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SOME TOPICS RELATING TO MODELLING OF DISPERSION IN BOUNDARY LAYER

by

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PREFACE

The several short notes comprising this special report were written by Dr. F. Pasquill* during the period December 1974-March 1975 while he was a Visiting Professor of Meteorology in the Department of Geosciences at North Carolina State University under support of a research grant from the Meteorology Laboratory of the Environmental Protection Agency. The six topics are all of major current interest in modeling of dispersion in the atmospheric boundary layer and are of such importance that they should be given early and wide publicity in a special report. Many of the topics continue and extend the discussion of items contained in the recently published book by Dr. Pasquill (Atmospheric Diffusion, 2nd Edition, John Wiley & Sons, Inc., 1974) and were the basis for a series of lectures presented during his recent visit to North Carolina State University and the Meteorology Laboratory.

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C	concentration (mass of material per unit volume of air)
E	rate of evaporation
$F(n)$	normalized one-dimensional spectral density
H	height of source
i	intensity of turbulence $\equiv \sigma_w/\bar{u}$ etc
k	von Kármán's constant
K	eddy diffusivity
ℓ	integral length-scale of turbulence
L	Monin-Obukhov length
n	frequency, cycles/sec.
p	pressure
$R(\xi)$	Lagrangian auto-correlation coefficient for time-lag ξ
s	time of travel of a particle reaching a given distance
S	normalized dispersion $\equiv \sigma_y/\sigma_v T$
t_L	Lagrangian integral time scale
T	temperature or time of travel
T_i	time of travel for $S = 1/2$
T'	normalized time of travel $\equiv T/t_L$
u,v,w	velocity components along axes x, y, z
u_e	equivalent advection velocity for diffusing material
u_*	friction velocity
x,y,z	rectangular coordinates, x along the mean wind and z vertical
x_m	downwind distance of ground-level maximum concentration from an elevated source
z_e	height at which $\bar{u}_e = \bar{u}(z)$
$\bar{X}$	mean distance of travel of particles in the x-direction after a given time
$\bar{Z}$	mean vertical displacement of particles after a given time
Z_m	vertical dimension of a plume of particles
α	exponent in power-law variation of wind with height
σ	standard deviation - of velocity component or of particle spread
λ_m	equivalent wavelength ($\equiv \bar{u}/n_m$) at which $nF(n)$ is a maximum

primes indicate departures from mean values.

ABSTRACT

This special report discusses six topics of major current interest in modeling of dispersion in the atmospheric boundary layer. These are the second-order closure modeling of turbulence, ~~and~~ crosswind dispersion and the properties of turbulence, wind direction fluctuation statistics over long sampling times, "local similarity" treatment of vertical spread from a ground source, representations of dispersion in terms of distance or time, and modeling for elevated sources.

SUMMARY

Section 1 contains a restatement of, and an attempt to clarify, an issue raised previously by the writer and F. B. Smith concerning use of the 2nd-moment equation for a passive material. The point in question is the feasibility of solving the equation to give the time rate-of-change of the vertical flux $(\frac{\partial F}{\partial t})$ near the surface of an effectively infinite, uniform, but time-dependent source. A practical example is the diurnal cycle of natural evaporation, and in this case it is easily demonstrated that the $\frac{\partial F}{\partial t}$ term is of a very small order relative to certain other terms. An analogous case is the distributed pollutant source with strength varying in the alongwind direction. Generally the issue is a very subtle matter, depending it seems on ensuring <u>physical</u> stability of the equation by adequate modelling of the terms. In current practice, however, certain 2nd-order closure procedures (e.g., that of Aeronautical Research Associates of Princeton, Inc.) do not depend essentially on evaluating $\frac{\partial F}{\partial t}$ (or the corresponding $\bar{u}\,\frac{\partial F}{\partial x}$ term) when this term is very small, but rather evaluate F from one of the much larger terms in which F has been introduced explicitly in the modelling procedure. In such cases, of course, the issue as originally raised does not apply.

In Section 2 the relation between the crosswind spread from a continuous point source and the properties of the crosswind component of turbulence in the atmospheric boundary layer is reviewed, especially in the light of some recent, unpublished analyses made available by the Department of Meteorology, Pennsylvania State University. A major point of interest is the implication that the Lagrangian correlation function is apparently substantially different

from a simple exponential form. The difference is in the sense of a much more rapid fall-off at short lag and a much slower fall at long lag. However, the precise form of the function remains in doubt, and the point is also re-emphasized that determination of the Lagrangian integral scale from dispersion data is subject to considerable uncertainty. Full resolution of both of these aspects requires continuing basic study.

Another aspect directly relevant to crosswind diffusion, namely the properties of the standard deviation (σ_θ) of the wind direction fluctuation over long sampling times, is considered in Section 3. Wind direction traces over three-hour periods near midday in sunny conditions at Cardington, England, have been analysed. Departures of 3-minute averages from a fitted linear trend were used to derive σ_θ as a function of sampling time. The growth curves differ widely, some approaching maximum value in about 30 min, while in others significant increase is maintained up to 90 min. The only evident orderly relation is a systematic decrease of the final σ_θ with wind speed. It is also noteworthy that the 3-hr linear trend was sometimes as much as three times σ_θ, which further emphasises the point that a simple 'climatology' of crosswind dispersion can hardly be expected for release times intermediate between a few minutes and the very long periods to which windrose statistics refer.

In Section 4 an attempt is made to develop a similarity treatment of the rate of vertical spread from a ground-level source in a thermally stratified boundary layer, without restriction to the surface-stress layer, and directly involving the measurable intensity and scale of the w- component of turbulence as a function of height. Suitably critical data on vertical spread and turbulence being at present unavailable, the similarity forms are used in an examination of estimates recently obtained from numerical solutions of the two-dimensional diffusion equation, employing K profiles specified in terms of the intensity and scale of turbulence. The rates of spread display orderly relations with the similarity variables over a 100-fold range associated with a wind range of thermal stratification. One immediately useful consequence is that the similarity relations provide an alternative and more convenient procedure for further calculations of vertical spread in lieu of further numerical solutions of the diffusion equation. The results also encourage the acquisition of critical observational data on which to determine the similarity relations more satisfactorily and more generally.

The alternatives of using time of travel or distance in describing dispersion are discussed in Section 5. For completely homogeneous flow, an argument is given in support of the equivalence which is usually assumed in terms of the mean wind speed $\bar{u}$. For boundary layer flow in which $\bar{u}$ is a function of height, identification of the time and distance descriptions requires an 'equivalent advecting speed' $\bar{u}_e$ which increases with distance as vertical spread increases. With certain assumptions, a rough estimate is obtained of $\bar{u}_e$ and hence of the height z_e at which $\bar{u}_e = \bar{u}(z)$. The result is used in further discussion of some of the crosswind dispersion data considered in Section 2 and of the difficulty of realistically inferring the magnitude of the Lagrangian time-scale.

Section 6 contains a brief review of the special problems relevant to the basic treatment of vertical dispersion from an elevated source. The present lack of an established working treatment for vertical spread that is of similar quality to those available for crosswind spread or for vertical spread from a ground-level source is noted. Some difference exists in U.K.

and U.S. practical systems for dealing with the distribution from elevated sources. Further work of a basic nature appears to be required to relate vertical spread (from an elevated source) more satisfactorily to boundary layer parameters.

1. ON THE '2ND-ORDER CLOSURE' APPROACH

This note is an attempt to proceed a little further in clarifying the issue raised by the writer and F. B. Smith in the note 'Some views on modelling dispersion and vertical flux' (Meteorological Office Met. 0.14 TDN 52, also in CCMS Proceedings of the Fifth Meeting of the Expert Panel on Air Pollution Modelling, 1974).

For the homogeneous time-dependent case of vertical flux of material (e.g., the non-steady quasi-uniform rate of evaporation from the ground), which is the time-analogue of the spatially varying but otherwise steady flux (e.g., the area source of pollution with non-uniform emission), the 2nd moment equation (e.g., see Donaldson, A.M.S. Workshop on Micrometeorology) is

$$\underset{(2)}{\frac{\partial \overline{w'C'}}{\partial t}} = -\underset{(3)}{\overline{w'^2}\,\frac{\partial \overline{C}}{\partial z}} - \underset{(5)}{\frac{\partial \overline{w'^2 C'}}{\partial z}} - \underset{(6+7)}{\frac{\overline{C'\partial p'}}{\rho \partial z}} + \underset{(8)}{g\,\frac{\overline{C'T'}}{T_o}}$$

$$+ \underset{(9)}{\nu\,\frac{\partial^2 \overline{w'C'}}{\partial z^2}} - \underset{(10)}{2\nu\,\frac{\overline{\partial w' \partial C'}}{\partial x_i \partial x_i}} \qquad (1.1)$$

The term numbers are as in T.D.N. 52 and the sum (6) + (7) follows directly if we assume $\rho = \rho_o$.

The issue arose from considering the relative magnitudes of terms (2) and (3) from experience of an extreme case of unsteadiness, i.e., the forenoon build-up of natural evaporation with strong insolation. Schematically this is known to be as in Fig. 1.1. For the discussion of TDN 52, specific data on estimates of E and the quantities in term (3)

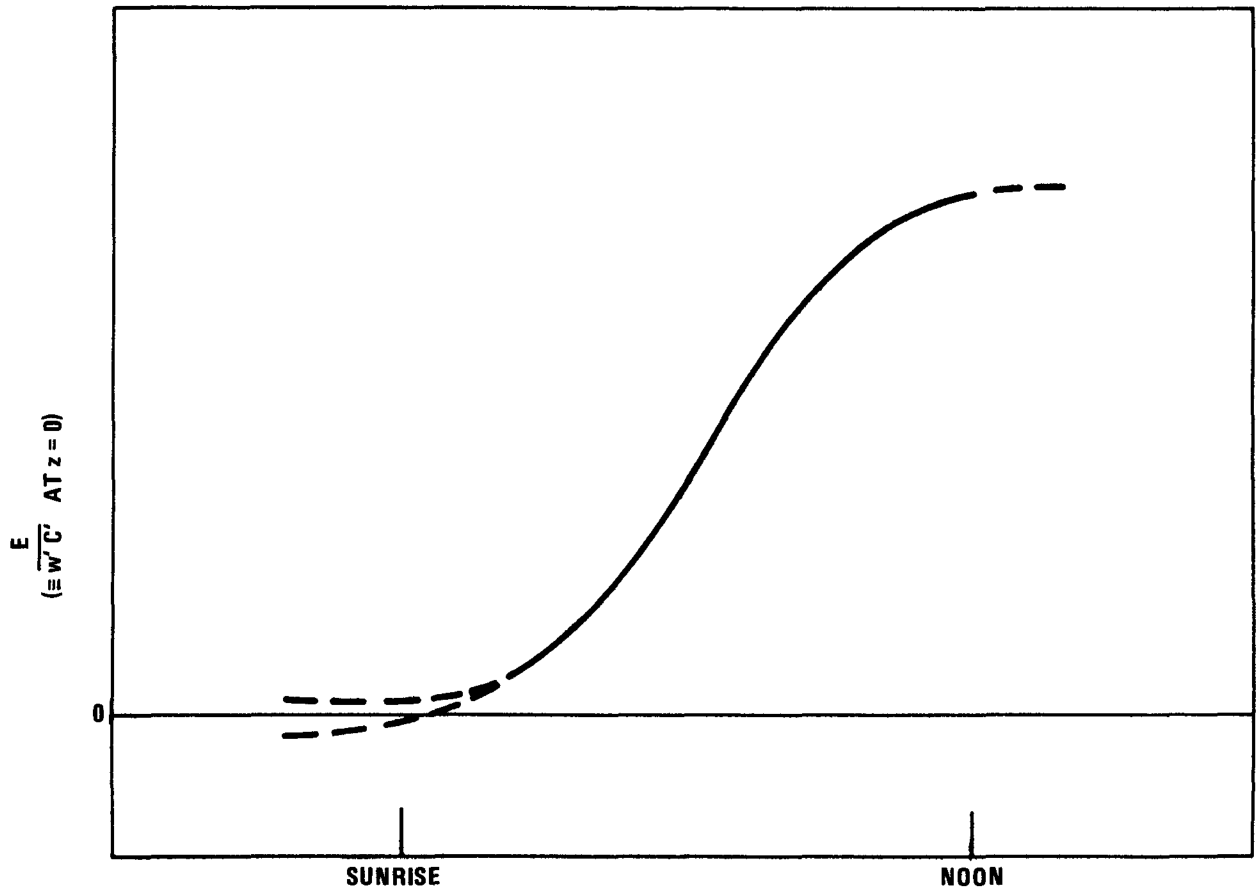

Figure 1.1. Forenoon build-up of natural evaporation.

were used, but the argument may be put in more general terms as follows, writing $\sigma_w^2 = \overline{w'^2}$:

$$\sigma_w^2 \frac{\partial \overline{C}}{\partial z} = \frac{\sigma_w^2}{K} \frac{K\partial \overline{C}}{\partial z} \qquad \text{(K = eddy diffusivity)}$$

$$= \frac{\sigma_w^2}{K} E \qquad \text{for small z} \qquad (1.2)$$

$$\simeq \frac{\sigma_w^2 E}{ku_* z} \qquad \simeq \frac{1}{k} \frac{\sigma_w^2}{u_*^2} \frac{u_* E}{z} \qquad (1.3)$$

The ratio σ_w^2/u_*^2 is known to be about 2, u_* to be about 1 m/sec, and for the present practical context we might take z = 10 m. Then term (3) becomes

$$\simeq \frac{1}{0.4} \frac{2}{10} E \text{ or } 0.5 E$$

At the time of <u>maximum</u> $\frac{\partial E}{\partial t}$, with sunrise to noon approximately 6 hours,

$$\frac{\partial E}{\partial t} \simeq \frac{E}{6 \times 3600} \text{ sec.}$$

so $\frac{\text{term (3)}}{\text{term (2)}} \simeq 0.5 \times 6 \times 3.6 \times 10^3 \simeq 10^4$.

Note that the ratio necessarily decreases with increasing z, but in setting z at 10 m we can scarcely be significantly overestimating the ratio for the practical matter of near-ground-level concentration.

Suppose now that the requirement is to solve the hierarchy of equations for the 1st and 2nd moments in order to derive $\frac{\partial \overline{w'C'}}{\partial t}$, then $\overline{w'C'}$ as a function of z and t, thence $\frac{\partial \overline{C}}{\partial t}$, thence $\overline{C}$ as a function of time. This would correspond to deriving pollutant concentration as a function of distance X over an area source.

From the foregoing numerical estimates it is clear that in any solution of the 2nd moment equation for $\frac{\partial \overline{w'C'}}{\partial t}$ the fractional error imposed by error in term (3) will be 10^4 times the latter (fractional error). At first sight it would appear therefore that the computing of $\frac{\partial \overline{w'C'}}{\partial t}$ with acceptable accuracy is doomed to failure, for modelling of terms on the R.H.S. of the 2nd moment equation with an accuracy better than 1 in 10^4 would seem unthinkable.

It has been pointed out by J. L. Lumley, however (in discussion and private correspondence), that the foregoing argument does not hold, and that the equation may be expected to be 'stable' as regards successive errors in $\frac{\partial \overline{w'C'}}{\partial t}$ provided the terms on the R.H.S. are <u>adequately modelled</u>. So, although the values of $\frac{\partial \overline{w'C'}}{\partial t}$ and of $\overline{w'C'}(t)$ may be wildly in error to start with, they may be expected to settle down to an 'acceptable' level of accuracy. Apparently this 'stability' is a common feature of balance equations of the type considered here, and is dependent on satisfactory modelling of the terms in a <u>physical</u> sense. The original issue then reduces to two further questions:

(a) accepting that it may not in fact be necessary to model the R.H.S. terms with the extreme accuracy indicated above, just <u>how accurately</u> must they be modelled to ensure acceptable accuracy in the term (2)?

(b) in the reiterative type of solution which would be followed, <u>how long</u> is required for the magnitude of $\overline{w'C'}$ to settle down to the adequately accurate value?

The answer to (a) presumably requires a progressive experience in the rational but necessarily 'trial and error' modelling of the various terms. For (b) a reasonable criteria would be some large number, say 10, times the characteristic time-scale of the turbulent mixing. It is not immediately obvious how, precisely, this time-scale is to be specified. Presumably it must be a Lagrangian time-scale t_L, presumably at a height representing the average depth of mixing for the material which has been released, and this height might typically be about 100 m. We might then argue roughly as follows from experience on the scale and spectral properties of the w- component in the atmosphere -- with t_E the 'fixed-point' integral-time scale, $t_L \simeq 4t_E \simeq 4z/\bar{u}$, i.e., approximately 1 minute for z = 100 m and wind speeds of practical interest.

On the foregoing figures the 'settling-down' time may be some tens of minutes, which may be acceptably small in relation to the diurnal change of evaporation. However, for the steady, spatially varying case over an area of dimension X the relevant time over which the source may change substantially is presumably $X/\bar{u}$, and with X say 10 km this time is 1 hour or less. In this case it is <u>not</u> so obvious that the solution for $\overline{w'C'}$ will have settled down during a typical transit time over an area source of pollution of typical size, and perhaps this means that the accuracy required in modelling the terms will be more stringent (in comparison with that for the case of diurnally varying rate of evaporation).

In practice it also appears that modelling of Eq. (1.1) will usually entail introduction of the flux $\overline{w'C'}$ explicitly in one of the larger terms on the R.H.S. (e.g., the procedure followed by Lewellen (1974)). When $\frac{\partial \overline{w'C'}}{\partial t}$ is very small, the solution for $\overline{w'C'}$ will be essentially determined by the modelled large term and the procedure will be tantamount to using a stationary form of the equation (irrespective of whether or not the small $\frac{\partial \overline{w'C'}}{\partial t}$ term is included). It is of course clear that in such circumstances doubts about the feasibility of adequately modelling Eq. (1.1) do not arise to anything like the degree originally envisaged in this note.

2. CROSSWIND DISPERSION AND THE PROPERTIES OF TURBULENCE

The relating of crosswind dispersion from a continuous point source to the flow properties in the atmospheric boundary layer has a fundamental basis in the Taylor (1921) treatment of diffusion by continuous movements. In the spectral form as historically developed, this treatment gives the following result (see Pasquill, 1974, hereafter referred to as Ref. A, p. 125):

$$\sigma_y^{2}(T) = \sigma_v^{2} T^{2} \int_0^{\infty} F_L(n) \frac{\mathrm{Sin}^2 \pi n T}{(\pi n T)^2} \, dn \qquad (2.1)$$

where $\sigma_y(T)$ is the crosswind r.m.s. displacement of particles from their mean position after time of travel T, σ_v is the r.m.s. v-component of the turbulence, and $F_L(n)$ is the normalized Lagrangian spectral density function in terms of frequency n, satisfying the integral relation $\int_0^{\infty} F_L(n)\, dn = 1$.

Eq. (2.1) may be rearranged into a form for a non-dimensional spread S, defined by $\frac{\sigma_y(T)}{\sigma_v T}$, as follows:

$$S^2 = \int_0^{\infty} \frac{F_L(n)}{t_L} \frac{\mathrm{Sin}^2 \pi (n t_L) \frac{T}{t_L}}{[\pi (n t_L) \frac{T}{t_L}]^2} \, d(n t_L) \qquad (2.2)$$

in which we have introduced a dimensionless frequency nt_L (t_L is the Lagrangian integral time-scale) and a dimensionless spectrum function $F_L(n)/t_L$. Note that in accordance with the cosine transform relation between F(n) and the correlation function R the term $F_L(n)/t_L$ is equivalent to $4F_L(n)/F_L(o)$. For a given spectrum function (2.2) is a function of $\frac{T}{t_L}$ only, i.e.,

$$S = f\left(\frac{T}{t_L}\right) \qquad (2.3)$$

which must have the general properties

$$f = 1 \qquad T \to 0$$

$$f = (2t_L/T)^{1/2} \qquad T \to \infty$$

and in which the behaviour at intermediate T is entirely determined by the shape of the spectrum.

For a prescription of f at intermediate T, two approaches have been followed:

(a) use of conjectured mathematical forms of the spectrum function $F_L(n)$ (or in practice the forms of the corresponding Lagrangian auto-correlation function $R(\xi)$, see p. 130 Ref. A)

(b) assumption of a simple form of similarity between Lagrangian and the (observable) fixed-point frequency spectrum (which is essentially Eulerian in form).

In the approach (b) suggested by Hay and Pasquill (see p. 135 Ref. A), the requirement reduces to deriving the variance of the turbulence component, as observed at a fixed point, after first averaging the turbulent fluctuation over periods T/β , where β is the ratio of the Lagrangian and "fixed-point" integral time-scales. In terms of the foregoing quantities

$$f = [\sigma_v]_{\infty,T/\beta} \Big/ [\sigma_v]_{\infty,0} \qquad (2.4)$$

where the subscripts ∞ refer to sampling time (length of record) and T/β or 0 to the averaging (smoothing) times. In practice finite sampling times (τ) may be used, with an error which may be argued to be small as long as $\tau > T$.

Another possible approach, which avoids the commital to particular correlogram form or the particular assumptions of the Hay-Pasquill approach, is

(c) use of observations of the variation of S with T (or with equivalent distance x) over a sufficient range to exhibit the large T limit, to derive t_L and so prescribe empirically the functional form of f.

An approach which is closely related to (c) has recently been adopted by R. Draxler at the Department of Meteorology, P.S.U. A practical difficulty in the application of (c) is that a convincing attainment of the large T form is very rare, and there are good reasons for not generally expecting the limit to be attained in practice. In Draxler's analysis the method was to characterize the time-scale by the time T_i at which S falls to 0.5 and to fit the variation of S with T/T_i to the form

$$f = \frac{1}{1 + a(\frac{T}{T_i})^{0.5}} \tag{2.5}$$

as a simple form consistent with the required limits of f and, of course, requiring

$$T_i/t_L = 2\,a^2 \tag{2.6}$$

Draxler's analysis shows that the data from all the available field studies on dispersion from a continuous point source may be represented by (2.5), albeit with very considerable scatter, and a broad specification of T_i/t_L (hence of t_L) is thereby provided.

In the process of study of Draxler's analysis, certain features have emerged which are outlined below.

<u>The form of R(ξ) implied by Draxler's form for S</u>

The Taylor relation for dispersion may be stated in differential form (see p. 124 Ref. A), and in the present notation and context

$$\frac{d\sigma_y^{\,2}(T)}{dT} = 2\sigma_v^{\,2}\int_0^T R(\xi)d\xi \tag{2.7}$$

or

$$\frac{d^2\sigma_y^{\,2}(T)}{dT^2} = 2\sigma_v^{\,2}\;R(\xi)\;,\quad \xi = T\;, \tag{2.8}$$

so yielding the Lagrangian auto-covariance $[\sigma_v^2 R(\xi)]$ from the 2nd differential of σ_y^2 (T). Eq. (2.8) may be written for Draxler's form (2.5), in terms of $T' = T/t_L$, as follows:

$$\sigma_y^2 = \frac{\sigma_v^2 \; T^2}{[1 + (\frac{T}{2t_L})^{1/2}]^2} \tag{2.9}$$

$$R(\xi) = \frac{1}{2\sigma_v^2} \frac{d^2\sigma_y^2}{d\,T^2} = \frac{1}{2} \frac{d^2}{d\,T^2} \left[\frac{T^2}{[1 + \left(\frac{T}{2t_L}\right)^{1/2}]^2} \right]$$

$$= \frac{1}{2} \frac{d^2}{dT'^2} \left[\frac{T'^2}{[1 + \left(\frac{T'}{2}\right)^{1/2}]^2} \right] \tag{2.10}$$

On differentiating this becomes:

$$R(\xi) = \frac{1}{(1 + T'^{1/2})^2} - \frac{7}{4} \frac{T'^{1/2}}{(1 + T'^{1/2})^3} + \frac{3}{4} \frac{T'}{(1 + T'^{1/2})^4} \tag{2.11}$$

where

$\xi' = \xi/2t_L$, $\quad \xi' = \xi/t_L$, $\quad t_L = \int_0^\infty R(\xi)\, d\xi$.

Values of $R(\xi)$ according to Eq. (2.11) are shown in Table 2.1 and Fig. 2.1, in comparison with $R(\xi)$ of simple exponential form, i.e., $R(\xi) = \exp(-\frac{\xi}{t_L})$. Note that $R(\xi)$ as in Eq. (2.11) initially falls off much more rapidly than the exponential form but is then maintained at finite (though very small) value for a much longer time. It is of interest to consider this difference in relation to an attitude which has previously been advocated -- namely that correlogram shape is of secondary importance as regards the S(T') function.

The S(T') Function for Different Correlogram or Spectrum Shapes

In earlier analyses of the significance of the form of the correlogram (p. 130, Ref. A), a certain range of shapes was found not to affect the S(T') function significantly, and it was accordingly concluded that σ_v and t_L would

Table 2.1. VALUES OF $R(\xi)$

ξ/t_L	Eq.(2.11)	$\exp(-\xi/t_L)$
0.1	0.471	0.905
0.2	0.360	0.819
0.4	0.254	0.670
1.0	0.139	0.368
2.0	0.078	0.135
4.0	0.040	0.018
10.0	0.014	5×10^{-5}

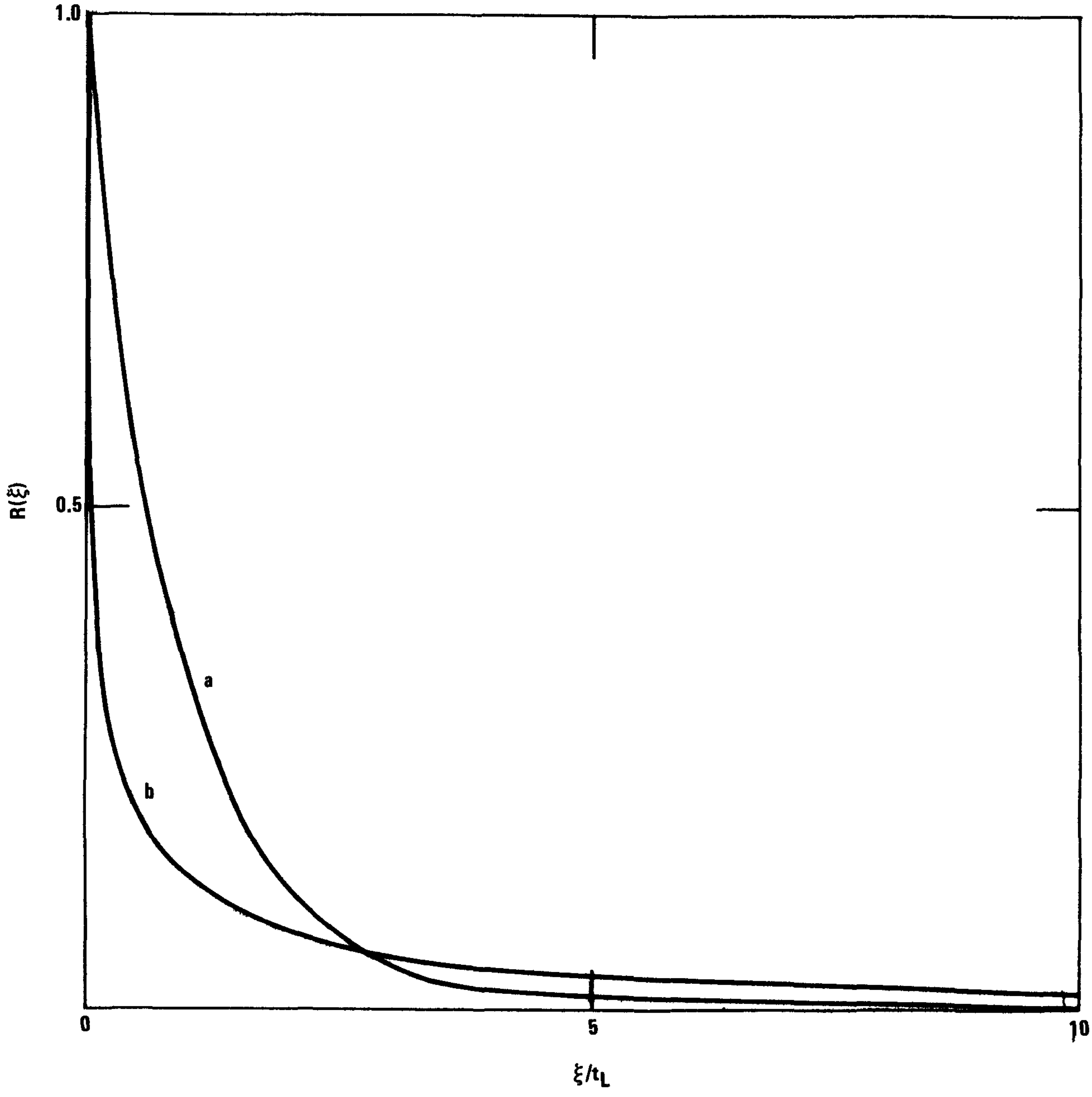

Figure 2.1. R(ξ) according to (a) exponential form and (b) Eq. (2.11).

be sufficient to determine σ_y. (Note that in the earlier analysis referred to dispersion was examined in terms of the dimensionless quantity $D = \sigma_y/\sigma_v t_L$, i.e., ST/t_L). However, among the forms of $R(\xi)$ adopted, the simple exponential form gave the most rapid initial fall. In view of the results in Table 2.1 and Fig. 2.1 we need to reconsider the point regarding the importance of correlogram shape.

Table 2.2 and Fig. 2.2 show values of S (= $\sigma_y/\sigma_v T$) against T' (= T/t_L) for the forms of $R(\xi)$ in Fig. 2.1 and also for a simple form of spectrum which had previously been found to fit certain 'fixed-point' data on the w- component and v- component fluctuations (p. 61, Eq. 2.115 & p. 70, Ref. A). In Lagrangian terms this spectrum is of the form

$$F(n) = \frac{4t_L}{(1 + 6\, t_L n)^{5/3}} \tag{2.12}$$

and its consideration in the present context is implicitly on the assumption of the Hay-Pasquill similarity in spectral shape. Also a single point on the S,T' curve has been calculated (by graphical integration of Eq. (2.1)) for another form of spectrum,

$$F(n) = \frac{4t_L}{(1 + 4t_L n)^2} \tag{2.13}$$

which in a Lagrangian context would be more acceptable than (2.12) in tending to n^{-2} at large n (see p. 89 Ref. A).

In comparison with the result for the simple exponential correlogram, both of the other curves show lower values of S; the maximum discrepancy being near T' = 2, as can be seen from the ratios included in Table 2.2. The single value based on Eq. (2.13) is S = 0.55 at T' = 4, which is virtually indistinguishable from the curve for Eq. (2.12), suggesting that the difference in high-frequency behaviour of these two forms of spectrum is unimportant in the present context.

Table 2.2. CALCULATED VALUES OF $S = \sigma_y/\sigma_v T$

T/t_L	Eq.(2.5) & (2.6)	for $R(\xi) = \exp(-\xi/t_L)$	for $F(n)$ in Eq.(2.12)
0.1	0.817		
0.4	0.691	0.938 (1.36)	0.848 (1.23)
0.8	0.613	--	0.777 (1.27)
1.0	--	0.860	--
1.2	0.564	--	0.724 (1.28)
2.0	0.500	0.755 (1.51)	0.650 (1.30)
4.0	0.414	0.615 (1.49)	0.543 (1.31)
7.0	--	0.495	--
10.0	--	0.424	--
12.0	0.290	0.390 (1.34)	0.368 (1.27)
20.0	--	0.308	--
40.0	--	0.221	--

[figures in parentheses are ratios of S to that for Eq.(2.5-2.6)]

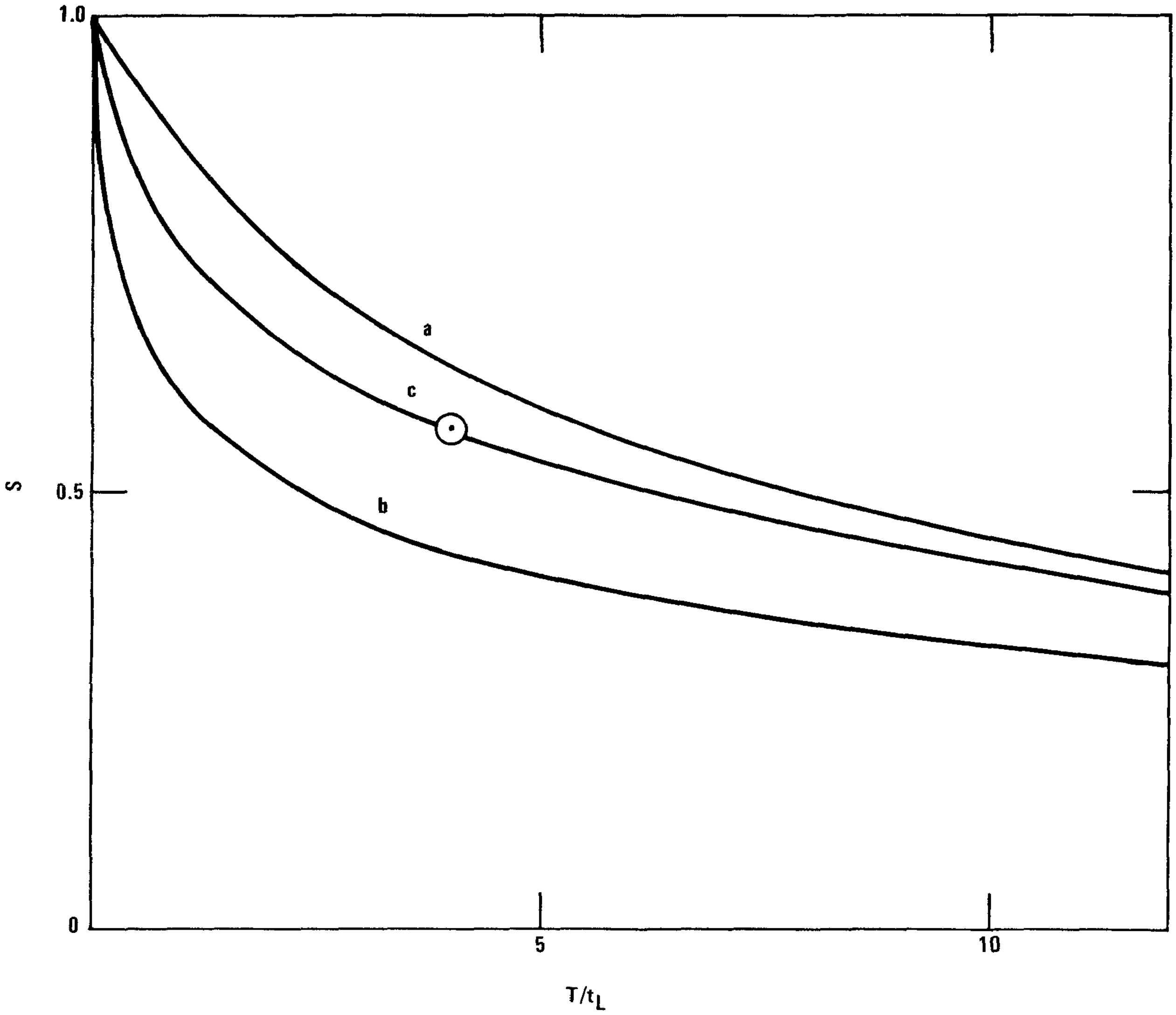

Figure 2.2. Normalized dispersion S as a function of T/t_L for (a) exponential correlogram, (b) Eq. (2.11), (c) Eq. (2.12). The single point 0 corresponds to Eq. (2.13).

The Nature of the Crosswind Dispersion Data

It has already been mentioned that the dispersion data exhibit large scatter, and it may be seen that this becomes important when considering the fit of the data to Eq. (2.9). The form of plotting used by Draxler was perforce in terms of the time T_i, and as noted previously consistency between his interpolation form Eq. (2.5) and Eq. (2.9) requires $T_i/t_L = 2a^2$. The magnitude of a is fairly sensitive to the magnitude adopted for S even at large T since

$$a = \left(\frac{1}{S} - 1\right)\left(\frac{T}{T_i}\right)^{-1/2}$$

The value of a obtained by Draxler, 0.9, is consistent with S = 0.27 at $T/T_i = 9$, whereas a value of S = 0.20 would lead to a = 1.3 and $T_i/t_L = 3.4$ (instead of 1.6). From the plot of data given by Draxler for 'ground releases in unstable conditions', reproduced here as Fig. 2.3, it may be seen that the relative merits of curves passing through the two values 0.27 and 0.20 are perhaps arguable, especially as for $T/T_i > 1$ the curve passing through S = 0.27 at $T/T_i = 9$ seems generally to be on the high side of the highest density of points.

Examination of the S,T Data from the 'Greenglow' and 'Hanford 30' Field Programmes

Data in neutral to moderately stable conditions obtained in the above-named programmes (which were included in Draxler's analysis) have been analyzed to give ensemble average values of S in the form $\sigma_y/\sigma_\theta x$, where σ_θ is the standard deviation of the wind direction fluctuation and $x = \bar{u}T$ has been assumed to apply ($\bar{u}$ was measured at a height of 7 ft.). The results are shown in Fig. 2.4. Drawn on Fig. 2.4 are curves corresponding to those of Fig.2.2, with different scales of T', chosen so as to give approximate fit to the observations. From these results it is clear that one might fit the data almost equally well in terms of Eq. 2.5-2.6 or Eq. 2.12, but with very different values of $\bar{u}t_L$ implied -- roughly 5 km or 1.6 km respectively. The fit provided by an exponential form of correlogram is poor and from the present data it must be concluded that the

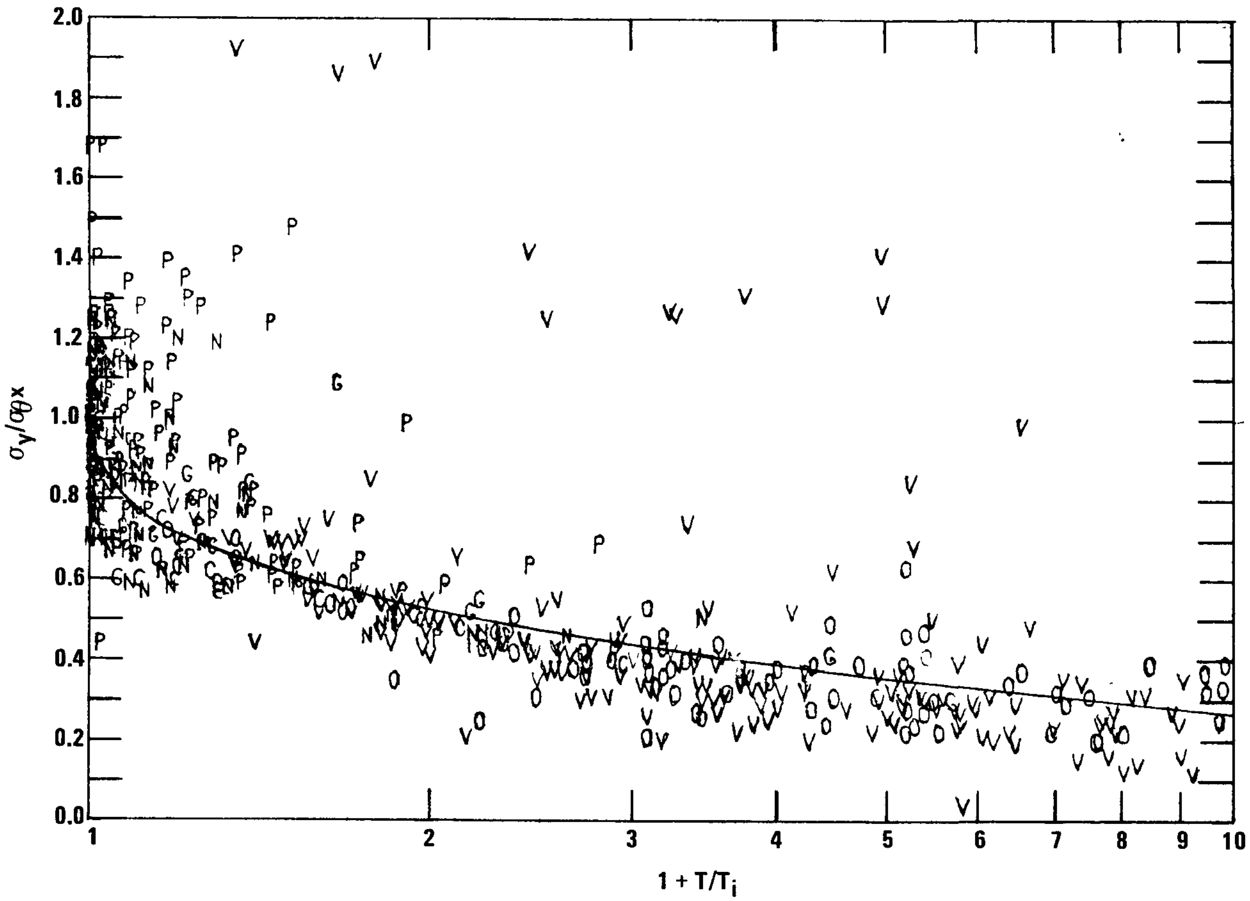

Figure 2.3. Horizontal dispersion-unstable-ground release (Draxler's Analysis).

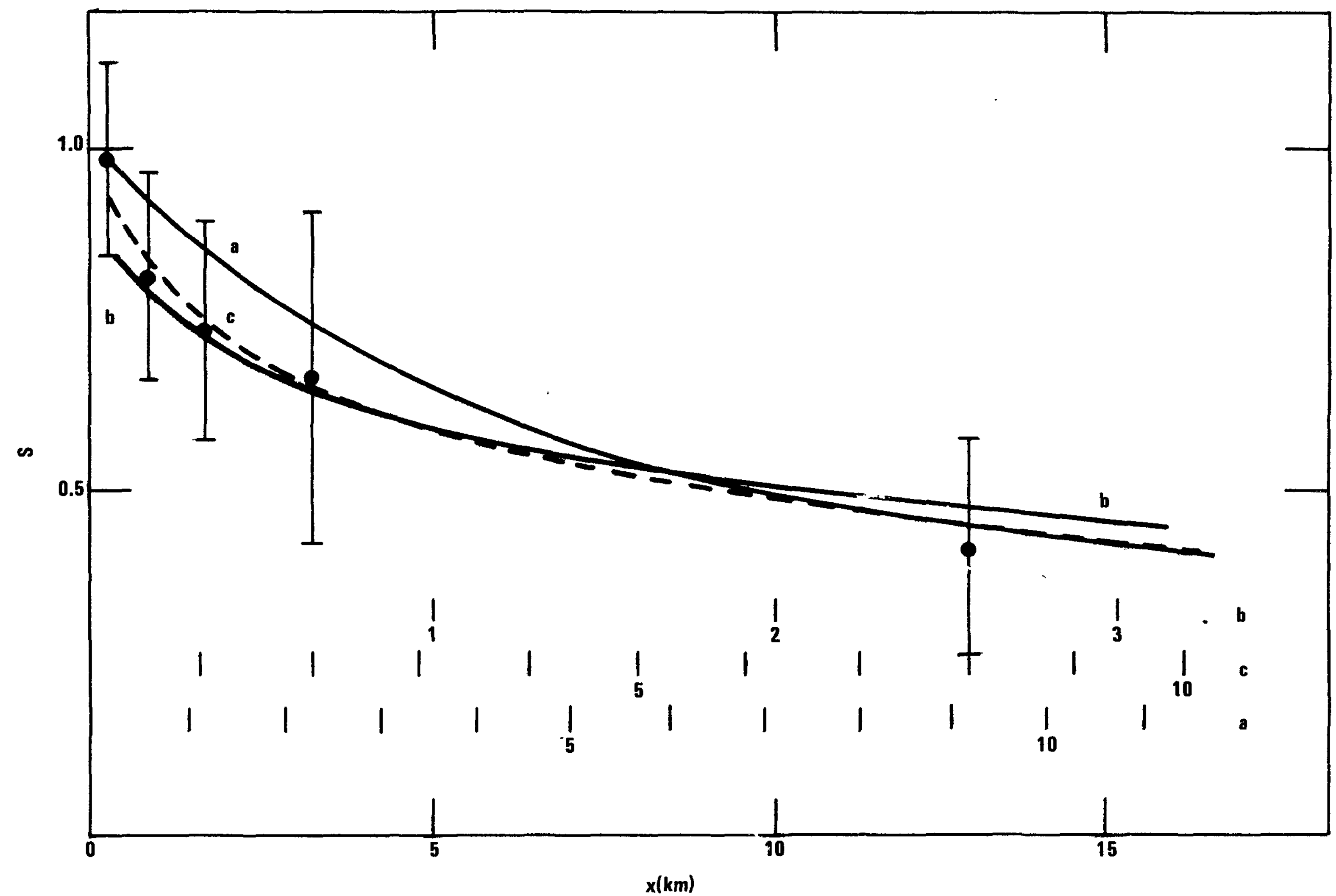

Figure 2.4. Normalized dispersion S against x (as observed in Greenglow and Hanford 30 series, vertical bars are extreme range) compared with calculated curves for T/t_L scales given. References a, b, c as in Figure 2.2. The data are for neutral to moderately stable flow--see Section 5 for further reference.

Lagrangian auto-correlation function is not of this simple form.

In the light of the curve shapes in Figure 2.4 there is no very clear-cut distinction between the interpolation form examined by Draxler and the form appropriate to the spectrum of Eq. (2.12). Further progress will probably depend on obtaining better insight into Lagrangian versus 'fixed-point' spectrum relations, and on making such comparisons as in Figure 2.4 with the added knowledge of the 'fixed-point' spectrum of the v- component (instead of σ_θ alone).

The situation is also complicated in another sense -- namely that in the sheared flow of the atmospheric boundary layer there is expected to be a contribution to σ_y from the interaction between vertical spread and the mean turning of wind with height. It has previously been argued (see p. 29 Ref. A), partly on the basis of the very data in Figure 2.3, that this interaction effect was not evident to an important degree over the range of distances included in Figure 2.3 (i.e., up to 12.8 km), but that it had become important at the extreme distance (25.6 km) included in the field measurements. When one is considering optimum fitting to S- data at large T it is clear from Figure 2.4 that minor uncertainties in the true value of S determined directly by the v- component of turbulence, <u>as distinct from the indirect effects of shear</u>, will sensitively influence the inferred magnitude of $\bar{u}t_L$.

The need to use the relation $x = \bar{u}T$ in interpreting dispersion measurements at a given distance downwind of a source has already been mentioned, and discussion of this point is continued in Section 5.

3. WIND DIRECTION FLUCTUATION STATISTICS OVER LONG SAMPLING TIMES

The application of the relations considered in Section 2 requires a prescription of the standard deviation σ_v of the crosswind component of turbulence or, as commonly used in practice, the standard deviation σ_θ of the wind direction fluctuation ($\sigma_v \simeq \sigma_\theta \bar{u}$). As σ_θ is not a commonly observed quantity there has been much interest in the development of 'climatological' relations for σ_θ in terms of other more easily specified parameters. Unfortunately the properties of the v- component and the customary boundary layer parameters do not show as much order as those of the w- component (see p. 81-84 of Ref. A). An outstanding feature is the relatively indeterminate nature of the low-frequency section of the spectrum, around a frequency of about 1 c/hr, a section which is intermediate between well-defined micrometeorological and macrometeorological sections, and usually termed mesometeorological.

The most obvious appearance of order seems to be in the high-frequency part of the micrometeorological spectrum. This is evident in relatively orderly relations for σ_θ for short sampling times (1 minute or so, see p. 81 of Ref. A). The properties of σ_θ for much longer sampling times (1 hour or more) have received little critical attention, and so it seemed desirable to carry out the pilot investigation described below.

A selection was made of ten 3-hr sections of the May 1973 wind direction records routinely made at a height of 44 m on a tower at Cardington, England. With one exception the sampling periods were near noon, with moderate to strong incoming solar radiation and with a good range of wind speed. Small reproductions of the 1200 Z surface synoptic charts are collected in Fig. 3.1. Average wind directions over consecutive 3-minute periods were extracted and these are displayed graphically in Fig. 3.2. For each 3-hr sample a linear regression of the 3-min average against time was calculated and departures

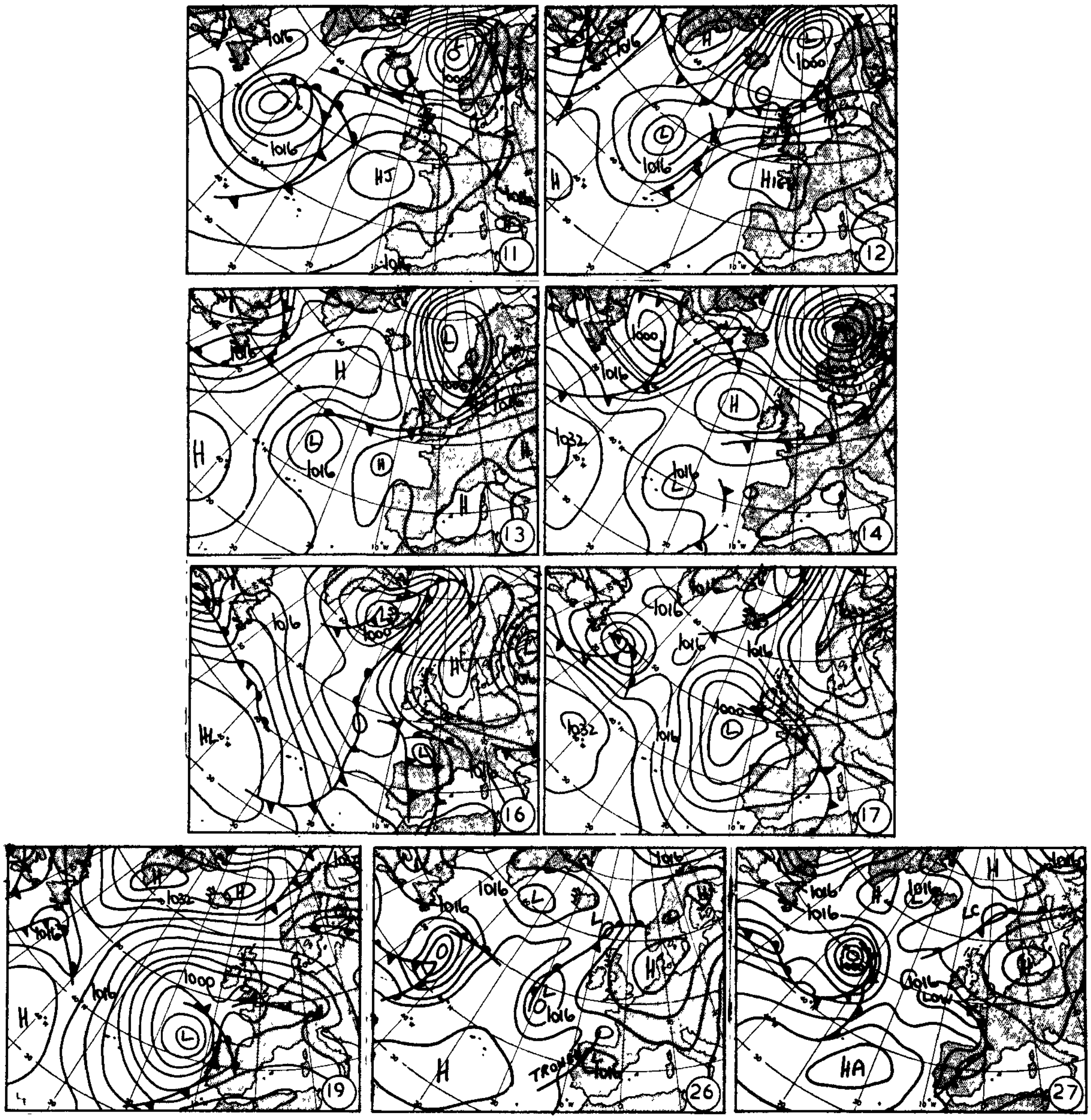

Figure 3.1. 1200 GMT surface charts, May 1973.

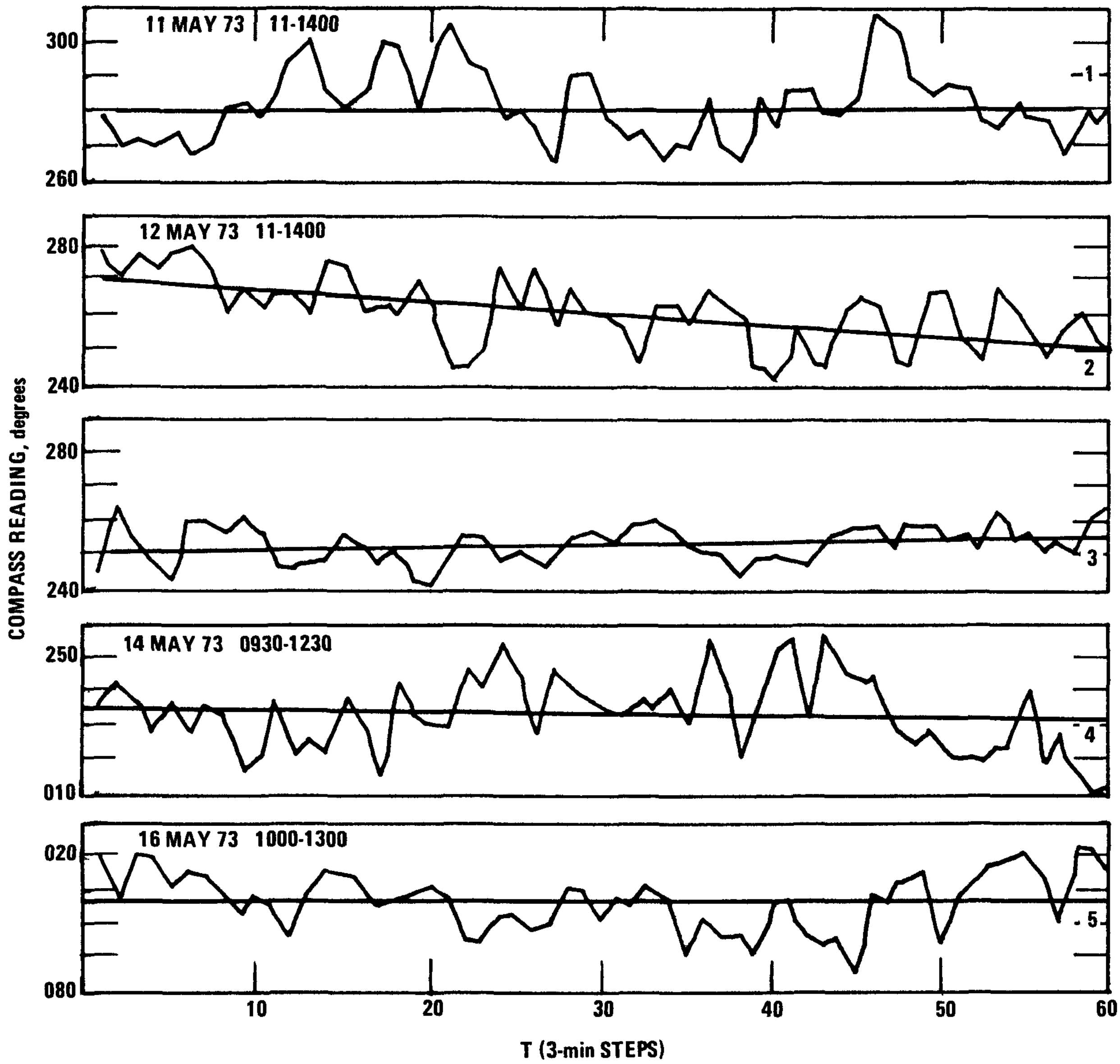

Figure 3.2. Three-minute average wind directions over 3-hr periods.

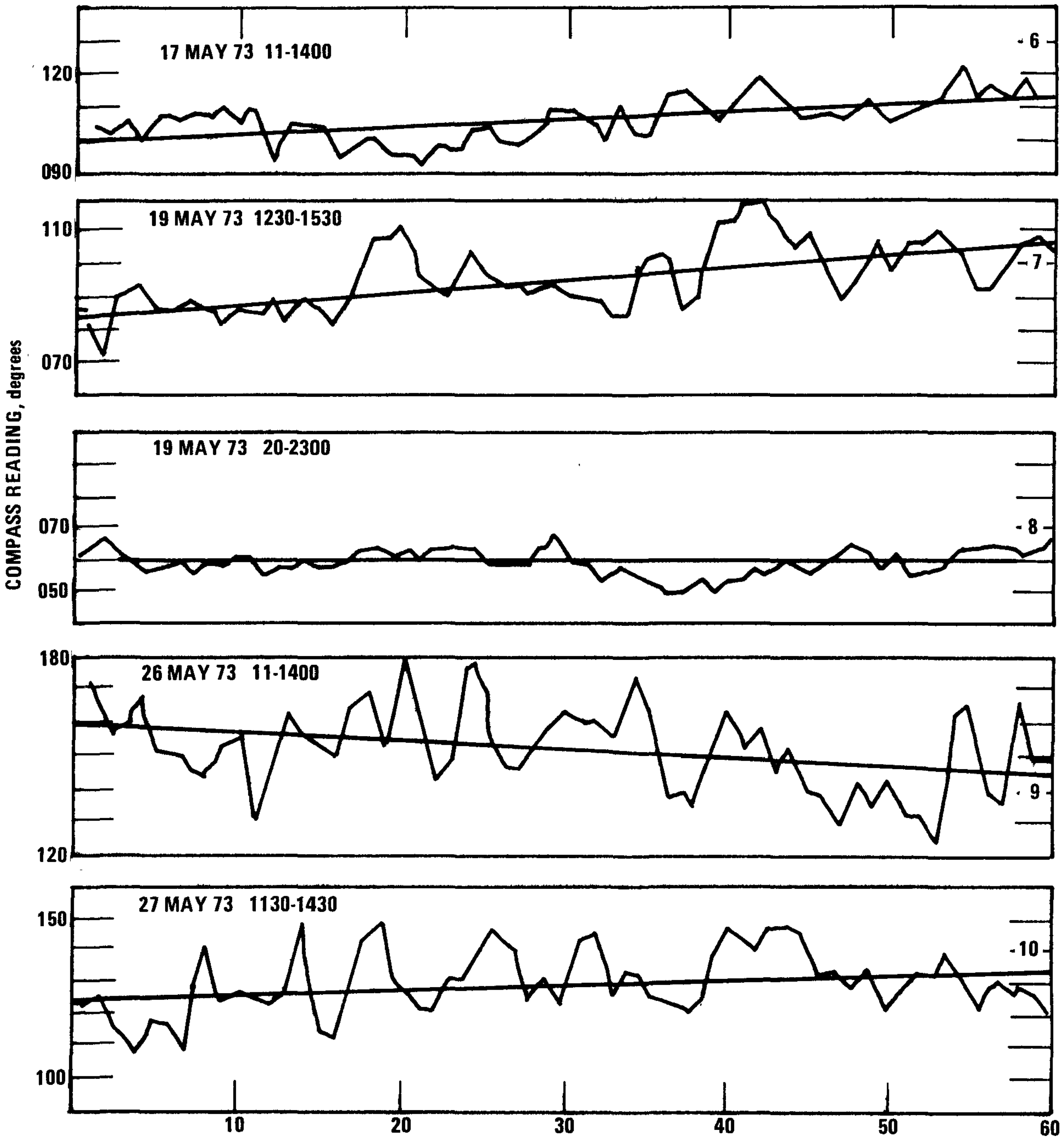

Figure 3.2.(continued). Three-minute average wind directions over 3-hr periods.

from the regression obtained. These 'fluctuations from the linear regression' were used to calculate standard deviations for sub-sampling-times ranging from 1/2 to 3 hours. The results, with background data including estimates of the geostrophic wind, are set out in Table 3.1, and graphs of the standard deviations (σ_θ) as functions of sampling time τ are in Fig. 3.3. In Table 3.1 the standard deviations are presented both in the original angular form (σ_θ) and in the form of σ_v, using the approximate conversion already noted.

The main features of the results are as follows:

1. The values of σ constitute a significant addition to those for a sampling time of 3 min (as for example summarized in pp. 81-83 of Ref. A). The addition would, of course, need to be made in terms of variance.

2. The additional variance represented by the 3-min averages sometimes appears mainly in the first 30 min of the extended sampling times but in others it appears progressively with further increase of sampling time up to 90 min. None of the samples gives any significant increase with increase of sampling time beyond 90 min.

3. With reference to the last point of 2. it should be remembered that a 'linear trend' has been 'extracted' from the basic variation of θ, and some of this trend would presumably appear as variance in much longer samples. Referring to Table 3.1 it will be seen that the 'trend' over 3 hrs may be as much as three times the standard deviation of the 3-min averages. In terms of short-range crosswind spread from a release over three

Table 3.1. SUMMARY OF CASE STUDIES OF WIND DIRECTION FLUCTUATION (STANDARD DEVIATIONS OF 3 MIN AVERAGES) WIND RECORDS AT ACTUAL HEIGHT OF 44m DURING MAY 1973

Case No.	1	2	3	4	5	6	7	8	9	10
Date	11/5	12/5	13/5	14/5	16/5	17/5	19/5	19/5	26/5	27/5
Time	11-1400	11-1400	11-1400	0930-1230	10-1300	11-1400	1230-1530	20-2300	11-1400	1130-1430
$\overline{u}$ m/sec	5.4	7.8	11.9	2.4	8.3	14.2	8.2		4.1	6.8
Rmw/cm^2	74	47	33	52	82	61	33		61	71
G/ $\overline{u}$	1.61	1.76	1.96	1.5	1.71	1.97	1.82		1.57	1.67
G	8.7	13.7	23.3	3.6	14.3	27.9	14.9		6.5	11.4
Mean direction	281	261	253	234	106	106	95	59	152	129

Sampling time τ	$\sigma(\tau)$ in (a) deg (b) m/sec																			
	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
1/2 hr	7.8	0.73	8.0	1.09	4.9	1.02	9.2	0.39	7.2	1.05	4.6	1.14	8.1	1.16	3.1	0.25	12.1	0.87	8.9	1.06
1 hr	10.1	0.96	8.1	1.11	5.4	1.12	11.1	0.47	8.5	1.24	5.1	1.28	8.4	1.21	3.9	0.31	12.3	0.88	9.0	1.07
1-1/2 hr	10.2	0.96	8.4	1.14	5.4	1.12	11.7	0.49	9.3	1.35	5.2	1.30	8.4	1.21	3.8	0.31	12.5	0.90	9.7	1.15
3 hr	10.3	0.97	8.4	1.14	5.4	1.13	11.7	0.50	9.3	1.36	5.3	1.31	8.4	1.22	4.0	0.33	12.5	0.90	9.7	1.15
Trend over 3 hrs (deg)	2		19		5		3		1		15		21		1		16		10	

hours this means that the arc of spread may contain a significant (if not dominant) contribution simply from the systematic swing of the wind over the three hours.

4. No variation with intensity of solar radiation is evident (presumably a consequence of the not very marked range of this property). However, an inverse relation between σ_θ and wind speed is clearly evident in Fig. 3.4. In terms of σ_v there appears to be a quite sharp increase with wind speed up about 7 m/sec, but the two highest wind speeds (near 12 and 14 m/sec) show no further increase in σ_v.

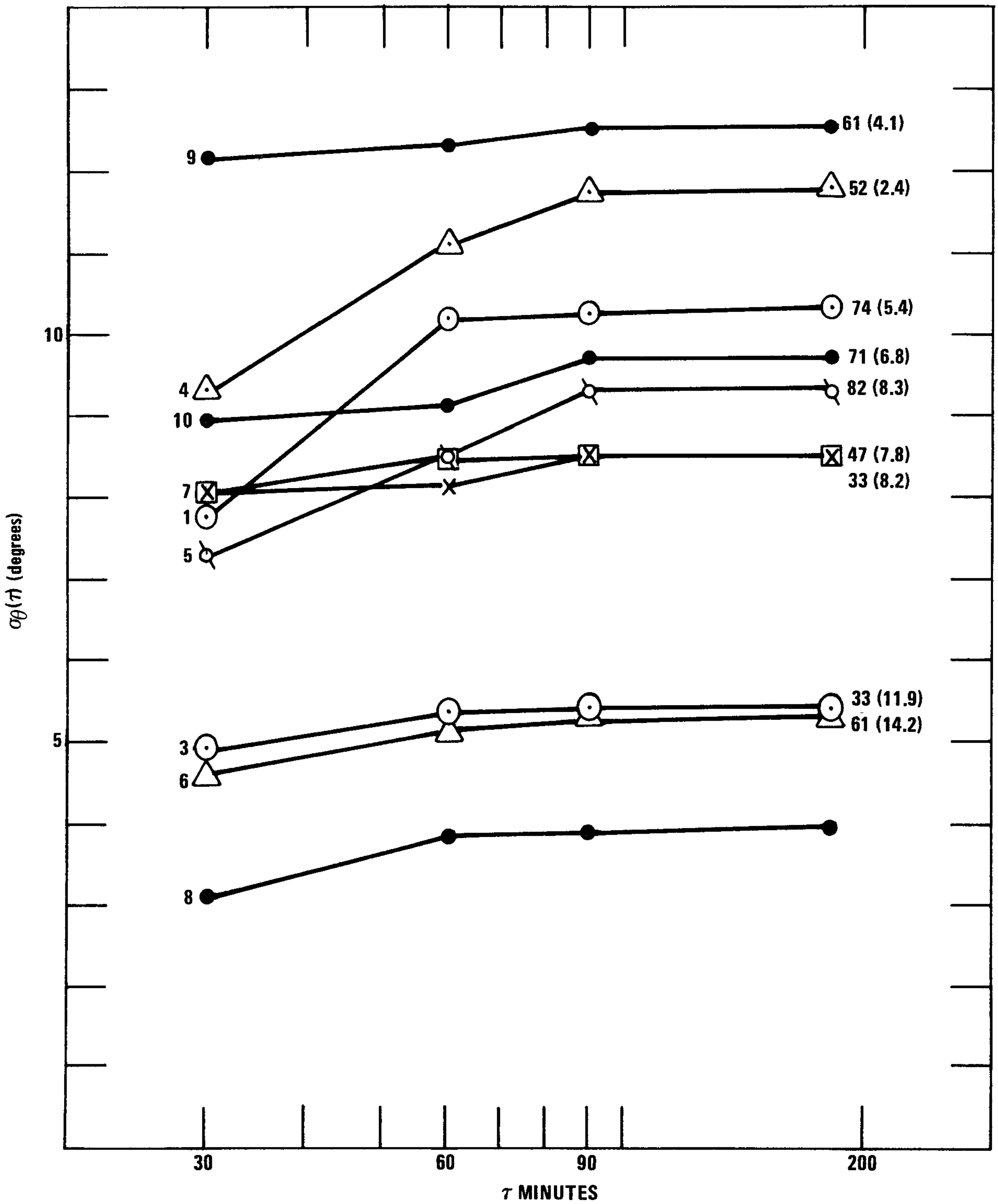

Figure 3.3. Standard deviation $\sigma_\theta(\tau)$ of the 3-min average wind directions (after removal of linear trend) as a function of sampling duration τ. Run numbers on left, wind speed m/sec at 44 m (in parentheses) and incoming solar radiation (mw/cm^2) on right of diagram.

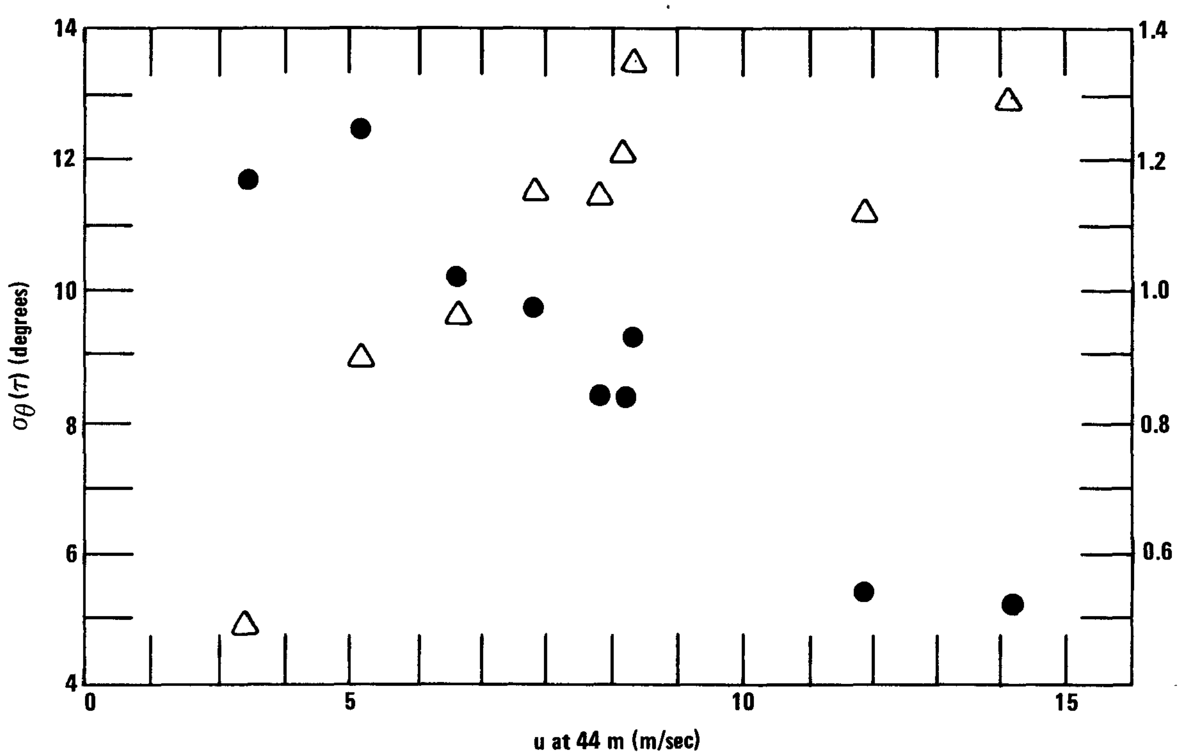

Figure 3.4. $\sigma_\theta(\tau)$ (●) and $\sigma_v(\tau)$ (▵) for sampling time τ = 90 min (excluding night observation No. 8).

4. 'LOCAL SIMILARITY' TREATMENT OF VERTICAL SPREAD FROM A GROUND SOURCE

In the 'so-called' Lagrangian similarity theory of diffusion, the basic principle is to express the rate of dispersion, say $d\overline{Z}/dt$ (where $\overline{Z}$ is the mean displacement of an ensemble of particles after a given travel time), in terms of the basic parameters of the turbulent boundary layer. A brief general review of the history and main developments has been given by the writer (Ref. A). The developments fall into two main sections -- the form of $d\overline{Z}/dt$ and the form for the resulting decay of concentration with distance downwind of a source -- and here the principal concern is with the former aspect.

For a stratified atmosphere the general starting point as adopted by Gifford (1962), following Batchelor's (1959) formulation of the problem for adiabatic flow, is

$$\frac{d\overline{Z}}{dt} = bu_* \, \Phi\left(\frac{\overline{Z}}{L}\right) \tag{4.1}$$

The constant b is now widely accepted as near 0.4 (arguments have indeed been advanced for equality with von Karman's constant), but the form of Φ has not been firmly established, and this is one of the outstanding limitations in the present stage of development of the theory. Another limitation is the restriction of the Batchelor-Gifford treatment to the surface-stress layer and consequently to vertical spreads no more than some tens of meters.

A Local Similarity Hypothesis for Unlimited Vertical Spread

A possible alternative which avoids some of the difficulties and limitations of the original approach is to postulate that the rate of increase of $\overline{Z}$, or alternatively of some measure Z_m of the extreme vertical displacement, is always determined by two local properties

(a) $\sigma_w(z)$

(b) the scale of turbulence $\ell(z)$ prevailing at $\overline{Z}$ or Z_m

as the case may be.

If $\frac{d\overline{Z}}{dt}$ is to be expressed in terms of the velocity and length scales now prescribed, the simplest dimensional hypothesis is

$$\frac{d\overline{Z}}{dt} = \sigma_w f_1 \left(\frac{\ell}{z}\right)_{\overline{Z}} \tag{4.2}$$

$$\frac{dZ_m}{dt} = \sigma_w f_2 \left(\frac{\ell}{z}\right)_{Z_m} \tag{4.3}$$

The use of the actual turbulent velocity σ_w instead of the friction velocity is not new and was indeed advocated by Panofsky (1975) at an early stage but was never followed up extensively. Its use has the obvious physical advantage of directly representing vertical mixing action. The feature which is new, and through which it is hoped to 'derestrict' the treatment as regards the extent of the vertical spread, is the introduction of the turbulent scale relative to the existing magnitude of spread. This has the essentially rational physical implication that the spreading action of a given turbulent velocity (σ_w) will be very different, for example, according as the equivalent length-scale of the turbulent fluctuation is small or similar to the existing magnitude of the spread. Physically the argument is analogous to that for the growth of a cluster of particles (which in size may be similar to or much larger than the eddies which are dispersing it).

Development of the Local Similarity Hypothesis Into a Form for Practical Test

We may rearrange Eq. (4.2) and (4.3) into a more practical form as follows, by taking the further step of assuming ℓ proportional to the spectral scale λ_m (defined by $\bar{u}/n_m$ where n_m is the frequency for peak magnitude of n S(n), and S(n) is the spectral energy density at frequency n). Then Eq. (4.2) and (4.3) may be rewritten

$$\frac{d\bar{Z}}{dt} = \sigma_w f_3 \left(\frac{\lambda_m}{z}\right)_{\bar{Z}} \tag{4.4}$$

$$\frac{dZ_m}{dt} = \sigma_w f_4 \left(\frac{\lambda_m}{z}\right)_{Z_m} \tag{4.5}$$

or writing $\sigma_w/\bar{u} = i$ and $\frac{d\bar{X}}{dt} \simeq \bar{u}(\bar{Z})$

$$\frac{1}{i}\frac{d\bar{Z}}{d\bar{X}} = f_3 \left(\frac{\lambda_m}{z}\right)_{\bar{Z}} \tag{4.6}$$

$$\frac{1}{i}\frac{dZ_m}{d\bar{X}} = f_4 \left(\frac{\lambda_m}{z}\right)_{Z_m} \tag{4.7}$$

(a more precise argument would recognize, following Batchelor (1964), that $\frac{d\bar{X}}{dt} = \bar{u}\ (c\bar{Z})$ etc. but this refinement will be omitted in the present considerations). If now we have specified i and λ_m as functions of height and are given the form of f_3 or f_4 (which would have to be determined empirically), Eq. (4.6) or (4.7) may be integrated numerically to give $\bar{Z}$ or Z_m as a function of $\bar{X}$.

An interesting additional relation follows from consideration of the neutral surface-stress layer, for which Eq. (4.1) may be put in the form (taking b = k)

$$\frac{d\bar{Z}}{d\bar{X}} = \left(\frac{K}{\bar{u}z}\right)_{\bar{Z}}$$

where $K(= ku_* z)$ is the eddy diffusivity. Equations (4.6) and (4.8) are formally equivalent if f_3 is a linear function (for then we have

$$i\lambda_m/z \propto K/\bar{u}z, \quad \text{i.e.} \quad K \propto \sigma_w \lambda_m ,$$

a form which may be argued from statistical theory considerations). This suggests that in general (i.e., irrespective of the extent of vertical mixing and of thermal stratification) we might expect

$$\frac{d\bar{Z}}{d\bar{X}} = f_5 \left(\frac{K}{\bar{u}z}\right)_{\bar{Z}} \tag{4.9}$$

$$\frac{dZ_m}{d\bar{X}} = f_6 \left(\frac{K}{\bar{u}z}\right)_{Z_m} \tag{4.10}$$

and it will be seen that these forms do indeed provide a simple generalization of estimates of vertical spread derived from the two-dimensional diffusion equation.

Preliminary Tests of the Local Similarity Predictions and Implications Regarding the Functional Forms

Observational data on vertical spread from a continuous source at ground-level in relation to the conditions of turbulence are available from the Prairie Grass project in U.S.A. (see p. 206 Ref. A) and from N. Thompson's field studies in stable conditions at short range (1965) and medium range (1966). Unfortunately, in none of these studies are the profiles of λ_m (in particular) immediately available, nor is it certain that such profiles could now be adequately extracted from the data records.

Pending the possible extraction of such profiles or, as may turn out to be necessary, the collection of new data, an interim analysis may be carried out using theoretical estimates of vertical spread derived by F. B. Smith (see Ref. A p. 363) from numerical solutions of the two-dimensional diffusion equation. As these solutions necessarily required specification of the K profiles the resulting data are immediately suitable for examination in terms of Eq. (4.9) and (4.10). Also, values of K used by Smith were derived explicitly from values of λ_m and imply values of σ_w, so with a little rearrangement of the data the examination may be conducted in terms of Eq. (4.6) and (4.7). It is noteworthy that a practical range of thermal stratification is covered by using parameters appropriate to neutral conditions, to strongly unstable conditions as represented by a Monin-Obukhov L of -7 m, and to moderately stable conditions represented by L = 4 m. In order to obtain such small magnitudes of L with realistic turbulent heat fluxes, a relatively light wind was assumed (geostrophic value 4 m sec^{-1}). The numerical details are collected in Table 4.1 and the final results are plotted in Figures 4.1 and 4.2.

From Figures 4.1 and 4.2 we can conclude that 'diffusion equation' values of vertical spread follow a near-universal relationship with 'similarity variables'. The present evidence, it will be noted, covers not only a wide range in stabilities but also a 100-fold range in vertical spread. The results have two immediate consequences:

(a) should we wish to adopt different K profiles from those used in the numerical solutions used here we may compute vertical spread ($\bar{Z}$ or Z_m) much more easily from the

Table 4.1. SIMILARITY ANALYSES OF VERTICAL SPREAD DATA FROM NUMERICAL SOLUTION OF THE 2-DIMENSIONAL DIFFUSION EQUATION (Units in metres and seconds)

L	-7	∞	+4	-7	∞	+4	-7	∞	+4	∞
x		30×10^3			3×10^3			300		300
σ_z	1200	320	63	280	78	17	27	12	3.7	1.6
Z_m	2850	688	135	602	168	36.6	58	25.8	7.9	3.44
$\bar{Z}$	954	254	50	223	62	13.5	21.5	9.5	2.92	1.27
$\frac{dZ_m}{dx} \times 10^3$	45.5	11.0	2.25	165	41.5	7.6	193	73	19.0	100
$\frac{d\bar{Z}}{dx} \times 10^3$	16.8	4.07	0.83	61	15.4	2.82	71	27	7.0	37
Data at Z_m										
K	105	5.4	0.26	103	7.5	2.24	13.5	1.8	0.102	0.25
u	4.0	4.0	4.0	3.95	3.8	3.6	3.6	3.08	2.3	2.18
$10^4 K/uZ_m$	102	19.6	4.8	430	118	18.2	647	226	56	330
λ_m	1000	500	104	982	385	52	210	75	11	10
$10^3 \sigma_w/\bar{u}$	263	27	6.3	264	51	13	179	78	40	113
$10 \frac{\bar{u}}{\sigma_w} \frac{dZ_m}{dx}$	1.73	4.07	3.57	6.25	8.14	5.85	10.8	9.36	4.75	8.81
λ_m/Z_m	0.39	0.73	0.77	1.63	2.29	1.42	3.62	2.91	1.39	2.91
Data at $\bar{Z}$										
K	105	8.15	0.26	74	4.2	0.142	4.4	0.67	0.049	0.092
u	3.98	3.92	3.8	3.88	3.45	2.7	3.28	2.62	1.70	1.71
$10^3 K/u\bar{Z}$	27.7	8.18	1.36	84.9	19.6	3.89	62.8	26.8	9.87	42.3
λ_m	1000	448	65	755	195	23	80	30	4.1	3.7
$10^2 \sigma_w/\bar{u}$	26.4	4.6	1.05	25.1	6.2	2.29	16.9	8.5	7.0	14.5
$10^2 \frac{\bar{u}}{\sigma_w} \frac{d\bar{Z}}{dx}$	6.4	8.8	7.9	24.3	24.7	12.3	42	32	10	25.5
$\lambda_m/\bar{Z}$	1.05	1.76	1.3	3.39	3.15	1.7	3.72	3.16	1.4	2.91

$\bar{u}$, λ_m and K - as used by F.B.S. for Z_0=.03m, V_g=4m/sec, taking $K=\varepsilon^{\frac{1}{3}}\lambda_m^{\frac{4}{3}}/15$

σ_z - as derived by F.B.S. from the numerical solutions.

Z_m *(height of cloud) and* $\bar{Z}$ *(mean height of particles), from* σ_z *assuming Gassian distribution* (i.e. Z_m/σ_z = 2.15, $\bar{Z}/Z_m$ = 0.37).

σ_w - as implied by $K = \frac{1}{10}\sigma_w\lambda_m$, which is consistent with foregoing expression for K in terms of ε.

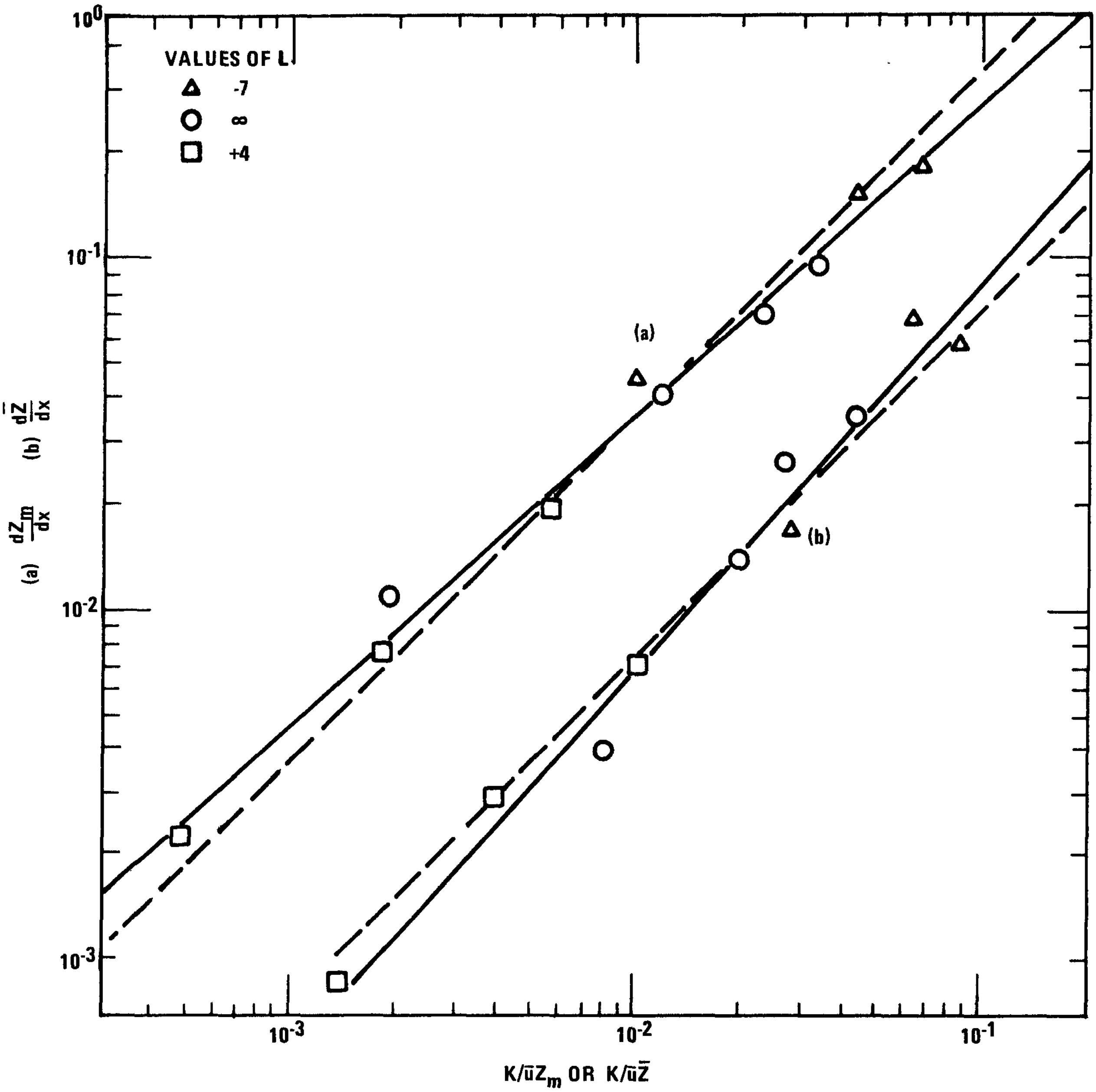

Figure 4.1. Rates of vertical spread dZ_m/dx or $d\bar{Z}/dx$ against $(K/\bar{u}z)$ at Z_m.

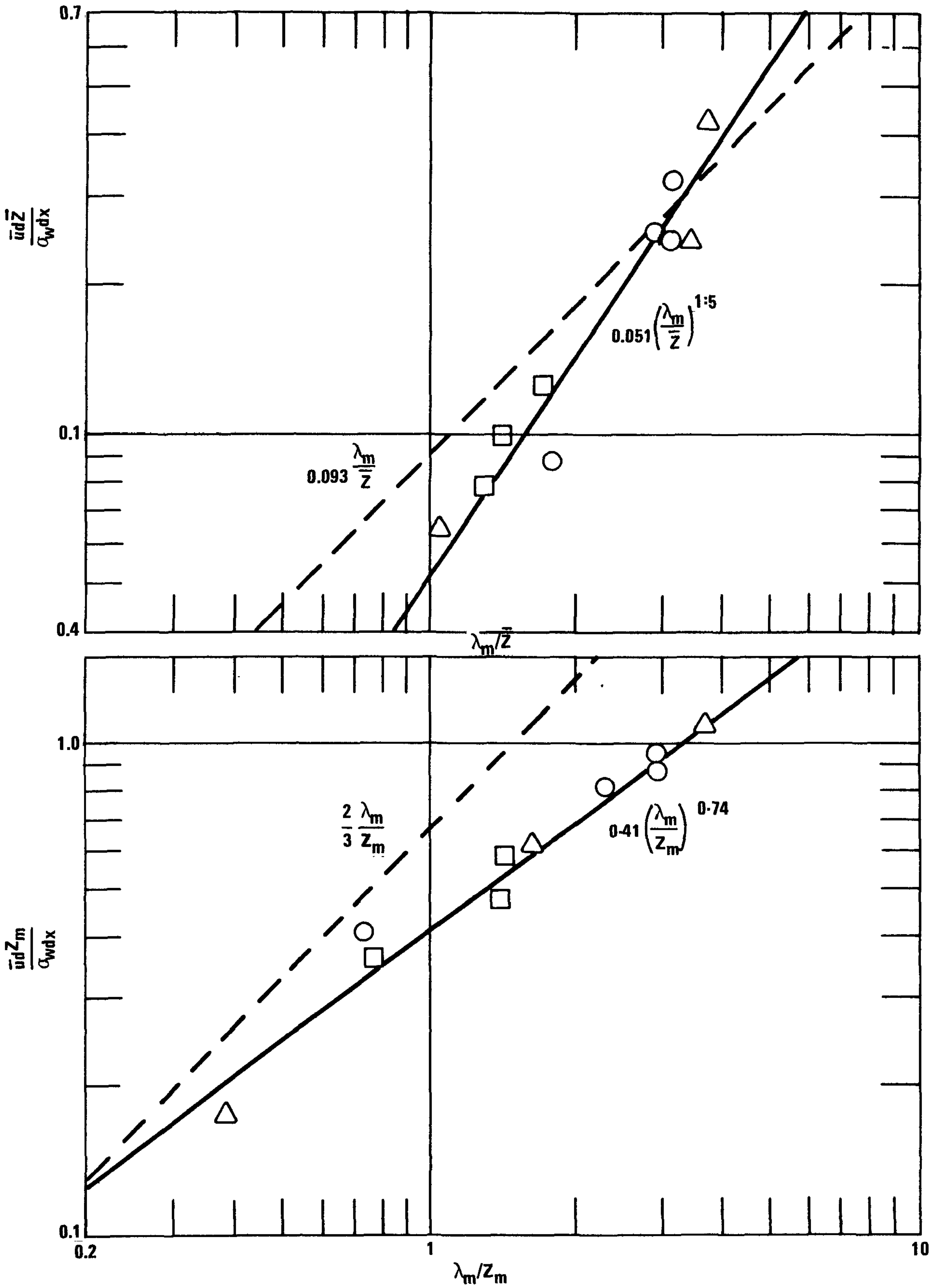

Figure 4.2. $\frac{\bar{u}d\bar{Z}}{\sigma_w dx}$ and $\frac{\bar{u}dZ_m}{\sigma_w dx}$ against (γ/z) at $\bar{Z}$ or Z_m.

similar relations here demonstrated and avoid the labor of further numerical solutions of the diffusion equation.

(b) The result encourages progress to the next step -- namely the relating of actual values of vertical spread to i and $\lambda/\bar{Z}$ or λ/Z_m, so determining the functions more directly and hopefully more generally.

5. REPRESENTATIONS OF DISPERSION IN TERMS OF DISTANCE OR TIME

The Formal Relation Between Time and Distance of Travel

In the description of the concentration field associated with a source of pollutant, we are normally concerned with source and reception positions fixed in space. This naturally leads to a consideration of dispersion parameters (e.g. σ_y or σ_z) as a function of distance of travel, rather than time of travel, even though in theoretical treatments we may well be thinking explicitly in terms of a velocity of dispersion ($d\bar{Z}/dt$ for example in Lagrangian similarity theory) and therefore in terms of dispersion after a given time. However, the latter is not the case in the diffusion equation approach, in which the representation may be entirely spatial (e.g., the conventional steady form of the two-dimensional conservation equation $\bar{u}\,\frac{\partial \bar{C}}{\partial x} = \frac{\partial}{\partial z} K \frac{\partial \bar{C}}{\partial z}$).

For the idealized case of a constant and uniform mean wind it is customary to relate distance (x) and time of travel (T) by writing

$$x = \bar{u}\,T \tag{5.1}$$

The physical significance of (5.1) requires careful consideration however.

In a homogeneous field of turbulence the alongwind (x) velocity at any position is $u = \bar{u} + u'$, with $\bar{u}$ identically the time-mean at the position, or the spatial mean. For an ensemble of passive particles which pass through the selected position in a long regular sequence (an idealized continuous point source), the following statements may be made.

(a) Over a travel time T each particle will have a mean speed $(u_p)_T$, and for T small $(u_p)_T = u$, and for the ensemble of particles the average of these mean speeds will be

$$[(u_p)_T] = [u] = [\bar{u} + u'] = \bar{u}. \tag{5.2}$$

If T is very large each particle will experience the whole spectrum of fluctuations and for each particle $(u_p)_T \approx \bar{u}$. It is accordingly a plausible hypothesis that Eq. (5.2) holds for all T and therefore the mean alongwind displacement $\bar{X}$ of the particles is

$$\bar{X} = [(u_p)_T]\, T = \bar{u}\, T \tag{5.3}$$

(b) Now consider the ensemble of particles as each achieves a constant alongwind displacement x after travelling a variable time s. Then if x (and s) are small

$$(u_p)_s = u$$

$$s = x/u = x/(\bar{u} + u')$$

$$= \frac{x}{\bar{u}} \left(1 - \frac{u'}{\bar{u}}\right) \quad \text{if } \frac{u'}{\bar{u}} \text{ is small}$$

and the ensemble average of s will be

$$[s] = \frac{x}{\bar{u}}$$

and for a long distance x each particle will have $(u_p)_s = \bar{u}$ by the same argument as in (a). Irrespective of x it is also a reasonable hypothesis that

$$[s] = \frac{x}{\bar{u}} \tag{5.4}$$

Thus in writing Eq. (5.1) in relation to dispersion there are two alternative interpretations of T

(c) T(= [s]) is the average time of travel of particles reaching the position x

(d) T is the particular time of travel after which the particles have an average displacement $\bar{X}$.

The question now to be considered is whether the dispersion characteristics (say lateral spread) apparent in these two different considerations (with $x = \bar{X}$ or $T = [s]$) are in any way different. One way of looking at this is to note that in the whole ensemble of particles considered as in (c) there will be a subset (A) which have near-constant time of travel equal to [s](=T). Likewise in (d) there will be a subset (B) with a near constant displacement equal to $\bar{X}(=x)$. These two subsets are by definition the same and will have the same dispersion characteristics. If each subset is an unbiased sample of the whole ensemble as regards (say) lateral spread (and there is no reason to expect otherwise in the conditions defined) then the dispersion properties of the two whole ensembles will also be identical.

The above argument, though not a formal proof, strongly supports the equivalence of dispersion properties in the time and distance coordinates as defined, for the case of <u>homogeneous</u> <u>flow</u>.

In the real case of a boundary layer wind field we may still be able to assume quasi-homogeneity in the horizontal, but $\bar{u}$ is now a function of z. The assumed relation between time and distance of travel may then be as in the Lagrangian similarity hypotheses, i.e.

$$\frac{d\bar{X}}{dt} = \bar{u}\ (c\bar{Z}) \tag{5.5}$$

where $\bar{Z}$ is the mean vertical displacement of the ensemble of particles, <u>each</u> of which has travelled for a given <u>time</u> T. Distance downwind x and mean downwind distance of travel $\bar{X}$ may then be related by making some simple assumptions as follows. Let

$$\bar{Z}(t) = at \tag{5.6}$$

$$\bar{u}(z) = bz^{\alpha} \tag{5.7}$$

Then substituting in Eq. (5.5) and integrating

$$\bar{X}(T) = \int_o^T b(act)^{\alpha}\ dt$$

$$= \frac{(ac)^{\alpha}\ b\ T^{(1+\alpha)}}{(1+\alpha)} \tag{5.8}$$

and on substituting (5.7) for a given reference height z_1 and rearranging,

$$\bar{X}(T) = \frac{(ac)^{\alpha}}{(1+\alpha)z_1^{\alpha}} (\bar{u}(z_1)\ T^{\alpha})\ T \qquad (5.9)$$

Thus we may define an 'equivalent homogeneous' $\bar{u}_e$ for which $\bar{X}(T)$ and $x(= \bar{u}_e[s])$ are identified, such that

$$\bar{u}_e = \frac{1}{(1+\alpha)} \left(\frac{ac}{z_1}\right)^{\alpha} T^{\alpha}\ \bar{u}(z_1) \qquad (5.10)$$

the important point being that $\bar{u}_e$ increases as T^{α}. Alternatively, by rearranging (5.8) and substituting (5.7) it can be seen that

$$\bar{u}_e = \bar{u} \text{ at } z_e = \frac{c\bar{Z}\ (T)}{(1+\alpha)^{1/\alpha}} \qquad (5.11)$$

On these arguments we should examine dispersion data (prescribed as f(x)) in terms of the statistical and similarity theories (which basically prescribe dispersion as f(T)) by using an equivalent advecting speed as in (5.10) or (5.11), always remembering however the special assumption made in (5.6). Suppose, for example, we take $\alpha = 0.15$ (a typical value for the atmospheric boundary layer wind profile), then in terms of Eq. (5.11)

$$z_e = \frac{c\bar{Z}\ (T)}{(1.15)^{1/0.15}}$$

$$= \frac{c}{2.5}\ \bar{Z}(T) \qquad (5.12)$$

and if we take the only estimate that has been made of c, i.e. 0.56 in neutral flow (see p. 119 of Ref. A) this becomes

$$z_e = 0.22\ \bar{Z}(T) \qquad (5.13).$$

The Physical Involvement of $\bar{u}$ in the Effects of Thermal Stratification on Dispersion

Apart from the formal relations considered above, there is complex involvement of wind speed in the effects of thermal stratification on the properties of turbulence. Thus, if we accept that the controlling parameter is z/L or, roughly, the value of the Richardson No. at z, then the grouping of dispersion data in terms of Ri will necessarily impose some grouping in wind speeds (since larger values of $|Ri|$ tend to be associated with small wind speeds). Thus the apparent effect of thermal stratification on say σ_y will tend to be different according as the time or distance representation is adopted. However, provided the time and distance coordinates are related consistently (as above) there is no obvious reason to expect, for example, that the two representations will lead to different specifications of the spatial distribution of pollutant concentration.

An interesting reflection on the foregoing point is provided by the following considerations of the continuous-point-source dispersion data reported by Fuquay, Simpson and Hinds. Fuquay *et al*. make much of the point that their normalized peak exposure (i.e. concentration x wind speed ÷ source strength) shows more variation with 'stability' (as represented by Ri at a given reference level) when considered in relation to time of travel (taken to be $x/\bar{u}$ with $\bar{u}$ at a given reference level) than when considered in relation to distance x downwind of the source. This result is obviously partly a reflection of the inter-relation between Ri and wind speed already mentioned; and it is difficult to see why on these grounds one should immediately infer (as do Fuquay *et al*.) that the dispersion v. time representation is basically preferable to dispersion v. distance. Fuquay *et al*. make no appeal to the preceding argument concerning the wind profile, and their conclusion that σ_y is a simple function of $\sigma_\theta \bar{u} (\simeq \sigma_v)$ and T, rather than of σ_θ and T, is to a large degree a statement of the obvious, following the Taylor approach of Section 2. Apart from the subtleties introduced by the wind profile they could equally logically have asserted that σ_y is a simple function of σ_θ and x, and not of σ_v and x! (i.e. in terms of Section 2, we may write $\sigma_y = \sigma_v \, Tf(T/t_L)$ or $(\sigma_v/\bar{u})$ $\bar{u} \, Tf(\bar{u}T/\bar{u}t_L)$ i.e. $\sigma_\theta x \, f(x/\bar{u}t_L)$).

The only important point would appear to be whether, for example, the testing of either of the foregoing forms in terms of observations at a distance x is obscured by the wind profile aspect. The point may be examined in two ways.

The Magnitude of $\sigma_y/\sigma_v T$ at Very Short Range

The mangitude of $\sigma_y/\sigma_v T$ theoretically tends to unity as T becomes very small. Failure to confirm values of unity in practice may stem from two causes:

(a) The time T is not short enough in relation to t_L -- this will always result in values lower than unity.

(b) The time T will normally be derived as $x/\bar{u}(z_1)$, i.e. using a wind speed at some convenient reference level, whereas the time $x/\bar{u}_e$ should be used. The direction of the effect (on the apparent $\sigma_y/\sigma_v T$ or $\sigma_y \bar{u}(z_e)/\sigma_v x$) will depend on the relative magnitudes of z_1 and the required z_e.

The Fuquay et al. data at x = 200 m in groups characterised by Richardson Number, are plotted against $\bar{u}$ (7 ft.) in Fig. 5.1. An overall average of the 'neutral-slightly stable' group ($0 < Ri < 0.08$) has already been derived as 0.98 (Section 2). Data in the more stable group are generally similar (apart from the isolated point at $\bar{u}$ near 0.9 m/sec), but those for the unstable group are relatively lower at the lower wind

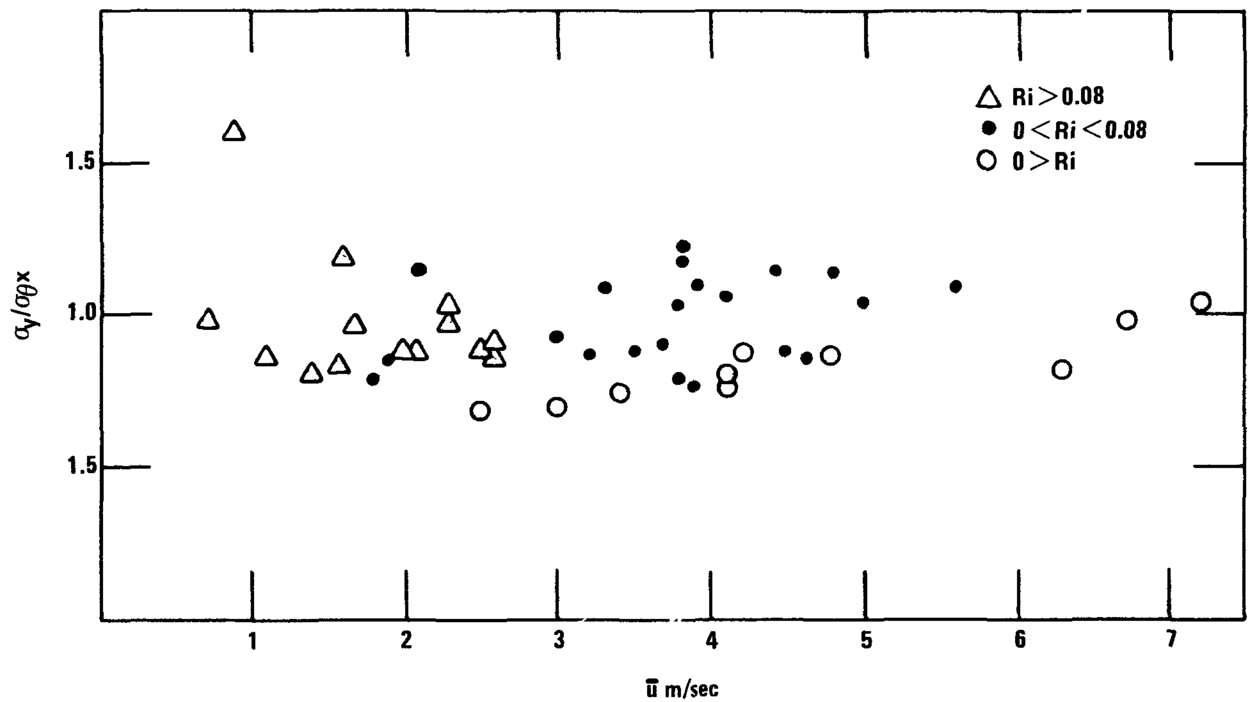

Figure 5.1. Crosswind spread at 200 m as a ratio to $\sigma_\theta x$ and as a function of ū (7 ft) for different values of Ri (data from Fuquay, et al, 1964).

speeds in this group. In order to examine the relevance of item (b) above it is necessary to know the magnitude of vertical spread and in the absence of observed values these may be estimated roughly (from p. 375 et seq of Ref. A), for z_o = 10 cm, x = 200 m and stability categories as stated, and taking $\bar{Z}/\sigma_z \simeq 0.78$

F = 5.5	σ_z = 4 m	or	$\bar{Z}$ = 3 m
E = 4.5	σ_z = 7 m		$\bar{Z}$ = 5.5 m
C = 3.5	σ_z = 10 m		$\bar{Z}$ = 8 m

According to the previous analysis $z_e = 0.22\ \bar{Z}$ and if this relation is adopted irrespective of stability the corresponding values of z_e are 0.7, 1.2 and 1.7 m.

In the Fuquay et al. data z_1 = 7 ft. or approximately 2 m, fortuitously a satisfactory correspondence to z_e in unstable condition and being somewhat high in the stable conditions. This means that $\bar{u}_e$ was probably overestimated in the stable conditions and this could be the explanation of the tendency to marginally higher values of $\sigma_y/\sigma_\theta x$ in the stable conditions. Admittedly, however, the argument cannot be made firm without begging the question as regards the actual magnitudes of T/t_L (i.e. in regard to the significance of their departures from zero) for the different conditions of stability.

The Magnitude of $\sigma_y/\sigma_v T$ at Long Range

At large x, when σ_y should tend to a variation with $x^{1/2}$, and if $z_e > z_1$ (as will usually be the case if z_1 is a customary low reference level), the quantity $\sigma_y\ \bar{u}(z_1)/\sigma_v x$ will tend to be an underestimate of the required $\sigma_y\ \bar{u}_e/\sigma_v x$ (for simplicity σ_v is being assumed independent of height). The time $x/\bar{u}(z_1)$ will correspondingly be an overestimate of the required time $x/\bar{u}_e$, but in view of the tendency to $\sigma_y \propto x^{1/2}$ the net effect will be that a $\sigma_y \bar{u}(z_1)/\sigma_v x$, $x/\bar{u}(z_1)$ observation will be an underestimate (of $\sigma_y \bar{u}_e/\sigma_v x$).

In analysing data such as those in Fig. 2.4 the ultimate interest is in evaluating t_L, which on the preceding arguments would be considered to be best approximated by

$$t_L = \sigma_y^2 \bar{u}_e/2\sigma_v^2 x$$

at large enough x. If $z_e > z_1$ it follows that use of $\bar{u}(z_1)$ will give an underestimate of t_L. As an example the effect may be estimated roughly for the case of Fig. 2.4, for which z_1 was 7 ft. According to the result in Eq. (5.13) z_e should be roughly $\bar{Z}/5$. In the absence of actual data $\bar{Z}$ may be estimated (from p. 375-6 of Ref. A) to be probably no less than say 40 m at x = 12.8 km (take z_o = 10 cm and P = 6). Hence the estimated value of z_e is 8 m and with z_1 near 2 m the ratio $\bar{u}_e/\bar{u}(z_1)$ could be considerable -- e.g. near 2 if α = 0.5 (a not unreasonably high exponent for a stable wind profile). Interpretation of dispersion data to yield estimates of t_L therefore needs to be carried out with careful attention to this aspect.

6. MODELLING FOR ELEVATED SOURCES

The basic treatment of the elevated source is still confronted with certain difficulties, and none of the principal approaches which apparently provide reasonably satisfactory interpretations of the ground-level source is obviously acceptable. In considering these difficulties (which are primarily concerned with vertical spread), it is helpful to consider the downward spread in three stages, in terms of the scale of turbulence ($\ell(z)$) and the height (H) of the source:

Stage 1. $\sigma_z < H$ [and therefore $< \ell(H)$]

Stage 2. $\sigma_z \simeq H$ [therefore $\simeq \ell(H)$]

Stage 3. $\sigma_z \gg H$ [therefore $\gg \ell(\sigma_z)$]

In brief the difficulties may be stated as follows:

(a) The Taylor statistical theory, while plausibly applicable to crosswind spread irrespective of the elevation of the source and to vertical spread in Stage (1), is in principle not applicable to vertical spread in Stages (2) and (3), on account of the systematic change (with height) of the scale of the vertical component.

(b) The gradient-transfer approach, while plausibly applicable to vertical spread from a ground-level source (hence to an elevated source at distances large compared with the distance of the ground-level maximum concentration, i.e. Stage (3)), is in principle not applicable to Stages (1) and (2) of vertical spread, i.e. those which determine the downwind position x_m of the ground-level maximum. This arises from the characteristic behaviour of a continuous source plume when the scale of turbulence is not small compared with the magnitude of the instantaneous plume spread.

With regard to (a), as expected, experience does support the applicability of the statistical theory for Stage (1) (see pp 198-202 Ref. A). However, it is surprising to find that certain experience with a moderately elevated source (see p. 203 of Ref. A.) is compatible with the statistical theory even in Stage (2), when the properties of the w- component at the height of release are employed as if applicable to the whole height range 0 - H. The cases in point were for H = 50 m and were characterised by at least a moderate degree of vertical mixing (i.e. the maximum value of x_m/H was 20), and for such cases it is plausible to consider that the effective scale of turbulence was not very different from that at z = 50 m (bearing in mind firstly that the variation of ℓ with z was unlikely to be much more rapid than $\ell \propto z$, and secondly that the ultimate importance of the smaller scales close to the ground would tend to be offset by scales even larger than that at z = H, such larger scales being effective in the downward-moving 'eddies' originating somewhat above z = H).

In respect of the application of these ideas on vertical spread to the evaluation of the concentration pattern downwind of an elevated continuous point source, it is worth noting that the pre-eminent requirements (additional of course to the source strength and wind speed) are

(c) the height H of the source (which really prescribes the vertical spread associated with the maximum concentration at ground level) and

(d) the crosswind spread σ_y at the distance of maximum concentration (x_m).

The foregoing statement may be formally demonstrated in terms of a simple plume model (see p. 274 Ref. A). Looked at in this way interest is focussed on the relation between <u>vertical spread</u> and $\underline{x_m}$. Also, as a further simplification, it may often be advantageous to think initially in terms of the concentration $\bar{C}(x,z)$ from an elevated continuous line source of infinite extent acrosswind (instead of $\bar{C}(x,y,z)$ from a continuous point source). In this case $\bar{C}(x,z)$ is dependent only on vertical dispersion, but since it is usually reasonable to regard the vertical and crosswind dispersion processes as acting independently on a continuous point source, we may write

$$\bar{C}(x,y,z) = \frac{1}{\sqrt{2\pi}\,\sigma_y} \bar{C}(x,z) \tag{6.1}$$

when the source strengths, respectively Q per unit time and Q per unit time and unit length, are identified. Eq. (6.1) assumes (reasonably) that the time-mean crosswind distribution is Gaussian. With this approach attention may be concentrated exclusively on the more complex vertical dispersion and the relatively simple effect of crosswind spread may be superimposed at a final stage.

Turning to the current state of the practical systems available for estimating the concentration pattern downwind of an elevated continuous point source, the following points are especially noteworthy:

(i) The Meteorological Office 1958 system of estimating σ_z (and σ_y) was basically formulated for a ground-level source, and no attempt was made to allow for the special properties of vertical spread in stages 1 and 2 above. This is the system which was reproduced as 'Pasquill-Gifford curves' and adopted in the Turner 'workbook'. The basic restriction to ground-level source also applies to the modification and extension of the 1958 system recently formulated by F. B. Smith (see p. 373 of Ref. A).

(ii) The ASME 'Recommended Guide for the prediction of the dispersion of airborne effluents' differs in principle from that in (i) in that it uses the direct (but necessarily limited) experience with a passive elevated source at Brookhaven National Laboratory. One feature of this which requires critical consideration is that, in line with the general trend in earlier work on diffusion, σ_y and σ_z are taken to have the same variation with distance x.

(iii) It has long been accepted that the growth with distance is fundamentally different for crosswind and vertical spread from a ground-level source. For the elevated source a systematic difference in the two growths was first directly demonstrated in Hogstrom's study with passive smoke in Sweden (see p. 278 of Ref. A). Moore's (1974) interpretation of the distribution of sulphur dioxide downwind of power stations in the U.K. recognizes such a difference and indeed adopts the simple forms $\sigma_y \propto x$, $\sigma_z \propto x^{1/2}$. Although it is probably not difficult to justify the latter assumption as roughly correct for relatively stable flow, it is open to question for unstable flow when the effective scale of turbulence may increase with height for heights in excess of that reached by hot plumes. (Note also that the T.V.A. analyses of 'hot plume' data which are quoted by F. Gifford in an unpublished draft show differences in the growth curves especially in relatively stable conditions.)

(iv) Moore's study also recognizes an important 'induced' spread of hot plumes in the stage of ascent, arising from the relative vertical motion of plume and ambient air.

Moore's interpretation of U.K. data on pollution distribution downwind of power stations is a commendable attempt to incorporate realistically the effects of both natural and induced spread. However, the natural spread is not related explicitly to the field of turbulence. Although the latter represents a very complex problem it may be that the time is 'ripe' for a further attempt - possibly by adapting statistical theory for stages 1 and 2 and matching to a similarity treatment of Stage 3 on the lines considered in Section 4.

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16. ABSTRACT

This special report discusses six topics all of major current interest in modelling of dispersion in the atmospheric boundary layer. These are the second-order closure modelling of turbulence, crosswind dispersion and the properties of turbulence, wind direction fluctuation statistics over long sampling times, 'local similarity' treatment of vertical spread from a ground source, representations of dispersion in terms of distance or time, and modelling for elevated sources.

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