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Project Summary

Persistence in Model Ecosystems

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Mathematical models aid in understanding environmental systems and in developing testable hypotheses relevant to the fate and ecological effects of toxic substances in such systems. Within the framework of *microcosm or laboratory ecosystem* modeling, some differential equation models, in particular, become tractable to mathematical analysis when the focus is on the problem of persistence. In this report, a hierarchy of microcosm-related models, the top level of which contains a nutrient-producer-grazer food chain, and general food chains are analyzed for persistence. The results, which take the form of inequalities involving model parameters, specify necessary conditions and sufficient conditions for continued presence of the model components throughout indefinite time intervals. These results can serve as a basis for preliminary evaluations of model performance.

This Project Summary was developed by EPA's Environmental Research Laboratory, Athens, GA, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).

Introduction

Persistence in mathematical representations of ecosystems is the analog for survival of organisms and continued presence of nutrients in the modeled system. Although exact solutions of the non-linear, differential equations describing growth-decay rates of substances or organisms in environmental systems are impossible to obtain, qualitative solutions for persistence are possible. The technique consists of constructing auxiliary functions and differential inequalities to determine necessary and sufficient conditions for persistence from model parameters.

This technique is applied to a variety of generally accepted ecological models, beginning with the basic chemostat, including cycling components, and finally including a top level predator. Necessary and sufficient conditions for persistence are obtained for the chemostat and cycling models. Only necessary conditions, however, are obtained for the full system.

General Lotka-Volterra food chain models are also investigated. Persistence of a top level predator is established for very general models. Persistence at the top level in a Lotka-Volterra model of general omnivory is also developed.

All the results, which have the form of inequalities involving model parameters, specify conditions for continued presence of the model components for an indefinite time. These results can serve as a basis for preliminary model evaluation and experimental design.

Summary

The mathematical analysis focusing on persistence in differential equation models results in criteria taking the form of inequalities involving model parameters. For the five-component, microcosm-related model and its subsystems, these results are expressed in terms of threshold values for the nutrient input rate. In the case of the food chain models, positivity of weighted sums of the intrinsic growth-decay rates constitute the conditions for persistence. Generally, the nutrient input level, the intrinsic growth rate of the food source, or the rate of supplementation of intermediate predator being sufficiently large guarantees persistence. The methods used consist of approximating the differential equations; in particular, linearization and the auxiliary function-comparison principle techniques are the main tools employed from the qualitative theory of ordinary differential equations.

In each of these models, it has been shown that solutions are bounded. A consequence of persistence for autonomous (right hand side of equations independent of t explicitly) systems, in this case, is that any solution with positive initial values must approach asymptotically either an equilibrium or a solution of oscillatory type in the interior of the feasible region. Furthermore, and possibly most significant, is the fact that persistence implies the existence of a recovery mechanism in the model. Solutions exhibiting small component densities at some time t tend to equilibrium or oscillatory type stable density configurations in the interior of the feasible region. That these models do not take into account stochastic effects contributes to this result.

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The complete report, entitled "Persistence in Model Ecosystems," (Order No. PB 82-196 916; Cost: $6.00, subject to change) will be available only from:

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