



Landfill and Surface Impoundment Performance Evaluation

LANDFILL AND SURFACE IMPOUNDMENT
PERFORMANCE EVALUATION MANUAL

SUBMITTED BY

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SUBMITTED TO

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Permit Writers Guidance Manual/Technical Resource Document

Preface

The land disposal of hazardous waste is subject to the requirements of Subtitle C of the Resource Conservation and Recovery Act of 1976. This Act requires that the treatment, storage, or disposal of hazardous wastes after November 19, 1980, be carried out in accordance with a permit. The one exception to this rule is that facilities in existence as of November 19, 1980 may continue operations until final administrative disposition is made of the permit application (providing that the facility complies with the Interim Status Standards for disposers of hazardous waste in 40 CFR Part 265). Owners or operators of new facilities must apply for and receive a permit before beginning operation of such a facility.

The Interim Status Standards (40 CFR Part 265) and some of the administrative portions of the Permit Standards (40 CFR Part 264) were published by EPA in the Federal Register on May 19, 1980. EPA will soon publish technical permit standards in Part 264 for hazardous waste disposal facilities. These regulations will ensure the protection of human health and the environment by requiring evaluations of hazardous waste management facilities in terms of both site-specific factors and the nature of the waste that the facility will manage.

The permit official must review and evaluate permit applications to determine whether the proposed objectives, design, and operation of a land disposal facility will be in compliance with all applicable provisions of the regulations (40 CFR 264).

EPA is preparing two types of documents for permit officials responsible for hazardous waste landfills, surface impoundments, and land treatment facilities: Permit Writers Guidance Manuals and Technical Resource Documents. The Permit Writers Guidance Manuals provide guidance for conducting the review and evaluation of a permit application for site-specific control objectives and designs. The Technical Resource Documents support the Permit Writers Guidance Manuals in certain areas (i.e. liners, leachate management, closure, covers, water balance) by describing current technologies and methods for evaluating the performance of the applicant's design. The information and guidance presented in these manuals constitute a suggested approach for review and evaluation based on best engineering judgments. There may be alternative and equivalent methods for conducting the review and evaluation. However,

if the results of these methods differ from those of the EPA method, their validity may have to be validated by the applicant.

In reviewing and evaluating the permit application, the permit official must make all decisions in a well defined and well documented manner. Once an initial decision is made to issue or deny the permit, the Subtitle C regulations (40 CFR 124.6, 124.7 and 124.8) require preparation of either a statement of basis or a fact sheet that discusses the reasons behind the decision. The statement of basis or fact sheet then becomes part of the permit review process specified in 40 CFR 124.6-124.20.

These manuals are intended to assist the permit official in arriving at a logical, well-defined, and well-documented decision. Checklists and logic flow diagrams are provided throughout the manuals to ensure that necessary factors are considered in the decision process. Technical data are presented to enable the permit official to identify proposed designs that may require more detailed analysis because of a deviation from suggested practices. The technical data are not meant to provide rigid guidelines for arriving at a decision. References are cited throughout the manuals to provide further guidance for the permit official when necessary.

FOREWORD

I would like to briefly discuss the philosophy upon which this Evaluation Procedures Manual is based. The problem of transport of liquids through hazardous waste landfills and surface impoundments is technically quite complicated. Moreover, the analytical techniques that are currently available do not allow the problem to be as comprehensively treated as would be desired. Nevertheless, I have tried to avoid an approach that reverts to empiricism and rules of thumb.

It is important to recognize that many evaluators will not possess strong technical backgrounds in the area of transport processes, thus this Evaluation Procedures Manual does not involve extremely complicated mathematics. I have used linearized versions of equations and have used solutions for simplified boundary conditions so that it is not necessary to evaluate overly complicated formulae.

Nevertheless, the analytical principles upon which the evaluation procedure is based are, in my opinion, sound ones and should provide the foundation for future analytical developments in the area.

This philosophical approach has three important implications:

- (1) As better analytical techniques are developed, it will be possible to modify the evaluation procedure in a rational and consistent manner. At the design level, acceptable configurations for landfills and surface impoundments will change gradually rather than abruptly. Thus the evaluator will not be placed in the awkward position of having to explain to the designer that a design that was acceptable last year is seriously out of compliance this year.
- (2) Engineering firms that design hazardous waste landfills and surface impoundments will be able to use more sophisticated analytical techniques if they desire. For example, they may wish to use nonlinear versions of equations or more comprehensive boundary conditions for equations, thereby introducing more realism into the analysis. However, because such analytical approaches are compatible with the approach being used by the evaluator, the designer will be able to explain the reason for differences in the results of the two analyses and be able to more easily convince the evaluator that the more progressive analytical approach yields an acceptable, though hopefully more economical, design.

- (3) The analytical approaches presented in this manual provide a quantitative basis upon which the evaluator and designer can discuss possible modifications so that an unacceptable design configuration can be transformed into one that is acceptable. This approach avoids the dilemma that the designer sometimes faces of (1) being told that a design violates an irrational rule of thumb criterion, but (2) being given no guidance on how to modify the design to comply with the intent of the requirements.

The burden is placed on the designer to provide a design that has quantitatively documented capabilities for containment of liquids. Designs cannot be rationalized based on statements that the design "has worked before" or that components whose effects have not been analyzed "contribute to retarding liquid movement." Designers must approach liquid routing through landfills and surface impoundments facilities as a discrete analysis task, and they will be required to substantiate their designs quantitatively. The Evaluation Procedures Manual implicitly provides techniques that can be used by the designer. A logical choice for the designer to make would be to use the same analytical procedures to arrive at the proposed design that the evaluator will be using in determining if the design is acceptable.

Chapters 2 and 3 describe the physical attributes of the facilities for which the evaluation procedure has been developed. Chapter 4 provides the analytical basis for the evaluation procedure. The fundamental physical-mathematical principles are presented and the resulting equations are given. However, intermediate steps that involve tedious algebra are not included in the present manual but are reserved for the technical support document.

Chapter 5 presents the detailed evaluation procedure and serves as a checklist. The experienced evaluator will use this chapter only. Appendices A and B present example evaluations. Appendix C lists symbols, and Appendix D gives sources of additional information.

In conclusion, I hope that this Evaluation Procedures Manual will provide a straight forward, analytically sound basis for the rational design of hazardous waste landfills and surface impoundments with respect to their ability to provide containment of liquids.

I would like to especially acknowledge the assistance of Mike Roulter, Dirk Brunner, Youssef Dakhouli, Jawed Umerani, Ruth Foltz and Susan DeHart. The contributions of Dr. Vincent T. Ricca are especially acknowledged.

Charles A. Moore
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1. INTRODUCTION

1.1 Purpose

This Evaluation Procedures Manual has been developed to describe the technical approach and to present equations for determining how the design of hazardous waste surface impoundments and landfills will function in controlling the quantity of liquids released to the environment.

The procedures described herein should allow an evaluator to determine the adequacy of designs for:

- (1) compacted clay liners or synthetic liners intended to impede the vertical flow of liquids,
- (2) sand or gravel drainage layers intended to convey liquids laterally into collection systems,
- (3) slopes on such liner systems, and
- (4) spacings of collector drains.

1.2 Relationship to Other Manuals

This procedures manual relates to other manuals as shown in Figure 1.1:

- (1) Hydrologic Simulation on Solid Waste Disposal Sites prepared by the U.S. Army Corps of Engineers, Waterways Experiment Station. This manual provides the analytical basis for determining the partitioning of rainfall into surface runoff and infiltration. The water that infiltrates is, in turn, partitioned into that which returns to the atmosphere through evapo-transpiration, that which is stored in the cover soil, and that which percolates downward into the landfill. The last of these components, percolation, becomes the principal input to the present manual because it is the inflow due to percolation that must be adequately controlled as it is routed through the landfill. The Hydrologic Simulation on Solid Waste Disposal Sites manual provides the inflow on a daily basis, based upon the CREAMS model developed by the U.S. Department of Agriculture. Alternate sources of this type of information could be provided by Use of the Water Balance Method for Predicting Leachate Generation from Solid Waste Disposal Sites (Fenn, Hanley and DeGeare, 1975) which is based upon the principles developed by Thornthwaite and Mather (1955 and 1957).

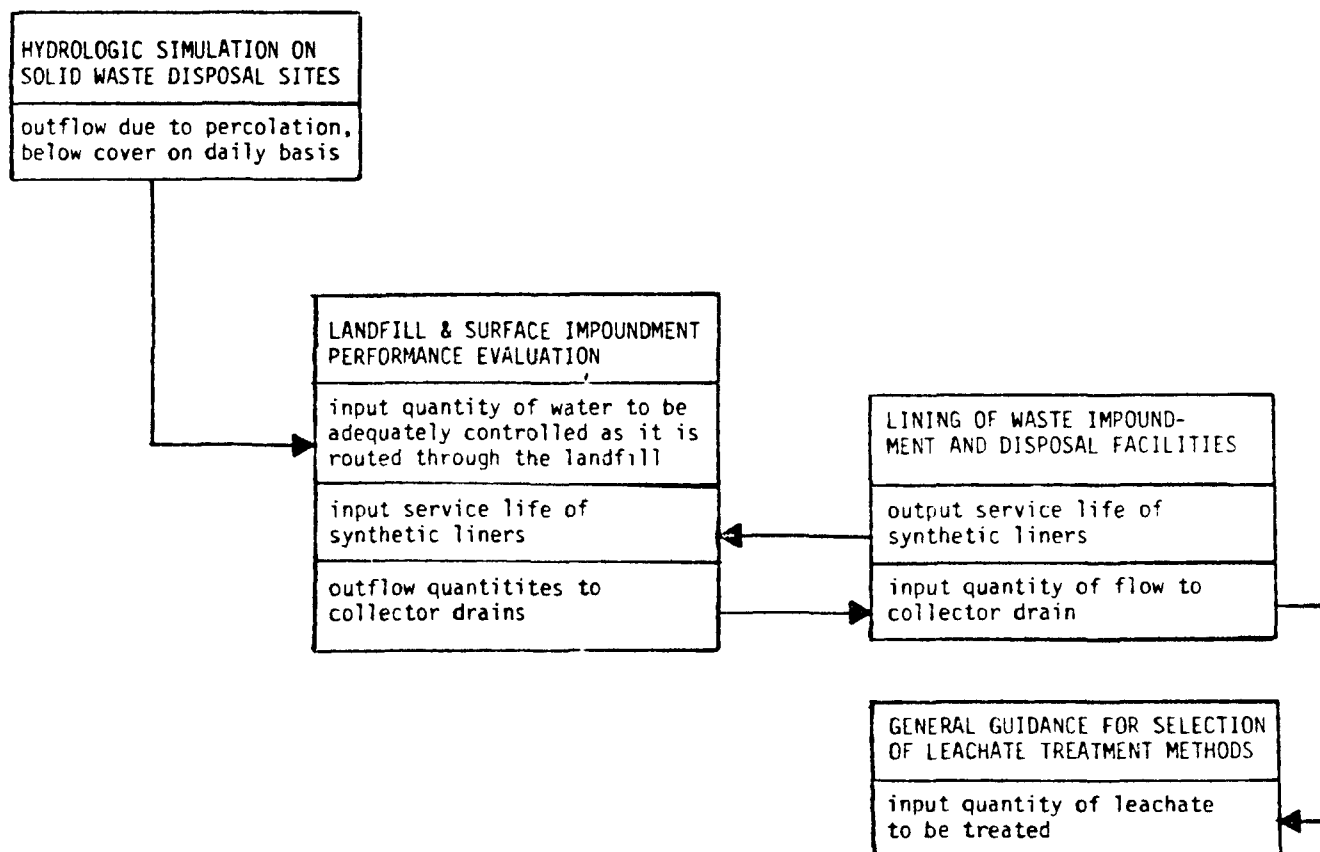


Figure 1.1 Relationship of this Procedures Manual to other Procedures Manuals.

- (2) Lining of Waste Impoundment and Disposal Facilities, by Henry Haxo of Matrecon and John Pacey of EMCON Associates. This manual relates to the present manual in two ways. First, it will provide information that can be used to determine the service life of synthetic liners. Second, the outflow quantities to collector drains from sand and gravel drainage layers determined according to procedures described in the present manual will become input quantities to the section of The Lining of Waste Impoundment and Disposal Facilities Manual that is concerned with sizing and layout of collector drains.
- (3) Hazardous Waste Leachate Management Manual by Monsanto Research. The outflow quantities to collector drains determined in the present manual will become an indirect input to the Hazardous Waste Leachate Management Manual.

2. APPLICABLE OPERATING CONDITIONS

This manual has been prepared with the assumption that the operating conditions for a hazardous waste landfill or surface impoundment meet the basic requirements of good engineering design. For example, in the case of landfills it is presumed that:

- (1) surface water run-on has been intercepted and directed from the site so that only the rainfall impinging directly on the landfill need be accounted for,
- (2) proper precautions have been taken to insure the integrity of cover soils so that erosion will not degrade cover performance,
- (3) synthetic liners have been properly installed so that their integrity is assured for their design life,
- (4) ground water flowing laterally into the landfill has been intercepted or otherwise diverted around the site,
- (5) artesian pressures in strata underlying the landfill have been relieved so that the hydrostatic head in the artesian aquifer lies below the base of the landfill,
- (6) during construction of the landfill water on the site has been controlled, and
- (7) the designs proposed for the components of the landfill itself form a properly functioning system.

In the case of hazardous waste lagoons it is presumed that:

- (1) inlet and outlet structures have been designed and rate of flow controlled so that there is no danger of scour of liners,
- (2) freeboard design and slope protection result in no detrimental wave action.

This manual is not intended to provide a means of proving that a poor design will not work. Its purpose, rather, is to allow the evaluator to confirm the adequacy of the design. It is the responsibility of the design engineer to propose an adequate design initially. Nevertheless, the principles used in the evaluation procedures described in this manual are in accord with the established principles of engineering analysis. Moreover, an attempt has been made to present the physical principles employed in these evaluation procedures in such a manner that a design engineer could, if he so desired, choose to use the same principles in arriving at a proposed design.

3. APPLICABLE DESIGN CONFIGURATIONS

It would not be practical or appropriate in this manual to specify exact configurations for hazardous waste landfill or surface impoundment designs. Rather, the approach followed here is to describe analytical procedures for evaluating the transport of liquids through simple modular configurations. With these analytical tools at his disposal, the evaluator can then interconnect several modules to simulate the specific configuration to be evaluated.

Figure 3.1 shows a hypothetical landfill cross-section including final cover, waste cells, intermediate or daily cover, and a drain and liner resting on undisturbed ground. Modules can be delineated as follows:

- (1) final cover with adjacent waste cell beneath,
- (2) intermediate cover with adjacent waste cells above and below, and
- (3) drain and liner system consisting (from top to bottom) of adjacent waste cell above, sand drain layer, synthetic liner and underlying undisturbed ground.

3.1 Functional Characteristics of Design Modules

Although Figure 3.1 represents only one of a variety of configurations that might be proposed, the modules will be examined in some detail to delineate the functional characteristics of each unit. This examination will form the basis for abstracting physical characteristics to be included in the modules for which analytical techniques will be presented. The configuration described here is hypothetical only and does not necessarily constitute a recommended design.

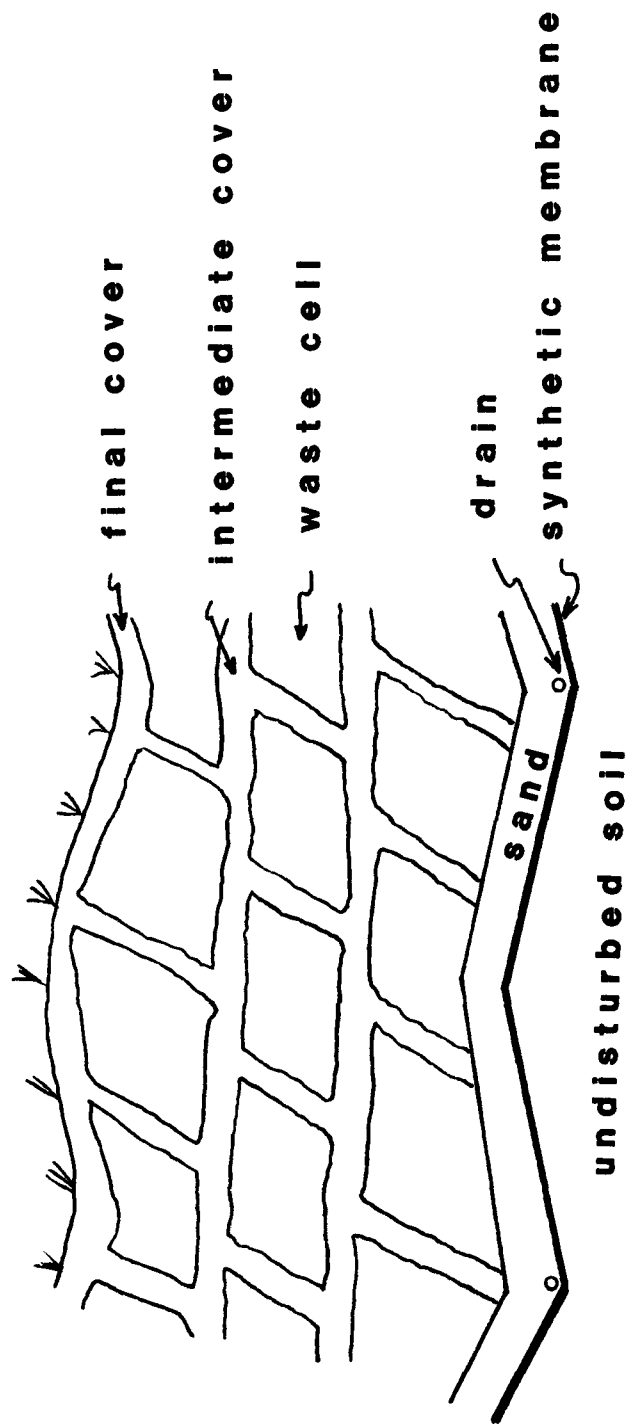


Figure 3.1 - Cross-section of landfill delineating typical containment modules.

The final cover shown in Figure 3.1 is redrawn in more detail in Figure 3.2a. It consists from bottom to top of:

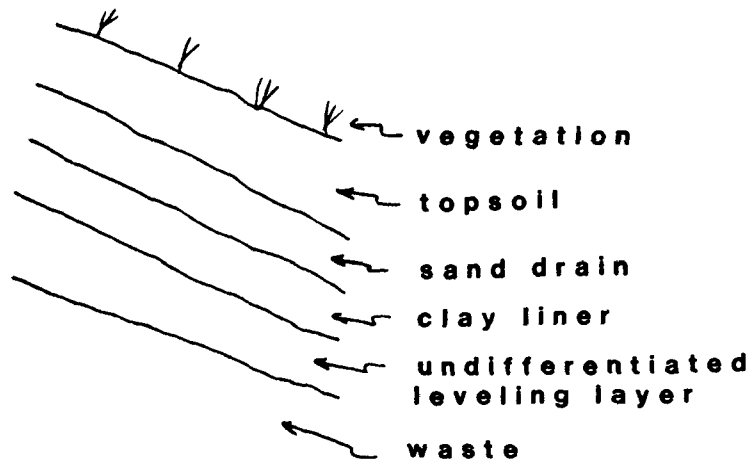
- (1) the waste,
- (2) an undifferentiated leveling layer whose purpose is to provide an even, reasonably firm base and controlled slope upon which to construct the final controlled cover,
- (3) a low permeability compacted clay liner to retard the rate of downward movement of liquid,
- (4) a high permeability sand or gravel drain to provide a horizontal pathway along which liquid collected on the clay liner can be transmitted to collector drains for diversion away from the landfill, and
- (5) a vegetated topsoil layer to provide both an opportunity for evapotranspiration to return liquid to the atmosphere and to provide a trafficable, erosion resistant surface upon which precipitation can be encouraged to flow horizontally for collection in drains and subsequent diversion away from or recirculation through the landfill.

The intermediate cover is drawn in more detail in Figure 3.2b. It consists of:

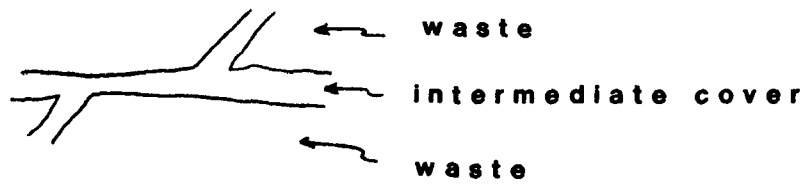
- (1) underlying waste,
- (2) a somewhat controlled layer of soil whose purpose is to provide a trafficable working surface and temporary diversion of water, and
- (3) overlying wastes.

The underlying drain and liner system is drawn in more detail in Figure 3.2c. It consists from bottom to top of:

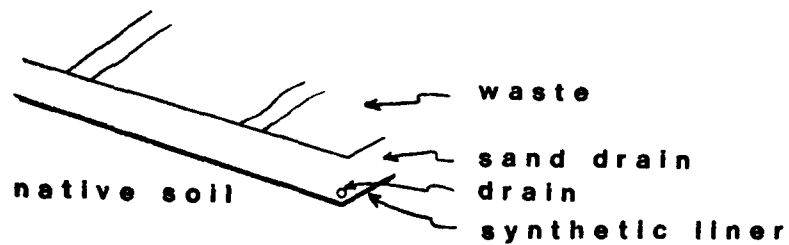
- (1) the undisturbed native soil to which the transmission of contaminated liquid must be controlled,
- (2) a very low permeability synthetic liner to restrict and control downward movement of liquids into the native soil,
- (3) a high permeability sand or gravel drain whose purpose is to provide a horizontal pathway along which liquid that collects on the synthetic liner can be transmitted to the collector drains for diversion away from the landfill, and
- (4) the overlying waste.



(a) detail of cover



(b) detail of intermediate cover



(c) detail of drain and liner system

Figure 3.2 - Detail views of modules constituting landfill cross-section of Figure 3.1.

3.2 Categorization of Functions of Design Units

The various layers described above perform differing functions with respect to the transmission of liquid through the landfill. Some are included for the purpose of controlling vertical flow of liquids because of their low permeability; e.g. (from top to bottom) the clay liner in the final cover, and the synthetic membrane in the bottom liner. The purpose of other layers of high permeability is to encourage flow to drains, e.g., the sand or gravel drain in the drain and liner system. Some layers are included because they are able to reduce the quantity of liquid available for leachate formation due to their evapo-transpiration properties; e.g., the topsoil. Finally, some layers serve functions not primarily concerned with their ability to control transmission of liquids; e.g., (from top to bottom) undifferentiated leveling layer, waste, and intermediate cover.

3.3 Definition of Units to be Included in the System for Control of Liquid Transmission

The above observations form the basis for the first axiom in evaluating the adequacy of containment in a hazardous waste surface impoundment:

Certain units within the design have as a primary purpose the control of liquid transmission. These units should be clearly delineated and their intended functions described by the designer in the design documents. When the adequacy of these units for performing their intended function is evaluated they should serve to control the transmission of liquids.

In submitting the facility plans the designer should:

- (1) have specifically referred to the unit as one intended for control of liquid transmission,
- (2) have described how the unit will function to achieve this control,
- (3) demonstrate that he has quantitatively assessed control capabilities of the unit in a rational manner, and
- (4) demonstrate that the unit can be reasonably expected to serve this function when constructed according to the specifications given in the design.

When reviewing these plans, units that do not meet all of these criteria should be considered only to provide a safety margin above the basic control requirements and should not be taken into consideration in evaluating the adequacy of the system to meet minimum control requirements.

3.4 Liquid Diversion Interfaces

It is evident from Figure 3.2 that many of the units that serve to control liquid transmission occur as modules consisting of an interface between two layers. There exist several distinct interfaces above which liquids are usually transmitted rapidly and in a horizontal direction, and below which liquids are usually transmitted slowly and in a vertical direction. It is the contrast in hydraulic transmissibility from high to low that accomplishes the change in flow direction from predominantly vertical to predominantly horizontal, thus diverting the flowing liquids to collector systems for subsequent conduction from the site. Examples are:

- (1) the interface between the atmosphere and the vegetative cover,
- (2) the interface between the sand drain and the compacted clay membrane in the final cover, and
- (3) the interface between the sand drain and the synthetic membrane in the bottom liner system.

3.5 Construction of Liquid Routing Diagrams

Figure 3.3 is a liquid routing diagram showing the several units comprising a landfill and clearly designating by the letters LTC those that are considered to be part of the Liquid Transmission Control system. Shown also is a series of lines and arrows that explain the routing of liquid on its course through the liquid transmission control system. The interfaces which are composed of high transmissibility units overlying low transmissibility units and which serve to divert flow direction from vertical to horizontal are designated by the symbol DI for Diversion Interface. This suggests the first step in the evaluation procedure:

If the designer has not already done so, construct a liquid routing diagram for the landfill. Designate the units that are considered to be part of the liquid transmission control system by the symbol LTC. Determine the location of diversion interfaces and label them DI. Use arrows to show the liquid transmission control mechanisms. If the designer has provided such a diagram, the evaluator should confirm that it represents an appropriate diagram for use in evaluating the proposed design.

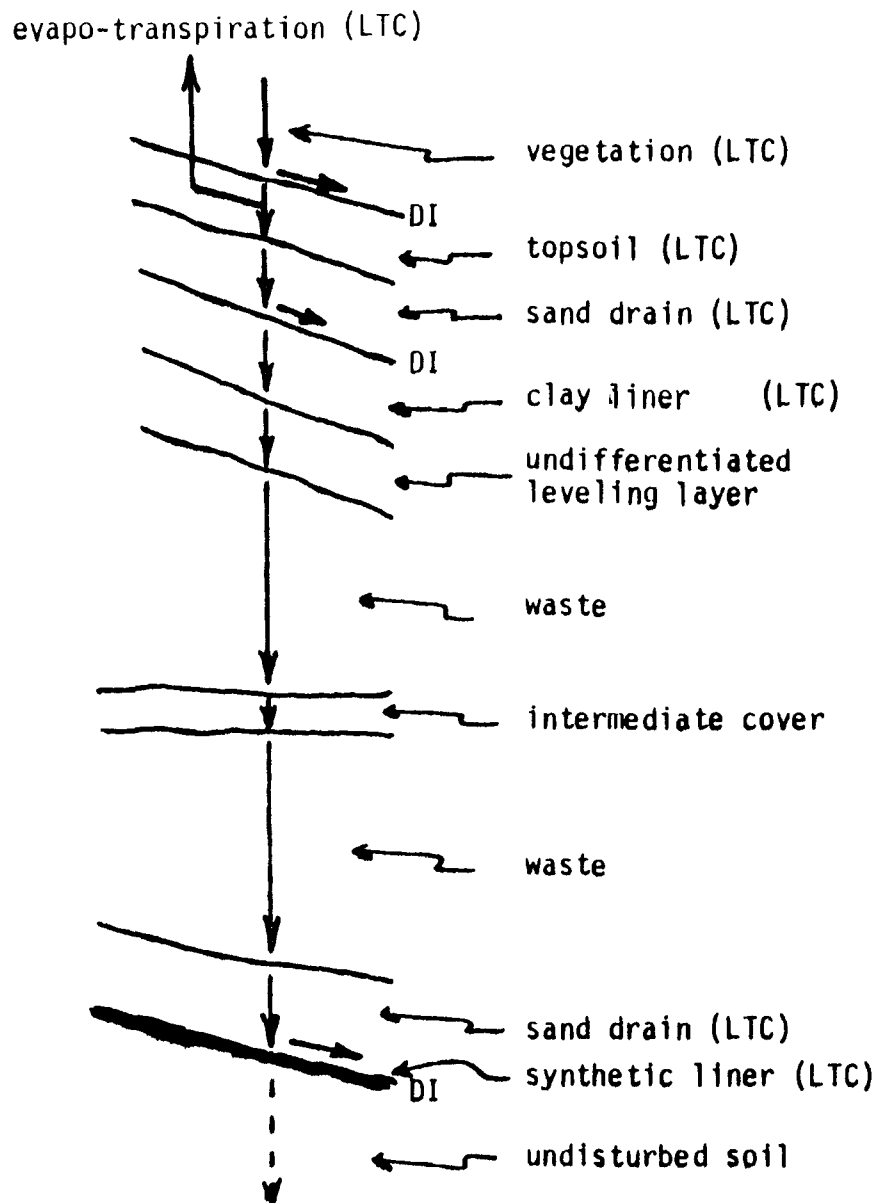


Figure 3.3 - Liquid routing diagram showing intended functions of components of leachate containment system.

4. OVERVIEW OF ANALYTICAL METHODS

4.1 Introduction

In section 3.4 it was observed that many modules intended to control transmission of liquids through landfills and surface impoundments consist of an interface overlain by a high transmissibility material (such as the atmosphere or a sand or gravel layer) and underlain by a low transmissibility material (such as compacted clay or synthetic membrane). The low transmissibility material diverts a portion of flow from vertical to horizontal at the interface and allows another portion of the flow to continue vertically through the liner. The more efficient the design, the larger will be the portion of liquid diverted to horizontal flow. The purpose of this chapter is to examine the principles that govern this process and to develop equations for quantitatively predicting the relative proportions between the flow that is diverted to a horizontal direction and the flow that continues to move vertically.

4.2 Horizontal Flow Through Sand and Gravel Drain Layers

Figure 4.1 shows the conditions assumed for calculating flow in sand and gravel drain layers. Figure 4.1a shows a layer of thickness $d(m)$ overlying a low permeability material. The system slopes symmetrically at an angle α (exaggerated in this drawing) down to drains spaced a distance $L(m)$ apart. The saturated permeability of the drain layer is $k_s(m/sec)$. Liquid impinges upon the system at a rate of $e(m/sec)$. The source of this liquid could be rainfall, recirculated leachate or liquid generated by the waste.

In the limiting case of $\alpha = 0$ (shown in Figure 4.1b) the shape of the water mound that accumulates in the drain layer has been given by Harr (1962) as

$$h = \left(\frac{e}{k_s} (L-x)x \right)^{1/2} \quad (4.2.1)$$

The maximum value of h occurs at $x = L/2$ for a horizontal liner and is given by

$$h_{\max} = \left(\frac{eL^2}{4k_s} \right)^{1/2} \quad (4.2.2)$$

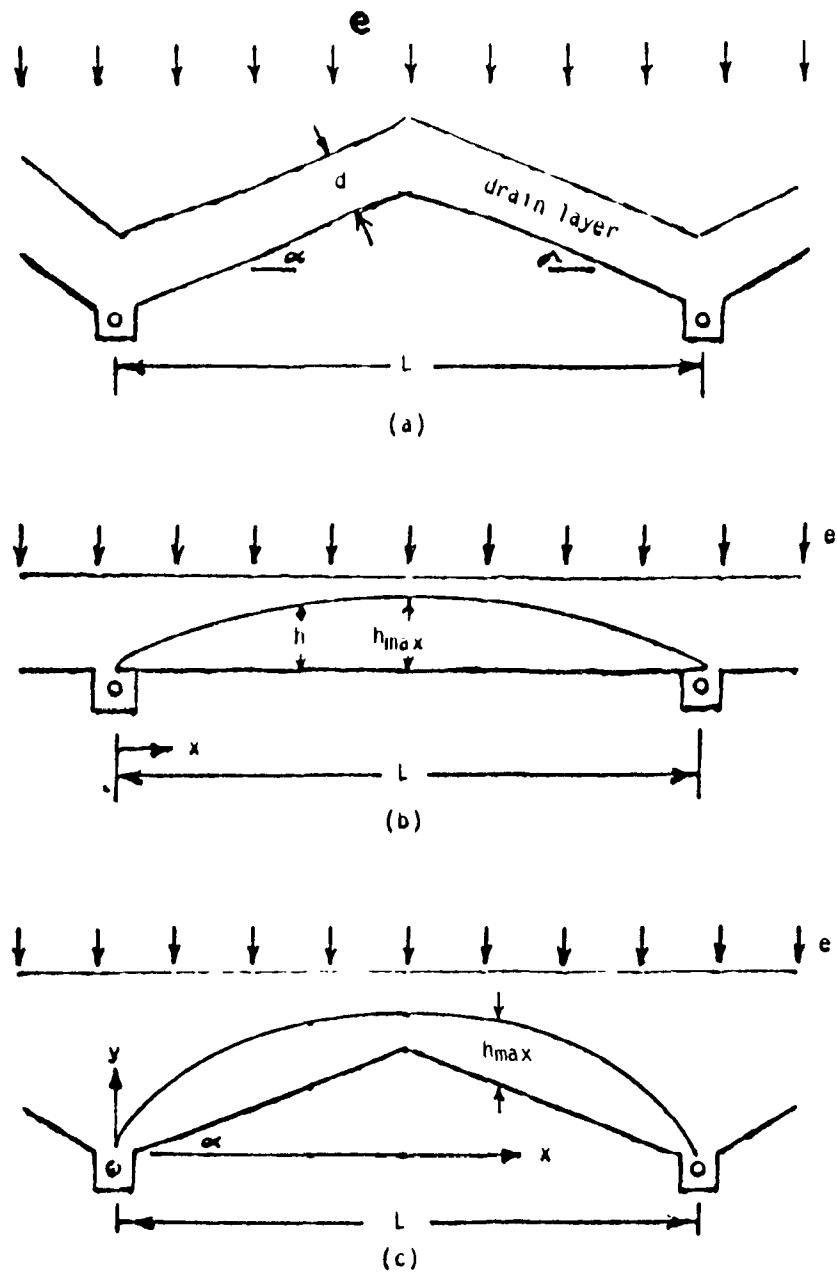


Figure 4.1 Geometry assumed for bounding solution for effectiveness of sand drains.

As an example, consider the case of a flat liner having drains placed 30 m (~ 100 ft.) apart in a sand having $k_s = 1 \times 10^{-3}$ cm/sec = 1×10^{-5} m/sec. Assume an annual rainfall of 100 cm/yr (39 in/yr) equally distributed in time so that $e = 3.17 \times 10^{-8}$ m/sec.

Thus

$$h_{\max} = \left(\frac{3.17 \times 10^{-8} \text{ m} (30)^2 \text{ m}^2 \text{ sec}}{4 \text{ sec } 1 \times 10^{-5} \text{ m}} \right)^{1/2} \quad (4.2.3)$$

$$= 0.84 \text{ m}$$

An upper bound on the quantity of liquid flowing into the drains (assuming no penetration into the liner) is given by

$$q = eL \quad (4.2.4)$$

$$= \frac{100 \text{ cm } 30 \text{ m}}{\text{yr } 100 \text{ cm}}$$

$$= 30 \text{ m}^3/\text{yr/m of drain}$$

$$= 8000 \text{ gallons/year/m of drain}$$

This calculation is based upon the fact that at steady-state all of the liquid that impinges on the drain layer must be carried by the collector pipes.

Obviously, it is impractical to design an open-topped, flat-bottomed landfill. Equation (4.2.2), however, provides a simple computational procedure for setting a bound on the thickness, d , required for the drain layer. Thus if d exceeds h_{\max} , there will be no danger of leachate overfilling the drain layer and rising into the hazardous waste.

The concept of a bound is often quite useful in engineering design, particularly when considered in conjunction with what might be called "other factors." Suppose, for example, that in the above calculations h_{\max} had been found to be 10 cm (4 in). The designer might then wish to specify that the drain layer be made 10 cm thick under the premise that

- (1) this thickness would satisfy the bound criterion,
- (2) usual construction procedures make it impractical emplacing a layer thinner than approximately 10 cm, and

- (3) the cost of placing the bounding thickness might not be significantly different from the cost of placing a lesser thickness based upon a more exact analysis.

The same rationale, of course, holds for the collector drain lines. Designing them to accommodate a bounding flow based on eq. (4.2.4) may be entirely feasible economically. Engineers often consider overdesign based upon bounding solutions to be acceptable practice in the case of one-time installations that are located in physically inaccessible places. This is particularly true when the impact of overdesign on the economics of the operation is small.

In the case of the example problem, the 100 cm per year of liquid impinging on the layer may be unrealistically high. If for example, only 10 cm of liquid impinged upon the drain layer because

- (1) the regional rainfall was less than 100 cm/yr,
- (2) only a fraction of the rainfall infiltrated,
- (3) losses occurred due to evapo-transpiration, and/or
- (4) a portion of the inflow was diverted by overlying control units,

then the bounding thickness would be only 0.27 m (10 in.). This thickness is within the realm of practicality. Alternatively, the designer could decrease L and/or choose a sand or gravel with a higher k_s (a k_s of 0.1 cm/sec is not impractical).

There are also other possibilities. Returning to Figure 4.1a, it can be seen that putting the drain system on a slope α not equal to 0 tends to accelerate the flow of water toward the collector system. Figure 4.1c shows the accumulation profile, and it is very much like that of Figure 4.1b, especially if α is small. (In fact, the situation of Figure 4.1b would be obtained by letting α approach 0.) The configuration with $\alpha \neq 0$ has some convenient properties when compared with $\alpha = 0$. The obvious one is that the hydraulic gradient toward the drain pipe is higher. This advantage, however, is not of greatest importance. The most significant advantage is that if the liquid were to cease impinging on the drain layer, the mound would completely drain out into the collector drain pipes in a finite amount of time if α is not equal to 0, whereas the drainage time for $\alpha = 0$ is infinitely long.

There will be a value for h_{\max} as shown in Figure 4.1c which is given by an expression similar to eq. (4.2.2):

$$h_{\max} = \frac{L\sqrt{c}}{2} \left[\frac{\tan^2 \alpha}{c} + 1 - \frac{\tan \alpha}{c} \sqrt{\tan^2 \alpha + c} \right] \quad (4.2.5)$$

where

$$c \equiv \frac{e}{k_s} \quad (4.2.6)$$

Therefore, the evaluator may wish to encourage the designer to adjust α , k_s and/or L to comply with the bounding criterion given in eq. (4.2.5). Figure 4.2 presents h_{\max}/L as a function of α for several values of $c = e/k_s$.

4.3 Vertical Flow Through Low Permeability Clay Liners

4.3.1 Physical Principles

The physical laws governing liquid moving downward through a low permeability clay liner are somewhat more complex than those governing liquid moving in sand and gravel drain layers. Because of the micropores that exist in clay soils, water moves not only by the hydraulic head caused by gravitational forces but also by capillary forces that tend to draw the liquid into the soil. Figure 4.3 illustrates this situation. The smaller the pore radius, the larger the capillary attraction force. Thus soils with a high clay content will have very small micropores and therefore very large capillary attraction forces. As the grain size of the soil increases, the capillary attraction force decreases; thus silty soils have lower capillary action than clays. Sandy soils have such large micropores that capillary attraction forces can reasonably be neglected as we in fact did in section 4.2.

The magnitude of the attraction due to capillary action is most conveniently measured in terms of a capillary potential, ψ . The units of ψ are in centimeters because it expresses the contribution of capillary attraction forces to the hydraulic head tending to move the liquid through the soil. Because pressure is conveniently positive, ψ is negative. Figure 4.4 shows an approximate range of values for ψ as a function of soil grain size. The purpose of this figure is not to give numerical values but rather to show that in clay sized soils the capillary potential becomes very large.

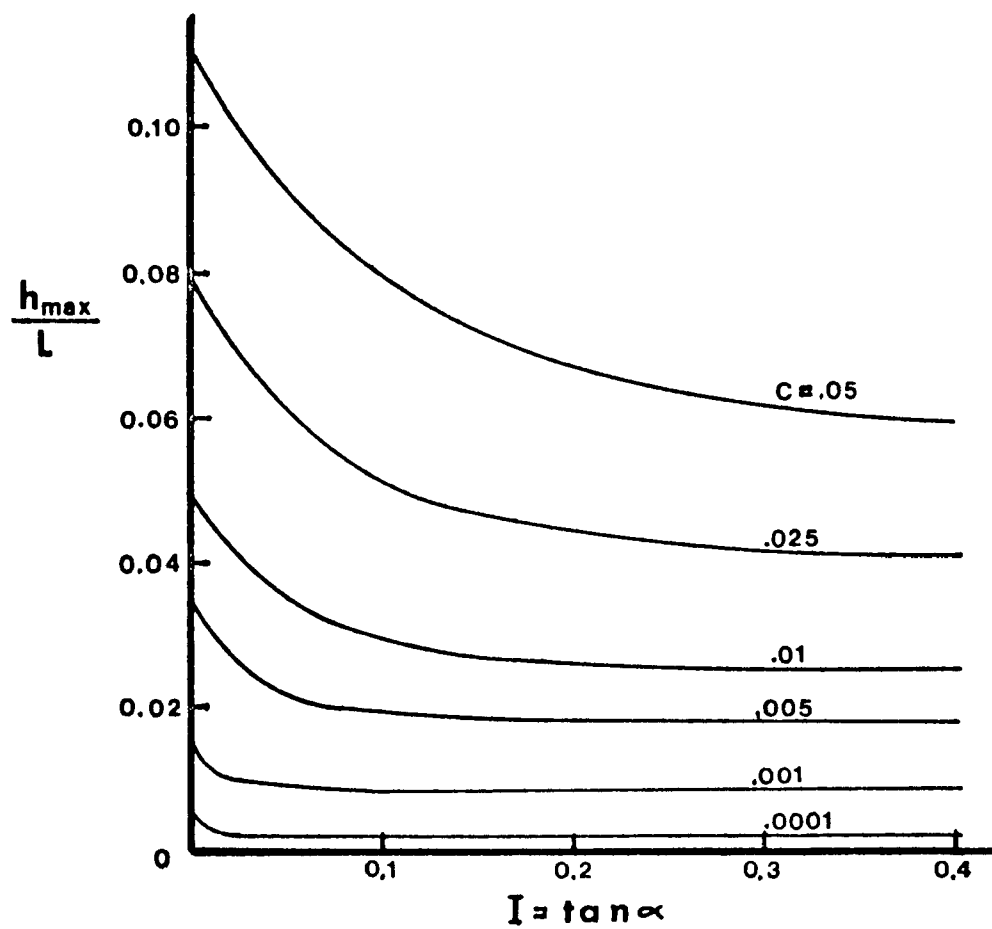


Figure 4.2 - Relationship between h_{\max}/L and $c = e/k_s$.

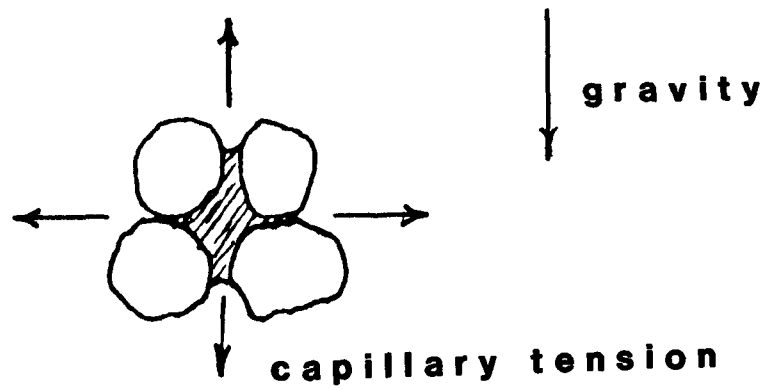


Figure 4.3 - Partially saturated soil showing menisci resulting in capillary tension in pore water.

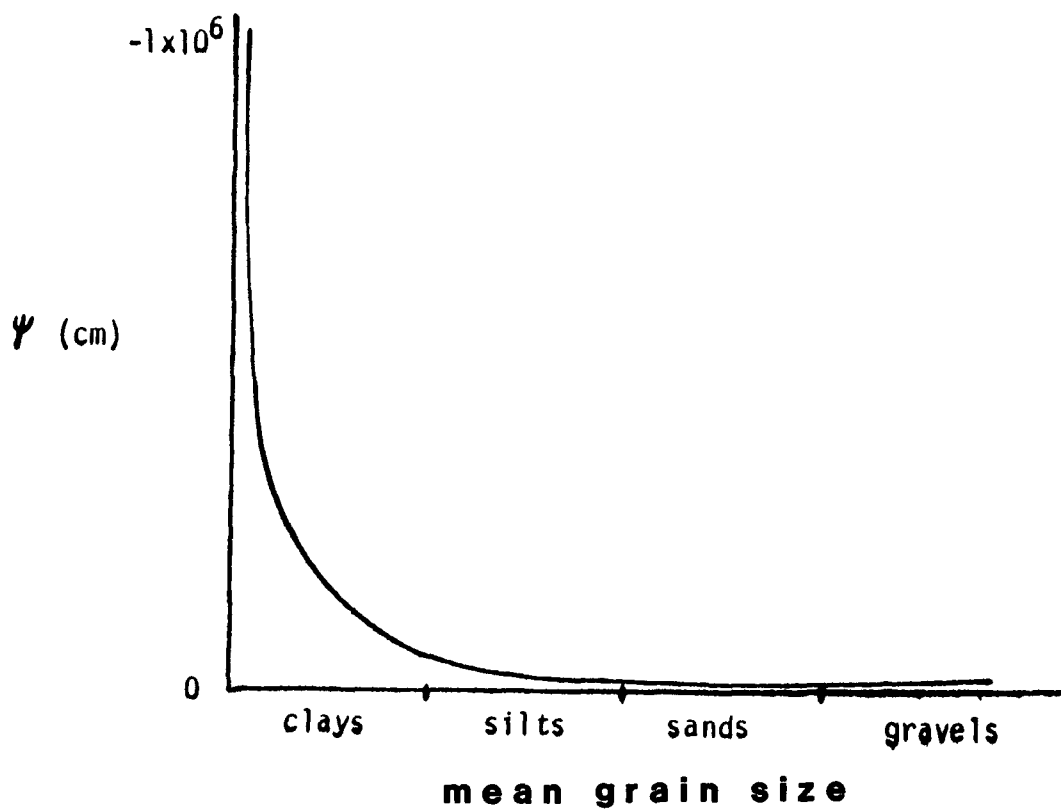


Figure 4.4 - Capillary potential versus mean soil grain size.

The real situation is even more complex. Because of the variety of different grain sizes present in soils, there will also be many different sizes of micropores. The smaller the micropore, the larger will be the capillary force attracting the water into the pore. As the moisture content of the soil increases, the smaller pores fill up with liquid and thus the capillaries tend to form in larger pores. The larger the pores within which the capillaries form, the smaller the value of the capillary potential. Thus, the higher the moisture content of the soil, the lower will be the capillary potential. Figure 4.5 shows the experimentally determined relationship between moisture content (θ) and capillary potential (ψ) for a popular research soil, the Yolo clay. Notice that the capillary potential passes through 7 orders of magnitude as the soil goes from dry to saturated. Moreover, when the soil is very dry the capillary forces are so large that they usually dominate gravitational forces; however, as moisture content nears saturation the capillary forces become so small that gravitational forces become dominant.

In practical applications, clay liners are usually compacted at a moisture content well below saturation. For the Yolo clay, the compaction moisture content might be around $\theta = 0.20$. From Figure 4.5 the capillary potential of this soil just after compaction would be of the order of $-1 \times 10^3 \text{ cm} = -10 \text{ m}$ (-33 feet). Thus, in order for gravitational forces to be of the same order of magnitude as the capillary forces, approximately 10 m of water would have to be standing on top of the liner. This situation would certainly be unacceptable in proposed landfill designs, and would not likely be encountered in lagoons.

This leads to two important axioms in evaluating compacted clay liners:

- (1) during the early stages of wetting of a compacted clay liner, capillary attraction forces will predominate over gravitational forces; and
- (2) as the clay liner becomes wetter the capillary forces decrease in importance; and, when the liner is saturated, these forces become negligible in comparison to gravitational forces.

During the life of a surface impoundment, the moisture content in the liner will gradually increase from the value at which the liner was compacted to the saturation water content. The process will, of course, not be uniform through the depth of the liner; rather, the top of the liner will become saturated very quickly and a wetting front will move downward through the liner. Figure 4.6 shows this process for the Yolo clay under conditions such that water is always present at the top of the liner but the depth of standing water is negligible. It can be seen, for example, that the wetting front moves on the order of 75 cm (30 in) in the first 10^6 sec (11.6 days). The saturated permeability of Yolo light clay is $1.2 \times 10^{-5} \text{ cm/sec}$.

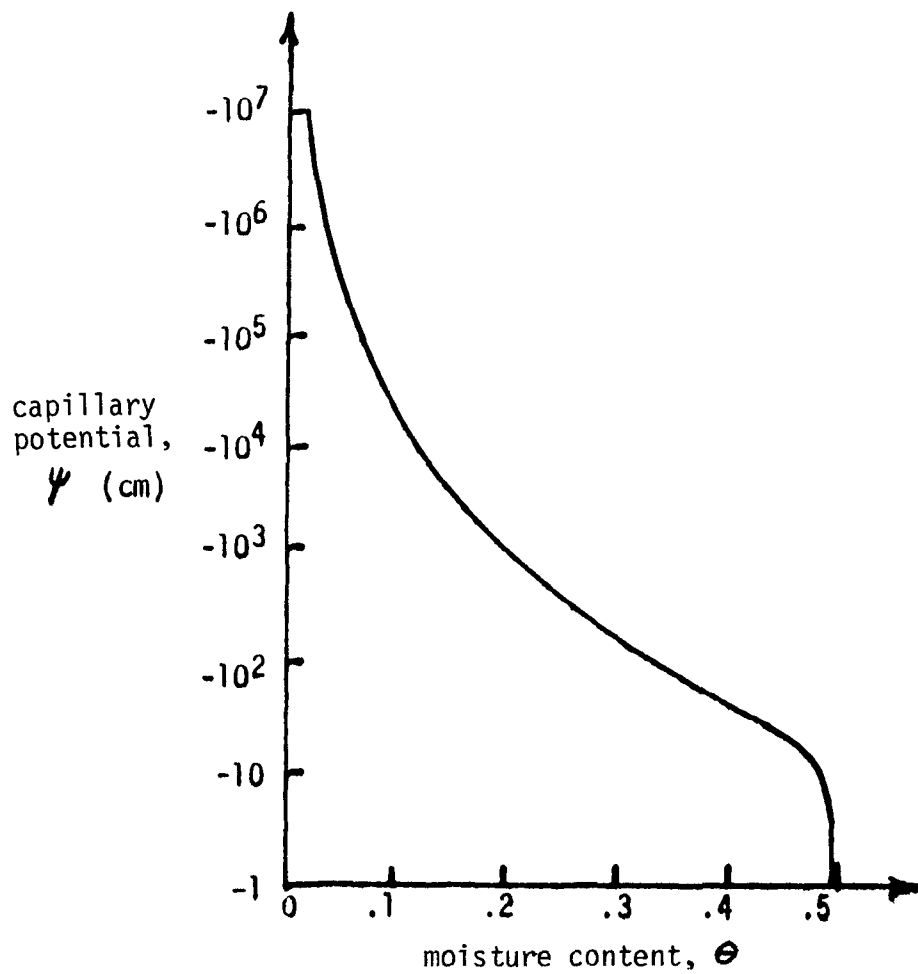


Figure 4.5 - Capillary potential as a function of water content for Yolo clay (from Philip, 1969).

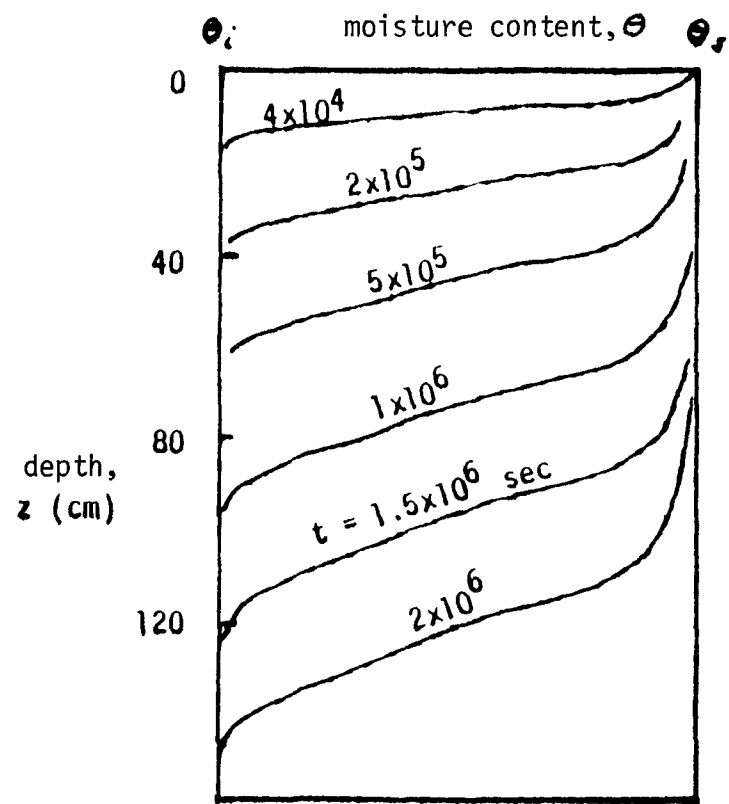


Figure 4.6 - Computed moisture profiles for infiltration into Yolo light clay (from Philip, 1969)

Assuming saturated flow under a unit hydraulic gradient, the liquid would require

$$\frac{75 \text{ cm}}{1.2 \times 10^{-5} \text{ cm/sec}} = 6.25 \times 10^6 \text{ sec} = 72 \text{ days}$$

to travel the same distance. Thus in this particular situation the rate of initial infiltration is 6.5 times as rapid as steady state saturated flow under a unit hydraulic gradient.

Philip (1969) showed that for infiltration into Yolo light clay under the above conditions, a time of 1×10^6 sec (11 days) is required before gravitational forces equal capillary forces in importance. We know then that throughout the 75 cm of infiltration shown above, capillary forces were dominant. (It should be noted that Yolo light clay would be a marginal liner material. The data for this soil have been used primarily because of their availability.)

The above discussion has important implications with respect to the evaluation of landfill liners:

- (1) If the evaluator is interested in determining the time required for saturation of a clay liner (which, in turn, represents the time to first appearance of leachate at bottom of liner), then the approach to analysis should be based upon the predominance of capillary forces.
- (2) If the evaluator is interested in determining the rate at which liquids will move through the liner on a long term (steady-state) basis, then the approach to analysis should be based upon saturated flow under gravitational forces.

4.3.2 Overview of Analytical Approaches

In this section the basic principles of analytical approaches will be described. The mathematics is complicated; however, it is important that the evaluator have an overview of these principles even though simplified mathematical equations will be given for application to the evaluation procedure.

The theory of infiltration is based upon the premise that Darcy's law holds for the transport process. Conceptually, Darcy's law states that the velocity of flow of water, U , is negatively proportional to the spatial change, $\nabla \phi$, of a driving potential function, ϕ . The constant proportionality is k . Thus

$$U = -k\Delta\phi. \quad (4.3.1)$$

For vertical flow in the z direction eq. (4.3.1) becomes

$$U = -k \frac{\partial \phi}{\partial z} \quad (4.3.2)$$

Applying the principle of conservation of mass and introducing eq. (4.3.2) yields an expression for the change in moisture content, θ , as a function of time, t :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial \phi}{\partial z} \right) \quad (4.3.3)$$

The driving potential, ϕ , consists of two parts:

- (1) the capillary potential, ψ , as previously defined, and
- (2) the gravitational potential, z , taken positive downward.

Thus

$$\phi = \psi - z \quad (4.3.4)$$

and eq. (4.3.3) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[k \frac{\partial (\psi - z)}{\partial z} \right] \quad (4.3.5a)$$

$$= \frac{\partial}{\partial z} \left[k \left(\frac{\partial \psi}{\partial z} - \cancel{\frac{\partial z}{\partial z}} \right) \right] \quad (4.3.5b)$$

$$= \frac{\partial}{\partial z} \left[k \frac{\partial \psi}{\partial z} - k \right] \quad (4.3.5c)$$

$$= \frac{\partial}{\partial z} \left(k \frac{\partial \psi}{\partial z} \right) - \frac{\partial k}{\partial z} . \quad (4.3.5d)$$

The principal difficulty with eq. (4.3.5) is that the dependent variable on the left side is θ , while the dependent variables on the right side are ψ and k . This can be remedied using the chain rule for differentiation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{d\psi}{d\theta} \frac{\partial \theta}{\partial z} \right) - \frac{dk}{d\theta} \frac{\partial \theta}{\partial z} \quad (4.3.6)$$

Note that the total differentials are used in the case of $d\psi/d\theta$ and $dk/d\theta$ because these relationships are functions of neither time, t , nor space, z . In fact $d\psi/d\theta$ has already been given in Figure 4.5 for the Yolo light clay. For completeness, Fig. 4.7 presents the relationship between k and θ from which $dk/d\theta$ can be determined.

It is notationally convenient to define two new parameters to represent two of the quantities in eq. (4.3.6):

$$D \equiv k \frac{d\psi}{d\theta} \quad (4.3.7)$$

and

$$K \equiv dk/d\theta \quad (4.3.8)$$

so that eq. (4.3.6) becomes

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D \frac{\partial \theta}{\partial z} \right) - K \frac{\partial \theta}{\partial z} \quad (4.3.9)$$

This equation has the desirable property that the dependent variable is moisture content, θ , on both sides. Thus it can be solved in conjunction with initial conditions and boundary conditions for a particular situation to give moisture content as a function of position and time.

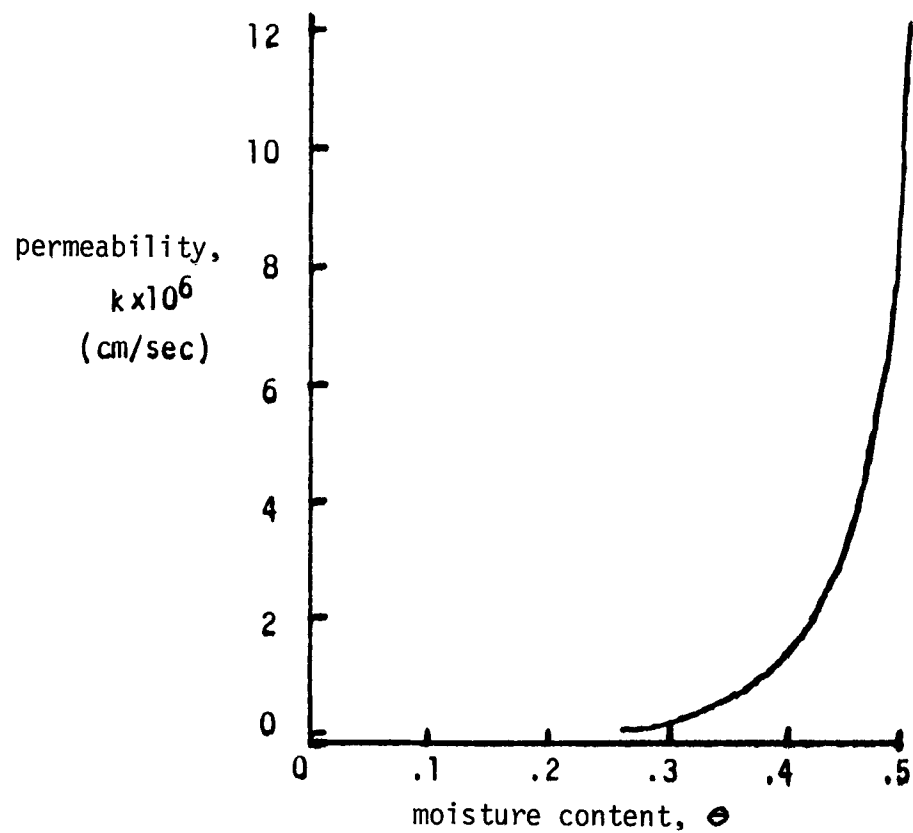


Figure 4.7 - Relationship between hydraulic conductivity and water content for Yolo light clay (from Philip, 1969)

Be assured, however, that eq. (4.3.9) is not very easily solved. Nevertheless, solution techniques are available (c.f. Philip, 1969, and Parlange, 1971a, b, c, 1972a, b, c, d, e, f), and the evaluator should be prepared to accommodate the innovative designer who has manipulated the undesirable mathematical attributes of eq. (4.3.9) into desirable design attributes for landfill liners.

However, for purposes of evaluating landfill liner designs it is necessary to transform eq. (4.3.9) into a more tractable form. In doing this it is important that the essential attributes of eq. (4.3.9) not be lost in a sea of empiricism. The basic form of eq. (4.3.9) must be retained, but it should be written in a way such that it can be solved with a reasonable amount of effort. An approach to accomplish this was described by Philip (1969) and involves linearizing the equation.

4.3.3 Linearized Approach to Analysis

The principal difficulty with eq. (4.3.9) is that the transport coefficients D and K are not constants; they vary greatly with position, z , and time, t . Both quantities can easily undergo excursions of value of several orders of magnitude. Nevertheless, it is possible to write eq. (4.3.9) in linearized form:

$$\frac{\partial \tilde{\theta}}{\partial t} = \frac{\partial}{\partial z} \left(D^* \frac{\partial \tilde{\theta}}{\partial z} \right) - K^* \frac{\partial \tilde{\theta}}{\partial z} \quad (4.3.10)$$

where the tilde implies approximation based upon the linearizing assumption of D^* and K^* constant. Under these conditions eq. (4.3.10) becomes

$$\frac{\partial \tilde{\theta}}{\partial t} = D^* \frac{\partial^2 \tilde{\theta}}{\partial z^2} - K^* \frac{\partial \tilde{\theta}}{\partial z} \quad (4.3.11)$$

Linearization, of course, has its price. First, the results of eq. (4.3.11) may be in error--for example $\tilde{\theta}$ may differ significantly from θ . Second, the values for D^* and K^* must be determined from laboratory experiments that approximate field conditions as closely as possible.

Nevertheless, integral properties such as infiltration rates and distance of advance of the wetting interface can be predicted with acceptable accuracy by means of the linearization technique.

4.3.4 Linearized Approach to Predicting Time of First Appearance of Leachate

In the early stages of infiltration (i.e., when the liner is wetting up) the first term on the right side of the eq. (4.3.11) will dominate. (Careful examination of the manipulations in eqs. 4.3.5a through d shows that the D^* term represents the capillary attraction part of the process.) The eq. (4.3.11) can be written

$$\frac{\partial \tilde{\theta}}{\partial t} \approx D^* \frac{\partial^2 \tilde{\theta}}{\partial z^2} \quad t \rightarrow 0 \quad (4.3.12)$$

We now impose the following initial and boundary conditions:

- (1) At initial time ($t=0$) assume that the moisture content is equal to θ_i throughout the depth of the liner:

$$\tilde{\theta} = \theta_i \quad \text{for } z > 0 \quad \text{and } t = 0 \quad (4.3.13)$$

(remember, z is positive downward).

- (2) At all times at the boundary ($z=0$) the moisture content is held at the saturation moisture content, θ_s :

$$\tilde{\theta} = \theta_s \quad \text{for } z = 0 \quad \text{and } t \geq 0. \quad (4.3.14)$$

The solution to eq. (4.3.12) for the initial and boundary conditions of eqs. (4.3.13) and (4.3.14), respectively, is:

$$\tilde{\theta} = \theta_i + (\theta_s - \theta_i) \operatorname{erfc} \left[\frac{z}{2\sqrt{D^*t}} \right] \quad (4.3.15)$$

where erfc is the complementary error function as given in Figure 4.8. This function can be called in most computer programming languages, and is tabulated in many books of mathematical tables (c.f., Crank, 1975).

Equation (4.3.15) is not particularly useful for the evaluation procedure because we are not especially interested in predicting moisture content profiles. More practical for the evaluator would be the ability to predict the time required for the wetting interface to penetrate the clay liner. This

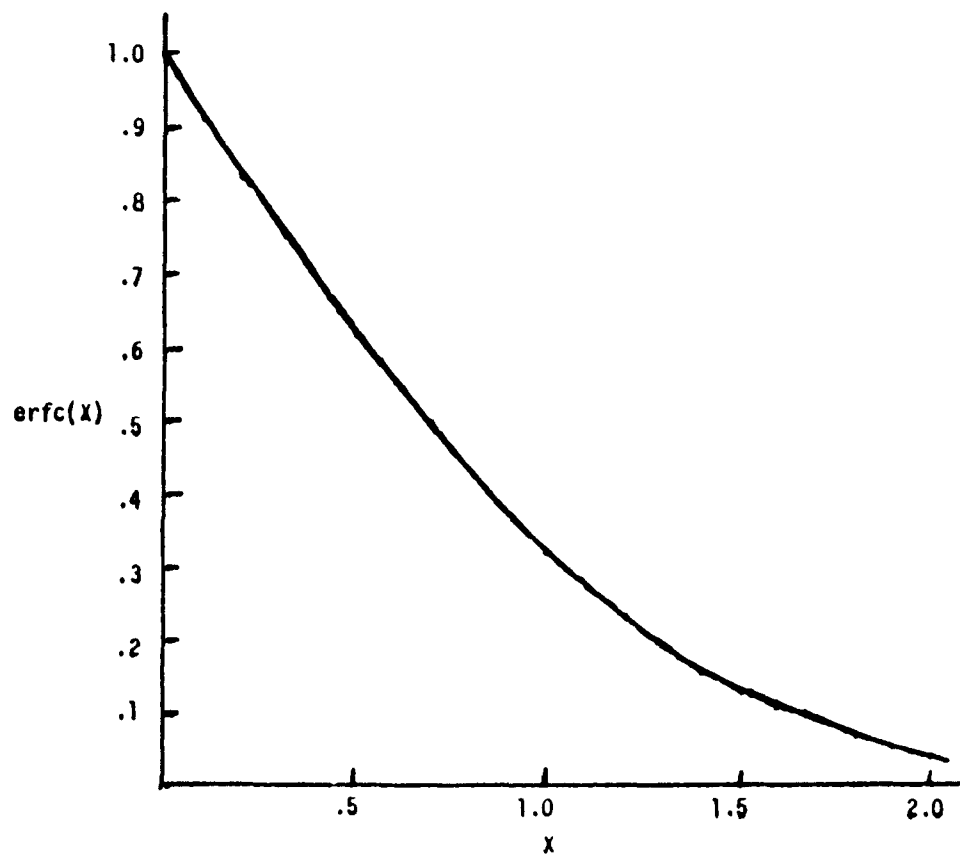


Figure 4.8 - Complementary error function.

can be done easily if a few simplifying approximations are made. These are illustrated in Figure 4.9. The principal simplification lies in the assumption that the soil extends below the depth represented by the thickness of the liner. Thus the "infinite depth" solution of eq. (4.3.15) is applicable. It is also assumed that the wetting interface has reached the bottom of the liner when an amount of moisture has entered the top boundary sufficient to saturate the soil to a depth, d , equal to the thickness of the liner. The relationship for the total amount of liquid entering up to time, t , is given by

$$M_t = 2(\theta_s - \theta_i) \sqrt{\frac{D^* t}{\pi}} \quad (4.3.16)$$

The quantity of liquid required to saturate the liner to a depth, d , is given by

$$M_t = (\theta_s - \theta_i) d \quad (4.3.17)$$

Equating (4.3.16) and (4.3.17) yields

$$t = \frac{\pi d^2}{4D^*} \quad (4.3.18)$$

Though only approximate, eq. (4.3.18) does yield a bounding value for use in evaluating the delay time that a clay liner provides.

It should be pointed out that for relatively thick liners or for soils having relatively low capillary potential, it would be more appropriate to apply eq. (4.3.11) without neglecting the last term. In this case the solution for the initial and boundary conditions given by eqs. (4.3.13) and (4.3.14), respectively, is given by Philip (1957) as

$$\bar{\theta} = \theta_i + \frac{\theta_s - \theta_i}{2} \left[\operatorname{erfc} \left(\frac{z - K^* t}{2\sqrt{D^* t}} \right) + \exp \left(\frac{K^{*2} z^2}{D^*} \right) \operatorname{erfc} \left(\frac{z + K^* t}{2\sqrt{D^* t}} \right) \right] \quad (4.3.19)$$

which is slightly more complicated than eq. (4.3.15); and in addition requires that the designer provide an appropriate value for K^* . In order to predict the time required for the wetting front to penetrate the liner, we write the expression equivalent eq. (4.3.16):

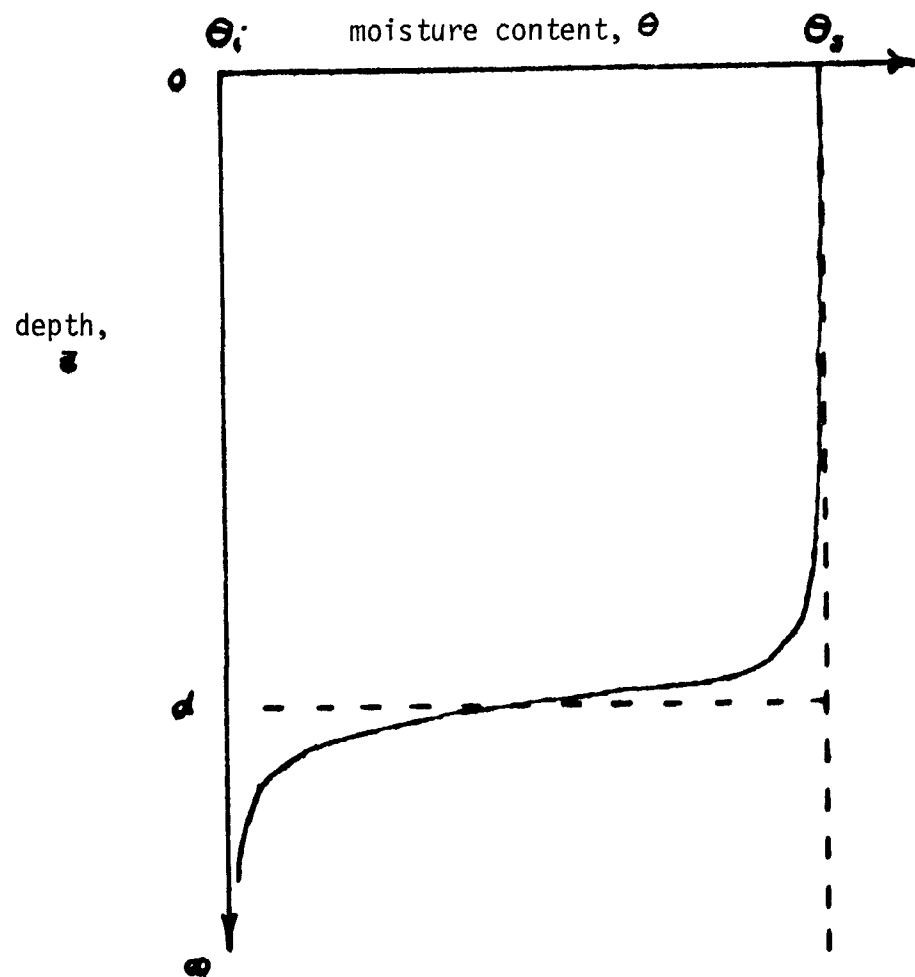


Figure 4.9 - Simplified geometry for calculating time for liquid to penetrate liner.

$$M_t = (\theta_s - \theta_i) \left[\frac{D^*}{K^*} \operatorname{erf} \left(\frac{K^*}{2} \sqrt{\frac{t}{D^*}} \right) + \frac{K^* t}{2} \operatorname{erfc} \left(- \frac{K^*}{2} \sqrt{\frac{t}{D^*}} \right) + \sqrt{\frac{D^* t}{\pi}} \exp \left(- \frac{(K^*)^2 t}{4 D^*} \right) \right] \quad (4.3.20)$$

which would produce the penetration time, t .

Introducing eq. (4.3.17) into eq. (4.3.20) gives:

$$d = \frac{D^*}{K^*} \operatorname{erf} \left(\frac{K^*}{2} \sqrt{\frac{t}{D^*}} \right) + \frac{K^* t}{2} \operatorname{erfc} \left(- \frac{K^*}{2} \sqrt{\frac{t}{D^*}} \right) + \sqrt{\frac{D^* t}{\pi}} \exp \left(- \frac{(K^*)^2 t}{4 D^*} \right) \quad (4.3.21)$$

Eq. (4.3.21) cannot be solved explicitly as was eq. (4.3.18); however, trial and error solution can be used to solve for the penetration time, t .

4.3.5 Linearized Approach to Predicting Leachate Flow When Liquid Has Poned on Liner

In the case of a surface impoundment lagoon where liquid has ponded on the top of the liner, eq. (4.3.6) still holds, however the boundary conditions become

$$\theta = \theta_i \quad \text{for } t = 0 \text{ and } z \geq 0 \quad (4.3.22a)$$

and

$$\psi = h \quad \text{for } z = 0 \text{ and } t \geq 0 \quad (4.3.22b)$$

For these mixed boundary conditions the most efficient solution procedure involves re-examining eq. (4.3.5d):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial \Psi}{\partial z} \right) - \frac{\partial k}{\partial z} \quad (4.3.5d)$$

with

$$D \equiv k \frac{d\Psi}{d\theta} \quad (4.3.7)$$

and

$$K \equiv \frac{dk}{d\theta} \quad (4.3.8)$$

Now from eq. (4.3.5d)

$$\frac{d\theta}{d\Psi} \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial \Psi}{\partial z} \right) - \frac{\partial k}{\partial z} \quad (4.3.23a)$$

$$\frac{k}{D} \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial \Psi}{\partial z} \right) - \frac{dk}{d\theta} \frac{d\theta}{d\Psi} \frac{\partial \Psi}{\partial z} \quad (4.3.23b)$$

$$\frac{k}{D} \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial \Psi}{\partial z} \right) + \frac{Kk}{D} \frac{\partial \Psi}{\partial z} \quad (4.3.23c)$$

$$\frac{\partial \Psi}{\partial t} = \frac{D}{k} \frac{\partial}{\partial z} \left(k \frac{\partial \Psi}{\partial z} \right) + K \frac{\partial \Psi}{\partial z} \quad (4.3.23d)$$

This diffusion equation in ψ can be linearized as was eq. (4.3.11):

$$\frac{\partial \tilde{\psi}}{\partial t} = D^* \frac{\partial^2 \tilde{\psi}}{\partial z^2} + K^* \frac{\partial \tilde{\psi}}{\partial z} \quad (4.3.24)$$

subject to the initial and boundary conditions:

$$\psi = \psi_i \text{ for } t = 0 \text{ and } z \geq 0 \quad (4.3.25a)$$

and

$$\tilde{\psi} = h_0 \text{ for } z = 0 \text{ and } t \geq 0 \quad (4.3.25b)$$

where ψ_i is the value of capillary potential corresponding to the initial moisture content, θ_i . The solution to eq. (4.3.24) subject to boundary conditions (4.3.25) is:

$$\tilde{\psi} = \psi_i + \frac{h_0 - \psi_i}{2} \left[\operatorname{erfc} \left(\frac{z - K^* t}{2\sqrt{D^* t}} \right) + \exp \left(\frac{(K^*)^2 z^2}{D^*} \right) \operatorname{erfc} \left(\frac{z + K^* t}{2\sqrt{D^* t}} \right) \right] \quad (4.3.26)$$

Eq. (4.3.26) can be used to predict the first appearance of leachate using the following approach. It will be assumed that the wetting interface has reached a distance, d , equal to the thickness of the liner when the moisture content at the base of the liner is $(\theta_i + \theta_s)/2$; or when

$$\psi = \psi \quad \left| \quad \frac{\theta_i + \theta_s}{2} \right.$$

Thus in eq. (4.3.26) set

$$\tilde{\psi} = \psi \quad \left| \quad \frac{\theta_i + \theta_s}{2} \right.$$

and $z = d$:

$$\Psi \left| \frac{\theta_i + \theta_s}{2} = \Psi_i + \frac{h_0 - \Psi_i}{2} \left[\operatorname{erfc} \left(\frac{d - K^* t}{2\sqrt{D^* t}} \right) + \exp \left(\frac{(K^*)^2 d}{D^*} \right) \operatorname{erfc} \left(\frac{d + K^* t}{2\sqrt{D^* t}} \right) \right] \right. \quad (4.3.27)$$

4.3.6. Linearized Approach to Predicting Leachate Flow Rates After Saturation

After the clay liner has become saturated, the flow process is dominated by gravitational forces, the capillary potential approaches the pressure head in the water, h_p , and the water content (which is at saturation) no longer changes with time. In this case in eq. (4.3.5a):

$$\frac{\partial \theta}{\partial t} \rightarrow 0 \quad (4.3.28)$$

and

$$k \rightarrow k_s \quad (4.3.29)$$

where k_s is the saturated hydraulic conductivity, more commonly referred to as the Darcy coefficient of permeability. Equation (4.3.5a) becomes

$$0 = \frac{\partial}{\partial z} \left[k_s \frac{\partial (h_p - z)}{\partial z} \right] \quad (4.3.30a)$$

$$= \frac{\partial}{\partial z} \left[k_s \frac{\partial h}{\partial z} \right] \quad (4.3.30b)$$

$$= k_s \frac{\partial^2 h}{\partial z^2} \quad (4.3.30c)$$

or

$$\frac{\partial^2 h}{\partial z^2} = 0 \quad (4.3.30d)$$

where

$$h = h_p - z \quad (4.3.31)$$

is the total hydraulic head (remember, z is positive downward). Equation (4.3.30d) is known as the Laplace equation.

The solution for flow velocity is simply obtained as

$$v = k_s \frac{\partial h}{\partial z} . \quad (4.3.32)$$

The total quantity of liquid passed by the clay liner in time, Δt , is given by

$$q = k_s \frac{\partial h}{\partial z} (A) \Delta t. \quad (4.3.33)$$

For the Yolo light clay with $k_s = 1.2 \times 10^{-5}$ cm/sec, a unit hydraulic gradient, an area of 1 square foot, and a time of 1 year:

$$\begin{aligned} q &= \frac{1.2 \times 10^{-5} \text{ cm}}{\text{sec}} \frac{1 \text{ ft}^2}{1 \text{ yr}} \frac{31.5 \times 10^6 \text{ sec}}{\text{yr}} \frac{\text{ft}}{30.5 \text{ cm}} \frac{7.48 \text{ gal}}{\text{ft}^3} \\ &= 93 \text{ gal/yr/ft}^2. \end{aligned} \quad (4.3.34)$$

4.4 Efficiencies of Liner-Drain Layer Systems

The efficiency of a liner-drain layer system is a quantitative measure of the proportion of liquid that moves through the drain layer and is collected by the drain collector lines, relative to the proportion that seeps into the liner. Wong (1977) proposed an approximate technique for quantifying this, based upon saturated Darcy flow in both the drain layer and the clay liner. Figure 4.10 describes the geometry assumed in his calculations.

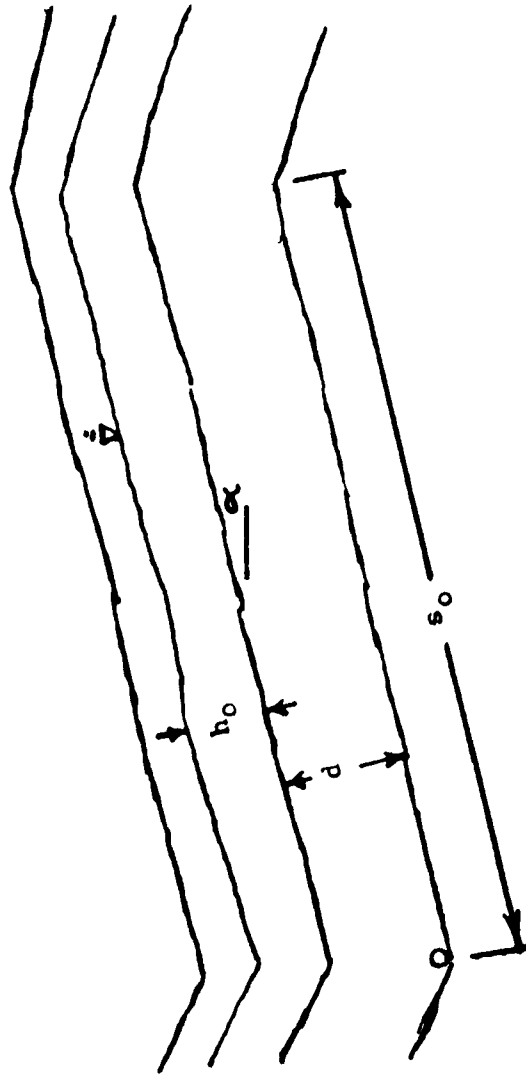


Figure 4.10 - Geometry for calculating efficiency of drain - liner systems using method proposed by Hong (1977).

The approach postulates that at some initial time a rectangular slug of liquid is placed upon the saturated liner to a depth h_0 . The liquid flows both horizontally along the slope of the system and vertically into the clay liner. The fraction of liquid moving into the collector drain system at time, t , is given by

$$\frac{s}{s_0} = 1 - \frac{t}{t_1} \quad (4.4.1)$$

and the fraction of liquid seeping into the clay liner is given by

$$\frac{h}{h_0} = \left(1 + \frac{d}{h_0 \cos \alpha}\right) e^{-Ct/t_1} - \frac{d}{h_0 \cos \alpha} \quad 0 \leq t \leq t_1, \quad (4.4.2)$$

where

$$t_1 \equiv \frac{s_0}{k_{s1} \sin \alpha} \quad (4.4.3)$$

$$C \equiv \left(\frac{s_0}{d}\right) \left(\frac{k_{s2}}{k_{s1}}\right) \cot \alpha \quad (4.4.4)$$

and

- s = length of saturated volume at time, t (cm)
- h = thickness of saturated volume at time, t (cm)
- s_0 = initial length of saturated volume = $L/2 \sec(\alpha)$ (cm)
- h_0 = initial thickness of saturated volume (cm)
- k_{s1} = saturated permeability of the material above clay liner (cm/sec)
- k_{s2} = saturated permeability of the clay liner (cm/sec)
- α = slope angle of the system ($^\circ$)
- d = thickness of the clay liner (cm).

Figure 4.11 shows the geometry at some time, t .

The efficiency of the liner is easily determined with reference to Figure 4.12 which graphs h/h_0 versus s/s_0 and t/t_1 . Equations (4.4.1) and (4.4.2) can be solved parametrically in t/t_1 , to yield the line shown on the figure. (The line is actually a curve; however, for practical liner-drain layer configurations it can be approximated as a straight

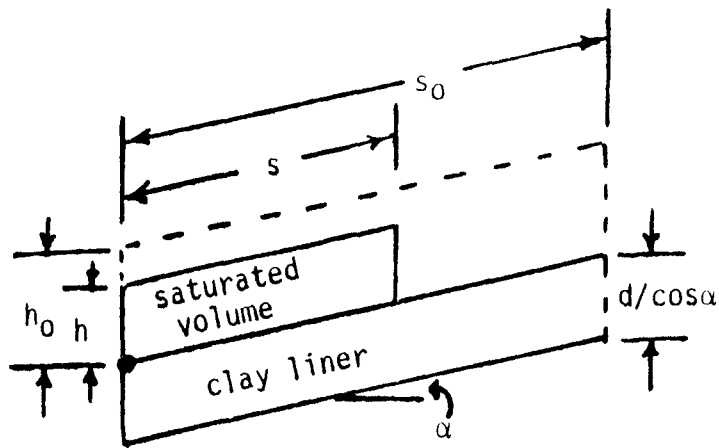


Figure 4.11 - Geometry for calculating efficiency of drain - liner systems. (after Wong, 1977)

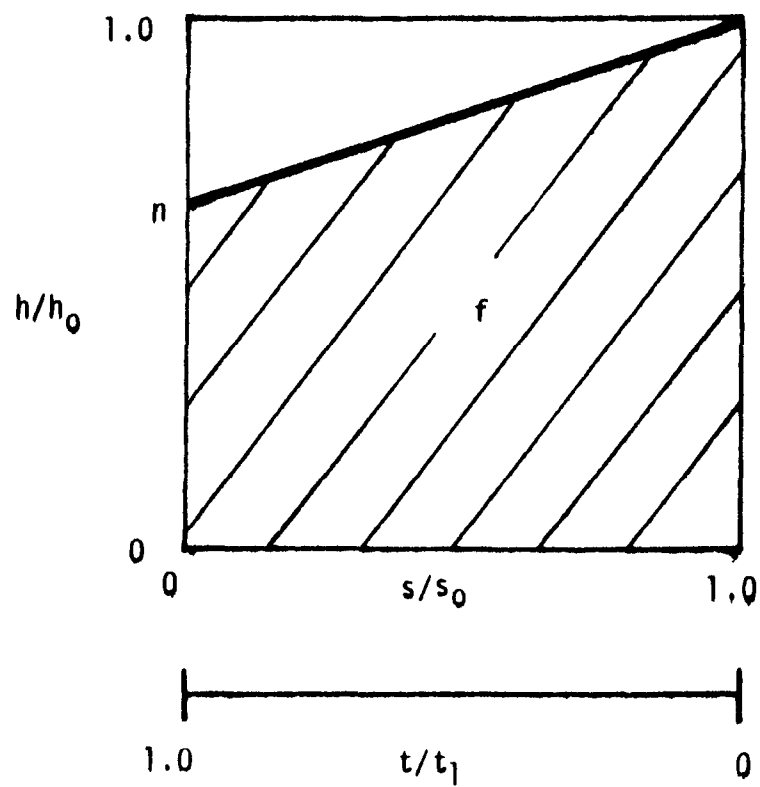


Figure 4.12 - Diagram for computing efficiency of drain - liner systems. (after Wong, 1977)

line.) In this case the efficiency of the system is given by the area labelled f in Figure 4.12. This area is most easily determined by calculating the value of h/h_0 when $t/t_1 = 1.0$ (or $s/s_0 = 0$). This value is called n and can be obtained by solving eq. (4.4.2) with $t/t_1 = 1.0$:

$$n = \left(1 + \frac{d}{h_0 \cos \alpha}\right) e^{-C} - \frac{d}{h_0 \cos \alpha} \quad (4.4.5)$$

The value of n can be either positive or negative; however, most efficient designs will have $n > 0$. The efficiency is given by either

$$f = \frac{1+n}{2} \quad \text{for } n \geq 0 \quad (4.4.6)$$

or

$$f = \frac{1}{2(1-n)} \quad \text{for } n \leq 0 \quad (4.4.7)$$

Thus the efficiency varies from 0 to 1.0.

The quantity of liquid draining out of the system is given by:

$$\underline{\text{amount collected in drains}} = f \times h_0 \quad (4.4.8)$$

and the quantity of liquid seeping into the liner is given by:

$$\underline{\text{amount seeping into liner}} = (1-f) \times h_0 \quad (4.4.9)$$

As an example, consider a system having a Yolo clay liner ($k_{s2} = 1.2 \times 10^{-5}$ cm/sec) that is 75 cm thick, overlain by a gravel layer having $k_{s2} = 0.1$ cm/sec. The entire system slopes at 4 ft/100 feet and the drain spacing is 30 m. Assume that an initial slug of liquid 10 cm thick impinges upon the system.

$$\begin{aligned}
\alpha &= 2.3^\circ \text{ (4 ft/100 ft)} \\
k_{s1} &= 0.1 \text{ cm/sec} \\
k_{s2} &= 1.2 \times 10^{-5} \text{ cm/sec} \\
d &= 75 \text{ cm} \\
s_0 &= L/2 \sec \alpha = 30 \text{ m/2 sec } (2.3^\circ) = 1500 \text{ cm}
\end{aligned}$$

From eq. (4.4.4),

$$c = \left(\frac{1500}{75} \right) \left(\frac{1.2 \times 10^{-5}}{.1} \right) \cot (2.3^\circ) \approx .06$$

From eq. (4.4.5),

$$\begin{aligned}
n &= 1 + \frac{75}{10 \cos(2.3^\circ)} e^{-.06} - \frac{75}{10 \cos(2.3^\circ)} \\
&= .507
\end{aligned}$$

and from eq. (4.4.6)

$$f = \frac{1 + .507}{2} = 75\%$$

Thus 75% of the liquid is diverted to the drains.

5. PROCEDURE FOR EVALUATING PROPOSED DESIGNS

5.1 Outline of Procedure

The procedure for evaluating proposed designs as shown in Figure 5.1 is as follows:

- (1) select a typical cross-section through the landfill from the design drawings (more than one cross-section may require evaluation);
- (2) construct a liquid routing diagram as shown in Figure 3.3;
- (3) isolate liquid diversion modules consisting of low permeability layers overlain by high permeability drain layers;
- (4) determine the amount of liquid impinging upon the first module;
- (5) calculate the bounding height of free standing liquid within the drain layer using eq. (4.2.2) or (4.2.5);
- (5a) compare the ratio of this height to the thickness of the drain layer with the ratio deemed to be appropriate for the site-specific conditions;

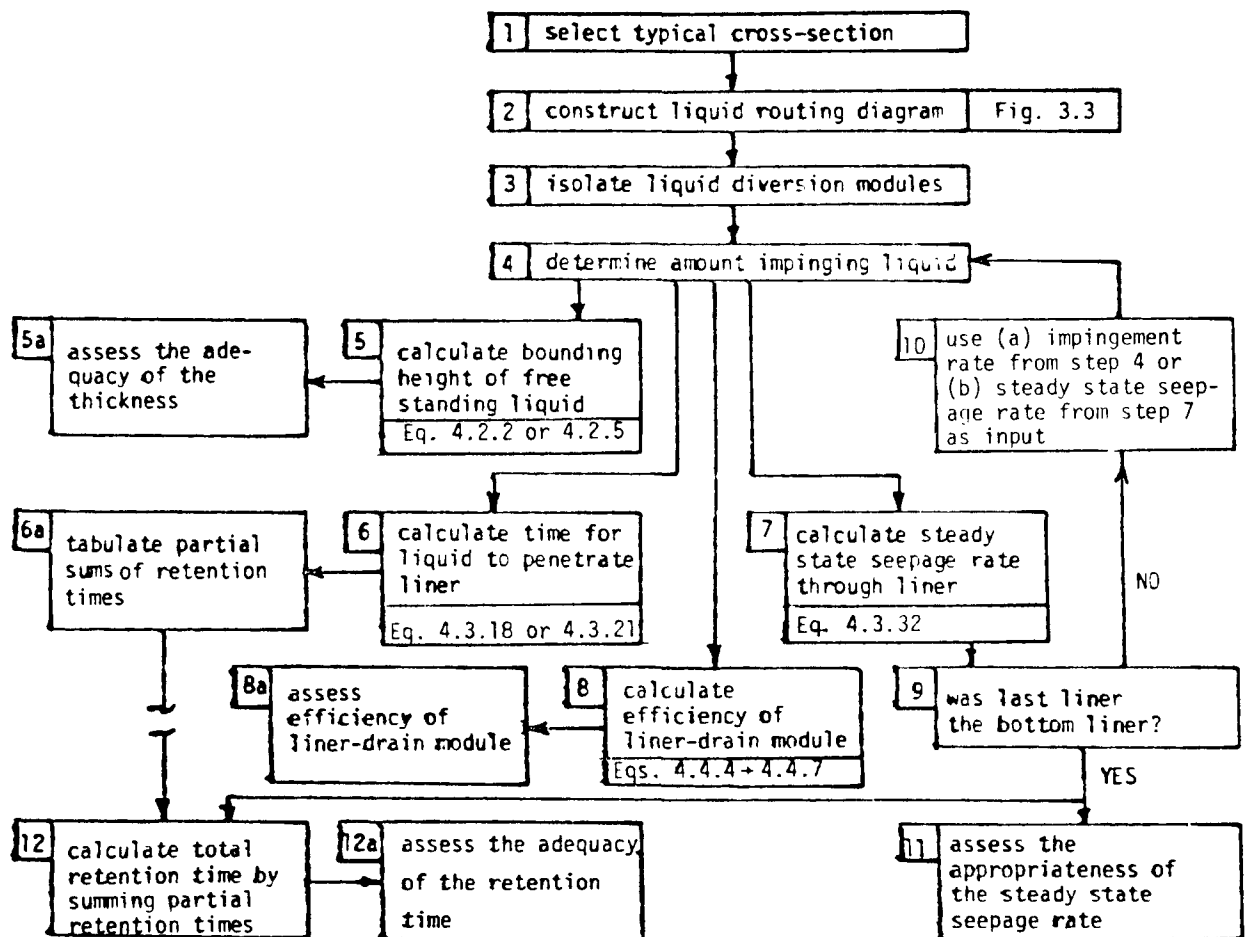


Fig. 5.1 - Flow chart of procedure used to evaluate containment.

- (6) calculate the time required for water to completely permeate each clay liner using eq. (4.3.18) or (4.3.21);
- (6a) tabulate the retention time for the modules;
- (7) calculate the steady-state seepage rate through the saturated liner using eq. (4.3.32);
- (8) calculate the efficiency of the liner-drain module using eqs. (4.4.4), (4.4.5), and (4.4.6) or (4.4.7);
- (8a) compare with the efficiency deemed to be appropriate for the site specific conditions;
- (9) determine whether or not all modules in the liquid routing diagram have been evaluated;
- (10) if there are more modules, use the lesser of (a) the impingement rate from step 4 or (b) the steady-state seepage rate calculated in step 7 as the impingement rate on the subsequent module;
- (11) if this module is the last module, compare the steady-state seepage rate calculated in step 7 with the steady-state seepage rate deemed to be appropriate for the site specific conditions;
- (12) if this module is the last module, also calculate the total retention time by adding the partial sums from step 6a for each module;
- (12a) compare the retention time calculated in step 12 with the retention time deemed to be appropriate for the site specific conditions.

Table 5.1 summarizes the equations used in the procedure.

5.2 Information Required to Use Evaluation Procedure

The input required to use the evaluation procedure is shown in Figure 5.2.

5.2.1 Input From Other Manuals

Input required to use the evaluation procedure is obtained from two other manuals:

- (1) The Hydrologic Simulation on Solid Waste Disposal manual provides input to step 4 (amount of liquid impinging on first module). This quantity is available on a daily basis as the quantity, Q . The evaluator may wish to express these values on a monthly average basis. Alternatively, monthly percolation can be determined using the Water Balance Method of Fenn, Hanley, and DeGeare (1975).

Table 5.1
Summary of Equations

<u>Eq. Number</u>	<u>Equation</u>	<u>Purpose</u>
4.2.2 (or 4.2.5)	$h_{\max} = \left(\frac{eL^2}{4k_s} \right)^{1/2}$	Predict maximum rise of liquid in sand drain
4.3.18 (or 4.3.21)	$t \approx \frac{\pi d^2}{4 D^*}$	Predict time to first apperance of liquid at base of clay liner
4.3.32	$v = k_s \frac{\partial h}{\partial z}$	Predict steady state seepage rate through saturated clay liner
4.4.4	$C = \frac{s_0}{d} \frac{k_{s2}}{k_{s1}} \cot \alpha$	Predict steady state seepage rate through saturated clay liner
4.4.5	$n = 1 + \frac{d}{h_0 \cos \alpha} e^{-C} - \frac{d}{h_0 \cos \alpha}$	Predict steady state seepage rate through saturated clay liner
4.4.6	$f = \frac{1+n}{2} \text{ if } n > 0$	Predict steady state seepage rate through saturated clay liner
or		
4.4.7	$f = \frac{1}{2(1-n)} \text{ if } n \leq 0$	Predict steady state seepage rate through saturated clay liner

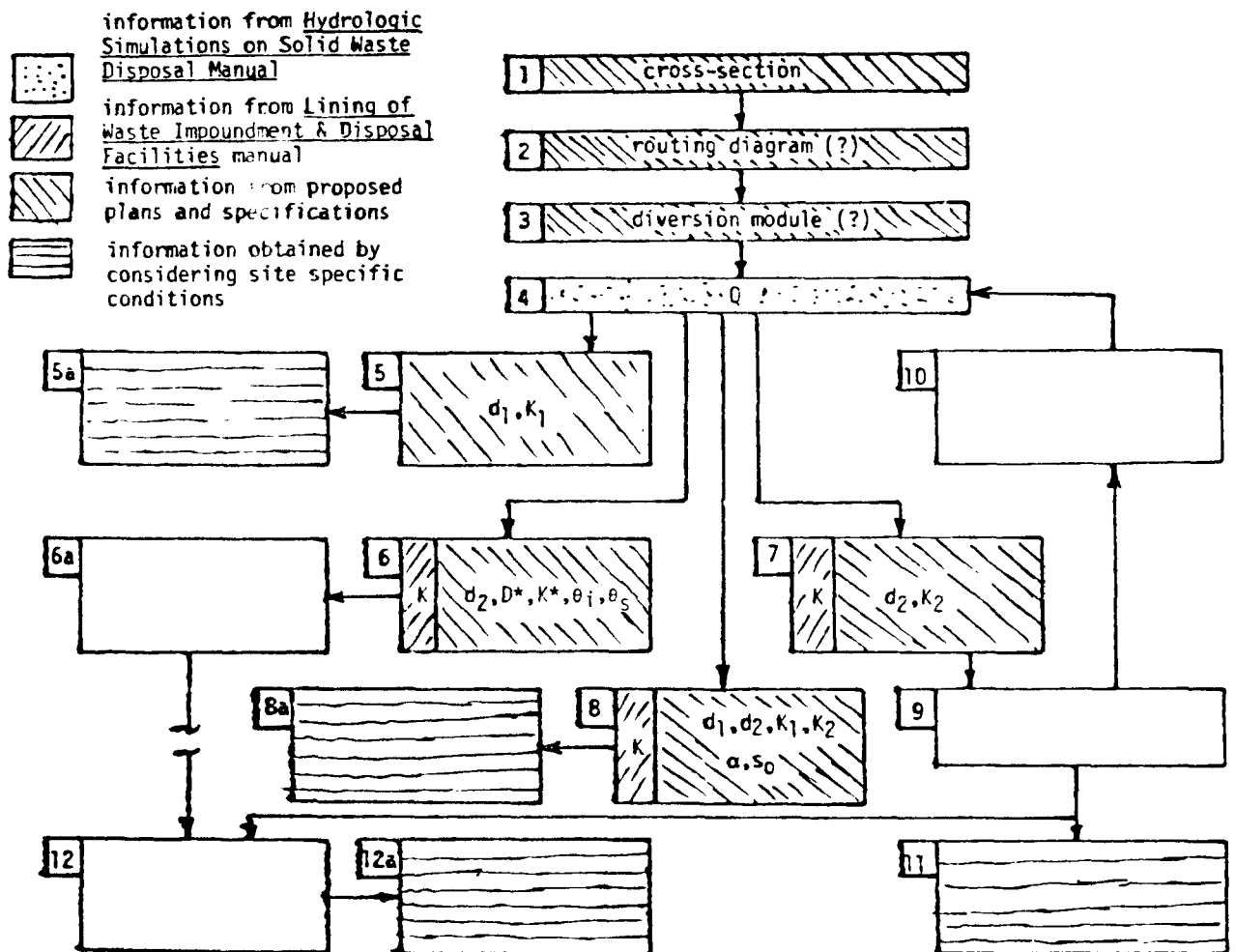


Figure 5.2 - Sources of input information for evaluation procedure.

- (2) The Lining of Waste Impoundment and Disposal Facilities manual provides effective permeability coefficients and design lives for synthetic membranes to be used as input in step 6, 7, and 8, respectively.

5.2.2 Input From Proposed Plans and Specifications

The evaluator will obtain several essential pieces of information from the proposed plans and specifications for the surface impoundment:

- (1) the typical cross-sections to be used in step 1 should be obtainable from the plans. These cross-sections will be used to construct the liquid routing diagrams of step 2. In addition, the designer may have provided liquid routing diagrams in these documents;
- (2) the liquid diversion modules may have been delineated by the designer in the specifications and may also be shown on the plans; and
- (3) the following information must be provided in the plans and specifications for each module:
 - (a) the thickness of the sand or gravel drain layer (to be used in steps 5 and 8);
 - (b) the thickness, d , of the clay liner (to be used in steps 6, 7, and 8);
 - (c) the coefficient of permeability, k_{s1} , for the sand or gravel drain layer (to be used in steps 5 and 8);
 - (d) the infiltration coefficients, D^* and K^* , for the clay liner (to be used in step 6);
 - (e) the initial compacted moisture content, θ_i , for the clay liner (to be used in step 6);
 - (f) the saturation water content, θ_s , for the clay liner (to be used in step 6);
 - (g) the coefficient of permeability, k_{s2} , for the saturated clay liner (to be used in steps 7 and 8);
 - (h) the slope, α , of the clay liner (to be used in step 8);
 - (i) the length, s_0 , from the high point of the liner to the drain (to be used in step 8).

5.3 Summary of Outputs of Evaluation Procedures

The output of the evaluation procedure will be used in the manner shown in Figure 5.3.

5.3.1 Outputs to Other Manuals

The output of the present procedure will be used as input to the Lining of Waste Impoundment and Disposal Facilities manual. Specifically, the quantity of liquid moving through sand and gravel drain layers and collected in drain lines as determined in step 8 will dictate the quantity of water to be carried by the collection pipes. Moreover, this quantity will become an indirect input to leachate treatment considerations.

5.3.2 Output to Determine Whether Proposed Design is Acceptable

The comparison of the proposed design's performance with conditions appropriate for the particular site (performed in steps 5a, 8a, 11, and 12a) will determine whether the proposed design is acceptable with respect to control of leachate.

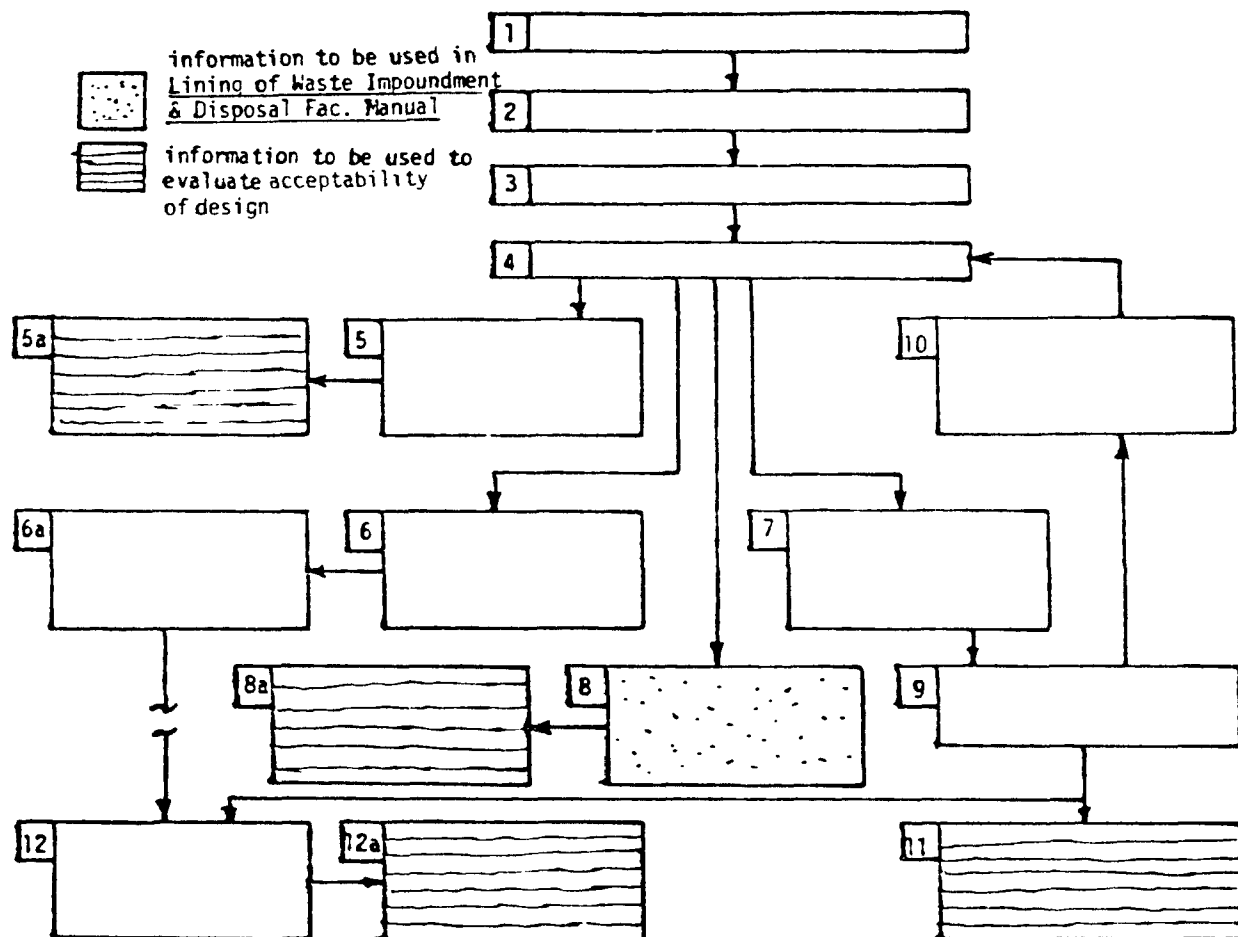


Figure 5.3 - Output information from evaluation procedure.

APPENDICES

APPENDIX A

Example Hazardous Waste Landfill

A landfill is to be located in a critical groundwater region. The landfill contains a section devoted to hazardous waste. This section is isolated by a sand drain layer and compacted clay liner both above and below the hazardous waste section. Figures A.1 and A.2 contain excerpts from the plans and specifications, respectively.

Solution to Hazardous Waste Landfill Problem

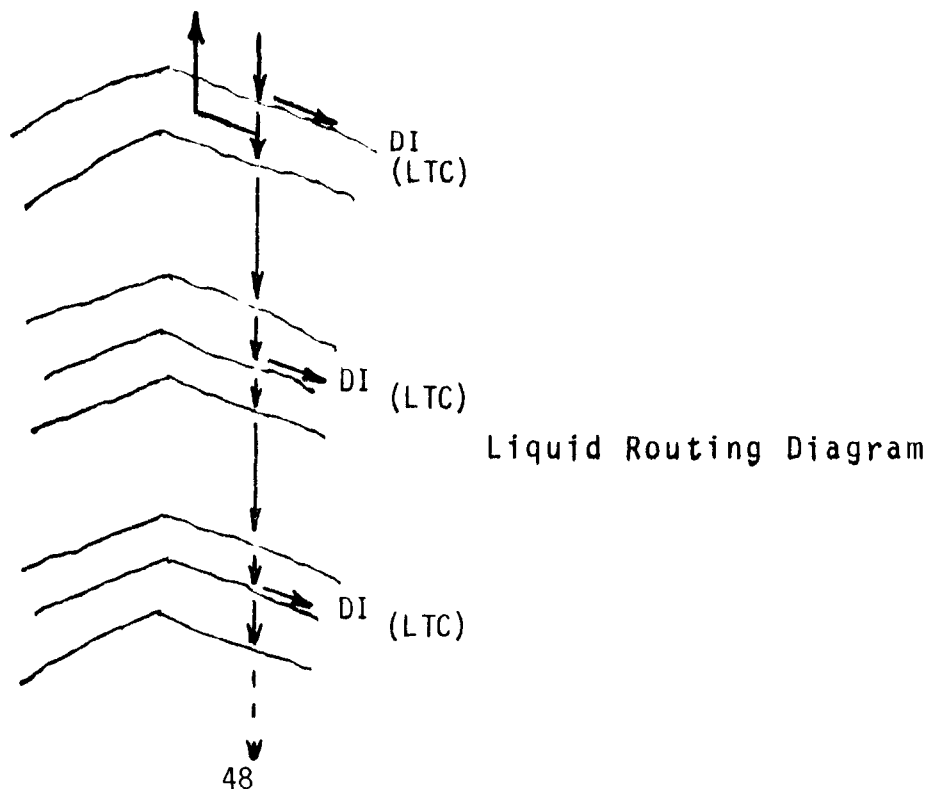
The solution procedure follows that recommended in the flow chart of Figure 5.1.

Step 1. Select typical cross-section.

The plans give only one typical cross-section (Figure A.1), so this cross-section is selected for analysis.

Step 2. Construct liquid routing diagram.

Excerpt 1 from the specifications deals with the leachate control system and will provide the basis for constructing the liquid routing diagram. The modules that are considered to be a part of this system are the vegetated cover, the intermediate sand drain and liner system, and the bottom sand drain and liner system. The intermediate cover was not included because the designer provided inadequate data to allow the evaluator to analyze its effects.



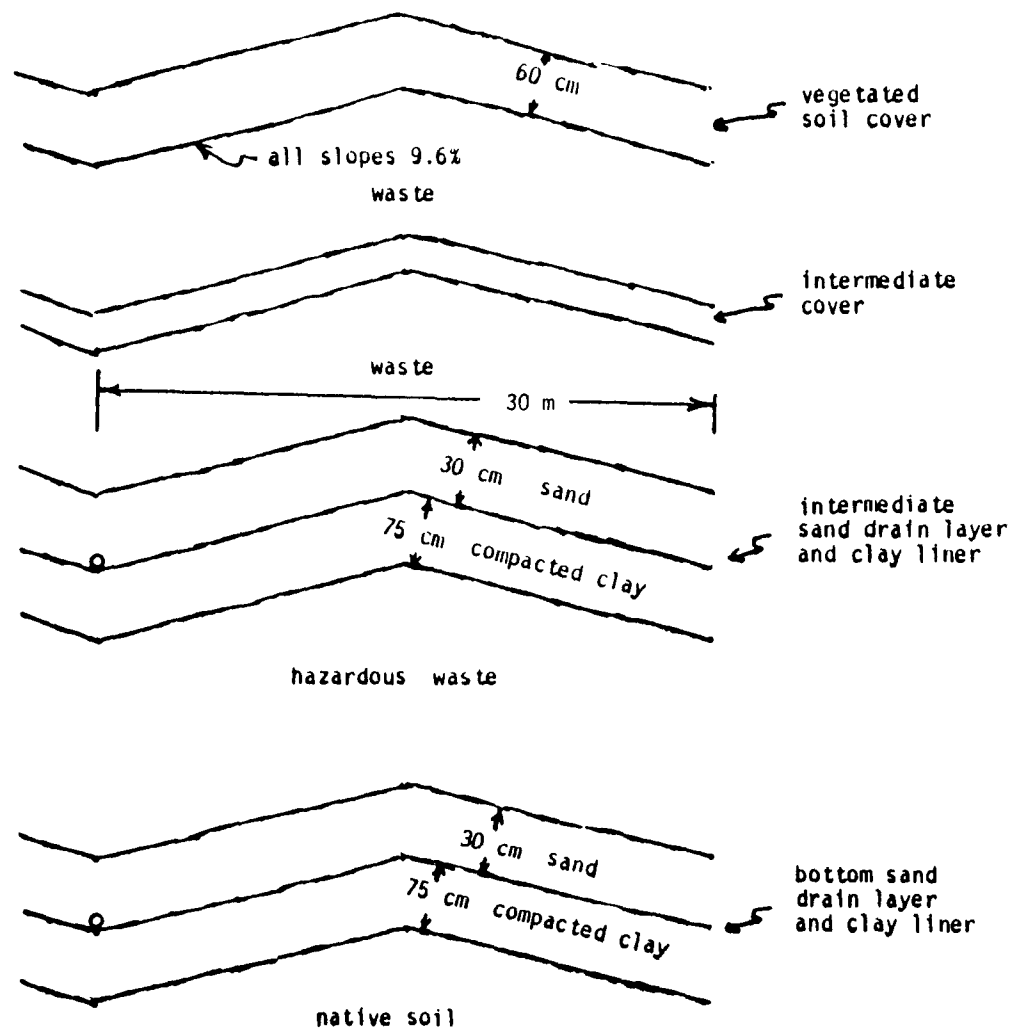


Figure A.1 - Excerpt from plans for proposed landfill.

-excerpt 1-

CONTROL OF LEACHATE

Leachate will be controlled by vegetated final cover, by an intermediate sand drain layer underlain by a compacted clay layer, and by a bottom liner overlain by sand. The clay for both liners is compacted at 2% above optimum water content and to 95% of Standard Proctor density. The sand layers are carefully placed at controlled thickness. Collector pipes are placed as shown on the plans. Intermediate cover will also provide some degree of control of liquid flow rates.

-excerpt 3-

LABORATORY TESTS ON CLAY

Laboratory permeability tests were performed on the clay compacted at 2% above optimum moisture content to field density. Water was initially allowed to diffuse into the sample under negligible hydrostatic head so that the linearized diffusivity, D^* , was determined to be 5×10^{-6} cm/sec. After D^* was established, back pressure was applied to hasten saturation of the sample. After saturation, the coefficient of permeability, k_s , was found to be 2×10^{-6} cm/sec.

-excerpt 2-

LABORATORY TESTS ON SAND

Laboratory permeability tests were performed on the sand compacted to the field density and then saturated. The coefficient of permeability was found to be 1×10^{-2} cm/sec.

Figure A.2 - Excerpts from specifications for proposed landfill.

Step 3. Isolate liquid diversion modules.

There are two liquid diversion modules: the intermediate sand drain and compacted clay layer and the bottom sand drain and compacted clay liner.

Step 4. Determine the amount of liquid impinging on the intermediate module.

To accomplish this, the results of a water balance calculation were used as input to the present analysis. The data for percolation below the cover for a typical year are:

MONTH	PRECIPITATION	PERCOLATION(cm)
January	0	0
February	0	0
March	12.21	1.83
April	6.48	3.25
May	9.47	2.51
June	10.6	0.05
July	9.27	0
August	9.04	0
September	8.56	0
October	5.18	0.61
November	1.7	1.6
December	0	0
Annual	72.51	9.85

Two values will be used for analysis:

$$e_{\text{Apr}} = \frac{3.25 \text{ cm}}{\frac{\text{mo}}{31} \frac{\text{day}}{24 \text{ hrs}} \frac{\text{hr}}{60 \text{ min}} \frac{\text{min}}{60 \text{ sec}}} \\ = 1.2 \times 10^{-6} \text{ cm/sec}$$

Step 5. Calculate bounding height of free standing liquid.

Excerpt 2 from the specifications shows that the sand to be used in the sand drain layers has a saturated permeability of $k_s = 1 \times 10^{-2} \text{ cm/sec}$.

Using eq. (4.2.5) for $L = 30 \text{ m}$, $k_s = 1 \times 10^{-2} \text{ cm/sec}$, $e = 1.2 \times 10^{-6} \text{ cm/sec}$ and $\alpha = 5.48^\circ$ (9.6%) gives:

$$c = \frac{1.2 \times 10^{-6}}{1 \times 10^{-2}} = 1.2 \times 10^{-4}$$

$$h_{\max} = \frac{30 \text{ m} \sqrt{1.2 \times 10^{-4}}}{2} \left[\frac{\tan^2 (5.48^\circ)}{1.2 \times 10^{-4}} + 1 - \frac{\tan (5.48^\circ)}{1.2 \times 10^{-4}} \sqrt{\tan^2 (5.48^\circ) + 1.2 \times 10^{-4}} \right]$$

$$h_{\max} = 8.2 \text{ cm}$$

which is less than the design value of 30 cm.

Step 5a. Because h_{\max} is less than 30 cm, the design is acceptable.

Step 6. Calculate time required for liquid to appear at bottom of liner.

Excerpt 3 from the specifications provides a value for linearized diffusivity of $D^* = 5 \times 10^{-5} \text{ cm}^2/\text{sec}$. Using this value in eq. (4.3.21) with $d = 75 \text{ cm}$ (from Figure A.1) gives

$$t = \frac{\pi}{4} \frac{d^2}{D^*} = \frac{(3.14)(75)^2 \text{ cm}^2}{4 (5 \times 10^{-6})} \frac{\text{sec}}{\text{cm}^2} \frac{\text{yr}}{3.1 \times 10^7 \text{ sec}} = 27.5 \text{ yrs.}$$

Step 6a. Tabulate partial sum.

Partial sum = 27.5 years.

Step 7. Calculate steady-state seepage rate through saturated liner.

Excerpt 3 from the specifications provides a value for the saturated coefficient of permeability of $k_s = 2 \times 10^{-6} \text{ cm/sec}$. Using this value in eq. (4.3.32) gives, for unit hydraulic gradient:

$$v = \frac{2 \times 10^{-5} \text{ cm (1)}}{\text{sec}}$$

$$= 2 \times 10^{-6} \text{ cm/sec}$$

as the steady-state seepage rate.

Step 8. Calculate efficiency of liner-drain module.

The required data were obtained as follows:

$$\begin{aligned} s_0 &= 15 \text{ m} \quad \cos(5.48^\circ) = 15.1 \text{ m} \quad (\text{plans, Fig. A.1}) \\ d &= 75 \text{ cm} \quad (\text{plans, Fig. A.1}) \\ k_{s2} &= 2 \times 10^{-6} \text{ cm/sec} \quad (\text{excerpt 3 from specs., Fig. A.2}) \\ k_{s1} &= 1 \times 10^{-2} \text{ cm/sec} \quad (\text{excerpt 2 from specs., Fig. A.2}) \\ \alpha &= 9.6^\circ = 5.48^\circ \quad (\text{plans, Fig. A.1}) \\ h_0 &= 3.25 \text{ cm} = \text{April percolation.} \end{aligned}$$

From eq. (5.1.4)

$$\begin{aligned} C &= \left(\frac{s_0}{d} \right) \left(\frac{k_{s2}}{k_{s1}} \right) \cot \alpha \\ &= \frac{15.1 \text{ m}}{75 \text{ cm}} \frac{100 \text{ cm}}{\text{m}} \frac{2 \times 10^{-6} \text{ cm/sec}}{1 \times 10^{-2} \text{ cm/sec}} \cot(5.48^\circ) \\ &= 0.042 \end{aligned}$$

From eq. (5.1.5)

$$\begin{aligned} n &= \left(1 + \frac{d}{h_0 \cos \alpha} \right) e^{-C} - \frac{d}{h_0 \cos \alpha} \\ &= \left(1 + \frac{75}{3.25 \cos(5.48^\circ)} \right) \exp(-.042) - \frac{75}{3.25 \cos(5.48^\circ)} \\ &= (1+23.18)(.958) - (23.18) = -.015 \end{aligned}$$

From eq. (4.4.7) since $n < 0$

$$f = \frac{1}{2(1-n)} = \frac{1}{2(1+0.015)} = 49\%$$

Step 8a. Compare with efficiency appropriate for the site.

While adequate site-specific information is not available for this site, the value of 49% is marginal. Values well in excess of 50% would be more acceptable.

Step 9. Was liner the bottom liner?

No, therefore go to step 10.

Step 10. Use steady-state seepage rate of 1.2×10^{-6} cm/sec from step 4 as amount of liquid impinging on bottom liner, then go to step 5 for bottom liner. (Note that the rate of 2×10^{-6} cm/sec from step 7 exceeded the available moisture.)

Step 5 and 5a. Same as for intermediate module.

Step 6. Same as for intermediate module.

Step 6a. Partial retention time = 27.5 years.

Steps 7, 8, and 8a. Same as for intermediate module.

Step 9. Was liner the bottom liner?

Yes, therefore go to Step 11.

Step 11. Compare steady-state seepage rate from Step 4 with rate appropriate for the site.

While adequate site-specific information is not available, 2×10^{-6} cm/sec could be acceptable.

Step 12. Calculate total retention time.

Total retention time = $27.5 + 27.5 = 55$ years.

Step 12a. Compare total retention time with that deemed to be appropriate for the site.

While site-specific information is not available, 55 years could be adequate.

APPENDIX B

Example Hazardous Waste Lagoon Problem

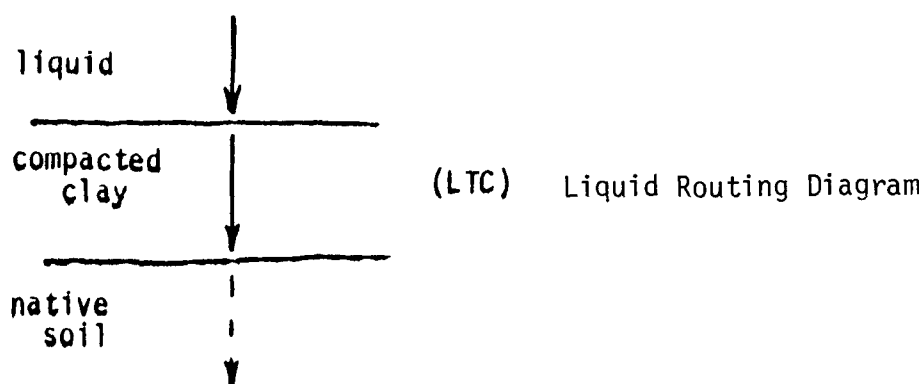
The solution procedure follows that are recommended in the flow chart of Figure 5.1. However, Steps 3, 5, 5a, 8 and 8a are not applicable.

Step 1. Select typical cross-section.

The plans give only one typical cross-section (Figure B.1), thus this cross-section is selected for analysis.

Step 2. Construct liquid routing diagram.

Excerpt 1 from the specifications states that the compacted clay liner constitutes the only leachate control system. Because there are no diversion layers the liquid routing diagram is quite simple.



Step 4. Determine amount of liquid impinging on the clay liner.

Amount consists of a static head of 2 meters.

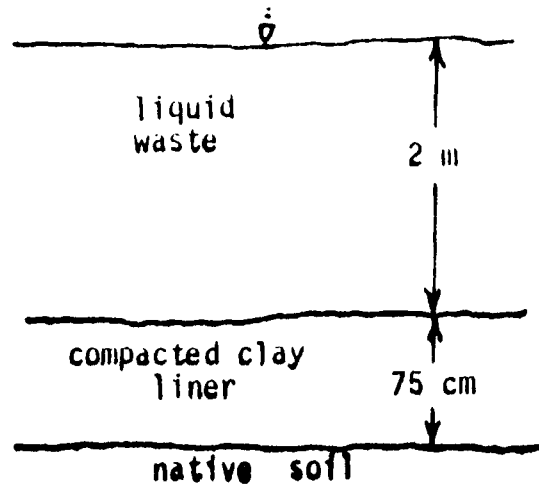


Figure B.1 - Excerpt from plans for proposed lagoon.

-excerpt 1-

CONTROL OF LEACHATE

Leachate will be controlled by a clay liner. The liner will be compacted at $\theta_i = .24$ moisture content and to 95% of Standard Proctor density. The saturated moisture content is $\theta_s = .495$.

-excerpt 2-

LABORATORY TESTS ON CLAY

Permeability tests were performed which yielded linearized transport coefficient values of $K^* = 2.3 \times 10^{-8}$ m/sec and $D^* = 1.3 \times 10^{-11}$ m²/sec. At a moisture content $\theta_i = .24$, the capillary potential was $\psi = .002$ cm = 2×10^{-5} m. At $\theta = (\theta_i + \theta_s)/2 = .37$, $\psi = 0.013$ cm. The saturated coefficient of permeability is $k_s = 1.2 \times 10^{-5}$ cm/sec.

Figure B.2 - Excerpts from specifications for proposed lagoon.

Step 6. Calculate time required for liquid to appear at bottom of liner.

The time required for first appearance of leachate is determined using eq. (4.3.27) with $\psi = 2 \times 10^{-5} \text{ m}$, $h = 2 \text{ m}$, $d = .75 \text{ m}$, $* = 2.3 \times 10^{-8} \text{ m/sec}$, $D^* = 1.3 \times 10^{-11} \text{ m}^2/\text{sec}$, ψ at $(\theta_i + \theta_s)/2 = 1.3 \times 10^{-4} \text{ m}$:

$$1.3 \times 10^{-4} = 2 \times 10^{-5} + \frac{2 - 2 \times 10^{-5}}{2} \operatorname{erfc} \frac{.75 - 2.3 \times 10^{-8} t}{2 \sqrt{1.3 \times 10^{-11} t}} \\ + \exp \frac{2.3 \times 10^{-8} (.75)}{1.3 \times 10^{-11}} \operatorname{erfc} \frac{.75 + 2.3 \times 10^{-8} t}{2 \sqrt{1.3 \times 10^{-11} t}}$$

Trial and error solution yields a retention time for the liner of 235 days.

Step 6a. Tabulate partial sum.

Partial sum = 235 days.

Step 7. Calculate steady-state seepage rate through saturated liner.

Excerpt 3 from the specification provides a value of the saturated coefficient or permeability of $k_s = 1.2 \times 10^{-5} \text{ cm/sec}$. This value can be used in eq. (4.3.32) with

$$\partial h = 200 \text{ cm} + 75 \text{ cm} = 275 \text{ cm}$$

and

$$\partial z = 75 \text{ cm}$$

to give

$$q = \frac{1.2 \times 10^{-5} \text{ cm}}{\text{sec}} \frac{275 \text{ cm}}{75 \text{ cm}} \frac{1 \text{ ft}^2}{30.5 \text{ cm}} \frac{1 \text{ yr}}{\text{yr}} \frac{\text{ft}}{\text{ft}^3} \frac{31.5 \times 10^6 \text{ sec}}{\text{yr}} \frac{7.48 \text{ gal}}{\text{ft}^3} \\ = 340 \text{ gal/ft}^2/\text{yr}$$

as the steady-state seepage rate.

Step 9. Was the liner the bottom liner ?

Yes, therefore go to Step 11.

Step 11. Compare the steady-state seepage rate from Step 7 with rate appropriate for the site.

Site-specific information is not available; however, the rate of 340 gal/ft²/yr is quite low.

Step 12. Calculate total retention time.

Total retention time is 235 days.

Step 12a. Compare with total retention time appropriate for the site.

Site-specific information is not available.

APPENDIX C

List of Symbols

c = constant defined by eq. (4.2.6)
 C = constant in eq. (4.4.4)
 D = diffusivity of partially saturated soil (cm^2/sec)
 D^* = linearized diffusivity (cm^2/sec)
 d = thickness of layer (m or cm)
 e = liquid impingement rate (m/sec)
 f = fraction of liquid collected by drain
 h = hydraulic head (cm)
 h_0 = initial liquid head on liner (cm)
 h_p = pressure head (cm)
 K = partially saturated permeability (cm/sec)
 K^* = linearized permeability (cm/sec)
 k_{s1} = saturated permeability of drain layer (cm/sec)
 k_{s2} = saturated permeability of clay layer (cm/sec)
 L = distance between drains (m)
 n = parameter defined in eq. (4.4.5) (cm)
 q = flow quantity (gallons/year)
 Q = percolation rate from cover (cm/sec)
 s = distance to trailing edge of draining liquid (cm)
 s_0 = distance from drain to break in slope (cm)
 t = time (sec)
 t_1 = time for complete drainage (sec)
 v = flow velocity (cm/sec)
 x = horizontal coordinate (cm)
 y = vertical distance (cm)
 z = vertical coordinate (cm)

α = slope angle of liner or drain layer ($^\circ$)
 ϕ = hydraulic driving potential
 ψ = capillary potential (cm)
 θ = volumetric water content
 θ_i = initial water content
 θ_s = saturation water content
 $\tilde{\theta}$ = water content predicted by linearized equations
 ∂ = partial differentiation
 ∇ = spacial gradient

APPENDIX D

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