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Project Summary

Upconing of a Salt/Fresh-Water Interface Below a Pumping Well

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Analytical solutions for the upconing of an abrupt salt-water/fresh-water interface beneath a pumping well and for the concentration profile across a moving interface are developed for two types of upconing problems. The first considers the position of the interface and the salinity of the pumped water for a specified pumping rate. The second type of problem addresses the pumping schedules to prevent salinization of a well or to reach a predetermined salinity in the pumped water.

An interactive Fortran computer code has been developed to obtain solutions to both types of problems. The user is provided with options to modify the definition of a given problem, and, therefore, can gain some insight into the effects of geometry and physical properties on the rate and extent of upconing and the salinization of a well.

This Project Summary was developed by EPA's Robert S. Kerr Environmental Research Laboratory, Ada, OK, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).

Introduction

The full report presents an analytical solution for the upconing of an abrupt salt-water/fresh-water interface below a pumping well. Dispersion phenomena arising from the displacement of a moving interface or a finite transition zone between the invading and displaced fluids can be superimposed on the analytical solution for the position of an abrupt interface. An interactive FORTRAN computer code has been developed that enables the user to modify parameters and to control the computational sequence. This interactive approach enables the user to gain insight into the effects of geometry and physical properties on the rate and extent of upconing and salinization of a well.

Mathematical Development

McWhorter (1972) presented the equations that describe the flow in saturated aquifers which are underlain by a zone of saline water and pointed out the difficulties in obtaining solutions to these problems. The complexity of the flow phenomenon has led many investigators to idealize the system as a fresh-water zone separated from an underlying salt-water zone by a sharp interface. In other words, the two fluids are assumed to be immiscible. Schmorak and Mercado (1969) followed this approach and accounted for the mixing of the two fluids by superimposing the effects of dispersion on the transient solution for the position of an abrupt interface.

For the case of upconing beneath a pumping well partially penetrating a relatively thick confined aquifer as shown in Figure 1, Schmorak and Mercado (1969) presented Bear and Dagan's solution for the position of the interface as a function of time and radial distance from the pumping well as

$$x(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d} \left[\frac{1}{(1+R^2)^{1/2}} - \frac{1}{[(1+\tau)^2+R^2]^{1/2}} \right] \quad (1)$$

where R and τ are dimensionless dis-

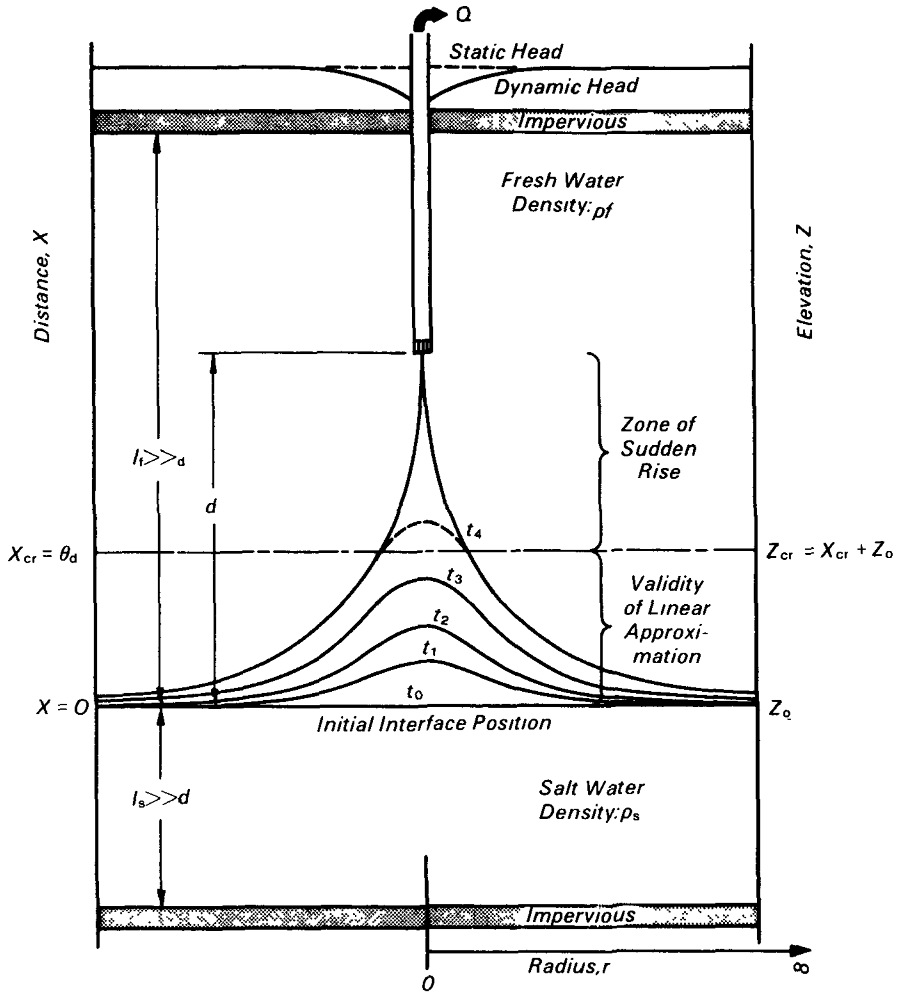

Figure 1. *Upconing of an abrupt interface below a pumping well*

tance and time parameters defined by

$$R = \frac{r}{d}\left(\frac{K_z}{K_x}\right)^{1/2} \qquad (2)$$

and

$$\tau = \frac{(\Delta\rho/\rho)K_z}{2\theta d}\, t \qquad (3)$$

Other notations are defined as follows (also refer to Figure 1):

d = distance from the bottom of the well to the initial interface elevation (L)
K_x,K_z = horizontal and vertical permeabilities, respectively (L/t)
Q = well pumping rate (L^3/t)
r = radial distance from well axis (L)
t = time elapsed since start of pumping (t)
X = rise of the interface above its initial position (L)
$\Delta\rho/\rho$ = dimensionless density difference between the two fluids, $(\rho_s - \rho_f)/\rho_f$
θ = porosity of the aquifer.

Application of the method of small pertubations restricts changes in the interface elevation to relatively small values. In terms of the physical problem, this restriction implies $d<<l_f$ and $d<<l_s$. Although the governing differential equations have been formulated for a confined aquifer, the results can be applied to unconfined systems if the drawdown is negligible compared to the saturated thickness of the freshwater zone.

The upconing process as treated above assumes that the two fluids are immiscible and that the interface between them is abrupt. Actually, the interface is diffuse and a transition zone exists between the two fluids in which the concentration varies from the concentration in one fluid to the concentration in the other fluid over a finite distance. This transition zone is related to dispersion processes which alter the concentration profile across the moving interface.

Bear and Todd (1960) approximated the concentration profile as a function of position, X; the "interface" position, $\bar{X}$; the equivalent total distance the interface is displaced $|\Delta\bar{X}|$, independent of direction; and the dispersivity, D_m. The concentration distribution is given by

$$\epsilon(X) = \frac{1}{2}\,\mathrm{erfc}\,\frac{X-\bar{X}}{(2\sigma_0^2 + 4D_m\,|\Delta\bar{X}|)^{1/2}} \qquad (4)$$

where $2\sigma_0$ is the initial width of the transition zone.

Superposition of Dispersion on the Upconing of an Abrupt Interface

The position of the interface as a function of time and radial distance from the well is evaluated using Equation 1, which assumes an abrupt interface between the two fluids. This elevation is assumed to correspond to $X|_{\epsilon=0.5}$, or the mean of the concentration distribution across the transition zone. The only difficulties in this approach occur for $\epsilon(X) = 0.0$ and $\epsilon(X) = 1.0$. Since

$$\epsilon(X) = 0 \text{ for } X \rightarrow \infty \qquad (5)$$

and

$$\epsilon(X) = 1.0 \text{ for } X \rightarrow 0 \qquad (6)$$

the transition zone would have an infinite width in theory. To overcome this physical impossibility, the width of the transition zone is arbitrarily set at five standard deviations. This range includes approximately 99 percent of the area under the concentration distribution curve.

Concentration in Pumped Water

The increase in concentration, or salinization, of pumped water is probably due to the intrusion of invading fluid above the critical depth. The linear approximation for the interface elevation is limited to elevations below the critical elevation and the dispersion concept should be limited to the zone below the critical depth. The complex mixing and flow phenomena above the critical depth, near the well screen, and within the well pipe are approximated empirically using the approach followed by Schmorak and Mercado (1969).

The average dimensionless concentration of the transition zone above the critical rise, $\bar{\epsilon}(X > X_{cr})$, is approximated as one-half the concentration at the critical depth, or

$$\epsilon(X > X_{cr}) = 0.5\ \epsilon(X_{cr}) \tag{7}$$

The concentration in the pumped water, ϵ_w, is determined from dilution of the average transition-zone concentration above the critical depth with displaced fluid, or

$$\epsilon_w = \varphi\bar{\epsilon}(X > X_{cr}) \tag{8}$$

where φ is an interception coefficient, or the fraction of transition zone fluid in the total volume pumped.

Computer Program

Two types of upconing problems are considered. The first involves the description of the expected interface elevation and the salinity of the pumped water as a function of time for a given pumping rate. The second problem addresses the maximum rate at which a well can be pumped without exceeding a specified salinity in the pumped water. Both types of problems are included in an interactive computer code. Data are required under two modes of operation—"Basic Input Data" and "Edit".

Basic input data are required to initiate a new problem using the UPCONE program. The data entries include the problem title, the physical properties of the aquifer and the two fluids, and the geometry of the system. The user is prompted for the required data through a series of input commands.

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "Edit" mode. The edit commands can be used to redefine the problem, execute elevation or pumping rate calculations, or terminate the program.

The program has been written in an unextended version of FORTRAN 77 and has been installed on microcomputers running under CP/M-80 and MS-DOS as well as variety of minicomputers and mainframe machines. The major modifications in code to implement the program on a given system is the assignment of logical devices. Guidelines for these types of modifications are clearly identified in the source code.

References

Bear J. and D. K. Todd. 1960. "The Transition Zone Between Fresh and Salt Waters in Coastal Aquifers." Contribution No. 29, Water Resources Center, University of California, Berkeley, CA.

McWhorter, D. B. 1972. "Steady and Unsteady Flow of Fresh Water in Saline Aquifers." Water Management Technical Report No. 20, Engineering Research Center, Colorado State University, Fort Collins, CO.

Schmorak, S. and A. Mercado. 1969. "Upconing of Fresh Water-Sea Water Interface Below Pumping Wells, Field Study," *Water Resources Research, Vol. 5, No. 6,* pp. 1290-1311.

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The complete report, entitled "Upconing of a Salt/Fresh-Water Interface Below a Pumping Well," (Order No. PB 85-215 341/AS; Cost: $11.50, subject to change) will be available only from:

National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
Telephone: 703-487-4650

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