United States
Environmental Protection
Agency

Office of Health and
Ecological Effects
Washington DC 20460

EPA-600/5-79-001d
February 1979

Research and Development

EPA

Methods Development for Assessing Air Pollution Control Benefits

Volume IV, Studies on Partial Equilibrium Approaches to Valuation of Environmental Amenities

OTHER VOLUMES OF THIS STUDY

Volume I, Experiments in the Economics of Air Pollution Epidemiology, EPA-600/5-79-001a.

This volume employs the analytical and empirical methods of economics to develop hypotheses on disease etiologies and to value labor productivity and consumer losses due to air pollution-induced mortality and morbidity.

Volume II, Experiments in Valuing Non-Market Goods: A Case Study of Alternative Benefit Measures of Air Pollution Control in the South Coast Air Basin of Southern California, EPA-600/5-79-001b.

This volume includes the empirical results obtained from two experiments to measure the health and aesthetic benefits of air pollution control in the South Coast Air Basin of Southern California.

Volume III, A Preliminary Assessment of Air Pollution Damages for Selected Crops within Southern California, EPA-600/5-79-001c.

This volume investigates the economic benefits that would accrue from reductions in oxidant/ozone air pollution-induced damages to 14 annual vegetable and field crops in southern California.

Volume V, Executive Summary, EPA-600/5-79-001e.

This volume provides a 23 page summary of the findings of the first four volumes of the study.

This document is available to the public through the National Technical Information Service, Springfield, Virginia 22161.

EPA-600/5-79-001d
February 1979

METHODS DEVELOPMENT FOR ASSESSING AIR POLLUTION CONTROL BENEFITS

Volume IV

Studies on Partial Equilibrium Approaches to Valuation of Environmental Amenities

by

Maureen L. Cropper, William R. Porter and Berton J. Hansen
University of California
Riverside, California 92502

Robert A. Jones and John G. Riley
University of California
Los Angeles, California 90024

USEPA Grant #R805059010

Project Officer
Dr. Alan Carlin
Office of Health and Ecological Effects
Office of Research and Development
U.S. Environmental Protection Agency
Washington, D.C. 20460

OFFICE OF HEALTH AND ECOLOGICAL EFFECTS
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
WASHINGTON, D.C. 20460

DISCLAIMER

This report has been reviewed by the Office of Health and Ecological Effects, Office of Research and Development, U.S. Environmental Protection Agency, and approved for publication. Approval does not signify that the contents necessarily reflect the views and policies of the U.S. Environmental Protection Agency, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.

PREFACE

The research studies presented in this volume emphasize some factors that are not completely treated in previous volumes. Most of the independent studies presented here tend to qualify the results of the experimental procedures set forth in earlier volumes. Each of them is therefore worthy of detailed attention.

ABSTRACT

The research presented in this volume explores various facets of the two central project objectives (the development of new experimental techniques for measuring the value of impovements in environmental amenities; the use of microeconomic methods to develop hypotheses on disease etiologies, and to value labor productivity and consumer losses due to air pollution-induced mortality and morbidity that have not been given adequate attention in the previous volumes. The valuations developed in these volumes have all been based on a partial equilibrium framework. W.R. Porter considers the adjustments and changes in underlying assumptions these values would require if they were to be derived in a general equilibrium framework. In a second purely theoretical paper, Robert Jones and John Riley examine the impact upon the aforementioned partial equilibrium valuations under variation in consumer uncertainty about the health hazards associated with various forms of consumption.

Two empirical efforts conclude the volume. M.L. Cropper employs and empirically tests a new model of the variations in wages for assorted occupations across cities in order to establish an estimate of willingness to pay for environmental amenities. The valuation she obtains for a 30 percent reduction in air pollution concentrations accords very closely with the valuations reported in earlier volumes.

The volume concludes with a report of a small experiment by W.R. Porter and B.J. Hansen intended to test a particular way to remove any biases that bidding game respondents have to distort their true valuations.

All of these studies tend to qualify the results of the experimental procedures discussed in earlier volumes. Further research will require: (1) an adequate specification of the mobility decision in response to degraded air quality; (2) consideration of relative price changes not directly related to air pollution as set forth in Chapter II and verified by Porter; and (3) how consumers evaluate a multitude of risks simultaneously, both in eating habits and pollution exposures where their economic and physical losses are uncertain.

CONTENTS

FIGURES

TABLES

CHAPTER I

INTRODUCTION TO VOLUME IV

The research presented in this volume explores various facets of the two central project objectives (the development of new experimental techniques for measuring the value of improvements in environmental amenities; the use of microeconomic methods to develop hypotheses on disease etiologies, and to value labor productivity and consumer losses due to air pollution-induced mortality and morbidity that have not been given adequate attention in the previous volumes. The valuations developed in these volumes have all been based on a partial equilibrium framework. W.R. Porter considers the adjustments and changes in underlying assumptions these values would require if they were to be derived in a general equilibrium framework. In a second purely theoretical paper, Robert Jones and John Riley examine the impact upon the aforementioned partial equilibrium valuations under variations in consumer uncertainty about the health hazards associated with various forms of consumption.

Two empirical efforts conclude the volume. M.L. Cropper employs and empirically tests a new model of the variations in wages for assorted occupations across cities in order to establish an estimate of willingness to pay for environmental amenities. The valuation she obtains for a 30 percent reduction in air pollution concentrations accords very closely with the valuations reported in earlier volumes.

The volume concludes with a report of a small experiment by W.R. Porter and B.J. Hansen intended to test a particular way to remove any biases that bidding game respondents have to distort their true valuations.

All of these studies tend to qualify the results of the experimental procedures discussed in earlier volumes. Further research will require: (1) an adequate specification of the mobility decision in response to degraded air quality; (2) consideration of relative price changes not directly related to air pollution as set forth in Chapter II and verified by Porter; and (3) how consumers evaluate a multitude of risks simultaneously, both in eating habits and pollution exposures where their economic and physical losses are uncertain.

CHAPTER II

PUBLIC GOODS DECISIONS WITHIN THE CONTEXT OF A GENERAL COMPETITIVE ECONOMY

by
William R. Porter

The purpose of this paper is to analyze the problem of public goods decision-making within the context of a general competitive economy for private goods. It is related to, but quite different from, recent works on the theory of value in economies with public goods.1/ The focal point of those works is the theoretical relationship between a Lindahl equilibrium and the core or Pareto optimum. Here we deal with the more mundane matter of what is involved in making a public goods production decision that will move the economy from its current equilibrium allocation to one that is Pareto superior. The theoretical techniques used are similar to allocation techniques for a planned economy,2/ however, the situation differs because private goods allocation here is accomplished in competitive markets.

There are two major types of problems involved in public goods decisions that are not encountered in private goods decisions. The first is to determine the proper concept of public good valuation, since the market does not provide one as it does in the case of private goods. The second is to obtain correct information about people's preferences concerning public goods in order to use the chosen valuation concept. Again the market normally does not provide this information, and the individuals usually have strong incentives to conceal or misrepresent their preferences.

The two problems are present when dealing with any public good (whether it is air pollution, public health, or national defense), therefore, although we are primarily interested in questions of environmental quality, the analysis and discussion will be presented in terms of an abstract public good.

The two problems are examined separately beginning with the determination of an appropriate valuation concept and a method of using that concept for decisionmaking when there is no problem of incorrect revelation of preferences. The framework for analysis is a general competitive economy model with public goods, but the ultimate object is to obtain results that will be useful in making real decisions on public goods allocation.

Many of the currently used concepts and methods of applied cost-benefit analysis have their theoretical foundations in partial equilibrium models. Therefore, it is quite possible that their use in a general economy having interactions among markets can lead to misallocation problems.

It has long been recognized by practitioners of cost-benefit analysis that the public good decision will have secondary effects on related markets therefore rendering the partial equilibrium methods inappropriate. However, this has not led to the development of general equilibrium methods for several reasons.

1. Many of the public good projects are small compared with the size of the overall economy, and therefore the secondary effects are thought to be small by comparison.

2. The possible complexity of a method that would try to model all the general equilibrium interactions would be unmanageable for applied work.

3. The tendency to separate the calculation of project benefits from those of project costs makes it seem that public good decisions deal more with the production of a scaler called net surplus rather than with the redistribution of vectors of commodities.

4. And among economists who have been interested in general economies with public goods and externalities, there has been an almost exclusive interest in the problems of existence of a competitive [Lindahl] equilibrium and its optimality properties, rather than in the problems facing the public decisionmaker of how to move from a non-optimal equilibrium to one that is Pareto superior.

This study uses the theoretical framework of a general competitive economy with public goods, however, the ultimate purpose is to obtain implications that will be useful in applications to real-world decision problems. We will look for ways in which the use of a general economy approach will yield results that are superior to the partial equilibrium methods. Therefore, efforts will be made to identify the types of errors that can arise when strictly partial equilibrium valuation methods are used in a general equilibrium economy. We will also propose ways in which the partial equilibrium methods can be modified in order to minimize the errors that are produced due to general equilibrium adjustments in the economy.

Before beginning the development of the basic model, we present the following example to illustrate the type of misallocation that can result from using partial equilibrium valuation measures in a general equilibrium context.

In a city plagued with air pollution, the property values in areas that are relatively free from pollution are quite high. The city government is considering a project that will uniformly reduce the average pollution levels throughout the city. It bases its acceptance of the project on whether the sum of people's valuations of the proposed pollution reduction exceeds the known cost of the project. The project is accepted, and the air pollution is reduced. After the pollution has been cleaned up, there is a general readjustment in property values resulting in large losses for the owners of the property that was previously "relatively free from pollution." These areas now have lower levels of pollution than before but they are not relatively so desirable. In view of the property value losses, these owners wish that the project had not been approved. If they could have anticipated the price changes that have occurred then their valuations would have been much lower and the project may not have been accepted.

The problem of unanticipated price changes due to the public good decision is more troublesome than is generally recognized for the following reasons.

1. It might be thought that the individuals could take the possibility of price changes into consideration when they evaluate the proposed public good project, however, there is really no way for the individual to do this since the new equilibrium prices after the project is completed depend on complex interaction of production technology and consumers' preferences which cannot be known by all individuals. Each person may be able to make a rough guess concerning the new prices, and that might reduce, but certainly would not eliminate, the possibility of misallocation due to imperfect price anticipation.

2. It is tempting to think that the problem is simply one of distribution where the losses of some are more than offset by the gains of others, and if the net surplus were appropriately redistributed then everyone would be better off than before. Unfortunately, movements from one general equilibrium to another are not so nicely behaved. It is entirely possible that even though the total apparent net surplus of the project, measured at the old equilibrium, is positive, the realized net surplus after the new equilibrium is reached is negative. Indeed, it is possible that everyone overvalued the public good project by assuming he could trade at the old prices.

3. The problem is not just one of using local measures of valuation for discrete changes. The difficulty is present even when discrete valuation measures are used. On the other hand, if the proposed public project is infinitesimal in size then the problem disappears.

In this air pollution example, it is important to note that the problem cannot be taken care of by using an estimate of the demand function for property. The property price change is simply used as an example, and it is important to realize that many other prices will change in a general adjustment. Furthermore, the estimate of the demand for property function will normally use data from a single equilibrium (in a cross-sectional study) which cannot reveal information about changes from one equilibrium to another.

To illustrate the problems of determining the proper level of public good production we examine a competitive market economy having two private goods and one public good. There are I consumers $i = 1, \ldots, I$, who each have constant endowment flows $\omega_i = (\omega_{i1}, \omega_{i2})$ of the two private goods and strictly quasi-concave utility functions $u^i(x_i, z)$ defined on their own consumption of private goods $x_i = (x_{i1}, x_{i2})$ and the amount available z of the public good. The level of public good z is produced according to the production function $z = f(y)$, where y is input of good 1.

Initially we assume that the government has perfect knowledge of the current market prices of private goods and the preferences of the individual consumers and is charged with the task of collecting the input of good 1 from the consumers in order to produce the proper level of the public good. (Note that the government's problem here is different than that of a central planner in that the private goods prices are determined in the market and are taken as given by the government).

We assume that the government's problem begins at a general equilibrium $[p,(x_i),z]$. Even though the level of the public good is not market determined and would not normally be thought of as a component of the general equilibrium, we include it here since it will be changing along with changes in the equilibrium prices p and allocation of private goods (x_i). The object is to specify a decision procedure that will use the collection of inputs of good 1 from consumers (taxation) and the production of the public good to bring about movement along a Pareto improving path toward a Pareto optimum. (Note that the tax used here is simply a flow of good 1 that is taken from each consumer independent of his own actions. In that sense it is a lump-sum tax).

A Continuous Path Method

In this simple model having only a single public good, the government's decision will deal only with the taxation problem since all of the proceeds of taxation must go into the single activity of public good production. The government's decision will be based on the individual marginal valuations of the public good defined as follows. At the equilibrium $[p,(x_i),z]$, person i's marginal valuation of the public good in terms of good 1 is:

$$v_i(x_i,z) = \frac{u_z^i(x_i,z)}{u_1^i(x_i,z)} = MRS_z \text{ for } 1 \qquad (2.1)$$

The marginal social valuation of the public good is defined as:

$$V(z) = \sum_{i=1}^{I} v_i \qquad (2.2)$$

The social cost of z units of the public good is:

$$C(z) = f^{-1}(z), \text{ where } f^{-1} \text{ denotes the inverse function of } f. \qquad (2.3)$$

The marginal social cost of the public good is:

$$C'(z) = [f^{-1}(z)]' \qquad (2.4)$$

Let s_i denote the total tax, in units of good 1, that person i is charged, and let γ_i be a non-negative weight that is assigned to person i, where $\sum_1 \gamma_i = 1$. The rate of change in the level of the public good is based on the magnitude of $[V(z) - C'(z)]$, which is called the net marginal social valuation of the public good. The rate of change is given by:

$$\dot{z} \equiv \frac{dz}{dt} = \alpha[V(z) - C'(z)], \quad \text{where } \alpha > 0. \tag{2.5}$$

Each person i's tax share is changed in such a way that he receives the share γ_i of the net social surplus resulting from the change. Therefore,

$$\frac{ds_i}{dz} = v_i - \gamma_i[V(z) - C'(z)], \quad \text{where } \gamma_i > 0 \text{ for all i, and } \sum_i = 1. \tag{2.6}$$

Summing over all individuals, we see that the sum of the tax changes is just sufficient to provide the necessary input C'(z) of good 1.

$$\sum_i \frac{ds_i}{dz} = \sum_i v_i - [V(z) - C'(z)]\sum_i \gamma_i$$
$$= V(z) - V(z) + C'(z) = C'(z). \tag{2.7}$$

No person is made worse off by the change, since each person's tax change is less than his own marginal valuation. Therefore, the procedure is continuously Pareto improving as long as the net marginal social valuation is nonzero.

The time rate of change in person i's tax is:

$$\dot{s}_i = \frac{ds_i}{dz} \cdot \frac{dz}{dt} = \alpha[v_i[V(z) - C'(z)] - \gamma_i[V(z) - C'(z)]^2]. \tag{2.8}$$

Equations (2.5) and (2.8) completely describe the time path of government action with respect to allocation in the economy. However, other reallocation is continuously occuring outside the domain of the government. As the level of the public good changes and taxes change, the consumers have incentive to adjust their private goods bundles through trade. Therefore, the government's actions are accompanied by continuously changing private goods prices. This fact is extremely important because if we think of an economy where private goods trading does not occur as the government changes taxes and the public good level, then the economy would not, in general, be at a Pareto optimum once the reallocation defined by (2.5) and (2.8) was complete.

The method of continuous government allocation in a three good economy can be easily generalized to more complicated economies having more private and public goods and a more general type of public good production function. However, the model just described is adequate to illustrate the main features involved in an optimal procedure of public good production and financing.

The continuous procedure summarized in equations (2.5) and (2.8) represents an extreme theoretical form for which we can guarantee that the economy will move in a continuously Pareto improving direction, but the model is very far from being applicable even in a real 3-good economy. It is important to note the massive informational and decisionmaking demands on both the government and the consumers in order to carry out the procedure.

a. The government must have continuous perfect information about each person's marginal valuation of the public good and about the marginal productivity of the public good production function.

b. The consumers must be continually in the private goods market offering and trading in order that the market can continuously find its new equilibrium. They must also be kept continuously up to date on their latest tax assessment so that they will know how much they have to trade.

The object is to develop procedures that are more applicable, but that will retain the optimality properties of the foregoing procedure. We will continue to use the model of a 3-good economy with public good production in order to examine the general equilibrium and Pareto optimality features of the problem. (It is clear that the Pareto optimality feature of public good production cannot be dealt with in a partial equilibrium framework, even though writers often use the terminology of general welfare economics when dealing with benefit-cost in partial equilibrium analysis).

The first step toward making the procedure applicable is to discretize the decision steps, since no real world decision procedure in economics can be carried out in a truly continuous fashion. In order to focus on the problems that are strictly associated with the discreteness of the procedure we will retain the assumption that the government has perfectly knowledge of people's valuations.

The use of a discrete decision procedure requires some additional definitions as follows. Beginning at some economy equilibrium $[p,(x_i),z]$, the government must decide on some discrete increment q in the public good that it will propose for production. Once the ocnsumers are informed of the proposal q they can form their own valuations of q in one of several ways whose merits will be discussed below.

Since good 1 is used for input into the production of any changes in z we will state all valuation in units of good 1.

C.V. Measure of Valuation

One of the most common ways of measuring person i's valuation of the proposed increment of the public good is to determine the maximum amount of good 1 he would be willing to give up in order to have the increment q produced. This measure is called (in certain contexts) the compensating variation (CV) associated with increment q. However, CV is usually defined in terms of a fixed nominal income and known prices, therefore it does not lend itself well to use in a general equilibrium context [see K-G. Maler, p. 126]. Under two different assumptions we consider the following CV measures.

Fixed Price Assumption

$$\nu_i^p = [\Delta x_{i1} | h_i(\hat{x}_{i1}-\Delta x_{i1},\hat{x}_{i2},z+q,p_1,p_2) = h_i(\hat{x}_{i1},\hat{x}_{i2},z,p_1,p_2)] \quad (2.9)$$

where h_i is the maximum utility function:

$$h_i(\omega_1,\omega_2,z,p_1,p_2) = \max \; u^i(x_{i1},x_{i2},z)$$
$$\text{s.t. } p_1x_{i1} + p_2x_{i2} = p_1\omega_1 + p_2\omega_2. \qquad (2.10)$$

ν_i^p measures the maximum amount of good 1 that person i would be willing to give up if he knew that after the increment q were produced he would be able to trade in the private goods market at the current prices p_1 and p_2. The problem with this measure is that the prices at which he will be able to trade after q is produced (if indeed it is produced) are not known at the time when γ_i^p is needed. By using current prices as the ones he will be able to trade at, he may overstate his valuation and end up at a utility level that is lower than his present level. This would destroy the Pareto-improving property of the allocation procedure. One way of avoiding this is to use the following conservative approach.

Fixed Utility Assumption

$$\nu_i^U = [\Delta x_{i1} \; u^i(\hat{x}_{i1}-\Delta x_{i1},\hat{x}_{i2},z+q) = u^i(\hat{x}_{i1},\hat{x}_{i2},z)] \qquad (2.11)$$

This measure assumes that the consumer will not be allowed to trade after he is taxed and the project is produced. Of course, if later he is able to trade then he will only do so if he is able to move to a preferred position. Therefore this method can never overstate the person's valuation of q, but it can understate the true valuation. An allocation procedure that is based on this measure will move only to Pareto superior points, but it may fail to move to some points that are Pareto superior.

E.V. Measure of Valuation

A frequently discussed measure of public good valuation is the minimum amount that a consumer would have to be given to make him as happy as he would be if he had the increment in the public good. The two EV measures that correspond to the CV measures given above are:

$$\mu_i^p = [\Delta x_{i1} \; h_i(\hat{x}_{i1}+\Delta x_{i1},\hat{x}_{i2},z,p_1,p_2) = h_i(\hat{x}_{i1},\hat{x}_{i2},\dot{z}+q,p_1,p_2)] \qquad (2.12)$$

$$\mu_i^U = [\Delta x_{i1} \; u^i(\hat{x}_{i1}+\Delta x_{i1},\hat{x}_{i2},z) = u^i(\hat{x}_{i1},\hat{x}_{i2},z+q)] \qquad (2.13)$$

Although the EV measures may have some theoretical interest in a partial equilibrium framework, it is clear from the expressions (2.12) and (2.13) above that they are not relevant to the type of public good allocation decision under consideration here. In order for the government to know whether to produce the increment q, it needs to know if the required resources for that production can be obtained without making someone worse-off. The difficulty with the EV measures is that they ask the consumers to compare two allocations that are technologically infeasible. The two allocations, as seen in (2.12) and (2.13) are $[(\hat{x}_{i1}+\Delta x_{i1},\hat{x}_{i2}),z]$ and $[(\hat{x}_{i1},\hat{x}_{i2}),z+q]$. It is clear that if the competitive allocation $[(\hat{x}_{i1},\hat{x}_{i2}),z]$ is both feasible and efficient, then the two allocations compared in the EV measure are either infeasible or inefficient except when $\Delta x_{i1}=0$, for all i, and when q=0. This fact renders the EV measures useless for decisionmaking in a general equilibrium context. Therefore we will use only CV measures in the following procedures.

Using one of the CV measures of valuation of the proposed increment q in the public good, the government decision procedure in the discrete framework is described below.

The marginal social valuation of the public good in the discrete case is:

$$V(z,q) = \sum_i \nu_i \tag{2.14}$$

The marginal social cost associated with a change from z to z+q of the public good is:

$$\Delta C = C(z+q) - C(z) \tag{2.15}$$

Therefore the net marginal social valuation is $[V(z,q) - \Delta C]$, and the government's decision rule will be to produce the increment q if $[V(z,q) - \Delta C] > 0$, and to not produce it otherwise. If it is to be produced then the necessary resources ΔC of good 1 are collected from the consumers according to the following formula:

$$\Delta s_i = \nu_i - \gamma_i[V(z,q) - \Delta C] \tag{2.16}$$

where Δs_i denotes the discrete change in person i's total tax and γ_i is person i's share of the net surplus, where $\sum_i \gamma_i = 1$ and $\gamma_i > 0$, $i = 1, \ldots, I$. Summing the tax changes over all consumers we see that:

$$\sum_i \Delta s_i = \Delta C \tag{2.17}$$

which is the needed amount of good 1 for input to produce the increment q.

Features of the Discrete Decision Process

Once the government has chosen which valuation measure to use, the process just described can be applied, and it is clearly more applicable than the previous continuous procedure since it will need only a finite amount of information for each proposed incremental change in the public good. The method works equally well for proposals where $q < 0$, therefore it can also be used to consider reductions in the public good level. Unfortunately the method has several weaknesses that detract somewhat from its greater degree of applicability. They are:

a. The procedure will, in general, stop before reaching a Pareto optimum, for any given q.

b. The procedure may cause reallocations that will make some consumers worse-off if the valuation measure ν^p is used. Therefore the procedure would not be Pareto-improving.

Both of these weaknesses can be eliminated through modification of the procedure, however, the modifications reduce the applicability by increasing the informational demands.

Problem (a) can be resolved by changing the size or the sign of q whenever a stop is encountered. As q becomes smaller the procedure requires more information per unit change in the public good, however, the government could make some judgment about how close is "close enough" to a Pareto optimum, in view of the cost of information for each decision.

Problem (b) can be eliminated by using ν^U rather than ν^P as the valuation measure. The difficulty with using ν^U, as mentioned earlier, is that it systematically understates the person's true valuation of the public good, given that there will be some trading possibilities in private goods if the project is approved. The valuation measure ν^U is based on the assumption that the consumers will not engage in private goods trade after the public good decision. To guarantee that the understatement is not preventing the detection of a possible Pareto improving move, the size of q must be reduced whenever a stop is encountered in order to see if there remain any possible Pareto improvements. The reduction in q increases the information requirements of the procedure.

A separate approach to this problem is to attempt to get accurate estimates of what the equilibrium prices will be if the size q proposal is approved. This is a difficult task since the prices will depend on market interactions that cannot be theoretically calculated without knowing all consumers' utility functions. Such information is equal in order of magnitude to that required in the continuous procedure. However, if rather than doing theoretical calculations of prices we allow a contingent claims market to operate then each consumer not only gets an accurate estimate of the future prices if the project is approved but he is able to hedge completely against possible loss due to price changes. The claims would be on private goods and they would be contingent on the approval of the increment q. Each person would have $(\hat{x}_{i1}-\nu_i, \hat{x}_{i2})$ units of contingent goods 1 and 2 to trade with, and would alter their valuations ν_i as the contingent goods market moved toward equilibrium. Once the contingent goods market reached an equilibrium the government could use the already described decision criteria to make the project approval and taxation decisions. The procedure would be guaranteed to move only to a Pareto superior allocation. If the project were not approved then the contingent claims would not be binding. Although this method requires the functioning of a competitive market for contingent claims, it uses an essentially decentralized procedure to determine accurate price estimates. It will be seen later that this type of contingent market can be very useful in applied procedures where the public good project is relatively large.

So far we have assumed that the government is able to get the consumers to reveal their correct valuations of public good changes. Unfortunately, whenever the consumers understand how their individual valuations are to be used for taxation purposes they have incentive to misrepresent their true valuations. This problem is widely referred to as the "free-rider" problem, and until recently it was thought to be unavoidable even in a purely theoretical model of an economy with public goods. Recent research has shown that it is possible to provide the proper incentives for individuals to submit accurate messages to the government concerning their true valuation functions.3/ This work is extremely important for theoretical development in this area, however, it is very far from a form that is applicable to actual public goods decision problems.

A different approach that also pays close attention to the individuals' incentives is one developed by Vernon Smith and tested by him and others in many experimental situations involving collective decisions.4/ This approach is not so fully developed theoretically, but it currently offers more promise in terms of application to public goods allocation problems in both a partial and a general economy framework. The method uses a system of bidding to overcome some of the distortionary effects of the free-rider problem.

In the following section we develop an extension of Vernon Smith's bidding mechanism that can be used to make Pareto improving decisions concerning public goods production in a general economy framework. The important thing about this method is that it does not require that the government know the consumers' preferences.

A Bidding Mechanism for Public Goods Decisions

In this section we develop an extension of Vernon Smith's Auction Mechanism for public good decisions to a general economy framework where private goods are traded in competitive markets, and the public good is produced by the government using private good inputs.

The bidding procedure developed here incorporates a market for contingent claims on private goods in order to avoid the type of unanticipated price changes that are associated with movements from one equilibrium to another. The claims are contingent on the approval of the public good project. Gambling on the outcome of the bidding procedure (by trading current goods for contingent claims) is prohibited since that would tend to bias people's bids and possibly cause some people to be worse off after the project decision. By trading in the contingent claims market each individual is able to determine the full value of his maximum willingness to pay for the public goods, and he can then form his bids in the same manner as in the partial equilibrium auction mechanism of Vernon Smith.

In Section 2.1 we examine the individual incentives in a partial equilibrium bidding procedure used to approve and finance a public good project. This procedure modifies Vernon Smith's Auction Mechanism5/ by: (1) adding an initial non-binding round of bidding used to determine if bidding should continue and to provide the group with an estimate of the net project surplus; and (2) including a positive and increasing stop-probability to induce the members to avoid a stalling strategy. Without analyzing all of the possible strategies that individuals could use we look at the type and the strength of the incentives that pull the group toward (or away from) a cooperative solution that is Pareto superior to the initial position. Section 2.2 develops the bidding procedure for an economy with two private goods and one public good. The public good is produced by the government using private good inputs obtained from consumers. The nature of the price uncertainty problem and its adverse effect on bidding decisions is explained. A market for contingent claims is designed to clear simultaneously with the bidding rounds in order to overcome the problems caused by price uncertainty. Section 2.3 gives the summary and concluding remarks.

2.1 Partial Equilibrium Procedure

The purpose of the bidding procedure described in this section is tc provide a framework within which a group can decide whether to approve the production of a given amount of a public good. The framework is based on the Auction Mechanism used in Smith for experiments in public good decisions.

The bidding procedure should enable the group to jointly approve and finance the production of public good projects that have a positive net surplus and to reject projects that do not. The procedure should not lead anyone into the position of being worse off after the decision, and it should provide the incentive and guidelines for quickly arriving at a cooperative Pareto superior solution when one exists. Although we will deal here with only a single discrete decision, it is clear that by using a sequence of such decisions the group could move toward a Pareto optimum.

Individual group members indicate their support for (opposition to) a project by submitting anonymous positive (negative) bids which establish the maximum amounts they can be assessed if the project is approved. Project approval occurs when the sum of the bids is at least as great as the project cost.

The total project cost is known to all, and after each round of bidding the sum of the bids is announced. As long as an individual's own project valuation is greater than his bid, he favors approval of the project. There are a finite number of bidding rounds, and if the project is not approved by the last round then it is judged infeasible and is abandoned. All potential gains from the project are lost if it is not approved by the last round. Members are not allowed individually to purchase small amounts of the public good.

If each person never bids higher than his true valuation then the method will never approve a project that makes anyone worse off, and in particular will not approve a project with a negative net social valuation. The procedure should then be considered successful if it is able to arrive at cooperative approval of projects having positive net valuations more frequently than other methods of unanimous social choice. Such a comparison can be made using experimental methods,6/ but cannot be done theoretically.

The fact that there is incentive for each member to keep his bid low in the hope that others will fill in the gap and cause the project to be approved may make it appear that this procedure has not really avoided the classic "free-rider" problem, and of course it hasn't entirely. However, it is important to recognize that the problem is greatly changed and is diminished in strength in this framework. In a contingent bidding procedure (one where bids are contingent on prcject acceptance) each person knows the amount of public good to be produced if his bid is accepted. Therefore he knows exactly what it is that he is valuing when he forms his bid. The same thing is not true in the case of private uncoordinated purchases of a public good or under systems of uncontingent donations toward production of a public good. As long as the sum of bids is less than the project cost, the incentive to free ride is offset by the incentive to increase the sum toward project approval. The strength of this incentive is diminished as one's bid gets close to his own project valuation. In the bidding procedure each person knows that he can signal a willingness to support the project without the fear that he will be left "holding the bag" if others don't cooperate sufficiently. Also the addition of bids for the same project corresponds to the way in which valuations must be added to determine the group value of a public good.

These features all tend to diminish the strength of the "free-rider" effect within this context. The results that Vernon Smith has obtained in experimental studies of his Auction Mechanism for public good decisions indicate that the free-rider effect is indeed diminished in such a context. The following modified auction mechanism was designed after observing the results of experiments conducted by Smith.

Project Apprcval

Consider a group of N individuals, indexed $i = 1, \ldots, N$, who will all be affected by the production of a public good project costing C. Person i has true valuation V^i for the proposed project. The following bidding procedure will be followed to determine if the project will be constructed and how much each person must pay toward the total cost C. There will be two stages of bidding composed of a total of T+1 rounds of bids. There will be only one round of bidding in Stage I. The purpose of this round of bidding is to determine whether or not the project will be considered further and to give everyone an estimate of the net project surplus, therefore the bids will be non-binding in terms of tax purposes.7/

Stage I (The Non-Binding Bids)

Each person anonymously submits his initial bid b_0^i. The decision rule for Stage I is: If $\sum_i b_0^i < C$, then stop bidding and abandon the project. If

If $\sum_i b_0^i > C$, then proceed to Stage II.

The purpose of Stage II is to decide on individual payments that will cover the total cost of the project. Each person determines his own bid of offered support for the project knowing that if the total of the bids is not high enough then the project may fail.

Stage II (The Binding Bids)

There will be at least one and at most T rounds of bidding in this stage. After each round in which the total bids fall short of cost there is a known probability that the procedure will be stopped and the project abandoned. The probability of this type of stop is t/T, where $t = 1, \ldots, T$ is the number of the round. The purpose of this increasing "stop" probability is to provide the incentive to the group to move quickly toward a solution.8/ At round $t = 1, \ldots, T$ the decision procedure will be:

If $\sum_i b_t^i > C$, then stop bidding, tax each member $b_t^i - 1/N(\sum_i b_t^i - C)$, and produce the public good.

If $\sum_i b_t^i \leq C$ and $\theta_t = 1$, then post the value $\sum_i b_t^i$ and proceed to the next round.

If $\sum_i b_t^i \leq C$ and $\theta_t = 0$, then stop bidding and do not produce the public good.

The distribution of θ_t is: $P(\theta_t = 0) = t/T$, $t = 1, \ldots, T$ and

$$P(\theta_t = 1) = 1 - P(\theta_t = 0)$$

The complete bidding procedure is explained to each member before round 0 of bidding.

There is no attempt made here to model completely the behavior or strategy of each individual. However, by looking at the situation from the point-of-view of a single agent we can get some idea of the incentive structure facing him. I will argue here that each person references his behavior to a commonly held notion of "fairness" which in this situation is defined as an equal sharing of the apparent gains. A person does not always feel obliged to abide by exact "fairness," and will at times attempt to get more than his "fair" share, and at other times be willing to accept less than his "fair" share in order to prevent the failure of the project.

Person i's true valuation of the public good is V^i. During Stage I of the bidding process he can bid any arbitrary value since he knows that he is not accountable for his bid in terms of future taxes, and no one else will ever know the value of his initial bid. However, he has incentive to make his initial bid close to his true valuation V^i. The reason for this is that if he overbids (i.e., bids $b_0^i > V^i$) in an attempt to help carry the project into Stage II then he is contributing to the overstatement of the apparent consumer surplus $(\sum_i b_0^i - C)$ associated with the project. An overstated apparent surplus will make it difficult to obtain joint approval in Stage II even if there is a large real surplus since unless he makes his Stage II bids greater than V^i (which would be foolish) then the other members must absorb his initial overbid believing that they are getting less than their fair share. On the other hand, if person i bids $b_0^i < V^i$ in an attempt to understate the apparent surplus so that he can get a larger share of the true surplus when the project is approved he increases the likelihood that the project will fail in round 0. Now it is certainly true that there may be some overbidding in Stage I for various possible reasons, however, if there are strong tendencies in one direction then this will result in a high proportion of failures in either Stage I or Stage II of the process. This high failure rate would presumably provide the incentive to correct this type of misbidding.

In Stage II person i is aware of the total apparent surplus $(\sum_i b_0^i - C)$ established in Stage I. If he takes this number as being the true surplus then his fair share is $1/N(\sum_i b_0^i - C)$ and his corresponding fair bid is $b_*^i = b_0^i - 1/N(\sum_i b_0^i - C)$. He knows that if everyone bids his fair bid that the project will be exactly approved on the first round and each will obtain an equal share of the apparent surplus. However, he may bid higher or lower than his fair bid depending on how urgently he wants the project approved and on what he believes that others will do. In general if he bids higher then he is contributing to rapid project approval, and if he bids lower he is attempting to get a larger share of the surplus while some socially beneficial projects will fail.

It was mentioned earlier that the procedure is designed to enlist everyone's support by giving each person a vested interest in the approval of the project. There is, of course, the possibility that one of the members derives his pleasure from foiling the plans of the others. There is no way that the procedure can offset this type of behavior if the person is determined to foil every project. Whether or not this type of behavior is frequent enough to cause problems for the method would most likely be brought out in experimental studies.

Project Size and Approval Determination

The two-stage bidding procedure can be extended to a procedure that determines both the size and approval of the public good project. This procedure takes advantage of the incentives present during the first stage to obtain information about the group valuation function of the public good.

Suppose that each of I members has the individual valuation function $V^i(Z)$, where $Z \geq 0$ is the level of the public good. Suppose that $C(Z)$ is the total cost of Z units of the public good. For convenience we assume that V^i is concave with $V^i(0) = 0$, for all i, and that C is convex and increasing with $C(0) = 0$.

Stage I

Each member anonymously submits a bid function $b_0^i(Z)$, knowing that the aggregate function $\sum_i b_0^i(Z) - C(Z)$ will be used to determine the project size to be considered for approval in Stage II. The project size $\bar{Z}$ is selected to maximize $\sum_i b_0^i(Z) - C(Z)$, and $\bar{Z}$, $\sum_i b_0^i(\bar{Z})$ and $C(\bar{Z})$ are announced to all members.

Stage II

This stage is handled exactly as in the previous procedure where $\bar{Z}$ = the project size, $\sum_i b_0^i(\bar{Z}) = \sum_i b_0^i$, and $C(\bar{Z}) = C$.

The interesting question here is whether there is incentive for the individual members to misrepresent their valuation functions $V^i(Z)$ in their Stage I bid functions $b_0^i(Z)$. The incentive for making one's initial bid function very close to one's true valuation function is the same as before, however in this case since the person cannot know what project size will be selected he is induced to bid "honestly" over the whole range. He wants the project to succeed in Stage I (i.e., to have the selected project to be $Z \neq 0$), but does not want the apparent surplus to be inflated so that approval is more difficult in Stage II.

2.2. Bidding Procedure for a General Economy

All of the previous sections rested on the assumption that people's valuations of a public good do not change as a result of the production of the public good. We assumed that the valuations were in units of money that the person is willing to give up to obtain the public good and that only money is required for the production of the public good. Of course, in reality, the production of a public good requires real resources which when demanded as inputs into public good production may affect the prices of all other goods. These price changes will alter both the money valuation and the real valuation of the public good, therefore raising some serious doubts about decision criteria that assume no changes take place. The difficulties are caused by the fact that changes in the level of the public good are associated with a movement from one general equilibrium to another, but at the time that agents are expected to make bids on such a change they do not know the prices that will prevail in the new equilibrium. Therefore, they are unable to know their own maximum willingness to pay for the proposed public good, and consequently they have inadequate basis for bidding. The following bidding procedure incorporates a market for claims that are contingent on project approval to provide the type of information needed by each agent. This contingent claims market allows the group to get close to the full valuation of the proposed public good and it protects each agent from ending up worse off after project approval due to unanticipated price changes. Therefore, by using this method the group will be more likely to find a Pareto superior solution if one exists since the element of price uncertainty will be removed, and we can be assured that projects will only be approved if they lead to Pareto superior allocations. The method uses the incentive structure of the previous section to induce members toward a cooperative decision. We will consider only the problem of project approval.

General Equilibrium Method

Consider an economy with two private goods and one public good. The public good is produced by the government using inputs of private good 1 obtained from the consumers. There are N consumers, indexed i = 1, . . .,N, who each have a utility function $u^i(x^i,z)$, where x^i is the consumer's vector of private goods and z is the amount of public good. The economy's initial resources of private goods is $\omega = \binom{\omega 1}{\omega 2}$, and there is initially no public good. The public good production function is $z = f(y)$, where y is the input of private good 1. There is no production of private goods, so the economy resource constraint is given by $\sum_i x^i + \binom{y}{0} \leq \omega$.

The public choice problem faced by this economy is whether to produce z units of the public good and if so how to distribute the taxes among the consumers to obtain the needed input. The total input of good 1 that is needed to produce z is denoted $C = f^{-1}(z)$. The society wants to approve this public good project if and only if it can do so in a Pareto improving way. The economy is assumed initially to be at the competitive equilibrium $[(x^i),0,p]$, where (x^i) is the allocation of private goods among the consumers, 0 is the current amount of public good, and $p = \binom{p1}{p2}$ is the equilibrium price vector. As before there will be T+1 rounds of bidding indexed t = 0, 1, . . .,T. There will be two stages of bidding consisting of the non-binding bids in Stage II. At each round of bidding a contingent claims market will be conducted, and the bids for that round become official when the market clears. No trading of uncontingent claims (i.e., contributing to possible non-approval of the project. He is never tempted to bid higher than V^i during Stage II since if the project is approved then he will suffer a net loss.

As t gets larger and closer to T (increasing the probability of a stop) the persons whose bids are much lower than their valuations have strong incentive to raise their bids in order to increase their bids since their gains would be small even if approval is accomplished. In this way the bidding procedure tends to put the greatest individual pressure for bid increases on those who are attempting to get the largest gains. It is they who have the largest vested interests in the project's success.

Ignoring the costs associated with conducting the bidding, the process will move only to Pareto superior points. This is true because no one will make a Stage II bid that is higher than his true valuation. Therefore, we know that the process will not move if there are no longer projects having a positive net surplus. So, in this partial equilibrium sense, the process will only move toward Pareto superior points and will not move from a Pareto optimum. However, there is the possibility that even though there is positive net surplus associated with a project that it will not be approved since the procedure may stop before approval is reached. It may seem wasteful that some projects having positive consumer surplus will fail due to a stop occurring before the cooperative solution is reached. However, if we imagine a procedure where, whenever there is a positive apparent surplus in Stage I, the Stage II bidding will continue until the group arrives at a cooperative solution, then we see that there is almost no incentive for the individuals to raise their bids up toward their valuations. By using a system that may cause a loss due to non-cooperative behavior at each round we provide some disincentive for holding out for a "free ride." The cost is that claims contingent on the failure of the project is allowed during the entire bidding procedure. This rule is used to prevent speculation on the success or failure of the project which might cause some members to end up worse off than originally. At the beginning of each round of bidding person i has x^i as his initial endowment of contingent claims. His choice of contingent claims at the end of round t is denoted $\mu_t^i = \begin{pmatrix} \mu_{1t}^i \\ \mu_{2t}^i \end{pmatrix}$. The current contingent claims prices are denoted ρ_1 and ρ_2. Person i's bid in round t is denoted b_t^i, and it represents the maximum amount of good 1 that he is willing to deliver to the government upon the approval of the project.

Stage I (The Non-Binding Bid)

Stage I will consist of one round of bids used only to determine if the project should be considered further. Since the contingent claims market in this round (and in other rounds) is competitive we will first look at the decision faced by the price taking agents. Given z, ρ_1 and ρ_2, person i chooses person i chooses a bid b_0^i and a contingent claims vector μ_0^i such that:

$$u^i(\hat{\mu}_0^i, z) \geq u^i(x^i, 0) \text{ and} \tag{2.18}$$

$$\hat{\mu}_0^i \text{ maximizes } u^i(\mu_0^i, z) \tag{2.19}$$

$$\text{subject to } \rho_1\mu_{10}^i + \rho_2\mu_{20}^i \leq \rho_1(x_1^i - b_0^i) + \rho_2 x_2^i$$

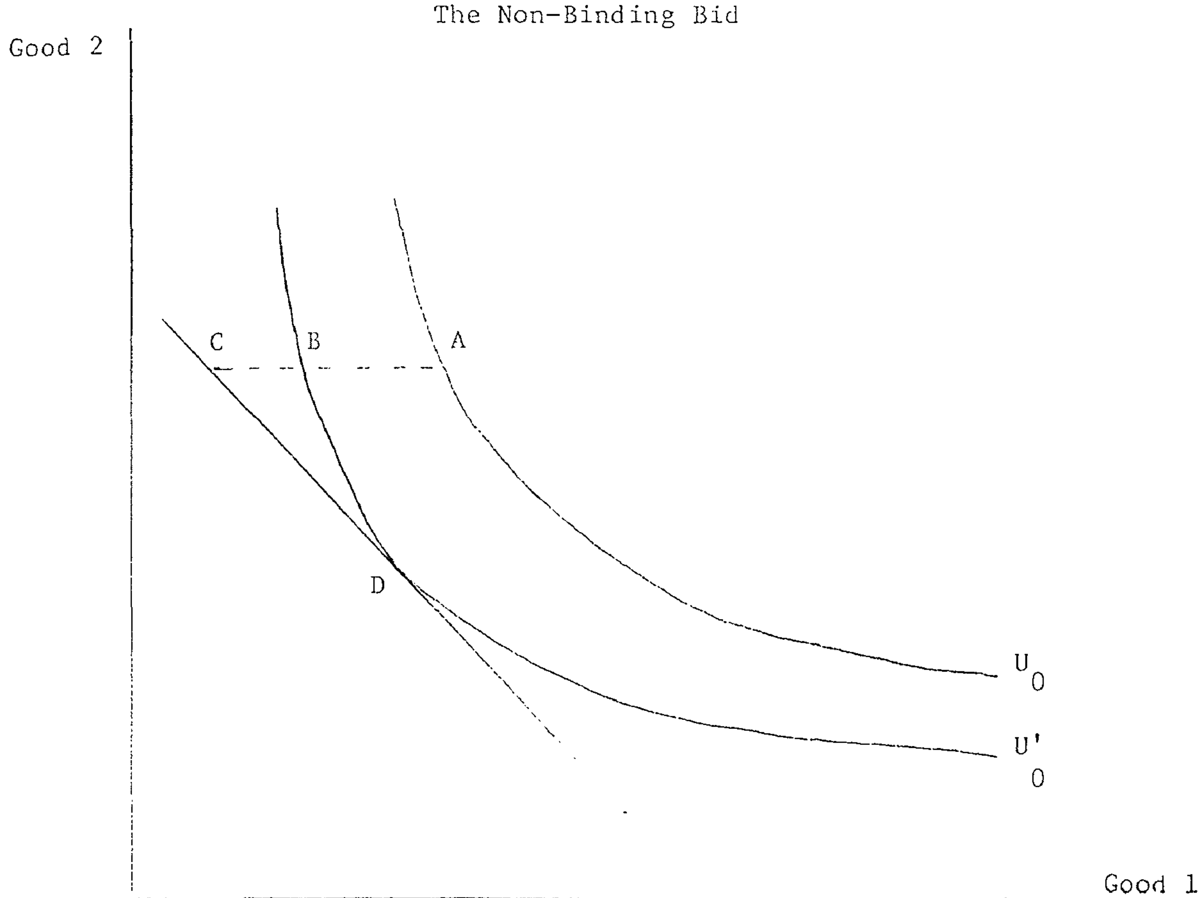
Let $\hat{b}_0^i$ denote the bid when (2.18) is an equality. Then $\hat{b}_0^i$ is the person's true maximum willingness to pay for the public good. In general, $\hat{b}_0^i$ is greater than the standard measure known as the compensating variation (CV), since the calculation of CV ignores price and trading considerations. Let q_0^i denote the compensating variation, in units of good 1, for z units of the public good. Mathematically, q_0^i satisfies the equation:

$$u^i(q_0^i, z) = u^i(x^i, 0) \tag{2.20}$$

Clearly $q_0^i \leq \hat{b}_0^i$, and except for a unique price ratio $q_0^i < \hat{b}_0^i$. This relationship is illustrated in the indifference curve diagram of Figure 2.1, where $U_0 = u^i(x^i, 0)$ denotes the indifference curve when there is zero public good, and U_0' denotes the indifference curve at the same utility level when there are z units of public good. q_0^i is the distance BA on the diagram, and $\hat{b}_0^i$ is the distance CA. The slope of the line CD indicates the price ratio for

Figure 2.1

The Non-Binding Bid



contingent claims. Therefore, we see that the contingent claims market allows the society to determine its full social valuation of the proposed public good, whereas CV measure does not because it doesn't allow for possible private goods trading. The Stage I bids become effective when the following market clearing condition holds:

$$\sum_i \mu_0^i + \begin{pmatrix} \sum_i b_0^i \\ 0 \end{pmatrix} = \omega \qquad (2.21)$$

The decision rule for Stage I is:

If $\sum_i b_0^i \leq C$, then abandon the project.

If $\sum_i b_0^i > C$, then post the values C and $\sum_i b_0^i$, and proceed to Stage II.

As in the partial equilibrium procedure each person here has some incentive to give an honest bid on round 0 since he knows that his bid will not be used to assign his tax and he has a vested interest in Stage I approval, but he realizes that an overstated apparent surplus will cause difficulty in Stage II approval.

Stage II (The Binding Bids)

Each person knows the value of the apparent consumer surplus established during round 0, therefore they each have some idea of their own fair bid $b_*^i = b_0^i - 1/N(\sum_i b_0^i - C)$. Also, each person is aware that the "stop" probability after round t is given by t/T. During round t with given values ρ_1 and ρ_2 person i chooses b_t^i and $\hat{\mu}_t^i$ such that:

$$u^i(\hat{\mu}_t^i, z) \geq u^i(x^i, 0), \text{ and} \qquad (2.22)$$

$$\hat{\mu}_t^i \text{ maximizes } u^i(\mu^i, z) \qquad (2.23)$$

$$\text{subject to } \rho_1\mu_1^i + \rho_2\mu_2^i \leq \rho_1(x_1^i - b_t^i) + \rho_2 x_2^i$$

The bids are effective once the prices ρ_1 and ρ_2 are such that the contingent claims market clears:

$$\sum \hat{\mu}_t^i + \begin{pmatrix} \sum_i b_t^i \\ 0 \end{pmatrix} = \omega \qquad (2.24)$$

Each person will bid in such a way that (2.22) is a strict inequality. The social decision rule in round t is:

If $\sum_i b_t^i > C$, then stop bidding, tax each member and produce the public good.

If $\sum_i b_t^i \leq C$ and $\theta_t = 1$, then post the value $\sum_i b_t^i$ and proceed to the next round.

If $\sum_i b_t^i \leq C$ and $\theta_t = 0$, then stop bidding and do not produce the public good.

The distribution of θ_t is:

$$P(\theta_t = 0) = t/T,\ t = 1, \ldots, T \text{ and}$$

$$P(\theta_t = 1) = 1 - P(\theta_t = 0).$$

This rule is exactly the same as in the partial equilibrium procedure except that here bids and the tax are in units of good 1 rather than money. If the project is approved in round t, then person j's holdings of the two goods after taxes is:

$$\begin{bmatrix} \hat{\mu}_{1t}^j + 1/N(\sum_i b_t^i - C) \\ \mu_{2t}^j \end{bmatrix}$$

This means that the contingent claims become real claims and if the sum of the bids is greater than the cost of producing z units of public good, then the households share the excess. Once the project has been approved, then the trading of private goods can resume.

It is clear from the description of the procedure that a project will only be approved if it leads to a Pareto superior allocation. Therefore, the procedure does guarantee that no one will be hurt as a result of unanticipated price changes.

Even though the general economy procedure was explained using a simple 3-good economy, it should be clear that there would be no theoretical problems involved in going to economies having n private goods, m public goods, and more general production sets for the public goods. The main feature that was introduced in order to use the partial equilibrium technique in a general economy was the market for contingent claims.

It is important to recognize the way that the contingent claims market is being used in this procedure to avoid a rather difficult problem concerning price expectations. The contingent claims market artificially creates a close approximation to the real market that will exist once the taxes are collected and the public good produced. With this market the agents are able to have accurate price expectations and therefore to accurately calculate their valuations of the public good. By prohibiting trades involving current (uncontingent) goods we avoid all of the problems caused by mixing people's preferences with their subjective probabilities that the project will be approved. Allowing only trade of contingent commodities once the project has been proposed separates the two types of markets so that gambling on the outcome of the project approval decision through trade is avoided. If this were allowed then the nature of the process would be altered considerably.

The use of contingent claims markets tends to conceal a severe problem in the applicability of the general economy procedure. We have assumed that the contingent claims market will clear simultaneously with each round of bidding without recognizing the substantial difficulty in finding the market clearing equilibrium in practice. Economists usually do not dwell on the difficulties involved in attaining the competitive equilibrium, so I will not do so here. However, in any application of this technique the problem would have to be dealt with.

2.3 Conclusions

By framing the public good decision within a general equilibrium model we are able to see clearly some of the problems associated with the use of the standard partial equilibrium techniques. Some of the features that are brought out in this framework are the following:

1. It emphasizes the fact that public good production is a reallocation process that moves the economy from one competitive equilibrium to another. This is especially important when dealing with projects that are not infinitesimal in size, since the discrete reallocation will lead to price changes that cannot automatically be anticipated. On the other hand, the partial equilibrium method views the government as a type of Marshallian firm whose actions will not have any effect on the rest of the economy.

2. The framework allows us to see clearly why the application of partial equilibrium methods of cost-benefit will not lead to allocations that are Pareto superior if the project is of discrete size.

3. The approach emphasizes the logical impossibility of separating costs from benefits and valuation from taxation and trade.

4. The inappropriateness of the EV measure for use in public goods decisions is made obvious by the technical infeasibility of the allocations it compares.

5. Changing the size of the project proposals brings out the tradeoff between information and allocative efficiency within this framework.

2.4 Recommendations

Based on the models developed in this report, there are several recommendations that can be made for avoiding the types of distortions caused by either unanticipated price changes or "the free-rider effect." They are:

1. Although it may not be practical to hold contingent markets for all commodities, it is conceivable that the government could organize markets for those goods that are highly likely to undergo substantial price changes. In the air pollution example, it would be useful to have a contingent market for real estate. Another likely candidate for contingent trading is any major input into the public good production. Thus, if the proposed project is to reduce air pollution by requiring (or prohibiting) the use of certain types of

fuels, then the government could organize contingent markets for various sources of energy among which there may be substantial substitution. The sponsorship of such markets would improve the valuation estimates of the public good project and it would allow consumers and producers to hedge against possible losses due to price uncertainty caused by the project. Furthermore, their existence would provide the means and the incentive for the public to stay informed about proposed public goods projects. The reason that the government should sponsor such markets rather than let them simply evolve due to normal market forces is to prevent the substantial danger of moral hazard that is present when people are allowed to gamble on the outcome of a decision they can influence. The government could insure that the contracts are only binding if the project is approved. The legal machinery required to enforce a contract that is contingent on a government decision would have to be developed very carefully since it is not now in existence and is not likely to develop on its own.

2. Another, less radical, suggestion for reducing the distortion caused by unanticipated price changes resulting from the public good decision is to have the government attempt to estimate the nature of important market interactions in supply and demand in order to calculate adjustments to the valuation and cost figures that are based on current prices. Econometric models for this type of estimation require more information than those used to estimate single supply or demand functions, however such techniques are currently in wide use and could be easily applied to this type of scheme.

3. The difficulty involved in applying the bidding mechanism to a real public good proposal depends on the exact nature of the public good. It is important in any application of this technique that the participant bidders realize the exact nature of the proposal, the current total of bids, and the fact that their own bid will be a binding obligation. If it is simply a number which they know will have no relationship to their tax, then it cannot provide a measure of their true valuation.

FOOTNOTES: CHAPTER II

[1] See Milleron for a survey to this literature.

[2] See Champsaur and Malinvaud for procedures for allocating public goods in a planned economy.

[3] See Groves and Ledyard for this result in a general equilibrium framework, and see Clarke, Groves and Loeb, and Tideman and Tullock for the result in partial equilibrium models.

[4] See Bohm, Ferejohn and Noll, Scherr and Babb, and Smith for descriptions and results of these experiments.

[5] Reported in Smith.

[6] It is clear that as the positive net surplus becomes smaller that there is less incentive for the members to cooperate. In experiments we could measure the approval rate as a function of the net surplus in order to determine how effective the method is.

[7] The usefulness of an initial round of non-binding bids is shown clearly by the experimental results reported in Smith. He designed this trial as a "practice trial" used to provide familiarity with the procedure but noted that it also provided the subjects with valuable information about the potential surplus available. I have made the continuation of the bidding contingent on obtaining a positive net surplus in the initial trial in order to provide disincentive to underbidding here.

[8] It is apparent in some of the experimental results reported in Smith that the bidding didn't get serious until the process got close to the last trial. Incorporating an increasing random stop probability makes each of the stage II rounds a potential last round. This should increase the seriousness of the bidding very early in the procedure.

REFERENCES

Bohm, P., "Estimating Demand for Public Goods: An Experiment," European Economic Review 3 (1972), 111-130.

Champsaur, P., "Neutrality of Planning Procedures in an Economy with Public Goods," Review of Economic Studies 43 (June 1976), 293-299.

Clarke, E.H., "Multipart Pricing of Public Goods," Public Choice 11 (1971), 17, 33.

Ferejohn, J. and R. Noll, "An Experimental Market for Public Goods: The PBS Station Program Cooperative," American Economic Review, Papers and Proceedings (May 1976), 267-273.

Groves, T. and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to the Free Rider Problem," Econometrica 45 (1977), 783-809.

Groves, T. and M. Loeb, "Incentives and Public Inputs," Journal of Public Economics 4 (1975), 211-226.

Mäler, K-G., Environmental Economics: A Theoretical Inquiry, Baltimore: Johns Hopkins University Press, 1974.

Malinvaud, E., "Prices for Individual Consumption, Quantity Indicators for Collective Consumption," Review of Economic Studies 120 (October 1972), 385-406.

Milleron, J-C., "Theory of Value with Public Goods: A Survey Article," Journal of Economic Theory 5 (1972), 419-477.

Scherr, B. and E. Babb, "Pricing Public Goods: An Experiment with Two Proposed Pricing Systems," Public Choice (Fall 1975), 35-48.

Smith, V.L., "Incentive Compatible Experimental Processes for the Provision of Public Goods," NBER Conference on Decentralization, April 23-25, 1976. Forthcoming in Research in Experimental Economics, edited by V.L. Smith, Greenwich, Conn., JAI Press.

Smith, V.L., "The Principle of Unanimity and Voluntary Consent in Social Choice," Journal of Political Economy 85 (1977).

Tideman, T. and G. Tullock, "A New and Superior Process for Making Social Choices," Journal of Political Economy 84 (1976), 1145-1159.

CHAPTER III

THE VALUE OF LEARNING ABOUT CONSUMPTION HAZARDS

by
Robert A. Jones

This report examines the implications of reducing uncertainty about the hazards associated with various forms of consumption. Section 3.1 focuses on the determinants of the dollar valuation of such a reduction in uncertainty, measured as the willingness to pay. The chapter begins with the simplest 'Marshallian' case and then successively generalizes the results at the cost of making Taylor's series approximations. It is shown that the value of reducing uncertainty is readily determined once estimates have been made of the ex-post shifts in demand associated with the information.

A major simplifying feature of the models in Section 3.1 is that all prices are exogenous. While this is perhaps a reasonable first approximation for many applications, it is surely inappropriate for non-produced commodities of uncertain quality. One important case is the adjustment of land prices to reflect differences in air quality in an urban environment. This case is the primary focus of Section 3.6. First the equilibrium location of a population with different incomes is described. It is shown that there is only a mild presumption in favor of location in the less hazardous areas by the more wealth. Optimal location of an identical population is then examined. Finally, it is shown that the expected value of research which reduces uncertainty about an environmental hazard may be fully reflected in land values.

Section 3.11 introduces time into the analysis, taking account of the fact that the prospect of future information will affect consumption decisions made prior to the receipt of the information. The central result is that if the possibly harmful effects of consuming a particular good depend on its accumulated consumption over the lifetime, then the prospect of receiving information about the maximum safe level of consumption reduces current consumption of that good.

3.1 The Value of Information

If a consumer is uncertain about the value of some parameter, for example the 'quality' of a particular product or the probability it will result in early death, he will in general be willing to pay to obtain a better estimate of the unknown parameter. In the following section we ask how much a consumer would be willing to pay for perfect information.

Formally, suppose uncertainty is captured by a parameter s and the utility of the consumer in state s is:

$$u = u(x(s);s) \tag{3.1}$$

where $x(s) = (x_1(s),\ldots,x_n(s))$ is consumption in state s.

To focus upon uncertainty about the quality of a product we assume that neither the price vector p nor income M are state dependent. Then with perfect information about the state provided at a cost of V, the consumer chooses x(s) to maximize u subject to his budget constraint. That is x(s) yields the solution of:

$$u(s) = \underset{x}{\text{Max}}\{u(x;s)\,|\,p'x \leq M - V\}. \tag{3.2}$$

Since the cost of obtaining the information is incurred prior to knowing the true state, anticipated benefit is a random variable u(s). Assuming that the consumer's preferences satisfy the von Neumann-Morgenstern axioms we can express the benefit as the expectation of this random variable, that is:

$$U^*(V) = \underset{s}{E}u(s)$$

$$= \int_{s\varepsilon S} u(s)dF(s) \tag{3.3}$$

where F(s) is the consumer's subjective probability distribution over the set of feasible states S.

Without the information, the consumer simply chooses x^o to maximize his expected utility. That is x^o yields the solution of:

$$U^o = \underset{x}{\text{Max}}\{\underset{s}{E}u(x;s)\,|\,p'x \leq M\} \tag{3.4}$$

Since x^o is a feasible solution to problem (3.3) when V = 0, $U^*(V) \geq U^o$ at V = 0. Moreover U*(V) is a non-increasing function of V. Therefore for some V* the expected utility associated with being perfectly informed at the time of purchase is equal to the expected utility in the absence of this information. V* is therefore the most the consumer would be willing to pay to be perfectly informed. That is, V* is the <u>reservation price</u> or <u>value</u> of perfect information.

In the following sections we derive expressions for V* under alternative assumptions about the utility function u(x;s). Section 3.2 considers the simple Marshallian case in which the marginal utility of expenditure on other goods is constant and independent of the state. This generates a particularly simple expression for the value of information. Section 3.3 introduces the more plausible situation in which marginal utility varies. After obtaining an expression for V* using the logarithmic utility function, a first order approximation is derived. The accuracy of this approximation is then discussed.

In Section 3.4 a first order approximation of the value of being perfectly informed is obtained for a general utility function u(x;s). The results are related to those of the previous two sections and several other special cases are then considered.

Finally, in Section 3.5 we turn to the value of becomming better informed rather than perfectly informed. A general definition of better information is provided and the first order approximation developed in section 1.3 is then extended.

3.2 Marshallian Analysis

Beginning with the simplest possible case suppost the utility associated with the consumption bundle x can be expressed as:

$$u(x_1, x_2, \ldots, x_n, s) = u_1(x_1; s) + y \tag{3.5}$$

where $y = \sum_2^n p_i x_i$ is expenditure on other goods. Suppose further that $S = \{1,2\}$, that is, s takes on two possible values with probabilities π_1 and π_2. Then expected utility:

$$U = \pi_1 u_1(x_1;1) + \pi_2 u_1(x_1;2) \tag{3.6}$$

The consumer faces a budget constraint:

$$p_1 x_1 + y = I$$

Since we are only dealing with uncertainty about the value of a single commodity we drop subscripts on x_1, p_1, and $u_1(x_1;s)$. Substituting for y in (3.5) we have:

$$U = \{\pi_1 u(x;1) + \pi_2 u(x,2)\} - px + M \tag{3.7}$$

Then the consumer chooses $x^o(p)$ to maximize (3.7).

At an interior option we therefore have:

$$\pi_1 \frac{\partial u}{\partial x}(x;1) + \pi_2 \frac{\partial u}{\partial x}(x;2) = p \tag{3.8}$$

Interpreting this in Marshallian terms, the function $p^o(x)$ defined by (3.8) is the price that would generate a demand of x.

Compare this with decisionmaking when the state of the world is known prior to trading:

$$u_s = u(x;s) - px + M - V$$

At an interior option

$$\frac{\partial u}{\partial x}(x,s) = p \tag{3.9}$$

Therefore the function $p^s(x) = \frac{\partial u}{\partial x}(x,s)$ is the perfect information Marshallian demand curves. These are depicted in Figure 1 for s = 1 and s = 2. Note that the incomplete information demand curve:

$$p^o(x) = \Sigma \pi_s p^s(s)$$

is simply a probability weighted average of the perfect information demand curves. With full information the consumer chooses either x^1 or x^2 at the prive p. With imperfect information the consumer chooses x^o where from (3.8)

$$p^o(x^o) = \Sigma \pi_s p^s(x^o) = p$$

In the latter case expected utility is, from (3.7).

$$U^o = \sum_s \pi_s (u(x^o;s) - px^o) + M$$

$$= \sum_s \pi_s \left(\int_0^{x^o} \frac{\partial u}{\partial x}(q;s)dq - px^o\right) + M$$

$$= \sum_s \pi_s \int_0^{x^o} (p^s(q) - p)dq + M$$

If the true state is known to be s utility is:

$$\int_0^{x^s} (p^s(q) - p)dq + M - V$$

Thus the expected utility with perfect information prior to trading is:

$$U^* = \sum_s \pi_s \int_0^{x^s} (p^s(q) - p)dq + M - V$$

Choosing V* so that U^o and U* are equal we have finally

$$V^* = \sum_s \pi_s \int_{x^o}^{x^s} (p^s(q) - p)dq \qquad (3.10)$$

For the two state case depicted in Figure 3.1, this can be rewritten as:

$$V^* = \pi_1 \int_{x^o}^{x^1} (p'(q) - p)dq + \pi_2 \int_{x^2}^{x^o} (p - p^2(q))dq$$

$$= \pi_1 (\text{AREA ABC}) + \pi_2 (\text{AREA ADE})$$

The value of perfect information is then equal to the expected net increase in consumer surplus.

Returning to the S state case, suppose we approximate the demand curves $p^s(x)$ by <u>parallel</u> linear demand curves of shape

$$\frac{dp^o}{dx}(x^o) = \sum \pi_s \frac{dp^s}{dx}(x^o)$$

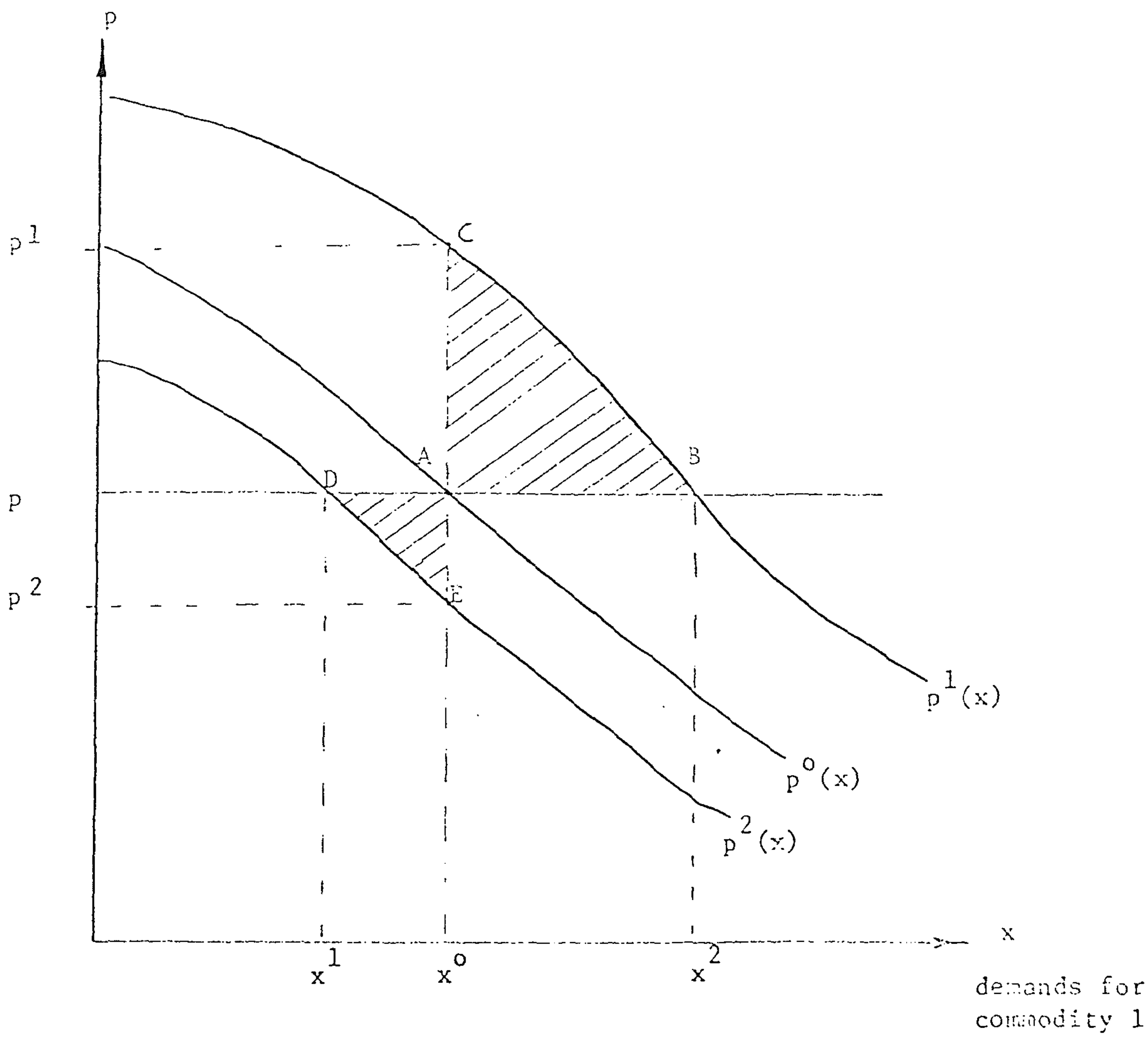
Substituting into (3.10) we then have

$$V^* \simeq \frac{\frac{1}{2}\sum \pi_s [p^s(x^o) - p^o(x^o)]^2}{\left|\frac{dp^o}{dx}\right|} \qquad (3.11)$$

$$= \text{var}[p^s(x^o)]/2\left|\frac{dp^o(x^o)}{dx}\right|$$

Figure 3.1

The Value of Information



$var\{p^s(x^o)\}$ is the variance of full information demand prices for the quality of x pruchased with imperfect information. $\frac{dp^o(x^o)}{dx}$ is the steepness of the incomplete information, inverse demand curve. The value of information is therefore an increasing function of the dispersion of demand prices and of the price sensitivity of demand.

3.3 Logarithmic Utility Functions

We now begin the process of relaxing the strong assumption of constant marginal utility. First we consider the issues for the special case in which the utility function takes on the simple form:

$$U = \sum_{i=1}^{n} \Theta_i \ln\beta_i x_i, \qquad \Theta_i, \ \beta_i \geq 0$$

where $\Theta = \Theta(s)$ and $\beta = \beta(s)$

In the absence of further information about the true state the consumer chooses a consumption bundle x^o yielding the solution of:

$$\text{Max}\{\underset{s}{E}u(x;s) | p'x \leq M\}$$

Note first that we can rewrite U as

$$U = \sum_i \Theta_i \ln\beta_i + \sum_i \Theta_i \ln x_i$$

Therefore x^o is the solution of

$$\text{Max}\{\underset{s}{E}\Sigma\Theta_i \ln x_i | p'x \leq M\}$$

It follows that information leading to a change in beliefs about the vector β but not α has no effect upon the optimal consumption bundle. In particular suppose the only uncertain parameter is β_1. For example a consumer might be uncertain about the quality per unit of a particular commodity. Then for the logarithmic case information about the true value of β has no effect upon the optimal consumption bundle x^o. Moreover the knowledge that β_1 will be known prior to the time of purchase has no effect upon the ex ante utility level. That is, the value of perfect information about β_1 is zero.

To generate a model in which information changes actions we therefore focus upon cases in which the vector $\Theta = (\Theta_1,\ldots,\Theta_n)$ is uncertain. Without further loss of generality we may set $\beta = (1,1,\ldots,1)$.

Consider the case in which

$$\Theta_1 = s$$

$$\Theta_i = (1 - s)\gamma_i \qquad i = 2,\ldots,n$$

$$\text{where } \sum_2^n \gamma_i = 1$$

Such a consumer is uncertain about his marginal valuation of commodity 1 relative to all other commodities but always spends his income on commodities 2,...,n in the same proportion. Given constant prices we may apply Hick's aggregation theorem and write the objective as

$$\underset{s}{\text{Max}}\{E(s\ln x_1 + (1 - s)\ln y)|p_1x_1 + y = M\} \tag{3.12}$$

In the absence of further information about the true state this problem reduces to the certainty equivalent problem:

$$\underset{x_1,y}{\text{Max}}\ \{\bar{s}\ln x_1 + (1 - \bar{s})\ln y\,|p_1x_1 + y \leq M\} \tag{3.13}$$

Solving we have:

$$U^o = \bar{s}\ln\bar{s} + (1 - \bar{s})\ln(1 - \bar{s}) - \bar{s}\ln p_1 + \ln M \tag{3.14}$$

Having paid V for perfect information about the true state the consumer chooses x(s) to yield the solution of:

$$\underset{x_1,y}{\text{Max}}\{s\ln x_1 + (1 - s)\ln y\,|p_1x_1 + y \leq M - V\}$$

Since this problem has exactly the form of problem (3.13) the solution u(s) takes the form of (3.14). We have

$$u(s) = s\ln s + (1 - s)\ln(1 - s) - s\ln p_1 + \ln(M - V)$$

Then the expected utility with full information prior to purchase is:

$$U^* = \underset{s}{E}\{s\ln s + (1 - s)\} - \bar{s}\ln p_1 + \ln(M - V) \tag{3.15}$$

The value of information V^* is then the level of V such that U^o and U^* are equal. Equating (3.14) and (3.15) and rearranging we have:

$$-\ln(1 - \frac{V^*}{M}) = \underset{s}{E}[s\ln s + (1 - s)\ln(1 - s)] - [\bar{s}\ln\bar{s} + (1 - \bar{s})\ln(1 - \bar{s})] \tag{3.16}$$

The first bracketed term is a strictly concave function and the second term is the value of this function at $\bar{s}$, the mean level of s. Then by Jensen's inequality this expression is necessarily positive. Expanding both sides using Taylor's approximation we also have,

$$\frac{V^*}{M} \approx \frac{1}{2}[\frac{1}{s} + \frac{1}{(1 - s)}]\text{var}(s) \tag{3.17}$$

$$= \text{var}(s)/2(1 - \bar{s})\bar{s}$$

It is interesting to compare this with the 'consumer surplus' estimate of the previous section. For the logarithmic utility function:

$$p^s(x) = sM/x$$

Substituting into (3.13) the Marshallian approximation can be written as

$$M\text{var}(s)2\bar{s}$$

Comparing this with (3.17) it follows that the Marshallian estimate of the value of perfect information is biassed downwards by a factor of $(1 - \bar{s})$. The two estimates differ because in the logarithmic case a change in s changes not only the demand curves for x_1 but also the damand for other goods y. When the triangles corresponding to those in Figure 1 are computed for both x_1 and y and the average areas are added together the resulting estimate of V* is indeed (3.17). All this suggests that the average area calculation is capable of further generalization. In Section 3.4 we shall see that this is indeed the case.

We conclude this section with a comparison of the exact value of information given by equation (3.16), with the approximation given by equation (3.17). Suppose s takes on two values $\bar{s} + \varepsilon$ and $\bar{s} - \varepsilon$ with equal probability.

Let

$$-\ln(1 - \frac{V^*}{M}) = A$$

Then

$$V^* = M(1 - e^{-A})$$

where from (3.16).

$$A = \frac{1}{2}[(\bar{s}+\varepsilon)\ln(1+\frac{\varepsilon}{\bar{s}})+(\bar{s}-\varepsilon)\ln(1-\frac{\varepsilon}{s})+(1-\bar{s}-\varepsilon)\ln(1-\frac{\varepsilon}{1-\bar{s}})+(1-\bar{s}-\varepsilon)\ln(1+\frac{\varepsilon}{1-\bar{s}})]$$

Also from (3.17) the approximation to the value of information can be expressed as:

$$V_a^* = \frac{M}{2}\varepsilon^2(\frac{1}{\bar{s}} + \frac{1}{1 - \bar{s}}).$$

Computational results are summarized in the following tables. Note that $V^*(\bar{s}) = V^*(1-\bar{s})$ and $V_a^* = V_a^*(1-\bar{s})$. Therefore the value of information for $\bar{s}$ = .7, .9 .99 can also be obtained from the two tables.

Comparison of these tables indicates that the approximation is remarkably good over the whole range of feasible values of $\bar{s}$. For example the mean difference between the ten computed values of V_a^* and V* expressed as a percentage of V*, is less than 6.5%. This is reason for having some confidence that the results developed in the next sections tield reasonably good approximations of V*.

3.4 General Utility Functions

We now consider the value of perfect information for any utility function u(x;s) which is twice differentiable in x and s and strictly quasi-concave in x. In contrast to the above discussion we allow not only x but also s to be a vector.

Suppose first that perfect information is provided at no cost. Then the consumer chooses x(p;s) yielding the solution of:

$$u(s) = \text{Max}\{u(x,s)\,|\,p'x \leq M\} \quad (3.18)$$

The expected utility thereby achieved is:

$$u^*(M) = \underset{s}{E}u(x(p,s);s)$$

Without the information the consumer chooses x^o to achieve an expected utility of:

$$u^o(M) = \underset{x}{\text{Max}}\{Eu(x,s)\,|\,p'x \leq M\}$$

Table 3.1

The Value of Perfect Information as a Percentage of Income

ε \ $\bar{s}$	.01	.10	.30	.50
.01	.696	.056	.024	.020
.10		7.215	2.387	1.994
.30			23.994	17.532
.50				50.000

Table 3.2

Approximation of the Value of Perfect Information
as a Percentage of Income

ε \ $\bar{s}$	.01	.10	.30	.50
.01	.505	.056	.024	.020
.10		5.556	2.417	2.000
.30			21.750	18.000
.50				50.000

Let s^o be that value of s so that:

$$x^o = x(p,s^o)$$

Then the increase in utility associated with having perfect information is:

$$u^*(M) - u^o(M) = \underset{s}{E}[u(x(p,s);s) - u(x(p,s^o);s)]$$

Expanding the right hand side according to Taylor's approximation we have

$$u^*(M) - u^o(M) = \underset{s}{E}[(x-x^o)'u_x + \frac{1}{2}(x-x^o)'u_{xx}(x-x^o) + (x-x^o)'u_{xs}(s-s^o)+\ldots] \quad (3.19)$$

Since x(p,s) is the solution of (18) it must satisfy the Kuhn-Tucker necessary conditions for the following Lagrangian:

$$L(x;\lambda;s)= u(x;s) + \lambda(m-p'x)$$

Assuming that x(p,s) is an interior solution we have:

$$L_x = u_x(x(p,s);s) - \lambda p=0 \quad (3.20)$$

Then the first term inside the bracket of expression (3.19) reduces to:

$$(x-x^o)'\lambda p = \lambda p'(x-x^o) = \lambda(p'x-p'x^o) = 0$$

Moreover, differentiating (3.20) with respect to both s and p we have:

$$u_{xx}x_p = \lambda I + p\lambda_p' \quad (3.21)$$

and

$$u_{xx}x_s + u_{xs} = p\lambda_s' \quad (3.22)$$

Linearizing the demand curves x(p;s) we have:

$$(x-x^o) \approx x_s(s-s^o) \quad (3.23)$$

Prior to the receipt of information x(p,s) is a random variable. Then actual demand $\tilde{x}$, can be thought of as a random drawing from the set $X = \{x|x = x(p,s);s \in S\}$. The Marshallian demand price vector associated with consumption vector x^o is therefore:

$$\tilde{p} = \{p:x(\tilde{p},s) = x^o\}$$

Then

$$\tilde{x} - x^o \approx x_p(\tilde{p}-p) \quad (3.24)$$

Utilizing (3.22) we can rewrite the third term in the bracket of (3.19) as follows:

$$(x-x^o)'u_{xs}(s-s^o) = (x-x^o)'(p\lambda_s' - u_{xx}x_s)(s-s^o)$$

$$= (x-x^o)'p\lambda_s'(s-s^o) - (x-x^o)'u_{xx}x_s(s-s^o)$$

The first term on the right hand side is zero since $p'x = p'x^o$. Then using the linear approximation (3.23) we have:

$$(x-x^o)'u_{xs}(s-s^o) \approx -(x-x^o)u_{xx}(x-x^o)$$

The increase in utility associated with having perfect information can therefore be approximated as follows:

$$u^*(M) - u^o(M) \simeq \frac{1}{2}\underset{s}{E}(\tilde{x}-x^o)'u_{xs}(s-s^o)$$

$$\simeq -\frac{1}{2}E(\tilde{x}-x^o)'u_{xx}(\tilde{x}-x^o) \qquad (3.25)$$

Substituting for u_{xx} and $\tilde{x}-x^o$ from (21) and (24) we have:

$$u^*(M) - u^o(M) \simeq -\frac{1}{2}E(x-x^o)'(\lambda I+p\lambda_p')(\tilde{p}-p)$$

$$= -\frac{1}{2}E\{(\tilde{p}-p)'(\tilde{x}-x^o)\lambda\}$$

From the first order conditions we have:

$$\frac{\partial u^*(M)}{\partial M} = \underset{s}{E}\lambda(s)$$

Therefore, ignoring the impact of variation across states in the marginal utility of income we have:

$$u^*(M) - u^o(M) = -\frac{1}{2}\frac{\partial u^*}{\partial M}E\{(\tilde{p}-p)'x_p(\tilde{p}-p)\} \qquad (3.26)$$

For the final step we note that the value of information is that level V* such that:

$$u^*(M-V^*) = u^o(M).$$

Taking first order approximation about V* = 0 we have:

$$u^*(M - V^*) \simeq u^*(M) - \frac{\partial u^*(M)}{\partial M}V^* \qquad (3.27)$$

Comparing (3.26) and (3.27) it follows that:

$$V^* \simeq -\frac{1}{2}E(\tilde{p}-p)'x_p(\tilde{p}-p) \qquad (3.28)$$

Suppose only the demand price of commodity 1 varies with s. Then:

$$V^* \simeq \frac{1}{2}\left|\frac{\partial x_1}{\partial p}\right|\text{var}(\tilde{p}_1-p_1)$$

Comparing this with expression (11) it follows that our approximation does correspond to that obtained in Section 3.2.

Similarly, for the logarithmic utility functions it is a straightforward exercise to show the approximation given (3.28) reduces to the expression obtained in Section 3.3.

3.5 The Value of Imperfect Information

The preceding sections were concerned with valuing information which eliminated all uncertainty about the effects of consuming various goods. V* represented what the consumer would pay for perfect information about s. But it is seldom feasible for research to eliminate all uncertainty about the characteristics of goods. Realistically, investigation only narrows the range in which the true characteristics lie, decreasing but not eliminating the dispersion of the consumer's probability distribution over s. In this section we ask how much a consumer would be willing to pay for such imperfect information.

The outcome of the research the consumer commissions, or <u>message</u> he receives, will be denoted by $\alpha \in A$ where A is the set of possible results. Before the research is conducted α is a random variable in the mind of the consumer. Its relation to the uncertain state of the world is embodied in a subjective joint probability distribution function $F(\alpha,s)$ over $A \times S$; $F(s)$, $F(\alpha)$, $F(s|\alpha)$ denote the associated marginal and conditional probability distributions. This pair $[A, F(\alpha,s)]$ is the <u>information structure</u> whose value we wish to determine.

If the information is provided at no cost, and if only s not the message itself affects his ultimate welfare, then upon receiving α the consumer chooses $x(p,\alpha) \equiv x$ to obtain conditional level of expected utility

$$E_{s|\alpha}\, u(x;s) = \underset{x}{\text{Max}}\{E_{s|\alpha}u(x;s) \mid p'\,x \leq M\} \tag{3.29}$$

Prior to the receipt of α, $\tilde{x}$ is a random variable, given α it is no longer random even though s may still be unknown. The anticipated level of expected utility prior to receipt of the message, depending both on the information structure and income, is:

$$u^*(M;A) = E_{s,\alpha}\, u(\tilde{x};s) = E_\alpha E_{s|\alpha} u(\tilde{x};s). \tag{3.30}$$

As before, the consumer chooses x^o without the information to achieve an expected utility of:

$$u^o(M) = \underset{x}{\text{Max}}\ \{E_s u(x;s) \mid p'x \leq M\}$$

and the increase in expected utility associated with having the information structure is:

$$u^*(M;A) - u^o(M) = E_\alpha E_{s|\alpha}\, [u(\tilde{x};s) - u(x^o;s)]. \tag{3.31}$$

Expanding the inner expectation of the right hand side in a Taylor series in x around $\tilde{x}$ yields:

$$E_{s|\alpha}\, [u(\tilde{x};s) - u(x^o;s)] = -E_{s|\alpha}\, [u(x^o;s) - u(\tilde{x};s)]$$
$$= -E_{s|\alpha}\, [\ (x^o - \tilde{x})'u_x + 1/2\ (x^o - \tilde{x})'u_{xx}\ (x^o - \tilde{x}) + \ldots] \tag{3.32}$$

Recalling that $\tilde{x}$ was the solution to (29), and forming the Langrangian

$$L(x,\lambda;a) = E_{s|\alpha}u(x;s) + \lambda(M-p'x),$$

x must satisfy the first order condition:

$$L_x = E_{s|\alpha}\, u_x(\tilde{x};s) - \tilde{\lambda}p = 0. \tag{3.33}$$

The scalar λ denotes the expected marginal utility of income conditional on research outcome α being received. Differentiating

with respect to p provides the additional relation (3.33)

$$[E_{s|\alpha} u_{xx}]\tilde{x}_p = \tilde{\lambda} I + p\tilde{\lambda}_p'. \tag{3.34}$$

Note that $\tilde{\lambda}$, x_p, $\tilde{x}$, $\tilde{\lambda}p$ are non-random once α is revealed. Substituting (3.33) into the first component of the right hand side of (3.32) tells us that:

$$E_{s|\alpha} (x^o-\tilde{x})'u_x = (x^o-\tilde{x})'E_{s|\alpha}u_x = (x^o-\tilde{x})'p\tilde{\lambda} = 0$$

since $x^{o'} p - \tilde{x}'p = M$ from the budget constraints.

Hence (3.31) is approximated by:

$$u^*(M;A) - u^o(M) \simeq E_\alpha E_{s|\alpha} [-1/2 \ (x^o-\tilde{x})'u_{xx}(x^o-\tilde{x})] \tag{3.35}$$

$$= -1/2 \ E_\alpha (x^o-\tilde{x})' [E_{s|\alpha}u_{xx}](x^o-\tilde{x})$$

Now define the Marshallian demand price vector $\tilde{p}$ associated with the consumption vector x^o conditional on message α being received as:

$$\tilde{p} = \{\tilde{p}: \ x(\tilde{p},\alpha) = x^o\}.$$

Linearly approximating the demand function for given α around p gives

$$(x^o-\tilde{x}) \approx \tilde{x}_p (\tilde{p}-p). \tag{3.36}$$

Substituting (3.36) into the right hand side of (3.35) yields:

$$-1/2 \ E_\alpha(x^o-\tilde{x})' [E_{s|\alpha}u_{xx}]\tilde{x}_p(\tilde{p}-p)$$

which can be written utilizing relation (34) as:

$$-1/2 \ E_\alpha (x^o-\tilde{x})' [\tilde{\lambda} I + p\tilde{\lambda}_p'] (\tilde{p}-p).$$

The $(x^o-\tilde{x})'p\tilde{\lambda}_p'(\tilde{p}-p)$ portion of their expression vanishes since $(x^o-\tilde{x})'p = p'x^o - p'\tilde{x} = M-M = 0$ from the budget constraints. Using (3.36) again on the remaining portion of the expression results in:

$$u^*(M;A)-u^o(M) \simeq -1/2 \ E_\alpha\tilde{\lambda}(\tilde{p}-p)'\tilde{x}_p(\tilde{p}-p). \tag{3.37}$$

Prior to receipt of the message the expected marginal utility of income is

$$\frac{\partial u^*(M;A)}{\partial M} = E_\alpha\tilde{\lambda}.$$

If the effect of messages on the slopes $\tilde{x}_p$ of the demand curves is negligible, and if we ignore any correlation between $\tilde{\lambda}$ and the remaining quadratic form in (3.37), then the expected gain in utility may be written almost precisely as in (3.26):

$$u^*(M;A)-u^o(M) = -1/2 \ \frac{\partial u^*}{\partial M} \ E_\alpha[(\tilde{p}-p)'x_p(\tilde{p}-p)]. \tag{3.38}$$

Analagously defining the value of the information structure as V^*_A for which:

$$u^*(M-V^*_A;A) = u^o(M),$$

one obtains a first order approximation to V^* of

$$V^* \approx -1/2\ E_\alpha(\tilde{p}-p)' x_p(\tilde{p}-p). \qquad (3.39)$$

Although it is an approximation, (3.39) provides a consistent estimate of the value of improving a consumer's estimate of s over a wide range of information structures. For example, if the research will provide perfect information, as when A coincides with S and $\alpha = s$, then (3.39) is identical to (3.28). If the research outcome in fact sheds no light on s, so that $x(p,\alpha) = x^o$ for all outcomes, then $\tilde{p} = p$ for all α and (3.39) indicates $V^*_A = 0$. More importantly, (3.39) makes it clear that research whose results would not change consumers' behaviour is valueless, even though it may significantly improve estimates of s in a purely statistical sense.

One final check on the plausibility of (3.39) as an approximate indicator of the value of imperfect information about the consequences of consuming various goods is to verify that information never has a negative value. Such a result must follow if the outcome of the research itself, as opposed to the true characteristics of goods s, has no direct effect on the consumer's utility. That (3.39) has this property can be demonstrated as follows. Assuming as we have that the slopes of the uncompensated demand curves as indicated by $x_p \equiv [\partial x_i/\partial p_j]$ are unaffected by the outcome of the research α, these slopes will be identical to those of the demand curves if no information was to be received. Using the Slutsky relation of conventional demand theory

$$\partial x_i/\partial p_j = \partial x_i^c/\partial p_j - x_j \partial x_i/\partial M$$

in which $x_i^c/\partial p_j$ is the slope of the income-compensated demand curve for good i with respect to the price of good j, we can express x_p as $x_p^c - x_m x'$ in which $x \equiv x^o$ is the consumption point at which the derivatives are evaluated. Inserting this expression for x_p into (3.39) gives us the alternate form

$$V^* \approx -1/2\ E_\alpha(\tilde{p}-p)'[x_p^c - x_M x^{o'}]\ (\tilde{p}-p).$$

But since $p'x^o = \tilde{p}'x^o = M$ from the budget constraints and definition of $\tilde{p}$, the second component of the inner bracketed expression becomes 0 when multiplied by $(\tilde{p}-p)$. Thus (3.39) can be alternately written as

$$V^* \approx -1/2\ E_\alpha(\tilde{p}-p)' x_p^c(\tilde{p}-p). \qquad (3.40)$$

The Stutsky matrix x_p^c is known to be symmetric and negative semidefinite. Hence the expectation of the quadratic form in (3.40) is non-positive and V* must be non-negative for all information structures.

3.6 Information and Price Adjustment

As analyzed in Section 3.1 of this report, information is valuable to the extent that consumption plans change with the message received. Loosely, the greater the optimal adjustment to the different messages the more an individual is willing to pay ex-ante for the provision of the information. Ignored, however, is the possibility that the receipt of information will have significant price effects.

Implicity in such a formulation is the assumption that prices are largely determined by cost conditions rather than the intersection of supply and demand curves. While this is a natural first approximation for a variety of applications it is particularly inappropriate for non-produced commodities of uncertain quality. One important case is the adjustment of land prices to reflect differences in air quality in an urban environment. It is this case that we shall focus on in the following sections.

We begin in Section 3.7 by illustrating the implications of price adjustment on the value of information for a simple exchange economy. It is shown that all agents in an economy may be made worse off by the announcement that the true quality of a product will be made known prior to trading. Essentially the anticipation of information introduces an additional distributive risk which reduces each individual's expected utility. It is shown that each agent would prefer to engage in a round of trading prior to the revelation of product quality, thereby insuring himself against an undesirable outcome.

The in Section 3.8 a simple urban model is developed in which a fixed number of individuals must be located in two regions. The equilibrium allocation of individuals is first examined. Simple sufficient conditions for higher income groups to locate in the preferred environment are established.

Surprisingly, it is shown that under non implausible alternative conditions both tails of the income distribution may locate in the preferred environment.

Section 3.9 asks what allocation of land and goods maximize a symmetric social welfare function. Starting with income equally distributed it is shown that optimization in general requires an income transfer from those living in one zone to those in the other. Under the conditions which imply that in equilibrium the rich will locate in the better environment, it is optimal to transfer income to those in the better environment from the remainder of the population! The intuition behind this paradoxical conclusion is then developed.

Finally, Section 3.10 focusses on the implications of conducting research to resolve uncertainty about the nature of the environmental hazard.

3.7 Information About Product Quality with Negative Social Value

Consider a two person economy in which aggregate endowments of two commodities, X and Y, are fixed and equal to unity. Both individuals have utility functions of the form:

$$u(x_i, y_i; \Theta) = (\Theta x_i)^{1/2} + y_i^{1/2} \quad i = 1,2$$

where Θ is a parameter reflecting the 'quality' of the product. Prior to trading Θ is unknown but both individuals believe that with equal probability Θ takes on the values 0 and 1.

Then the expected utility of agent i is:

$$U^\circ(x_i,y_i) = \underset{\Theta}{E}u(x_i,y_i;\Theta) = \frac{1}{2}x_i^{1/2} + y_i^{1/2} \qquad (3.41)$$

Without loss of generality we may set the price of y equal to unity. Then each agent chooses (x_i,y_i) to maximize U° subject to a budget constraint

$$px_i + y_i \leq p\bar{x}_i + \bar{y}_i$$

where $(\bar{x}_i,\bar{y}_i)$ is the agent's endowment.

Since U° is strictly concave the following first order condition yields the global maximum.

$$\frac{\partial U^\circ}{\partial x_i} \bigg/ \frac{\partial U^\circ}{\partial y_i} = \frac{1}{4}x_i^{-1/2} \bigg/ \frac{1}{2}y_i^{-1/2} = \frac{1}{2}\left(\frac{y_i}{x_i}\right)^{1/2} = p$$

Then:

$$\frac{y_i}{x_i} = 4p^2 \qquad i = 1,2 \qquad (3.42)$$

It follows that:

$$1 = \frac{\Sigma y_i}{\Sigma x_i} = 4p^2$$

Thus the equilibrium price of x is 1/2 and from (3.42) $y_i = x_i$, $i = 1,2$. Suppose $(\bar{x}_1,\bar{y}_1) = (1,0)$ and $(\bar{x}_2,\bar{y}_2) = (0,1)$. Then from the budget constraint it is a straightforward matter to show that:

$$(x_1,y_1) = (1/3,1/3) \text{ and } (x_2,y_2) = (2/3,2/3)$$

From (3.41) the expected utility of the agents is given by:

$$U_1^\circ = \frac{3^{1/2}}{2} \qquad U_2^\circ = \left(\frac{3}{2}\right)^{1/2}$$

Next suppose that research is to be conducted which will reveal the true state prior to any trading. If $\Theta = 0$ the endowment of agent 1 is valueless hence there can be no trade ex post. Then:

$$u_1(\Theta=0) = 0 \quad \text{and} \quad u_2(\Theta=0) = 1$$

If $\Theta = 1$ each agent has an ex-post utility function:

$$u_i = x_i^{1/2} + y_i^{1/2}$$

Applying an almost identical argument to that made above, it can be shown that for such preferences the equilibrium price of x is unity and both agents consume half the aggregate endowment. Then:

$$u_1(\Theta=1) = 2^{1/2} = u_2(\Theta=1)$$

Prior to the revelation of the information both agents place an equal probability on the two possible states. Thus expected utility levels with the information are:

$$U_1^* = \frac{1}{2}u_1(\theta=0) + \frac{1}{2}u_1(\theta=1) = \left(\frac{1}{2}\right)^{1/2}$$

and

$$U_2^* = \frac{1}{2}u_2(\theta=0) + \frac{1}{2}u_2(\theta= 1) = \frac{1 + 2^{1/2}}{2}$$

Then $(U_1^*)^2 - (U_2^\circ)^2 = 1/2 - 3/4 < 0$

and $(U_2^*)^2 - (U_2^\circ)^2 = \frac{3 + 2\sqrt{2}}{4} - \frac{6}{4} < 0$

The prospect of information prior to trading therefore creates a distributive risk which reduces the expected utility of every agent!

Each agent would therefore like to insure himself against such risk. It follows that there are potential gains to opening the commodity market prior to the announcement of the true state. Since the future spot price of X relative to Y, $\hat{p}$, is independent of individual endowments it follows from the above analysis that $\hat{p} = 0$ if $\theta = 0$ and $\hat{p} = 1$ if $\theta - 1$, that is:

$$\hat{p}(\theta) = \theta; \ \theta = 0,1$$

If the spot price of X is p, agent i can select bundles (x_i,y_i) satisfying

$$px_i + y_i = p\bar{x}_i + \bar{y}_i \tag{3.43}$$

When the state is announced the agent then makes a second round of exchanges subject to the contraint:

$$\hat{p}(\theta)x_i(\theta) + y_i(\theta) = \hat{p}(\theta)x_i + y_i \qquad \theta = 1,2. \tag{3.44}$$

But if $\theta = 0$ the future spot price $\hat{p}(\theta) = 0$. It follows that there will be no trading after the announcement, that is:

$$(x_1(0),y_1(0)) = (x_i,y_i)$$

if $\theta = 1$ the future spot price, $\hat{p}(\theta) = 1$. Given the symmetry of the indifference curves each agent will trade in such a way as to equalize his spending on the two commodities.

Then $$(x_i(1),y_i(1)) = \left(\frac{x_i + y_i}{2} \ , \ \frac{x_i + y_i}{2}\right)^{1/2}$$

Expected utility of agent i is therefore

$$U(x_i,y_i) = \frac{1}{2}y_i^{1/2} + \left(\frac{x_i + y_i}{2}\right)^{1/2}$$

With a spot price of p, agent i chooses x_i and y_i to maximize $U(x_i,y_i)$ subject to his budget constraint(3.43). The first order condition for expected utility maximization is therefore:

$$\frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial y_i}} = \frac{\left(\frac{x_i}{y_i}+1\right)^{-1/2}}{2^{-1/2}+\left(\frac{x_i}{y_i}+1\right)^{-1/2}} \qquad (3.45)$$

It follows that $\frac{x_i}{y_i}$ is the same for both agents, hence equal to $\frac{\Sigma x_i}{\Sigma y_i} = 1$. Then from (3.45) $p = 1/2$. From the budget constraint (3.43) it follows that

$$(x_1,y_1) = (1/3,1/3) \text{ and } (x_2,y_2) = (2/3,2/3)$$

But this is exactly the consumption achieved by each agent in the absence of the information. Therefore the prior trading just eliminates the undesired utility risk, and the expected value of the information is zero.[1/]

A central feature of this and the earlier results is that agents correctly anticipate the price implications of the state revealing message. If consumers are unaware of these implications the analysis of section 1 applies. Each will therefore place a positive value on the information.

Of course it is a long leap from this simple example to a general proposition. However it does seem reasonable that there will, in general, be a tendency for price adjustments to offset the anticipated gains associated with better information. Thus except in cases where there are solid ground for arguing that prices are cost determined, the expressions for the value of information developed in Section 3.1 seem likely to overstate true value.

3.8 Urban Location and Land Values with Environmental Hazards

One very important case in which price adjustments to changes in information are central, is that of urban location. To illustrate the issues we shall consider a city which consists of two zones.

The utility of any individual living in the second zone is a concave function $U(x,y)$ of the area of his residence x and expenditure on other commodities y. If provided the same bundle of commodities in the environmentally affected first zone his utility drops to $U(x,y)-s$. That is, s is the loss in utility associated with living in the "smoggy" first zone.

Suppose each individual purchases land from some outside landowner and all have identical incomes.[2/] Let P_i be the price of a unit of land in zone i. For those locating in the second zone the utility level achieved is:

$$V(p_2,I) = \underset{x,y}{\text{Max}}\{U(x,y)\,|\,P_2x + y = I\} \qquad (3.46)$$

Similarly for those locating in the first zone the utility level achieved is:

$$V(p_1,I) - s = \underset{x,y}{\mathrm{Max}}\{U(x,y)\,|\,p_1x + y = I\} \quad -s. \tag{3.47}$$

In the absence of constraints on land purchases, the value of land in the "smoggy" zone must fall until utility is equated in the two zones. This is depicted in Figure 3.2.

At the level of an individual consumer, one measure of the cost of the smog is the extra income H that a person living in the second zone would have to be given in order to make him willing to move at constant prices. In formal terms this is the Hicksian compensation required to maintain the utility level of an individual in the smoggy zone at the higher land value P_2, that is:

$$V(P_2,\ I + H) = V(P_1,\ I) = V(P_2,\ I) + s \tag{3.48}$$

This is also depicted in Figure 3.2.

With this background we can now ask which individuals live where, if incomes are not equally distributed. For expositional ease we shall restrict our attention to utility functions that are homothetic. Suppose that income is distributed continuously. Then for some income level I° individuals will be indifferent between living in the two zones. We therefore have:

$$V(P_2,\ I^\circ) = V(P_1,\ I^\circ) - s$$

An individual with income $I > I^\circ$ locates in the smog free zone if and only if:

$$V(P_2,\ I) > V(P_1,\ I) - s$$

Consider Figure 3.2. Those with incomes of I° are indifferent between C_1 and C_2 and hence between C_1' and C_2. Then:

$$V(P_2,\ I^\circ) = V(P_2,\ I^\circ + H^\circ) - s. \tag{3.49}$$

Moreover given our assumption that those with incomes of I locate in the smog free zone, they must prefer D_2 to D_1, and hence prefer D_2 to D_1'. Then:

$$V(P_2,\ I) > V(P_2,\ I + H) - s \tag{3.50}$$

Combining (3.49) and (3.50) the higher income group prefer zone 2 if and only if:

$$V(P_2,\ I + H) - V(P_2,\ I) < V(P_2,\ I^\circ + H^\circ) - V(P_2,\ I^\circ) \tag{3.51}$$

For the special case of homothetic preferences depicted in Figure 3.3 we also have:

$$\frac{OC_1'}{OC_1} = \frac{OD_1'}{OD_1}$$

Moreover, $$\frac{OD_1}{OC_1} = \frac{I}{I_\circ} \text{ and } \frac{OD_1'}{OC_1'} = \frac{I + H}{I_\circ + H_\circ}$$

Figure 3.2

Urban Location and Land Values

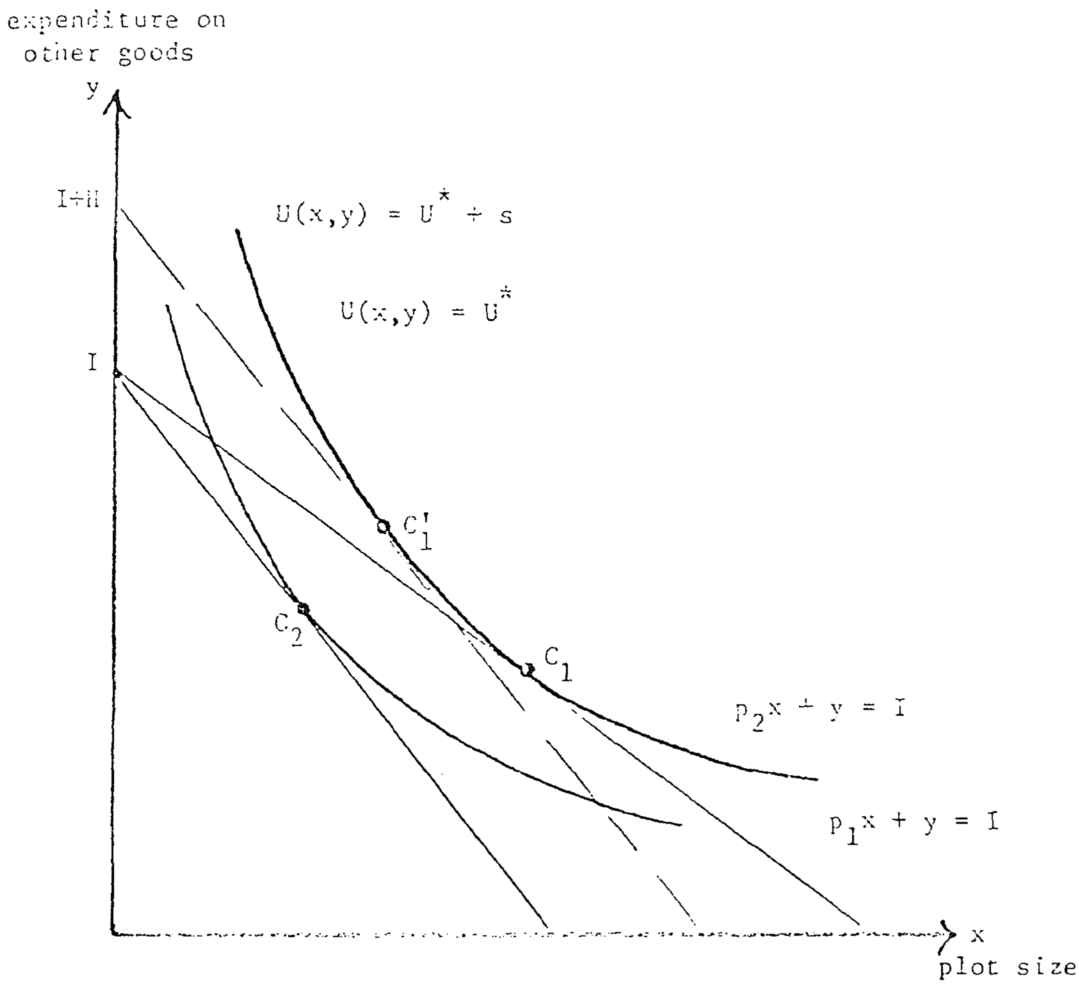
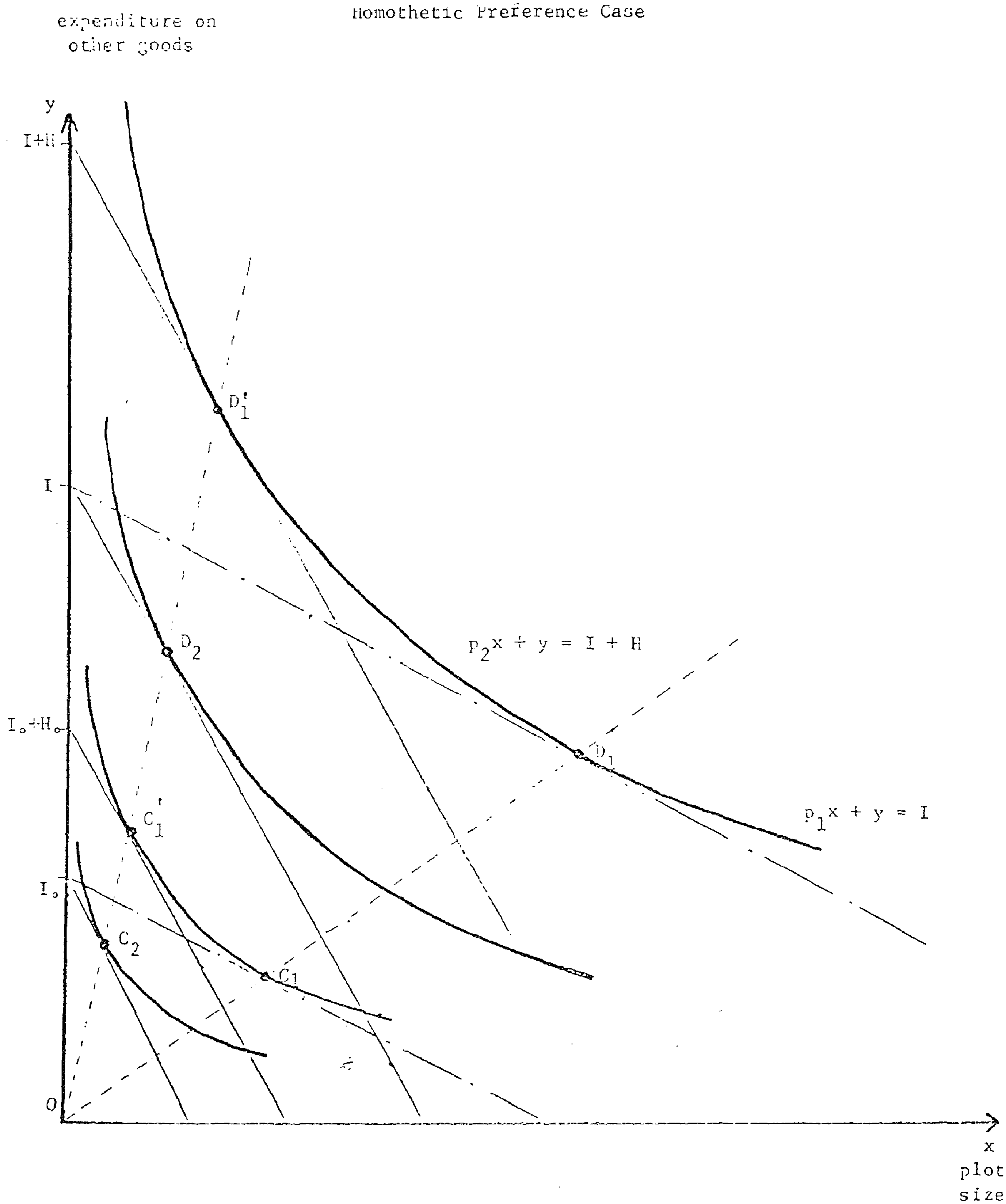


Figure 3.3

Homothetic Preference Case



It follows immediately that:

$$\frac{H_\circ}{I_\circ} = \frac{H}{I}$$

We may therefore rewrite the necessary and sufficient condition (3.51) as

$$V(P_2,(\frac{I_\circ + H_\circ}{I_\circ})I) - V(P_1, I) < V(P_2,I_\circ + H_\circ) - V(P_2,I_\circ) \qquad (3.51)'$$

Note that the left and right hand sides of (3.51)' are equal for $I = I_\circ$. Then a sufficient condition for all those with higher incomes to prefer zone 2 is that the left hand side of (3.51)' be decreasing in I, that is:

$$\frac{1}{I_\circ}[(I_\circ + H_\circ)V_I(P_2,I_\circ + H_\circ) - I_\circ V_I(P_2,I_\circ] < 0 \qquad (3.52)$$

In turn a sufficient condition for inequality (3.52) to hold for the required $H_\circ$ is that it should hold for any $H_\circ$. But this is the case if:

$$\frac{\partial}{\partial I}[IV_I(P_2,I)] < 0$$

that is:

$$\frac{-IV_{II}}{V_I} > 1 \qquad (3.53)$$

Thus with homothetic preferences a sufficient condition for the higher income groups to prefer the smog free zone is that the income elasticity of the marginal utility of income be greater than unity. Conversely, if each of the above inequalities is reversed, it follows that with homothetic preferences a sufficient condition for the higher income groups to prefer the smoggy region is that the elasticity of marginal utility be less than unity.

We now note that this elasticity is also the coefficient of relative aversion to income uncertainty. Arrow (1971) has argued that the latter must be in the neighborhood of unity and increasing in income. Accepting this conclusion it follows that there is no clear presumption that income and environmental quality will be positively correlated. Indeed if relative risk aversion is less than unity for low incomes, and rises above unity as income increases it is possible for an equilibrium configuration with high and low income groups sharing the smog-free region and middle income groups in the smoggy region.

Of course this conclusion is very much dependent upon the underlying assumptions. Suppose that instead of entering additively, the environmental affects are multiplicative. That is, with the environment affected by an amount s, utility is:

$$u_1(x,y)u_2(s)$$

where $u_2(0) = 1$ and $u_2'(s) < 0$.

Each consumer chooses x, y and his location to maximize the utility or, equivalently, the logarithm of this utility, that is:

$$\ln u_1(x,y) - \ln u_2(s).$$

Setting $U(x,y) = \ln u_1(x,y)$ the problem becomes equivalent to the one already analysed. Therefore higher income groups will live in the smog free areas if the relative risk aversion of an individual with a utility function $\ln U_1(x,y)$ exceeds unity. Since ln $(\cdot)$ is a strictly concave function, this individual's relative risk aversion exceeds that of an individual with a utility function $U_1(x,y)$. Therefore the sufficient condition is weakened and the presumption that higher income individuals will live in the less environmentally affected area is strengthened.

3.9 Optimal Urban Location

In the previous section we considered some of the positive implications of intra urban environmental differences. It turns out that there are also rather puzzling normative implications, at least if one adopts the usual approach of maximizing a symmetric social welfare function. Suppose that initially all individuals have the same income. Some locate in the smog-free zone and the rest in the smoggy zone. A naive view might be that those living in the smog should be compensated by an income transfer from those in the smog free zone. Not so, an economist would almost certainly respond. If individuals are free to move from one zone to the other, land values will adjust to equalize utilities.

While the response is correct as far as it goes, it does not necessarily follow that the sum of all the utilities, or indeed any symmetric function of each utility, is maximized as a result. For expositional ease we shall consider only the Benthamite welfare function. Let a_i be the are of zone i, n_i the number assigned to this zone, $\bar{n}$ the total population and $\bar{y}$ the total income. We seek to maximize the utility sum:

$$W = \sum_{i=i}^{2} n_i[U(\frac{a_i}{n_i}, y_i) - s_i]$$

subject to the constraints:

$$n_1 + n_2 = \bar{n}; \; n_1y_1 + n_2y_2 = \bar{y}$$

To solve we form a Lagrangian

$$L = W + \lambda(\bar{n} - n_1 - n_2) + \mu(\bar{y} - n_1y_1 - n_2y_2)$$

Necessary conditions for a maximum are therefore,

$$\frac{\partial L}{\partial y_i} = n_i(U_{y_i} - \mu) = 0. \qquad (3.54)$$

and

$$\frac{\partial L}{\partial n_i} = U(x_i, y_i) - s_i - x_iU_{x_i} - \lambda - \mu y_i = 0 \qquad (3.55)$$

where $x_i = a_i/n_i$.

Suppose that the optimal distribution of land and individuals is $(x(s_i), y(s_i)) \quad i = 1,2$

Differentiating the two first order conditions with respect to s we have:

$$\frac{d}{ds}(U_y) = 0$$

and

$$U_x x'(s) + U_y y'(s) - 1 - x'(s)\, U_x - x\frac{d}{ds}(U_x) - \mu\frac{dy}{ds} = 0$$

Substituting for μ from (3.54) this reduces to:

$$\frac{d}{ds}(U_x) = -\frac{1}{x} \tag{3.57}$$

Writing out the derivatives in (3.56) and (3.57) we therefore have,

$$\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix} \begin{bmatrix} x'(s) \\ y'(s) \end{bmatrix} = -\frac{1}{x} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Applying Cramer's rule yields:

$$x'(s) = -\frac{1}{x}\frac{U_{yy}}{|H_u|} \text{ and } y'(s) = -\frac{1}{x}\frac{U_{xy}}{|H_u|} \tag{3.58}$$

where H_u is the Hessian matrix of the function $U(x,y)$. Given the concavity of U the principal minors of H_u must alternate in sign thus $x'(s) > 0$. It follows that the optimal plot size is larger for those located in the smoggy zone.

Furthermore, substituting from (3.58) we also have:

$$\begin{aligned} \frac{dU}{ds} &= x'(s)U_x + y'(s)U_y \\ &= -\frac{1}{x}\frac{(U_x U_{yy} - U_y U_{xy})}{|H_u|} \\ &= -\frac{1}{x}\frac{\begin{vmatrix} U_x & U_{xy} \\ U_y & U_{yy} \end{vmatrix}}{|H_u|} \end{aligned} \tag{3.59}$$

Consider an individual located in zone i facing a land price of P_i, and having an income of I. Given that he is to remain in this zone, he chooses a consumption (x_i, y_i) yielding the solution of:

$$\text{Max}\{U(x_i, y_i) \mid P_i x_i + y_i = I\}$$

Introducing the Lagrangian λ (equal to the marginal utility of income) the following first order conditions must be satisfied:

$$U_x = \lambda p_i$$

$$U_y = \lambda$$

Suppose income I were increased. Differentiating the first order conditions we have:

$$\begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix} \begin{bmatrix} x_i'(I) \\ y_i'(I) \end{bmatrix} = \lambda'(I) \begin{bmatrix} p_i \\ 1 \end{bmatrix} = \frac{\lambda'(I)}{\lambda(I)} \begin{bmatrix} U_x \\ U_y \end{bmatrix}$$

Then applying Cramer's rule:

$$\frac{dx_i}{dI} = \frac{1}{\lambda} \frac{d\lambda}{dI} \frac{\begin{vmatrix} U_x & U_{xy} \\ U_y & U_{yy} \end{vmatrix}}{|H_u|} \tag{3.60}$$

Combining (3.59) and (3.60) we have:

$$\frac{dU}{ds} = \frac{\frac{-1}{x}\frac{dx_i}{dI}}{\frac{1}{\lambda}\frac{d\lambda}{dI}} = \frac{E(x_i, I)}{E(\lambda, I)}$$

The expected utility of an individual residing in zone i is $U(x_i, y_i) - s_i$. Therefore the change in expected utility as the smog level s increases is

$$\frac{dU}{ds} - 1 = \frac{E(x_i, I)}{E(\lambda, I)} - 1 \tag{3.61}$$

Therefore if the right hand side is positive for any price P_i and income level I, it is optimal for those in the smoggy zone to have a higher utility. Conversely, if the right hand side is always negative it is optimal to transfer income to those in the less smoggy zone!

For the special case of homothetic preferences examined in the previous section $E(x_i, I)=1$. Therefore in such cases it is optimal to transfer income to those in the less smoggy zone if and only if the income elasticity of marginal utility exceeds unity. Thus the condition obtained in section 2.2 ensuring that the higher income groups will locate in the less smoggy zone also ensures that for a population with equal incomes, the utility sum is maximized with a transfer of income to those in the less smoggy zone!

Such paradoxical results have already been noted in the urban literature by Mirrlees (1972) Riley (1974) and others, although the usual emphasis has been on the implications of differential transportation costs. Recently Arnott and Riley (1977) have attempted to explain the origin of these results as a production asymmetry. While their analysis does not carry over directly, to this more complicated case the basic issues are the same.

Suppose we begin with incomes equally distributed, as in Figure 3.2. Since land is cheaper in the smoggy zone plot sizes are larger, unless land is a Giffen good. That is, C_1 lies to the right of C_2. Moreover, if land is a normal good C_1' is above and to the right of C_2. Arnott and Riley note that for a normal good the marginal utility of income rises with a Hicks compensated fall in the price of the good. That is, the marginal utility of income rises around the curve from C_1' to C_1. With diminishing marginal utility of income marginal utility falls in moving from C_2 to C_1'. If the latter effect outweighs the former (and this will be the case with a sufficiently high income elasticity of marginal utility) marginal utility is lower at C_1 than at C_2. Maximization of any differentiable symmetric social welfare function therefore requires a transfer of income from those in the low marginal utility, smoggy zone to those in the less smoggy zone.

3.10 Uncertain Environmental Quality and the Prospect of Better Information

In the previous two sections we analysed the implications of environmental quality differences for property values and locational choice. Given the simple formulation of the model, none of the results are changed if s is reinterpreted as the expected utility loss associated with a polluted environment. We now consider the implications for property values of conducting research which would resolve the uncertainty about the hazards of the pollution. For expositional ease we consider the case in which the polluted region is small relative to the unpolluted region. Then to a first approximation land value and hence utility in the latter is unaffected by such information. Continuing with our assumption of a perfectly elastic response to any utility differential, it follows that expected utility in the two regions will be fixed at some level $\bar{U}$. Then prior to any consideration of research resolving uncertainty about the environmental hazard, the consumption bundle in the "rest of the world" C_0 and in the affected region C_1 yield the same expected utility level. This is depicted in Figure 3.4. Now suppose it is announced that research will reveal the true level of s. For simplicity suppose this takes one of two values s_0 (=0) and s_1. If s = 0 the utility level of individuals in regions 1 rises to $\bar{U} + E(s)$. This attracts individuals into the region and the price of land is bid up. Eventually the price of land reaches P_0 and outsiders no longer gain from relocation. Similarly, if $s = s_1$ the utility of those in region 1 is $\bar{U} + E(s) - s_1 < \bar{U}$. Individuals therefore leave until the price of land falls to the point where the utility differential is eliminated. Assuming individuals own their own homes, those remaining in region 1 have ex-post budget constraints:

$$P_1(s)x + y = P_1(s)x_1 + y_1$$

Final consumption is therefore dependent upon the true state s. This is also depicted in Figure 3.4. Note that in both states we have:

$$U(C_1(s)) > U(C_1)$$

In anticipation of the release of the information about s, expected utility in region 1 is therefore:

$$\underset{s}{E}(U(C_1(s)) - s) = \underset{s}{E}U(C_1(s)) - E(s) > U(C_1) - E(s) = \bar{U}$$

Figure 3.4

Uncertainty and Better Information

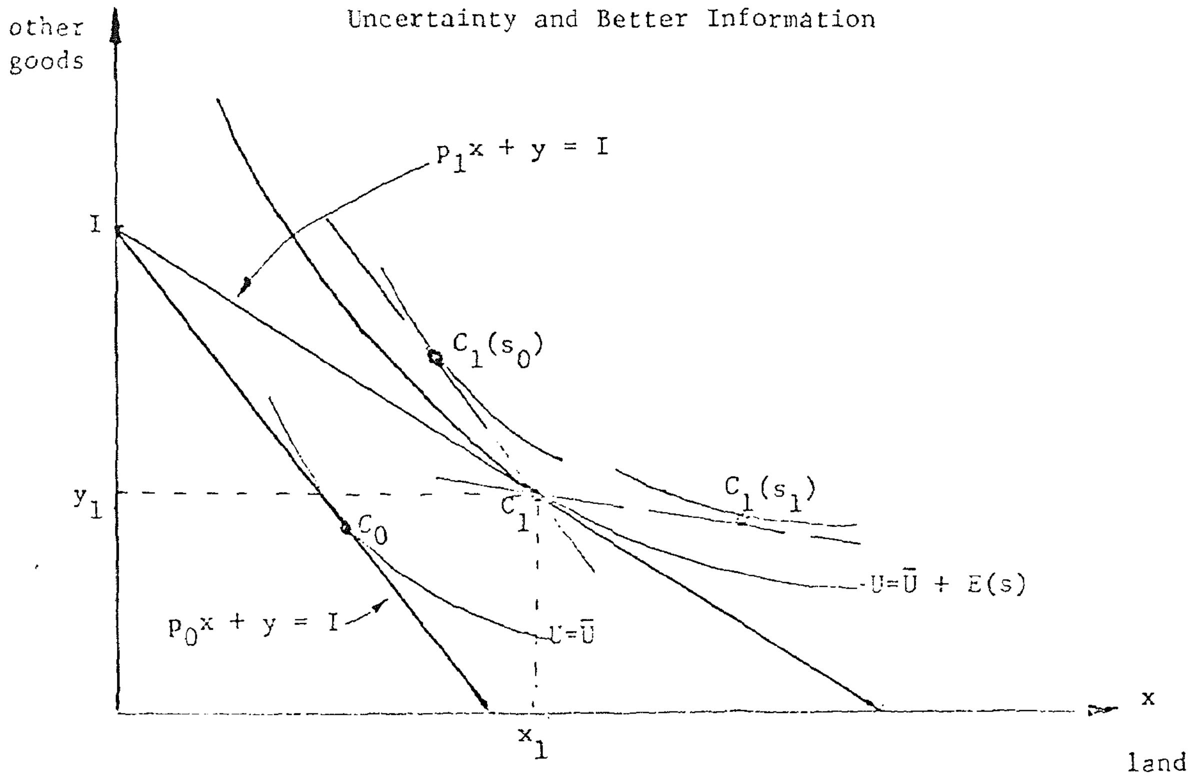
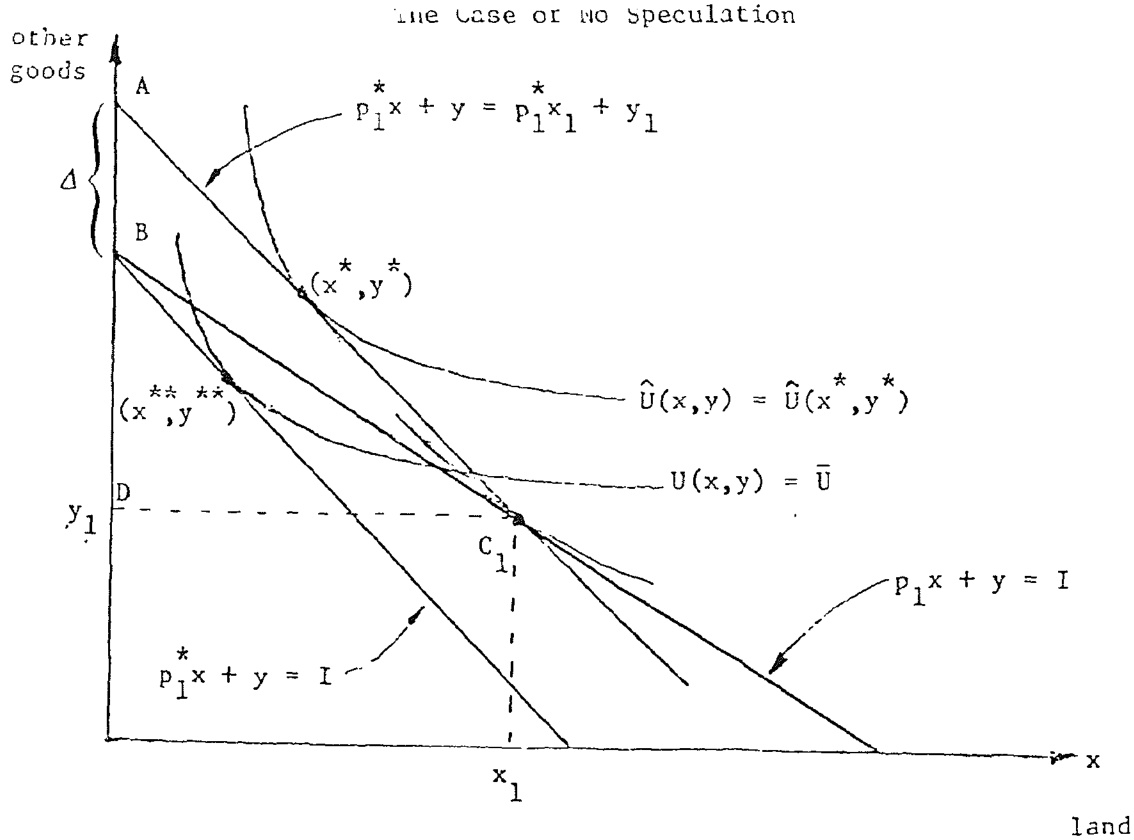


Figure 3.5

The Case of No Speculation



Therefore all homeowners in region 1 are made strictly better off by the announcement of the proposed research. As a result outsiders will wish to relocate in region 1. The value of land is therefore bid up to some level p^* where the expected utility achieved by relocation once again falls to $\bar{U}$.

The budget constrainst of those initially in region 1 and those moving into the region are depicted in Figure 3.5 under the assumption that the price of land jumps too quickly for significant speculative activity.

Suppose the former group chooses a bundle (x^*, Y^*) and the latter (x^{**}, y^{**}). Each group of course anticipates retrading at a later point. Since both face an expected loss due to the environmental hazard of E(s) we can write the utility differential as:

$$\hat{U}(x^*, y^*) - \hat{U}(x^{**}, y^{**})$$

where $\hat{U}(x,y) - \underset{p_1}{E}V(p_1, p_1 x + y)$ is the derived utility function for both groups.

Of course there is no simple relationship between the indifference curves for the derived utility function U(x,y) and the underlying function U(x,y). However it must be the case that those entering the region have the expected utility level $\bar{U}$. That is: $U(x^{**}, y^{**}) = \bar{U}$.

It follows that $\hat{U}(x^*, y^*) - \hat{U}(x^{**}, y^{**})$ is the gain in expected utility for those located initially in region 1. Consider again Figure 3.5. In order for those entering region 1 to achieve as high a utility level as the initial land owners, it would be necessary to increase the income of each from I to I+Δ. Thus Δ is a measure of the dollar valuation of the information. Note that $AD = p_1^* x_1$ and $BD = p_1 x_1$. Therefore the value of information to each individual initially located in region 1 is:

$$\Delta = (P_1^* - P_1) x_1$$

Aggregating over the whole region, the total value of the information is equal to the increase in the value of the land in the region.

Unfortunately it is difficult to visualize how one might make a quantitative prediction of the extent of this revaluation without working back to the underlying preferences. In a later draft we intend to illustrate how this might be done for the Cobb-Douglas case.

3.11 Precautionary Response to the Prospect of Information

Section 3.1 explores the value to an individual of receiving either perfect or partial information about product quality prior to making any consumption decisions. Consumption decisions were binding once made and could not be altered if subsequent information about s arrived. It is generally the case, however, that once an individual (or society) does choose to acquire additional information about some good it takes some time to produce it through experimentation and research. In the meantime current consumption decisions must still be made, although future consumption plans may be appropriately revised upon receipt of the experimental outcomes.

This chapter examines the impact of the prospect of future information on decisions made prior to the receipt of that information. The basic result is that if the possibily harmful effects of consuming a particular good depend on its accumulated consumption over one's lifetime, then the prospect of receiving information about the maximum safe level of consumption reduces current consumption of that good. Moreover the sooner the information is anticipated the larger is the reduction in the optimal current consumption.

Sections 3.12 and 3.13 characterize an agent's response to the prospect of learning in a two period context: the agent must rely on prior beliefs when choosing first period consumption but may receive additional information before choosing second period consumption. Section 3.14 examines the agents' response to variations in the expected time of arrival of the information in a continuous time framework.

3.12 When Learning Prospects Do Not Affect Current Actions

We first examine circumstances in which the prospect of future information about a good has no effect on current consumption decisions. Let x_1 and x_2 denote an individual's consumption in two successive periods of the good with uncertain characteristics s. Expenditures on all other goods in the two periods will be denoted by y_1 and y_2. Ignoring rate of time preference considerations and adopting the constant marginal utility of income assumption of section 1.1, let us assume the agent's "lifetime" utility has the form:

$$U(x_1,x_2,y_1,y_2;s) \equiv u(x_1,x_2;s) + y_1 + y_2.$$

Without loss of generality we may choose the units of x so that its price is 1 and, ignoring interest rate considerations, write the agent's budget constraint as $x_1 + x_2 + y_1 + y_2 \leq M$.

This section examines the case in which lifetime utility is additively separable in the two periods' consumptions. The consumer's objective is thus to maximize the expected value of:

$$U = u_1(x_1;s) + u_2(x_2;s) + y_1 + y_2 \qquad (3.62)$$

subject to $x_1 + x_2 + y_1 + y_2 \leq M$.

If no further information is forthcoming the agent relies on his prior probabilistic beliefs about s to choose x_1 and x_2 maximizing:

$$E_sU \equiv E_su_1(x_1;s) + E_su_2(x_2;s) + M - x_1 - x_2 \qquad (3.63)$$

Assuming concavity and differentiability of u_1 and u_2, the necessary first order conditions for a maximum are:

$$E_s\frac{\partial u_1}{\partial x_1} = 1, \qquad E_s\frac{\partial u_2}{\partial x_2} = 1. \qquad (3.64)$$

The maximizing levels of consumption are denoted x_1^o and x_2^o, and the ex ante level of expected utility is:

$$U^o(M) = \underset{x_1, x_2}{\text{Max}}\ E_s[u_1(x_1;s) + u_2(x_2;s) + M - x_1 - x_2]. \qquad (3.65)$$

If perfect information about s is forthcoming after x_1 is chosen but before x_2 is chosen, then x_2 can be adjusted according to s revealed. The level of expected utility attainable becomes:

$$U^*(M) = \underset{x_1, x_2(s)}{\text{Max}}\ E_s[u_1(x_1;s) + u_2(x_2(s);s) + M - x_1 - x_2(s)].$$

The maximum principle of dynamic programming permits this to be rewritten as:

$$U^*(M) = \underset{x_1}{\text{Max}}\ E_s\ \underset{x_2}{\text{Max}}[u_1(x_1;s) + u_2(x_2;s) + M - x_1 - x_2] \qquad (3.66)$$

$$= \underset{x_1}{\text{Max}}\ E_2\{u_1(x_1;s) + M - x_1 + \underset{x_2}{\text{Max}}\ [u_2;s) - x_2]\}$$

$$= \underset{x_1}{\text{Max}}\ E_s[u_1(x_1;s) + M - x_1] + E_s \underset{x_2}{\text{Max}}[u_2(x_2;s) - x_2].$$

The last two equalities follow from additive structure of the utility function. The first order conditions for an interior maximum are:

$$E_s \frac{\partial u_1}{\partial x_1} = 1, \qquad \frac{\partial u_2}{\partial x_2} = 1 \text{ for all s.}$$

Denoting the optimum consumption levels by x_1^* and $x_2^*(s)$, it follows from a comparison of (3.64) and (3.67) that $x_1^o = x_1^*$. The prospect of learning the true value of s before x_2 is chosen leaves unaltered the optimal current level of consumption of the risky good x_1.

3.13 Utility Affected by Accumulated Consumption

In the study of environmental hazards, what is usually uncertain is the effect of consuming particular goods, ingesting contaminants or continuing polluting activities over long periods of time. Individuals and policy-makers are concerned about the potential effect of current activities on welfare in the future. Such potential effects are ruled out at the start by the additive separability of section 3.1. One way to capture such concerns is to suppose that lifetime utility depends in part on the accumulated consumption of x over time. Hence let us assume the agent's utility function is of the form:

$$U = u_1(x_1) + u_2(x_2) + y_1 + y_2 + v(x_1 + x_2;s). \qquad (3.68)$$

To focus on cumulative consumption effect and yet maintain notational simplicity we have further assumed that the immediate effects of consuming x are known: i.e., u_1 and u_2 are independent of s.

How might this cumulative consumption term $v(x_1 + x_2;s)$ be interpreted and what properties might it have? In environmental problems the concern is often that continuing an activity at high levels over long periods may ultimately have a large negative impact on welfare, although lower levels of activity may be tolerated without ill effects. Examples include the ingestion of cumulative toxins such as heavy metals, exposure to carcinogenic substances, and continued pollution of water bodies leading to eutrophication. The accumulated level of contamination which may be tolerated without harmful effects, however, is generally not known for certain. The essential structure of these situations is captured by a $v(x_1 + x_2;s)$ of the form:

$$v(x_1 + x_2;s) = \begin{cases} 0 \text{ if } x_1 + x_2 \leq s \\ -\alpha \text{ if } x_1 + x_2 > s. \end{cases} \tag{3.69}$$

The potential loss α is assumed very large, but finite. It is interpreted as the cost of clean-up, cure or compensation if cumulative consumption exceeds the initially uncertain "safe level" s. The agnet's prior beliefs about s are represented by a probability density function f(s).

The following analysis shows that the prospect of receiving perfect information about s before x_2 is chosen, compared with no information, reduces the optimal current compensation x_1 of the risky good.

First, suppose no further information is forthcoming. Neither x_1 nor x_2 may be chosen contingent on s. The maximum level of expected lifetime utility attainable is:

$$U^\circ(M) \equiv \underset{\substack{x_1,x_2 \\ y_1,y_2}}{\text{Max}}\; E_s[u_1(x_1) + u_2(x_2) + y_1 + y_2 + v(x_1 + x_2;s)] \tag{3.70}$$
$$\text{subject to } x_1 + x_2 + y_1 + y_2 \leq M.$$

Substituting the budget constraint and form of v from (3.69) into (3.70) yields:

$$U^\circ(M) = \text{Max}[u_1(x_1) + u_2(x_2) + M - x_1 - x_2 - \alpha \int_0^{x_1+x_2} f(s)ds]. \tag{3.71}$$

Denoting by x_1^o and x_2^o the maximizing values of x_1 and x_2, the first order conditions for an interior maximum are:

$$u_1'(x_1^o) = 1 + \alpha f(x_1^o + x_2^o) \tag{3.72}$$
$$u_2'(x_2^o) = 1 + \alpha f(x_1^o + x_2^o).$$

Next, suppose that s is revealed to the agent prior to x_2 being chosen. In contrast with (3.70), the maximum expected utility attainable is:

$$U^*(M) \equiv \underset{\substack{x_1, x_2^s \\ y_1, y_2^s}}{\text{Max}}\; E_s[u_1(x_1) + u_2(x_2^s) + y_1 + y_2^s + v(x_1 + x_2^s; s)]$$

$$\text{subject to } x_1 + x_2^s + y_1 + y_2^s \leq M \text{ for all } s.$$

Eliminating the budget constraint through substitution and utilizing the maximum principle yields:

$$U^*(M) = \underset{x_1}{\text{Max}}\; E_s \underset{x_2}{\text{Max}}[u_1(x_1) + u_2(x_2) + M - x_1 - x_2 + v(x_1 + x_2; s)]. \tag{3.74}$$

The inner maximum with respect to x_2 takes both s and x_1 as given; hence its maximizer $x_2^*(x_1,s)$ is a function of both variables.

This second period reaction function $x_2^*(x_1,s)$ must be determined before the optimal initial consumption level x_1^* can be characterized. First, let us define $\overline{x}_2$ to be the level of x_2 that would be consumed if the good had no harmful consumption effects (i.e., $v = 0$ for all x_1,x_2). If such were the case then consumption would rise to where $u_2'(\overline{x}_2) = 1$, and the consumer surplus realized from second period consumption of x would be $u_2(\overline{x}_2) - \overline{x}_2$. Second, let us make more precise the meaning of α being "large." By large we mean that α exceeds $u_s(\overline{x}_2) - \overline{x}_2$, implying that once s is revealed the agent would reduce x_2 to 0, if necessary, to avoid the penalty of exceeding the safe cumulative consumption level. Of course if it turns out that previous consumption x_1 already exceeded s, and if negative consumption is ruled out, then nothing more can be lost and $x_2 = \overline{x}_2$ will be chosen. The structure of v thus leads to the second period reaction function:

$$x_2^*(x_1,s) = \begin{cases} \overline{x}_2 & \text{if } s < x_1 \\ s - x_1 & \text{if } x_1 \leq s \leq x_1 + \overline{x}_2 \\ \overline{x}_2 & \text{if } x_1 + \overline{x}_2 < s. \end{cases} \tag{3.75}$$

Turning our attention to the outer maximum of (3.74), x_1 is chosen to attain expected utility:

$$\begin{aligned} U^*(M) &= \underset{x_1}{\text{Max}}\; E_s[u_1(x_1) + u_2(x_2^*) + M - x_1 - x_2^* + v(x_1 + x_2^*; s)] \\ &= \underset{x_1}{\text{Max}}\{u_1(x_1) + M - x_1 + E_s[u_s(x_2^*) - \alpha\int_0^{x_1} f(s)ds\}. \end{aligned} \tag{3.76}$$

Note that the penalty α is incurred only if s turns out to be less than x_1. The first order condition for an interior maximum is:

$$u_1'(x_1^*) = 1 - E_s[u_2'(x_2^*) - 1]\,\frac{\partial x_2^*}{\partial x_1} + \alpha f(x_1^*). \qquad (3.77)$$

From (3.75) $\partial x_2^*/\partial x_1$ is 0, except when $x_1 \leq s \leq x_1 + \bar{x}_2$ in which case it is -1. Substituting x_2^* from (3.75) into (3.77) gives:

$$u_1'(x_1^*) = 1 + \alpha f(x_1^*) + \int_{x_1^*}^{x_1^*+\bar{x}_2} [u_2'(s-x_1^*) - 1]f(s)ds. \qquad (3.78)$$

The last term is the expected increase in second period consumer surplus obtainable if one unit less x had been consumed in the first period, and is non-negative.

Finally we may compare first period consumptions with and without the prospect of information. Solving both (3.72) and (3.78) for 1 and equating provides:

$$u_1'(x_1^o) - \alpha f(x_1^o + x_2^o) = u_1'(x_1^*) - \alpha f(x_1^*) - \int_0^{\bar{x}_2} [u_2'(\delta)-1]f(x_1^*+\delta)d\delta.$$

The integral in (3.78) has been rewritten using the change of variable $\delta = s - x_1^*$. One further assumption is needed to obtain unambiguous results: assume the density function f(s) is <u>non-increasing</u>. The exponential and uniform distributions would have this property, for example.

Our objective is to demonstrate that $x_1^o \geq x_1^*$. If $x_1^o = x_1^*$ then the right hand side of (3.79) would generally be smaller than the left hand side both because $f(x_1^o) \geq f(x_1^o + x_2^o)$ and because the right hand side integral is non-negative. But the derivative of the right hand side of (3.79) with respect to x_1 is negative at x_1^* -- that is the necessary second order condition for the maximum in (3.76). Hence x_1^* must be <u>less</u> than x_1^o for (3.79) to hold. The prospect of learning the true value of s reduces the optimal consumption level of the risky good until the information arrives.

The previous section demonstrated that if consuming a good is risky because the safe level of cumulative consumption might be exceeded (resulting in large loss), then the prospect of learning the safe level part way through one's life reduces initial consumption of that good. But the comparison between receiving such information and not is, in essence, a comparison between receiving information before and after second period consumption is chosen. It was the prospect of receiving information <u>sooner</u> which curbed initial consumption until the safe level was revealed. This section casts the problem in a continuous time framework, in which information may arrive at any point during the agent's life, to show that the sooner is perfect information forthcoming the lower is the optimal pre-information consumption level.

Let the units in which time is measured be such that the agent lives one period, λ denote the time at which s is revealed, and x_t, y_t represent the rate of consumption of x and other goods at time $0 \leq t \leq 1$. Assuming the agent's instantaneous utility function for consuming x is identical at all points in time, the lifetime utility function analogous to (3.68) is:

$$U = \int_0^1 u(x_t)dt + \int_0^1 y_t dt + v(\int_0^1 x_t dt;s). \qquad (3.80)$$

The agent's budget constraint is:

$$\int_0^1 (x_t + y_t)dt \leq M. \qquad (3.81)$$

The maximum level of expected utility attainable with wealth M if s is revealed at time λ is:

$$U^*(M,\lambda) = \max_{x_t^s, y_t^s} E_s[U] \quad \text{subject to } \int_0^1 (x_t^s + y_t^s)dt \leq M \qquad (3.82)$$

$$\text{and } \left.\begin{array}{l} x_t^s = x_t^{s'} \\ y_t^s = y_t^{s'} \end{array}\right\} \begin{array}{l} \text{for } t < \lambda \\ \text{for all } s,s'. \end{array}$$

The second constraint expresses the fact that if s is unknown before time λ then consumption prior to that time cannot be contingent on s. From the form of the utility function and absence of positive interest it can be readily shown that x_t is constant before and after time λ, although the levels in these two intervals may differ. Denoting by x_1 and x_2^s the constant consumption levels before and after time λ, eliminating the budget constraint through substitution and integrating over these intervals $[0,\lambda]$, $[\lambda,1]$ yields the more tractable expression:

$$U^*(M,\lambda) = \max_{x_1, x_2^s} E_s[\lambda u(x_1) + (1-\lambda)u(x_2^s) + M - \lambda x_1 - (1-\lambda)x_2^s + v(\lambda x_1 + (1-\lambda)x_2^s;s)]. \qquad (3.83)$$

The loss $v(\cdot,s)$ from cumulative consumption $\lambda x_1 + (1-\lambda)x_2^s$ exceeding s is α as in section 3.2. The prior probability density function on s is f(s).

The optimal initial level of consumption x_1^* may now be characterized using the same analysis as employed in the previous section. Let $\overline{x}$ denote the optimal rate of consumption of x if the good had no harmful cumulative consumption effects: i.e., $u'(\overline{x}) = 1$. If α is sufficiently large then the optimal level of x_2 as a function of x_1, s and λ is given by the reaction function:

$$x_2^*(x_1,s,\lambda) = \begin{cases} \bar{x} \text{ if } s < \lambda x_1 \\ \dfrac{s - \lambda x_1}{1 - \lambda} \text{ if } \lambda x_1 \le s \le \lambda x_1 + (1-\lambda)\bar{x} \\ \bar{x} \text{ if } \lambda x_1 + (1-\lambda)\bar{x} < s. \end{cases} \tag{3.84}$$

Applying the maximum principle, x_1 is chosen to attain expected utility:

$$U^*(M,\lambda) = \underset{x_1}{\text{Max}}\ E_s[\lambda u(x_1) + (1-\lambda)u(x_2^*) + M - \lambda x_1 - (1-\lambda)x_2^* + v(\lambda x_1 + (1-\lambda)x_2^*;s)] \tag{3.85}$$

$$= \underset{x_1}{\text{Max}}\{\lambda u(x_1) + M - \lambda x_1 + (1-\lambda)E_s[u(x_2^*) - x_2^* - \alpha \int_0^{\lambda x_1} f(s)ds\}.$$

Again, the penalty α is incurred only if cumulative consumption λx_1 prior to s being revealed exceeds s.

The first order condition of an interior maximum with respect to x_1 in (3.85) is:

$$\lambda[u'(x_1^*) - 1] + (1-\lambda)E_s[u'(x_2^*) - 1]\frac{\partial x_2^*}{\partial x_1} - \alpha\lambda f(\lambda x_1^*) = 0 \tag{3.86}$$

From (3.84) it is clear that $\partial x_2^*/\partial x_1 = 0$ except when $\lambda x_1 \le s \le \lambda x_1 + (1-\lambda)\cdot \bar{x}$, in which case $\partial x_2^*/\partial x_1 = -\lambda/(1-\lambda)$. Making these substitutions into (3.86) gives us:

$$u'(x_1^*) - 1 - \int_{\lambda x_1^*}^{\lambda x_1^* + (1-\lambda)\bar{x}} [u'(\frac{s - \lambda x_1^*}{1 - \lambda}) - 1]f(s)ds - \alpha f(\lambda x_1^*) =$$

$$u'(x_1^*) - 1 - \int_0^{(1-\lambda)\bar{x}} [u'(\frac{\delta}{1-\lambda}) - 1]f(\alpha+\lambda x_1^*)d\delta - \alpha f(\lambda x_1^*) = 0$$

The change of variable $\delta = (s-\lambda x_1^*)$ was used to rewrite the integral in the latter expression. The second order condition for (3.87) to indicate a maximum (used later) is:

$$u''(x_1^*) - \lambda \int_0^{(1-\lambda)\bar{x}} [u'(\frac{\delta}{1-\lambda}) - 1]f(\delta+\lambda x_1^*)d\delta - \alpha\lambda f'(\lambda x_1^*) < 0 \tag{3.88}$$

We are now in a position to answer how x_1^* varies with λ. If $\lambda = 1$, so that no information about s arrives before all consumption decisions have been made, then (3.87) reduces to:

$$u'(x_1^*) = 1 + \alpha f(x_1^*). \tag{3.89}$$

That is x is consumed at a rate where its immediate marginal utility of consumption just equals the marginal utility of the other goods foregone plus the marginal expected utility loss from increasing the likelihood that s is exceeded. Totally differentiating (3.87) with respect to λ yields the relation between x_1^* and λ

$$[u''(x_1^*) - \lambda \int_0^{(1-\lambda)\bar{x}} [u'(\tfrac{\delta}{1-\lambda}) - 1] f(\delta+\lambda x_1^*) d\delta - \alpha\lambda f'(\lambda x_1^*)] \frac{dx_1^*}{d\lambda} \tag{3.90}$$

$$= -\bar{x}[u'(\bar{x}) - 1] f(\cdot) + \int_0^{(1-\lambda)\bar{x}} u''(\tfrac{\delta}{1-\lambda}) \frac{\delta}{(1-\lambda)^2} f(\cdot) d\delta$$

$$+ \int_0^{(1-\lambda)\bar{x}} [u'(\tfrac{\delta}{1-\lambda}) - 1] x_1^* f'(\cdot) d\delta + \alpha x_1^* f'(\lambda x_1^*).$$

The expression multiplying $dx_1^*/d\lambda$ on the left side of (3.90) is negative from the second order condition (3.88). The first term on the right side is zero from the definition of $\bar{x}$, the second negative since u is assumed concave, the third and fourth negative under the assumption that f(s) is non-increasing introduced in section 3.2. Hence it follows that $dx_1^*/d\lambda \geq 0$: The sooner knowledge of s is anticipated (smaller is λ), the smaller is the optimal initial consumption of the risky good.

FOOTNOTES: CHAPTER III

[1] It should be noted that this result is not a general one. If individuals assign different probabilities to the two states or have different preferences, at least one individual will have a higher expected utility with trading before and after announcement of the true state. Moreover, by an appropriate redistribution of income both can be made better off in our ex-ante sense.

[2] Alternatively the city owns the land and reimburses rents in excess of the agricultural value of the land in a lump sum manner.

REFERENCES

Arnott, R. and J.G. Riley, "Asymmetrical Production Possibilities, the Social Gains from Inequality and the Optimum Town," Scandinavian Journal of Economics 79 (1977).

Arrow, K.J., Essays in the Theory of Risk-Bearing, Markham, 1971, chapters 3 and 12.

Blackwell, D., "Equivalent Comparisons of Experiments," Annals of Mathematics and Statistics 24 (June 1953), 265-272.

Bradford, D. and H. Kelejian, "The Value of Information for Crop Forecasting in a Market System," ECON INC. Discussion paper, (August 1975).

DeGroot, M.H., "Uncertainty, Information and Sequential Experiments," Annals of Mathematical Statistics 32 (June 1962).

Hirshleifer, J., "The Private and Social Value of Information and the Reward to Inventive Activity," American Economic Review 61 (September 1971).

Jones, R.A. and J. Ostroy, "Flexibility and Uncertainty," UCLA Discussion Paper #73 (September 1977).

Marschak, J., "Role of Liquidity Under Complete and Incomplete Information," American Economic Review (May 1949), 182-195.

Marschak, J. and K. Miyasawa, "Economic Comparability of Information Systems," Int. Econ. Review 9 (June 1969), 137-174.

Mirrlees, J.A., "The Optimum Town," Swedish Journal of Economics 74, 1972.

Riley, J.G., "Optimal Residential Density and Road Transportation," Journal of Urban Economics 2 (April 1974).

CHAPTER IV

THE VALUATION OF LOCATIONAL AMENITIES: AN ALTERNATIVE TO THE HEDONIC PRICE APPROACH

by
Maureen L. Cropper

It is widely recognized that the process of urbanization creates both positive and negative externalities. The important question from the viewpoint of welfare economics is what value consumers place on these externalities. If consumers regard large cities as yielding net disutility then a regression of wages on population and population density will indicate how much individuals must be compensated for living in urban areas. This figure, as suggested by Tobin and Nordhaus, may be used to adjust welfare measures for the trend toward urbanization. Alternatively, this information may be used to determine optimal city size (Henderson, Tolley). Even if cities on net yield positive utility the valuation of particular disamenities is useful for public decisionmaking. This has led to a large number of studies (Getz and Huang, Hoch and Drake, Mayer and Leone, Rosen 1977) which have computed hedonic prices for locational amenities such as crime, pollution, congestion, and local public goods.

The purpose of this paper is not simply to add to a growing empirical literature, but to present an alternative method of valuing locational amenities. In the studies cited above, marginal valuations of amenities are obtained by regressing the wage rate in city i on the level of amenities in that city. This equation is usually interpreted as an equilibrium locus of wage-amenity combinations since, if workers are mobile, wage rates should adjust to reflect differences in site-specific amenities. According to the theory of hedonic prices (Rosen 1974, 1977) the gradient of the wage-amenity locus represents consumers' marginal willingness to pay for amenities evaluated at market equilibrium.

In this paper valuations of environmental goods are obtained by estimating labor supply functions for various occupations, under the assumption that the supply of labor will be lower in cities where disamenities are high. The labor supply functions to be estimated are derived from a model of locational choice in which workers select not only the city in which they live but their housing site within the city. Conditions for equilibrium in the land market in each city lead to an equation in which the real acceptance wage for each occupation in city i is a function of employment in that occupation and the level of amenities in the city. By specifying explicitly the form of individuals' utility functions it is possible to relate the coefficients of the labor supply function to the coefficients of the utility function, which in turn may be used to compute willingness to pay.

The novelty of this approach is that it explicitly considers the spatial character of individual cities. By ignoring the spatial dimension of the problem, previous studies have been forced to assume that individuals within each city are exposed to the same level of amenities, regardless of where they live. In our model it is possible to find assumptions about the geographic distribution of amenities, and about utility functions, which allow the acceptance wage to be expressed as a function of the level of amenities at a single location within the city; or, when this is not possible, to assess the bias introduced by measuring amenities at a single point.

The spatial model also allows us to determine precisely what is meant by the "value of reducing crime" or the "value of improving air quality." Under the assumptions below the labor supply function captures the value which individuals place on amenities both at their residence and at their work site. The coefficients may therefore be used to estimate the maximum willingness to pay for an equal proportionate change in an amenity throughout the city.

The theoretical model which underlies the valuation of amenities is presented in section I below. In order to obtain reliable estimates of willingness to pay one must take account of factors affecting the demand for labor which allow firms to compensate workers for urban disamenities. This is accomplished in section I by developing a model in which industries expand in cities where locational amenities -- proximity to input and output markets, low property tax rates -- are favorable. In section II the empirical counterpart of this model is developed and labour supply functions are estimated for nine one-digit occupations using data from the 1970 Census of Population. The labor supply functions indicate which amenities are most important in consumer location decisions and whether they are valued equally by all occupational groups. The regression results are used in section III to illustrate how marginal valuations of amenities may be inferred from the coefficients of the labor supply function.

4.1 An Equilibrium Model of Urban Location

To keep the notation simple the model below is presented for the case of a single occupation and two industries, one of which produces for home consumption and the other for export. Generalization to the case of several occupations and industries is considered in section I.C.

The model used to justify our valuation of amenities consists of a large number of cities, each one of which contains a business district surrounded by residential areas. Below, it is assumed that each city is circular with the business district at the city center; however, our results continue to hold as long as all industry is located in a single area and residential districts are indexed by their distance from this area.

Within each city live identical workers who can costlessly migrate from one city to another, but who must work in the city in which they reside. Outside of cities live landowners who rent land within the city boundaries to workers and firms, the capital owners who own the capital equipment used by firms.

For simplicity it is supposed that the size of the CBD and the boundary of the city are both fixed. Thus what is analyzed is a short-run situation where the period of analysis is long enough to allow workers to move freely from one city to another but not long enough to allow the size of the city to adjust to this migration. This short-run equilibrium persists until the city re-zones agricultural areas as residential districts and provides them with various public services (sewers, water, electricity). Since it is unlikely that real-world data reflect a long-run equilibrium situation, the assumption that the city boundary is fixed does not seem inappropriate for empirical work.

For the purposes of empirical work it is also convenient to assume that the land in the city center is located at a single point in space so that no distinctions need be made among locations in the CBD. This may be defended on the grounds that land in the CBD of a city is usually small relative to the total area of the city. All land in the center of city i is thus assumed to rent at the same price. The spatial character of the rest of the city is acknowledged by expressing the rent on land in residential areas as a function $r_i(k)$ of k, the distance of the annulus from the boundary of the CBD.

A. Assumptions Regarding Workers

We shall assume that workers in all cities are identical and work a fixed number of hours in the CBD of the city in which they live at a wage of w_i per period. Each period the worker makes a fixed number of trips from his home to the CBD. In urban location models it is customary to assume that the cost of commuting from the residence to the CBD is an increasing function of distance traveled but does not depend on the worker's income. This assumption, however, is incompatible with the log-linear utility function employed below, which implies that a constant fraction of income is spent on transportation. To be consistent with that utility function transportation is treated as another good which the individual purchases, and commuting costs are not subtracted from income. The disutility associated with commuting is instead captured by including the term $k^{-\xi}$ in the utility function.

It is assumed that each worker receives utility from the size of his residential site, q, from the quality of local goods consumed, x, and from y, the amount of imports consumed. Utility is also received from site-specific amenities, which may vary from one location to another within the city.

In general, the fact that individuals in the same city are exposed to different levels of crime, pollution, and even temperature, leads to problems of aggregation when cities are the units of observation in empirical work. This poses no problem here as long as the value of each amenity at location k can be expressed as the product of the value of the amenity measured in the CBD and a dispersion function which describes how the amenity varies with distance from the point of measurement. In the case of industrial pollution, for example, emissions are generated in the CBD and spread to other parts of the city. Pollution at location k can therefore be written $P_i a_i(k)$ where P_i is pollution measured in the CBD and $a_i(k)$ is a function which is decreasing in k.

Following this approach we denote by $A_i a_i(k)$ the level of amenities which the individual experiences at his housing site, k. (For convenience, only a single $A_i a_i(k)$ is included in the utility function.) The level of amenities in the CBD, A_i, enters the utility function separately since most amenities which are consumed at home are enjoyed at the work site also.

Since the individual takes locational amenities as given, utility

$$U_i = Bq^{\beta} x^{\alpha_1} y^{\alpha_2} A_i^{\eta} [A_i a_i(k)]^{\delta} k^{-\xi}, \qquad \alpha_1+\alpha_2+\beta = 1 \tag{4.1}$$

will vary, for constant q, x, and y, according to the city and neighborhood in which the individual lives. For any location (i,k) the individual can determine his maximum utility be choosing q, x and y to maximize (4.1) subject to the constraint:

$$w_i = r_i(k)q + P_{1i}x + P_{2i}y, \tag{4.2}$$

where the prices of land, local goods, and imports are all taken as given. The utility maximization problem yields demand functions for residential land and for x and y. These can, in turn be substituted into (4.1) to yield the indirect utility function:

$$V_i(k) = Cw_i r_i(k)^{-\beta} P_{1i}^{-\alpha_1} P_{2i}^{-\alpha_2} A_i^{\eta+\delta} a(k)^{\delta} k^{-\xi}, \qquad C = B\beta^{\beta}\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}, \tag{4.3}$$

which gives the level of utility in each neighborhood of each city as a function of site-specific amenities, income and prices.

The fact that individuals are free to choose their residence implies that in equilibrium the level of utility $V_i(k)$ must be identical in all locations. Furthermore, if city i is small relative to the size of the country, $V_i(k)$ may be regarded as exogenously determined and hence $V_i(k)=V^*$ for all i and k. Worker mobility thus implies that rents, wages and the prices of local goods must adjust to compensate for differences in amenities across locations. The extent of this adjustment depends on how much individuals value amenities, as reflected by the coefficient $\eta+\delta$.

It might at first appear that $\eta+\delta$ could be inferred by solving the locational equilibrium condition $V_i(k)=V^*$ for w_i and estimating the resulting equation using data across cities. Unfortunately this leads to an equation involving land prices and amenities, which vary within, as well as across, cities. This problem is solved, however, if (4.3) is used to derive the supply function for labor.

In order to obtain the labor supply function (4.3) may be solved explicitly for $r_i(k)$ to give each individual's maximum willingness to pay for land at location k,

$$r_i(k) = (C/V^*)^{1/\beta} w_i^{1/\beta} P_{1i}^{-\alpha_1/\beta} P_{2i}^{-\alpha_2/\beta} A_i^{(\eta+\delta)/\beta} a(k)^{\delta/\beta} k^{-\xi/\beta} \tag{4.4}$$

Since land will be sold to the highest bidder (4.4) also represents the equilibrium rent function in city i. Now for the land market to be in equilibrium the population (labor force) in city i must be such that the demand for land at distance k from the CBD equals the supply. Equivalently, if $2\pi k dk$ is the fixed supply of land at distance k, then the number of persons living in ring k, n(k), must satisfy:

$$n(k) = \frac{2\pi k dk r_i(k)}{\beta w_i} \tag{4.5}$$

Substituting for $r_i(k)$ from (4.4) and integrating from k=0 to $k-\bar{k}_i$, the fixed boundary of the city, yields the number of workers in the city as a function of amenity levels and the wage,

$$N_i = \int_0^{\bar{k}_i} n(k)dk = M w_i^{(1-\beta)/\beta} P_{1i}^{-\alpha 1/\beta} P_{2i}^{-\alpha_2/\beta} A_i^{(\eta+\delta)/\beta} f_i(\bar{k}_i), \tag{4.6}$$

where $M = 2\pi\beta^{-1}(C/V^*)^{1/\beta}$ and $f_i(\bar{k}_i) = \int_0^{\bar{k}_i} k^{1-\xi/\beta} a_i(k)^{\delta/\beta} dk$.

Equation (4.6) is the supply function of labor in city i, which may be used to estimate the coefficient of amenities in the utility function. For purposes of estimation, however, it is convenient to write the labor supply function in the form:

$$(w_i/P_i)^* = c + \frac{\beta}{1-\beta} N_i^* - \frac{\eta+\delta}{1-\beta} A_i^* - F_i(\bar{k}_i), \quad P_i \equiv (\alpha_1+\alpha_2)/(1-\beta), \tag{4.7}$$

where asterisks denote logarithms of the variables. The variable on the left-hand side of (4.7) is the real acceptance wage -- the money wage in city i divided by a price index in which all commodities except residential land are weighted by the fraction of the budget spent on each. The acceptance wage is an increasing function of N_i since, if land is fixed, an increase in population will raise rents and thus the income necessary to maintain V^*. Amenities such as sunshine and clean air enter equation (4.7) with negative coefficients, while disamenities, for which individials must be compensated, increase the acceptance wage.

Note that due to the multiplicative nature of utility only the value of amenities in the CBD appears in the supply function. The dispersion function $a_i(k)$ which captures the fact that individuals in each city are exposed to different levels of amenities, is subsumed in $F_i(\bar{k}_i)$. Since in the short run $\bar{k}_i$ is exogenous, we shall regard the $\bar{k}_i$ as independent drawings from a probability density function. $F_i(\bar{k}_i)$ can then be regarded as an error term which is independently though not identically distributed for all cities. If, however, the dispersion functions are identical in all cities, then the error terms $F(\bar{k}_i)$ will be independently and identically distributed for all i.

The coefficient of amenities in the utility function can therefore be estimated by regressing the real wage in city i on employment and on amenities in city i. In order to obtain consistent estimates of $\eta+\delta$, however, it is necessary to first identify factors which determine the demand for labour in each city.

B. Assumptions Regarding Firms

Rather than develop a model which explicitly treats firm migration we assume that there is a production function for industry X and for industry Y in each city. Differences in natural resource endowments, transportation costs and locational amenities lead to differences in production costs among cities which, in turn, explain the growth of industry in each city.

For city i the production function of the export industry may be written:

$$Y_i = D_2 N_{2i}^a L_{2i}^b K_{2i}^c S_{2i}^d E_{2i}^f, \qquad a+b+c < 1, \qquad (4.8)$$

where L_{2i} denotes land and other raw material inputs, N_{2i}, labor inputs, K_{2i}, capital goods, S_{2i}, pollution generated by the industry and E_{2i} environmental goods which affect the production process. The latter might include climate or the level of air pollution in the city. Population, N_i, may also enter the production function as a proxy for agglomeration economies if these are relevant for industry Y.

We shall assume that industry Y behaves as a price-taker in all markets. Thus given output price, input prices, and a tax on effluents, the industry determines profit-maximizing levels of inputs L, N and K and a level of emissions, S. Industry X behaves analogously.

Although each industry regards input and output prices as exogenous, the wage, the price of land in the CBD, and the price of local goods are determined by equilibrium conditions in product and factor markets in city i. Equating the aggregate demand for land in the CBD to the size of the CBD, the aggregate demand for labor to the right-hand side of (4.6) and the supply of X to the aggregate demand for X yields a system of three equations which may be solved for the price of land, the wage, and the price of X. The equilibrium level of employment (population) may be found by substituting the equilibrium wage into (4.6) and the quantity of local goods produced obtained by substituting P_{1i} into the aggregate demand function for X.

Environmental goods which depend on output or on population are also determined by market equilibrium conditions. The level of pollution in the CBD of city i may be expressed as a function of industrial emissions, $S_{1i}+S_{2i}$, and weather conditions in the CBD. Crime, which depends on population and on the wage, must also be regarded as endogenous.

In the model outlined here the size of industry in city i, and hence the demand for labor, depends on the parameters of the production function and on input and output prices. For the purposes of empirical work, however, it is the exogenous factors which determine the size of industry that are important. These enter the model through the variable E_i and by affecting the prices of capital goods, natural resources, and the price of exports.

As indicated above the output of industry Y is sold in national markets at a price $\bar{p}$ which may be regarded as exogenous to each city. The price received by firms in city i, however, will fall short of $\bar{p}$ by the cost of shipping Y to market. Since shipping costs depend on the distance of city i from the central market and on the intervening topography, one would expect the demand for labor to be higher in cities close to output markets which have access to cheap sources of transportation.

The prices of natural resources and capital goods, which are assumed to be traded in centrally located markets, may also be regarded as exogenous to firms in city i. The delivered cost of these inputs (and hence the damand for labor) depends on the proximity of the city to input markets and on the feasibility of using low-cost means of transportation, e.g., water v. air.

Finally, the demand for labor should be higher in areas where land prices are low. Although the price of land in the CBD is endogenous to city i, it is affected by the size of the CBD and by the property tax rate, both of which are determined by the government in the short run and are treated as exogenous in our model.

C. Generalization to Several Occupations

The model of sections A and B, although locically consistent, is based on assumptions which are difficult to accept in empirical work. By treating all workers as identical the model ignores variations in skill levels and job experience which explain a large proportion of variation in wages across cities. The model also imposes the stringent requirement that all individuals have identical preferences. These assumptions may be relaxed by estimating labor supply functions for separate occupational groups; however, it must first be demonstrated that the coefficients of the disaggregated labor supply functions have the same interpretation as the coefficients of equation (4.7).

Suppose in the model above that there are several classes of workers, with each class possessing different skills or years of job experience. This means that a distinction will have to be drawn among categories of labor in the production functions for X and Y, with each type of labor entering the production function with a different coefficient. There will as a result be a separate demand function for each type of labor; however, as long as factor markets are perfectly competitive, generalization to several occupational groups is straightforward.

Deriving the supply functions for labor presents more difficulties. Suppose for simplicity that members of each occupational group are identical and work a fixed number of hours in the CBD at the wage paid to their group. While workers within each group have the same tastes, it seems reasonable to allow preferences for consumption goods and amenitities to differ among groups. The indirect utility function for each group will thus be of the form;

$$V_{ij}(k) = C_j w_{ij} r_i(k)^{-\beta_j} P_{1i}^{-\alpha 1_j} P_{2i}^{-\alpha 2_j} A^{\eta_j+\delta_j} a(k)^{\delta_j} k^{-\xi_j} \qquad (4.9)$$

where parameters are subscripted to allow for differences in tastes among groups.

As in the case of a single category of labor, the labor supply function for each occupational group is derived from that group's location decision. In locational equilibrium all members of the occupational class must experience the same utility regardless of the neighborhood or city in which they live. Thus V_{ij} must be constant for all i and k and equal to V_j^*. (If each city is small and open, the V_j^* can be considered exogenous to the city.) This equilibrium condition is used to determine where in each city members of group j will live. The group's labor supply function is then derived by summing the number of persons in each neighborhood, n(k), across all neighborhoods k in which members of the group reside.

The crucial step in the above procedure is determining the spatial distribution of occupational groups within each city. Equilibrium in the land market requires that land at each location be sold to the highest bidder. To determine the bid function for each occupation the locational equilibrium condition [(4.9) with $V_{ij}=V_j^*$] may be solved for $r_{ij}(k)$, group j's maximum willingness to pay for land at each location. Under certain assumptions these bid functions, if plotted against k, will be downward-sloping and will intersect any number of times. Each city will thus be divided into neighborhoods which are segregated on the basis of occupation, with neighborhood boundaries determined by the intersections of the $r_{ij}(k)$'s. Summing the number of persons per annulus, n(k), across all k at which group j resides (K_{ij}) yields group j's supply function for labor,

$$N_{ij} = M_j w_{ij}^{(1-\beta_j)/\beta_j} P_{1i}^{-\alpha 1_j/\beta_j} P_{2i}^{-\alpha 2_j/\beta_j} A_i^{(\eta_j+\delta_j)/\beta_j} \int_{k \varepsilon K_{ij}} k^{1-\xi_j/\beta_j} a(k)^{\delta_j/\beta_j} dk. \qquad (4.10)$$

The trouble with this procedure is that the boundaries of the group j neighborhoods, which are determined by the intersections of the $r_{ij}(k)$'s cannot be treated as exogenous but themselves depend on w_{ij}. The integral on the left-hand side of (4.10) cannot therefore be regarded as a random error term, and omitting it from the equation will bias the coefficients of N_{ij} and A_i in (4.11).

$$(w_{ij}/P_i)^* = c_j^* + \frac{\beta_j}{1-\beta_j} N_{ij}^* - \frac{\eta_j+\delta_j}{1-\beta_j} A_i^* + \varepsilon_{ij}^*. \qquad (4.11)$$

How serious this problem is depends on the extent to which current neighborhood boundaries depend on current wages and levels of amenities. To the extent that they do not the limits of integration in the supply function may be regarded as independent of w_{ij} and A_i, and the integral in (4.10) may be treated as a random error term.

4.2 Empirical Specification and Estimation of the Model

The model of section I implies that one may value urban amenities by estimating labor supply functions of the form (4.11). To illustrate this approach supply function were estimated for one-digit occupational categories using data from the 1970 Census of Population. The results of these regressions are presented below following a description of our empirical model.

A. Specification of the Labor Supply Function

To estimate equation (4.11) one must find empirical counterparts to the amenities A_i which influence consumer location decisions. One group of variables found to be important in previous studies are the amenities and disamenities associated with urbanization. Most regressions, for example, include air pollution, crime and congestion (population density) as measures of the disamenities of urban life while using some index of availability of goods and services (number of sports franchises, number of TV stations) to capture the advantages offered by large cities. In the context of our urban location model all amenities and disamenities associated with urban scale should be treated as endogenous variables. Our small sample size (n=28), however, makes it difficult to treat more than one or two variables as endogenous. Scale amenities must therefore be treated as exogenous, causing simultaneous equations bias, or must be omitted from the equation altogether.

To resolve this problem air pollution, measured by the arithmetic mean of sulfur dioxide, is included in the labor supply function as an endogenous variable. Crime is also included but is treated as exogenous on the grounds that crime rates are affected by law enforcement practices, by the racial composition of the population, and even by climate (Hoch), all of which are exogenous to the model of section I. The only measure of urban amenities explicitly included in the regression equation is availability of health facilities -- number of hospital beds per 100,000 and number of doctors per 100,000. Unlike other measures of availability of goods and services these variables are not very responsive to variations in income and can more reasonably be regarded as exogenous.

Scale amenities which are omitted from the labor supply function will be captured in part by the endogenous employment variable, N_{ij}. In equation (4.11) this variable represents the effect of land prices on wages and is expected to have a positive coefficient. If, however, N_{ij} enters the utility function as a proxy for scale amenities then its coefficient should be wirtten $(\beta-\gamma)/(1-\beta)$ where γ represents the net effect of scale amenities. If the amenities of urban life outweigh the disamenities then the sign of employment may actually be negative.

Other factors which are likely to affect location decisions are climate and scenic beauty. Although these variables can truly be regarded as exogenous, high correlation between individual amenity measures, together with a small sample, makes it difficult to include all relevant variables in the regression equation. Of the one dozen climate variables considered, only the two most significant, average July temperature and wind velocity, appear in the final equation. These variables should therefore be regarded as proxies for the amenities of climate, and their individual coefficients should be interpreted with caution.

A similar situation arises in the case of scenic amenities. Scenic amenities, which may be measured by proximity to the ocean or to the mountains, are closely related to the availability of recreational facilities (beaches, parks, skiing). Unfortunately the measure of recreational facilities used in our empirical work, number of national parks, state parks and national forests within 100 miles of each city, was highly correlated with a dummy variable = 1 if the city was located on the ocean and with a dummy variable indicating the availability of beaches. To avoid collinearity problems only a single variable, the coastal dummy, was retained in the final equation. Its coefficient should therefore be interpreted as a proxy for both recreational and aesthetic amenities.

An additional category of amenities to be considered is employment opportunities within each city. In our theoretical model employment opportunities are captured entirely by the wage rate w_{ij}. In reality, markets are imperfect and individuals must consider the probability of being unemployed. For married males the relevant variables are the unemployment rate in the individual's own occupation as well as some indicator of employment opportunities for women. If the ratio of females to males in the labor force were identical in all cities, then the ratio of females to males actually employed would indicate the availability of jobs for women. This variable, first suggested by Getz and Huang, appears in one set of regressions reported below. An alternate measure of employment opportunities, which is more in the spirit of our model, is the real median earnings of women in each city. This is included in the labor supply functions of blue collar males, as reported in Table II.

While both measures of employment opportunities for women are significant for some occupations, the unemployment rate for males is not and has been deleted from the labor supply function. The poor performance of the unemployment rate is probably due to the fact that aggregate unemployment is of little significance to members of specific occupations. Unemployment rates for one-digit occupations are, unfortunately, unavailable for the year 1970.

B. Identification of the Labor Supply Function

The model of Section I implies that the labor supply function must be estimated as part of a simultaneous equation system in which the real wage, employment, and air pollution are endogenously determined. Exogenous variables in the system which affect the location of industry but not of workers may be used to identify the labor supply function. The discussion in I.B suggests at least three such variables -- availability of raw material inputs, proximity to output markets, and availability of cheap transportation. The empirical counterparts of these are used as excluded exogenous variables in the 2SLS estimation of (4.11).

Availability of raw materials is measured by the value of farm products, the number of acres of commercial timberland and by value added in mining, all measured for the state in which the SMSA is located. Proximity to other cities is measured by the percent of goods (by weight) shipped at least 500 miles from the SMSA and by the percent of goods shipped within 100 miles of th SMSA boundary. High values of the former variable should indicate that a city is isolated from output markets, whereas high values of the latter should indicate the reverse. A dummy variable equal to 1 if the city is a port is included to indicate availability of cheap transportation.

Finally, as noted at the beginning of section I, the size of each city is regarded as fixed in our model on the grounds that we are dealing with a short-run equilibrium situation. Since land prices will affect the growth of industry, city size (in acres) and the effective property tax rate are both included as excluded exogenous variables in the estimation of the labor supply function.

C. Estimation of the Labour Supply Function

The labor supply functions presented in Tables I-III have been estimated using 1970 Census of Population data for 28 of the 39 cities for which BLS Cost of Living indexes are available. (A list of these cities and a description of data sources appear in the Appendix). In each of the regressions the dependent variable is the median earnings of all males who worked 50-52 weeks in 1969. The wage variable in each case is deflated by the BLS intermediate budget cost of living index, with the price of housing removed from the index, as indicated in I.A.

By including only those individuals who worked for the entire year, and by estimating labor supply functions for specific occupations one is able to control for some of the factors other than amenities which account for inter-city variation in wage rates. Median earnings, however, may vary due to differences in union membership, in educational levels and in years of job experience. Since data on union membership and on the ratio of union to non-union wages are available by region for one-digit occupations it is possible to adjust the earnings variable using the formula:

$$w_{observed} = (1-a)w_{non-union} + aw_{union}, \qquad (4.12)$$

where a represents the percentage of workers in unions. The non-union wage, obtained by solving (12), is the dependent variable in the regressions for blue-collar occupations.

To test the significance of human capital factors and racial discrimination in explaining variation in wages, median earnings in each occupations (undeflated by the cost of living index but adjusted for union membership) were regressed on the average age of workers in the occupation, on the percent of non-whites in the occupation and on the average school years completed by all males in the SMSA. In all cases the years of schooling variable, which is unavailable by occupation, was, not surprisingly, insignificant. The average age of the workforce, however, was positively related to the money wage for all occupations and was significant at the .05 level in all but two cases. Percent non-white was highly significant, with the expected negative sign, for laborers and service workers, the only two occupations employing a high percent of non-whites.

In the context of our model it seems most appropriate to treat average age and percent non-white as exogenous variables which affect the productivity of labor, as perceived by firms. Average age and percent non-white are therefore included as exogenous variables in estimating the labor supply function, the former for all occupations except managers and the latter for laborers and service workers only.

Finally, wage rates may vary across cities due to disequilibrium movements in workers and firms not allowed for in the model of section I. For example, an increase in the demand for labor in city i will put upward pressure on the wage rate and should be accompanied by an inflow of workers into the city. To allow for this possibility the net migration rate is included as an explanatory variable in one set of regressions.

4.3 Empirical Results

An important question to be answered by our empirical model is which groups of variables are most important in individuals' location decisions. A related question is whether these variables are the same for all occupational groups. To answer these questions Table 4.1 presents regression results for nine occupations with the same set of variables appearing in each equation.

In examining these results one must be careful to interpret individual variables as proxies for groups of amenities. Viewed in this way scenic amenities (coastal dummy), scale amenities (employment), and the availability of health facilities seem to be the most important factors in location

Table 4.1

Estimated Labor Supply Functions

(n = 28)	All Earners	Professional Workers	Non-Farm Managers	Sales Workers	Clerical Workers
Constant	6.0536***	4.8012***	5.9472***	4.3032**	4.089***
	(1.4295)	(1.7002)	(1.6454)	(1.9612)	(1.5067)
Employment	0.0273**	0.0257	0.0342**	0.0365**	0.0233*
	(0.0160)	(0.0196)	(0.0185)	(0.0206)	(0.0164)
SO_2	0.0219*	0.0231	0.0255*	0.0151	0.0209
	(0.0161)	(0.0193)	(0.0180)	(0.0204)	(0.0170)
July temperature	-0.4397***	0.0392	-0.0768	0.0386	-0.2247*
	(0.1327)	(0.1584)	(0.1525)	(0.1815)	(0.1397)
Wind velocity	-0.1087**	-0.1545**	-0.1507**	-0.0855	-0.0737
	(0.0576)	(0.0674)	(0.0658)	(0.0781)	(0.0608)
Doctors/100,000	-0.1381**	-0.1065	-0.1008	-0.0031	-0.1156*
	(0.0681)	(0.0807)	(0.0739)	(0.0941)	(0.0721)
Hospital beds/100,000	-0.0651**	-0.0376	-0.0228	-0.0637*	-0.0380
	(0.0338)	(0.0399)	(0.0392)	(0.0469)	(0.0355)
Crimes/100,000	+0.0743**	0.1070**	0.0785**	0.0503	0.0709**
	(0.0349)	(0.0422)	(0.0403)	(0.0472)	(0.0367)
Female/Male Employment	-0.0613	0.0335	0.0956	0.0094	-0.1900*
	(0.1206)	(0.1441)	(0.1394)	(0.1651)	(0.1277)
Coastal Dummy	-0.0639***	-0.0192	-0.0690**	-0.0938***	-0.0462**
	(0.0249)	(0.0303)	(0.0286)	(0.0338)	(0.0265)
R^2	.7429	.5916	.6047	.5499	.5637

(continued)

Note: All variables are in natural logarithms.

*** = Significant at .01 level, one-tailed test.

** = Significant at .05 level, one-tailed test.

* = Significant at .10 level, one-tailed test.

Table 4.1
(continued)

(n = 28)	Craftsmen	Operatives	Non-Farm Laborers	Service Workers
Constant	4.3419** (1.7232)	2.4042* (1.5790)	6.4412*** (1.4466)	7.8618*** (2.2571)
Employment	0.0360** (0.0200)	0.0014 (0.0163)	0.0344** (0.0143)	0.0386* (0.0242)
SO_2	0.0185 (0.0192)	0.0242* (0.0179)	0.0340** (0.0150)	0.0488** (0.0250)
July temperature	-0.4680*** (0.1583)	-0.4339*** (0.1441)	-0.8984*** (0.1332)	-0.8524*** (0.2117)
Wind velocity	-0.0904 (0.0695)	-0.0352 (0.0635)	0.0314 (0.0575)	0.0342 (0.0899)
Doctors/100,000	-0.1241* (0.0814)	-0.0401 (0.0736)	-0.1657** (0.0680)	-0.2321** (0.1071)
Hospital beds/ 100,000	-0.0439 (0.0407)	-0.1021*** (0.0368)	0.0014 (0.0338)	-0.0267 (0.0529)
Crimes/100,000	0.0496 (0.0414)	0.0048 (0.0374)	0.0385 (0.0347)	0.0832 (0.0554)
Female/Male Employment	-0.3349** (0.1459)	-0.5548*** (0.1356)	-0.2327** (0.1217)	-0.0220 (0.1905)
Coastal Dummy	-0.0869*** (0.0299)	-0.0410* (0.0262)	-0.0293 (0.0250)	-0.0054 (0.0398)
R^2	.7508	.7998	.8873	.7366

Note: All variables are in natural logarithms.

*** = Significant at .01 level, one-tailed test.

** = Significant at .05 level, one-tailed test.

* = Significant at .10 level, one-tailed test.

decisions: Each of these variables consistently has the expected sign and is asymptotically significant at the 0.10 level or better in six out of nine regressions.

The behaviour of employment is of particular interest since it is this variable which represents the effects of city size. In all occupations the coefficient of employment is positive, which would seem to imply that individuals must be compensated for living in large cities. One must, however, be cautious in drawing this conclusion. The coefficient of employment in the labor supply function β_1, depends not only on γ, the coefficient of city size in the utility function, but on β, the proportion of income spent on the housing site. Specifically,

$$\beta_1 = (\beta-\gamma)/(1-\beta). \tag{4.13}$$

Given $\hat{\beta}$ and $\hat{\beta}_1$, equation (4.13) may be solved for $\hat{\gamma}$, which is clearly increasing in both variables. The smallest value of $\hat{\gamma}$ implied by Table I occurs when $\hat{\beta}_1 = .0014$. Note that even if $\hat{\beta}$ were only .03, $\hat{\gamma}$ would still be positive (although small) indicating that cities yield net amenities to consumers. This conclusion, however, must be qualified by the fact that crime and air pollution, two disamenities partially associated with city size, are included separately in the regression equation and are often significant and positive.

One must also be cautious in interpreting the variable doctors/ 100,000, which may represent amenities other than health facilities. The coefficient of this variable is particularly large for laborers and service workers, groups for whom scenic amenities do not appear to be significant. Conversely, in cases where MD's is insignificant the coastal dummy is significant. This suggests that MD's/100,000 may act as a proxy for scenic amenities, an hypothesis which is not unreasonable if doctors take part of their income in the form of locational amenities. This hypothesis is also strengthened by casual inspection: San Francisco, Denver and New York are among the cities with the highest number of doctors per capita, whereas Wichita, Kansas is the sample minimum.

Of the remaining variables, crime is significant in five equations and is clearly more important for white-collar than for blue-collar workers. Air pollution, measured here by sulfur dioxide, has the expected positive sign for all occupations but seems to be more significant for blue-collar occupations. If this result appears surprising, it should be remembered that blue-collar workers are more mobile than highly-paid white-collar workers, whose location decisions are likely to depend on job-related amenities. Pollution and other locational amenities are therefore more likely to appear significant in the labor supply functions for blue-collar occupations.

This reasoning may explain why climate variables do not appear to be very significant for white-collar workers. (The two exceptions in the case of wind velocity are most likely due to the effect of wind on air quality.) For blue-collar workers average July temperature is highly significant and appears as an amenity in all cases. The extremely large coefficients of temperature may be due to the variable acting as a proxy for other climate variables or, since July temperature is higher in Southern cities, as a proxy for the large supply of unskilled labor often used to explain the lower level of wages in the South.

The remaining variable in the supply function, the ratio of female to male employment, is the more significant of the two measures of employment opportunities for women. As indicated in Table I this variable is not significant in the supply functions for highly-paid white-collar workers but is significant for clerical workers and for most blue-collar occupations, implying that the importance of employment opportunities varies inversely with the husband's income. It is interesting to note that these results are similar to those of Getz and Huang, who find female/male employment to be highly significant in labor supply functions estimated from the same set of data.

The results of using median earnings for women in place of female/male employment are reported in Table II. Female earnings is significant for only two occupations (operatives and laborers) but has a market effect on the coefficients of other variables whenever it is included in the equation. In general the coefficients of other amenities increase in absolute value and in significance. This may be the result of high pairwise correlations between female earnings and employment, crime, and doctors per 100,000 which are not present when female/male employment is used. For this reason the results presented in Table I should be viewed as more reliable.

To test the possibility that wage data reflect disequilibrium movements of workers, the equations in Table I were re-estimated with net migration included in non-log form. The net migration variable was significant only for while-collar occupations and these results are reported in Table 4.3. In all cases net migration has a positive sign, suggesting that wages for white-collar workers are higher in some cities due to an increase in the demand for labor to which workers have not fully adjusted. Adding net migration to the equation does not drastically alter the conclusions of Table 4.1, but does affect the relative importance of the pollution and employment variables. Sulfur dioxide is now significant in three out of four white-collar occupations, whereas employment is significant only in the aggregate labor supply function. This result is probably due to the positive correlation between employment and air pollution, which makes it difficult to separate the effects of the two variables.

The Valuation of Environmental Amenities -- An Illustration

We shall not illustrate, using the results of Table 4.1 - 4.3, how valuations of locational amenities can be inferred from the coefficients of the labor supply function. In the model of section I a given percentage change in A_i in the CBD of city i implies an equal percentage change in the amenity throughout the city. The amount an individual is willing to pay for this change may be defined as the largest amount of income one can take away from the individual without altering his utility. If the change in A_i is so small that it does not affect prices in city i then willingness to

Table 4.2

Labor Supply Functions of Blue-Collar Workers

(n = 28)	Craftsmen	Operatives	Non-Farm Laborers	Service Workers
Constant	8.1298*** (1.0665)	8.9434*** (1.0216)	9.2910*** (0.8401)	8.1321*** (1.2702)
Employment	0.0527** (0.0219)	0.0228 (0.0182)	0.0443*** (0.0150)	0.0453** (0.0254)
SO_2	0.0231 (0.0214)	0.0402** (0.0212)	0.0410*** (0.0155)	0.0447** (0.0242)
July temperature	-0.5682*** (0.1709)	-0.5964*** (0.1658)	-0.9733*** (0.1339)	-0.8525*** (0.2017)
Wind velocity	-0.1234* (0.0776)	-0.0890 (0.0746)	0.0095 (0.0600)	(0.0276) (0.0900)
Doctors/100,000	-0.2128*** 0.0780	-0.1704** (0.0754)	-0.2221*** (0.0611)	-0.2390*** (0.0918)
Hospital beds/ 100,000	-0.0319 (0.0453)	-0.0921** (0.0435)	0.0038 (0.0353)	-0.0264 (0.0530)
Crimes/100,000	0.0684 (0.0525)	0.0639 (0.0505)	0.0642* (0.0409)	0.0804 (0.0617)
Median Earnings, Females	-0.1659 (0.1743)	-0.4299*** (0.1637)	-0.1989* (0.1355)	-0.0119 (0.2153)
Coastal Dummy	-0.0817** (0.0333)	-0.0290 (0.0308)	-0.0265 (0.0259)	-0.0088 (0.0391)
R^2	.6849	.7155	.8764	.7380

*** = Asymptotically significant at the .01 level.

** = Asymptotically significant at the .05 level.

* = Asymptotically significant at the .10 level.

Table 4.3

Labor Supply Functions of White-Collar Workers

(n = 27)	All Earners	Professional Workers	Non-Farm Managers	Sales Workers	Clerical Workers
Constant	5.3028***	3.6120**	5.0071***	3.4306*	3.4875**
	(1.4216)	(1.5818)	(1.6746)	(1.9937)	(1.5794)
Employment	0.0277*	0.0118	0.0206	0.0322	0.0050
	(0.0188)	(0.0215)	(0.0224)	(0.0244)	(0.0200)
SO_2	0.0263*	0.0332*	0.0388**	0.0232	0.0313*
	(0.0188)	(0.0210)	(0.0216)	(0.0240)	(0.0210)
July temperature	-0.4879***	-0.0478	-0.1421	-0.0176	-0.2622**
	(0.1305)	(0.1474)	(0.1537)	(0.1825)	(0.1449)
Wind velocity	-0.0704	-0.0882*	-0.0937*	-0.0382	-0.0304
	(0.0593)	(0.0659)	(0.0698)	(0.0823)	(0.0661)
Doctors/100,000	-0.1488**	-0.0831	-0.0802	-0.0086	-0.0790
	(0.0689)	(0.0755)	(0.0825)	(0.0989)	(0.0769)
Hospital beds/ 100,000	-0.0513*	-0.0346	-0.0253	-0.0523	-0.0477
	(0.0363)	(0.0393)	(0.0437)	(0.0524)	(0.0398)
Crimes/100,000	0.0702**	0.0887**	0.0639*	0.0450	0.0570*
	(0.0343)	(0.0385)	(0.0406)	(0.0482)	(0.0382)
Female/Male Employment	-0.1831*	-0.1618	-0.0539	-0.1300	-0.2756
	(0.1298)	(0.1447)	(0.1538)	(0.1822)	(0.1454)
Coastal Dummy	-0.0718***	-0.0315***	-0.0794***	-0.1014***	-0.0491**
	(0.0243)	(0.0275)	(0.0286)	(0.0338)	(0.0273)
Net Migration	0.0022**	0.0036***	0.0030**	0.0026*	0.0017
	(0.0012)	(0.0014)	(0.0014)	(0.0017)	(0.0013)
R^2	.7873	.7037	.6599	.6086	.5947

*** = Asymptotically significant at the .01 level.

** = Asymptotically significant at the .05 level.

* = Asymptotically significant at the .10 level.

pay, Δw_i, is defined implicitly by:

$$Cw_i r_i(k)^{-\beta} P_{1i}^{-\alpha 1} P_{2i}^{-\alpha 2} A_i^{\eta+\delta} a(k)^{\delta} k^{-\xi} =$$

$$C(w_i - \Delta w_i) r_i(k)^{-\beta} P_{1i}^{-\alpha 1} P_{2i}^{-\alpha 2} (A_i + \Delta A_i)^{\eta+\delta} a(k)\ k^{-\xi}. \qquad (4.14)$$

This can be simplified to:

$$\Delta w_i = w_i[1-(1+k)^{-(\eta+\delta)}], \qquad k \equiv \Delta A_i / A_i \qquad (4.15)$$

where k denotes the proportional change in A_i.

Willingness to pay can thus be computed solely from knowledge of income and the exponent of amenities in the utility function. To estimate $\eta+\delta$ from the coefficient of A_i in the labor supply function, $-(\eta+\delta)/(1-\beta)$, requires knowledge of β, the proportion of income spent on the residential housing site. If employment acts as a proxy for scale amenities β cannot be inferred from the coefficient of N_i, however, valuations of A_i can be computed for alternate values of β.

To illustrate the use of (4.15), willingness to pay for one-, ten-, and twenty-percent changes in selected amenities are shown in Table 4.4 for an individual whose yearly income is $9,000. These figures are based on results reported in Table 4.1, and, in view of the discussion above, should be interpreted with caution.

Table 4.4 implies that an individual with the same preferences as a manager would be willing to pay between 0.68% and 0.80% of his income for a 10% reduction in the total crime rate. Since the cost on insuring one's possessions against theft is already included in the cost of living index, this valuation represents the phychic disutility attached to crime. These figures correspond closely to valuations of crime obtained by Rosen (1977), who estimates that individuals would be willing to pay between 0% and 1.16% of their income for a comparable reduction in the crime rate. The coefficient of violent crime in the labor supply functions estimated by Getz and Huang, 0.05, also suggests that our estimates of willingness to pay are reasonable.

The value placed on a reduction in sulfur dioxide, although low by comparison with crime, is higher than the figure obtained by Ridker and Henning in their important study of air pollution in the St. Louis SMSA. By regressing property value by census tract (1960) on site-specific amenities, Ridker and Henning estimate that a permanent decrease in SO_2 by approximately 30% would raise the value of an average home by $245. Based on figures in Table 4.1 the present discounted value of a 30% reduction in SO_2, calculated for a person earning the median income in St. Louis in 1960, is between $418 and $489, or roughly twice the figure cited by Ridker and Henning. One reason for this discrepancy is that under the

Table 4.4

Valuations of Environmental Amenities

	Crime (Managers)			Sulfur Dioxide (Laborers)			July Temperature (Operatives)		
	-1%	-10%	-20%	-1%	-10%	-20%	+1%	+10%	+20%
.19	$5.86	$61.2	$129	$2.50	$26.1	$55.1	$31.1	$294	$554
.10	6.51	68.0	143	2.77	29.0	61.2	34.6	326	613
.05	6.87	71.8	151	2.92	30.6	64.6	36.5	344	646

NOTE: All figures represent annual values of willingness to pay, computed for an individual with an income of $9,000.

assumptions of I.A. our figures capture willingness to pay for reductions in pollution at the work site and at home, whereas the property value approach measures willingness to pay at the residence only. Furthermore, part of our estimate may represent willingness to pay for a reduction in suspended particulates. Particulates, being highly correlated with sulfur dioxide, are omitted from the labor supply function to avoid problems of multicollinearity.

The least reliable estimates in Table 4.4 are those for summer temperature. In Tables 4.1 - 4.3 July temperature appears as an amenity, with individuals willing to give up income for above-average temperatures. Since the coefficient of temperature for laborers and service workers likely represents the effects of lower skill levels in the South, the estimates in Table 4.4 are computed using the more moderate coefficient for operatives. If evaluated at the sample geometric mean, 75°F, this figure implies that an individual earning \$9000 is willing to pay between \$294 and \$344 per year for an increase in average temperature from 75° to 82.5°F. While not unreasonable, this figure is higher than valuations implied by hedonic price regressions (see Meyer and Leone) and should be regarded as purely illustrative.

In the case of a dichotomous amenity, e.g., the coastal dummy, equation (4.15) no longer applies and willingness to pay must be calculated from

$$\Delta w_i = w_i(1-e^{\zeta}) \qquad (4.16)$$

where ζ is the coefficient of the dichotomous amenity in the utility function. Using (4.16) Table 4.1 implies that a manager will give up between \$660 and \$770 if his income is \$12,000. This figure, of course, must be regarded as approximate since the coastal dummy reflects other scenic amenities as well.

Finally, equation (4.15) may be used to infer how much of the husband's earnings a family would be willing to give up in order to increase the earning opportunities for the wife. Theory suggests that a family should not give up an equal amount of the husband's earnings if the shadowprice of the wife's time at home exceeds that of the husband. In Table 4.2 the highest significant coefficient of female earnings is -0.43, obtained for operatives. This implies that a male operative will relinquish at most 4% of his earnings for a 10% increase in real female earnings. If this figure should seem small, recall that it is based on the behavior of all operatives, some of whom are not married or do not have working wives.

4.4 Conclusion

This paper has presented a method of valuing environmental amenities using a model which describes the location of workers within as well as among cities. This allows us explicitly to deal with the fact that individuals within the same city are exposed to different levels of amenities. As long as individuals have log-linear utility functions the value of an amenity to an individual located anywhere in the city can be computed from the coefficients of an aggregate labor supply function which includes the level of the amenity measured at a single point within the city.

To illustrate the proposed method of valuing amenities labor supply functions were estimated for nine occupations using data from the 1970 Census of Population. The results of these regressions are of interest quite apart from the problem of valuing amenities since they indicate which groups of variables are important in inter-urban location decisions. Based on the signs and asymptotic significance levels of the regression coefficients crime and scenic amenities, measured here by a coastal dummy variable, seem to be the most important environmental goods in the location decisions of white-collar workers. Pollution (SO_2) is significant for three out of four blue-collar occupations, and is important for white-collar workers if net migration is included in the equation. Employment opportunities for females, whether measured by median real earnings of females or by the ratio of female workers to male workers, seems to be an important consideration in the location decisions of blue-collar workers, as does the availability of health facilities (MD's/100,000, hospital beds/100,000). Surprisingly, climate variables do not seem very important, especially for white-collar workers, although this conclusion must be qualified by the fact that it is hard to separate the effects of climate from other variables.

The original motive for this paper was to place a value on the amenities and disamenities associated with urbanization. Subject to certain qualifications, willingness to pay for reductions in crime and air pollution are presented in section 3.3 above. While one would not want to place too much confidence in the figures, it is clear that certain groups of individuals must be compensated for these urban disamenities. The same, however, cannot be said for the other effects of city size. For all occupations the coefficient of the urban scale variable is positive, which appears to indicate that urbanization yields not disutility. One cannot, however, regard the coefficient of employment as the marginal value of city size. The latter, as shown sbove, is very likely positive, indicating that the effects of urbanization not captured by other variables yield positive utility.

CHAPTER V

VALUATION REVEALING GUESSES: A REPORT ON THE EXPERIMENTAL TESTING OF A NON-MARKET VALUATION PROCEDURE

by

William R. Porter and Berton J. Hansen

This paper describes a survey method that can be used to measure the public's valuation of a public good. In its simplest form, the method attempts to determine the aggregate valuation of a public good (or change in a public good) by a group of consumers. It is designed to provide each respondent with strong incentive to (a) consider the valuation question seriously and (b) to disclose unbiased information about the public good valuation.

The method consists of asking each surveyed respondent to guess as close as possible to the "true average valuation" of the others in the group. Before guessing each person is told that if his guess is within α% of the actual average of the other peoples' guesses that he will be paid a large prize of β dollars. The change of winning the price provides each respondent with the incentive to attempt seriously to guess the average guesses of others, and since his most important information about others' true valuations is his own valuation, his guess will, if properly interpreted, reveal unbiased information about his own true valuation of the public good.

The underlying hypothesis in such a technique is that people base their guesses about the average of a characteristic in others on the level of that characteristic in themselves plus a partial but unbiased belief about their own relative position in the group.

Now since it is impossible to test such a hypothesis for a characteristic like people's true valuation of a public good, we have designed and conducted an experiment in guessing about the average of a measurable, but not commonly known, characteristic of members in a well-defined group. The results of this experiment were used in designing and interpreting a survey method of public good valuation.

5.1 Description of the Experiment-in-Guessing

A random sample of students drawn from the population of students at the University of California, Riverside (enrolled during the Winter Quarter of 1978) were sent copies of the attached letter.

The students who responded to the letter were scheduled for individual appointments during weekday mornings where they were read the following instructions and questions:

Procedure During Interview of "Experiment in Guessing"

[Establish identity of interviewee and close door for privacy].
[Record student control number __________].

The questions I will ask you are related to the amount of money that is usually carried by UCR students. Your answers will be strictly confidential.

First, will you please count the amount of money (U.S. currency and coins only) that you are now carrying. [Record amount __________(M)]

Second, what is the average amount of money that you carry in the morning of a school day? [Record amount __________(A)]

In the following question you will have an opportunity to win $50.00. Therefore, please pay close attention to what I will ask you to do, and do not answer until you are sure that you understand the situation.

You are one member of a group of 20 UCR students who will answer this question. Each of you will guess a number based upon a clue that I will give to all of you. The one member of the group who guesses closest to the average of the 20 guesses will win $50.00. Here is the clue: The number guessed should be close to the amount of money that an average UCR student carries in the morning of a school day. [If the student indicates that he does not understand, then tell him: "You are to guess as close as possible to the average guess of the others, realizing that all of you have been given the same clue." Reread the clue].

What is your guess? [Record amount __________(G)]

Thank you very much. That concludes the interview. As soon as we calculate the averages for each group, we will notify the winners. That will be in approximately 3 weeks. Thank you again for your help.

A total of 107 students were interviewed, and upon completion of the interviews the averages were calculated, and the winners were notified and paid their prizes in cash.

The objective of the experiment was to see if there was a systematic relationship between the value of a person's guess G_i about others' average behavior and his idea of his own average behavior N_i. The idea being that in the analogous public good method we would be attempting to measure the unknown ΣN_i by using the known ΣG_i. Therefore the fundamental question is: What is the nature of the random distribution of ΣG_i about the true value ΣN_i, and how does that distribution change as the sample size n gets large?

Our purpose in asking the first question in the procedure concerning

M_i was to focus each respondent's attention on the exact amount of money he currently was carrying so that he could more accurately form a judgment about the average amount he normally carries, N_i. It also provided an objectively measurable quantity ΣM_i as a check on the accuracy of beliefs about one's average behavior.

The characteristic -- the average amount of money that one carries -- was chosen for the experiment because it is something (like one's own valuation of a public good) that is known by each about himself but is very imperfectly known by each about others. Therefore when asked to guess about the average of this characteristic in others, it is natural to use one's own best knowledge (of oneself) plus some idea of one's relative position.

The results of the experiment provide a strong indication that people do base their guesses about others on knowledge about themselves and that their aggregate guesses are very accurate estimates of the average true value of the characteristic. The statistical results are presented below.

(Student number 25 was removed from the sample because his money carrying behavior was so extremely different than the other students that we could not expect their guesses to take account of his behavior. Student number 25 was carrying \$423.87 at the time of the interview and he said that he carries an average of \$150.00 each day).

Mean value of "Average Amount Carried":

$$\overline{N} = \frac{1}{106} \sum_{i=1}^{106} N_i = 5.6715$$

Mean value of "Average Guess":

$$\overline{G} = \frac{1}{106} \sum_{i=1}^{106} G_i = 5.8075$$

Suppose we assume that the average amount carried by a student is a random variable

(1) $N_i = \mu + \eta_i$, where μ is the "true" average amount carried by the entire population and η_i has a normal $(0,\sigma_\eta^2)$ distribution.

Suppose each student's guess G_i is a random variable defined by

(2) $G_i = N_i + \varepsilon_i$, where ε_i has a normal $(0,\sigma_\varepsilon^2)$ distribution.

Then we can write

(3) $G_i = \mu + \omega_i$, where $\omega_i \sim N(0,\sigma_\omega^2)$ and $\sigma_\omega^2 = \sigma_\eta^2 + \sigma_\varepsilon^2 + 2\text{cov}(\eta,\varepsilon)$

In a procedure where we do not know the value N_i (such as in the public good case), then using equation (3) we can use the observations on the G_i's and our knowledge of the distribution of ω_i to estimate the value of μ.

Suppose we consider the measured $\overline{N}$ to be the true population mean μ, then the estimate of the variance of ω_i calculated from the data is:

$$\delta^2_\omega = (3.812)^2$$

Using this estimate we can calculate the sample sizes that are required to achieve various levels of accuracy in the measurement of μ. Let $R_\beta(\alpha)$ denote the sample size required to be $(100\beta)\%$ certain that $\overline{G}$ is within $(100\alpha)\%$ of μ. Therefore $R_\beta(\alpha)$ is the smallest integer n required to guarantee that

$$P[\frac{1}{n}\sum_{i=1}^{n} G_i - \mu \leq \alpha\mu] \geq \beta$$

The following table shows selected values of $R_\beta(\alpha)$.

Table 5.1

α	$R_{.80}(\alpha)$	$R_{.90}(\alpha)$	$R_{.95}(\alpha)$
.10	75	123	174
.05	297	489	695
.01	7,425	12,225	17,355

By analogy, if the value of σ^2_ω is similar, then these numbers indicate that using a guessing technique for public good valuation will allow us to be 90% certain of obtaining a measure that is within 10% of the true social value by interviewing as few as 123 randomly selected consumers in the area. Of course, the value of σ^2_ω may not be the same, however, we will obtain an estimate of σ^2_ω as the interviews proceed, and it is possible to use a sequential technique to determine when the sample size is sufficient for a given level of accuracy.

A very important property of this procedure is that it provides a measure of the accuracy of the estimate obtained, and it is impossible to say the same thing of previously used methods. Further experimental studies are needed to substantiate the unbiasedness property of this type of procedure, however the results of our own experiment indicate that the method is quite promising.

Based on the results of the guessing experiment we propose the following method of determining the public good valuation by a specific group.

Public Good Guessing Procedure

For each of the selected respondents:

1. Describe the exact proposed change in the public good from level A to level B in a way that enables the respondent to form a clear conception of the difference.

2. Define a person's true valuation of the change from A to B as: the most that a person would be willing to pay per month in order to have B rather than A if that were the only way he could obtain B.
Alternative Definition: The amount per month that would be just slightly more than a person would be willing to pay in order to have B rather than A if that were the only way he could obtain B.

3. Read the following statement to the respondent:

   You are one member of a group of _____ people selected from (describe the population) who will guess a number based upon a clue that will be given to all of you. If your guess is within α% of the average of the guesses of the others then you will receive a price of β dollars. The clue is that your guess should be close to the average true valuation of the (described) public good by the people in the (described) population. What is your guess?

The above method of having each respondent attempt to guess the average of others' guesses where each knows that the others are given the same clue and are also trying to guess the average of the guesses is designed to avoid bias that originates from strategic behavior. To see that this is a potential problem, consider the following two-stage guessing procedure.

Two-Stage Procedure

Ask each member of a selected group the following questions:

1. What is your true valuation of the change from B to A?

2. You are one member of a group of _____ persons who have been asked the preceding question. If you can guess within α% of the average of the others' stated valuations (given in their answers to question 1.), then you will win a prize of β dollars. What is your guess?

The potential bias in the two-stage procedure originates with the possibility of strategic behavior in response to the first question. Since the respondent is offered no incentive to answer truthfully to question (1), [indeed, it is impossible here to use a prize as incentive for truthfulness since the respondent knows that there is no method of verification] it is natural for him to consider the effect his response will have on either a project approval or a project financing decision. As soon as he forms a belief about this relationship then he is rational to give a stated valuation that he believes will influence the outcome in his favor. The fact that his subjective belief about the relationship may be incorrect does not alter the fact that it is costless for him to overstate or understate his valuation in the direction of his own perceived interest, and therefore, he probably will. When he is asked to guess the average stated valuations of the others, he will immediately realize that they also had incentive to distort their responses; hence, in order to win the prize, he must guess in the direction of their distortions rather than toward what he believes is their true average valuation. To argue that people are too unsophisticated to go quickly through this complicated chain of reasoning when responding to such seemingly hypothetical questions is to ignore the fact that even ordinarily dull people become quite suspicious when their own self interest may be involved. The result of this is that the average guess in the Two-Stage Procedure is likely to be biased in an unpredictable direction.

In contrast to the Two-Stage Procedure, the proposed Public Good Guessing Procedure offers no net incentive for strategic behavior. Each person has incentive to guess a number that is as close as possible to the average of the guesses by others. If each believes that the others are trying (as the clue suggests) to guess close to the true average valuation, then he will seriously attempt to guess near what they believe is the true average valuation. Neither he nor they have any incentive for over or under bidding; therefore, the average of the guesses is likely to be close to the average of the true valuations. The results of the guessing experiment suggest that this is indeed the case. Any incentive to state a guess that will strategically affect the outcome of the public good decision is offset by the incentive to win the cash prize, if the prize is high enough.

5.2 Incentive Structure of the Proposed Public Good Guessing Procedure

In contrast to the Two-Stage Procedure, the proposed Public Good Guessing Procedure does not reference people's guesses to previously stated valuations or bids. Instead, it uses a simultaneous guessing method having only the given clue, "the average true valuation of the public good by the people in the described population," as a common reference point. Each respondent knows that none of the respondents can exactly know the "average true valuation;" however, each has incentive (in the form of the prize) to attempt to guess what other people think this value is, since the prize is won by guessing close to the average guess of others. The respondents will use strategic behavior; however, in this case (if the prize is large enough), the objective of the strategic behavior will be to win the cash prize rather than to affect the outcome of the public good decision. The Guessing Experiment conducted at the University of California, Riverside, indicated that if the respondents do use strategic behavior to win the prize, then their aggregate guesses will accurately reveal their aggregate true valuation of the public good. Therefore, we see that rather than attempting to eliminate strategic behavior, the proposed method redirects the respondent's strategy in a way the reveals public good valuation.

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO.	2.
3. RECIPIENT'S ACCESSION NO.	
4. TITLE AND SUBTITLE Methods Development for Assessing Air Pollution Control Benefits: Volume IV, Studies on Partial Equilibrium Approaches to Valuation of Environmental Amenities	5. REPORT DATE February 1979
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Maureen L. Cropper, William R. Porter, Berton J. Hansen, Robert A. Jones, and John G. Riley	8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Wyoming Laramie, Wyoming 82071	10. PROGRAM ELEMENT NO. 1HA616 and 630
	11. CONTRACT/GRANT NO. R805059-01
12. SPONSORING AGENCY NAME AND ADDRESS Office of Health and Ecological Effects Office of Research and Development U.S. Environmental Protection Agency Washington, DC 20460	13. TYPE OF REPORT AND PERIOD COVERED Interim Final, 10/76-10/78
	14. SPONSORING AGENCY CODE EPA-600/18
15. SUPPLEMENTARY NOTES	

16. ABSTRACT

The research presented in this volume of a five volume study of the economic benefits of air pollution control explores various facets of the two central project objectives that have not been given adequate attention in the previous volumes. The valuations developed in these volumes have all been based on a partial equilibrium framework. W.R. Porter considers the adjustments and changes in underlying assumptions these values would require if they were to be derived in a general equilibrium framework. In a second purely theoretical paper, Robert Jones and John Riley examine the impact upon the aformentioned partial equilibrium valuations under variation in consumer uncertainty about the health hazards associated with various forms of consumption.

Two empirical efforts conclude the volume. M.L. Cropper employs and empirically tests a new model of the variations in wages for assorted occupations across cities in order to establish an estimate of willingness to pay for environmental amenities. The valuation she obtains for a 30 percent reduction in air pollution concentrations accords very closely with the valuations reported in earlier volumes. The volume concludes with a report of a small experiment by W.R. Porter and B.J. Hansen intended to test a particular way to remove any biases that bidding game respondents have to distort their true valuations.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
Economic analysis Air pollution	Economic benefits of pollution control	13B

18. DISTRIBUTION STATEMENT Release unlimited	19. SECURITY CLASS *(This Report)* Unclassified	21. NO. OF PAGES 101
	20. SECURITY CLASS *(This page)* Unclassified	22. PRICE

EPA Form 2220-1 (9-73)

☆U.S. GOVERNMENT PRINTING OFFICE: 1979 O—620-007/3763 REGION 3-1

United States
Environmental Protection
Agency
Washington DC 20460

RD-683

Official Business
Penalty for Private Use
$300

Special Fourth-Class Rate
Book
Postage and Fees Paid
EPA
Permit No. G-35