



## *Project Summary*

# Potential Flow Model for Gaussian Plume Interaction With Simple Terrain Features

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The theory of turbulent plumes embedded within potential flow fields is discussed for flows modified by special complex terrain situations. Both two- and three-dimensional isolated terrain obstacles are considered. Concentration estimates are evaluated using a Gaussian solution to the appropriate diffusion equation; dispersion coefficients are modified to account for terrain-induced kinematic constraints, and plume centerline trajectory is obtained from a stream line of the potential flow. Specific limitations to the theory and its applicability are reviewed.

A computer algorithm is developed and documented to perform these calculations. Dispersion estimates and ground-level concentrations are given for a variety of meteorological situations. Parameters of the problem include obstacle height, effective source height, distance between source and obstacle, crosswind aspect ratio of the obstacle, and atmospheric stability. The potential flow theory, originally applicable to neutral flows, is extended by an empirical approximation to slightly stable flows. Additionally, an interpolation scheme is proposed for objects of arbitrary crosswind aspect ratio between the limiting cases of a hemisphere and a half-circular cylinder. Model computations are compared to laboratory experimental results and to field measurements.

*This Project Summary was developed by EPA's Environmental Sciences Research Laboratory, Research Triangle Park, NC, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).*

### Introduction

This study has been motivated by the requirement for a treatment of plume dispersion in complex terrain that is practical for regulatory use, yet retains much of the essential physics of the problem. For regulatory purposes, the modeling approach should emphasize cases with potential for high ground-level concentrations. These include:

- stable conditions and low wind speeds, under which direct plume impact or blocking by nearby terrain obstacles may occur, and
- neutral or slightly stable conditions and moderate or high wind speeds, under which the plume centerline trajectory passes over and close to the terrain surface.

This report addresses the development of modeling methods for neutral and slightly stable conditions. The general approach employed follows the theory of turbulent plumes embedded within potential flow fields. The theory is applied to the calculation of ground-level concentrations using a Gaussian form of solution to the diffusion equa-

tion. Streamfunctions proper to potential flow over a cylinder (aspect ratio =  $\infty$ ) and to potential flow over a sphere (aspect ratio = 1) form cornerstones of the model. These are extended to describe flows over terrain features of intermediate crosswind aspect ratio by a weighting of the two limiting streamfunctions. This weighting scheme was derived in part using results from wind tunnel experiments for flows over obstacles of intermediate aspect ratio.

In addition, although the model is strictly applicable only to neutral flows, an empirical approximation scheme is included to define streamline lowering caused by increased stratification. The empirical basis for this portion of the model is derived from stratified tow-tank experiments.

Other restrictions in the use of the model have not been addressed. These limitations to the model arise from the neglect of boundary layer phenomena such as flow separation, unsteady wake effects, time-dependent effects of stability (e.g., lee wave generation), and surface heating effects. In addition, the theory is applicable in a strict mathematical sense only for thin plumes.

The report discusses the rationale for selecting particular modeling approaches, provides full technical documentation for the algorithms developed, and presents, for a number of specific test situations, the results of comparisons between model calculations and laboratory and field observations.

## Approach

In complex terrain, moderate or high wind speeds with neutral or slightly stable stratifications often result in high pollutant concentrations because, as the plume is transported over terrain features, it is forced to pass close to the terrain surface. Physical mechanisms relevant to these conditions include terrain-induced alteration of the plume centerline trajectory and kinematic constraints on horizontal and vertical dispersion.

The modeling approach used here applied potential flow theory to a Gaussian point source model. The model incorporates a theory for turbulent plumes embedded within potential flow fields based on solutions to the diffusion equations describing flow fields over two-dimensional and three-dimensional axisymmetric terrain obstacles. Qualitatively, these solutions are of Gaussian form, with crosswind and vertical dis-

persion coefficients evaluated as line integrals of the velocity field along the plume centerline trajectory.

Evaluating the terrain-influenced dispersion coefficients for this model requires specifying the crosswind and the vertical diffusivities. To compare model calculations with analogous flat terrain situations, an approximation scheme was implemented, using the PGT dispersion coefficients as a calibrating scale. Qualitatively, the diffusivity at a given distance from the source along the plume centerline streamline is taken as that for the same transit time in flat terrain. The consequence of this assumption is that model calculations of dispersion coefficients reduce to flat terrain values in the limits of large downwind distances or small obstacles.

To account for the effects of stability, an approximation consistent with laboratory observations is adopted to lower the height of the neutral streamline within two obstacle heights of the obstacle center by an amount determined by the height of the obstacle and the Froude number.

A second approximation is derived to account for an obstacle with arbitrary crosswind aspect ratio. In this case, two- and three-dimensional streamlines are weighted to provide an intermediate plume centerline trajectory.

## Results

Model computations indicate that maximum concentrations vary significantly with obstacle size, effective stack height, and relative distance between the stack and the obstacle. Comparisons of model predictions with available observations test the model performance for a limited number of possible combinations of these and other factors. Table 1 summarizes the range of model parameters involved in the comparisons. Note that  $\lambda$  is the crosswind aspect ratio of the obstacle,  $Fr$  is the Froude number

characterizing the importance of density stratification in defining the flow,  $X_s/a$  is the distance between the stack and the obstacle normalized by the obstacle height, and  $H_s/a$  is the effective stack height normalized by the obstacle height.

The "smooth tunnel" comparison and the "tow tank" comparisons to the model under conditions that minimize the influence of processes not contained in the model. They show that the model is able to predict the observed maximum surface concentration within a factor of two (generally overpredicting depending on the interpretation of the observed plume properties in the absence of the obstacle).

The "rough tunnel" comparisons include the effects of a strong boundary layer on flow over obstacles with triangular cross section. Model predictions of maximum surface concentrations are again within a factor of two (overprediction) for obstacles of aspect ratio 1. However, as the crosswind aspect ratio increases, the concentration predictions fall below the observations. The data indicate that the plume size is significantly enhanced upstream and over hills with the larger aspect ratios. The deformations included in the potential flow field approximation are only partly responsible. A better understanding of plume dispersion in a deforming boundary layer flow is likely to be needed to describe these experiments more accurately.

Plume interactions with two terrain features, a ridge and an isolated mound near the Widows Creek Steam Electric Power Plant in Alabama have been modeled. Meteorological conditions used in the model correspond to several hours selected from nine months of hourly  $SO_2$  and meteorological data collected by the TVA Air Quality Branch. The hours selected for comparison were derived by matching hours of high measured  $SO_2$  concentrations on the two terrain features with neutral-to-stable atmospheric conditions. In addi-

**Table 1.** Range of Model Parameters Evaluated in Laboratory and Field Tests of the Potential Flow Model

Comparison Study	$\lambda$	$Fr$	$X_s/a$	$H_s/a$
Smooth Tunnel	1	999	3.7	0.4, 0.
Tow Tank	1	0.97	3.7	0.4
Rough Tunnel	1, 2, 3, $\infty$	999	3.7	0.5, 1.
Widows Creek	$4 \infty$	1.3-1.7	32	>1
		0.4-1.0	12	>1

**Table 2.** Comparison of Observed Concentrations ( $\mu\text{g}/\text{m}^3$ ) at Widows Creek and Predicted Concentrations Based on the Potential Flow Model with Buoyant Plume Enhancement

Julian Day	Multilayer Plume Height (m)	Observed Concentration	Complex Terrain Model Predictions		
			Stability C	Stability D	Stability E
<i>Ridge Impact</i>					
3	422	393	982	270	—
40	402	550	729	157	—
190	320	576	—	4,599	3,354
230	370	603	—	12,038	7,711
<i>Mound Impact</i>					
4	301	1,179	—	1,529	531
166	373	367	—	99	16
222	392	393	—	2,123	246

tion, only those hours with nearly coincident vertical temperature and velocity profiles were considered. Of the seven hours selected, four are associated with impacts on the ridge, and three are associated with impacts on the isolated mound.

A comparison of observed and calculated concentrations for the two most appropriate dispersion parameter classes are given in Table 2. Most of the observations fall between the model predictions for the two dispersion parameter classes.

Uncertainties in the meteorological conditions at plume height, and in the emissions from the facility, cloud the comparison of model predictions with observations. Given the range of data, reasonable combinations of assumptions can produce good correspondence between the concentrations in six of the seven cases. This does not, however, constitute an adequate evaluation of the model because of the uncertainties underlying these assumptions. The three greatest uncertainties lie in the specification of the dispersion parameters, the final plume height, and the actual emission rate.

### Conclusions

These preliminary assessments suggest that with verification and refinement, the approach may be applicable to the following situations:

- isolated, single terrain obstacles of arbitrary height, of cross-section approximately circular in a plane parallel to the wind direction, and of arbitrary aspect ratio in the crosswind direction; and
- neutral to slightly stable stratifications.

A number of limitations arise mainly from physical effects that are not

described by the potential flow model. The model should not be applied to the following situations:

- stable flow cases in which the plume may directly impact the hill;
- dispersion cases dominated by surface boundary layer effects;
- unstable cases (e.g., strongly convective situations) for which potential theory is unsuitable; and
- cases dominated by wake effects.

The range of suitable applications of the model is also limited by the theoretical approximations made, and by the limited configurations studied experimentally. These limitations include:

- the "thin plume" approximation ( $\sigma_z/n_s < 1.0$  where  $n_s$  is the height of the plume centerline above the hill crest);
- the surface boundary layer effects not considered in the model;
- the Gaussian behavior assumed in the model in contrast to non-Gaussian behavior seen in some of the experimental results;
- restriction of the available experiments to a small number of stack heights, and to fixed configuration of obstacle relative to stack; and
- the hill shapes used in the laboratory experiments, as compared to the spherical hill shape assumed in the theoretical calculations.

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*The complete report, entitled "Potential Flow Model for Gaussian Plume Interaction with Simple Terrain Features," (Order No. PB 81-171 837; Cost: \$17.00, subject to change) will be available only from:*

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