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OPTIMUM MECHANICAL AERATION SYSTEMS FOR RIVERS AND PONDS

ENVIRONMENTAL PROTECTION AGENCY • WATER QUALITY OFFICE

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OPTIMUM MECHANICAL AERATION SYSTEMS FOR RIVERS AND PONDS

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ABSTRACT

The total annual cost of providing supplemental aeration of streams and lakes by tested and untested aeration equipment is estimated. Analytical and empirical equations are presented for the determination of operating characteristics of the various devices used to aerate natural bodies of water. For the example stream evaluated in this study, the most economical means of artificial aeration generally possible was found to be mechanical aerators which generate a highly turbulent white-water surface. For the example lake evaluated, the most economical technique for the continual input of oxygen into a lake was found to be diffused aeration using air bubbles; whereas the most economical technique for rapid input of oxygen, operating only while the lake is being destratified, was found to be a hybrid system consisting of a large diameter ducted propeller which draws water from the lake bottom and discharges it at the surface where it is aerated by a mechanical aerator.

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Key Words: Mechanical Aeration*, Aeration Efficiency*, Aeration Devices, Aeration Methods

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FOREWORD

At the present time there is a considerable concern about nutrients, particularly phosphates and nitrates, from properly treated sewage effluents and from farming operations, being introduced into our water systems. With these nutrients plant growth develops and the water becomes fouled by the oxygen demand of this plant growth when it decays.

On the other side of the picture, farms for growing catfish are being developed. These fish develop most rapidly when grown in water rich in nutrients (and also warmed). The great fishing areas of the world, such as the Grand Banks and off the coast of Peru, are areas where the ocean water contains many chemical nutrients which allow marine growth that provides feed for the fish.

On the broad scale, it might be wondered why the addition of nutrients into the water, so long as there are no poisons, shouldn't make the stream more valuable and desirable. The answer might well be that this would be true so long as the level of dissolved oxygen in the stream or lake were maintained. This report is concerned with economical methods for maintaining high levels of dissolved oxygen in quiet rivers and ponds.

In this report the annual cost of adding oxygen to streams and lakes is estimated for a variety of existing and new aerating devices. In order that it will be possible to determine the capacity of the machinery required for a given application, expressions are presented which describe mathematically the operating characteristics of the various aeration devices. Where possible, these expressions are developed in analytical form so that the optimum operating conditions can be made apparent.

One of the interesting conclusions that can be drawn from these expressions is that the maximum volume of lake water that can be set into circulation by one pump is limited to a cylindrical "cell" equal in depth to the water depth and in diameter to a value equal to about four times the water depth in the absence of vertical temperature gradients. If, on the other hand, vertical temperature gradients do exist, extremely large volumes of water may be influenced by a single pump until such time as the body of water is destratified.

A methodology for estimating the total annual cost as the sum of capital and operating cost for aeration devices is presented. The report concludes with a section in which the total annual cost for increasing the dissolved oxygen level of a typical stream and lake is estimated.

The economic evaluation leads to several interesting conclusions. Included in these is the observation that diffused aeration of streams will not be economically competitive with other methods even if the diffused system can be designed to achieve a much higher ratio of oxygen captured by the water to oxygen supplied than has ever been demonstrated for comparable depths. It is also shown that the cost of the most economical stream aerating devices can be further decreased by about a factor of 1.5 if recirculation is successfully inhibited. A means of accomplishing this task is suggested in the report.

Although diffused aeration is shown not to be the optimum aeration technique for water of limited depth such as a stream, this technique is shown to increase in its economic competitive position as the depth increases.

I. CONCLUSIONS

1. For slow moving streams the five most economical ways to increase the DO level through a given limited range in order of lowest total annual cost are 1) venting of hydraulic turbines (if such an arrangement is possible at the site in question), 2) white-water generators if recirculation can be successfully inhibited, 3) lift-drop aerators if recirculation can be successfully inhibited, 4) free fall over a dam (if such construction is possible at a given site, and 5) white-water generators without provisions to inhibit recirculation of water through the equipment. Of the five methods only the last one can be applied without qualifications since methods 1 and 4 are site-dependent and methods 2 and 3 depend on equipment modifications that have not yet been proven.

2. If recirculation can be successfully inhibited in white-water generators used in streams, their total annual cost can be expected to decrease by approximately a factor of 1.5.

3. The total annual cost for large diameter ducted propellers is about a factor of 1.4 greater than the corresponding cost for white-water generators used in stream application without provisions for inhibiting recirculation. However, this conclusion is based only on the analytically estimated performance of sub-surface aerators without any experimental data and the estimate could well be off by a factor comparable to 1.4.

4. Diffused aeration of streams (depth $\sim$ 4 feet) using air bubbles will lead to higher annual cost than aeration by means of white-water generators, lift-drop generators, or dams even when the diffused aeration system has been carefully designed so that the capture coefficient (f_{co}) is considerably higher ($\sim$ .5) than has been achieved in practice up to the present time and the total pressure drop in the system is maintained at a low level ($\sim$ 15 psi).

5. The annual cost of aerating a slow moving stream by means of diffused aeration with pure oxygen bubbles is the highest of all methods considered in this report if the capture coefficient is assumed equal to measured laboratory values. However, since almost 90% of the total annual cost is operating cost, the capital cost is low and this method may be the most economical for short term application in a given situation.

6. The volume of lake water that can be set into circulation by a

single pump, in the absence of vertical temperature gradients, is limited to a "cell" equal in depth to the water depth and in diameter to a value equal to approximately four times the water depth. This "cell" volume is not substantially influenced by the slope of the bottom unless the slope is extreme ($\sim 45^{\circ}$). If vertical temperature gradients do exist, a considerable volume of water can be set into circulation by a single pump until the lake is destratified and the circulation "cell" is established.

7. If oxygen is to be added continuously throughout an entire lake, diffused aeration appears to have a total annual cost which is substantially lower than the cost for sub-surface aerators.

8. If oxygen is to be added to a lake only during the time required to destratify the lake, a hybrid system, consisting of one sub-surface circulator and a white-water generator appears to be capable of transferring oxygen far more economically than sub-surface aerators alone or diffused aeration alone. In addition this method allows the use of one white-water generator at several sites in a given season.

II. RECOMMENDATIONS

1. That field tests be conducted with white-water generators that have been modified so as to substantially inhibit recirculation. It is recommended that the modification consist of the addition of an "L" shaped draft tube placed so as to draw in water at some distance upstream from the location of the surface agitating blades.

2. That field or laboratory tests be conducted to check validity of the predicted operating characteristics of large diameter ducted propellers as aeration devices for streams.

3. That field or laboratory tests be conducted to establish the feas ibility of a hybrid system (consisting of a large diameter ducted propeller and a white-water generator) for the rapid aeration of stratified lakes during the destratification phase of circulation.

4. That analytical and experimental studies be conducted to establish the feasibility of using a ducted propeller to skim large quantities of cool, oxygen-rich water off a quiescent lake surface at the time of day when the surface water temperature is a minimum and pump it to the bottom. This technique offers the possibility of maintaining a thermocline while increasing the DO level of hypolimnion water.

III. INTRODUCTION

The introduction of organic material into our rivers and ponds is a major factor in water quality control. Conventional methods of oxidizing sewage use spray ponds, aeration beds (sewage sprayed over a bed of rocks), surface agitators and bubble percolation. These are highly efficient aeration methods and can reduce the BOD of large volumes of waste in a short time and within a limited area. However, these methods of aeration which were developed for sewage disposal involve a high investment and operating cost.

This study is directed toward the economic improvement of the dissolved oxygen level of a quiescent stream or pond which is already polluted. In this case it is not necessary to accomplish the improvement in quality within a very short time nor in a very limited area, and the simplicity and reliability of operation and the investment and operating cost of the equipment are primary considerations.

The amount of oxygen that may be dissolved is limited by temperature and the amounts of dissolved colloidal material present. At sea level and at a temperature of 20^oC (68^oF), the saturation value of dissolved oxygen in pure water is 9 mg/liter. The oxygen saturation value as a function of temperature is given in Table I as adapted from Ref. 1. Atmospheric oxygen provides the major source of dissolved oxygen replenishment in water. It is often agreed that a minimum dissolved oxygen content of 5 ppm is required for healthy aquatic life. If the dissolved oxygen content of natural streams falls substantially below this level, not only does the aquatic life deteriorate but the aerobic bacteria that effect the decay of the organic material are replaced by the anaerobic bacteria that generate odorous gases in the decomposition of the organic material.

The amount of oxygen dissolved in the stream water is the result of the balance between those mechanisms which supply oxygen to the stream; primarily, diffusion of oxygen from the atmosphere into the water and photosyntheses, and the mechanisms which remove oxygen from the water; namely, bio-oxidation of organic waste (including the organic load imposed by dead algae), oxygen consumption by direct chemical reaction, and support of aquatic life.

Dissolved oxygen content is not the only factor that defines water quality. However, it is closely related to the odor of the water - the most obvious and objectionable factor. Generally the introduction into a stream of a pollutant having a high biological oxygen demand is

TABLE I

Dissolved Oxygen Saturation Values
for Distilled Water, mg/liter (Ref. 1)

Temperature, °C	*0.0*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
0	14.65	14.61	14.57	14.53	14.49	14.45	14.41	14.37	14.33	14.29
1	14.25	14.21	14.17	14.13	14.09	14.05	14.02	13.98	13.94	13.90
2	13.86	13.82	13.79	13.75	13.71	13.68	13.64	13.60	13.58	13.53
3	13.49	13.46	13.42	13.38	13.35	13.31	13.28	13.24	13.20	13.17
4	13.13	13.10	13.06	13.03	13.00	12.96	12.93	12.89	12.86	12.82
5	12.79	12.76	12.72	12.69	12.66	12.62	12.59	12.56	12.53	12.49
6	12.46	12.43	12.40	12.36	12.33	12.30	12.27	12.24	12.21	12.18
7	12.14	12.11	12.08	12.05	12.02	11.99	11.96	11.93	11.90	11.87
8	11.84	11.81	11.78	11.75	11.72	11.70	11.67	11.64	11.61	11.58
9	11.55	11.52	11.49	11.47	11.44	11.41	11.38	11.36	11.33	11.30
10	11.27	11.24	11.22	11.19	11.16	11.14	11.11	11.08	11.06	11.03
11	11.00	10.98	10.95	11.93	10.90	10.87	10.85	10.82	10.80	10.77
12	10.75	10.72	10.70	10.67	10.65	10.62	10.60	10.57	10.55	10.52
13	10.50	10.48	10.45	10.43	10.40	10.38	10.36	10.33	10.31	10.33
14	10.26	10.24	10.22	10.19	10.17	10.15	10.12	10.10	10.08	10.06
15	10.03	10.01	9.99	9.97	9.96	9.92	9.90	9.88	9.86	9.84
16	9.82	9.79	9.77	9.75	9.73	9.71	9.69	9.67	9.65	9.63
17	9.61	9.58	9.56	9.54	9.52	9.50	9.48	9.46	9.44	9.42
18	9.40	9.38	9.36	9.34	9.32	9.30	9.29	9.27	9.25	9.23
19	9.21	9.19	9.17	9.15	9.13	9.12	9.10	9.08	9.06	9.04
20	9.02	9.00	8.98	8.97	8.95	8.93	8.91	8.90	8.88	8.86
21	8.84	8.82	8.81	8.79	8.77	8.75	8.74	8.72	8.70	8.68
22	8.67	8.65	8.63	8.62	8.60	8.58	8.56	8.55	8.53	8.52
23	8.50	8.48	8.46	8.45	8.43	8.42	8.40	8.38	8.37	8.36
24	8.33	8.32	8.30	8.29	8.27	8.25	8.24	8.22	8.21	8.19
25	8.18	8.16	8.14	8.13	8.11	8.10	8.08	8.07	8.06	8.04
26	8.02	8.01	7.99	7.98	7.96	7.95	7.93	7.92	7.90	7.89
27	7.87	7.86	7.84	7.83	7.81	7.80	7.78	7.77	7.75	7.74
28	7.72	7.71	7.69	7.68	7.66	7.65	7.64	7.62	7.61	7.59
29	7.58	7.56	7.55	7.54	7.52	7.51	7.49	7.48	7.47	7.45
30	7.44	7.42	7.41	7 40	7.38	7.37	7.35	7.34	7.32	7.31

objectionable because it rapidly depletes the available dissolved oxygen and leads to the objectionable septic condition. Likewise the introduction of nutrients such as phosphates into the water is generally considered undesirable because they lead to aquatic growth which, when decaying after growth, imposes a heavy organic load on the water that might generate septic conditions. However, marine life feeds upon the aquatic growth and it is well known that the best fishing grounds are those where currents rich in minerals allow growth to provide feed for the fish. For this reason, if it is possible to control the dissolved oxygen by artificial means, there may be benefits in the form of increased production of fish and marine life.

If sufficient time is available, the oxygen content of water in contact with air will reach its saturated or equilibrium value. Dissolved oxygen values below the saturation level are an indication of the presence of organic pollution from sewage, organic industrial waste, agricultural land runoff, forest and natural land runoff or oxygen demands exerted during respiration, and die-off of over-abundant blooms of algae and aquatic plant life. Because dissolved oxygen levels provide a gross appraisal of many factors taking place in stream water, universal use has been made of this parameter for waste treatment plant design, stream standards for protection of aquatic life, abatement criteria for determining low flow augmentation requirements, and evaluation of stream assimilative capacity in connection with permissible stream loadings.

Robert K. Davis [2] performed some economic studies of various processes in terms of the level in pounds (BOD_5)(1) removed or offset daily. A very lengthy publication on this subject was also issued by the U. S. Army Engineer District in Baltimore [3]. The results of these works and the works referenced in these publications show that the most economic means of raising DO in quiet rivers and streams are by diffused aeration and mechanical aeration.

Table II (on the following page) which shows this is based on Davis' paper [2].

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(1) The biochemical oxygen demand of polluted water is a measure of the oxygen required to stabilize decomposable organic matter by aerobic bacterial action. Incubation for 5 days at 20°C (BOD_5) is the standard that is used during river basin field studies.

TABLE II

Cost of Offsetting 20,000 lbs. BOD Daily [Ref. 2]

Process	Cost
Microstraining	$12,000,000
Step Aeration	13,000,000
Chemical Precipitation	18,000,000
Powdered Adsorption	43,000,000
Granular Adsorption	95,000,000
Effluent Distribution	19,000,000
Diffused Aeration	2,200,000
Mechanical Aeration	1,700,000

The cost information developed for the various processes was based on the capital costs, maintenance costs, and operating costs. The capital costs are amortized over a 50-year period. Effluent distribution, the two carbon processes (granular and powdered adsorption), and the combination (step aeration followed by microstraining) require the most capital investment. (This is also the case for low-flow augmentation.) Chemical precipitation involves a low capital investment but high operating costs. Both effluent distribution and carbon adsorption have very large power requirements. It is obvious from the cost data that the reaeration devices are the least costly of the alternative processes.

Although the cost of mechanical or diffused aeration devices is far lower than the alternatives, the total cost of numerous devices for reducing the BOD in countless estuaries throughout the country runs into the billions of dollars. This means that it is most important to develop highly economical and efficient devices. It is the purpose of this study to develop 1) the detailed criteria for design and 2) basic designs of devices that will reduce the biological oxygen demand in streams and ponds at minimum costs.

IV. METHODS OF INDUCING AERATION

Any mechanical aeration scheme must consist of two fundamental steps. The oxygen must be captured by the water at some gas-water interface and the captured oxygen must then be distributed throughout the body of water. If the gas is air, then the rate at which oxygen is captured at the air-water interface can be estimated by calculating the rate at which oxygen molecules strike the air-water interface and then multiplying this rate by a capture coefficient, that is, the ratio of oxygen molecules that are captured by the water per unit time when they strike to the total number that strike per unit time. The total number of oxygen molecules that strike the interface per unit time per unit area is shown in Appendix A (See Eq. A-3) to be given by the expression

$$N = \frac{P}{\sqrt{2\pi m k T}} \quad \frac{\text{No.}}{\text{cm}^2\,\text{sec}} \qquad \text{(Eq. 1)}$$

where N = number of molecules striking interface per second
P = partial pressure of oxygen, dynes/cm^2
k = Boltzman Constant - $138^{o} \times 10^{-6}$ erg/^{o}K
T = absolute gas temperature, ^{o}K
m = mass of an oxygen molecule, grams

If the oxygen being captured by the water is supplied from the atmosphere with a partial pressure of 0.21 atmosphere and a temperature of 20^{o}C, then the rate at which oxygen strikes the surface is found from Eq. 1 to be 6.30 #m/ft^{2}sec. Of this incident flux of oxygen molecules, some will rebound off the interface and return to the atmosphere and some (a fraction β) will be captured by the water at the interface and is then available to the water below the surface if it can reach that region by either molecular diffusion (in the absence of any turbulence at the air-water interface) or by eddy diffusion (if sufficient turbulence is present). The fraction of incident molecules that is captured (the capture coefficient β) is a function of the dissolved oxygen concentration (DO) at the surface, and will be a maximum when the surface DO is zero and will be zero when the DO at the surface corresponds to the saturation value. In Appendix A β is estimated to be given by the expression

$$\beta = .74 \left(\frac{C_s \quad C}{C_w}\right) \qquad \text{(Eq. 2)}$$

where C = concentration of DO at the surface, mg/liter

C_s = saturation concentration of oxygen, mg/liter

C_w = concentration of water in mg/liter

The rate at which oxygen is captured at the air-water interface can now be found by multiplying Eq. 1 by Eq. 2. Thus

$$\frac{\dot{M}_{O_2}}{A} = 4.65 \times 10^{-6} (C_s - C) \ \#m/ft^2 sec \qquad \text{(Eq. 3)}$$

where C_s and C are to be expressed in mg/liter

As an example, consider the capture of atmospheric oxygen at the air-water interface at the instant the surface DO level is zero and the water temperature is 20°C (C_s = 9 mg/liter). This rate is given by Eq. 3 as 4.17×10^{-5} $\#m/ft^2 sec$.

The order of magnitude for the time required to saturate a newly exposed water surface with an initial zero DO level can be established by computing the time required to capture sufficient oxygen to approach (within 99%) saturating the upper layer where dynamic capture of oxygen molecules is the dominant phenomenon rather than molecular diffusion, say a layer ten molecules thick. When this is done with the use of Eq. 3, the time is found to be 30×10^{-6} sec. or 30 microseconds and the total oxygen accumulated in the water during this period is 1.25×10^{-9} $\#m_{O_2}/ft^2$.

In order to establish the relative importance of the two mechanisms involved in the aeration process it is necessary to compare the surface capture rate of oxygen with the rate of the molecular diffusion or the eddy diffusion by means of which the oxygen initially captured at the surface is transported to the water below the surface. However, before doing this, it will be helpful to very briefly review the physics of the gas absorption into a liquid, that is, oxygen into water.

When considering the diffusion of a gas of limited solubility, like oxygen, into water, two extreme cases can be readily identified for specifying the condition of the water, namely: 1) stagnant water at one end of the spectrum and 2) extremely turbulent water with a broken "white" surface on the other end of the spectrum. In the case of oxygen being absorbed into stagnant water, the surface water when first exposed to the atmosphere will capture oxygen at a rate given by Eq. 3; however, once at the air-water interface the only mechanism for the oxygen to move into the liquid is by molecular diffusion. For this case, the rate of diffusion is given by Fick's law, namely:

$$\dot{M}_{O_2} = D_L A \frac{\partial C}{\partial z}$$

where $\dot{M}_{O_2}$ = time rate of oxygen mass transfer

D_L = diffusivity coefficient for oxygen diffusing in water

A = surface area of interface

$\frac{\partial C}{\partial z}$ = concentration gradient in direction of diffusion

Eq. 4 can be expressed in terms of C, t, D_L and z only by considering the conservation of oxygen for a differential slice of the water as shown below.

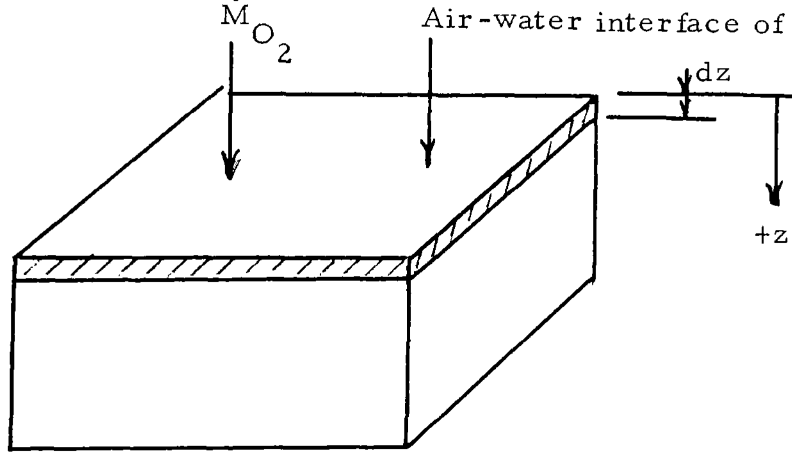

Fig. 1 - Diffusion of Oxygen into Water

Since oxygen is conserved, we can write for the differential control volume:

$$(-D_L A \frac{\partial C}{\partial z}) - \{- D_L A \frac{\partial C}{\partial z} + \frac{\partial}{\partial z}[- D_L A \frac{\partial C}{\partial z}]\ dz\} = A(\frac{\partial C}{\partial t})\ dz \qquad \text{(Dq. 5)}$$

Eq. 5 reduces to the following form when second order terms are neglected:

$$D_L (\frac{\partial^2 C}{\partial z^2}) = (\frac{\partial C}{\partial t}) \qquad \text{(Eq. 6)}$$

For an infinitely deep pool of water initially at a uniform dissolved oxygen content of C_o except at the interface where the dissolved oxygen content is at the saturation values (C_s), Eq. 6 can be integrated and the results used to evaluate the concentration gradient ($\partial C/\partial z$) at the surface in Eq. 4 to obtain the mass flow rate at the surface as given below in Eq. 7: (see Ref. 4)

$$\frac{\dot{M}_{O_2}}{A} = \sqrt{\frac{D_L}{\pi t}}\,(C_s - C_o) \qquad \text{(Eq. 7)}$$

where t = time elapsed from time zero (2)

In order to make comparisons between the various models to be developed in this report, as well as with data reported in the open literature, it will be convenient to rewrite the right hand side of Eq. 7 as the product of a coefficient and a "driving force," thus Eq. 7 can be expressed as:

$$\frac{\dot{M}_{O_2}}{A} = K_L\,(C_s - C_o) \qquad \text{(Eq. 8)}$$

where K_L = liquid film coefficient = $\sqrt{\frac{D_L}{\pi t}}$ for this case.

If we again consider the atmosphere at 20°C to be the source of oxygen, then Eq. 8 can be used to predict the rate of oxygen flow into a semi-infinite pool of water (which is initially at zero DO) as a function of time. A summary of the results of such a calculation are given below for convenience.

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(2) Although any consistent set of units may of course be used in Eq. 7, it has become the custom to speak of C in terms of mg/liter. Unfortunately such units do not conveniently fit into Eq. 7. However, since a mg/liter is equivalent to one part per million by weight, it follows that one mg/liter of dissolved oxygen is equivalent to 62.4 x 10^{-6} $\#m_{O_2}/ft^3$. Thus a convenient form of Eq. 7 is

$$\frac{\dot{M}_{O_2}}{A} = 62.4 \times 10^{-6}\sqrt{\frac{D_L}{\pi t}}\,(C_s - C_o)\ \#m/ft^2\text{-sec}$$

where D_L is in ft^2/sec

t is in sec

C_s, C_o are in mg/liter

TABLE III

Oxygen Transfer Rate and Total Accumulated Oxygen as a Function of Elapsed Time for Molecular Diffusion

Time	M_{O_2}/A, #m/ft^2 sec	Accumulated oxygen in water, #m/ft^2 of surface
1 microsecond	5.18×10^{-5}	1.036×10^{-10}
1 millisecond	1.61×10^{-6}	3.21×10^{-9}
1 second	5.18×10^{-8}	1.036×10^{-7}
1 hour	8.55×10^{-10}	6.21×10^{-6}
1 day	1.74×10^{-10}	3.05×10^{-5}
1 month	3.19×10^{-11}	1.68×10^{-4}

Two observations are to be made in regard to Table III. First, when a "new" surface is initially exposed, oxygen from the atmosphere will tend to saturate the upper layer or interface and oxygen will be withdrawn from this layer by molecular diffusion into the main body. When the new surface is initially exposed to the atmosphere, the rate at which O_2 would be withdrawn from a saturated surface and zero DO water exceeds the rate at which the atmosphere replenishes it. After an exposure time of about 2 microseconds, the two rates become equal and for longer exposure times the atmosphere will replenish the interface oxygen at a faster rate than it will be removed by diffusion. Thus the interface can be assumed to be at the saturation conditional except for the short initial period of high transfer by diffusion into the main body. Since the processes that will be of interest in the present study will be at least of the order of microseconds, it can be safely assumed that the interface is always saturated. Some references are available in the literature to support this conclusion that the interface will remain saturated by atmospheric replenishment except at the very high diffusion rates [5].

The second observation to be made in regard to Table III is that the total mass of oxygen accumulated by a quiescent body of water, say a pond in summer time, by molecular diffusion, which has its surface renewed only once a day (for example, due only to daytime heating and nighttime cooling) is 3.05×10^{-5} #m_{O_2}/ft^2, whereas the total amount of oxygen that could be accumulated by the same body due to molecular diffusion alone could be increased to 4.49×10^{-3} #m_{O_2}/ft^2 if the surface were to be replaced each second.

Before leaving the discussion of Table III it is helpful to calculate the time it will take for a one-foot deep pool of water to increase in DO from a uniform value of zero to an average value of 1 mg/liter. This can be done by combining solutions for two semi-infinite slabs as given in Eq. 8 or by use of nondimensional solutions given in Ref. 6. When the calculations are performed, the time is found to be 2.4 years.

As we move from the end of the spectrum where the water is stagnant to where the water is moving, but moving so slowly that it is in laminar flow, Eq. 8 remains valid if the water is deep enough so that the velocity profile is essentially square. As the velocity is increased or the water is made more shallow so that the flow remains laminar but with a pronounced velocity-depth profile, Eq. 8 must be modified. This modification will not alter the oxygen flow rates substantially. A stream will generally be in laminar flow if the Reynolds Number based on the hydraulic diameter is less than 2000, that is, when

$$\frac{V\, D_{HY}\rho}{\mu} < 2000 \qquad \text{(Eq. 9)}$$

where V = average flow velocity, ft/sec

D_{HY} = hydraulic diameter, ft

= 4 times the flow cross section divided by the wetted parameter

$\sim$ 4 times the depth for a river which is much wider than it is deep

ρ = density of water, slugs/ft^3

μ = absolute viscosity of water, $\#_f/\text{ft}^2$

Likewise a stream will generally be in turbulent flow if the Reynolds Number is above 4000. Between a Reynolds Number of 2000 and 4000 the stream will be in a transition mode of flow.

As the stream velocity is increased sufficiently to just leave the laminar flow regime, a thin laminar layer of water may form if the coupling between the air flow in the atmosphere and the water flow is favorable. If this condition exists, then the rate at which oxygen will enter the water is fixed by the rate at which the oxygen can move across the thin laminar layer by molecular diffusion in view of the fact that once the oxygen has penetrated the laminar layer, it will be transported by the turbulence eddies which is extremely more rapid than molecular diffusion. The physical situation is depicted in Fig. 2.

The situation depicted in Fig. 2 is the well-known "liquid film" model

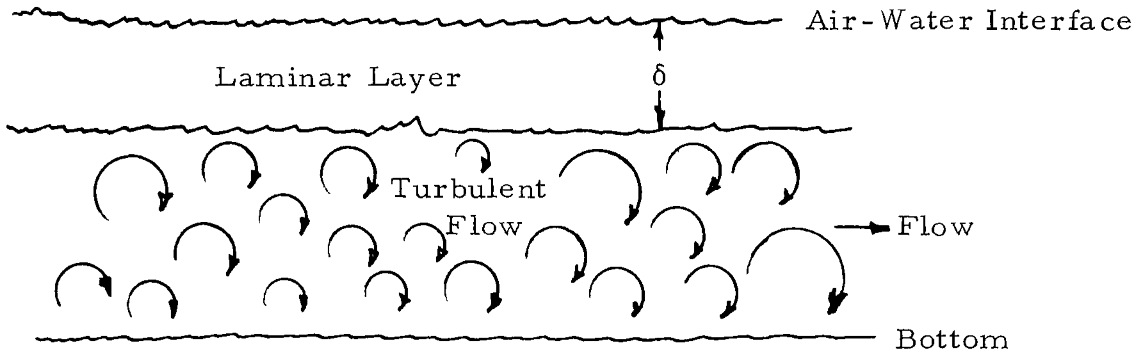


Fig. 2 - Oxygen Transfer under conditions where the liquid film controls the mass flow rate

for diffusion of a slightly soluble gas into a liquid originally proposed by Lewis and Whitman [7].

If the assumption is made that sufficient mixing takes place in the turbulent flow zone to make the DO uniform in this region at any time, then the rate of flow of oxygen from the atmosphere to the water is again given by Fick's Law, Eq. 4:

$$\dot{M}_{O_2} = - D_L A \frac{\partial C}{\partial z}$$

For a thin layer this equation can be simplified by replacing the concentration gradient, $\partial C/\partial z$, with $\Delta C/\Delta z$ where

$$\frac{\Delta C}{\Delta z} = \frac{C - C_s}{\delta} \qquad \text{(Eq. 10)}$$

where C = uniform oxygen concentration in turbulent zone
C_s = surface oxygen concentration, to be taken as the saturation value
δ = thickness of laminar zone

Substituting Eq. 9 into Eq. 4 yields:

$$\frac{\dot{M}_{O_2}}{A} = \frac{D_L}{A} (C_s - C) \qquad \text{(Eq. 11)}$$

Again as in the case of Eq. 7, it is convenient to rewrite Eq. 11 as a product of a coefficient (the liquid film coefficient) and a "driving-force" (the dissolved oxygen deficient), thus Eq. 11 can be expressed:

$$\frac{\dot{M}_{O_2}}{A} = K_L (C_s - C) \qquad \text{(Eq. 12)}$$

where $K_L = D_L/\delta$ for this case

Although it is difficult to model the quiescent layer and thereby arrive at a value of δ, a conservative estimate (conservative in the sense that it will predict values of δ on the high side and the reaeration rates on the low side) can be made by replacing the air with a smooth solid boundary and then comparing the stream flow to a flow in a smooth pipe the diameter of which is equal to four times the hydraulic radius. For such a model the thickness of the laminar sublayer (δ) adjacent to the smooth walls is given by the relationship [8]:

$$\delta = \frac{5\mu}{\rho}\sqrt{\frac{\rho}{\tau_o}} \text{ ft} \qquad \text{(Eq. 13)}$$

where ρ = fluid density in slugs/ft^3
μ = fluid viscosity in $\#_f$ sec/ft^2
τ_o = fluid shear stress at the walls in $\#_f$/ft^2

The shear stress at the wall of a pipe may in turn be expressed as a function of the pipe friction factor, f_p, for fully developed turbulent flow and is given in the expression:

$$\tau_o = \frac{1}{4} f_p \frac{\rho V^2}{2} \qquad \text{(Eq. 14)}$$

where V = flow velocity in ft/sec
f_p = friction factor for smooth pipe, turbulent flow and is a function of Reynolds Number [8]

Thus Eq. 13 can be rewritten with the aid of Eq. 14 to yield:

$$\delta = \frac{5D}{Rey_{HY}} \frac{\sqrt{8}}{\sqrt{f_p}} \qquad \text{(Eq. 15)}$$

where $Rey_{HY} = \frac{\rho V D_{HY}}{\mu}$

Before calculating an example, it should be pointed out that the present model can have validity only in or near the transition region. When the stream turbulence is increased sufficiently beyond the transition zone, the existence of a laminar or quiescent layer at the air-water interface can no longer be assumed to represent the actual flow situation. Rather at sufficiently high levels of turbulence, the eddies will penetrate to the air-water interface and thus provide a far more rapid mechanism for transporting surface captured oxygen away from the interface than is provided by molecular diffusion in the present model.

For an example of an application of the present model, consider a two foot deep stream initially at zero DO (air at 20°C) and moving at Reynolds Number 2000, 4000, and 40, 000. For this case Eq. 12 and Eq. 15 yield an oxygen flow rate from the atmosphere to the water as given in Table IV.

TABLE IV

Oxygen Transfer Rate across a Thin Quiescent Layer of Surface Stream Water

Quiescent, Rey_{HY}	V (River Velocity) ft/sec	Surface Stream Water M_{O_2}/A, #m/ft^2 sec
2, 000	.003	4.76×10^{-11}
4, 000	.006	1.068×10^{-10}
40, 000	.06	7.93×10^{-10}

As the stream velocity is increased sufficiently, the turbulent eddies will penetrate to the air-water interface. As a result the previous process by which oxygen captured at the air-water interface was transported to the bulk of the water; namely, molecular diffusion across the quiescent layer, will be replaced by the more rapid process of eddy diffusion. To describe this process, a surface replacement theory has been proposed and developed by several workers, most notably by Higbie [4] and Danckwertz [9]. The replacement theory was proposed to overcome the difficulty of accepting the existence of a laminar or quiescent layer under conditions of strong turbulent flow in the main body of water. In his 1951 paper, Danckwertz points out:

> "The fictitious nature of the "liquid film" is probably widely suspected; nevertheless, it is constantly referred to as though it actually existed. This may be regarded, for many purposes, as a harmless and convenient useage, as measured absorption rates appear to conform to the expression:
>
> $$\frac{M}{A} = K_L (C_s - C_o)$$

> where K_L, the liquid-film mass-transfer coefficient, is constant for a given liquid and gas under given conditions. However, if the film is in fact an unrealistic one, it may lead to erroneous results if it is used as the basis of theories which seek to relate K_L to the conditions of operation."

In place of a laminar surface layer which always contains the same liquid, Danckwertz assumed the existence of a laminar layer of non-fixed identity, that is, he assumed the liquid surface layer was continually being replaced with fresh liquid (liquid from the turbulent zone). Before this replacement theory can be applied to streams and ponds, a method must be available for estimating the rate of surface renewal (r) and the coefficient K_L. Considerable effort has been devoted to this task, notably by O'Connor and Dobbins [10, 11, 12].

In their 1956 paper O'Connor and Dobbins developed an expression for the coefficient K_L and two expressions for the surface renewal, one applicable to isotropic turbulence and one applicable to non-isotropic turbulence. Later in 1958 O'Connor [11] pointed out that the equation developed for the isotropic case was the more generally significant one and that the equation for non-isotropic turbulence could be omitted from consideration. The equation for the liquid film coefficient given by O'Connor and Dobbins for water deeper than .04 cm is (See Eq. 27, Ref. 10):

$$K_L = \sqrt{D_L r} \qquad \text{(Eq. 16)}$$

and their expression for the surface renewal rate for isotropic turbulence is (See Eq. 36, Ref. 10):

$$r = \frac{u}{H} \qquad \text{(Eq. 17)}$$

where u = average stream velocity
H = average stream depth

In the development of Eq. 16 the authors point out that two assumptions are required. First, that the concentration is uniform throughout the depth of water and second, that the concentration does not change with time. The first assumption will be satisfied if the river flow is turbulent. The second assumption will be satisfied if the time required to renew the interface surface (1/r) is short compared to the time necessary to make a substantial change in the DO concentration.

Using Eq. 16 and Eq. 17 the rate of oxygen transfer as given by the renewal theory becomes:

$$\frac{\dot{M}_{O_2}}{A} = K_L (C_s - C) \qquad \text{(Eq. 18)}$$

where $K_L = \sqrt{\frac{D_L u}{H}}$ for this case

C = bulk average DO concentration

In 1964 Dobbins [12] developed an analytical expression for the surface renewal rate to be used in place of the semi-empirical expression given in Eq. 17. In Dobbins' analytical expression the energy dissipated per unit time per unit mass of water must be determined as it plays a dominant role in the expression. Although the analytical expression is more general than Eq. 17 and as a result yields correlations over a wider range of conditions, its use in general is hampered by its lack of simplicity. In addition it should be noted that the analytical expression and Eq. 17 yield about the same degree of agreement between predicted and measured values of the liquid film coefficient for natural streams but the use of the analytical expression results in far more accurate predictions of aeration in laboratory channels as may be seen from Fig. 3 (3) as adapted from Ref. 12.

Because of the simplicity and reasonably good agreement that Eq. 17 provides for natural waterways, we shall use this equation rather than the more general analytical expression given by Dobbins.

It is helpful to compare the results of the previous example used in the situation where a laminar surface layer might be assumed (low, but turbulent Reynolds numbers) to the results given by the surface renewal theory. Thus we again consider a two foot deep stream

---

(3) In making an observation on the oxygen transfer into a natural body of water, a question arises as to the surface area to be used in the calculation. Either the true interfacial area A_s can be used or the horizontal projected interfacial area A_o. The two areas are related by the following product of area and coefficient:

$$K_L A_s = K_L' A_o$$

where K_L' = apparent liquid film coefficient

K_L = actual liquid film coefficient

The value of K_L' is plotted in Fig. 3.

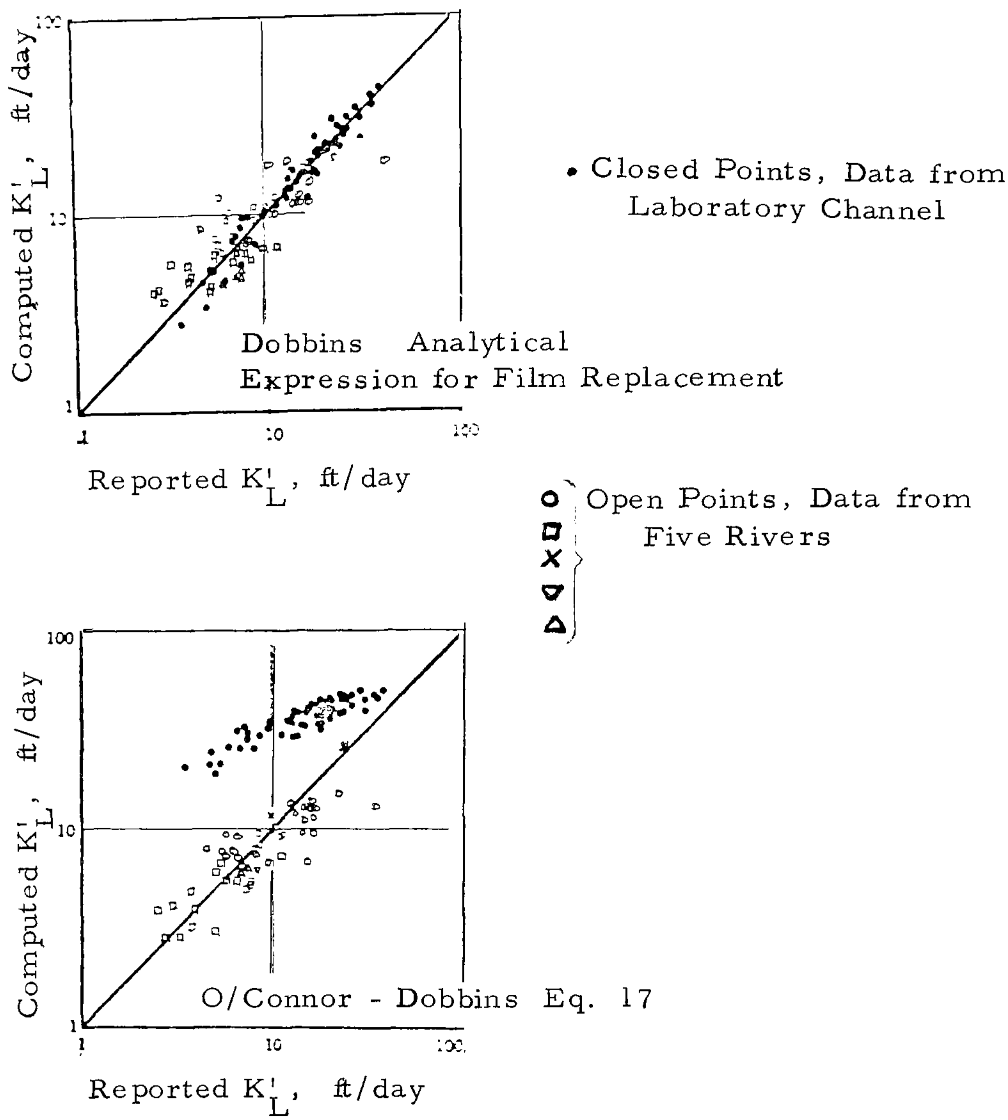


Fig. 3 Comparison of Reported and Calculated Apparent Liquid Film Coefficient (Adapted from Ref. 12)

initially at zero DO (air at 20°C) but the stream is now assumed to move at Reynolds Numbers of 4,000, 40,000, 400,000 and 4,000,000. For this case the oxygen transfer rate may be found with the use of Eq. 18. A summary of these results is given in Table V.

TABLE V

Oxygen Transfer Rates for Turbulent Rivers

Rey_{HY}	u River Velocity ft/sec	$\frac{\dot{M}_{O_2}}{A}$ #m/ft^2 sec
4,000	.006	5.07×10^{-9}
40,000	.06	1.60×10^{-8}
400,000	.6	5.07×10^{-8}
4,000,000	6.0	1.60×10^{-7}

All of the oxygen transfer rates discussed so far, namely, (a) oxygen capture at a "new" surface, (b) oxygen transport by molecular diffusion across a thin laminar layer and finally (c) oxygen transport by eddy diffusion at a surface being renewed at a finite rate, can be expressed in convenient units as was previously discussed in references to Eq. 7, by the equation:

$$\frac{\dot{M}}{A} = 62.4 \times 10^{-6} K_L (C_s - C) \ \#m_{O_2}/ft^2 \ sec \qquad \text{(Eq. 19)}$$

where $K_L = \sqrt{D_L/\pi t}$ in ft/sec for Case a

$K_L = \sqrt{D_L/\delta}$ in ft/sec for Case b

$K_L = \sqrt{(D_L u)/H}$ in ft/sec for Case c

C_s = saturation level of DO in mg/liter

C = DO level in mg/liter

It should be pointed out that the value of the liquid film coefficient K_L for Case c must reach the value corresponding to surface capture (or 4.17×10^{-5} #m_{O_2}/ft^2 sec) as an upper limit when the surface renewal rate increases from a finite rate to an infinite rate.

In Fig. 4 the three cases of oxygen transfer as given by Eq. 19 are

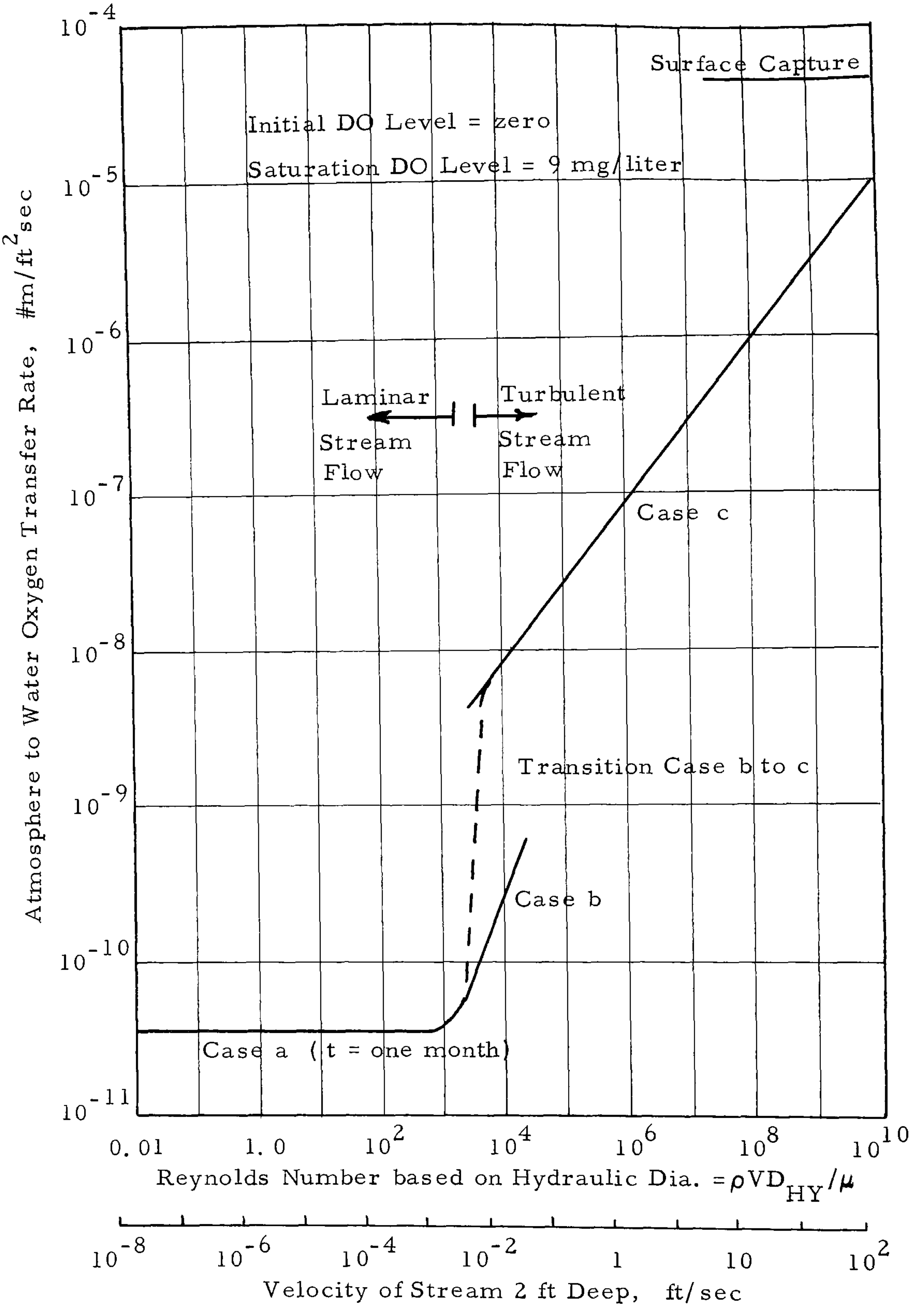


Fig. 4 - Oxygen Transfer Rate for Various Rate Control Situations

compared for convenience. The example used to construct Fig. 4 is the previously used two foot deep river at zero DO and a value of C_s equal to 9 mg/liter. In Fig. 4 the value of t selected for Case a is one month.

Two significant observations can be made by inspecting Fig. 4. First, it is noted that the rate at which oxygen is captured at the surface is several orders of magnitude faster than the rate at which eddy diffusion in a naturally flowing stream can carry the oxygen into the bulk of the water even when the stream velocity is at its upper limit of about 10 ft/sec. Second, if turbulence is induced near the surface of a quiescent stream in order to destroy a thin laminar layer or in order to increase the surface renewal rates, the rate of oxygen transfer from the atmosphere to the upper layer of water can probably be increased by orders of magnitude.

In addition to the transfer of oxygen from the atmosphere to the water at the air-water interface, it may also be considered desirable to inject air or pure oxygen at the bottom of the water and let it diffuse into the water as the bubble rises upward. Likewise aeration may be accomplished by spraying the water into the air.

The engineering and economic characteristics of the various aeration devices will be discussed in detail in the following sections.

It should be noted that the liquid film coefficient is known to depend on the physical and chemical characteristics of the water as well as on the flow field. In particular, the film coefficient K_L undergoes significant variations with water temperature and the presence of surface-active agents. These two variations are discussed briefly below.

The liquid film coefficient is known to increase with temperature. Correlations for this variation of the form

$$K_L(T) = K_{L(20^{\circ}C)} \, C_1^{\,T-20^{\circ}C} \qquad \text{(Eq. 20)}$$

where T = water temperature in $^{\circ}C$

C_1 = a constant coefficient reported in the literature to vary between 1.016 to 1.037 [13]

If surface-active agents are present, their molecules will orient themselves on the air-water interface and create a resistance to molecular diffusion of oxygen across the interface. The resulting decrease in the liquid film coefficient has been measured and can be as high as a factor of two.

V. DISSOLVED OXYGEN CONCENTRATION IN QUIESCENT STREAMS AND PONDS

Both streams and ponds experience a hydrodynamic annual cycle which may contain substantial periods of time during which conditions are unfavorable for natural reaeration. If the oxygen demand on the stream or pond water remains high during these same periods of low natural reaeration, the DO level may be depressed below the level required to prevent a nuisance condition from being generated.

From Fig. 4 it is obvious that the periods of low natural reaeration occur when the flow rate drops. The severity, length and time of year of minimum river flow rate is highly dependent on the particular site under study as may be seen from the sample of annual hydrographs shown in Fig. 5 [from Ref. 14]. In general it is not unusual to find a variation of two orders of magnitude or more between the minimum and maximum stream or river flow rate during the year. For example, the Mississippi River at St. Paul, Minnesota, has a minimum flow rate of 632 cfs, a maximum of about 176,000 cfs and an annual average of 9,800 cfs [15]. Although the velocity of the stream will vary with depth as well as flow rate, stream velocities will range between 0.01 to 10 ft/sec. From Fig. 4 it is seen that the natural oxygen transfer rate also varies by about a factor of 1000 over this range of velocities. Although the natural reaeration rate may be sufficient high to maintain the DO at acceptable levels during periods of modest to high flow, it may not remain so during the period of low flow. During these periods artificial stream aeration could be used to supplement the natural aeration in order to offset the BOD load in the stream sufficiently to maintain the required DO level.

Lakes and ponds located in regions where the water temperature can fall below the value corresponding to maximum density (4°C) experience an annual cycle which consists of two periods of vigorous vertical circulation, one in early winter when the water freezes and one in the spring when it melts. The oxygen transfer rate during these two periods is very high and the entire body of water is often assumed to reach the saturated DO level during these periods [16]. Between these two periods if the body of water is of sufficient depth, it may divide into an upper layer (the epilimnion) of warm rather turbulent water and a lower layer (the hypolimnium) of cold relatively undisturbed water. The two zones are separated at the so-called thermocline, or the location of maximum temperature gradient. Since oxygen can be transported across the thermocline only by molecular diffusion, which is very slow, very little oxygen will be supplied to the

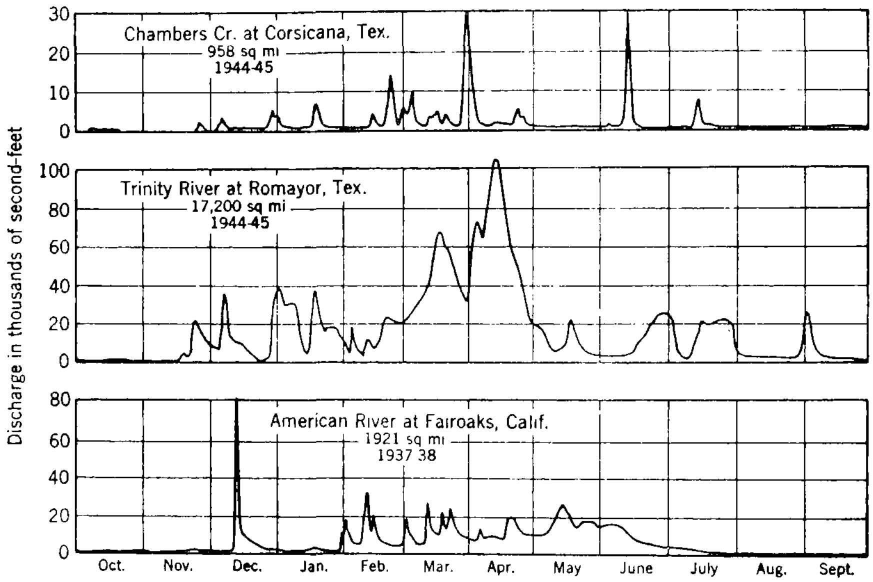


Fig. 5 - Annual Hydrographs for Three Stations (after Ref. 14)

hypolimnium after the thermocline is established. On the other hand, a shallow body of water will exhibit no such division into two layers by a thermocline.

The depth at which a thermocline will form during the summer is dependent on several factors including the wind mixing, evaporative cooling, back radiation, and solar and atmospheric radiation. Depending on the relative interaction of these factors the thermocline may be found at a depth of approximately 10 to 50 feet.

During mid-summer the wind velocities are at their minimum and solar radiation is at its maximum. Under these conditions a well-sheltered pond may become quiescent with the result that the atmospheric oxygen transfer rate may decrease to values approaching that shown as Case a in Fig. 4.

VI. DEVICE OPERATING CHARACTERISTICS

In this section the mode of operation of existing and new aeration devices is discussed and their operating characteristics are presented. Where possible the operating characteristics are developed analytically in order to provide an insight into how these characteristics might be optimized.

Sub-Surface Devices

Based on the fact that the rate of oxygen capture at the air-water interface is orders of magnitude faster than either molecular diffusion across a thin quiescent layer of thickness that might be anticipated in a stream, or the eddy diffusion that results from the usual turbulence present in a body of water, it can be concluded that oxygen transfer can be induced efficiently (that is, at high values of the ratio of mass of oxygen transferred to energy expended) by circulating the water with a sub-surface device so that "new" or "fresh" surfaces are rapidly presented to the atmosphere where they can quickly capture oxygen and subsequently distribute this oxygen throughout the body of water as a result of the induced circulation. The objective of the device is thus two-fold: first to enhance the rate of surface renewal at the air-water interface, and second to induce as large a circulation as possible. Both objectives are to be achieved at minimum power. In addition to the efficiency with which the device induces oxygen transfer, it will also be convenient to compute the oxygen flux density for the device, that is, the rate at which the device will cause oxygen to flow across unit surface area per unit time.

Since it is desired to set large volumes of water into circulation with as low an expenditure of energy as possible, the use of large diameter slow moving propellers is indicated. That this should be the choice may readily be seen by forming the ratio of momentum flux to kinetic energy flux across the device, namely, $[(\rho AV)V]/[(\rho AV)(V^2/2)]$ and noting that the ratio will be high for low fluid velocities and that the mass flow can be simultaneously kept high if the area is made large.

The analytical technique employed to estimate the operating characteristics of sub-surface devices was to simulate the device by a series of two-dimensional potential flow functions so arranged as to satisfy the boundary conditions at the bottom of the body of water and at the air-water interface. Once the appropriate potential functions had been established, they could be used to compute the flow field. Since the flow velocity at a point will decrease exponentially to zero as the point is moved further away from the potentially modeled sub-surface device, the limit of the zone of induced circulation must be defined at some low value of the velocity - in this case 1% of the centerline velocity was selected. Once the flow field has been calculated, the oxygen transfer rate can be estimated by application of the renewal theory as given by Eq. 18.

As a first approximation the following assumptions are made: the bottom of the body of water is horizontal, no side walls, no density gradients, no benefit from induced flow that persists beyond the zone of active circulation, and no oxygen consumption processes take place in the water. Using the above assumptions, three generalized sub-surface aerator configurations were simulated by two-dimensional potential flow theory. The three configurations are summarized below and are shown schematically in Fig. 6.

Case I

This generalized configuration consists of a vertical duct through which the water is forced to flow upward and is then allowed to discharge somewhat below the air-water interface. After discharging the water will flow away from the vertical centerline of the duct near the bottom of the body of water. The flow was simulated by infinite series of sources and sinks placed along the vertical axis of the duct.

For this configuration the duct width has been made small compared to the water depth so that Case I can be used to model a large ducted propeller in a deep body of water, for example, a 20-foot wide duct in a 100-foot deep pool of water. It should be pointed out that Case I can also be used to demonstrate the behavior of a small width duct placed in a shallow body of water. Although the last situation can be ruled impractical, it will allow Case I to be used to quantitatively demonstrate the influence of duct size.

Case II

Geometrically this configuration is the same as Case I; however, in contrast with Case I the duct width for Case II has been made large compared to the depth of the water. As a result Case II can be used to model wide ducts in shallow water. The flow was simulated by several infinite series of sources and sinks placed along lines parallel to the vertical axis of the duct.

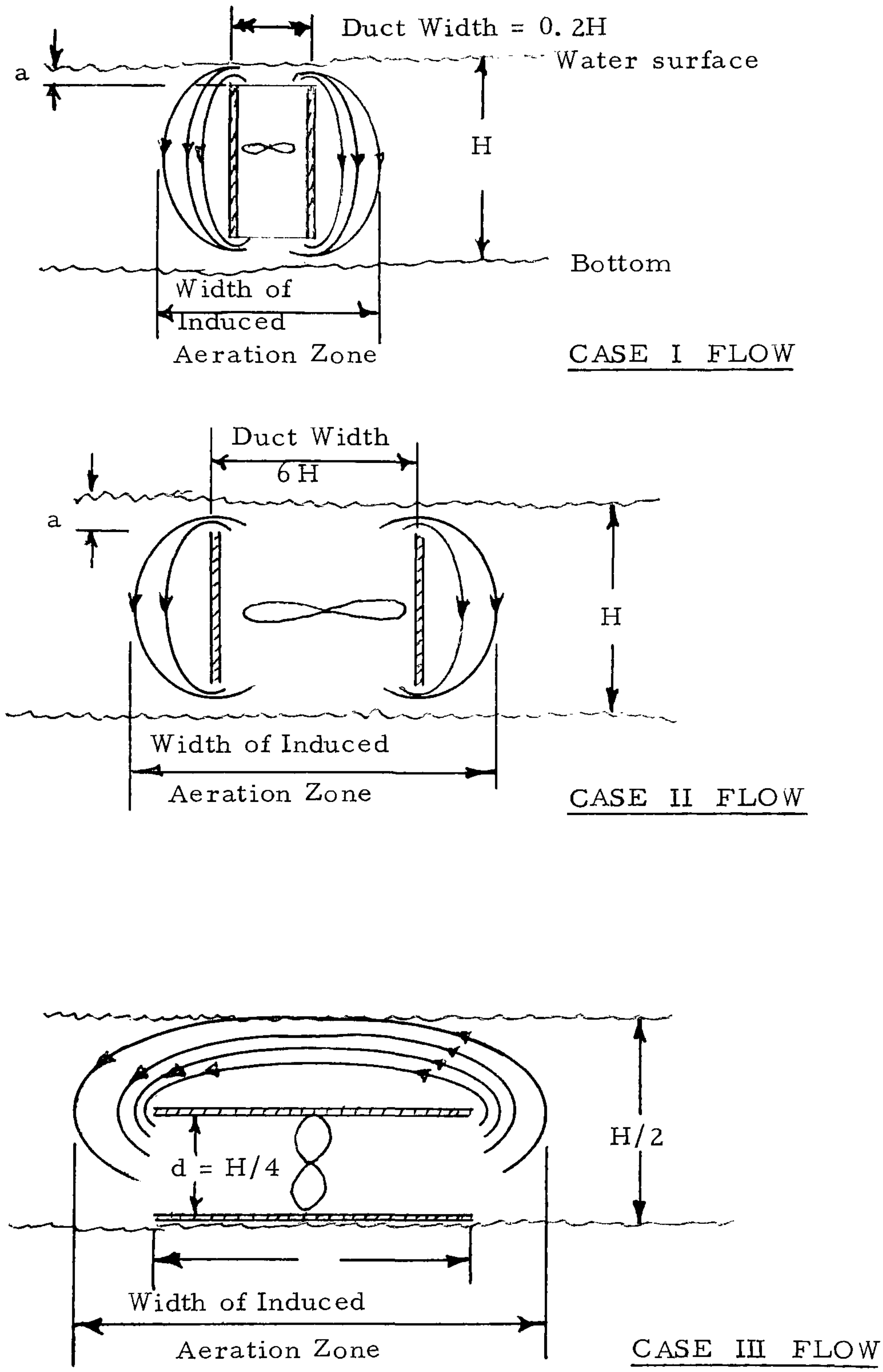


Fig. 6 - Schematic Sketches of Subsurface Aerators

Case III

In this configuration water is forced to flow through a horizontal duct and completes its circuit by passing between the outside top of the duct and the air-water interface. The duct may be placed on the bottom of the body of water as shown in Fig. 6 or it may be suspended at some distance below the air-water interface. The flow was simulated by two infinite sets of sources and sinks placed along vertical lines passing through the duct outlet and inlet respectively.

The duct length has been made long compared to the water depth in order to use this case for application to shallow water such as a stream or river.

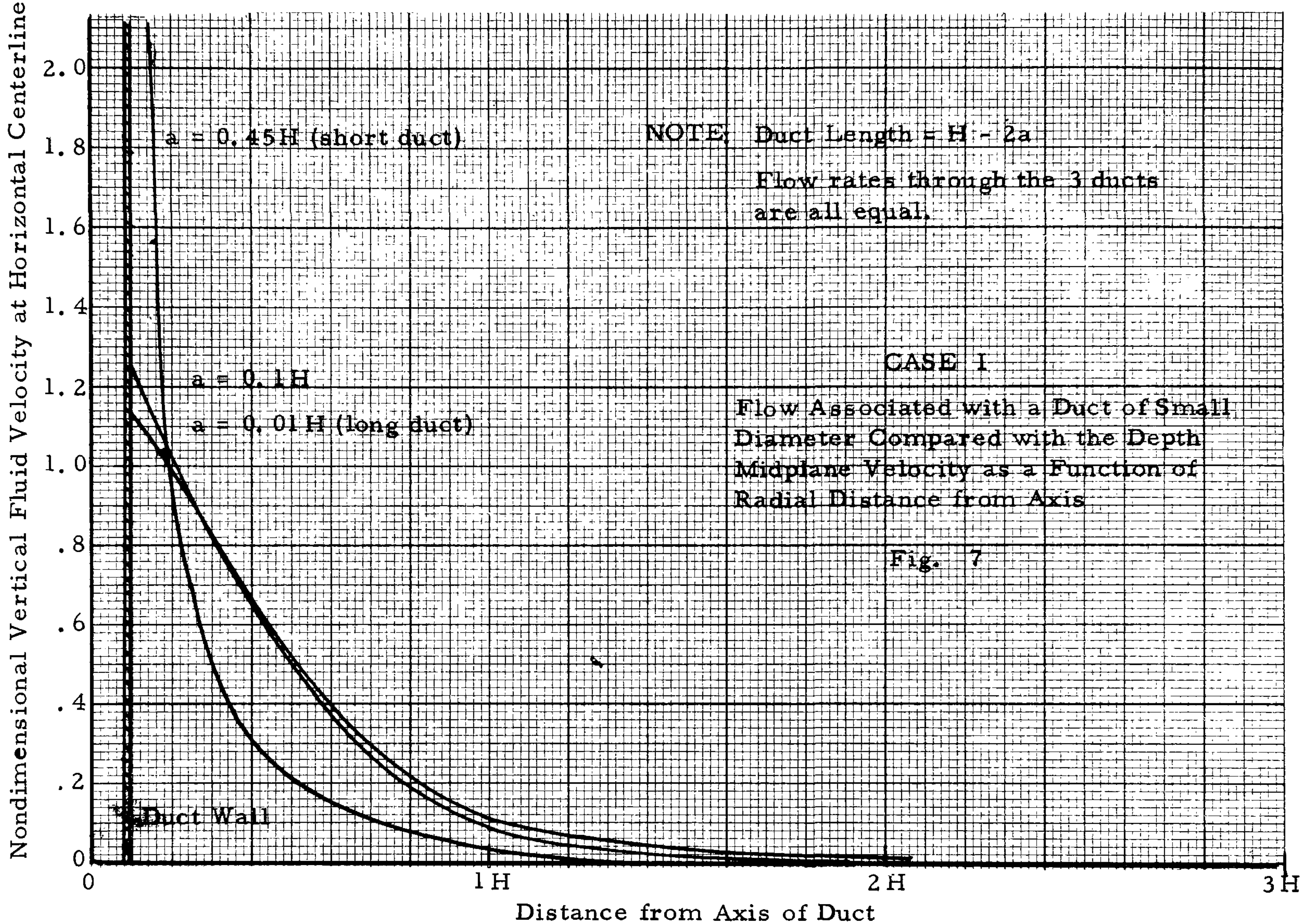
The details of the potential flow calculations and a discussion of the assumptions are given in Appendix B. The results of these calculations are discussed below in a brief fashion for convenience.

CASE I

The size of the circulation zone induced by the sub-surface device shown as Case I in Fig. 6 can be established by computing the vertical velocity along the horizontal line that passes through the mid-height point of the duct. From the symmetry of the device, it is noted that there will be no horizontal component of velocity along this line and thus the strength of the vertical component is a measure of the intensity of circulation at any given point. The vertical velocity along this horizontal centerline is shown in Fig. 7.

As mentioned previously, the velocity will decrease exponentially with distance from the duct and some arbitrary low velocity will have to be assumed to represent the end of the circulation zone. In this case the end of the zone was selected to be $\pm$ 2 H for a = .1 H and a = .01 H (where the velocity has decreased to about 1% of its value near the wall of a long duct as shown in Fig. 7).

An inspection of Fig. 7 shows that although the zone of circulation decreases as the duct length is made shorter, the decrease is not significant for values of "a" between 0.01 and 0.1. In addition it is shown in Appendix B, based on a constant circulation zone width of 4H, that the rate of oxygen transfer from the atmosphere to the water also decreases with decreasing duct length as shown by Eq. 21.



CASE I

Flow Associated with a Duct of Small Diameter Compared with the Depth Midplane Velocity as a Function of Radial Distance from Axis

Fig. 7

$$\dot{M}_{O_2} = 50.6 \times 10^{-6}(C_s - C)V^{1/2}H^{1/2} \quad \#m_{O_2}/\text{hr per foot of device} \quad (\text{For } a = 0.01H) \qquad \text{(Eq. 21A)}$$

$$\dot{M}_{O_2} = 47.4 \times 10^{-6}(C_s - C)V^{1/2}H^{1/2} \quad \#m_{O_2}/\text{hr per foot of device} \quad (\text{For } a = 0.1H) \qquad \text{(Eq. 21B)}$$

$$\dot{M}_{O_2} = 18.8 \times 10^{-6}(C_s - C)V^{1/2}H^{1/2} \quad \#m_{O_2}/\text{hr per foot of device} \quad (\text{For } a = .45H) \qquad \text{(Eq. 21C)}$$

where C_s = DO saturation concentration in mg/liter
C = DO concentration in mg/liter
V = fluid velocity in duct, ft/sec
H = water depth, ft

It can be concluded from Eq. 21 that higher oxygen transfer rates can be achieved by having the duct extend close to the water surface and that the lack of any duct around the propeller would seriously degrade the performance of the device.

The oxygen capture efficiency, η, for this device was estimated in Appendix B and is given by Eq. 22 below in pounds of oxygen captured per shaft hp-hr expended.

$$\eta = 0.143\,(C_s - C)H^{-1/2}V^{-5/2} \quad \#m_{O_2}/\text{hp-hr} \quad (\text{For } a = 0.01) \qquad \text{(Eq. 22A)}$$

$$\eta = 0.134\,(C_s - C)H^{-1/2}V^{-5/2} \quad \#m_{O_2}/\text{hp-hr} \quad (\text{For } a = 0.1) \qquad \text{(Eq. 22B)}$$

$$\eta = 0.053\,(C_s - C)H^{-1/2}V^{-5/2} \quad \#m_{O_2}/\text{hp-hr} \quad (\text{For } a = 0.45) \qquad \text{(Eq. 22C)}$$

where C_s, C are in mg/liter
H in ft
V in ft/sec

CASE II

The vertical velocity along the horizontal line that passes through the duct mid-height point may again, as in Case I, be used to measure the zone of induced circulation. This velocity is plotted in Fig. 8 as a function of distance from the vertical centerline. Again using this plot as a guideline the circulation zone is assumed to terminate at $\pm$ 5H for total width of 10H. Based on the results of Case I, only one value of "a" was selected, namely, a = 0.1 H.

In a manner similar to that used for Case I, the oxygen transfer rate and oxygen capture efficiency were estimated in Appendix B and are given by Eq. 23 and Eq. 24 respectively:

$$\dot{M}_{O_2} = (.007)(C_s - C)V^{1/2}H^{1/2} \quad \#m_{O_2}/\text{hr per foot of device} \qquad \text{(Eq. 23)}$$

$$\eta = 0.657\,(C_s - C)H^{-1/2}V^{-5/2} \quad \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. 24)}$$

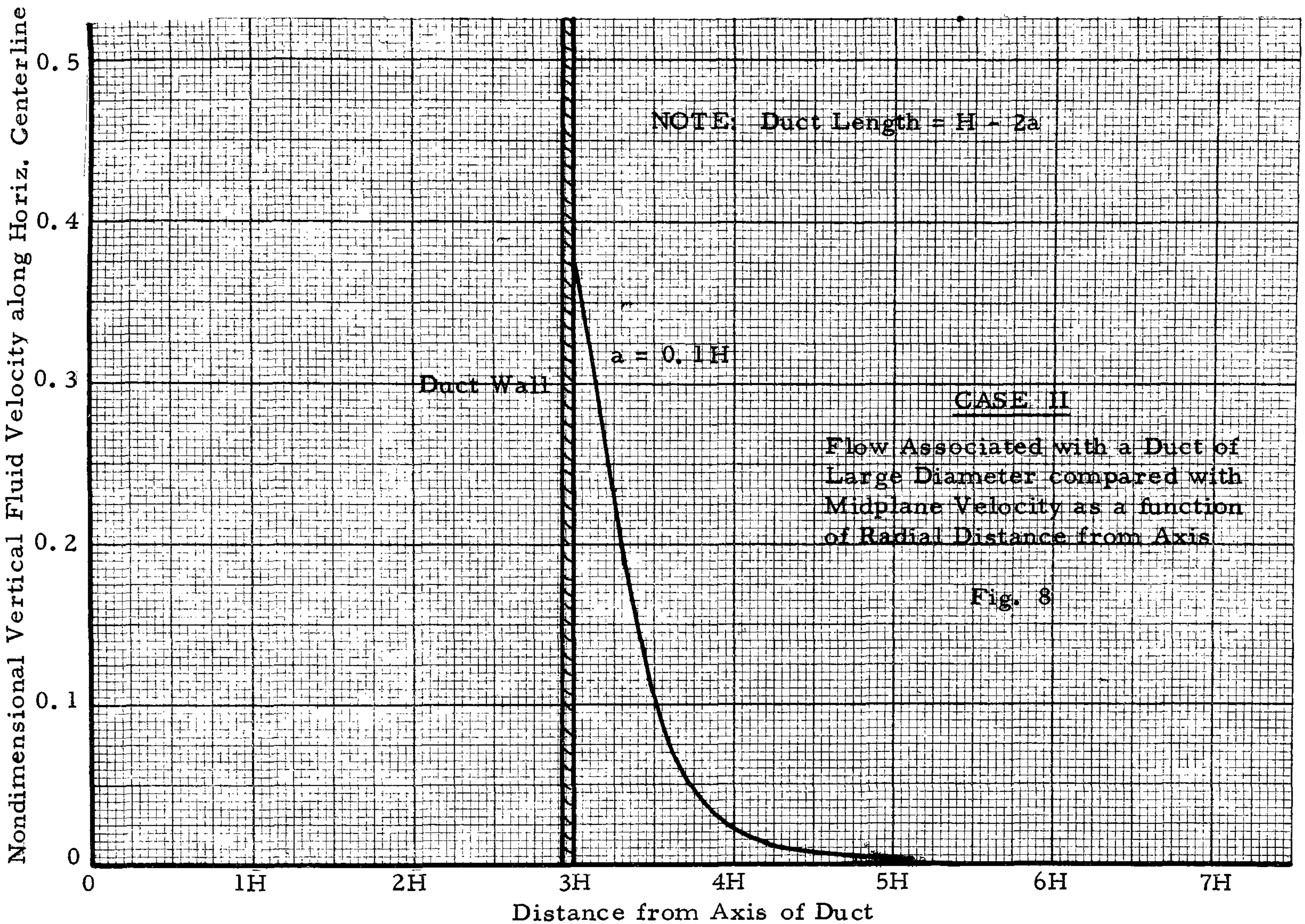
where C_s, C are in mg/liter
V in ft/sec
H in ft

CASE III

The zone of induced circulation for this configuration may be estimated by computing the magnitude of the horizontal velocity component at the air-water interface as a function of distance away from a vertical line that passes through the mid-length point of the duct. This velocity is shown in Fig. 9 for one value of b, namely b = 10H. From Fig. 9 it is seen that the zone of active circulation is approximately equal to the width of the duct. The potential flow equations developed in Appendix B indicate that the size of the circulation zone and the oxygen transfer rate per unit length (that is, unit depth into the paper as shown in the schematic given in Fig. 6) decrease as the duct width is decreased similar to the manner previously discussed for Case I. Again the conclusion can be drawn that omission of a duct around the propeller would seriously degrade the performance of the device.

The oxygen transfer rates and capture efficiencies for Case III were estimated in Appendix B and are given below in Eq. 25 and Eq. 26 respectively:

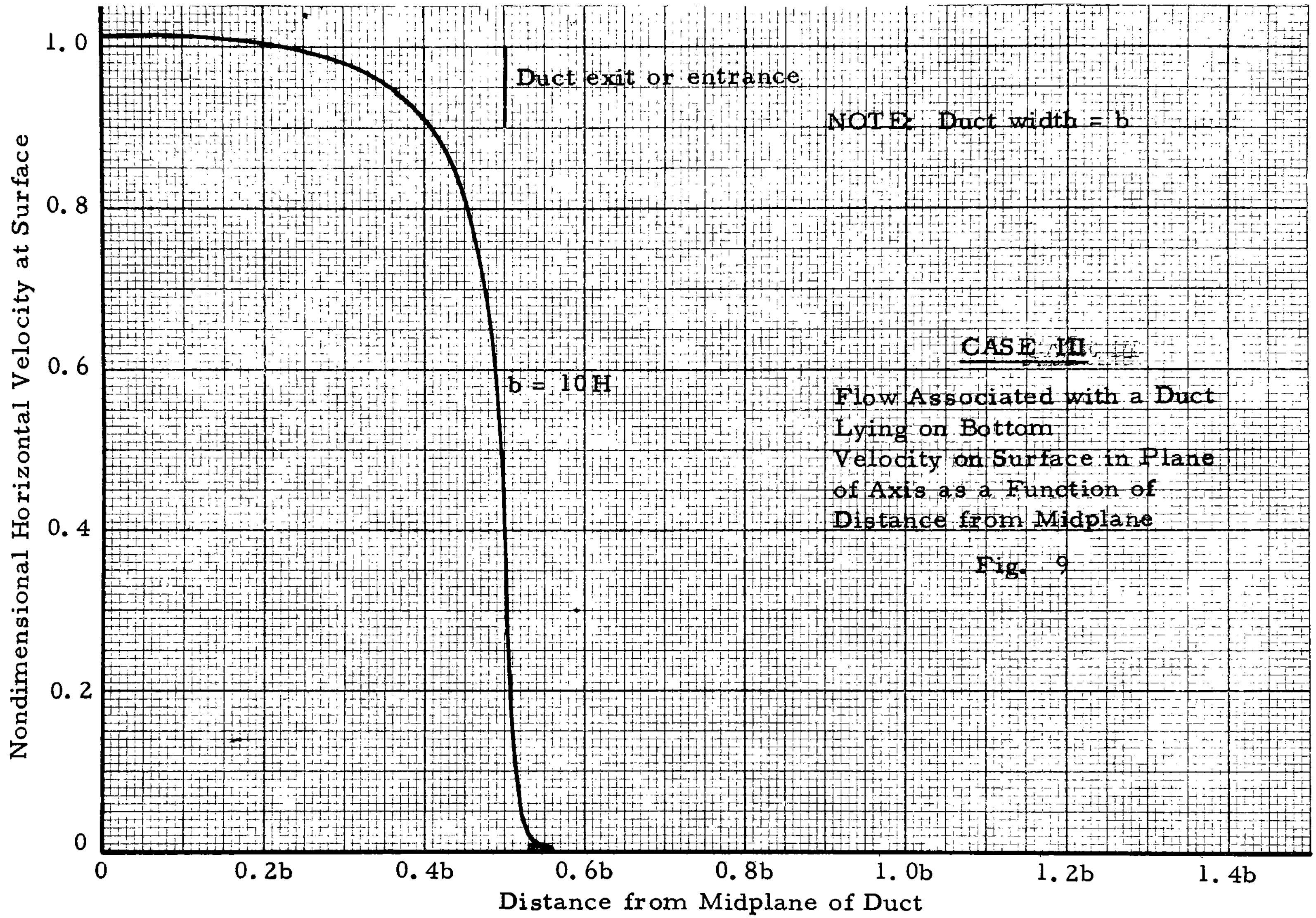
$$\dot{M}_{O_2} = .000524\,(C_s - C)V^{1/2}H^{-1/2} \quad \#m_{O_2}/\text{hr} \qquad \text{per foot of device (For } b = 10H,\ d = \frac{H}{4}\text{)} \qquad \text{(Eq. 25)}$$



CASE II

Flow Associated with a Duct of Large Diameter compared with Midplane Velocity as a function of Radial Distance from Axis

Fig. 8



CASE III

Flow Associated with a Duct Lying on Bottom Velocity on Surface in Plane of Axis as a Function of Distance from Midplane

Fig. 9

$$\eta = 0.589\,(C_s - C)H^{-1/2}V^{-5/2} \quad \#m_{O_2}/\text{hp-hr} \quad (\text{For } b = 10H,\ d = \frac{H}{4}) \qquad \text{(Eq. 26)}$$

where C_s and C are in mg/liter

V = duct velocity in ft/sec

H = twice the water depth in ft

Although a two-dimensional potential flow model was used in order to develop expressions for the oxygen transfer rates and capture efficiencies, it does not seem unreasonable at this stage of analysis to assume that the performance of an actual three-dimensional device (that is, a circular duct) can be estimated by multiplying Eqs. 21, 23, and 25 by the diameter to obtain the transfer rate per machine and to use Eqs. 22, 24 and 26 as given.

If in addition to the induced flow a natural flow exists in the body of water which tends to transport the water past the sub-surface aeration, some benefit in the transfer rate and capture efficiency will be derived from the turbulence which was initiated at the aeration station and which will require some finite time to dissipate.

The use of a two-dimensional model to simulate an actual device which is circular tends to overestimate the transfer rates and capture efficiencies whereas neglecting the benefit from the turbulence that persists beyond the aeration station tends to underestimate the same value.

The influence of a sloping stream or pond bottom on the circulation pattern (and hence on the oxygen transfer rates and capture efficiencies) may be investigated by making suitable alterations to the potential flow functions developed for the case of a horizontal bottom.

For an infinitely deep body of water the flow discharging from the outlet of a vertical pipe at a distance "a" below the surface (as in Case I) can be simulated by two sources, one a distance "a" below the interface and one a distance "a" above the interface as shown in Fig. 10-a. The source above the interface is the so-called mirror image source used to cancel flow across the interface and thus satisfy an imposed boundary condition. On the other hand, if the water has a finite depth, H, the flow at the duct entrance must be simulated with a sink. In order to satisfy the boundary condition on the bottom, a second sink must be added a distance "a" below the bottom as shown in Fig. 10-b. However, these two new sinks must have mirror image sinks relative to the air-water interface and likewise the two sources must have

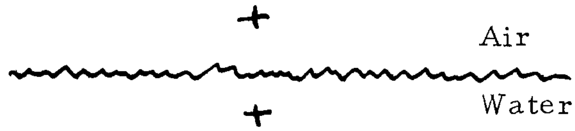


Fig. 10-a

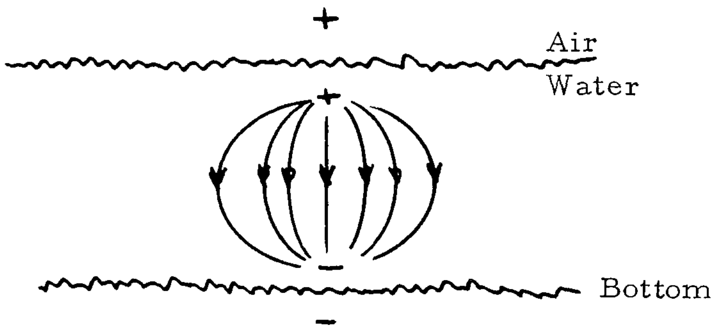
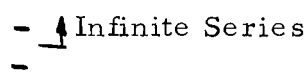


Fig. 10-b



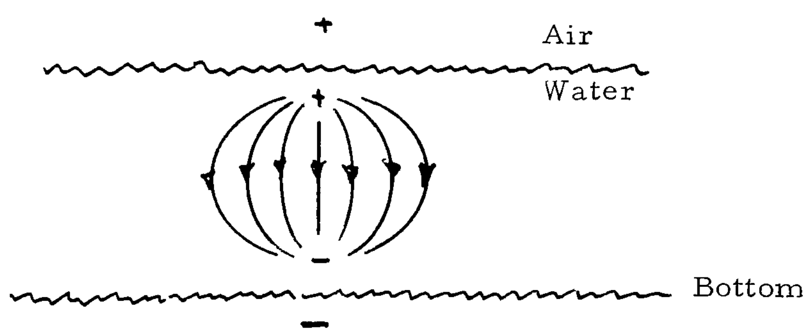


Fig. 10-c

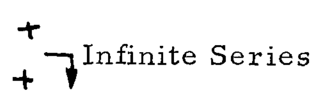


Fig. 10 - Source-Sink Arrangement to Simulate Circulation Pattern Induced by a Vertical Duct and Propeller in a Body of Water with a Horizontal Bottom

mirror image sources relative to the bottom for boundary conditions to be satisfied. The problem of simulating flow in a vertical duct located between the air-water interface and the bottom leads therefore to the use of an infinite series of sources and one of sinks all placed along a vertical line as shown in Fig. 10-c.

If the vertical line along which the sources and sinks are located is regarded as the circumference of a circle of infinite radius, then the same technique can be adapted for the simulation of a bottom with a slope. This technique can be applied only to a discrete number of bottom slope angles but this presents no difficulty in estimating the influence of circulation patterns. The method consists of placing the sources and sinks along the circumference of a circle as shown schematically in Fig. 11 whose radius is given by the expression

$$r_o = \frac{H}{\tan \alpha}, \text{ ft}$$

where α is the angle that the bottom makes with the horizontal
H is the water depth, ft

The size and intensity of the flow field created by a source and sink located near the top and bottom of a body of water with a bottom that slopes at an angle α can be conveniently characterized by moving away from the vertical centerline of the body of water along the "mid-line" (see insert a and b on Fig. 12) and calculating the velocity at right angles to the mid-line. A plot of this velocity is shown in Fig. 12 for three bottom slopes, namely, 0^o, 11^o and 45^o.

An inspection of Fig. 12 shows that the zone of circulation is extended to larger distances for small angles ($\sim 11^o$) with a subsequent decrease in the mid-line velocity near the centerline of the pond as the case must be since the source and sink strength are maintained constant for the three cases shown in Fig. 12. When the slope angle is large ($\sim 45^o$), the zone of circulation is decreased and the mid-line velocity near the pond centerline increased to compensate. However, the change in the size or intensity of the flow field is not significant even for angles as large as 45^o. Thus it may be concluded that the influence of the bottom slope on the circulation pattern can be neglected unless the slope is very steep ($\sim 45^o$) in which case the zone of circulation is decreased by the confining "side-wall" formed by the sloping bottom.

The influence of a vertical density gradient on the potential flow patterns induced by sub-surface devices is important during the period in

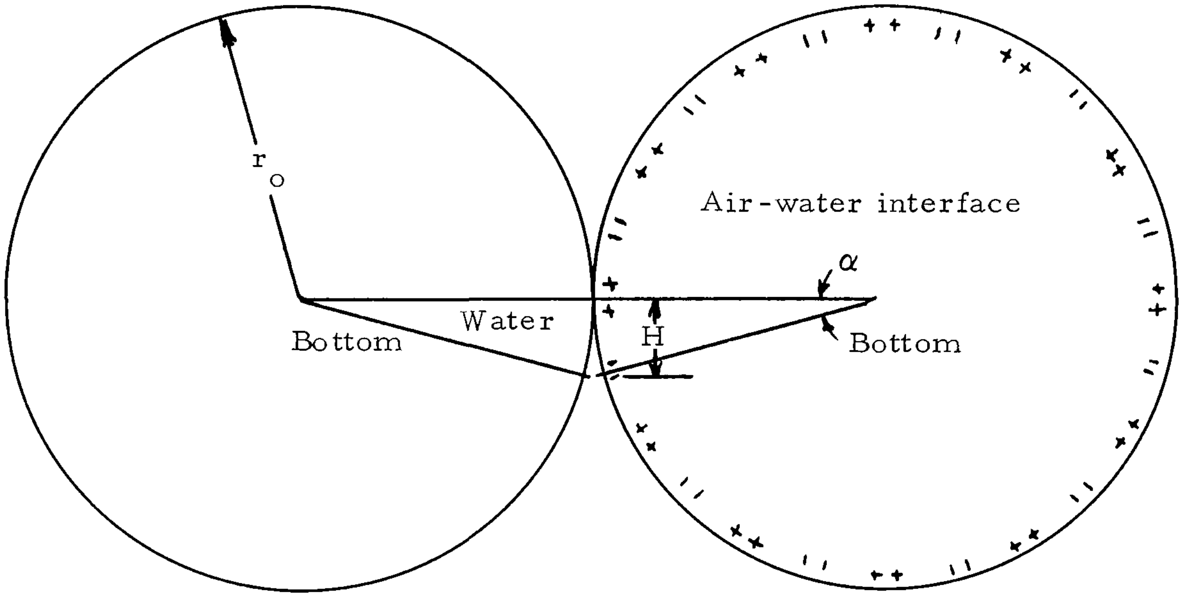


Fig. 11 - Source-Sink Arrangement to Simulate Circulation pattern induced by a Vertical Duct and Propeller in a Body of Water with a Sloping Bottom

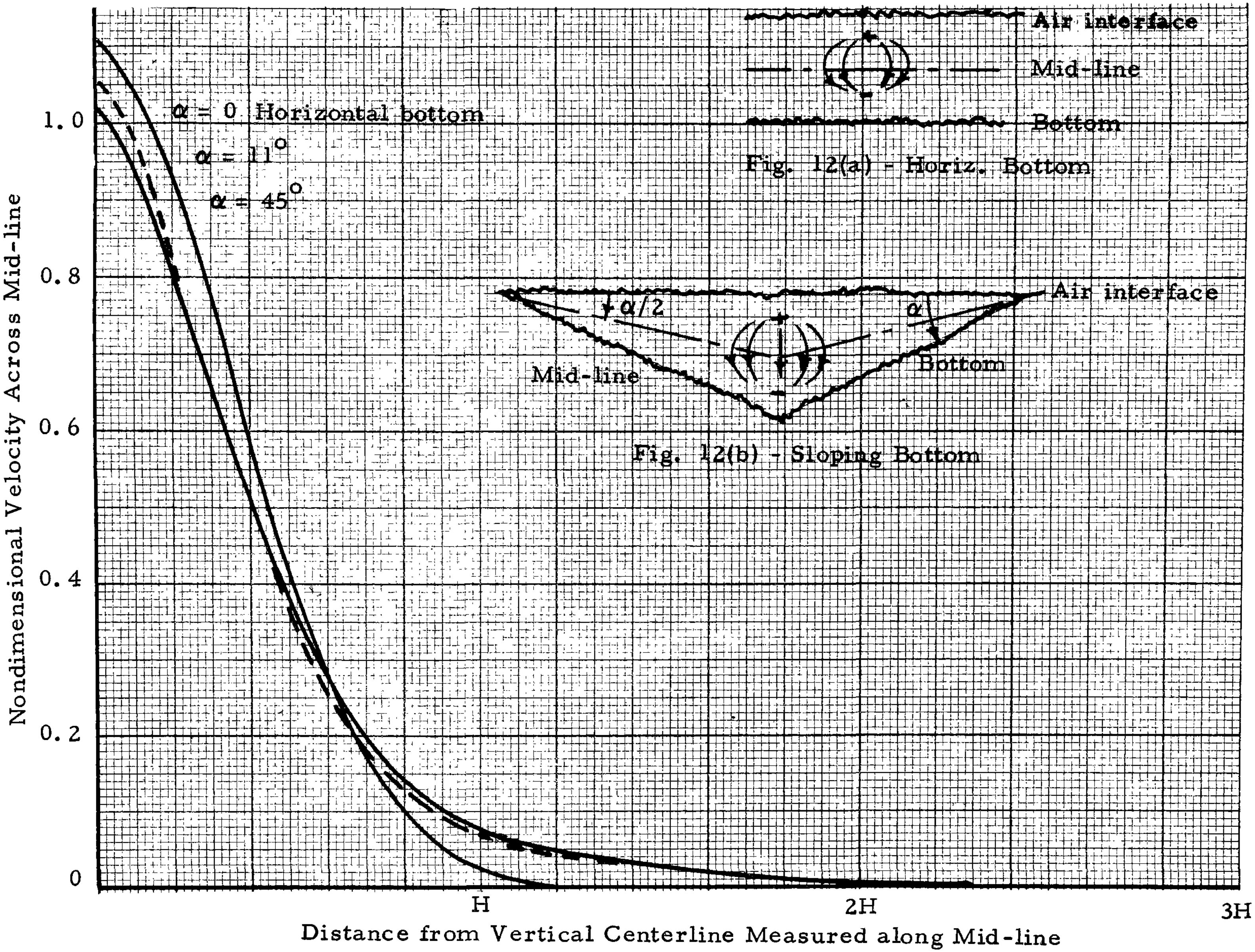


Fig. 12 - Velocities with Sloping Bottom

which the sub-surface aeration device is first placed in operation. As the operating time increases any initial density gradients will be destroyed and the final circulation pattern will be approximated by the potential flow fields previously discussed.

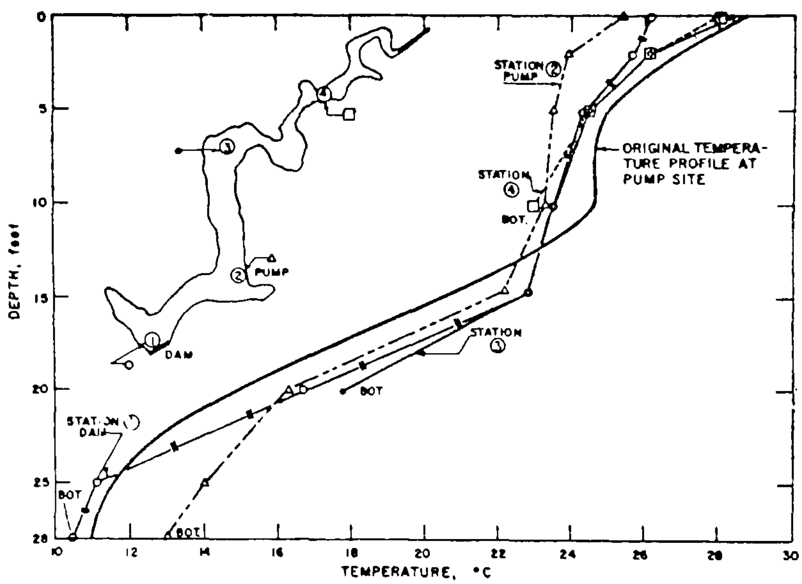
A literature survey was conducted on the influence of vertical temperature gradients on potential flow patterns; however, very little material has been found in the open literature on this topic and the task of obtaining analytical solutions does not seem feasible at present. Although it seems possible to develop a computer solution using numerical techniques, the necessary effort and cost to do so appear excessively high compared to the contribution that would result to the overall program. As a result we have decided to develop approximate order-of-magnitude expressions to deal with temperature gradient effects and to describe the situation phenomenonologically in as much detail as possible. These guidelines will enable us to estimate the rate at which a temperature gradient is destroyed by the sub-surface devices as well as the limit to the volume that will be affected.

Destratification of impoundments has received considerable attention in recent years because of its adverse influence on the quality of water stored below the thermocline [17]. One of the earliest experimental efforts to destratify a body of water was reported by Hooper, Ball and Tanner in 1952 [18]. In their scheme water was pumped from the hypolimnion to the surface of the lake by an on-shore centrifugal pump. Extensive tests with mechanical pumping as a destratification technique have more recently been reported by Irwin, Symons and Robeck [19]. Their scheme consisted of pumping cold water from near the bottom up through a vertical duct surrounding the pump and discharging it near the air-water interface. The raft-mounted equipment of Irwin <u>et al</u> introduced far less hydraulic losses than the earlier shore-mounted equipment of Hooper <u>et al</u>.

When the cold hypolimnion water is discharged near the top, it mixes with the warmer epilimnion water to produce a jet of water with a temperature (and hence density) between the extreme values associated with water originally at the very top and very bottom. As a result this jet will descend to a water depth where the density is equal to that in the jet. Since it is then in a neutrally buoyant configuration, the jet will not descend any deeper but will continue to move in a horizontal plane away from the vertical duct because of the influence of the water being continually pumped through the duct. Such stratified flows have been studied experimentally and analytically because of their importance to water quality in impounded water and in the design of power plant cooling water intake structures [20,21,22]. If there is substantially no mixing of the epilimnion and hypolimnion waters across the thermocline, then there must be a drainage of cold hypolimnion water along the bottom into the duct intake of such magnitude to equal the rate of water addition to the epilimnion. Thus if the duct is located at the deepest point of the body of water, these two counter flow currents will continue (and the thermocline location will continue to drop) until the lake is completely mixed. As the mixing nears completion, the counter-flow currents will deform into the potential flow pattern previously calculated. In contrast to the potential flow currents, the counter-current stratified flows will be slower but will extend to far greater distances from the duct. Precisely how far away from the ducts that the stratified flow will persist is difficult to assess analytically. However, from several experimental field projects it is known that these distances are many orders of magnitude greater than the diameter of the zone of circulation predicted by potential flow theory for the same device operating in water with no vertical density gradients. For example, in Ref. 19, the experimental destratification of a long narrow lake, Lake Vesuvius in Ohio, is discussed. This lake is about 16,000 feet long and not more than 800 feet wide, with a maximum depth of about 30 feet and a surface of 105 acres. The pump was located approximately 3000 feet from one end in about 30 feet of water. For the duct diameter of 1 foot used in the experiment, potential flow theory as developed for Case I indicates a radius of about 50 feet for the zone of circulation whereas the data presented in Ref. 19 indicated that the stratified flow was felt as strongly at least 10,000 feet away from the duct as within 50 feet (See Fig. 12) but with a time lag of a few days.

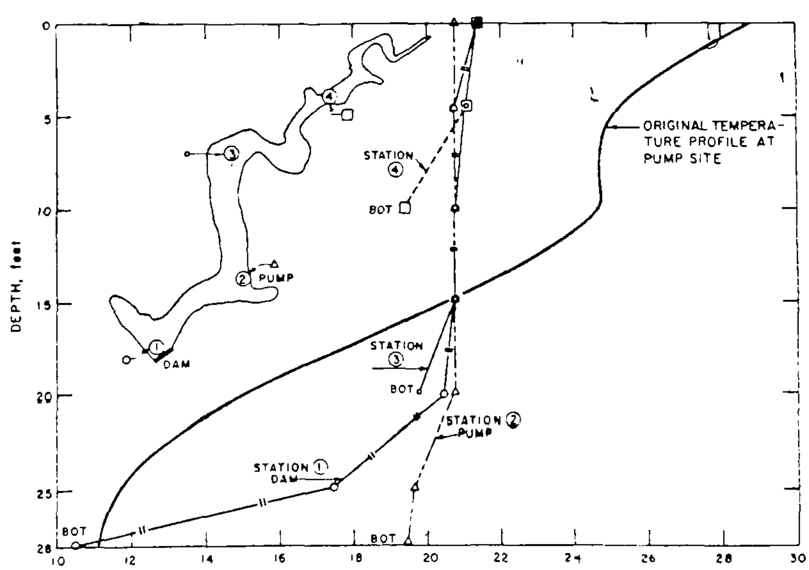
The pumping at Lake Vesuvius was stopped after 208 hours at which time the temperature at any given depth was almost the same at any location on the lake. Since the pumping was discontinued, no evidence is available from this experimental work on the transition from stratified flow to a flow pattern that can be approximated by potential flow theory.

However, Paul D. Uttermark has commented on the mechanical pumping of a very similar lake in Wisconsin [23]. Uttermark pointed out that the Wisconsin Conservation Department had been continuously pumping a 100-acre, 28 foot deep lake with a very high volume flow rate pump. He noted that in order to draw water from any great distance (over 100 feet) into the pump, the body of water must be stratified. Once the lake was mixed, a mixing cell developed in the immediate area of the pump. As Uttermark stated:



After 2. 5 Days of Pumping

Fig. 13. Temperature Profiles at Various Stations in Vesuvius Lake (from Ref. 19)



After 8. 5 Days of Pumping

Fig. 14. Temperature Profiles at Various Stations in Vesuvius Lake (from Ref. 19)

> "We found that during the summer of 1967 we created a mixing cell in the lake. We were able to pump water very rapidly in one location but the rest of the lake was not affected at all . . . The horizontal temperature differences from inside this highly mixed area to just 100 feet away were about 5°F. In the mixing cell, however, we had a very uniform temperature from the surface of the lake to the bottom."

In the lake discussed by Uttermark pumping had been conducted continuously for sufficient time to pass from stratified counter-current flow through the transition phase into a region that can be approximated by potential flow. As noted by the author, the fact that a "cell" developed is most likely the result of an equilibrium being established between the pumping rate (which destroys stratification) and continuous surface heating of the lake during the summer (which induces stratification).

The nature of this equilibrium configuration will depend on the pumping rate and the net heating of the lake. The influence of these factors can be easily noted by considering two extreme cases. First, consider a strongly stratified lake in the summer and a small pumping rate. As cold water is discharged near the surface it mixes and settles to a level of equal density. Since the flow rate is low, the density of the jet will be dominated by the epilimnion water. If the pumping rate is sufficiently low, stratified flow may continue all summer. Second, consider a lake which is only slightly stratified (as might exist after a few days of pumping an initially strongly stratified lake) and a very high pumping rate. When the colder water from the bottom is discharged near the top, its density can be changed only slightly because of the relative abundance of colder water. As a result the buoyant forces are insufficient to overcome the suction exerted at the duct intake and a "cell" or potential source-sink flow is established.

It should be noted that the oxygen transfer rate for stratified flow (for the same DO deficit) can not be greater than for potential flow. Although the flow extends over a considerably larger area for the stratified case, it does so at a strata below the surface and at very low velocities such that no additional surface renewal can be anticipated at the greater distances.

For the present study it is of interest to be able to predict the approximate time required to pass from stratified flow to the potential flow condition. This is most readily accomplished by examining the energy required to destratify a body of water. The energy required to totally mix a lake can be estimated as follows:

Assume a stratified lake may be represented to two separate, homogeneous strata of temperatures, densities and thicknesses, T_1, ρ_1, a and T_2, ρ_2, b. In the stratified state the gravity center of each layer lies in the layer midplane. The gravity center of the total stratified system lies somewhere between, at a distance X from the bottom. After mixing, the gravity center lies in the midplane of the lake. The work required to mix the lake is then the change in potential between the stratified and mixed conditions. This quantity is frequently termed the "stability" of the lake and is the work required to raise the total water weight the distance

$$\epsilon = \frac{a+b}{2} - X \text{ , ft} \qquad \text{(Eq. 28)}$$

The distance X may be computed by equating to zero moments taken about the stratified systems' gravity center.

$$X = \frac{\rho_2 a(b + \frac{a}{2}) + (\rho_1 b^2)/2}{\rho_1 b + \rho_2 a} \qquad \text{(Eq. 29)}$$

The distance between mixed and stratified centroids is then:

$$\epsilon = \frac{(ab)/2(\rho_1 - \rho_2)}{a\rho_2 + b\rho_1} \text{ , ft} \qquad \text{(Eq. 30)}$$

The work required for mixing, W, is

$$W = A(a+b)\, g \rho_3 \epsilon \text{ , } \#_f\text{-ft} \qquad \text{(Eq. 31)}$$

where ρ_3 is the density of the mixture, slugs/ft^3

A is the lake surface area, ft^2

g is the acceleration due to gravity, 32.2 ft/sec^2

Equations 30 and 31 can be combined to yield

$$W = \frac{A\rho_3(a+b)\, ab(\rho_1 - \rho_2)\, g}{2(a\rho_2 + b\rho_1)} \text{ , } \#_f\text{-ft} \qquad \text{(Eq. 32)}$$

Since $\rho_1 \approx \rho_2 \approx \rho_3$, this can be approximated by

$$W = \frac{1}{2} A\, a\, b(\rho_1 - \rho_2) g \text{ , } \#_f\text{-ft} \qquad \text{(Eq. 33)}$$

In terms of lake volume, I, Eq. 33 becomes

$$W = \frac{1}{2} I \frac{ab}{(a+b)} (\rho_1 - \rho_2) g \quad \#_f\text{-ft} \qquad \text{(Eq. 34)}$$

where I is the lake volume in ft^3

Note that this analysis assumes the lake is of uniform depth, which is not usually the case. Since the epilimnium thickness, a, is probably uniform across most of the lake, an average value of b should be used.

The time required to mix a lake can be estimated by equating the product of power input to the lake and time to the total work required. Assuming the pumping power is held constant, this condition can be expressed as:

$$t = \frac{W}{(DE)\, P} \qquad \text{(Eq. 35)}$$

where DE = "destratification efficiency" or the ratio of energy input to the device to the energy required to shift the center of gravity, expressed as a fraction.

P = input power, $(\#_f\text{-ft})/\text{sec}$

t = pumping time required to destratify, sec

Since the "destratification efficiency" (DE) involves losses due to pump efficiency, friction in the duct, and kinetic energy of the flow as well as the induced mixing pattern, it will be dependent on the water velocity in the duct. Lower duct velocities will give higher efficiencies although exit velocities must be high enough to induce sufficient mixing near the surface to impart a buoyancy to the water from the hypolimnium. Higher duct velocities will yield lower efficiency first because of duct and pump losses and second, if the duct velocities are sufficiently high, the overabundance of cold water may cause recirculation of the pumped water to the duct inlet.

Based on experimental work with equipment that had a high duct velocity (8.3 ft/sec), Symons, Irwin and Robeck [24] reported a DE of 0.0014 (or 0.14%) for a 96 acre 25 foot deep (average) lake destratification (Boltz Lake in Kentucky). As the authors noted, kinetic energy losses in this equipment were high due to the excessive duct velocity and it would not seem unreasonable to anticipate values of DE at least an order of magnitude higher for low velocity equipment.

Equations 34 and 35 may be combined to obtain an expression for the time required to destratify a lake, namely,

$$t = \frac{I\,ab\,(\rho_1 - \rho_2)\,g}{2\,(DE)\,P\,(a+b)}, \text{ sec} \qquad \text{(Eq. 36)}$$

To establish the order of magnitude of the time given by Eq. 36, consider a 100-acre stratified lake, of average depth equal to 30 feet, a = 15 ft, b = 15 ft, $T_1 = 70^oF$ (ρ_1 = 1.938 slugs/ft^3, $T_2 = 50^oF$ (ρ_2 = 1.940 slugs/ft^3), which is to be destratified by a 10 hp mechanical pump. For these conditions Eq. 36 predicts a time of 160 hours for an assumed DE of 0.01.

In Table VI is given a summary of the approximate destratification times from Ref. 19 for a 16 hp pump with a flow rate of 2,800 gpm.

TABLE VI

Measured Lake Destratification Time

Lake	Area (acres)	Average Depth (ft)	Approx Time to Destratify (hrs)
Stewart Hollow	8	15	37.5
Caldwell	10	10	8.0
Pine	14	7	35.0
Vesuvius	105	12	208.0

From Eq. 36 and Table VI it can be concluded that the time required for a typical sub-surface aerator to pass through the stratified flow mode of operation will be of the order of one week or less. During this period the liquid film coefficient and active oxygen capture surface area will not be greater than the corresponding values when the device is operating in the potential flow mode; however, the DO deficit near the duct outlet may be considerably higher during the initial phase of stratified flow.

Diffused Aeration

The injection of air (or pure oxygen) at the bottom of a body of water in order to supply oxygen for biological reactions has for some time been applied to waste treatment plants where the BOD demand is substantial. More recently it has been applied to rivers and reservoirs [25, 26, 27]. This technique of transferring oxygen into the water consists of injecting air or oxygen bubbles at the bottom of the body of water. The bubbles rise and as they do oxygen diffuses from the bubble surface into the water. The diffusion process may be controlled by the relatively slow molecular diffusion process if the turbuelnce in the water is sufficiently low or by the more rapid process of eddy diffusion if sufficient turbulence exists.

For a single bubble rising in water the rate of oxygen transfer is given by Eq. 8, namely

$$\frac{\dot{M}_{O_2}}{A} = K_L(C_s - C) \qquad \text{(Eq. 8)}$$

where C is the uniform DO concentration at any given time (See Eq. 7 for a discussion of the units)

In practice bubbles will be generated rapidly at a number of orifices or diffuser plugs (porous plugs). Since the residence time of the bubbles in the water may be quite small, only some fraction of the oxygen originally in the bubble may have diffused into the water before the bubble reaches the air-water interface. As shown by Ippen and Carver [28], the mass fraction of oxygen originally in the bubble which is captured (f_{CD}) can be found by dividing the transfer rate for all bubbles by the oxygen pumping rate which for air bubbles is:

$$f_{CD} = \frac{K_L A_T[(C_s - C) \times 62.4 \times 10^{-6}]}{\dot{Q}(.21\gamma_a)} \qquad \text{(Eq. 37)}$$

where $\dot{Q}$ = volume flow rate of air, ft^3/sec

γ_a = mass density of air, $\#m/ft^3$

A_T = surface area of all bubbles in the water at a given instant, ft^2

C_s, C = dissolved oxygen concentrations, mg/liter

K_L = liquid film coefficient, ft/sec

Since the surface area of all the bubbles must equal the product of the surface area of one bubble times the number of bubbles generated per unit time times the residence time of the bubble in the water, Eq. 37 can be expressed as the product of three nondimensional terms, namely,

$$f_{CD} = (28.6)\left(\frac{H}{d}\right)\left(\frac{K_L}{V_T}\right)\left[\frac{(C_s - C) \times 62.4 \times 10^{-6}}{\gamma_a}\right] \qquad \text{(Eq. 38)}$$

where d = bubble diameter

V_T = bubble terminal velocity

For a given body of water the magnitude of the depth (H) is fixed as well as the value of the DO deficit (C_s-C), with the result that one is free to select only the size of the bubble for once this has been fixed the value of V_T and K_L also become determined.

The relation between the bubble size (d) and the terminal velocity (V_T) is readily found by noting that when gas is introduced near the bottom of the water in the form of bubbles, the bubbles will experience a period of acceleration until they reach a terminal velocity as shown in Fig. 15.

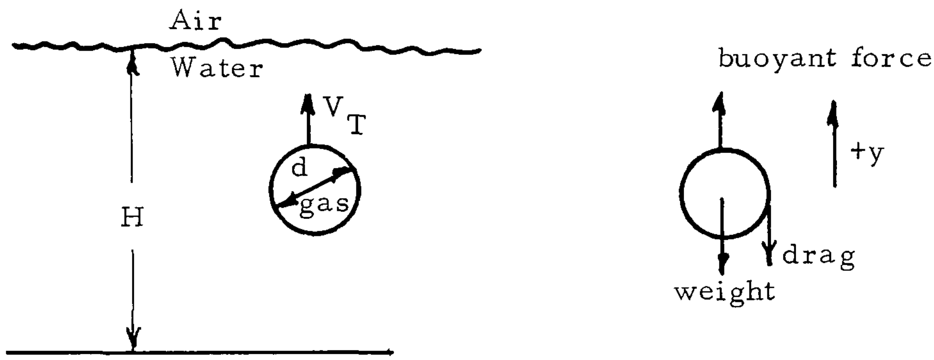


Fig. 15 - Rise of a Bubble in Water

When the bubble has reached its terminal velocity, the net force on the bubble will be zero, thus

$$-\frac{\pi}{6}(d^3)\rho_a g + \left(\frac{\pi}{6}d^3\right)\rho_w g - C_D\left(\frac{\pi}{4}d^2\right)\rho_w \frac{V_T^2}{2} = 0 \qquad \text{(Eq. 39)}$$

where d = diameter of the air bubble, ft

ρ_a = density of air, slugs/ft^3

ρ_w = density of water, slugs/ft^3

g = acceleration due to gravity, 32.2 ft/sec^2

C_D = drag coefficient, which is defined as

$$F_{DRAG} = C_D A\,[(\rho V^2)/2]$$

V_T = terminal velocity of bubble

In order to solve Eq. 39 for the terminal velocity, it is necessary to know the value of the drag coefficient C_D. The value of this coefficient is known analytically for low Reynolds number flow (Stokes Law) and is also known experimentally over a wide range of flow conditions.

In general as the bubble diameter is increased the terminal velocity also increases. Thus small diameter bubbles are associated with low Reynolds numbers and large diameter bubbles with high Reynolds numbers. As the Reynolds number is decreased, the resident time will increase and this by itself would result in an increase in the capture coefficient; however, as the Reynolds number is decreased, the viscous forces increase relative to the inertia forces and a thicker layer of water can be anticipated to remain with the bubble as it rises which by itself will decrease the capture rate since the oxygen must diffuse across this thicker exposed layer. As a result of these considerations it would be helpful to know the capture coefficient over a range of bubble diameters and hence Reynolds numbers for application in streams and ponds. In particular it would be helpful to have an analytical expression for the most efficient region of operation (which will be shown to be low Reynolds number flow) to serve as a guideline for optimization.

From experimental investigations on the terminal velocity of bubbles rising in water, it is known [28] that bubbles behave like solid spheres only up to Reynolds numbers of about 70. For Reynolds numbers up to 1.0, the relation between the drag coefficient (C_D) and the Reynolds number for solid spheres is known to be

$$C_D = \frac{24}{\left(\frac{\rho_w V_T d}{\mu_w}\right)} \approx \frac{24}{\left(\frac{V_T d}{\nu_w}\right)} \qquad \text{(Eq. 40)}$$

where ρ_w = density of water, slugs/ft^3

μ_w = absolute viscosity of water, $\#_f$-sec/ft^2

ν_w = kinematic viscosity of water, ft^2/sec

For the limiting case of Reynolds number = 1.0 (bubble diameter ~ 0.12 mm), Eq. 39 can be solved for the bubble terminal velocity with the use of Eq. 40. Thus

$$V_T = \frac{d^2 g \left(1 - \frac{\rho_a}{\rho_w}\right)}{18\nu}, \text{ ft/sec} \qquad \text{(Eq. 41)}$$

Since the critical Reynolds number for a sphere is greater than 100, 000, the flow at these low Reynolds numbers will be laminar. The bubble will creep through the water with a residence time of $t_r = H/V_T$. Since it is anticipated that at this very low Reynolds number the bubble will drag a layer of water with it as it rises, the appropriate liquid film coefficient is that given by Eq. 8, namely,

$$K_L = \sqrt{\frac{D_L}{t\pi}} \qquad \text{(Eq. 8)}$$

where $\quad t$ = exposure time for the fluid layer

It should be pointed out that although the expression for K_L given in Eq. 8 will decrease with time, as $1/\sqrt{t}$, K_L will be used only when it is integrated over time to determine the total oxygen transfer from the bubble. Since the integration of

$$\int_0^{t_r} dt/\sqrt{t}$$

is equal to $2\sqrt{t_r}$, and since it will be somewhat more convenient to use a constant value of K_L which yields the correct results compared to the exact integration procedure, the value of t to be used in Eq. 8 will be taken as $(1/4)t_r$ or $(1/4)(H/V_T)$. Thus the film coefficient can be taken as

$$K_L = 2\sqrt{\frac{D_L}{\pi \frac{H}{V_T}}} \qquad \text{(Eq. 42)}$$

Thus the liquid film coefficient, K_L, can be expressed by combining Eq. 41 and Eq. 42.

$$K_L = \frac{2d^2 D_L\, g(1 - \frac{\rho_a}{\rho_w})}{9H\nu_w \pi} \qquad \text{(Eq. 43)}$$

If Eq. 43 is used to predict the liquid film coefficient for small diameter bubbles, then the capture coefficient can be readily computed by substituting Eq. 43 into Eq. 38 to obtain

$$f_{co} = \frac{138}{d^2}\sqrt{\frac{D_L \nu_w H}{g(1-\rho_a/\rho_w)}}\left[\frac{(C_s - C) \times 62.4 \times 10^{-6}}{\gamma_a}\right] \qquad \text{(Eq. 44)}$$

An inspection of Eq. 44 clearly shows the desirability of as small a bubble as possible in order to increase the capture coefficient. However, the rapid generation of smaller and smaller bubbles at the diffuser plug will require progressively higher pressure drops across the plug and hence progressively higher compression power for a fixed mass flow rate of gas. In addition, as the number of bubbles generated per unit time at one diffuser plug increases, there is a tendency for adjacent bubbles to coalesce into large diameter bubbles.

When the flow rate is sufficiently low both Ippen et al [29] and Maier [30] have shown that the diameter of a bubble formed at an orifice under water is 10 to 11 times larger in diameter than the orifice. As the gas emerges from the orifice it has a buoyancy which tends to make it rise; however, this force is resisted by a shear force across the orifice opening. As the gas flow rate is increased above some threshold limit, the bubble diameter increases and the bubbles leave in a chain-like array. Thus to produce bubbles of very small size one must either use very small openings or provide a means to shear the bubble off the face of the orifice before it has grown to its natural 10-11 orifice diameters. Obviously to obtain the 0.12 mm diameters needed to produce Reynolds numbers of 1.0, orifice diameters of 0.012 mm (~ .0005") may not be practical in view of clogging difficulties. However, it should be possible to approach these small bubble sizes by use of large orifices and devices designed to increase the shear force at the orifice face. Such work has been conducted by many workers including Maier [30], Langelier [31] and Zieminski et al [32].

In considering the diffusion process for bubbles, note must be taken that C_s increases with pressure in accord with Henry's Law and hence with depth. Thus for modes depths (< 200 ft) and air bubbles,

$$C_s = \left(C_{s \text{ at normal atmospheric conditions}} \right) \frac{.21 \left[14.7 + \frac{P_{HYD}}{144} \right]}{.21 \times 14.7 \text{ psia}} \qquad \text{(Eq. 45)}$$

where P_{HYD} = hydrostatic pressure at depth z ft

$= \rho g z$

The usual practice is to evaluate the gas bubble pressure and bubble size at the mid-depth location.

If pure oxygen is used in place of air, Eq. 37 is applicable if the factor of 0.21 is omitted in the denominator. The remaining equations are valid with the corresponding assumptions with the result that the capture coefficient for pure oxygen is given by the expression

$$f_{co} = \frac{29.1}{d^2} \sqrt{\frac{D_L \nu_w H}{g(1 - \rho_{O_2}/\rho_w)}} \left[\frac{(C_s - C) \times 62.4 \times 10^{-6}}{\gamma_{O_2}} \right] \qquad \text{(Eq. 46)}$$

where ρ_{O_2} = density of oxygen

γ_{O_2} = mass density of oxygen, $\#m/ft^3$

It should be noted here that although Eq. 44 and Eq. 46 differ very little in notation form, there may exist a significant difference in the cor rect value of the saturation concentration C_s to be assigned to each equation in view of the fact that at a given temperature C_s increases linearly with the partial pressure of the oxygen in accord with Henry's Law as given by Eq. 45. For example, if the same size air and oxygen bubbles are generated at some point, C_s for the pure oxygen bubbles will be 1/.21 times greater than for the air bubbles. As a result the capture coefficient should be the same magnitude for air and pure oxygen bubbles formed at the same depth and of the same size.

In order to establish the order of magnitude of the capture coefficient for very small bubbles (Reynolds number = 1.0, bubble diameter ~ .12mm), it is helpful to consider the 2-foot deep body of water initially at zero DO and with C_s at the surface equal to 9 mg/liter. For this case the use of Eq. 44 indicates a capture coefficient of 0.820. This example indicates the possibility of high capture efficiency if the bubble diameter can be made sufficiently small and it shows that a limit exists on how small the diameter need be for a given depth. It should be noted that in any prolonged application of diffused aeration that the water may saturate with N_2 but still have an O_2 deficit with the result that the partial pressure of O_2 in the bubble may decrease substantially as the bubble rises and becomes almost pure N_2. Under these conditions Eq. 44 is not valid in its present form but would have to be corrected to show how $\dot{M}_{O_2}$ decreases as the bubble rises due to a falling off in the magnitude of C_s.

No data could be found in the literature relative to the capture coefficient for bubbles small enough to produce Reynolds numbers of the order of 1.0. Ippen and Carver [28] reported measured capture coefficients as a function of depth for larger bubbles (Reynolds number range of approximately 300 to 900). The values reported by Ippen and Carver are reproduced in Fig. 16 for convenience. From Fig. 16 it is noted that for our previous example of a 2-foot deep body of water at zero DO, an 11-fold increase in bubble diameter (from 0.12 to 1.32mm) results in a decrease of the calculated capture coefficient of 0.82 to a measured value of 0.15 or a decrease by a factor of 5.5. The desirability of producing small bubbles is clear.

Zieminski [32] reported capture coefficients from 0.29 to 0.33 for a pilot model diffuser system specifically designed to produce small bubbles by a shearing action at a depth of about four feet. The size of the bubbles generated in this experimental work was not reported. For the same depth Ippen and Carver measured a capture coefficient of about 0.22 for 1.32 mm bubbles.

From Fig. 16 it is also noted that the capture coefficients for air and pure oxygen bubbles of the same size tend to become equal only as the water depth is increased. The authors imply that the reason for the difference in f_{co} at low water depth is the influence of nitrogen diffusing from the water into the bubble at the diffuser plug, an influence which is diminished as the column height is increased since activity at the diffuser plug is then overshadowed by the longer main diffusion process that takes place between the diffuser plug and the atmosphere-water surface.

As mentioned before, increasing the flow rate through one diffuser plug will result in the coalescence of bubbles and hence no further improvement in the capture coefficient. It is difficult to establish a general criteria for the onset of this event. In order to establish a guideline we might assume the fraction of volume below the air-water interface occupied by air or oxygen bubbles, η_o, should not exceed perhaps a value of 0.001. Thus the maximum air or oxygen volume flow rate for a body of water of depth H would be

$$\frac{\dot{Q}}{A} = 0.001\, V_T \quad , \ ft^3/ft^2\ sec$$

where A = water surface area at atmosphere-water interface, ft^2
V_T = terminal velocity of bubbles, ft/sec

In order to compute the maximum mass flow rate of oxygen into the diffuser system at low Reynolds numbers, it is necessary only to multiply Eq. 47 by $.21\gamma_a$ for air bubbles and by γ_{O_2} for pure oxygen bubbles and substitute Eq. 41 for V_T. Thus for air

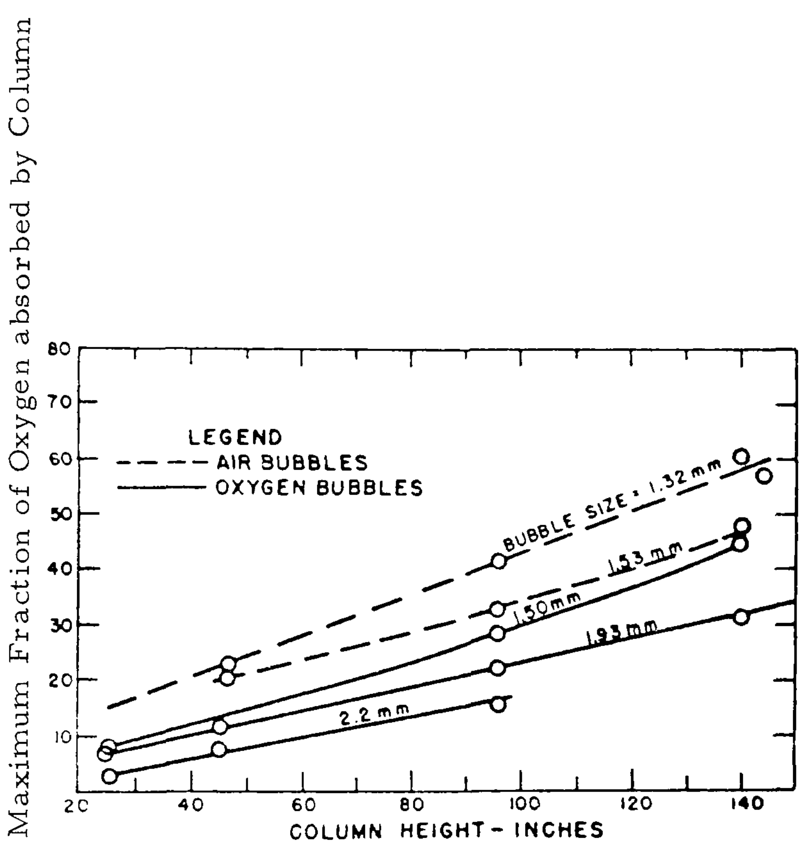


Fig. 16 - Measured Effect of Bubble Size and Column Height on Oxygen Capture Coefficient for Water with an initially Low DO (~ 0) Level (From Ref. 16)

$$\left(\frac{\dot{M}_{O_2}}{A}\right)_{\substack{\text{into}\\ \text{diffuser}}} = \frac{1.167 \times 10^{-5} \gamma_a d^2 g (1 - \rho_a/\rho_w)}{\nu_w}, \ \#m/ft^2 sec \qquad \text{(Eq. 48)}$$

and likewise for pure oxygen bubbles

$$\left(\frac{\dot{M}_{O_2}}{A}\right)_{\substack{\text{into}\\ \text{diffuser}}} = \frac{5.56 \times 10^{-5} \gamma_{O_2} d^2 g (1 - \rho_{O_2}/\rho_w)}{\nu_w}, \ \#m/ft^2 sec \qquad \text{(Eq. 49)}$$

When Eq. 48 or Eq. 49 is multiplied by the appropriate expression for capture coefficient (and provided $f_{co} \lessapprox 1.0$), the expression for the rate of oxygen transfer by diffused aeration is given by the following equations for air bubbles with Reynolds numbers ≤ 1.0:

$$\left(\frac{\dot{M}_{O_2}}{A}\right)_{\substack{\text{into}\\ \text{water}}} = \frac{0.00161 \sqrt{g 1 - \rho_a/\rho_w) D_L H} \left[(C_s - C) \times 62.4 \times 10^{-6}\right]}{\sqrt{\nu_w}}, \ \#m_{O_2}/ft^2 sec \qquad \text{(Eq. 50)}$$

where D_L is in ft^2/sec

H is in ft

C_s, C are in mg/liter

g is in ft/sec

ν_w is in ft^2/sec

and likewise for pure oxygen bubbles:

$$\left(\frac{\dot{M}_{O_2}}{A}\right)_{\substack{\text{into}\\ \text{water}}} = \frac{0.00161 \sqrt{1 - \rho_{O_2}/\rho_w) D_L H} \left[(C_s - C) \times 62.4 \times 10^{-6}\right]}{\sqrt{\nu_w}}, \ \#m_{O_2}/ft^2 sec \qquad \text{(Eq. 51)}$$

It is interesting to note from Eq. 50 and Eq. 51 that the oxygen transfer rate per unit surface area does not depend on the bubble diameter if the fraction of volume occupied by the bubbles is to be held at some constant magnitude. However, the capture coefficient and hence the oxygen capture efficiency does depend on the bubble size as will be discussed later.

Although the two above equations for the transfer rate of oxygen to water from air bubbles and pure oxygen bubbles respectively appear very similar, again it should be pointed out that the value of C_s is a function of the partial pressure of oxygen in the bubble in accord with Henry's Law as given by Eq. 45. As a result if air and oxygen bubbles are generated at the same place and same size, the value of C_s for the pure oxygen bubbles will be 1/.21 times greater than for the air bubbles. Thus if all other factors were to remain equal, the oxygen transfer rate for pure oxygen bubbles should be some 4.7 times greater than for air bubbles generated at the same pressure and diameter.

Eq. 50 can be used to establish the order of magnitude of the maximum oxygen transfer per unit air-water interface for air bubbles at low Reynolds number flow. When this is done for C_s = 9 mg/liter and $C = 0$ at a depth of 2 feet, $(\dot{M}_{O_2}/A)_{max}$ is found to be 3.60×10^{-7} #m/ft^{2}sec. It is helpful to compare this value with the values given in Fig. 4.

The energy required to form a unit mass of air bubbles of uniform diameter, D, and at a depth, H, must be determined in order to establish the overall efficiency of the diffused aeration technique. The required energy will be composed of the energy necessary to increase the pressure to the hydrostatic pressure at the point of release, the pressure drop through the diffuser plug and the work required to create new water-gas surfaces. Of the three, the last can be neglected compared to the first two for bubbles of interest in this study. The second work term is a function of the gas flow rate through the diffuser plug and must go to zero as the flow rate goes to zero in the limit.

If the pressure required to force the gas through the plug and associated piping is taken as a constant value = ΔP_p, then the required energy per #m of oxygen for air bubbles based on 100% efficient adiabatic compression for air is

$$W_a = \frac{1}{.233} C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right], \ \#_f\text{-ft}/\#m_{O_2} \qquad \text{(Eq. 52)}$$

where P_1 = atmospheric pressure, $\#_f/\text{ft}^2$

$P_2 = \rho_w g H + P_1 + \Delta P_p$, $\#_f/\text{ft}^2$

$k = 1.4$

T_1 assume 530°R

C_p = specific heat of air, $\#_f$-ft/#m

W_a = work to compress air per pound of O_2, $\#_f\text{-ft}/\#m_{O_2}$

Thus

$$W_a = 426{,}000 \left[\left(\frac{\Delta P_p}{P_1} + 1 + \frac{H}{33.9}\right)^{.286} - 1\right], \quad \#_f\text{-ft}/\#m_{O_2} \qquad \text{(Eq. 53)}$$

The oxygen capture efficiency, η, is the reciprocal of the above expression multiplied by the capture coefficient, f_{co}, and compression efficiency:

$$\eta = \frac{2.34 \times 10^{-6} (f_{co})(\eta_{comp})}{\left[\frac{\Delta P_p}{P_1} + 1 + \frac{H}{33.9}\right]^{.286} - 1}, \quad \#m_{O_2}/\#_f\text{-ft} \qquad \text{(Eq. 54)}$$

or in the usual dimensions

$$\eta = \frac{4.65 f_{co} (\eta_{comp})}{\left[\frac{\Delta P_p}{P_1} + 1 + \frac{H}{33.9}\right]^{.286} - 1}, \quad \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. 55)}$$

It should be noted from Eq. 55 that it is essential to reduce the frictional pressure drop in the system (ΔP_p) to as low a value as possible in order to achieve high oxygen transfer efficiencies. For example, at a depth of 10 feet, a capture coefficient of 0.5 and a compressor efficiency of 80%, the capture efficiency given by Eq. 55 is 24.2 $\#m_{O_2}$/hp-hr for a zero ΔP_p, and 7.9 $\#m_{O_2}$/hp-hr for a 15 psi friction drop in the system.

If Eq. 44 is taken as the appropriate value of f_{cc} for low Reynolds numbers, then the oxygen capture efficiency becomes

$$\eta = \frac{644}{\left[\frac{\Delta P_p}{P_1} + 1 + \frac{H}{33.9}\right]^{.286} d^2} \sqrt{\frac{D_1 \nu_w H}{g(1 - \rho_a/\rho_w}} \left[\frac{(C_s - C) \times 62.4 \times 10^{-6}}{\gamma_a}\right] \qquad \text{(Eq. 56)}$$

Whipple et al [25] conducted extensive tests on a diffused aeration system placed in a seven-foot deep pool of a river and reported values of η from 0.62 to 1.28 $\#m_{O_2}$.hp-hr (referred to standard conditions of zero initial DO, 20°C) and a capture coefficient, f_{co}, in the range 0.0198 to 0.0415. Eq. 55 yields an efficiency 1.6 and 3.4 $\#m_{O_2}$/hp-hr for this depth at the measured capture coefficient of 0.0198 and 0.0415 respectively for an assumed frictional pressure drop of zero. The bubble size was not reported in this study but it can be assumed that no specific design was incorporated to produce very small bubbles.

Other diffusion experiments in which no attempt was made to efficiently produce small bubbles have resulted in similar oxygen transfer efficiencies of approximately 0.5 to 1.5 $\#m_{O_2}$/hp-hr for depths of 10 feet and referred to zero DO level. Zieminski, Vermillion and St. Leger [32], under controlled laboratory conditions demonstrated a maximum value of η = 5.9 $\#m_{O_2}$/hp-hr at a depth of 10 feet. The energy expended included the work to drive the device used to shear the bubbles off before they grew to the "natural" size. The size of the bubbles was not reported.

The Penberthy Company has stated that their jet aerators have operated at η = 4.85 $\#m_{O_2}$/hp-hr. In this device air is passed through large diameter nozzles (~ 5/8 in), mixed with pumped water in a swirl chamber and then the bubbles are finely divided and mixed as the air-water mixture is discharged through an exit diffuser into the main body of water at a depth H. The power includes both the power for the air compressors and the water pump.

Although air or oxygen bubbles will set up circulation of water as they rise from the bottom and as a result their influence on inducing atmospheric oxygen capture at the surface similar to mechanical sub-surface devices must be considered, it is known from experience with air-lift pumps [33] that even when the bubbles are forced to flow up through an optimum size vertical duct that the efficiency (ratio of water power to air power) of diffused air pumping is approximately only 50% which is substantially below the level that can be achieved with a mechanical pump. In addition it is known that the efficiency falls off rapidly as the duct diameter is increased beyond the optimum size.

For example, Bernhardt [34] reported on experimental work in the Wahnback Reservoir in Germany in which a duct was placed around and above the diffuser plug in order to aerate the hypolimnium without destroying the stratification. His duct was 6.6 ft in diameter, 70 ft long, had a measured flow rate of 67.9 cfs of water, and air was supplied by a 36.5 kw (40 hp) compressor. If the entrance and exit losses are assumed to each be equal to one velocity head and the frictional losses along the duct are accounted for, then the power required to produce this flow would be

$$P = \frac{\dot{m}}{550}\left[\frac{V^2}{2g_c} + h_f\left(\frac{1}{d}\right)\left(\frac{V^2}{2g_c}\right)\right] = .986 \text{ hp} \qquad \text{(Eq. 57)}$$

where $\dot{m}$ = mass flow rate, #m/sec

h_f = dimensionless friction factor = .012 [8]

g_c = the dimensional constant 32.2 #m-ft/$\#_f$-sec^2

or approximately 1 hp at 100% pump efficiency. To overcome this 1 hp requirement, he had to use a compressor input power of 49 hp.

Although the above example may be somewhat biased against diffused aeration as a pumping device because of the depth of the Wahnback (4), it does in general point out the order of magnitude argument. As a result it will be assumed that atmospheric oxygen capture at the atmosphere-water interface for diffused aeration can be neglected compared to the direct diffusion of oxygen from the rising bubbles.

Before considering the relative merits of mechanical and diffused aeration pumping when the body of water is stratified, it will be convenient for a future discussion to compare the power required to pump the water in the Wahnback Reservoir with the power of a mechanical pump of equal capacity. Because of the very low head on such a pump (~ .13 ft), a propeller type pump would be the most efficient. To the best of our knowledge, mechanical pumps with a propeller have not been built for pumping water at these very low heads (1 ft or less). However, from experience with marine propellers it would seem possible to construct one with an efficiency of at least 50%. If this were the case, the power required for a mechanical (propeller type) pump to do the same pumping as was accomplished with diffused air in the Wahnback Reservoir would be about 2 hp. It is interesting to note that the Aero-Hydraulic Gun (which is a high volume very low head positive displacement pump that utilizes a very large single air bubble (a foot or more in diameter) as a piston), would require approximately 10 hp to do the same pumping job [35].

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(4) As the water depth is increased, mechanical pumps will display an ever increasing advantage over diffused aeration devices in terms of their ability to pump and circulate water in view of the fact that as the water depth is increased, the air pressure must be increased accordingly, whereas the static head on the pump remains the same and only the frictional loss in the duct will increase, which in turn can be minimized by maintaining a low velocity in the duct.

If the body of water is initially stratified, the question of the destratification efficiency of a diffused aeration system vs a mechanical pump arises. Here it is not simply a question of which device is a more efficient pump for the amount of water entrained in the rising jet from the diffuser as a function of depth is an important consideration. Unlike the ducted mechanical pump which must depend on mixing of the cold water near the surface so as to produce a jet which will be buoyant at some level below the surface, the diffused aeration system will produce continual mixing from the diffuser plug upward. Symon et al [26] reported the measured destratification efficiency (DE) for diffused aeration, mechanical pumps and the Aero-Hydraulic Gun. A summary given by Symon is reproduced in Table VII for convenience.

As noted by Symon et al the DE shown in Table VII should not be itself be used to conclude that diffused aeration is a more efficient technique than mechanical pumps for destratification because the pump used in these experiments produced very high (8.4 ft/sec) duct velocities as noted previously.

Water Spray Aeration

In this technique the water is sprayed into the air where it captures atmospheric oxygen on the surface of the drop. The oxygen subsequently diffuses into the drop and eventually falls back to the main body of water.

If, as a first approximation, it is assumed that the turbulent transport within the drop of water is negligible, then all oxygen transport from the drop's surface to the interior must be by molecular diffusion. Under these conditions the oxygen transfer rate from the atmosphere will be given by Eq. 8.

$$\frac{\dot{M}_{O_2}}{A} = K_L(C_s - C) \qquad \text{(Eq. 8)}$$

where

$$K_L = \sqrt{\frac{D_L}{\pi t}} \qquad \text{(See Eq. 7 for note on units)}$$

Again since we are interested in the total oxygen accumulated by molecular diffusion in the drop after a residence time in the air, the appropriate "constant" K_L to be used is found by noting

TABLE VII

Comparison of Destratification Efficiencies of Various Studies (from Ref. 26)

Lake Reference	Dates of Mixing	Method	Volume acre-ft	Change in Stability Uncorr (U) Corr. (C) kw-hr	Total Energy Input kw-hr	Destratification Efficiency, DE Calculated over entire mixing period, %	Total Energy Input/Unit Volume Mixing kw-hr/ acre ft
Test	6/2-6/7	Diffused	2,930	27 (C)	1,760	1.5	0.6
Lake 1	6/21-6/24	air		23 (C)	1,680	1.4	0.6
	7/15-7/20	pump		33 (C)	2,200	1.5	0.7
	8/18-8/23			16 (C)	2,860	0.6	1.0
	1966					1.2+	0.7+
Test	(5/16-5/19,	Diffused					
Lake 2	5/20-5/23)	air	4,600	7 (C)	3,285	0.2	0.7
	6/10-6/15	pump		24 (C)	2,740	0.9	0.6
	7/1-7/8			35 (C)	3,740	0.9	0.8
	7/26-8/3			28 (C)	3,550	0.8	0.8
	9/8-9/13			8 (C)	2,540	0.3	0.5
	1966					0.6+	0.7+
Vesuvius	9/3-9/17/64	Mech.	1,260	6.7 (U)	1,862	0.4*	1.5
Boltz	8/6-9/10/65	Pump	2,930	28 (C)	14,300	0.2	4.9
King Geo.	7/13-8/17/66	"	18,375	23.8 (U)	32,640	0.1*	1.8
Wolford	4/18-4/25/62	Dif. Air	2,500	14.8 (U)	2,570	0.6*	1.0
Wahnbach	6/9-9/9/61	"	33,740	400 (U)	104,800	0.4*	3.1
Cox Hollow	6/30-7/18/66	Aefo-hyd Gun	1,190	7.9 (U)	2,140	0.4*	1.8
						0.5 (avg)	2 (avg)

*These data may be somewhat in error as the change in stability was not corrected for any natural change in stability that may have occurred during the mixing operation.

+Average for all mixings in 1966.

These data are intended as a rough guide only and are influenced by many factors: shape of lake, time of year, meteorologic condition (temperature, sunlight, wind)

$$\frac{\dot{M}_{O_2}}{A} = \int_0^{t_r} \frac{\dot{M}_{O_2}}{A}\,dt = 2\sqrt{\frac{D_L}{\pi}}\, t_r^{1/2}\,(C_s - C) \qquad \text{(Eq. 58)}$$

Thus the "constant" value of K_L is taken as

$$K_{L_{const.}} = 2\sqrt{\frac{D_L}{\pi t_r}} \qquad \text{(Eq. 59)}$$

The relation between the residence time (t_r) and the maximum height reached by the water above the air-water interface (h) can readily be found in the absence of drag as

$$t_r = 2^{3/2}\sqrt{\frac{h}{g}}\ , \text{ sec} \qquad \text{(Eq. 60)}$$

where h = spray height, ft

g = acceleration due to gravity, ft^2/sec

This residence time is independent of the angle of spray.

In the above discussion it has been assumed that the water leaves the nozzle as drops which is not the actual case. In operation the water leaves the nozzle as a sheet and then disintegrates into drops.

The bubble surface area generated per unit time is given by the expression

$$A = \frac{\dot{Q}_w\,(\pi d^2)}{\left(\frac{\pi d^3}{6}\right)}\ , \ ft^2 \qquad \text{(Eq. 62)}$$

where $\dot{Q}_w$ = water volume flow rate through spray nozzle, ft^3/sec

d = drop diameter

The above expression can be simplified to the following:

$$A = \frac{6 Q_w}{d} \text{ , } ft^2 \qquad \text{(Eq. 63)}$$

The oxygen transfer rate can now be found by combining Eq. 58, Eq. 60 and Eq. 63:

$$\dot{M}_{O_2} = \frac{11.38}{d} \; \frac{\dot{Q}_w D_L^{1/2} h^{1/4} \left[(C_s - C) \times 62.4 \times 10^{-6} \right]}{g^{1/4}} \text{ , } \#m_{O_2}/sec \qquad \text{(Eq. 64)}$$

where C_s, C are in mg/liter

For a given water flow rate through the spray nozzle Eq. 64 indicates that the oxygen transfer rate varies inversely with the drop diameter.

In order to create the spray, energy must be invested in creating new interfacial surfaces, imparting an initial velocity to the water and to overcome viscous losses in the equipment. The first of these required energies is equal to the product of the surface tension and the created area. It will be small compared to the second term unless the drops are made very small (of the order of 10 molecular diameters). The second term is readily accounted for; however, the third term depends on the flow rate and specific hardware and care must be taken to minimize this loss as it can be very high. As a first approximation it can be assumed that the flow rates are made low enough so that the viscous losses are negligible. Under this assumption the power required for a water volume flow rate of $\dot{Q}_w$ is given by the expression

$$P_w = \dot{Q}_w \rho_w g h \text{ , } \#_f\text{-ft/sec} \qquad \text{(Eq. 65)}$$

The oxygen transfer efficiency can now be found by dividing Eq. 64 by Eq. 65 to obtain

$$\eta = \frac{14.03 \, D_L^{1/2} \, (C_s - C)}{d h^{3/4} \rho_w g^{5/4}} \text{ , } \#m_{O_2}/hp\text{-}hr \qquad \text{(Eq. 66)}$$

where C_s, C are in mg/liter

D_L is in ft^2/sec

Although Eq. 66 has a tendency to underestimate the transfer efficiency because it does not account for the additional capture of oxygen that will take place due to the turbulence created at the area where the spray falls back into the main body of water, it also has a tendency to overestimate the capture efficiency because it does not include the energy necessary to overcome viscous losses in the spray nozzle and associated fittings.

The most common pressure nozzle for generating a spray is the "hollow-cone" nozzle which consists of a whirl chamber and an orifice. The water is introduced into the whirl chamber through tangential ducts and is thereby set into vigorous rotation. The water leaves the whirl chamber and exits through an orifice as a conical sheet which is subsequently broken up into drops by interacting with the air. Although the drop size and size distribution are a function of the specific hardware, it is noted in Ref. 36 that sprays of inductrial interest have drop sizes less than 1 mm (.039") in diameter and often less than .2mm.

The hollow-cone nozzle has been used at capacities up to 200 gallons/minute for spray cooling ponds. Based on Eq. 64 and Eq. 66 a nozzle of this capacity, with h = 10 feet, and producing drops of average diameter at the low end of the range or 0.20 mm, would induce an oxygen transfer rate of 0.17 $\#m_{O_2}$/hr at an oxygen capture efficiency of 0.33 $\#m_{O_2}$/hp-hr, and would consume 0.5 hp not including mechanical losses.

Lueck, Blabaum, Wiley and Wisniewski [37] reported the operating characteristics of a spray aeration unit in which the spray was produced by a propeller and nozzle as shown in Fig. 17. This machine was specifically designed for instream aeration and had a flow rate of 250,000 gph, a power of 19 hp, and produced a spray approximately 8 to 10 ft high and 35 ft in diameter. The authors reported an instream oxygen transfer rate from 4.6 to 0.2 $\#m_{O_2}$/hp-hr, at an initial DO level of less than 1 mg/liter and a temperature of about 26°C.(C_s = 8.0 mg/liter).

A comparison of the performance of this spray device and the results predicted by the analytical model used in this section can be made only if a drop diameter is assumed. Lueck et al indicated the droplets produced by the unit were "rather large in size" and they suggested that the device be redesigned so that "the spray can be atomized more." It is not possible to establish the approximate droplet size from this report, but if their implication of a "coarse" spray is taken to mean the mean between 1 and .2 mm diameter of industrial sprays, then the present model would predict an oxygen capture rate of 1.13 $\#m_{O_2}$/hr, a capture efficiency of 0.12 $\#m_{O_2}$/hp-hr and a total power of 9.4 hp without considering mechanical losses. As noted previously

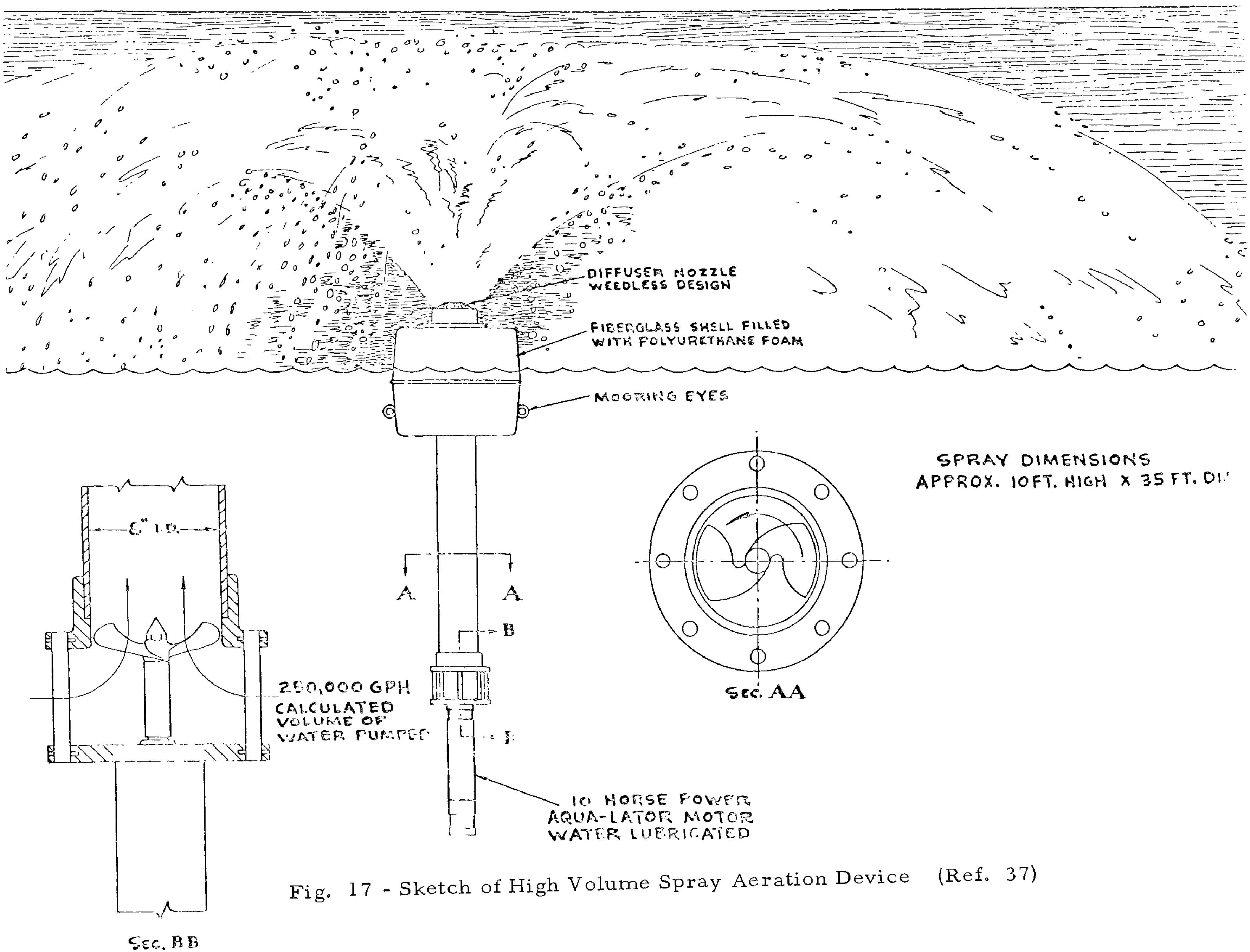


Fig. 17 - Sketch of High Volume Spray Aeration Device (Ref. 37)

the present analytical model does not account for the capture of oxygen that results from the turbulence induced at the area where the water falls back into the main body of water. This oxygen transfer can be substantial compared to the amount of oxygen transferred to the water while it is in the air. For example, in a later section on Weirs, Dams and Cascades it will be shown that for a carefully designed weir equal to the spray height in the previous example (9 feet) and operating with the same initial conditions, the measured oxygen transfer efficiency is about 1. 2 $\#m_{O_2}$/hp-hr.

Although the analytical model indicates that the spray aerator performance could be improved by reducing the diameter of the drops, the increase in energy required to do so would probably be prohibitively large. That the viscous power losses which have been neglected become large as the drop size is decreased from 1 mm can be seen from an estimate given in Ref. 36 in which it is noted that the viscous losses encountered in atomizing 1 gram of water in 1 second to 0. 001 mm diameter amount to a power of 100 to 10, 000 hp.

Because of the high power required to rapidly create small diameter bubbles and because diffusion rates are considerably higher in the initial stage after surface formation, it would seem to be more economical to create new surfaces than to create a fine spray with a long residence time. Such a step has been accomplished in a number of commercial surface aerators such as the Bird-Simplex High Intensity Aeration Process. This type of device is discussed in the next section and is classified as "White Water Generators."

White Water Generators

White water generators are designed to induce transfer of oxygen from the atmosphere to the water by rapidly exposing "new" water surfaces as a result of energetically agitating the air-water interface. In addition, they are usually equipped with a submerged pump or draft tube for the purpose of circulating the water. Circulation patterns established by sub-surface pumps have been previously discussed.

Unlike spray aeration devices considered in the previous section, white-water generaters are not designed to lift the water appreciably above the surface with the result that the residence time of the agitated water in the air is small. However, the amount of surface generated per unit time is high. For a fresh surface the rate of oxygen capture at any time is given by Eq. 8, or

$$\frac{\dot{M}_{O_2}}{A} = K_L (C_s - C) \qquad \text{(See Eq. 7 for note on units)}$$

where $K_L = \sqrt{\dfrac{D}{\pi t}}$

t = time elapsed from instant of surface generation

It should be noted that Eq. 8 is based on previously unexposed water surfaces whereas the machanical aerator used in a river or stream (with low O_2 uptake) is very likely to pick up water in its surface blades that has just been agitated and not yet transferred its oxygen to the main body of water. In addition it is quite difficult to estimate the active surface area produced by this device.

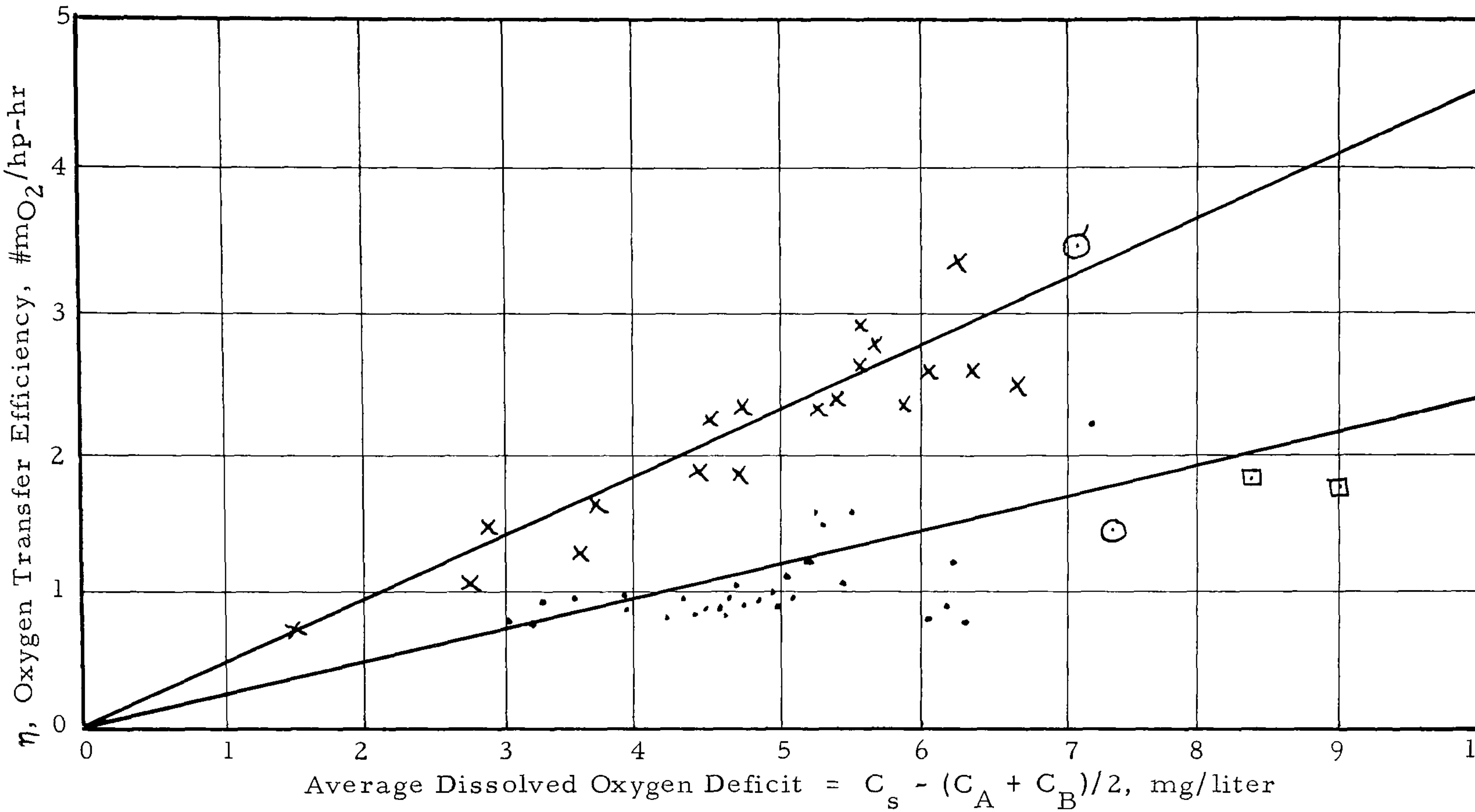
Because of the above considerations, it was considered not possible to develop a reliable analytical model of the white-water generators and as a result experimental results had to be used directly. From Eq. 8 it is noted that the oxygen transfer rate and hence oxygen transfer efficiency should increase linearly with the oxygen deficit, $(C_s - C)$. Oxygen transfer efficiencies measured in the laboratory and in the field are shown in Fig. 18. Each of the two major sets of data shown in Fig. 19 (University of Minnesota laboratory flow-through test and the field test on the Passaic River) show approximately the same linear trend for the efficiency as a function of DO deficit but the slopes of the line drawn through each set differ by about a factor of two with the slope for the flow-through laboratory test being higher than that for the river field test. In the case of the field test both Whipple et al [25] and Kaplousky et al [38] have demonstrated that the oxygen transfer efficiency increases substantially with increased river flow rates. For example, consider the addition of oxygen to the Passaic River during times of high flow compared to times of low flow as shown in Table VIII from Ref. 25.

A straight line drawn through the two major sets of data shown in Fig. 18 can be expressed as Eq. 67 and Eq. 68 for the flow-through and river test respectively.

$$\eta = 0.46\,(C_s - \bar{C}) \;, \quad \#m_{O_2}\cdot\text{hp-hr} \qquad \text{(Eq. 67)}$$

where C_s, $\bar{C}$ are in mg/liter

$$\eta = 0.24\,(C_s - \bar{C}) \;, \quad \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. 68)}$$



Chicago Canal (High River Flow Rate), Ref. 38

Passaic River, Ref. 25

Univ. of Minn., Flow-through Laboratory Test, Ref. 15

Aerated Lagoons, Ref. 39

C_A, DO above aerator; C_B, DO below aerator; η = Efficiency based on Shaft HP

Fig. 18 - Comparson of Various Measured Oxygen Transfer Efficiencies for White-water Generators

TABLE VIII

SUMMARY OF STEADY-STATE FIELD TEST DATA FOR MECHANICAL AERATOR - (From Ref. 25)

Date	Flow Q (CFS)	Water Temp. (°C)	Shaft Power (HP)	Dissolved Oxygen (ppm) Upstream	Dissolved Oxygen (ppm) Downstream	Oxygen Added (lbs.O_2/hr.)
8/9/67	1620	23.5	90.0	1.12	1.58	167.4
8/31	520	22.0	83.4	2.50	3.35	99.3
7/9/68	159	24.5	79.5	4.60	6.00	50.0
7/10	147	24.0	84.1	3.60	4.45	34.4
7/12	133	25.0	71.7	4.10	6.43	69.6
7/16	111	26.0	80.8	3.00	5.94	73.3
7/16	110	28.6	78.2	3.20	5.35	53.1
7/18	134	27.5	82.4	2.70	4.30	48.2
7/18	128	28.9	77.1	2.90	4.85	56.1
7/23	113	29.4	76.6	2.60	5.00	60.9
7/24	110	26.2	75.2	1.90	4.10	54.4
7/26	125	25.4	71.6	1.20	4.00	78.6
7/29	92	24.8	74.6	2.20	5.10	59.9
7/30	91	23.0	85.7	3.00	6.00	61.3
7/31	93	24.0	85.8	2.10	5.00	60.6
8/1	93	26.0	80.7	2.40	4.50	43.9
8/6	106	25.0	88.3	1.50	4.50	71.4
8/7	110	26.8	81.0	1.70	5.20	86.5
8/7	104	25.0	82.2	1.40	4.70	77.1
8/9	99	26.0	79.3	0.60	4.50	86.7
8/9	98	27.0	78.0	1.60	4.50	63.8
8/22	85	24.9	81.6	1.40	4.50	59.2
8/27	130	23.0	73.1	1.60	3.50	55.5

* DO samples taken 1000 ft. upstream, and 2000 ft. downstream of the aerator.

Weirs, Dams and Cascades

It can be anticipated that the highly turbulent flow at the base of a weir, dam or cascade effectively generates "new" water surfaces and hence induces oxygen transfer from the atmosphere by surface capture. The effectiveness of such flows in aerating the water has been documented by several works. Gameson [40] in an extensive field study on weirs in England showed experimentally that the increase in DO across a free weir (that is, one in which the water falls freely and does not adhere to the face of the structure) was given by the expression

$$\frac{(C_s - C_A)}{(C_s - C_B)} = 1 + 0.152\, a_1 h \qquad \text{(Eq. 69)}$$

where C_s = saturation DO, mg/liter

C_A = DO above weir, mg/liter

C_B = DO below weir, mg/liter

a_1 = an experimental coefficient
= 0.85 for sewage
= 1.00 moderately polluted water
= 1.25 slightly polluted water

h = height of fall, ft

In the experimental study Gameson demonstrated that the head on the weir did not influence the change in DO across the weir (at least for a head in the range 6-13 inches) and that the majority of the oxygen transfer took place at the splash area and not in the falling water. In addition, the experimental work indicated that turbulent sloping channels provide less aeration than weirs for the same total head loss.

Cameron, Van Dyke and Ogden [41] extended the field work of Gameson with laboratory experiments on a weir where the water temperature could be controlled. They showed that Eq. 69 could be expressed as given below so as to include the influence of temperature.

$$\frac{(C_s - C_A)}{(C_s - C_B)} = 1 + 0.11\, a_1 h (1 + 0.046T) \qquad \text{(Eq. 70)}$$

where T = water temperature in ^{o}C

Gannon [42] found good agreement between the change in DO predicted by Eq. 70 and the measured values for treated domestic waste passing over a 3.1 foot weir.

If a natural drop occurs in a stream or if such a drop can be created by a permanent structure, then no additional power would be required to achieve the increase in DO given by Eq. 70 for flow over a weir. If on the other hand the water must be raised, the energy to do this may be estimated by neglecting viscous losses and kinetic energy losses compared to the energy required to elevate the water. When this is done, the oxygen transfer efficiency for weirs (4) is found to be:

$$\eta = \frac{1.98}{h}\left[1 - \frac{1}{1 + .11\,ah(1 + .046T)}\right](C_s - C_A) \quad , \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. 71)}$$

where C_s, C_A are in mg/liter

h is in ft

T is in $^{\circ}C$

For example, if the water temperature is $20^{\circ}C$, C_A is zero, and h = 10; then $\eta = 1.30\ \#m_{O_2}/\text{hp-hr}$ and $(C_s - C_A)/(C_s - C_B) = 3.64$

Measurements have been made of the DO above and below a number of dams in the northeast United States [43]. Table IX shows a comparison of the predicted oxygen capture efficiency as given by Eq. 71 for weirs and measured values for flow over dams as given in Ref. 43.

TABLE IX

Comparison of Measured and Calculated Oxygen Transfer Efficiency for Dams

h (ft)	Temp ($^{\circ}C$)	η, $\dot{M}_{O_2}$/hp-hr: $\eta = \dot{M}_{O_2}(\text{measured})/Q\gamma h$	η = Eq. 71
4	20.5	0.95	2.30
5	20.5	0.73	2.02
5	22.5	0.88	1.98
6	20.5	1.20	1.82
8	22.0	1.35	1.48
8	22.5	1.37	1.46
9	27.5	0.92	1.25
11	19.0	0.95	1.22
20	27.0	0.74	0.67

(4) If the water must be pumped to raise its elevation, it can readily be done with a propeller type fan and then allowed to splash back down on an apron. Such a device will be referred to as a lift-drop aerator.

Note: In Ref. 43 the initial DO is said to always be less than 2 mg/liter and predominantly zero. In Table IX the DO is assumed to be zero in all cases.

Although the transfer efficiency predicted for free weir fall by Eq. 71 yields results that are more than a factor of 2 higher than the measured values for the 4- and 5-foot dams, it must be noted that the dams studied were studied in an "as found" condition which may have been far from the optimum condition for the effective aeration on which Eq. 71 is based.

The influence of the depth of the water at the base of the dam can readily be seen by comparing two experimental programs discussed in Ref. 43. In the first program water passing over dams on the Mohawk River and splashing into relatively deep water at the base was studied. When the results obtained for dams up to 15 feet high were extrapolated, it was found that a dam of about 30 feet would saturate zero DO water passing over it. In the second study a number of dams in New England with water splashing onto relatively shallow water or stone or concrete aprons were studied. When this data was extrapolated, it was found that a dam of only approximately 18.5 feet was needed to saturate zero DO water.

Although some uncertainties exist in the available data, it appears that Eq. 71 can be used as a valid model for calculating the oxygen transfer efficiency for weirs and dams provided that the structure is such that the water falls free rather than adhering to the face of the structure and provided that the water is allowed to fall on shallow water or a masonry apron.

A comparison of Eq. 70 and Eq. 71 reveals that for the same total elevation drop (h) a series of weirs (that is, a cascade) will produce a more efficient oxygen transfer than a single weir. For example, using water with an initial DO level of zero, a single weir with $h = 9$ feet, $T = 20^oC$, will increase the DO to 6.32 mg/liter at an efficiency of 1.39 $\#m_{O_2}$/hp-hr, whereas a cascade of three 3-foot weirs would increase the DO to 7.42 mg/liter at an efficiency of 1.63 $\#m_{O_2}$/hp-hr.

Hydraulic Turbine Aeration by Venting

Air may be made to flow into the exhaust end of a hydraulic power turbine by simply cutting a hole through the casing and installing a control valve provided that this section of the turbine has been designed to operate at pressures below one atmosphere. When air is allowed to enter the turbine, the water flow rate and power (as well as power per unit water flow rate) decrease. The maximum oxygen transfer rates were estimated in Ref. 44 to correspond to ΔDO of about 1.8 mg/liter and the oxygen transfer efficiency was reported to be approximately 2.6 to 4.1 $\#m_{O_2}$/hp-hr.

This technique is limited to select sites. The cost of modifying existing turbines will depend considerably on the type of machine. This technique, like diffused aeration, does offer the possibility of injecting pure oxygen rather than air.

U-Tube Aeration

The U-Tube aeration method is an oxygen transfer process introduced in the Netherlands and consists of inducing or injecting air into water as it starts down one leg of pipe formed into a U shape. The arrangement has two features which enhance the transfer of O_2. First, the water passing down the U-tube drags the air bubbles along (if the water velocity is high enough), thus increasing the bubble residence time. Second, as the bubbles pass down the tube, the pressure increases and C_s increases according to Henry's Law.

Speece, Adams and Wooldridge [45] have presented considerable experimental data on the operation of U-tubes up to 40 feet deep and 60 inches in diameter. They showed that water could be completely saturated with oxygen (at a total pressure of 1 atm) by use of these tubes and that the oxygen transfer efficiency for water with an initial DO of about zero could be made as high as 3.7 $\#m_{O_2}$/hp-hr for 40 foot deep tubes and as high as 1.8 $\#m_{O_2}$/hp-hr for 10 foot deep tubes.

Brush Aerators

Brush aeration, which consists of a rapidly rotating cylindrical brush that projects water across the surface, have been widely used in sewage treatment tanks in Europe. Under the high oxygen uptake conditions that exist, measured oxygen transfer rates of about 3.0 $\#m_{O_2}$/hp-hr have been reported for low DO water [46]. It should be noted that in the sewage treatment tanks, which are usually made about 10 to 15 feet deep and 15 to 30 feet wide, the brushes are able to establish a circular flow pattern in order to continually aerate "new" water. However, in a natural body of water it is unlikely that any substantial circulation pattern will be developed by the brushes because of the lack of side walls in close proximity to the brush. As a result the oxygen transfer efficiency for these devices would probably decrease considerably when they are placed in natural bodies of water.

VII. OPTIMUM ECONOMIC SELECTION

Streams

When considering the use of supplemental aeration to prevent the development of critically low levels of dissolved oxygen in the most economical way, it must be noted that for any type of aeration equipment the oxygen transfer rate and oxygen transfer efficiency depend directly on the oxygen deficit. As a result, it will be most economical to add the oxygen when the DO has fallen to the lowest acceptable level and in addition it will economical to increase the DO to some value less than saturation at a given location. As the water flows away from the aeration station its DO will again decrease if the organic load in the water is high enough to cause bacteria to consume more oxygen than is replenished by natural aeration. If this is the case, a second aeration station will have to be installed when the DO again falls to its lowest acceptable level. The process will have to be repeated until the organic material in the water is consumed. Whipple [25] has demonstrated the use of a computer simulation technique for determining the number and spacing of supplemental aeration stations required for a given river flow and organic loading. The lowest acceptable DO level is usually taken as 4 mg/liter.

The economic selection will be based on the lowest combined yearly capital cost and operating cost. The capital cost will be reduced to a yearly value by assuming an equipment lifetime and an interest rate for the cost of money used to purchase the equipment.

Although the optimum selection of aeration equipment will be influenced by the characteristics of the particular stream, the procedure can be demonstrated by looking at one aeration station where the DO level has reached its minimum acceptable level of 4 mg/liter and it is desired to increase the DO to 6 mg/liter. As an example we shall select a 200-foot wide river, 4 feet deep, moving at a slow velocity of 0.05 ft/sec and at a temperature of 20°C. From Fig. 4 it is seen that the natural aeration rate for this example is given by Case C (See Eq. 18). The volume flow of water and the required supplemental oxygen transfer rate for this example are 40 cfs and 18 $\#m_{O_2}$/hr respectively.

In searching for the economic optimum equipment, all types of aeration equipment discussed in the last section will be considered in order starting with sub-surface devices.

Sub-surface Devices

Since the water is shallow, the most efficient of the two shallow water configurations will be selected for evaluation, namely the large diameter ducted propeller previously listed as Case II. The oxygen transfer and transfer efficiency for Case II are given by Eq. 23 times 6H, the effective width (See Fig. 8), and Eq. 24 respectively, or

$$\dot{M}_{O_2} = (.007)(C_s - C)V^{1/2}H^{1/2}(6H) \quad \#m_{O_2}/hr \qquad \text{(Eq. 72)}$$

$$\eta = 0.657\,(C_s -)V^{-5/2}H^{-1/2} \quad \#m_{O_2}/hp\text{-}hr \qquad \text{(Eq. 24)}$$

where C_s, C are in mg/liter

H is in ft

V is in ft/sec

Since the effective width of a Case II device has been estimated at 10H or 40 feet, a minimum of five such devices would have to placed across the stream to aerate all the flow simultaneously. Each unit would have to add 3.60 $\#m_{O_2}/hr$. However, if the number of units is selected at too low a value, the duct velocity will have to be high in order to achieve the necessary oxygen transfer with the result that the transfer efficiency will be prohibitively low.

In addition to the power required to pump against the total hydraulic head on the propeller, power will be required to overcome friction in the bearings of the propeller. If this frictional loss is assumed to be a constant 0.1 hp, then the overall transfer efficiency becomes

$$\eta_T = \frac{\eta_o\,\eta_p}{\left[\dfrac{1}{.657(C_s - C)V^{-5/2}H^{-1/2}} + \dfrac{0.1\ hp}{\dot{M}_{O_2}}\right]} \qquad \text{(Eq. 73)}$$

where η_o = efficiency of device used to drive sub-surface aerator shaft, assume 80%

η_p = propeller efficiency, assume 50%

Although the efficiency based on shaft power will increase as the power per unit decreases, the frictional loss becomes more important as the power per unit is decreased. As a result there will be an optimum number of units that will make the overall efficiency maximum as shown in Table X.

TABLE X

Total Power Variation with Number of Sub-surface Units

Number of units	V ft/sec	η_T	P_{unit} hp	P_{total} hp
5	7.19	.00387	930	4650
10	1.79	.120	15	150
15	.79	.785	1.5	22.5
20	.44	1.89	.48	9.6
25	.29	2.31	.31	7.8
30	.20	2.22	.27	8.1

From Table X the optimum number of units can be taken as 25. The capital cost of each unit was estimated to be $1334 in Appendix C. The The total initial or capital costs are itemized below.

Cost of Aeration Units @ $1334 each	$33,350
Cost of Hydraulic Pump and Drive	1,000
Cost of Hydraulic Piping	1,000
Installation Cost of Aeration Units including anchors (@ $50 each)	1,250
Electrical Power Supply (based on 500 ft to nearest utility wire)	1,500
Site Preparation	2,000
Shelter	1,000
Sub-Total	41,100
Contingencies and Engineering (@ 20%)	8,220
Total Initial Cost	$49,320

The estimated annual operating costs are listed below.

Electric Power (@ .01/kw-hr, for 168 full days(5)) = (7.8)(168)(24)(.745)(.01)	234
Maintenance @ 1% of initial cost = (.01)(.8 x 49,320)	384

(5) Based on the work of Whipple [25], it is assumed that the aeration units will run for three months at 24 hours/day and for five months at 12 hours/day.

Personnel @ 4 hours a week for 32 weeks, \$10/hr = (128)(10.00)	\$ 1,280
Total Annual Operating Cost	\$ 1,898

The total annual cost can now be estimated by taking a basic interest rate of 8%. For a life of 20 years, this results in a capital recovery rate (interest plus amortization) of 10.185% and a capital cost per year of .10185 x \$49,320 or \$5,010. Thus the total annual cost is

Capital cost per year	\$ 5,010
Annual Operating Cost	1,898
Total Annual Cost	\$ 6,908

Since the annual operating cost is smaller than the annual capital cost, it is helpful to determine the total annual cost when the number of subsurface units is decreased. For example, if the number of units is decreased from 25 to 15, the initial capital cost will be

Cost of Aeration Units (@ \$1334)	\$20,000
Cost of Hydraulic Pump and Drive	900
Cost of Hydraulic Piping	600
Installation cost of Aeration Units (@ \$60 each)	900
Electrical Power Supply (based on 500 ft to nearest utility wire)	1,500
Site Preparation	2,000
Shelter	1,000
Subtotal	26,000
Contingencies and Engineering (@ 20%)	5,360
Total initial cost	\$32,169

The estimated annual operating costs are listed below:

Electric Power (@ %.01/kw-hr, for 168 full days)	685
Maintenance (@ 1% of initial cost)	258
Personnel (@ 4 hrs/wk for 32 weeks, \$10/hr)	1,280
Total Annual Operating Cost	2,223

For an interest rate of 8% and a life expectancy of 20 years, the total annual cost will be the sum of \$3,270 and \$2,223 or \$5,493.

It should be noted from Table X that a further decrease from 15 to 10 units will result in a substantial increase in the power and cost.

Diffused Aeration Systems

For completeness both air bubbles and pure oxygen bubbles will be evaluated.

Air Bubbles

In order not to possibly omit the optimum aeration equipment, three values of oxygen transfer efficiency will be used to evaluate the economics of this method. First, the most conservative estimate would be to select the average of the measured values reported by Whipple et al [25], namely, 0.85 $\#m_{O_2}$/hp-hr at standard conditions or 0.376 $\#m_{O_2}$/hp-hr for the average condition of the present example. Second, the value of 4.85 $\#m_{O_2}$/hp-hr reported by the Penberthy Co. for jet aerators at standard conditions or 2.17 $\#m_{O_2}$/hp-hr for the average condition of this example. Finally, the most optimistic of the three, a value of 7.8 $\#m_{O_2}$/hp-hr calculated by means of Eq. 55 for standard conditions and a hypothetical system capable of producing small enough bubbles (Reynolds number ~ 1.0) so that the capture coefficient (f_{co}) can be assumed to be as high as 1/2 and the total frictional pressure drop in the system as low as 15 psi. The above value corresponds to 3.52 $\#m_{O_2}$/hp-hr for the conditions of the present problem. Since the above three efficiencies do not take into account the efficiency of the device that drives the compressor, they must be revised. The revised values of η are tabulated below on the assumption of an electric drive of 90% efficiency together with the total input power to raise the DO from 4 to 6 mg/liter.

TABLE XI

Oxygen Transfer Efficiencies and Power for Diffused Aerators

		η $\#m_{O_2}$/hp-hr	P_{hp}
Case 1	Measured in Stream	0.339	53.2
Case 2	Penberthy	1.943	9.25
Case 3	$f_{co} = 1/2$, $\Delta P_p = 15$ psi	3.17	5.68

The accuracy of a cost estimate for an aeration system can be enhanced by experience with on-site operation. Whipple et al [25] made cost estimates for diffused and mechanical aeration in a river with characteristics similar to the present example in 1969. It is felt that their estimates would be more reliable than independent ones not based on actual operating experience. Based on Whipple's data for an 80 hp electrically driven air blower with an initial capital cost of $51,000, the capital cost for Case 1, 2 and 3 would be $34,000, $8,850(6) and $3,610 respectively.

The estimated annual operating costs are listed below.

	Case 1	Case 2	Case 3
Electric Power			
(@ $.01 kw-hr for 168 full days)	$1,600	$ 277	$ 171
Maintenance			
(@ 3% initial cost)	1,020	177	110
Personnel			
(@ 12 hours/week, 32 weeks) $10/hr	3,840	3,840	3,840
Total Annual Operating Cost	$6,460	$4,294	$4,121

If the life time of this equipment is assumed to be 10 years and the interest rate is again taken at 8%, the capital recovery rate becomes 14.09%. Based on the above, the total annual cost is computed below.

	Case 1	Case 2	Case 3
Capital Cost per Year	$4,790	$1,248	$ 509
Annual Operating Cost	6,460	5,542	4,630
Total Annual Cost	$11,250	$5,542	$4,630

Pure Oxygen Bubbles

Again in order to cover the entire possible range, two values of the capture coefficient will be evaluated for diffusion of pure oxygen bubbles. First, the most conservative value will be that measured by Ippen and Carver [29] for a depth of four feet and bubble size of 1.50 mm, namely $f_{co} = .13$ for standard conditions or 0.0578 for the present condition. Second, a value of $f_{co} = .50$ for smaller bubbles if they could be generated at standard conditions or .222 for the conditions in the present problem.

In order to make cost estimates, it will be assumed that the oxygen is stored under pressure and that no additional work need be done on it to cause it to pass through the diffuser system. Under this assumption the capital cost will consist only of an oxygen storage tank, gas

(6) Estimated value for air blowers alone increased by 50% to account for the fact that the energy input is by a combination of air blowers and water pumps.

pressure regulating system, the diffuser plugs and associated piping. This capital cost is estimated below.

	Case 1 $f_{co} = .0578$	Case 2 $f_{co} = .222$
Cost of O_2 storage tank (a 3-day supply)	$19,900	$ 5,700
Cost of O_2 pressure regulator	400	400
Cost of piping and diffuser plugs	1,000	1,000
Installation cost of diffuser unit	500	500
Site preparation	2,000	2,000
Shelter	1,000	1,000
Sub-total	24,800	10,600
Contingencies and Engineering @ 20%	4,960	2,120
Total Initial Cost	$29,760	$12,120

The estimated annual operating costs are listed below.

Maintenance (@ 1% of initial cost)	200	87
Personnel (@ 8 hours/week for 32 weeks, $10/hr)	2,560	2,560
Cost of pure oxygen (@ $35/ton delivered, 3320 #m/day for Case 1 and 863 #m/day for Case 2, 168 full days)	21,900	5,700
Total Annual Operating Cost	$24,660	$ 8,347

If it assumed that this equipment has a life-time of 20 years and the same interest rate of 8% is again used, the capital recovery rate will be 10.185%. Based on this rate the total annual costs are given below.

Capital Cost per year	$ 3,030	$ 1,235
Annual Operating cost	24,660	8,347
Total Annual Cost	$27.690	$ 9,582

Water Spray Aeration and White Water Generation

In view of the consideration that for the same horsepower a spray aerator and a white water aerator will have virtually the same initial capital cost as well as the same life and maintenance cost, they will differ substantially only as a result of their different oxygen transfer efficiency, which is lower for the spray units that have been tested as previously noted.

In order to cover a wide range of operation in the search for optimum equipment, three values of the transfer efficiency will be evaluated, one for spray devices and two for white-water generators. The value for spray devices will be taken as the mean of the measured range reported by Lueck et al [37], namely, 0.67 #m_{O_2}/hp-hr at standard conditions or 0.298 #m_{O_2}/hp-hr for the conditions of the present example. The two values selected for white-water generators are the ones given by Eq. 68 and Eq. 67 (and shown in Fig. 18) for the river test reported by Whipple et al [25] and the University of Minnesota flow-through laboratory test [15] respectively. These two values are 2.16 and 4.14 at standard conditions or 0.96 and 1.84 for the present conditions respectively. Since the above efficiencies are based on shaft power, they must be corrected to include the efficiency of the drive unit. Based on the assumption that the drive is a directly coupled electric motor with an efficiency of 90%, the corrected transfer efficiency and the total required input power is given below for the three cases.

TABLE XII

Oxygen Transfer Efficiency and Power for Spray Aerator and White-Water Generators

	Case 1 Spray Aerator	Case 2 River Test	Case 3 Flow Through Test
Overall oxygen transfer efficiency #m_{O_2}/hp-hr	0.268	0.865	1.66
Total required input horsepower to increase DO from 4 to 6 mg/liter	67.2	20.8	10.8

As in the case of capital cost for diffused aeration, the capital cost for spray aerators and white-water generators will be based on Whipple's capital cost data for white-water generators, namely, $44,000 per 75 hp electric drive unit. Based on the above the initial capital cost for Cases 1, 2 and 3 is $39,500, $12,000 and $6,350 respectively.

The estimated annual operating costs are listed below.

	Case 1 Spray	Case 2 River Test	Case 3 Flow Through Test
Electric Power (@ $. 01 kw-hr for 168 full days)	$2, 020	$ 624	$ 324
Maintenance (@ 3% of initial cost)	950	293	153
Personnel (@ 4 hours/week, 32 weeks @ $10/hr)	1, 280	1, 280	1, 280
Total Annual Operating Cost	$4, 250	$2, 197	$1, 757

If it is assumed that this equipment has a life-time of 10 years and the interest rate is 8%, the capital recovery rate is 14. 09%. Based on this rate, the total annual costs are given below.

Capital cost per year	5, 560	1, 720	895
Annual operating cost	4, 250	2, 197	1, 757
Total Annual Cost	$10, 810	$ 3, 917	$ 2, 652

Weirs, Dams and Cascades

If a natural drop in the river is available for the construction of a dam, then there will be no operating cost for energy and the cost of such aeration reduces to the initial capital cost of construction and the annual maintenance cost. On the other hand, if no natural drop in elevation is available, the water can be pumped to a higher elevation and allowed to splash down on a masonry apron or pad - a "lift-drop" aerator. The capital cost of such "lift-drop" aerators will be virtually the same as for spray and white-water generators and power will of course have to be supplied. The total annual cost of each of these two systems (natural drop and lift-drop) is estimated below.

Natural Drop

The height of the drop that must be available in order to increase the DO from 4 to 6 mg/liter can be calculated from Eq. 70. When this is done, the drop is found to be 2. 53 ft.

The initial capital costs are given below for the case where a natural drop is available in the stream for the construction of a dam.

Cost of dam material (concrete at \$20/cu yd for a 200' x 8' x 4' dam)	\$ 4,740
Cost of forms and pouring concrete (assumed equal to material cost)	4,740
Excavation and site preparation (@ \$10/cu yd, 200' x 50' x 4')	14,850
Cost of temporary coffer dams	4,000
	28,330
Contingencies and Engineering @ 20%	5,660
Total Initial Cost	\$33,990

The estimated annual operating costs are listed below.

Maintenance (@ 1% of initial cost)	283
Personnel (@ 4 hours every two weeks @ \$10/hr for 32 weeks)	640
Total Annual Operating Cost	923

If the life-time of this equipment is taken as 40 years with an interest rate of 8%, the capital recovery rate will be .0838. Thus the total annual cost will be

Capital cost per year	\$ 2,850
Annual operating cost	923
Total annual cost	\$ 3,773

Lift-Drop Aerator

If no natural drop in elevation is available, then the water can be pumped and allowed to splash back onto an apron. The efficiency for this operation can be estimated from Eq. 71. For a drop of 2.53 feet, a value of $\eta = 1.565$ $\#m_{O_2}$/hp-hr is obtained. However, in order that a fair comparison can be made between this type device and other devices operating in a stream where some of the water will be recirculated through the device before it is mixed with the main body (for example, a comparison with white-water generators), this value must be reduced by some factor to account for recirculation. From Eq. 67 and Eq. 68 or from Fig. 18 this factor is seen to be approximately 2.0 for white-water generators. Since the two types of devices are similar, a factor of 2 can be assumed for this case also.

For the purpose of evaluating the economics, two values of η will be used to represent devices in which recirculation is and is not inhibited, namely, 1.565 and 0.782 #m/hp-hr respectively. These two values correspond to a total input power of 12.8 and 25.6 hp respectively if the drive efficiency is taken as 90%.

Assuming the capital cost for the Lift-Drop device is the same as for white-water generators, the capital cost will be $7,500 and $15,000 respectively for the two values of η.

The estimated annual operating costs are listed below for the two cases.

	Case I $\eta = 1.565$	Case II $\eta = 0.782$
Electric Power (@ $0.01/kw-hr for 168 full days)	$ 385	$ 769
Maintenance (@ 3% of initial cost)	180	360
Personnel (24 hrs/wk, 32 weeks, @ $10/hr)	1,280	1,280
Total Annual Operating Cost	1,845	2,409

If this equipment is assumed to have a life-time of 10 years and the interest rate is 8%, then the annual capital cost will be $1,055 and $2,110 for the two cases. The annual cost is given below.

Capital Cost per year	1,055	2,110
Annual Operating Cost	1,845	2,409
Total Annual Cost	$2,900	$4,519

Hydraulic Turbine Aeration be Venting

The initial capital cost of providing a vent for a new turbine at the design stage is insignificant. However, the cost of providing a vent in existing equipment may run into several thousand dollars. In addition to the vent, an air flow control must be provided.

If the initial capital cost is taken at $1000 to cover the control system and $3000 for a vent in existing equipment, and if the equipment life is assumed to be 20 years, then the annual capital cost will be $406 for an interest rate of 8%.

The total loss in electric power produced by the turbulence because of air venting may be estimated by taking the mean value of η reported

Lakes

When the supplemental aeration of lakes is to be evaluated for optimum economics, the same procedure that was used for streams can be employed. However, two additional factors must be considered. First, whether the lake is initially stratified or not; second, the time span in which the DO level must be improved from its initial value to the final desired value.

It is helpful to recall a few previously discussed points concerning the flow patterns induced in lakes. If the lake is stratified with high DO water in the epilimnion and low DO water in the hypolimnion and no appreciable organic load remains in the hypolimnion, then it might suffice to simply mix the water vertically. It has been shown that this can be done from a single location with one pump.

If on the other hand it is necessary to continue to add O_2 after the water has been mixed vertically, the influence of even the most effective circulator (the large diameter ducted propellers) can only circulate the water within a cylindrical "cell" equal in depth to the water depth and in diameter to about four times the depth for a deep lake.

The economic selection of equipment will be demonstrated by an example in which a 100-foot deep stratified lake with a thermocline at a depth of 20 feet, a DO level in the epilimnion of 6 mg/liter and a DO in the hypolimnion of 4 mg/liter is to be raised to a DO level of 6 mg/liter everywhere. Thus the amount of oxygen that must be added per acre of surface is 435 $\#m_{O_2}$. If the time in which the DO increase must take place is assumed to be three months, then this corresponds to 0.198 $\#m_{O_2}$/hr per acre. If the lake surface area is taken at 100 acres, then the total oxygen flow rate will be 19.8 $\#m_{O_2}$/hr.

A sub-surface aerator, a diffused aeration system with air bubbles, and a hybrid system consisting of a white-water generator and a ducted propeller will be evaluated

Sub-Surface Aerators

Since the water is deep, the ducted propeller with diameter = .2H previously listed as Case I will be selected as the sub-surface device. The oxygen transfer and transfer efficiency for Case I are given by Eq. 21B times 4H and Eq. 22B respectively, or

$$\dot{M}_{O_2} = 47.4 \times 10^{-6} (C_s - C)\, V^{1/2} H^{1/2} (4H) \;\; \#m_{O_2}/hr \qquad \text{(Eq. 21B)}$$

$$\eta = 0.134\, (C_s - C)\, H^{-1/2} V^{-5/2} \;\; \#m_{O_2}/hp\text{-}hr \qquad \text{(Eq. 22B)}$$

where C_s, C are in mg/liter

H is in ft

V is in ft/sec

The last equation, however, must be corrected for drive efficiency, propeller efficiency and friction loss in the bearings similar to the manner used to develop Eq. 73. Thus the expression for overall effic iency becomes

$$\eta_T = \frac{\eta_d \; \eta_p}{\left[\dfrac{1}{.134 (C_s - C)\, V^{-5/2} H^{-1/2}} + \dfrac{0.1 \text{ hp}}{\dot{M}_{O_2}} \right]} \qquad \text{(Eq. 74)}$$

where η_d = efficiency of device used to drive sub-surface aeration shaft, assume 80%

η_p = propeller efficiency, assume 50%

Since the effective diameter of a Case I unit has been estimated at 4H ft, a minimum of approximately 34 units will be required for continuous aeration (that is, to continue to circulate all the water after destratification). At this number of units, each unit would have to add 0.582 $\#m_{O_2}/hr$. For this transfer rate the duct velocity and oxygen transfer efficiency are found to be 0.77 ft/sec and 0.04 $\#m_{O_2}/hp\text{-}hr$ respectively. The total power required is thus equal to 19.87/.04 or 494 hp.

The initial capital cost and annual operating costs are given below.

Initial Capital Cost	
Cost of aeration units @ $2844/ea, See App. C)	$ 96,500
Cost of hydraulic pump and drive	20,000
Cost of hydraulic piping (based on 2,090' x 2,090' surface area of 7 x 2,090 @ $1.00/ft)	14,630
Installation cost of aeration units Including anchor (@ $100/ea)	3,400

Electric power supply (based on 500' to nearest utility wire	$ 1,500
Site preparation	2,000
Shelter	1,000
	119,030
Contingencies and Engineering @ 20%	23,806
Total Initial Cost	$142,836
Annual Operating Cost	
Electric power (@ $0.01/kw-hr, 91.5 full days)	8,900
Maintenance (@ 1% of initial cost = (.01)(.8 x 142,836)	1,190
Personnel (@ 4 hrs/wk, 12 weeks, $10/hr)	480
Total Annual Operating Cost	$ 9,760

If the interest rate is taken at 8% and the life-time of the equipment at 20 years, then the total annual cost will be the sum of $14,500 and $9,760, or $24,260.

Diffused Aeration - Air Bubbles

In view of the fact that no comprehensive experimental data could be found relative to the oxygen transfer efficiency for water depths of the order of 100 feet, it will be necessary to estimate the capture coefficient and subsequently the efficiency from Eq. 55, or

$$\eta = \frac{4.65 f_{co} \eta_{comp}}{\left[\frac{\Delta P_p}{p_1} + 1 + \frac{H}{33.9}\right]} \quad \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. 55)}$$

From Fig. 16 and the previous discussion of this data, the value of f_{co} will approach unity and may be assumed to be unity without introducing serious error. If the frictional pressure drop is assumed to be 15 psi, the compressor efficiency is assumed to be 80% and the electric drive efficiency is taken at 90%, then the overall transfer efficiency will be 2.56 $\#m_{O_2}$/hp-hr. The total required power will be 7.75 hp.

The initial capital cost and the annual operating cost are estimated below.

Initial Capital Cost	
Cost of compressor and drive	$ 4,950
Cost of air pipes (spaced 200 feet apart for a 2,090' x 2,090' surface = 23,000 ft at $1.00/ft)	23,000
Installation cost of piping (@ $1.00/ft)	23,000
Electric power supply (based on 500 feet to nearest utility wire)	1,500
Site preparation	2,000
Shelter	1,000
	55,450
Contingencies and Engineering (@ 20%)	11,090
Total Initial Cost	$66,540
Annual Operating Cost	
Electric power (@ $0.01/kw-hr for 91.5 full days)	127
Maintenance (@ 1% of initial cost)	554
Personnel (@ 4 hrs/wk for 12 weeks, $10/hr)	480
Total Annual Operating Cost	$1,161

If the interest rate is taken at 8% and the life-time of the equipment at 20 years, the total annual cost will be the sum of $6,770 and $1,161 or $7,931.

Hybrid System

It is interesting to compare the above results for diffused aeration and sub-surface aeration with a hybrid system consisting of one sub-surface device used to produce stratified counter-current flow and a white water generator placed at the outlet of the sub-surface device and operated only during the time interval in which the lake is being destratified. In order to estimate the cost, one of the 34 sub-surface units will be selected. Since each of the 34 units had a horsepower input of 14.5 hp, the time to destratify the lake is found to be 313.2 hours from Eq. 36 if a destratification efficiency (DE) of 2.5% is assumed for an epilimnion temperature of 70°F and a hypolimnion temperature of 50°F. The white-water generator must therefore induce a transfer of 139 $\#m_{O_2}$/hr for 313.2 hours in order to increase the initial lake water from a DO level of 4 to 6 mg/liter. Since the water that goes through the white-water generator will not be circulated through it again until it is well mixed because of the stratified flow, the appropriate value of η is taken as that measured in the University of Minnesota flow-through test, namely, 4.14 #m_{O_2}/hp-hr at standard conditions or 1.84 for the present conditions based on shaft power. If the drive is assumed to be electric at 90% efficiency, the total input power for the white-water generator would be 139 [.9(1.89)] or 83.7 hp. Based on Whipple's data the initial capital cost of the white-water generator installation alone would be $49,100. In addition to this capital cost the sub-surface initial cost, as given below, must be included.

Cost of one sub-surface device	$ 2,844
Cost of hydraulic pump and drive	2,000
Cost of hydraulic piping (based on 1/2 x 2,090 at $1.00/ft)	1,045
Installation cost of one unit	500
	6,389
Contingencies and Engineering (@ 20%)	1,278
Sub-surface total	7,667
White-water generator total	49,100
Total Initial Capital Cost of Hybrid System	$56,767

The annual operating costs are estimated below:

Electric Power (@ $0.01/kw-hr for 156.6 hrs (83.7 + 14.5)(156.6)(.745)(.01)	114
Maintenance (@ 3% of initial cost)	1,370
Personnel (16 hrs/cycle for 1 cycle, @ $10/hr)	160
Total Annual Operating Cost	$1,644

If the interest rate is taken at 8% and the life-time of the equipment is taken at 20 years, then the total annual cost will be the sum of $5,760 and $1,644, or $7.404.

In view of the fact that the white-water generator represents a major portion of the capital cost and since this equipment can be made relatively portable, it may be desirable to design the hybrid system with a permanent sub-surface device but with a portable white-water generator that could be moved easily from one lake to another, thus reducing the total annual cost per lake.

A summary of the cost for each of the three systems for supplemental aeration of the 100-acre lake is given in Table XIV.

TABLE XIV

Summary of Annual Cost of Aeration Systems for Lakes

Type of Aerator	Comments	Total Annual Cost
Hybrid, Sub-Surface plus White-Water Generator	System in operation only during the 313.2 hours required to destratify the lake	\$ 7,404
Sub-Surface Aerators	Parallel pipes spaced 200 ft apart, with an assumed capture coefficient of unity, and an assumed frictional pressure drop of 14.7 psi. System in continuous operation for 3 months to achieve required ΔDO of 2 mg/liter	7,931
Diffused Aeration	34 units in continuous operation for 3 months to achieve required ΔDO of 2 mg/liter	24,260

In regard to the question of whether destratification is desirable or not in a given lake, it should be noted that for the example lake used in this section, the hybrid system would destratify the lake in about two weeks (based on an assumed DE of 2.5%), the sub-surface aeration would destratify the lake in less than one day at the same value of DE, and the diffused aeration system would require about four weeks to destratify the lake if it operated at the same value of DE.

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X. NOMENCLATURE

a	a given or fixed distance, ft
a_1	an empirical coefficient
A	area, ft^2
A'	apparent surface area, ft^2
b	a given or fixed distance, ft
C	concentration, mass per unit volume, see text for units
C_1	a constant
C_p	specific heat, $\#_f$-ft/#m
C_d	drag coefficient
d	width or diameter, ft
D_L	diffusion coefficient, ft^2/sec
D_{HY}	hydraulic diameter, ft
DE	destratification efficiency, fraction
f_{co}	oxygen capture coefficient
f_p	pipe friction factor
F	force, $\#_f$
g	acceleration of gravity, 32 ft/sec^2
g_c	dimensional constant = 32.2 #m-ft/$\#_f$-sec^2
h	height, ft
H	water depth, ft
I	Volume, ft^3
k	Boltzmann constant or ratio of specific heats
K_L	liquid film coefficient, ft/sec
K'_L	apparent liquid film coefficient, ft/sec
m	mass of a molecule, grams
$\dot{m}$	mass flow rate, #m/sec
$\dot{M}$	mass flow rate, #m/sec
N	number of molecules striking unit surface per unit time
p	gas pressure or partial pressure, psi or psf

P power, $\#_f$-ft/sec or hp

$\dot{Q}$ volume flow rate, ft^3/sec

r surface renewal rate, 1/sec

r_o radius along which potential functions are located, ft

t time

T temperature, ^oF

u average stream velocity, ft/sec

V velocity, ft/sec

W work or energy, $\#_f$-ft

x distance along x axis, ft

z distance along z axis, ft

α angle between horizontal and water bottom or spray direction, degrees

β surface capture coefficient

γ mass density, #m/ft^3

δ film thickness, ft

ϵ distance between centroids, ft

η efficiency factor

μ absolute viscosity, $\#_f$-sec/ft^2

ν kinematic voscosity, ft^2/sec

ρ mass density, slugs/ft^3

τ_o wall shear stress, ft/sec

$\#_f$ force expressed in pounds

#m mass expressed in pounds

Subscripts

a air

A above

B below

Comp Compression

Subscripts (cont.)

N_2	nitrogen
O_2	oxygen
p	frictional
r	residence
s	saturation
t	total or overall
T	terminal
w	water

APPENDIX A

OXYGEN CAPTURE AT AIR-WATER INTERFACE

OXYGEN CAPTURE AT AIR-WATER INTERFACE

If the air in the vicinity of the water surface is assumed to have a Maxwellian equilibrium velocity distribution, then the number of oxygen molecules from the air that strike unit area of the interface per unit time can be found by integrating the velocity distribution with respect to orientation and speed. When this operation is carried out, the number of oxygen molecules that leave the air and strike the interface is given by the expression

$$\dot{N} = n \left(\frac{kT}{2\pi m}\right)^{1/2} \qquad \text{(Eq. A-1)}$$

where n = number density of oxygen molecules, no/cm^3

k = Boltzmann constant, 1.380×10^{-16} erg/^oK

T = absolute gas temperature, ^oK (assume 20^oC)

m = mass of oxygen molecules, grams

Since air will behave essentially as a perfect gas at modest pressures and temperatures, the perfect gas equation of state can be introduced into Eq. A 1, namely,

$$P_{O_2} = nkt \qquad \text{(Eq. A-2)}$$

where P_{O_2} = partial pressure of oxygen in air, assume to be 0.21 atmospheres

When Eq. A-2 is substituted in Eq. A-1, the expression for the number of oxygen molecules that strike unit area of water surface per unit time becomes

$$N = \frac{P}{\sqrt{2\pi m k T}} \qquad \text{(Eq. A-3}$$

The mass of oxygen that leaves the air and strikes the water per unit surface area per unit time is readily found by multiplying Eq. A-3 by the mass of an oxygen molecule. When this is done and the resulting expression evaluated for a temperature of 20^oC and a partial oxygen pressure of 0.21 atm, the rate of oxygen flow to the water surface is found to be

$$\dot{M}_{O_2} = 6.30\ \#m_{O_2}/ft^2\ sec \qquad \text{(Eq. A-4)}$$

Some of the molecules that strike the surface will be captured by the water and some will rebound back into the air. Given time (and in the absence of diffusion of oxygen from the interface to the main body of water), the surface will come to a dynamic equilibrium where the flux of oxygen molecules leaving the water and returning to the air will equal the flux of oxygen molecules leaving the air and striking the water. This dynamic equilibrium may be characterized by means of the apparent fraction of the oxygen molecules that leave the air, strike the water surface and are captured by the water. This fraction of incident molecules which is captured by the water (β) is a function of the degree to which the liquid interface is saturated with oxygen. As the dissolved oxygen level at the interface approaches saturation, β must approach zero. The magnitude of β may be approximated by equating it to the ratio of volume available to oxygen molecules in the liquid interface region to the total volume of the region at the time the molecules strike the surface. This ratio can be expressed as

$$\frac{\left(\frac{1}{m_{O_2}}\right)(A.\ N.)\left(\frac{\pi}{4}D_{O_2}^{2}\right)(C_s - C)}{\left(\frac{1}{m_{H_2O}}\right)(A.\ N.)\left(\frac{\pi}{4}D_{H_2O}^{2}\right)(C_w)} \qquad \text{(Eq. A-5)}$$

where m = mass of a molecule

A. N. = Avogardro number

D = molecular collision diameter

C_s = saturation concentration of DO, mg/liter

C = concentration of DO at the interface, mg/liter

C_w = concentration density of water, mg/liter

When Eq. A-5 is evaluated, the expression for β reduces to

$$\beta = 0.74 \times 10^{-6} (C_s - C) \qquad \text{(Eq. A-6)}$$

Eq. A-6 and Eq. A-4 can now be combined to determine the mass of oxygen captured per unit time per unit surface area, namely,

$$M_{O_2(\text{captured})} = 6.30\beta \ \# m_{O_2}/ft^2 sec \qquad \text{(Eq. A-7)}$$

or

$$\dot{M}_{O_{2\,(captured)}} = 4.65 \times 10^{-6} (C_s - C) \ \#m_{O_2}/ft^2 sec \qquad \text{(Eq. A-8)}$$

where C_s, C are in mg/liter

APPENDIX B

POTENTIAL FLOW SIMULATION OF SUB-SURFACE DEVICES

POTENTIAL FLOW SIMULATION OF SUB-SURFACE DEVICES

The flow induced by a sub-surface device may be approximated by a family of potential flow functions. Once the flow field has been determined, the rate of surface renewal can be estimated and hence the rate of oxygen transfer from the atmosphere to the water can be estimated.

The flow patterns for various device configurations (See Fig. 6 for a schematic of the three flow cases) will first be computed based on the assumption that the water is of uniform density, the bottom is horizontal and the side walls are infinitely far away. The influence of a vertical temperature and a sloping bottom and side walls is discussed in the body of the report. Each of the three flow cases is developed separately below.

Case 1 FLOW

The flow pattern induced by a propeller and a vertical duct, for a duct width which is small compared to the water depth, can be approximated by a combination of an infinite series of sources and an infinite series of sinks placed along the vertical centerline of the duct as shown in Fig. B-1. The sources and sinks are so placed that no flow takes place across the bottom or across the air-water interface. Hence these two boundaries are treated as flat plates.

The complex potential function for all the sources, F_+, can be expressed in the x, y cartesian coordinate system (or z complex coordinate system) as

$$F_+ = \sum_{n=0}^{\infty} \{-\frac{Q}{2\pi} \ell n\,[z - in(2H-a)] - \frac{Q}{2\pi} \ell n\,[z - in(2H+a)]\}$$

$$+ \sum_{n=1}^{\infty} \{-\frac{Q}{2\pi} \ell n\,[z + in(2H-a)] - \frac{Q}{2\pi} \ell n\,[z + in(2H+a)]\} \quad \text{(Eq. B-1)}$$

where Q = constant strength of the sources, ft^3 per ft length of device

Eq. B-1 can be expressed in closed form as a hyperbolic function, namely

$$F_+ = -\frac{Q}{2\pi} \ell n\,[\sinh \frac{\pi z}{(2H-a)}] - \frac{Q}{2\pi} \ell n\,[\sinh \frac{\pi z}{(2H+a)}] \quad \text{(Eq. B-2)}$$

In a similar fashion the complex potential function for all the sinks,

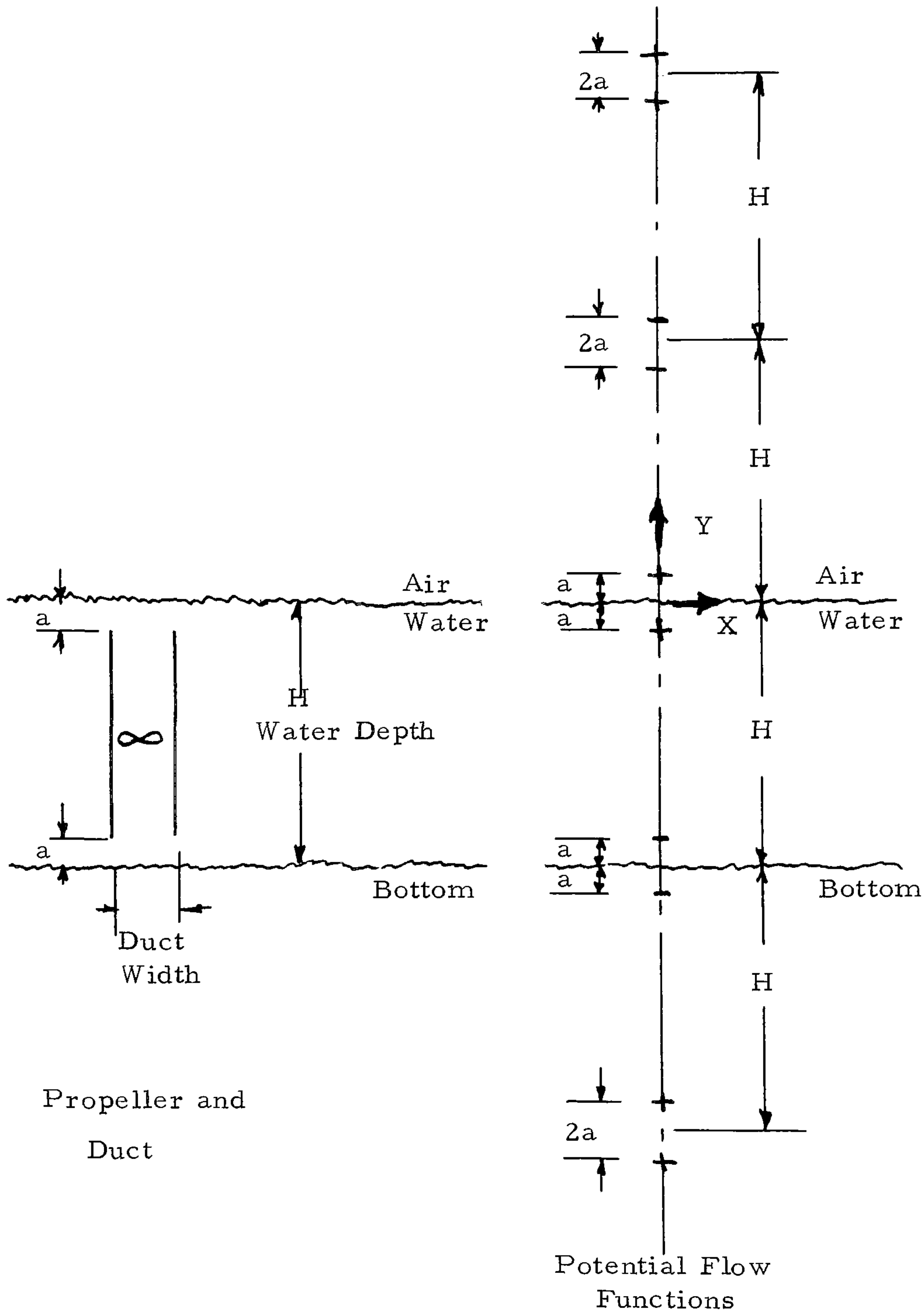


Fig. B-1 - Distribution of Sources and Sinks to Simulate Flow induced by a Propeller in a Duct whose Width is Small compared to the Water Depth

F_-, can be expressed in the x, y cartesian coordinate system (or z complex coordinate system) as

$$F_- = \frac{Q}{2\pi} \ln[\sinh \frac{\pi(z+iH)}{(2H-a)}] + \frac{Q}{2\pi} \ln[\sinh \frac{\pi(z+iH)}{(2H+a)}] \quad \text{(Eq. B-3)}$$

where Q = constant strength of the sinks, ft^3/sec per ft length of device

The complex function for the source-sink flow between the air-water interface and the bottom can now be found by adding Eq. B-2 and Eq. B-3 to obtain

$$F = -\frac{Q}{2\pi} \ln[\sinh \frac{\pi z}{(2H-a)}] - \frac{Q}{2\pi} \ln[\sinh \frac{\pi z}{(2H+a)}]$$

$$+ \frac{Q}{2\pi} \ln[\sinh \frac{\pi(z-iH)}{(2H-a)}] + \frac{Q}{2\pi} \ln[\sinh \frac{\pi(z+iH)}{(2H+a)}] \quad \text{(Eq. B-4)}$$

The velocity in the x-direction, u, and the velocity in the y-direction, v, can now be determined at any point by noting that the following relation exists between F, u and v:

$$\frac{\partial F}{\partial z} = -u + iv \quad \text{(Eq. B-5)}$$

From Eq. B-5 and Eq. B-4, the velocity components are found to be

$$u = \frac{Q}{4H}\{\coth\frac{\pi x}{2H}\}\left\{\frac{[\cot\frac{\pi(y-a)}{2H}]^2+1}{[\cot\frac{\pi(y-a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2} + \frac{[\cot\frac{\pi(y+a)}{2H}]^2+1}{[\cot\frac{\pi(y+a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2}\right\}$$

$$-\frac{Q}{4H}\{\coth\frac{\pi x}{2H}\}\left\{\frac{[\cot\frac{\pi(y+H-a)}{2H}]^2+1}{[\cot\frac{\pi(y+H-a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2} + \frac{[\cot\frac{\pi(y+H+a)}{2H}]^2+1}{[\cot\frac{\pi(y+H+a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2}\right\}$$

(Eq. B-6)

$$v = -\frac{Q}{4H}\left\{\frac{\cot\frac{\pi(y-a)}{2H} - \cot\frac{\pi(y-a)}{2H}[\coth\frac{\pi x}{2H}]^2}{[\cot\frac{\pi(y-a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2} + \frac{\cot\frac{\pi(y+a)}{2H} - \cot\frac{\pi(y+a)}{2H}[\coth\frac{\pi x}{2H}]^2}{[\cot\frac{\pi(y+a)}{2H}]^2+[\coth\frac{\pi x}{2H}]^2}\right.$$

(cont.)

$$+\frac{Q}{4H}\left\{\frac{\cot\frac{\pi(y+H-a)}{2H}-\cot\frac{\pi(y+H-a)}{2H}\left[\coth\frac{\pi x}{2H}\right]^2}{\left[\cot\frac{\pi(y+H-a)}{2H}\right]^2+\left[\coth\frac{\pi x}{2H}\right]^2}\right.$$

$$\left.+\frac{\cot\frac{\pi(y+H+a)}{2H}-\cot\frac{\pi(y+H+a)}{2H}\left[\coth\frac{\pi x}{2H}\right]^2}{\left[\cot\frac{\pi(y+H+a)}{2H}\right]^2+\left[\coth\frac{\pi x}{2H}\right]^2}\right\} \qquad \text{(Eq. B-7)}$$

The magnitude of the circulation zone can now be determined by plotting v along the horizontal line that passes through the mid-height point (See Fig. 7). From the symmetry u will be zero along this line and hence v serves as a measure of the intensity of circulation at a given point.

Once a duct width has been selected, the volume flow rate through the duct per unit length of device [= (width)(v)] can be related to the strength of the individual sources and sinks per unit length of device (Q) by estimating what fraction of the water that emanates from the source near the top of the water and passes down to the sink at the bottom by taking a path outside of the duct walls, and hence which fraction passes down to the sink at the bottom by taking a path inside the duct walls. This estimate was made by plotting the vertical component of velocity at various depths as a function of horizontal distance from the vertical centerline. For a = 0.01H these two ratios were estimated at 0.174 and 0.826 respectively, whereas for a = 0.10H and a = 0.45H, the corresponding ratios were o.236, 0.764 and 0.700, 0.300 respectively.

The liquid film coefficient K_L and hence the oxygen transfer rate from the atmosphere to the water can now be estimated from the flow field. The liquid film coefficient for this type flow is given by Eq. 16 as

$$K_L = \sqrt{D_L r} \qquad \text{(Eq. B-8)}$$

where D_L = diffusion coefficient for O_2 in water, $2.65 \times 10^{-8}\ ft^2/sec$

r = surface renewal rate

The surface renewal rate can be approximated by means of Eq. 17 as

$$r = \left(\frac{\Delta u}{\Delta y}\right)_{\text{upper water layer}} \qquad \text{(Eq. B-9)}$$

If the right hand side of Eq. B-9 is evaluated by taking the difference in the x-direction velocity at the surface and at mid-depth, then the liquid film coefficient at some distance x from the duct centerline becomes

$$K_L = \sqrt{D_L \left(\frac{u_{surface}}{H/2}\right)} \qquad \text{(Eq. B10)}$$

Since the zone of circulation is shown to be $-2H < x < +2H$ in the report, the oxygen transfer rate per unit length of device is given by the expression

$$\dot{M}_{O_2} = \int_{x=-2H}^{x=+2H} \sqrt{D_L \left(\frac{u_{surface}}{H/2}\right)} \, (C_s - C) \, dx \qquad \text{(Eq. B-11)}$$

where $u_{surface}$ is given by Eq. B-6 when y is made equal to zero.

When Eq. B-11 is evaluated, the final expression for the transfer rate of oxygen for Case I Flow becomes

$$\dot{M}_{O_2} = C_1 (10^{-6})(C_s - C) V^{1/2} H^{1/2} \quad \text{\#m/hr per ft length of device} \qquad \text{(Eq. B-12)}$$

where C_s, C are in mg/liter

V = duct velocity for the assumption that the duct width is equal to 0.2H

C_1 = 50.6 for a = 0.01H

C_1 = 47.4 for a = 0.1H

C_1 = 18.8 for a = 0.45H

H = water depth, ft

In order to estimate the energy that must be supplied to the propeller to produce a flow with velocity V in the duct, it is noted that in steady operation the required energy will be equal to the energy dissipated in frictional heating as the water enters the duct at the bottom, moves through the duct, exits from the duct at the top and the loss of kinetic energy as the water flow in the external circuit from the top to the bottom of the duct. Since it is anticipated to operate the device at velocities of a few ft/sec or less, the entrance losses and losses along the duct can be neglected compared to the exit loss and kinetic energy loss in the external circuit. These two losses together at most should equal the kinetic energy of fluid in the duct just before it passes through the exit of the duct. Assuming the required energy is equal to a maximum value of one kinetic energy head, the oxygen transfer efficiency can be found by dividing Eq. B-12 by the required power, namely

$$P = \frac{\rho \dot{Q} V^2}{2g} \qquad \text{(Eq. B-13)}$$

where $\dot{Q}$ = volume flow rate through the duct,
= .2HV per ft length of device

When Eq. 12 is divided by Eq. 13, the oxygen transfer efficiency is found to be

$$\eta = \frac{C_2 (C_s - C)}{H^{1/2} V^{5/2}} \quad \#m_{O_2}/\text{hp-hr} \qquad \text{(Eq. B-14)}$$

where $C_2 = 0.143$ for $a = 0.01$
$C_2 = 0.134$ for $a = 0.10$
$C_2 = 0.053$ for $a = 0.45$

Case II FLOW

The flow pattern induced by a propeller and a vertical duct, for a duct width which is wide compared to the water depth, can be approximated by a combination of several infinite series of sources and several infinite series of sinks placed along lines parallel to the vertical centerline of the duct as shown in Fig. B-2. The sources and sinks are so placed that no flow takes place across the bottom or across the air-water interface. As a first approximation eleven infinite series of sources and eleven infinite series of sinks were selected to characterize the flow as shown in Fig. B-2.

The complex potential for the infinite series of sources and sinks along any one vertical line parallel to the duct centerline is given by Eq. B-4. Thus the components of velocity (u and v) at any point are given by adding the contribution of each of the eleven series together. The contribution of each individual series is given by Eq. B-6 for the x-direction component of velocity and by Eq. B-7 for the y-direction component of velocity.

Proceeding in the same manner as for Case I Flows, the surface

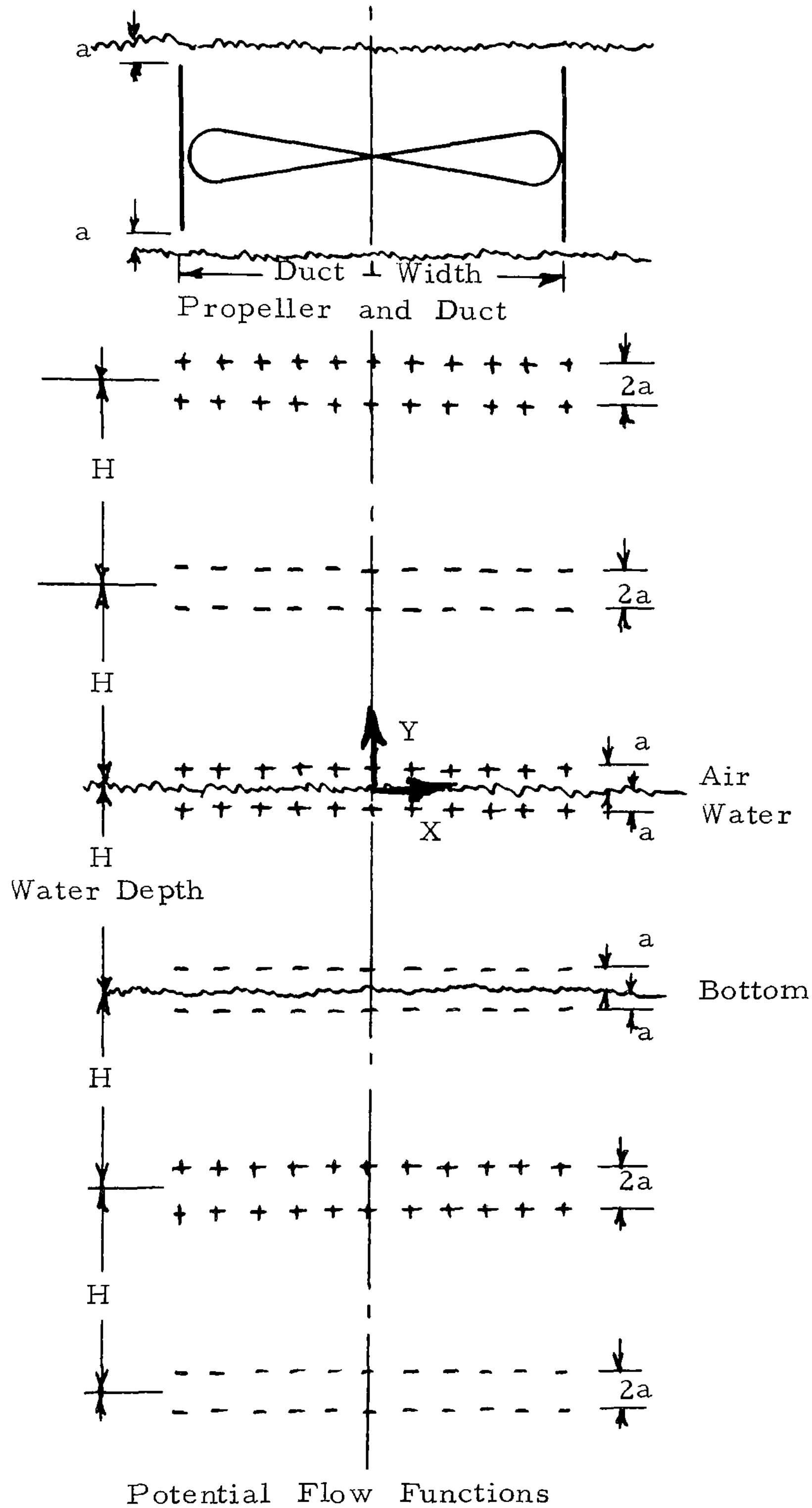


Fig. B-2 Distribution of Sources and Sinks to Simulate the Flow Induced by a Propeller in a Vertical Duct whose Width is Large compared to the Water Depth renewal rate can be approximated from Eq. B-9 as

$$r = \frac{u_{surface}}{a} \qquad \text{(Eq. B-15)}$$

Since in the text of the report the zone of circulation is shown to be $-5H < x < +5H$ for a duct width of six times the water depth, the oxygen transfer rate per unit length of device is given by the expression

$$\dot{M}_{O_2} = \int_{x=-5H}^{x=+5H} \sqrt{D_L \left(\frac{u_{surface}}{a}\right)} (C_s - C)\, dx \qquad \text{(Eq. B-16)}$$

When Eq. B-16 is evaluated for the total contribution of the eleven infinite series, the final expression for oxygen transfer becomes

$$\dot{M}_{O_2} = 7000 \times 10^{-6} (C_s - C) V^{1/2} H^{1/2} \quad \text{\#m/hr per ft length of device} \qquad \text{(Eq. B-17)}$$

where C_s, C are in mg/liter

V = duct velocity, ft/sec

H = water depth, ft

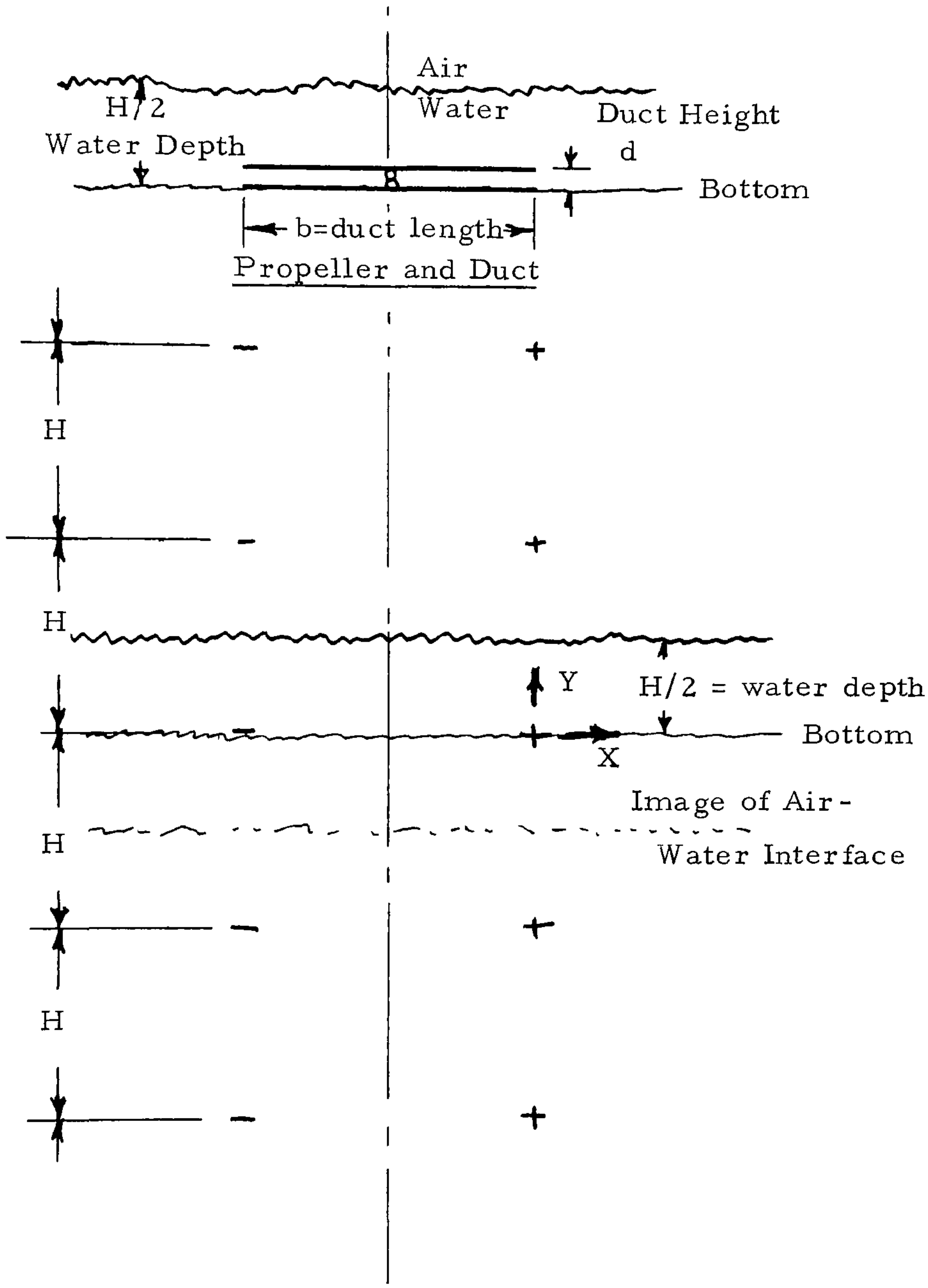
If Eq. B-17 is divided by Eq. B-13, the expression for the oxygen transfer efficiency becomes

$$\eta = \frac{0.657 (C_s - C)}{h^{1/2} H^{5/2}} \quad \text{\#m}_{O_2}\text{/hp-hr} \qquad \text{(Eq. B-18)}$$

Case III FLOW

The flow pattern induced by a horizontal ducted propeller for a duct height which is small compared to the water depth can be approximated by the combination of an infinite series of sources and an infinite series of sinks. The sources are placed along a vertical line that passes through the duct outlet and the sinks are placed along a parallel vertical line that passes through the duct inlet as shown in Fig. B-3.

The complex potential function for the infinite series of sources with respect to the x, y cartesian coordinate system (or the z complex coordinate system) is given by the expression



Potential Flow Functions

Fig. B-3 - Distribution of Sources and Sinks to Simulate Flow Induced by a Propeller in a Horizontal Duct

$$F_+ = \sum_{n=0}^{\infty} \{-\frac{Q}{2\pi} \ell n [z-inH]\} + \sum_{n=1}^{\infty} \{-\frac{Q}{2\pi} \ell n [z+inH]\} \qquad \text{(Eq. B-19)}$$

Eq. B-19 can be expressed in closed form by means of a hyperbolic function, thus

$$F_+ = -\frac{Q}{2\pi} \ell n [\sinh \frac{\pi z}{H}] \qquad \text{(Eq. B-20)}$$

In the same manner the complex potential function for the infinite series of sinks can be expressed in closed form as

$$F_- = \frac{Q}{2\pi} \ell n [\sinh \frac{\pi(z+b)}{H}] \qquad \text{(Eq. B-21)}$$

where b = length of duct, ft

The total complex potential function for the two infinite series is given by adding Eq. B-20 and Eq. B-21, namely

$$F = -\frac{Q}{2\pi} \ell n [\sinh \frac{\pi z}{H}] + \frac{Q}{2\pi} \ell n [\sinh \frac{\pi(z+b)}{H}] \qquad \text{(Eq. B-22)}$$

The velocity components (a and v) can now be found at any point with the aid of Eq. B-5, thus

$$u = \frac{Q}{2H} \left\{ \frac{[\cot \frac{\pi y}{H}]^2 \coth \frac{\pi x}{H} + \coth \frac{\pi x}{H}}{[\cot \frac{\pi y}{H}]^2 + [\coth \frac{\pi x}{H}]^2} \right\} - \frac{Q}{2H} \left\{ \frac{[\cot \frac{\pi y}{H}]^2 \coth \frac{\pi(x+b)}{H} + \coth \frac{\pi(x+b)}{H}}{[\cot \frac{\pi y}{H}]^2 + [\coth \frac{\pi(x+b)}{H}]^2} \right\} \qquad \text{(Eq. B-23)}$$

$$v = \frac{-Q}{2H} \left\{ \frac{\cot \frac{\pi y}{H} - \cot \frac{\pi y}{H} [\coth \frac{\pi x}{H}]^2}{[\cot \frac{\pi y}{H}]^2 + [\coth \frac{\pi x}{H}]^2} \right\} + \frac{Q}{2H} \left\{ \frac{\cot \frac{\pi y}{H} - \cot \frac{\pi y}{H} [\coth \frac{\pi(x+b)}{H}]^2}{[\cot \frac{\pi y}{H}]^2 + [\coth \frac{\pi(x+b)}{H}]^2} \right\} \qquad \text{(Eq. B-24}$$

Since it is anticipated that this configuration might be applied to a stream, the situation requires b > H. For this case the x-direction component of velocity as given by Eq. B-23 does not vary much with distance in the y-direction for a given value of x between x=0 and x=-b. As a result the velocity-depth profile will be similar to that found in a natural stream and hence the surface renewal rate can be estimated from Eq. B-25 (See. Eq. 36, Ref. 10).

$$r = \frac{0.1 u_{ave}}{0.1(H/2)} \qquad \text{(Eq. B-25}$$

where u_{ave} = average of x-direction velocity over the depth at a given location

Since the zone of circulation is shown to correspond approximately to the duct width for b > H, the oxygen transfer rate per unit length of device is given by the expression

$$\dot{M}_{O_2} = \int_{x=0}^{x=-b} \sqrt{D_L \left(\frac{u_{surface}}{\frac{H}{2} - d}\right)} \, (C_s - C) \, dx \qquad \text{(Eq. B-26)}$$

When Eq. B-26 is evaluated with the use of Eq. B-23 for d = H/4, the expression for the oxygen transfer rate per unit length of device becomes

$$\dot{M}_{O_2} = C_1(10^{-6})(C_s - C) V^{1/2} H^{1/2} \quad \text{\#m/hr per ft length of device} \qquad \text{(Eq. B-27)}$$

where C_s, C are in mg/liter

V = duct velocity, ft/sec

H = twice the water depth, ft

C_1 = 524 for b = 10H, d = .5H

C_2 = 262 for b = 5H, d = .5H

The oxygen transfer efficiency can again be estimated by assuming the required power input must be equivalent to the dissipation of one velocity head. Thus the transfer efficiency, η, can be found by dividing Eq. B-27 by Eq. B-13 to obtain

$$\eta = \frac{C_2(C_s - C)}{H^{1/2} V^{5/2}} \qquad \# m_{O_2}/\text{hp-hr} \qquad \text{(Eq. B-28)}$$

where $C_2 = 0.589$ for $b = 10H$, $d = .5H$

$C_2 = 0.214$ for $b = 5H$, $d = .5H$

APPENDIX C

COST ESTIMATE OF SUB-SURFACE AERATORS

COST ESTIMATE OF SUB-SURFACE AERATORS

The capital cost for Case I and Case II devices are estimated in this Appendix.

CASE II DEVICE

Fig. C-1 shows a sketch of a sub-surface device designed for a stream depth of four feet. The cost estimate for this device is given below.

Material Cost

Part	Material	Weight	Cost
Top Ring	Alum. 6'' x .5''	208#	$ 208
Bottom Ring	Alum. 6'' x .5''	208#	208
Side Channels	Alum. 4'' x .180''	150#	150
Propeller Braces	Alum. 4'' x .180''	38#	38
Side Wall	Fiberglass 4' x 75' x .25''	Area=300 ft^2 @ \$.50/ft^2	150
Fan (fabricated)	Fiberglass	-	200
Hydraulic Motor	-	-	80
Total Material Cost			$1,034
Fabrication Cost			300
Total Unit Cost			$1,344

CASE I DEVICE

Case I differs from Case II primarily in the fact that the duct length is long in the Case I device and short in the Case II device. Case I flow could be created by the same unit shown in Fig. C-1 with the addition of a long duct held up by buoyant material in the form of a number of collars. If the cost of this extension is taken at \$0.25/ft^2, then the total cost of a Case I unit for a 100-foot deep lake would be \$1,334 + $(.25)(96)(\pi \times 20)$ or \$2844.

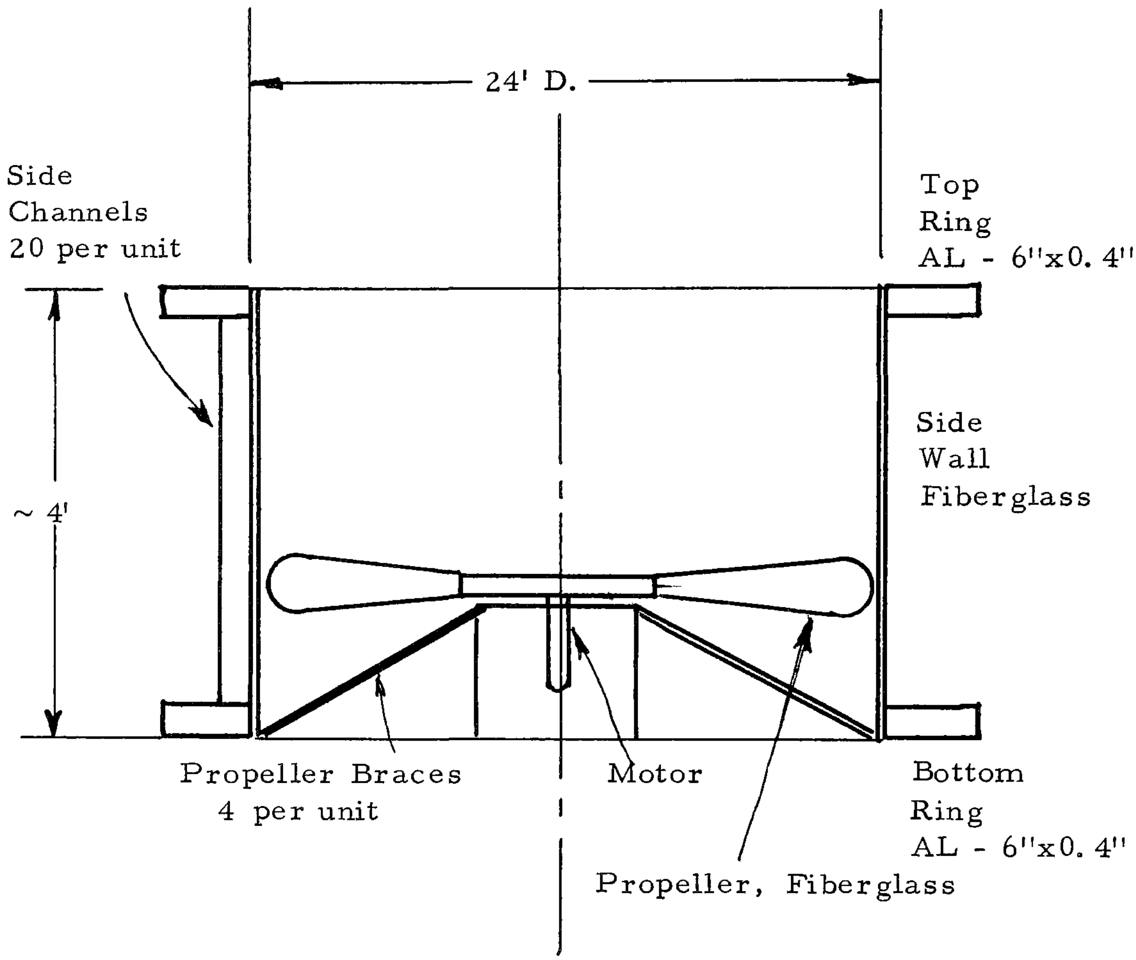


Fig. C-1 - Sub-Surface Circulating Device - Propeller in a Vertical Duct

BIBLIOGRAPHIC:	ACCESSION NO.
Hogan, W.T., Reed, F.E. and Starbird, A.W., Mechanical Aeration Systems for Rivers and Ponds, Littleton Research & Engineering Corp., Final Rept. FWQA Contract 14-12-576, 11/70 ABSTRACT A study of methods of increasing the dissolved oxygen in rivers and ponds. Analytical and empirical relations establish operating characteristics of tested and untested aerating devices. From estimates of cost and efficiency, the most economical methods of aerating rivers and ponds are determined.	KEY WORDS Mechanical Aeration Aeration Efficiency Aeration Rivers Ponds Economic Prediction Hydraulic Engineering Water Quality Control

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27 *Abstract*

The total annual cost of providing supplemental aeration of streams and lakes by tested and untested aeration equipment is estimated. Analytical and empirical equations are presented for the determination of operating characteristics of the various devices used to aerate natural bodies of water. For the example stream evaluated in this study, the most economical means of artificial aeration generally possible was found to be mechanical aerators which generate a highly turbulent white-water surface. For the example lake evaluated, the most economical technique for the continual input of oxygen into a lake was found to be diffused aeration using air bubbles; whereas the most economical technique for rapid input of oxygen, operating only while the lake is being destratified, was found to be a hybrid system consisting of a large diameter ducted propeller which draws water from the lake bottom and discharges it at the surface where it is aerated by a mechanical aerator.

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