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Research Laboratory
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Research and Development

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Project Summary

A Mathematical Model of Electrostatic Precipitation (Revision 3)

M.G. Faulkner and J.L. DuBard

The objectives of this research program were: to upgrade the model of electrostatic precipitation developed under the sponsorship of the Environmental Protection Agency (EPA), to make the computer program which performs the calculations required by the model more user oriented, and to fully document computer program subroutines that perform fundamental calculations or utilize numerical techniques.

Model improvements include: a new method of calculating solutions to the electric field equations, a dynamic method of describing the effects of rapping reentrainment, a procedure for calculating effluent opacity, and a routine that checks the input data. Revision 3 of the model calculates efficiency in about 10% of the time required by Revision 1. The option to use input data expressed entirely in the metric system is provided, as are options to terminate the calculation after the calculation of V-I curves and to use an internal data set to estimate precipitator efficiency based on the resistivity of the dust particles.

This Project Summary was developed by EPA's Industrial Environmental Research Laboratory, Research Triangle Park, NC, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).

Introduction

Revision 3 of the electrostatic precipitator (ESP) performance model developed by Southern Research Institute (SRI) for the EPA has been completed. Since the model was released in 1975, it has been widely used to study and troubleshoot existing ESPs and to validate proposed new ESPs. The two revisions to the original model increased its usefulness and convenience of operation. Revision 3 further increases the model's utility by offering a significant reduction in required computation time plus several new features.

Collection Processes Used in the Model

The model predicts ESP performance by first performing the collection efficiency calculation under ideal conditions and then correcting the results for non-ideal conditions and unmodeled effects. For the calculation, the ESP is divided into sections (permitting different electrical conditions in each section) and then further divided into a maximum of 99 incremental lengths on the order of the ESP's wire-to-wire spacing. Efficiency is calculated sequentially on each incremental length to determine total collection efficiency. To compensate for the different charging and collection rates of large and small particles, the input dust load is divided into a maximum of 20 size bands, each handled separately in the efficiency calculation.

The ESP model uses the Deutsch equation to calculate the collection efficiency for each particle size band in each incremental length:

$$\eta = 1 - \exp(-Aw/Q) \quad (1)$$

where

η=the fractional collection efficiency,
A=the collection plate area,

W=the migration velocity of the charged particles, and
Q=the gas volume flow rate.

The migration velocity is calculated from:

$$w=\frac{qE_pC}{6\pi au} \qquad (2)$$

where
q=the charge on the particle,
E_p=the electric field at the plate,
C=the Cunningham slip correction factor,
a=the radius of the particles, and
u=the viscosity of the gas.

Thus the calculation of collection efficiency requires calculating the charge on each size particle in each incremental length and of the electric field at the plate. This can be done simply by estimation formulas or more rigorously by using a more complete charging theory and a detailed analysis of the field at all points in the ESP. The more rigorous process yields better accuracy over a wide range of applications and is used more frequently. However, it involves many lengthy calculations and requires much more computer time than the estimation process. The changes resulting in reduced computer time for Revision 3 affect the electric field and particle charge calculations of the rigorous calculation.

Features of the ESP Model

Table 1 summarizes ESP model features including the changes due to the various revisions. These features are discussed below in the order in which they are presented in Table 1.

1. The estimation procedure for calculating particle charges and the electric field at the plate remains unchanged.

2. The rigorous calculation of the electric field at the plate has been changed in Revision 3. Although the general procedure used to solve the field equations remains the same, the convergence of the iterative technique used to achieve a solution is accelerated using the dominant eigenvalue method. This method was previously applied to various chemical processes and to the electric field calculation.

The procedure used in the rigorous calculation of the field at the plate consists of simultaneously solving two field equations relating the electric potential and the space charge density at every point inside the ESP:

$$\frac{\Delta^2 V}{\Delta x^2}+\frac{\Delta^2 V}{\Delta y^2}=-\frac{\rho}{\varepsilon_0}, \text{ and} \qquad (3)$$

$$\rho^2=\varepsilon_0\left(\frac{\Delta V}{\Delta x}\frac{\Delta\rho}{\Delta x}+\frac{\Delta V}{\Delta y}\frac{\Delta\rho}{\Delta y}\right) \qquad (4)$$

where
V=the potential,
x=the coordinate from wire to plate,
y=the coordinate along the gas flow,
ρ=the space charge density, and
ε_0=the permittivity of free space.

In the model, the calculation is performed at every point on the grid shown in Figure 1. The initial estimate of the potential at each grid point, V_{ij}, is made using Cooperman's solution to the field equations. Following this, the space charge density at each grid point, ρ_{ij}, is calculated from V_{ij}. V_{ij} is then recalculated from ρ_{ij}. The process of alternately calculating ρ_{ij} and V_{ij} continues until the change in V_{ij} between iterations is less than a preset value at every grid point. At this point, the calculated current density at the plate is compared to the measured current density. If the values differ by more than 1%, the space charge at the corona is adjusted and the calculation of ρ_{ij} and V_{ij} begins again. If these values are within 1%, the potential solution is considered to have converged and the value of the electric field at the plate is obtained.

In Revision 3, the convergence rate has been accelerated by the dominant eigenvalue method. Using the new procedure, two criteria are applied to the calculated potential after each iteration: (1) at least five iterations must have taken place since the last acceleration step, and (2) the change in the potential between the last two iterations must be less than a present value at every grid point. These criteria ensure that each acceleration step has sufficient time to produce a stable solution before a new acceleration step is applied. If both criteria are met, the potential is adjusted at every point by a factor derived from the potential changes due to the last two iterations. The potential adjustment has the form:

$$V_{ij}^{(n+1)}=V_{ij}^{(n-1)}+\alpha(V_{ij}^{(n)}-V_{ij}^{(n-1)})/(1-\lambda) \qquad (5)$$

where
$V_{ij}^{(n+1)}$=the adjusted potential for use in the next iteration,
$V_{ij}^{(n-1)}$=the potential calculated in the previous iteration,
α=a damping factor, $0<\alpha<1$, used to prevent instability,
$V_{ij}^{(n)}$=the potential at point i,j calculated in the last iteration, and
λ=the dominant eigenvalue.

λ is given by:

$$\lambda=\|\Delta V^{(n)}\|/\|\Delta V^{(n-1)}\|, \qquad (6)$$

where

$$\|\Delta V^{(n)}\|=[\Sigma_{ij}(V_{ij}^{(n)}-V_{ij}^{(n-1)})^2]^{1/2}. \qquad (7)$$

Following the adjustment, the mutual calculation of ρ_{ij} and V_{ij} is resumed until either the potential converges or the conditions for another acceleration step are met. Since the convergence criteria are not altered by this process, the accuracy of the calculation remains high. The effect of a potential adjustment is greater than the effects of many iterations. In the ESP model, including the dominant eigenvalue method reduced the number of V-ρ iterations required for the complete efficiency calculation from 16,000 to 1,400 in a study of a power plant, a 91% reduction.

Table 1 shows the inclusion of an analytic approximation to the electric field in Revision 2. This approximation offered almost the same calculation accuracy as the rigorous calculation while using much less computer time. With the incorporation of the dominant eigenvalue procedure in Revision 3, the

Table 1. ESP Model Revision Summary

Process	*Revision*
Estimation Procedure	*1,2,3*
Electric field calculation	
rigorous calculation	*1,2*
same with dominant eigenvalues (faster)	*3*
analytic approximation	*2*
Particle charging calculation	
Simpson's rule integration	*1*
Gaussian integration (faster)	*2,3*
Correction for non-ideal effects	
sneakage, velocity distribution, non-rapping reentrainment	*1,2,3*
empirical rapping calculation	*1,2,3*
dynamic rapping calculation	*3*
Opacity calculation	*3*
Option of metric input data	*3*
Option to reduce printed output	*3*
Internal data set	*3*
V-I curve generation	*1,2,3*
Option to stop after printing V-I curve	*3*
Routine to check input data	*3*

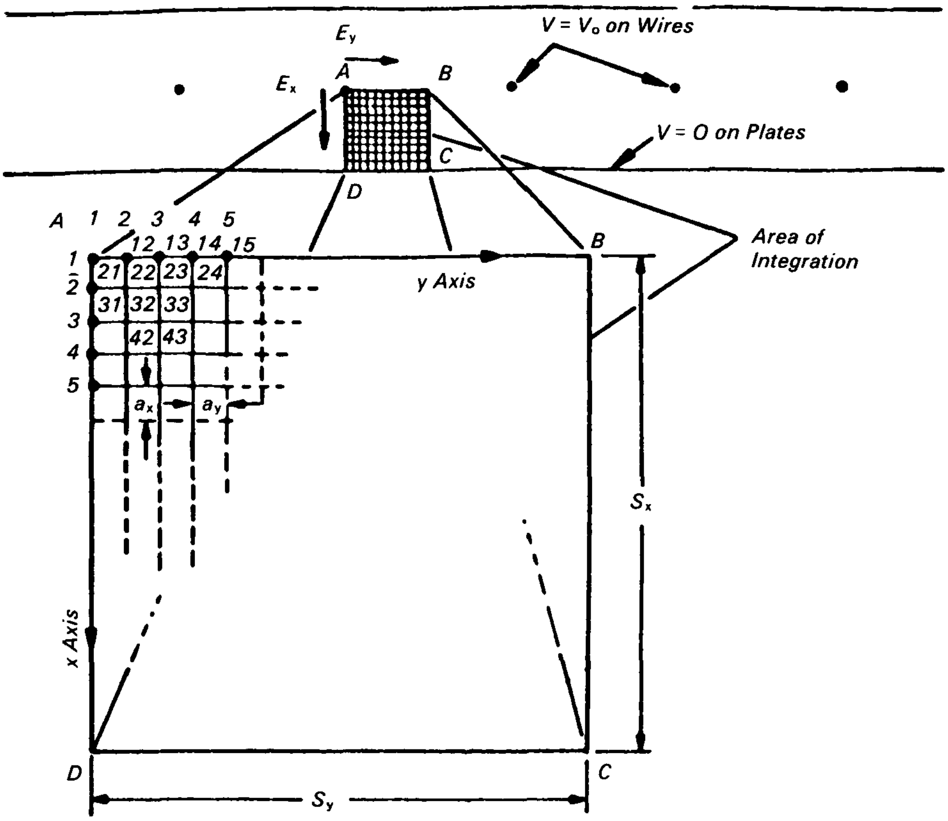
rigorous field calculation is now as fast at the approximation. Consequently, the approximation procedure was not included in Revision 3.

3. The speed of the particle charging routine is also reduced. In order to calculate the charge on each size particle in each incremental length using the more exact ionic charging theory, many integrations are required. The change to the Gaussian integration scheme in Revision 2 greatly reduced the integration time.

The net effect of the changes to the electric field calculation and the particle charging routine is a reduction in computer time of about 90% with no loss in accuracy.

4. As previously mentioned, after the ideal collection efficiency is calculated, the model corrects the efficiency for various non-ideal effects. Corrections for sneakage, non-rapping reentrainment, and non-uniform gas velocity are applied by adjusting the migration velocity for each size particle by a factor based on theoretical and experimental studies of a pilot scale precipitator. This method of correction has not been changed in Revision 3.

Also unchanged is the empirical calculation of losses due to rapping reentrainment. This efficiency correction is based on the amount of mass collected in the last section of the ESP. A fraction of the collected mass is fit to a log-normal size distribution and added by size bands to the ESP penetration dust predicted by the ideal efficiency calculation. The collection efficiency is then recomputed based on the combined effluent. The log-normal size distribution used to describe the reentrained particles has default parameters of a 6 μm mass median diameter with a standard deviation of 2.5. Other sized distribution data may be used in addition to these values if desired. However, the total mass of the reentrained particles is fixed by a relationship based on data derived from a study of rapping reentrainment in six power plants. The subroutine which performs these operations requires a negligible amount of computer time relative to the ideal calculation and occurs automatically unless the new dynamic rapping routine is selected.



S_y = one half wire-to-wire spacing
S_x = wire-to-plate spacing
$a_x a_y$ = increment sizes for integration
V_o = applied voltage
E_x = component of electric field perpendicular to plate
E_y = longitudinal component of electric field
j = average current density

Figure 1. *Nomenclature used in the numerical analysis of the electrical conditions in wire-plate precipitators.*

The new dynamic rapping routine utilizes a different process to calculate the effects of rapping reentrainment. This routine keeps track of dust layer growth at every point in the ESP as a function of time. At a user specified time of rap, the collected dust is removed from the rapped ESP increments. A specified fraction of the removed dust is fit to a log-normal size distribution and stored for the calculation of reentrainment. The entire collection efficiency calculation is then repeated, except that the reentrained dust particles are reintroduced into the gas flow during the calculation. The mechanism for this is that the number of particles in each size band per cubic meter of gas which are to be reentrained due to the rap of increment i is added to the number of particles/size band/cubic meter already present in the gas flow immediately before the calculations for increment i+1 begin. Thus the efficiency in increment i+1 is calculated on an increased amount of dust due to the reentrainment from the preceding increment. Inherent in this mechanism is the assumption that the reentrained particles instantaneously acquire the charge found on particles of the same size at that point in the gas flow.

The calculation of rapping reentrainment effects using the dynamic rapping routine is very lengthy because the ideal collection efficiency must be calculated at least twice for each rap: the first time, to find the dust layer thickness at the time of the rap; and the second, to calculate the effects of the reentrainment. Depending on the complexity with which the reentrained dust is specified, up to seven additional efficiency calculations may be required. Therefore it is recommended that the estimation procedure be used to reduce computer time. The dynamic rapping calculation is very flexible in that the user may vary the times of the raps, the rapping sequence, the fraction of the collected dust that gets reentrained, the size distribution and the density of the reentrained dust, and the duration of the rapping puff. This flexibility should be of use when comparing different rapping schemes. At present, no experimentally supported data set, such as that used in the empirical rapping correction, exists for the dynamic rapping calculation. A study, to determine if such a data set can or should be developed, would be valuable. Currently, the data for dynamic rapping must be matched to each application. Figure 2 shows the results of using dynamic rapping on power plant data.

5. A plume opacity calculation has been added to Revision 3 of the model. This routine calculates differential and total extinction efficiencies and the total opacity of the ESP outlet plume calculated by the model, using:

$$\text{Opacity} = 1 - \exp(-EL) = 1 - \exp(-NAQ_{ext}L), \qquad (8)$$

where

E=the extinction coefficient,
L=the pathlength of the light beam,
N=the particle concentration,
A=the projected area of the particle, and
Q=the extinction efficiency.

Figure 3 shows the extinction efficiency as a function of the particle size parameter, $x = \pi D/\lambda$, for four complex indices of refraction. D and λ are the particle diameter and the wavelength of light beam, respectively. The calculation, based on the Mie theory, is performed at 10 wavelengths weighted according to the photopic response of the human eye. EPA-approved opacity monitors must simulate this color response, which is maximum at $\lambda = 0.55$ µm and has a width at half maximum of 0.1 µm. The required input data are stack diameter and complex index of refraction. A single wavelength-dependent index of refraction or up to 10 values of wavelength-independent indices of refraction may be entered. Alternatively, the refractive index may be omitted, in which case the calculation is performed using the default values of 1.5 and 1.5-0.1 i.

6. To facilitate the use of the model in other countries, the data entry routine has been modified to receive data in metric units if desired. However, unless all-metric data is specified, the data must be in the mixed metric/English units used in previous versions of the model.

7. The option of reducing the amount of printed output data is provided. This will be of use where only data summaries are of interest.

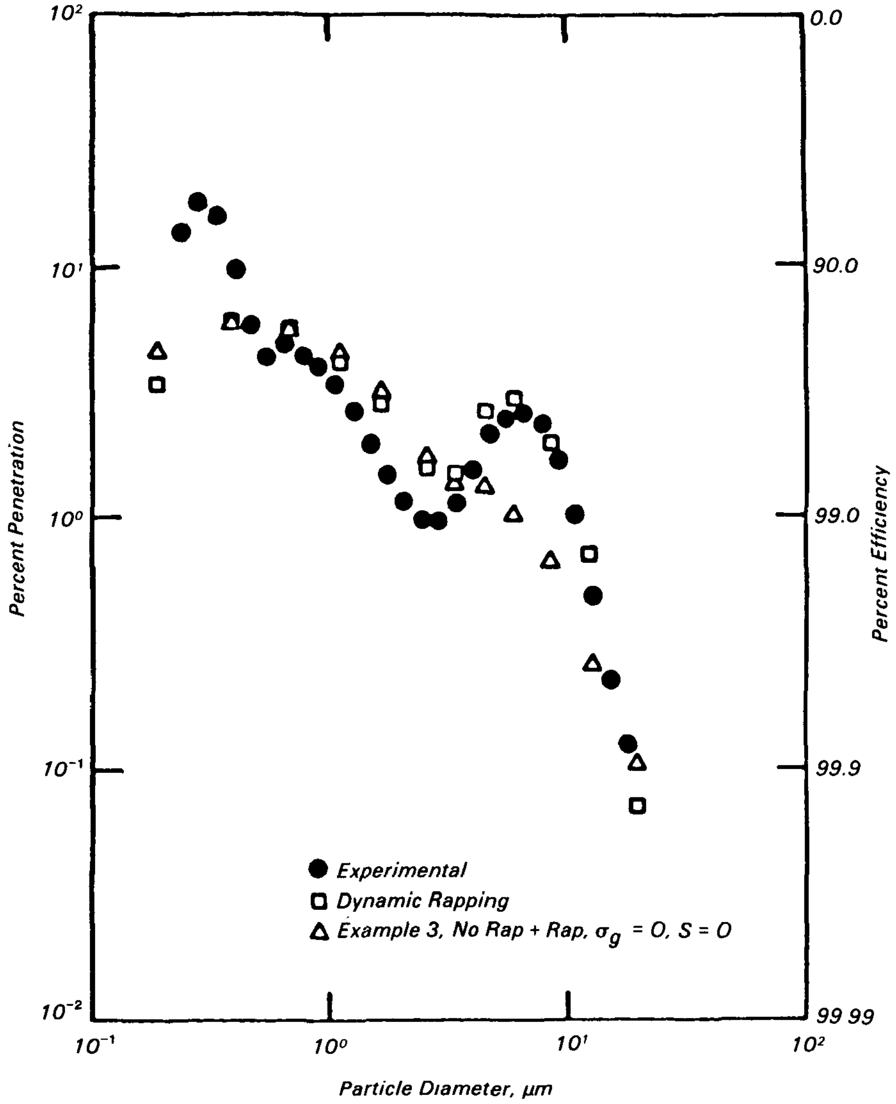


Figure 2. Comparison of standard and dynamic rapping reentrainment calculations

8. A built-in data set is provided in Revision 3. This data set contains nearly complete data based on a "typical" power plant. The only input data required are the resistivity of the collected particle and the collection plate spacing. From this, the model calculates a set of V-I curves and operating points for an ESP based on a study of 17 cold-side utility fly ash precipitators. The length of the ESP is varied so that efficiency may be calculated for SCA values of 40 to 158 m^2/m^3/sec (200-800 ft^2/1000 acfm). This routine will be of use for verifying the operation of the model and for establishing a starting point for a more detailed examination of ESP design.

9. The model may be used to generate V-I curves. The ability to calculate V-I curves has been in the model since Revision 1 as part of a routine that calculates the operating conditions and electric fields in the ESP if the actual operating conditions are not available. This is a very lengthy calculation which may be more frequently used in Revision 3 due to the significantly reduced computer time resulting from the dominant eigenvalue procedure. In addition, a switch now terminates the program after the V-I curve is printed, if only V-I data are required. Alternatively, if the ESP for which V-I curves are desired is similar to a utility fly ash ESP, the V-I curves may be generated in the routine which calculates the internal data set. Again, a switch terminates the program after the V-I curves are printed.

10. A routine for checking the input data has been included in Revision 3. This

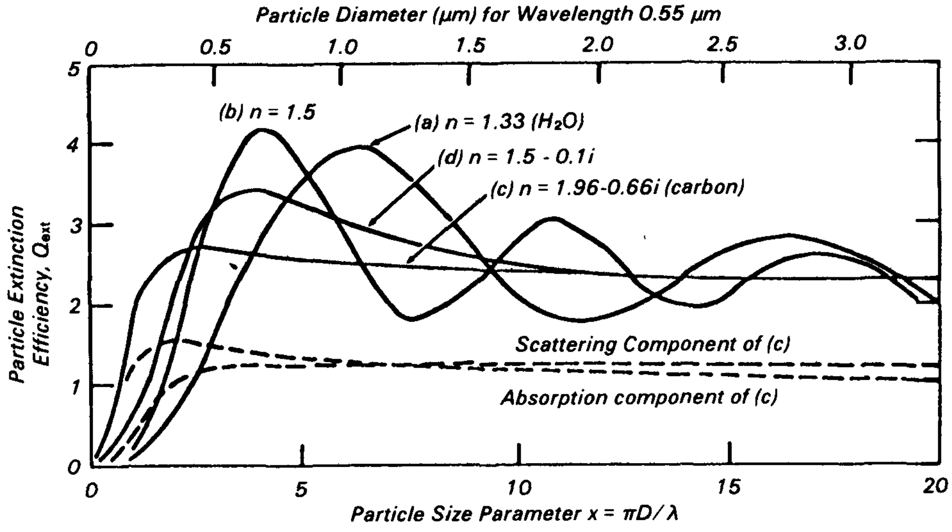


Figure 3. *Extinction efficiency as a function of particle size parameter.*

routine checks each item of input data for variables which are out of the allowed range (e.g., having too many size bands) and for operating conditions which have allowed but unusually large or small values, such as found in a laboratory scale ESP. If anomalies in the data are discovered, diagnostic messages will be printed, stating that an error has been made, or (in the second case) that certain values are unusual and should be checked.

Computer Related Data

Revision 3 requires only 12% more memory than Revision 1. On the SRI Digital Equipment Cooperation (DEC) System 20 computer, Revision 3 uses 206 kilobytes of 7-bit memory. To facilitate model usage on computers with less memory, the code is marked so that some of the less essential functions may be easily removed. The minimum program length, with all of the marked statements removed, is 135 kilobytes, which is about 65% of the entire program.

The data format for Revision 3 is largely the same as for previous versions of the model. Old data sets may be used without changes to yield the same results in less time. Only the additions (e.g., opacity, dynamic rapping) require additional input data. Program users may convert Revisions 1 and 2 of the ESP model to the latest version by making changes in the main routine and seven subroutines (CMAN, EFLD1, EFLD2, CHARGN, RATE, ADJUST, and PRTINP), deleting the data block and subroutine EFLD3 (Revision 2), and adding the seven new subroutines.

Conclusions

The EPA/SRI ESP model has proven to be useful in the design and evaluation of ESPs. Revision 3 of the model offers greater utility through greater flexibility and added features such as opacity calculation and dynamic rapping. Revision 3 is also much faster than previous model versions, which results in significantly reduced computer costs that should encourage wider use of the model. As with Revision 1, Revision 3 is complete in two volumes. Volume I describes the physical processes being modeled, the algorithms used, and the necessary input data, and lists the FORTRAN code. Volume II is a user's manual that contains complete explanations of the input and output data, examples of programs which illustrate the various options available, and a tutorial of the effects of varying some of the computer operating parameters, with supporting examples.

Recommendations

Although the mathematical model of ESP in this report represents a significant improvement over the previous version, additional work would improve the fundamental basis and user-oriented aspects of the model. Research of value would include:

1. Theoretical and experimental studies of the effects of particles on the electrical conditions to better describe the effect on the electric field distribution.
2. Theoretical and experimental studies of electrical breakdown mechanisms in the collected particulate layer to acquire the capability of theoretical prediction of when electrical breakdown will ensue for a given value of dust resistivity.
3. Since the model underpredicts field-measured collection efficiencies for fine particles without the use of empirical correction factors, theoretical and experimental studies could remove the empiricism or explain the discrepancy. These studies could include a reevaluation of the theories now used in the model and an examination of now-neglected effects; e.g., particle charging near corona wires and phenomena due to the gas flow field. There is evidence that free electrons may play a role in particle charging in negative coronas for temperatures of 150°C to 350°C. A charging theory that accounts for this effect would be of value.
4. Theoretical and experimental studies on the mechanisms involved in separating the dust layer from the collection plates during a rap and on the quantity and nature of the dust reentrained into the gas flow. This could provide an analytical method determining the nature of the reentrained dust used in dynamic rapping and reduce dependence on empirically derived data. This change could increase the usefulness of the model in optimizing the electrical operating conditions and the rapping schedule and intensity of an ESP.
5. Investigations and implementation of alternative numerical techniques to make the computer program run significantly faster.

M. G. Faulkner and J. L. DuBard are with Southern Research Institute, Birmingham, AL 35255.
Leslie E. Sparks *is the EPA Project Officer (see below).*

The complete report consists of two volumes, entitled "A Mathematical Model of Electrostatic Precipitation (Revision 3):"
"Volume I. Modeling and Programming," (Order No. PB 84-212 679; Cost: $34.00)
"Volume II. User's Manual," (Order No. PB 84-212 687; Cost $28.00)

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