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Atmospheric Dispersion Parameters in Plume Modeling

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ATMOSPHERIC DISPERSION PARAMETERS IN PLUME MODELING

by

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ABSTRACT

A brief survey is given of the present position in the specification of atmospheric dispersion parameters for use in estimating pollutant concentration from a continuous point release.

The theoretical indications of the distribution to be expected across a time-mean plume are recalled, with particular reference to the existence of the Gaussian form. Observational evidence, especially as regards the vertical distribution from a surface release, is also recalled, and the practical significance of departure from an assumed Gaussian form is noted.

The use of the Taylor statistical theory in the generalized estimation of crosswind spread in quasi-ideal boundary layer flow is briefly summarized. Recent considerations of the behaviour of the crosswind component of turbulence in the surface layer and new developments from laboratory modeling of horizontal dispersion in convective mixing are noted.

A brief survey is given of the achievements of gradient-transfer theory and Lagrangian similarity theory in calculating vertical spread from a surface release. New tests against previous dispersion data underline inadequacies in the present approaches in very unstable conditions. Promising developments from the laboratory modeling of a convectively mixed layer and from the 2nd-order-closure modeling of the turbulent fluctuation equations are summarized.

The assimulation of theory and experience into practical systems for the specification of σ_y and σ_z is briefly reconsidered. For σ_y a practical procedure based on wind direction fluctuation data is reaffirmed. For σ_z a new format which may be envisaged for future composite curves is suggested. Finally, the inherent limitations of practical systems for estimating concentration levels are reiterated.

CONTENTS

FIGURES

TABLES

LIST OF SYMBOLS

The usual subscript notation is used to indicate, for example, axis of reference (x,y,z), velocity component involved (u,v,w). A zero subscript is used to denote a ground-level value or an initial value. A subscript g denotes a geostrophic value.

f	Coriolis parameter
c_p	specific heat of air at constant pressure
H	vertical heat flux (H_o surface value)
k	von Kármán's constant
K	eddy diffusivity
L	Monin-Obukhov length $-\frac{u_*^3}{H}\frac{c_pT}{kg}$
Q	source strength, rate of emission (point source) or rate of emission per unit length (line source)
$R(\xi)$	Lagrangian correlation coefficient for time-lag ξ
Ri	Richardson number
Ro	Surface Rossby number V_g/fz_o
t_L	Lagrangian time scale
T	absolute temperature or time of travel
v,w	velocity components along axes y,z
u_*	friction velocity $\equiv(\tau_o/\rho)^{1/2}$
w_*	free convective velocity scale $(gH_oz_i/\rho c_pT)^{1/3}$
x,y,z	rectangular coordinates, x along mean wind y across mean wind and z vertical
z_o	roughness length
X	distance of travel downwind of release position
Z	vertical displacement of particle
z_i	mixing depth
z_r	release height
ε	rate of dissipation of turbulent kinetic energy per unit mass of air

θ wind direction

λ_m spectral scale represented by equivalent wavelength at which product of frequency and spectral density is a maximum

ρ air density

σ^2 variance

ϕ Monin-Obukhov universal function

τ horizontal shearing stress or duration of sampling or release

SECTION 1

INTRODUCTION

The purpose of this report is to take stock of the present position in the technique of representing, for air quality modeling, the properties of crosswind and vertical spread from a single point source, as a function of distance downwind and of the conditions of flow. Reviewing of this nature has been continuously in process for many years, and several easily accessible earlier accounts will be referenced. In the present account the concern will be partly with some brief reminders of well-established features and partly with a summary of recent developments.

It should be remembered at the outset that the reliability of the estimates which are variously made of the familiar properties σ_y and σ_z must be judged in two respects:

(a) The accuracy, reproducibility, and representativeness of the available full-scale measurements of dispersion, and

(b) The validity of the theoretical frameworks within which the results of such measurements may be understood and from which those inevitably incomplete results may be generalized.

As regards (b) we have long been essentially dependent, for practical applications, on two classical approaches, the gradient-transfer theory, requiring appropriate specifications of the eddy diffusivity field, and the statistical theory as initiated by G. I. Taylor, requiring knowledge of certain statistical properties of the flow turbulence. For details of the background of these approaches, references 1 and 2 may be consulted. More recently, useful and promising theories have been developed in the area of similarity argument and in the whole sophisticated field of the higher-moment turbulent fluctuation equations and their solution through 2nd-order-closure assumptions and hypotheses.

We shall be concerned (specifically in Sections 3 and 4) with the guidance provided by all the foregoing approaches in the prescription of σ_y and σ_z. However, the full description of dispersion also requires a knowledge of the shapes of the distribution of concentration of material crosswind and vertically, and it seems appropriate to begin (Section 2) with some discussion of this aspect. Finally, in Section 5, consideration is given to the assimilation of practical experience and theoretical development into the working formulae and graphs which are currently advocated for estimation of the dilution of air pollutants.

SECTION 2

THE SHAPE OF THE DISTRIBUTION OF CONCENTRATION FROM A SOURCE

One of the most widely used mathematical models of dispersion from a continuous point source contains the assumption that the crosswind and vertical distributions are of Gaussian form. Various theoretical and empirical features with bearing on this aspect are now summarized briefly.

IMPLICATIONS OF THE CLASSICAL PARABOLIC EQUATION

For simplicity, and for consistency with the view followed later in this discussion, attention is first confined to the vertical spread and to a surface release, though the implications of this particular subsection will apply equally well to elevated release and to crosswind spread if the gradient-transfer assumption is considered to apply in those respects also. The appropriate equations, in either the one-dimensional time-dependent form (relevant to an instantaneous plane source of infinite extent) or the two-dimensional steady form (relevant to a continuous line source of infinite extent crosswind), and the vertical distribution characteristics which follow from these equations, with certain assumptions about $\bar{u}$ and K, are summarized in Table I.

The first four lines of the table, with steady K, are well-known results, and the fifth and sixth lines, for arbitrarily time-dependent K, are straightforward extensions of the steady-state results. The format of the results and the selection of cases are such as to bring out certain important points as follows:

(a) A Gaussian form follows only when r_1 (or r_2) = 2. This includes as a special case (m = n = o) the so-called Fickian case of constant K (and constant $\bar{u}$). It is noteworthy, however, that generally the condition m = n (with $n-m < 2$ as required for the solution to be valid) is sufficient. Near the ground the K profile index n is typically in the region of unity, and falls substantially below

TABLE 1. FORMS OF VERTICAL DISTRIBUTION AND VERTICAL SPREAD IMPLIED BY SOLUTIONS OF THE CLASSICAL PARABOLIC EQUATION OF DIFFUSION*

Source†	K	u	Form of B in $C(T,z)/C(T,0)$ or $C(X,z)/C(X,0) = EXP(-B)$	Characteristic Scale of Vertical Spread
I.A.S.	K (const.)		$z^2/4KT$	$2(KT)^{1/2}$
C.C.L.S.	K (const.)	u (const.)	$uz^2/4KX$	$2(KX/u)^{1/2}$
I.A.S.	$K z^{n}$‡		$z^{r_1}/r_1^2K_1T$, $r_1 = 2-n$	$(r_1^2K_1T)^{1/r_1}$
C.C.L.S.	K_1z^n	u_1z^m	$u_1z^{r_2}/r_2^2K_1X$, $r_2 = m-n+2$	$(r_2^2K_1X/u_1)^{1/r_2}$
I.A.S.	$K_1(t)z^n$		$z^{r_1}/r_1^2s_1$, $s_1 = \int_0^T K_1(t)dt$	$(r_1^2s_1)^{1/r_1}$
C.C.L.S.	$K_1(x)z^n$	u (const.)	$uz^{r_1}/r_1^2s_2$, $s_2 = \int_0^X K_1(x)dx$	$(r_1^2s_2/u)^{1/r_1}$

* $\frac{\partial \overline{c}}{\partial t}$ (or $\overline{u}\,\frac{\partial \overline{c}}{\partial x}$) $= \frac{\partial}{\partial z}(K\frac{\partial \overline{c}}{\partial z})$

† I.A.S. = Instantaneous area source of infinite extent at z = 0.

C.C.L.S. = Continuous crosswind line source of infinite extent at x = 0.

‡ K_1 = K at unit height, $K_1(t)$ time-dependent K at unit height, $K_1(x)$ x-position dependent K at unit height

unity only in stable conditions or with increase in the depth of layer embraced. As the corresponding wind profile index n is typically in the range 0.1 to 0.5, the larger values occurring only with stable conditions and at low heights, it is evident that $m = n$ is likely to occur only in a combination of the latter conditions. Otherwise, solution of the diffusion equation for a surface release in the atmospheric boundary layer implies a decidely non-Gaussian vertical distribution.

(b) With K and u steady, when a Gaussian distribution does follow from the diffusion equation, it is necessarily associated with the magnitude of the spread growing as $T^{\frac{1}{2}}$ or $X^{\frac{1}{2}}$.

(c) For a homogeneous time-dependent K or a steady spatially varying K, it appears, from lines 5 and 6 of the Table, that a Gaussian distribution will not generally be associated with $T^{\frac{1}{2}}$ or $X^{\frac{1}{2}}$ forms of growth. Note especially that if $K_1(T) \propto T^{\alpha}$ (or X^{α}), with positive and $r = 2$, it follows that the growth of the spread will behave as $T^{(1+\alpha)/2}$ (or $X^{1+\alpha)/2}$). We see then that, according to the gradient-transfer treatment, it is formally possible to have a combination of non parabolic growth and Gaussian distribution if there are appropriate time or space variations in the eddy diffusivity K, as might possibly occur in practice from the diurnal cycle and from systematic changes in roughness or surface heating.

THE SPECIAL CASE OF STEADY HOMOGENEOUS TURBULENCE

A well-known result of the G. I. Taylor statistical theory, brought out in more detail by Batchelor[3], is that the spread of particles released serially from a point grows linearly with time of travel initially, then progressively less rapidly, tending ultimately to $T^{\frac{1}{2}}$. Batchelor concluded that the particle displacements have a Gaussian probability distribution at all values of T, though for reasons which must be different for different ranges of T. With such a universal Gaussian behaviour it then follows that the diffusion equation does provide a description of the dispersion process for all T (or X), , provided K varies appropriately with T (or X), indeed in a manner which satisfies

$$K = \frac{1}{2}\ \frac{d\,\sigma^2}{d\,T} \text{ or } \frac{\overline{u}}{2}\ \frac{d\,\sigma^2}{d\,X}$$

Obvious similarities exist between this result and that discussed in the foregoing subsection, but--it cannot be emphasized too strongly--these are to be regarded only in a formal sense and not in a meaningful physical sense. In the foregoing subsection the variations in K are to be envisaged as arising from variations in the fluid properties. On the other hand, the result for steady homogeneous turbulence is (logically) to be regarded as a consequence of a dispersive action that is not gradient-transfer in nature. The crucial point is that except at large enough T or X the displacements of the particles are caused by turbulent motions of a scale larger than the whole cross section of the plume of particles.

It may be noted that several workers in dispersion modeling (e.g., Fortak[4]) have drawn attention to the result that the conventional Gaussian plume formula, with arbitrary variation of σ with time or distance of travel, is a solution of the diffusion equation with K defined as above. Although there is no objection to using this result for homogeneous, steady turbulence merely as a convenient formality, the important point is that the apparent K's (associated with a certain nonparabolic growth of σ), implicitly constant with height but varying systematically with T or X, cannot then simply be used as genuine gradient-transfer K's in some other context, e.g., in the treatment of the vertical transfer of the particles to an absorbing ground. In the latter case, logically, the appropriate K must increase with height and be quasi-constant in time and space.

CONFORMITY TO GAUSSIAN DISTRIBUTION IN REALITY AND THE SIGNIFICANCE TO ESTIMATES OF CONCENTRATION

A list of sources of observational evidence for the existence or absence of Gaussian shape in the distribution of dispersed material is given in Table 2.

Outstanding departures from the Gaussian form are as follows:

Value of Exponent r

For vertical spread at short range from a surface release, the exponent r in the exponential form is near 1.5 irrespective of thermal stratification.

TABLE 2. OBSERVATIONAL EVIDENCE FOR EXISTENCE OF GAUSSIAN SHAPE IN TIME-MEAN DISTRIBUTIONS

	Nature of Dispersion	Gaussian?	Reference
1	Crosswind spread from a continuous point source	Yes	(1) pp 173 & 227
2	Vertical spread from an elevated continuous source of passive particles (before ground becomes effective)	Yes	(5)
3	Vertical spread in first kilometer downwind of a surface release	No ($r \simeq 1.5$)	(6)
4	Vertical spread from near-surface release in laboratory convectively-mixed layer	Only at very early stage	(7)
5	Vertical distribution of power station plume	Only in lower half	(8)

The significance of the precise value of r to the magnitude of the ground-level concentration for specified σ_z, wind speed, and source strength is shown in Table 3; clearly the departure from Gaussian becomes of practical importance only when r is less than 1.5 or much larger than 2.

Non-Gaussian Intermediate Stage

A certain intermediate stage in the mixing in a laboratory convectively mixed layer (even before the distribution is obviously modified by the presence of the upper boundary to mixing) exhibits non-Gaussian form. Deardorff and Willis'[7] study shows an early-stage Gaussian vertical distribution followed by the appearance of an elevated maximum, which progressively rises through the mixed layer before the final condition of uniformity with height is achieved. This means that the concentration at ground level transiently "undershoots" the value which would be calculated (given σ_z) on the existence and degree of this effect in the real atmospheric mixed layer has yet to be provided.

Non-Gaussian Aspects of Instantaneous Distribution

Hamilton's[8] lidar observations of the time-mean vertical distribution of a power-station plume apparently may be fitted to a Gaussian shape over the lower half of the distribution, which, of course, is the significant half as regards the development of the ground-level concentration to is maximum value. It is, however, highly questionable whether such an approximation obtains in the instantaneous distribution, the property that will determind short-term concentrations in the path of the elevated plume. The fact that the growth of a rising hot plume is initially dominated by an induced internal circulation leads one to expect a tendency to a flat rather than a peaked distribution in the cross section of the plume, but evidence for this is not immediately available.

TABLE 3. VALUES OF $2\bar{u}C(X,0)\sigma_z/Q$ FOR A TWO-DIMENSIONAL (INFINITE LINE SOURCE) MODEL WITH $\bar{u}$ INVARIANT WITH HEIGHT AND $C(X,z)/C(X,0) = \exp -bz^r$

r	1.0	1.5	2.0	2.5
$2\bar{u}C(X,0)\sigma_z/Q$	1.37	0.96	0.8	0.73

N.B. These figures are for unbounded vertical diffusion; for a source at ground-level they should be doubled. They may be taken as equivalent to $\int_{-\infty}^{+\infty} C(x,y,o)dy$ from a point source.*

*See p. 350, Ref. 1 for further details

SECTION 3

GENERALIZED ESTIMATION OF CROSSWIND SPREAD IN QUASI-IDEAL BOUNDARY LAYER FLOW

In the atmospheric boundary layer both the magnitude and scale of the crosswind component (v) of turbulence are found to change only slowly with height (in contrast to the pronounced increase with height of the scale of the vertical component). Accordingly, when also there are no sharp changes with time or position, we may treat the flow as quasi-homogeneous as regards the lateral dispersive action of the turbulence.

For the lateral spread of a continuous point source (and indeed the vertical spread when the plume is elevated), the physical irrelevance of gradient-transfer action has been discussed on many occasions, and the purely formal quality of the representation in terms of the parabolic diffusion equation has been noted in the previous section. The Lagrangian form of Monin-Obukhov similarity argument, although originally including lateral as well as vertical spread, has been seriously questioned for the former aspect of dispersion (p. 119 of Ref. 1). In an Eulerian sense it has become evident that the v-component is not a simple function of z/L (where L is the Monin-Obukhov length), and it would be most surprising if the Lagrangian v-properties differed from the Eulerian in this respect. A possible rationalization of the behaviour of the v-component in the surface layer in convective conditions has recently been proposed by Panofsky et al.[9]. Their analysis of several sets of data on σ_v/u_* over uniform surfaces with friction velocity u_* demonstrates a universal dependence, not on z/L, but on z_i/L, where z_i is the effective convective mixing depth. We will be noting the special significance of their result to lateral spread later in this section. More generally, i.e. including neutral and stable flows, there are complexities in σ_v-properties to be expected from synoptic-scale changes in the flow and from mesoscale topographical influences, the latter especially in stable conditions and even in unstable conditions in light winds.

THE TAYLOR STATISTICAL THEORY

Of the three working theories hitherto available to us, the practical adaptation of the famous Taylor theory of diffusion by continuous movements is the one most clearly suited to the estimation of σ_y for a continuous point source. The developments to date have already been extensively reviewed (p. 123 et seq, p. 185 et seq, Ref. 1, and p. 6 et seq of Ref. 2), and only the main points need to be emphasized here.

Determination of Crosswind Spread

In accordance with the assumption of quasi-homogeneous conditions, the spread σ_y is related to σ_v and time of travel downwind of the source in the forms

$$\sigma_y(T)/\sigma_v t_L = f(T/t_L) \tag{1}$$

$$\text{with } f(T/t_L) \rightarrow T/t_L \text{ as } T \rightarrow 0 \tag{2}$$

$$\text{or } \sigma_y(T) \rightarrow \sigma_v T \text{ as } T \rightarrow 0 \tag{3}$$

$$\text{and } f(T/t_L) \rightarrow (2T/t_L)^{\frac{1}{2}} \text{ as } T \rightarrow \infty \tag{4}$$

$$\text{or } \sigma_y(T) \rightarrow \sigma_v (2Tt_L)^{\frac{1}{2}} \text{ as } T \rightarrow \infty \tag{5}$$

t_L being the Lagrangian integral time scale. This means that in principle, neglecting wind direction turning with height (to which reference will be made later), crosswind spread is determined by σ_v (which is measurable, and to some extent describable in boundary-layer climatological terms), and by t_L (which is not easily measurable and for which the theoretical and observational background is only partially helpful).

Effect of the τ-T Relationship

Rigorously, the adaptation of the Taylor treatment requires that σ_v have the limiting value associated with effectively infinite sampling time (of the turbulence) and corresponding effectively infinite release or sampling time of the material. On a qualitative argument, however, (p. 136, Ref. 1) the result may be expected to be valid for sampling (or release) time τ and time of travel T when $T < \tau$. As T increases beyond τ, the argument is that the different properties of cluster growth (as distinct from time-mean plume growth) become increasingly relevant and ultimately dominant.

Inadequacy of the X-T Relationship

In strictly homogeneous conditions, including uniformity of wind speed, the foregoing σ_y, T relations may be converted to σ_y, X form by substituting $X = \bar{u}T$. However, for real boundary layer flow, even though an assumption of quasi-homogeneity may be acceptable as regards the properties of the v-component, the variation of mean wind speed with height makes the simple X, T relation inadequate. A practical solution is to replace $\bar{u}$ by an equivalent advecting speed, u_e, which increases with T (as vertical spread increases), and with certain assumptions a rough estimate of u_e is derivable in terms of the wind profile[10].

PRACTICAL ADAPTATION OF THE TAYLOR THEORY AT VARIOUS RANGES OF T

In principle, and qualifications including those noted in Determination of Crosswind Spread and Effect of the τ-T Relationship above, the method provides for estimates of σ_y in a way that takes into account the properties of turbulence for any surface roughness and any thermal stratification. The practical utility and limitations are most conveniently considered in well-defined ranges of T/t_L.

$T \ll t_L$.

It has been demonstrated that in this range σ_y approaches the simple limit in Eq. 3, i.e.

$$\sigma_y(T) = \sigma_v T \qquad (6)$$

or approximately

$$\sigma_y(X) \simeq \frac{\sigma_v}{u_e} X \simeq \sigma_\theta X \qquad (6a)$$

noting that σ_θ, which is the standard deviation of wind direction fluctuation, should strictly be taken as a function of vertical spread, in accordance with the definition of u_e. Thus at short enough time or distance the only requirement is an estimate of σ_θ. Abundant evidence supports rough agreement with the very simple form in Eq. (6a), but in more precise terms the crucial point is the behaviour of $f(T/t_L)$ in Eq. (1) at small T/t_L; this will be apparent in the more general considerations which follow.

<u>Intermediate Range of T</u>

For this range the form of $f(T/t_L)$ needs to be specified, basically from the form of the Lagrangian auto-correlation function $R(\xi)$, and earlier considerations (p. 130 of REf. 1) had led to the impression that $f(T/t_L)$ was insensitive to such variations of the shape of $R(\xi)$ as were originally considered likely. A recent analysis of dispersion data by Draxler[11] has reopened this question, and it now has to be considered that the initial reduction of $R(\xi)$ with time-lag ξ may be considerably more rapid than $\exp-\xi/t_L$[2,10], which was the sharpest fall hitherto considered. It is noteworthy that a form which has a sharper fall and fits selected dispersion data is provided by

(a) The Hay-Pasquill hypothesis of a simple scale relation between Lagrangian (moving particle and Eulerian fixed point) turbulent fluctuations (see p. 135 of Ref. 1), and

(b) An empirical form of the shape of the (Eulerian) v-spectrum (see p. 70 of Ref. 1).

Full application on the foregoing lines, in terms of a specification of the actual v-spectrum, is considered to be the most satisfactory approach, but the requirement for a relatively sophisticated measurement and analysis of the time-lapse fluctuation of v or θ is obviously a practical difficulty. One reasonable practical solution would appear to be the use of an empirical generalization of good quality σ_y data accompanied by σ_θ data, and a collection of such data has been assembled by the writer[12]. According to this data, $\sigma_y/X\sigma_\theta$ follows a simple function of X, largely irrespective of roughness and thermal stratification, with departures (for individual samples of data) which are within a factor of 1.5 at short range (< 1 km) and 2.0 at longer range (10 km). With reference to the remark at the end of the paragraph or $T \; t_L$, note that according to the various U.S. tests $\sigma_y/\sigma_\theta X$ is on average detectably below unity even at a distance as short as 100 m.

<u>Very Large T</u>

If the flow were ideally homogeneous, the specification of σ_y at very large T (or corresponding X) would require only an estimate of $t_L^{\frac{1}{2}}$ in addition to σ_θ. Unfortunately, another aspect of departure from homogeneity then becomes effective--namely, the turning of mean wind direction with height--as a result of which there is, through the process of vertical spreading, a contribution to the crosswind spread additional to that directly associated with σ_v. Certain more or less elaborate theoretical treatments of this hear effect are already available (see pp. 156 and 229 of Ref. 1). With these and some empirical guidance, first impressions are that the additional contribution becomes important only for distances of travel exceeding 5-10 km.

LATEST DEVELOPMENTS RELEVANT TO THE MODELING OF CROSSWIND DISPERSION

Laboratory Modeling

Discussion of the laboratory model of Deardorff and Willis for dispersion in a convectively mixed planetary boundary layer is considered in more detail in relation to vertical dispersion in the next section. In one of the latest reports of this work[13], the implication of the model in respect to horizontal dispersion is also considered. From an ensemble of seven experiments with the water tank having side-dimension/mixing depth z_i ratio 4.0, and with the convective scaling velocity w_* near 1 m sec^{-1}, Willis and Deardorff derive statistics of lateral displacement of nonbuoyant particles released on a line near the bottom of the tank. As in the work on vertical dispersion, these are expressed in similarity terms, σ_y/z versus $t_* = w_* t/z_i$), identification with distance of travel in a wind being achieved through the relation $X = Ut$ with U constant. They compare their laboratory results with field data obtained in Idaho for distances of travel up to 3200 m from a continuous point source, making plausible estimates of the likely magnitudes of w_* and z_i during the field tests. A remarkable degree of agreement--within about 10% in the ensemble averages--is found. In noting this very encouraging initial success in verification of the full-scale applicability of laboratory modeling in convective conditions, two qualifications deserve special mention.

The σ_y t curves found in the laboratory tank contain a distinct inflection, tion, with $d\sigma_y/dt$ temporarily reduced and then restored. In full scale distance terms this appears in the 3- to 4-km range. Willis and Deardorff[13] ascribe the feature to the delayed appearance of horizontal spread from thermal outflows at the top of the mixed layer. It is noteworthy that this effect has not yet been detected in full-scale measurements.

The second point concerns the stipulation of the full-scale conditions, especially of wind speed, for which the laboratory (windless) results may be reliably adopted. In this connection Willis and Deardorff prescribe an upper limit of 12 m sec^{-1}. For all lower wind speeds the implication is that the horizontal dispersive action is essentially controlled by buoyancy forces and not by the mean shear, and in this respect some further guidance is now available from the recent generalizations about the behaviour of the magnitude of the surface layer v-component in convective conditions. Panofsky et al.'s[9] form for σ_v, in terms of u_*,L and z_i, may be used to prescribe the combinations of wind speed and heat flux which, for given z_0 and z_i, result in a σ_v which is dominantly (say to the extent of 90%) a consequence of the heat flux. Details are set out in Table 4 for z_0 20 cm (a moderate roughness intermediate between smooth plains and urban complexes) and for z_i = 1500 m (a mixing depth typical of afternoon conditions). Note that in such circumstances a surface wind speed of even 4 m sec^{-1} requires a vertical heat flux near 500 w m^{-1} to meet the foregoing criterion of dominance of the buoyancy contribution to σ_v. This is a very strong sensible heat flux, unlikely to occur except over dry terrain and with the highest sun in low latitudes. From this standpoint it seems that the 12 m sec^{-1} limit prescribed by Willis and Deardorff[13] may require unrealistically high heat fluxes, or, alternatively, a very much smoother surface or much larger z_i.

SECOND-ORDER CLOSURE MODELING

A growing effort is being devoted to the use of 2nd-moment equations, in which the gradient-transfer assumption is avoided, at least in the 1st-moment equations such as those considered in Table 1. The progress of the technique is more comprehensive in relation to vertical transfer as will be discussed in Section 4. The writer is unaware of any crucial examinations of the success of the technique in relation to crosswind spread per se.

TABLE 4. WIND SPEED CONDITIONS FOR APPLICABILITY OF THE CONVECTIVE-LIMIT FORM FOR σ_v/u_*

Taking $\sigma_v/u_* = (12 - z_i/2L)^{1/3}$ Ref. 9

$\rightarrow (-z_i/2L)^{1/3}$ for large $(-z_i/L)$

it follows that

$$u_*(-Z_i/2L)^{1/3}/\sigma_v \geq 0.9 \text{ when } -z_i/L \geq 65$$

Associated values of surface heat flux

H_o and $\overline{u}(10m)$ for $-z_i/L = 65$, $z_i = 1500m$, $z_o = 0.2m$

$\overline{u}(10m)$ m sec^{-1}	2	2.5	3	3.5	4
H_o wm^{-2}	67	130	220	350	520

SECTION 4

GENERALIZED EXAMINATION OF VERTICAL SPREAD σ_z IN QUASI-IDEAL BOUNDARY LAYER FLOW

THE GRADIENT-TRANSFER AND SIMILARITY TREATMENTS OF VERTICAL SPREAD FROM A SURFACE RELEASE

The fundamental acceptability of the gradient-transfer relation for turbulent transfer is often seriously questioned (e.g., see Corrsins's discussion,[14]). Empirically, however, the method is undoubtedly successful in certain applications, a success which Corrsin refers to as "largely fortuitous and certainly surprising."

It is now a familiar notion that time-mean spread from a continuous point source is initially dominated by turbulent motions of scale that are large compared to the cross section of the plume of particles, when the concern is with lateral spread or even with vertical spread when the plume is clear of the ground. This type of scale relation, which in an obvious physical sense is the very opposite of that implied in a gradient-transfer process, does not exist, however, in the vertical spread action when passive particles originate at the boundary. The point is simply that at any stage in the vertical growth of the plume the effective turbulent motions are constrained in scale by the presence of the underlying boundary, the effective scale being dependent on height. This presumably is the essential reason for some success in the K-treatment of the ground-level infinite crosswind line source.

Also noteworthy at this point is the formal consistency between the K-treatment, using the familiar momentum-transfer analogy, and the Lagrangian similarity treatment, for vertical spread as a function of time in the surface-stress region of the neutral boundary layer (p. 117 of Ref. 1). Associated with this is a simple relation between the rate of vertical spread and the eddy diffusivity, in the form

$$d\overline{Z}/dt = K(\overline{Z})/\overline{Z} \tag{7}$$

where $\overline{Z}$ is the mean displacement of particles at a given time after release at the surface. The result is exact for the neutral surface-stress layer (p. 118 of Ref. 1) and has been found to be a good approximation in other thermal stratifications and at greater heights[2,15].

In Table 5 a list is given of applications of the mutually consistent gradient-transfer and similarity approaches that have led to useful explicit formulations of the growth of σ_z with time or distance. Unfortunately, observational data for the critical testing of the theoretical results are still largely confined to short range, notably the early Porton-Cardington data at 100 and 229 m (neutral conditions) and the Prairie Grass data at distances up to 800 m (for a wide range of stratification). Note, however, that in the latter observations only those at 100 m include <u>measurement</u> of the vertical distribution of condentration. Those at other distances are confined to ground level and provide only indirect estimates of σ_z.

A reexamination of the Prairie Grass data in relation to some of the foregoing methods has recently been attempted by the writer, using the indirect estimates of σ_z derived from the concentration measurements at ground level (basically those given by Cramer[16], but with an adjustment allowing for the variation of wind speed with height neglected in Cramer's analysis). Full details are assembled in Appendix A, and a summary of the principal results follows:

(a) The magnitudes of $d\overline{Z}/dt$ implied by the Cramer-type analysis of the Prairie Grass data do not exactly support a similarity relation with $\overline{Z}$ and the Monin-Obukhov L, as conjectured by Gifford[22], though systematic discrepancies from such a relation are not large.

(b) Predicted curves which follow from the assumption that $K = ku_*z/\phi_H$ with the different estimates available for ϕ_H, embrace the range of the data on $d\overline{Z}/dt$ in unstable conditions. In stable conditions the one predicted curve presented tends to be a slight overestimate.

(c) A predicted curve based on a similarity hypothesis consistent with $K = a\sigma_w\lambda_m$, and evaluated using latest estimate of the

TABLE 5. GRADIENT-TRANSFER AND SIMILARITY CALCULATIONS OF VERTICAL SPREAD FROM A SURFACE RELEASE

No.	Equations* and Parameters Used	Stability	Reference
1	CCLS eq., $K(z) = K_1 z^n$, $\overline{u}(2) = \overline{u}_1 z^{(1-n)}$ fitted to surface-stress similarity law	Neutral	Calder[17]
2	$d\overline{Z}/dt = ku_*$, $d\overline{X}/dt = \overline{u}(c\overline{Z})$, first with c = 1, then with c = 0.6 as estimated by Chatwyn	Neutral	Batchelor[18] Pasquill[2,19]
3	CCLS eq., $K = ku_* z/\phi_m$ with $\phi_m = 1 + aRi$	Stable	Tyldesley[20]
4	$d\overline{Z}/dt = ku_*/\phi_H$, $d\overline{X}/dt = \overline{u}(\overline{Z})$, ϕ_H from Businger	All	Chaudrey and Meroney[15]
5†	CCLS eq., $K = \frac{1}{15} \varepsilon^{1/3} \lambda_m^{4/3}$	All	Smith and Matthews[21]
6‡	$d\overline{Z}/dt = a\left[\sigma_w \lambda_m / z\right]_{\overline{Z}}$, $\frac{d\overline{x}}{dt}$ as in 2, a = 0.154	All	Pasquill[12] & App. A of this report
7§	As 2 for $\overline{Z} \leq a_1 u_*/f$, $d\sigma^2/dt = 2ka_1 u_*^2/f$ for $a_1 u_*/f \leq \overline{Z} \leq a_2 u_*/2f$	Neutral	App. B of this report

* CCLS - Continuous Crosswind Line Source

† Using tentative generalization from limited ε, λ_m profile data for whole depth of boundary layer

‡ σ_w, λ_m profiles from Minnesota 1973 boundary layer experiment and other recent generalizations

§ $a_1 a_2$ constants obtained in terms of Rossby Surface Number Similarity, $a_2 u_*/f$ being depth of boundary layer properties σ_w and λ_m, provides a rough fit in unstable conditions though with a curve shape which in detail differs from that based on $K = ku_* z/\phi_H$. For stable conditions the $K = a\sigma_w\lambda_m$ prediction is a gross overestimate. In the latter connection it may be significant that the present evaluation uses an empirical generalization[23] in which σ_w/u_* increases with increasing z/L.

(d) At large $\overline{X}/-L$ the data show a tendency to a growth more rapid than the $(\overline{Z}/-L)^{1/3}$ form predicted on simple similarity grounds, assuming a regime of free convection with the determining parameters height and surface heat flux.

Clearly, several interesting features still need more satisfactory interpretation, but in the meantime the foregoing results provide at least a useful empirical step in that, for example, the observed stability dependence of the function $\Phi(Z/L) = (1/ku_*)(d\overline{Z}/dt)$ may now plausibly be applied to a surface roughness different from that of the Prairie Grass site. In principle this Φ may be converted to the function $d\overline{Z}/d\overline{X}$ by recombining it with the functional form for $d\overline{X}/dt$ as determined by the wind profile (see note (c) of Table A-2, Appendix A). Integration may then be carried out, if necessary numerically, to give $\overline{Z}$ as a function of distance $\overline{X}$ for any specified z_0 and L.

For the consideration of the growth of spread at much longer range from the source (several kilometres and beyond), no definitive direct measurements of σ_z are yet available. In any case, the knowledge of the flow characteristics in the upper part of the boundary layer, required for proper interpretation and generalization of such dispersion measurements, has only recently begun to accumulate. As noted in Table 5 (Methods 5 and 6) some of these new data have recently been applied in constructing new estimates of σ_z.

In nonconvective conditions, for which the gradient-transfer assumption seems least open to objection, the characteristics of the boundary layer (including its depth) should be seen within the framework of surface Rossby Number similarity. This aspect, which has been brought out specifically in 2nd-order closure treatments[24], was not incorporated in Method 5 of Table 5. An examination of the implications of simplified assumptions about the neutral

K-profile, within the framework of Surface Rossby Number similarity (Method 7 of Table 5) is reported in Appendix B. With the existing uncertainties about the K-profile in the upper nine-tenths of the boundary layer, this type of analysis, even if carried out with complete mathematical rigor, can at present provide no more than an interim solution. However, in one example which will be noted later in this section, the results are encouraging in being remarkably consistent with a completely independent 2nd-order closure calculation. Furthermore, they do bring out the schematic way in which any extension of the σ_z curve (beyond the well-established short-range Lagrangian-similarity form) must reflect the important control exerted on boundary-layer depth by geostrophic wind and surface roughness.

In principle the procedure of Appendix B may also be considered for the stable boundary layer. However, at first sight the shape of the K-profile (as suggested, for example, by the analysis of Businger and Arya,[25]) seems unlikely to be suitably approximated by the simple two-layer form adopted in Appendix B. Instead, the numerically more demanding finite-difference solution of the diffusion equation would probably be desirable.

DEARDORFF'S MIXED-LAYER SIMILARITY APPROACH AND LABORATORY MODELING

Deardorff's approach[6] is that the structure and transfer properties in a capped convectively mixed layer are determined completely by the surface heat flux H_0 and the depth of mixing z_i. The latter parameter is the obvious characteristic length scale, and on dimensional grounds Deardorff defines a characteristic velocity scale

$$w_* = (gH_0 z_i/\rho c_p T)^{1/3} \tag{8}$$

From these considerations Deardorff argues that the properties of dispersion of non buoyant particles, say, the σ_z growth with time, should obey a universal form

$$\sigma_z/z_i = f(t_*) \tag{9}$$

where t_* is a dimensionless time $w_* t/z_i$. The form of the function f has been estimated from both numerical modeling and from the laboratory measurements in a heated water tank referred to briefly in Section 2.

The most recent publication[13] of the tank data provides a vertical spread curve, $\overline{(Z')^2}^{\frac{1}{2}}$ against t_* where Z' is the displacement from heights of release z_r, for the following conditions:

$$
\begin{aligned}
z_r &= 0.067\ z_i \\
z_i &= 28.7\ \text{cm} \\
\text{tank width}/z_i &= 4 \\
w_* &\simeq 1\ \text{cm sec}^{-1}
\end{aligned}
$$

and with the following properties

- Slight acceleration up to $\overline{(Z')^2}^{\frac{1}{2}}/z_i \simeq 0.4$, $t_* \simeq 0.75$,
- Bending over to a maximum $\overline{(Z')^2}^{\frac{1}{2}}$ near 0.6, $t_* \simeq 1.5$,
- Slow descent to the asymptotic value of 0.5 at t_* near 3.

For the range $0.2 \leq t_* \leq 0.8$ Willis and Deardorff[13] give $\overline{(Z')^2}^{\frac{1}{2}}/z_i \propto t_*^{1.15}$

In the equivalent Gaussian plume form with idealized reflection of the distribution at the lower boundary, the familiar σ_z is equal to $\overline{(Z')^2}^{\frac{1}{2}}$ only at small and large values of σ_z/z_r, and Willis and Deardorff include a graph of the general relation between these two specifications of the vertical spread. Using this graph their vertical spread has been converted to σ_z and found to fit a linear form

$$\sigma_z/z_i = 0.61\ t_* \tag{10}$$

with a discrepancy less than 3% over the range $0.2 < t_* < 0.8$ or $0.1 < \sigma_z/z_i < 0.5$. Adopting an effective advecting speed u_e this may be written in distance (X) terms as

$$\sigma_z/z_i = 0.61(w_*/u_e)(X/z_i) \tag{11}$$

Reference has already been made to a striking feature in Willis and Deardorff's[13] dispersion data, namely, the appearance of a non-gaussian distribution with a progressively elevated maximum. It is interesting to note, however, that this departure from Gaussian shape of vertical profile does not appear at t_* values below 0.5 or $\sigma_z/z_i < 0.3$). As an essentially uniform vertical distribution is achieved at $t_* = 3$, it follows that the 'undershoot' in surface concentration (compared with that for a Gaussian shape and idealized reflection from the upper boundary) is within the range $0.5 < t_* < 3$. Also, it appears from their Figure 10 that the surface concentration at $t_* = 1.56$ is roughly one-half the magnitude appropriate to a uniform vertical distribution.

One important feature which remains to be settled is the sensitivity of the vertical spread properties to the height of release. Willis and Deardorff[13] are careful to emphasize that their result applies to a specific value of z_r. Naturally, one would expect the distribution to be insensitive to z_r when σ_z/z_r is sufficiently large, and they estimate this condition to be satisfied for $t_* < 00.5$. On this basis we may expect the Gaussian-type vertical spread, for all z_r/z_i (0.067).

$$\sigma_z/z_i = = 0.61\ t_* = 0.61(w_*/u_e)(X/z_i),\ 0.5 < t_* < 0.8 \qquad (12)$$

For $t_* < 0.5$ some dependence on $z_r z_i$ is to be suspected but has yet to be specified.

Another important feature to be kept in mind (as in the brief discussion relating to crosswind spread) is the condition under which the laboratory results may be considered directly applicable to an atmospheric mixed layer with wind, with full-scale magnitudes of z_i and w_*. Willis and Deardorff[13] plausibly take the view that the upper limit of wind speed for such direct scaling should be associated with a sufficiently large magnitude of z_i-L and for this they adopt 10. In other words, the stimulation is that when the negative Monin-Obukhov length is less than one-tenth of the mixing depth, the mixed layer properties are controlled by the similarity laws we have just discussed. On this basis, they estimate[13] an upper-limit wind speed of 12 m sec^{-1}, but this does need qualification in respect to roughness and heat flux. Thus with z_i = 1000 m, hence L < 100 m, the criterion is met at

$\overline{U}(10)m = 12$ m sec^{-1}, over a relatively smooth surface (say z_0 = 3 cm) only when $H_0 > 550$ mw m^{-2}, an unusually large heat flux. The wind condition must, of course, be even more restrictive over the rough surfaces more likely to be of interest in urban pollution. If for this case we take z_0 = 1 m, and take a heat flux of 300 mw m^{-2} as more typical of the range of conditions of interest, even allowing for artificial heating, the wind speed limit is 3.7 m sec^{-1}! In general, of course, the limit depends on the roughness and intensity of surface heating, but the foregoing example leaves no doubt that the 12 m sec^{-1} value limit constitutes an overstatement of the applicability of the laboratory results.

SECOND-ORDER CLOSURE MODELS

In the gradient-transfer models which have been considered so far, the continuity equation relating rate of change of concentration to turbulent flux divergence is put into the "mean quantity forms appearing in Table 1 by substituting

$$\text{Flux} = K \times \text{gradient.}$$

Second moments which represent this flux (e.g., $\overline{w'C'}$ where w' and C' have the usual meaning--turbulent fluctuations from the mean values) can, however be expressed in full, without the gradient-transfer assumption, for the momentum, heat, and material content properties. Solution of these equations requires "closure," by making certain assumptions for simplifying (modeling) the more complex terms, including 3rd moments. The idea is that such assumptions, which may or may not be of the gradient-transfer type, should not introduce errors as large as might arise from making the gradient-transfer assumption in the simple equations for $d\overline{C}/dt$, etc.

Considerable literature has accumulated on the evolution of this approach, application of which entails a large effort in numerical solution. The necessity for such a development, as well as its potential, have been discussed in a general article by Lumley[26]. The main areas of failure which must be expected in the simple gradient-transfer model and for which we may expect the 2nd-order-closure system to provide more correct treatment are as follows:

(a) Violation of scale-relation requirements, in a more subtle fashion than that already noted as regards relative scales of turbulence and plume, for example, as a consequence of time-dependent changes in the turbulence characteristics.

(b) The special action of buoyancy-driven vertical mixing. This point is taken up in detail in Lumley's article. One of the distortions which this imposes on the vertical distribution expected from gradient-transfer condition has emerged in the Willis and Deardorff[13] laboratory data discussed in the foregoing subsection, namely, the appearance of a markedly non-Gaussian distribution (obviously associated with a counter-gradient transfer) after the vertical spreading has progressed to a certain state. However, on this last specific point note that the "certain stage" corresponds to a substantial magnitude of σ_z/z_i, about 0.3!

(c) The vertical spread from an elevated source as distinct from a surface release. This is the vertical analog of the crosswind dispersion problem, though with the additional complication of a variation (in the vertical) of the scale of turbulence. This means that neither the statistical theory nor the gradient-transfer theory is applicable for vertical spread from an elevated source, though for a surface-release the second theory has been provided with some empirical verification and will be given some further support.

EARLY ACHIEVEMENTS OF THE 2ND-ORDER CLOSURE TECHNIQUE

Some specific achievements in the context of predicting dispersion from sources have already been reported in the literature. The first and most extensive application is that by Lewellen and Teske[24,27,28] using the 2nd-order closure assumptions advocated by Donaldson[29]. These latter assumptions provide expression of unknown terms in relation to characteristic velocity and length scales and in some respects seem tantamount to a gradient-transfer assumption, but, of course, then only at the 2nd-order. A brief summary of the achievements of the Lewellen-Teske work especially relevant to the present discussion follows:

(a) For a ground release the vertical spread growth with distance in a neutral atmosphere is included in Figure 1 in comparison with curves which follow from methods 5 and 7 of Table 5. There is a very reasonable degree of agreement which adds support to the validity of the simple gradient-transfer method, for distances of travel (> 1 km) for which no observational test has so far been developed.

(b) In a free-convection mixed layer[27] the vertical distributions exhibit a remarkable similarity to those observed in the laboratory by Willis and Deardorff[13]. The progressive elevation of the level of the concentration maximum is well reproduced, but the "undershooting" of the surface concentration is not so obvious as in the laboratory data.

(c) Although the expected reduction in σ_z with stability is reproduced, the particular conditions adopted do not exhibit any significant change from the spread in neutral flow at distances in the region of 0.1 km, and the result cannot yet be confirmed in terms of the Prairie Grass data, which do show a substantial reduction of the vertical spread at 0.1 km downwind.

(d) The potential of the approach in the treatment of the elevated source is displayed in a limited way. Interestingly, for a neutral atmosphere the 2nd-order closure calculations show that vertical spread is smaller from a source elevated at some hundreds of metres than from a surface release, by a factor near two at the range corresponding to $\sigma_z \simeq z_r/2$. The reduction is (not surprisingly) associated with a fall in σ_w with height.

Another recent application is that of Zeman and Lumley[30] (using a closure scheme differing from that of Donaldson) for the case of the surface input of pollution into a mixing layer growing as a result of surface heating. From their calculations they derive effective eddy diffusivities which have a vertical profile broadly resembling those derived empirically by Crane and Panofsky[31] from observations of a morning buildup of carbon monoxide pollution in Los Angeles. In these particular calculations, although the

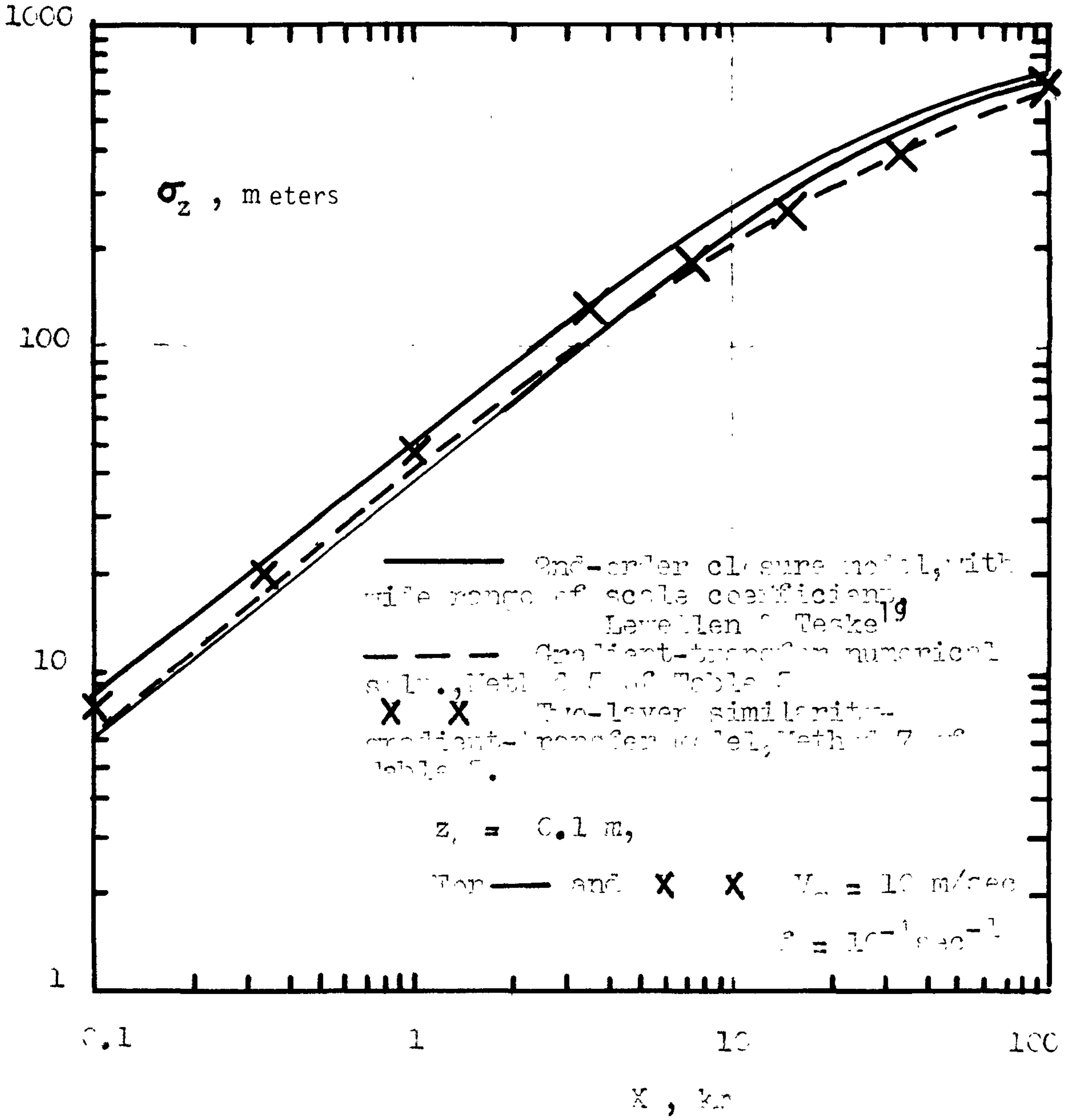

Figure 1. Vertical spread from a surface release in a neutral boundary layer.

effective values of K became large in the middle of the mixed layer, they do not become effectively infinite (as implied by Crane and Panofsky's analysis) or negative as found in Deardorff and Willis'[32] laboratory study of dispersion from an instanteous release.

SECTION 5

THE PRESENT POSITION AND PROSPECTS IN PRACTICAL SYSTEMS

For the practical estimation of pollutant concentration arising from a single continuous point source, two systems in particular have been in popular use for many years--the Pasquill-Gifford (PG) curves incorporated in the Turner Workbook[33], and the Smith-Singer (SS) curves incorporated in the publication[34] by the American Society of Mechanical Engineers.

The PG curves were based on the understanding and experience accumulated by the late 1950's on dispersion from surface releases, but they have been adopted as practical approximations for elevated releases also. Their most popular feature is the presentation of the σ_y,σ_z, estimates in terms of stability categories prescribed in terms of routine meteorological data. The method has been reviewed in the general context of dispersion-estimate systems by Gifford[35] and in the specific context of the conventional Gaussian-plume model by Weber[36]. The need for improvement and updating has been under consideration for some time, and this was the motivation for Weber's review and for the companion report by the writer[12] on specific requirements for modification of the parameters and procedure in the Turner Workbook.

The Smith-Singer curves are based on observations of ground-level concentration from an elevated (z_r = 100 m) release of passive material at the Brookhaven National Laboratory. Estimates of σ_y and σ_z are given in groups designated by the width of the wind direction trace and also in formulae in terms of σ_A and σ_E, the standard deviations of the direction and inclination of the wind.

It is evident from the progress discussed in the foregoing sections that the basis for revision of the practical systems is still far from complete. However, certain obvious evidence points to desirable changes in the PG curves.

One interim revision has already been suggested[12] for the estimation of σ_y. The main points of this are firstly the reminder of the applicability of the Workbook σ_y parameter to a sampling (or release) time of 3 minutes; secondly, the reaffirmation of the value of wind direction fluctuation data; and thirdly, a plausible generalization for variation with distance. Recent progress in generalizing about the behaviour of σ_v (or the corresponding wind direction fluctuation σ_θ) in the atmospheric boundary layer is directly relevant to prospects of a climatological basis for σ_y, but certain features of wind direction variation are not readily included in such a climatology (notably synoptic and topographically induced variations).

As regards σ_z, certain details cannot immediately be settled, but the essentials of a new emerging pattern are already clear enough to suggest a new format along lines indicated in Figure 2. Explanatory notes are included with the diagram. At this stage the determining "flow parameters" are all in their basic form: z_0, L, w_*, z_i. For the future, not only is there the task of confirming and completing the detail of the σ_z,X curves branching off the basic neutral form, but also a parallel requirement for updating the representation of the second and third of those parameters in more convenient meteorological terms.

Note that the sections of curve given for unstable conditions are in fact based on elevated source (laboratory) data, using the principle that the distinction between the concentration distributions from surface and elevated releases must progressively diminish to a negligible amount as σ_z/z_r increases. Deardorff and Willis[32] suggest that the approximation is adequate at $\sigma_z/z_r \approx 5$, and this means that the curves in Figure 2 are appropriate for all z_r/z_i less than about 0.07. Not only is confirmation of this estimate needed, but also the effect of z_r at shorter distances and in other stabilities has yet to be convincingly evaluated. One immediate assurance that the use of surface-release predictions is not severely misleading for moderate elevations of source in neutral conditions is contained in Table 6. This compares the similarity prediction for $z_0 = 1$ m with the Smith-Singer estimates for a release at 100 m, at downwind distances of 0.1-10. Note also that for the unstable conditions Willis and Deardorff[13] have demonstrated good agreement between their tank values of σ_z and the

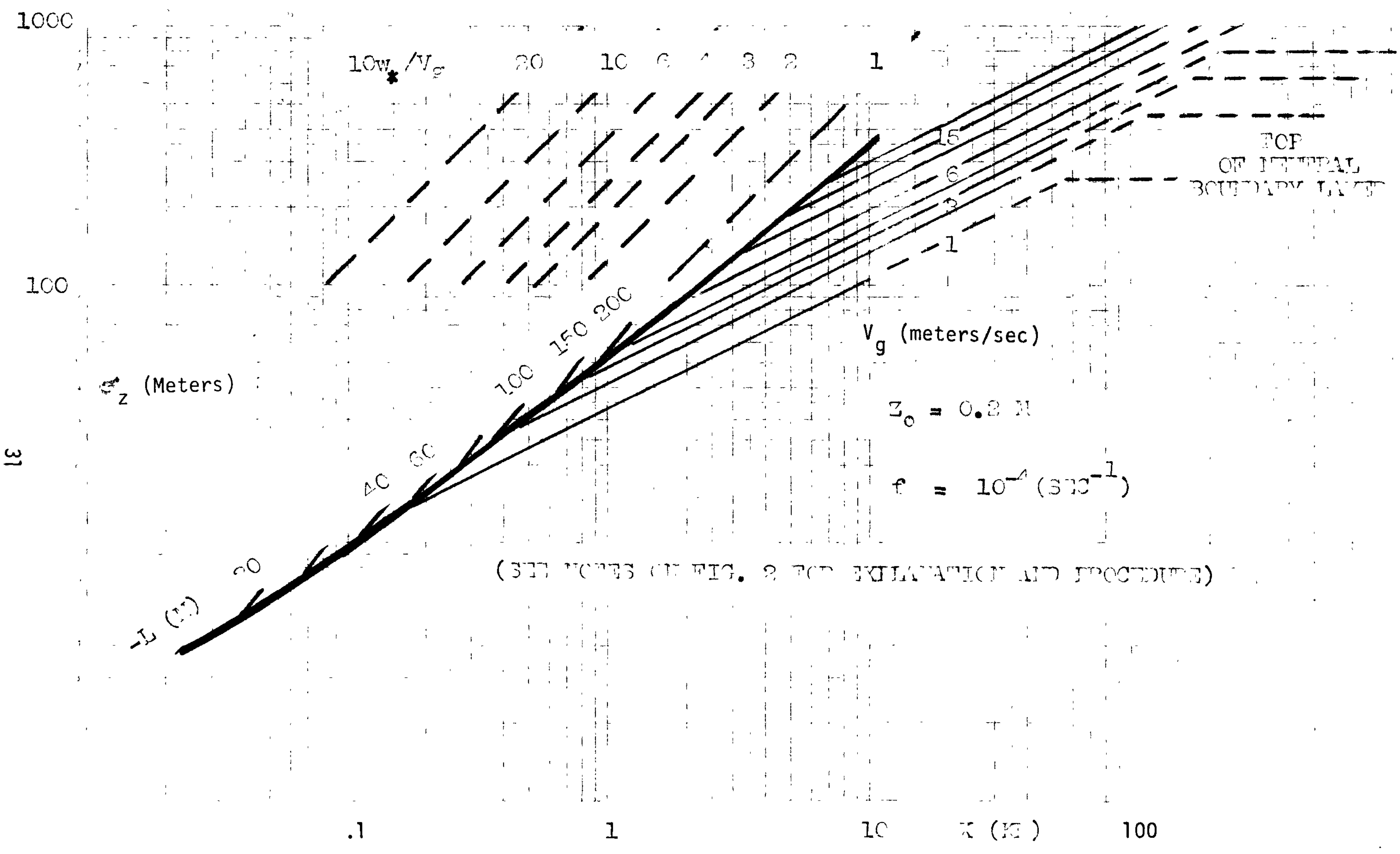

Figure 2. Envisaged new format of σ_z curves.

TABLE 6. COMPARISON OF CALCULATED VERTICAL SPREAD FROM A SURFACE RELEASE WITH THE SMITH-SINGER CURVE FOR HEIGHT OF RELEASE 100 M, IN NEUTRAL FLOW

Distance downwind (km)	0.1	1	10
σ_z (Smith-Singer) m	8	50	280
σ_z (calculation by method 7 of Table 5, with z_0 1 m	18	79	302*

* At this distance the calculated σ_z increases slowly with the geostrophic wind speed V_g, and the value given is for V_g = 10 m/sec.

Smith-Singer estimates, when the data are scaled in terms of z_i and w_*.

Even when the idealized relations for σ_z are settled, the calculation of concentration distribution will always be subject to a whole chain of uncertainties:

(a) Source strength and position,

(b) Departure from the idealized relations between σ_y,σ_z and the basic flow parameters, and

(c) Uncertainties in the basic flow parameter values implied by routine meteorological data.

Features especially relevant to (b) are the controls exerted by topography on the flow pattern and the effects of site geometry on the behaviour of a plume near the release point. Discussion of such features will be found in recent articles by Gifford[35,37] and Egan[38]. An important component of (c) is the mean wind direction specification, especially as regards individual short-term averages of concentration when the plume is narrow.

Errors in the ground-level position and magnitude of the maximum concentration from an elevated source arise especially from uncertainties in the effective height of source, the estimated growth of vertical spread, and the wind direction. As a consequence, individual estimates of concentration at a particular receptor near the ensemble-average position of the maximum may be grossly in error (see discussion p. 303 of Ref. 1).

Generally, the error statistics for concentration estimates must be expected to vary widely, according to the particular conditions of source, site geometry, terrain, and airflow characteristics. From the experience accumulated in dispersion studies and air pollution surveys, it is possible to make realistic estimates of the best levels of accuracy which may be achieved even in the simplest conditions likely to be encountered in practice. An early exercise on these lines has been described by the writer (p. 398 of Ref. 1). It is scarcely necessary to emphasize that laboratory-type accuracy cannot be expected. Accuracies in the region of $\pm$ 10 percent may be possible in the most ideal circumstances of source, terrain, and airflow, but for many circumstances of practical interest there seems at present no basis for expecting uncertainties less than several ten's of percent statistically or factors of twc individually.

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APPENDIX A

SIMILARITY-EMPIRICAL CONSIDERATIONS OF VERTICAL SPREAD AT SHORT RANGE FROM A SURFACE RELEASE

DATA FROM PROJECT "PRAIRIE GRASS"

The "Prairie Grass" measurements included measurements of vertical distribution at a distance of 100 m from the source. Magnitudes of σ_z derived from these data have been put together with additional measurements in stable conditions in England by Pasquill (Ref. 1, p 207, Fig. 4.14).

Indirect estimates of σ_z have also been derived by Cramer[2] at greater distances, up to 800 m, from the "Prairie Grass" data by using the measurements of peak concentration and crosswind spread, and substituting these in the conventional Gaussian plume formula. Also, from this Gaussian formula (in which implicitly the wind is invariant with height)

$$C(x,o,o) \propto 1/\bar{u}\, \sigma_y \sigma_z \tag{A-1}$$

and if C, σ_y are measured as a function of distance the relative variation of σ_z automatically follows and may be converted into absolute values of σ_z by adopting the directly measured value at 100 m. The results for σ_z are given as a function of distance, for various magnitudes of the standard deviation of wind direction fluctuation σ_A, in Figure 11 of Cramer's paper, and corresponding exponents in the power-law variation of C, σ_y, and σ_z with distance over distance intervals 50-100, 100-200, 200-400, and 400-800 m are given in Tables 4a and 4b of that paper.

The foregoing procedure is questionable on account of the variation of wind with height, and a more acceptable form of Eq. (A-1) is

$$C(x,o,o) \propto 1/\, u_e\, \sigma_y\, \sigma_z \tag{A-2}$$

where u_e is an effective wind speed, increasing with σ_z and therefore with distance. An expression for u_e may be obtained by correctly writing the conservation equation with $\bar{u}$ a function of height ($\propto z^n$ for convenience).

With CIC(x,0) denoting the crosswind integrated concentration at the surface, and assuming a Gaussian distribution in the vertical, the conservation equation for a rate of release Q is

$$Q = CIC(x,0) \int_0^{\infty} \bar{u}(z) \exp(-z^2/2\sigma_z^2)dz . \tag{A-3}$$

Taking

$$\bar{u}(z) = \bar{u}(z_1)(z/z_1)^n \tag{A-4}$$

$$Q = 2^{(n-1)/2} \sigma_z^{(1+n)} \bar{u}_1 z_1^{-n} \Gamma((1+n)/2) \ CIC(x,0) \tag{A-5}$$

which with $z_1 = \sigma_z$ is of the form corresponding to Eq. (A-2), i.e.,

$$Q = (\pi/2)^{\frac{1}{2}} \sigma_z u_e CIC(x,0) \tag{A-6}$$

if we take

$$u_e = (2/\pi)^{\frac{1}{2}} 2^{(n-1)/2} \Gamma((1+n)/2) \bar{u}(\sigma_z) . \tag{A-7}$$

The ratio $u_e/\bar{u}(\sigma_z)$ is of course unity for the case of wind constant with height (n = 0). As n is increased to unity (which embraces the range found in practice), the ratio falls to 0.798.

For the present purpose of adjusting the original Cramer estimates of σ_z, the essential point is that we may now take u_e directly proportional to $\bar{u}(\sigma_z)$, i.e., to σ_z^n, for a given wind profile and neglecting the (slow) variation of n with height in practice. Accordingly, if in Eq. (A-2) we take $C(x,0,0) \propto x^{-c}$ and $\sigma_y \propto x^a$, it follows that

$$x^{-2} \propto 1/(x^a \sigma_z^n \sigma_z)$$

and therefore

$$\sigma_z \propto x^b \text{ with } b = (c - a)/1+n) \tag{A-8}$$

instead of b = c - a as given by Cramer. Starting with the directly measured values of σ_z(100 m) and Cramer's values of b, the foregoing result has been used to derive adjusted values of σ_z in a step-by-step procedure set out in Table A-1. Note that the original estimates at 800 m are reduced by a fraction ranging from only 0.02 in the most unstable conditions to nearly 0.6 in the most stable conditions.

The adjusted empirical values of σ_z have been used to derive a dimensionless dispersion-velocity function

$$\Phi(\bar{Z}/L) = (1/ku_*)(d\bar{Z}/dt) \tag{A-9}$$

in which $\bar{Z}$ is the mean height reached by particles at a given time after leaving the surface. This form was first postulated by Gifford[3], with a constant b in place of the von Karman constant k, and identity of b and k was first argued by Ellison (see Ref. 1, p 117). Details of the derivation of $\Phi(\bar{Z}/L)$ are summarized in Table A-2, and the results are shown graphically in Figure A-1.

In accordance with the setting of b = k = 0.4 in earlier considerations of Lagrangian similarity (see Ref. 1, pp. 118 and 206), $\Phi(\bar{Z}/1)$ tends to unity as $\bar{Z}/L$ tends to zero, and this is reaffirmed in the trend of the empirical values presented here. However, as the function departs from unity, increasing in unstable flow and decreasing in stable flow, similarity in terms of u_* and L is not entirely supported, in that the set of points for different magnitudes of L/z_0 do not collapse onto a single curve. Although the departures from a single curve are not large, compared say with the scatter in the empirical specification of the Monin-Obukhov velocity profile function Φ_M, it may be significant that a systematic effect of L/z_0 is clearly evident.

SIMILARITY HYPOTHESES ON MONIN-OBUKHOV LINES

Form of Φ have already been suggested in terms of the Monin-Obukhov functions for transfer of momentum and heat (see Table 5 of the text), either explicitly, or implicitly through gradient transfer considerations. These forms are reproduced graphically in Figure A-1. Note that the form in terms of Φ_M seriously underestimates the effect of stable flow, a result which is partly to be expected from the fact that Tyldesley's setting of the constant α was determined by fitting to Thompson[4] data, and these data (see Ref 1, p 207) do show a smaller reduction in σ_z than do the "Prairie Grass" data.

TABLE A-1. ADJUSTMENT OF CRAMER'S 1957 ESTIMATES OF σ_z FROM PRAIRIE GRASS DATA

Lengths in metres, z_o = 0.008							
σ_z(100m)	10	8	6	4.5	3	2	
Ri(2m)	-0.5	-0.27	-0.09	0	0.06	0.12	(1)
L	-4	-7.4	-22.2		23.5	7.1	(2)
Cramer's σ_A(deg)	21.5	16	11.5	8	5	3	(3)
Cramer's b (100-200)	1.43	1.24	1.07	1.0	0.9	0.8	(4)
Est. n	0.1	0.1	0.1	0.16	0.24	0.34	(5)
Corrd. b	1.30	1.13	0.97	0.86	0.73	0.60	(6)
Adjusted σ_z(200)	24.6	17.5	11.8	8.16	4.97	3.03	(7)
b (200-400)	1.93	1.66	1.47	1.20	0.90	0.90	
Est. n	0.1	0.1	0.1	0.13	0.29	0.42	
Corrd. b	1.75	1.51	1.34	1.06	0.70	0.63	
Adjusted σ_z(400)	82.8	49.9	29.8	17.0	8.1	4.7	
b (400-800)	2.46	2.08	1.75	1.30	0.80	0.40	
Est. n	0.1	0.1	0.1	0.12	0.35	0.45	
Corrd. b	2.24	1.89	1.59	1.16	0.59	0.28	
Adjusted σ_z(800)	391	185	89.7	38.0	12.2	5.7	
Cramer's Original σ_z(800)	400	200	110	50	23	14	

TABLE A-1. ADJUSTMENT OF CRAMER'S L(%& ESTIMATES OF σ_z FROM PRAIRIE GRASS DATA

Explanatory Notes

(1) From "eye fit" curve, Fig. 4.14, p 207 of Pasquill[1]

(2) Taking $Ri(z) \doteq z/L$ in unstable conditions

$$Ri(z) = (z/L)(1 + 4.8z/L)^{-1} \text{ in stable conditions}$$

(3) Interpolating for $\sigma_z(100)$ on Fig. 11 of Cramer[2]

(4) Interpolating in accordance with σ_A in Cramer's Tables 4a and 4b

(5) In $u \propto z^n$, for unstable conditions $n = 0.1$ is taken as a reasonable approximation. In stable conditions a power law was fitted to the similarity form specified in Table A-3, for $z_0 = 0.8$ cm, over the height interval corresponding to Cramer's $\overline{Z}$ values, where $\overline{Z} = 0.776\ \sigma_z$, the latter relation being appropriate to a vertical distribution form $\exp-(bz^s)$ with $s = 1.5$.

(6) Estimated corrected b is $b/(1+n)$ as argued in text

(7) Adjusted $\sigma_z(200)$ = "directly" measured $\sigma_z(100) \times 2^b$
Adjusted $\sigma_z(400)$ = Adjusted $\sigma_z(200) \times 2^b$
etc.

TABLE A-2. ESTIMATES OF $(1/ku_*)(d\bar{Z}/dt)$ FROM ADJUSTED VERTICAL SPREAD ESTIMATES IN TABLE 1

x	$\bar{Z}$	$d\bar{Z}/dx$*	$k\bar{u}/u_*$†	$(1/ku_*)(d\bar{Z}/dt)$‡	$\bar{Z}/L$
		(L -4,	L/z_0	-500)	
141.4	12.17	0.1119	5.505	3.85	- 3.04
282.2	35.03	0.2168	6.178	8.29	- 8.76
565.7	139.7	0.5532	7.026	24.29	-34.9
		(L -7.4,	L/z_0	-925)	
141.4	9.17	0.0733	5.62	2.57	- 1.24
282.8	22.93	0.1225	6.169	4.72	- 3.10
565.7	74.52	0.2490	6.941	10.80	-10.07
		(L -22.2,	L/z_0	-2775)	
141.4	6.52	0.0447	5.85	1.63	- 0.29
282.8	14.53	0.0688	6.36	2.73	- 0.65
565.7	40.10	0.1127	6.952	4.90	- 1.81
		(L 23.5,	L/z_0	2937.5)	
141.4	3.00	0.0155	5.78	0.56	0.128
282.8	4.92	0.0122	6.46	0.49	0.209
565.7	7.71	0.00804	7.18	0.36	0.328
		(L 7.1,	L/z_0	887.5)	
141.4	1.91	0.00810	5.70	0.29	0.269
282.8	2.92	0.00651	6.52	0.27	0.412
565.7	4.01	0.00199	7.26	0.090	0.565

Explanatory Notes:

* In accordance with the analysis of Table A-1, assume over 2:1 intervals of distance that

$$\bar{Z}(x)/\bar{Z}_1 = (x/x_1)^b$$

and that $\bar{Z} = 0.776\ \sigma_z$. Considering $\bar{Z}$ and $d\bar{Z}/dx$ at the geometric mean of the distances x_1 and $2x_1$, it follows that

$$d\bar{Z}/dx = b\bar{Z}/2^{\frac{1}{2}}\, x_1$$

$$\bar{Z} = 2^{b/2}\bar{Z}_1 .$$

TABLE A-2. ESTIMATES OF (1ku$_*$)(d$\overline{Z}$/dt) FROM ADJUSTED VERTICAL SPREAD ESTIMATES IN TABLE L

[1] For 0.6 $\overline{Z}$/L, evaluated from the wind profile equation given in Table 3, and assuming $u \propto z^{0.1}$ for -z/L greater than 1.0.

[2] Invoking the similarity assumption

$$d\overline{X}/dt = u(c\overline{Z})$$

with c = 0.6 (see item 2 of Table 5 of main text for reference).

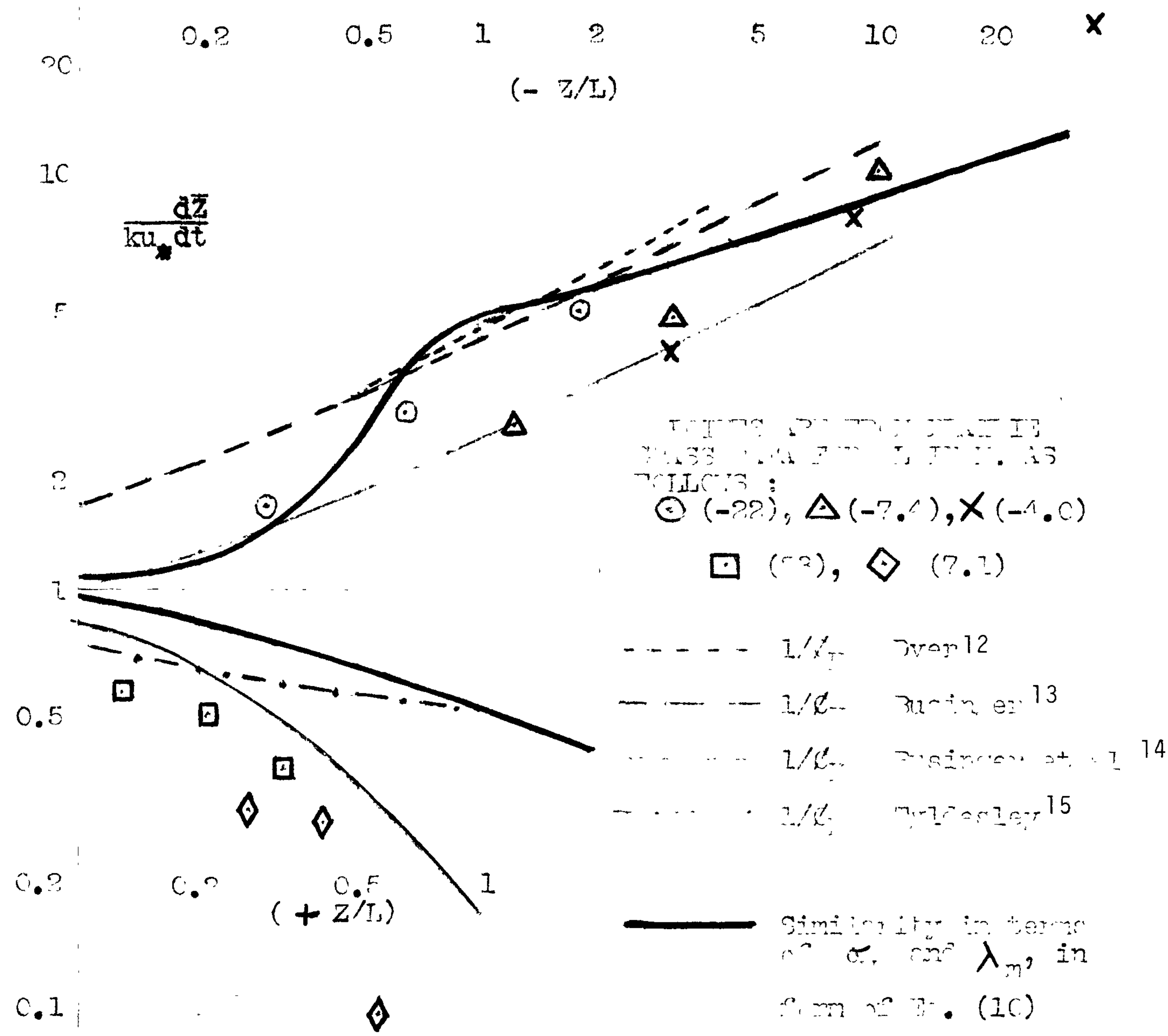

Figure A-1. Vertical diffusion similarity functions.

The form $\Phi = 1/\phi_H$ advocated by Chaudrey and Meroney[5] does provide an encouraging fit, though there is ambiquity as to the precise numerical values to be adopted for ϕ_H in unstable flow, and the one set of values considered in stable flow overestimates Φ somewhat.

SIMILARITY IN TERMS OF THE INTENSITY AND SCALE OF TURBULENCE

As an alternative to the original form of Lagrangian similarity treatment, the writer[6,7] has suggested a simple local similarity approach in terms of the intensity and scale of the vertical component of turbulence, both regarded as functions of height.

In the practical form to be considered here, the dispersion velocity is written as follows:

$$d\overline{Z}/dt = a \left[\sigma_w \lambda_m / z\right]_{\overline{z}} \qquad \text{(A-10)}$$

in which λ_m is the equivalent wavelength at which the product of frequency and spectral density of the vertical component is a maximum. It turns out that to a useful approximation the foregoing relation is the same as that which follows from the assumption of a gradient-transfer relation with

$$K = a\, \sigma_w \lambda_m \ . \qquad \text{(A-11)}$$

On identifying Eq. (A-11) with the Monin-Obukhov form of K_M in the special case of neutral flow, and inserting the latest estimates of σ_w/u and λ_m/z (see later) the magnitude of the constant a is 0.154. (Alternatively, note that this magnitude is consistent with the result $d\overline{Z}/dt = 0.4\ u_*$.)

The latest estimates of the boundary layer parameters σ_w/u_* and λ_m/u_* as a function of z/L are listed in Table A-3, with the magnitude of the dispersion function Φ which then follows from a combination of Eq. (A-9) and A-10). Also included are values of the velocity profile function ψ, from which $d\overline{Z}/dt$ may be converted to $d\overline{Z}/d\overline{X}$, where $\overline{X}$ is the mean distance of travel of the particles.

As can be seen from Figure A-1, the function Φ derived in Table A-3 shows a growth in unstable flow which, over the total range of $\overline{Z}/L$ covered, is similar to that found in the "Prairie Grass" measurements of dispersion. However, there are differences in detail, notably the very sharp increase in the

TABLE A-3. MAGNITUDES OF BOUNDARY LAYER PARAMETERS AND DIMENSIONLESS DISPERSION VELOCITY $\Phi(Z/L)$ DEFINED IN EQ. (A-9) AND (A-10)

z/L	ψ	σ_w/u_*	λ_m/z	$\Phi(\bar{Z}/L)$
-10		4.084	6.0	9.423
- 5		3.276	6.0	7.557
- 2		2.487	6.0	5.737
- 1	-1.032	2.064	6.0	4.763
- 0.5	-0.707	1.764	3.5	2.373
- 0.2	-0.354	1.520	2.0	1.170
- 0.1	-0.185	1.419	2.0	1.093
- 0.05	-0.082	1.362	2.0	1.047
- 0.02	-0.024	1.325	2.0	1.017
- 0.01	-0.009	1.313	2.0	1.010
0	0	1.3	2.0	1.000
0.01	0.047	1.32	(1.97)	(1.000)
0.02	0.094	1.34	(1.94)	(1.000)
0.05	0.235	1.37	(1.90)	(1.000)
0.1	0.47	1.40	1.8	0.970
0.3	1.41	1.57	1.25	0.755
0.5	2.35	1.70	1.0	0.653
1	4.7	1.97	0.67	0.507
2	(9.4)	2.36	0.45	0.407

$k\bar{u}/u_* = \ln(z/z_0) + \psi(z/L)$

ψ for -ve values of z/L, Dyer and Hicks[9]

ψ for +ve values of z/L, 4.7z/L, Merry and Panofsky[8]

$\sigma_w/u_* = 1.3 \left[1.0 + 3.0(z/-L)\right]^{1/3}$ for -ve values of z/L, Panofsky et al[10] published

$= 1.3 \left[1.0 + 2.5(z/L)\right]^{1/3}$ for +ve values of z/L, based on Merry and Panofsky[8]

λ_m/z Based on Kaimal et al[11] 1972. The "refined" estimates in parentheses are consistent with the general shape of the λ_m, z curve, taking the neutral value as 2.0, and are used rather than rounded values of 2.0 to avoid values greater than 1.0 for Φ.

"similarity theory" value fof $\overline{Z}/L$ in the region of -0.5, which does not appear in the empirical values. This sharp increase is associated with the sharp increase of λ_m with height in that range of z/L. On the other hand, in the most unstable conditions the empirical values show a much greater rate of growth than the "similarity theory" values, the latter necessarily tending to a limiting growth as $(\overline{Z}/-L)^{1/3}$ in accordance with the limits of σ_w/u_* and λ_m/z in Table A-3. Note also that this $(\overline{Z}-L)^{1/3}$ limiting growth also follows from the similarity hypothesis that in free-convective transfer the dispersion velocity is determined uniquely by the surface heat flux and the height $(\overline{Z})$, so something is clearly calling for explanation in the observed growth of Φ at large values of $(\overline{Z}-L)$.

In stable conditions the "similarity theory" function falls away from unity much more slowly than is found empirically. It may be significant here that a determining factor in the similarity function is the increase of σ_w/u_* with z/L, now claimed by Merry and Panofsky[8] and included in Table A-3.

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APPENDIX B

A SIMPLE TWO-LAYER MODEL FOR ESTIMATING VERTICAL SPREAD BEYOND THE SURFACE-STRESS LAYER IN NEUTRAL FLOW

The equivalence of the Lagrangian similarity result, $d\bar{Z}/dt = ku_*$, and gradient-transfer with $K = ku_*z$ has already been noted. These laws, however, apply only over a shallow surface layer (the surface-stress layer) and for the higher parts of the boundary layer a different form of K-profile must apply. There have been several guides to the form of this profile, and one of the latest, due to Businger and Arya[1], is shown in Figure B-1 in the dimensionless form K/u_*h versus z/h where h is the depth of the boundary layer. This profile was derived in a treatment in which von Karman's constant was taken to be 0.35; bearing this figure in mind the reader can observe that the profile follows the surface-stress layer form very closely up to $z/h = 0.05$ and to a reasonable approximation even up to $z/h = 0.1$. Thereafter, there is the non familiar bending of the curve to form a maximum K, in this case with $K/u_* = 0.033$ at $z/h = 0.25$, and then a continuous falloff at greater heights. Note also, as an indication of the present uncertainties, that this maximum K is only about half that previously estimated by Townsend[2] ($K/u_*h = 0.067$).

Notwithstanding the uncertainties it is now suggested that a useful general guide to the σ_z growth curve throughout a substantial depth of the atmospheric neutral boundary layer may be obtained from a simple model with the following properties:

(a) The surface-stress-layer result for $\frac{d\bar{Z}}{dt}$ and K is taken to apply to $\bar{Z}_1 = h/12$.

(b) K is taken to be constant at the surface-stress-layer value $K(\bar{Z})$ for $h/12 \leq z \leq h/2$.

(c) In accordance with Surface-Rossby-Number similarity, the boundary layer depth h is a_2u_*/f, with a_2 prescribed as a function of Surface-Rossby-Number Ro ($= V_g/fz_0$) following a treatment by F. B. Smith[3].

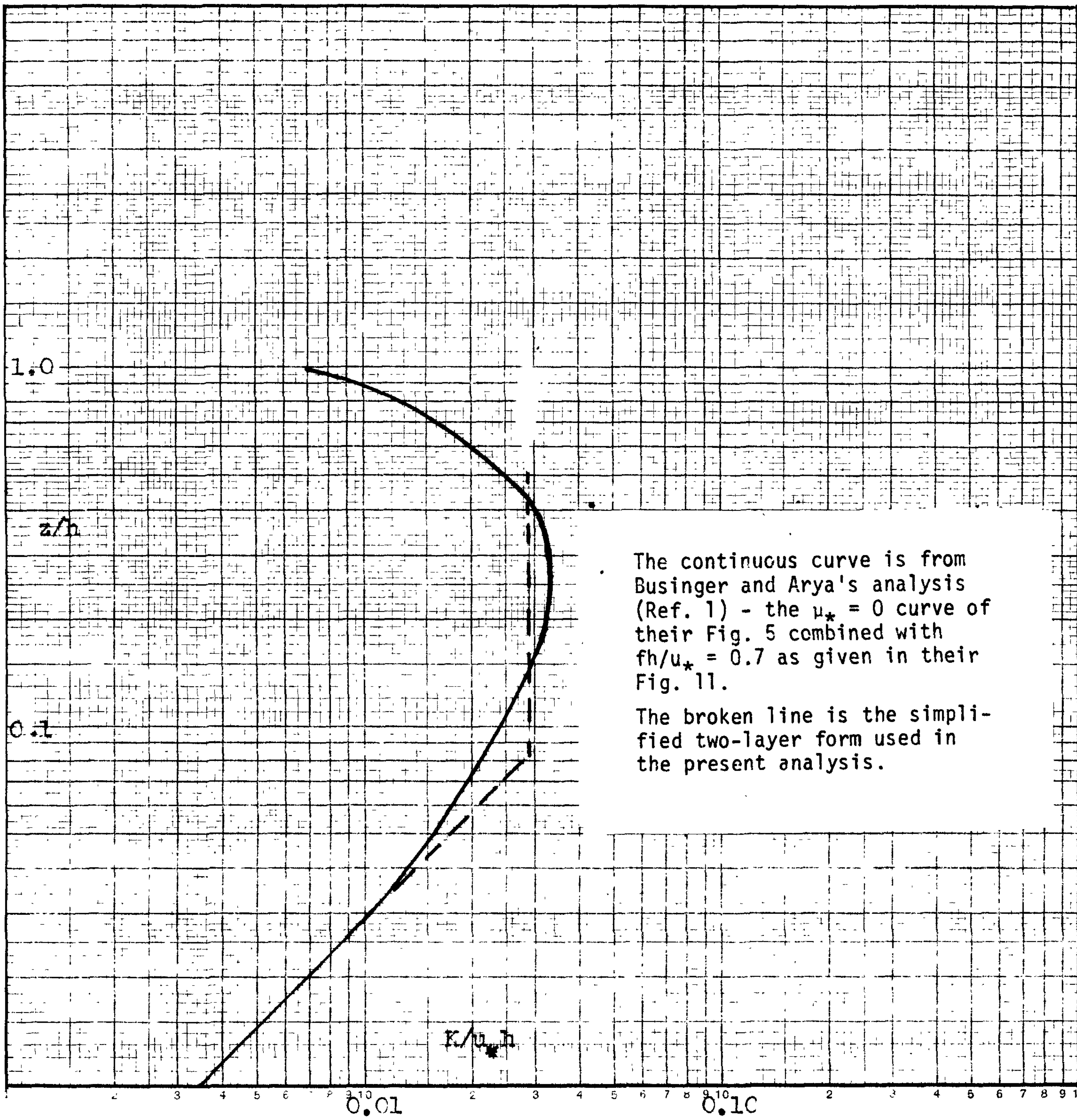

Figure B-1. K-profile in a neutral boundary layer of depth h.

A general solution of the two-dimensional equation of diffusion with this two-layer form of K cculd be obtained numerically, but a short-cut calculation adequate for bringing out the essential properties of the σ_z growth may be carried out as follows: For the lower layer ($\overline{Z} \leq \overline{Z}_1$)

$$d\overline{Z}/dt = ku_* \tag{B-1}$$

$$d\overline{X}/dt = \overline{u}(c\overline{Z}) \tag{B-2}$$

$$k\overline{u}(z)/u_* = \ln(z/z_0). \tag{B-3}$$

These equations may be integrated to give $\overline{X}$ as a function of $\overline{Z}$, and the solution which follows from taking the lower limit of integration at $\overline{Z} = z_0$ is

$$k^2\overline{X}' = \overline{Z} \quad \ln\overline{Z}' - (1 - \ln c)(1 - 1\overline{Z}') \tag{B-4}$$

where $\overline{X}' = \overline{X}/z_0$ and $\overline{Z}' = \overline{Z}/z_0$ (see Ref. 4, p. 117). This result is physically unacceptable when $\overline{Z}'$ is near 1.0, since with c less than 1.0 (approximately 0.6) $d\overline{X}/dt$ actually becomes negative. This unacceptable result may be eliminated simply by making the lower limit of integration z_0/c, in which the result is

$$k^2\,\overline{X}' = \overline{Z}'(\ln c\overline{Z}' - 1) + 1/c. \tag{B-5}$$

If K were everywhere constant ($= K_c$) we could write

$$\overline{Z}^2 = 2(\overline{Z}/\sigma_z)^2\, K_c T \tag{B-6}$$

or

$$d\overline{Z}/dt = (\overline{Z}/\sigma_z)^2\, K_c/\overline{Z}. \tag{B-7}$$

and for the Gaussian distribution implicit in this result the magnitude of $(\overline{Z}/\sigma_z)^2$ is 0.63 (see Ref. 4, p 205), whereas in the surface-stress layer

$$d\overline{Z}/dt = K\overline{Z})/\overline{Z}. \tag{B-8}$$

For an idealized K profile as defined in (a) and (b) above, Eqs. (B-7) and (B-8) for $d\overline{Z}/dt$ are strictly acceptable only for small $\overline{Z}$ and large $\overline{Z}$ respectively, and for intermediate values there should be a progressive change in the coefficient applied to $K(\overline{Z})\overline{Z}$ as $\overline{Z}$ graws. In the simplified model proposed here a discontinuous change from Eq. (B-8) to Eq. (B-7) at $\overline{Z} = \overline{Z}_i$ will be adopted as a useful overall approximation.

From Eq. (B-6), substituting $K_c = ku_*\overline{Z}_i$ and writing $(\overline{Z}/\sigma_z)^2 = r$,

$$\overline{Z}^2 - \overline{Z}^2_i = 2r^2k\, u_*\overline{Z}_i\ (T - T_i) \tag{B-9}$$

with T_i and T the travel times corresponding to $\overline{Z}_i$ and $\overline{Z}$, applicable for $\overline{Z}$ up to say $6\overline{Z}_i$ in accordance with (b) above. For conversion to a $\overline{Z}$, $\overline{X}$ relation Eq. (B-9) should be combined with Eq. (B-2) and an appropriate value of c. However, since a large proportion of the total change of wind speed with height occurs in the bottom one-tenth of the boundary layer, it will be adequate for present purposes to take with Eq. (B-9) a constant value of $\overline{X}/T$ equal to the wind speed at say $z/h = 1/6$. This constant "effective wind speed" u_e may be represented as a fraction p of the geotrophic wind speed, p being drivable as a function of Ro from F. B. Smith's[3] model. Writing $\overline{Z}$ and $\overline{X}$ in the dimensionless forms above, Eq. (B-9) then becomes

$$\overline{Z}'^2 - \overline{Z}_1'^2 = 2r^2ku_*\overline{Z}_1'(\overline{X} - \overline{X}_1')/pV_g \tag{B-10}$$

and at large $\overline{Z}/\overline{Z}_1$

$$\overline{Z}'^2 = 2r^2ku_*\overline{Z}'_i\overline{X}'/pV_g. \tag{B-11}$$

Given r and k Eqs. (B-10) and (B-11) are determined by a_2, u_*/V_g, p, and V_g/fz_0, the first three of these all being functions of the last (the Surface Rossby Number) and determinable from Smith's model. Table B-1 gives, for a practical range of the Surface Rossby Number, the magnitudes of the foregoing parameters, of $\overline{Z}_1$, $\overline{X}_1$, and of the coefficients A and B in the $\overline{Z}$, $\overline{X}$ relations are specified in the notes below the table.

The resulting data for $\overline{Z}/z_0$ as a function of $\overline{X}/z_0$ are shown graphically in Figure B-2. There is a sharp discontinuity at the junctions of the surface-stress-layer curve (Eq. (B-5) with the Ro-dependent sections (Eq. (B-10), at $\overline{Z} = \overline{Z}_1$. This is a consequence of the discontinuity in $d\overline{Z}/dt$ according to Eq. (B-8) and Eq. (B-7). Note also that the limiting form in Eq. (B-11) is an adequate approximation to Eq. (B-10) except near the discontinuity for the smaller values of Ro. Finally, it is emphasized that the curves are taken only to $\overline{Z} = h/2$, and even at this stage there will be some error (overestimating $\overline{Z}$) if, as expected, there is a reduction of K in the top part of the boundary layer.

TABLE B-1. PARAMETERS AND RESULTS IN THE ESTIMATION OF VERTICAL SPREAD FROM A SURFACE RELEASE, BEYOND THE SURFACE-STRESS LAYER, IN NEUTRAL FLOW

1	Ro	10^4	3×10^4	10^5	3×10^5	10^6	3×10^6	10^7
2	a_2	0.320	0.308	0.290	0.275	0.260	0.247	0.236
3	u_*/V_g	0.080	0.070	0.062	0.054	0.0475	0.043	0.0385
4	u_e/V_g	0.717	0.790	0.858	0.870	0.886	0.904	0.910
5	$\bar{Z}'_1$	21.33	53.9	149.8	371.3	1029	2655	7572
6	$\bar{X}'_1$	207.6	835.2	3.277 $\times10^3$	1.023 $\times10^4$	3.490 $\times10^4$	1.058 $\times10^5$	3.512 $\times10^5$
7	A	3.775 $\times10^2$	1.200 $\times10^3$	4.093 $\times10^3$	1.180 $\times10^4$	3.790 $\times10^4$	1.102 $\times10^5$	3.534 $\times10^5$
8	B	1.098	1.555	2.341	3.416	5.285	7.996	12.73

Notes

1. Ro = Surface Rossby No. = V_g/fz_o
2. $a_2 = hf/u_*$ where h is depth of boundary layer
4. $u_e = d\bar{X}/dt$, taken to be u at $z = h/6$, $= pV_g$, evaluated from the interpolation form $ku(z)/u_* = \ln(z/z_o) - z/h$
5. $\bar{Z}'_1 = h/12z_o$
6. $\bar{X}'_1$ = value of $\bar{X}'$ for $\bar{Z}' = \bar{Z}_1$ in Eq. (B-5)
7. Coefficient A in $\bar{X}' - \bar{X}'_1 = A\left[(\bar{Z}'^2/\bar{Z}'^2_1) - 1\right]$ (see Eq. (B-10))
8. Coefficient B in $Z' = B\,X'^{\frac{1}{2}}$ (see Eq. (B-11)) (for both A and B, k = 0.4 and $\bar{Z}/\sigma_z = 0.796$, the latter figure being appropriate to a Gaussian distribution)

2,3,4. Data all in accordance with F. B. Smith's[3] neutral boundary layer model, with unpublished minor adjustments.

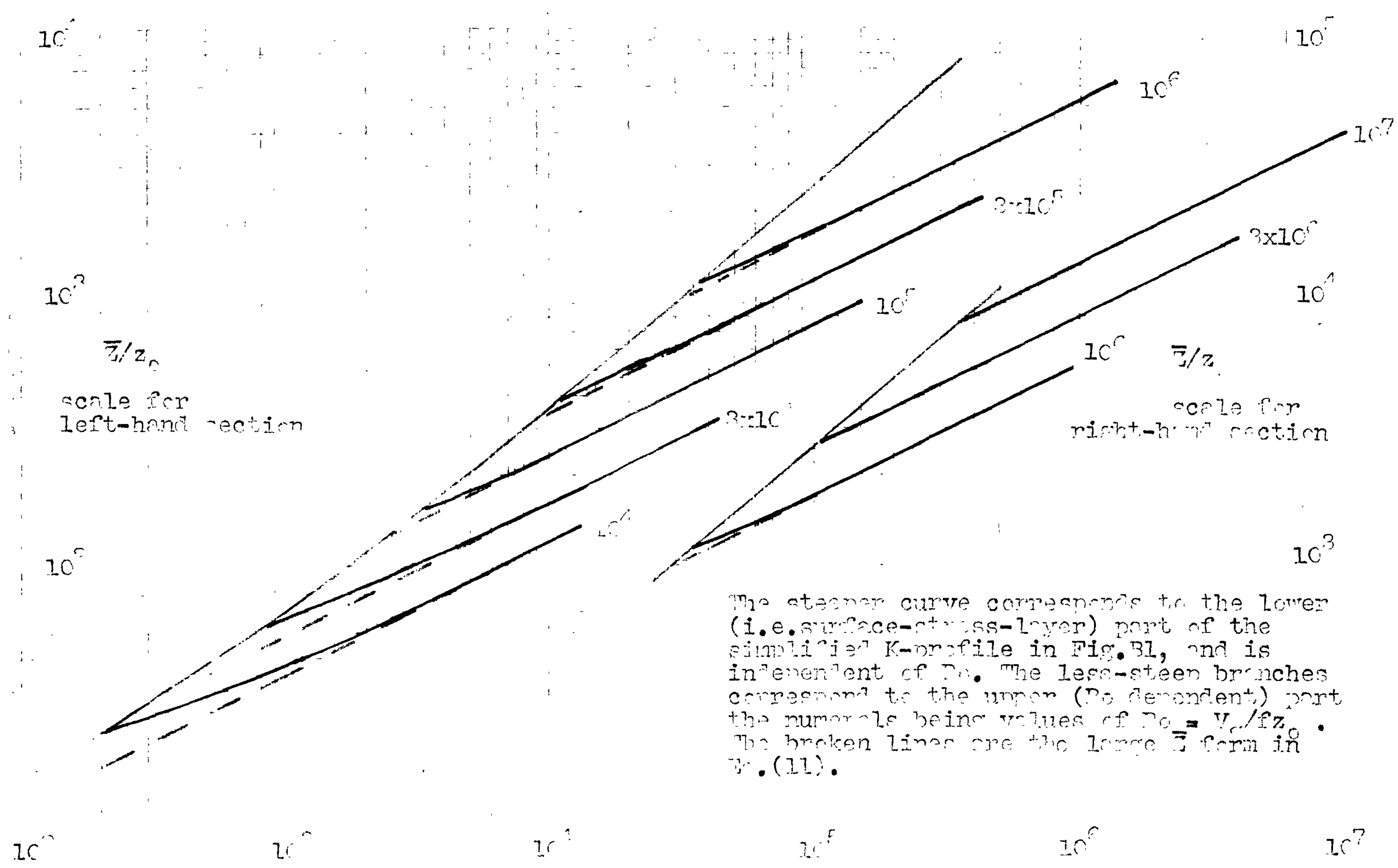

Figure B-2. Vertical spread curves for a neutral boundary layer with Surface-Rossby-Number similarity properties as in Table B-1.

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16. ABSTRACT

A survey of the underlying foundations of the present systems for specification of the atmospheric dispersion parameters for a continuous point source leads to the conclusion that the basis for revision of the current systems is still far from complete, but certain obvious evidence points to desirable changes. The author's previous recommendations on the crosswind spread are reiterated, and a suggested revision for vertical spread as a function of surface roughness, stability, and for the unstable case, the convective velocity scale and mixing depth, is offered.

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