interfacial profile equation, that interface must lie between "momentum layers"* (Carstens[18]). However, the interface found from this experiment may be called the interface between "temperature layers." In fact, it was

*The momentum layers are the layers with alternative flow direction, that is, a layer is referred to the one between 0 points in the velocity distribution.

assumed that these two interfaces were very close to each other. According to Carstens[18], they do not necessarily coincide with each other. An attempt to see the coincidence between two interfaces was not made with equipment measuring velocity. However, visual observations of the movement of small bubbles in the flow suggested that these interfaces were practically identical, as far as the experiment in this study is concerned.

As for the averaging of temperature in the layers, no other method seems to be more justified.

Two methods of finding the value of the stratified critical depth have been attempted and the results are shown in Sec. 7.3. Both methods use the profiles obtained in the experiment to estimate both the stratified critical depth and the interfacial friction factor. The interfacial and free surface profile equations in non-dimensional form are used for the computation. (See Sec. 5.3 for the equations.) The following parameters are substituted into these equations: (1) estimated stratified critical depth, (2) estimated interfacial friction factir, (3) bottom friction factor obtained by the method described in the bottom friction factor, (4) lower layer depth at the point in the flume where the two layers are considered to be fully established in the profile obtained by the experiment, (5) total depth at that point, and (6) density difference.

Estimated stratified critical depth is expressed in the form of the stratified critical depth calculated by the definition multiplied by some constant. Similarly, estimated interfacial friction factor takes the form of the value obtained by the method shown earlier multiplied by a constant. Numerically integrated interfacial profiles are compared with the profiles obtained in the experiment to find the combination of the stratified critical depth and the interfacial friction factor which produce a calculated interfacial profile best fit to the profile in the data. (See the Appendix C for the computer program.)

In the first method (method 1), it is assumed that the density difference could be different than the one obtained by the method described at the beginning of Sec. 7.2. Therefore, density difference is expressed by

$$(\Delta\rho/\rho)/{}_m h_c{}^3 \tag{253}$$

where ${}_m h_c$ = multiple for the stratified critical depth. It gives

$$h_c = {}_m h_c \cdot {}_o h_c \tag{254}$$

where ${}_o h_c$ = the stratified critical depth calculated by its definition.

In the second method (method 2), we assume that the ignored momentum coefficient, $\beta$, is the factor responsible for the smaller stratified critical depth. For this method,

$${}_m h_c = \beta^{1/3} \tag{255}$$

The result is shown in Sec. 7.3.

## 7.3 RESULTS OF THE EXPERIMENT AND THE DATA ANALYSIS

### The Number of the Experiments Conducted

Some preliminary experiments were not directed to produce specific profiles. They are classified into certain profiles from observation. Experiments done without the service of the cool water control valve because of its malfunction are included in the total number of experiments.

Table 1 . NUMBER OF EXPERIMENTS CONDUCTED

| Profile | Number | Number used for analysis |
|---|---|---|
| S1 | 4 | 1 |
| S2 | 19 | 3 |
| S3 | 9 | 4 |
| M1 | 6 | 2 |
| M2 | 4 | 3 |
| M3 | 2 | 2 |
| H1 | 2 | 2 |
| H2 | 1 | 1 |
| Total | 47 | 18 |

Experimental Data and Its Analysis

Data and their analyses for different profiles are presented in the following order: S1, S2, S3, M1, M2, M3, H1, and H2 profiles. In every profile classification, each experiment used for the analysis is presented by:

1. Observation of the experiment for the profile;
2. An experimental photograph and a figure of vertical temperature distribution curves at several points along the flume;
   [Each figure, table, and computer print-out is marked by a profile identification, as S1-3/30. "S1" is profile designation described in Fig. 7. "3/30" is the date of the experiment (in 1973).

   The number seen in photos is Expt. No. It does not agree with the number of the experiments actually conducted: this number was introduced to identify the photos after a part of the preliminary experiment was conducted. The photos are not clear since they include the reflection of items located on the other side of the room and the shadow of the railing for the probe bar.

   The figure includes an interfacial line obtained by connecting inflection points of the temperature distribution curve, and virtual stratified normal depth line and stratified critical depth line obtained by the method 1. These lines are shown only for the portion used for the analysis. The longitudinal distance in the figures of the vertical temperature distribution is shorter than that in the photos. Each flume section is 1.22 meters (4 feet) long. Therefore, only the 7.32 meter (24 foot) length of the flume is shown in the figures. Further, depth is very much exaggerated compared to the longitudinal distance in the figures].
3. A table of experimental data and results of the methods 1 and 2 for comparison between them;
   [Lower layer depth and total depth are obtained from temperature distribution curves. The results of the method 1 and 2 are chosen from the print-outs of interfacial profile computation which follow this table.]

4. Computer printouts of interfacial profile for the method 1 and 2. [A printout expressed in inches for depth and in feet for distance along the flow precedes a printout expressed in a non-dimensional form. The computer program shown in Appendix C gives printouts showing both dimensional and non-dimensional length on one sheet. They are, however, shown in separate pages as described, obtained by modifying the program slightly.]

Explanation of symbols and legend used in these figures and table follows.

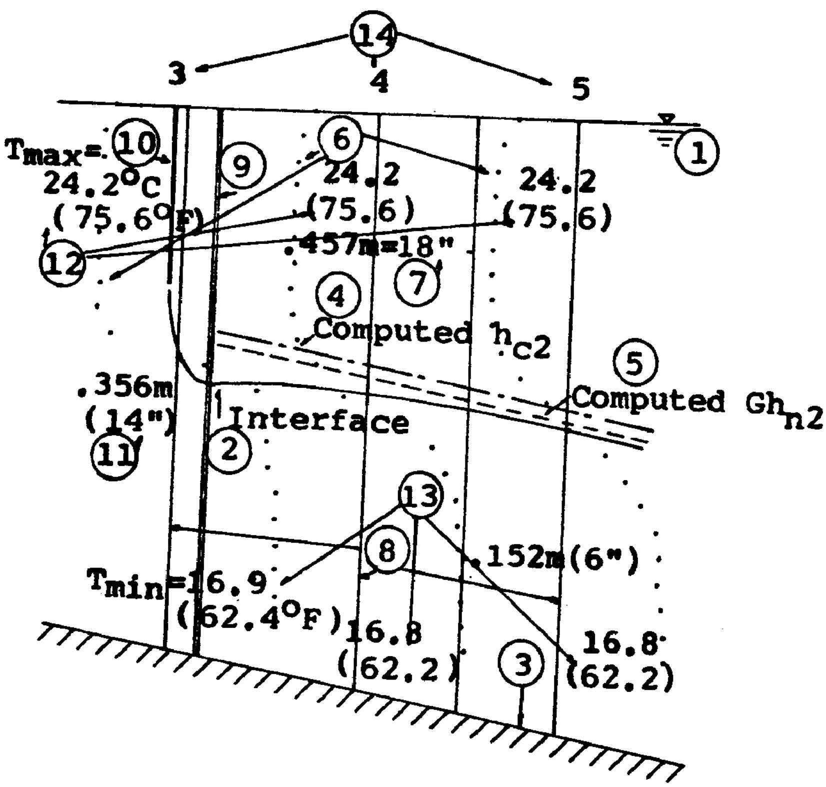



Fig. 11 Definition Sketch for Temperature Distribution Curve

[1] = free surface;

[2] = interface obtained by connecting inflection points in the temperature distribution curve;

[3] = bottom;

[4] = stratified critical depth line calculated by the method 1. The stratified critical depth line from method 2 is not shown to ensure simplicity;

[5] = virtual stratified normal depth line calculated by the method 1;

[6] = temperature of water measured at 25.4 millimeter (one inch) intervals of the depth at the center of each flume section except when the gate is used. Since the gate was inserted at the center of flume section 3, the measurement was taken 76.2 mm (3") behind the gate for the section. Measurement was also taken at at least one point between sections 3 and 4, but their curves are not shown in the figure because of the limited space;

[7] = to show depth from the bottom for reference;

[8] = point of the flume where temperature measurement was taken;

[9] = starting point of the calculation;

[10] = gate;

[11] = gate opening;

[12] = highest water temperature in a curve, in $^oC$ (in $^oF$);

[13] = lowest water temperature in a curve, in $^oC$ (in $^oF$); and

[14] = name of a flume section, which is at the center of the section in the figure.

Symbols are listed alphabetically. Parentheses and brackets after explanation are for the tables; the values in parentheses are in British units; the values in brackets are non-dimensional.

Symbols expressed in capital letters (usually shown in parentheses) are for computer printouts. If "I" is found in these symbols, they are in expressed in inches.

$\beta$ (=BETA)= momentum coefficient obtained by method 2;

DTDHI = difference between total depth and horizontal drawn through the free surface at the starting point, in inches;

DX = integration interval, in feet;

$dh_2/dx$(=HP) = interfacial slope ($dh_2/dx_N$);

$dh_2/dx_N$(=HPN) = non-dimensionalized $dh_2/dx$
$=dh_2/(dxS_b)$ for positive slope flow ($dh_2/dx_N$= $dh_2/dx$ for horizontal slope flow);

$\Delta\rho/\rho$(=DELTA) = ${}_o(\Delta\rho/\rho)/{}_mh_c{}^3$ in method 1;

$f_b$(=FB) = bottom friction factor estimated by the method in Sec. 7.2;

$f_i$ (=FI) = interfacial friction factor obtained by method 1 or 2 = ${}_mf_i \cdot {}_of_i$, in which ${}_mf_i$ is substituted by trial and error;

$Gh_{n2}$(=GHNI) = virtual stratified normal depth found by method 1 or 2, in meters (in inches) [non-dimensional];

$Gh_{n2N}$(=GHNN) = non-dimensionalized $Gh_{n2}$ = $Gh_{n2}/h_{c2}$;

H(=TDI) = total depth, in meters (in inches) [non-dimensional; if there are two separated by a comma, the former is for method 1, and the latter for method 2];

$H_N$(=TDN) = non-dimensionalized H = $H/h_{c2}$;

HP = see $dh_2/dx$;

HPN = see $dh_2/dx_N$;

$h_2$(=HI) = lower layer dpeth, in meters (in inches) [non-dimensional; the former is for method 1 and the latter for method 2];

$h_{2N}$(=HN) = non-dimensionalized $h_2$=$h_2/h_{c2}$;

$h_{c2}$(=HC2I) = stratified critical depth obtained by method 1 or 2, in meters (in inches) = ${}_mh_c \cdot {}_oh_{c2}$, in which ${}_mh_c$ is substituted by trial and error;

${}_mf_i$(=MFI) = see $f_i$;

${}_mh_c$(=MHC) = see $h_{c2}$;

${}_o\Delta\rho/\rho$(=ODELTA) = relative density difference, or the ratio of density difference between two layers to mean density of two layers, calculated from $T_1$ and $T_2$;

${}_of_i$ = interfacial friction factor estimated in Sec. 7.2;

${}_oh_{c2}$ (=OHC2I) = stratified critical depth calculated by its definition $h_{c2}{}^3=q^2/[({}_o\Delta\rho/\rho)g]$, in meters (in inches), ±%=estimated maximum error;

q(=Q) = flow rate, in $m^3$/sec (in cfs);

$S_b$(=SB) = slope of channel bottom;

$T_0$ = average air temperature during the experiment, in °C (in °F);

$T_1$(=T1) = averaged temperature in the upper layer, in °C (in °F);

$T_2$(=T2) = averaged temperature in the lower layer, in $^oC$ (in $^oF$);

$T_S$ = average temperature of water to be supplied into the flume, in $^oC$ (in $^oF$), subscript 1 for the upper layer and 2 for the lower layer;

TDI = see H;

TDN = see $H_N$;

x(=X) = distance along the flow from the starting point of the computation in meters (in feet) [non-dimensional for method 1, $x_N$ for method 2]; and

$x_N$(=XN) = non-dimensionalized x

= $S_b x/h_{c2}$ for positive slope flow

= $x/h_{c2}$ for horizontal slope flow.

·S1 profile

The profiles designated as S1 were produced with the slope considered to be steep and the flow rate low. The observation, the photograph, and the temperature distribution of both S1-3/30 shown here and the other three experiments have the following general tendencies.

The lower layer begins very markedly, but gradually becomes ambiguous and deeper toward downstream, and disappears. Meanwhile, another interface which is definitely situated above the original interface and almost horizontal appears, gradually becomes clear, and finally becomes the only interface at the end of the flume.

The latter interface may have the characteristics predicted for the S1 profile. If this assumption is correct, the internal jump, which was expected to occur from the analogy to open-channel flow, is replaced by the gradual change in the depth of the lower layer for this two-layered flow.

S1-3/30

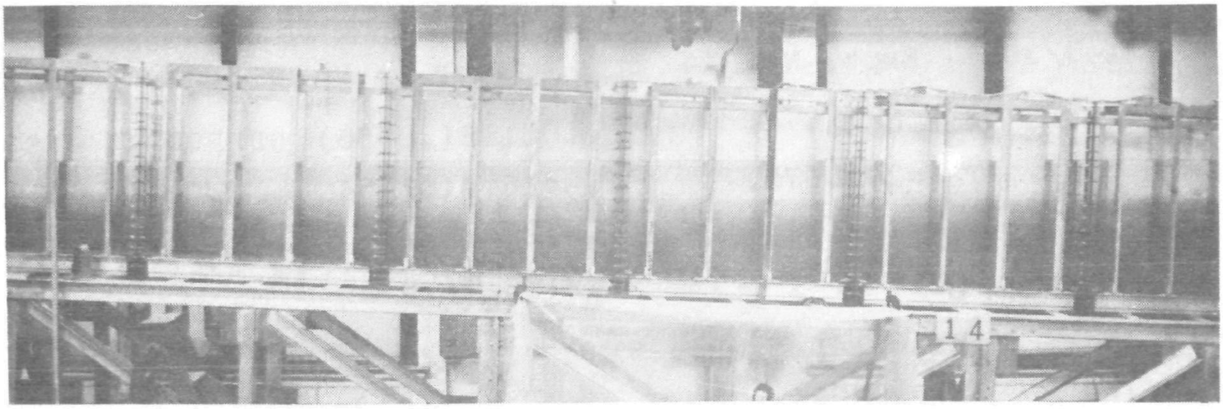
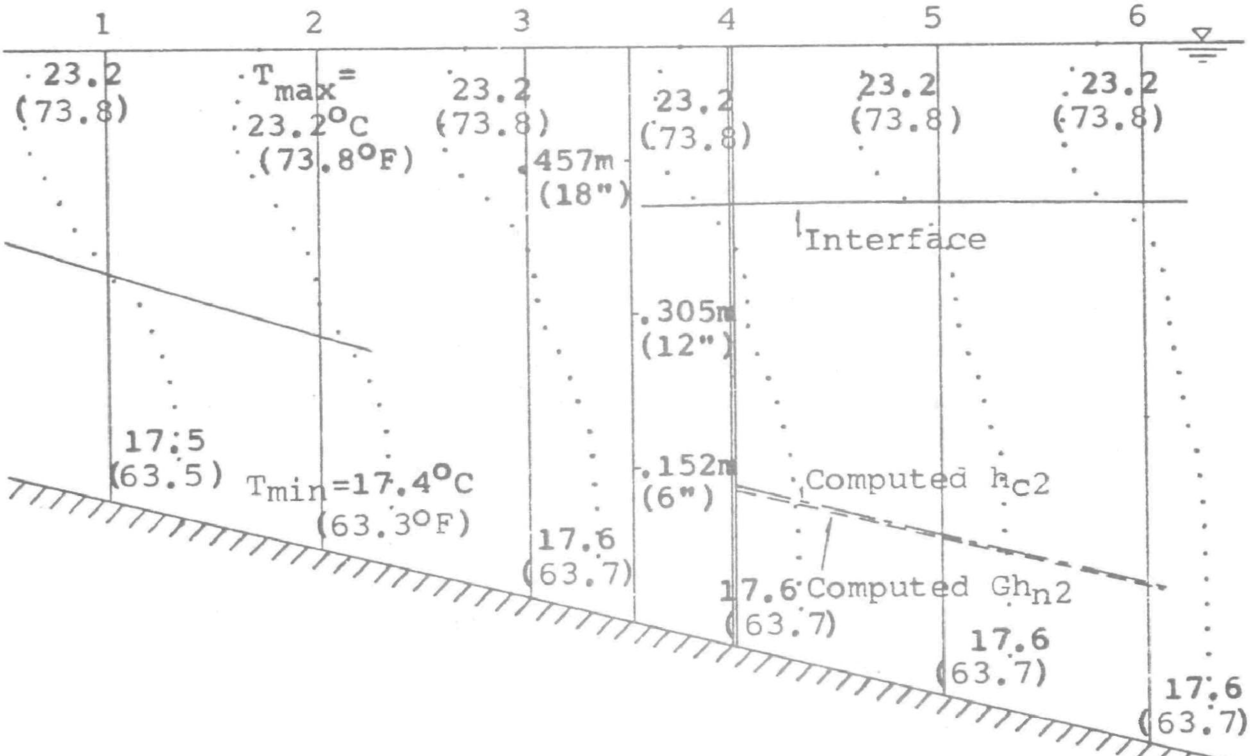


Fig. 12. Experiment S1-3/30.




$S_b$= 0.0408

Fig. 13. Temperature distribution curve for S1-3/30.

Table 2. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR S1-3/30.

S1-3/30 Expt. No. 14

$S_b$=.0408 q=.00158(.0556)

$T_{S1}$=24.3(75.7) $T_{S2}$=16.4(61.5) $T_0$=20.0(68.0)

$T_1$=22.7(72.9) $T_2$=18.4(65.1) $_o\Delta\rho/\rho$=.000912

$f_b$=.272 $f_b/(8S_b)$=.833

$_oh_{c2}$=.145(5.69)±28%

$_of_i$=.0402

| x | 0.00 | 1.22(4.00) [.316,.334] | 2.44(8.00) [.632,.667] |
|---|---|---|---|
| $h_2$ | .437(17.2) [2.77,2.93] | .488(19.2) [3.10,3.27] | .533(21.0) [3.39,3.58] |
| H | .592(23.3) [3.75,3.96] | .643(25.3) [4.08,4.31] | .693(27.3) [4.40,4.65] |

Method 1

$h_{c2}$=.157(6.20) $_mh_c$=1.090 $\Delta\rho/\rho$=.000704

$f_i$=.0100 $_mf_i$=0.250 $f_i/f_b$=.0369

| | | | |
|---|---|---|---|
| $h_2$ | .437(17.2)[2.77] | .487(19.2)[3.09] | .536(21.1)[3.41] |
| $Gh_{n2}$ | .155(6.10)[.984] | .155(6.12)[.987] | .156(6.14)[.990] |
| $dh_2/dx$ | .04090[1.002] | .04086[1.001] | .04083[1.001] |
| H | .591(23.3)[3.75] | .640(25.2)[4.07] | .690(27.2)[4.38] |

Method 2

$h_{c2}$=.149(5.87) $_mh_c$=1.032 β=1.10

$f_i$=.0161 $_mf_i$=0.400 $f_i/f_b$=.0591

| | | | |
|---|---|---|---|
| $h_2$ | .437(17.2)[2.93] | .487(19.2)[3.26] | .536(21.1)[3.60] |
| $Gh_{n2}$ | .146(5.73)[.976] | .146(5.76)[.981] | .147(5.70)[.986] |
| $dh_2/dx$ | .04092[1.003] | .04087[1.002] | .04084[1.001] |
| H | .591(23.3)[3.96] | .640(25.2)[4.29] | .690(27.2)[4.63] |

**Table 3. INTERFACIAL PROFILE CALCULATION FOR S1-3/30.**

S1-3/30 ACC= 4.0 METHOD 1

| | | |
|---|---|---|
| T1= 22.7C= 72.9F | T2= 18.4C= 65.1F | Q= 5.560E-02 |
| SB= 4.080E-02 | FB= 2.719E-01 | ODELTA= 9.120E-04 |
| OHC2I= 5.688E 00 | OFI= 4.015E-02 | .125*FB/SB= 8.330E-01 |
| HC2I= 6.200E 00 | MHC= 1.090 | DELTA= 7.042E-04 |
| FI= 1.004E-02 | MFI= 0.250 | FI/FB= 3.692E-02 |

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.720E 01 | 6.097E 00 | 2.325E 01 | 4.090E-02 | 0.00E-01 |
| 2.000E 00 | 1.818E 01 | 6.108E 00 | 2.423E 01 | 4.087E-02 | -1.34E-05 |
| 4.000E 00 | 1.916E 01 | 6.119E 00 | 2.521E 01 | 4.086E-02 | 3.81E-06 |
| 6.000E 00 | 2.014E 01 | 6.129E 00 | 2.619E 01 | 4.084E-02 | 5.72E-06 |
| DOUBLED INTERVAL DX= 4.0000E 00 | | | | | |
| 8.000E 00 | 2.112E 01 | 6.140E 00 | 2.717E 01 | 4.083E-02 | -9.54E-07 |
| INTERFACE BECOMES HORIZONTAL | | | | | |

S1-3/30 METHOD 2

| | | |
|---|---|---|
| HC2I= 5.872E 00 | MHC= 1.032 | BETA= 1.100 |
| FI= 1.606E-02 | MFI= 0.400 | FI/FB= 5.907E-02 |

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.720E 01 | 5.730E 00 | 2.325E 01 | 4.092E-02 | 0.00E-01 |
| 2.000E 00 | 1.818E 01 | 5.745E 00 | 2.423E 01 | 4.089E-02 | 1.49E-06 |
| 4.000E 00 | 1.916E 01 | 5.760E 00 | 2.521E 01 | 4.087E-02 | 1.91E-05 |
| 6.000E 00 | 2.014E 01 | 5.774E 00 | 2.619E 01 | 4.085E-02 | 2.00E-05 |
| DOUBLED INTERVAL DX= 4.0000E 00 | | | | | |
| 8.000E 00 | 2.112E 01 | 5.789E 00 | 2.717E 01 | 4.084E-02 | 1.43E-05 |
| INTERFACE BECOMES HORIZONTAL | | | | | |

## Table 3. CONTINUED.

S1-3/30 ACC= 4.0 NON-DIMENSIONALIZED METHOD 1

T1= 22.7C= 72.9F T2= 18.4C= 65.1F Q= 5.560E-02
SB= 4.080E-02 FB= 2.719E-01 ODELTA= 9.120E-04
OHC2I= 5.688E 00 OFI= 4.015E-02 .125*FB/SB= 8.330E-01

HC2I= 6.200E 00 MHC= 1.090 DELTA= 7.042E-04

FI= 1.004E-02 MFI= 0.250 FI/FB= 3.692E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 2.774E 00 | 9.835E-01 | 3.750E 00 | 1.002E 00 |
| 1.579E-01 | 2.932E 00 | 9.852E-01 | 3.908E 00 | 1.002E 00 |
| 3.159E-01 | 3.091E 00 | 9.869E-01 | 4.066E 00 | 1.001E 00 |
| 4.738E-01 | 3.249E 00 | 9.886E-01 | 4.224E 00 | 1.001E 00 |
| DOUBLED INTERVAL DXN= 3.1587E-01 | | | | |
| 6.317E-01 | 3.407E 00 | 9.903E-01 | 4.382E 00 | 1.001E 00 |
| INTERFACE BECOMES HORIZONTAL | | | | |

S1-3/30 METHOD 2

HC2I= 5.872E 00 MHC= 1.032 BETA= 1.100

FI= 1.606E-02 MFI= 0.400 FI/FB= 5.907E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 2.929E 00 | 9.758E-01 | 3.960E 00 | 1.003E 00 |
| 1.668E-01 | 3.097E 00 | 9.784E-01 | 4.127E 00 | 1.002E 00 |
| 3.335E-01 | 3.264E 00 | 9.809E-01 | 4.293E 00 | 1.002E 00 |
| 5.003E-01 | 3.431E 00 | 9.834E-01 | 4.460E 00 | 1.001E 00 |
| DOUBLED INTERVAL DXN= 3.3354E-01 | | | | |
| 6.671E-01 | 3.598E 00 | 9.859E-01 | 4.627E 00 | 1.001E 00 |
| INTERFACE BECOMES HORIZONTAL | | | | |

.S2 profile

These profiles were produced on the steep slope and with a fairly high flow rate. In most of the experiment, the two layers start with a fairly unstable zone of approximately 8 to 12 feet, which may be called the "frontal zone." This frontal zone is followed by the lower layer with almost constant depth. The temperature distribution curve shows that the lower layer depth increases gradually toward the end of the flume.

It seems that this profile reaches the virtual stratified normal depth very quickly after the unstable frontal zone without having much of the descending curve which is characteristic of the S2 curve predicted.

The question is then how the gradual increase in the lower layer depth can be explained. The following was attempted, although it did little to explain this effect.

If the lower layer depth is equal to the virtual stratified normal depth at this portion of the flow, we may calculate the value of $f_i$ by substituting $Gh_{n2}$ in place of $h_2$ in $G_H$ (Eq. 94) and using the values of $h_2$, H, $\Delta\rho/\rho$, and $f_b$. Unfortunately, the accuracy of the value of $\Delta\rho/\rho$ is not enough to verify the fact that $\Delta\rho/\rho$ decreases along the flow, which can be expected from the decrease in temperature differenc between the two layers. Therefore, the values of $f_i$ are calculated, using the constant stratified critical depth. However, this trial finds a very rapid increase in $f_i$ (doubled between the two sections) which may be considered as unlikely to occur. Then, the accuracy of the estimate of the value of $f_b$ may be questioned.

Another question is what happens if the increase in the lower layer depth continues. Since the length of the flume is limited, the experiment did not find any definite answer. From the S1 experiment, it may be stated that the S2 flow would eventually become the profile similar to the S1 flow.

S2-4/30

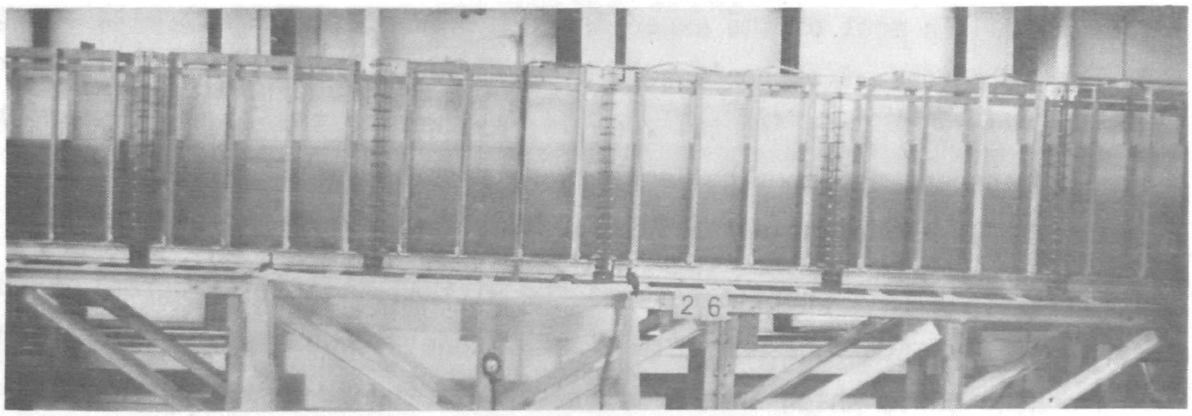
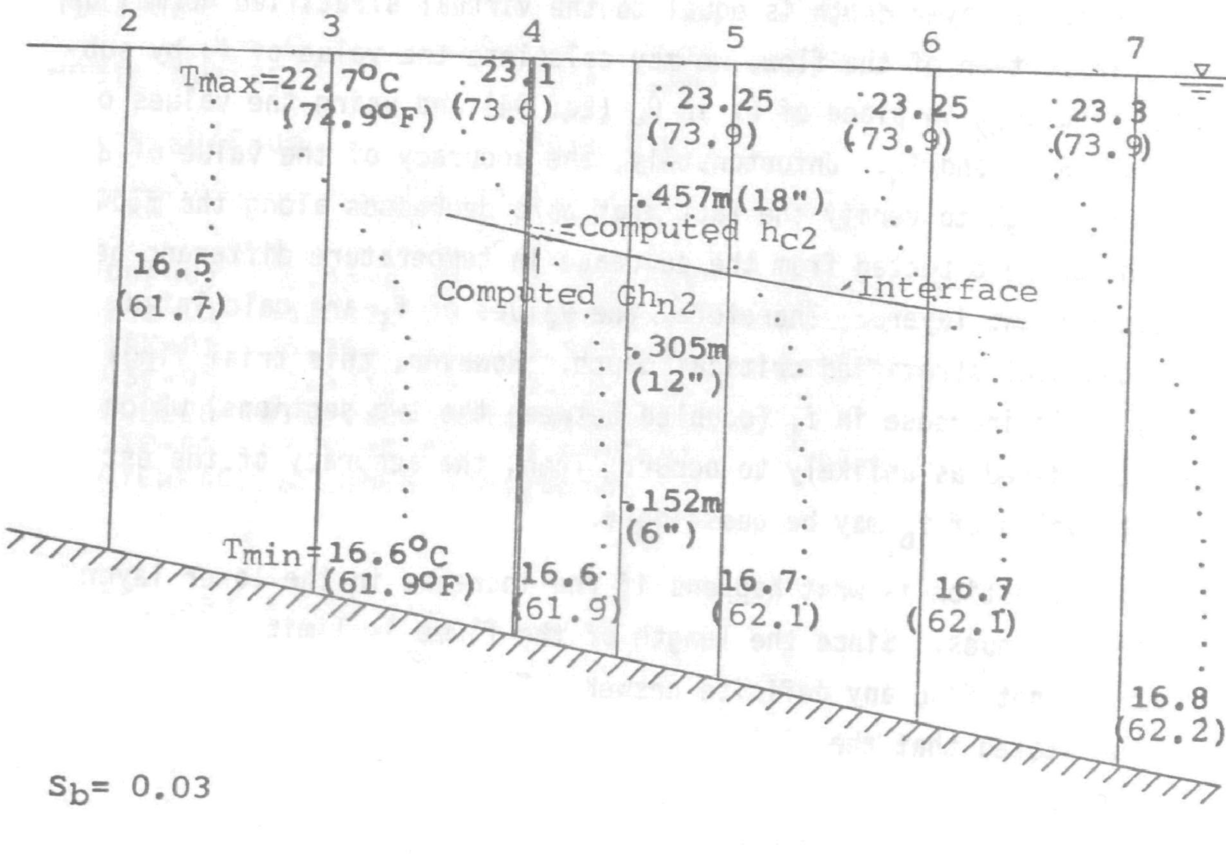



Fig. 14. Experiment S2-4/30.




$S_b$= 0.03

Fig. 15. Temperature distribution curve for S2-4/30.

Table 4. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR S2-4/30.

S2-4/30 Expt. No. 26

| | | |
|---|---|---|
| $S_b$=.03 | q=.00631(.2228) | |
| $T_{S1}$=24.6(76.3) | $T_{S2}$=16.5(61.7) | $T_0$=20.4(68.7) |
| $T_1$=21.8(71.2) | $T_2$=17.1(62.8) | ${}_o\Delta\rho/\rho$=.000944 |
| $f_b$=.211 | $f_b/(8S_b)$=.881 | |

${}_oh_{c2}$=.359(14.1)±26%

${}_of_i$=.0278

| x | 0.00 | 1.22(4.00) [.902,.914] | 2.44(8.00) [1.80,1.83] |
|---|---|---|---|
| $h_2$ | .399(15.7) [.983,.995] | .401(15.8) [.989,1.00] | .409(16.1) [1.01,1.02] |
| H | .574(22.6) [1.41,1.43] | .610(24.0) [1.50,1.52] | .645(25.4) [1.59,1.61] |

Method 1

| | | |
|---|---|---|
| $h_{c2}$=.406(15.98) | ${}_mh_c$=1.130 | $\Delta\rho/\rho$=.000654 |
| $f_i$=.00444 | ${}_mf_i$=0.160 | $f_i/f_b$=.0210 |

| | |
|---|---|
| $h_2$ | .399(15.70)[.983] |
| $Gh_{n2}$ | .398(15.66)[.980] |
| $dh_2/dx$ | -.0042[-.140] |
| H | .574(22.6)[1.41] |

Method 2

| | | |
|---|---|---|
| $h_{c2}$=.401(15.78) | ${}_mh_c$=1.116 | $\beta$=1.39 |
| $f_i$=.0350 | ${}_mf_i$=1.260 | $f_i/f_b$=.166 |

| | |
|---|---|
| $h_2$ | .399(15.70)[.995] |
| $Gh_{n2}$ | .398(15.66)[.992] |
| $dh_2/dx$ | -.0149[-.495] |
| H | .574(22.6)[1.43] |

Table 5. INTERFACIAL PROFILE CALCULATION FOR S2-4/30.

```
S2-4/30      ACC= 4.0                                          METHOD 1

T1= 21.8C= 71.2F       T2= 17.1C= 62.8F              Q= 2.228E-01
SB= 3.000E-02          FB= 2.114E-01          QDELTA= 9.440E-04
OHC2I= 1.414E 01       OFI= 2.777E-02     .125*FB/SB= 8.808E-01

HC2I= 1.598E 01         MHC= 1.130              DELTA= 6.542E-04

FI= 4.443E-03           MFI= 0.160              FI/FB= 2.102E-02

     X          HI          GHNI        TDI          HP        DTDHI

0.000E-01 1.570E 01 1.566E 01 2.260E 01 -4.209E-03  0.00E-01
  H APPROACHES GHN
```

```
S2-4/30                                                        METHOD 2

HC2I= 1.578E 01         MHC= 1.116                   BETA= 1.390

FI= 3.499E-02           MFI= 1.260              FI/FB= 1.655E-01

     X          HI          GHNI        TDI          HP        DTDHI

0.000E-01 1.570E 01 1.566E 01 2.260E 01 -1.486E-02  0.00E-01
  H APPROACHES GHN
```

Table 5. CONTINUED.

```
S2-4/30        ACC= 4.0         NON-DIMENSIONALIZED           METHOD 1

T1= 21.8C= 71.2F            T2= 17.1C= 62.8F                Q= 2.228E-01
SB= 3.000E-02               FB= 2.114E-01          ODELTA= 9.440E-04
OHC2I= 1.414E 01            OFI= 2.777E-02     .125*FB/SB= 8.808E-01

HC2I= 1.596E 01               MHC= 1.130              DELTA= 6.542E-04

FI= 4.443E-03                 MFI= 0.160              FI/FB= 2.102E-02

     XN            HN            GHNN          TDN            HPN

0.000E-01     9.826E-01     9.801E-01     1.414E 00     -1.403E-01
  H APPROACHES GHN

S2-4/30                                                        METHOD 2

HC2I= 1.578E 01               MHC= 1.116                BETA= 1.390

FI= 3.499E-02                 MFI= 1.260              FI/FB= 1.655E-01

     XN            HN            GHNN          TDN            HPN

0.000E-01     9.949E-01     9.924E-01     1.432E 00     -4.954E-01
  H APPROACHES GHN
```

·S3 profile

This profile was produced with the use of the gate. The interface dips down slightly after the gate to have the portion which may be called vena contracta (after the term used in open-channel flow), then curves up, and finally the lower layer depth becomes almost constant.

When the gate opening was set to the lower layer depth found in the preliminary calculation of a profile, that flow which occurred in the flume was totally different from the expected one; that is, the lower layer depth was much larger than calculated. This may be due to the turbulence in the flow following the gate produced by the gate.

S3-5/14

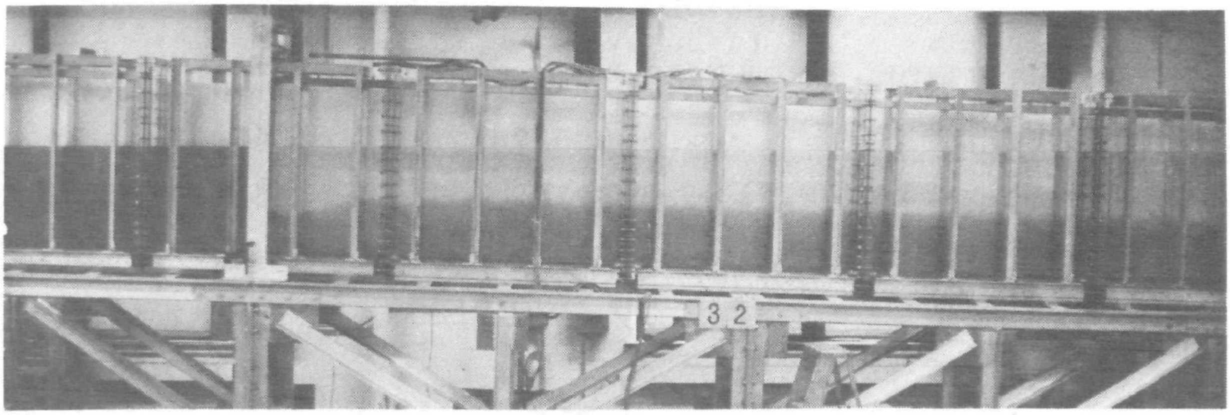


Fig. 16. Experiment S3-5/14.

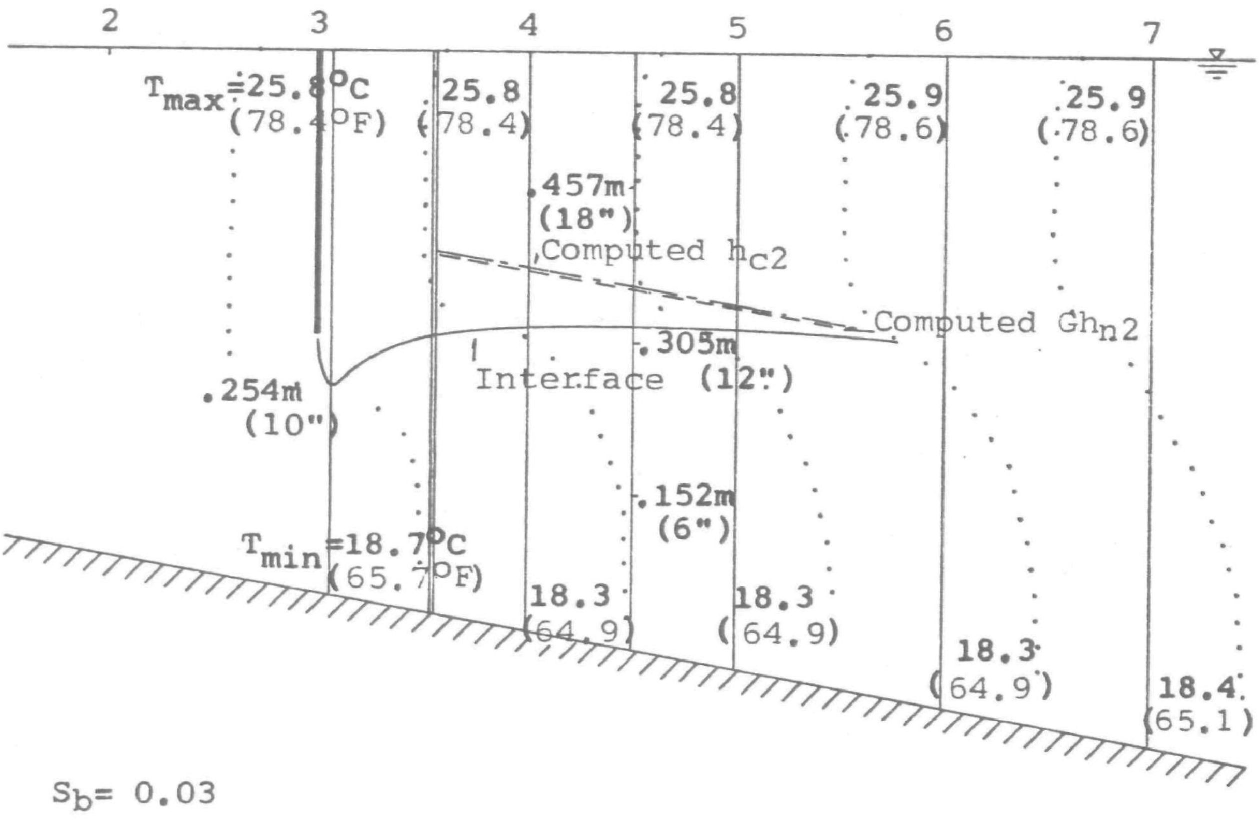



Fig. 17. Temperature distribution curve for S3-5/14.

Table 6. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR S3-5/14.

S3-5/14 Expt. No. 32

| | | |
|---|---|---|
| $S_b$=.03 | q=.00378(.1337) | |
| $T_{S1}$=26.4(79.5) | $T_{S2}$=18.0(64.4) | $T_0$=22.8(73.0) |
| $T_1$=25.6(78.1) | $T_2$=19.4(66.9) | $_o\Delta\rho/\rho$=.00154 |
| $f_b$=.208 | $f_b/(8S_b)$=.867 | |

$_oh_{c2}$=.217(8.54)±15%

$_of_i$=.0301

| x | 0.00 | 1.22(4.00) [.102,.101] | 2.44(8.00) [.204,.203] |
|---|---|---|---|
| $h_2$ | .274(10.8) [.766,.760] | .320(12.6) [.894,.887] | .351(13.8) [.979,.971] |
| H | .559(22.0) [1.56,1.55] | .594(23.4) [1.66,1.65] | .632(24.9) [1.76,1.75] |

Method 1

| | | |
|---|---|---|
| $h_{c2}$=.358(14.1) | $_mh_c$=1.650 | $\Delta\rho/\rho$=.000343 |
| $f_i$=.0135 | $_mf_i$=0.450 | $f_i/f_b$=.0651 |

| | | | |
|---|---|---|---|
| $h_2$ | .274(10.8)[.766] | .309(12.2)[.864] | .344(13.5)[.961] |
| $Gh_{n2}$ | .355(14.0)[.993] | .356(14.0)[.995] | .357(14.1)[.997] |
| $dh_2/dx$ | .0288[.960] | .0287[.956] | .0276[.919] |
| H | .558(22.0)[1.56] | .594(23.4)[1.66] | .631(24.8)[1.76] |

Method 2

| | | |
|---|---|---|
| $h_{c2}$=.361(14.2) | $_mh_c$=1.663 | $\beta$=4.60 |
| $f_i$=.406 | $_mf_i$=13.50 | $f_i/f_b$=1.954 |

| | | | |
|---|---|---|---|
| $h_2$ | .274(10.8)[.760] | .306(12.0)[.847] | .338(13.3)[.935] |
| $Gh_{n2}$ | .350(13.8)[.970] | .354(14.0)[.982] | .358(14.1)[.993] |
| $dh_2/dx$ | .0254[.845] | .0259[.863] | .0267[.891] |
| H | .558(22.0)[1.55] | .594(23.4)[1.65] | .631(24.8)[1.75] |

**Table 7. INTERFACIAL PROFILE CALCULATION FOR S3-5/14.**

S3-5/14 ACC= 3.0 METHOD 1

T1= 25.6C= 78.1F T2= 19.4C= 66.9F Q= 1.337E-01
SB= 3.000E-02 FB= 2.080E-01 QDELTA= 1.540E-03
OHC2I= 8.543E 00 QFI= 3.010E-02 .125*FB/SB= 8.667E-01

HC2I= 1.410E 01 MHC= 1.650 DELTA= 3.428E-04

FI= 1.354E-02 MFI= 0.450 FI/FB= 6.512E-02

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.080E 01 | 1.399E 01 | 2.195E 01 | 2.879E-02 | 0.00E-01 |
| 2.000E 00 | 1.149E 01 | 1.401E 01 | 2.267E 01 | 2.876E-02 | 9.24E-06 |
| 4.000E 00 | 1.218E 01 | 1.402E 01 | 2.339E 01 | 2.868E-02 | 3.43E-05 |
| 6.000E 00 | 1.287E 01 | 1.404E 01 | 2.411E 01 | 2.846E-02 | 5.82E-05 |
| 8.000E 00 | 1.354E 01 | 1.405E 01 | 2.483E 01 | 2.757E-02 | 1.00E-04 |
| 1.000E 01 | 1.430E 01 | 1.407E 01 | 2.555E 01 | 3.345E-02 | 1.12E-04 |

IT IS NO LONGER A S3 PROFILE FROM THIS POINT

S3-5/14 METHOD 2

HC2I= 1.421E 01 MHC= 1.663 BETA= 4.600

FI= 4.063E-01 MFI=13.500 FI/FB= 1.954E 00

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.080E 01 | 1.378E 01 | 2.195E 01 | 2.535E-02 | 0.00E-01 |
| 2.000E 00 | 1.141E 01 | 1.387E 01 | 2.267E 01 | 2.561E-02 | 8.49E-04 |
| 4.000E 00 | 1.203E 01 | 1.395E 01 | 2.339E 01 | 2.589E-02 | 1.61E-03 |
| 6.000E 00 | 1.265E 01 | 1.403E 01 | 2.411E 01 | 2.622E-02 | 2.29E-03 |
| 8.000E 00 | 1.329E 01 | 1.411E 01 | 2.483E 01 | 2.673E-02 | 2.91E-03 |
| 1.000E 01 | 1.395E 01 | 1.420E 01 | 2.555E 01 | 2.952E-02 | 3.45E-03 |
| 1.200E 01 | 1.464E 01 | 1.430E 01 | 2.627E 01 | 2.377E-02 | 3.95E-03 |

IT IS NO LONGER A S3 PROFILE FROM THIS POINT

**Table 7. CONTINUED.**

S3-5/14 ACC= 3.0 NON-DIMENSIONALIZED METHOD 1

T1= 25.6C= 78.1F T2= 19.4C= 66.9F Q= 1.337E-01
SB= 3.000E-02 FB= 2.080E-01 ODELTA= 1.540E-03
OHC2I= 8.543E 00 OFI= 3.010E-02 .125*FB/SB= 8.667E-01

HC2I= 1.410E 01 MHC= 1.650 DELTA= 3.428E-04

FI= 1.354E-02 MFI= 0.450 FI/FB= 6.512E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 7.662E-01 | 9.925E-01 | 1.557E 00 | 9.596E-01 |
| 5.108E-02 | 8.152E-01 | 9.937E-01 | 1.608E 00 | 9.588E-01 |
| 1.022E-01 | 8.641E-01 | 9.948E-01 | 1.659E 00 | 9.561E-01 |
| 1.532E-01 | 9.128E-01 | 9.959E-01 | 1.710E 00 | 9.486E-01 |
| 2.043E-01 | 9.606E-01 | 9.969E-01 | 1.762E 00 | 9.189E-01 |
| 2.554E-01 | 1.015E 00 | 9.983E-01 | 1.813E 00 | 1.115E 00 |

IT IS NO LONGER A S3 PROFILE FROM THIS POINT

S3-5/14 METHOD 2

HC2I= 1.421E 01 MHC= 1.663 BETA= 4.600

FI= 4.063E-01 MFI=13.500 FI/FB= 1.954E 00

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 7.601E-01 | 9.701E-01 | 1.545E 00 | 8.448E-01 |
| 5.067E-02 | 8.032E-01 | 9.759E-01 | 1.596E 00 | 8.537E-01 |
| 1.013E-01 | 8.467E-01 | 9.817E-01 | 1.646E 00 | 8.630E-01 |
| 1.520E-01 | 8.907E-01 | 9.875E-01 | 1.697E 00 | 8.739E-01 |
| 2.027E-01 | 9.353E-01 | 9.934E-01 | 1.748E 00 | 8.911E-01 |
| 2.534E-01 | 9.820E-01 | 9.997E-01 | 1.799E 00 | 9.839E-01 |
| 3.040E-01 | 1.030E 00 | 1.006E 00 | 1.849E 00 | 7.922E-01 |

IT IS NO LONGER A S3 PROFILE FROM THIS POINT

·M1 profile

The M1 profile was obtained with a small slope and a small flow rate. The interface was almost parallel to the free surface in the experiment.

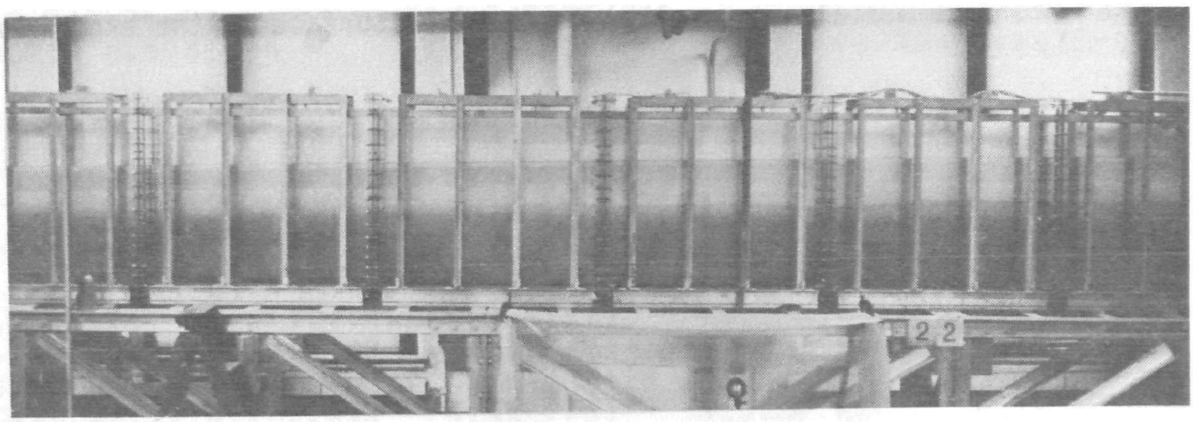


Fig. 18. Experiment M1-4/21.

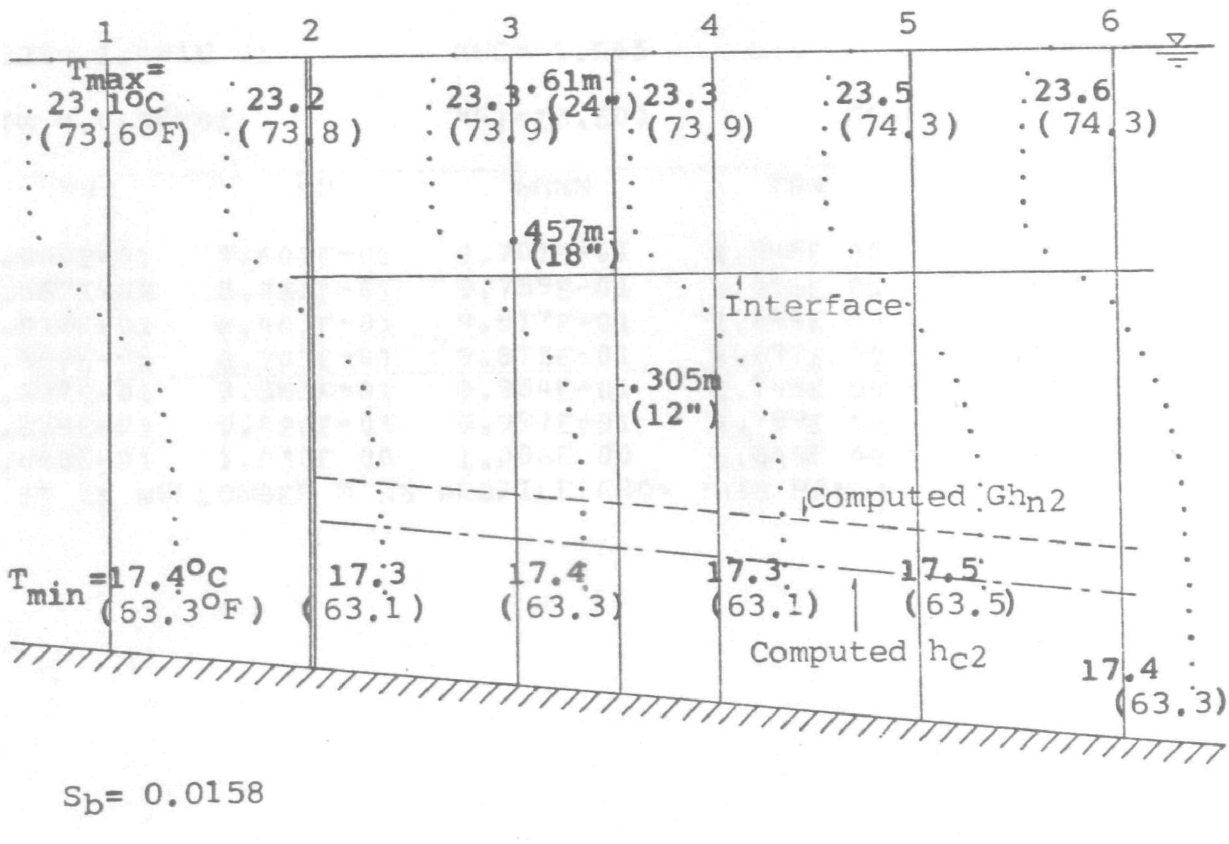



Fig. 19. Temperature distribution curve for M1-4/21.

Table 8. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR M1-4/21.

M1-4/21 Expt. No. 22

| | | |
|---|---|---|
| $S_b$=.0158 | q=.00158(.0556) | |
| $T_{S1}$=24.1(75.4) | $T_{S2}$=16.3(61.3) | $T_0$=21.9(71.4) |
| $T_1$=23.1(73.6) | $T_2$=18.1(64.6) | ${}_o\Delta\rho/\rho$=.00106 |
| $f_b$=.255 | $f_b/(8S_b)$=2.02 | |

${}_o h_{c2}$=.137(5.39)±23%

${}_o f_i$=.0394

| x | 0.00 | 2.44(8.00) [.268,.269] | 4.88(16.0) [.536,.538] |
|---|---|---|---|
| $h_2$ | .389(15.3) [2.71,2.71] | .424(16.7) [2.96,2.96] | .462(18.2) [3.22,3.23] |
| H | .604(23.8) [4.21,4.22] | .645(25.4) [4.49,4.51] | .683(26.9) [4.76,4.77] |

Method 1

| | | |
|---|---|---|
| $h_{c2}$=.144(5.66) | ${}_m h_c$=1.050 | $\Delta\rho/\rho$=.000918 |
| $f_i$=.00788 | ${}_m f_i$=0.200 | $f_i/f_b$=.0309 |

| | | | |
|---|---|---|---|
| $h_2$ | .389(15.3)[2.71] | .425(16.7)[2.96] | .462(18.2)[3.22] |
| $Gh_{n2}$ | .187(7.36)[1.30] | .187(7.36)[1.30] | .187(7.37)[1.30] |
| $dh_2/dx$ | .0148[.937] | .0150[.952] | .0152[.962] |
| H | .604(23.8)[4.21] | .643(25.3)[.643] | .681(26.8)[4.75] |

Method 2

| | | |
|---|---|---|
| $h_{c2}$=.143(5.64) | ${}_m h_c$=1.048 | β=1.15 |
| $f_i$=.0236 | ${}_m f_i$=0.600 | $f_i/f_b$=.0926 |

| | | | |
|---|---|---|---|
| $h_2$ | .389(15.3)[2.71] | .425(16.7)[2.97] | .462(18.2)[3.22] |
| $Gh_{n2}$ | .187(7.35)[1.30] | .187(7.38)[1.31] | .188(7.41)[1.31] |
| $dh_2/dx$ | .0148[.936] | .0150[.951] | .0152[.961] |
| H | .604(23.8)[4.22] | .643(25.3)[4.49] | .681(26.8)[4.76] |

**Table 9. INTERFACIAL PROFILE CALCULATION FOR M1-4/21.**

M1-4/21 ACC= 4.0 METHOD 1

T1= 23.1C= 73.6F T2= 18.1C= 64.6F Q= 5.560E-02
SB= 1.580E-02 FB= 2.552E-01 QDELTA= 1.063E-03
QHC2I= 5.386E 00 QFI= 3.940E-02 .125*FB/SB= 2.019E 00

HC2I= 5.655E 00 MHC= 1.050 DELTA= 9.183E-04

FI= 7.880E-03 MFI= 0.200 FI/FB= 3.088E-02

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.530E 01 | 7.348E 00 | 2.380E 01 | 1.480E-02 | 0.00E-01 |
| 4.000E 00 | 1.601E 01 | 7.353E 00 | 2.456E 01 | 1.493E-02 | -2.00E-05 |
| 8.000E 00 | 1.673E 01 | 7.358E 00 | 2.532E 01 | 1.504E-02 | -8.58E-06 |
| 1.200E 01 | 1.746E 01 | 7.364E 00 | 2.608E 01 | 1.513E-02 | -1.34E-05 |
| DOUBLED INTERVAL | | DX= 8.0000E 00 | | | |
| 1.600E 01 | 1.818E 01 | 7.369E 00 | 2.683E 01 | 1.521E-02 | -2.86E-06 |
| 2.400E 01 | 1.965E 01 | 7.380E 00 | 2.835E 01 | 1.533E-02 | -1.24E-05 |
| 3.200E 01 | 2.113E 01 | 7.391E 00 | 2.987E 01 | 1.542E-02 | -6.68E-06 |
| 4.000E 01 | 2.261E 01 | 7.402E 00 | 3.138E 01 | 1.549E-02 | -6.68E-06 |
| DOUBLED INTERVAL | | DX= 1.6000E 01 | | | |
| 4.800E 01 | 2.410E 01 | 7.413E 00 | 3.290E 01 | 1.554E-02 | -1.62E-05 |
| 6.400E 01 | 2.709E 01 | 7.435E 00 | 3.593E 01 | 1.562E-02 | -1.05E-05 |
| 8.000E 01 | 3.009E 01 | 7.457E 00 | 3.897E 01 | 1.566E-02 | -3.81E-06 |
| 9.600E 01 | 3.311E 01 | 7.479E 00 | 4.200E 01 | 1.570E-02 | 0.00E-01 |
| 1.120E 02 | 3.612E 01 | 7.502E 00 | 4.504E 01 | 1.572E-02 | 0.00E-01 |
| DOUBLED INTERVAL | | DX= 3.2000E 01 | | | |
| 1.280E 02 | 3.914E 01 | 7.524E 00 | 4.807E 01 | 1.574E-02 | 0.00E-01 |
| 1.600E 02 | 4.519E 01 | 7.568E 00 | 5.414E 01 | 1.576E-02 | -1.53E-05 |
| 1.920E 02 | 5.124E 01 | 7.611E 00 | 6.020E 01 | 1.577E-02 | 0.00E-01 |
| 2.240E 02 | 5.730E 01 | 7.654E 00 | 6.627E 01 | 1.578E-02 | 0.00E-01 |
| DOUBLED INTERVAL | | DX= 6.4000E 01 | | | |
| 2.560E 02 | 6.336E 01 | 7.697E 00 | 7.234E 01 | 1.578E-02 | 0.00E-01 |
| 3.200E 02 | 7.548E 01 | 7.781E 00 | 8.447E 01 | 1.579E-02 | 0.00E-01 |

INTERFACE BECOMES HORIZONTAL

## Table 9. CONTINUED.

M1-4/21    ACC= 4.0    METHOD 2

HC2I= 5.643E 00    MHC= 1.048    BETA= 1.150

FI= 2.364E-02    MFI= 0.600    FI/FB= 9.263E-02

| X | HT | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.530E 01 | 7.351E 00 | 2.380E 01 | 1.479E-02 | 0.00E-01 |
| 4.000E 00 | 1.601E 01 | 7.365E 00 | 2.456E 01 | 1.492E-02 | 2.15E-06 |
| 8.000E 00 | 1.673E 01 | 7.378E 00 | 2.532E 01 | 1.502E-02 | 2.00E-05 |
| 1.200E 01 | 1.745E 01 | 7.392E 00 | 2.608E 01 | 1.511E-02 | 3.72E-05 |
| DOUBLED INTERVAL | DX= 8.0000E 00 | | | | |
| 1.600E 01 | 1.818E 01 | 7.405E 00 | 2.683E 01 | 1.519E-02 | 2.86E-05 |
| 2.400E 01 | 1.965E 01 | 7.433E 00 | 2.835E 01 | 1.531E-02 | 5.25E-05 |
| 3.200E 01 | 2.112E 01 | 7.460E 00 | 2.987E 01 | 1.540E-02 | 6.10E-05 |
| 4.000E 01 | 2.260E 01 | 7.488E 00 | 3.138E 01 | 1.547E-02 | 8.01E-05 |
| DOUBLED INTERVAL | DX= 1.6000E 01 | | | | |
| 4.800E 01 | 2.409E 01 | 7.516E 00 | 3.290E 01 | 1.552E-02 | 8.30E-05 |
| 6.400E 01 | 2.708E 01 | 7.571E 00 | 3.593E 01 | 1.560E-02 | 9.54E-05 |
| 8.000E 01 | 3.008E 01 | 7.626E 00 | 3.897E 01 | 1.565E-02 | 1.08E-04 |
| 9.600E 01 | 3.308E 01 | 7.680E 00 | 4.200E 01 | 1.568E-02 | 1.37E-04 |
| 1.120E 02 | 3.610E 01 | 7.734E 00 | 4.504E 01 | 1.570E-02 | 1.37E-04 |
| DOUBLED INTERVAL | DX= 3.2000E 01 | | | | |
| 1.280E 02 | 3.911E 01 | 7.788E 00 | 4.807E 01 | 1.572E-02 | 1.53E-04 |
| 1.600E 02 | 4.516E 01 | 7.892E 00 | 5.414E 01 | 1.575E-02 | 1.53E-04 |
| 1.920E 02 | 5.121E 01 | 7.995E 00 | 6.020E 01 | 1.576E-02 | 1.68E-04 |
| 2.240E 02 | 5.726E 01 | 8.095E 00 | 6.627E 01 | 1.577E-02 | 1.68E-04 |
| DOUBLED INTERVAL | DX= 6.4000E 01 | | | | |
| 2.560E 02 | 6.332E 01 | 8.192E 00 | 7.234E 01 | 1.578E-02 | 1.68E-04 |
| 3.200E 02 | 7.544E 01 | 8.381E 00 | 8.447E 01 | 1.578E-02 | 1.83E-04 |

INTERFACE BECOMES HORIZONTAL

Table 9. CONTINUED.

M1-4/21 ACC= 4.0 NON-DIMENSIONALIZED METHOD 1

T1= 23.1C= 73.6F T2= 18.1C= 64.6F Q= 5.560E-02
SB= 1.580E-02 FB= 2.552E-01 ODELTA= 1.063E-03
OHC2I= 5.386E 00 OFI= 3.940E-02 .125*FB/SB= 2.019E 00

HC2I= 5.655E 00 MHC= 1.050 DELTA= 9.183E-04

FI= 7.880E-03 MFI= 0.200 FI/FB= 3.088E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 2.705E 00 | 1.299E 00 | 4.208E 00 | 9.365E-01 |
| 1.341E-01 | 2.832E 00 | 1.300E 00 | 4.343E 00 | 9.448E-01 |
| 2.682E-01 | 2.959E 00 | 1.301E 00 | 4.477E 00 | 9.517E-01 |
| 4.023E-01 | 3.087E 00 | 1.302E 00 | 4.611E 00 | 9.575E-01 |
| DOUBLED INTERVAL DXN= 2.6821E-01 | | | | |
| 5.364E-01 | 3.216E 00 | 1.303E 00 | 4.745E 00 | 9.624E-01 |
| 8.046E-01 | 3.475E 00 | 1.305E 00 | 5.013E 00 | 9.702E-01 |
| 1.073E 00 | 3.736E 00 | 1.307E 00 | 5.281E 00 | 9.759E-01 |
| 1.341E 00 | 3.998E 00 | 1.309E 00 | 5.549E 00 | 9.803E-01 |
| DOUBLED INTERVAL DXN= 5.3642E-01 | | | | |
| 1.609E 00 | 4.261E 00 | 1.311E 00 | 5.818E 00 | 9.836E-01 |
| 2.146E 00 | 4.790E 00 | 1.315E 00 | 6.354E 00 | 9.883E-01 |
| 2.682E 00 | 5.321E 00 | 1.319E 00 | 6.891E 00 | 9.914E-01 |
| 3.218E 00 | 5.854E 00 | 1.323E 00 | 7.427E 00 | 9.934E-01 |
| 3.755E 00 | 6.387E 00 | 1.326E 00 | 7.963E 00 | 9.949E-01 |
| DOUBLED INTERVAL DXN= 1.0728E 00 | | | | |
| 4.291E 00 | 6.921E 00 | 1.330E 00 | 8.500E 00 | 9.959E-01 |
| 5.364E 00 | 7.990E 00 | 1.338E 00 | 9.573E 00 | 9.973E-01 |
| 6.437E 00 | 9.061E 00 | 1.346E 00 | 1.065E 01 | 9.981E-01 |
| 7.510E 00 | 1.013E 01 | 1.353E 00 | 1.172E 01 | 9.986E-01 |
| DOUBLED INTERVAL DXN= 2.1457E 00 | | | | |
| 8.583E 00 | 1.120E 01 | 1.361E 00 | 1.279E 01 | 9.989E-01 |
| 1.073E 01 | 1.335E 01 | 1.376E 00 | 1.494E 01 | 9.993E-01 |

INTERFACE BECOMES HORIZONTAL

## Table 9. CONTINUED.

M1-4/21 ACC= 4.0 NON-DIMENSIONALIZED METHOD 2

HC2I= 5.643E 00 MHC= 1.048 BETA= 1.150

FI= 2.364E-02 MFI= 0.600 FI/FB= 9.263E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 2.711E 00 | 1.303E 00 | 4.218E 00 | 9.360E-01 |
| 1.344E-01 | 2.838E 00 | 1.305E 00 | 4.352E 00 | 9.440E-01 |
| 2.688E-01 | 2.965E 00 | 1.307E 00 | 4.487E 00 | 9.507E-01 |
| 4.032E-01 | 3.093E 00 | 1.310E 00 | 4.621E 00 | 9.564E-01 |
| DOUBLED INTERVAL DXN= 2.6880E-01 | | | | |
| 5.376E-01 | 3.222E 00 | 1.312E 00 | 4.755E 00 | 9.612E-01 |
| 8.064E-01 | 3.482E 00 | 1.317E 00 | 5.024E 00 | 9.688E-01 |
| 1.075E 00 | 3.743E 00 | 1.322E 00 | 5.293E 00 | 9.745E-01 |
| 1.344E 00 | 4.005E 00 | 1.327E 00 | 5.562E 00 | 9.789E-01 |
| DOUBLED INTERVAL DXN= 5.3760E-01 | | | | |
| 1.613E 00 | 4.269E 00 | 1.332E 00 | 5.831E 00 | 9.823E-01 |
| 2.150E 00 | 4.798E 00 | 1.342E 00 | 6.368E 00 | 9.871E-01 |
| 2.688E 00 | 5.330E 00 | 1.351E 00 | 6.906E 00 | 9.902E-01 |
| 3.226E 00 | 5.863E 00 | 1.361E 00 | 7.443E 00 | 9.924E-01 |
| 3.763E 00 | 6.397E 00 | 1.371E 00 | 7.981E 00 | 9.940E-01 |
| DOUBLED INTERVAL DXN= 1.0752E 00 | | | | |
| 4.301E 00 | 6.932E 00 | 1.380E 00 | 8.519E 00 | 9.951E-01 |
| 5.376E 00 | 8.002E 00 | 1.399E 00 | 9.594E 00 | 9.966E-01 |
| 6.451E 00 | 9.074E 00 | 1.417E 00 | 1.067E 01 | 9.975E-01 |
| 7.526E 00 | 1.015E 01 | 1.434E 00 | 1.174E 01 | 9.981E-01 |
| DOUBLED INTERVAL DXN= 2.1504E 00 | | | | |
| 8.602E 00 | 1.122E 01 | 1.452E 00 | 1.282E 01 | 9.985E-01 |
| 1.075E 01 | 1.337E 01 | 1.485E 00 | 1.497E 01 | 9.990E-01 |
| INTERFACE BECOMES HORIZONTAL | | | | |

·M2 profile

This profile was the most clearly observed in the flume. The lower layer depth decreases along the flow after the unstable frontal zone. The higher the flow rate, the steeper the slope of the interface became.

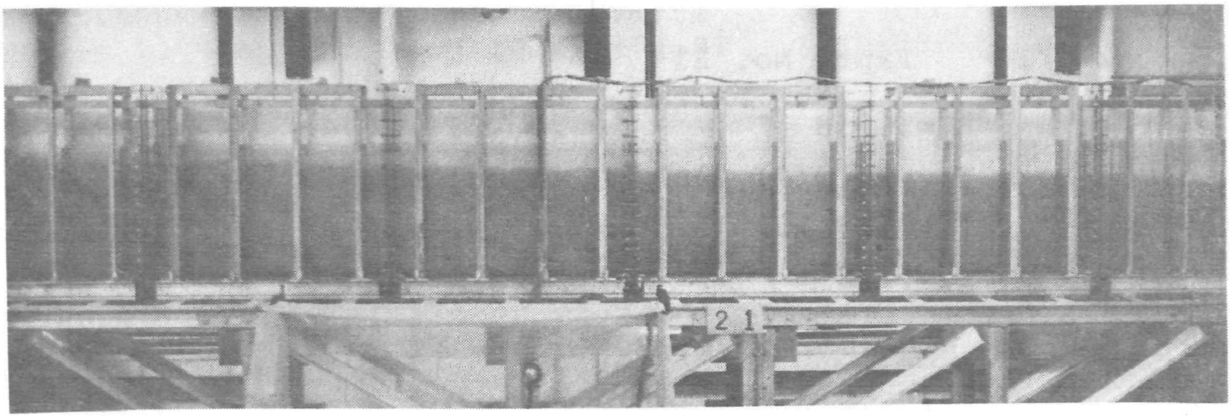


Fig. 20. Experiment M2-4/18.

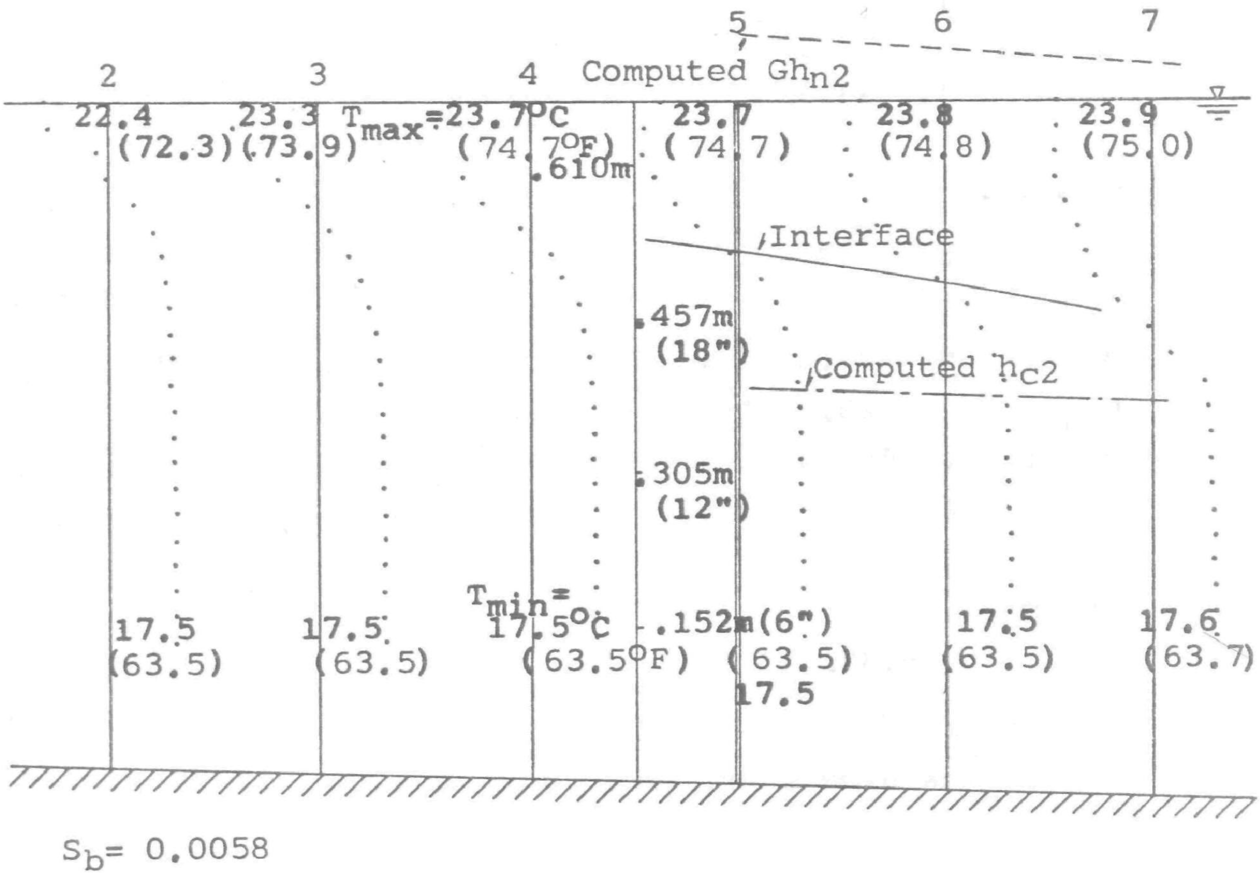



Fig. 21. Temperature distribution curve for M2-4/18.

## Table 10. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR M2-4/18.

M2-4/18 Expt. No. 21

| | | |
|---|---|---|
| $S_b$=.0058 | q=.00757(.2674) | |
| $T_{S1}$=25.2(77.4) | $T_{S2}$=17.7(63.9) | $T_0$=23.2(73.8) |
| $T_1$=22.8(73.0) | $T_2$=17.8(64.0) | $_o\Delta\rho/\rho$=.00105 |
| $f_b$=.225 | $f_b/(8S_b)$=4.84 | |

$_oh_{c2}$=.392(15.4)±20%
$_of_i$=.0274

| x | 0.00 | .610(2.0) [.0090,.0090] | 1.22(4.0) [.0181,.0181] |
|---|---|---|---|
| $h_2$ | .523(20.6) [1.34,1.34] | .511(20.1) [1.31,1.31] | .498(19.6) [1.27,1.27] |
| H | .676(26.6) [1.73,1.73] | .678(26.7) [1.73,1.73] | .683(26.9) [1.74,1.74] |

### Method 1

| | | |
|---|---|---|
| $h_{c2}$=.392(15.4) | $_mh_c$=1.000 | $\Delta\rho/\rho$=.00105 |
| $f_i$=.0206 | $_mf_i$=0.750 | $f_i/f_b$=.0915 |

| | | | |
|---|---|---|---|
| $h_2$ | .523(20.6)[1.34] | .511(20.1)[1.31] | .498(19.6)[1.27] |
| $Gh_{n2}$ | .742(29.2)[1.90] | .736(29.0)[1.88] | .730(28.7)[1.86] |
| $dh_2/dx$ | -.0185[-3.19] | -.0209[-3.60] | -.0245[-4.22] |
| H | .676(26.6)[1.73] | .679(26.7)[1.73] | .683(26.9)[1.74] |

### Method 2

| | | |
|---|---|---|
| $h_{c2}$=.392(15.4) | $_mh_c$=1.000 | $\beta$=1.00 |
| $f_i$=.0206 | $_mf_i$=0.750 | $f_i/f_b$=.0915 |

The same entry as in Method 1.

**Table 11. INTERFACIAL PROFILE CALCULATION FOR M2-4/18.**

M2-4/18 ACC= 3.0 METHOD 1

T1= 22.8C= 73.0F T2= 17.8C= 64.0F Q= 2.674E-01
SB= 5.800E-03 FB= 2.247E-01 ODELTA= 1.048E-03
OHC2I= 1.542E 01 OFI= 2.740E-02 .125*FB/SB= 4.843E 00

HC2I= 1.542E 01 MHC= 1.000 DELTA= 1.048E-03

FI= 2.055E-02 MFI= 0.750 FI/FB= 9.146E-02

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 2.060E 01 | 2.922E 01 | 2.660E 01 | -1.853E-02 | 0.00E-01 |
| 2.000E 00 | 2.013E 01 | 2.897E 01 | 2.674E 01 | -2.089E-02 | 9.27E-05 |
| 3.999E 00 | 1.959E 01 | 2.874E 01 | 2.688E 01 | -2.445E-02 | 1.85E-04 |
| 5.999E 00 | 1.894E 01 | 2.851E 01 | 2.702E 01 | -3.044E-02 | 2.63E-04 |
| 7.999E 00 | 1.808E 01 | 2.828E 01 | 2.716E 01 | -4.326E-02 | 3.71E-04 |
| 9.999E 00 | 1.654E 01 | 2.797E 01 | 2.730E 01 | -1.173E-01 | 4.49E-04 |

HALVED INTERVAL DX= 1.0000E 00
GHN-CUBE BECOMES NEGATIVE

M2-4/18 METHOD 2

HC2I= 1.542E 01 MHC= 1.000 BETA= 1.000

FI= 2.055E-02 MFI= 0.750 FI/FB= 9.146E-02

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 2.060E 01 | 2.922E 01 | 2.660E 01 | -1.853E-02 | 0.00E-01 |
| 2.000E 00 | 2.013E 01 | 2.897E 01 | 2.674E 01 | -2.089E-02 | 9.27E-05 |
| 3.999E 00 | 1.959E 01 | 2.874E 01 | 2.688E 01 | -2.445E-02 | 1.85E-04 |
| 5.999E 00 | 1.894E 01 | 2.851E 01 | 2.702E 01 | -3.044E-02 | 2.63E-04 |
| 7.999E 00 | 1.808E 01 | 2.828E 01 | 2.716E 01 | -4.326E-02 | 3.71E-04 |
| 9.999E 00 | 1.654E 01 | 2.797E 01 | 2.730E 01 | -1.173E-01 | 4.49E-04 |

HALVED INTERVAL DX= 1.0000E 00
GHN-CUBE BECOMES NEGATIVE

**Table 11. CONTINUED.**

M2-4/18 ACC= 3.0 NON-DIMENSIONALIZED METHOD 1

T1= 22.8C= 73.0F T2= 17.8C= 64.0F Q= 2.674E-01
SB= 5.800E-03 FB= 2.247E-01 ODELTA= 1.048E-03
OHC2I= 1.542E 01 OFI= 2.740E-02 .125*FB/SB= 4.843E 00

HC2I= 1.542E 01 MHC= 1.000 DELTA= 1.048E-03

FI= 2.055E-02 MFI= 0.750 FI/FB= 9.146E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 1.336E 00 | 1.895E 00 | 1.725E 00 | -3.194E 00 |
| 9.026E-03 | 1.305E 00 | 1.879E 00 | 1.734E 00 | -3.602E 00 |
| 1.805E-02 | 1.270E 00 | 1.864E 00 | 1.743E 00 | -4.216E 00 |
| 2.708E-02 | 1.228E 00 | 1.849E 00 | 1.752E 00 | -5.249E 00 |
| 3.610E-02 | 1.172E 00 | 1.834E 00 | 1.761E 00 | -7.459E 00 |
| 4.513E-02 | 1.073E 00 | 1.814E 00 | 1.770E 00 | -2.022E 01 |

HALVED INTERVAL DXN= 4.5136E-03
GHN-CUBE BECOMES NEGATIVE

M2-4/18 METHOD 2

HC2I= 1.542E 01 MHC= 1.000 BETA= 1.000

FI= 2.055E-02 MFI= 0.750 FI/FB= 9.146E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 1.336E 00 | 1.895E 00 | 1.725E 00 | -3.194E 00 |
| 9.026E-03 | 1.305E 00 | 1.879E 00 | 1.734E 00 | -3.602E 00 |
| 1.805E-02 | 1.270E 00 | 1.864E 00 | 1.743E 00 | -4.216E 00 |
| 2.708E-02 | 1.228E 00 | 1.849E 00 | 1.752E 00 | -5.249E 00 |
| 3.610E-02 | 1.172E 00 | 1.834E 00 | 1.761E 00 | -7.459E 00 |
| 4.513E-02 | 1.073E 00 | 1.814E 00 | 1.770E 00 | -2.022E 01 |

HALVED INTERVAL DXN= 4.5136E-03
GHN-CUBE BECOMES NEGATIVE

·M3 profile

This profile utilized the gate. At the first glance, it is very similar to the S3 profile. However, one notable difference between them found during the experiment was that the wave was seen after the lower layer reached the largest depth for this profile, whereas no significant wave was seen in the S3 profile. Since the theory predicts the discontinuity at the point where the interface meets the stratified critical depth, this profile may be considered to be M3 profile.

The interface calculation could not produce a profile very close to the profile obtained in the experiment. This may be due to the short length of the curve available for the analysis. Unfortunately, the data with a gradual ascending curve was not obtained.

M3-5/18

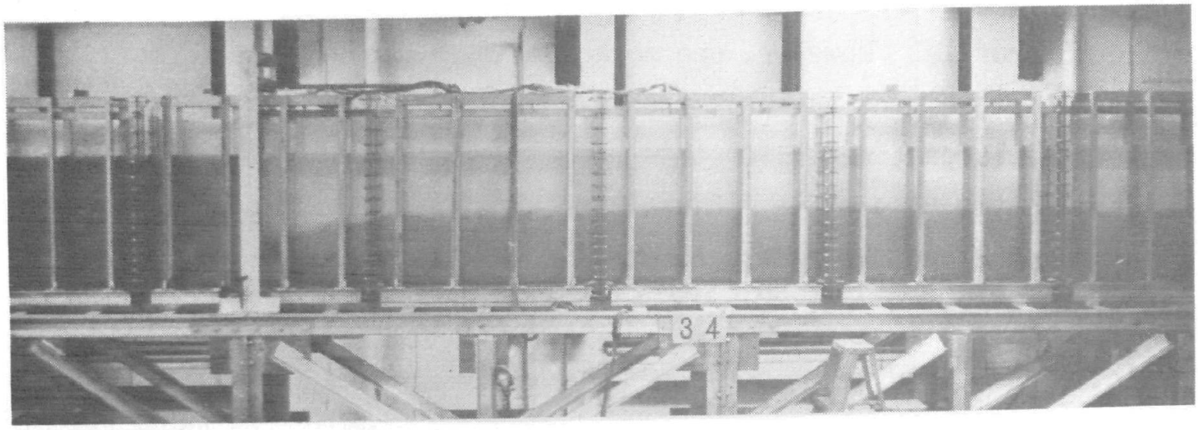


Fig. 22. Experiment M3-5/18.

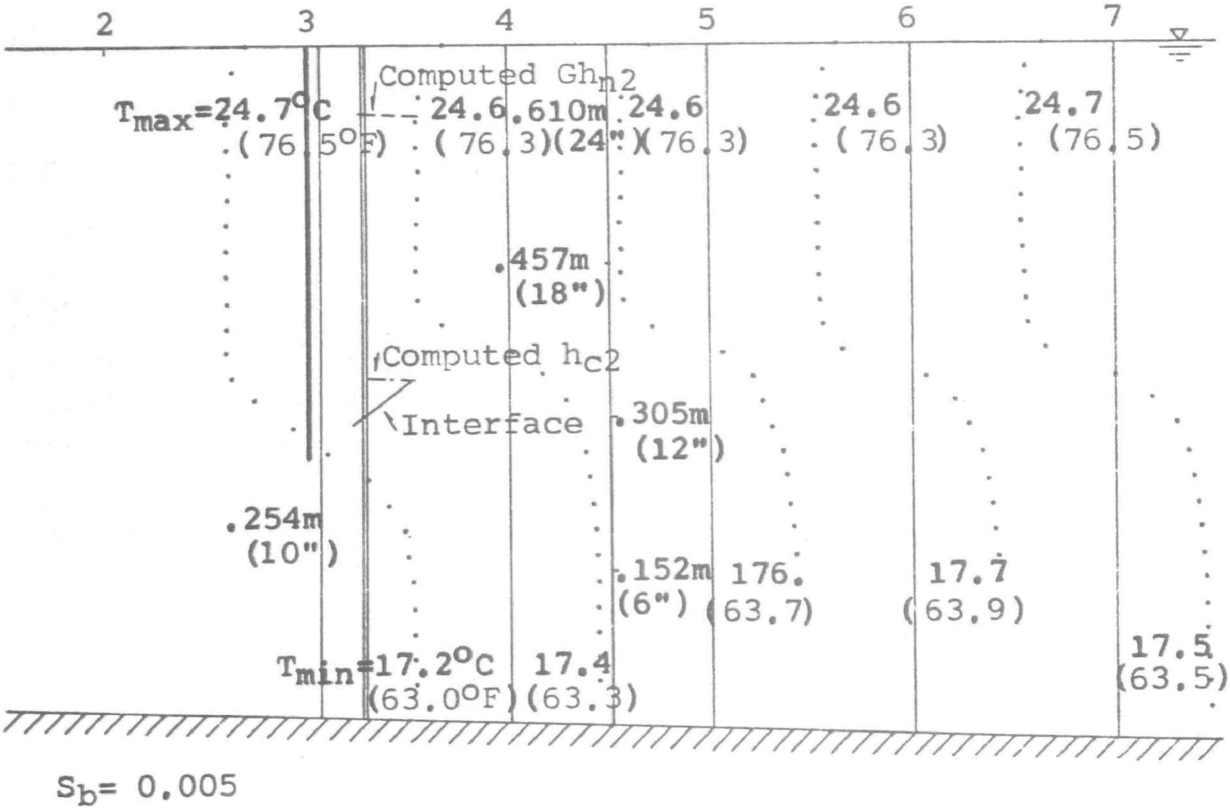



Fig. 23. Temperature distribution curve for M3-5/18.

Table 12. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR M3-5/18.

M3-5/18    Expt. No. 34

| | | |
|---|---|---|
| $S_b$=.005 | q=.00378(.1337) | |
| $T_{S1}$=25.1(77.2) | $T_{S2}$=17.0(62.6) | $T_0$=21.8(71.2) |
| $T_1$=24.3(75.7) | $T_2$=17.8(64.0) | ${}_o\Delta\rho/\rho$=.00141 |
| $f_b$=.210 | $f_b/(8S_b)$=5.24 | |

${}_oh_{c2}$=.223(8.79)±14%

${}_of_i$=.0305

| x | 0.00 | .075(.25) [.0011,.0012] | .152(.50) [.0023,.0023] |
|---|---|---|---|
| $h_2$ | .300(11.8) [.895,.911] | .307(12.1) [.917,.934] | .317(12.5) [.947,.965] |
| H | .668(26.3) [1.99,2.03] | .668(26.3) [1.99,2.03] | .669(26.3) [1.99,2.03] |

Method 1

| | | |
|---|---|---|
| $h_{c2}$=.335(13.2) | ${}_mh_c$=1.500 | $\Delta\rho/\rho$=.000418 |
| $f_i$=.0107 | ${}_mf_i$=0.350 | $f_i/f_b$=.0509 |

| | | | |
|---|---|---|---|
| $h_2$ | .300(11.8)[.895] | .307(12.1)[.917] | .317(12.5)[.946] |
| $Gh_{n2}$ | .599(23.6)[1.79] | .600(23.6)[1.79] | .600(23.5)[1.79] |
| $dh_2/dx$ | .0881[17.6] | .108[21.6] | .159[31.8] |
| H | .668(26.3)[1.99] | .668(26.3)[2.00] | .669(26.3)[2.00] |

Method 2

| | | |
|---|---|---|
| $h_{c2}$=.329(13.0) | ${}_mh_c$=1.474 | $\beta$=3.20 |
| $f_i$=.232 | ${}_mf_i$=7.600 | $f_i/f_b$=1.11 |

| | | | |
|---|---|---|---|
| $h_2$ | .300(11.8)[.911] | .307(12.1)[.933] | .318(12.5)[.967] |
| $Gh_{n2}$ | .560(22.0)[1.70] | .562(22.1)[1.71] | .566(22.3)[1.72] |
| $dh_2/dx$ | .0850[17.0] | .111[22.2] | .216[43.2] |
| H | .668(26.3)[2.03] | .668(26.3)[2.03] | .669(26.3)[2.03] |

Table 13. INTERFACIAL PROFILE CALCULATION FOR M3-5/18.

M3-5/18 ACC= 3.0 METHOD 1

T1= 24.3C= 75.7F T2= 17.8C= 64.0F Q= 1.337E-01
SB= 5.000E-03 FB= 2.096E-01 ODELTA= 1.412E-03
OHC2I= 8.794E 00 OFI= 3.047E-02 .125*FB/SB= 5.240E 00

HC2I= 1.319E 01 MHC= 1.500 DELTA= 4.184E-04

FI= 1.066E-02 MFI= 0.350 FI/FB= 5.088E-02

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.180E 01 | 2.360E 01 | 2.630E 01 | 8.811E-02 | 0.00E-01 |
| 1.250E-01 | 1.194E 01 | 2.360E 01 | 2.631E 01 | 9.637E-02 | -5.56E-06 |
| 2.499E-01 | 1.209E 01 | 2.361E 01 | 2.631E 01 | 1.079E-01 | -1.11E-05 |
| 3.749E-01 | 1.226E 01 | 2.362E 01 | 2.632E 01 | 1.258E-01 | -1.67E-05 |
| 4.998E-01 | 1.248E 01 | 2.363E 01 | 2.633E 01 | 1.590E-01 | -7.00E-06 |
| 6.248E-01 | 1.277E 01 | 2.364E 01 | 2.634E 01 | 2.629E-01 | -1.26E-05 |
| HALVED INTERVAL DX= 6.2500E-02 | | | | | |
| 6.873E-01 | 1.308E 01 | 2.366E 01 | 2.634E 01 | 9.406E-01 | -7.72E-06 |
| 7.498E-01 | 1.391E 01 | 2.371E 01 | 2.634E 01 | -1.351E-01 | -2.87E-06 |
| SIGN OF INTERFACIAL SLOPE IS CHANGED | | | | | |

M3-5/18 METHOD 2

HC2I= 1.296E 01 MHC= 1.474 BETA= 3.200

FI= 2.316E-01 MFI= 7.600 FI/FB= 1.105E 00

| X | HI | GHNI | TDI | HP | DTDHI |
|---|---|---|---|---|---|
| 0.000E-01 | 1.180E 01 | 2.204E 01 | 2.630E 01 | 8.497E-02 | 0.00E-01 |
| 1.248E-01 | 1.193E 01 | 2.208E 01 | 2.631E 01 | 9.521E-02 | 1.80E-05 |
| 2.496E-01 | 1.209E 01 | 2.214E 01 | 2.632E 01 | 1.108E-01 | 3.60E-05 |
| 3.745E-01 | 1.227E 01 | 2.220E 01 | 2.632E 01 | 1.387E-01 | 5.40E-05 |
| 4.993E-01 | 1.253E 01 | 2.229E 01 | 2.633E 01 | 2.159E-01 | 8.73E-05 |
| HALVED INTERVAL DX= 6.2500E-02 | | | | | |
| 5.615E-01 | 1.274E 01 | 2.237E 01 | 2.633E 01 | 4.196E-01 | 1.09E-04 |
| 6.237E-01 | 1.268E 01 | 2.234E 01 | 2.634E 01 | 3.303E-01 | 1.30E-04 |
| 6.859E-01 | 1.314E 01 | 2.252E 01 | 2.634E 01 | -5.072E-01 | 1.67E-04 |
| SIGN OF INTERFACIAL SLOPE IS CHANGED | | | | | |

Table 13. CONTINUED.

M3-5/18 ACC= 3.0 NON-DIMENSIONALIZED METHOD 1

T1= 24.3C= 75.7F T2= 17.8C= 64.0F Q= 1.337E-01
SB= 5.000E-03 FB= 2.096E-01 ODELTA= 1.412E-03
OHC2I= 8.794E 00 OFI= 3.047E-02 .125*FB/SB= 5.240E 00

HC2I= 1.319E 01 MHC= 1.500 DELTA= 4.184E-04

FI= 1.066E-02 MFI= 0.350 FI/FB= 5.088E-02

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 8.945E-01 | 1.789E 00 | 1.994E 00 | 1.762E 01 |
| 5.684E-04 | 9.050E-01 | 1.789E 00 | 1.994E 00 | 1.927E 01 |
| 1.137E-03 | 9.166E-01 | 1.790E 00 | 1.995E 00 | 2.158E 01 |
| 1.705E-03 | 9.298E-01 | 1.790E 00 | 1.995E 00 | 2.516E 01 |
| 2.274E-03 | 9.458E-01 | 1.791E 00 | 1.996E 00 | 3.181E 01 |
| 2.842E-03 | 9.683E-01 | 1.792E 00 | 1.997E 00 | 5.258E 01 |
| HALVED INTERVAL DXN= 2.8428E-04 | | | | |
| 3.126E-03 | 9.914E-01 | 1.794E 00 | 1.997E 00 | 1.881E 02 |
| 3.410E-03 | 1.054E 00 | 1.797E 00 | 1.997E 00 | -2.703E 01 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

M3-5/18 METHOD 2

HC2I= 1.296E 01 MHC= 1.474 BETA= 3.200

FI= 2.316E-01 MFI= 7.600 FI/FB= 1.105E 00

| XN | HN | GHNN | TDN | HPN |
|---|---|---|---|---|
| 0.000E-01 | 9.106E-01 | 1.701E 00 | 2.029E 00 | 1.699E 01 |
| 5.779E-04 | 9.210E-01 | 1.704E 00 | 2.030E 00 | 1.904E 01 |
| 1.156E-03 | 9.328E-01 | 1.708E 00 | 2.031E 00 | 2.215E 01 |
| 1.734E-03 | 9.471E-01 | 1.713E 00 | 2.031E 00 | 2.775E 01 |
| 2.312E-03 | 9.666E-01 | 1.720E 00 | 2.032E 00 | 4.317E 01 |
| HALVED INTERVAL DXN= 2.8938E-04 | | | | |
| 2.600E-03 | 9.831E-01 | 1.726E 00 | 2.032E 00 | 8.392E 01 |
| 2.888E-03 | 9.784E-01 | 1.724E 00 | 2.032E 00 | 6.606E 01 |
| 3.176E-03 | 1.014E 00 | 1.738E 00 | 2.033E 00 | -1.014E 02 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

·H1 profile

As predicted, H1 profile is very similar to the M2 profile.

H1-4/23

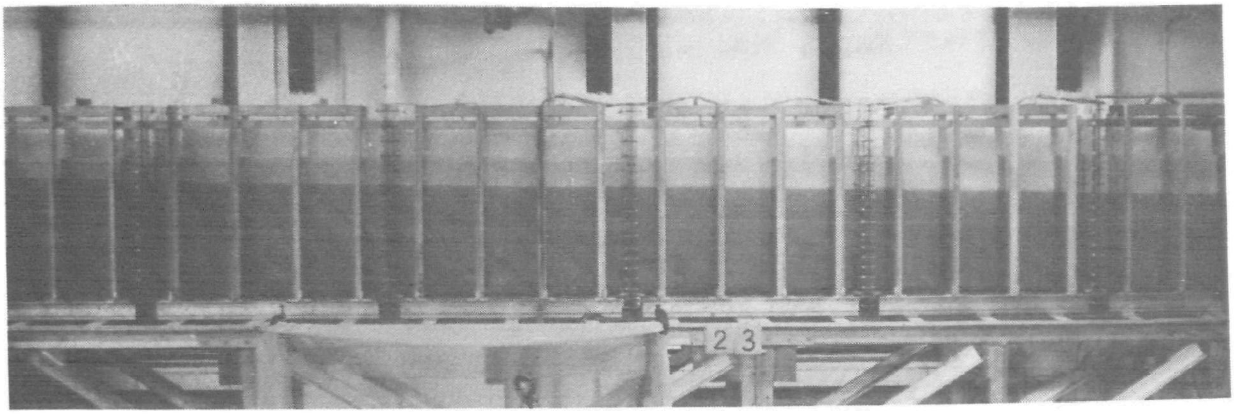



Fig. 24. Experiment H1-4/23.

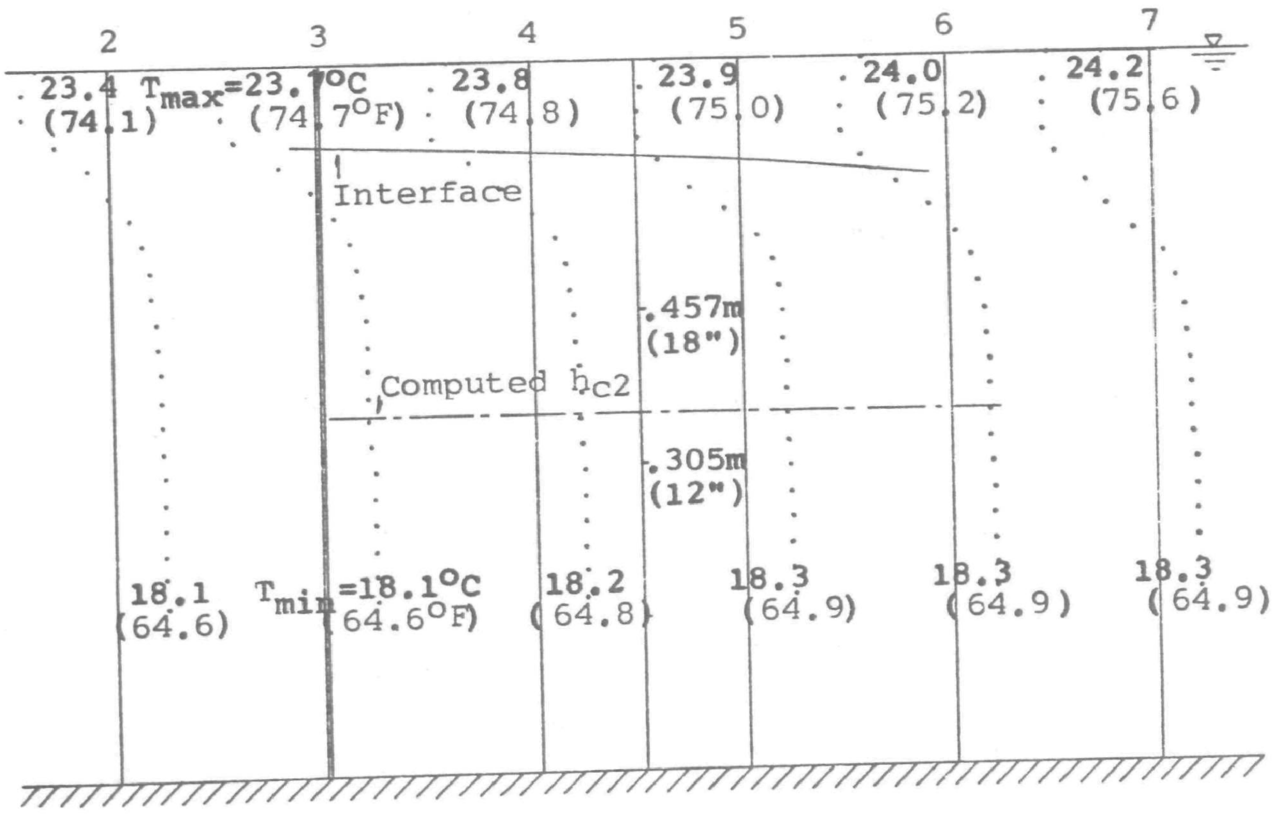



Fig. 25. Temperature distribution curve for H1-4/23.

## Table 14. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR H1-4/23.

H1-4/23 Expt. No. 23

$q$=.00504(.1782)

$T_{S1}$=25.1(77.2) $T_{S2}$=17.8(64.0) $T_0$=24.4(75.9)

$T_1$=23.4(74.3) $T_2$=18.6(65.5) $_o\Delta\rho/\rho$=.00107

$f_b$=.255

$_oh_{c2}$=.297(11.7)±20%

$_of_i$=.0310

| x | 0.00 | 1.22(4.00) [3.42,3.42] | 2.44(8.00) [6.84,6.84] |
|---|---|---|---|
| $h_2$ | .622(24.5) [1.75,1.75] | .612(24.1) [1.72,1.72] | .599(23.6) [1.68,1.68] |
| H | .719(28.3) [2.01,2.01] | .719(28.3) [2.01,2.01] | .719(28.3) [2.01,2.01] |

Method 1

$h_{c2}$=.356(14.0) $_mh_c$=1.200 $\Delta\rho/\rho$=.000618

$f_i$=.00620 $_mf_i$=0.200 $f_i/f_b$=.0243

| | | | |
|---|---|---|---|
| $h_2$ | .622(24.5)[1.75] | .611(24.1)[1.72] | .600(23.6)[1.68] |
| $dh_2/dx$ | -.00872 | -.00916 | -.00970 |
| H | .719(28.3)[2.01] | .719(28.3)[2.01] | .719(28.3)[2.01] |

Method 2

$h_{c2}$=.356(14.0) $_mh_c$=1.200 $\beta$=1.73

$f_i$=.0372 $_mf_i$=1.200 $f_i/f_b$=.146

| | | | |
|---|---|---|---|
| $h_2$ | .622(24.5)[1.75] | .611(24.1)[1.72] | .600(23.6)[1.68] |
| $dh_2/dx$ | -.00895 | -.00905 | -.00924 |
| H | .719(28.3)[.719] | .719(28.3)[2.01] | .719(28.3)[2.01] |

**Table 15. INTERFACIAL PROFILE CALCULATION FOR H1-4/23.**

H1-4/23 ACC= 3.0 METHOD 1

| | | |
|---|---|---|
| T1= 23.5C= 74.3F | T2= 18.6C= 65.5F | Q= 1.782E-01 |
| SB= 0.000E-01 | FB= 2.551E-01 | ODELTA= 1.067E-03 |
| OHC2I= 1.169E 01 | OFI= 3.099E-02 | .125*FB= 3.189E-02 |
| HC2I= 1.403E 01 | MHC= 1.200 | DELTA= 6.175E-04 |
| FI= 6.198E-03 | MFI= 0.200 | FI/FB= 2.430E-02 |

| X | HI | TDI | HP | DTDHI |
|---|---|---|---|---|
| 0.000E-01 | 2.450E 01 | 2.825E 01 | -8.718E-03 | -1.53E-05 |
| 2.000E 00 | 2.429E 01 | 2.825E 01 | -8.928E-03 | -1.53E-05 |
| 4.000E 00 | 2.407E 01 | 2.825E 01 | -9.161E-03 | -1.53E-05 |
| 6.000E 00 | 2.385E 01 | 2.825E 01 | -9.419E-03 | -1.53E-05 |
| DOUBLED INTERVAL | DX= 4.0000E 00 | | | |
| 8.000E 00 | 2.362E 01 | 2.825E 01 | -9.704E-03 | -1.53E-05 |
| 1.200E 01 | 2.314E 01 | 2.825E 01 | -1.037E-02 | -1.53E-05 |
| 1.600E 01 | 2.262E 01 | 2.825E 01 | -1.120E-02 | -1.53E-05 |
| DOUBLED INTERVAL | DX= 8.0000E 00 | | | |
| 2.000E 01 | 2.206E 01 | 2.825E 01 | -1.227E-02 | 1.53E-05 |
| 2.800E 01 | 2.074E 01 | 2.825E 01 | -1.560E-02 | 3.05E-05 |
| 3.600E 01 | 1.893E 01 | 2.825E 01 | -2.350E-02 | 6.10E-05 |
| HALVED INTERVAL | DX= 4.0000E 00 | | | |
| 1.600E 01 | 2.262E 01 | 2.825E 01 | -1.120E-02 | -1.53E-05 |
| 2.000E 01 | 2.206E 01 | 2.825E 01 | -1.227E-02 | -1.53E-05 |
| 2.400E 01 | 2.144E 01 | 2.825E 01 | -1.366E-02 | -1.53E-05 |
| 2.800E 01 | 2.074E 01 | 2.825E 01 | -1.560E-02 | 1.53E-05 |
| 3.200E 01 | 1.992E 01 | 2.825E 01 | -1.850E-02 | 1.53E-05 |
| 3.600E 01 | 1.893E 01 | 2.825E 01 | -2.350E-02 | 4.58E-05 |
| 4.000E 01 | 1.757E 01 | 2.825E 01 | -3.516E-02 | 3.05E-05 |
| HALVED INTERVAL | DX= 2.0000E 00 | | | |
| 4.200E 01 | 1.656E 01 | 2.825E 01 | -5.239E-02 | 3.05E-05 |
| 4.400E 01 | 1.394E 01 | 2.825E 01 | 1.711E 00 | 3.05E-05 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

## Table 15. CONTINUED.

H1-4/23 ACC= 3.0 METHOD 2

HC2I= 1.403E 01 MHC= 1.200 BETA= 1.730

FI= 3.719E-02 MFI= 1.200 FI/FB= 1.458E-01

| X | HI | TDI | HP | DTDHI |
|---|---|---|---|---|
| 0.000E-01 | 2.450E 01 | 2.825E 01 | -8.950E-03 | -1.53E-05 |
| 2.000E 00 | 2.428E 01 | 2.825E 01 | -8.985E-03 | 7.63E-05 |
| 4.000E 00 | 2.407E 01 | 2.825E 01 | -9.045E-03 | 1.37E-04 |
| 6.000E 00 | 2.385E 01 | 2.825E 01 | -9.129E-03 | 1.98E-04 |
| DOUBLED INTERVAL DX= 4.0000E 00 | | | | |
| 8.000E 00 | 2.363E 01 | 2.825E 01 | -9.237E-03 | 2.59E-04 |
| 1.200E 01 | 2.318E 01 | 2.825E 01 | -9.526E-03 | 3.81E-04 |
| 1.600E 01 | 2.271E 01 | 2.825E 01 | -9.920E-03 | 5.04E-04 |
| DOUBLED INTERVAL DX= 8.0000E 00 | | | | |
| 2.000E 01 | 2.223E 01 | 2.825E 01 | -1.044E-02 | 6.26E-04 |
| 2.800E 01 | 2.116E 01 | 2.825E 01 | -1.201E-02 | 8.54E-04 |
| 3.600E 01 | 1.988E 01 | 2.825E 01 | -1.493E-02 | 1.05E-03 |
| 4.400E 01 | 1.816E 01 | 2.825E 01 | -2.222E-02 | 1.30E-03 |
| HALVED INTERVAL DX= 4.0000E 00 | | | | |
| 4.800E 01 | 1.687E 01 | 2.825E 01 | -3.407E-02 | 1.40E-03 |
| 5.200E 01 | 2.441E 01 | 2.825E 01 | -8.960E-03 | 1.51E-03 |
| 5.600E 01 | 2.397E 01 | 2.825E 01 | -9.077E-03 | 1.65E-03 |
| HALVED INTERVAL DX= 2.0000E 00 | | | | |
| 4.600E 01 | 1.758E 01 | 2.825E 01 | -2.641E-02 | 1.36E-03 |
| 4.800E 01 | 1.687E 01 | 2.825E 01 | -3.406E-02 | 1.40E-03 |
| 5.000E 01 | 1.585E 01 | 2.825E 01 | -5.566E-02 | 1.46E-03 |
| HALVED INTERVAL DX= 1.0000E 00 | | | | |
| 4.500E 01 | 1.789E 01 | 2.825E 01 | -2.406E-02 | 1.33E-03 |
| 4.600E 01 | 1.758E 01 | 2.825E 01 | -2.641E-02 | 1.36E-03 |
| 4.700E 01 | 1.725E 01 | 2.825E 01 | -2.956E-02 | 1.39E-03 |
| 4.800E 01 | 1.687E 01 | 2.825E 01 | -3.406E-02 | 1.40E-03 |
| 4.900E 01 | 1.642E 01 | 2.825E 01 | -4.127E-02 | 1.43E-03 |
| 5.000E 01 | 1.585E 01 | 2.825E 01 | -5.566E-02 | 1.46E-03 |
| 5.100E 01 | 1.494E 01 | 2.825E 01 | -1.163E-01 | 1.50E-03 |
| HALVED INTERVAL DX= 5.0000E-01 | | | | |
| 5.150E 01 | 1.398E 01 | 2.825E 01 | 2.295E 00 | 1.50E-03 |
| SIGN OF INTERFACIAL SLOPE IS CHANGED | | | | |

Table 15. CONTINUED.

```
H1-4/23        ACC= 3.0        NON-DIMENSIONALIZED          METHOD 1

T1= 23.5C= 74.3F          T2= 18.6C= 65.5F               Q= 1.782E-01
SB= 0.000E-01             FB= 2.551E-01             ODELTA= 1.067E-03
OHC2I= 1.169E 01          OFI= 3.099E-02           .125*FB= 3.189E-02

HC2I= 1.403E 01             MHC= 1.200               DELTA= 6.175E-04

FI= 6.198E-03               MFI= 0.200               FI/FB= 2.430E-02

     XN                HN                 TDN                 HPN

0.000E-01          1.747E 00          2.014E 00          -8.718E-03
1.711E 00          1.731E 00          2.014E 00          -8.928E-03
3.422E 00          1.716E 00          2.014E 00          -9.161E-03
5.133E 00          1.700E 00          2.014E 00          -9.419E-03
   DOUBLED  INTERVAL   DXN= 3.4217E 00
6.843E 00          1.684E 00          2.014E 00          -9.704E-03
1.027E 01          1.649E 00          2.014E 00          -1.037E-02
1.369E 01          1.613E 00          2.014E 00          -1.120E-02
   DOUBLED  INTERVAL   DXN= 6.8435E 00
1.711E 01          1.572E 00          2.014E 00          -1.227E-02
2.395E 01          1.478E 00          2.014E 00          -1.560E-02
3.080E 01          1.349E 00          2.014E 00          -2.350E-02
   HALVED   INTERVAL   DXN= 3.4217E 00
1.369E 01          1.613E 00          2.014E 00          -1.120E-02
1.711E 01          1.572E 00          2.014E 00          -1.227E-02
2.053E 01          1.528E 00          2.014E 00          -1.366E-02
2.395E 01          1.478E 00          2.014E 00          -1.560E-02
2.737E 01          1.420E 00          2.014E 00          -1.850E-02
3.080E 01          1.349E 00          2.014E 00          -2.350E-02
3.422E 01          1.253E 00          2.014E 00          -3.516E-02
   HALVED   INTERVAL   DXN= 1.7109E 00
3.593E 01          1.180E 00          2.014E 00          -5.239E-02
3.764E 01          9.934E-01          2.014E 00           1.711E 00
   SIGN OF INTERFACIAL SLOPE IS CHANGED
```

Table 15. CONTINUED.

```
H1-4/23      ACC= 3.0        NON-DIMENSIONALIZED          METHOD 2
HC2I= 1.403E 01              MHC= 1.200                   BETA= 1.730
FI= 3.719E-02                MFI= 1.200                   FI/FB= 1.458E-01
   XN                 HN                 TDN                  HPN
0.000E-01         1.746E 00          2.013E 00           -8.950E-03
1.710E 00         1.731E 00          2.013E 00           -8.985E-03
3.420E 00         1.715E 00          2.013E 00           -9.045E-03
5.131E 00         1.700E 00          2.013E 00           -9.129E-03
  DOUBLED INTERVAL   DXN= 3.4204E 00
6.841E 00         1.684E 00          2.013E 00           -9.237E-03
1.026E 01         1.652E 00          2.013E 00           -9.526E-03
1.368E 01         1.619E 00          2.013E 00           -9.920E-03
  DOUBLED INTERVAL   DXN= 6.8408E 00
1.710E 01         1.584E 00          2.013E 00           -1.044E-02
2.394E 01         1.507E 00          2.013E 00           -1.201E-02
3.078E 01         1.417E 00          2.013E 00           -1.493E-02
3.762E 01         1.294E 00          2.013E 00           -2.222E-02
  HALVED  INTERVAL   DXN= 3.4204E 00
4.104E 01         1.202E 00          2.013E 00           -3.407E-02
4.447E 01         1.739E 00          2.013E 00           -8.960E-03
4.789E 01         1.708E 00          2.013E 00           -9.077E-03
  HALVED  INTERVAL   DXN= 1.7102E 00
3.933E 01         1.253E 00          2.013E 00           -2.641E-02
4.104E 01         1.202E 00          2.013E 00           -3.406E-02
4.275E 01         1.130E 00          2.013E 00           -5.566E-02
  HALVED  INTERVAL   DXN= 8.5510E-01
3.848E 01         1.275E 00          2.013E 00           -2.406E-02
3.933E 01         1.253E 00          2.013E 00           -2.641E-02
4.019E 01         1.229E 00          2.013E 00           -2.956E-02
4.104E 01         1.202E 00          2.013E 00           -3.406E-02
4.190E 01         1.170E 00          2.013E 00           -4.127E-02
4.276E 01         1.130E 00          2.013E 00           -5.566E-02
4.361E 01         1.065E 00          2.013E 00           -1.163E-01
  HALVED  INTERVAL   DXN= 4.2755E-01
4.404E 01         9.965E-01          2.013E 00            2.295E 00
  SIGN OF INTERFACIAL SLOPE IS CHANGED
```

·H2 profile

This profile is almost the same as the M3 profile.

H2-5/23

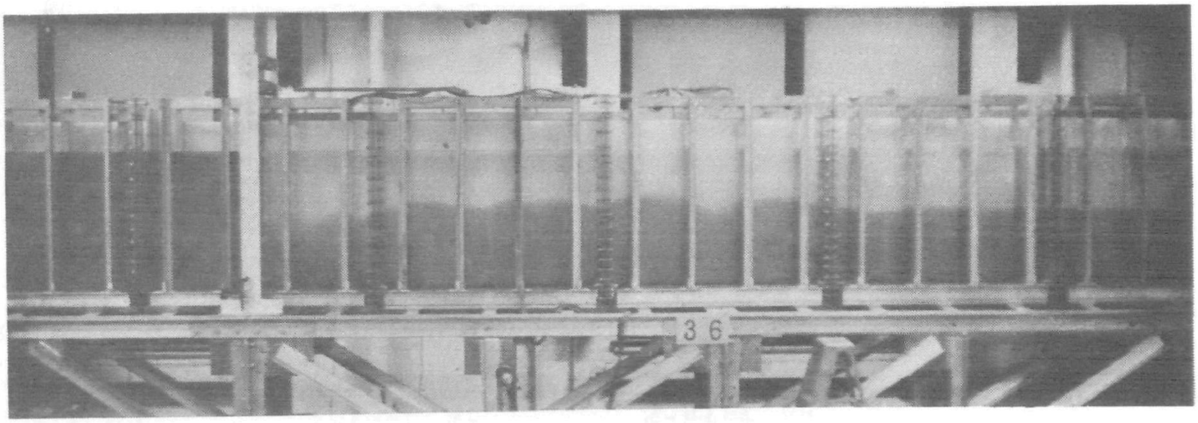



Fig. 26. Experiment H2-5/23.

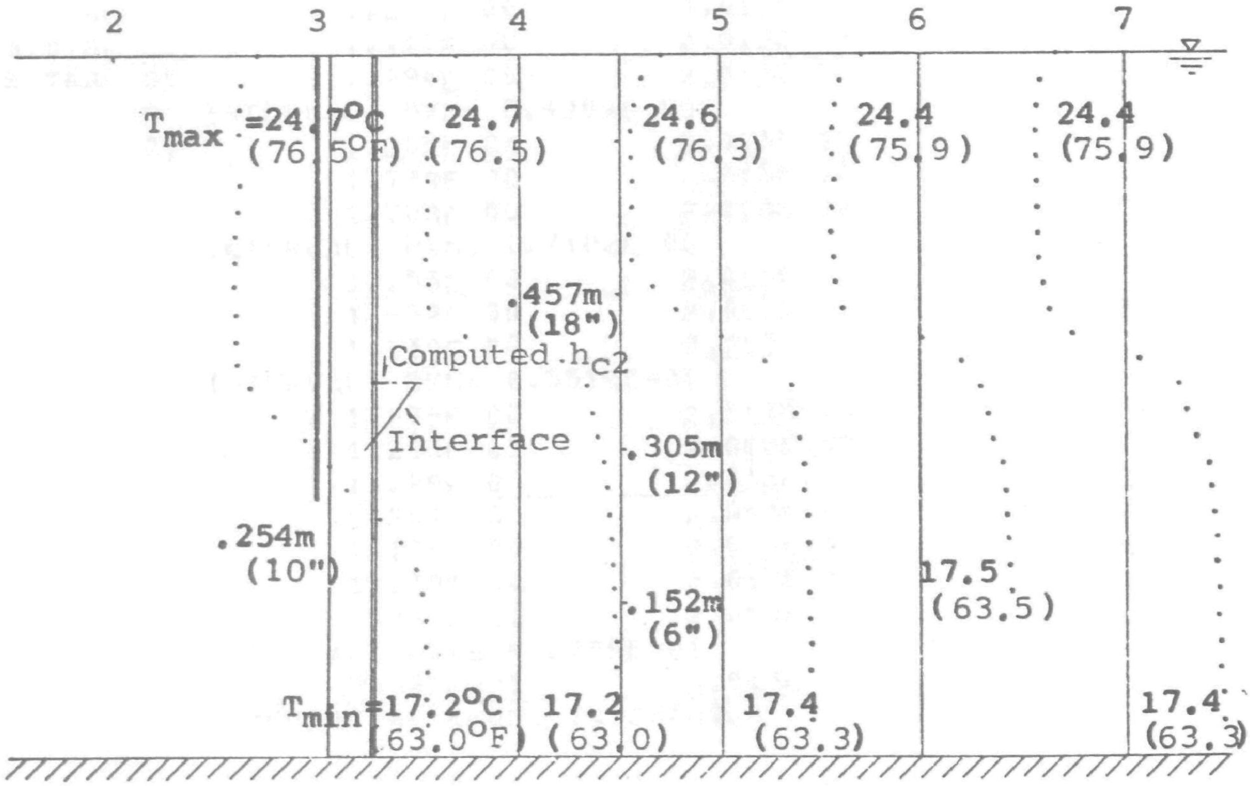



Fig. 27. Temperature distribution curve for H2-5/23.

Table 16. EXPERIMENTAL DATA AND RESULTS OF METHODS 1 AND 2 FOR H2-5/23.

H2-5/23 Expt. No. 36

$q=.00442(.1560)$

$T_{S1}=24.9(76.8)$ $T_{S2}=16.9(62.4)$ $T_0=21.8(71.2)$

$T_1=24.4(75.9)$ $T_2=17.9(64.2)$ ${}_o\Delta\rho/\rho=.00142$

$f_b=.210$

${}_oh_{c2}=.247(9.73)\pm 14\%$

${}_of_i=.0296$

| x | 0.00 | .152(0.5) [.411,.411] | .228(.75) [.617,.617] |
|---|---|---|---|
| $h_2$ | .317(12.5) [.857,.857] | .345(13.6) [.932,.932] | .363(14.3) [.980,.980] |
| H | .692(27.3) [1.87,1.87] | .692(27.3) [1.87,1.87] | .692(27.3) [1.87,1.87] |

Method 1

$h_{c2}=.371(14.6)$ ${}_mh_c=1.500$ $\Delta\rho/\rho=.000420$

$f_i=.0592$ ${}_mf_i=2.000$ $f_i/f_b=.282$

| | | | |
|---|---|---|---|
| $h_2$ | .317(12.5)[.857] | .337(13.3)[.910] | .354(13.9)[.954] |
| $dh_2/dx$ | .107 | .165 | .316 |
| H | .692(27.3)[1.87] | .692(27.3)[1.87] | .692(27.3)[1.87] |

Method 2

$h_{c2}=.371(14.6)$ ${}_mh_c=1.500$ $\beta=3.375$

$f_i=.474$ ${}_mf_i=16.00$ $f_i/f_b=2.25$

| | | | |
|---|---|---|---|
| $h_2$ | .317(12.5)[.857] | .338(13.3)[.911] | .356(14.0)[.960] |
| $dh_2/dx$ | .108 | .172 | .377 |
| H | .692(27.3)[1.87] | .692(27.3)[1.87] | .692(27.3)[1.87] |

**Table 17. INTERFACIAL PROFILE CALCULATION FOR H2-5/23.**

H2-5/23 ACC= 4.0 METHOD 1

T1= 24.4C= 75.9F T2= 17.9C= 64.2F Q= 1.560E-01
SB= 0.000E-01 FB= 2.102E-01 ODELTA= 1.419E-03
OHC2I= 9.730E 00 OFI= 2.960E-02 .125*FB= 2.628E-02

HC2I= 1.459E 01 MHC= 1.500 DELTA= 4.204E-04

FI= 5.920E-02 MFI= 2.000 FI/FB= 2.816E-01

| X | HI | TDI | HP | DTDHI |
|---|---|---|---|---|
| 0.000E-01 | 1.250E 01 | 2.725E 01 | 1.074E-01 | -1.53E-05 |
| 1.250E-01 | 1.267E 01 | 2.725E 01 | 1.158E-01 | -1.53E-05 |
| 2.500E-01 | 1.285E 01 | 2.725E 01 | 1.268E-01 | -1.53E-05 |
| 3.750E-01 | 1.305E 01 | 2.725E 01 | 1.419E-01 | -1.53E-05 |
| 5.000E-01 | 1.328E 01 | 2.725E 01 | 1.648E-01 | -1.53E-05 |
| 6.250E-01 | 1.355E 01 | 2.725E 01 | 2.057E-01 | -1.53E-05 |
| 7.500E-01 | 1.393E 01 | 2.725E 01 | 3.155E-01 | -1.53E-05 |
| HALVED INTERVAL DX= 6.2500E-02 | | | | |
| 8.125E-01 | 1.423E 01 | 2.725E 01 | 5.724E-01 | -1.53E-05 |
| 8.750E-01 | 1.137E 01 | 2.725E 01 | 7.401E-02 | -1.53E-05 |
| 9.375E-01 | 1.143E 01 | 2.725E 01 | 7.509E-02 | -1.53E-05 |
| HALVED INTERVAL DX= 3.1250E-02 | | | | |
| 7.812E-01 | 1.406E 01 | 2.725E 01 | 3.900E-01 | -1.53E-05 |
| 8.125E-01 | 1.423E 01 | 2.725E 01 | 5.721E-01 | -1.53E-05 |
| 8.437E-01 | 1.770E 01 | 2.725E 01 | -6.057E-02 | -1.53E-05 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

Table 17. CONTINUED.

H2-5/23 ACC= 4.0 METHOD 2

HC2I= 1.459E 01 MHC= 1.500 BETA= 3.375

FI= 4.736E-01 MFI=16.000 FI/FB= 2.253E 00

| X | HI | TDI | HP | DTDHI |
|---|---|---|---|---|
| 0.000E-01 | 1.250E 01 | 2.725E 01 | 1.081E-01 | -1.53E-05 |
| 1.250E-01 | 1.267E 01 | 2.725E 01 | 1.172E-01 | 1.53E-05 |
| 2.500E-01 | 1.285E 01 | 2.725E 01 | 1.293E-01 | 4.58E-05 |
| 3.750E-01 | 1.306E 01 | 2.725E 01 | 1.462E-01 | 7.63E-05 |
| 5.000E-01 | 1.330E 01 | 2.725E 01 | 1.724E-01 | 1.22E-04 |
| 6.250E-01 | 1.359E 01 | 2.725E 01 | 2.214E-01 | 1.53E-04 |
| 7.500E-01 | 1.401E 01 | 2.725E 01 | 3.773E-01 | 2.14E-04 |
| HALVED INTERVAL DX= 6.2500E-02 | | | | |
| 8.125E-01 | 1.447E 01 | 2.725E 01 | 1.771E 00 | 2.14E-04 |
| 8.750E-01 | 1.308E 01 | 2.725E 01 | 1.482E-01 | 2.14E-04 |
| 9.375E-01 | 1.320E 01 | 2.725E 01 | 1.602E-01 | 2.14E-04 |
| HALVED INTERVAL DX= 3.1250E-02 | | | | |
| 7.812E-01 | 1.417E 01 | 2.725E 01 | 5.227E-01 | 2.14E-04 |
| 8.125E-01 | 1.448E 01 | 2.725E 01 | 1.853E 00 | 2.14E-04 |
| 8.437E-01 | 1.512E 01 | 2.725E 01 | -4.209E-01 | 2.14E-04 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

**Table 17. CONTINUED.**

H2-5/23 ACC= 4.0 NON-DIMENSIONALIZED METHOD 1

T1= 24.4C= 75.9F T2= 17.9C= 64.2F Q= 1.560E-01
SB= 0.000E-01 FB= 2.102E-01 QDELTA= 1.419E-03
OHC2I= 9.730E 00 OFI= 2.960E-02 .125*FB= 2.628E-02

HC2I= 1.459E 01 MHC= 1.500 DELTA= 4.204E-04

FI= 5.920E-02 MFI= 2.000 FI/FB= 2.816E-01

| XN | HN | TDN | HPN |
|---|---|---|---|
| 0.000E-01 | 8.565E-01 | 1.867E 00 | 1.074E-01 |
| 1.028E-01 | 8.679E-01 | 1.867E 00 | 1.158E-01 |
| 2.055E-01 | 8.804E-01 | 1.867E 00 | 1.268E-01 |
| 3.083E-01 | 8.941E-01 | 1.867E 00 | 1.419E-01 |
| 4.111E-01 | 9.098E-01 | 1.867E 00 | 1.648E-01 |
| 5.139E-01 | 9.286E-01 | 1.867E 00 | 2.057E-01 |
| 6.166E-01 | 9.542E-01 | 1.867E 00 | 3.155E-01 |
| HALVED INTERVAL DXN= 5.1387E-02 | | | |
| 6.680E-01 | 9.751E-01 | 1.867E 00 | 5.724E-01 |
| 7.194E-01 | 7.793E-01 | 1.867E 00 | 7.401E-02 |
| 7.708E-01 | 7.832E-01 | 1.867E 00 | 7.509E-02 |
| HALVED INTERVAL DXN= 2.5694E-02 | | | |
| 6.423E-01 | 9.631E-01 | 1.867E 00 | 3.900E-01 |
| 6.680E-01 | 9.751E-01 | 1.867E 00 | 5.721E-01 |
| 6.937E-01 | 1.212E 00 | 1.867E 00 | -6.057E-02 |
| SIGN OF INTERFACIAL SLOPE IS CHANGED | | | |

## Table 17. CONTINUED.

H2-5/23 ACC= 4.0 NON-DIMENSIONALIZED METHOD 2

HC2I= 1.459E 01 MHC= 1.500 BETA= 3.375

FI= 4.736E-01 MFI=16.000 FI/FB= 2.253E 00

| XN | HN | TDN | HPN |
|---|---|---|---|
| 0.000E-01 | 8.565E-01 | 1.867E 00 | 1.081E-01 |
| 1.028E-01 | 8.680E-01 | 1.867E 00 | 1.172E-01 |
| 2.055E-01 | 8.807E-01 | 1.867E 00 | 1.293E-01 |
| 3.083E-01 | 8.948E-01 | 1.867E 00 | 1.462E-01 |
| 4.111E-01 | 9.110E-01 | 1.867E 00 | 1.724E-01 |
| 5.139E-01 | 9.309E-01 | 1.867E 00 | 2.214E-01 |
| 6.166E-01 | 9.596E-01 | 1.867E 00 | 3.773E-01 |
| HALVED INTERVAL DXN= 5.1388E-02 | | | |
| 6.680E-01 | 9.914E-01 | 1.867E 00 | 1.771E 00 |
| 7.194E-01 | 8.962E-01 | 1.867E 00 | 1.482E-01 |
| 7.708E-01 | 9.041E-01 | 1.867E 00 | 1.602E-01 |
| HALVED INTERVAL DXN= 2.5694E-02 | | | |
| 6.423E-01 | 9.709E-01 | 1.867E 00 | 5.227E-01 |
| 6.680E-01 | 9.918E-01 | 1.867E 00 | 1.853E 00 |
| 6.937E-01 | 1.036E 00 | 1.867E 00 | -4.209E-01 |

SIGN OF INTERFACIAL SLOPE IS CHANGED

Comment on the Result of Methods 1 and 2

Calculation either by method 1 or 2 can produce different profiles close to the profiles obtained in the experiment. This fact may elucidate that the interfacial profile equations derived in Sec. 5 can successfully predict the existence of the different profiles which occurred in the flume.

When looking at the summary of the values of ${}_{m}h_{c}$ and ${}_{m}f_{i}$ (Table 18), the following statements can be made:

1. The values of ${}_{m}h_{c}$ from the methods 1 and 2 are close.
2. Most of the values of ${}_{m}h_{c}$ are within the maximum values of ${}_{m}h_{c}$ which could happen by error (second column of the table 18) for the profiles that did not use the gate.
3. Most of the values of ${}_{m}h_{c}$ are larger than 1.
4. Most of the values of ${}_{m}f_{i}$ for the method 1 are smaller than 1, whereas most of the values of ${}_{m}f_{i}$ for the method 2 are larger than 1.

From 1, 2, and 3, it can be stated that, although variation of ${}_{m}h_{c}$ may be explained by the error in the calculated stratified critical depth, there must be some factor decreasing the value of the calculated stratified critical depth because most of ${}_{m}h_{c}$ are larger than 1.

Methods 1 and 2 were used to increase the calculated value of the stratified critical depth; however, nothing conclusive has been obtained.

According to Keulegan[16] and Bata and Knezevich[13], the estimated magnitude of $f_i$ for a small channel and for small velocities is smaller than the coefficient obtained for smooth-walled channel. If this is true, method 1 is a suitable method to correct the stratified critical depth. However, this method tends to give similar values to the $f_i$ for the cases when the gate must have introduced more turbulence and thus should have larger $f_i$. Method 2 seems to give larger $f_i$ for the cases involving the gate than for the cases awithout the gate. However, the values of $f_i$ on the whole are fairly large.

Therefore, as far as the data obtained in this experimental system are concerned, they do not offer a solid way to predict the profiles occur-

Table 18. VALUES OF $_mh_c$ AND $_mf_i$

| Profile Identification | Error in $_oh_{c2}$ | $_mh_c$ M 1 | $_mh_c$ M 2 ($\beta$) | $_mf_i$ M 1 | $_mf_i$ M 2 |
|---|---|---|---|---|---|
| S1-3/30 | ±28% | 1.090 | 1.032(1.10) | 0.250 | 0.400 |
| S2-4/02 | ±35% | 1.020 | 1.007(1.02) | 1.450 | 1.730 |
| S2-4/11 | ±25% | 1.150 | 1.150(1.52) | 0.400 | 2.000 |
| S2-4/30 | ±26% | 1.130 | 1.116(1.39) | 0.160 | 1.260 |
| S3-5/02 | ±24% | 1.650 | 1.749(5.35) | 0.005 | 3.900 |
| S3-5/04 | ±15% | 1.400 | 1.401(2.75) | 0.150 | 6.400 |
| S3-5/09 | ±16% | 1.450 | 1.409(2.80) | 0.500 | 8.200 |
| S3-5/14 | ±15% | 1.650 | 1.663(4.60) | 0.450 | 13.50 |
| M1-3/26 | ±27% | 1.600 | 1.518(3.50) | 0.850 | 7.500 |
| M1-4/21 | ±23% | 1.050 | 1.048(1.15) | 0.200 | 0.600 |
| M2-3/21 | ±41% | 1.160 | 1.310(2.25) | 0.700 | 1.800 |
| M2-3/23 | ±35% | 1.490 | 1.710(5.00) | 0.800 | 5.250 |
| M2-4/18 | ±20% | 1.000 | 1.000(1.00) | 0.750 | 0.750 |
| M3-5/18 | ±14% | 1.500 | 1.474(3.20) | 0.350 | 7.600 |
| M3-5/21 | ±15% | 0.900 | 0.951(0.86) | 9.000 | 10.50 |
| H1-4/23 | ±20% | 1.200 | 1.200(1.73) | 0.200 | 1.200 |
| H1-4/25 | ±20% | 1.050 | 1.051(1.16) | 1.000 | 1.450 |
| H2-5/23 | ±14% | 1.500 | 1.500(3.38) | 2.000 | 16.00 |

ring in this flume.

## SECTION VIII

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## SECTION IX

## SYMBOLS

(See Sec. 7.3 and Appendices for the symbols used in computer programs).

b = width;

f = friction factor;

g = acceleration of gravity;

h = depth of a layer;

i = with an under bar, unit vector of x direction;

j = with an under bar, unit vector of y direction;

k = with an under bar, unit vector of z direction;

l = longitudinal spacing of roughness element;

m = transversal spacing of roughness element;

n = with an under bar, outwardly directed unit normal vector;

p = with an under bar, unit vector in the perpendicular direction;

q = flow rate per unit width;

r = height of roughness element;

s = width of roughness element;

t = time;

v = velocity;

w = mean velocity of points on the surface of a volume;

x = longitudinal distance (in the direction of the flow); and

= with an under bar, spatial coordinates;

y = transversal distance (in the direction of the width of the flow);

z = vertical distance (in the direction normal to x);

A = area, either cross sectional of surface; and

= adverse slope flow;

B = term defined in Eq. 165;

C = constant in Eq. 237;

D = term defined in Eq. 171;

E = term defined in Eqs. 172, 174, and 187; and

= elevation;

G = factor related to the stratified normal depth for the lower layer;

H = total depth; and

= horizontal slope flow;

I = with doube under bars, unit tensor;

K = constant in Eq. 240;

L = horizontal distance;

M = mild slope flow; and

= method;

P = pressure; and

= positive slope flow (= S and M);

R = hydraulic radius;

Re= Reynolds number;

S = slope; and

= steep slope flow;

T = with double under bars, stress tensor; and

= temperature

U = average velocity in a layer; and

= upper layer analysis;

V = volume;

W = wetted perimeter which does not include the bottom, or wall;

$\alpha$ = term defined in Eqs. 238 and 239;

$\beta$ = momentum coefficient;

$\gamma$ = angle defined in Fig. 10;

$\delta$ = angle defined in Fig. 2;

$\varepsilon$ = angle defined in Fig. 3;

$\zeta$ = angle defined in Fig. 4;

$\theta$ = angle between the x axis and the horizontal;

$\mu$ = coefficient of viscosity;

$\nu$ = kinematic viscosity;

$\rho$ = density, but $(\rho_1+\rho_2)/2$ in Eq. 76;

$\sigma$ = shear stress;

$\tau$ = stress which has taken consideration into the turbulence;

$\Phi$ = arbitrary scalor or tensor;

$\psi$ = arbitrary vector or tensor;

$\Delta$ = difference;

$\nabla$ = gradient vector operator;

D/Dt = material derivative;

< > = average of a variable integrated over lamina;

' (prime) = fluctuating component of a variable when considering the turbulence;

$\bar{}$(upper bar) = mean value of a variable when considering the turbulence;

_(under bar) = vector;

$_=$(double under bars) = tensor;

Subscripts

a = arbitrary control;

b = bottom;

c = critical;

e = entrance and exit;

f = friction;

i = interface;

m = material; and

= multiple;

n = normal;

o = original;

q = equivalent;

r = circulation;

s = surface;

w = wetted perimeter which does not include the bottom, or wall;

x = x component of a vector;

y = y component of a vector;

z = z component of a vector;

H = horizontal slope flow;

L = hypolimnetic flow;

M = momentum coefficient included;

N = non-dimensionalized by dividing with the critical depth;

P = positive slope flow;

S = water to be supplied into the flume;

U = upper layer analysis;

0 = atmosphere;

1 = upper layer; and

2 = lower layer.

# SECTION X

# APPENDICES

A. Computer Program "H2PROFIL" and an Example of the Print-out

B. Computer Program "ESTMFBFI" and an Example of the Print-out

C. Computer Program "STRAFLOW"

## APPENDIX A

COMPUTER PROGRAM "H2PROFIL" AND AN EXAMPLE OF THE PRINT-OUT.

This program is to calculate the H2 profile in the open-channel flow in a rectangular experimental flume, using the 4th-order Milne predictor-corrector method. This is written in WATFIV. Generally, comments will precede statements, except for GO TO statements.

### Symbols

Symbols used in the program are shown in parentheses. "0" in variables expressing length usually indicates initial values.

ACC = exponent in "UP" (see below), which is the upper limit to be used to determine an integration interval. ("DN" is the lower limit.);

DX(=HH) = interval of an integration;

FQ = equivalent friction factor for the bottom and the wall;

HC = critical depth, in feet;

Q = flow rate, in cfs;

X = longitudinal distance from a starting point, in feet;

XF = limit of X to terminate a calculation;

Y(=H) = depth, in feet;

YI(=H) = depth, in inches;

YID(-HID) = difference between depth and the initial depth, in inches; and

YP(=HP) = slope of the free surface.

### Data Card Specification

1st card includes Q and HO in FORMAT (2E10.3).

2nd card includes FQ in FORMAT (E10.3). FQ is positive. When going to next data, let FQ have negative value. To terminate computation, let FQ be 0.

```
/PROGRAM H2PROFIL,NOEXT
C
C     *** DECLARATIONS AND INITIALIZATIONS
      REAL ACC/5./, XF/16./,
     1     AA(4)/.5,.2928932,1.707107,.1666667/,
     2     BB(4)/1.,.2928932,1.707107,.3333333/,
     3     CC(4)/.5,.2928932,1.707107,.5/
      DIMENSION Y(5,2), YP(5,2), YOUT(2), YPOUT(2), QQ(2)
      DATA (YP(K,1), K=1,5)/5*1./
      UP= 29./10.**ACC
      DN= UP/600.
C
C     *** LIST OF FORMATS
  100 FORMAT(2E10.3)
  110 FORMAT(E10.3)
  200 FORMAT('1',///////
     1     /' ',T47,'H2 PROFILE',T74,'ACC=',F4.1)
  210 FORMAT('-',T22,'Q=',1PE10.3,T42,'HC=',F10.3)
  220 FORMAT(' ',T22,'FQ=',1PE10.3/
     1     /' ',T26,'X',T40,'YP',T52,'Y',T64,'YI'
     2     ,T76,'YID'/)
  230 FORMAT(' ',T22,1PE9.3,E14.3,2E12.3,E13.3)
  240 FORMAT(' ',T27,'INTERVAL IS HALVED   DX= ',1PE10.4)
  250 FORMAT(' ',T27,'INTERVAL IS DOUBLED  DX= ',1PE10.4)
  260 FORMAT('1',' ')
C
C     *** READ Q AND H0 AND PRINT TITLE
  300 READ 100, Q, H0
      H0I= H0*12.
      HC= (Q**2/32.17)**.3333333
      PRINT 200, ACC
C
C     *** TO CHANGE FQ
  310 READ 110, FQ
      IF(FQ) 300,1000,320
C     *** PRINT Q, HC, AND FQ AND GIVE INITIAL VALUES
  320 PRINT 210, Q, HC
      PRINT 220, FQ
      Y(1,1)= 1.
      Y(1,2)= H0
      HH= 1.
      IN= 0
      GO TO 900
C     *** GO TO INTEGRATION
```

```
C       *** COMPUTE SLOPE OF FREE SURFACE
  400 YP(K,2)= FQ*(1.+2.*Y(K,2))/8./(1.-(Y(K,2)/HC)**3)
C       *** TO PRINT INITIAL VALUES
      IF(IN.EQ.0) GO TO 999
      GO TO (910,920,950,960), L1
C       *** GO BACK TO INTEGRATION
C
C       *** TO PRINT OUTPUT
  600 X= YOUT(1)-1.
      HP= YPOUT(2)
      IF(IN.EQ.0) HP0= HP
      H= YOUT(2)
      HI= 12.*H
      HID= H0I-HI
      PRINT 230, X, HP, H, HI, HID
      IN= 1
C
C       *** TO TERMINATE A CALCULATION
      IF(X.GT.XF) GO TO 310
      IF(HP*HP0.LT.0.) GO TO 310
      GO TO (910,940), L2
C       *** GO BACK TO INTEGRATION
C
C
C       *** INTEGRATION - 4TH ORDER RUNGE-KUTTA METHOD
C                            WITH GILL'S CONSTANT
C       *** INITIALIZE QQ
  900 K= 1
      DO 905 I=1,2
  905 QQ(I)= 0.
      L1= 1
      L2= 1
      GO TO 400
C       *** GO TO COMPUTE THE SLOPE OF FREE SURFACE
C
C       *** IF K=4, GO TO THE PREDICTOR-CORRECTOR PART
  910 IF(K.EQ.4) GO TO 930
      K= K+1
      DO 915 I=1,2
      Y(K,I)= Y(K-1,I)
  915 YP(K,I)= YP(K-1,I)
      J= 1
      L2= 1
```

```
C      *** IF J=5, GO TO SET THE OUTPUT OF THE INTEGRATION
  920 IF(J.EQ.5) GO TO 999
      DO 925 I=1,2
      DD= HH*YP(K,I)
      RR= AA(J)*DD-BB(J)*QQ(I)
      Y(K,I)= Y(K,I)+RR
  925 QQ(I)= QQ(I)+3.*RR-CC(J)*DD
      J= J+1
      L1= 2
      GO TO 400
C      *** GO TO COMPUTE THE SLOPE OF FREE SURFACE
C
C
C      *** INTEGRATION - 4TH ORDER MILNE PREDICTOR-
C                        CORRECTOR METHOD
C      *** INITIALIZE PMC
  930 PMC= 0.
      MIL= 0
C
C      *** GET MODIFIED Y WITH PREDICTOR
  940 K= 5
      Y(5,1)= Y(4,1)+HH
      PRED= Y(1,2)+1.333333*HH*(2.*YP(2,2)
     1      -YP(3,2)+2.*YP(4,2))
      Y(5,2)= PRED-.9655172*PMC
      L1= 3
      GO TO 400
C      *** GO TO COMPUTE THE SLOPE OF FREE SURFACE
C      *** GET IMPROVED Y WITH CORRECTOR
  950 COR= Y(3,2)+HH/3.*(YP(3,2)+4.*YP(4,2)+YP(5,2))
      PMC= PRED-COR
      Y(5,2)= COR+.0344828*PMC
C
C      *** TEST IF THE INTEGRATION INTERVAL IS O.K.
      TEST= ABS(1.-PRED/COR)
      IF(TEST.GT.UP) GO TO 970
      IF(TEST.LT.DN) GO TO 980
C
C      *** NO CHANGE IN INTERVAL
      L1= 4
      GO TO 400
C      *** GO TO COMPUTE THE SLOPE OF FREE SURFACE
```

```
C      *** DETERMINE Y AND YP FOR THE INTEGRATION OF
C          NEXT POINT
  960 DO 965 I=1,2
      DO 965 K=1,4
      Y(K,I)= Y(K+1,I)
  965 YP(K,I)= YP(K+1,I)
      K= 5
      MIL= 1
      L2= 2
      GO TO 999
C      *** GO TO SET THE OUTPUT OF AN INTEGRATION
C
C      *** HALVING INTERVAL
  970 HH= .5*HH
      PRINT 240, HH
C      *** IF MIL=0, GO TO THE RUNGE-KUTTA PART
      IF(MIL.EQ.0) GO TO 900
      DO 975 I=1,2
  975 Y(1,I)= Y(4,I)
      GO TO 900
C      *** GO TO THE RUNGE-KUTTA PART
C
C      *** DOUBLING INTERVAL
  980 HH= 2.*HH
      PRINT 250, HH
      DO 985 I=1,2
  985 Y(1,I)= Y(3,I)
      GO TO 900
C      *** GO TO THE RUNGE-KUTTA PART
C
C      *** TO SET THE OUTPUT OF AN INTEGRATION
  999 YOUT(1)= Y(K,1)
      YOUT(2)= Y(K,2)
      YPOUT(2)= YP(K,2)
      GO TO 600
C      *** GO TO PRINT THE OUTPUT
C
C      *** TO ELIMINATE COMPUTER MESSAGE OUT OF
C          LAST PRINTED PAGE
 1000 PRINT 260
      STOP
      END
```

H2 PROFILE ACC= 5.0

Q= 1.114E-01 HC= 7.280E-02
FQ= 1.350E-01

| X | YP | Y | YI | YID |
|---|---|---|---|---|
| 0.000E-01 | -3.054E-04 | 3.294E-01 | 3.953E 00 | 0.000E-01 |
| 1.000E 00 | -3.062E-04 | 3.291E-01 | 3.949E 00 | 3.670E-03 |
| 2.000E 00 | -3.069E-04 | 3.288E-01 | 3.945E 00 | 7.349E-03 |
| 3.000E 00 | -3.077E-04 | 3.285E-01 | 3.942E 00 | 1.104E-02 |
| INTERVAL IS DOUBLED | | DX= 2.0000E 00 | | |
| 4.000E 00 | -3.084E-04 | 3.282E-01 | 3.938E 00 | 1.474E-02 |
| 6.000E 00 | -3.100E-04 | 3.276E-01 | 3.931E 00 | 2.216E-02 |
| 8.000E 00 | -3.115E-04 | 3.269E-01 | 3.923E 00 | 2.962E-02 |
| INTERVAL IS DOUBLED | | DX= 4.0000E 00 | | |
| 1.000E 01 | -3.131E-04 | 3.263E-01 | 3.916E 00 | 3.711E-02 |
| 1.400E 01 | -3.163E-04 | 3.250E-01 | 3.901E 00 | 5.222E-02 |
| 1.800E 01 | -3.196E-04 | 3.238E-01 | 3.885E 00 | 6.748E-02 |

## APPENDIX B

COMPUTER PROGRAM "ESTMFBFI" AND AN EXAMPLE OF THE PRINT-OUT.

This program is to obtain, from $q/\nu$ and h, the estimate of $f_b$ value, and the value of $f_i$ which is used as a starting point for the trial and error method. This is written in WATFIV. Generally comments will precede statements, except for GO TO statements.

Symbols

H = depth, in feet;
HI = depth, in inches;
HIO = starting value of depth for a computation, in inches;
SC = shape correction factor (see the program for the definition); and
QN = $q/\nu$.
For the definitions of other terms, see the program under statement number 300, 320, and 570.

Data Card Specification

Only one card includes profile identification, $q/\nu$, and HIO in FORMAT (A7, 3X, E7.3, 3X, F4.1). To terminate computation, it is necessary to have a card starting with FINISH and having 0s for $q/\nu$ and HIO.

```
/PROGRAM ESTMFBFI,NOEXT
C
C      *** DECLARATIONS AND INITIALIZATIONS
       CHARACTER PROFIL*7
       REAL FQ(2), FQP(2), RW(2), TRW(2),
     1      RB(2), FB(2), D(2),
     2      FQ0/.05/, DFQ0/.05/
C
C      *** LIST OF FORMATS
  100  FORMAT(A7,3X,E7.3,3X,F4.1)
  200  FORMAT('1',//////
     1        /' ',T22,A7,T35,'Q/NU=',1PE10.3/
     2        /' ',T23,'HI',T32,'FB+2H*FW',T49,'FB'
     3        ,T62,'REB',T76,'REW'/)
  210  FORMAT(' ',T22,F4.1,1P4E14.3)
  220  FORMAT('+',T40,'*')
  230  FORMAT(' ',T22,F4.1,5X,'NO SOLUTION')
  240  FORMAT('+',T43,'BETWEEN FQ=0. AND 5.')
  241  FORMAT('+',T42,', SINCE D= CONSTANT')
  242  FORMAT('+',T42,', SINCE D DOES NOT CONVERGE')
  243  FORMAT('+',T42,', SINCE RB .LE. 0.')
  244  FORMAT('+',T42,', SINCE FB .LE. 0.')
  250  FORMAT('-',T23,'HI',T49,'FT',T76,'REI'/)
  260  FORMAT(' ',T22,F4.1,T45,1PE9.3,T73,E9.3)
  270  FORMAT('1',' ')
C
C
C      *** READ PROFILE IDENTIFICATION, Q/NU, AND HI0
  300  READ 100, PROFIL, QN, HI0
C      *** TO TERMINATE COMPUTATION
       IF(PROFIL.EQ.'FINISH')  GO TO 999
C      *** PRINT PROFILE IDENTIFICATION AND Q/NU
       PRINT 200, PROFIL, QN
       QF= QN**0.2
       HI= HI0
C
C      *** FOR NEW HI, DETERMINE VARIABLES RELATED TO H
  320  H= HI/12.
       HF= 0.223**0.8*H
       QNH= QN/H
       H2= 2.*H
       P= 1.+H2
       IF(H.GT.0.5) GO TO 330
       SC= 0.8727*(ALOG(P)-H)
       GO TO 350
  330  SC= 0.8727*(ALOG(P/H2)-0.5)
```

```
C      *** INITIAL APPROXIMATION OF FQ
   350 FQ(1)= FQ0
       DFQ= DFQ0
       J= 1
       GO TO 560
C      *** GO TO CALCULATE D(2) AND D(1)
C
C      *** FIND IF THE CURVE RISES OR FALLS WHEN FQ
C          INCREASES AND THEN DECIDE INITIAL DIRECTION
C          OF INCREMENT OF FQ(DFQ)
   360 DD= D(2)-D(1)
C      *** IF DD= 0, THIS D-FQ CURVE IS CONSTANT
C          AND GO TO PRINT 'NO SOLUTION'
       IF(DD.EQ.0.) GO TO 786
       DFQ= -SIGN(1.,DD)*SIGN(DFQ,D(1))
       J= 2
       IF(DFQ*DFQ0.GT.0.) GO TO 400
       GO TO 560
C      *** SINCE POSITIVE INCREMENT WAS NOT RIGHT,
C          GO TO CALCULATE NEW D(2)
C
C      *** CHECK IF THE SIGN OF D(2) AND D(1) DIFFERS
   400 IF(D(2)*D(1)) 440,420,430
C      *** IF EITHER D(1) OR D(2) .EQ.0., THIS IS
C          A SOLUTION AND GO TO PRINTING
   420 IF(D(2)) 760,750,760
C      *** D(2) AND D(1) HAVE THE SAME SIGN
   430 K= 1
       GO TO 450
C      *** D(2) AND D(1) HAVE THE DIFFERENT SIGN
   440 K= 2
C      *** IF DAD= ABS(D(2))-ABS(D(1)).GE.0.,
C          NO SOLUTION FOR K= 1 AND DFQ CAN BE SHORTENED
C          FOR K= 2
   450 IF(DAD.GE.0.) GO TO (787,460), K
C      *** PREVIOUS VARIABLES WITH I= 2 ARE SET TO I= 1
       FQ(1)= FQ(2)
       D(1)= D(2)
       FQP(1)= FQP(2)
       RW(1)= RW(2)
       RB(1)= RB(2)
C      *** IF K= 1, STILL HAVE TO USE THE SAME DFQ AND
C          GO TO CALCULATE NEW D(2)
       IF(K.EQ.1) GO TO 550
C      *** HALVE DFQ WITH THE SIGN OF DAD
   460 DFQ= .5*DFQ*SIGN(1.,DAD)
       GO TO 550
C      *** GO TO CALCULATE D(2)
```

```
C      *** SHORTEN DFQ FOR FQ CLOSE TO THE LIMIT OF FQ
  500 DFQ= DFQ/10.
      GO TO 560
C      *** GO BACK TO CALCULATE D(2)
C
C      *** IF FQ IS BEYOND THE LIMIT OF FQ, NO SOLUTION
  550 IF(FQ(1).GT.4.999999.OR.FQ(1).LT..000001) GO TO 785
C
C      *** DEFINE NEW FQ(2)
  560 FQ(2)= FQ(1)+DFQ
C      *** TO GET THE SOLUTION CLOSE TO
C             THE LIMIT OF FQ
      IF(FQ(2).GT.5..OR.FQ(2).LT..0000001) GO TO 500
C
C      *** CALCULATE D AND NECESSARY VARIABLES
  570 DO 600 I=1,2
C      *** TO AVOID REPEATED CALCULATION
      IF(I.EQ.1.AND.J.EQ.2) GO TO 600
      FQP(I)= FQ(I)*P
      RW(I)= HF/QF/FQP(I)**.8
      TRW(I)= 1.-2.*RW(I)
      RB(I)= H*TRW(I)
      IF(RB(I).LT..0000001) GO TO 788
      FB(I)= FQP(I)*TRW(I)
      IF(FB(I).LT..0000001) GO TO 789
      D(I)= 1./SQRT(FB(I))-SC-2.03*ALOG10(RB(I))-3.1
  600 CONTINUE
C
C      *** TO CHECK IF THERE IS A SOLUTION BETWEEN
C             THE LIMITS OF FQ
      IF(FQ(2).EQ.4.999999) GO TO 610
      IF(FQ(2).EQ..000001) GO TO 620
      GO TO 630
  610 IF(DD*D(2).LT.0.) GO TO 785
      GO TO 630
  620 IF(DD*D(2).GT.0.) GO TO 785
C
C      *** TO CHECK WHICH FQ IS CLOSER TO THE SOLUTION
  630 DAD= ABS(D(2))-ABS(D(1))
C      *** TO OBTAIN A SOLUTION
C             BY THE LIMIT OF D
      IF(DAD.GE.0.0) GO TO 640
      IF(ABS(D(2)).LT..0001) GO TO 750
      L= 1
      GO TO 650
  640 IF(ABS(D(1)).LT..0001) GO TO 760
      L= 2
```

```
C       *** TO OBTAIN A SOLUTION
C           BY THE LIMIT OF DFQ
  650 IF(ABS(DFQ).LT.1.E-9) GO TO (745,755), L
      GO TO (360,400), J
C       *** GO BACK TO THE ITERATION
C
C       *** SET THE SOLUTION
C       *** J= 3 IS TO PRINT '*' AFTER A SOLUTION
C           TO INDICATE THAT THIS SOLUTION IS OBTAINED
C           BY THE LIMIT OF DFQ
  745 J= 3
  750 AFQ= FQ(2)
      AFQP= FQP(2)
      ARW= RW(2)
      ARB= RB(2)
      AFB= FB(2)
      GO TO 770
  755 J= 3
  760 AFQ= FQ(1)
      AFQP= FQP(1)
      ARW= RW(1)
      ARB= RB(1)
      AFB= FB(1)
C       *** CALCULATE REYNOLDS NUMBER AND PRINT OUTPUT
  770 REB= QNH*ARB
      REW= QNH*ARW
      PRINT 210, HI, AFQP, AFB, REB, REW
C
C       *** TO PRINT '*' MENTIONED ABOVE
      IF(J.NE.3) GO TO 800
      PRINT 220
      GO TO 800
C       *** TO PRINT THE REASON WHY THERE IS NO SOLUTION
  785 J= 4
      GO TO 790
  786 J= 5
      GO TO 790
  787 J= 6
      GO TO 790
  788 J= 7
      GO TO 790
  789 J= 8
  790 PRINT 230, HI
      IF(J.EQ.4) PRINT 240
      IF(J.EQ.5) PRINT 241
      IF(J.EQ.6) PRINT 242
      IF(J.EQ.7) PRINT 243
      IF(J.EQ.8) PRINT 244
```

```
C       *** INCREASE HI
  800 HI= HI+.1
C       *** GO BACK TO CALCULATE MORE OUTPUT
      IF(HI.LE.HI0+1.95) GO TO 320
C
C
C       *** PRINT HEADING FOR FI TABLE
      PRINT 250
C       *** SET HI TO HI0
      HI= HI0
C
C       *** CALCULATE FI AND REYNOLDS NUMBER
C           AND PRINT THEM
  900 REI= .5*QN/(1.+HI/12.)
      FI= .223/REI**.25
      PRINT 260, HI, FI, REI
C
C       *** INCREASE HI
      HI= HI+.1
C       *** TO CALCULATE MORE OUTPUT AND GO TO NEXT DATA,
C           IF FI TABLE IS DONE
      IF(HI-HI0-1.95) 900,300,300
C
C       *** TO TAKE COMPUTER MESSAGE OUT OF
C           LAST PRINTED PAGE
  999 PRINT 270
      STOP
      END
```

S1-3/30    Q/NU= 4.950E 03

| HI | FB+2H*FW | FB | REB | REW |
|---|---|---|---|---|
| 18.1 | 2.643E-01 | 1.373E-01 | 2.572E 03 | 7.883E 02 |
| 18.2 | 2.650E-01 | 1.372E-01 | 2.564E 03 | 7.867E 02 |
| 18.3 | 2.656E-01 | 1.371E-01 | 2.556E 03 | 7.851E 02 |
| 18.4 | 2.663E-01 | 1.371E-01 | 2.548E 03 | 7.834E 02 |
| 18.5 | 2.670E-01 | 1.370E-01 | 2.539E 03 | 7.818E 02 |
| 18.6 | 2.677E-01 | 1.369E-01 | 2.531E 03 | 7.802E 02 |
| 18.7 | 2.684E-01 | 1.368E-01 | 2.523E 03 | 7.786E 02 |
| 18.8 | 2.691E-01 | 1.367E-01 | 2.515E 03 | 7.770E 02 |
| 18.9 | 2.698E-01 | 1.367E-01 | 2.507E 03 | 7.754E 02 |
| 19.0 | 2.705E-01 | 1.366E-01 | 2.500E 03 | 7.738E 02 |
| 19.1 | 2.712E-01 | 1.365E-01 | 2.492E 03 | 7.722E 02 |
| 19.2 | 2.719E-01 | 1.364E-01 | 2.484E 03 | 7.707E 02 |
| 19.3 | 2.726E-01 | 1.363E-01 | 2.476E 03 | 7.691E 02 |
| 19.4 | 2.733E-01 | 1.363E-01 | 2.469E 03 | 7.675E 02 |
| 19.5 | 2.740E-01 | 1.362E-01 | 2.461E 03 | 7.659E 02 |
| 19.6 | 2.747E-01 | 1.361E-01 | 2.453E 03 | 7.643E 02 |
| 19.7 | 2.754E-01 | 1.361E-01 | 2.446E 03 | 7.628E 02 |
| 19.8 | 2.761E-01 | 1.360E-01 | 2.438E 03 | 7.612E 02 |
| 19.9 | 2.768E-01 | 1.359E-01 | 2.431E 03 | 7.596E 02 |
| 20.0 | 2.775E-01 | 1.358E-01 | 2.423E 03 | 7.581E 02 |

| HI | FI | RET |
|---|---|---|
| 18.1 | 3.979E-02 | 9.867E 02 |
| 18.2 | 3.982E-02 | 9.834E 02 |
| 18.3 | 3.985E-02 | 9.802E 02 |
| 18.4 | 3.989E-02 | 9.770E 02 |
| 18.5 | 3.992E-02 | 9.738E 02 |
| 18.6 | 3.995E-02 | 9.706E 02 |
| 18.7 | 3.999E-02 | 9.674E 02 |
| 18.8 | 4.002E-02 | 9.643E 02 |
| 18.9 | 4.005E-02 | 9.612E 02 |
| 19.0 | 4.008E-02 | 9.581E 02 |
| 19.1 | 4.011E-02 | 9.550E 02 |
| 19.2 | 4.015E-02 | 9.519E 02 |
| 19.3 | 4.018E-02 | 9.489E 02 |
| 19.4 | 4.021E-02 | 9.459E 02 |
| 19.5 | 4.024E-02 | 9.429E 02 |
| 19.6 | 4.028E-02 | 9.399E 02 |
| 19.7 | 4.031E-02 | 9.369E 02 |
| 19.8 | 4.034E-02 | 9.340E 02 |
| 19.9 | 4.037E-02 | 9.310E 02 |
| 20.0 | 4.040E-02 | 9.281E 02 |

APPENDIX C

COMPUTER PROGRAM "STRAFLOW"

This program is to calculate profiles of the interface and the free surface of the two-layered flow, using 4th-order Milne predictor-corrector method. This is written in WATFIV. Generally, comments will precede statements, except for GO TO statements.

Symbols

"O" in variables usually indicates initial values. "N" in variables indicates non-dimensionalized variables. Variables of length is written in feet. If "I" is included in the variables, they are in inches.

ACC = exponent in "UP" (see below), which is the upper limit to be used to determine an integration interval ("DN" is the lower limit).;

BETA = momentum coefficient;

BF = FACTOR/BETA;

DELTA = ODELTA/MHC**3 for method 1;

DTDH = difference between the total depth and the horizontal drawn through the free surface at the starting point;

DX = an integration interval;

FACTOR = .125*FB/SB for the S and M profiles and = .125*FB for the H profile;

FB = bottom friction factor;

FI = MFI*OFI, in which MFI is substituted by trial and error;

GHN = virtual stratified normal depth;

H = depth of the lower layer;

HC2 = MHC*OHC2, in which MHC is substituted by trial and error;

HP = slope of the interface;

ODELTA = $\Delta\rho/\rho$ calculated from T1 and T2;

OFI = originally estimated interfacial friction factor;

OHC2 = stratified critical depth for the lower layer analysis calculated according to the definition (see under statement number 300 in the program. 32.17 is the value of gravitational acceleration in ft/sec$^2$.);

Q = flow rate, in cfs;

SB = slope of the channel bottom;

T1 = mean temperature in the upper layer ("C" in Celsius and "F" in Fahrenheit);

T2 = mean temperature in the lower layer;

TD = total depth of the flow;

TDFAC = in method 1, .125*FI*DELTA/SB for the M and S profiles and = .125*FI*DELTA for the H profile. For method 2, the above TDFAC is divided by BETA;

X = longitudinal distance from a starting point; and

XF = limit of X to terminate a calculation;

Data Card Specification

1st card is for profile identification, as S1-3/30, punched from the 1st column. 1st column indicates the slope-M, S, or H. 2nd column is either 1, 2,, or 3. Any identification can be put in from column 3 to 7.

2nd card includes DXO, FB, HOI, TDOI, and OFI in FORMAT (5E10.3).

3rd card includes SB, TC(1) = T1 in Celsius, TC(2) = T2 in Celsius, Q, ODELTA, and OHCI in FORMAT (6E10.3). If OHCI is not known, let it be 0, and it will be calculated.

4th card shows method, either 1 or 2. When going to next data, let method be 0.

5th card includes either MHC or BETA, depending on the method. Initial value, limit value, and increment are punched in FORMAT (3E10.3).
If only one value is used, it may be punched at the initial value; if a value smaller than that value is used for the limit value in this case, increment should be zero.

6th card includes MFI in a similar way to the 5th card.

```
/PROGRAM STRAFLOW,NOEXT
C
C     *** DECLARATIONS AND INITIALIZATIONS
      CHARACTER SLOPE*1, IDENT*1, DATE*5
      INTEGER PROFIL
      REAL MFI, MFI0, MFIL, MFID, MHC, MHC0, MHCL, MHCD,
     1       XF/1000./,
     2       AA(4)/.5,.2928932,1.707107,.1666667/,
     3       BB(4)/1.,.2928932,1.707107,.3333333/,
     4       CC(4)/.5,.2928932,1.707107,.5/
      DIMENSION Y(5,3), YP(5,3), YOUT(3), YPOUT(3),
     1              GHNNC(5), QQ(3), TC(2), TF(2),
     2              PRED(3), COR(3), PMC(3)
      DATA (YP(K,1), K=1,5)/5*1./
C
C     *** LIST OF FORMATS
  100 FORMAT(A1,A1,A5,F4.1)
  110 FORMAT(5E10.3/6E10.3)
  120 FORMAT(I1)
  130 FORMAT(3E10.3/3E10.3)
  200 FORMAT('1',//////' ')
  201 FORMAT(' ',/' ')
  202 FORMAT(' ',T13,A1,A1,A5)
  203 FORMAT('+',T25,'ACC=',F4.1)
  204 FORMAT('+',T65,'METHOD ',I1)
  210 FORMAT('0',T13,'T1=',F5.1,'C=',F5.1,'F',T35,'T2='
     1         ,F5.1,'C=',F5.1,'F',T61,'Q=',1PE10.3
     2         /' ',T13,'SB=',E10.3,T35,'FB=',E10.3
     3         ,T56,'ODELTA=',E10.3
     4         /' ',T13,'OHC2I=',E10.3,T35,'OFI=',E10.3)
  220 FORMAT('+',T52,'.125*FB/SB=',1PE10.3)
  221 FORMAT('+',T55,'.125*FB=',1PE10.3)
  230 FORMAT('0',T13,'HC2I=',1PE10.3,T37,'MHC=',0PF6.3)
  231 FORMAT('+',T83,'HC2=',1PE10.3)
  232 FORMAT('+',T57,'DELTA=',1PE10.3)
  233 FORMAT('+',T62,'BETA=',F6.3)
  235 FORMAT('+',T102,'HC2/SB=',1PE10.3)
  236 FORMAT('+',T121,'BF=',1PE10.3)
  237 FORMAT('+',T106,'BF=',1PE10.3)
  240 FORMAT('0',T13,'FI=',1PE10.3,T37,'MFI=',0PF6.3)
  241 FORMAT('+',T57,'FI/FB=',1PE10.3)
  250 FORMAT('0',T17,'X',T27,'HI',T36,'GHNI',T46,'TDI'
     1         ,T58,'HP',T67,'DTDHI',T87,'XN',T98,'HN'
     2         ,T107,'GHNN',T117,'TDN',T128,'HPN'/)
  251 FORMAT('0',T17,'X',T30,'HI',T42,'TDI',T56,'HP'
     1         ,T67,'DTDHI',T87,'XN',T101,'HN',T113,'TDN'/)
```

```
260 FORMAT(' ',T15,'HALVED  INTERVAL  DX= ',1PE10.4
   1       ,T85,'HALVED  INTERVAL  DXN= ',E10.4)
261 FORMAT(' ',T15,'DOUBLED INTERVAL  DX= ',1PE10.4
   1       ,T85,'DOUBLED INTERVAL  DXN= ',E10.4)
270 FORMAT(' ',T12,1P4E10.3,E11.3,E10.2
   1       ,T83,E9.3,1X,3E10.3,E11.3)
271 FORMAT(' ',T9,1P4E13.3,E12.2,T83,E9.3,1X,2E13.3)
280 FORMAT(' ',T15,'H APPROACHES GHN')
281 FORMAT(' ',T15,'INTERFACE BECOMES HORIZONTAL')
282 FORMAT(' ',T15,'SIGN OF INTERFACIAL SLOPE '
   1       'IS CHANGED')
283 FORMAT(' ',T15,'XF=',1PE10.3,' IS EXCEEDED')
284 FORMAT(' ',T15,'H BECOMES NEGATIVE')
285 FORMAT(' ',T15,'H EXCEEDS TOTAL DEPTH')
286 FORMAT(' ',T15,'INTERFACE BECOMES TOO STEEP')
287 FORMAT(' ',T15,'FREE SURFACE BECOMES TOO STEEP')
288 FORMAT(' ',T15,'GHN-CUBE BECOMES NEGATIVE')
289 FORMAT(' ',T15,'IT IS NO LONGER A ',A1,A1
   1       ,' PROFILE FROM THIS POINT')
290 FORMAT(' ',T15,'CONTINUE TO THE NEXT PAGE')
291 FORMAT(' ',T15,'CONTINUED FROM THE PREVIOUS PAGE'/)
292 FORMAT(' ',T15,'COMPUTATION IS TERMINATED')
299 FORMAT('1',' ')
C
C
C      *** READ PROFILE IDENTIFICATION
300 READ 100, SLOPE, IDENT, DATE, ACC
C      *** TO TERMINATE COMPUTATION
       IF(SLOPE.EQ.'F') GO TO 2000
C      *** READ DATA
       READ 110, DX0, FB, HOI, TDOI, OFT,
     1          SB, TC(1), TC(2), Q, ODELTA, OHCI
C      *** TO GIVE INDEX
       IF(SLOPE.EQ.'M') PROFIL= 1
       IF(SLOPE.EQ.'S') PROFIL= 1
       IF(SLOPE.EQ.'H') PROFIL= 2
       IF(SLOPE.EQ.'M'.AND.IDENT.EQ.'1') NOMPRO= 1
       IF(SLOPE.EQ.'M'.AND.IDENT.EQ.'2') NOMPRO= 2
       IF(SLOPE.EQ.'M'.AND.IDENT.EQ.'3') NOMPRO= 3
       IF(SLOPE.EQ.'S'.AND.IDENT.EQ.'1') NOMPRO= 4
       IF(SLOPE.EQ.'S'.AND.IDENT.EQ.'2') NOMPRO= 5
       IF(SLOPE.EQ.'S'.AND.IDENT.EQ.'3') NOMPRO= 6
       IF(SLOPE.EQ.'H'.AND.IDENT.EQ.'1') NOMPRO= 7
       IF(SLOPE.EQ.'H'.AND.IDENT.EQ.'2') NOMPRO= 8
C      *** TO CALCULATE OHCI
       IF(OHCI.EQ.0.) OHCI= 12.*((Q**2/ODELTA/32.17)
     1                     **.3333333)
```

```
C      *** TO DETERMINE UPPER AND LOWER LIMIT FOR
C          TEST IN THE PREDICTOR-CORRECTOR METHOD
       UP= 29./10.**ACC
       DN= UP/600.
C      *** TO CONVERT TEMPERATURE FROM CELSIUS
C          TO FAHRENHEIT
       DO 310 II=1,2
  310  TF(II)= 1.8*TC(II)+32.
C      *** TO BE USED TO SKIP PRINTING DATA
       IP= 0
C      *** TO BE USED TO PRINT OUTPUT FROM BOTH METHODS
C          IN ONE PAGE WHEN OUTPUT IS SHORT
       IPWP= 0
C
C      *** READ METHOD
  350  READ 120, METHOD
C      *** GO TO NEXT DATA
       IF(METHOD.EQ.0) GO TO 300
C      *** PRINT PROFILE IDENTIFICATION AND METHOD
       IF(IPWP.EQ.1) GO TO 351
       PRINT 200
       PRINT 202, SLOPE, IDENT, DATE
       PRINT 203, ACC
       PRINT 204, METHOD
       GO TO 352
  351. PRINT 201
       PRINT 202, SLOPE, IDENT, DATE
       PRINT 204, METHOD
  352  IF(IP.NE.0) GO TO 370
C      *** PRINT DATA
       PRINT 210, TC(1), TF(1), TC(2), TF(2), Q,
     1            SB, FB, ODELTA,
     2            OHCI, OFI
       IP= 1
C
C      *** DEFINE VARIABLES WHICH IS NOT CHANGED
C          WHILE CHANGING MHC(BETA) AND MFI
       OHC= OHCI/12.
       FACTOR= .125*FB
       IF(PROFIL.EQ.1) GO TO 360
       PRINT 221, FACTOR
       DO 355 K=1,5
  355  GHNNC(K)= 0.
       GO TO 370
  360  FACTOR= FACTOR/SB
       PRINT 220, FACTOR
```

```
  370 IF(METHOD.EQ.2) GO TO 380
C     *** READ MHC AND MFI FOR METHOD 1
      READ 130, MHC0, MHCL, MHCD,
     1             MFI0, MFIL, MFID
      MHC= MHC0
      GO TO 390
C     *** READ BETA AND MFI FOR METHOD 2
  380 READ 130, BETA0, BETAL, BETAD,
     1             MFI0, MFIL, MFID
      BETA= BETA0
  390 FI0= MFI0*OFI
C
C     *** FOR NEW MHC OR BETA, CHANGE VARIABLES
C         RELATED TO HC AND PRINT THEM AND
C         NON-DIMENSIONALISE VARIABLES RELATED TO LENGTH
  400 FI= FI0
      MFI= MFI0
      IF(METHOD.EQ.1) GO TO 410
      MHC= BETACR= BETA**.3333333
      FACTOR= BF= FACTOR/BETA
  410 HC= MHC*OHC
      HCI= 12.*HC
      HN0= H0I/HCI
      TDN0= TD0I/HCI
      PRINT 230, HCI, MHC
      PRINT 231, HC
      IF(METHOD.EQ.2) GO TO 420
      DELTA= ODELTA/MHC**3
      PRINT 232, DELTA
      GO TO 430
  420 PRINT 233, BETA
      DELTA= ODELTA
  430 IF(PROFIL.EQ.2) GO TO 440
      XFAC= HC/SB
      DXN= DX0/XFAC
      PRINT 235, XFAC
      IF(METHOD.EQ.1) GO TO 450
      PRINT 236, BF
      GO TO 450
  440 DXN= DX0/HC
      IF(METHOD.EQ.1) GO TO 450
      PRINT 237, BF
C
C     *** FOR NEW MFI, CHANGE VARIABLES RELATED TO
C         FI AND PRINT THEM
  450 FRICS= FI/FB
```

```
      TDFAC= .125*FI*DELTA
      IF(METHOD.EQ.2) GO TO 460
      IF(PROFIL.EQ.2) GO TO 470
      TDFAC= TDFAC/SB
      GO TO 470
  460 TDFAC= TDFAC/BETA
      IF(PROFIL.EQ.2) GO TO 470
      TDFAC= TDFAC/SB
  470 PRINT 240, FI, MFI
      PRINT 241, FRICS
C     *** PRINT HEADING
      IF(PROFIL.EQ.2) GO TO 480
      PRINT 250
      GO TO 490
  480 PRINT 251
C
C     *** GIVE INITIAL VALUES
  490 Y(1,1)= 1.
      Y(1,2)= HNO
      Y(1,3)= TDNO
      HH= DXN
C     *** TO BE USED TO COUNT THE LINES OF PRINTED OUTPUT
      ILC= 0
C     *** TO BE USED TO COUNT THE NUMBER OF MESSAGE
C          INDICATING INTERVAL CHANGE
      ILCM= 0
      GO TO 900
C     *** GO TO INTEGRATION
C
C     *** TO COMPUTE SLOPE OF INTERFACE(YP(K,2))
C          AND FREE SURFACE(YP(K,3))
  500 IF(PROFIL.EQ.2) GO TO 510
C          FOR S AND M PROFILE
      GHNNC(K)= FACTOR*(1.+FRICS/(1.-Y(K,2)/Y(K,3)))
      IF(GHNNC(K).LT.0.) GO TO 838
      YP(K,2)= (Y(K,2)**3-GHNNC(K))/(Y(K,2)**3-1.)
      YP(K,3)= 1.+TDFAC/(Y(K,2)**2*(Y(K,3)-Y(K,2)))
      GO TO 520
C          FOR H PROFILE
  510 YP(K,2)= FACTOR*(1.+FRICS/(1.-Y(K,2)/Y(K,3)))
     1          /(1.-Y(K,2)**3)
      YP(K,3)= TDFAC/(Y(K,2)**2*(Y(K,3)-Y(K,2)))
C     *** TO PRINT INITIAL VALUES
  520 IF(ILC.EQ.0) GO TO 999
      GO TO (910,920,950,960), L1
C     *** GO BACK TO INTEGRATION
```

```
C      *** DIMENSIONALIZE VARIABLES AND PRINT OUTPUT
  600 XN= YOUT(1)-1.
      HN= YOUT(2)
      HI= HN*HCI
      TDN= YOUT(3)
      TDI= TDN*HCI
      HPN= YPOUT(2)
      IF(ILC.EQ.0) HPN0= HPN
      TDPN= YPOUT(3)
      IF(PROFIL.EQ.2) GO TO 610
      X= XN*XFAC
      GHNN= GHNNCR**.3333333
      GHNI= GHNN*HCI
      HP= HPN*SB
      DTDHI= TDI-TD0I-XN*HCI
      IF(ILC.EQ.0) DTDHI= 0.
      PRINT 270, X, HI, GHNI, TDI, HP, DTDHI,
     1           XN, HN, GHNN, TDN, HPN
      GO TO 620
  610 X= XN*HC
      DTDHI= TDI-TD0I
      PRINT 271, X, HI, TDI, HPN, DTDHI,
     1           XN, HN, TDN
  620 LC= 1
      ILC= ILC+1
C      *** ILCT IS TOTAL COUNT OF LINES PRINTED
      ILCT= ILC+ILCM
C      *** TO GO TO PRINT IN THE NEXT PAGE
      IF(ILCT.EQ.38) GO TO 800
C      *** TO TERMINATE A PROFILE CALCULATION WHEN
C          IT IS PRINTED IN 2 PAGES
      IF(ILCT.EQ.78) GO TO 840
C
C      *** TESTS TO TERMINATE A CALCULATION AND
C          GIVE THE REASON
  700 IF(PROFIL.EQ.2) GO TO 710
      IF(ABS(HI/GHNI-1.000).LT..003) GO TO 830
      IF(ABS(HPN-1.000).LT..001) GO TO 831
  710 IF(HPN*HPN0.LT.0.) GO TO 832
      IF(X.GT.XF) GO TO 833
      IF(HI.LT.0.) GO TO 834
      IF(HI.GT.TDI) GO TO 835
      IF(ABS(HPN).GT.1.E5) GO TO 836
      IF(ABS(TDPN).GT.1.E5) GO TO 837
```

```
      GO TO (750,752,754,756,758,760,762,764), NOMPRO
  750 IF(HI.GE.GHNI.AND.GHNI.GE.HCI) GO TO 790
      GO TO 839
  752 IF(GHNI.GE.HI.AND.HI.GE.HCI) GO TO 790
      GO TO 839
  754 IF(GHNI.GE.HCI.AND.HCI.GE.HI) GO TO 790
      GO TO 839
  756 IF(HI.GE.HCI.AND.HCI.GE.GHNI) GO TO 790
      GO TO 839
  758 IF(HCI.GE.HI.AND.HI.GE.GHNI) GO TO 790
      GO TO 839
  760 IF(HCI.GE.GHNI.AND.GHNI.GE.HI) GO TO 790
      GO TO 839
  762 IF(HI.GE.HCI) GO TO 790
      GO TO 839
  764 IF(HCI.GE.HI) GO TO 790
      GO TO 839
  790 GO TO (910,940), L2
C     *** GO BACK TO INTEGRATION
  800 PRINT 290
      PRINT 200
      PRINT 291
      PRINT 202, SLOPF, IDENT, DATE
      PRINT 230, HCI, MHC
      PRINT 240, FI, MFI
      IF(PROFIL.EQ.2) GO TO 810
      PRINT 250
      GO TO 820
  810 PRINT 251
  820 GO TO (700,998), LC
C     *** GO BACK TO CALCULATION AFTER HAVING CHANGED
C         INTERVAL
  830 PRINT 280
      GO TO 850
  831 PRINT 281
      GO TO 850
  832 PRINT 282
      GO TO 850
  833 PRINT 283, XF
      GO TO 850
  834 PRINT 284
      GO TO 850
  835 PRINT 285
      GO TO 850
  836 PRINT 286
      GO TO 850
  837 PRINT 287
      GO TO 850
```

```
  838 PRINT 288
      GO TO 850
  839 PRINT 289, SLOPE, IDENT
      GO TO 850
  840 PRINT 292
C     *** TO PRINT OUTPUT FROM BOTH METHODS
C         IN ONE PAGE WHEN OUTPUT IS SHORT
  850 IF(ILCT.LT.13) GO TO 870
C
C     *** TO CHANGE MFI
      MFI= MFI+MFID
      FI= MFI*OFI
      IF(MFI.LT.MFIL) GO TO 450
C
C     *** TO CHANGE MHC OR BETA
      IF(METHOD.EQ.2) GO TO 860
      MHC= MHC+MHCD
      IF(MHC-MHCL) 400,400,350
  860 BETA= BETA+BETAD
      IF(BETA-BETAL) 400,400,350
C
  870 IPWP= 1
      GO TO 350
C
C
C     *** INTEGRATION - 4TH ORDER RUNGE-KUTTA METHOD
C                          WITH GILL'S CONSTANT
C     *** INITIALIZE QQ
  900 K= 1
      DO 905 I=1,3
  905 QQ(I)= 0.
      L1= 1
      L2= 1
      GO TO 500
C     *** GO TO COMPUTE THE SLOPE OF INTERFACE
C         AND FREE SURFACE
C
C     *** IF K=4, GO TO THE PREDICTOR-CORRECTOR PART
  910 IF(K.EQ.4) GO TO 930
      K= K+1
      DO 915 I=1,3
      Y(K,I)= Y(K-1,I)
  915 YP(K,I)= YP(K-1,I)
      J= 1
      L2= 1
```

```
C       *** IF J=5, GO TO SET THE OUTPUT OF THE INTEGRATION
  920   IF(J.EQ.5) GO TO 999
        DO 925 I=1,3
        DD= HH*YP(K,I)
        RR= AA(J)*DD-BB(J)*QQ(I)
        Y(K,I)= Y(K,I)+RR
  925   QQ(I)= QQ(I)+3.*RR-CC(J)*DD
        J= J+1
        L1= 2
        GO TO 500
C       *** GO TO COMPUTE THE SLOPE OF INTERFACE
C           AND FREE SURFACE
C
C       *** INTEGRATION - 4TH ORDER MILNE PREDICTOR-
C                         CORRECTOR METHOD
C       *** INITIALIZE PMC
  930   DO 935 I=2,3
  935   PMC(I)= 0.
        MIL= 0
C
C       *** GET MODIFIED Y WITH PREDICTOR
  940   K= 5
        Y(5,1)= Y(4,1)+HH
        DO 945 I=2,3
        PRED(I)= Y(1,I)+1.333333*HH*(2.*YP(2,I)
     1          -YP(3,I)+2.*YP(4,I))
  945   Y(5,I)= PRED(I)-.9655172*PMC(I)
        L1= 3
        GO TO 500
C       *** GO TO COMPUTE THE SLOPE OF INTERFACE
C           AND FREE SURFACE
C
C       *** GET IMPROVED Y WITH CORRECTOR
  950   DO 955 I=2,3
        COR(I)= Y(3,I)+HH/3.*(YP(3,I)
     1          +4.*YP(4,I)+YP(5,I))
        PMC(I)= PRED(I)-COR(I)
  955   Y(5,I)= COR(I)+.0344828*PMC(I)
C
C       *** TEST IF THE INTEGRATION INTERVAL IS O.K.
        AMAX= 0.
        DO 957 I=2,3
        TEST= ABS(1.-PRED(I)/COR(I))
        IF(AMAX.GE.TEST) GO TO 957
        AMAX= TEST
  957   CONTINUE
        IF(AMAX.GT.UP) GO TO 970
        IF(AMAX.LT.DN) GO TO 980
```

```
C      *** NO CHANGE IN INTERVAL
       L1= 4
       GO TO 500
C      *** GO TO COMPUTE THE SLOPE OF INTERFACE
C             AND FREE SURFACE
C      *** DETERMINE Y AND YP FOR THE INTEGRATION OF
C            NEXT POINT
   960 DO 965 I=1,3
       DO 965 K=1,4
       Y(K,I)= Y(K+1,I)
   965 YP(K,I)= YP(K+1,I)
       K= 5
       MIL= 1
       L2= 2
       GO TO 999
C      *** GO TO SET THE OUTPUT OF AN INTEGRATION
C
C      *** HALVING INTERVAL
   970 HH= .5*HH
       INT= 1
       GO TO 990
C      *** GO TO CALCULATE DX
C      *** IF MIL=0, GO TO THE RUNGE-KUTTA PART
   972 IF(MIL.EQ.0) GO TO 900
       DO 975 I=1,3
   975 Y(1,I)= Y(4,I)
       GO TO 900
C      *** GO TO THE RUNGE-KUTTA PART
C
C      *** DOUBLING INTERVAL
   980 HH= 2.*HH
       INT= 2
       GO TO 990
C      *** GO TO CALCULATE DX
   982 DO 985 I=1,3
   985 Y(1,I)= Y(3,I)
       GO TO 900
C      *** GO TO THE RUNGE-KUTTA PART
C
C      *** CALCULATE DX
   990 IF(PROFIL.EQ.2) GO TO 992
       DX= HH*XFAC
       GO TO 994
   992 DX= HH*HC
```

```
C       *** PRINT CHANGED INTERVAL
  994 IF(INT.EQ.2) GO TO 996
      PRINT 260, DX, HH
      GO TO 997
  996 PRINT 261, DX, HH
C       *** COUNT PRINTED LINES
  997 LC= 2
      ILCM= ILCM+1
      ILCT= ILC+ILCM
C       *** TO GO TO NEXT PRINTING PAGE
      IF(ILCT.EQ.38) GO TO 800
C       *** TO TERMINATE COMPUTATION OF A PROFILE
  998 IF(ILCT.EQ.82) GO TO 840
      GO TO (972,982), INT
C       *** GO TO SET A STARTING VALUE FOR RUNGE-KUTTA PART
C
C       *** TO SET THE OUTPUT OF AN INTEGRATION
  999 YOUT(1)= Y(K,1)
      GHNNCB= GHNNC(K)
      DO 1005 I=2,3
      YOUT(I)= Y(K,I)
 1005 YPOUT(I)= YP(K,I)
      GO TO 600
C       *** GO TO PRINT THE OUTPUT
C
C       *** TO ELIMINATE COMPUTER MESSAGE OUT OF
C           LAST PRINTED PAGE
 2000 PRINT 299
      STOP
      END
```

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*16. Abstract* The "hypolimnetic flow" is a two-layered flow with the upper layer stagnant. This report presents the possibility of different flow regimes for the hypolimnetic flow which may be determined from the parameters of slope of channel bottom, flow depth, flowrate, density difference of water in the two layers, and channel roughness. The analysis is limited to the steady-state case of the hypolimnetic flow and the "upper layer analysis" in which the lower layer is stagnant. Interfacial profile equations which predict possible existence of ten different flow regimes for the hypolimnetic flow and two regimes for the upper layer analysis were obtained from the equations of continuity and momentum for two-layered flow. Experimental apparatus, consisting of a large scale open-channel tilting flume and water supply and control systems capable of circulating water of two different temperatures was designed and constructed. The experiment employed observations of a dyed flow layer and measurements of vertical temperature distributions in the flume. It was found that eight different flow regimes could be generated in the flume for the hypolimnetic flow on the positive and horizontal slopes.

*17a. Descriptors* *Stratified flow, *Flow profiles, *Interfaces, *Research facilities, Continuity equation, Density currents, Fluid friction, Free surfaces, Hydraulics, Impounded waters, Momentum equation, Non-uniform flow, Saline water - fresh water interfaces, Steady flow, Thermal stratification.

*17b. Identifiers* *Hypolimnetic flow regimes, *Experimental open-channel flume, *Interfacial profile equations.

*17c. COWRR Field & Group* 08C

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