A TECHNIQUE FOR CALCULATING OVERALL EFFICIENCIES OF PARTICULATE CONTROL DEVICES

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LIST OF ABBREVIATIONS

A	Electrostatic precipitator collecting surface, ft^2
B_c	Width of gravity settling chamber, ft
C	Gravity settling chamber empirical factor, dimensionless
C'	Venturi scrubber correlation coefficient, dimensionless
D	Particle diameter, microns
D_c	Cut size, microns
D_m	Mean particle diameter, microns
esu	Electrostatic units
E_T	Overall collection efficiency of a single control device
E_{T_n}	Overall collection efficiency of n control devices operated in series
F	Electrostatic precipitator migration velocity constant, $microns^{-1}$
$f(D)$	Frequency size distribution of particles
g	Gravitational acceleration, ft/sec^2
L	Venturi scrubber: liquid injection rate, $ft^3/1000\ ft^3$
L_c	Length of gravity settling chamber, ft
M_{c_i}	Total mass of particles collected by a control device, lb
M_i	Total mass of particles entering a control device, lb
N_c	Number of parallel chambers in gravity settling chamber
$Q(D)$	Size collection efficiency of a control device
V	Gas volumetric flow rate, ft^3/sec
V_t	Venturi scrubber: gas stream throat velocity, ft/sec
$W(D)$	Migration velocity of a particle of diameter D in an electrostatic precipitator, ft/sec
$Y(D)$	Cumulative size distribution of particles
α	Cyclone empirical size efficiency parameter, $micron^{-1}$
β	Slope of a semilogarithmic particle cumulative distribution, $micron^{-1}$
μ	Gas viscosity, lb/ft-sec
ρ	Gas density, lb/ft^3
ρ_p	Particle density, lb/ft^3
W_i	Width of the cyclone gas inlet, ft
N_t	Number of revolutions the gas stream makes in the cyclone
V_i	Inlet gas velocity to cyclone, ft/sec
E_p	Charging field in electrostatic precipitator, kV/cm
E_0	Collecting field in electrostatic precipitator, kV/cm
K	Dielectric constant of particles collected in electrostatic precipitator

ABSTRACT

A generalized mathematical technique is developed to calculate the overall collection efficiency of particulate control devices. Equipment operating parameters and the size distribution of the particles in the inlet gas stream are used in the calculation. The technique is successively applied to efficiency calculations for settling chambers, cyclones, venturi scrubbers, and electrostatic precipitators. Extension of this mathematical method is made to encompass control devices operated in series. Specific examples are also included to illustrate the technique when it is applied to single and multiple devices.

Key words: control efficiency, particle size, settling chamber, cyclone, venturi scrubber, electrostatic precipitator, mathematical technique

A TECHNIQUE FOR CALCULATING OVERALL EFFICIENCIES OF PARTICULATE CONTROL DEVICES

BASIC DEVELOPMENT

Before the effectiveness of a given particulate control device can be determined, it is necessary that its overall collection efficiency be calculated under existing operating conditions. Collection efficiency is a function of such operating conditions as temperature, pressure, and velocity of the gas stream entering the device; dimensions of the control device; and statistical size distribution of the incoming particles. For most devices it is possible to determine either the theoretical or empirical functional relationship between efficiency and particle size. That is:

$$\text{Size collection efficiency} = Q(D) \tag{1}$$

Where: D = diameter of particle, microns

In many cases, the cumulative size distribution of the particles in the incoming gas stream can be readily determined from a few measurements. The cumulative distribution is the function used to calculate the mass or weight fraction of particles in a given sample having diameters greater than a stated diameter, D:

$$\text{Cumulative size distribution} = Y(D) \tag{2}$$

Once known, the cumulative distribution can be used to determine the frequency distribution, which is used to calculate the masses of particulates that fall within the various ranges of size (size-fractions).

The following mathematical development shows how the frequency size distribution, f(D), can be found from the cumulative distribution, Y(D). Consider a particulate sample of mass, M(lb), with diameters ranging from 0 to ∞. By definition, Y(D) equals the mass fraction of particles larger than diameter D.

$$Y(D) = \int_D^\infty f(x)dx = -\int_\infty^D f(x)dx \tag{3}$$

This relationship holds because Y(D) is the integral of f(D). Differentiating Y(D) with respect to D:

$$d/dD\ [Y(D)] = d/dD\ [-\int_{\infty}^{D} f(x)dx] = -f(D) \qquad (4)$$

Now, consider all the particles in the size interval D to D + dD. The mass of these particles collected by the device, dM_C, would equal the mass of incoming particles in the interval, Mf(D) dD, multiplied by the collection efficiency at size D, Q(D):

$$dM_C = Mf(D)\ Q(D)\ dD \qquad (5)$$

To determine the overall collection efficiency, E_T, one would simply integrate dM_C from 0 to ∞ and divide by the total incoming mass, M:

$$E_T = \frac{1}{M}\int_0^{\infty} dM_C = \int_0^{\infty} f(D)Q(D)dD \qquad (6)$$

This is the basic equation for overall collection efficiency. Equation 6 can be further simplified by the substitution of f(D). For many, if not most, particulate streams, the weight distribution very closely follows a log-normal curve, which is a normal distribution skewed toward very small or very large particle sizes. Substitution of the log-normal distribution function into equation 6 would result in an expression that probably could not be integrated to yield a closed-form function. Either numeric or graphic integration would have to be used to calculate the overall efficiency (see Example 1). A simpler approach can be taken if one recognizes that for values of Y(D) ranging from about 0.15 to 0.85, the distribution approximates an exponential of the form:

$$Y(D) = e^{-\beta D} \qquad (7)$$

Where: β = slope of the semilog distribution, $micron^{-1}$

The parameter β can be obtained by plotting the log of the cumulative distribution of particle size-fractions against the particle diameters. These size-fraction measurements can be obtained from one or more particle separation procedures such as sieving (for coarse particles), elutriation, and sedimentation (for fine particles). (It must be emphasized that equation 7 deviates from the actual weight distribution for relatively small and relatively large particles.)

Recalling equation 4: f(D) = -d/dD [Y(D)]. Then, substituting equation 7 into equation 4:

$$f(D) = -d/dD\ [Y(D)] = -d/dD\ [e^{-\beta D}] = \beta e^{-\beta D} \qquad (8)$$

Substitution into equation 6 yields:

$$E_T = \beta\int_0^{\infty} e^{-\beta D}\ Q(D)\ dD \qquad (9)$$

APPLICATIONS TO SINGLE CONTROL DEVICES

To obtain efficiencies, the above equations can be applied to some of the commonly used particulate control devices; e.g., gravity settling chambers, cyclones, venturi scrubbers, and electrostatic precipitators.

GRAVITY SETTLING CHAMBERS

According to R. T. Shigehara,[1] the size efficiency of a settling chamber is based on Stoke's settling law and such parameters as the spatial dimensions of the device and the characteristics of the gas stream:

$$Q(D) = \frac{[Cg(\rho_p - \rho)L_C B_C N_C]D^2}{18\mu V} = [k]D^2 \qquad (10)$$

Where:

- C = dimensionless empirical factor (use 0.5 if no other information is available)
- g = gravitational acceleration, ft/sec^2
- ρ_p = particle density, lb/ft^3
- ρ = gas density, lb/ft^3
- μ = gas viscosity, lb/ft-sec
- V = volumetric flow rate, ft^3/sec
- B_C = chamber width, ft
- L_C = chamber length, ft
- N_C = number of parallel chambers: 1 for a simple chamber and N trays + 1 for a Howard setting chamber

For a given set of operating conditions, the terms in brackets are considered to be constants. Thus, substitution into equation 9 yields the overall collection efficiency:

$$E_T = \beta\int_0^\infty kD^2 \, e^{-\beta D} \, dD \qquad (11)$$

This integral must be evaluated in two parts: first, between the limits of $D = 0$ and $D = \sqrt{1/k}$ and second, between the limits of $D = \sqrt{1/k}$ and $D = \infty$. This is necessary because the size efficiency remains at a constant value of 100 percent for all particles having diameters greater than or equal to $\sqrt{1/k}$. Thus:

$$E_T = \left(\beta\int_0^{\sqrt{1/k}} kD^2 \, e^{-\beta D} \, dD\right) + \left(\beta\int_{\sqrt{1/k}}^\infty e^{-\beta D} \, dD\right) = \frac{2k}{\beta^2}\left(1 - e^{-\beta/\sqrt{k}}[1 + \beta/\sqrt{k}]\right) \qquad (12)$$

Example 1:

A settling chamber, operated as the primary cleaner on a heating plant spreader stoker, has the following parameters:

Empirical factor, C = 0.60

Particle density, ρ_p = 150 lb/ft^3

Number of parallel chambers, N_c = 1 (simple chamber)

Gas viscosity, $\mu = 1.68 \cdot 10^{-5}$ lb/ft-sec at 400° Fahrenheit, 1 atmosphere

Gas density, ρ = 0.0462 lb/ft^3

Volumetric flow rate, V = 120 acf/sec

Chamber length, L_c = 20 ft

Chamber width, B_c = 10 ft

Gravitational constant, g = 32.2 ft/sec^2

Slope of the semilogarithmic particle distribution, β =0.025micron^{-1}

Substituting these values into equation 10:

$$k = \frac{Cg\ (\rho_p - \rho)\ L_cB_cN_c}{18\mu V} = \left[\frac{(0.60)(32.2\ \text{ft/sec}^2)(150\ \text{lb/ft}^3)(20\ \text{ft})}{(18)(1.68 \cdot 10^{-5}\ \text{lb/ft-sec})(120\ \text{ft}^3/\text{sec})}\right] \cdot \left[\frac{(10\ \text{ft})(1)}{(30.48 \cdot 10^4\ \text{microns/ft})}\right]$$

$$k = 1.719 \cdot 10^{-4}\ \text{micron}^{-2}$$

And: $$E_T = \frac{2k}{\beta^2}\left(1 - e^{-\beta/\sqrt{k}}\ [1 + \beta/\sqrt{k}]\right) = 0.3131 \text{ or } 31.31 \text{ percent}$$

It is interesting to compare this calculated efficiency with the value obtained from a numerical integration method. In Figure 1 the cumulative distribution function, Y(D), is plotted against the particle diameter. As the dotted lines on the graph indicate, the weight fraction of particles falling between 10 and 20 microns, $\Delta Y(D)$, is 0.1723. The average size collection efficiency for this size interval corresponds to the average particle size, 15 microns:

$$Q(15) = kD_{AV}^2 = (1.719 \cdot 10^{-4}\ \text{micron}^{-2})\ (15\ \text{microns})^2 = 0.0387 \text{ or } 3.87 \text{ percent} \quad (13)$$

From this equation, Table 1 can be constructed.

Table 1. AVERAGE OVERALL COLLECTION EFFICIENCIES

Size range, microns	D_{AV}, microns	$\Delta Y(D)$	$Q(D_{AV})$	$Q(D_{AV}) \cdot \Delta Y(D)$
0 to 5	2.5	0.1175	0.00107	0.00013
5 to 10	7.5	0.1037	0.00967	0.00100
10 to 20	15.0	0.1723	0.0387	0.00667
20 to 30	25.0	0.1341	0.1074	0.01440
30 to 40	35.0	0.1045	0.2106	0.02201
40 to 50	45.0	0.0814	0.3481	0.02834
50 to 60	55.0	0.0634	0.5200	0.03297
60 to 70	65.0	0.0493	0.7263	0.03581
70 to 76	73.0	0.0242	0.9161	0.02217
>76	--	0.1496	1.0000	0.14960

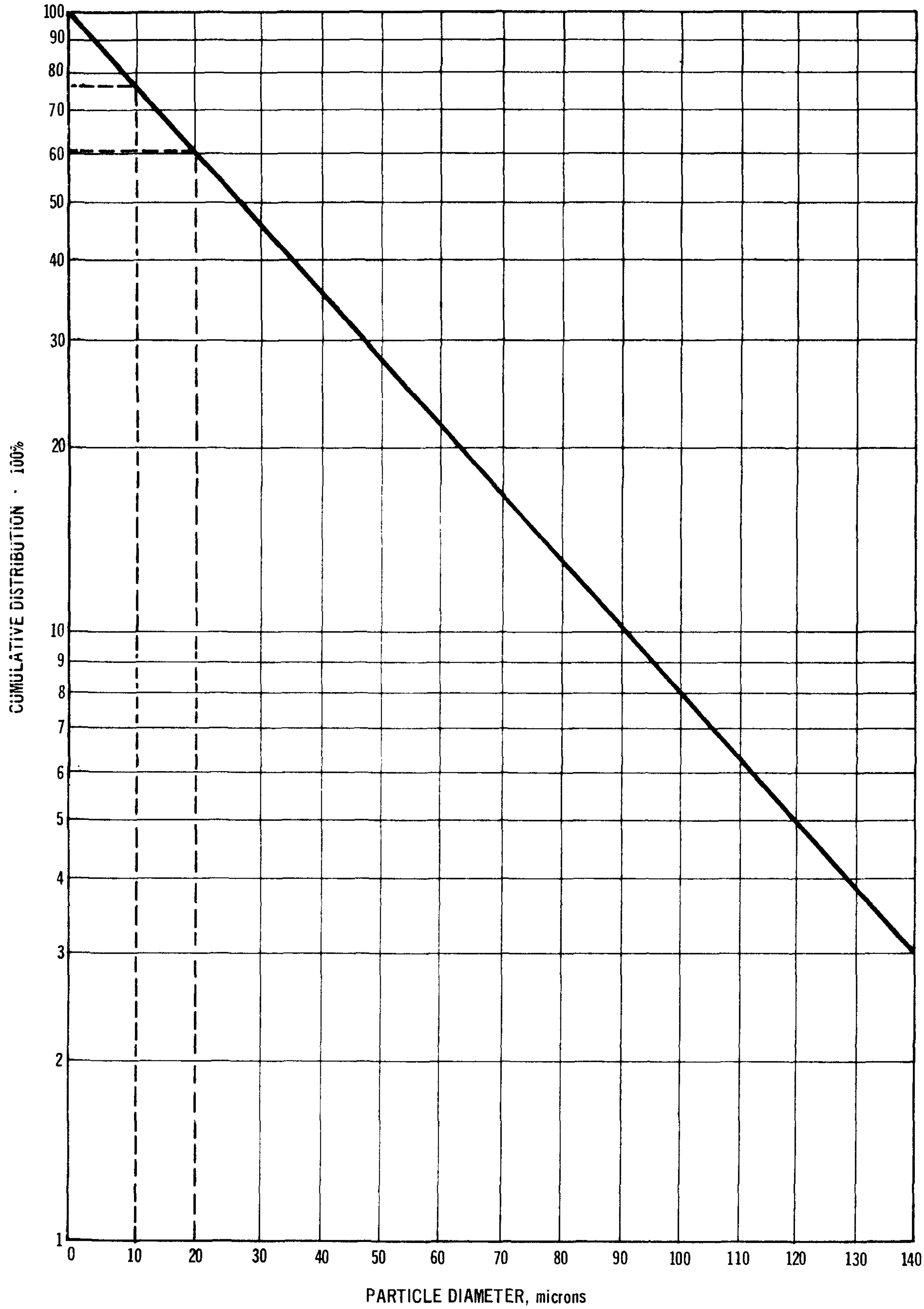

Figure 1. Cumulative distribution versus particle diameter.

The overall efficiency is given by the sum of the products of $\Delta Y(D)$ and $Q(D_{AV})$:

$$E_T = 0.3131 \text{ or } 31.31 \text{ percent} \quad (14)$$

Note that this is exactly equal to the calculated value.

CYCLONES

Unlike gravity settling chambers, no useful theoretical expression has ever been developed for cyclones that ties operating parameters, equipment dimensions, etc. to particle size to yield a size efficiency function. However, several attempts have been made at deriving empirical functions. For example, Gallaer[2] determined that the cyclone size efficiency curve, when plotted on semilogarithmic paper, results in a straight line, the equation of which is:

$$Q(D) = 1 - e^{-\alpha D} \quad (15)$$

Where: α = a constant for the particular cyclone in question, micron^{-1}

Direct substitution of equation 15 into the basic equation results in:

$$E_T = \beta\int_0^{\infty}(e^{-\beta D})(1 - e^{-\alpha D})dD = \frac{\alpha}{\alpha + \beta} \quad (16)$$

Grove[3,4] developed a method for replacing the constants α and β in this expression with two commonly measured particle parameters, the cut size, D_c, expressed in microns, and the mean particle diameter, D_m, also given in microns. By definition, the cut size is the diameter corresponding to a size efficiency of 50 percent, or:

$$Q(D) = 0.50 = e^{-\beta D_c} \quad (17)$$

The cut size can, in turn, be calculated from the following expression:

$$D_c = \sqrt{\frac{9\mu W_i}{2\pi N_t V_i(\rho_p - \rho)}} \quad (18)$$

Where:

W_i = width of the cyclone gas inlet, ft

N_t = number of revolutions the gas stream makes in the cyclone (5 to 10 is typical)

V_i = inlet gas velocity, ft/sec

ρ_p = particle density, lb/ft^3

ρ = gas density, lb/ft^3

μ = gas viscosity, lb/ft-sec

The mean particle diameter is defined as the size above which 50 percent of the particles lie or where the cumulative distribution equals 0.50:

$$Y(D) = 0.50 = e^{-\beta D_m} \quad (19)$$

Combining equations 16 and 17, and solving for α yields:

$$\alpha = \beta\left(\frac{D_m}{D_c}\right) \quad (20)$$

Finally, substitution of this expression for α into the E_T relationship yields:

$$E_T = \frac{D_m}{D_m + D_c} = \frac{1}{1 + (D_c/D_m)} \qquad (21)$$

The reciprocal efficiency, $1/E_T$, is plotted against the ratio $D_c/D_m = \beta/\alpha$ in Figure 2.

Example 2:

A dry cyclone is used as a secondary cleaner following the gravity settling chamber in Example 1. The device has the following operating characteristics:

Width of gas inlet, W_i = 1 ft
Number of revolutions, N_t = 10
Inlet gas velocity, V_i = 41.4 ft/sec

Substitution of these parameters into equation 18 yields:

$$\text{Cut size } (D_c) = \sqrt{\frac{9\ (1.68\cdot 10^{-5}\ \text{lb/ft-sec})\ 1\ \text{ft}}{2\pi\cdot 10\ (150\ \text{lb/ft}^3)(41.4\ \text{ft/sec})}} \cdot 30.48\cdot 10^4\ \text{microns/ft}$$

$$= 6\ \text{microns}$$

Because the size distribution is semilogarithmic, equation 19 can be rearranged to yield:

$$\text{Mean particle diameter } (D_m) = \frac{\ln 2}{0.025\ \text{micron}^{-1}} = 28\ \text{microns}$$

Hence, the efficiency of the cyclone alone would be:

$$E_T = \frac{1}{1 + (\beta/\alpha)} = \frac{1}{1 + (D_c/D_m)} = \frac{1}{1 + (6/28)} = 82.37\ \text{percent}$$

VENTURI SCRUBBERS

The collection mechanism of venturi scrubbers is based on the impingement of the particles on a spray of liquid (usually water) droplets. The efficiency of collection is a function of water droplet size, velocity and viscosity of the gas stream, and rate of liquid injection to the scrubber, as well as particle diameter and density. The combined work of Ranz and Wong[5] has resulted in a theoretical size efficiency function, similar in form to that of the cyclone:

$$Q(D) = 1 - \exp\left[-C'LD\sqrt{\frac{V_t \rho_p}{18 D_w \mu}}\right] \qquad (22)$$

Where:

C' = dimensionless correlation coefficient (varies from 1 to 2; 1.5 is a reasonable estimate)

L = liquid injection rate, $ft^3/1000\ ft^3$

V_t = gas stream velocity through venturi throat, ft/sec

ρ_p = particle density, lb/ft^3

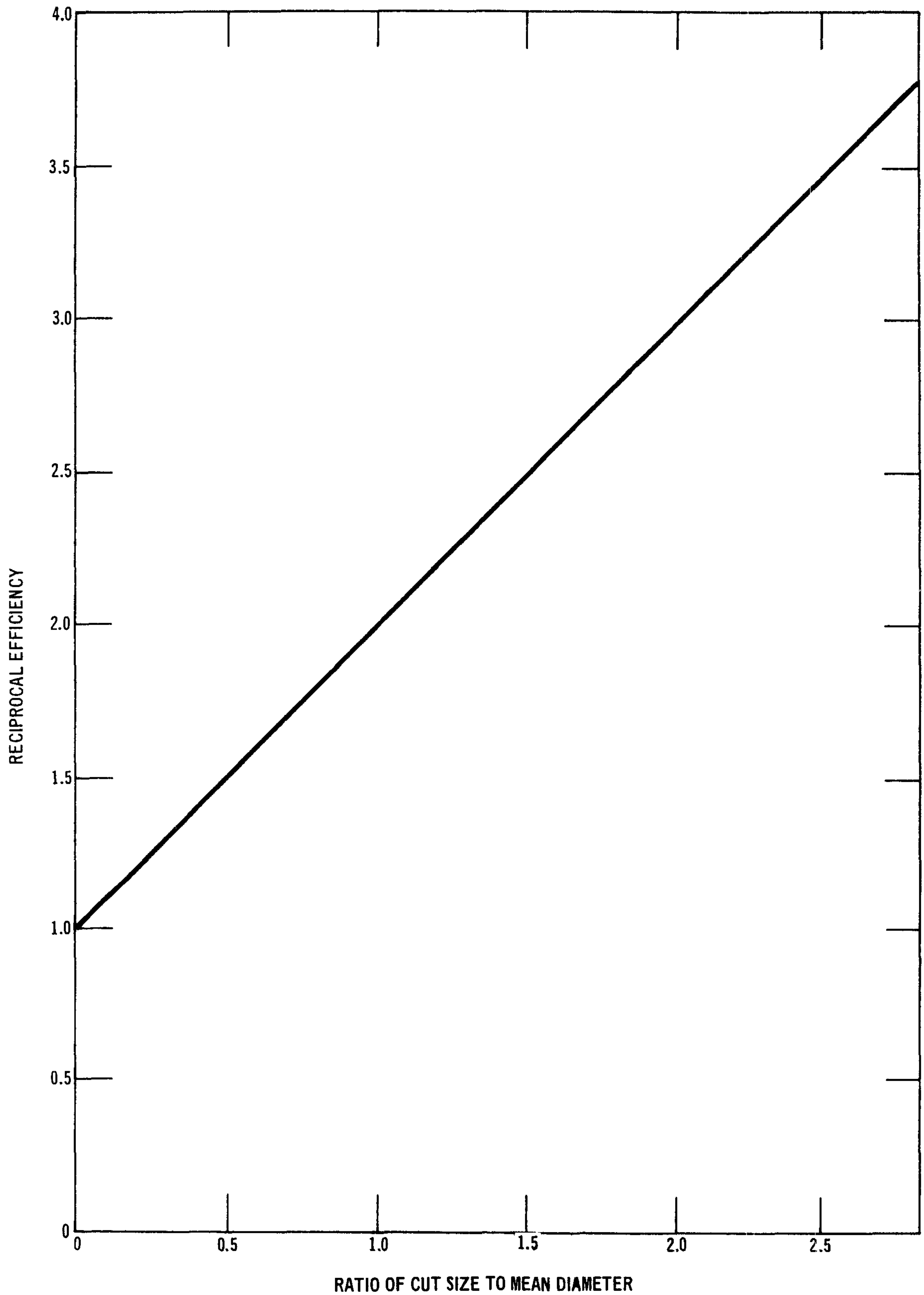

Figure 2. Reciprocal efficiency versus ratio of cut size to mean diameter for cyclone.

D_w = average liquid droplet diameter, microns

μ = gas viscosity, lb/ft-sec

D = particle diameter, microns

For a given venturi, all of the terms above become constants, except D, so that equation 22 can be simplified to:

$$Q(D) = 1 - e^{-\gamma D} \tag{23}$$

$$\text{Where:} \quad \gamma = C'L\sqrt{\frac{V_t \rho_p}{18 D_w \mu}} \tag{24}$$

The average liquid droplet diameter can, in turn, be calculated from the following relationship:

$$D_w = \frac{1.6 \cdot 10^4}{V_t} + 28.5\ L^{1.5} \tag{25}$$

Substitution into the basic equation results in an expression for overall efficiency identical in form to that of the cyclone:

$$E_T = \frac{\gamma}{\gamma + \beta} = \frac{1}{1 + (\beta/\gamma)} \tag{26}$$

Refer to Figure 3 for a graphic representation of equation 26.

Example 3:

Because the spreader stoker described in Example 1 emitted fairly large quantities of small-diameter particulates, it became necessary to install a venturi water scrubber to follow the settling chamber and cyclone. The operating parameters of this device were:

Correlation coefficient, C'=1.40

Liquid injection rate, L = 1.50 $ft^3/1000\ ft^3$

Throat velocity, V_t = 250 ft/sec

Average liquid droplet diameter, D_w = 100 microns

From equation 24:

$$\gamma = C'L\sqrt{\frac{V_t \rho_p}{18\ D_w \mu}}$$

$$\gamma = (1.40)(1.50)\sqrt{\frac{(250\ ft/sec)(150\ lb/ft^3)}{18 \cdot 100\ microns\ (30.48 \cdot 10^4\ microns/ft)(1.68 \cdot 10^{-5}\ lb/ft\text{-}sec))}}$$

$$\gamma = 1.40\ microns^{-1}$$

The overall efficiency for the venturi is calculated from equation 26:

$$E_T = \frac{1}{1 + (\beta/\gamma)} = \frac{1}{1 + (0.025/1.40)} = 98.25 \text{ percent}$$

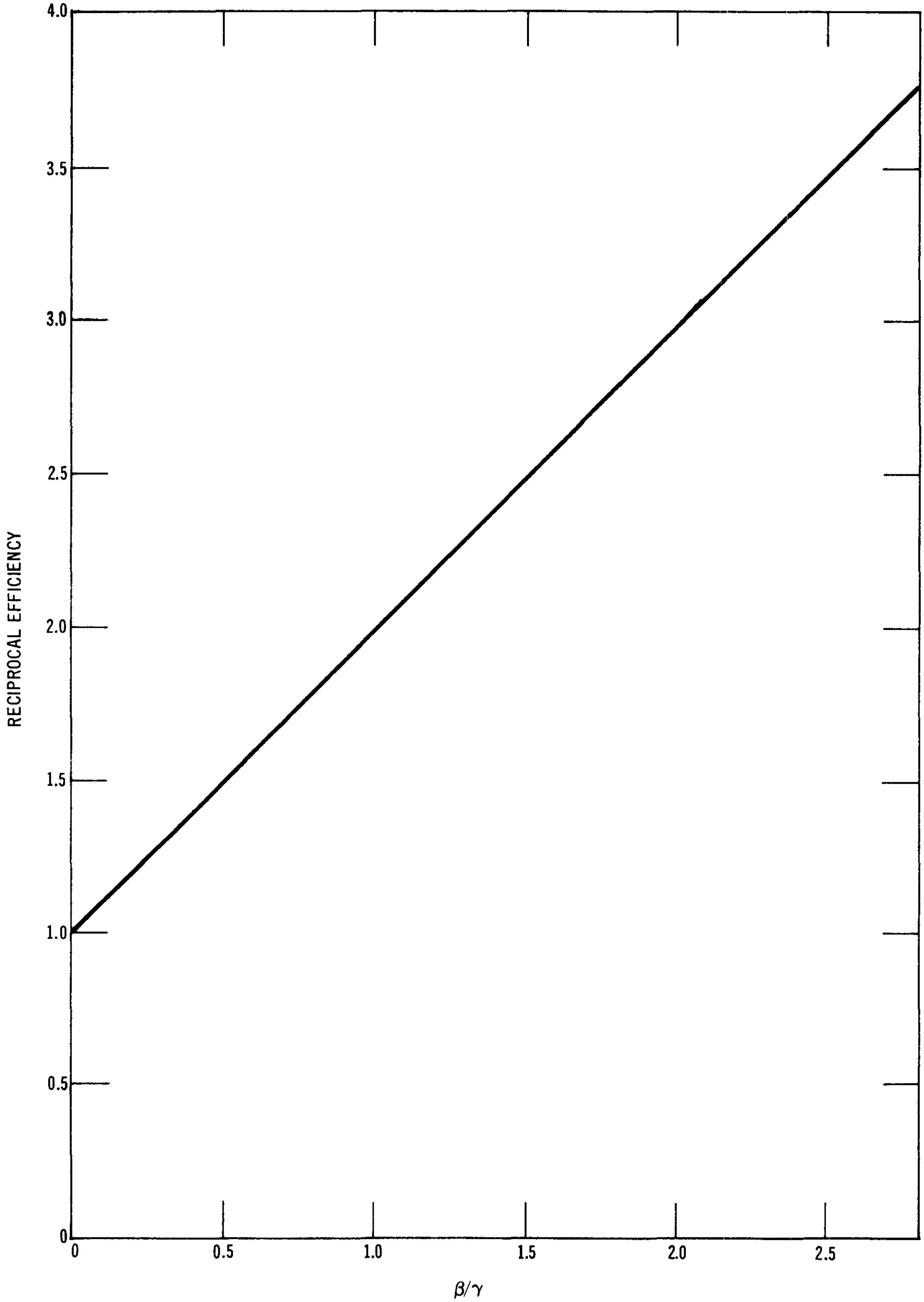

Figure 3. Reciprocal efficiency versus β/γ for venturi scrubber.

ELECTROSTATIC PRECIPITATORS

According to Engelbrecht,[6] the size collection efficiency of an electrostatic precipitator also follows the familiar exponential function:

$$Q(D) = 1 - \exp\left(\frac{-AW(D)}{V}\right) \tag{27}$$

Where:

A = collecting surface of the precipitator, ft^2

V = volumetric flow rate of the gas stream through the device, ft^3/sec

$W(D)$ = migration velocity of a particle of diameter D in the precipitator, ft/sec

This migration velocity is, in turn, dependent on several operating parameters, as well as on the particle size:

$$W(D) = \left(\left[\frac{E_o E_p}{12\pi\mu}\right]\left[1 + 2\left\{\frac{K-1}{K+2}\right\}\right]\right)D = (F)D \tag{28}$$

Where:

E_p = precipitator peak effective electrical charging field

E_o = average precipitator collecting field

K = dielectric constant of collected particle (for minerals, K = 1 to 10; for water, K = 80)

μ = gas stream viscosity, poise or g/cm-sec

According to White,[7] the factor $1 + 2\{K-1/K+2\}$, which has dimensions of $g/cm\text{-}sec^2\text{-}esu^2$, is introduced to account for the electric field distortion due to dielectric particles. For large values of K, the factor approaches 3, the value for a conducting particle.

Substitution of W(D) into equation 20 yields:

$$Q(D) = 1 - \exp\frac{\{-AF\cdot D\}}{V} = 1 - e^{-\delta D}, \tag{29}$$

since every term except D is a constant for a given set of operating conditions. Applying the basic equation to this case results in the familiar:

$$E_T = \frac{\delta}{\delta + \beta} = \frac{1}{1 + \beta/\delta} \tag{30}$$

(see Figure 4).

Example 4:

Referring to the preceding examples, suppose that for economic reasons an electrostatic precipitator was chosen as the third control device to be used in conditioning the stoker tail gas stream. The parameters of this device were:

Peak effective charging field, E_p = 1.92 kV/cm

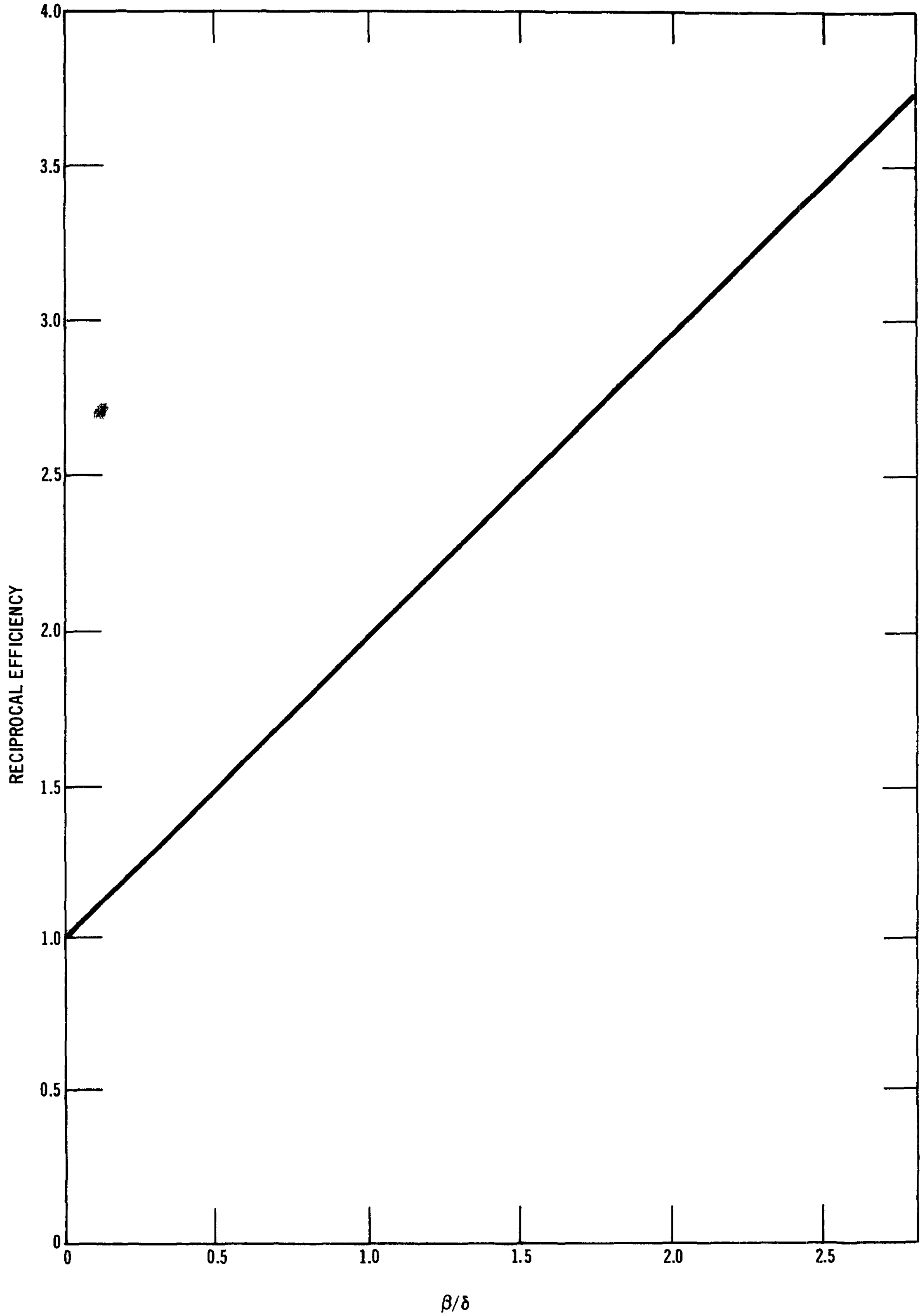

Figure 4. Reciprocal efficiency versus β/δ for electrostatic precipitator.

Average effective collecting field, E_0 = 1.15 kV/cm

Dielectric constant of particle, K = 7

Gas stream viscosity, $\mu = 1.68 \cdot 10^{-5}$ lb/ft. - sec

Precipitator collecting surface, A = 2,000 ft^2

Gas stream volumetric flow rate, V = 120 ft^3/sec

Therefore:

$$F = \frac{(1.92\ kV/cm)(1.15\ kV/cm)(3.33\ esu\text{-}cm/kV)^2}{(12\pi)(1.68 \cdot 10^{-5}\ lb/ft\text{-}sec)(14.88\ g\text{-}ft/lb\text{-}cm)} \left[1 + 2\left\{\frac{7-1}{7+2}\right\}\right] g/cm\text{-}sec^2\text{-}esu^2$$

$F = 0.269\ sec^{-1}$

According to equation 29:

$$\delta = \frac{AF}{V} = \frac{(2{,}000\ ft^2)(0.269\ sec^{-1})}{(120\ ft^3/sec)(30.48 \cdot 10^4\ microns/ft)} = 0.333\ microns^{-1}$$

Thus, the overall efficiency is:

$$E_T = \frac{1}{1 + \beta/\delta} = \frac{1}{1 + (0.025/0.333)} = 93.0 \text{ percent}$$

APPLICATIONS TO CONTROL DEVICES OPERATED IN SERIES

Because it is necessary in certain operations to remove a higher percentage of the incoming particulates than is possible with a single control device, two or more devices are often operated in series. Van Der Kolk[8] evaluated the overall efficiency of series-connected cyclones. The following development extends his argument to cases where different devices are operated in series. First, consider n devices arranged as illustrated in Figure 5:

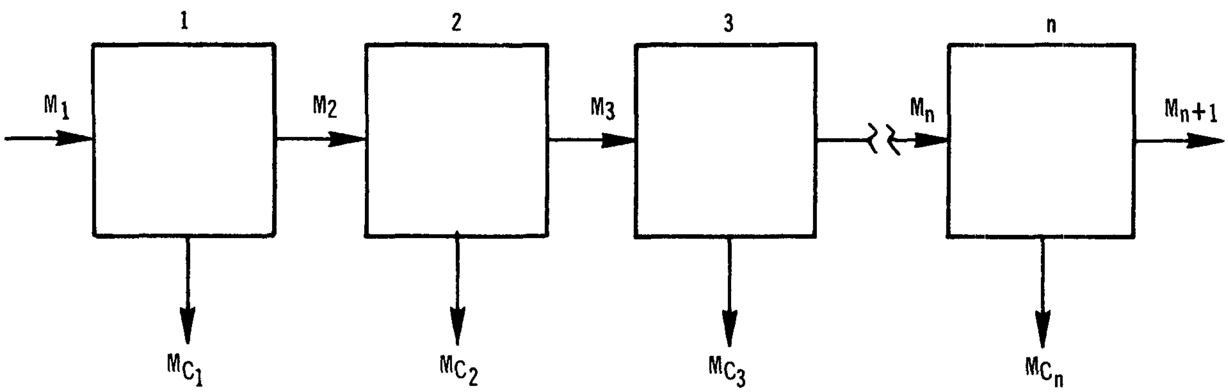

Figure 5. n control devices operated in series.

In this arrangement, the outlet stream of device 1 becomes the inlet stream of device 2, and so forth.

Now, let the total mass of particulates entering each device be designated as M_1 for device 1, M_2 for device 2, etc., and the respective masses collected be M_{c_1}, M_{c_2} . . . M_{c_n}.

Again, consider the differential size interval D to D + dD. The mass of particulate entering device 1 in this size interval is:

$$dM_1 = M_1 f(D) dD \tag{31}$$

The amount collected by device 1, dM_{c_1}, simply equals its size efficiency, $Q_1(D)$, times the entering quantity, or:

$$dM_{c_1} = M_1 Q_1(D) f(D) dD \tag{32}$$

The amount leaving device 1 and entering device 2 is:

$$dM_2 = dM_1 - dM_{c_1} = M_1[1 - Q_1(D)] f(D) dD \tag{33}$$

Extending this argument to the n^{th} device:

$$dM_n = M_1[1 - Q_{n-1}(D)]\,[1 - Q_{n-2}(D)] \; . \; . \; . \; [1 - Q_1(D)]\, f(D) dD \tag{34}$$

$$dM_{c_n} = M_1Q_n(D)[1 - Q_{n-1}(D)]\,[1 - Q_{n-2}(D)]\ .\ .\ .\ [1 - Q_1(D)]f(D)dD \qquad (35)$$

The total mass collected by n devices would be:

$$dM_{c_T} = \sum_{i=1}^{n} dM_{c_i} = dM_{c_1} + dM_{c_2} + \ .\ .\ .\ + dM_{c_n}, \qquad (36)$$

or, by equation 35:

$$dM_{c_T} = M_1f(D)dD\ \{Q_1(D) + Q_2(D)\ [1 - Q_1(D)] + \ .\ .\ .\ + Q_n(D)\ [1 - Q_{n-1}(D)]\ [1 - Q_{n-2}(D)]\ .\ .\ .\ [1 - Q_1(D)]\} \qquad (37)$$

Integration of equation 36 over all particle sizes (0 to ∞) and division by the inlet mass, M_1, yields the overall collection efficiency for the n devices:

$$E_{T_n} = \frac{1}{M_1}\int_0^{\infty}\left[\sum_{i=1}^{n} dM_{c_i}\right] \text{ or}$$

$$E_{T_n} = \int_0^{\infty}\{Q_1(D) + Q_2(D)[1 - Q_1(D)] + \ .\ .\ .\ + Q_n(D)\ [1 - Q_{n-1}(D)]\ .\ .\ .\ [1 - Q_1(D)]\}\{f(D)dD\} \qquad (38)$$

Application of equation 38 to a specific case illustrates its properties. For example, consider the series arrangement of a cyclone, followed by a venturi. In this case, the respective size efficiency functions are:

$$Q_1(D) = 1 - e^{-\alpha D} \text{ (cyclone)} \qquad (39)$$

$$Q_2(D) = 1 - e^{-\gamma D} \text{ (venturi)} \qquad (40)$$

Substituting these into equation 38, and assuming that the frequency distribution is represented by equation 8:

$$E_{T_2} = \int_0^{\infty}\{1 - e^{-\alpha D} + e^{-\alpha D} + [1 - e^{-\gamma D}]\}\ \beta e^{-\beta D}\ dD \qquad (41)$$

Simplifying, and substituting for α:

$$E_{T_2} = \frac{\alpha + \gamma}{\alpha + \beta + \gamma} = \frac{\beta D_m + \gamma D_c}{\beta(D_m + D_c) + \gamma D_c} \qquad (42)$$

Example 5:

Again consider the spreader stoker and its various control devices described in Examples 1 through 4. Suppose it were necessary to know the overall collection efficiency of the cyclone and venturi operated in series. This can be done by merely substituting the previously calculated values for α, β, and γ into equation 42:

$$E_{T_2} = \frac{(0.025 \text{ micron}^{-1})(28 \text{ microns}) + (1.40 \text{ microns}^{-1})(6 \text{ microns})}{(0.025 \text{ micron}^{-1})(28 + 6 \text{ microns}) + (1.40 \text{ microns}^{-1})(6 \text{ microns})} = 98.38 \text{ percent}$$

As this calculation shows, the operation of these two control devices results in an overall efficiency of 98.38 percent--only 0.13 percent higher than that obtained when the venturi is used alone.

As equation 40 indicates, the overall efficiency for the devices is a function of their operating parameters, k, α, and γ, and the frequency distribution parameter, β.

In conclusion, one important point should be emphasized. That is, there is no assurance that the operating parameters (α, γ, δ, and k) for control devices operated singly would remain constant for the same devices arranged in a series. For instance, experience has shown that the cyclone parameter α is directly dependent on, among other things, the inlet dust concentration. This concentration, in turn, varies with the number of devices used and the operating configuration. Therefore, the equations developed for series operations cannot be used to accurately calculate the control efficiency unless the respective equipment parameters remain constant. In any case, actual field testing at operating conditions should be performed to adequately evaluate the system.

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