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Robert S. Kerr Environmental
Research Laboratory
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Project Summary

Demonstration of the Analytic Element Method for Wellhead Protection

Hendrik M. Haitjema, Otto D.L. Strack, and Stephen R. Kraemer

A new computer program has been developed to determine time-of-travel capture zones in relatively simple geohydrological settings. The W*h*AEM package contains an analytic element model that uses superposition of (many) closed form analytical solutions to generate a ground-water flow solution. W*h*AEM consists of two executables: the preprocessor GAEP, and the flow model CZAEM. W*h*AEM differs from existing analytical models in that it can handle fairly realistic boundary conditions such as streams, lakes, and aquifer recharge due to precipitation.

The preprocessor GAEP is designed to simplify the procedures for getting data into a ground-water model; specifically it facilitates the interactive process of ground-water flow modeling that precedes capture zone delineation. The flow model CZAEM is equipped with a novel algorithm to accurately define capture zone boundaries by first determining all stagnation points and dividing streamlines in the flow domain. No models currently in use for wellhead protection contain such an algorithm.

This Project Summary was developed by USEPA's Robert S. Kerr Environmental Research Laboratory, Ada, OK, to announce key findings of the Research Project that is fully documented in separate reports and supporting software (See Project Report ordering information at the back).

Introduction

The delineation of capture zones requires precise determination of streamlines. In most numerical methods, such accurate determination is difficult because the velocities are computed on the basis of values of piezometric heads that are known only at the nodes of a mesh. This deficiency stimulated the development of a number of computer models which implement elementary analytic solutions for ground-water flow problems. These analytic models are capable of producing more or less approximate shapes of the boundaries of capture zones for any given time; e.g., the USEPA's original wellhead protection model WHPA. However, even when the velocity components are known precisely, accurate determination of the boundaries of capture zones still requires that both stagnation points and dividing streamlines are known.

The delineation of capture zones in complex settings is currently done either by the use of discrete numerical models or analytic element models. The discrete numerical models, such as MODFLOW/MODPATH of the US Geological Survey, and analytic element models, such as Strack's MLAEM, require detailed knowledge of the setting and advanced modeling expertise to run; and they do not currently have advanced algorithms for delineation of capture zones.

The Wellhead Analytic Element Model (W*h*AEM) falls between the two aforementioned technologies. It does contain an advanced algorithm for determining capture zones for any well at any time based on precise knowledge of the locations of all stagnation points and dividing streamlines. It has features that make the inclusion of open or closed, head-specified boundaries possible (for example to model streams), but lacks the power of advanced discrete numerical or analytic element models. The authors believe that the newly developed model will serve ground-water professionals who wish to determine capture zones in relatively simple geohydrological settings.

W*h*AEM consists of two executables: the preprocessor GAEP and the flow model CZAEM (see Figure 1). In order to facilitate data entry in the computer program CZAEM (Capture Zone Analytic Element Model), a separate computer program was developed called GAEP, (Geographic Analytic Element Preprocessor). This program makes it possible for most users not familiar with the input structure of analytic element models to concentrate on modeling aspects, rather than on the intricacies of preparing input data files.

W*h*AEM was developed under a cooperative agreement between EPA and Indiana University and the University of Minnesota.

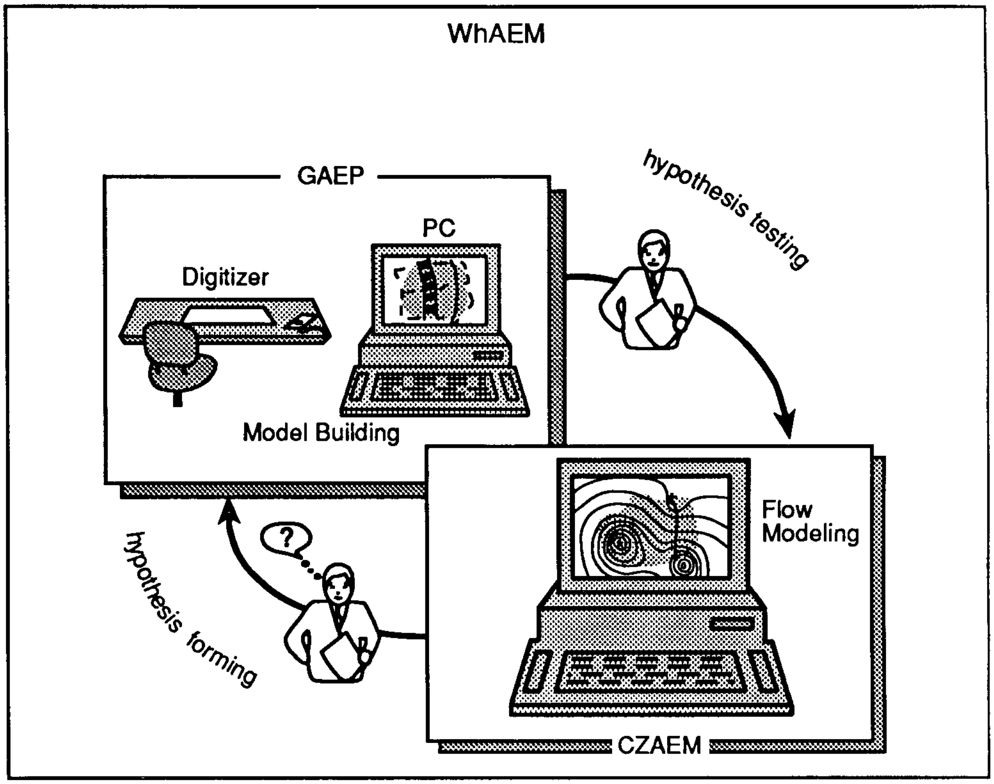

Figure 1. The modeling process using WhAEM.

The Analytic Element Method

The analytic element method was developed at the end of the seventies by Otto Strack at the University of Minnesota. For a detailed description of the method, the reader is referred to *Groundwater Mechanics*, O.D.L. Strack, 1989, Prentice Hall, while a brief review follows.

This new method avoids the discretization of a ground-water flow domain by grids or element networks. Instead, only the boundaries of the surface water and aquifer features in the domain are discretized and entered into the model. Each of these boundary segments is represented by closed form analytic solutions—the analytic elements. The comprehensive solution to a complex, regional ground-water flow problem is obtained by superposition of all analytic elements in the model, from a few hundred to thousands.

Traditionally, modeling ground-water flow by use of analytic functions was considered to be limited to homogeneous aquifers of constant transmissivity. However, by formulating the ground-water flow problem in terms of appropriately chosen discharge potentials rather than piezometric heads, the analytic element method becomes applicable to both confined and unconfined flow conditions as well as to heterogeneous aquifers.

The analytic elements are chosen to best represent certain hydrologic features. For instance, stream sections are represented by line-sinks; lakes or wetlands may be represented by areal sink distributions. Streams and lakes that are not fully connected to the aquifer are modeled by area sinks with resistance to flow between surface water and the aquifer. Discontinuities in aquifer thickness or hydraulic conductivity are modeled by use of line doublets (double layers). Other analytic elements may be used for special features, such as drains, fractures, and slurry walls.

The analytic element method differs fundamentally from most classical numerical models, for instance:

1. The solution is analytical; no interpolation is required to compute heads or velocities at any point in the domain. This makes the method insensitive to scale, allowing contour plots and streamlines to be produced in areas varying in size from several square feet to hundreds of square miles.
2. Since the velocity field is calculated analytically, inaccuracies in capture zone boundaries and isochrones of travel times are due solely to approximations made in the conceptual model and its implementation in the program; they are not the consequence of a model grid resolution and the associated numerical (approximate) differentiation process.
3. The aquifer is unbounded in the horizontal plane; there are no artificial model boundaries that may influence the solution.

W*h*AEM

Time-of-travel capture zone delineation is conducted by ground-water flow modeling. Modeling in this context is an interactive process of data acquisition, data analysis, and running a computer model. Initial modeling results prompt changes in the conceptual model, which in turn will lead to new modeling results. In the absence of definite data, hypothesis testing and sensitivity analysis will be important aspects of the modeling process. As a result, the modeler will usually make frequent changes to the input data file (an ASCII file of program instructions) during the modeling process.

W*h*AEM was designed to facilitate this process. GAEP greatly improves the process of data entry, especially when used in combination with USGS topographic maps and a digitizer (optional). The GAEP generated electronic background map becomes the template for "on-screen" design of the ground-water flow model. GAEP communicates with the flow solver CZAEM through an ASCII file that contains the command script for delineating capture zones.

Program CZAEM

CZAEM is a single layer model for simulating steady flow in homogeneous aquifers. The mathematical framework underlying the model is based on the Dupuit-Forchheimer assumption, where the vertical resistance to flow is negligible, such as for shallow aquifer flow. The implementation of the analytic element method in CZAEM is elementary, supporting only a few basic analytic elements. These elements can be used to simulate river boundaries, streams, lakes, wells, uniform flow, and uniform infiltration over a circular area.

Line-sinks are used to model river boundaries, streams, and lakes. Line-sinks are mathematical functions that simulate a constant rate of extraction along a line. The sink densities (strengths) of the line-sinks in the model are determined such that the heads at the center of the line-sinks are equal to specified values (usually chosen to equal the water levels in the streams or lakes). The accuracy with which the ground-water inflow (or outflow) along a stream can be modeled improves with a finer subdivision of the stream in line-sink segments.

The well function (Thiem equation) is used to model wells with given discharge (pumping rate). Unlike numerical models, the piezometric head distribution and the velocity field near a well remain accurate, since there is no discretization of the aquifer by a grid or element network.

A special function, the "pond" function, is used to model areal recharge due to precipitation. Since CZAEM models steady state flow, this recharge rate is a yearly average. The "pond" function is a circular element with an areal "source" density equal to the recharge rate. The circular pond overlays the domain of interest, the well field and surrounding surface waters, to simulate the desired aquifer recharge.

The uniform flow function may be used to replace the combined effects of areal recharge and surface water boundaries, similar to the WHPA program. Since W*h*AEM allows the explicit representation of these boundary conditions, the uniform flow approximation is less often used in applications to field problems.

The computer program CZAEM is an elementary analytic element model with the capability to generate capture zones of wells. The program has the following modules:

1. AQUIFER, for the input of aquifer data.
2. GIVEN, for the input of uniform flow and areal recharge.
3. REFERENCE, for the input of the head at one point in the aquifer.
4. WELL, for the input and implementation of wells.
5. LINE-SINK, for the input and implementation of line-sinks.
6. GRID, for the generation of a grid of piezometric heads to be contoured.
7. PLOT, for piezometric contour plotting.
8. TRACE, for streamline tracing.
9. CAPZONE, for the generation of time-of-travel capture zones.
10. CURSOR, for cursor controlled interaction with elements.
11. CHECK, for monitoring the values of parameters.
12. IO, for binary write and save of solutions.
13. PSET, for sending graphics to output devices.

Capzone Module

The capzone module has been designed to define the capture zone boundaries as well as the travel time isochrones inside these capture zone boundaries, called timezones for arbitrary arrangements of wells and line-sinks (stream boundaries). Rather than merely tracing a number of streamlines from the well, the capzone module logic first determines all stagnation points in the flow domain, determines whether they are connected to the wells by streamlines, and uses them as the basis for defining the capture zone boundaries. Under multiple well scenarios, one well may have several different capture zones, termed subzones, which have their own travel time isochrones.

Figure 2 illustrates capture zones for five different wells surrounded by two stream branches and a tributary (solid lines). The dashed lines are background map features for orientation purposes. The capture zones are pear shaped and of finite extent: the wells receive all their water from areal recharge.

In Figure 3, travel time isochrones are presented for two wells in a uniform flow field. Notice that the isochrones wrap all the way around the well; between the well and its stagnation point, all travel times between zero and infinity occur. Also notice that the capture zone of the right-hand well wraps around the capture zone of the other well, causing a discontinuity in the residence times across the latter capture zone envelope.

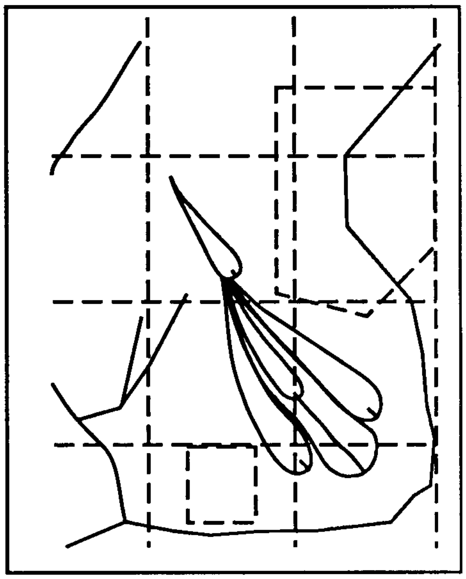

***Figure 2.** Capture zone envelopes for five wells in a regional setting defined by streams and areal recharge*

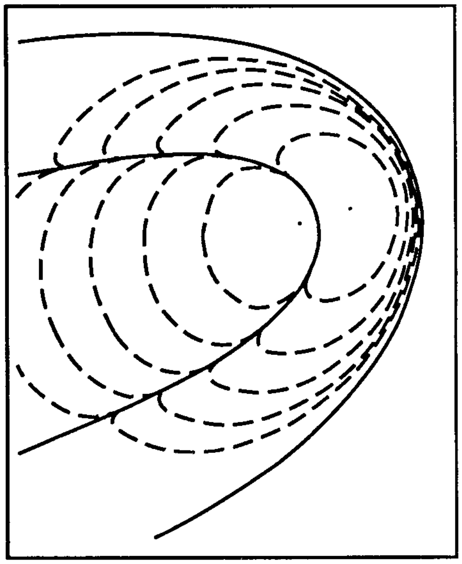

***Figure 3.** Travel time isochrones for two wells in a uniform flow field.*

Figure 4 shows capture subzones for a well near a river. The well is receiving 80% of its discharge from the far-field and 20% from the river.

The module CAPZONE has been written specifically for this project. The source code (FORTRAN) of this module is available from USEPA and contains documentation to facilitate its inclusion in other ground-water models.

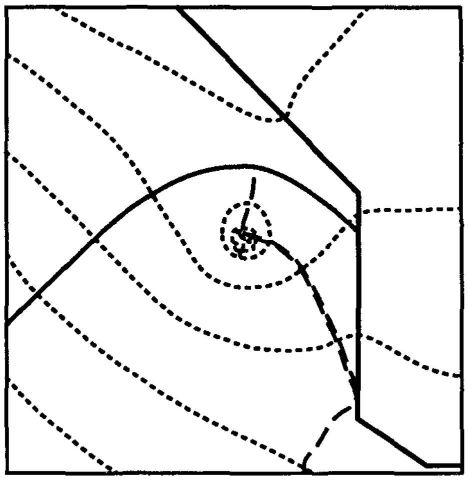

Figure 4. Subzones for a well near a river. The dotted lines represent hydraulic head contours.

Preprocessor GAEP

The absence of a grid or element network in analytic element models makes it unnecessary to translate hydrography data (stream locations and associated stream levels) into cell specific data, which is the main function of preprocessors for numerical models. In contrast, the stream levels and geometry are entered into the analytic element model, through simple commands which are in an ASCII file. These commands can be input directly from the keyboard (command line mode) or read in from an ASCII file (batch mode) (see Figure 5). The line-sinks representing streams and lakes are represented in the file by their end coordinates and a specified head at the center (stream level). The traditional procedure for creating such an input data file is to sketch the line-sink layout on a map, write the stream elevations near these line-sinks, and use a digitizer to produce the coordinate pairs listed in Figure 5. The syntax of the various commands in Figure 5 must be consistent with the requirements of CZAEM, or one or more errors occur when the file is read by CZAEM. Any change in the input data, as part of the interactive modeling procedure requires editing of the input file, whereby it is often necessary to revisit the topographical map and digitize new line-sinks.

To facilitate this process, the preprocessor GAEP separates the digitizing activities from the creation of analytic elements (e.g. line-sinks) to be included in the input data file for CZAEM. GAEP, therefore, has two functions:

1. Creation of a digital map of all streams, lakes, wetlands, well locations, and background map information (roads, city boundaries, outlines of surface geological features, etc.). The surface water features have associated with them the water levels as reported on the topographical map (intersections of elevation contours with the stream beds).
2. Creation of an input data file for CZAEM with all aquifer data and all analytic elements (line-sinks and wells) needed for the model.

The first activity, creation of the digital map, is a routine procedure that does not require any modeling expertise or hydrological knowledge. The digital map is saved on disk for future use by the modeler. A digital map, as displayed on screen by GAEP, is reproduced in Figure 6. By pointing at a feature with the mouse, its name is displayed for easy identification.

The modeler will use GAEP and a digital map previously saved on disk to create a CZAEM input data file. To represent a stream by line-sinks, the modeler merely points at the stream with the mouse and selects it by "clicking" the mouse button. By moving the pointer over the stream and clicking on intended line-sink end points, a string of line-sinks is created with heads computed at their centers using the stream elevations stored with the digital map. In Figure 7, a string of line-sinks is illustrated. The numbers printed near the line-sinks represent the average stream elevations (heads at the center of the line-sinks). GAEP will also prompt for aquifer data; and when instructed to create the CZAEM input data file, write an ASCII file to disk that can be read directly by CZAEM. In fact, the GAEP generated input data file will introduce the data, solve the problem, and create a grid with piezometric heads. The modeler then enters the TRACE and CAPZONE modules in CZAEM to generate capture zone boundaries.

```
AQUIFER
BOTTOM 330
THICKNESS 100
PERMEABILITY 350
POROSITY 0.20
RETURN
AQUIFER
REFERENCE     0    656160     410
RETURN
*Rain Element
SINKDISK
DISCHARGE
  21983   -6193   43881   -24755   -0.00411
RETURN
*TabletItemID:     wabash east
LINESINK
HEAD
 -13409  -18389 -12778 -10574  395.2 [we1]
 -12778  -10574  -8046  -5825  396.1 [we2]
  -8046   -5825  -5927  -1257  396.8 [we3]
  -5927   -1257  -2322    367  397.2 [we4]
```

Figure 5. Part of a CZAEM input data file

Conclusions and Recommendations

The deliverable of this project consists of an analytic element modeling package for simulating steady flow in homogeneous aquifers, with the primary objective to delineate capture zones in settings with streams, rivers, lakes, infiltration and wells. New algorithms have been developed for the accurate delineation of capture zone boundaries. These algorithms are implemented in the computer code CZAEM. The algorithms make accurate delineation of capture zone boundaries possible. A preprocessor program, GAEP, has been developed to facilitate the entry of field data into CZAEM. GAEP simplifies the process of modeling considerably. Specifically, GAEP separates the time consuming (but routine) task of digitizing hydrography data from the creation of conceptual models and subsequent analytic element input data files. With GAEP, the modeler is free to concentrate on interpretation of modeling results rather than the details of data modification and entry into CZAEM.

The W*h*AEM package is documented in various ways. The primary documentation is contained in a program manual, which includes installation instructions, program descriptions and a tutorial for the integrated use of GAEP and CZAEM. Reference manuals for both GAEP and CZAEM are provided in the W*h*AEM manual. A tutorial for stand-alone use of the program CZAEM is available as a separate document. Finally, both GAEP and CZAEM codes support on-line help.

The W*h*AEM package should only be used by ground-water hydrologists qualified to address wellhead protection delineation problems. W*h*AEM is designed to assist hydrologists in learning as much as possible about the geohydrological problems they face. Model predictions must always be interpreted critically, with the simplifying assumptions in mind. For complex geohydrological settings, it may be necessary to apply a more powerful model than W*h*AEM, requiring more experience from the ground-water flow modeler.

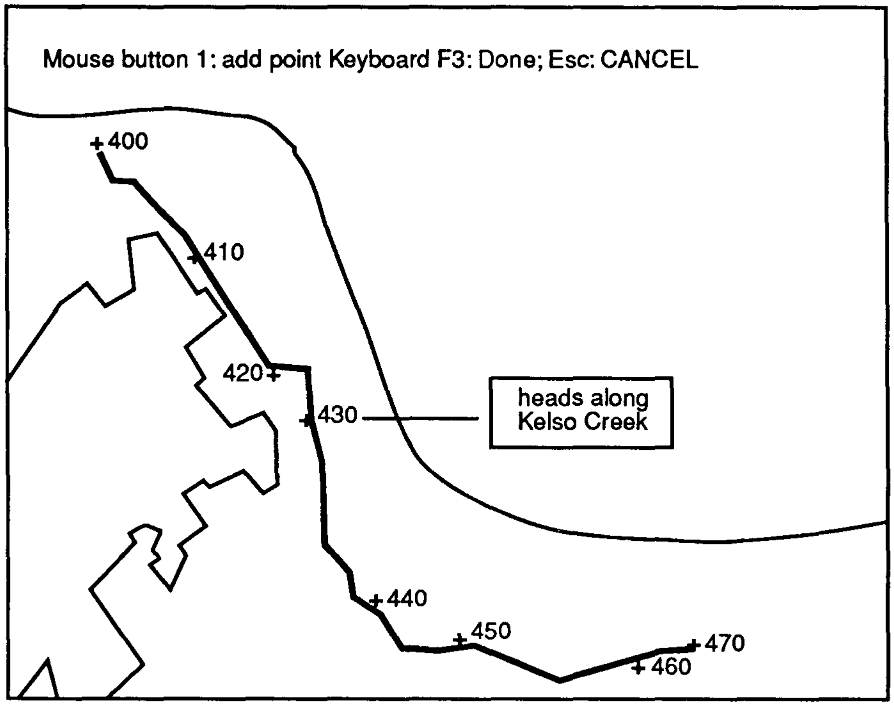
Mouse button 1: add point Keyboard F3: Done; Esc: CANCEL

Figure 6. Kelso Creek with elevation marks where contours cross the stream

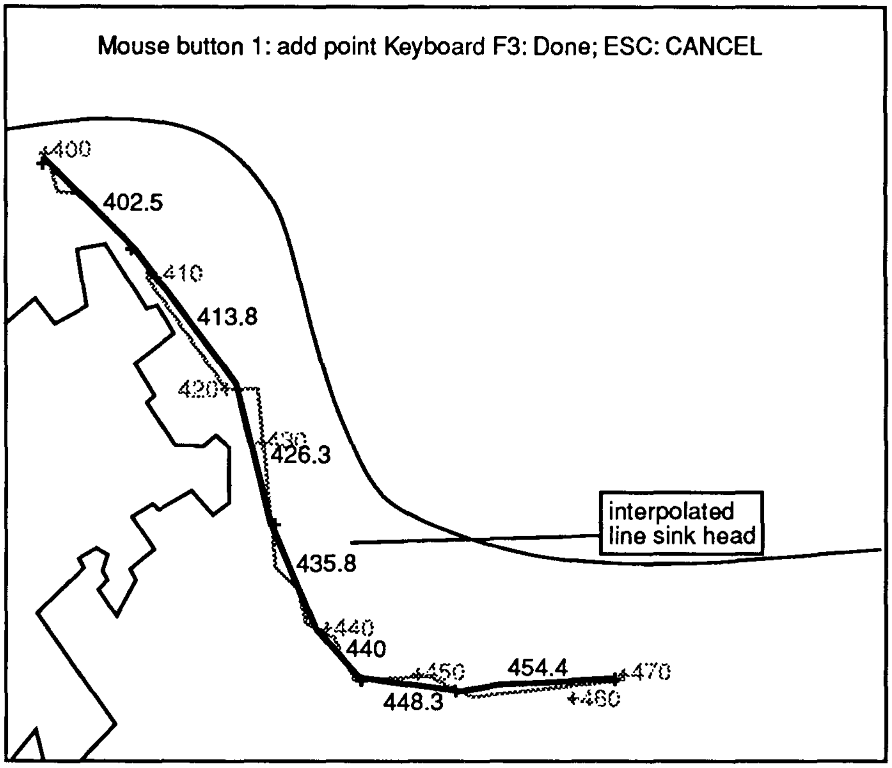
Mouse button 1: add point Keyboard F3: Done; ESC: CANCEL

Figure 7. Line-sinks created along Kelso Creek.

Hardware Requirements

- 386 or 486 PC
- 2.5 MB RAM
- MS compatible mouse
- digitizer (optional)
- printer (optional)

Software Requirements

- MS DOS version 5.0 or higher
- Windows 3.1 (optional)

Hendrik M. Haitjema is with the School of Public and Environmental Affairs, Indiana University, Bloomington, IN 47405.
Otto D.L. Strack is with the University of Minnesota, Civil and Mineral Engineering, Minneapolis, MN 55455.
***Stephen R. Kraemer** (also the EPA Project Officer see below), is with the U.S. Environmental Protection Agency, Ada, OK 74820*

The complete report consists of two volumes, entitled "CZAEM User's Guide: Modeling Capture Zones of Ground Water Wells Using the Analytic Element Method," (Order No. PB95-194189; Cost: $19.50, subject to change) and "WhAEM: Program Documentation for the Wellhead Analytic Element Model," (Order No. PB95-167375; Cost $27.00, subject to change) will be available only from:

National Technical Information Service
5285 Port Royal Road
Springfield, VA 22161
Telephone: 703-487-4650

The EPA Project Officer can be contacted at:

Robert S. Kerr Environmental Research Laboratory
U.S. Environmental Protection Agency
Ada, OK 74821

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