United States
Environmental Protection
Agency
Research and Development

Robert S. Kerr Environmental
Research Laboratory
Ada OK 74820

EPA/600/S2-91/065 Jan. 1992

EPA

# Project Summary

# The RETC Code for Quantifying the Hydraulic Functions of Unsaturated Soils

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**This summary describes the RETC computer code for analyzing the soil water retention and hydraulic conductivity functions of unsaturated soils. These hydraulic properties are key parameters in any quantitative description of water flow into and through the unsaturated zone of soils. The program uses the parametric models of Brooks-Corey and van Genuchten to represent the soil water retention curve, and the theoretical pore-size distribution models of Mualem and Burdine to predict the unsaturated hydraulic conductivity function from observed soil water retention data. Some of the analytical expressions used for quantifying the soil water retention and hydraulic conductivity functions are presented. A brief review is also given of the nonlinear least-squares parameter optimization method used for estimating the unknown coefficients in the hydraulic models. The program may be used to predict the hydraulic conductivity from observed soil water retention data assuming that an estimate of the saturated hydraulic conductivity is available. The program also permits one to fit analytical functions simultaneously to observed water retention and hydraulic conductivity data, or to predict the hydraulic curves from specified model parameters, if no retention and conductivity data are available. The actual report, summarized here, serves as both a user manual and reference document.**

***This Project Summary was developed by EPA's Robert S. Kerr Environmental Research Laboratory, Ada, OK, to announce key findings of the research project that is fully documented in a separate report of the same title (see Project Report ordering information at back).***

## Introduction

Interest in the unsaturated (vadose) zone has dramatically increased in recent years because of growing evidence and public concern that the quality of the subsurface environment is being adversely affected by industrial, municipal, and agricultural activities. Computer models are now routinely used in research and management to predict the movement of water and chemicals into and through the unsaturated zone of soils. Unfortunately, current technology of developing sophisticated numerical models for water and solute movement in the subsurface seems to be well ahead of our ability to accurately estimate the unsaturated soil hydraulic properties. While a large number of laboratory and field methods have been developed over the years to measure the soil hydraulic functions, most methods are relatively costly and difficult to implement.

Accurate in situ measurement of the unsaturated hydraulic conductivity has remained especially cumbersome and time-consuming. One alternative to direct measurement of the unsaturated hydraulic conductivity is to use theoretical methods which predict the conductivity from more easily measured soil water retention data. Methods of this type are generally based on statistical pore-size distribution models, a large number of which have appeared in the soil science and petroleum engineer-

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ing literature during the past several decades (see *Mualem* [1986] for a review). Implementation of these predictive conductivity models still requires independently measured soil water retention data. Measured input retention data may be given either in tabular form or by means of closed-form analytical expressions which contain parameters that are fitted to the observed data.

Except for simplifying the prediction of the hydraulic conductivity, the use of analytical functions for hydraulic properties in soil water flow studies is attractive for other reasons. For example, analytical functions allow for a more efficient representation and comparison of the hydraulic properties of different soils and soil horizons. They are also more easily used in scaling procedures for characterizing the spatial variability of soil hydraulic properties across the landscape. And, if shown to be physically realistic over a wide range of water contents, analytical expressions provide a method for interpolating or extrapolating to parts of the retention or hydraulic conductivity curves for which little or no data are available. Analytical functions also permit more efficient data handling in unsaturated flow models.

The purpose of this summary is to outline the contents of a research report which documents the RETC (RETention Curve) computer program for describing the hydraulic properties of unsaturated soils. The program may be used to fit several analytical models to observed water retention and/or unsaturated hydraulic conductivity or soil water diffusivity data. The RETC code is a descendent of the SOHYP code previously documented by *van Genuchten* [1978]. New features in RETC include (i) a direct evaluation of the hydraulic functions when the model parameters are known, (ii) a more flexible choice of hydraulic parameters to be included in the parameter optimization process, (iii) the possibility of evaluating the model parameters from observed conductivity data rather than only from retention data, or simultaneously from measured retention and hydraulic conductivity data, and (iv) user friendly program preparation.

## Parametric Models for the Soil Hydraulic Functions

Water flow in variably-saturated soils is traditionally described with the Richards equation:

$$C\frac{\partial h}{\partial t}=\frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}-K\right) \qquad (1)$$

where $h$ is the soil water pressure head (with dimension L), $t$ is time (T), $z$ is soil depth (L), $K$ is the hydraulic conductivity ($LT^{-1}$), and $C$ is the soil water capacity ($L^{-1}$) approximated by the slope ($d\theta/dh$) of the soil water retention curve, $\theta(h)$, in which $\theta$ is the volumetric water content ($L^3L^{-3}$). The solution of the Richards equation requires knowledge of the unsaturated soil hydraulic functions $\theta(h)$ and $K(h)$ or $K(\theta)$. This paper discusses the parametric models for $\theta(h)$ and $K(\theta)$ used in the RETC code.

### Soil Water Retention Models

One of the most popular functions for describing $\theta(h)$ has been the equation of *Brooks and Corey* [1964], further referred to as the BC-equation:

$$S_e=\frac{\theta-\theta_r}{\theta_s-\theta_r}=\begin{cases}(\alpha h)^{-\lambda} & (\alpha h>1)\\ 1 & (\alpha h\leq 1)\end{cases} \qquad (2)$$

where $S_e$ is the effective degree of saturation, also called the reduced water content ($0 \leq S_e \leq 1$), $\theta_r$ and $\theta_s$ are the residual and saturated water contents, respectively; $\alpha$ is an empirical parameter ($L^{-1}$) whose inverse is often referred to as the air entry value or bubbling pressure, and $\lambda$ is a pore-size distribution parameter affecting the slope of the retention function. For notational convenience, $h$ and $\alpha$ are taken positive for unsaturated soils (i.e., $h$ denotes suction). Following *van Genuchten and Nielsen* [1985], $\theta_r$ and $\theta_s$ in this study are viewed as being essentially empirical parameters in a soil water retention function. Because of its simple form, (2) has been used in numerous unsaturated flow studies. The BC-equation has been shown to produce fairly accurate results for many coarse-textured soils characterized by relatively narrow pore- or particle-size distributions (large $\lambda$-values). Results have generally been less accurate for fine-textured and undisturbed, structured field soils because of the absence of a well-defined air-entry value for these soils.

Several continuously differentiable (smooth) equations have been proposed to improve the description of soil water retention near saturation. Most of these functions are mathematically too complicated to be easily incorporated into predictive pore-size distribution models for the hydraulic conductivity, or lack a simple inverse relationship which makes them less attractive for many soil water studies. A related smooth function with attractive properties is the equation of *van Genuchten* [1980], further referred to as the VG-equation:

$$S_e=[1+(\alpha h)^n]^{-m} \qquad (3)$$

where $\alpha$, $n$, and $m$ are empirical constants affecting the shape of the retention curve.

Figures 1 and 2 show calculated retention curves based on (3) for various values of $m$ and $n$. Plots are given in terms of the reduced pressure head, $\alpha h$. The curves in Figure 1 are for two values of $m$, whereas in Figure 2 the product $mn$ was kept constant at an arbitrary value of 0.4. This last feature causes all curves to approach a limiting curve at low values of $S_e$. The limiting curve follows from (3) by removing the factor 1 from the denominator. This shows that the VG- and BC-functions become equivalent at low $S_e$ when $\lambda = mn$. The same limiting curve also appears when $n$ in (3) is allowed to go to infinity, while simultaneously decreasing $m$ such that the product, $mn$, remains at 0.4. As shown in Figure 2, the limiting BC-equation exhibits a sharp break in the curve at the air entry value $h_a=1/\alpha$. For finite values of $n$ the curves remain smooth and more or less sigmoidally-shaped on a semilogarithmic plot (Figure 2a). Note, however, that the curves become markedly nonsigmoidal on the regular $\theta(h)$ plot (Figure 2b), especially when $n$ is relatively small.

Figure 2 also demonstrates the effect of imposing various restrictions on the permissible values of $m$ and $n$. Again, when $n \rightarrow \infty$ (while keeping the product $mn$ constant), the limiting curve of Brooks and Corey with a well-defined air entry value appears (Figure 2a,b). When $m=1-1/n$, as suggested by *van Genuchten* [1980] for the Mualem-based conductivity prediction, and keeping $mn$ at 0.4, the retention function is given by the curve designated $n = 1.4$ in Figure 2. Similarly, when $m=1-2/n$ for the Burdine-based conductivity equation, the retention function is given by the curve $n=2.4$ in Figure 2. The variable $m,n$ case leads to mathematical expressions for $K$ which may be too complicated for routine use in soil water flow studies. Imposing one of the three restrictions will, for a given value of $mn$, fix the shape of the retention curve at the wet end when $S_e$ approaches unity.

The results of fitting (3) to retention data of four different soils are shown in Table 1. The examples were previously discussed by *van Genuchten and Nielsen* [1985]. The table contains fitted parameter values and the calculated sum of squares, SSQ, of the fitted versus observed water contents. The SSQ values reflect the relative accuracy of the retention models in describing the observed data. For Weld silty clay loam, the BC-equation ($n \rightarrow \infty$) matches the data equally well as the variable $m,n$ case, whereas the VG-curves associated with the restrictions $m=1-1/n$ and $m=1-2/n$ produced relatively poor results. This situation is different for Touchet

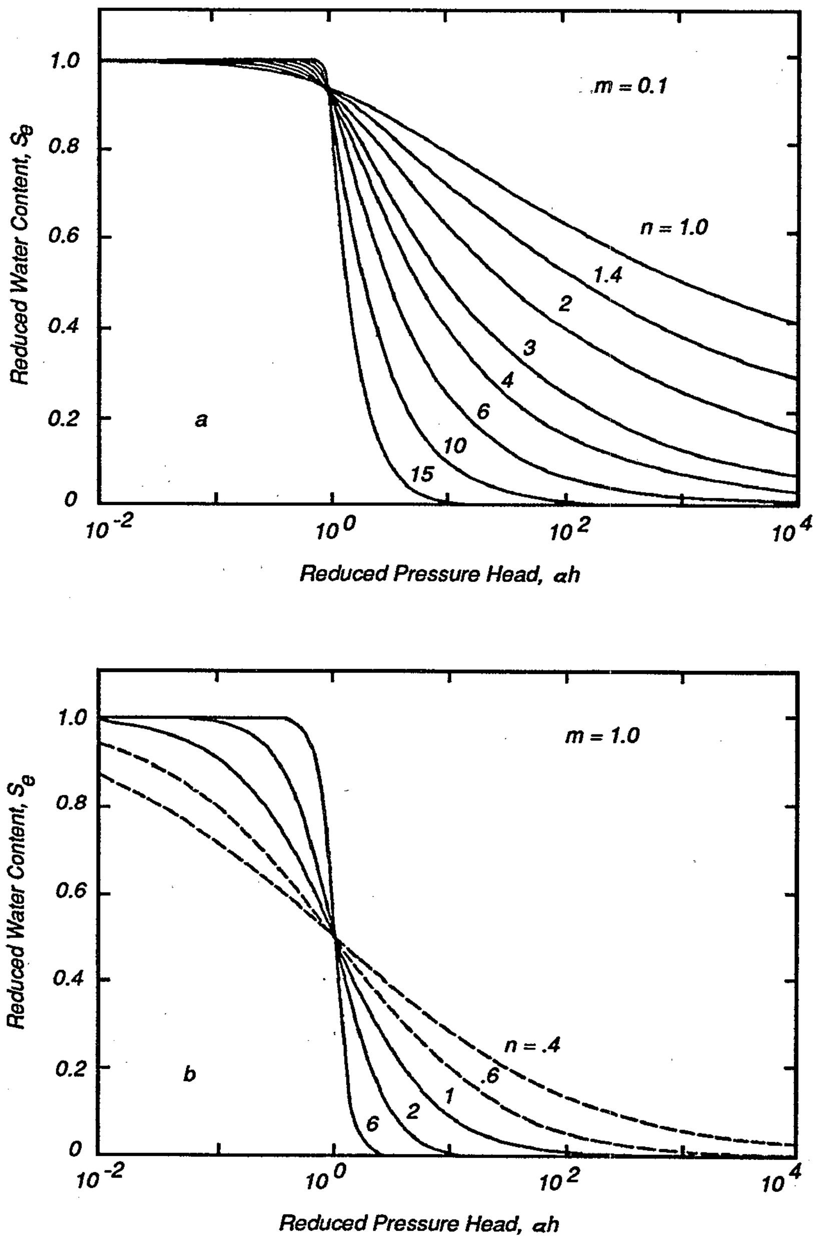



Figure 1. *Soil water retention curves based on (3) for various values of* n *assuming* m=0.1 *(a) and* m=1.0 *(b).*

silt loam and G.E. No. 2 sand where the BC-equation produces an unacceptable fit while the VG-equation with restricted *m,n* values produces results which are essentially identical to those for the general case when *m* and *n* are independent. Finally, there is a progressively better fit to the Sarpy loam data going from the BC limiting curve via the restricted cases *m*=1-2/*n* and *m*=1-1/*n*, to the more general case of variable *m,n*.

From these results, and many other examples not further discussed here, we conclude that (3) with variable *m,n* gives an excellent fit to observed retention data for most soils [*van Genuchten et al.*, 1991]. The only exceptions are certain structured or aggregated soils characterized by very distinct bimodal pore-size distributions. Of the three cases with restricted *m* values, *m*=1-1/*n* seems to perform best for many but not all soils, while the BC-equation generally performs best for selected coarse-textured and/or repacked, sieved soils with relatively narrow pore-size distributions. Although the variable *m,n* case produced always superior results, its use is not necessarily recommended when only a limited range of retention data (usually in the wet range) is available. Unless augmented with laboratory measurements at relatively low $S_e$ values, such data sets may not lead to accurate and description of the retention curve in the dry range. Keeping both *m* and *n* variable may then lead to uniqueness problems in the parameter estimation process.

## Mualem's Hydraulic Conductivity Model

The model of *Mualem* [1976] for predicting the relative hydraulic conductivity, *K*, can be written as

$$K(S_e) = K_s S_e^{\ell}\left[\frac{f(S_e)}{f(1)}\right]^2 \qquad (4)$$

with

$$f(S_e) = \int_0^{S_e} \frac{1}{h(x)}\,dx \qquad (5)$$

where $K_s$ is the hydraulic conductivity at saturation, and $\ell$ is a pore-connectivity parameter estimated by *Mualem* [1976] to be about 0.5 as an average for many soils. Substituting the inverse of (3) into (5) and integration leads to the following expression for *K*:

$$K(s_e) = K_s S_e^{\ell}[I_\zeta(p,q)]^2 \qquad (6)$$

where $I_\zeta(p,q)$ is the Incomplete Beta function and

$$p = m + 1/n, \quad q = 1 - 1/n, \quad \text{and } \zeta = S_e^{1/m} \qquad (7)$$

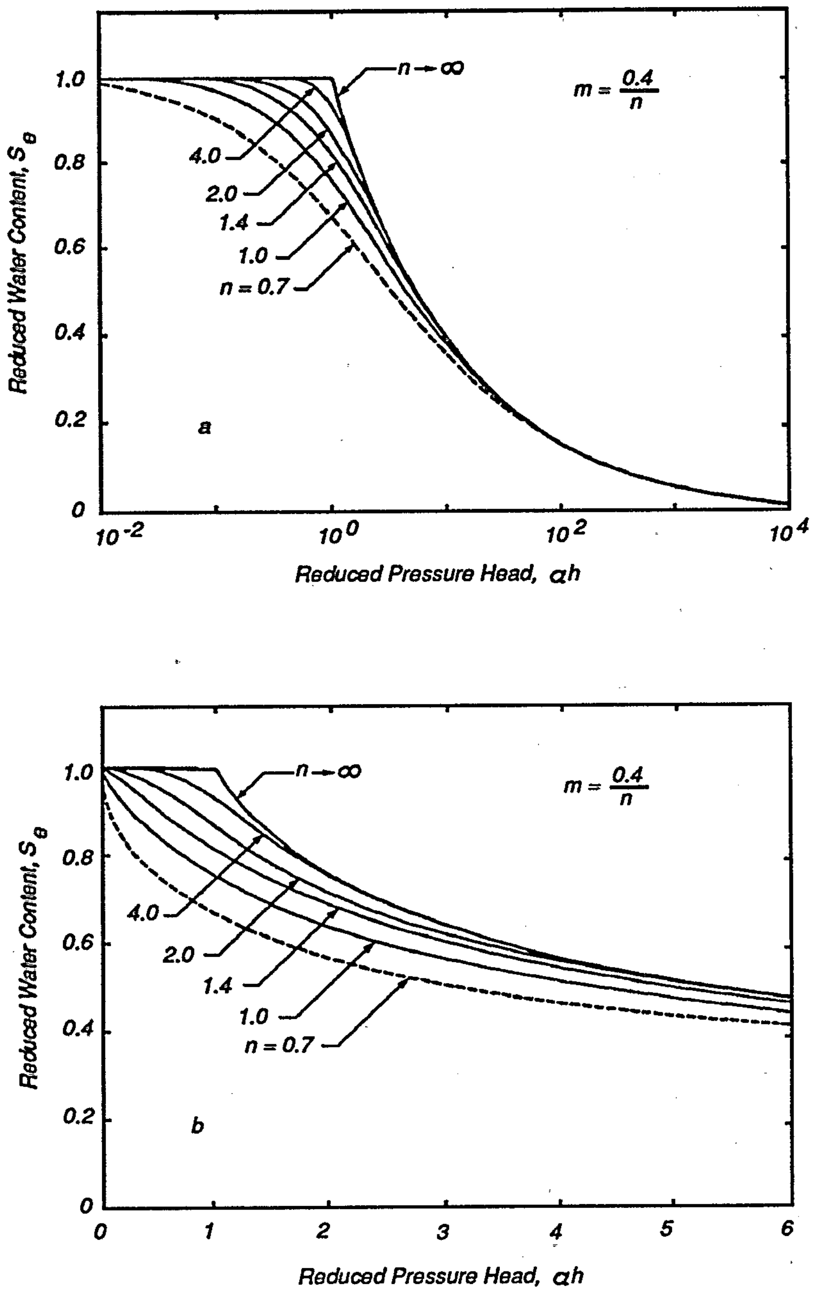



*Figure 2.* *Semilogarithmic (a) and regular (b) plots of soil water retention curves based on (3) with mn=0.4.*

Equation (6) holds for the general case of independent $m$ and $n$ in (3). Simpler expressions for $K$ may be obtained when the permissible values for $m$ and $n$ are restricted such that $k=m-1+1/n$ becomes an integer [*van Genuchten*, 1980]. The simplest case arises when $k=0$, which leads to the restriction $m=1-1/n$. Equation (6) reduces then to the form

$$K(S_e) = K_s S_e^{\ell}[1 - (1 - S_e^{1/m})^m]^2 \qquad (8)$$

When the BC retention function, Eq. (2), is substituted into (5), the hydraulic conductivity function according to Mualem becomes

$$K(S_e) = K_s S_e^{\,\ell+2+2/\lambda} \qquad (9)$$

Figure 3 shows calculated curves for the relative hydraulic conductivity, $K_r=K/K_s$, as a function of the reduced pressure head, $\alpha h$, and the reduced water content, $S_e$, according to (6) for variable $m,n$ with the product $mn$ fixed at 0.4. The conductivity curves in Figure 3a remain smooth, except for the limiting case when $n \to \infty$. Figures 3a and 3b show that the hydraulic conductivity decreases when $n$ becomes smaller, and that $K_r$ becomes identical to zero when $n=1$. No solution for the predicted hydraulic conductivity function exists when $n<1$. To avoid this complication, it is helpful to invoke the restriction $m=1-1/n$. Figure 4 shows a general dimensionless plot of $K_r(S_e)$ when the restriction $m=1-1/n$ is implemented. The curves in this figure are based on (8) for selected values of $m$, using a value of 0.5 for the pore-connectivity parameter $\ell$ as suggested by *Mualem* [1976].

The predictive equations for $K$ used thus far assume that $K_s$ is a well-defined and easily measured soil hydraulic parameter. This assumption is probably correct for many repacked, coarse-textured and other soils characterized by relatively narrow pore-size distributions. However, direct field measurement of $K_s$ is generally very difficult for undisturbed and especially structured field soils. Also note that the hydraulic conductivity near saturation is determined primarily by soil structural properties which are known to be subject to considerable spatial variability in the field. This is in contrast to soil textural properties which generally are less variable and have a more dominant effect on $K$ in the dry range. The rapid decrease of the predicted hydraulic conductivity near saturation when $n$ is relatively small (cf. Figure 3) is intuitively realistic. It suggests that $K$ near saturation is determined by only a very few large macropores or cracks which may

*Table 1. Fitted soil hydraulic parameters for the retention curves [after* van Genuchten and Nielsen, *1985]*

| *Type of curve* | $\theta_r$ ($cm^3/cm^3$) | $\theta_s$ ($cm^3/cm^3$) | $\alpha$ (1/cm) | *n* (-) | $\lambda$, $m^{\dagger}$ (-) | *SSQ* ($10^{-5}$) |
|---|---|---|---|---|---|---|
| | | | Weld silty clay loam | | | |
| *variable* m, n | 0.116 | 0.469 | 0.0173 | 61.54 | m=0.0308 | 18 |
| m=1-1/n | 0.159 | 0.496 | 0.0136 | 5.45 | (m=0.816) | 487 |
| m=1-2/n | 0.155 | 0.495 | 0.0143 | 5.87 | (m=0.659) | 425 |
| n→∞ | 0.116 | 0.465 | 0.0172 | - | λ=1.896 | 21 |
| | | | Touchet silt loam | | | |
| *variable* m, n | 0.081 | 0.524 | 0.0313 | 3.98 | m=0.493 | 14 |
| m=1-1/n | 0.102 | 0.526 | 0.0278 | 3.59 | (m=0.721) | 17 |
| m=1-2/n | 0.082 | 0.524 | 0.0312 | 3.98 | (m=0.497) | 14 |
| n→∞ | 0.018 | 0.499 | 0.0377 | - | λ=1.146 | 367 |
| | | | G. E. No. 2 sand | | | |
| *variable* m, n | 0.091 | 0.369 | 0.0227 | 4.11 | m=4.80 | 24 |
| m=1-1/n | 0.057 | 0.367 | 0.0364 | 5.05 | (m=0.802) | 34 |
| m=1-2/n | 0.0 | 0.370 | 0.0382 | 4.51 | (m=0.557) | 56 |
| n→∞ | 0.0 | 0.352 | 0.0462 | - | λ=1.757 | 354 |
| | | | Sarpy loam | | | |
| *variable* m, n | 0.051 | 0.410 | 0.0127 | 1.11 | m=0.886 | 60 |
| m=1-1/n | 0.032 | 0.400 | 0.0279 | 1.60 | (n=0.374) | 99 |
| m=1-2/n | 0.012 | 0.393 | 0.0393 | 2.45 | (m=0.185) | 199 |
| n→∞ | 0.0 | 0.380 | 0.0444 | - | λ=0.387 | 539 |

† *Values for* m *in parentheses were calculated from the fitted* n*-values.*

have little relation to the overall pore-size distribution that determines the general shape of the predicted conductivity curve at intermediate water contents. Thus, both theoretical and experimental considerations suggest that $K_s$ should not be used as a matching point for the hydraulic conductivity models [*Luckner et al.*, 1989]. Instead, it seems more accurate to match the predicted and observed unsaturated hydraulic conductivity functions at a water content somewhat less than saturation. The same holds for the saturated water content, $\theta_s$, which is best regarded as an empirical parameter to be used in the context of a specific water retention model, and hence must be fitted to observed unsaturated soil water retention data points.

Equations (6) and (8) assume that the predicted and observed hydraulic functions can be matched at saturation using the measured value of the saturated hydraulic conductivity, $K_s$. If more conductivity data are available, the RETC program permits one to include these additional data directly in the hydraulic parameter estimation process. In that case the program also allows one to estimate the parameters $\ell$ and $K_s$. The parameter estimation analysis of the retention and hydraulic conductivity data may be carried out consecutively or simultaneously. An important advantage of the simultaneous fit is that observed hydraulic conductivity data, if available, can be used to better define soil water retention parameters (and vice-versa).

## *Burdine's Hydraulic Conductivity Model*

The model of *Burdine* [1953] can be written in a general form as follows

$$K(S_e) = K_s S_e^{\ell} \frac{g(S_e)}{g(1)} \qquad (10)$$

in which

$$g(S_e) = \int_0^{S_e} \frac{1}{[h(x)]^2} dx \qquad (11)$$

where, as in (4), the pore-connectivity parameter $\ell$ accounts for the presence of a tortuous flow path. A variety of values have been suggested for $\ell$; *Burdine* [1953] assumed a value of 2.

Results analogous to those for Mualem's model can be derived also for Burdine's model. For independent *m* and *n* the hydraulic conductivity function can be written as

$$K(S_e) = K_s S_e^{\ell} I_{\zeta}(r,s) \qquad (12)$$

where

$$r = m + 2/n \quad and \; q = s - 2/n \qquad (13)$$

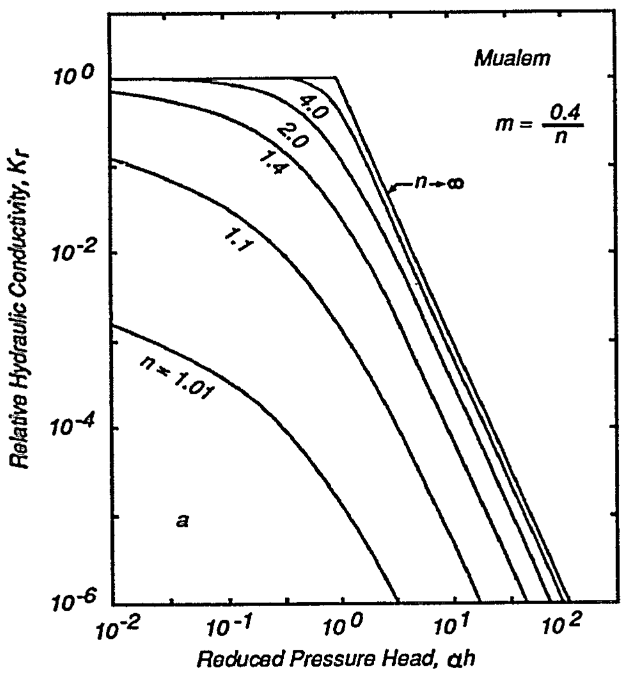
Equation (12) for variable *m,n* may again be simplified by imposing restrictions on the permissible values on *m* and *n*. The restriction *m*=1-2/*n* leads to [*van Genuchten*, 1980]:

$$K(S_e) = K_s S_e^{\ell} [1 - (1 - S_e^{1/m})^m] \qquad (14)$$

For completeness we also give the conductivity expressions when the BC limiting retention curve (i.e., for $n \to \infty$ with the product $\lambda = mn$ remaining finite) is used in conjunction with Burdine's model:

$$K(S_e) = K_s S_e^{\ell + 1 + 2/\lambda} \qquad (15)$$

Figure 5 shows calculated curves for the relative hydraulic conductivity, $K_r$, as a function of the reduced pressure head, $\alpha h$, and the reduced water content, $S_e$, as given by (12) for the variable *m,n* case. Notice that, similarly as in Figure 3a for Mualem's model, the Burdine-based expressions remain smooth functions of the pressure head as long as *n* is finite. One important difference between Figure 3 and 5 is that the Burdine-based equations hold only for *n* >2, while the Mualem-based formulations are valid for all *n*>1. Since many soils have *n*-values which are less than 2 (including



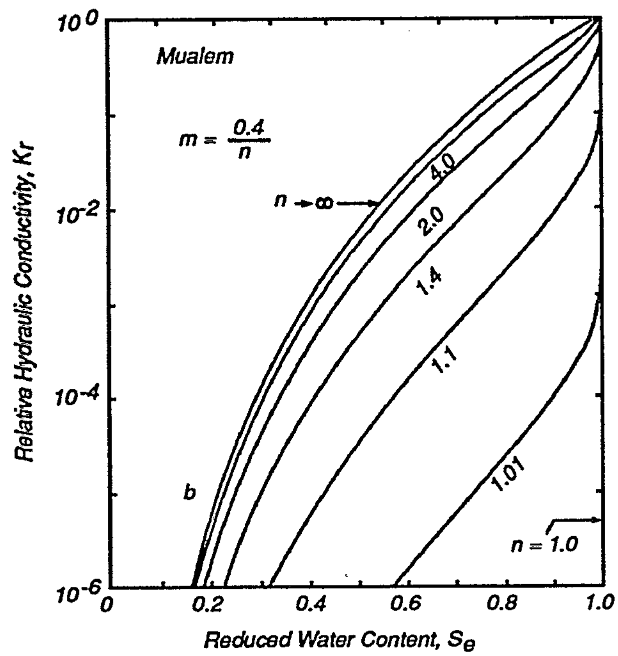




*Figure 3.* *Calculated curves for the relative hydraulic conductivity versus reduced pressure head (a) and reduced water content (b) as predicted from the retention curves in Figure 2 using Mualem's model with $\ell$=0.5.*

Sarpy loam, see Table 1), the Burdine-based models have less applicability than the Mualem-based expressions given in this report. Finally, Figure 6 gives a dimensionless plot of the Burdine-based conductivity function, $K(S_e)$, assuming $\ell = 2$ and $m$=1-2/$n$. As in Figure 4, the value of $m$ is bounded by $0<m<1$, which in this case follows from the requirement that $n>2$ and the restriction that $m$=1-2/$n$.

## Parameter Estimation

The soil water retention curve, $\theta(h)$, according to (3) contains five potentially unknown parameters: $\theta_r$, $\theta_s$, $\alpha$, $n$, and $m$. The predictive equation for $K$ introduces $\ell$ and $K_s$ as two additional unknowns. Hence, the soil hydraulic functions contain a maximum of seven independent parameters. The model parameters are represented here schematically by the parameter vector b = ($\theta_r$, $\theta_s$, $\alpha$, $n$, $m$, $\ell$, $K_s$). The RETC code may be used to fit any one, several, or all of these parameters simultaneously to observed data.

The most general formulation arises when the parameters $m$ and $n$ are assumed to be independent. The hydraulic conductivity function is then given by (6) or (12) when the predictive models of Mualem and Burdine are used, respectively. The restrictions $n\to\infty$ (i.e., the BC-function), $m$=1-1/$n$ and $m$=1-2/$n$ will reduce the maximum number of independent parameters from seven to six. In addition to imposing restrictions on $m$ and $n$, the RETC user can keep one or more of the other coefficients (e.g.,$\theta_s$, $\ell$, or $K_s$) constant during the parameter optimization process, provided that an estimate of those coefficients is available. For example, the model parameter vector reduces to **b** = ($\theta_r$, $\theta_s$, $\alpha$, $n$) when the Mualem restriction $m$=1-1/$n$ is implemented and only retention data are used in the optimization.

RETC uses a nonlinear least-squares optimization approach to estimate the unknown model parameters from observed retention and/or conductivity or diffusivity data. The aim of the curve-fitting process is to find an equation that maximizes the sum of squares associated with the model, while minimizing the residual sum of squares, SSQ. The residual sum of squares reflects the degree of bias (lack of fit) and the contribution of random errors. SSQ will be referred to as the objective function O(**b**) in which **b** represents the unknown parameter vector. RETC minimizes O(**b**) iteratively by means of a weighted least-squares approach based on Marquardt's maximum neighborhood method [*Marquardt*, 1963]. During each iteration step, the elements $b_j$ of the parameter

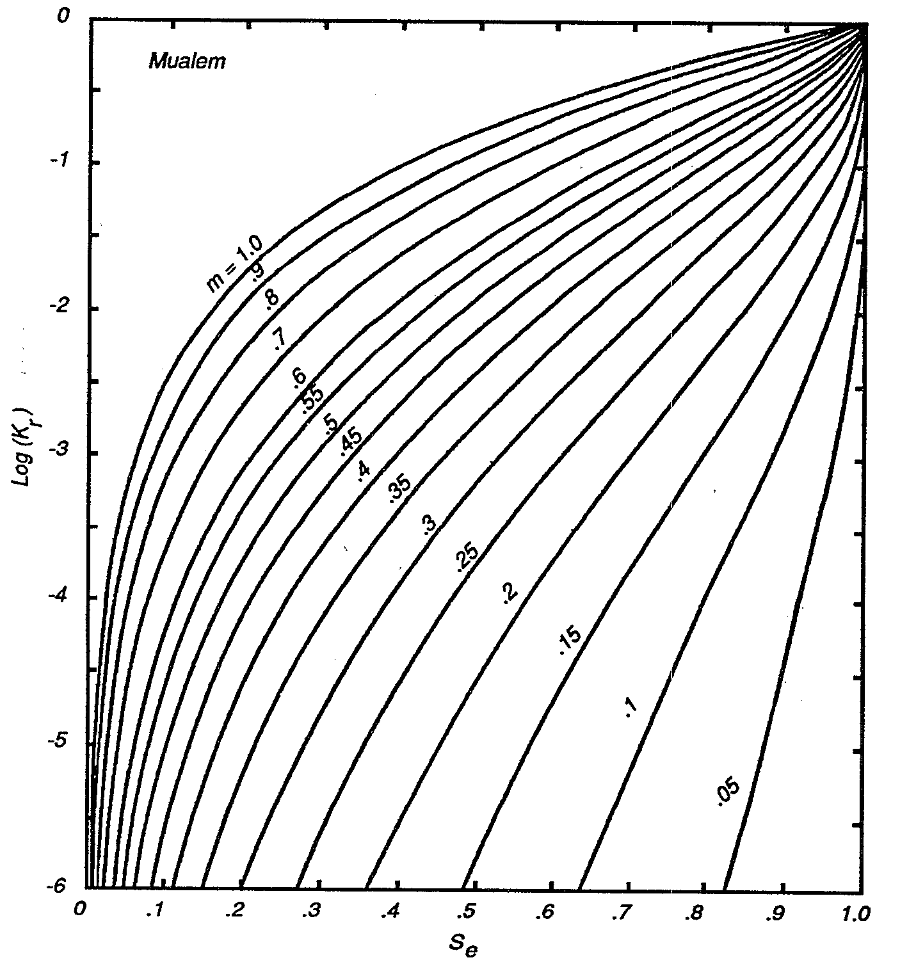



*Figure 4.* *Dimensionless semilogarithmic plot of the relative hydraulic conductivity, $K_r$, versus reduced water content, $S_e$, for various values of* m. *The curves were predicted from (3) using Mualem's model with $\ell$=0.5 and assuming* $m=1-1/n$.

vector **b** are updated sequentially, and the model results are compared with those obtained at the previous iteration level. RETC offers the option to print, among other information, O(**b**) for each iteration.

When both retention and conductivity data are available, the objective function is given by

$$O(\mathbf{b}) = \sum_{i=1}^{N} \{w_i[\theta_i - \hat{\theta}_i(b)]\}^2 \qquad (16)$$

$$+ \sum_{i=N+1}^{M} \{w_i W_1 W_2 [Y_i - \hat{Y}_i(\mathbf{b})]\}^2$$

where $\theta_i$ and $\hat{\theta}_i$ are the observed and fitted water contents, respectively, $Y_i$ and $\hat{Y}_i$ are the observed and fitted conductivity data, $W_1$ and $W_2$ are weighing factors as explained below, $N$ is the number of retention data points and $M$ is the total number of observed retention and conductivity data points. The weighing factors $w_i$ reflect the reliability of the measured data points, and ideally should be set equal to the observation error. Because reliable estimates for $w_i$ are often lacking, these weighing factors are frequently set equal to unity.

The weighing factor $W_1$ in (16) allows one to place more or less weight on the hydraulic conductivity data in their entirety, relative to the soil water retention data. Because conductivity data usually show considerably more scatter than water content data, and generally are also less precise, it is often beneficial to assign relatively less weight to the conductivity data in (16).

The parameter $W_2$ is introduced to ensure that proportional weight is given to the two different types of data in (16), i.e., $W_2$ corrects for the difference in number of data points and also eliminates, to some extent, the effect of having different units for $\theta$ and $K$. The value for $W_2$ is calculated internally in the program according to

$$W_2 = (M-N)\sum_{i=1}^{N} w_i\theta_i / N \sum_{i=N+1}^{M} w_i|Y_i| \qquad (17)$$

The effect of (17) is to prevent one data type in (16) from dominating the other type solely because of larger numerical values.

Because for most conductivity data the observation error depends on the magnitude of the measured data, the implicit assumption of statistically independent observation errors (made if all $w_i$ are set to 1) is violated. To partly correct this problem, RETC has the option of implementing a logarithmic transformation, $Y_i = \log(K_i)$, in (16) before carrying out the parameter estimation process. We recommend the use of a logarithmic transformation unless special accuracy of the conductivity function in the wet range is required. In that case one may decide to use the untransformed data since these put relatively more weight on the higher $K$ values.

Except for well-defined data sets covering a wide range of $\theta$ and/or $K$ data, it is important to limit as much as possible the number of parameters to be included in the parameter optimization process. The RETC output includes a matrix which specifies the degree of correlation between the fitted coefficients in the different hydraulic models. We suggest to always perform a "backward" type of regression, i.e., by initially fitting all parameters and then fixing certain parameters one by one if these parameters exhibit high correlations. The most frequent cases of correlation occur between $m$, $n$, and $\ell$ if no restrictions are placed on $m$ and $n$, and between $n$ and $\ell$ if one of the restrictions on $m$ and $n$ is imposed. We also recommend using one of the restrictions on $m$ and $n$, unless the observed data show little scatter and cover a wide range of pressure head and/or hydraulic conductivity data. It is also possible to fix the residual water content at zero, or some other value, while keeping $m$ and $n$ independent, or assuming $m$=1-1/$n$. Fixing $\theta_r$ is especially appropriate when few data at relatively low water contents or pressure heads are available. $K_s$ and $\theta_s$ are both very susceptible to experimental errors and should ordinarily not be fixed. In some cases the results may even improve when the "saturated" water content is deleted entirely from the parameter optimization analysis.

An important measure of the goodness of fit is the $r^2$ value for regression of the observed versus fitted values. The $r^2$ value is a measure of the relative magnitude of the total sum of squares associated with the fitted equation; a value of 1 indicates a perfect correlation between the fitted and observed values. RETC provides additional statistical information about the fitted pa-

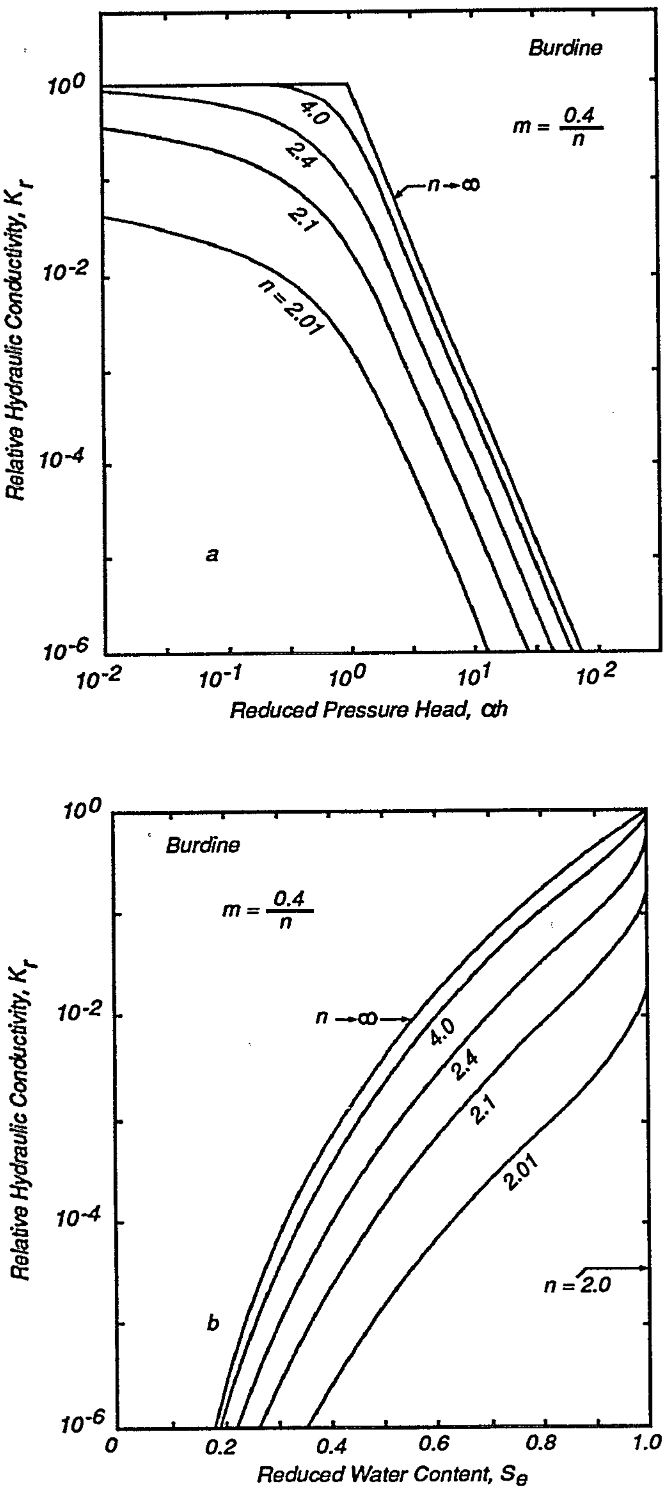



*Figure 5. Calculated curves for the relative hydraulic conductivity versus reduced pressure head (a) and reduced water content (b) as predicted from the retention curves in Figure 2 using Burdine's model with $\ell = 2$.*

rameters such as mean, standard error, T-value, and lower and upper bounds of the 95% confidence level around each fitted parameter $b_j$. It is desirable that the real value of the target parameter always be located in a narrow interval around the estimated mean as obtained with the optimization program.

Finally, because of possible problems related to convergence and parameter uniqueness we recommend routinely re-running the program with different initial parameter estimates to make sure that the program converges to the same global minimum in the objective function. Although RETC will not accept initial estimates that are out of range, it is ultimately the user's responsibility to select meaningful initial estimates.

## The RETC User Guide

### *Program Options*

The RETC code provides several options for describing or predicting the hydraulic properties of unsaturated soils. The code may be used to fit any one, several, or all of the six or seven unknown parameters simultaneously to observed data. RETC can be applied to four broad classes of problems as outlined below.

A. *The direct (or forward) problem.* RETC may be used to calculate the unsaturated soil hydraulic functions if the model parameter vector $\mathbf{b} = (\theta_r, \theta_s, \alpha, n, m, \ell, K_s)$ is specified by the user. Values for  and $K_s$ are not needed when only the retention function is being calculated. The direct problem, which bypasses the optimization part of RETC, is being executed whenever this option is specified, or when no observed data are given in the input file.

B. *Predicting K(h) from observed θ (h) data.* This option permits one to fit the unknown retention parameters (with or without restricted *m,n* values) to observed soil water retention data. The fitted retention parameters are subsequently used to predict the hydraulic conductivity functions by making use of the models of Mualem or Burdine. This case assumes that the initial estimates for $\ell$ and $K_s$ remain unaltered during the parameter optimization process.

C. *Predicting θ (h) from observed K(h) data.* In some instances experimental conductivity data may be available but no observed retention data. RETC may then be used to fit the unknown hydraulic coefficients to observed conductivity data. Once the unknown coefficients are determined, the retention function may be calculated. This option is also needed when a consecutive fitting procedure is followed for the retention and hydraulic conductivity

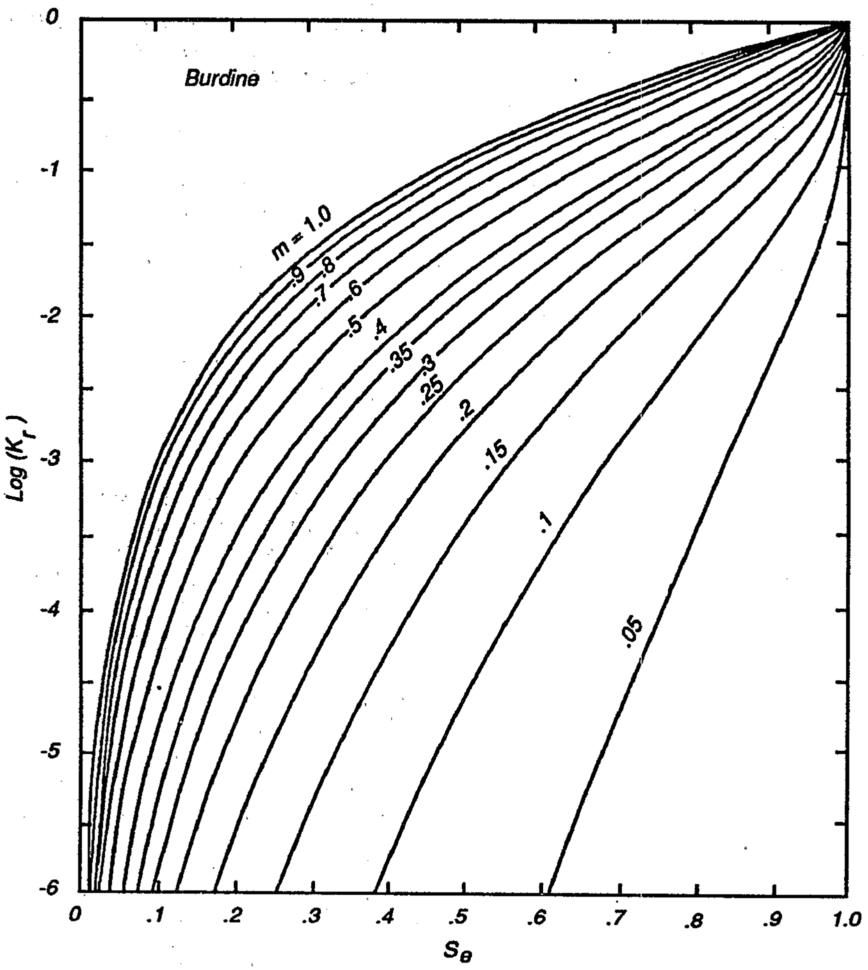



*Figure 6.* *Dimensionless semilogarithmic plot of the relative hydraulic conductivity,* $K_r$*, versus (3) reduced water content,* $S_e$*, for various values of* m. *The curves were predicted from using Burdine's model with* $\ell=2$ *and assuming* $m=1-2/n$.

data, i.e., when some of the hydraulic parameters are first fitted to observed soil water retention data, followed by a fit of $\ell$ and/or $K_s$ to observed conductivity data.

D. *Simultaneous fit of θ (h) and K(h) data.* This option results in a simultaneous fit of the model parameters to observed water retention and hydraulic conductivity data.

## *Code Structure and Program Preparation*

RETC consists of a main program, three subroutines (MODEL, MATINV, and PRINT), and two functions (GAMMA and BINC). Most mathematical manipulations associated with the least-squares calculations are carried out in MAIN. Of the two functions, GAMMA evaluates the Gamma (Γ) function, and BINC the Incomplete Beta Function. Of the three subroutines, MATINV performs matrix inversions needed for the least-squares analysis, while subroutine MODEL calculates the soil water retention and/or hydraulic conductivity/diffusivity functions as determined by the input variables METHOD and MTYPE. Subroutine PRINT provides an optional listing of the calculated hydraulic properties at the end of the optimization.

In addition to the above subroutines and functions, the code contains several interface subroutines that display control settings. The program is written in Fortran 77 and can be run on any IBM-PC compatible machine. While not required, a numeric coprocessor is recommended for increased accuracy and speed. The control file RETC.CTL, and example data input and output files are given in the report. The examples correspond to the four types of problems (A through D) summarized previously.

The control variables in RETC.CTL determine the operation of RETC. These variables can be changed interactively; a listing of variable options and current settings are displayed on the screen prior to execution of a problem. The user must specify the names of input, output, and plotting files. The data input file, which can be created with a text editor, should contain the independent and dependent variable with the accompanying $w_i$ on each line. RETC keeps track of the number of retention and conductivity points while reading the input file. The output file contains observed and fitted hydraulic data, as well as statistical information regarding the optimization procedure. The user can also request that detailed hydraulic curves are calculated from the fitted or specified model parameters. The results are written to the main output file, as well as to separate files to facilitate plotting or their direct input into numerical codes for simulating variably-saturated water flow.

## Summary and Conclusions

This summary outlines the RETC computer program for evaluating the hydraulic properties of unsaturated soils. The soil water retention curve, *θ (h)*, in the program can be represented by the equations of Brooks-Corey or van Genuchten, while the unsaturated hydraulic conductivity, *K(h)*, is formulated in terms of the statistical pore-size distribution models of Mualem and Burdine. The RETC code is shown to be useful for a variety of applications including (i) predicting the unsaturated hydraulic properties from previously estimated soil hydraulic parameters (the forward problem), (ii) predicting the unsaturated hydraulic conductivity functions from observed retention data, and (iii) quantifying the hydraulic properties by simultaneous analysis of a limited number of soil water retention and hydraulic conductivity data points.

The complete report serves as both a user manual and reference document. Detailed information is given about the computer program, along with instructions for data input preparation. A large part of the report consists of listings of the source code, sample input and output files, and several illustrative examples. The accompanying software should lead to a more accurate and convenient method of analyzing the unsaturated soil hydraulic properties. The information appears especially useful for theoretical and applied scientists, engineers, and others, concerned with the movement of water and chemicals into and through the unsaturated (vadose) zone.

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*U.S. Government Printing Office: 1992 — 648-080/60041

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*The complete report consists of one volume, entitled "The RETC Code for Quantifying the Hydraulic Functions of Unsaturated Soils," (Order No. PB92-119668/AS; Cost: $19.00) and one disk (Order No. PB92-501329/AS; Cost: $90.00). Both costs are subject to change. The volume and disk will be available only from:*

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