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THE "GAUSSIAN-PLUME" MODEL WITH LIMITED VERTICAL MIXING

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by

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PREFACE

Restriction of the vertical spread of pollution plumes, as when the atmospheric surface mixing-layer is of limited depth, is an important effect that must be quantitatively estimated in air quality modeling. On a recent visit to the U.S.A., Dr. F. Pasquill kindly agreed to document some considerations of this question in relation to the practical application of "Gaussian-plume" modeling, thus extending the discussion in his recent book Atmospheric Diffusion. The publication of this material as an EPA report is made in the interests of wide dissemination of the information to air quality modelers.

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ABSTRACT

Application of the "Gaussian'plume" model for atmospheric dispersion from an elevated source in a mixing layer of limited depth normally involves consideration of multiple reflections of the plume between the upper and lower boundaries. The present analysis considers some simple approximation formulae that should be useful in practical applications.

SECTION 1

INTRODUCTION

In the use of the "Gaussian Plume Model" an important requirement is to make satisfactory allowance for the limiting of vertical spread by an elevated stable layer in the atmosphere. Physically it is to be expected that in the simple case of a mixing layer of constant depth the vertical distribution downwind of a source will ultimately be uniform, though of course there will continue to be a variation of concentration acrosswind. On the other hand, for a source at relatively low level, there will be some distance downwind within which the upper boundary of the mixed layer has no limiting effect on the vertical spread and, given the dispersion parameter σ_z, the progress of vertical spread may then be treated as if the upper boundary did not exist. Between these two relatively simple regimes the vertical distribution will have a complex form. The present interest is to review briefly (in the context of the Gaussian Plume Model) the estimation of the limits of this intermediate regime and the procedure for evaluating ground-level concentration within it.

SECTION 2

SOME ESSENTIAL POINTS IN THE HISTORY OF THE PROBLEM

(a) In the British Meteorological Office system of dispersion estimates (1) a simple and somewhat intuitive rule was suggested, in which the prescribed vertical spread was used until the effective "total" spread (as represented by 2.15 σ_z) became equal to the depth of mixing h', at some distance downwind d_1. It was then assumed that at distance $2d_1$ the stage of uniform vertical distribution would be reached, the concentration thereafter being given by substituting in the Gaussian plume formula a <u>constant</u> vertical spread in accordance with

$$2.15\sigma_z \approx 2h' \quad (\text{ or to be exact } 1.715h') \qquad (1)$$

Having derived concentrations for distances d_1 and $2d_1$ the concentration-distance graph over the intermediate distances was completed by interpolation.

(b) The procedure in (a) was offered as a simple way of representing the essential consequence of multiple reflections of a plume between upper and lower boundaries. A complete expression for these multiple reflections, in which the effect of the upper boundary is represented in exactly the same way as is the ground in the familiar treatment of the elevated source without limitation of upward vertical spread, was given by Bierly and Hewson (2). Their equation is reproduced in the Turner Workbook (3) [Eq. (5.8), p. 36 therein].

(c) A compact form of the multiple-reflection equation, for the special case of the <u>ground-level</u> concentration, has been stated by the writer (4) in the context of "box models". For the crosswind integrated concentration (CWID) from a continuous point source (formally equivalent to the concentration from an infinite line source acrosswind) the ground-level value is given by the series expression

$$(2/\pi)^{\frac{1}{2}} (Q/u\sigma_z) \sum \exp -(Z_n^2/2\sigma_z^2) \qquad (2)$$

where $Z_n = H \pm 2nh'$, with $n = 0,1,2 \ldots$, H the height of the source and Q the source strength (per unit time for a point source or per unit time and unit length in the case of a line source). In the further discussion the infinite line source form will be adopted, the foregoing expression then being the magnitude of the ground-level concentration C(x,0). For a point source the concentration C(x,y,0) is given by

$$C(x,y,0)/C(x,0) = (2\pi)^{-\frac{1}{2}} \sigma_y^{-\frac{1}{2}} \exp -(y^2/2\sigma_y^2) \qquad (3)$$

In the absence of an upper boundary to the mixed layer the series at (2) becomes, with n = 0

$$C(x,0) = (2/\pi)^{\frac{1}{2}} (Q/u\sigma_z) \exp -(H^2/2\sigma_z^2) \qquad (4)$$

which with Eq. (3) gives the well-known form for an elevated point source with no limitation in vertical spread. Consecutive reflections from an upper boundary at h' are represented by n = 1,2,3 etc. Bierly and Hewson's more extensive expression [for C(x,0,z)] is easily seen to reduce to the expression at (2) above on setting z to zero, and applying (3) with $y = 0$.

SECTION 3

EVALUATION OF THE MULTIPLE-REFLECTION EQUATION

It is convenient to normalize the concentration C(x,0) in the reflection case, in terms of the ultimate constant value

$$C(\infty,0) = Q/uh' \qquad (5)$$

and to write the dimensionless form

$$C(x,0)uh'/Q = (2/\pi)^{\frac{1}{2}} (h'/\sigma_z) \sum \exp -(Z_n^2/2\sigma_z^2) \qquad (6)$$

which may be evaluated in terms of σ_z/h' and H/h'. The number of terms required to give adequate convergence of the series increases with the magnitude of σ_z/h', but experience with the calculation soon shows that the constant magnitude $C(\infty,0)$ is very closely approximated, for all H/h' between 0 and 1, at the stage $\sigma_z/h' \approx 1.0$ and then requires terms up to n = 1 or 2 only. In Table 1 values of the dimensionless concentration are given for intervals of 0.1 in H/h' and for $\sigma_z h'$ ranging from 0.01 to 1.4, values very much less than unity being omitted as of no practical importance.

SECTION 4

THE UPPER LIMIT OF σ_z/h' FOR WHICH Eq.(4) IS AN ADEQUATE APPROXIMATION TO Eq.(6)

The form appropriate to unlimited upward spread, Eq.(4), is of course the $n = 0$ term of Eq.(6). Table 2 shows the fractional contribution of this term to the total and so represents the fraction of the correct Eq.(6) value provided by Eq.(4). Note the rapid decrease to 0.5 (actually 0.49 for $\sigma_z/h' = 1.0$) as H/h' approaches 1.0, which is associated with the $n = 0$ and -1 terms approaching equality and together representing the total effect (except for $\sigma_z/h' = 1.0$, at which stage the n +1 and -2 terms are just becoming effective, representing a fractional contribution of 0.02). The range for adequate approximation from Eq.(4) depends on the magnitude of H/h' and for example the practical limit of $\sigma_z/h' = 0.47$, as adopted in the rule summarized in (a) of Section 2, is obviously acceptable for H/h' up to say 0.7. The latter magnitude of H/h' may encompass most of the cases of practical interest, but there may be interest in plumes rising to near the top of the mixed layer, for which the full form of Eq.(6) is required below a limit in σ_z/h' smaller than 0.47.

SECTION 5

THE LOWER LIMIT OF σ_z/h' FOR WHICH Eq.(5) IS AN ADEQUATE APPROXIMATION TO Eq.(6)

It has already been noted that the "uniform vertical distribution" form becomes a close approximation when σ_z/h' is near unity. It is indeed within 1.4% when σ_z/h' is 1.0 and the discrepancy decreases rapidly as σ_z/h' increases. Even for σ_z/h' as low as 0.8 the discrepancy is at most 8.5%, depending on the magnitude of H/h'. In relation to the working rule recalled in (a) of Section 2, note that the distance limit ($2d_1$) specified there is twice the distance at which $\sigma_z/h' = 0.47$ and accordingly corresponds to

$$\sigma_z/h' = 0.47 \times 2^s \qquad \text{if} \qquad \sigma_z \propto x^s$$

the latter power law form being expected at least over modest ranges of x (e.g. see Ref. 4 p. 375). Thus the corresponding limit in σ_z/h' is 0.8 say when the exponent s is about 0.8, which is an expected value in near-neutral conditions. For unstable (or stable) conditions s is expected to be larger (or smaller). It appears therefore that the old rule is reasonably satisfactory except perhaps in stable conditions. Even then, with s = 0.58 (as estimated for stability category F, see Ref. 4 p. 375) the discrepancy is at most 15 - 20%, for H/h' near zero or unity, and decreases as H/h' approaches 0.5 and, of course, as distance is increased beyond $2d_1$.

SECTION 6

THE INTERMEDIATE RANGE NOT ADEQUATELY REPRESENTED BY EITHER Eq.(4) OR Eq.(5)

The data in Table 1 may be used to interpolate, graphically if needs be, for the normalized concentration at values of σ_z/h' intermediate between the limits specified in 4. and 5. above. Alternatively a power-law fit to Eq.(6) between these limits of σ_z/h' may be adopted, and applied in practice by taking a linear interpolation in the concentration-distance graph between the corresponding limits of distance, as in the rule (a) of Section 2. This equivalence in the σ_z/h' and "distance" representations is assured if σ_z is a simple power function of x, as we expect at any rate over modest ranges of x, and the range involved here is very small in practical terms (i.e. about 2:1). The question remaining at issue is the choice of optimum values of the limits in σ_z/h'. To be precise these depend on the magnitude of H/h' but for a single rule-encompassing the magnitudes $0 < H/h' \leq 0.7$ the best compromise would appear to be

Eq.(4) $0 < \sigma_z/h' \leq 0.5$

Eq.(5) $\sigma_z/h' \geq 0.85$

Linear interpolation of log C v. log x for the distances corresponding to $0.5 < \sigma_z/h' < 0.85$

The basis for this procedure is evident in Fig. 1 where for a representative selection of H/h' the normalized concentration is shown plotted against σ_z/h' on a log-log basis. In the linear interpolations shown between the above limits of σ_z/h' the greatest discrepancy in relation to the correct curve is in the case of H/h' = 0.7 and is then only about 6%. These limits for the interpolation procedure are very similar to those implied in the old rule, namely $\sigma_z/h' = 0.47$ and (in near-neutral conditions) about 0.8 (but greater or less in unstable or stable flow).

It is emphasized that the foregoing procedure is advocated only for $H/h' \leq 0.7$. For $H/h' = 1.0$ it is evident from the data in Section 4. and Table 2 that the following applies, exactly for small values of σ_z/h' and with a discrepancy of only 2% at $\sigma_z/h' = 1.0$

$$C(x,0) = \begin{cases} \text{twice the value given by Eq.(4)} & 0 \leq \sigma_z/h' \leq 1.0 \\ \text{that given by Eq.(5)} & \sigma_z/h' > 1.0 \end{cases}$$

For values of H/h' between 0.7 and 1.0 there would appear to be no alternative to the use of values as in Table 1, preferably with smaller intervals in H/h' especially for $\sigma_z/h' < 0.5$.

Finally it may be recalled that the specially sudden transition between the regimes in accordance with Eq.(4) and (5) for $H/h' = 0$ has already been noted in a recent review of the procedure of the Turner Workbook. There (i.e. in Section 3(d)(iii) of Ref. 5) a new procedure was recommended which amounts to

$$\begin{array}{ll} \text{Eq.(4)} & \sigma_z/h' \leq 0.8 \\ \text{Eq.(5)} & \sigma_z/h' > 0.8 \end{array}$$

The maximum discrepancy from Eq.(6) is at $\sigma_z/h' = 0.8$ and there amounts to 8.5%.

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TABLE 1

NORMALIZED CONCENTRATION ACCORDING TO Eq.(6)

The entries are C(x,0)uh'/Q, i.e. ground-level concentration normalized with reference to the "uniform vertical distribution value" Q/uh', for an infinite crosswind line source of strength Q per unit length and time at height H within a mixed layer of constant depth h' and with a constant wind speed u .

H/h'→ σ_z/h' ↓	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.01	79.79										
0.02	9.89	1.487 (-4)									
0.03	26.60	1.028 (-1)									
0.04	19.95	0.877									
0.05	15.96	2.159	5.35 (-3)								
0.06	13.30	3.317	5.14 (-2)								
0.08	9.97	4.566	0.438	8.83 (-3)							
0.10	7.979	4.839	1.080	8.86 (-2)	2.677 (-3)	2.974 (-5)	1.215 (-7)				
0.2	3.989	3.520	2.420	1.295	0.540	1.753 (-1)	4.43 (-2)	8.74 (-3)	1.336 (-3)	1.599 (-4)	2.974 (-5)
0.3	2.660	2.516	2.130	1.613	1.093	0.663	0.360	1.750 (-1)	7.69 (-2)	3.274 (-2)	2.059 (-2)
0.4	1.995	1.933	1.760	1.506	1.211	0.915	0.652	0.442	0.292	2.042 (-1)	1.753 (-1)
0.5	1.597	1.566	1.476	1.338	1.168	0.986	0.808	0.653	0.533	0.458	0.432
0.6	1.340	1.323	1.274	1.198	1.103	0.998	0.894	0.801	0.727	0.679	0.663
0.8	1.085	1.081	1.069	1.050	1.026	1.000	0.974	0.950	0.931	0.919	0.915
1.0	1.014	1.014	1.012	1.008	1.004	1.000	0.996	0.991	0.988	0.986	0.986
1.2	1.002	1.002	1.001	1.001	1.001	1.000	0.999	0.999	0.999	0.998	0.998
1.4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Figures in parentheses are exponents (of 10) and refer to the number above.

TABLE 2

THE FRACTIONAL CONTRIBUTION OF THE $n = 0$ TERM IN Eq.(6)

H/h' → / σ_z/h' ↓	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.90	0.50
0.4	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.92	0.78	0.50
0.5	1.00	1.00	1.00	1.00	0.99	0.98	0.96	0.92	0.83	0.69	0.50
0.6	0.99	0.99	0.99	0.98	0.98	0.94	0.90	0.84	0.75	0.64	0.50
0.8	0.92	0.92	0.90	0.89	0.86	0.82	0.77	0.72	0.65	0.58	0.50
1.0	0.79	0.78	0.77	0.76	0.73	0.70	0.67	0.63	0.59	0.54	0.49

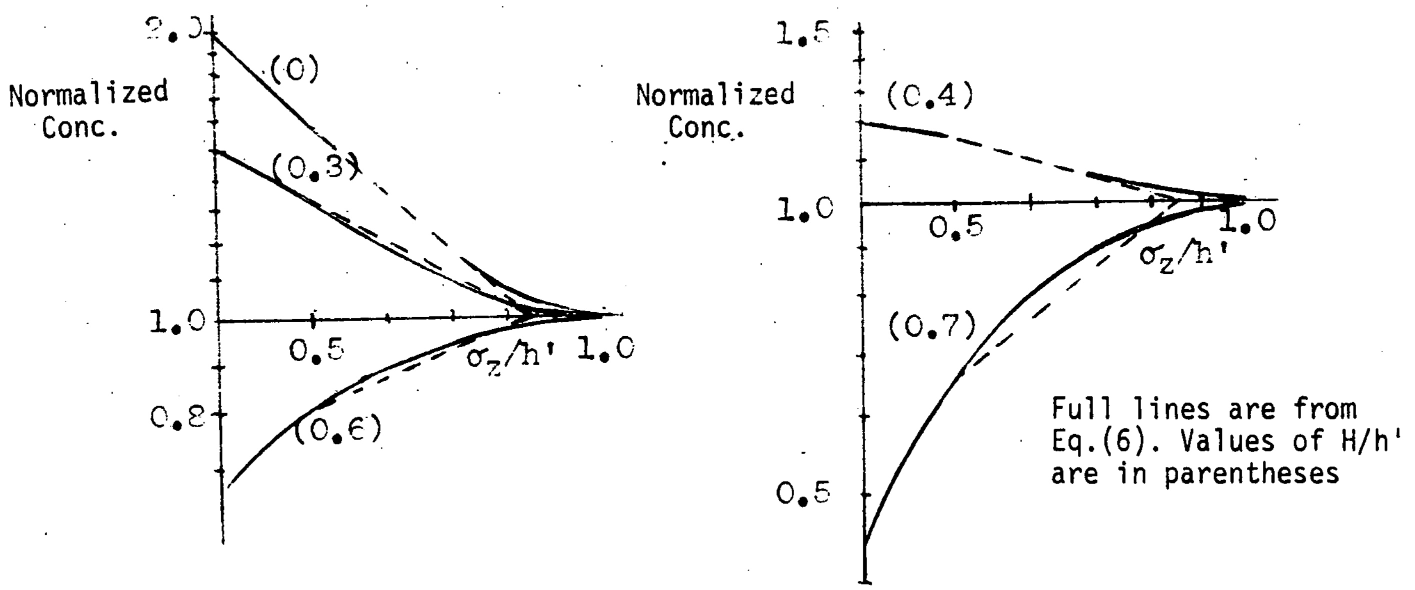

Fig. 1. Plots of normalized concentration according to Eq. (6), demonstrating adequacy of linear interpolation between σ_z/h' = 0.5 and 0.85

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