



## Project Summary

# Applications of Decision Theory Techniques in Air Pollution Modeling

Robert G. Lamb and Saroj K. Hati

**This study applies methods of operations research to two basic areas of air pollution modeling: (1) the generation of wind fields for use in models of regional scale transport, diffusion and chemistry and (2) the application of models in studies of optimal pollution control strategies. The work is illustrated in the context of a hypothetical problem in which optimal sites and emissions control plans are sought for two new power plants. The study addresses all aspects of the problem starting with stochastic specification of the wind fields in the region of interest, proceeding to the development of simple models that relate pollutant emissions to both short- and long-term averaged concentration, and concluding with the incorporation of game theory concepts into mathematical methods of finding optimum solutions to multi-objective problems. Five objectives are considered in this study: minimization of plant operating cost and minimization of given short-period maximum and long-period averaged concentrations of each of two pollutants. The optimization procedure attempts to fulfill these objectives jointly while complying with specified constraints on the overall system.**

**This study emphasizes concepts and technique development rather than the solution of actual problems.**

***This Project Summary was developed by EPA's Atmospheric Sciences Research Laboratory, Research Triangle Park, NC, to announce key findings of the research project that is fully documented in a separate report of the same***

***title (see Project Report ordering information at back).***

### Introduction

This study applies methods of operations research to two aspects of air pollution modeling: (1) the generation of wind fields for use in models of regional scale transport, diffusion and chemistry and (2) the application of diffusion models in studies of optimal pollution control strategies, particularly the siting of new industries. The first application represents an attempt to implement an idea developed by one of the authors (R. G. Lamb) in an earlier study of the formulation of wind fields for use in regional scale air pollution models. It was shown in that study that the physical laws of fluid motion and a set of discrete meteorological observations do not uniquely define the wind field. Rather they delineate an infinite set of functions any one of which is a possible description of the atmospheric flow that existed during the period the observations were made. This inherent uncertainty should be accounted for explicitly in air pollution model applications so that decision makers can use model outputs properly. This study develops a technique that affects this type of model enhancement.

Since models are used extensively to assess the effectiveness of proposed solutions to air quality problems, it would be advantageous to have a means by which a computer could work with models to find optimal solutions to given problems. Moreover, in the area of policy analysis, cost/benefit ratios

must often be estimated for a myriad of hypothetical policy options. For example, what is the cost of the additional emissions controls needed on a given source to achieve compliance with a given secondary standard; and how does this cost compare with that of the controls that one would find necessary were the cost and the degree of violation of the secondary standard treated as objectives to be minimized jointly? Answering questions of this kind has not been practical heretofore because techniques have not been available for applying air pollution models to multicriteria optimization problems. This study uses concepts of game theory to develop a technique of this kind.

### Ensembles of Wind Fields

If the horizontal wind ( $u, v$ ) in a shallow layer of the atmosphere is represented at any instant  $t_1$  by a complex Fourier series containing  $2(2M+1)^2$  amplitudes, compatibility with the principle of mass conservation reduces the number of independent amplitudes to  $4M(M+1)$ . In other words, the mathematical description of the entire wind field ( $u, v$ ) in a given region at time  $t_1$ , is represented by a single point in a  $4M(M+1)$  dimensional phase space in which each coordinate axis is associated with one of the independent Fourier amplitudes. If at time  $t_1$ , one has  $J$  observations of the wind, say  $(u_j, v_j)$ ,  $j = 1, \dots, J$ , made at arbitrary locations  $(x_j, y_j)$ ,  $j = 1, \dots, J$ , these define a hyperplane of dimension  $4M(M+1) - 2J$  in the phase space. Each point on the hyperplane is a *possible* description of the flow at time  $t_1$ .

We say that it is a possible description because it satisfies all observations exactly and it satisfies mass conservation. However, if the flow descriptions represented by individual points on the hyperplane were examined you would find that most of them are improbable flows in the sense that they contain features that are not normally observed in the atmosphere. For example, it is possible for a flow field to contain an intense vortex between the wind observing sites without violating either the velocities measured at the wind stations  $(u_j, v_j)$  or mass conservation.

Thus we need a means of using additional information, such as climatological data or empirical knowledge of atmospheric motion, to diminish as much as possible the highly underdetermined nature of the flow specification. We pro-

pose to achieve this by assigning to each of the possible flows a weight  $\rho$ , which we call a probability, whose value is a measure of how consistent that particular flow is with additional information. The specific information that one chooses to use and the manner in which the estimates of  $\rho$  are associated with it are purely subjective decisions that become an intrinsic part of the model in which the wind fields are ultimately used.

One method of estimating the flow probabilities  $\rho$ , which is illustrated in the report, makes use of observed kinetic energy spectra. For example, aircraft measurements made in the upper troposphere indicate that both the  $u$  and  $v$  components have energy spectral densities that are proportional to the  $-5/3$  power of the wave number for wave numbers  $|k|$  greater than about  $5 \cdot 10^{-5}$   $\text{rad m}^{-1}$ , i.e., wave lengths smaller than about 500 km; and energy density proportional to  $|k|^{-3}$  for longer wave length perturbations. These observations lead us to the hypothesis:

*The probability of occurrence of atmospheric flow field described by a given point on the hyperplane is directly proportional to the degree to which the Fourier transform associated with that point satisfies the energy spectra characteristics cited above.*

Locating the points on the hyperplane that satisfy the energy conditions best is achieved by defining an optimization problem whose solutions are the flow fields we seek and applying methods of operations research to solve the problems. The report describes this process in some detail, showing how nonlinear programming and related techniques can be used to generate a collection of points  $\underline{x}_i$ ,  $i = 1, \dots, I$ , from the flow field hyperplane. To each point  $\underline{x}_i$ , which represents a complete wind field description, a probability  $\rho_i$  is assigned whose value is determined by how well, relative to the other  $(I-1)$  members of the set, its energy spectrum conforms to the given specifications.

The process just described yields an ensemble of  $I$  wind descriptions applicable to the single instant  $t_1$ . By repeating the process for each of the  $N$  hours, say, that meteorological measurements are available in a given period  $T$ , we obtain  $N$  ensembles. By selecting one member from each ensemble, we can produce a sequence of functions to describe the temporal evolution of the

flow field. We assign probabilities to each of the  $I^N$  possible sequences under the following hypothesis:

*The probability of a given sequence of flow fields is directly proportional to the degree to which that sequence satisfies the principle of momentum conservation.*

The report describes in detail how dynamic programming and Bellman's principle of optimality can be used to implement this hypothesis. The final result is a set of function sequences and their associated probability values that provide descriptions of the winds ( $u, v$ ) over the given region during the period  $T$ . An air pollution model driven by each member of this set of flow descriptions in turn will produce an ensemble of concentrations fields for each specified distribution of species sources. The differences that exist among the members of the concentration ensemble reflect the uncertainty created by our inability to specify atmospheric motion exactly. The underlying philosophy of the flow field ensemble approach is that this inherent uncertainty should be dealt with explicitly in air pollution models for them to be used meaningfully in decision making studies.

### Model Application and Game Theory

Selecting sites for new industries that produce air pollutants is a task that requires crucial tradeoffs between capital and operating costs on the one hand and the requirement to satisfy air quality standards on the other hand. Ideally, one would like to find plant locations where air quality standards can be met at minimal cost. In this study we focus on the problem of siting power plants, in which case the costs considered include fuel cost, the cost of transporting fuel from given sources (such as coal mines) to the given plant sites and the cost of emissions controls (such as flue gas desulfurization and fly ash precipitation). Both primary and secondary air quality standards on one or more pollutants are considered. Optimal tradeoffs among the air quality and cost objectives are sought within the space of feasible solutions using concepts of cooperative game theory.

Game theory has been developed for both cooperative and noncooperative games. In the latter, each player acts independently to maximize his own gain. In cooperative games, players form

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coalitions under the expectation that working together each can achieve an outcome that is better than that achievable in fully competitive play. The measure of success of cooperative play is embodied in the concept of pareto-optimality which involves a single objective function constructed from the objectives of each player. This study uses these concepts to develop a general procedure for finding solutions to air quality problems that involve multiple objectives. The technique is demonstrated using a simple, hypothetical problem in which sites for two new power plants are sought that jointly minimize operating cost—fuel plus emissions controls—and peak and long-term averaged concentrations of two pollutants, a total of five objectives in all.

### **Conclusions**

New methods have been developed for use in two areas of air pollution modeling: generation of wind fields for regional scale models, and applications of models in multi-criteria optimization studies. The first technique differs fundamentally from existing methods in that it produces ensembles of flows rather than a single flow specification. In this regard it is consistent with the fact that physical laws and discrete meteorological data do not uniquely specify the wind field. Moreover, it allows total use of meteorological observations in contrast to the limited utilization that is achieved in applications where the bulk of the meteorological inputs are generated by a meteorological model.

The newly developed method of treating multicriteria optimization problems should enhance the value of air pollution models in cost/benefit, land use and other studies where one is interested in optimal tradeoffs among policy options. Although the method is quite general in its applicability, it can be expensive to operate, computationally, unless the air quality models are expressible in closed forms.

*The EPA authors Robert G. Lamb and Saroj K. Hati are with the Atmospheric Sciences Research Laboratory, Research Triangle Park, NC 27711.*

*The complete report, entitled "Applications of Decision Theory Techniques in Air Pollution Modeling," (Order No. PB 86-216 793/AS; Cost: \$11.95, subject to change) will be available only from:*

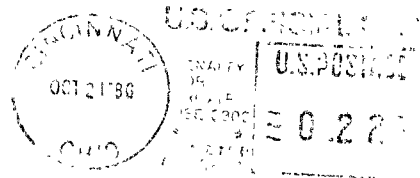
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*The EPA authors can be contacted at:*

*Atmospheric Sciences Research Laboratory  
U.S. Environmental Protection Agency  
Research Triangle Park, NC 27711*

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