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A User's Manual for Three-Dimensional Heated Surface Discharge Computations

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A USER'S MANUAL FOR THREE-DIMENSIONAL HEATED SURFACE DISCHARGE COMPUTATIONS

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ABSTRACT

In February 1971, a report by K. D. Stolzenbach and Donald R. F. Harleman entitled "An Analytical and Experimental Investigation of Surface Discharges of Heated Water" was published. (Technical Report No. 135, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, M.I.T.; also published as Water Pollution Control Research Series Report No. 16130 DJU 02/71 by the Water Quality Office, Environmental Protection Agency, Washington, D.C.) The report described above presented a literature review of previous analytical and experimental research on heated surface discharges, the development of a predictive theory for calculating three-dimensional temperature distributions in the near-field region and verification of the theory based on laboratory work at M.I.T. and elsewhere. The above report also contained a listing of the computer program as originally developed.

Subsequent work and experience in using the computer program for the calculation of temperature distributions have resulted in modifications and improvements in the program. In addition, a number of runs covering a wide range of input parameters have been carried out. These runs were done with two objectives: (1) to explore the limits of applicability of the theory and (2) to prepare design charts showing the important characteristics of the near field temperature distribution for heated surface discharges. These charts will enable the designer to make rapid estimates of surface isotherms and vertical thickness of surface jets.

This report presents a review of the theoretical background for the three-dimensional temperature prediction model, a detailed discussion of the revised computer program and a case study illustrating the procedure for optimizing the design of a surface discharge channel. The revised computer program flow chart, program listing and a sample of the input and output data are given in the appendices.

One difference between the revised computer program presented in this report and the original computer program of February 1971 deserves further comment. The original program contained the possibility of considering a sloping bottom in the receiving water. The sloping bottom, was assumed to extend downward

in a linear slope from the bottom of the discharge channel. In the revised computer program it is assumed that the bottom of the receiving water does not interfere with the development of the surface jet. Thus the revised program corresponds to $S_x = \infty$ in the original program. The reasons for eliminating the bottom slope effect in the revised program are two-fold: (1) the mathematical model, as originally constituted, did not adequately predict the point of separation of the heated jet from the bottom or lateral spreading when the jet is in contact with the bottom; (2) from the standpoint of environmental impact, it may be desirable to accept the depth of the receiving water as a constraint and to design the surface discharge channel to minimize interference of the heated jet with the bottom.

It is recognized that almost none of the operating power plants that employ surface discharge schemes have been designed to minimize bottom impact. This fact makes it difficult to compare field data from many existing plants with temperature predictions based on the mathematical model presented in this report.

Further analytical and experimental studies of bottom interference with heated surface jets are presently underway in the M.I.T. Parsons Laboratory and will be the subject of a future technical report.

Inquiries relating to the availability of the program source deck should be directed to Professor D.R.F. Harleman, Room 48-335, M.I.T., Cambridge, Massachusetts 02139.

TABLE OF CONTENTS

LIST OF FIGURES

Figure		Page

I. Introduction

1.1 Characteristics of Heated Discharges

The discharge of heated condenser water into natural bodies of water can be broadly classified into two groups: surface discharges and submerged discharges. The latter class includes single port submerged discharges and multi-port diffusers. From the standpoint of minimizing the environmental impact of thermal effluents, the surface discharge has a number of advantages when compared with submerged discharges: (1) By careful design of a surface discharge it is possible to avoid temperature rises and high velocities along the bottom of the receiving water. (2) The time of travel of organisms entrained at the condenser water intake is short. (3) Heat dissipation from the surface of the receiving water is high because of the tendency of the discharge to form a stratified surface layer.

A typical open cycle system withdraws water from a natural water body through an intake structure and passes the flow through the turbine condensers where it undergoes a temperature rise before being returned to the receiving water through a discharge structure. In-plant technological considerations dictate condenser temperature rises of from 10-30°F with correspondingly large cooling water flows depending upon the amount of rejected heat.

The use of natural water bodies and coastal waters for disposal of waste heat must take into account the effect upon the environment of the flows and temperature rises induced in the receiving water. There is general agreement that water temperature increases which approach the sub-lethal range of impaired biological activity should be avoided. Practically all regulatory agencies provide this type of protection through controls on both maximum temperatures and allowable temperature rises and by designating the size of mixing zones for various types of receiving waters. In addition to environmental considerations the intake structure must be located and designed to prevent significant recirculation of heated water. Separation of the intake and discharge or the use of selective withdrawal structures are the most common techniques.

The temperature distribution induced in the receiving water by a heated discharge is determined by the characteristics of the discharge structure and by the local ambient heat transfer processes. Close to the point of discharge the momentum of the discharged water creates jet-like mixing of the heated and ambient water. Within this "near field" region the temperature and velocity of the discharge decrease because of dilution by entrained water. The magnitude and extent of the dilution is determined primarily by the nature of the initial discharge flow, its submergence, velocity, dimensions and temperature rise above ambient. Mixing increases with increasing discharge momentum and decreases with increasing temperature rise. The greater the submergence of the discharge below the water surface, the lower the temperature rise at the surface will be after mixing. Mixing may also be affected by the presence of physical obstructions which tend to block the supply of dilution water. Surface discharges entrain a flow at least equal to the discharge flow in the near field, often up to twenty times as much.

Beyond the near field mixing region the discharge velocity and turbulence level are of the order of ambient values. In this "far field" region further entrainment does not occur and the temperature distribution is determined by natural turbulent convection and diffusion. Ultimately all of the rejected heat contained in the discharge passes to the atmosphere through the water surface, a process driven by the elevated surface temperatures. These far field heat transfers are highly variable, being determined by local water currents, wind, and meteorological conditions.

Consideration of either environmental impact or recirculation on discharge design must start with the near field temperature distribution. Far field processes are generally an order of magnitude less efficient in reducing temperatures and the far field temperature distribution tends to be more dependent upon the total amount of waste heat rather than the discharge design. In contrast, a wide range of dilutions is achievable in the near field through diverse types of discharge structures. It has been common practice to analyze near field temperatures by constructing scale models of the discharge structure. There is a pressing need for analytical models which relate the discharge characteristics to the flow and temperature distribution in the receiving water. Furthermore, since the analysis must almost always be performed in advance of actual plant construction, the analytical model must be totally predictive, containing no undetermined phenomenalogical coeffients.

This report describes the basis, structure, and use of a predictive model of the three-dimensional behavior of surface discharges of heated water. Emphasis is placed upon the assumptions underlying the theory, the scope of its validity, the nature of its limitations, and the proper application to actual discharges. For similar treatments of submerged discharges, the reader is referred to references 1 and 2.

1.2 Surface Discharges

The theory presented herein considers a discharge of heated water from a rectangular open channel at the surface of an ambient body of water of infinite extent in which a current may be flowing (see Figure 1.1). The three-dimensional temperature distribution depends upon the mixing between the discharged and ambient water and upon the rate of heat transfer to the atmosphere at the water surface.

Experimental investigations of three-dimensional buoyant surface jets have been reported by Tamai (3), Wiegel (4), Jen (5), Stefan (6) and Hayashi (7). Buoyant surface discharges are distinguished from non-buoyant turbulent jets by lateral gravitational spreading and by reduction of vertical entrainment such as described by Ellison and Turner (8) for the two-dimensional case. The net result of these two processes is that the velocity and temperature distributions are much wider than deep with the increased surface area raising the possibility (as suggested by Hayashi) for significant surface heat loss. Previous analytical treatments (Hoopes (9), Motz (10)) of heated surface discharges have failed to take into account, in a single three-dimensional theory, the roles of buoyancy, initial channel shape, turbulent entrainment, and surface heat loss upon the temperature distribution.

In this treatment the discharge is assumed to be a free turbulent jet with a well defined turbulent region in which velocity and temperature are related to centerline values by similarity functions. An unsheared core region is accounted for. Turbulent entrainment is represented by entrainment coefficients as first introduced by Morton, Taylor, and Turner (11) and applied to other buoyant jet problems by Morton (12) and Fan (1) among others. A major contribution of this work is the treatment of lateral buoyant spreading by incorporating an assumed distribution for the lateral velocity into the set of integrated governing equations. Surface heat loss is assumed to be determined by a single heat loss coefficient as defined by Edinger (13). In the presence of a cross current in the receiving water the jet is deflected by entrainment of ambient lateral momentum.

The basic philosophy behind the formulation of the theory is that the

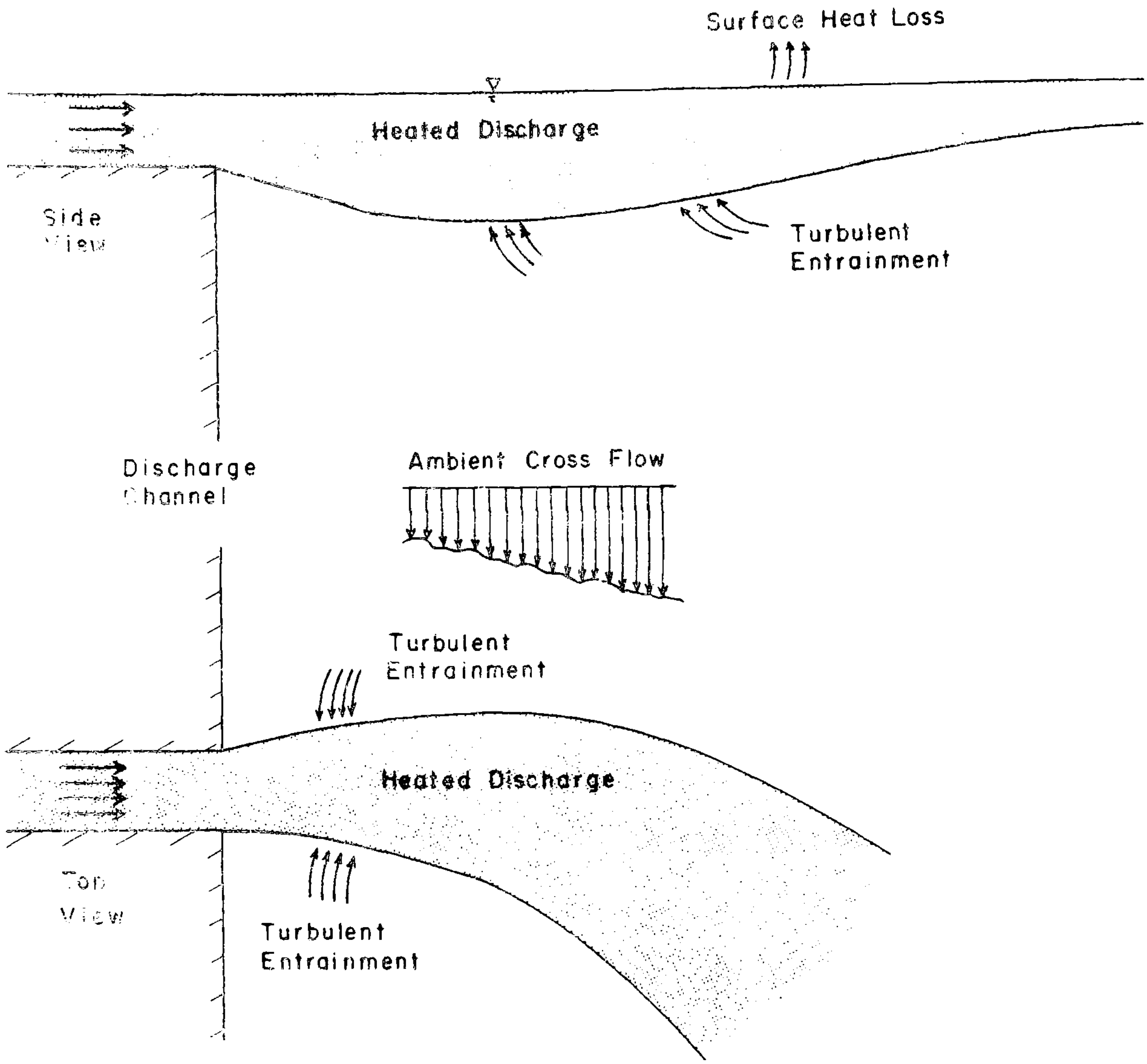

Fig. 1-1 Schematic of Heated Discharge

solution should match known non-buoyant jet behavior as the buoyancy terms go to zero. The effects of buoyancy are obtained by use of the basic equations and some judicious assumptions about the jet structure. In this way, the theory contains no undetermined coefficients and comparison of the theory with observations involves no curve fitting.

Because of the dependency of the treatment upon jet theory, the predictions of the model are valid only in the region where turbulence resulting from the discharge dominates ambient turbulent processes, i.e. in the near field.

II. The Mathematical Model

2.1 Basic Assumptions

The model considers a discharge Q_o of heated water at temperature T_o and density ρ_o from a rectangular open channel of depth h_o, width $2b_o$, and initial angle θ_o at the surface of a receiving body of water at temperature T_a, density ρ_a and of large extent laterally and longitudinally. It is assumed that the bottom of the receiving water does not interfere with the vertical development of the surface jet. A non-uniform current V may be present in the receiving water. It is assumed that this current is parallel to the shoreline; however, its magnitude may vary in the offshore direction as shown in Figure 2.1. Far from the jet the water surface, η, is uniformly at $z = 0$. The flow in the receiving water is characterized by its velocity, density, pressure and temperature. These four variables are related by equations expressing conservation of mass, momentum and heat and an equation of state. The solution of the equations must be developed by setting certain terms in the basic equations equal to zero on the basis of the following assumptions:

a) Steady flow: $\frac{\partial}{\partial t} = 0$

b) Large Reynolds number: viscous terms negligible

c) Boussinesq approximation: density gradients only important in pressure terms

d) Hydrostatic pressure: $p = -\int_{\eta}^{z} \rho g dz$

e) No jet induced motion at large depths: $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$ as $z \rightarrow -\infty$

f) Boundary layer flow: $\frac{\partial}{\partial x} << \frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z}$

g) Small density differences: $\frac{\rho_a - \rho_o}{\rho_a} << 1$

h) Mild jet curvature: $V/u_o < 1$

With the above assumptions the basic equations of mass, momentum and heat conservation may be simplified to the following form:

Mass Conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (2.1)$$

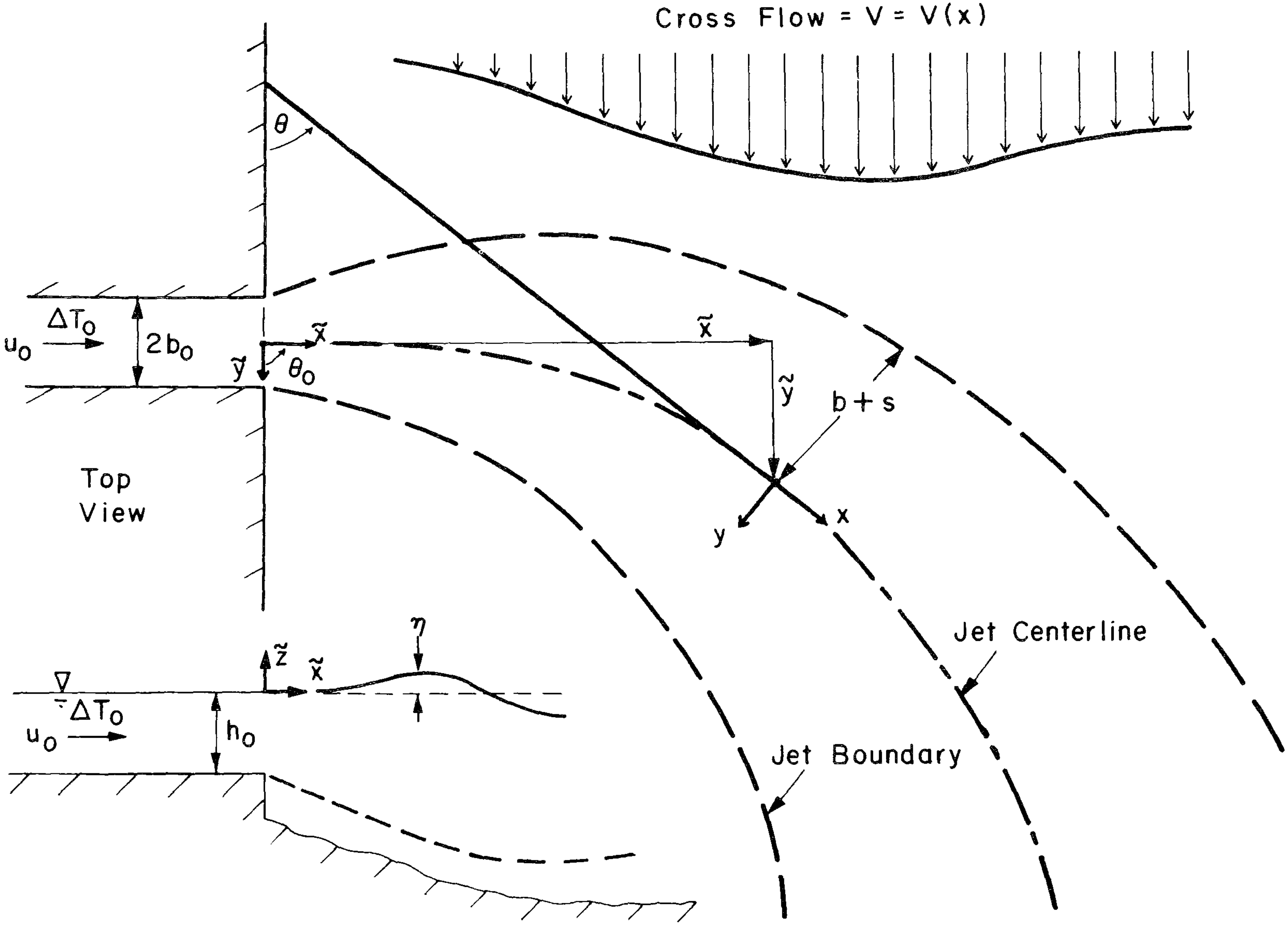

Fig. 2.1 Coordinate Definitions

x-Momentum Conservation

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = \beta g \int_z^{-\infty} \frac{\partial T}{\partial x}\, dz - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z} \qquad (2.2)$$

y-Momentum Conservation

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial wv}{\partial z} = \beta g \int_z^{-\infty} \frac{\partial T}{\partial y}\, dz + u^2 \frac{\partial \theta}{\partial x} \qquad (2.3)$$

Heat Conservation

$$\frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} = - \frac{\partial v'T'}{\partial y} - \frac{\partial w'T'}{\partial z} \qquad (2.4)$$

where:

x,y,z = coordinate directions relative to the jet centerline (see Figure 2.1)

u,v,w = mean velocity components

u',v',w' = turbulent fluctuating velocity components

T = mean temperature

T' = turbulent fluctuating temperature

θ = centerline deflection angle (see Figure 2.1)

g = gravitational acceleration

β = $\frac{1}{\rho}\frac{\partial \rho}{\partial T}$ where ρ = density

2.2 Discharge Structure

The governing equations (2.1-2.4) assumed for heated surface discharges may not be solved without further manipulation since the turbulent transfer terms are not determined. The technique used to develop the solution is to assume a structure for the velocity and temperature within the discharge, and boundary conditions at the outer edges, leaving as unknowns only certain values such as the centerline velocity and temperature. The governing equations may then be integrated over a cross-section perpendicular to the discharge centerline This procedure eliminates the unknown turbulent terms and yields a set of first order differential equations which may be solved for the variables describing the discharge behavior.

The assumed structure of the discharge is shown in Figure 2.2. The longitudinal velocity and temperature distributions are taken to be as follows where η is the water surface elevation and u_c and $\Delta T_c = T_c - T_a$ are the centerline velocity and temperature rise above ambient at $z = \eta$, $y = 0$:

$$
\begin{aligned}
&u = u_c + V\cos\theta \quad \Delta T = \Delta T_c && 0<|y|<s && -r<z<\eta \\
&u = u_c f(\zeta_z) + V\cos\theta \quad \Delta T = \Delta T_c t(\zeta_z) && 0<|y|<s && -(r+h)<z<-r \\
&u = u_c f(\zeta_y) + V\cos\theta \quad \Delta T = \Delta T_c t(\zeta_y) && s<|y|<s+b && -r<z<\eta \\
&u = u_c f(\zeta_y) f(\zeta_z) + V\cos\theta \quad \Delta T = \Delta T_c t(\zeta_y) t(\zeta_z) && s<|y|<s+b && -(r+h)<z<-r
\end{aligned}
\tag{2.5}
$$

where $\zeta_y = \dfrac{|y|-s}{b}$ and $\zeta_z = \dfrac{-z-r}{h}$

The lengths r and s pertain to the initial core region and h and b to the turbulent region of the jet (see Figure 2.2).

The particular forms of the similarity functions are assumed to be as follows:

$$
\begin{aligned}
f(\zeta) &= (1 - \zeta^{3/2})^2 \\
t(\zeta) &= 1 - \zeta^{3/2}
\end{aligned}
\tag{2.6}
$$

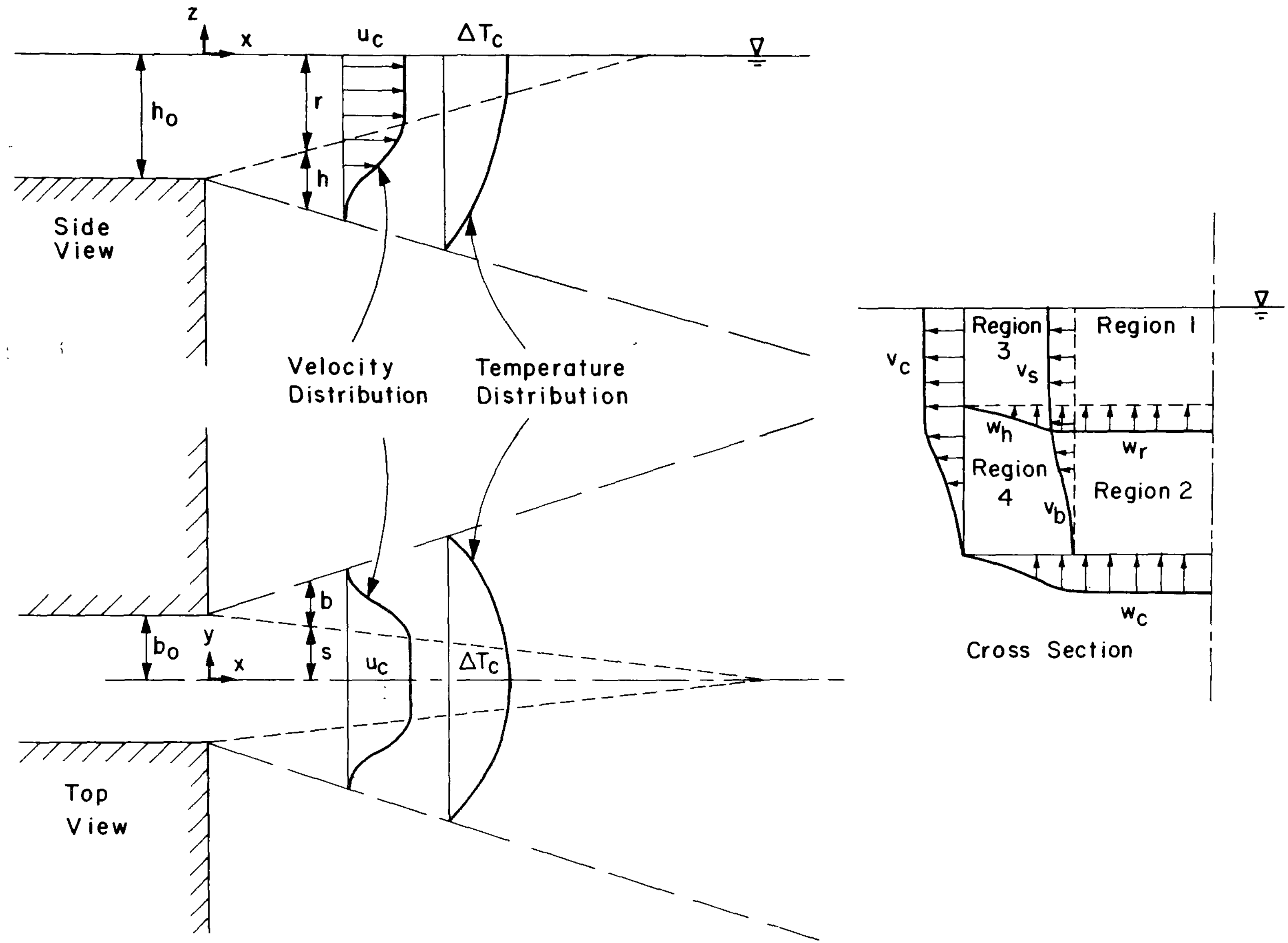

Fig. 2.2 Discharge Structure

The above functions were proposed by Abramovich (14). They have the desirable properties that a distinct jet boundary is defined at $\zeta = 1.0$ and that the velocity and temperature distributions are not identical but are related by $t = f^{1/2}$ as implied by Taylor's vorticity transfer theory.

The y momentum equation (2.3) expresses a balance between the lateral density gradients and lateral convective motions. It may be shown that for buoyant jets these lateral movements and thus the pressure gradient may be of the same order as convective transport in the x direction. The balance between lateral turbulent fluctuating pressures and Reynolds stresses, which is found in non-buoyant jets, is of second order and is neglected here. Since there are finite (but second order) lateral velocities in a non-buoyant jet, the lateral velocity v in equation (2.3) must be intrepeted as the buoyant spreading velocity in excess of the non-buoyant value. To enable an integration of equation (2.3) over the jet cross section, the distribution of the lateral spreading velocity, v, must be specified. The lateral velocity is geometrically related to the longitudinal velocity by:

$$\frac{v}{u} = \tan\phi \qquad (2.7)$$

where ϕ is the lateral jet streamline angle from the centerline in excess of the non-buoyant value. A distribution for u has already been given; it thus remains to choose a reasonable distribution for ϕ such that the following conditions are satisfied: (1) $\tan\phi = 0$ at $y = 0$ and (2) $\tan\phi = (\frac{db}{dx} - \varepsilon)$ at $y = b+s$ where ε is the lateral spreading rate of a <u>non-buoyant</u> jet under the same conditions (cross flow, channel size, etc.). Since the gravitational spread is induced by the lateral temperature gradient, the y dependence of $\frac{\partial T}{\partial y}$ is used to distribute $\tan\phi$ between $y = 0$ and $|y| = b + s$. The result is:

$$v = \pm\left(\frac{db}{dx} - \varepsilon\right) u \zeta_y^{1/2} \qquad s<|y|<s + b \text{ (v has the sign of y)}$$

$$v = 0 \qquad \text{elsewhere} \qquad (2.8)$$

The above distribution for v insures that for a non-buoyant jet in which $\frac{db}{dx} = \varepsilon$ the gravitational spreading velocity is identically zero everywhere.

Depending upon whether r or s are non-zero at a given centerline distance, the jet cross section will have 1, 2, or 4 regions (see Figure 2.2) on each side of the centerline (y = 0). To permit integration of the governing equations over each region separately, the velocities and the turbulent transfe of heat and momentum are specified at the boundaries of the regions. At the centerline of the jet, symmetry implies that there is no net transfer of mass, momentum or heat:

$$v = u'v' = v'T' = 0 \qquad y = 0 \tag{2.9}$$

At the boundaries between the regions the velocities are assumed to be:

$$\begin{array}{lll} u'w' = 0 & & \\ w = w_r & 0<|y|<s & \\ w = w_h f(\zeta_y) & s<|y|<s+b & \left.\right\} z = -r \\ w = 0 & s+b<|y| & \end{array}$$

$$\begin{array}{llll} u'v' = 0 & & & \\ v = \pm v_s & -r < z < \eta & \text{(v has the sign of y)} & \\ v = \pm v_b f(\zeta_z) & -(r+h)< z< -r & & \left.\right\} |y| = s \\ v = 0 & z < -(r+h) & & \end{array} \tag{2.10}$$

The internal velocities w_r, w_h, v_s, and v_b may be determined as part of the solution to the integrated governing equations but are of little interest in themselves.

The water surface is a boundary which permits no transfer of mass or momentum:

$$\left.\begin{aligned} u\frac{\partial\eta}{\partial x} + v\frac{\partial\eta}{\partial y} &= w \\ \overline{u'w'} &= 0 \end{aligned}\right\} \quad z = \eta \tag{2.11}$$

The transfer of heat through the water surface is assumed to be proportional to the surface temperature rise above ambient:

$$\overline{w'T'} = k(T - T_a) \qquad z = \eta \tag{2.12}$$

The coefficient of heat loss, k, has units of velocity and is thus a kinematic quantity. It is related to the surface heat exchange coefficient, K, defined by Edinger (13) by:

$$k = \frac{K}{\rho c} \tag{2.13}$$

where ρ = density and c = specific heat of water. The determination of k(or K) for a particular case is discussed in a later section.

The outer boundaries of the jet is where entrainment of ambient water occurs but across which no heat is transferred. The boundary conditions are:

$$\begin{aligned} &\overline{u'w'} = \overline{w'T'} = 0 \\ &w = w_e - V\cos\theta\,\frac{dh}{dx} && 0<|y|<s && z = -(r+h) \\ &w = w_e f(\zeta_y) - V\cos\theta\,\frac{dh}{dx} && s|<y|<s+b \\ & && && (2.14) \\ &\overline{u'v'} = \overline{v'T'} = 0 \\ &v = \pm v_e + V\cos\theta\,\frac{db}{dx} && -r<z<\eta && y = s+b \\ &v = \pm v_e f(\zeta_z) + V\cos\theta\,\frac{db}{dx} && -(r+h)<z<-r \end{aligned}$$

(v has the sign of y)

The velocities, w_e and v_e, are manifestations of the entrainment of ambient fluid into the turbulent region. In plane and axisymmetric non-buoyant jets, it is known that the entrainment velocity is proportional to the local center-line velocity. Note that Equation 2.8 gives $v = 0$ at $|y| = s + b$ where the boundary condition 2.14 specifies $|v| = v_e$. This is because the entrainment velocity v_e is an order of magnitude less than the spreading velocity v and its contribution to the integration of equation (2.3) is assumed to be balanced by the turbulent terms. which were neglected in the y equation. Ellison and Turner (8) have demonstrated that the vertical entrainment is a function of the gross Richardson number, $\frac{\beta g \Delta T_c h}{u_c^2}$, in two-dimensional jets. In this study the entrainment velocities are assumed to be given by

$$
\begin{aligned}
\frac{v_e}{u_c} &= \alpha_y \\
\frac{w_e}{u_c} &= \alpha_z \exp \left\{ -C \frac{\beta g \Delta T_c h}{u_c^2} \right\}
\end{aligned}
\tag{2.15}
$$

Ellison and Turner's data indicate a value of C = 5.0 as appropriate. The entrainment coefficients, α_y and α_z, are to be determined such that the solution for the non-buoyant case ($T_o = T_a$) agrees with the experimental observations that the growth of a non-buoyant turbulent region is symmetrical:

$$
\begin{aligned}
\frac{db}{dx} &= \frac{dh}{dx} = \varepsilon \\
\frac{ds}{dx} &= \frac{dr}{dx}
\end{aligned}
\tag{2.16}
$$

For non-buoyant jets discharging into a quiescent receiving water the spreading rate, ε_o, is constant. In these cases, Abramovich gives $\varepsilon_o = .22$ for the similarity functions, f and t, used here.

The boundary condition at $x = 0$ is related to the discharge channel geometry, the flow rate Q_o, and the initial discharge temperature T_o.

$$\left.\begin{aligned} r &= h_o \\ s &= b_o \\ h &= b = 0 \\ u &= u_o = \frac{Q_o}{2h_o b_o} + V\cos\theta_o \\ \Delta T_c &= \Delta T_o \\ \theta &= \theta_o \\ \tilde{x} &= \tilde{y} = 0 \end{aligned}\right\} \quad x = 0 \tag{2.17}$$

2.3 Integration of the Equations

With the velocity and temperature distributions and boundary conditions stated in the previous section, the equations of motion may be integrated over the y-z cross section of the jet. The x momentum and mass conservation equations are integrated over each of the four possible jet regions on each side of the centerline plane. (Integration over both sides of the jet results in redundant equations because of the assumed symmetry.) This yields eight equations. The y-momentum and heat equations are integrated over the entire half-jet cross-section, yielding two more equations. With this choice of integrating limits, the terms in the integrated y-equation represent a balance between the lateral gravitational force and the lateral spreading of the discharge. Another equation is generated from the y-equation by integrating it over the entire cross section of the jet on both sides of the centerline. In this case the lateral spreading terms all drop out, being anti-symmetrical, and the remaining terms give the rate of deflection of the jet in the presence of a cross current. The equation set, completed by a simple geometrical relationship between the coordinates (x,y) referred to the jet centerline and the fixed centerline coordinates $(\tilde{x},\tilde{y})$ is given in Table 2.1. Further details in the derivation of these equations are given in Stolzenbach and Harleman (15).

Region 1: continuity $rs \frac{d}{dx} [u_c + V\cos\theta] + rv_s - sw_r = 0$

Region 2: continuity $s \left[\frac{d}{dx} [h(u_c I_1 + V\cos\theta)] + (u_c + V\cos\theta) \frac{dr}{dx} + w_r - \alpha_{sz} u_c\right] + v_b h I_1 = 0$

Region 3: continuity $r \left[\frac{d}{dx} [b(u_c I_1 + V\cos\theta)] + (u_c + V\cos\theta) \frac{ds}{dx} - v_s + \alpha_y u_c\right] - w_r b I_1 = 0$

Region 4: continuity $\frac{d}{dx} \left[hb(u_c I_1{}^2 + V\cos\theta)\right] + (u_c I_1 + V\cos\theta) \left[b \frac{dr}{dx} + h \frac{ds}{dx}\right] + (w_h - \alpha_{sz} u_c) I_1 b$

$- (v_b - \alpha_y u_c) I_1 h = 0$

Region 1: x momentum $(u_c + V\cos\theta)[2rs \frac{d}{dx} [u_c + V\cos\theta] + rv_s - sw_r] + \beta g\ s\ [\frac{d}{dx} (\Delta T_c \frac{r^2}{2}) + I_3 r \frac{d}{dx} (\Delta T_c h)] = 0$

Region 2: x momentum $s \left[\frac{d}{dx} [h\ (u_c{}^2 I_2 + 2V\cos\theta u_c I_1 + V^2\cos^2\theta)] + [u_c + V\cos\theta]^2 \frac{dr}{dx}\right.$

$\left. + w_r [u_c + V\cos\theta] - \alpha_{sz} u_c V\cos\theta + \beta g\ [I_4 \frac{d\Delta T_c h^2}{dx} + I_3 \Delta T_c h \frac{dr}{dx}]\right] + v_b h\ (u_c I_2 + V\cos\theta I_1) = 0$

Table 2.1 Integrated Equations for Deflected Buoyant Jets

Region 3: x momentum

$$r\left[\frac{d}{dx}\left[b(u_c^2 I_2 + 2V\cos\theta u_c I_1 + V^2\cos^2\theta)\right] + [u_c + V\cos\theta]^2 \frac{ds}{dx} - v_s\,[u_c + V\cos\theta] + \alpha_y u_c V\cos\theta\right] - w_h b\,[u_c I_2 + V\cos\theta I_1] + \beta g\left[\frac{I_3}{2}\,\frac{d\Delta T_c b r^2}{dx} + I_3^2 r\,\frac{d\Delta T_c bh}{dx} + \Delta T_c\left(\frac{r^2}{2} + I_3 hr\right)\frac{ds}{dx}\right] = 0$$

Region 4: x momentum

$$\frac{d}{dx}\left[hb(u_c^2 I_2^2 + 2V\cos\theta u_c I_1^2 + V^2\cos^2\theta)\right] + [u_c^2 I_2 + 2V\cos\theta u_c I_1 + V^2\cos^2\theta]\left[b\,\frac{dr}{dx} + h\,\frac{ds}{dx}\right] + [w_h b - v_b h]\,[u_c I_2 + V\cos\theta I_1] - u_c I_1\,[\alpha_{sz} b - \alpha_y h]V\cos\theta + \beta g\left[I_3 I_4\,\frac{d\Delta T_c h^2 b}{dx} + I_4 \Delta T_c h^2\,\frac{ds}{dx} + I_3^2\,\Delta T_c hb\,\frac{dr}{dx}\right] = 0$$

Jet y momentum

$$\frac{d}{dx}\left[\frac{db}{dx} - \varepsilon\right][u_c^2 b I_6(r + hI_2) + 2V\cos\theta u_c b I_5(r + hI_1) + V^2\cos^2\theta b(r+h)] - \beta g\,\Delta T_c\left(\frac{r^2}{2} + I_3 rh + I_4 h^2\right) = 0$$

Table 2.1 (cont'd) Integrated Equations for Deflected Buoyant Jets

Jet Heat

$$\frac{d}{dx}\left[u_c \Delta T_c (s + bI_7)\ (r + hI_7) + V\cos\theta \Delta T_c\ (s + bI_3)\ (r + hI_3)\right]$$

$$+\ k\Delta T_c\ (s + bI_3) = 0$$

Jet Bending

$$\left[u_c^{\ 2}\ (s + bI_2)\ (r + hI_2) + 2V\cos\theta u_c\ (s + bI_1)\ (r + hI_1)\ +\right.$$

$$\left.V^2\cos^2\theta(s+b)\ (r + h)\right]\ \frac{d\theta}{dx} - u_c V\sin\theta\left[-\alpha_{sz}(s + bI_1) + \alpha_y (r + hI_1)\right] = 0$$

Jet x position

$$\frac{d\tilde{x}}{dx} - \sin\theta = 0$$

Jet y position

$$\frac{d\tilde{y}}{dx} - \cos\theta = 0$$

$$I_1 = \int_0^1 f(\zeta)\, d\zeta = \int_0^1 (1 - \zeta^{3/2})^2\, d\zeta = .4500$$

$$I_2 = \int_0^1 f^2(\zeta)\, d\zeta = \int_0^1 (1 - \zeta^{3/2})^4\, d\zeta = .3160$$

$$I_3 = \int_0^1 t(\zeta) d\zeta = \int_0^1 (1 - \zeta^{3/2}) d\zeta = .6000$$

$$I_4 = \int_0^1 \int_\zeta^1 t(\zeta)\, d\zeta d\zeta = \int_0^1 \int_\zeta^1 (1 - \zeta^{3/2})\, d\zeta d\zeta = .2143$$

$$I_5 = \int_0^1 f(\zeta)\, \zeta^{1/2}\, d\zeta = \int_0^1 (1 - \zeta^{3/2})^2\, \zeta^{1/2}\, d\zeta = .2222$$

$$I_6 = \int_0^1 f^2(\zeta)\, \zeta^{1/2} d\zeta = \int_0^1 (1 - \zeta^{3/2})^4\, \zeta^{1/2}\, d\zeta = .1333$$

$$I_7 = \int_0^1 f(\zeta)\, t(\zeta)\, d\zeta = \int_0^1 (1 - \zeta^{3/2})^3\, d\zeta = .3680$$

Table 2.1 (cont'd) Integrated Equations for Deflected Buoyant Jets

The values of the non-buoyant ($\Delta T_o = 0$) entrainment coefficients which satisfy equations 2.16 in addition to Table 2.1 are:

$$
\begin{aligned}
\alpha_y &= -(I_1 - I_2)\varepsilon_o && \text{for} && s > 0 \\
\alpha_y &= -\frac{I_1 \varepsilon_o}{2} && \text{for} && s = 0 \\
\alpha_z &= (I_1 - I_2)\varepsilon_o && \text{for} && r > 0 \\
\alpha_z &= \frac{I_1 \varepsilon_o}{2} && \text{for} && r = 0
\end{aligned}
\tag{2.18}
$$

where I_1 and I_2 are integration constants as given in Table 2.1.

2.4 Solution of the Equations

The thirteen equations in Table 2.1 are a first order system of dimensional differential equations in x for the variables: u_c, ΔT_c, h, b, r, s, θ, $\tilde{x}$, $\tilde{y}$, v_s, v_b, w_r, and w_h. The actual solution proceeds first by writing the equations in a dimensionless form by normalizing each variable by the characteristic values: u_o, $\Delta T_o = T_o - T_a$, and $\sqrt{h_o b_o}$. The solution is then determined by the following dimensionless parameters:

$$\mathbb{F}_o = \frac{u_o}{\sqrt{\beta g \Delta T_o h_o}} = \text{initial densimetric Froude number}$$

$$A = h_o/b_o = \text{aspect ratio}$$

$$k/u_o = \text{heat loss parameter}$$

$$V/u_o = \text{cross flow parameter}$$

The computer program which solves the equations is described in Section IV along with instructions for its use. The output consists of values of u/u_o, $\Delta T_c/\Delta T_o$, $h/\sqrt{h_o b_o}$, $b/\sqrt{h_o b_o}$, $r/\sqrt{h_o b_o}$, $s/\sqrt{h_o b_o}$, θ, $\tilde{x}/\sqrt{h_o b_o}$ & $\tilde{y}/\sqrt{h_o b_o}$ as a function of $x/\sqrt{h_o b_o}$. In addition to these variables which describe the jet structure (u_c, ΔT_c, . . ., etc.) other interesting dependent quantities which are functions of x may be defined. In the buoyant jet the vertical entrainment is a function of the local densimetric Froude number (an inverse Richardson number), $\mathbb{F}_L$:

$$\mathbb{F}_L = \frac{u_c}{\sqrt{\beta g \Delta T_c h}} \tag{2.19}$$

where u_c, ΔT_c, and h are local values at a given distance from the origin. The total flow in the jet may be determined by integrating the x velocity, u, over the jet cross section. The ratio of the flow at a given x to the initial flow is the jet dilution, D, or Q as labeled by the output,

$$D = \frac{u_c(r + I_1 h)(s + I_1 b) + V\cos\theta(r + h)(s + b)}{(u_o + V\cos\theta_o)h_o b_o} \tag{2.20}$$

Similarly the effect of surface heat loss may be evaluated by calculating the ratio of convected excess heat flow in the jet to the initial excess heat flow, HT.

$$HT = \frac{u_c \Delta T_c(r + I_7 h)(s + I_7 b) + V\cos\theta\Delta T_c(r + h)(s + b)}{\Delta T_o(u_o + V\cos\theta_o)\, h_o b_o} \tag{2.21}$$

Finally a dimensionless time of travel along the jet centerline from the end of the discharge channel to x is computed.

$$TM = \frac{u_o}{\sqrt{h_o b_o}} \int_o^x \frac{dx}{u_c} \tag{2.22}$$

The structure of a heated surface discharge is shown for a particular theoretical calculation in Figures 2.3 and 2.4. The main features are:

1) A core region in which the centerline velocity is constant and the centerline temperature rise decreases very slightly. The dilution, D, and the local densimetric Froude number, $\mathbb{F}_L$, do not vary greatly in this region. The magnitude of $\mathbb{F}_L$ is much larger than $\mathbb{F}_o$ because the initial depth of the turbulent region, h, is zero. There is no significant surface heat loss in the core region.

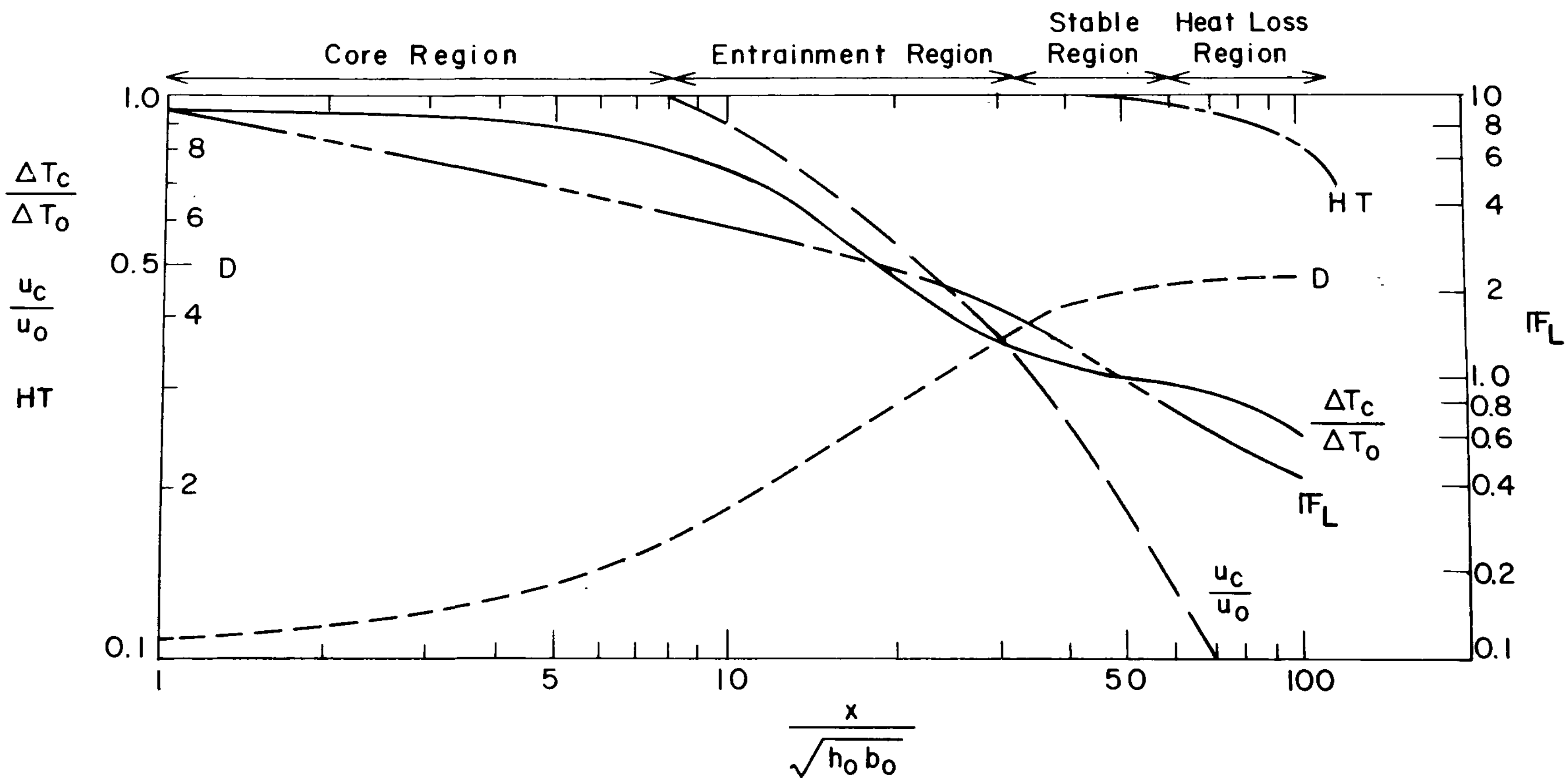

Fig. 2.3 Calculated Surface Discharge Parameters: $\mathbb{F}_o = 4.4$, $A = 0.35$, $k/u_o = 4.2 \times 10^{-5}$, $V/u_o = 0$

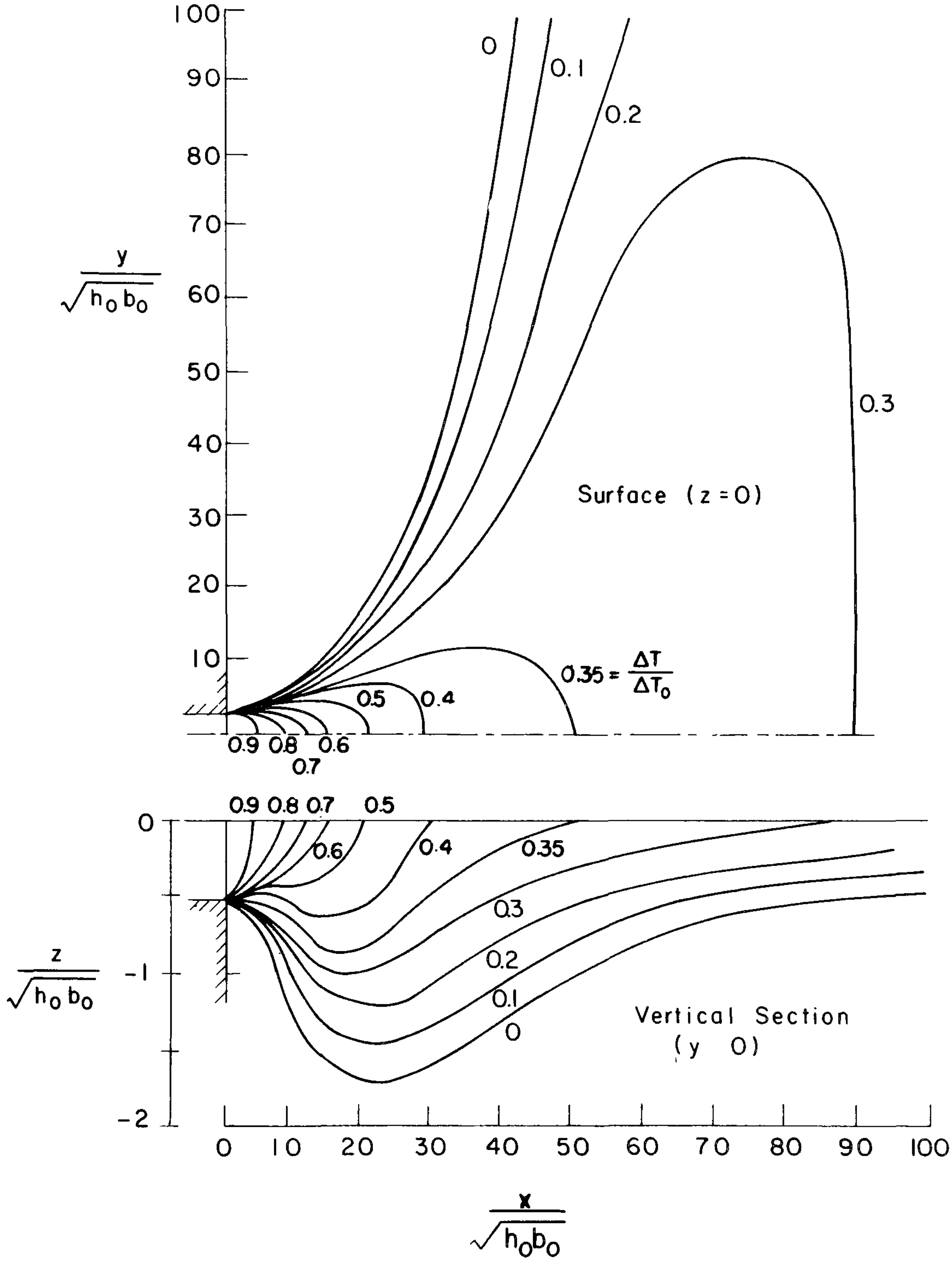

Fig. 2.4 Calculated Isotherms of $\Delta T / \Delta T_0$ · $\mathbb{F}_0$ - 4.4, A =0.35, k/u_0 4.2 x 10^{-5}, V/u_0 0

2) An entrainment region in which the centerline velocity and temperature drop sharply, approximately as $1/x$, as in a non-buoyant jet. The jet spreads vertically by turbulent processes. The lateral growth is dominated by gravitational spreading at a much greater rate than the vertical turbulent spread. Because of this large ratio of lateral to vertical spread, the jet reaches a maximum depth beyond which the bottom boundary rises to maintain mass conservation (see Figure 2.4). Local densimetric Froude numbers in this region decrease rapidly and the dilution rises sharply as a result of entrainment (see Figure 2.3). Surface heat loss remains negligible in this region, i.e., HT = 1.0.

3) A stable region in which vertical entrainment is inhibited by vertical stability as indicated by the local densimetric Froude number which is of order one or less. The jet depth continues to decrease because of lateral spreading. The small jet depths reduce the lateral entrainment, and the dilution and centerline temperature remain relatively constant in this region. The centerline velocity, however, drops sharply as a consequence of the large lateral spread. The surface temperature pattern is dominated by the wide, constant temperature stable region.

4) A heat loss region marking the end of the stable region. The lateral spread is sufficiently large to allow significant surface heat transfer and the temperature begins to fall again. Once surface heat loss becomes significant, the rate of temperature decrease is very rapid. However, at this point the centerline velocity is so low that the discharge may no longer be considered as a jet. In general it may be stated that surface heat loss, as determined by the value of k/u_o, is not an important factor in reducing the discharge temperature within the region of jet-like entrainment. Beyond the stable region, temperature distribution is controlled by passive diffusion processes acting upon the buoyant plume.

5) The primary effect of a current in the receiving water is to deflect the discharge without significantly affecting the dilution processes up to the point where the jet centerline velocity excess, u_c, is reduced to the magnitude of the cross current component in the centerline direction. For $u_c \leq V\cos\theta$ the ambient water may have a turbulence level comparable to the discharge turbulence and the basic assumptions of the theory will not be valid. This region is properly considered part of the far field.

The dilution and corresponding centerline temperature achieved in the stable region are of interest since the stable region constitutes a significant portion of the surface temperature distribution. Figure 2.5 is a plot of the dilution and surface temperature in the stable region as a function of $\mathbb{F}_o$ with values of A indicated. It is clear that the ultimate stable dilution in a buoyant jet depends primarily upon $\mathbb{F}_o$ and to a lesser extent upon A.

The maximum depth of the discharge, $h_{max}/\sqrt{h_o b_o}$, is also of interest and may be determined from the theoretical calculations. Figure 2.6 indicates that the maximum vertical penetration of the jet is a function of $\mathbb{F}_o$ with a very small dependence upon A.

For quick calculation involving the maximum jet penetration, centerline dilution, or centerline temperature rise, the results from Figure 2.5 and 2.6 can be condensed into three simplified formulas involving a new parameter, $\mathbb{F}'_o$, defined by

$$\mathbb{F}'_o = \mathbb{F}_o A^{1/4} = \frac{u_o}{\sqrt{g\left(\frac{\Delta\rho_o}{\rho_a}\right)(h_o b_o)^{1/2}}} \qquad (2.23)$$

$\mathbb{F}'_o$ is merely a "Froude number" whose characteristic length is the scaling length, $(h_o b_o)^{1/2}$, rather than a depth. The numerical results of Figures 2.5 and 2.6 are then approximated by

$$\left(\frac{\Delta T_o}{\Delta T_c}\right)_s = \sqrt{(\mathbb{F}'_o)^2 + 1} \simeq \mathbb{F}'_o \quad (\text{for } \mathbb{F}'_o > 3) \qquad (2.24)$$

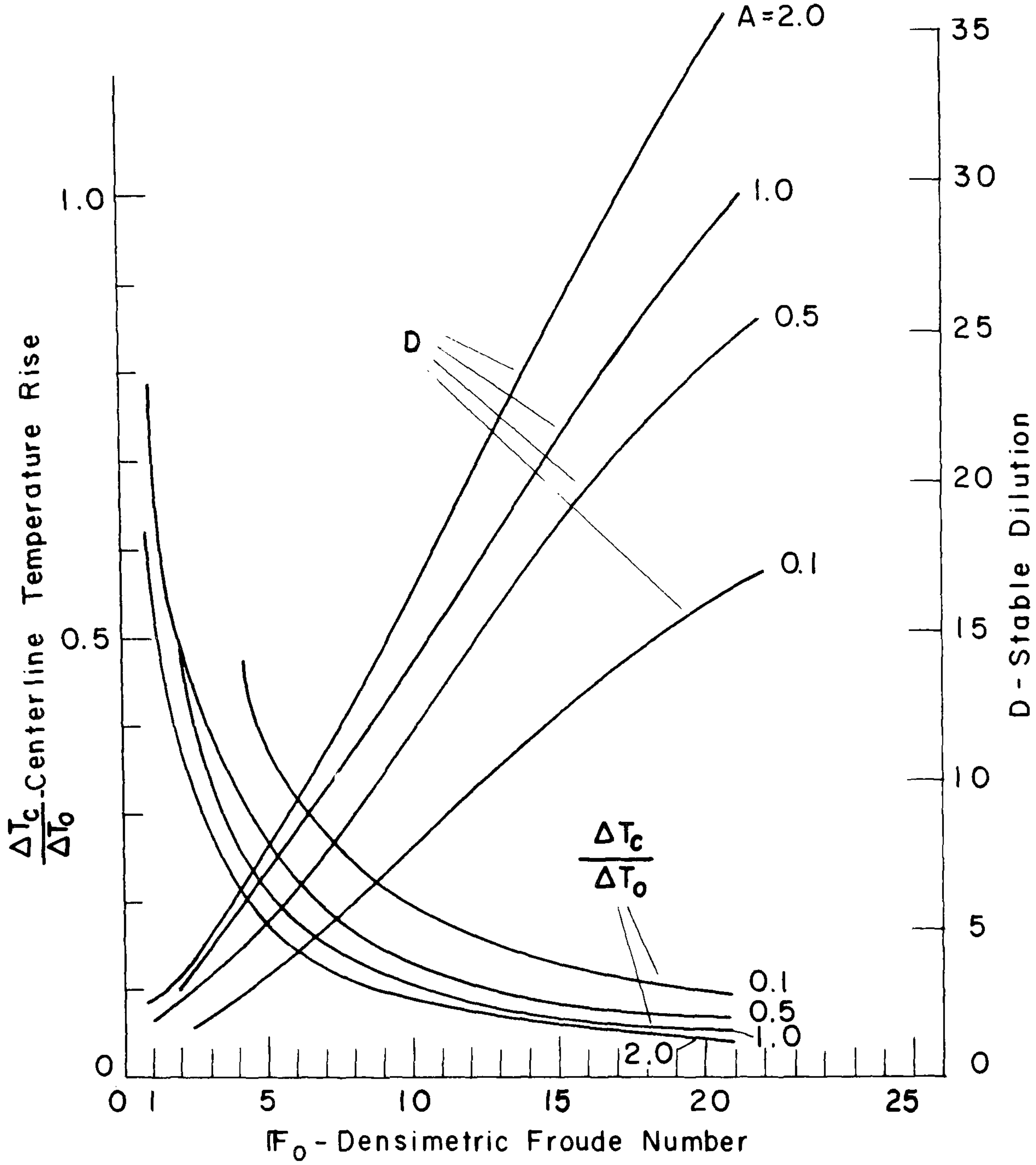

Fig. 2.5 Centerline Temperature Rise $\Delta T_c/\Delta T_o$ and Dilution, D, in the Stable Region for $k/u_o = V/u_o = 0$.

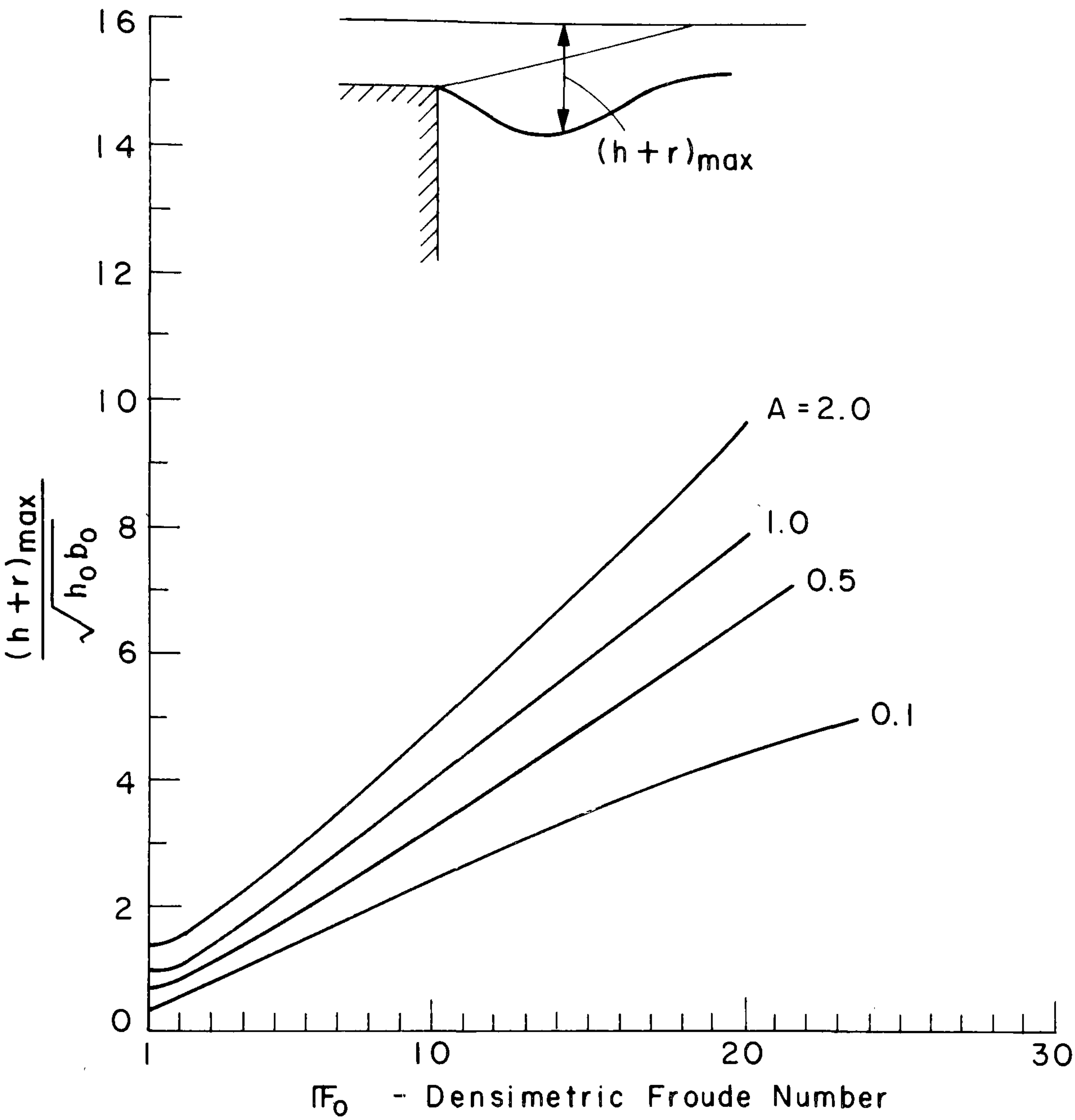

Fig. 2.6 Maximum Vertical Depth of the Jet, $\frac{(h+r)_{max}}{\sqrt{h_o b_o}}$ for $k/u_o = V/u_o = 0$

$$D_{stable} = 1.4\sqrt{(\mathbb{F}_o')^2 + 1} \simeq 1.4\ \mathbb{F}_o'\ (\text{for } \mathbb{F}_o' > 3) \qquad (2.25)$$

$$\frac{(h + r)_{max}}{(h_o b_o)^{1/2}} = 0.42\ \mathbb{F}_o' \qquad (2.26)$$

For $\mathbb{F}_o' > 3$, $h_{max} = (h + r)_{max}$ and (2.26) reduces to

$$\frac{h_{max}}{(h_o b_o)^{1/2}} = 0.42\ \mathbb{F}_o' \qquad (2.27)$$

Equations 2.24 and 2.25 relate to properties of the jet in the stable region (stable centerline temperature rise and stable dilution respectively) and (2.26) relates to a maximum property of the jet. For the spatial history of the discharge or for information relating to lateral spreading or centerline deflection under a current, use of the generalized plots of the theoretical solution is suggested. Figures 2.7a to 2.7i give the basic calculated discharge properties for a range of values of $\mathbb{F}_o$, A, and V/u_o with k/u_o set equal to zero. As discussed previously, a non-zero value of k/u_o will have only secondary effect upon the temperature distribution and setting $k/u_o = 0$ will always yield a slightly conservative result (i.e. higher temperatures). The next two chapters discuss the computer program and its application to actual discharge design.

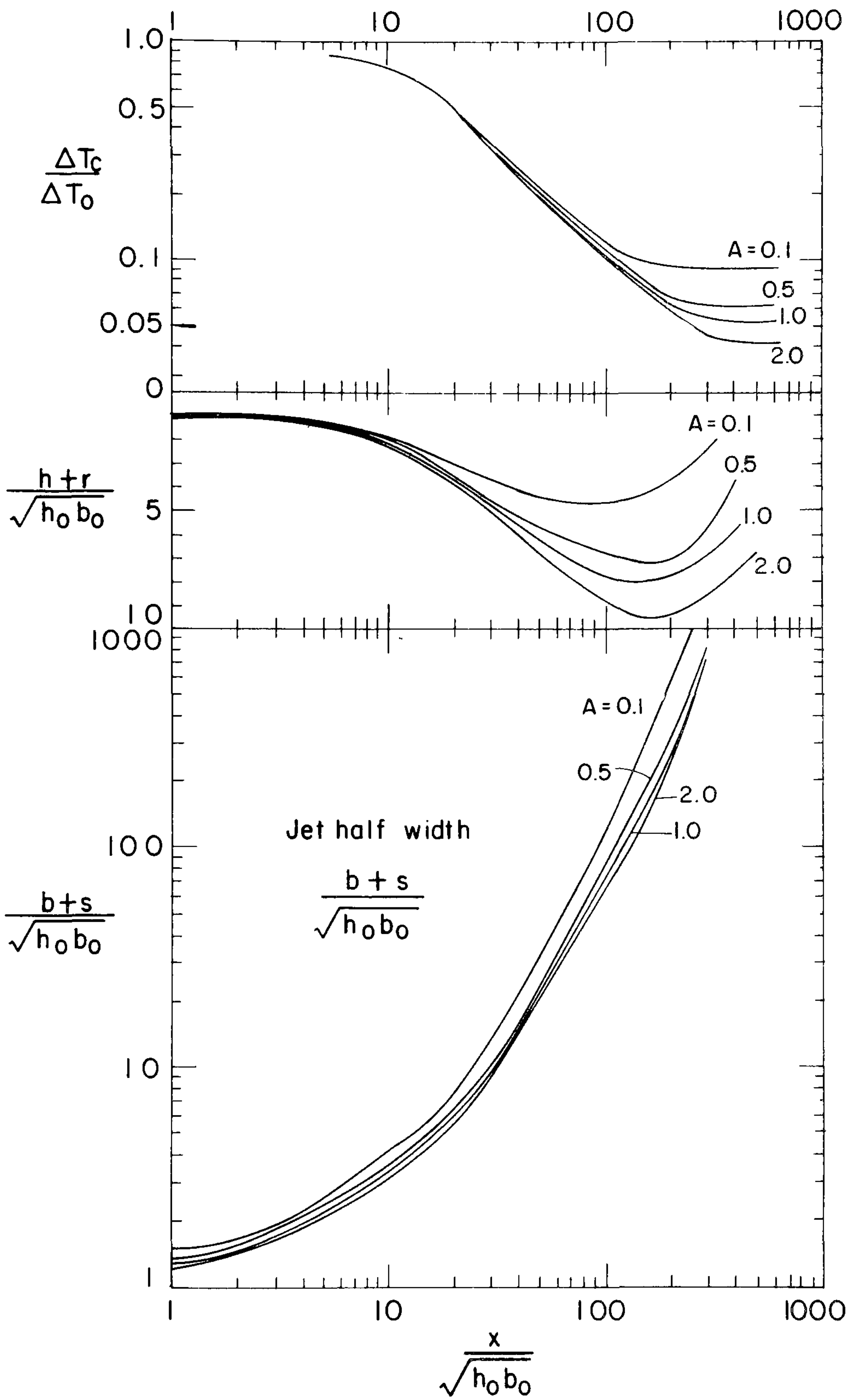

Fig. 2.7a Jet Parameters for $\mathbb{F}_0 = 20.0$, $V/u_o = 0$, $k/u_o = 0$

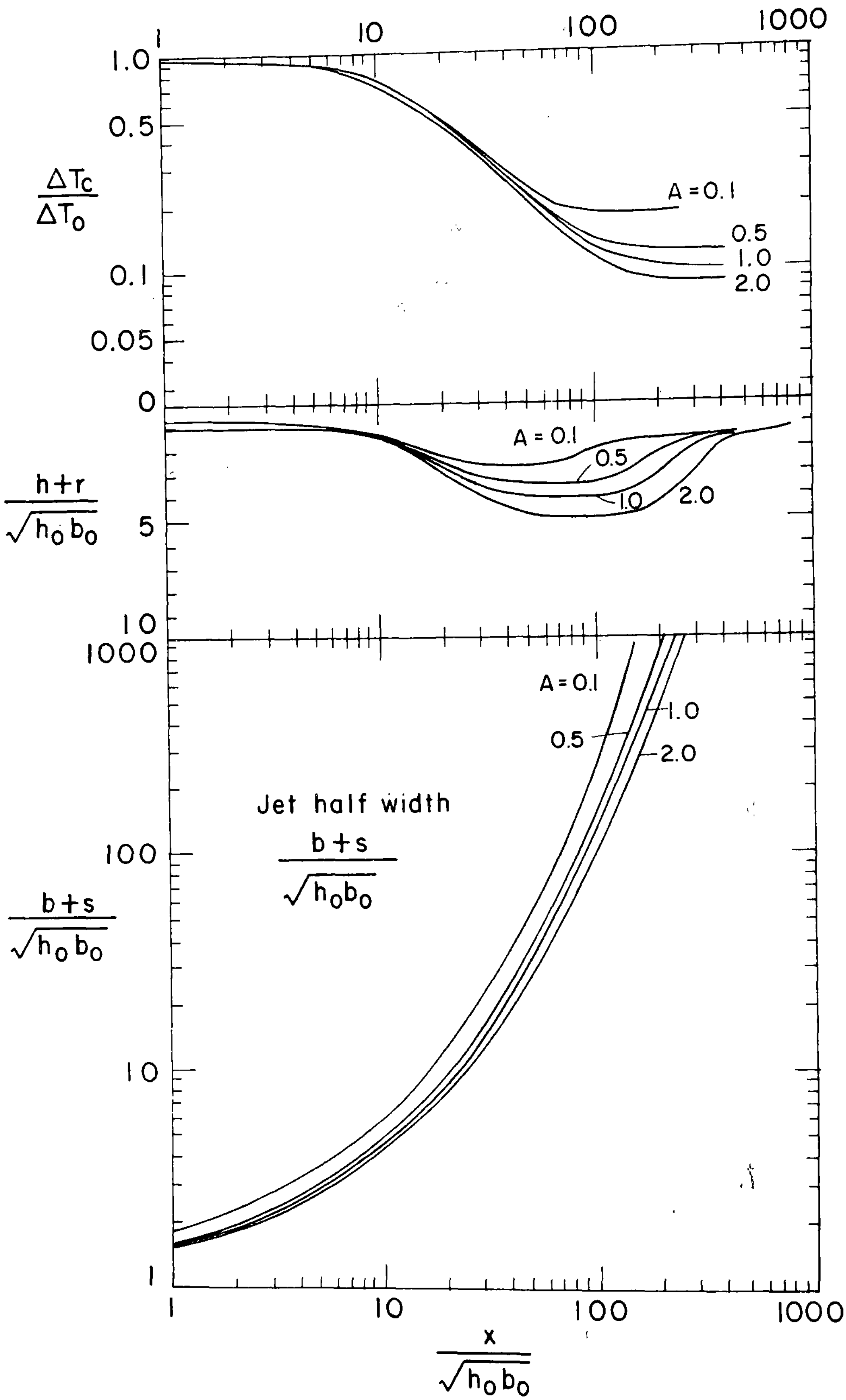

Fig. 2.7b Jet Parameters for $\mathbb{F}_0 = 10.0$, $V/u_0 = 0$, $k/u_0 = 0$

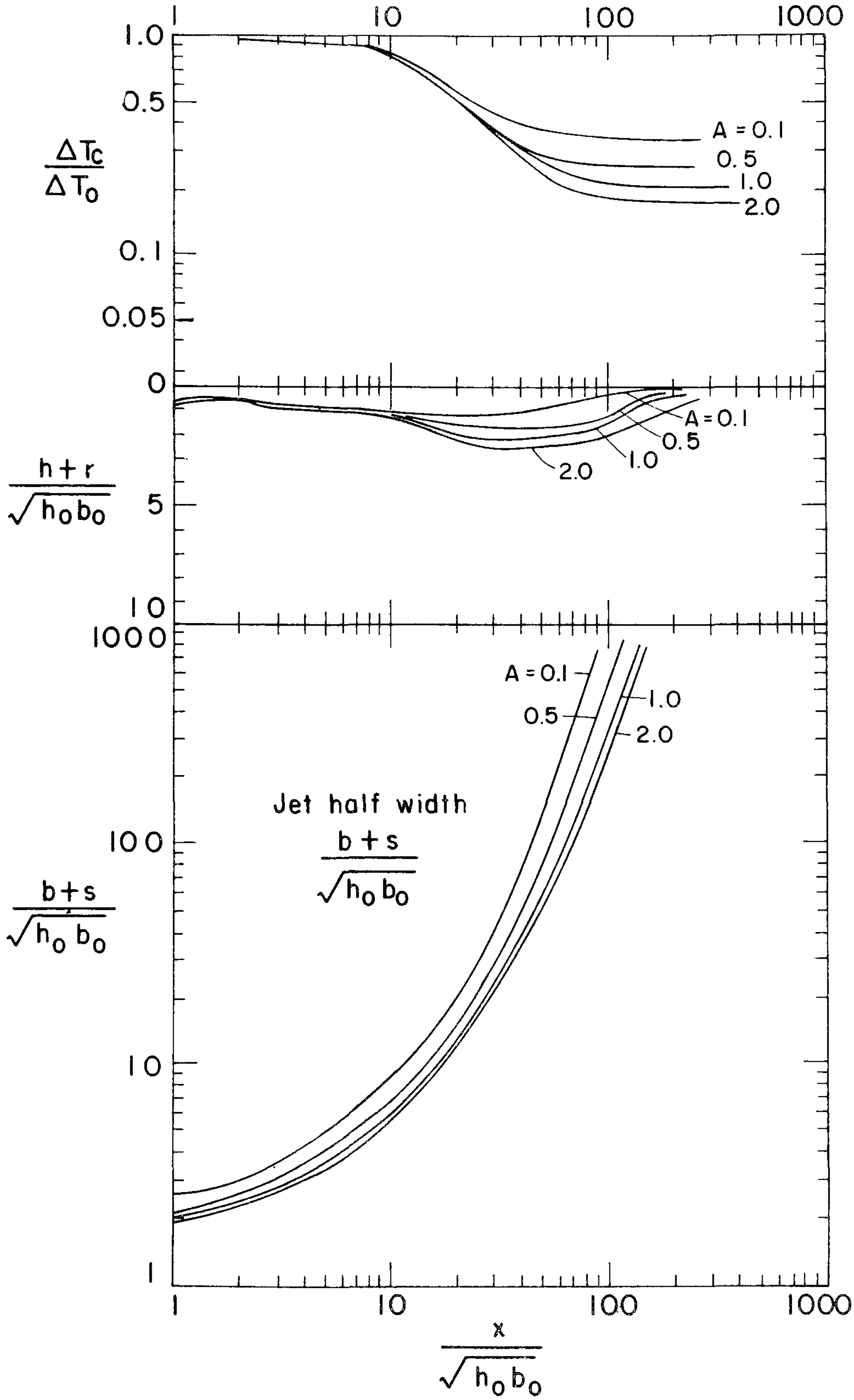

Fig. 2.7c Jet Parameters for $\mathbf{F}_o = 5.0$, $V/u_o = 0$, $k/u_o = 0$

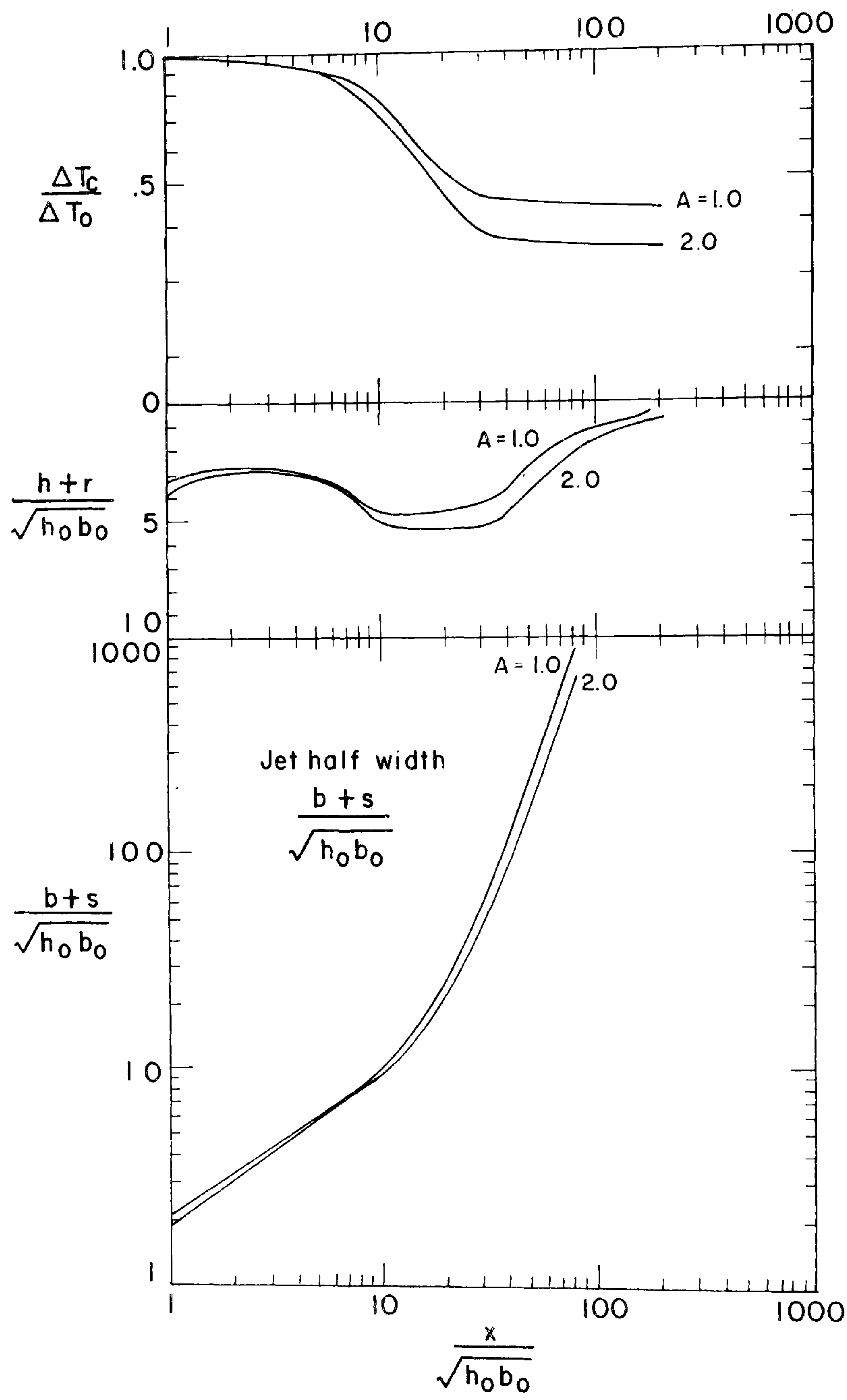

Fig. 2.7d Jet Parameters for $\mathbb{F}_0 = 2.0$, $V/u_0 = 0$, $k/u_0 = 0$

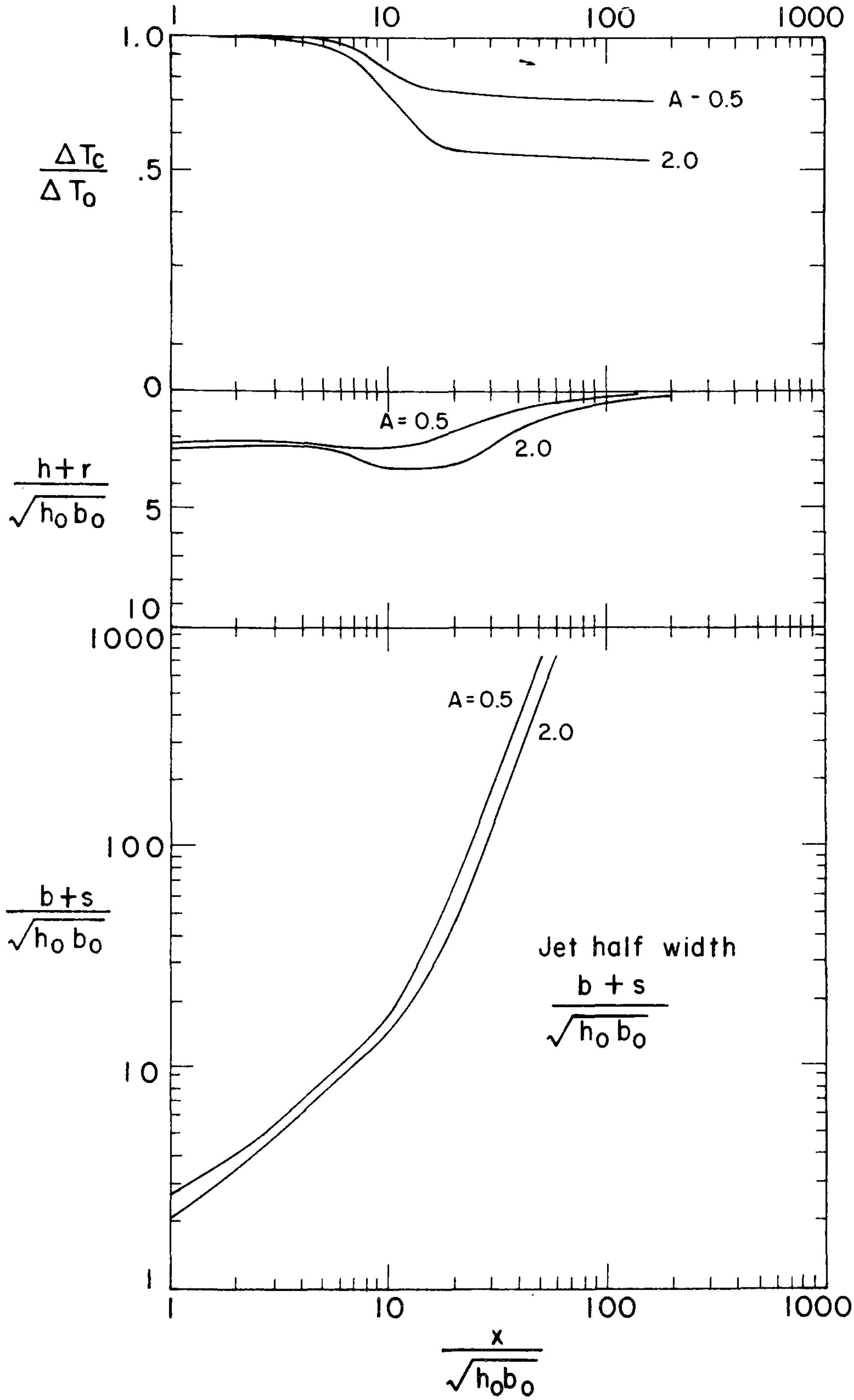

Fig. 2.7e Jet Parameters for $\mathbb{F}_0 = 1.0$, $V/u_0 = 0$, $k/u_0 = 0$

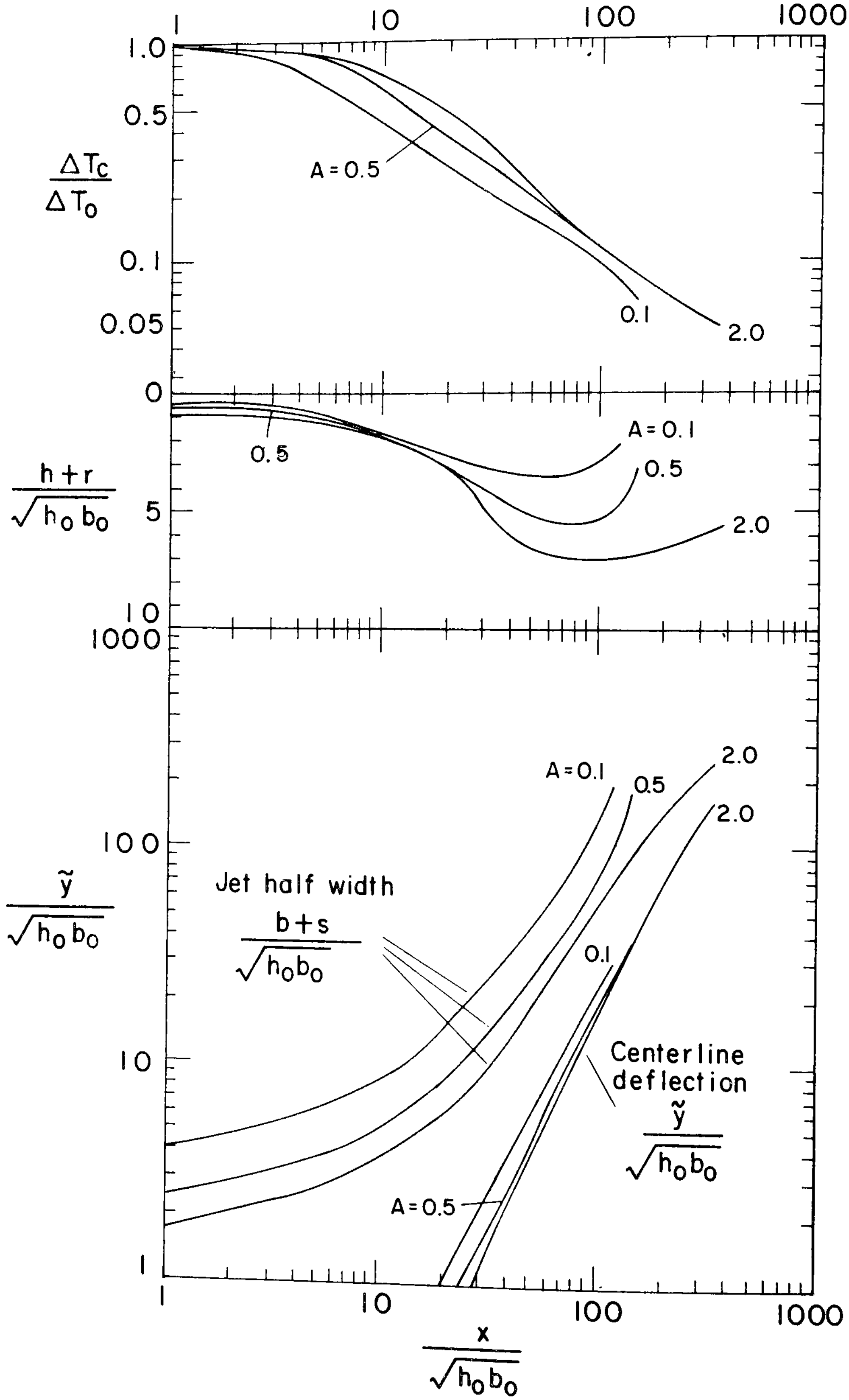

Fig. 2.7 f Jet Parameters for $\mathbb{F}_0 = 20.0$, $V/u_0 = 0.025$, $k/u_0 = 0$

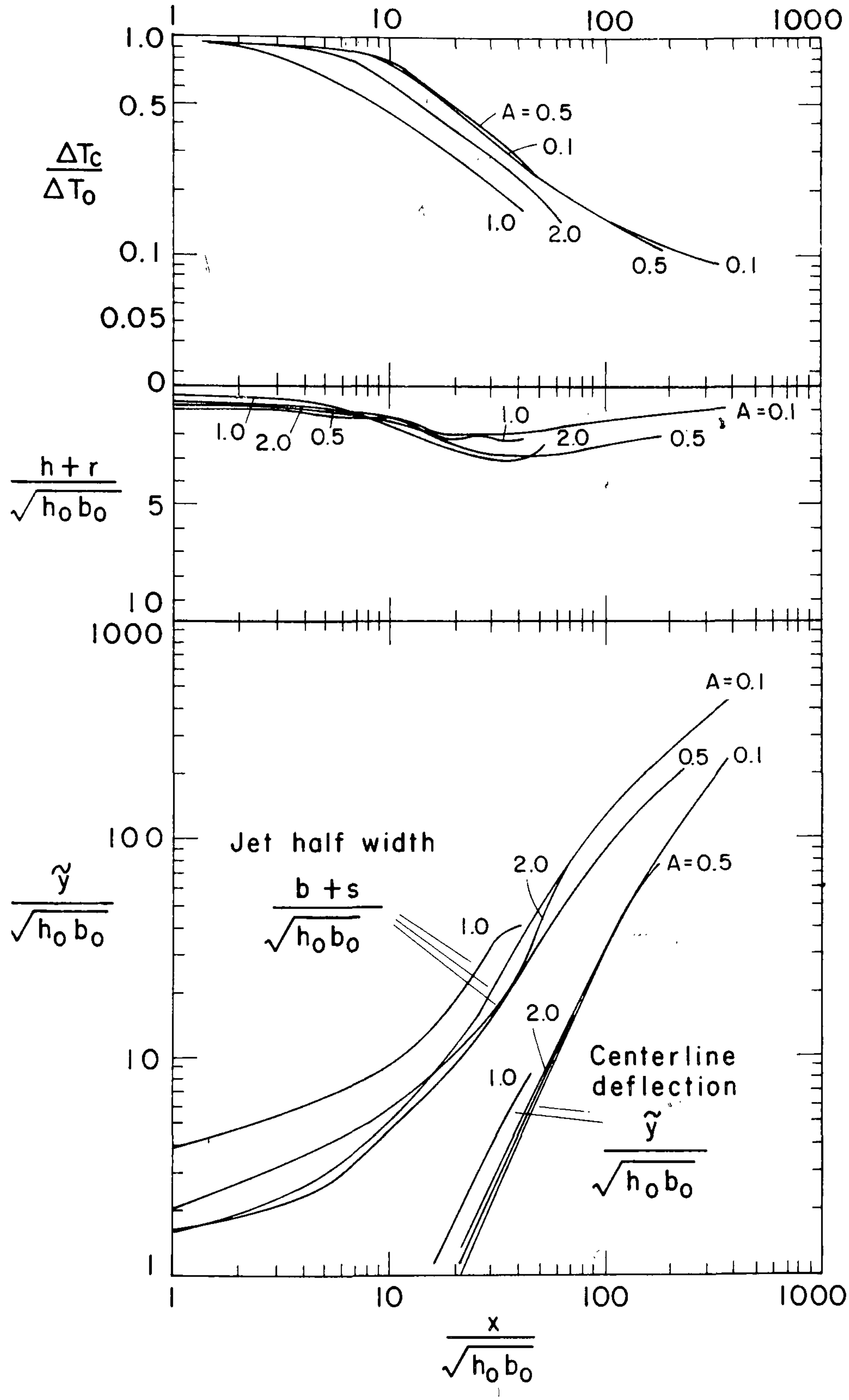

Fig. 2.7g Jet Parameters for $\mathbb{F}_0 = 10.0$, $V/u_0 = 0.05$, $k/u_0 = 0$

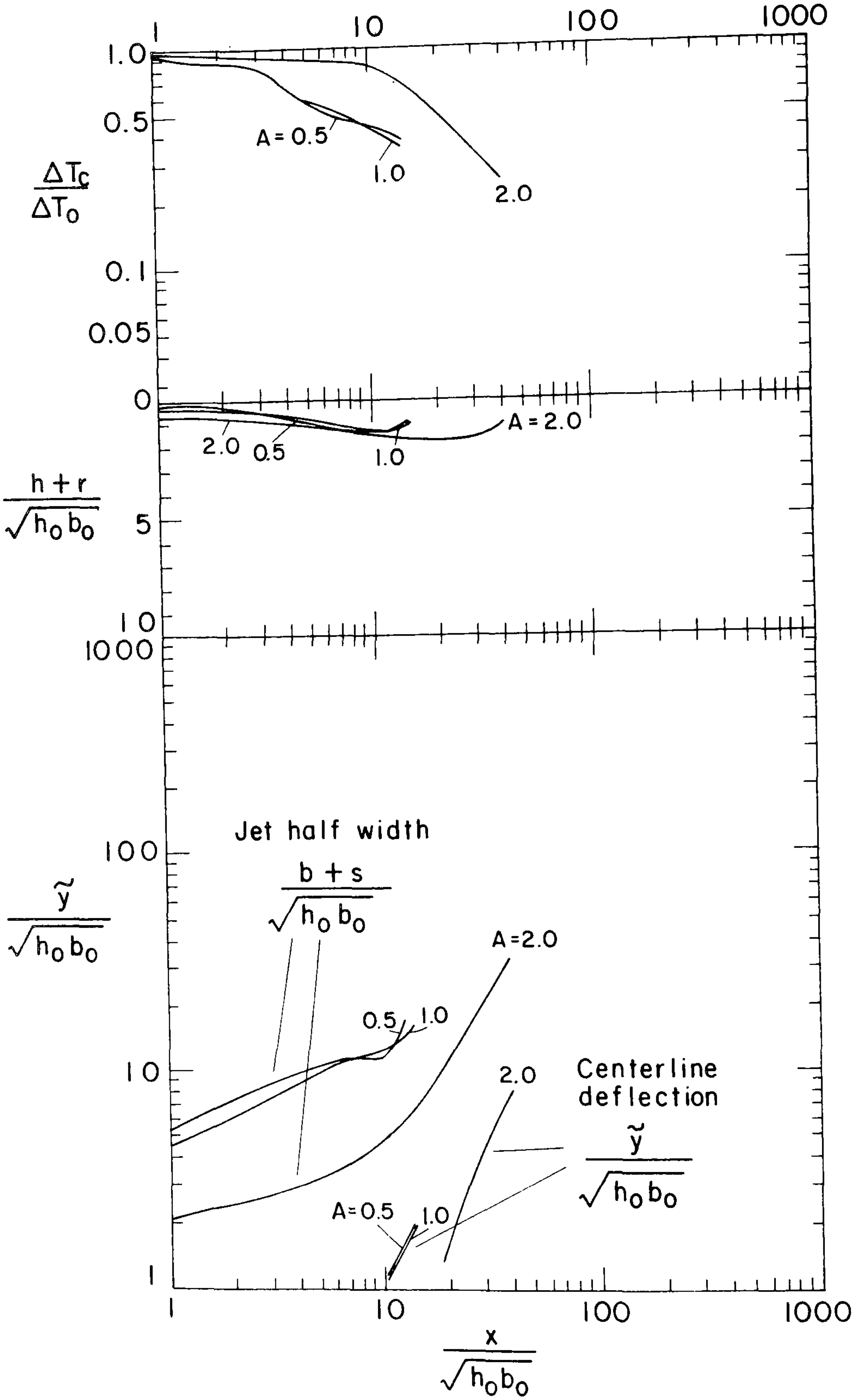

Fig. 2.7h Jet Parameters for $\mathbb{F}_0 = 5.0$, $V/u_0 = 0.1$, $k/u_0 = 0$

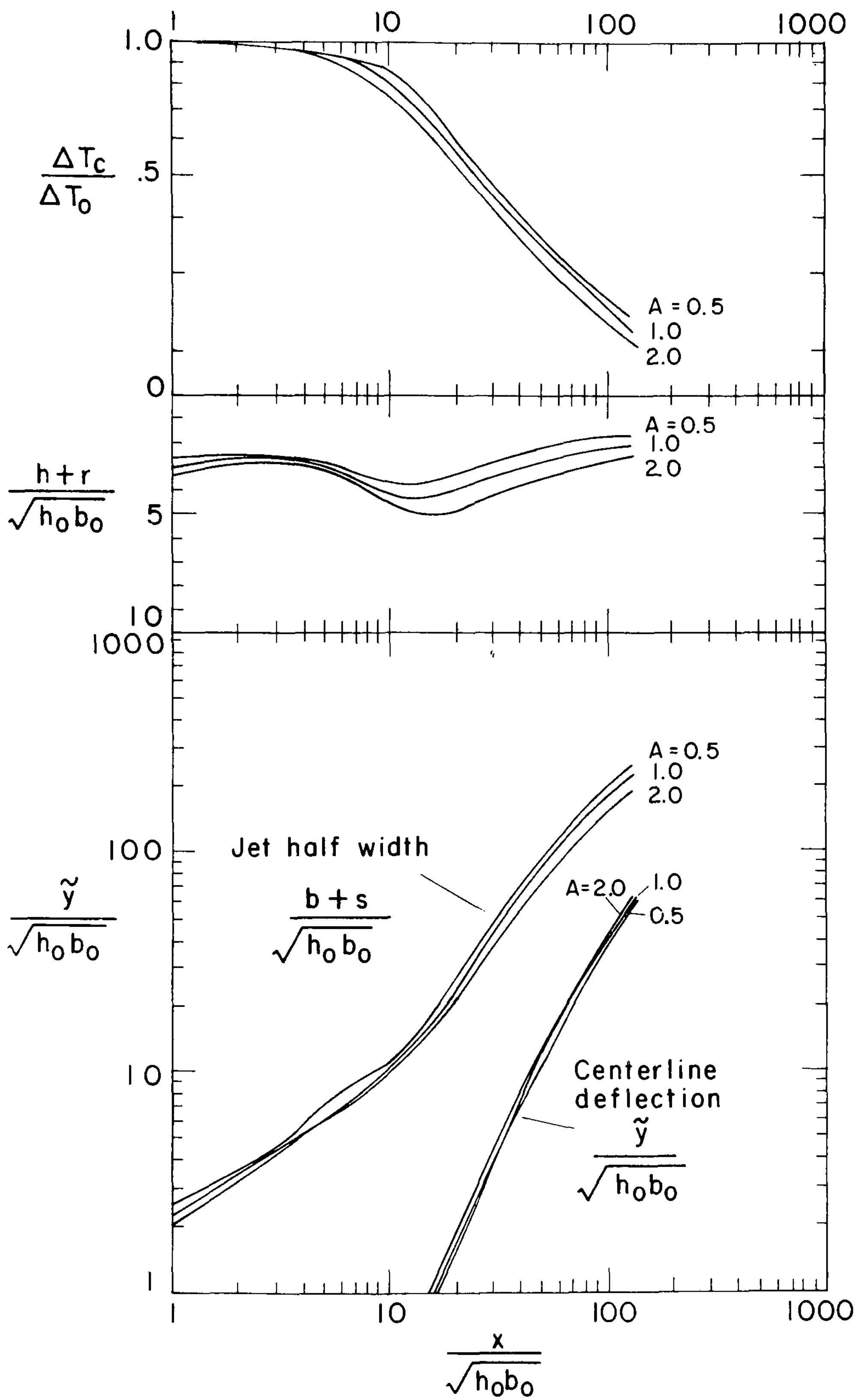

Fig. 2.7i Jet Parameters for $\mathbb{F}_0 = 2.0$, $V/u_0 = 0.1$, $k/u_0 = 0$

III. The Program

This chapter is a detailed description of the program for computing the solution to the heated surface discharge equations. The solution method is described including the basic equations, and the computational scheme. Lastly, the computer program input and output format are discussed.

3.1 Dimensionless Equations

The computations are performed with the following dimensionless variables:

$$
\begin{aligned}
\bar{x} &= x/\sqrt{h_o b_o} \\
\bar{\tilde{x}} &= \tilde{x}/\sqrt{h_o b_o} \\
\bar{\tilde{y}} &= \tilde{y}/\sqrt{h_o b_o} \\
\bar{u} &= u_c/u_o \\
\overline{\Delta T} &= \Delta T_c/\Delta T_o \\
\bar{h} &= h/\sqrt{h_o b_o} \\
\bar{b} &= b/\sqrt{h_o b_o} \\
\bar{r} &= r/\sqrt{h_o b_o} \\
\bar{s} &= s/\sqrt{h_o b_o} \\
\bar{V} &= V/u_o \\
&\theta
\end{aligned}
\tag{3.1}
$$

The equation set in Table 2.1 is reduced by eliminating the internal velocities v_s, u_b, w_r and w_h as follows:

a) Sum all the mass conservation equations to form a total mass conservative equation:

$$\frac{d}{d\bar{x}}\left[\bar{u}(\bar{s}+\bar{b}I_1)(\bar{r}+\bar{h}I_1) + \bar{V}\cos\theta\ (\bar{s}+\bar{b})(\bar{r}+\bar{h})\right] - \bar{u}\left[\alpha_{sz}(\bar{s}+\bar{b}I_1) - \alpha_y(\bar{r}+\bar{h}I_1)\right] = 0 \tag{3.2}$$

b) Sum all the momentum conservation equations to form a total momentum conservation equation:

$$\frac{d}{d\bar{x}}\left[\bar{u}^2(\bar{s}+\bar{b}I_2)(\bar{r}+\bar{h}I_2) + 2\bar{u}\bar{V}\cos\theta\,(\bar{s}+\bar{b}I_1)(\bar{r}+\bar{h}I_1) + \bar{V}^2\cos^2\theta\,(\bar{s}+\bar{b})(\bar{r}+\bar{h})\right.$$

$$\left. + \mathbb{F}_o^{-2}A^{-1/2}\,\overline{\Delta T}\,(\bar{s}+\bar{b}I_3)(\tfrac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4)\right] - \bar{u}\bar{V}\cos\theta\left[\alpha_{sz}(\bar{s}+\bar{b}I_1) - \alpha_y(\bar{r}+\bar{h}I_1)\right] = 0 \qquad (3.3)$$

c) The total heat equation:

$$\frac{d}{d\bar{x}}\left[\bar{u}\,\overline{\Delta T}\,(\bar{s}+\bar{b}I_7)(\bar{r}+\bar{h}I_7) + \overline{\Delta T}\,\bar{V}\cos\theta\,(\bar{s}+\bar{b}I_3)(\bar{r}+\bar{h}I_3)\right]$$

$$+ \left[\frac{k}{u_o}\right]\overline{\Delta T}\,(\bar{s}+\bar{b}I_3) = 0 \qquad (3.4)$$

d) The y-momentum spreading equation:

$$\frac{d}{d\bar{x}}\left[\left[\frac{d\bar{b}}{d\bar{x}} - \varepsilon\right]\left[\bar{u}^2\bar{b}I_6(\bar{r}+\bar{h}I_2) + 2\bar{u}\bar{V}\cos\theta\,\bar{b}I_5(\bar{r}+\bar{h}I_1) + \bar{V}^2\cos^2\theta\,\bar{b}(\bar{r}+\bar{h})\right]\right]$$

$$- \mathbb{F}_o^{-2}A^{-1/2}\,\overline{\Delta T}\,(\tfrac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2I_4) = 0 \qquad (3.5)$$

e) The y-momentum deflection equation:

$$\frac{d\theta}{d\bar{x}} + \frac{\bar{u}\bar{V}\sin\theta\left[\alpha_{sz}(\bar{s}+\bar{b}I_1) + \alpha_y(\bar{r}+\bar{h}I_1)\right]}{\bar{u}^2(\bar{s}+\bar{b}I_2)(\bar{r}+\bar{h}I_2)+2\bar{u}\bar{V}\cos\theta(\bar{s}+\bar{b}I_1)(\bar{r}+\bar{h}I_1)+\bar{V}^2\cos^2\theta\,(\bar{s}+\bar{b})(\bar{r}+\bar{h})} = 0 \qquad (3.6)$$

f) Combine the region 1 mass and momentum conservation equations to eliminate v_s and w_r:

$$(\bar{u}+\bar{V}\cos\theta)\ \frac{d}{d\bar{x}}\ (\bar{u}+\bar{V}\cos\theta) + \mathbb{F}_o^{-2}\ A^{-1/2}\left[\frac{1}{2}\ \bar{r}\ \frac{d\overline{\Delta T}}{d\bar{x}} + \overline{\Delta T}\ \frac{d\bar{r}}{d\bar{x}} + I_3 \frac{d\overline{\Delta T}\ \bar{h}}{d\bar{x}}\right] = 0$$

(3.7)

g) Combine the momentum and mass conservation equations from regions 1, 2, and 3 to eliminate v_s, v_b, w_t and w_h:

$$(\bar{r}\bar{b} + \bar{s}\bar{h})\ \left[(\bar{u}I_2+\bar{V}\cos\theta I_1)\ \frac{d\bar{u}}{d\bar{x}} + (\bar{u}(2I_1 - \frac{I_2}{I_1}) + \bar{V}\cos\theta)\ \frac{d\bar{V}\cos\theta}{d\bar{x}}\right]$$

$$+ \bar{u}\bar{V}\cos\theta\ (I_1 - \frac{I_2}{I_1})\ (\bar{s}\ \frac{d\bar{h}}{d\bar{x}} + \bar{r}\ \frac{d\bar{b}}{d\bar{x}}) + \bar{u}\ (1 - \frac{I_2}{I_1})\ \frac{d}{d\bar{x}}\ [\bar{r}\bar{s}\ (\bar{u}+\bar{V}\cos\theta)]$$

$$+ \mathbb{F}_o^{-2}\ A^{-1/2}\left[\bar{s}(I_4\ \frac{d\overline{\Delta T}\ \bar{h}^2}{d\bar{x}} + I_3\overline{\Delta T}\ \bar{h}\ \frac{d\bar{r}}{d\bar{x}}) + \frac{1}{2}I_3\ \frac{d\overline{\Delta T}\ \bar{b}\bar{r}^2}{d\bar{x}} + I_3{}^2\bar{r}\ \frac{d\overline{\Delta T}\ \bar{b}\bar{h}}{d\bar{x}} + \right.$$

$$\left.\overline{\Delta T}(\frac{1}{2}\ \bar{r}^2 + \bar{r}\bar{h}I_3)\ \frac{d\bar{s}}{d\bar{x}}\right] + \bar{u}(\alpha_{sz}\bar{s} - \alpha_y\bar{r})\ \frac{I_2}{I_1} = 0 \qquad (3.8)$$

h) The geometrical relationships:

$$\frac{d\bar{\tilde{x}}}{d\bar{x}} - \sin\theta = 0$$

(3.9)

$$\frac{d\bar{\tilde{y}}}{d\bar{x}} - \cos\theta = 0$$

Using the similarity functions defined in the theory, the integration constants have the values:

$$
\begin{aligned}
I_1 &= .4500 \\
I_2 &= .3160 \\
I_3 &= .6000 \\
I_4 &= .2143 \\
I_5 &= .2222 \\
I_6 &= .1333 \\
I_7 &= .3680
\end{aligned}
\tag{3.10}
$$

The entrainment coefficients are as follows assuming $\varepsilon_o = .22$:

$$
\begin{aligned}
\alpha_z &= .0295 \quad \bar{r} > 0 \qquad \alpha_{sz} = \alpha_z \exp\left[-5\,\frac{\overline{\Delta T}\,\bar{h}}{\bar{u}^2}\,\mathbb{F}_o^{-2}\,A^{-1/2}\right] \\
\alpha_z &= .0495 \quad \bar{r} = 0 \\
\alpha_y &= -.0295 \quad \bar{s} > 0 \\
\alpha_y &= -.0495 \quad \bar{s} = 0
\end{aligned}
\tag{3.11}
$$

It should be noted that the computer program may be used for other choices of similarity functions and spreading rate than those assumed here by simply changing the program statements which specify the values of I_1-I_7 and ε_o. The entrainment coefficients are automatically computed from these variables using equations (2.18)

The above <u>nine</u> equations may be solved for the $\bar{x}$ derivatives of $\bar{\tilde{x}}$, $\bar{\tilde{y}}$, $\bar{u}$, $\overline{\Delta T}$, $\bar{h}$, $\bar{b}$, $\bar{r}$, $\bar{s}$, and θ. (Note that $\bar{V}$ is given as an input and thus is known. The "initial" conditions at $\bar{x} = 0$ are:

$$
\begin{aligned}
\bar{\tilde{x}} = \bar{\tilde{y}} &= 0 & \bar{h} = \bar{b} &= 0 \\
\bar{u} &= 1 - \bar{V}\cos\theta_o \;(\bar{V} \text{ taken at origin}) & \bar{r} &= A^{1/2} \\
\overline{\Delta T} &= 1 & \bar{s} &= A^{-1/2} \\
& & \theta &= \theta_o
\end{aligned}
\tag{3.12}
$$

3.2 The Computational Scheme

The computer program is conceptually simple. After all inputs are read the variables are initialized to their values at the discharge origin. The solution proceeds by advancing along the discharge centerline (increasing $\bar{x}$) using the calculated derivatives of each variable to calculate its behavior. The differential equation technique is a fourth order Runge-Kutta scheme which has been taken, in modified form, from the IBM Scientific Subroutine Package. The program consists of a main program and five subroutines:

MAIN: This program reads the input data, sets up the initial conditions for each calculation, and calls the subroutine SRKGS which performs the calculations.

SRKGS: This subroutine is a modified version of the IBM Scientific Subroutine Package DRKGS for solving a system of differential equations. The form of the equation system is:

$$[a_{ij}] \quad \frac{dy_j}{dx} = c_i \tag{3.13}$$

where a_{ij} is a coefficient matrix, y_j is a vector of the variables ($\bar{u}$, $\overline{\Delta T}$, etc.) and c_i is a vector of the constants in the i equations. Subroutine SRKGS advances the solution by successively calling subroutine FCT which solves the system of equations for $\frac{dyj}{dx}$. The results of the calculations are periodically printed by calling subroutine OUTP.

FCT: This subroutine uses the current values of the variables, computes the coefficient matrix, a_{ij}, the vector c_i, and solves the resulting system of linear equations for $\frac{dyj}{dx}$ by calling routine SGELG. The following details concerning this routine may be of interest:

a) To reduce the equation set to first order the equation $\frac{d}{d\bar{x}}\left[\frac{d\bar{b}}{d\bar{x}}\right] = \frac{d^2\bar{b}}{d\bar{x}^2}$ is added where $\frac{d^2\bar{b}}{d\bar{x}^2}$ is computed from the y-spreading Equation (3.5) and $\frac{d\bar{b}}{d\bar{x}}$ becomes a variable.

b) Because of their relatively simple form, the y-deflection Equation (3.6) and the geometrical relationships (3.9) are not included in the matrix solved by SGELG but are solved separately within FCT.

c) When an ambient current is present, ($V \neq 0$) the value of ε is computed by setting $\overline{\Delta T} = 0$ and solving a reduced set of equations from which the heat equation (3.4) is omitted and the y-spreading equation (3.5) is replaced by $\frac{db}{dx} = \frac{dh}{dx}$. The full buoyant equation set is then solved using the calculated value of ε. When there is no ambient current, ($V=0$) the spreading rate is $\varepsilon_o = .22$, and the full equation set is solved directly.

SGELG: This subroutine is a modified form of the IBM Scientific Subroutine Package routine DGELG. It solves the system of linear equations by Gauss-Jordan reduction.

OUTP: The values of the calculated variables (y_j) are printed in formatted form. Certain other quantities of interest are also printed (see Section 3.4).

CROSS: This subroutine is called by the other routines whenever the velocity of the cross flow is required. The routine uses the input values V_1 to V_8 to compute the cross flow as a function of $\bar{\bar{x}}$. This subroutine may be modified by any user to any particular functional form with the only requirement being that the routine place the value of the cross flow at the current value of $\bar{\bar{x}}$ in V and the value of $\frac{d\bar{V}}{d\bar{x}}$ in DV. The cross flow function used in the present version is described in the next section.

The computations for a particular case will be terminated for one of the following reasons:

1) the limit on $\bar{x}$ specified by the input has been reached (see next section). This is the normal termination.
2) $\bar{u} \leq \bar{V} \cos\theta$: This termination occurs when the centerline velocity excess is reduced to the same magnitude as the ambient current in which case the basic assumptions of the theory are not valid.
3) $\bar{u} \leq .02$: The limiting value .02 for $\bar{u}$ is an arbitrarily chosen small number below which the assumptions of jet-like behavior are not valid.

4) The total dimensionless momentum, $M = \bar{u}^2(\bar{r} + \bar{h}I_2)(\bar{s} + \bar{b}I_2) + 2\bar{u}\bar{V}\cos\theta(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}I_1) + \bar{V}^2 \cos^2\theta(\bar{r} + \bar{h})(\bar{s} + \bar{b}) + \mathbb{F}_o^{-2}A^{-1/2}\,\overline{\Delta T}\,(\bar{s} + \bar{b}I_3)\,(\frac{1}{2}\,\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2 I_4)$, which should be constant for $\frac{V}{u_o} = 0$ and nearly constant for $\frac{V}{u_o}$ near zero has deviated from its initial value by more than 25%. This termination indicates that the computation is accumulating a large numerical error.

3.3 Input Formats

	Input Data	Format
	Number of cases to be calculated	I3
One Set for each Calculation	$\mathbb{F}_o$, A, k/u_o, θ_o, $\bar{x}_L$, ERR, STEP	2F10.5, F10.7, 4F10.5
	V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8	8F10.5

where $\mathbb{F}_o$ = initial densimetric Froude number $= \dfrac{u_o}{\sqrt{\dfrac{\Delta\rho_o}{\rho_a}\, gh_o}} = \dfrac{u_o}{\sqrt{\beta\Delta T_o gh_o}}$

A = aspect ratio = h_o/b_o

k/u_o = surface heat loss parameter

θ_o = initial angle (in degrees) between the discharge centerline and the boundary of the ambient region

$\bar{x}_L$ = the value of $x/\sqrt{h_o b_o}$ at which the program should terminate for this case

ERR = maximum allowable average error in the variables at each time step

STEP = interval of $\bar{x}$ at which variable values should be printed

V_1-V_8 = constants describing the cross flow. In the present program version only V_1 - V_5 are used as follows

$$\bar{V} = V_1 + V_2 \exp\left[- V_3\left[V_4\bar{\bar{x}} - V_5\right]^2\right]$$

$$\frac{d\bar{V}}{d\bar{\bar{x}}} = 2(V_4\bar{\bar{x}} - V_5)V_2V_3V_4 \exp\left[- V_3(V_4\bar{\bar{x}} - V_5)^2\right]$$

A sample input listing is given in Appendix III.

3.4 Output Format

The output is paged for each calculation with the basic input variables printed on the heading of each page. The values of the variables are printed in column form under the following headings

$$X = \bar{x}$$
$$H = \bar{h}$$
$$B = \bar{b}$$
$$R = \bar{r}$$
$$S = \bar{s}$$
$$\mathbb{F}_L = \mathbb{F}_o A^{1/4} / \left(\frac{\overline{\Delta T}\ \bar{h}}{\bar{u}^2} \right)^{1/2}$$
$$Q = \bar{u}(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}I_1) + \bar{V}\cos\theta \quad (\bar{r} + \bar{h}) \quad (\bar{s} + \bar{b})$$
$$M = \bar{u}^2(\bar{r} + hI_2)(\bar{s} + \bar{b}I_2) + 2\bar{u}\bar{V}\cos\theta(\bar{r} + \bar{h}I_1)(\bar{s} + \bar{b}\bar{I}_1)$$
$$+ \quad \bar{V}^2\cos^2(\bar{r} + \bar{h})(\bar{s} + \bar{b}) + \mathbb{F}_o^{-2} A^{-1/2}\ \overline{\Delta T}(\bar{s}+\bar{b}I_3)(\tfrac{1}{2}\bar{r}^2 + \bar{r}\bar{h}I_3 + \bar{h}^2 I_4)$$
$$U \quad \bar{u}$$
$$T = \overline{\Delta T}$$
$$HT = \overline{\Delta T}\ [\bar{u}(\bar{r} + \bar{h}I_7)(\bar{s} + \bar{b}I_7) + \bar{V}\cos\theta(\bar{r} + \bar{h}) \quad (\bar{s} + \bar{b})]$$
$$V = \bar{V}$$
$$XP = \tilde{x}$$
$$YP = \tilde{y}$$
$$THD = \theta$$
$$TM = \int_o^{\bar{x}} \frac{d\bar{x}}{\bar{u}}$$

An example of program output is shown in Appendix III.

IV. Applications

The theoretical model presented in the preceding sections permits determination of the behavior of a heated surface discharge as a function of relatively few controlling parameters: $\mathbb{F}_o$, h_o/b_o, k/u_o, V/u_o. The geometry of actual field configurations are rarely as simple as those assumed in the theory and judgement must be applied in the schematization of the discharge to a form for which the dimensionless parameters may be given. Similarly, the output of the program must be interpreted in the light of the basic assumptions of the model. The following sections discuss in detail the generation of input data for the computations and the construction of the temperature distribution from the program output. Finally, a case study is presented as an example of the use of the theory and computer program.

4.1 Schematization

This section is a discussion of the data requirements and schematization techniques for preparing input to the program. The following are the physical data needed as input to the theoretical calculations.

The ambient temperature, T_a, is assumed in the theory to be constant in space and time. Actual ambient receiving water temperatures are often stratified vertically or horizontally and may be unsteady due to wind, tidal action or diurnal variations in solar heating. The natural stratification of the ambient water may be increased by accumulation of heat at the water surface if the discharge is in a semi-enclosed region. The value of the ambient temperature, for a given initial temperature rise, ΔT_o, determines the initial density difference between the discharged and ambient water. Also the effectiveness of the entrainment of ambient water in reducing the discharge temperatures is a function of the ambient stratification.

If the temperature differences resulting from ambient stratification in the vicinity of the discharge are the same magnitude as the initial discharge temperature rise, the theoretical model of this study should not be applied without further development to account for the ambient stratification. The theory is valid if an ambient temperature, T_a, may be chosen which is representative of the receiving water temperatures, that is, if the temporal or spatial variations in ambient temperature do not differ from T_a by more than a few degrees Fahrenheit.

The initial discharge temperature rise, ΔT_o, is determined from the discharge temperature, T_o, and the chosen ambient temperature, T_a. The value of ΔT_o will be equal to the temperature rise through the condensers only if the intake temperature is equal to T_a. Actual discharge temperatures should be relatively steady unless the power plant has several different condenser designs in which case the discharge temperature will vary with the plant load. Use of the theory requires that a constant value of ΔT_o be specified, the choice being based on the likely steady value of the discharge temperature.

The initial relative density difference, $\Delta\rho_o/\rho_a$ may be related to T_a and ΔT_o by $\Delta\rho_o/\rho_a = \beta\Delta T_o$ where β is a function of T as given in Figure 4.1 for fresh water. The curve of β vs. T is not linear but only a small error will be introduced if T is set equal to $T_a + \Delta T_o/2$. A more accurate value of $\Delta\rho_o/\rho_a$ may be obtained by using tabulated values of water density vs. temperature, or in the case of salt water, water density vs. salinity and temperature.

The initial condenser water discharge velocity, u_o, is a function of the power plant condenser water pumping rate and the discharge channel area.

The discharge channel geometry is important since the theory uses the square root of one half of the discharge channel flow area as a scaling length. Calculation of the discharge densimetric Froude number, $\mathbb{F}_o$, requires specification of the initial depth, h_o; and the aspect ratio, A, requires both h_o and the initial width, b_o. The following procedure is suggested for any channel shape:

a) Let h_o be the actual maximum discharge channel depth so that calculation of $\mathbb{F}_o$ is not affected by the schematization.

b) Let b_o be such that the correct discharge channel area is preserved:

$$b_o = \frac{\text{channel area}}{2h_o} \tag{4.1}$$

Then the aspect ratio is given by:

$$A = \frac{h_o}{b_o} = \frac{2h_o^2}{\text{channel area}} \tag{4.2}$$

As an example, the aspect ratio of a circle is $8/\pi = 2.55$.

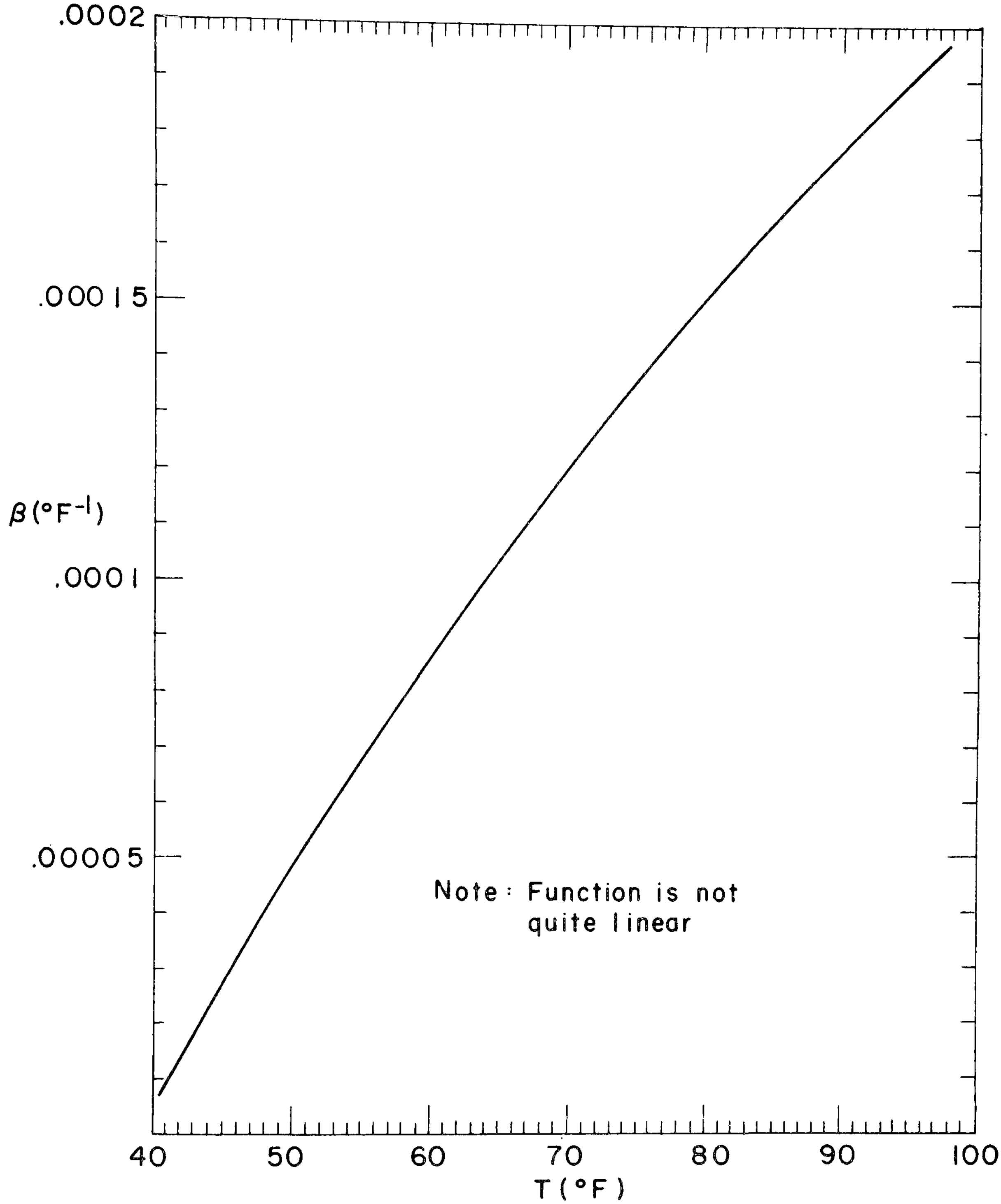

Fig. 4-1 Coefficient of Thermal Expansion for Fresh Water $\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial T}$ (°F^{-1}) as a Function of Temperature T(°F)

The discharge channel geometry may vary with time if the elevation of the receiving water changes due to tidal motion or other causes. In this case separate calculations for each elevation of the receiving water must be made, assuming that the steady state theory predicts the instantaneous temperature distribution.

Very complicated arrangements of the discharge channel relative to the boundaries of the ambient receiving waters are beyond the capabilities of the theory of this study. If the discharge channel is nearly at right angles to the solid boundaries, the theory may be used as developed, or if the discharge is directed parallel to a straight boundary, the discharge may be schematized by assuming that the solid boundary is the centerline of a jet whose discharge channel is twice the width of the actual discharge (see Figure 4.2). The width b_o is then twice the width calculated by the procedure discussed previously. However, if it is not clear whether the jet will entrain water from one or both sides or if irregularly shaped solid boundaries will deflect the jet or distort it from the form assumed in the theory, a meaningful schematization is not possible.

The densimetric Froude number $\mathbb{F}_o$ and the aspect ratio A of the surface discharge channel should be chosen to be consistent with the bottom topography of the receiving water body. If optimum dilution is to be obtained, the vertical development of the surface jet should not be limited by the bottom of the receiving water. This is generally considered to be a desirable objective by the aquatic or marine biologist inasmuch as it avoids exposure of benthic organisms to high velocities and temperature rises. Designs which contain minimum interaction between the surface jet and the bottom topography are shown on the left side of Figure 4.3. Examples of surface jets in which there are substantial interactions in relation to the bottom topography are shown on the right side of Figure 4.3. In the latter case, vertical entrainment and dilution are reduced by the interference of the jet and the bottom.

If the discharge channel densimetric Froude number, $\mathbb{F}_o$, is less than unity, a wedge of ambient water will intrude into the discharge channel and the heated flow will be forced to obtain a densimetric Froude number of unity at the discharge point (see Figure 4.4). The depth of the heated flow in the presence of a wedge, $h_o{}^*$, is given by:

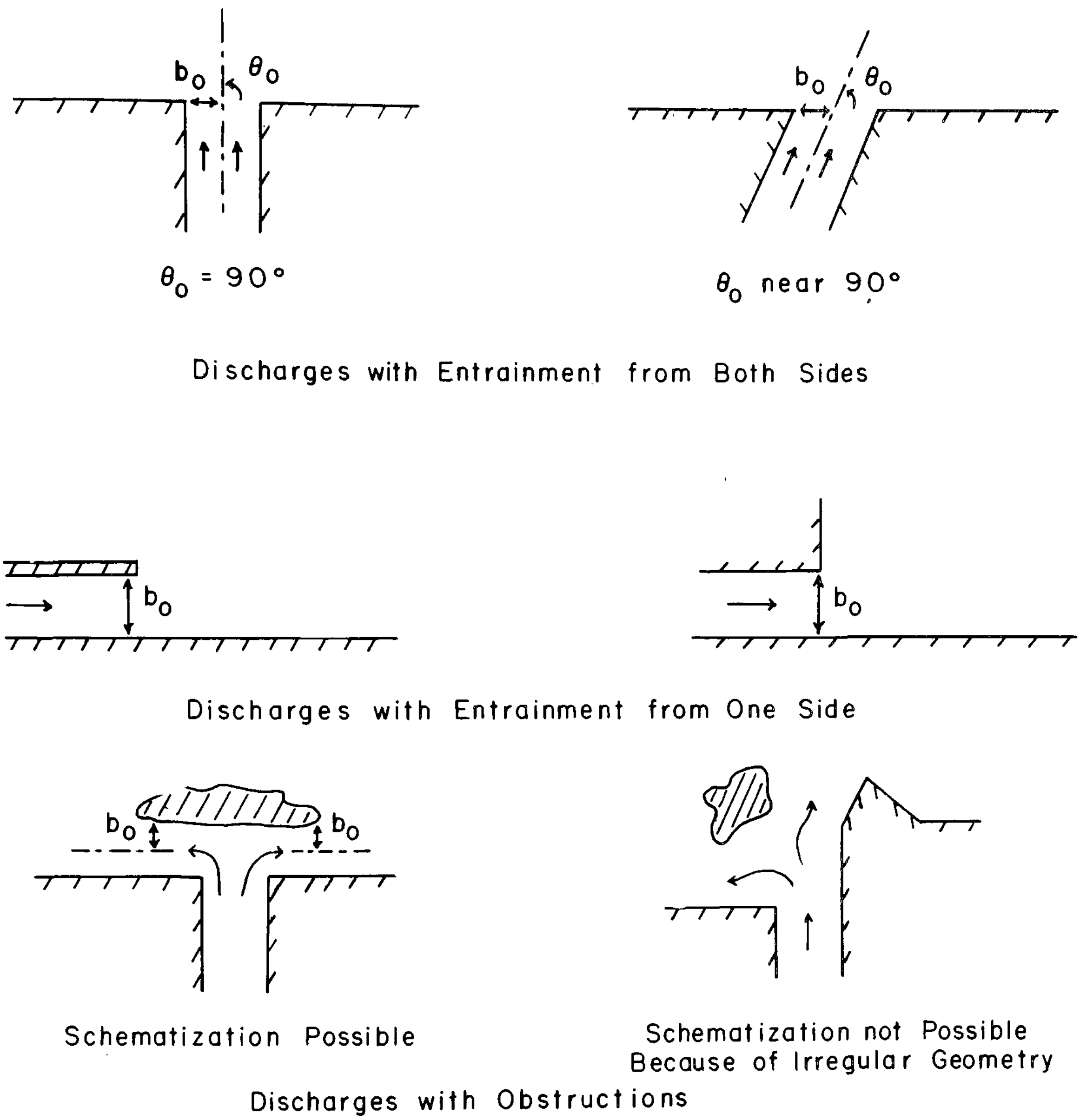

Fig. 4.2 Discharge Channel Schematization

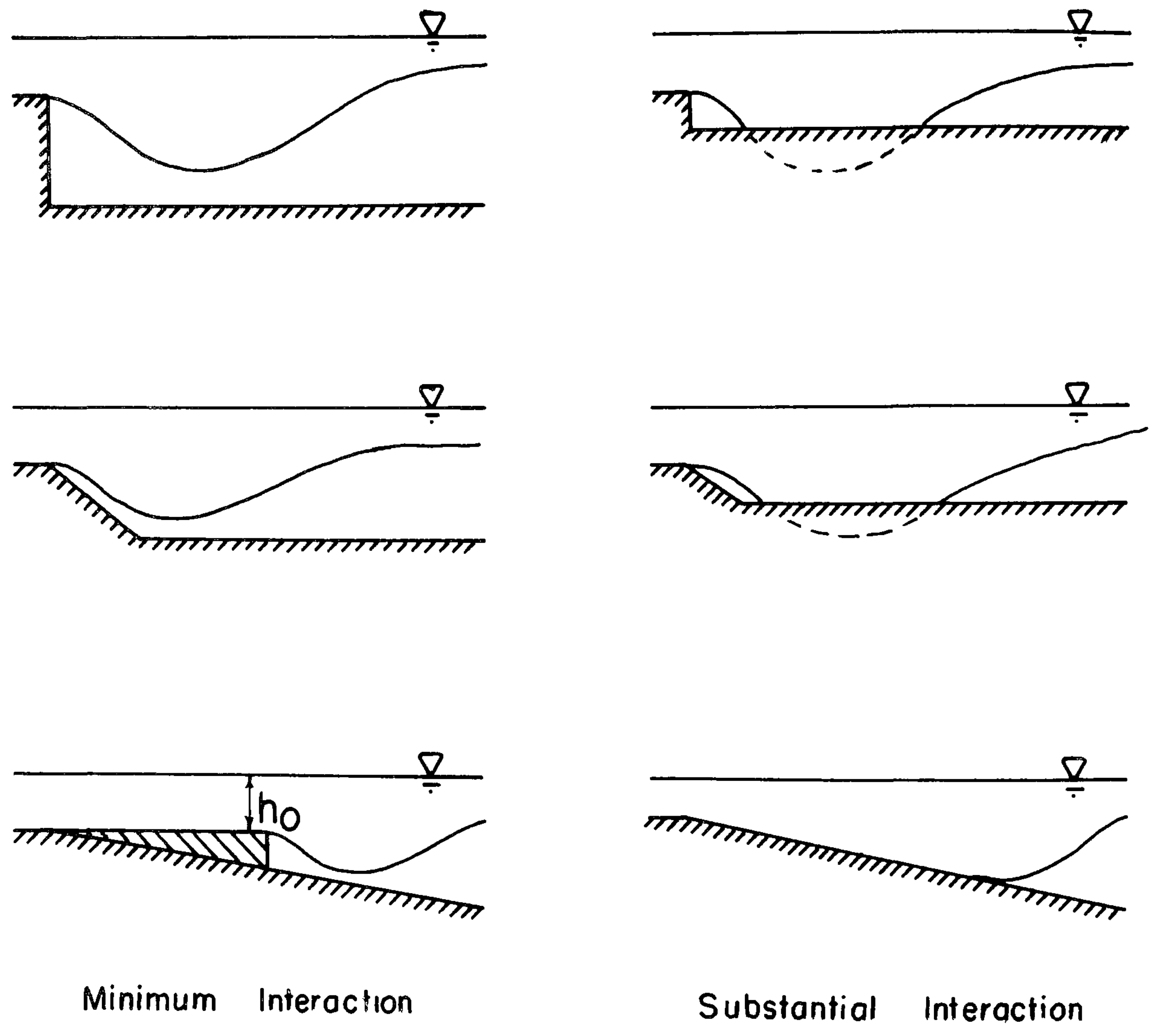

Fig. 4.3 Limitations on Maximum Jet Thickness by Bottom Topography

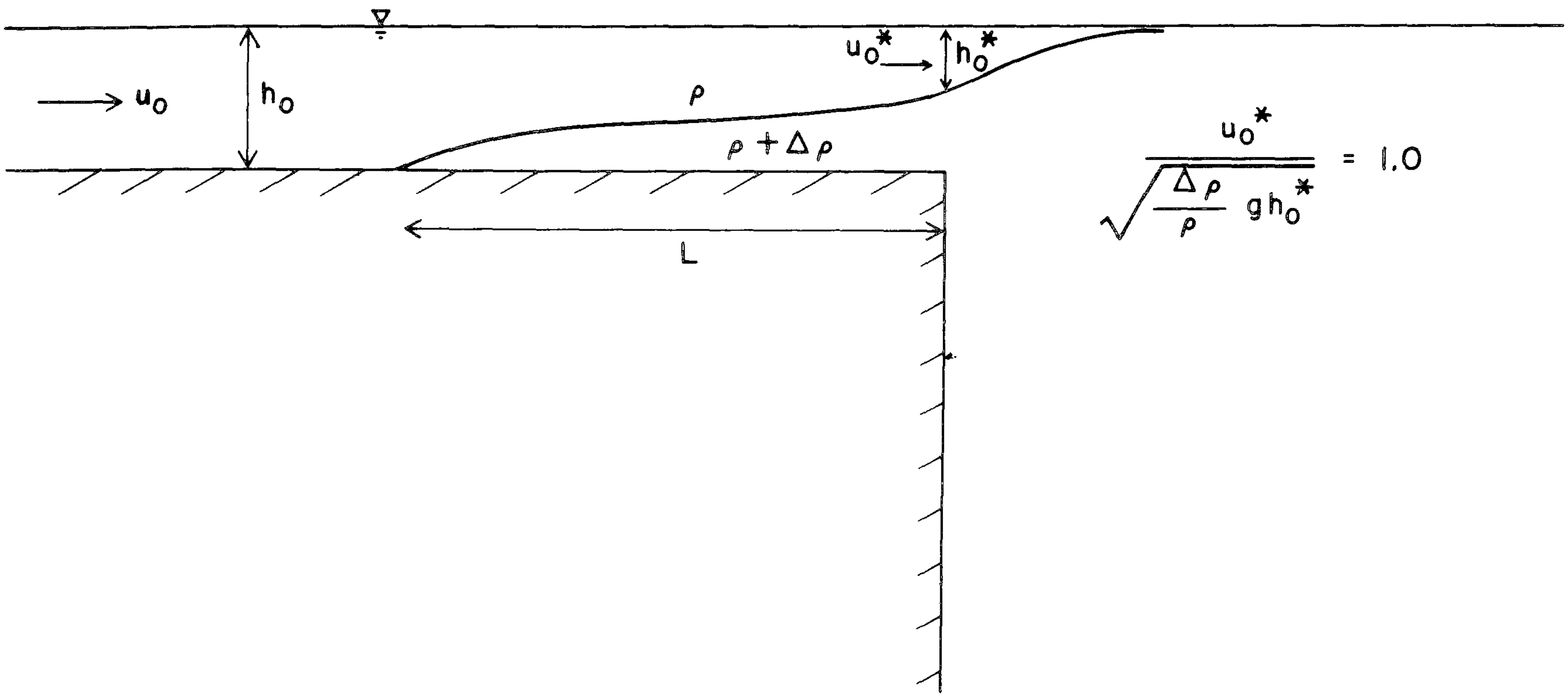

Fig. 4.4 Two Layer Flow in the Discharge Channel

$$\frac{h_o^*}{h_o} = \mathbb{F}_o^{2/3} \tag{4.3}$$

where h_o and $\mathbb{F}_o$ are based on the channel dimensions. A flow area may be calculated based on the depth h_o^* and the aspect ratio calculated as described previously in this section.

The surface heat loss coefficient, k, may be estimated from local meteorological variables, principally wind speed, and the ambient water temperature. Care should be taken to use values for k which are appropriate for local conditions. Edinger (13) and Ryan (16) discuss methods for determining values of $K = \rho ck$.

The cross flow velocity, V, in the receiving water may be measured directly or estimated from flow measurements in the case of a channel flow. The theory accepts as input values of V/u_o as a function of $\tilde{x}/\sqrt{h_o b_o}$ (see Chapter 3).

Once the schematization of the discharge configuration is achieved, the theoretical calculation is performed by the computer program described in Chapter 3. The inputs to the program for each calculation are $\mathbb{F}_o$, $A = h_o/b_o$, k/u_o and V/u_o (as a function of $\tilde{x}/\sqrt{h_o b_o}$).

It should be noted that the theoretical model may not necessarily be applied to any arbitrary set of input parameters. In general the value of k/u_o has little effect; however, depending upon the value of the other parameters, $\mathbb{F}_o$, A, and V/u_o, and upon the specified error size, ERR, the program may fail to generate a solution. This problem may take one of the following forms:

1) Little or no advancement of solution, i.e. the step size remains very small.
2) Abrupt discontinuities noticeable either in the values of the variables or in the total mass, momentum, or heat.
3) Unstable behavior in one or more variables, culminating usually in an overflow error.

The chance of the program encountering one of the above problems increases with decreasing $\mathbb{F}_o$ and A and increasing V/u_o. The value of ERR produces no consistent result except that too large a value of ERR most often yields unstable behavior or discontinuities, and too small a value may prevent advancement.

The causes of this anomolous behavior in certain cases are not totally understood at the present time, but are thought to be related to the following:

1) accumulation of round-off error where the solution should be stable, i.e. a totally numerical problem,
2) genuinely unstable solutions, probably caused by the determinant of the differential equation coefficient matrix being zero or near zero.

The first of these could probably be solved by paying greater attention to round off error generation than is now done. To the extent that the second cause has no physical significance it may also be desirable to eliminate this problem by numerical manipulation. However, the governing equations for the heated discharge are similar to open channel flow equations which possess critical flow points at which the equations strictly have no solution and the coefficient matrix is zero. It is not clear to what extent the degeneracy of the three-dimensional heated discharge equations corresponds to critical flow behavior; the answer will only come by observing actual discharges in the laboratory and field. Thus for the time being the limitation on the computational range of this theory must be accepted.

Of course the values of $\mathbb{F}_o$, A, and V/u_o are pre-determined by the case being considered, leaving ERR as the only free parameter. Figures 2.7a to 2.7i indicate the range of each variable and the various combinations of values for which calculations may be successfully performed. Table 4.1 gives a rough guideline as to the best value of ERR to be used for different cases. In general if the solution is not advancing, a larger ERR might be tried and if the solution appears unstable a smaller value should be tried. Experience has shown that the behavior of the solution may also depend upon the type of computer used. The results quoted herein are based on calculations done on an IBM 360 either -65 or -75. Users employing different systems than this are encouraged first of all to try several values of ERR in an attempt to make an unsuccessful calculation work and secondly to take up the challenge of illuminating more clearly than is done here the properties of the governing equations, the physical relevance of these properties, and their efficient numerical treatment.

Table 4.1

Guideline to Choice of Maximum Error Value, ERR

$\mathbb{F}_o$	A 0.1 - 0.5	A 0.5 - 1.0	A 1.0 - 1.5	A 2.0 - ∞
1 - 2	.005 with no crossflow .05 with cross-flow	.005	.005	.005
2 - 5	.005 with no crossflow .05 with cross-flow	.005	.01	.01
5 - 10	.01	.01	.01	.01
10 - ∞	.01	.01	.01	.01

4.2 Use of the Program Output

An example of the program output is shown in Appendix III. All of the outputs are in dimensionless form and must be transformed by the following steps

1) Multiply $\tilde{x}$, $\tilde{y}$, $\bar{x}$, $\bar{r}$, $\bar{s}$, $\bar{b}$, and $\bar{h}$ by $\sqrt{h_o b_o}$
2) Multiply $\bar{u}$ by u_o
3) Multiply $\overline{\Delta T}$ by ΔT.
4) Multiply TM by $\sqrt{h_o b_o}/u_o$

The calculated isotherms may be constructed as follows:

1) Locate the centerline using $\tilde{x}$ and $\tilde{y}$.
2) Assign values of $T_c = T_a + \Delta T_c$ along the centerline.
3) Plot the boundaries of the core and turbulent regions using s, b, r and h.
4) Assign temperatures within the discharge using the assumed temperature distribution (Equation 2.5).

Figure 4.5 may be used as an aid in the construction of isotherm contours; lines of constant $\zeta_y = \frac{y-s}{b}$ or $\zeta_z = \frac{z-r}{h}$ are plotted and then the temperature contours are drawn by referring to the figure. The values of temperature rise above ambient are expressed as fractions of ΔT_o, the initial temperature rise.

If the surface area within a given temperature rise ΔT_* is of interest, the following formula may be used:

$$A_* = 2h_o b_o \int_0^{\bar{x}_*} \left[\bar{s} + \bar{b}\ (1 - \overline{\Delta T_*}/\overline{\Delta T})^{2/3}\right] d\bar{x} \qquad (4.4)$$

where

$$\left.\begin{aligned} \bar{s} &= s/\sqrt{h_o b_o} \\ \bar{b} &= b/\sqrt{h_o b_o} \\ \overline{\Delta T} &= \Delta T_c/\sqrt{h_o b_o} \end{aligned}\right\} \quad \text{As given in the program output as a function of } \bar{x}.$$

$\overline{\Delta T_*} = \Delta T_*/\Delta T_o$

$\bar{x}_* =$ the value of $x/\sqrt{h_o b_o}$ where $\overline{\Delta T} = \overline{\Delta T_*}$

A simple computer program may be written which performs the above integration numerically.

Finally, the travel time along the centerline to a given temperature, $T_a + \Delta T_*$, is given by TM $(\bar{x}_*)\frac{\sqrt{h_o b_o}}{u_o}$.

It is important to note that although the theory calculates the temperature distribution out to very small values of temperature rise above ambinet, these extreme regions of the discharge will be subject to distortion by random ambient processes especially wind stresses.

The calculated temperature distribution must be interpreted with this in mind and no critical significance should be given to the <u>exact</u> calculated position of isotherms of small temperature rise.

4.3 Case Study - Heated Surface Discharge into a Receiving Water Body of Finite Depth

Consider a proposed discharge into a large body of water at ambient temperature T_a with a uniform depth H as shown on Figure 4.6. A constraint is

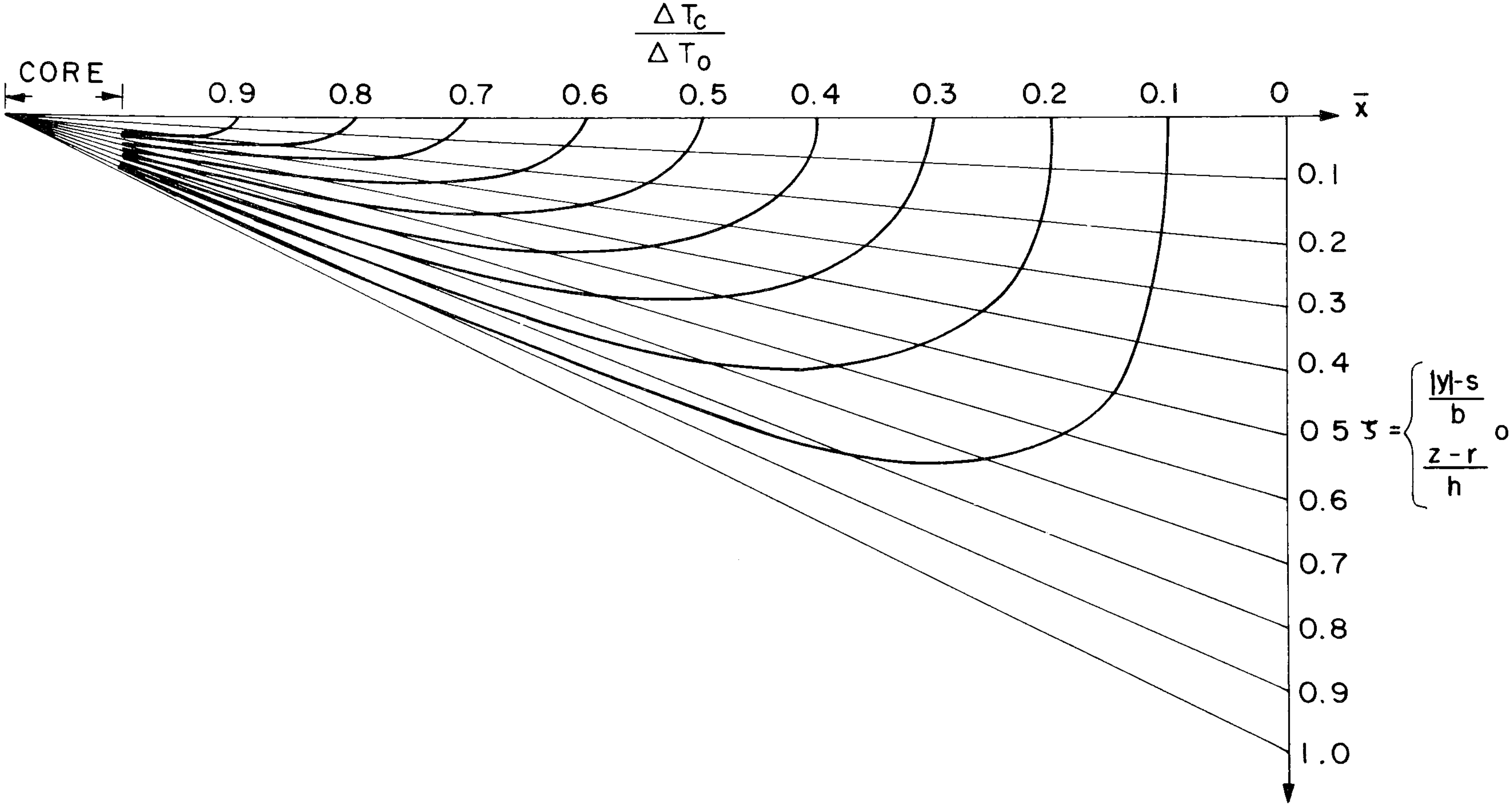

Fig. 4.5 Temperature Distribution Plotting Aid

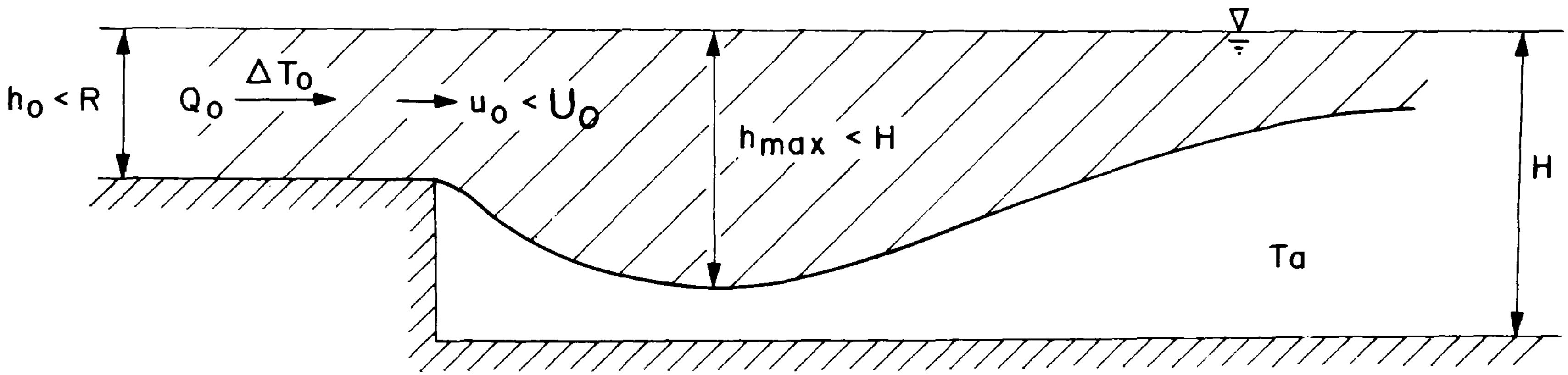

Fig. 4.6 Schematic of Case Study Problem

placed upon the discharge velocity such that $u_o \leq U_o$. Because the theory applies only to discharges which are not influenced significantly by solid boundaries in the receiving water, it is desirable that $h_{max} \leq H$ where h_{max} is the maximum vertical penetration of the discharge (see Figure 4.6). The choice of h_{max} as the "critical" boundary of the jet is an arbitrary one but is probably conservative considering the assumed structure of the jet which sets $u = 0$ at that point. Entrainment should not be significantly affected by contact with the bottom at the point of maximum depth only.

A third constraint may be imposed by local topography. For instance h_o must be less than or equal to the water depth, and $2b_o$ may be constrained by some critical width. Thus in general $h_o < R$ and $b_o < S$.

The discharge is characterized by an initial temperature rise, ΔT_o, and a condenser water flow, Q_o. The problem treated in this case study is to design a discharge channel for maximum ultimate (stable) dilution (see Figure 2.5) while meeting all the imposed constraints. This case study will illustrate the generation of temperature contours and isotherm areas from the computer output in addition to treating the more specific, but often relevant, problem of optimizing a channel design with one or more constraints imposed.

The definition of $\mathbb{F}_o'$ and the following approximate formulas may be recalled from Chapter 2 for $\mathbb{F}_o'$ greater than 3.

$$\mathbb{F}_o' = \mathbb{F}_o A^{1/4} = \frac{u_o}{(g\beta\Delta T_o)^{1/2}(h_o b_o)^{1/4}} \tag{4.5a}$$

$$= \frac{Q_o}{2(g\beta\Delta T_o)^{1/2}(h_o b_o)^{5/4}} \tag{4.5b}$$

$$= \frac{2^{1/4} u_o^{5/4}}{(g\beta\Delta T_o)^{1/2} Q_o^{1/4}} \tag{4.5c}$$

$$\left(\frac{\Delta T_o}{\Delta T_c}\right)_s \approx \mathbb{F}_o' \tag{4.6a}$$

$$D_s \simeq 1.4\ \mathbb{F}'_o \tag{4.6b}$$

$$\frac{h_{max}}{(h_o b_o)^{1/2}} \simeq 0.42\ \mathbb{F}'_o \tag{4.7}$$

Equation (4.5) defines $\mathbb{F}'_o$, (4.6a and b) relates centerline dilution and overall dilution to $\mathbb{F}'_o$ and (4.7) relates the maximum jet penetration to $\mathbb{F}'_o$. Note that for a given $\Delta\rho_o/\rho$ and Q_o both the maximum stable dilution and the maximum penetration are dependent only on discharge velocity u_o and not on the aspect ratio. This provides freedom in selecting channel geometry.

The constraint that $h_m \leq H$ may be restated using (4-7) as

$$\mathbb{F}'_o \leq \frac{6.8\ H^{5/3}(g\beta\Delta T_o)^{1/3}}{Q_o^{2/3}} \tag{4.8}$$

and from (4.5c) the constraint that $u_o \leq U_o$ may be restated as

$$\mathbb{F}'_o \leq \frac{1.19\ U_o^{5/4}}{(g\beta\Delta T_o)^{1/2} Q_o^{1/4}} \tag{4.9}$$

Because ultimate dilution increases monotonicly with $\mathbb{F}'_o$ (see Eqn. 4.6), the channel should be designed with $\mathbb{F}'_o$ equal to the smaller of the two expressions in (4.8) and (4.9). From (4.5b) it may be shown that

$$h_o b_o = \frac{Q_o^{4/5}}{2^{4/5}\mathbb{F}'^{4/5}_o (g\beta\Delta T_o)^{2/5}} \qquad \text{and} \tag{4.10}$$

$$u_o = \frac{Q_o}{2h_o b_o} \tag{4.11}$$

where $\mathbb{F}'_o$ is picked from (4.8 and 4.9). If there is a constraint on either the width or depth of the discharge structure, this constraint may be included with (4.10) to determine h_o and b_o. If there is a constraint on both width and depth such that $h_o < R$ and $b_o < S$ and the product $RS < h_ob_o$ as determined by (4.10), then a discharge channel cannot be designed with the desired constraints.

Example: Design a discharge channel no more than 12 feet deep to achieve maximum stable dilution of a heated discharge of 2,000 cfs and 15°F temperature rise into a water body 31 ft. deep at an ambient temperature of 70°F. Maximum discharge velocity is $U_o = 6$ ft/sec. The area within the 4°F surface isotherm should be calculated.

1. The value of β is taken from Figure 4-1 using T = 77.5°F, $\beta = .000144°F^{-1}$ and $(g\beta\Delta T_o) = .069$ ft/sec^2.

2. Using (4.8) the constraint on depth is

$$\mathbb{F}'_o \leq \frac{6.8(31)^{5/3}(.069)^{1/3}}{2000^{2/3}} = 5.3$$

Using (4.9) the constraint on velocity is

$$\mathbb{F}'_o \leq \frac{(1.19)(6)^{5/4}}{(.069)^{1/2}(2000)^{1/4}} = 6.4$$

Hence $\mathbb{F}'_o$ should equal 5.3 and the jet should just touch the bottom.

3. From (4.10) and (4.11)

$$h_ob_o = \frac{2000^{4/5}}{2^{4/5}(5.3)^{4/5}(.069)^{2/5}} = 192 \text{ ft}^2,$$

$$(h_ob_o)^{1/2} = \text{scaling length} = 13.9 \text{ ft.}$$

and $$u_o = \frac{2000}{2(192)} = 5.2 \text{ ft/sec}$$

4. No constraint has been placed on b_o so any number of combinations of h_ob_o will be satisfactory. As an example, $h_o = 11$ ft. and $b_o = 17.5$ feet satisfies the requirement on channel depth and results in a $\mathbb{F}_o \cong 6.0$; aspect ratio, $A \cong 0.6$; and $\mathbb{F}'_o = 5.3$.

5. Theoretical calculation (computer output) for these values are shown in Appendix III.
6. Figure 4.5 is used to plot the isotherms in two planes (horizontal at the surface and vertical at the jet centerline) in Figure 4.7.
7. Equation 4.4 is used to calculate the area within the 4° isotherm with $\overline{\Delta T}_* = 4/15 = .27$ and $\bar{x}_* = 45$. The computations are organized as follows:

$\bar{x}$	$\bar{s}$	$\bar{b}$	$\overline{\Delta T}$	$\Delta T_*/\Delta T$	$(1-\overline{\Delta T}_*/\overline{\Delta T})^{2/3}$	$\bar{s}+\bar{b}(1-\overline{\Delta T}_*/\overline{\Delta T})^{2/3}$	$\sum\left[\bar{s}+\bar{b}(1-\Delta T_*/\Delta T)^{2/3}\right]\Delta\bar{x}$
1.131	1.52	.585	.971	.278	.805	1.99	2.25
2.62	1.37	1.16	.940	.287	.798	2.30	5.67
5.24	1.44	2.39	.884	.305	.784	3.31	14.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10.5	1.48	5.27	.692	.390	.719	5.27	39.3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
41.9	0	41.5	.282	.957	.122	5.05	267.

The first and last column may be multiplied by the scaling length, $\sqrt{h_o b_o} = 13.9$ ft. and plotted against each other to produce the 74° isotherm shown in Fig. 4.7. The integration in Equation 4.4 results in an $A_* = 2(192)\ (267) = 103{,}000$ sq. ft. or 2.4 acres.

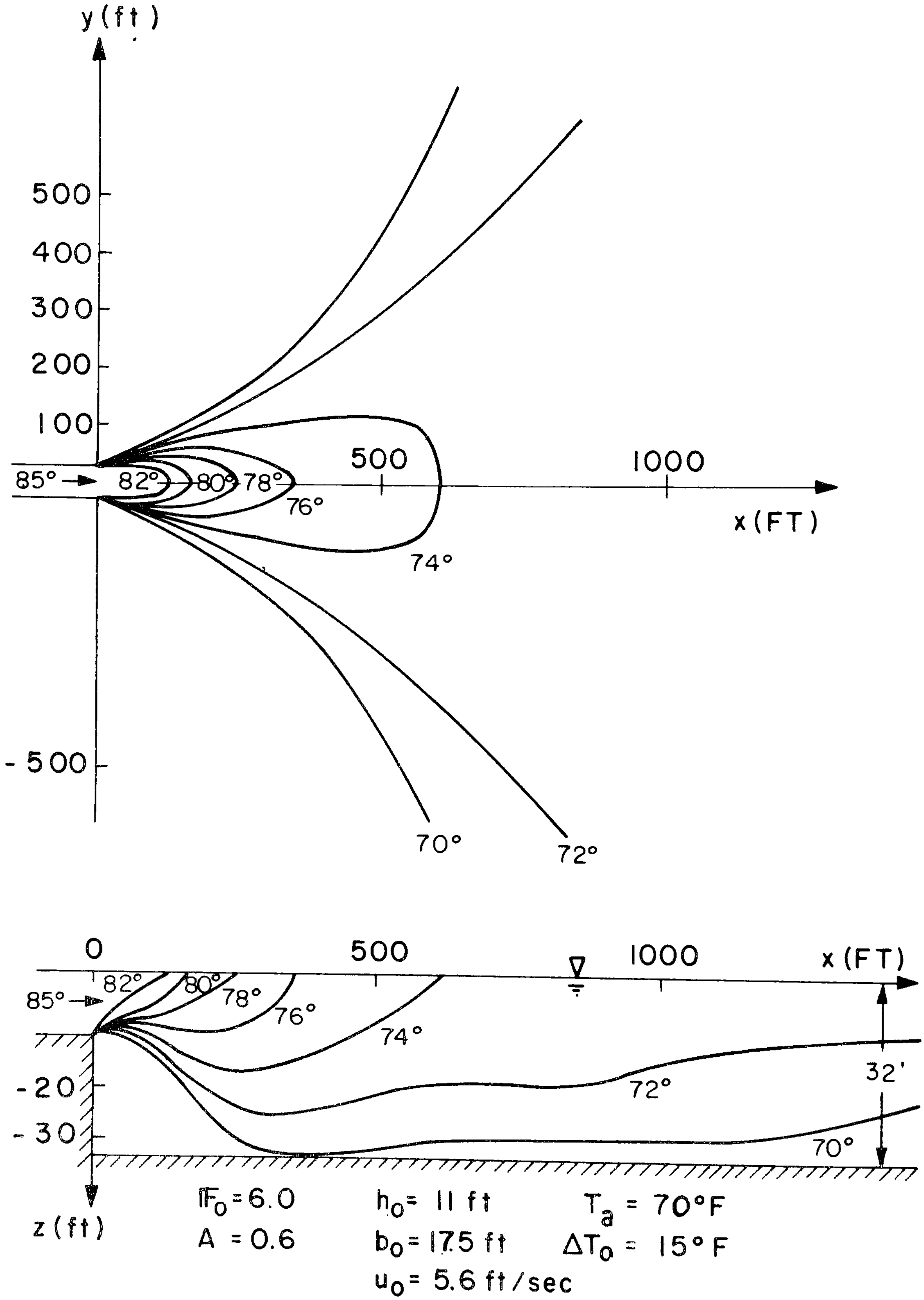

Fig. 4.7 Calculated Contours for the Case Study Example

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*Also published as Water Pollution Control Research Series Report No. 16130 DJU 02/7. by the Water Quality Office, Environmental Protection Agency, Washington, D. C.

List of Symbols

A - discharge channel aspect ratio, h_o/b_o

a_{ij} - coefficient matrix in the governing differential equation set

b - horizontal surface distance from core boundary to jet boundary

b_o - one half the width of rectangular discharge channel

C - coefficient in the exponent of Ellison and Turner's vertical entrainment velocity function

c_{ij} - vector of the constants in the governing differential equation jet

D - ratio of flow in the jet to the initial flow = dilution

D_s - dilution in the stable region of the heated discharge

$\frac{db}{dx}$ - lateral spread of the turbulent region of a jet

ERR - maximum allowable average roundoff error for each step in the numerical computation

$\mathbb{F}_o$ - densimetric Froude number of the discharge channel = $\dfrac{u_o}{\sqrt{\frac{\Delta\rho_o}{\Delta\rho} g h_o}}$

$\mathbb{F}_L$ - local densimetric Froude number in the jet = $\dfrac{u_c}{\sqrt{\frac{\Delta\rho}{\rho} g h}}$

$\mathbb{F}'_o$ - a characteristic Froude number = $\mathbb{F}_o A^{1/4}$

f - similarity function for velocity = $(1 - \zeta^{3/2})^2$

g - acceleration of gravity

HT - ratio of heat flow in the jet to the initial discharge heat flow

H - maximum allowable vertical penetration of the jet

h - vertical centerline distance from core boundary to jet boundary

h_o - depth of the discharge channel

h^*_o - depth of the heated flow at the point of discharge if a cold water wedge is present

h_{max} - maximum value of h obtained in a heated discharge

I_1 - $\int_o^1 f(\zeta)d\zeta = .4500$

I_2 - $\int_o^1 f^2(\zeta)d\zeta = .3600$

I_3 - $\int_o^1 t(\zeta)d\zeta = .6000$

I_4 - $\int_0^1 \int_\zeta^1 t(\zeta)d\zeta d\zeta = .2143$

I_5 - $\int_0^1 f(\zeta)\zeta^{1/2}d\zeta = .2222$

I_6 - $\int_0^1 f^2(\zeta)\zeta^{1/2}d\zeta = .1333$

I_7 - $\int_0^1 f(\zeta)t(\zeta)d\zeta = .3680$

K - surface heat loss coefficient

k - kinematic surface heat loss coefficient

p - pressure

Q_o - discharge channel flow

R - maximum allowable depth of discharge channel

r - vertical distance from the jet centerline to the boundary of the core region

S - maximum allowable half-width of discharge channel

s - horizontal distance from the jet centerline to the boundary of the core region

STEP - increment in $x/\sqrt{h_o b_o}$ at which numerical output is printed

T - temperature

T_a - ambient temperature

T_c - jet centerline surface temperature

T_o - temperature of the heated flow in the discharge channel

T_* - a particular temperature of interest

ΔT - temperature rise above ambient in the jet, $T - T_a$

ΔT_c - surface temperature rise above ambient at the jet centerline, T_c-T_a

ΔT_o - temperature difference between the discharge and the ambient water, T_o-T_a

ΔT_* - T_*-T_a

TM - dimensionless time of travel along the jet centerline,

$$\frac{u_o}{\sqrt{h_o b_o}} \int_o^x \frac{dx}{u_c} .$$

t - similarity function for temperature = $(1-\zeta^{3/2})$

u,v,w - velocity components in the coordinate system relative to the centerline of a deflected jet

$\tilde{u},\tilde{v},\tilde{w}$ - velocity components in the fixed coordinate system

U_o - maximum allowable velocity in the discharge channel

u_o - velocity in the discharge channel

u_o^* - velocity of the heated flow at the point of discharge if a cold water wedge is present

u_c - surface centerline jet velocity

V - ambient crossflow velocity

V_1-V_8 - ambient crossflow velocity components as input to the numerical program

v_e - lateral velocity of the entrained flow at the jet boundary

v_c - lateral velocity in the jet at $y = s+b$ and $-(h+r)<z<\eta$

v_h - lateral velocity in the jet at $y = s$ and $-(h+r)<z<-r$

v_s - lateral velocity in the jet at $y = s$ and $-r<z<\eta$

w_c - vertical velocity in the jet at $z = -(r+h)$ and $0<y<s+b$

w_b - vertical velocity in the jet at $z = -r$ and $b<y<s+b$

w_e - vertical velocity in the jet at $z = -r$ and $0<y<s$

x,y,z - coordinate direction relative to the centerline of a deflected jet

$\tilde{x},\tilde{y},\tilde{z}$ - fixed coordinate direction

x_* - value of x where $\Delta T = \Delta T_*$

x_L - value of $x/\sqrt{h_o b_o}$ at which the numerical computation terminates

y_j - vector of the variables $(\overline{u},\overline{\Delta T}$, etc.) in the governing differential equation set

α_y - lateral entrainment coefficient in non-buoyant and buoyant jets

α_z - vertical entrainment coefficient in a non-buoyant jet

α_{sz} - vertical entrainment coefficient in a buoyant jet

β - coefficient of thermal expansion of water

ε - spread, $\frac{db}{dx}$, of the turbulent region of a non-buoyant jet

ε_o - spread, $\frac{db}{dx}$, of the turbulent region in an undeflected non-buoyant jet

ζ_y - dimensionless width of the turbulent region of a jet, $\frac{|y|-s}{b}$

ζ_z - dimensionless depth of the turbulent region of a jet, $\frac{z-r}{h}$

ζ - either of ζ_y or ζ_z

θ - angle between the jet centerline (x axis) and the $\breve{y}$ axis

θ_o - angle between the discharge channel centerline and the $\tilde{y}$ axis

ρ - density of water

ρ_a - density of the ambient water

ρ_o - density of the heated discharge

$\Delta\rho$ - difference between the ambient water density and the water density, $\rho_a - \rho$

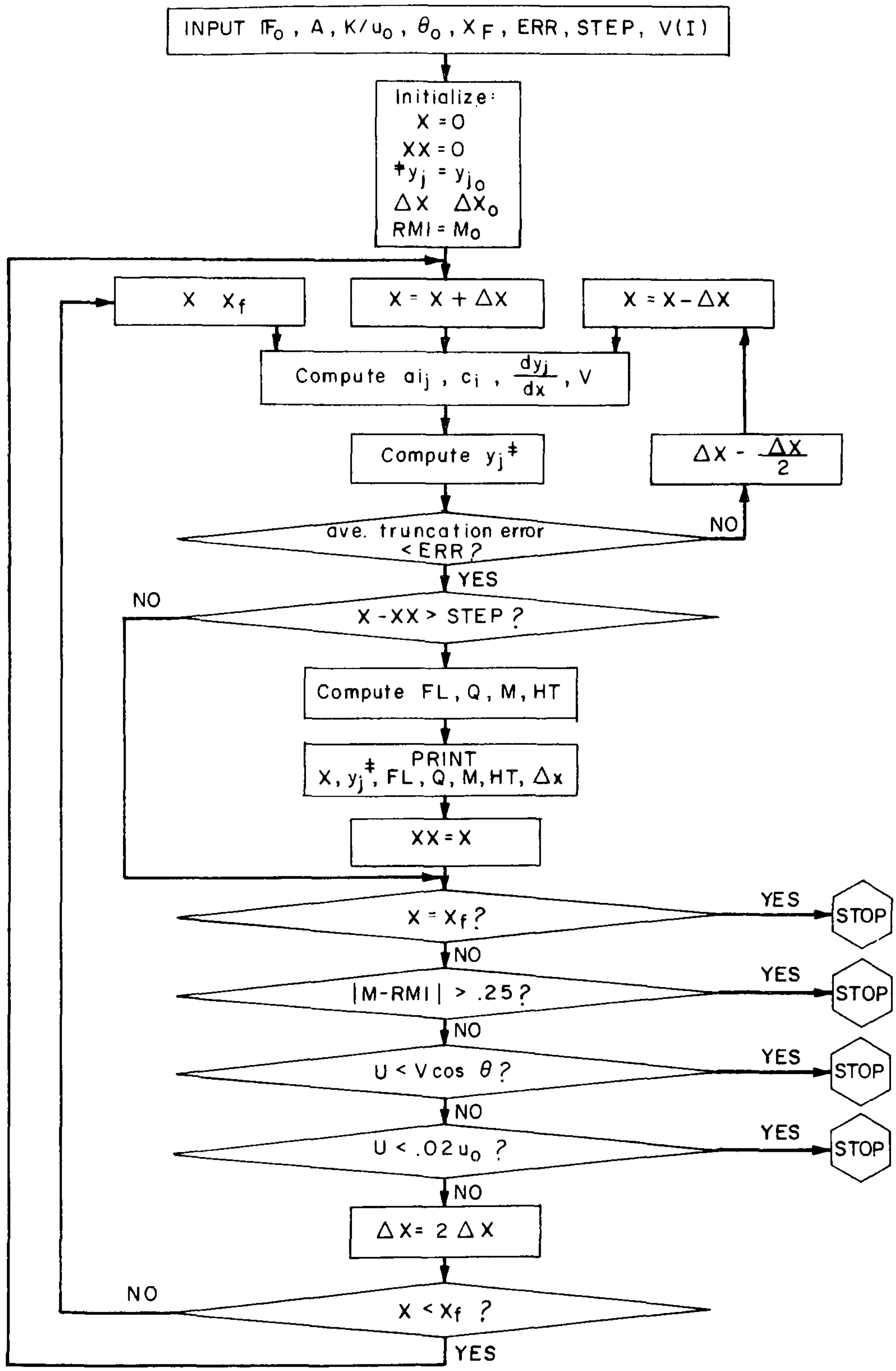
$\Delta\rho_o$ - difference between the ambient water density and the density of the heated flow in the discharge channel, $\rho_a - \rho_o$

superscript ' - indicates a turbulent fluctuating quantity

superscript $^-$ - indicates a dimensionless quantity. Note that variables printed by the computer are dimensionless but are capitalized.

subscript s - indicates a jet property in the stable region

Appendix I Flow Chart for Solution of $\left[a_{ij}\right] \frac{dy_j}{dx} \quad c_i$



‡y_j include U, T, B, H, R, S, $\frac{dB}{dx}$, θ, XP, and YP; $\frac{dB}{dx}$ is not printed

```
      DOUBLE PRECISION DERY,Y,PRMT,X,TH,XP,YP,U,B,T,H,BX,S,R,XLIM,AUX,XX
      DOUBLE PRECISION A,C,DCOS,DSIN,DELX
      REAL I1,I2,I3,I4,I5,I6,I7,I8,M
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),A(10,10),C(10),VEL(8)
      EQUIVALENCE (Y(8),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
      EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,A,C,DELX
      COMMON EPSB,G,DV,IER,WIR,WI,VI,KMI,M,FR,AS,E
      COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      EPS=0.22
      I1=.4500
      I2=.3160
      I3=.6000
      I4=.2143
      I5=.2220
      I6=.13333
      I7=.3680
      RR=5
      READ (RR,100) KK
  100 FORMAT (I3)
      K=1
   10 READ(RR,101) FR,AS,E,THID,XLIM,ERR,STEP
  101 FORMAT (2F10.5,F10.7,4F10.5)
      READ (RR,102) (VEL(I),I=1,8)
  102 FORMAT (8F10.5)
      THI=3.14159*THID/180.0
      R=AS**.5
      S=1./R
      H=.0001
      XX =-2.0
      B=H
      T=1.0
      BX=EPS
      TH=THI
```

Appendix II Program Listing

```
      XP=0.                                                             PGM10037
      YP=0.                                                             PGM10038
      CALL CROSS                                                        PGM10039
      U=1.0-V*COS(THI)                                                  PGM10040
      IER=0                                                             PGM10041
      TM=0                                                              PGM10042
      XI=0                                                              PGM10043
      NPAGE=0                                                           PGM10044
      NLINE=50                                                          PGM10045
      REG=1                                                             PGM10046
      FI=(1./(FR**2))*S                                                 PGM10047
      RMI=1.0+R*FI*.5                                                   PGM10048
      PRMT(1)=0.                                                        PGM10049
      PRMT(2)=XLIM                                                      PGM10050
      PRMT(3)=.00001                                                    PGM10051
      PRMT(4)=ERR                                                       PGM10052
      IF (VEL(1)) 2,1,2                                                 PGM10053
    1 NDIM=7                                                            PGM10054
      VFG=2                                                             PGM10055
      GO TO 3                                                           PGM10056
    2 NDIM=10                                                           PGM10057
      VFG=1                                                             PGM10058
    3 DO 4 N=1,NDIM                                                     PGM10059
      DERY(N)=1./NDIM                                                   PGM10060
    4 CONTINUE                                                          PGM10061
      CALL SRKGS                                                        PGM10062
      IF(K-KK) 7,8,8                                                    PGM10063
    7 K=K+1                                                             PGM10064
      GO TO 10                                                          PGM10065
    8 CALL EXIT                                                         PGM10066
    9 STOP                                                              PGM10067
      END                                                               PGM10068
```

```
      SUBROUTINE SRKGS
      DOUBLE PRECISION PRMT,Y,DERY,AUX,A,B,C,X,XEND,H,AJ,BJ,CJ,R1,R2,
     1DELT,DABS,XLIM,XX,AUN,CUN,DCOS,DSIN,DELX
      REAL I1,I2,I3,I4,I5,I6,I7,I8,R
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION A(4),B(4),C(4),AUN(10,10),CUN(10)
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),VEL(3)
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,AUN,CUN,DELX
      COMMON EPSB,G,DV,IER,WIR,WI,VI,RMI,M,FR,AS,E
      COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      DO 1 I=1,NDIM
    1 AUX(8,I)=.06666666666666666667D0*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)=0.D0
      CALL FCT
C
C     ERROR TEST
      IF(H*(XEND-X))38,37,2
C
C     PREPARATIONS FOR RUNGE-KUTTA METHOD
    2 A(1)=.5D0
      A(2)=.29289321881345248D0
      A(3)=1.7071067811865475D0
      A(4)=.16666666666666667D0
      B(1)=2.D0
      B(2)=1.D0
      B(3)=1.D0
      B(4)=2.D0
      C(1)=.5D0
      C(2)=.29289321881345248D0
      C(3)=1.7071067811865475D0
      C(4)=.5D0
C
```

```
C        PREPARATIONS OF FIRST RUNGE-KUTTA STEP                          SRKG0037
         DO 3 I=1,NDIM                                                   SRKG0038
         AUX(1,I)=Y(I)                                                   SRKG0039
         AUX(2,I)=DERY(I)                                                SRKG0040
         AUX(3,I)=0.D0                                                   SRKG0041
       3 AUX(6,I)=0.D0                                                   SRKG0042
         IREC=0                                                          SRKG0043
         H=H+H                                                           SRKG0044
         IHLF=-1                                                         SRKG0045
         ISTEP=0                                                         SRKG0046
         IEND=0                                                          SRKG0047
C                                                                        SRKG0048
C                                                                        SRKG0049
       4 IF((X+H-XEND)*H)  7,6,5                                         SRKG0050
C        START OF A RUNGE-KUTTA STEP                                     SRKG0051
       5 H=XEND-X                                                        SRKG0052
       6 IEND=1                                                          SRKG0053
C                                                                        SRKG0054
C        RECORDING OF INITIAL VALUES OF THIS STEP                        SRKG0055
       7 CALL OUTP                                                       SRKG0056
      86 IF(PRMT(5))40,8,40                                              SRKG0057
       8 ITEST=0                                                         SRKG0058
       9 ISTEP=ISTEP+1                                                   SRKG0059
C                                                                        SRKG0060
C                                                                        SRKG0061
C        START OF INNERMOST RUNGE-KUTTA LOOP                             SRKG0062
         J=1                                                             SRKG0063
         DO 91 I=1,NDIM                                                  SRKG0064
         AUX(3,I)=0.D0                                                   SRKG0065
      91 AUX(6,I)=0.D0                                                   SRKG0066
      10 AJ=A(J)                                                         SRKG0067
         BJ=B(J)                                                         SRKG0068
         CJ=C(J)                                                         SRKG0069
         DO 11 I=1,NDIM                                                  SRKG0070
         R1=H*DERY(I)                                                    SRKG0071
         R2=AJ*(R1-BJ*AUX(6,I))                                          SRKG0072
```

```
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF(J-4)12,15,15
   12 J=J+1
      IF(J-3)13,14,13
   13 X=X+.5D0*H
   14 CALL FCT
      GO TO 10
C     END OF INNERMOST RUNGE-KUTTA LOOP
C
C     TEST OF ACCURACY
   15 IF(ITEST)16,16,20
C
C     IN CASE ITEST=0 TESTING OF ACCURACY IS IMPOSSIBLE
   16 DO 17 I=1,NDIM
   17 AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
   18 IHLF=IHLF+1
      X=X-H
      H=.5D0*H
      DO 19 I=1,NDIM
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
   19 AUX(6,I)=AUX(3,I)
      GO TO 9
C
C     IN CASE ITEST=1 TESTING OF ACCURACY IS POSSIBLE
   20 IMOD=ISTEP/2
      IF(ISTEP-IMOD-IMOD)21,23,21
   21 CALL FCT
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
   22 AUX(7,I)=DERY(I)
      GO TO 9
```

```
C                                                                       SRKG0109
C        COMPUTATION OF TEST VALUE DELT                                 SRKG0110
   23 DELT=0.D0                                                         SRKG0111
      DO 24 I=1,NDIM                                                    SRKG0112
   24 DELT=DELT+AUX(8,I)*DABS(AUX(4,I)-Y(I))                            SRKG0113
      IF(DELT-PRMT(4))28,28,25                                          SRKG0114
C                                                                       SRKG0115
C        ERROR IS TOO GREAT                                             SRKG0116
   25 GO TO 26                                                          SRKG0117
   26 DO 27 I=1,NDIM                                                    SRKG0118
   27 AUX(4,I)=AUX(5,I)                                                 SRKG0119
      ISTEP=ISTEP+ISTEP-4                                               SRKG0120
      X=X-H                                                             SRKG0121
      IEND=0                                                            SRKG0122
      GO TO 18                                                          SRKG0123
C                                                                       SRKG0124
C        RESULT VALUES ARE GOOD                                         SRKG0125
   28 DO 50 K=1,6                                                       SRKG0126
      IF (Y(K)) 25,50,50                                                SRKG0127
   50 CONTINUE                                                          SRKG0128
  281 CALL FCT                                                          SRKG0129
      DO 29 I=1,NDIM                                                    SRKG0130
      AUX(1,I)=Y(I)                                                     SRKG0131
      AUX(2,I)=DERY(I)                                                  SRKG0132
      AUX(3,I)=AUX(6,I)                                                 SRKG0133
      Y(I)=AUX(5,I)                                                     SRKG0134
   29 DERY(I)=AUX(7,I)                                                  SRKG0135
      IF(PRMT(5))40,30,40                                               SRKG0136
   30 DO 31 I=1,NDIM                                                    SRKG0137
      Y(I)=AUX(1,I)                                                     SRKG0138
   31 DERY(I)=AUX(2,I)                                                  SRKG0139
      IREC=IHLF                                                         SRKG0140
      IF(IEND)32,32,39                                                  SRKG0141
C                                                                       SRKG0142
C        INCREMENT GETS DOUBLED                                         SRKG0143
   32 IHLF=IHLF-1                                                       SRKG0144
```

```
      ISTEP=ISTEP/2                                                     SRKG0145
      H=H+H                                                             SRKG0146
   33 IMOD=ISTEP/2                                                      SRKG0147
      IF(ISTEP-IMOD-IMOD)4,34,4                                         SRKG0148
   34 GO TO 35                                                          SRKG0149
   35 IHLF=IHLF-1                                                       SRKG0150
      ISTEP=ISTEP/2                                                     SRKG0151
      H=H+H                                                             SRKG0152
      GO TO 4                                                           SRKG0153
C                                                                       SRKG0154
C     RETURNS TO CALLING PROGRAM                                        SRKG0155
   37 IHLF=12                                                           SRKG0156
      GO TO 39                                                          SRKG0157
   38 IHLF=13                                                           SRKG0158
   39 CALL OUTP                                                         SRKG0159
   40 RETURN                                                            SRKG0160
      END                                                               SRKG0161
```

```
      SUBROUTINE OUTP
      DOUBLE PRECISION DERY,Y,PRMT,X,TH,XP,YP,U,B,T,H,BX,S,R,XLIM,AUX
      DOUBLE PRECISION A,C,XX,DCOS,DSIN,DELX
      REAL I1,I2,I3,I4,I5,I6,I7,I8,M
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),A(10,10),C(10),VEL(8)
      EQUIVALENCE (Y(8),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
      EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,A,C,DELX
      COMMON EPSB,G,DV,IER,WIR,WI,VI,RMI,M,FR,AS,E
      COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      P=6
      CALL CROSS
      GO TO (100,98,108,118), REG
   98 S=0.0
  100 IF(R-.01)101,101,109
  101 R=0
      REG=REG+2
  109 GO TO (110,105,110,105), REG
  108 R= 0.0
  110 IF(S-.01)111,111,105
  111 S=0.
      REG=REG+1
      GO TO 105
  118 R=0.0
      S=0.0
  105 CONTINUE
  120 IF(X-XX-STEP) 170,170,121
  121 XX=X
  122 IF (NLINE-50) 150,140,140
  140 NPAGE=NPAGE+1
  141 NLINE=0
      WRITE (P,202) NPAGE
  202 FORMAT(1H1,24HBUOYANT JET CALCULATIONS,10X,5HPAGE ,I2)
      WRITE (P,2021)
```

```
2021 FORMAT(/2X,13HFROUDE NUMBER,8X,12HASPECT RATIO,7X,
    118HANGLE OF DISCHARGE,6X,14HROUNDOFF ERROR,7X,4HXLIM,14X,
    214HHEAT LOSS COEF)
      WRITE (P,203) FR,AS,THID,ERR,XLIM,F
 203  FORMAT(1X,4(F9.5,12X),F10.5,11X,F8.7//)
      WRITE (P,300) (I,VEL(I),I=1,8)
 300  FORMAT(8(3X,3HVEL,I1,1H=,F8.3)//)
      WRITE (P,199)
 199  FORMAT(4X,1HX,8X,1HH,8X,1HB,8X,1HR,8X,1HS,8X,2HFL,7X,
     11HQ,7X,1HM,5X,1HU,5X,1HT,4X,2HHT,5X,1HV,6X,2HXP,
     27X,2HYP,5X,3HTHD,6X,2HTM/)
 150  NLINE=NLINE+1
      THD=180.0*TH/3.14159
      FL=T*H/(U*U)
      FL=FL/(AS**.5)
      FL=ABS(FL)
      FL=FL**.5
      FL=FR/FL
      Q=U*(S+B*I1)*(R+H*I1)+V*DCOS(TH)*(S+B)*(R+H)
      M=U*U*(S+B*I2)*(R+H*I2)+Q*V*DCOS(TH)
      M=M+U*V*DCOS(TH)*(S+B*I1)*(R+H*I1)
      M=M+(R*R*.5+I3*H*R+I4*H*H)*(S+B*I3)*T*FI
      HT=U*T*(S+B*I7)*(R+H*I7)+T*V*DCOS(TH)*(S+B*I3)*(R+H*I3)
      DELX=X-XI
      DTM=2*DELX/(U+UI)
      TM=TM+DTM
  95  IF(NDIM-7)96,96,97
  96  XP=X
  97  CONTINUE
      WRITE (P,200) X,H,B,R,S,FL,Q,M,U,T,HT,V,XP,YP,THD,TM
 200  FORMAT(7(1X,D8.3),5(1X,F5.3),2(1X,D8.3),1X,F5.1,2X,D8.3)
      XI=X
      UI=U
      IF(ABS(M-RMI)-.25) 180,180,158
 180  IF(U-V*DCOS(TH))160,160,159
 159  IF(U-0.02) 160,160,170
```

```
158 WRITE (P,400)
400 FORMAT(/10X,43HMOMENTUM HAS EXCEEDED BOUNDS RUN TERMINATES)
    GO TO 190
160 WRITE (P,401)
401 FORMAT(/10X,48HJET VELOCITY HAS BECOME TOO SMALL RUN TERMINATES)
190 PRMT(5)=1.0
170 RETURN
    END
```

```
      SUBROUTINE CROSS
      DOUBLE PRECISION DERY,Y,PRMT,X,TH,XP,YP,U,B,T,H,BX,S,R,XLIM,AUX,XX
      DOUBLE PRECISION AUN,CUN,DCOS,DSIN,DELX
      REAL I1,I2,I3,I4,I5,I6,I7,I8,M
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),VEL(8)
      DIMENSION AUN(10,10),CUN(10)
      EQUIVALENCE (Y(8),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
      EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,AUN,CUN,DELX
      COMMON EPSB,G,DV,IER,WIR,WI,VI,RMI,M,FR,AS,E
      COMMON THIO,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      A=XP*VEL(4)
      A=A-VEL(5)
      V=-A*2*VEL(3)*VEL(4)
      A=A*A
      A=VEL(3)*A
      A=VEL(2)*EXP(-A)
      DV=V*A
      V=VEL(1)+A
      RETURN
      END
```

```
      SUBROUTINE FCT
      DOUBLE PRECISION DERY,Y,PRMT,X,TH,XP,YP,U,B,T,H,BX,S,R,XLIM,AUX,XX
      DOUBLE PRECISION A,C,DCOS,DSIN,DELX
      REAL I1,I2,I3,I4,I5,I6,I7,I8,M
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),A(10,10),C(10),VEL(8)
      EQUIVALENCE (Y(8),TH),(Y(9),XP),(Y(10),YP),(Y(3),B),(Y(1),U)
      EQUIVALENCE (Y(2),T),(Y(4),H),(Y(7),BX),(Y(6),S),(Y(5),R)
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,A,C,DELX
      COMMON EPSB,G,DV,IER,WIR,WI,VI,RMI,M,FR,AS,E
      COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      DO 50 I=1,10
      C(I)=0.
      DERY(I)=0.
      DO 50 II=1,10
      A(I,II)=0.
   50 CONTINUE
  101 FLAG=VFG
      WI=EPS*U
      VI=-WI
      EPSB=EPS
      ARG=5*T*H/(U*U*FR*FR)
      ARG=ARG/(AS**.5)
      IF(ARG)10100,10100,10101
10100 ARG=0.
10101 IF(ARG-100.)10103,10103,10102
10102 WIR=0.
      GO TO 10104
10103 WIR=EXP(-ARG)
10104 GO TO (1011,1012), FLAG
 1011 G=0.
      BXS=BX
      BX=0.
      GO TO 1013
 1012 G=FI*T
```

```
      WI=WI*WIR
 1013 GO TO (102,103,104,105), REG
  102 WI=WI*(I1-I2)
      VI=VI*(I1-I2)
      GO TO 110
  103 WI=WI*(I1-I2)
      VI=VI*I1/2
      GO TO 110
  104 WI=WI*I1/2
      VI=VI*(I1-I2)
      GO TO 110
  105 WI=WI*I1/2
      VI=VI*I1/2
  110 H1=R+H*I1
      B1=S+B*I1
      A(1,1)=H1*B1
      A(1,2)=U*I1*B1
      A(1,3)=U*B1
      A(1,4)=U*H1
      C(1)=WI*B1-VI*H1-U*I1*H1*BX
      H3=R+H*I3
      B3=S+B*I3
      H2=R+H*I2
      B2=S+B*I2
      HG=R*R/2+R*H*I3+H*H*I4
      A(2,1)=2*U*H2*B2
      A(2,2)=U*U*I2*B2+G*B3*(R*I3+2*H*I4)
      A(2,3)=U*U*B2+G*B3*(R+H*I3)
      A(2,4)=U*U*H2+G*HG
      A(2,5)=FI*B3*HG
      C(2)=-(U*U*I2*H2+G*I3*HG)*BX
      B7=S+B*I7
      H7=R+H*I7
      A(3,1)=G*B7*H7
      A(3,2)=U*G*I7*B7
      A(3,3)=U*G*B7
```

```
      A(3,4)=U*G*H7
      A(3,5)=U*B7*H7*FI
      C(3)=-E*G*B3-U*G*I7*H7*BX
      GO TO (1,2,3,4), REG
    1 A(5,1)=U
      A(5,2)=G*I3
      A(5,3)=G
      A(5,5)=FI*(R/2+H*I3)
      GO TO 5
    2 A(5,4)=1.
      GO TO 5
    3 A(5,3)=1.0
      GO TO 5
    4 A(5,4)=1.
      A(4,3)=1.0
      GO TO 6
    5 I8=1.-I2/I1
      A(4,1)=U*((S*H+R*B)*I2+R*S*I8)
      A(4,2)=G*(2*S*H*I4+R*B*I3*I3)
      A(4,3)=U*U*S*I8+G*(S*H*I3+R*B*I3)
      A(4,4)=U*U*R*I8+G*(R*R/2+H*R*I3)
      A(4,5)= FI*(S*H*H*I4+B*R*R*I3/2+B*H*R*I3*I3)
      C(4)=U*(VI*R-WI*S)*I2/I1-G*(R*R/2+H*R*I3)*I3*BX
    6 GO TO (120,131), VFG
  120 CALL CROSS
      DERY(9)=DSIN(TH)
      DERY(10)=DCOS(TH)
      VST=V*DERY(9)
      VCT=V*DERY(10)
      DT=U*U*(S+B*I2)*(R+H*I2)+2*U*VCT*(S+B*I1)*(R+H*I1)
     1+VCT*VCT*(R+H)*(S+B)
      DT=-VST*(WI*(S+B*I1)-VI*(R+H*I1))/DT
      DERY(8)=DT
      H0=R+H
      DT=VST*DT-DERY(10)*DV*DERY(9)
      B0=S+B
```

```
      A(1,2)=A(1,2)+VCT*B0                                              FCTN0109
      A(1,3)=A(1,3)+VCT*B0                                              FCTN0110
      A(1,4)=A(1,4)+VCT*H0                                              FCTN0111
      C(1)=C(1)-VCT*H0*BX+BC*H0*DT                                      FCTN0112
      A(2,1)=A(2,1)+2*VCT*H1*B1                                         FCTN0113
      A(2,2)=A(2,2)+VCT*(2*U*I1*B1+VCT*B0)                              FCTN0114
      A(2,3)=A(2,3)+VCT*(2*U*B1+VCT*B0)                                 FCTN0115
      A(2,4)=A(2,4)+VCT*(2*U*H1+VCT*H0)                                 FCTN0116
      C(2)=C(2)-VCT*(2*U*I1*H1+VCT*H0)*BX+2*(U*H1*B1+VCT*B0*H0)*DT      FCTN0117
     1+VCT*(WI*B1-VI*H1)                                                FCTN0118
      A(3,2)=A(3,2)+G*VCT*I3*B3                                         FCTN0119
      A(3,3)=A(3,3)+G*VCT*B3                                            FCTN0120
      A(3,4)=A(3,4)+G*VCT*H3                                            FCTN0121
      A(3,5)=A(3,5)+FI*VCT*H3*B3                                        FCTN0122
      C(3)=C(3)-G*VCT*I3*H3*BX+G*B3*H3*DT                               FCTN0123
      GO TO (7,8,8,9),REG                                               FCTN0124
    7 A(5,1)=A(5,1)+VCT                                                 FCTN0125
      C(5)= (U+VCT)*DT                                                  FCTN0126
      GO TO 9                                                           FCTN0127
    8 A(4,1)=A(4,1)+VCT*I1*(R*B+S*H)                                    FCTN0128
      A(4,2)=A(4,2)+S*U*VCT*(I1-I1/I2)                                  FCTN0129
      A(4,3)=A(4,3)+S*U*VCT*I8                                          FCTN0130
      A(4,4)=A(4,4)+R*U*VCT*I8                                          FCTN0131
      C(4)=C(4)-R*U*VCT*(I1-I1/I2)*BX                                   FCTN0132
     1+((U*(2*I1-I2/I1)+VCT)*(S*H+R*B)+R*S*U*I8)*DT                     FCTN0133
    9 GO TO (130,131),FLAG                                              FCTN0134
  130 A(1,2)=A(1,2)+(U*I1*H1+VCT*H0)                                    FCTN0135
      A(2,2)=A(2,2)+(U*U*I2*H2+2*U*VCT*I1*H1+VCT*VCT*H0)                FCTN0136
      GO TO (10,10,10,11),REG                                           FCTN0137
   10 A(4,2)=A(4,2)+R*U*VCT*(I1-I2/I1)                                  FCTN0138
   11 CALL SGELG(5)                                                     FCTN0139
      EPSB=C(2)                                                         FCTN0140
      BX=BXS                                                            FCTN0141
      FLAG=2                                                            FCTN0142
      G=FI*T                                                            FCTN0143
      WI=WI*WIR                                                         FCTN0144
```

```
      GO TO 110                                                         FCTN0145
  131 DB=BX-EPSB                                                        FCTN0146
      A(6,1)=DB*2*U*B*H2*I6                                             FCTN0147
      A(6,2)=DB*U*U*B*I2*I6                                             FCTN0148
      A(6,3)=DB*U*U*B*I6                                                FCTN0149
      A(6,6)=U*U*B*H2*I6                                                FCTN0150
      C(6)=G*HG-U*U*I6*DB*H2*BX                                         FCTN0151
      GO TO (132,133),VFG                                               FCTN0152
  132 A(6,1)=A(6,1)+2*B*VCT*I5*H2*DB                                    FCTN0153
      A(6,2)=A(6,2)+VCT*B*(2*U*I5*I1+VCT)*DB                            FCTN0154
      A(6,3)=A(6,3)+VCT*B*(2*U*I5+VCT)*DB                               FCTN0155
      A(6,6)=A(6,6)+VCT*B*(2*U*I5*H1+VCT*H0)                            FCTN0156
      C(6)=C(6)-DB*VCT*(2*U*I5*H1+VCT*H0)*BX+2*DB*B*(U*I5*H1+VCT        FCTN0157
     1*H0)*DT                                                           FCTN0158
  133 CALL SGELG(5)                                                     FCTN0159
      DERY(1)=C(1)                                                      FCTN0160
      DERY(2)=C(5)                                                      FCTN0161
      DERY(3)=BX                                                        FCTN0162
      DERY(4)=C(2)                                                      FCTN0163
      DERY(5)=C(3)                                                      FCTN0164
      DERY(6)=C(4)                                                      FCTN0165
      DERY(7)=C(6)                                                      FCTN0166
      RETURN                                                            FCTN0167
      END                                                               FCTN0168
```

```
      SUBROUTINE SGELG(M)
      DOUBLE PRECISION AD,AI,XX,DABS,DCOS,DSIN,SAVE,S,R,XLIM,AUX
      DOUBLE PRECISION DERY,Y,PRMT,X,TH,XP,YP,U,B,T,H,BX
      DOUBLE PRECISION A,PIV,TB,TOL,PIVI,DELX
      REAL MUN
      INTEGER P,RR,REG,VFG,FLAG
      DIMENSION PRMT(5),Y(10),AUX(8,10),DERY(10),AI(100),R(10),VEL(8)
      DIMENSION A(100),AD(10,10)
      DIMENSION SAVE(10)
      EQUIVALENCE (AI(1),AD(1,1))
      COMMON DERY,Y,PRMT,X,XLIM,AUX,XX,AI,R,DELX
      COMMON EPS8,G,DV,IER,WIR,WI,VI,RMI,MUN,FR,AS,E
      COMMON THID,THI,ERR,VEL,I1,I2,I3,I4,I5,I6,I7,I8,EPS,V,BXS,DT,FI
      COMMON P,RR,NPAGE,NLINE,REG,NDIM,VFG,FLAG,IHLF,STEP,TM,XI
      N=1
      EPSS=.1E-16
C     SCALE THE MATRIX
      DO 54 I=1,M
      AMAX=R(I)
      DO 52 J=1,M
      DA=DABS(AD(I,J))
      IF(DA-AMAX)52,52,51
   51 AMAX=DA
   52 CONTINUE
      DO 53 J=1,M
      AD(I,J)=AD(I,J)/AMAX
   53 CONTINUE
      R(I)=R(I)/AMAX
   54 CONTINUE
C     SAVE THE Y EQUATION ROW
      MM=M+1
      DO 999 I=1,MM
      SAVE (I)=AD(MM,I)
  999 CONTINUE
C     CONVERT MATRIX TO COLUMN FORM
      IJ=0
```

```
      NM=0
      DO 1100 K=1,M
      DO 1000 L=1,M
      IJ=IJ+1
      NM=NM+1
 1000 A(IJ)=AI(NM)
 1100 NM=NM+10-M
      IF(M)23,23,1
C
C        SEARCH FOR GREATEST ELEMENT IN MATRIX A
    1 IER=0
      PIV=0.D0
      MM=M*M
      NM=N*M
      DO 3 L=1,MM
      TB=DABS(A(L))
      IF(TB-PIV)3,3,2
    2 PIV=TB
      I=L
    3 CONTINUE
      TOL=EPSS*PIV
C        A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I)
C
C
C        START ELIMINATION LOOP
      LST=1
      DO 17 K=1,M
C
C        TEST ON SINGULARITY
      IF(PIV)23,23,4
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 PIVI=1.D0/A(I)
      J=(I-1)/M
      I=I-J*M-K
```

```
      J=J+1-K                                                           SGEL0073
C     I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT               SGEL0074
C                                                                       SGEL0075
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R      SGEL0076
      DO 8 L=K,NM,M                                                     SGEL0077
      LL=L+I                                                            SGEL0078
      TB=PIVI*R(LL)                                                     SGEL0079
      R(LL)=R(L)                                                        SGEL0080
    8 R(L)=TB                                                           SGEL0081
C                                                                       SGEL0082
C     IS ELIMINATION TERMINATED                                         SGEL0083
      IF(K-M)9,18,18                                                    SGEL0084
C                                                                       SGEL0085
C     COLUMN INTERCHANGE IN MATRIX A                                    SGEL0086
    9 LEND=LST+M-K                                                      SGEL0087
      IF(J)12,12,10                                                     SGEL0088
   10 II=J*M                                                            SGEL0089
      DO 11 L=LST,LEND                                                  SGEL0090
      TB=A(L)                                                           SGEL0091
      LL=L+II                                                           SGEL0092
      A(L)=A(LL)                                                        SGEL0093
   11 A(LL)=TB                                                          SGEL0094
C                                                                       SGEL0095
C     ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A               SGEL0096
   12 DO 13 L=LST,MM,M                                                  SGEL0097
      LL=L+I                                                            SGEL0098
      TB=PIVI*A(LL)                                                     SGEL0099
      A(LL)=A(L)                                                        SGEL0100
   13 A(L)=TB                                                           SGEL0101
C                                                                       SGEL0102
C     SAVE COLUMN INTERCHANGE INFORMATION                               SGEL0103
      A(LST)=J                                                          SGEL0104
C                                                                       SGEL0105
C     ELEMENT REDUCTION AND NEXT PIVOT SEARCH                           SGEL0106
      PIV=0.D0                                                          SGEL0107
      LST=LST+1                                                         SGEL0108
```

```
      J=0                                                               SGEL0109
      DO 16 II=LST,LEND                                                 SGEL0110
      PIVI=-A(II)                                                       SGEL0111
      IST=II+M                                                          SGEL0112
      J=J+1                                                             SGEL0113
      DO 15 L=IST,MM,M                                                  SGEL0114
      LL=L-J                                                            SGEL0115
      A(L)=A(L)+PIVI*A(LL)                                              SGEL0116
      TB=DABS(A(L))                                                     SGEL0117
      IF(TB-PIV)15,15,14                                                SGEL0118
   14 PIV=TB                                                            SGEL0119
      I=L                                                               SGEL0120
   15 CONTINUE                                                          SGEL0121
      DO 16 L=K,NM,M                                                    SGEL0122
      LL=L+J                                                            SGEL0123
   16 R(LL)=R(LL)+PIVI*R(L)                                             SGEL0124
   17 LST=LST+M                                                         SGEL0125
C     END OF ELIMINATION LOOP                                           SGEL0126
C                                                                       SGEL0127
C                                                                       SGEL0128
C     BACK SUBSTITUTION AND BACK INTERCHANGE                            SGEL0129
   18 IF(M-1)23,22,19                                                   SGEL0130
   19 IST=MM+M                                                          SGEL0131
      LST=M+1                                                           SGEL0132
      DO 21 I=2,M                                                       SGEL0133
      II=LST-I                                                          SGEL0134
      IST=IST-LST                                                       SGEL0135
      L=IST-M                                                           SGEL0136
      L=A(L)+.5D0                                                       SGEL0137
      DO 21 J=II,NM,M                                                   SGEL0138
      TB=R(J)                                                           SGEL0139
      LL=J                                                              SGEL0140
      DO 20 K=IST,MM,M                                                  SGEL0141
      LL=LL+1                                                           SGEL0142
   20 TB=TB-A(K)*R(LL)                                                  SGEL0143
      K=J+L                                                             SGEL0144
```

```
      R(J)=R(K)                                                         SGEL0145
   21 R(K)=TB                                                           SGEL0146
   22 GO TO (223,221), FLAG                                             SGEL0147
  221 MM=M+1                                                            SGEL0148
      DO 222 I=1,M                                                      SGEL0149
      R(MM)=R(MM)-SAVE(I)*R(I)                                          SGEL0150
  222 CONTINUE                                                          SGEL0151
      R(MM)=R(MM)/SAVE(MM)                                              SGEL0152
  223 DO 24 I=1,100                                                     SGEL0153
      A(I)=0.                                                           SGEL0154
   24 CONTINUE                                                          SGEL0155
      RETURN                                                            SGEL0156
C                                                                       SGEL0157
C                                                                       SGEL0158
C        ERROR RETURN                                                   SGEL0159
   23 IER=-1                                                            SGEL0160
      RETURN                                                            SGEL0161
      END                                                               SGEL0162
```

Input:

```
  1
6.0     0.6     0.0     90.0    500.0   0.01    1.0
0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
```

Output:

BUOYANT JET CALCULATIONS PAGE 1

FROUDE NUMBER	ASPECT RATIO	ANGLE OF DISCHARGE	ROUNDOFF ERROR	XLIM	HEAT LOSS COEF
6.00000	0.60000	90.00000	0.01000	500.00000	.0

VEL1= 0.0 VEL2= 0.0 VEL3= 0.0 VEL4= 0.0 VEL5= 0.0 VEL6= 0.0 VEL7= 0.0 VEL8= 0.0

X	H	B	R	S	FL	Q	M	U	T	HT	V	XP	YP	THD	TM
.0	.100D-03	.100D-03	.775D 00	.129D 01	.528E 03	.100E 01	1.014	1.000	1.000	1.000	0.0	.0	.0	90.0	.0
.131D 01	.190D 00	.585D 00	.521D 00	.152D 01	.124E 02	.109E 01	1.014	1.005	0.971	1.000	0.0	.131D 01	.0	90.0	.131E 01
.262D 01	.378D 00	.116D 01	.450D 00	.137D 01	.890E 01	.118E 01	1.014	1.004	0.940	1.000	0.0	.262D 01	.0	90.0	.261E 01
.524D 01	.724D 00	.239D 01	.218D 00	.144D 01	.664E 01	.138E 01	1.014	1.006	0.884	1.000	0.0	.524D 01	.0	90.0	.522E 01
.786D 01	.104D 01	.377D 01	.293D-01	.153D 01	.574E 01	.161E 01	1.014	1.007	0.828	1.000	0.0	.786D 01	.0	90.0	.782E 01
.918D 01	.119D 01	.450D 01	.0	.152D 01	.517E 01	.177E 01	0.995	0.931	0.759	0.984	0.0	.918D 01	.0	90.0	.918E 01
.105D 02	.134D 01	.527D 01	.0	.148D 01	.465E 01	.196E 01	0.997	0.847	0.692	0.986	0.0	.105D 02	.0	90.0	.107E 02
.131D 02	.161D 01	.692D 01	.0	.130D 01	.391E 01	.232E 01	1.000	0.726	0.597	0.988	0.0	.131D 02	.0	90.0	.140E 02
.157D 02	.185D 01	.873D 01	.0	.100D 01	.342E 01	.265E 01	1.002	0.645	0.535	0.990	0.0	.157D 02	.0	90.0	.178E 02
.210D 02	.221D 01	.129D 02	.0	.195D 00	.283E 01	.321E 01	1.002	0.539	0.457	0.990	0.0	.210D 02	.0	90.0	.267E 02
.236D 02	.224D 01	.153D 02	.0	.103D 00	.267E 01	.347E 01	0.999	0.492	0.424	0.987	0.0	.236D 02	.0	90.0	.317E 02
.249D 02	.227D 01	.166D 02	.0	.0	.258E 01	.360E 01	0.996	0.471	0.409	0.985	0.0	.249D 02	.0	90.0	.345E 02
.262D 02	.228D 01	.180D 02	.0	.0	.250E 01	.373E 01	0.996	0.449	0.394	0.985	0.0	.262D 02	.0	90.0	.373E 02
.275D 02	.228D 01	.196D 02	.0	.0	.244E 01	.387E 01	0.997	0.429	0.381	0.985	0.0	.275D 02	.0	90.0	.403E 02
.288D 02	.227D 01	.211D 02	.0	.0	.238E 01	.400E 01	0.997	0.411	0.368	0.985	0.0	.288D 02	.0	90.0	.434E 02
.315D 02	.225D 01	.246D 02	.0	.0	.227E 01	.426E 01	0.997	0.380	0.346	0.985	0.0	.315D 02	.0	90.0	.501E 02
.367D 02	.220D 01	.324D 02	.0	.0	.211E 01	.476E 01	0.997	0.329	0.310	0.985	0.0	.367D 02	.0	90.0	.648E 02
.419D 02	.215D 01	.415D 02	.0	.0	.197E 01	.523E 01	0.997	0.290	0.282	0.985	0.0	.419D 02	.0	90.0	.818E 02
.472D 02	.209D 01	.516D 02	.0	.0	.186E 01	.568E 01	0.997	0.260	0.260	0.985	0.0	.472D 02	.0	90.0	.101E 03
.524D 02	.204D 01	.629D 02	.0	.0	.177E 01	.609E 01	0.997	0.235	0.242	0.985	0.0	.524D 02	.0	90.0	.122E 03
.629D 02	.202D 01	.884D 02	.0	.0	.151E 01	.683E 01	0.994	0.189	0.215	0.984	0.0	.629D 02	.0	90.0	.171E 03
.734D 02	.211D 01	.124D 03	.0	.0	.107E 01	.707E 01	0.995	0.134	0.208	0.984	0.0	.734D 02	.0	90.0	.236E 03
.839D 02	.195D 01	.174D 03	.0	.0	.868E 00	.715E 01	0.994	0.104	0.206	0.983	0.0	.839D 02	.0	90.0	.325E 03
.105D 03	.156D 01	.324D 03	.0	.0	.658E 00	.720E 01	0.991	0.070	0.204	0.981	0.0	.105D 03	.0	90.0	.565E 03
.126D 03	.125D 01	.552D 03	.0	.0	.543E 00	.723E 01	0.988	0.052	0.202	0.978	0.0	.126D 03	.0	90.0	.909E 03
.147D 03	.102D 01	.864D 03	.0	.0	.470E 00	.723E 01	0.985	0.040	0.202	0.974	0.0	.147D 03	.0	90.0	.136E 04
.168D 03	.359D 00	.126D 04	.0	.0	.418E 00	.722E 01	0.981	0.033	0.201	0.971	0.0	.168D 03	.0	90.0	.194E 04
.210D 03	.641D 00	.235D 04	.0	.0	.349E 00	.720E 01	0.973	0.024	0.200	0.963	0.0	.210D 03	.0	90.0	.342E 04
.252D 03	.506D 00	.382D 04	.0	.0	.304E 00	.716E 01	0.965	0.018	0.200	0.956	0.0	.252D 03	.0	90.0	.542E 04

JET VELOCITY HAS BECOME TOO SMALL RUN TERMINATES

Appendix III Input and Output

SELECTED WATER RESOURCES ABSTRACTS

INPUT TRANSACTION FORM

1. Report No. | 2. | 3. Accession No. **W**

4. Title
A User's Manual for Three-Dimensional Heated Surface Discharge Computations

5. Report Date

6.

8. Performing Organization Report No.

7. Author(s)
K.D. Stolzenbach, E. Eric Adams, and Donald R.F. Harleman

10. Project No.

9. Organization
Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

11. Contract/Grant No.
16130 DJU

13. Type of Report and Period Covered

12. Sponsoring Organization
Environmental Protection Agency

15. Supplementary Notes
Environmental Protection Agency Report Number EPA-R2-73-133, January 1973.

16. Abstract

The temperature distribution induced in an ambient body of water by a surface discharge of heated condenser cooling water must be determined for evaluation of thermal effects upon the natural environment, for prevention of recirculation of the heated discharge into the cooling water intake, for improved design of laboratory scale models and for insuring that discharge configurations meet legal temperature regulations. This report presents a review of the theoretical background for a three-dimensional temperature prediction model, a detailed discussion of the computer program and a case study illustrating the procedure for optimizing the design of a surface discharge channel. Flow chart, program listing and a sample of the input and output data are given in the appendices. The model presented here includes modifications of the report by Keith D. Stolzenbach and Donald R. F. Harleman, published in February 1971 entitled, "An Analytical and Experimental Investigation of Surface Discharges of Heated Water."

17a. Descriptors

Waste heat disposal, heated surface discharge, turbulent buoyant jets, temperature prediction, thermal pollution.

17b. Identifiers

17c. COWRR Field & Group 05G

18. Availability

19. Security Class. (Report)

20. Security Class. (Page)

21. No. of Pages

22. Price

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Abstractor

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