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Project Summary

Analysis of Persistence in Model Ecosystems: Deterministic and Stochastic Food Web Models

Thomas C. Gard

Mathematical models aid in understanding environmental systems and in developing testable hypotheses relevant to the fate and ecological effects of toxic substances in such systems. Within the framework of microcosm or laboratory ecosystem modeling, some differential equation models, in particular, become tractable to mathematical analysis when the focus is on the problem of persistence.

In this report, a microcosm-related, nutrient-producer-grazer, chemostat-chain model and general food web models are analyzed for persistence. The results, which take the form of inequalities involving model parameters, specify sufficient conditions for continued presence of the model components throughout indefinite time intervals. These results can serve as a basis for a preliminary evaluation of model performance.

This Project Summary was developed by EPA's Environmental Research Laboratory, Athens, GA, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).

Introduction

Persistence in mathematical representations of ecosystems in the analogue for survival of organisms and continued presence of nutrients in the modeled system. Although exact solutions of the nonlinear differential equations describing growth-decay rates of the substances or organisms are impossible to obtain, qualitative investigations for persistence are possible. The technique consists of constructing auxiliary functions and differential inequalities to determine conditions for persistence from model parameters.

This technique is applied to a variety of generally accepted ecological models. Specifically, previous results for a nutrient-producer-grazer chemostat-chain have been improved; sufficient as well as necessary conditions for persistence are obtained. General Lotka-Volterra type deterministic and stochastic food web models are investigated. Sufficient conditions for persistence of a top-level predator are established for models that can represent arbitrary numbers of trophic levels and species per trophic level and arbitrary degrees of intraspecific competition and omnivory.

All the results, which take the form of inequalities involving model parameter bounds, specify conditions for continued presence of the model components for an indefinite time. These results can serve as a basis for preliminary model evaluation and experimental design.

Summary

The mathematical analysis focusing on persistence in differential equation models of current and potential interest to the U.S. Environmental Protection Agency results in criteria taking the form of inequalities involving model parameters.

For the chemostat-chain model, the main result is expressed in terms of a threshold value for the nutrient input rate. In the case of deterministic food web models, positivity of minimum weighted net growth rates constitutes the persistence conditions. For stochastic food web models, these net growth rates must exceed corresponding weighted sums of the random fluctuation intensities. Generally, the nutrient input level or a sufficiently large total intrinsic growth rate of the food sources guarantees persistence.

The methods used consist of approximating the differential equations; in particular, the auxiliary function-comparison principle techniques are the main tools employed from the qualitative theory of ordinary and stochastic differential equations. The extension of the results previously reported to the case of general food web models demonstrates the robust nature of this approach to the problem of ecosystem model stability.

Thomas C. Gard is with University of Georgia, Athens, GA 30602.
***Harvey W. Holm** is the EPA Project Officer (see below).*
The complete report, entitled "Analysis of Persistence in Model Ecosystems: Deterministic and Stochastic Food Web Models," (Order No. PB 84-226 984; Cost: $7.00, subject to change) will be available only from:
National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
Telephone: 703-487-4650
The EPA Project Officer can be contacted at:
Environmental Research Laboratory
U.S. Environmental Protection Agency
Athens, GA 30614

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