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# Project Summary

# Plume 2D: Two-Dimensional Plumes in Uniform Ground Water Flow

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**A closed-form analytical solution for two dimensional plumes was incorporated in an interactive computer program. The assumption of an infinite aquifer depth and uniform source mass rate and source location was overcome by using the principal of superposition in space and time. The source code was written in a subset of FORTRAN 77 and can be compiled with FORTRAN IV, FORTRAN 66 as well as FORTRAN 77. As a result, the code is nearly independent of hardware and operating system. The model can be solved for either vertically or horizontally averaged conditions.**

***This Project Summary was developed by EPA's Robert S. Kerr Environmental Research Laboratory, Ada, OK, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).***

## Introduction

Relatively simple analytical methods can often be used to evaluate ground-water contamination problems, depending upon the complexity of the system and the availability of field data. Analytical models can also serve as valuable tools in developing parameters for more sophisticated numerical models. Although the numerical evaluation of an analytical solution to a ground-water problem may be mathematically complex, analytical models are well suited for interactive use on digital computers. Many analytical solutions to ground-water contamination problems can be coded on programmable hand-held calculators. In general, very few input parameters are required to define a given problem, and numerical results can be calculated in a few seconds.

This report presents analytical solutions to two ground-water pollution problems—two-dimensional plumes in uniform ground-water flow. An interactive computer code has been developed which enables the user to modify the definition of a given problem, and thus gain some insight into the effects of various parameters on the extent of a contaminant plume.

## Model Formulation

The differential equation describing the conservation of mass of a component in a saturated, homogenous aquifer with uniform, steady flow in the x-direction can be written as

$$R_d \frac{\partial C}{\partial t} + V^* \frac{\partial C}{\partial x} = D_x^* \frac{\partial^2 C}{\partial x^2} - D_y^* \frac{\partial^2 C}{\partial y^2} - R_d \lambda \qquad (1)$$

where

$C$ = component mass per unit volume of fluid phase
$D_x^*$ = dispersion coefficient in x-direction
$D_y^*$ = dispersion coefficient in y-direction
$R_d$ = retardation coefficient
$V^*$ = average interstitial velocity in x-direction
$x,y$ = rectangle coordinates
$\lambda$ = first-order decay constant.

The retardation coefficient accounts for partitioning of the component between the fluid and solid phases using a linear adsorption isotherm and is defined as

$$R_d = 1 + \frac{\rho_B}{\theta} K_d \qquad (2)$$

where

$\rho_B$ = bulk density of the rock
$\theta$ = effective porosity
$K_d$ = distribution coefficient for a linear adsorption isotherm.

Closed-form analytical solutions for the two types of ground-water contamination problems shown in Figure 1 are included in this report. The first is a vertically-averaged solution which describes a contaminant plume in the x-y, or horizontal, plane (Figure 1a). The second is a horizontally-averaged solution describing a contaminant plume in a vertical plane (Figure 1b).

The vertically-averaged solution applies to an aquifer of infinite areal extent and finite depth. The contaminant is assumed to be well mixed over the saturated thickness. The source of contamination is a vertical line source located at the origin of a coordinate system in the x-y plane. The conceptual model is similar to an injection well which fully penetrates the saturated zone or a finite vertical segment of the aquifier. Wilson and Miller (1978) have also applied this solution down-gradient from a contaminant source at the water table. For a relatively thin saturated zone, vertical dispersion will tend to mix the contaminant vertically. The concentration distribution can be considered as being two-dimensional in a horizontal plane at distances downstream of the source for the concentration distribution to become uniform with depth. For a continuous source of strength $M_0'$ at the origin, the vertically-averaged solution is (Hunt, 1978; Wilson and Miller, 1978)

$$C = \frac{M_0' \, \mathrm{EXP}\left(\frac{V_x^*}{2D_x}\right)}{4\pi\theta(D_x D_y)^{0.5}} W(U,B) \qquad (3)$$

where

$$U = \frac{\left(\frac{V_x^*}{D_x^*}\right)^2 + \frac{D_x^*}{D_y^*}\left(\frac{V_y^*}{D_x^*}\right)^2}{\frac{4\,V^2\,t}{R_d D_x^*}} \qquad (4)$$

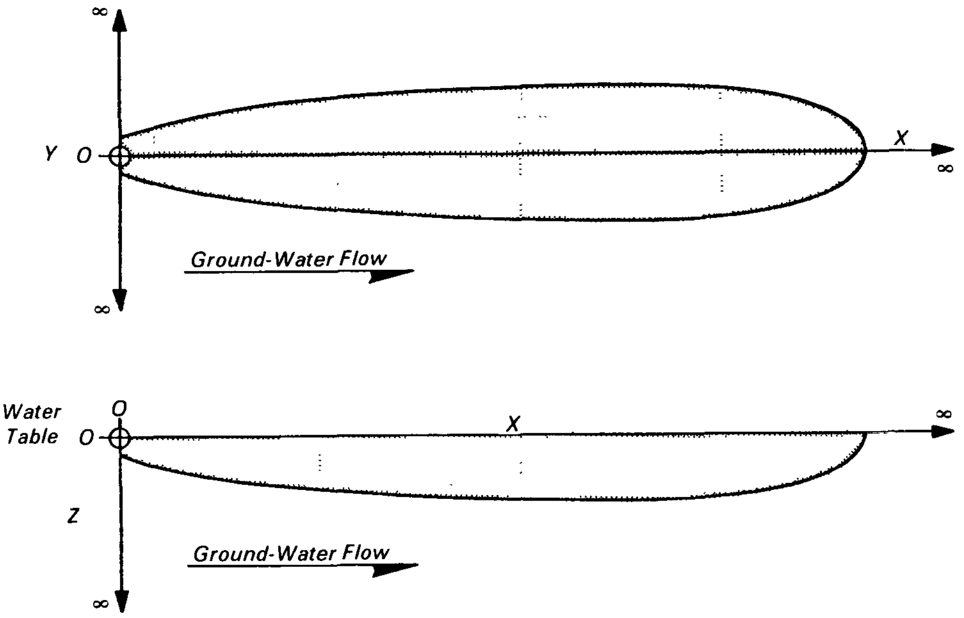

***Figure 1.*** *Coordinate systems for (A) vertically averaged solution and (B) horizontally averaged solution.*

and

$$B = \frac{1}{2}\left[\left(\frac{V_x^*}{D_x^*}\right)^2 + \frac{Dx}{Dy}\left(\frac{V_y^*}{D_x^*}\right)^2\right]^{1/2}$$

$$\left[1 + \frac{4\,D_x^*\,R_d\lambda}{V^{*2}}\right]^{1/2} \qquad (5)$$

The function W(U,B) is defined as

$$W(U,B) = \int_u^{\infty} \frac{1}{\xi}\,\mathrm{EXP}\left(-\xi - \frac{B^2}{4\xi}\right)d\xi \qquad (6)$$

where $\xi$ is a dummy integration variable. This function is often referred to as the "well function for leaky artesian aquifers" (Hantush, 1956). The corresponding steady-state solution of Equation 1 is

$$C = \frac{M_0' \, \mathrm{EXP}\left(\frac{V_x^*}{2D_x^*}\right)}{2\pi\theta\,(D_x^* D_y^*)^{0.5}} K_0(B) \qquad (7)$$

where $K_0(B)$ is the modified Bessel function of the second kind of order zero.

The horizontally-averaged solution is based on the conceptual model shown in Figure 1b. A line source is located at the water table and normal to the direction of ground-water flow. A problem which might fit this conceptual model is seepage from a trench.

The closed-form analytical solution follows directly from the vertically-averaged solution. Since the water table represents a no-flow boundary passing through the origin, the horizontally-averaged solution can be written directly as

$$C = \frac{M_0' \, \mathrm{EXP}\left(\frac{V_x^*}{2D_x^*}\right)}{2\pi\theta\,(D_x^* D_z^*)^{0.5}} W(U,B) \qquad (8)$$

The steady-state solution is

$$C = \frac{M_0' \, \mathrm{EXP}\left(\frac{V_x^*}{2D_x^*}\right)}{\pi\theta\,(D_x^* D_z^*)^{1/2}} K_0(B) \qquad (9)$$

Equations 5 and 7 and 8 and 9 can be used to calculate concentrations in contaminant plumes under the following assumptions and limitations:

1. The ground-water regime is completely saturated.
2. All aquifer properties are constant and uniform throughout the problem domain.
3. The ground-water flow is horizontal, continuous, and uniform throughout the aquifer.

4. The aquifer is infinite in extent.
5. The contaminant source is a line located at the origin of the coordinate system.
6. The mass flow rate of the source is constant.
7. At zero time the concentration in the aquifer is zero.

The assumptions of an infinite aquifer and uniform source rates can be overcome by using the principles of superposition in space and time. Superposition can also be used to include multiple sources.

## Computer Program

The closed-form analytical solutions for the two-dimensional plumes as presented above have been incorporated in an interactive computer program. The source code has been written in a subset of FORTRAN 77 and can be compiled with FORTRAN IV, FORTRAN 66, as well as FORTRAN 77 compilers. As a result, the code is almost entirely independent of hardware and operating systems. Those changes which may be required to implement the code on a given system, such as assigning logical devices are clearly identified.

The program has been developed for interactive use and requires input data under two modes of operation—"Basic Input Data" and "Edit." The basic input data are required to initiate a new problem. The user is prompted for the required data through a series of input commands.

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "edit" mode. The two character edit commands listed in Table 1 can be used to redefine the problem, run the calculations, and terminate the program.

The program has been written to require a minimum of machine resources and will run on both 8 and 16 bit microcomputers under CP/M, MS-DOS, and PC-DOS as well as larger minicomputers and mainframe machines.

*Table 1.* Edit Commands

| *Command* | *Variable changed/Execution* |
|---|---|
| *ST* | *Saturated Thickness* |
| *PO* | *Porosity* |
| *VX* | *New Seepage Velocity* |
| *RD* | *Retardation Coefficient* |
| *DE* | *Decay Constant* |
| *DX* | *X-Dispersion Coefficient* |
| *DY* | *Y-Dispersion Coefficient* |
| *DZ* | *Z-Dispersion Coefficient* |
| *RT* | *Source Rate Schedule* |
| *OB* | *Observation Points* |
| *XC* | *X-Coordinates* |
| *YC* | *Y-Coordinates* |
| *ZC* | *Z-Coordinates* |
| *TC* | *Observation Times* |
| *CS* | *Change Solution/Sources* |
| *MU* | *Menu of Edit Commands* |
| *LI* | *List input data* |
| *RN* | *Run* |
| *NP* | *New Problem* |
| *DN* | *Done* |

## Summary

The models and computer codes developed in this project are intended to serve as additional tools in the analysis of ground-water contamination problems. The user must select the best tool for the problem at hand based on a sound understanding of the principles of ground-water hydrology, the physical problem, and the limitations of the mathematical model(s). Unfortunately, these computer programs cannot substitute for an understanding of the processes and mechanisms of solute transport in ground-water systems or sound judgement based on training and experience.

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*The complete report, entitled "PLUME 2D: Two-Dimensional Plumes in Uniform Ground Water Flow," (Order No. PB 85-214 450/AS; Cost: $11.50, subject to change) will be available only from:*

*National Technical Information Service*
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