air pollution training institute

Effective Stack Height

Plume Rise

UNITED STATES ENVIRONMENTAL PROTECTION AGENCY

SI:406

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Effective Stack Height

Plume Rise

1974

UNITED STATES ENVIRONMENTAL PROTECTION AGENCY
Office of Air and Waste Management
Office of Air Quality Planning and Standards
Control Programs Development Division
Air Pollution Training Institute

Objectives

EFFECTIVE STACK HEIGHT/PLUME RISE

OBJECTIVES

To introduce the student visually to elevated pollutant emissions and to present the physical principles most important in determining the rise of a plume.

To present the background assumptions of several commonly used equations for computing plume rise together with major advantages and disadvantages or limitations of each.

To show visually several meteorological and topographical conditions which limit the use of any plume rise equation.

To require the calculation of effective stack height by three common equations so that the student realizes what input data is needed; he can then compare the different answers obtained by using the same data. The second exercise treats Briggs' equations in more depth, while the third presents current EPA calculation procedures.

To present an in-depth discourse on the development of the Briggs' equations together with recent developments and modifications suggested by Briggs and the Meteorology Laboratory, EPA.

Introduction

EFFECTIVE STACK HEIGHT/PLUME RISE

Effective Stack Height/Plume Rise is a self-instructional package designed by the Air Pollution Training Institute, Environmental Protection Agency. An Air Pollution Training Institute Certificate of Completion will be awarded if the learner achieves a satisfactory level on the problem sets included in the package. The suggested involvement time is eight hours.

The package contains:

Plume Rise/Effective Stack Height; a work manual

Plume Rise; a text by Gary A. Briggs, Ph.D., Research Meteorologist Atmospheric Turbulence and Diffusion Laboratory, NOAA

Plume Rise; an audio tape presentation by Briggs

Effective Stack Height; an audio-slide presentation by James L. Dicke, Meteorologist, Meteorology Laboratory, Air Pollution Training Institute, EPA

A complete listing of the components is on page 7.

The package consists of three exercises. Exercise one is made up of a narrated slide series and an APTI article, both entitled Effective Stack Height and both writen by Mr. Dicke. Exercise two is made up of the text Plume Rise and an audio tape presentation by Dr. Briggs with accompanying lecture notes in the work manual. Exercise three contains a summary of Dr. Briggs' lastest analyses and the current EPA calculation procedures as stated by D. Bruce Turner, Environmental Applications Branch, Meteorology Laboratory, EPA. Problem sets conclude each exercise.

Those who desire a certificate should complete each component in the order of presentation in the exercise. Problem sets may be submitted individually or the three sets may be submitted on completion of the package. All problem sheets must be returned to receive credit. Adequate space is provided for calculations. Extra paper may be used if additional space is required, but these calculations must also be submitted to receive credit toward a certificate.

A critique form is provided in the appendix. It will take but a moment to complete the basic questions. The Air Pollution Training Institute welcomes evaluation. For extended comments, please use the back of the critique form. As an effort towards maintaining a viable and current package, it will be necessary for the participant to complete the critique before a certificate will be awarded.

Pre-addressed envelopes are provided. If these are not enclosed in the package, please mail your problem sets and critique to:

Air Pollution Training Institute
National Environmental Research Center
Research Triangle Park, North Carolina 27711
Attention: Plume Rise Instructor SI - 406

COMPONENTS

●THE PACKAGE

●LEARNER RESPONSE TO THE PACKAGE

1. Submission of Problem Set One to APTI
2. Submission of Problem Set Two to APTI
3. Submission of Problem Set Three to APTI
4. Critique of the package (submit with Problem Set Three)

●AIR POLLUTION TRAINING INSTITUTE
RESPONSIBILITY TO THE LEARNER

1. Instructor grading of the Problem Sets; return of evaluation to the learner with set of correct answers
2. Instructor certifying satisfactory completion
3. Registrar issuing Certificate of Completion to learner

Exercise One

A. EFFECTIVE STACK HEIGHT
An audio-slide presentation by James L. Dicke

B. EFFECTIVE STACK HEIGHT
James L. Dicke

C. PROBLEM SET ONE

Please complete the exercise in the given order. Upon completion of components A and B, please work the problems in set one and forward to the Air Pollution Training Institute. A pre-addressed envelope is provided. A critique of your calculations will be returned with a copy of the correct calculations. Please be certain that your name and address is on each sheet. ALL CALCULATIONS MUST BE RETURNED TO RECEIVE CREDIT.

Air Pollution Training Institute
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A

EFFECTIVE STACK HEIGHT

James L. Dicke *

Component A, "Effective Stack Height", is an audio-slide presentation. It is made up of 54 35mm slides with an accompanying 50 minute narration on cassette tape. The slides and cassette tape are standard format and should be suitable for any 35mm slide projector and audio cassette recorder-playback unit. For your convenience, a cued script of the audio portion of the presentation is included in the appendix.

When you have completed the audio-slide series, please turn your attention to Mr. Dicke's article on effective stack height (component B). Problem Set One, which may be forwarded toward credit for an Air Pollution Training Institute certificate, completes the exercise.

*James L. Dicke, Meteorologist, Meteorology Laboratory and Air Pollution Training Institute, Environmental Protection Agency

EFFECTIVE STACK HEIGHT

James L. Dicke *

In any consideration of concentrations downwind from a source, it is desirable to estimate the effective stack height, the height at which the plume becomes level. Rarely will this height correspond to the physical height of the stack.

A high velocity of emission of the effluents and a temperature higher than that of the atmosphere at the top of the stack will act to increase the effective stack height above the height of the actual stack. The effect of aerodynamic downwash, eddies caused by the flow around buildings or the stack, and also the evaporative cooling of moisture droplets in the effluent may cause lowering of the plume to the extent that it may be lower than the physical stack height.

EFFECT OF EXIT VELOCITY AND STACK GAS TEMPERATURE

A number of investigators have proposed formulas for the estimation of effective stack height under given conditions: Davidson (1949), Sutton (1950), Bosanquet et al. (1950), Holland (1953), Priestley (1956).

A recent comparison of actual plume heights and calculations using six of the available formulas was made by Moses and Strom (1961). The formulas used were Davidson-Bryant, Holland, Scorer, Sutton, Bosanquet-Carey-Halton, and Bosanquet (1957). They found that "There is no one formula which is outstanding in all respects." The formulas of Davidson-Bryant, Holland, Bosanquet-Carey-Halton, and Bosanquet (1957) appear to give satisfactory results for many purposes. It must be pointed out that the experimental tests made by Moses and Strom used stack gas exit velocities less than 15 m/sec and that temperatures of the effluent were not more than that of the ambient air.

Stewart, Gale, and Crooks (1958) compared effective stack heights for the Harwell reactor emitting radioactive Argon with computations using the formula of Bosanquet et al. (1950). The temperature of the gases was 50°C above that of the ambient air and stack gas velocity was 10 m/sec. At low wind speeds, agreement between formula and plume height were quite good. At wind speeds greater than 6 m/sec and distances greater than 600 meters from the stack the formula underestimates effective stack height.

Two of the formulas for estimation of effective stack height are given below. Both the Davidson-Brvant formula and Holland's formula frequently underestimate the effective stack height. Therefore, a slight safety factor is frequently made by using the following formulas.

*James L. Dicke, Meteorologist, Meteorology Laboratory, and Air Pollution Training Institute, Environmental Protection Agency

Effective Stack Height/Plume Rise

The Davidson-Bryant formula is:

$$\Delta H = d\left(\frac{v_s}{u}\right)^{1/4}\left(1 + \frac{\Delta T}{T_s}\right) \qquad (1)$$

Where:

ΔH = the rise of the plume above the stack

d = the inside stack diameter

v_s = stack gas velocity

u = wind speed

ΔT = the stack gas temperature minus the ambient air temperature (^{o}K)

T_s = the stack gas temperature (^{o}K)

Any consistent system of units for ΔH, d, v_s, and u may be used. It is recommended that v_s and u be in meters/sec and d in meters which will give ΔH in meters.

The Holland stack rise equation is:

$$\Delta H = \frac{v_s d}{u}\left(1.5 + 2.68 \times 10^{-3}\, p \frac{\Delta T}{T_s} d\right) \qquad (2)$$

Where:

ΔH = the rise of the plume above the stack (meters)

v_s = stack gas velocity (m/sec)

d = the inside stack diameter (meters)

u = wind speed (m/sec)

p = atmospheric pressure (mb)

T_s = stack gas temperature (^{o}K)

ΔT = as in equation (1) and 2.68×10^{-3} is a constant having units of ($m^{-1}\ mb^{-1}$).

It is recommended that the result from the above equation be used for neutral conditions. For unstable conditions a value between 1.1 and 1.2 times that from the equation should be used for ΔH. For stable conditions a value between 0.8 and 0.9 times that from the equation should be used for ΔH.

Since the plume rise from a stack occurs through some finite distance downwind, these formulas should not be applied when considering effects near the stack. Note that these formulas do not consider the stability of the atmosphere but only the ambient air temperature. Actually, stability should have some effect upon the plume rise.

Lucas et al., 1963, have tested Priestley's theory on stack rise at two power stations. These investigators write the formula for stack rise:

$$z_{max} = \alpha \frac{Q^{1/4}}{u} \qquad (3)$$

Where:

z_{max} = effective rise above the top of the stack (feet)

α = a variable affected by lapse rate and topography ($ft^2\ MW^{-1/4}\ sec^{-1}$)

Q = the rate of heat emission from the stack in megawatts (MW)

u = the wind speed (ft/sec)

They determined α to be 4900 and 6200 for the 2 power stations under neutral conditions. Clarke (1968) states that if the exit velocity of the stack gases exceeds 2000 fpm (23mph), no rain will enter the stack. As an example, he found from a ten year summary of U.S. Weather Bureau wind data for Chicago that 98% of the days had a maximum wind velocity $\leq$ 20 mph.

EFFECT OF EVAPORATIVE COOLING

In the washing of effluent gases to absorb certain gases before release to the atmosphere, the gases are cooled and become saturated with water vapor. Upon release from the absorption tower further cooling is likely due to contact with cold surfaces of ductwork or stack. This causes condensation of water droplets in the gas stream. Upon release from the stack, the water droplets evaporate withdrawing the latent heat of vaporization from the air and consequently cooling the plume, causing it to have negative buoyancy, thereby reducing the stack height. (Scorer 1959).

The practice of washing power plant flue gases to remove sulfur dioxide is practiced at Battersea and Bankside power stations near London, where frequent lowering of the plumes to ground level is observed.

EFFECT OF AERODYNAMIC DOWNWASH

The influence of the mechanical turbulence around a building or stack may significantly alter the effective stack height. This is especially true under high wind conditions when the beneficial effect of high stack gas velocity is at a minimum and the plume is emitted nearly horizontally. The region of disturbed flow surrounds a building generally to twice its height and 5 to 10 times its height downwind. Most of the knowledge about the turbulent wakes around stacks and buildings have been gained through wind tunnel studies. Sherlock and Stalker (1940), (1941), Rouse (1951), Sherlock (1951), Sherlock and Leshner (1954), (1955), Strom (1955-1956), Strom et al. (1957), and Halitsky (1961), (1962), (1963). By using models of building shapes and stacks the wind speeds required to cause downwash for various wind directions may be determined.

In the use of a wind tunnel the meteorological variables that may most easily be taken into account are the wind speed and the wind direction (by rotation of the model within the tunnel). The plant factors that may be taken into consideration are the size and shape of the plant building, the shape, height, and diameter of the stack, the amount of emission, the stack gas velocity, and perhaps the density of the emitted effluent. The study of the released plume from the model stack has been done by photography (Sherlock and Lesher, 1954), decrease in light beam intensity (Strom, 1955), and measurement of concentrations of a tracer gas (Strom et al., 1957, Halitsky, 1963).

By determining the critical wind speeds that will cause downwash from various directions for a given set of plant factors, the average number of hours of downwash annually can then be calculated by determining the frequency of wind speeds greater than the critical speeds for each direction (Sherlock and Lesher, 1954). It is assumed that climatological data, representative of the site considered, are available.

It is of interest to note that the maximum downwash about a rectangular structure occurs when the direction of the wind is at an angle of 45 degrees from the major axis of the structure and that minimum downwash occurs with wind flow parallel to the major axis of the structure (Sherlock and Lesher, 1954).

It has been shown by Halitsky (1961), (1963) that the effluent from flush openings on flat roofs frequently flows in a direction opposite to that of the free atmosphere wind due to counter-flow along the roof in the turbulent wake above the building. In addition to the effect of aerodynamic downwash upon the release of air pollutants from stacks and buildings, it is also necessary to consider aerodynamic downwash when exposing meteorological instruments near or upon buildings so that representative measurements are assured.

In cases where the pollution is emitted from a vent or opening on a building and is immediately influenced by the turbulent wake of the building, the pollution is quite rapidly distributed within this turbulent wake. An initial distribution may be assumed at the source with horizonal and vertical variances of δy^2 and δ_z^2 in the form of a binormal distribution of concentrations. These variances are related to the building width and height.

The resulting equation for concentrations from this source has $(\sigma_y{}^2 + \delta_y{}^2)^{1/2}$ in place of σ_y and $(\sigma_z{}^2 + \delta_z{}^2)^{1/2}$ in place of σ_z in the point source equations.

EFFECT OF VERY LARGE POWER PLANTS

A power generating plant in the range of 1000-5000 megawatt capacity emits heat to such an extent that its own circulation pattern will be set up in the air surrounding the plant. It is doubtful that extrapolation of dispersion estimates from existing smaller sources can be applied to these large plants. Fortunately, the effluent plume will rise far above the ground and surface concentrations of pollutants downwind will increase by only a rather small amount most of the time. Such large plants will usually be engineered to minimize the effects of the two preceding topics. The "2 1/2" rule will tend to eliminate downwash and the "4/3's" rule for stack gas velocity, i.e. v_s should exceed $\bar{u}$ by at least a third, will tend to eliminate entrainment of the effluent into the wake of the stack. (Pooler, 1965)

Three weather conditions, however, can still bring ground level fumigations: high winds, inversion breakup, and a limited mixing layer with light winds. The climatology of these conditions will determine the magnitude and frequency of the pollution problem. Pooler (1965) has presented nomograms for estimating groundlevel SO_2 concentrations for these three situations together with effective stack height formulas. Briggs (1965) has also presented a plume rise model which has been compared with data from various TVA power plants. Pooler (1967) introduced a slight modification to one of Briggs's equation and suggests this latter equation be substituted for the Pooler (1965) inversion breakup fumigation equation. Thus the basic equation for this important weather condition is:

$$\Delta H = \left[2.31 \frac{v_s r^2 T_A}{u} \left(\frac{\Delta T}{T_s}\right) \Big/ \frac{d\theta}{dz}\right]^{1/3} \quad (4)$$

Where: u = wind speed (m/sec)

r = inside stack radium (m)

T_A = ambient air temperature (^oK)

$\frac{d\theta}{dz}$ = potential temperature lapse rate

Other symbols are defined as in equation (2) above.

A correction term for the additive effect of multiple-stack sources is also presented in Pooler (1967).

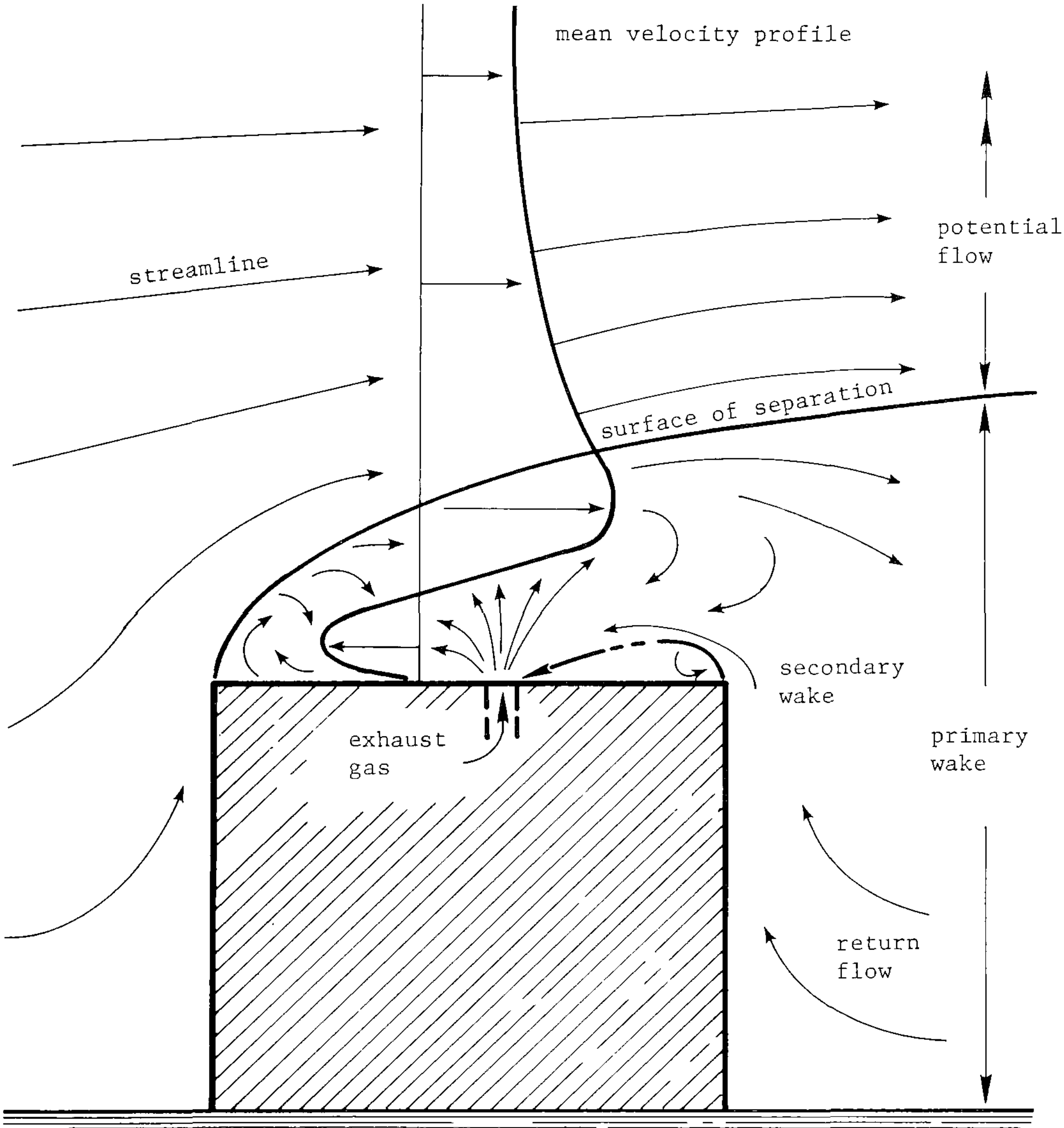
Other industries are also utilizing tall stacks for pollutant dispersion and anticipate results similar to the low measured ground level concentrations found near tall British power plant stacks. A case in point is the 1250 foot stack constructed in 1971 at a cost of $5.5 million to serve the Inco Copper Cliff smelter in the Sudbury district of Ontario, Canada. This gigantic stack is 116 feet in diameter at the base, tapering to just under 52 feet in diameter at the top. The interior diameter is 45 feet.

STATE OF THE ART

Considerable research is being conducted to further quantify the dilution effects of tall stacks and to develop better models for predicting the dispersion of power plant effluent in complex terrain and meteorological regimes. An extensive series of field experiments is being conducted, called the Large Power Plant Effluent Study (LAPPES), near Indiana, Pennsylvania and a report has been published by Schiermeier and Niemeyer (1968). Field measurements include determining plume geometry by laser-radar, in-plume and ground level SO_2 concentrations, vegetation damage and meteorological conditions during the experiments.

A summary of recent European studies dealing with plume rise and stack effluent dispersion is contained in the July 1967 issue of the journal Atmospheric Environment.

In addition another recent publication which contains a specific chapter on calculating effective stack height is the ASME Recommended Guide for the Prediction of the dispersion of Airborne Effluents(1973).



Typical flow pattern around a cube with one face normal to the wind.

Effective Stack Height/Plume Rise

A comprehensive literature survey in this field was conducted by NAPCA (EPA) and incorporated into an annotated bibliography of over 200 references. The publication, "Tall Stacks, Various Atmospheric Phenomena and Related Aspects" (1969), includes articles published through mid-1968.

A very recent series of EPA publications deal with air pollution aspects of emission sources. Of particular interest is OAP Pub. No. AP-96, "Electric Power Production - A Bibliography with Abstracts". Section D on air quality measurements and Section E on atmospheric interactions contain many specific references to plume rise determination, plume behavior, and pollutant concentrations associated with this class of sources.

Briggs in his publication, Plume Rise (1969), has presented both a critical review of the subject and a series of equations applicable to a wide range of atmospheric and emission conditions. These equations are being employed by an increasing number of meteorologists and are used almost exclusively within EPA. An important result of this study is that the rise of buoyant plumes from fossil-fuel plants with a heat emission of 20 megawatts (MW) - 4.7×10^6 cal/sec - or more can be calculated from the following equations under neutral and unstable conditions.

$$\Delta H = 1.6\ F^{1/3}\ u^{-1}\ x^{2/3} \qquad (5)$$

$$\Delta H = 1.6\ F^{1/3}\ u^{-1}\ (10\ h_s)^{2/3} \qquad (6)$$

where:

- ΔH = plume rise
- F = buoyancy flux
- u = average wind at stack level
- x = horizonal distance downwind of the stack
- h_s = physical stack height

Equation (5) should be applied out to a distance of $10\ h_s$ from the stack; equation (6) at further distances.

The buoyancy flux term, F, may be calculated from:

$$F \approx \frac{g\ Q_H}{\pi c_p \rho T} \simeq 3.7 \times 10^{-5} \left[\frac{m^4/sec^3}{cal/sec}\right] Q_H \qquad (7)$$

where:

g= gravitational acceleration
Q_H= heat emission from the stack, cal/sec
c_p= specific heat of air at constant pressure
ρ= average density of ambient air
T= average temperature of ambient air

Alternatively, if the stack gases have nearly the same specific heat and molecular weight as air, the buoyancy flux may be determined from:

$$F= \frac{\Delta T}{T_s} \quad g\ v_s\ r^2 \qquad (8)$$

where the notation has been previously defined.

In stable stratification with wind equation (5) holds approximately to a distance $x = 2.4\ u\ s^{-1/2}$ where:

$$s = \frac{g}{T}\ \frac{\partial\theta}{\partial z}\ \text{, a stability parameter} \qquad (9)$$

$\frac{\partial\theta}{\partial z}$ = lapse rate of potential temperature

Beyond this point the plume levels off at about

$$\Delta H = 2.4 \left(\frac{F}{u\ s}\right)^{1/3} \qquad (10)$$

However, if the wind is so light that the plume rises vertically, the final rise can be calculated from:

$$H = 5.0\ F^{1/4}\ s^{-3/8} \qquad (11)$$

For other buoyant sources, emitting less than 20 MW of heat, a conservative estimate will be given by equation (5) up to a distance of:

$$x = 3x^{*} \qquad (12)$$

where:

$$x^{*} = 0.52 \left[\frac{\sec^{6/5}}{\text{ft.}^{6/5}}\right] F^{2/5}\ h_s^{\ 3/5} \qquad (13)$$

Effective Stack Height/Plume Rise

This is the distance at which atmospheric turbulence begins to dominate entrainment.

Anyone who is responsible for making plume rise estimates should familiarize himself thoroughly with Briggs' work.

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Exercise Two

D. <u>PLUME RISE</u>
   Gary A. Briggs

E. PLUME RISE
   An audio presentation with supplementary
   lecture materials by G.A. Briggs

F. PROBLEM SET TWO

Please complete the exercise in the given order. Upon completion of components D and E, please work the problems in set two and forward to the Air Pollution Training Institute. A pre-addressed envelope is provided. A critique of your calculations will be returned with a copy of the correct calculations. Please be certain that your name and address is on each sheet. ALL CALCULATIONS MUST BE RETURNED TO RECEIVE CREDIT.

Air Pollution Training Institute
National Environmental Research Center
Research Triangle Park, North Carolina 27711
Attention: Plume Rise Instructor SI - 406

D

PLUME RISE

Dr. Gary A. Briggs *

Component D, Plume Rise, is a separate 80 page text.

Briggs [1] has recommended the following correction be made in Eq. 5.7, p. 58:

The constant 2.9 should be 2.4

1. Briggs, G. A. 1972. Discussion on Chimney Plumes in Neutral and Stable Surroundings. Atmos. Environ. 6:507-510, July 1972.

When you have completed the text, please turn to Component E, a taped lecture by Dr. Briggs. The audio cassette is standard format and should be suitable for any audio cassette recorder-playback unit. Please do not attempt to use the lecture cassette without referring to the supplementary written material.

Problem Set Two, which may be forwarded toward credit for an Air Pollution Training Institute certificate, completes the exercise.

*Dr. Gary A. Briggs, Research Meteorologist
Atmospheric Turbulence and Diffusion Laboratory,
National Oceanic and Atmospheric Administration

PLUME RISE

Gary A. Briggs *

Component E is an audio tape cassette of a lecture by Dr. Briggs. The cassette is standard format and should be suitable for any audio recorder-playback unit.

Please do not attempt to use the lecture cassette without referring to the following supplementary material.

$F_m \equiv$ momentum Flux/ $\pi\rho$

$= (\rho_o/\rho)\ w_o^2\ r_o^2$

$F \equiv$ buoyancy Flux/ $\pi\rho$ ← good

$= g\ (1-\rho_o/\rho)\ w_o\ r_o^2$ ← better

$= g\ (1-m_o/m)\ w_o\ r_o^2\ (T/T_o)$ ← best*

$+ g\ Q_H/(\pi\ C_p\ \rho\ T)$

$u \equiv$ Avg. wind speed at plume height

$=$ Avg. wind speed at stack height

$s \equiv (g/T)\ \overline{\partial\Theta/\partial z}$

$= (g/T)\ (\Delta T/\Delta z\ +\ 1°C/100m)$

From top of stack to top of plume

$$* \quad \frac{g\ \dot{Q}_H}{\pi\ C_p\ \rho\ T} = 8.9\ \frac{m^4}{sec^3}\ \frac{1013\ mb}{P}\ \frac{\dot{Q}_H}{MW}$$

*Dr. Gary A. Briggs, Research Meteorologist
Atmospheric Turbulence and Diffusion Laboratory,
National Oceanic and Atmospheric Administration.

Should heat of condensation be included in Q_H?

Usually not, but --

1. Calculate rise Δh on basis of no condensation.
2. Calculate max. volume flux $V_{max} = \pi\ u\ (0.5\ \Delta h)^2$
   ($u \geq 0.2\ F^{1/4}\ s^{1/8} = 0.08\ (F/sec)^{1/4} \approx 0.5$ m/sec)
3. Calculate moisture capacity of entrained air:

   $$Q_{cap} = \left(q_{sh} - \overline{q_a}\right) V_{max} ,$$

   where q_{sh} is the saturation specific humidity at the height of the plume ($h = h_s + \Delta h$) and $\overline{q_a}$ is the average specific humidity through the plume rise layer. q_{sh} is a function of the ambient temperature at height h.
4. Subtract Q_{cap} from efflux rate of water vapor, Q_{wv}.

   If remainder is positive, assume condensation adds (Q_{wv} - Q_{cap}) L to heat emission Q_H, where L is the heat of fusion.
5. Recalculate buoyancy parameter F and new Δh.
6. Start at Step 2 again, using new value of Δh. Repeat calculation until desired degree of convergence is obtained.

Should evaporative cooling be included in Q_H?

Yes!

To be conservative, assume complete evaporation of water droplets in plume. Subtract Q_{wl} L from the heat emissions, where Q_{wl} is the efflux rate of liquid water.

If the total buoyancy turns out to be negative, so is the plume rise -- the plume falls to the ground, and:

$$V_{max} = 0.2\ u\ h_s^2 \quad ; \quad X = \frac{Q}{V_{max}} = \frac{Q}{0.2\ u\ h_s^2}$$

To be more exact, follow the procedure outlined for heat of condensation calculation, except:

1. Use appropriate formula for V_{max}:

   $V_{max} = 0.8\ u\ \Delta h^2$ if the buoyancy is positive

   $V_{max} = 0.2\ u\ h_s^2$ if the buoyancy is negative

2. If $Q_{wv} - Q_{cap}$ is negative, assume that evaporation subtracts $(Q_{cap} - Q_{wv})$ L, but no more than Q_{wl} L, from the heat emission

3. If resultant buoyancy ever becomes negative, terminate the calculation.

"Rule of lowest plume rise": consequences for buoyant plumes

1. $\Delta h = 5.0\ F^{1/4}\ s^{-3/8}$ or $\Delta h = 2.4 \left(\frac{F}{us}\right)^{1/3}$?

"Calm" formula gives lowest rise if $u < \left(\frac{2.4}{5.0}\right)^3 F^{1/4}\ s^{1/8}$.

At most, upper limit is about 0.5 m/sec ($F \approx 10^3\ \frac{m^4}{sec^3}$,

$s \approx 10^{-3}\ sec^{-2}$).

Therefore, for practical purposes, "calm" formula only applies to zero wind speed.

2. $\Delta h = 1.6\ F^{1/3}\ u^{-1}\ (3x^*)^{2/3}$ or $\Delta h = 2.4 \left(\frac{F}{us}\right)^{1/3}$?

"Neutral" formula gives lowest rise if $s < \left(\frac{2.4}{1.6}\right)^3 \left(\frac{u}{3x^*}\right)^2$,

or if $\pi\ u\ s^{-1/2} > 7x^*$.

Buoyant plume rise in stable conditions

Since $\Delta h = 2.4 \left(\frac{F}{us}\right)^{1/3}$ applies very widely to buoyant plumes in stable air, it is seen that Δh is not very sensitive to variations in u and s, particularly since they tend to be negatively correlated: on windy nights, the stability is small; wind speed increases with height, but stability decreases. Hence, "ballpark" estimates of Δh in such cases need not include u and s. On the basis of TVA and Bringfelt data, I recommend:

$$\Delta h = 17\ F^{1/3} \quad \text{(very stable or high wind)}$$

$$\Delta h = 35\ F^{1/3} \quad \text{(slightly stable or low wind)}$$

... where Δh is in m, and F is in m^4/sec^3

Buoyant plume rise in neutral conditions

"Plume Rise" recommends the "2/3 law" cut off at a distance x_{max} as a simple approximation to buoyant plume rise in neutral conditions, where $x_{max} = 10\ h_s$ (only for fossil fuel plants with $Q_H > 20$ MW) or $x_{max} = 3x^*$, with x^* being a function of F and h_s given by Eq. 4.35.

Recent examination shows that, within the range of present data, a simple cut-off distance proportional to $\sqrt{F}$ works just as well. This also avoids a problem in the case of very small source heights. Specifically,

$$x_{max} = 63\ \text{m}\ \sqrt{F/(\text{m}^4/\text{sec}^3)}$$

$$= 1250\ \text{ft}\ \sqrt{Q_H/(10^6\ \text{cal/sec})}$$

This results in a "final" rise

$$\Delta h = 25\ (\text{sec-m}^{-2/3})\ F^{2/3}/u$$

> Note: the $x_{max} = 10\ h_s$ formula was NOT intended to be applied to gas turbine plumes.

Comparison of 10 h_s, 3x* (Eq. 4.35) and 1250 $\sqrt{Q_H}$ for Table 5.1 of "Plume Rise".

SOURCE	$10h_s$ (ft)	3x* (ft)	1250 $\sqrt{Q_H}$ (ft)
Harwell	N.A.	1110	1310
Bosanquet	N.A.	1450	1550
Darmstadt	N.A.	1140	1150
Duisburg	N.A.	2100	1700
Tallawarra	N.A.	2040	2140
Lakeview	4930	4890	4250
Earley	N.A.	1450	1550
	2500	2300	2700
Castle Donington	4250	4500	4300
	4250	5100	5000
Northfleet	4900	4200	3500
	4900	5000	4300
Shawnee	2500	2400	2900
Colbert	3000	2920	3250
Johnsonville	4000	4200	4100
Widows Creek	5000	5700	5100
Gallatin	5000	5800	5100
	5000	4400	3650
Paradise	6000	7100	5800

Exercise Three

G. SOME RECENT ANALYSES OF PLUME RISE OBSERVATIONS
Gary A. Briggs

H. ESTIMATION OF PLUME RISE
D. Bruce Turner

I. PROBLEM SET THREE

Please complete the exercise in the given order. Upon completion of components G and H, please work the problems in set three and forward to the Air Pollution Training Institute. A pre-addressed envelope is provided. A critique of your calculations will be returned with a copy of the correct calculations. Please be certain that your name and address is on each sheet. ALL CALCULATIONS MUST BE RETURNED TO RECEIVE CREDIT.

Air Pollution Training Institute
National Environmental Research Center
Research Triangle Park, North Carolina 27711
Attention: Plume Rise Instructor SI - 406

G

SOME RECENT ANALYSES OF PLUME RISE OBSERVATIONS

Gary A. Briggs *

Introduction

Good estimates of plume rise are required to predict the dispersion of continuous gaseous emissions having large buoyancy or a high efflux velocity. The rise of such emissions above their source height often account for a considerable reduction of the concentration experienced at the ground.

According to a recent critical survey on the subject, several dozen programs of plume rise observations have been carried out and the results published. This alone does not solve the problem, however. The quality of these observations varies considerably, and in some cases important parameters were not measured. The picture is also blurred by the presence of turbulence in the atmosphere, which causes the plume rise to fluctuate rapidly in many situations. The great number of empirical plume rise formulas in the literature reflect these uncertainties. Each formula is based on an analysis of one or more sets of observations, but each time a different style of analysis or a different collection of observations is used, a different empirical formula results. When applied to new situations, the predictions of these formulas sometimes differ by a factor of ten. Obviously, great care in the analysis of available observations is required.

The present paper is a summary of some of the analyses of observations made in my recent critical review on plume rise and in my doctoral dissertation. Both of these works include extensive comparisons of observations with formulas; care was taken to categorize the data according to the type of source and the meteorological conditions, and to weight the data according to the quality and quantity of observations they represent.

* Dr. Gary A. Briggs, Research Meteorologist
Atmospheric Turbulence and Diffusion Laboratory,
National Oceanic and Atmospheric Administration.

Atmospheric Turbulence and Diffusion Laboratory Contribution Number 38. This paper was presented at the 1970 International Air Pollution Conference of the International Union of Air Pollution Prevention Associations and portions have been extracted for publication by the Journal of the Air Pollution Control Association.

Momentum Conservation and the "1/3 Law" for Jets

One of the major findings of researchers in the field of plume rise is that the radius of a plume bent over in a wind is approximately proportional to the rise of the plume centerline above its source height.[1,2,3,4] This is true for a considerable distance downwind of the source, at least several stack heights. Mathematically, we can express this by

$$r = \gamma z \tag{1}$$

where r is a characteristic plume radius, γ is a constant (dimensionless), and z is the rise of the plume centerline. Surprisingly, this simple relationship accounts very well for the great bulk of observed plume rises, when it is used with appropriate conservation assumptions.

For instance, no outside forces act on a non-buoyant plume (jet) rising through unstratified surroundings, so we might expect that the total flux of vertical momentum in the plume is conserved. If the plume is only slightly inclined above the horizontal, is nearly the same density as the ambient air, ρ, and has a horizontal component of motion nearly equal to the mean wind at that height, u, then the flux of mass is approximately $\rho\pi r^2 u$. The flux of vertical momentum is then $w(\rho\pi r^2 u)$, where $w = u\,dz/dx$, the vertical velocity of the centerline of a plume segment moving downwind at a speed u; x is the distance downwind of the source. We then have

$$w\,u\,r^2 = u^2\gamma^2 z^2 dz/dx = F_m = \text{constant}, \tag{2}$$

where F_m is the initial vertical momentum flux divided by $\pi\rho$. For a jet having the same density as the ambient air, which must be true if it is non-buoyant, F_m is given by

$$F_m = w_o^2 r_o^2 = w_o^2 D^2/4, \tag{3}$$

where w_o is the mean efflux velocity, r_o is the radius of the stack, and D is its diameter, assuming a circular, vertical source. Integration of Equation 2 yields the prediction that

$$z^3 = (3F_m / \gamma^2 u^2)x$$

$$\Delta h = z = (\beta w_o^2 D^2/4\gamma^2 u^2)^{1/3}\ x^{1/3} \tag{4}$$

$$\Delta h/D = (3 / 4\gamma^2)^{1/3}\ R^{1/3}\ (x/D)^{1/3},$$

where $R = w_o / u$, the ratio of the efflux velocity to the wind speed. In this paper, the "plume rise", Δh, is identified with the height of the plume centerline above the source ($\Delta h = z$).

The above prediction that the rise of a jet is proportional to the one-third power of distance downwind, the "1/3 law", is very well confirmed by the available observations on jets. These are plotted in Figure 1 for the data in which R = 2, 4, 8, 16, and 40. (The code identifying the six experiments is given in Reference 2.) Surprisingly, the dotted lines representing the "1/3 law" give fair agreement with observations even in the upper-left part of the figure, where the plumes are more nearly vertical than horizontal (the derivation of Equation 4 utilizes the assumption that the plumes are only slightly inclined). However, the data indicate a stronger dependence on R than the two-thirds power. Specifically, the dotted lines represent Equation 4 with

$$\gamma = 1/3 + R^{-1} \tag{5}$$

Thus, it appears that the "entrainment constant", γ, varies with the ratio of efflux velocity to wind speed for a jet. This turns out not to be true for a buoyant plume. Substituting this expression for γ into Equation 4, we have finally

$$\Delta h/D = 1.89 \left(\frac{R}{1+3R^{-1}}\right)^{2/3} (x/D)^{1/3} \quad . \tag{6}$$

Buoyancy Conservation and the "2/3 Law" for Buoyant Plumes

If a buoyant plume is rising through unstratified surroundings and it neither gains nor loses buoyancy through radiation, ordinarily the total flux of buoyancy is conserved (for an exemption, see Reference 5). Applying Newton's Second Law to a segment of a bent-over plume moving downwind with the mean speed of the wind, we find that the rate of vertical momentum flux increase equals the buoyancy flux:

$$d(w\ u\ r^2)/dt = u d(w\ u\ r^2)/dx = b\ u\ r^2 \quad , \tag{7}$$

where b is a characteristic buoyant acceleration of the plume and bur^2 is the buoyancy flux divided by $\pi\rho$. The initial value of bur^2 is given by

$$F = g(1-\rho_o / \rho)\ w_o r_o^2 \quad , \tag{8}$$

where g is gravitational acceleration and ρ_o is the density of the effluent at the stack. A better determination of F, that accounts for alteration of buoyancy due to dilution with ambient air, is given by

$$F = g(1-m_o / m)\ (T/T_o) w_o r_o^2 + gQ_H / (\pi c_p \rho T) \quad , \tag{9}$$

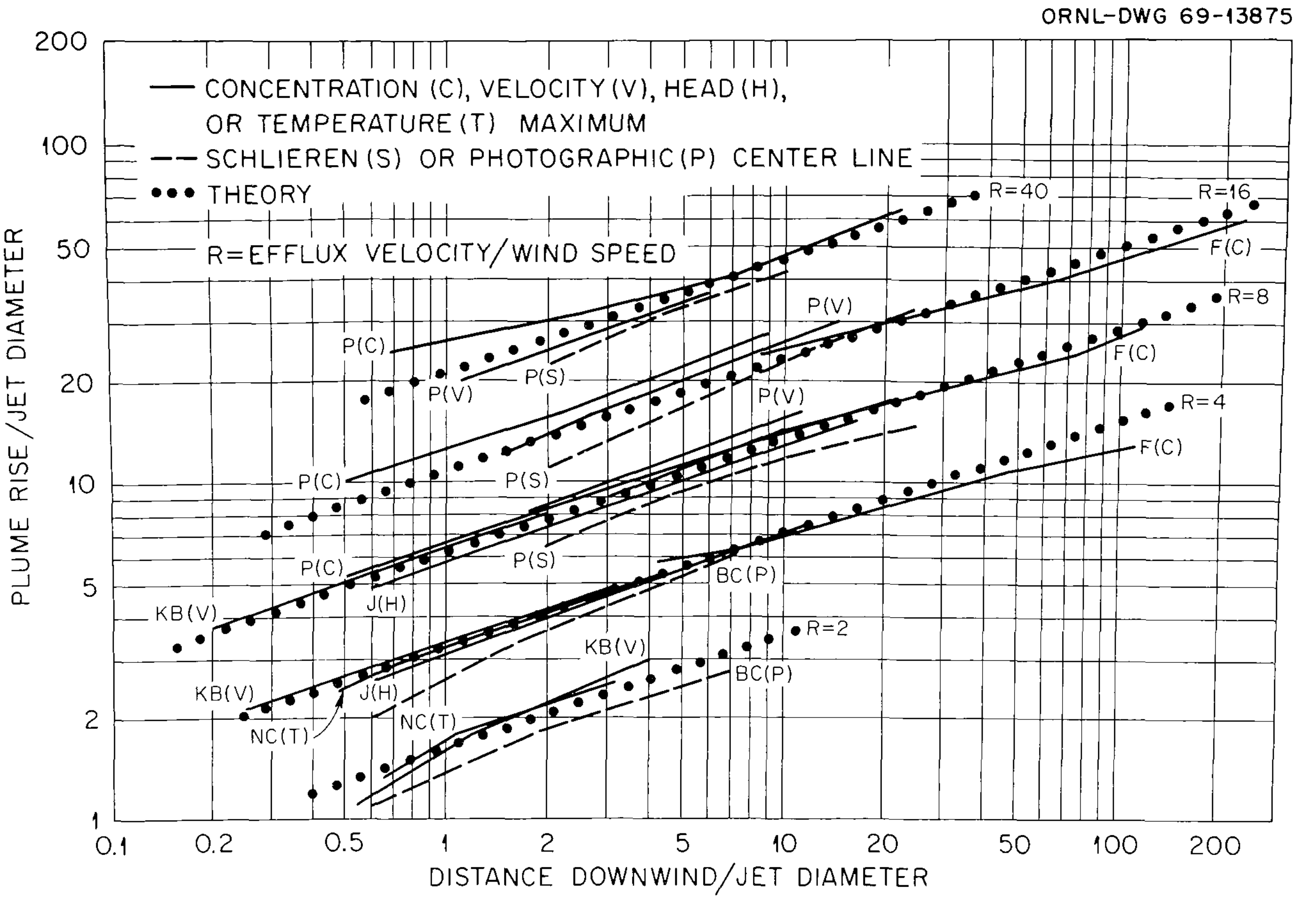

Figure 1. Rise of jet centerlines versus distance downwind for R = 2, 4, 8, 16, and 40. (dotted lines are Equation 6)(see Reference 2 for code identifying experiments)

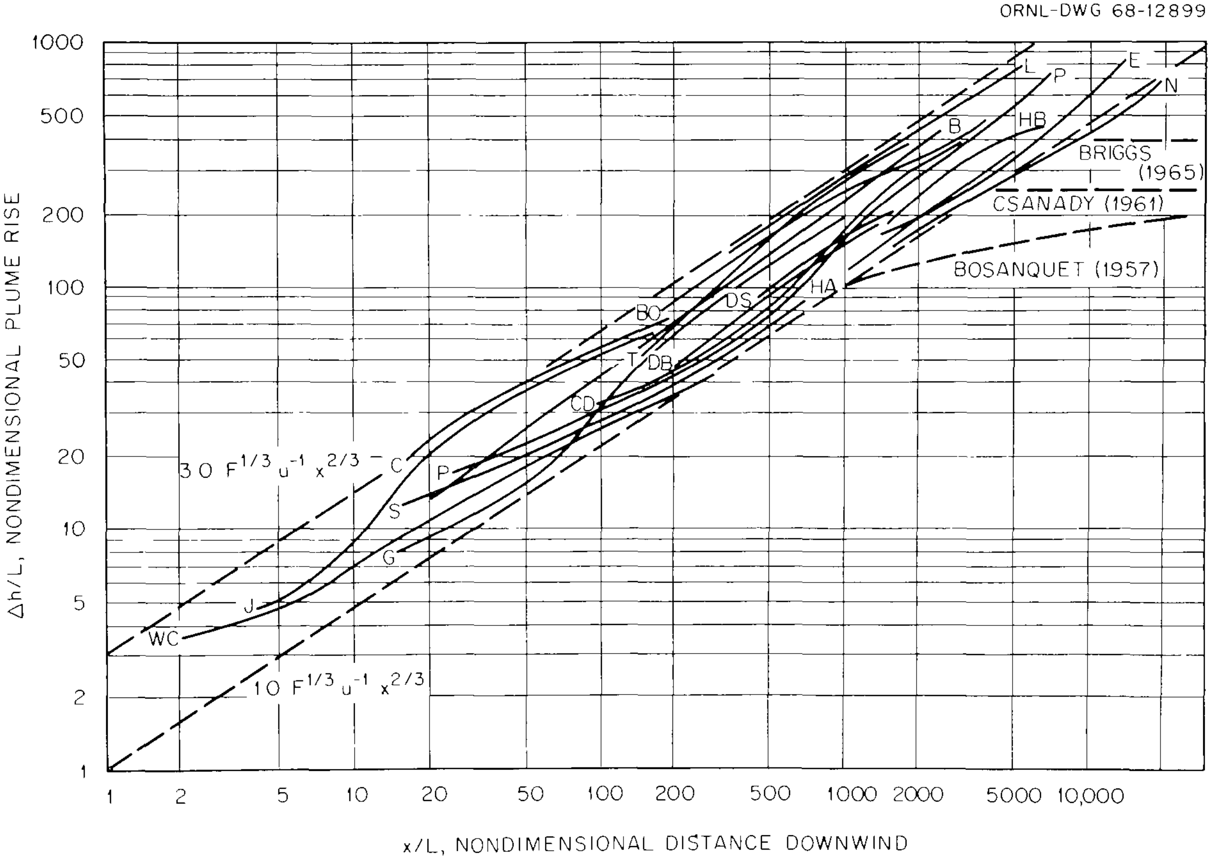

Figure 2. Buoyant plume rise versus distance downwind, both nondimensionalized by $L = F/u^3$ (see Reference 1 for code identifying source)

where m is the mean ambient molecular weight (28.9), T is the ambient absolute temperature, c_p is the specific heat capacity of air (0.24 cal/gm -°K), and Q_H is the heat emission; subscript "o" denotes values for the efflux gas, instead of the ambient air. The quantity $g/(\pi c_p \rho T)$ is just a constant, 3.7 10^{-5} (m^4/cal-sec^2), times the ratio of standard sea level pressure to the ambient pressure. If the effluent has considerable latent heat due to water vapor and condensation of the plume is likely to occur near the source, as would be expected in cold or wet weather, this latent heat may be inclined in the determination of Q_H; otherwise, only dry heat should be considered.

If the process producing the effluent is uniform, F is proportional to the rate of production. For instance, for modern fossil fuel power plants, F is about 1.5 m^4/sec^3 times the megawatts per stack generated.

When $bur^2 = F =$ constant, Equation (7) integrates to

$$wur^2 = F_m + F\,x/u \tag{10}$$

This relation implies that buoyancy becomes more important than the initial momentum flux when $x > uF_m / F$. For a hot effluent with about the same heat capacity and mean molecular weight as air, this occurs at $x = uw_o/(g(T_o/T-1))$, a distance of the order of 5 seconds times the wind speed for most hot plumes. Then the effect of F_m quickly becomes negligible, and for the region in which Equation (1) applies, we have

$$z^3 = (3F/2\gamma^2 u^3)x^2$$

$$\Delta h = z = (3F/2\gamma^2)^{1/3}\, u^{-1}\, x^{2/3} \tag{11}$$

$$\Delta h/L = (3/2\gamma^2)^{1/3}\, (x/L)^{2/3}$$

Where $L = F/u^3$, a characteristic length for the rise of buoyant plumes.

Most of the observations available on buoyant plume rise approximate this "2/3" of rise with distance downwind. This is illustrated in Figure 2, which is a superposition of curves hand drawn through scatter diagrams of $\Delta h/L$ versus x/L for 16 individual sources[1] (the sources are identified in Reference 1). Only data for stable atmospheric conditions are omitted. There is considerably greater scatter about the "2/3 law" in this figure than there is about the "1/3 law" for jets in Figure 1. The difference is probably due to the fact that all the experiments represented in Figure 1 were made under controlled conditions in wind tunnels or modeling channels, while all the observations shown in Figure 2 were made on plumes from real stacks in the atmosphere. This introduces the possibility of aerodynamic effects caused by buildings near the stack and uneven terrain, and also

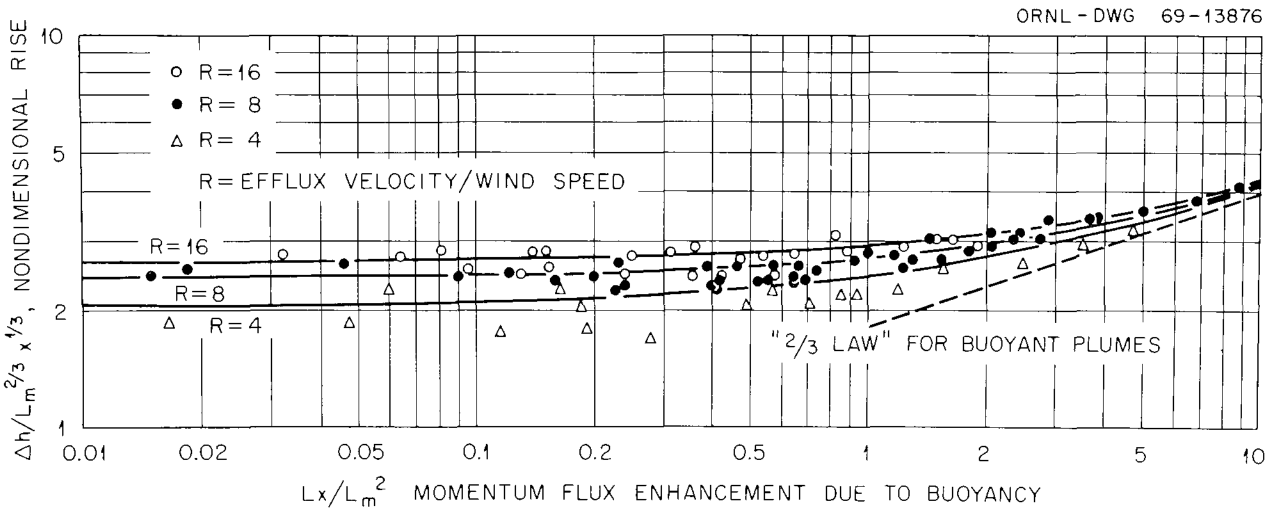

Figure 3. Nondimensionalized rise of model plume centerlines versus ratio of buoyancy-induced momentum to initial momentum. (solid lines are Equation 13)

assures greater fluctuations about the mean plume rise due to large turbulent eddies in the atmosphere. Atmospheric turbulence should also lead to more rapid mixing of plumes with ambient air, and therefore a downward departure from the "2/3 law" should occur at some point downwind; however, Figure 2 offers no particular support for this expectation. This means either that L does not correlate well with the distance of downward departure (leveling off), or that "leveling off" occurs at greater values of x/L than those measured up until now.

There is no evidence that γ is dependent on R for buoyant plumes[6], at least when R > 1.2. Below this value, downwash of the plume into the low pressure region in the wake of the stack is likely to occur. This reciprocal wind speed relationship predicted by Equation 11 with γ constant is well established for buoyant plumes in neutral conditions. An analysis[1] of photographed plume diameters and concurrent plume rises of TVA plumes from single stacks showed that $\gamma \doteq 0.5$. Bringfelt[3] obtained similar results, finding an average value of γ of 0.53 for eleven plumes in slightly stable or windy conditions and 0.46 for ten plumes in strongly stable or weak wind conditions. The behavior of buoyant plumes in stable conditions is well predicted by $\gamma = 0.5$, as will be shown, but the optimum fit[1] to the "2/3 law" at large distances downwind in neutral and unstable conditions corresponds closer to $\gamma = 0.6$, or

$$\Delta h = 1.6\, F^{1/3}\, u^{-1}\, x^{2/3} \tag{12}$$

Transition to Buoyancy-Dominated Rise

Equation 10 implies that a transition from the "1/3 law" for momentum-dominated rise to the "2/3 law" for buoyancy-dominated rise occurs as Fx/uF_m grows from small to large values. This prediction is confirmed by Figure 3, which plots observations of plume rise modeled in a channel by Fan.[7] The rises are divided by

$$L_m^{2/3}\, x^{1/3} = F_m^{1/3}\, u^{-2/3}\, x^{1/3}$$

so there should be no variation with $Fx/uF_m = Lx/L_m^2$ in the momentum-dominated region. This is seen to be approximately true in the left-hand side of the figure, where $Lx/L_m^2 < 0.3$. However, there is a separation of the points for different values of R in this region; this is easily accounted for by letting $\gamma = 1/3 + R^{-1}$, as was done for jets. There is a clear upswing and some convergence of the points in the right-hand side of the figure, and these appear to asymptotically approach the line representing the "2/3 law", Equation 11, with $\gamma = 0.5$.

The simplest way to describe this transition mathematically is to integrate Equation 10, after substituting $r = \gamma z$ and $w = u\,dz/dx$, and then substitute the empirical value of γ for jets in the momentum term ($\gamma = 1/3 + R^{-1}$) and the empirical value for γ for buoyant plumes in the buoyancy term (γ = 0.5). The result is

$$\Delta h = z = 3^{1/3} \left(\frac{F_m x}{(1/3 + R^{-1})^2 u^2} + \frac{F x^2}{2(0.5)^2 u^3} \right)^{1/3} . \qquad (13)$$

This equation is represented as solid lines in Figure 3, for R = 4, 8, and 16. It is seen to describe the transition region fairly well.

Stability-Limited Rise

When a plume rises in a stable environment, it entrains air and carries it upward into regions of relatively warm ambient air. Eventually, the plume's buoyancy becomes negative and its rise is terminated. If heat is conserved, that is, the motion is adiabatic, the rate at which each plume element loses temperature relative to the ambient temperature is just its rate of rise times the ambient potential temperature gradient (potential temperature, Θ, is the temperature that air would acquire if it were compressed adiabatically to a standard pressure, usually the mean pressure at sea level; the potential temperature gradient is just the real temperature gradient plus the adiabatic lapse rate, i.e., $\partial\Theta/\partial z = \partial T/\partial z + 1\,°C/100m$ in our lower atmosphere). The resulting decay of the buoyancy flux is expressed by

$$d(bur^2)/dt = u\,d(bur^2)/dx = -w\, sur^2 \qquad (14)$$

where $s = (g/T)\ \partial\Theta/\partial z$.

If we differentiate Equation 7 with respect to t and substitute in equation 14, we find that

$$d^2(wur^2)/dt^2 = u^2 d^2(wur^2)/dx^2 = -s(wur^2) \quad . \qquad (15)$$

This equation establishes the fact that $s^{-1/2}$ is a characteristic time for the decay of the momentum flux. If s is positive and approximately constant with height, the momentum flux is a harmonic function of $s^{1/2}$ t:

$$wur^2 = F_m \cos(s^{1/2} t) + s^{-1/2} F \sin(s^{1/2} t). \qquad (16)$$

Since r always increases, the plume centerline behaves like a damped harmonic oscillator. If the wind speed is constant with height, a jet ($F \doteq 0$) reaches its maximum rise at $x = ut = (\pi/2)\, us^{-1/2}$, and a buoyant plume ($F_m \doteq 0$) reaches its maximum rise at $x = \pi us^{-1/2}$.

The above conclusions are based on conservation assumptions alone, and do not

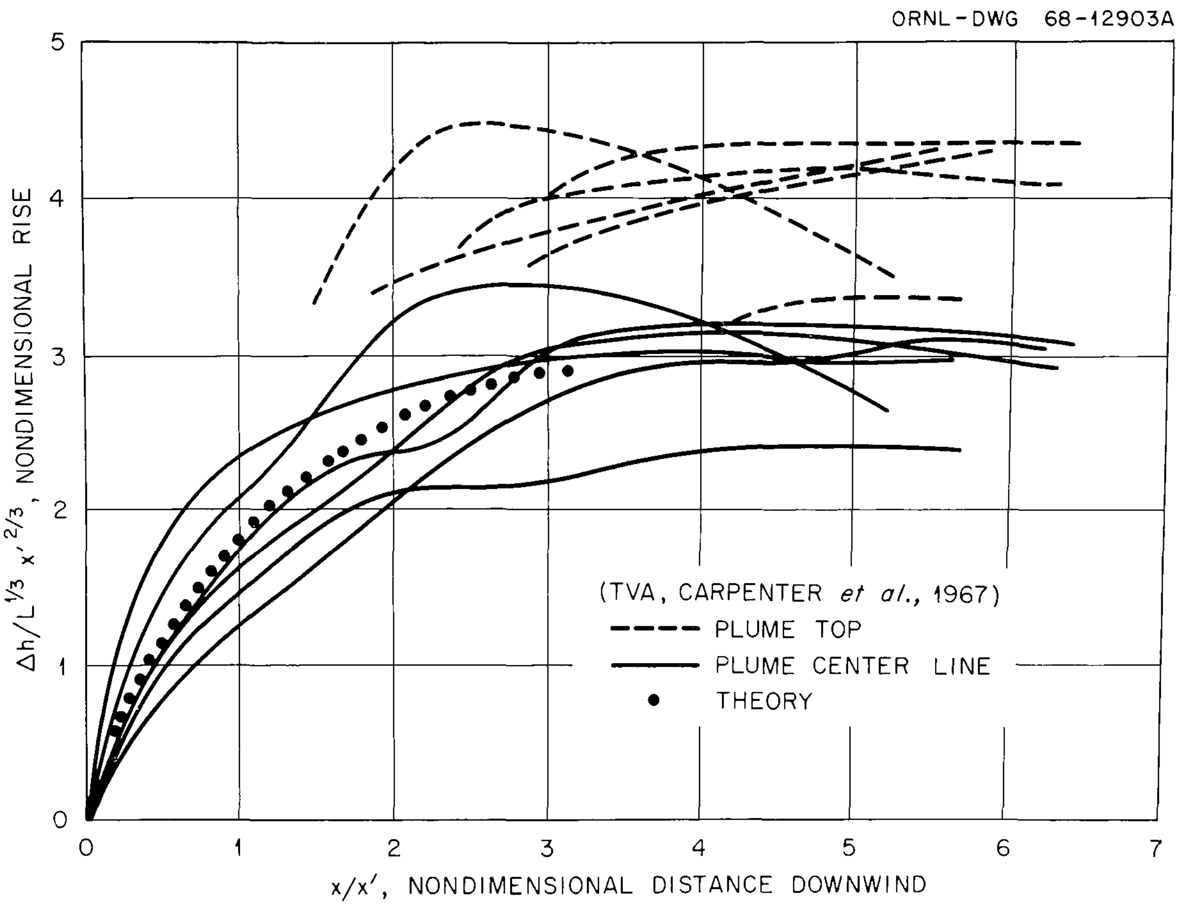

Figure 4. Nondimensionalized rise versus nondimensionalized distance downwind for single-stack TVA plumes in stable conditions. (dotted line is Equation 18)

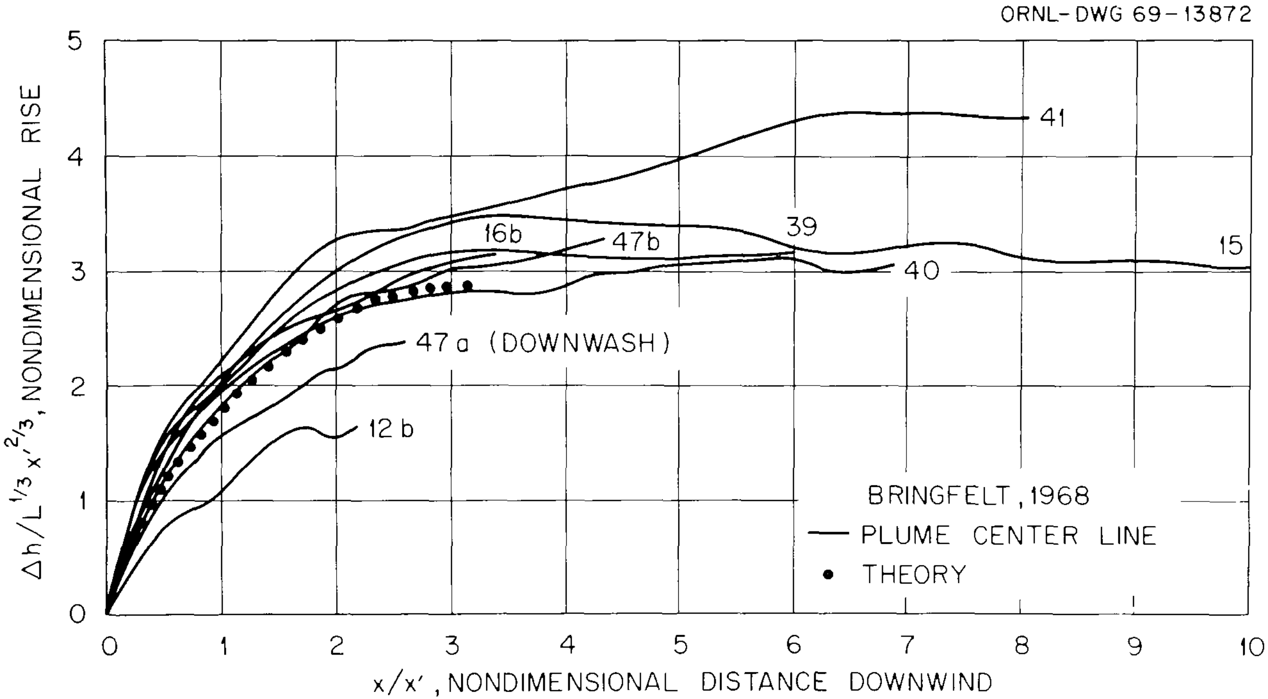

Figure 5. Nondimensionalized rise versus nondimensionalized distance downwind for plumes measured by Bringfelt in stable conditions. (dotted line is Equation 18)

depend on the behavior of r. Equation 1, $r = \gamma z$, is still a good approximation for r up to the point of maximum rise; obviously, it cannot apply beyond this point, as it would imply a shrinking plume. With u constant, $w = u dz/dx$, and $r = \gamma z$, Equation 16 can easily be integrated. For a jet we find that

$$\Delta h = z = \left(\frac{3F_m}{\gamma^2 u s^{1/2}}\right)^{1/3} (\sin (x/x'))^{1/3}$$

$$= 3\ (1 + 3R^{-1})^{-2/3} \left(\frac{F_m}{u\ s^{1/2}}\right)^{1/3} \text{(maximum rise)} \quad , \tag{17}$$

where $x' = us^{-1/2}$ and $\gamma = 1/3 + R^{-1}$. For a buoyant plume we find that

$$\Delta h = z = \left(\frac{3F}{\gamma^2 us}\right)^{1/3} (1- \cos\ (x/x'))^{1/3}$$

$$= 2.9 \left(\frac{F}{us}\right)^{1/3} = 2.9\ L^{1/3}\ x'^{2/3} \quad \text{(maximum rise)} \ . \tag{18}$$

There are sufficiently detailed data to verify Equation 18, which is shown as dotted lines in Figure 4 and 5. Both these figures show plume rise, divided by $L^{1/3}\ x'^{2/3}$, versus x/x'. To determine s, the measured potential temperature gradients were averaged throughout the layer of plume rise (from the top of the stack to the top of the plume). The first figure shows centerlines of TVA plumes from single stacks that were observed to level off in stable air. It also shows the observed rises of plume tops. The second figure shows the longest plume centerlines observed in very stable air by Bringfelt.[3] Both of these figures give excellent support to the prediction that the maximum rise is obtained at $x = \pi x'$. There is only a little evidence of oscillation beyond this point; evidently, most plume centerlines experience considerable damping through continued mixing beyond this point. The leveled-off plume rises, which range from 140 to 290m for the TVA data and from 60 to 160m for the Bringfelt data, seem to be well approximated by Equation 18 on the average. For the TVA data, the scatter about the predicted rise seems to be greatest in the rising stage, which

Table 1.
Ratios of Calculated to Observed Plume Rises in Stable Conditions

Formula	Bringfelt	TVA	Bringfelt and TVA
Holland	0.33 ± 73%	0.81 ± 07%	0.72 ± 39%
Priestley	0.74 ± 22%	0.44 ± 05%	0.47 35%
Bosanquet	1.09 ± 24%	1.22 ± 12%	1.20 ± 18%
Schmidt, m = 0	0.29 ± 07%	0.28 ± 24%	0.28 16%
Schmidt, m = 1/2	0.94 ± 27%	0.85 ± 25%	0.90 ± 27%
Equation 18	0.89 ± 07%	0.96 ± 08%	0.93 ± 08%

approximates the "2/3 law" when $x \lesssim 2x'$. There is less scatter in this stage in the Bringfelt data, which utilized more representative wind speed measurements and were taken during much more stable conditions.

Reference 2 also compares these observations with other formulas for buoyant plume rise in stable conditions, namely; the Holland formula,[9] minus 20% as suggested for stable conditions; Priestley's equation,[10] reduced to the case of a buoyant, point source; Bosanquet's formula,[11] similarly reduced; and Schmidt's formula,[12] with his parameter "m" set equal to 0 and 1/2. The centerline plume rises at a standard distance $x = 5x'$ were interpolated and averaged for periods in which there were at least five photographs of the plume at this distance. This yielded five periods each from the TVA and the Bringfelt observations. The ratios of calculated to observed plume rises were then calculated for each formula and each period. The median ratio and the average percentage deviation from the median ratio for each formula are shown in Table 1.

The TVA heat emissions were substantially higher than those observed by Bringfelt, so the high percentage deviation exhibited by the Holland and Priestley formulas may be because of too much and too little predicted dependence of rise on heat emission, respectively. Clearly, Equation 18 for maximum rise gives the most consistently good predictions for buoyant, stability-limited plume rise.

Turbulence-Limited Rise

The simple plume rise model outlined in the preceding section, based on $r = \gamma z$ and conservation assumptions, succeeds in predicting the approximate rise behavior of all available observations of plumes bent over in a wind. It is very similar to the successful model for nearly vertical plumes suggested by G.I. Taylor in 1945 and later developed by Morton, Taylor, and Turner. However, as it stands, it predicts unlimited rise in neutral ($s = 0$) and unstable environments ($s < 0$). This is contrary to the expectations of many plume rise observers, some of whom have assumed that the plume rise is the rise of the plume at the point that it becomes hard to follow. Yet, no observations made so far show any leveling-off tendencies, except in stable conditions.[1]

Nevertheless, it is quite reasonable to expect more rapid growth of the plume radius in neutral and unstable conditions, due to the presence of considerable environmental turbulence. This, in turn, leads to a reduced rise velocity and perhaps to a limited plume rise, at least in neutral conditions. The question is how to account for the enhanced growth of the plume radius. One way is to assume that only eddies of the same order of size as the plume radius are effective at mixing ambient air into the plume, and that these eddies are predominantly in the inertial subrange of the atmospheric turbulence spectrum. This part of the spectrum is adequately characterized by the eddy energy dissipation rate, ε, and eddies of the order of r in size have velocities of the order of $\varepsilon^{1/3}\, r^{1/3}$. This suggests the relationship

$$dr/dt = \beta\varepsilon^{1/3} r^{1/3} , \tag{19}$$

where β is a constant (dimensionless). This should not apply at small distances, where r is small and w is large; Equation 1 gives a faster growth rate at first ($r = \gamma z$ implies that $dr/dt = \gamma w$). In References 1 and 2, I developed a model identical to the one outlined so far, except for the assumption that Equation 19 applies instead of Equation 1 beyond the distance at which $\beta\varepsilon^{1/3}\, r^{1/3}$ becomes equal to γw. Since this model is based on an inertial range atmospheric turbulence entrainment assumption, I call this the "IRATE" plume rise model.[2]

For rise in neutral conditions, in which $bur^2 = F$ = constant, the "IRATE" model predicts a very gradual leveling of the plume centerline beyond the distance that $\beta\varepsilon^{1/3}\, r^{1/3}$ becomes equal to γw, designated by x^*. For a buoyant plume, the "2/3 law", Equation 11, applies to the first stage of rise. The distance of transition to the second stage is then given by

$$x^* = (2/3)^{7/5}\, (\gamma F)^{2/5}\, u^{3/5}\, (\beta\varepsilon^{1/3})^{-9/5} \tag{20}$$

If ε is approximately constant above the height at which this transition occurs, the second stage rise ($x > x^*$) is given by

$$\Delta h = (3F/2\gamma^2)^{1/3}\, u^{-1}\, x^{*\,2/3} \left[\frac{55}{16} - \frac{243}{32}\;\frac{(x/x^* + 5/8)}{(x/x^* + 5/4)^2}\right]. \tag{21}$$

While this is not a very simple formula, note that it is just the "2/3 law" times a function of x/x^*. A final rise, equal to 55/16 times the rise at the transition point, is approached, but only at a great distance; 90% of the asymptotic rise is achieved at $x = 20x^*$, and x^* can be as large as a kilometer for a very buoyant or very high source. It is unlikely that the maximum ground concentration occurs well before this point, especially since Equation 19 predicts an extremely large radius at $x = 20x^*$. If $\gamma = 0.5$ and the bottom of the plume is taken to be a distance $(\Delta h - r)$ above the source height, we find that the plume bottom begins to descend at about $x = 2x^*$ and spreads down to the source height $(r = \Delta h)$ at $x = 5x^*$. Since the growth of the plume radius is quite rapid at this point, the highest ground concentration should ordinarily occur in this neighborhood. It therefore seems prudent to use the rise at $x = 5x^*$, 2.3 times the rise at the transition point, as the "final" plume rise in neutral conditions. This rise is the same as that given by the "2/3 law" with $x = 3.5x^*$, and Equation 21 deviates from the "2/3 law" by only 11% at $x = 3.5x^*$. This suggests a much more practical prediction procedure for buoyant plume rise in neutral conditions:

$$\Delta h = (3F/2\,\gamma^2)^{1/3}\, u^{-1}\, x^{2/3} \qquad \text{when } x < 3.5x^*$$

$$\Delta h = (3F/2\,\gamma^2)^{1/3}\, u^{-1}\, (3.5x^*)^{2/3} \quad \text{when } x > 3.5x^*. \qquad (22)$$

With this approach, in a simple way we recognize the observed fact that plume rise is substantially a function of distance, yet we have a usable "final" rise formula.

In order to use the formulas based on the "IRATE" model, an estimate of x^* is needed; this requires values of β and ε for substitution into Equation 20. In Reference 1, β was conservatively estimated to be about unity, on the basis of observations on the growth rates of puffs and particle clusters (in order to infer the value of β from Equation 19, it was necessary to estimate values of ε, as this quantity was not measured in the puff and particle experiments). It is well known that $\varepsilon = 2.5u^{*3}/\bar{z}$ in the neutral surface layer,[13] but this layer extends only to a height of the order of $10^{-2}u^*/f$ in neutral conditions (u^* is the friction velocity, $\bar{z}$ is the height above the ground, and f is the Coriolis parameter); in mid-latitudes, this height is of the order of 10 seconds times the wind speed. However, most plumes rise to heights above this layer, where ε is less dependent on wind speed and height.

In a convective mixing layer, such as exists in the lowest few hundred to few thousand meters on any sunny, non-windy day, the average value of ε is about $(1/2)\, gH/(c_p \rho T)$, where H is the heat flux transported upward from the ground;[2] a fairly strong heat flux, 1 cal/cm^2-min, corresponds to $\varepsilon = 30\ \text{cm}^2/\text{sec}^3$; the eddy dissipation rate is relatively constant with height, except near the ground, where the neutral surface layer expression dominates. Less is known about the variation of ε above the surface layer in neutral conditions.

If ε ceased to decrease with height at the top of this layer, it would be proportional to $fu*^2$, which is approximately proportional to fu^2. In Reference 1, measurements of ε at heights from 15 to 1200m were shown to fit $\varepsilon \propto u$ slightly better than $\varepsilon \propto u^2$ or ε = constant (measurements made in stable or convective conditions were excluded from this analysis). This result was especially convenient for application to the IRATE plume rise model, as $\varepsilon \propto u$ cancels out the wind speed in Equation 20; above the neutral surface layer, x* is approximately independent of u. However, the great variation in ε at these heights, of the order of ±50%, can be expected to account for considerable variations in the plume rise.

In the above analysis, definite decrease of ε values with height above the ground was also noted. Above a height somewhere between 100 and 300m the variation becomes much less. The best fit in the 15 to 300m range was given by

$$\varepsilon = 0.068\ (\text{m/sec})^2\ u/\bar{z}\quad . \tag{23}$$

Substituted in Equation 20, this gives

$$x^* = 2.16\ \text{m}(F/\text{m}^4/\text{sec}^3)^{2/5}\ (\bar{z}/\text{m})^{3/5}\quad . \tag{24}$$

In References 1 and 2, it was suggested that conservative values of x* and Δh would result by evaluating ε at the source height, h_s. Thus, $\bar{z} = h_s$, but no more than $\bar{z}$ = 300m, was substituted in Equation 24; for the present, let us designate this estimation of x* by x^*_1. A simpler estimate of x* is suggested by a plot of <u>total</u> plume height, $h_s^1 + \Delta h$, versus buoyancy flux. Using TVA[8], CERL[14,15] Bringfelt[16] and other observations, a quite conservative value for $\bar{z}$ is given by

$$\bar{z} = 22\ \text{m}\ (F/\text{m}^4/\text{sec}^3)^{3/8} \le 100\ \text{m}\quad . \tag{25}$$

The 100m maximum value of $\bar{z}$ may be overly conservative when the stack height itself is greater than 100m, but consider also the fact that ε does not diminish very much with height above this elevation. The predicted final value for Δh only depends on $\bar{z}^{0.4}$, and the scatter in the few plume rise data at large distances renders tentative any conclusion about the best evaluation of x* for these purposes. Equations 23 and 25 give a particularly simple way to evaluate x*, and can even be applied to ground sources without difficulty. Let us designate this estimate of x* by x_2^*:

$$\begin{aligned} x_2^* &= 14\text{m}\ (F/\text{m}^4/\text{sec}^3)^{5/8} && \text{when } F < 55\ \text{m}^4/\text{sec}^3 \\ x_2^* &= 34\text{m}\ (F/\text{m}^4/\text{sec}^3)^{2/5} && \text{when } F > 55\ \text{m}^4/\text{sec}^3\quad . \end{aligned} \tag{26}$$

A number of plume rise formulas for buoyant plumes in neutral conditions were compared with all available observations in Reference 1; TVA[8] and CERL[14,15] data each comprised about one-third of the data analyzed. The more recent observations by Bringfelt[3,16] were added to the comparisons in Reference 2. A similar analysis is summarized in Table 2. Since the relationship $\Delta h \propto u^{-1}$ seemed well verified[1] for neutral conditions, the average value of $u\Delta h$ for each source was calculated at the greatest distance downwind that was represented by at least three 30-120 min periods of observations with at least five Δh determinations each. The Bringfelt data had to be handled differently, because there was only one period of observation for many of the sources; accordingly, these are weighted only one-third as much as the "Reference 1" observations in the last column of Table 2. When appropriate measurements were available, it was required that $x \leq 2x'$, in order to exclude cases of stability-limited plume rise. Cases of probable downwash, terrain effects, etc. were eliminated in "select" set of observations in Reference 1, and agreement with almost every formula improved. Of 25 periods chosen by Bringfelt for analysis,[3] only 12 are selected here (periods 7, 8, 11b, 15, 18, 27a, 27b, 31, 29, 41, 47a, and 27b); the periods rejected greatly increase the scatter in the ratios of calculated to observed values for every formula tested, tending to obscure the comparison. Table 2 shows the median value of the ratio of calculated to observed plume rises, and the average percentage deviation of these ratios from the median, for eight different formulas of the $\Delta h \propto u^{-1}$ type. The last column combines the two sets of select data, with appropriate weighting.

Of the first three formulas, which are empirical, it is seen that the Moses and Carson[18] formula in which $\Delta h \propto Q_H^{1/2}$ gives the most consistent fit to the select data; the fit would be optimized by multiplying by a correction factor of 2. The next three formulas are based on the Priestley[10] model, the first being the asymptotic prediction of the first stage[19] that $\Delta h \propto Q_H^{1/4} u^{-1} x^{3/4}$.

This formula and the "2/3 law", Equation 11, are very similar, and neither predict any "final" rise; yet, both of these formulas give better agreement than the empirical formulas. The scatter in the Bringfelt data makes it difficult to conclude that any one of the six formulas is superior, as about ± 15% seems to be the lowest possible scatter. The differences between the last five formulas are also slight in the Reference 1 data, except in the select set. In this set, as well as in the weighted select data, it is seen that Equation 22 gives the best fit; this equation is simply the "2/3 law", Equation 11, terminated at a distance $x = 3.5x^*$. The second estimate for x^*, x_2^*, as given by Equation 26, seems to have a slight edge over $x^* = x_1^*$; the amount of scatter and the scarcity of data at large values of x/x^* makes this comparison of $x^* = x_1^*$ and $x^* = x_2^*$ inclusive. Which value of x^* to be preferred is mostly just a matter of convenience.

When it is not clear whether plume rise is turbulence-limited or stability-limited, an analysis[2] of the IRATE model with both factors included shows that Equation 18 or Equation 22, whichever gives the <u>lowest</u> rise in a given

Table 2.

Ratios of Calculated to Observed Plume Rises in Neutral Conditions

Formula	Reference 1 (all data)	Reference 1 (select)	Bringfelt (select)	Weighted, Select Data
Holland (9)	0.44 ± 37%	0.47 ± 26%	0.26 ± 32%	0.40 ± 35%
Stümke (17)	0.79 ± 27%	0.72 ± 24%	0.82 ± 35%	0.74 ± 29%
Moses and Carson (18)	0.54 ± 34%	0.48 ± 19%	0.53 ± 28%	0.48 ± 23%
Priestley (10, 19)	1.44 ± 26%	1.41 ± 18%	1.42 ± 15%	1.41 ± 17%
Lucas, et al. (14)	1.36 ± 21%	1.24 ± 22%	1.36 ± 13%	1.35 ± 19%
Lucas (20)	1.18 ± 20%	1.16 ± 14%	1.12 ± 17%	1.16 ± 15%
Equation 11 *	1.17 ± 23%	1.17 ± 12%	1.08 ± 13%	1.17 ± 13%
Equation 22 * ($x^* = x_1^*$)	1.12 ± 21%	1.13 ± 08%	0.99 ± 15%	1.11 ± 10%
Equation 22 * ($x^* = x_2^*$)	1.12 ± 17%	1.13 ± 06%	1.00 ± 13%	1.11 ± 09%

* with $\gamma = 0.5$

situation, offers a good approximation of the very complicated prediction that results when both ε and s are greater than zero. In unstable conditions, there is no strong evidence that the average plume rise differs much from its value at the same wind speed in neutral conditions, but the rise is much more variable.[1,2]

Summary and Conclusions

Direct analysis of plume rise observations and several comparisons of observations with a number of empirical and theoretical formulas have shown that very satisfactory predictions of plume rise are given by a rather spare physical-mathematical model. This model was briefly outlined here and is more rigorously developed in Reference 2; it basically consists of the assumptions that momentum, buoyancy, and potential temperature are conserved, that the horizontal component of motion of plume elements is essentially equal to the mean wind speed, u, and that $r = \gamma z$ in a first stage of rise and $dr/dt = \beta\varepsilon^{1/3} r^{1/3}$ in a second stage of rise (r is the characteristic plume radius, z is the rise of the plume centerline above the source height, ε is the eddy dissipation rate of ambient atmospheric turbulence, and γ and β are dimensionless constants). Empirical guidance is used in evaluating γ, β, and ε.

The assumptions that $r = \gamma z$ and that momentum is conserved in a non-buoyant plume (jet) in unstratified surroundings lead to a simple "1/3 law" of rise that fits a large variety of observed jet center lines:

$$\Delta h/D = 1.89 \left(\frac{R}{1+3R^{-1}} \right)^{2/3} (x/D)^{1/3} , \qquad (6)$$

where x is the distance downwind, D is the stack diameter, and R is the ratio of efflux velocity to wind speed. To derive Equation 6, one must assume that $\gamma = 1/3 + R^{-1}$. On the other hand, there is no evidence that γ is a function of R for buoyant plumes. The assumptions that buoyancy is conserved and that the initial plume momentum is negligible for a very buoyant plume in unstratified surroundings lead to the often-cited "2/3 law" of rise:

$$\Delta h = 1.6 F^{1/3} u^{-1} x^{2/3} , \qquad (11)$$

where F is the initial buoyancy flux divided by $\pi\rho$; complete expressions for F are given by Equations 8 and 9. The constant in Equation 11 is based on the best fit to data shown in Table 2, and corresponds to $\gamma = 0.6$. Only equations that include the second stage entrainment assumption that $dr/dt = \beta\varepsilon^{1/3} r^{1/3}$ give a better fit to observations of the rise of hot plumes in near-neutral conditions. For plumes in which both momentum and buoyancy are significant, Equation 13 gives a semi-empirical transition between Equations 6 and 11.

Buoyancy becomes the dominant factor for most hot plumes at a distance downwind of the order of five seconds times the wind speed.

The assumption that the potential temperature of entrained air is conserved leads to the prediction that a buoyant plume attains a maximum rise at a distance $x = \pi u s^{-1/2}$ in stable air ($s = (g/T)\partial\theta/\partial z$, g is gravitational acceleration, T is the absolute ambient temperature, θ is the ambient potential temperature, and $\partial\theta/\partial z = \partial T/\partial z + 1°\,C/100m$). This prediction is very well confirmed by plots of plume rise versus distance in stable conditions ($\partial\theta/\partial z$ and u are averaged from the top of the stack to the top of the plume). These plots also indicate that the "2/3 law" is approximated when $x < 2us^{-1/2}$ and that the plume centerlines level off at a height

$$\Delta h = 2.9 \left(\frac{F}{us}\right)^{1/3} \tag{18}$$

This corresponds to the maximum rise given by $r = \gamma z$ with $\gamma = 0.5$. Equation 18 gives substantially better agreement with observations than other formulas tested for buoyant, stability-limited rise. It should be noted that in very light winds the well-proven[1,2] formula of Morton, Taylor, and Turner best applies if it gives a lower plume rise than Equation 18:

$$\Delta h = 5.0\, F^{1/4}\, s^{-3/8} \tag{27}$$

In neutral conditions, a limited rise results only after the second stage entrainment assumption is utilized. A good approximation to the complete prediction for buoyant plumes in neutral conditions is given by

$$\begin{aligned} \Delta h &= 1.6\, F^{1/3}\, u^{-1}\, x^{2/3} && \text{when } x < 3.5x^* \\ \Delta h &= 1.6\, F^{1/3}\, u^{-1}\, (3.5x^*)^{2/3} && \text{when } x > 3.5x^*\ , \end{aligned} \tag{22}$$

where x* is the distance of transition from the first stage to the second stage of rise. This equation gives a somewhat better fit to observations than any other formula tested when x* is estimated by:

$$\begin{aligned} x^* &= 14\text{m}\, (F/\text{m}^4/\text{sec}^3)^{5/8} && \text{when } F < 55\ \text{m}^4/\text{sec}^3 \\ x^* &= 34\text{m}\, (F/\text{m}^4/\text{sec}^3)^{2/5} && \text{when } F > 55\ \text{m}^4/\text{sec}^3\ . \end{aligned} \tag{26}$$

This equation for x^* should be considered tentative, since it is based on limited empirical determinations of β and ε, and there is too much scatter in the few observed plume rises at large values of x/x^* to make any strong conclusions about x^*. Equations 22 and 26 apply satisfactorily to the mean rise in unstable conditions as well, and also in slightly stable conditions if they give a lower rise than Equation 18.

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H

ESTIMATION OF PLUME RISE

D. Bruce Turner *

The Environmental Application Branch has used the equations of Dr. Gary Briggs for a number of years to estimate plume rise. Dr. Briggs has revised the equation several times.(1,2,3)

The following procedures are consistent with the way in which the Meteorology Laboratory calculates Briggs' plume rise:

The following symbols are used:

π	A Constant - 3.14
g	Graviational Acceleration = 9.80 m sec^{-2}
T	Ambient Air Temperature, ^oK
u	Average wind speed at stack level, m sec^{-1}
v_s	Stack gas exit velocity, m sec^{-1}
d	Top inside stack diameter, m
T_s	Stack gas exit temperature, ^oK
V_f	Stack gas volume flow, $m^3 sec^{-1}$
F	Buoyancy flux parameter, $m^4 sec^{-3}$
x^*	Distance at which atmospheric turbulence begins to dominate entrainment, m
ΔH	Plume rise above stack top, m.
x	Downwind distance from the source, m.
x_f	Distance downwind to final rise, m.
$\partial\theta/\partial z$	Vertical potential temperature gradient of atmosphere, $^oK\ m^{-1}$
s	Restoring acceleration per unit vertical displacement for adiabatic motion in the atmosphere - a stability parameter, sec^{-2}

If T is not given, we have been using:

T = 293^{o}K (20^{o}C) for design calculations

$$V_f = \frac{\pi}{4} v_s d^2 = 0.785\ v_s d^2 \qquad (1)$$

$$F = \frac{g}{\pi} V_f \left(\frac{T_s - T}{T_s}\right) = 3.12\ V_f\left(\frac{T_s - T}{T_s}\right) \qquad (2)$$

*D. Bruce Turner, Chief, Environmental Applications Branch
Meteorology Laboratory, Environmental Protection Agency

May, 1973

For unstable or neutral conditions:

$$x^* = 14\ F^{5/8} \quad \text{For F less than 55} \tag{3}$$

$$x^* = 34\ F^{2/5} \quad \text{For F greater than or equal to 55} \tag{4}$$

The distance of the final rise is: $x_f = 3.5\ x^*$ (5)

The final plume rise:

$$\Delta H = \frac{1.6\ F^{1/3}\ (3.5\ x^*)^{2/3}}{u} \tag{6}$$

For x less than the distance of final rise:

$$\Delta H = \frac{1.6\ F^{1/3}\ x^{2/3}}{u} \tag{7}$$

For stable conditions, $\partial\theta/\partial z$ is needed

If $\partial\theta/\partial z$ is not given use:

$0.02\ {}^{o}K\ m^{-1}$ for stability E

$0.035\ {}^{o}K\ m^{-1}$ for stability F

$$s = g\ \frac{\partial\theta/\partial z}{T} = 9.806\ \frac{\partial\theta/\partial z}{T} \tag{8}$$

Calculate

$$\Delta H = 2.4 \left(\frac{F}{us}\right)^{1/3} \tag{9}$$

and

$$\Delta H = \frac{5\ F^{1/4}}{s^{3/8}} \quad \text{(plume rise for calm conditions)} \qquad (10)$$

Use the smaller of these two ΔH's

This is the final rise.

The distance to final rise is:

$$x_f = \frac{3.14\ u}{s^{1/2}} \qquad (11)$$

If you want to calculate rise for a downwind distance x less than x_f, this is given by

$$\Delta H = \frac{1.6\ F^{1/3}\ x^{2/3}}{u} \qquad (12)$$

which is the same equation used for unstable and neutral conditions.

Although (under stable conditions) the plume begins to rise according to the 2/3 power with distance, it does not continue the same rate of rise to the distance of final rise, x_f, given by equation (11). Therefore equation (12) will give a ΔH higher than the final rise at distances beyond about 2/3 x_f. It is therefore recommended that when using equation (12), the result be compared with the final rise and the smaller value used. In effect then, for determining the plume rise at a distance, x, (during stable conditions) the minimum value of the three values of ΔH determined by equations (9), (10) and (12) should be used.

Problem set three (component I) follows this article. An Air Pollution Training Institute certificate will be awarded upon satisfactory completion of the three problem sets and the return of a completed critique. All calculations must be returned to the Air Pollution Training Institute. A set of answer sheets will be returned to the learner.

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Appendix

CRITIQUE FORM

EFFECTIVE STACK HEIGHT SCRIPT

[1]EFFECTIVE STACK HEIGHT

A cued script of James L. Dicke's audio tape in Component A.

The topic I am going to discuss is effective stack height and its calculation. Before proceeding any further, we should define exactly what is meant by effective stack height. [2]Effective stack height is the height at which the plume centerline from a smoke stack becomes essentially level. To make sure that everyone understands, let's go through that again. The effective stack height is the height above ground at which the plume centerline becomes essentially level. [3]This is true whether we are talking about the plume from a large stack or [4]the emission from a small pipe or vent near rooftop, with an effective stack height not much above the top of the building.

[5]Effective stack height, which we will denote by capital H, is equal to the physical height of the stack, small h, plus the rise of the plume above the stack, delta h. [6]In this diagram, the effective stack height is reached near the right end of the slide, where the centerline of the plume is leveling out. When an effective stack height is specified, a virtual origin for the plume has been assumed. It is assumed that the plume is not emitted from the stack, but from some point above and possibly upwind of the stack. This virtual origin is such that a plume with dimensions similar to those of the real plume results at the distance where the effective stack height is reached. [7]Here you can see the virtual origin is slightly behind the stack. It is at height H, which is equal to the physical stack height, small h, plus the plume rise, delta h.

[8]Next, we should answer the question, "Why do we need to know the effective stack height?" Primarily, the effective stack height is required because of its effect on the ground-level concentration of contaminants. Its practical use in the diffusion equation can be demonstrated to you in other sessions. As the effective stack height increases, the concentration at ground level becomes less due to the fact that there is more atmosphere in which to dilute the pollutants. Also, as effective stack height increases, the distance to the maximum concentration is moved further downstream from

the source and the maximum concentration becomes smaller.

[9]As you can see from the example, if we assume that emissions from both stacks are equivalent, the concentration at some point downwind from the lower stack will be greater than that from the taller stack, since the emissions from the taller stack have a greater volume of the atmosphere in which to dilute before they significantly affect the ground. Also, the maximum concentration, due to the lower stack, will be greater and closer to the source than that for the taller stack, which will have a smaller maximum concentration with the point of maximum concentration off the slide some place to the right. [10]A practical example is considered in this slide taken near Brilliant, Ohio. Suppose that all three sets of stacks have equivalent emissions. It is apparent that the largest concentration will be caused by the middle plant. The plant on the right will cause the smallest concentrations since it has the greatest effective stack height.

[11]Again, effective stack height is equal to the physical stack height plus plume rise. Usually, the physical stack height can be measured or is already known. Therefore, we will be mainly concerned with calculating the plume rise.

[12]Most stacks have some plume rise, though there are cases where plume rise is zero or negative. The momentum and buoyancy of gases in a plume cause the plume to rise, sometimes to several times the physical stack height. As you can see in this slide, the effective stack height, which is near the top of the slide, is at least three times the physical stack height. [13]With no wind the plume might rise straight up until it reaches a level where there is some wind, and then level out quite rapidly, or expending all its buoyancy and momentum, it just remains stationary, forming a cloud. When there is some wind, the plume will gradually slope upward and over until it levels out. [14]In any case, we are interested primarily in the height at which the plume becomes level at some point downstream from the source.

[15]If there is no buoyancy or momentum, the plume becomes level almost immediately, especially if wind speeds are moderate. As you can see in this case, the plume does not rise at all after leaving the stack, but becomes level immediately. [16]Here plume rise is zero, and effective stack height is equal to the physical stack height. [17]However, in cases of very strong winds or improper stack design, aerodynamic downwash on the lee side of the stack can cause the plume to lower below the top of the stack. [18]In fact, where there are short stacks, or the effluent is being emitted right at rooftop, aerodynamic downwash on the lee side of the building is common. [19]Here plume rise can be thought of as being negative and, as a result, the effective stack height is less than the physical stack height. Another factor which can contribute to the lowering of a plume below the physical stack height is evaporative cooling of the moisture droplets in the plume. We will consider mainly cases where there is some plume rise, but we will discuss aerodynamic downwash and evaporative cooling briefly near the end of this session.

[20]We are interested in cases where there is rise of the plume. [21]Most of the plume-rise equations are obtained either empirically or through a theoretical derivation. The empirical equations are obtained by arranging the stack and atmospheric parameters so that visual observations or physical measurements of plume rise will be duplicated by the equation. Theoretical equations are derived from physical principles including dimensional analysis. However, both types of equations are dependent on data collected in a laboratory, a wind tunnel or in the field, since the theoretical equations usually include at least one constant which is dependent on sampled data.

All the equations which have been derived to date are subject to certain criticisms. These criticisms boil down to the fact that there is no one equation which is universally accurate and reliable. No one equation applies to all sizes of sources and under all atmospheric conditions. Some equations give a generally good estimate of plume rise, but only when the equation is used with the specific stack and under the atmospheric conditions for which the plume-rise equation was derived. Also, there is a general lack of plume-rise data on a large variety of sources and atmospheric conditions; this makes it difficult to come up with a widely applicable equation. Also, even when sufficient data are available for obtaining an equation, there is still the problem of little independent data with which to check the equation.

22 In the past 20 years there have been many attempts at expressions for plume-rise - twenty equations and more. The first two references which you see give a very good evaluation of plume-rise equations and demonstrate which equations give the most reasonable results. Some additional factors which affect plume rise are also discussed. The third reference, by Stern, gives a good review of plume-rise equations to 1968. These references are among those listed in Part B.

23 Plume rise is a result of the buoyancy and momentum of the stack effluent and the manner in which they are affected by the atmosphere.
24 The individual parameters which affect buoyancy and momentum with regard to the stack are: the stack diameter at the top of the stack in meters; stack gas velocity, meters per second; stack gas temperature, degrees Kelvin; and the heat content of the effluent, frequently expressed as calories per second. We will consider units only in the MGS system.
25 The individual atmospheric parameters which affect the buoyancy and momentum are: wind speed, meters per second; potential temperature lapse rate - degrees Kelvin per 100 meters; atmospheric stability - whether unstable, neutral or stable; atmospheric temperature - degrees Kelvin; and atmospheric pressure in millibars. In this session we will assume the mean molecular weight of the atmosphere and the stack gas are essentially the same.

26 Various combinations of these elements contribute individually to the momentum and buoyancy. The parameters which contribute to momentum are the stack gas exit velocity, the stack diameter at the top, and the atmospheric wind speed. The parameters which contribute to buoyancy are the stack gas exit temperature, the atmospheric temperature, potential temperature lapse

rate, and the heat content of the effluent. Usually, the buoyancy term is expressed either as a temperature difference between the gas and the atmosphere or as the heat content of the effluent.

27 Two of the plume-rise equations which frequently appear are the Bryant-Davidson and the Holland equations. These are of interest because they are widely used; they provide simple computation schemes; and they are conservative, thus providing a safety factor. By conservative, I mean that, if anything, they underestimate the amount of plume rise; this allows an over-calculation of the pollutant concentration at the ground. Thus, by using these equations, a ground level concentration will never be underestimated if the source conditions are similar to those for which the equation was derived. From the health and safety standpoint this is the desirable situation.

The Bryant-Davidson equation has, as you see here, the momentum term on the left and the buoyancy term on the right. The momentum term is based on wind tunnel experiments and the buoyancy term was added later to take buoyancy into consideration. The authors state that this equation should be applied only to stacks of moderate or great height; however, the stacks referred to are actually rather small by today's standards. Other problems with the equation are, there is no allowance for different atmospheric stabilities, and the equation is based on wind tunnel data rather than on data collected in the field.

28 The Holland equation, which we will go through rather thoroughly, is presented here. The first portion of the equation on the right is the momentum term, the other, with the temperature difference, is the buoyancy term. As you can see, there is a correction allowable for unstable and for stable conditions. The equation, as we see it here, is for neutral stability conditions. To apply it to unstable atmospheric conditions, it is appropriate to multiply the answer by 1.1 or 1.2; for stable atmospheric conditions, the effective stack height should be multiplied by 0.9 or 0.8. The data on which the Holland equation are based are: physical stack heights from approximately 50 to 60 meters, diameters in the range of 2 to 4 meters, stack gas exit velocities from 2 to 20 mps, exit temperatures from about 350 to 450°K, atmospheric wind speeds from approximately 0.5 to over 9 mps, and all atmospheric stabilities. The criticisms of this equation, which, I might mention, apply to many of the equations we have seen, are: it is empirical; the coefficient 2.68×10^{-3} has units which are per meter per millibar; the data used in obtaining the equation are limited; and the equation is conservative. The last item, as I stated earlier, is actually desirable.

29 The Bosanquet equation is the complex expression which you see here. The terms A, X, and X_o require separate calculations and the functions f_1 and f_2 are evaluated from tables. As you might imagine, this equation is rather difficult and time consuming to use. Although it gives a generally good approximation of plume rise, the approximations are consistently on the high side.

30 The Lucas equation was developed by the Central Electricity Research Laboratory in Great Britain. It is the first of several equations we will consider that does not have a momentum term, only a buoyancy term. In these equations it is assumed that the upward momentum of a plume is negligible compared to its buoyancy. The Lucas equation, as you can see, is expressed as the heat content of the effluent divided by wind speed multiplied by an empirical coefficient, where the coefficient depends on the source being considered. This coefficient is variable for different source types and also can vary for sources of the same type. In other words, two power plants may have different coefficients. The approximations given by this equation are consistently on the high side; also this equation is not very sensitive to changes in stack diameter.

31 Now, having covered the background, we are prepared to discuss some of the more commonly used plume-rise equations and to look at some of their desirable and undesirable qualities. The Stümke equation is a supposed improvement of the Holland equation. The first term on the right hand side

of the equation can be considered the momentum term, stack gas exit velocity times diameter divided by wind speed; and the second term, which expresses the temperature difference between the gas and the atmosphere, can be thought of as the buoyancy term. This equation gives a good approximation of plume rise, but it predicts a bit high for some of the observed data. However, for a widely applicable equation, it probably gives as good a result as any other equation.

32 The CONCAWE equation also has a European background. It was developed by a group concerned with emissions from oil refineries. The equation gives reasonably good estimates of plume rise, but it is not recommended for use with large power plants.

33 Briggs has derived an equation from dimensional considerations. The general equation states that the plume rise is equal to the buoyancy flux, F, to the 1/3 power, times the downward distance, to the 2/3 power, divided by the wind speed measured at stack height. The first equation should be applied out to distances up to ten times the physical stack height. At greater distances the plume is assumed to have leveled off and if there is no change in wind speed or stability, the plume centerline will remain at an essentially constant height. In addition, these two equations should be applied only to sources emitting at least 20 megawatts of heat or at least 5 million calories per second.

Under stable conditions a stability parameter, s, is introduced, proportional to the potential temperature lapse rate.

If there is no wind, the above equations become meaningless and thus the last equation should be used.

34 In this slide we can see how to calculate the buoyancy flux and stability parameter terms. The buoyancy flux is equal to the temperature difference between the stack gases and the atmosphere divided by the stack gas temperature times the acceleration of gravity, the stack gas exit velocity and the square of the stack radius, or it may be approximated by a constant times the heat emission expressed in calories per second. The stability parameter 's' is equal to the acceleration of gravity divided by the atmospheric temperature at stack height multiplied by the lapse rate of potential temperature through the layer in which the plume is dispersing. Anyone who is responsible for making plume rise estimates should be thoroughly familiar with Briggs' work.

The last three equations we have considered, the Lucas equation, the CONCAWE equation, and the Briggs equation, have been getting quite a bit of discussion recently. The series of Briggs equations are considered to be the most up-to-date equations and those most applicable to large power plants, which are the source of many air pollution problems. However, as I indicated, they are not all-encompassing, and do have certain deficiencies.

35 One other thing we should consider with regard to plume-rise equations is some of the work done by Moses and Carson. They have taken the basic form of many plume-rise equations with a momentum term to some power times a coefficient plus a buoyancy term to some power times a coefficient plus a constant. They have taken this equation, used much of the available plume-rise data, and by using regression techniques, have determined values for the coefficient. They have done this in three ways for the equation as you see here, with $C_5 = 0$, and with just the buoyancy term. In most cases the plume-rise equations which resulted are accurate to within approximately 30 meters. One other item that should be noted about this technique is that for some of their evaluations they got a negative coefficient for C_2; this, in effect, says that the momentum of a plume detracted from the plume rise, which may be unrealistic. This indicates the problems involved in deriving an equation where it is fitted only to describe the data, without proper consideration of the physical realities.

36 Now, let us take a look at the results that several of these equations give in predicting plume rise. We will consider the Lucas, the Briggs, and the Holland equations, as applied to heat emission data from power plants. As you can see, the Lucas equation overpredicts for most of the data. The

Briggs equation goes through approximately the middle of the data. The Holland equation underpredicts for all the data except for sources with very large heat emissions. Remember, however, the original data used to develop the equation did not include heat emissions of this magnitude. Therefore, if the Lucas equation is used, you would be predicting more plume rise than actually occurred, the Briggs equation would give a good average, and with the Holland equation, you would be obtaining an underestimate except for very large sources. Also, another problem with plume rise data is demonstrated here. These plume-rise observations are supposedly for the same stack under similar atmospheric conditions, and you can see the wide range of plume rises which are obtained.

[37]Now, let us consider an example in which we can use the Holland equation to calculate plume rise. [38]These data are from the TVA's Shawnee power plant near Paducah, Kentucky. The stack gas exit velocity is 14.7 mps, the diameter of the stack at the top is 4.27 meters, the height of the stack is 77 meters, its exit temperature is 416 degrees Kelvin. The atmospheric parameters are wind speed five meters per second, atmospheric temperature 288 degrees Kelvin, and pressure 1,000 millibars. In evaluating this equation you can see the computed plume rise is 63 meters. The effective stack height is equal to the physical stack height plus plume rise. The physical stack height was 77 meters and the calculated plume rise was 63 meters, giving us an effective stack height for this plant and these atmospheric conditions of 140 meters. [39]A similar calculation using the Briggs equation results in a plume rise of 158 meters. This is 2 and 1/2 times the rise we calculated using the Holland equation and, based on our previous discussion, is about what you might expect.

[40]We have discussed various ways to estimate effective stack height through plume-rise computations. It should be pointed out that there are adverse effects of meteorology and terrain which can make these plume-rise calculations unrealistic. The conditions which can adversely affect plume rise are elevated inversions, irregular terrain, and changing thermal regimes.

[41]If there is an inversion based at some level above the ground where the temperature of the air starts increasing with height, the inversion base will act as a lid on the rise of the effluent, not allowing it to penetrate through the inversion. [42]Some actual scenes in the New York City area when there were data to indicate an elevated inversion based at 1,500 feet, shows the effect of inversions on plume rise. Here we see a plume which appears to be rising and gradually leveling out. Then, all of a sudden, its top becomes level, as if sheared off, or as if somebody put their hand down on top of it. [43]Another good example is this slide where, in the upper left hand corner, the plume, which is relatively small, can be seen to be rising gradually, then suddenly flattens out.

[44]In cases of complex terrain, under reasonably stable flow on the windward side of a hill, the plume actually rises over the hill instead of impacting into the side of the hill. This is where a plume-rise calculation would indicate that the plume should impact on the side of the hill where, in fact, it rises over the hill. On the downwind side, the turbulence induced by the terrain causes the plume to be downwashed, so an effective stack height calculation here would be of no use since the plume is actually lowered by induced turbulence and is not allowed to rise naturally.[45]In cases of complex terrain and highly unstable meteorological conditions, as you can see here, a calculation of an effective stack height would be of no use in calculating the impact on the hill to the right, since the plume is looping and has a much stronger impact on the hill than a calculation using an effective stack height would indicate. The plume impinges right on the hill.

[46]In this case, we have a plume emitted over grassy terrain behind the hangers. An effective stack height is established then, as the plume moves out over the concrete runways where the convective turbulence is much greater, the plume seems to be lifted to a new effective stack height.

[47]Now we will consider what can be called negative plume rise, consisting of evaporative cooling of the effluent and aerodynamic downwash. A good

example of the negative plume rise caused by evaporative cooling is what happened to a power plant in Great Britain several years ago. In this case the effluent was put through a spray tower to absorb gases. The gases were thus cooled and saturated with water vapor. Contact with the cold surfaces of the duct work caused further cooling and condensation of the water vapor. When the effluent was released into the atmosphere, the condensed water droplets evaporated, withdrawing the latent heat of vaporization from the surrounding plume and air, thus causing the plume to cool below the atmospheric temperature. The plume thus had negative buoyancy and the effective stack height was reduced to below the physical stack height. In this case, the plume actually came right down to the ground. For this plant the adverse effect caused by evaporative cooling resulted in ground-level concentrations which were greater than before anything had been done to the stack effluent.

48 With regard to aerodynamic downwash, eddies which are the result of mechanical turbulence around a building or low stack can affect the effective stack height. They cause the plume to be downwashed. This is especially so when the wind speed is high, momentum is small, and the plume is emitted horizontally. Most knowledge about this situation has come from wind tunnel studies such as those conducted by Halitsky at New York University. The results of these investigations show that maximum downwash around a rectangular building occurs when the wind direction is at a 45° angle to the major axis of the structure and is a minimum when it is parallel to this axis. Maximum downwash would occur when the wind blows from one corner of the building to the opposite corner, and the least downwash would be when the wind was blowing parallel to the building. **49** Also, it has been shown that effluents from flush openings on rooftops frequently flow in a direction opposite to the wind, due to counterflow induced by turbulence along the roof in the turbulent wake above the building. **50** The region of disturbed flow extends up to twice the building height and 5 to 10 times its height downwind as indicated by the streamlines. **51** Two rules of thumb which are commonly used to prevent downwash are (1) the stack gas exit velocity should be 1.5 times the average wind speed to prevent downwash in the wake of the stack, and (2) a stack should be 2.5 times the height of the building adjacent to the stack, to overcome building turbulence. These items should be kept in mind whenever one deals with a situation where downwash could be taking place, although there is no really good quantitative way to handle all situations. **52**Here is an example of a power plant which has short stacks certainly less than two times the height of the adjacent building, and you can see the inferior stack which is subject to aerodynamic downwash. **53**Downwash is not readily apparent here but I have seen the situation when the effluent came down directly on top of the river, flowed across the river right above its surface and up the hill on the opposite side. **54**Again, in this instance, to assume an evalated emission from this source would be a mistake.

Plume Rise

G. A. Briggs

AEC CRITICAL REVIEW SERIES

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G. A. Briggs

Air Resources Atmospheric Turbulence and Diffusion Laboratory
Environmental Science Services Administration
Oak Ridge, Tennessee

Prepared for
Nuclear Safety Information Center
Oak Ridge National Laboratory

U.S. ATOMIC ENERGY COMMISSION
Office of Information Services
1969

Available as TID-25075 from

National Technical Information Service
U. S. Department of Commerce
Springfield, Virginia 22151

Library of Congress Catalog Card Number: 72-603261

Printed in the United States of America
USAEC Technical Information Center, Oak Ridge, Tennessee

November 1969; latest printing, December 1972

FOREWORD

Scientists and technologists have been concerned in recent years about the "explosion" of original literature engendered by the staggering volume of research and development being undertaken throughout the world. It has proved all but impossible for scientific workers to keep up with current progress even in quite narrow fields of interest. Automated retrieval systems for identifying original literature pertinent to the interests of individuals are being developed. These systems are only a partial solution, however, because the original literature is too large, too diverse, too uneven in quality, to fully satisfy by itself the information needs of scientists.

In this situation of vastly expanding knowledge, there is increasing recognition of the valuable role that can be played by critical reviews of the literature and of the results of research in specialized fields of scientific interest. Mr. Briggs's study, the third published in the AEC Critical Review Series, is an excellent example of this genre.

This review is also significant as a further step in the unceasing effort of the AEC to assure that nuclear plants operate safely. *Plume Rise* is a much needed addition in a field in which a meteorologist must choose from over 30 different plume-rise formulas to predict how effluents from nuclear plants are dispersed into the atmosphere. Mr. Briggs presents and compares all alternatives, simplifies and combines results whenever possible, and makes clear and practical recommendations.

The Atomic Energy Commission welcomes any comments about this volume, about the AEC Critical Review Series in general, and about other subject areas that might beneficially be covered in this Series.

CONTENTS

SYNOPSIS

The mechanism of plume rise and dispersion is described in qualitative terms with emphasis on possible aerodynamic, meteorological, and topographical effects. Plume-rise observations and formulas in the literature are reviewed, and a relatively simple theoretical model is developed and compared with other models. All available data are used to test the formulas for a number of idealized cases.

The inverse wind-speed relation, $\Delta h \propto u^{-1}$, is shown to be generally valid for the rise of a hot plume at a fixed distance downwind in near-neutral conditions. Nine formulas of this type are compared with data from sixteen different sources, and the best agreement is obtained from the "$\frac{2}{3}$ law," $\Delta h = 1.6F^{1/3} u^{-1} x^{2/3}$, modified by the assumption that a ceiling height is reached at a distance of ten stack heights downwind. The term F is proportional to the heat emission. In uniform stratification buoyant plumes are seen to follow the $\frac{2}{3}$ law until a ceiling height of $2.9\ (F/us)^{1/3}$ is approached, where s is proportional to the potential temperature gradient. In calm conditions the formula $\Delta h = 5.0F^{1/4} s^{-3/8}$ is in excellent agreement with a wide range of data.

Formulas of a similar type are recommended for nonbuoyant plumes on the basis of much more limited data.

1 INTRODUCTION

The calculation of plume rise is often a vital consideration in predicting dispersion of harmful effluents into the atmosphere, yet such a calculation is not straightforward. The engineer or meteorologist must choose from more than thirty different plume-rise formulas, and a casual search through the literature for help in choosing is likely to be confusing. The purpose of this survey is to present an overall view of the pertinent literature and to simplify and combine results whenever possible, with the objective of setting down clear, practical recommendations.

The importance of stack height and buoyancy in reducing ground concentrations of effluents has been recognized for at least 50 years.[1] In a 1936 paper Bosanquet and Pearson[2] showed that under certain conditions the maximum ground concentration depends on the inverse square of stack height, and experience soon confirmed this relationship.[3] Later the stack height in this formula came to be replaced by the "effective stack height," which was defined as the sum of the actual stack height and the rise of the plume above the stack. Since smoke plumes from large sources of heat often rise several stack heights above the top of the stack even in moderately high winds, plume rise can reduce the highest ground concentration by an order of magnitude or more.

In spite of the importance of plume rise in predicting dispersion, there is much controversy about how it should be calculated. A recent symposium on plume behavior,[4] held in 1966, summarizes the current state of affairs. Lucas expressed a desire for better agreement between empirical results and stated flatly, "There are too many theoretical formulae and they contradict one another!" Spurr lamented, "The argument for and against different plume rise formulae can be discussed clinically by physicists and theoreticians, but the engineer who has to apply the formulae is obliged to make a choice." He then compared five recent formulas for a specific example and concluded that the results varied by a factor of 4 in the calculated maximum ground concentration. Even worse examples were given in the same symposium.

There are reasons for the lack of agreement. Different techniques for measuring plume height and wind speed can account for some of the disparity in the data, but the differences in the results are due primarily to the different concepts of what constitutes effective stack height. A recent paper by Slawson and Csanady states:

> With an ostrich-like philosophy, the effective stack height is often defined to be the point where the plume is just lost sight of. It is then not very surprising to find that the observed thermal rise of the plume depends, for example, on a power of the heat flux ranging from ¼ to 1.0, influenced by a number of factors including, presumedly, the observer's eyesight.†

It was natural for early plume-rise observers to assume that a smoke plume leveled off in all conditions and that the plume was near the height of leveling off when it was inclined only slightly above the horizontal; subsequent observations suggest otherwise. The early formula of Holland,[6] sometimes called the Oak Ridge formula, was based on photographic data that followed the plumes only 600 ft downwind,[7] yet recent data of the Tennessee Valley Authority (TVA) show plumes still rising at 1 and even 2 miles downwind. Over this distance even a small inclination above the horizontal becomes important. The plume height normally of greatest concern is that above the point of maximum ground concentration, and it seems logical to define this as the effective stack height, as suggested by Lucas.[4] A major difficulty with this definition is that none of the present observations goes that far downwind. In practice we must choose formulas for plume rise on the basis of agreement with data on hand and, at the same time, be aware of the limitations of the data.

General plume behavior, which is discussed briefly in the next chapter, has been described in greater detail in other publications. The textbook by Sutton[8] first reviewed all aspects of diffusion, including plume rise. Pasquill[9] surveyed the subject in considerably more detail and on the basis of more data than was previously available. The first edition of *Meteorology and Atomic Energy*[10] adequately covered the qualitative aspects of plume rise and diffusion, but the new edition[11] is quantitatively more up-to-date. An excellent survey by Strom[12] reviewed all aspects of plume behavior, including the potential for modeling dispersion. Smith briefly reviewed the main qualitative considerations in plume rise and diffusion[13] and more recently discussed the practical aspects of dispersion from tall stacks.[14] The practical experience of TVA has been described by Thomas,[15] by Gartrell,[16] and by Thomas, Carpenter, and Gartrell.[17] The British experience with diffusion from large power plants and their tall-stack policy has been analyzed by Stone and Clark.[18]

Several attempts have been made at setting down definite procedures for calculating diffusion, including the plume rise. The first, primarily concerned with dust

†Ref. 5, page 311.

deposition, was by Bosanquet, Carey, and Halton.[19] Hawkins and Nonhebel[7] published a procedure based on a revised formulation for plume rise by Bosanquet.[20] More recently, Nonhebel[21] gave detailed recommendations on stack heights, primarily for small plants, based on the Bosanquet plume-rise formula and the Sutton diffusion formula.[8,9] Many of these recommendations were adopted in the British *Memorandum on Chimney Heights,*[22] which has been summarized by Nonhebel.[23] Scorer and Barrett[24] outlined a simple procedure applicable to long-term averages. A CONCAWE† publication[25,26] presented a method for determining stack height for a plant built on flat, open terrain with a limited range of gas emissions; this method included a formula for plume rise based on regression analysis of data. The American Society of Mechanical Engineers (ASME)[27] has prepared a diffusion manual with another formula for plume rise. The implications of this formula and the CONCAWE formula are discussed in Ref. 28. Further discussions of plume-rise questions can be found in Refs. 29 to 33.

†CONCAWE (Conservation of Clean Air and Water, Western Europe), a foundation established by the Oil Companies' International Study Group for the Conservation of Clean Air and Water.

2 BEHAVIOR OF SMOKE PLUMES

Plume dispersion is most easily described by discussing separately three aspects of plume behavior: (1) aerodynamic effects due to the presence of the stack, buildings, and topographical features; (2) rise relative to the mean motion of the air due to the buoyancy and initial vertical momentum of the plume; and (3) diffusion due to turbulence in the air. In reality all three effects can occur simultaneously, but in the present state of the art they are treated separately and are generally assumed not to interact. This is probably not too unrealistic an assumption. We know that undesirable aerodynamic effects can be avoided with good chimney design. Clearly the rise of a plume is impeded by mixing with the air, but there is not much agreement on how important a role atmospheric turbulence plays. It is known that a rising plume spreads outward from its center line faster than a passive plume, but this increased diffusion rate usually results in an only negligible decrease of ground concentrations.

The following sections discuss the three aspects of plume diffusion. Symbols and frequently used meteorological terms are defined in Appendixes B and C.

DOWNWASH AND AERODYNAMIC EFFECTS

Downwash of the plume into the low-pressure region in the wake of a stack can occur if the efflux velocity is too low. If the stack is too low, the plume can be caught in the wake of associated buildings, where it will bring high concentrations of effluent

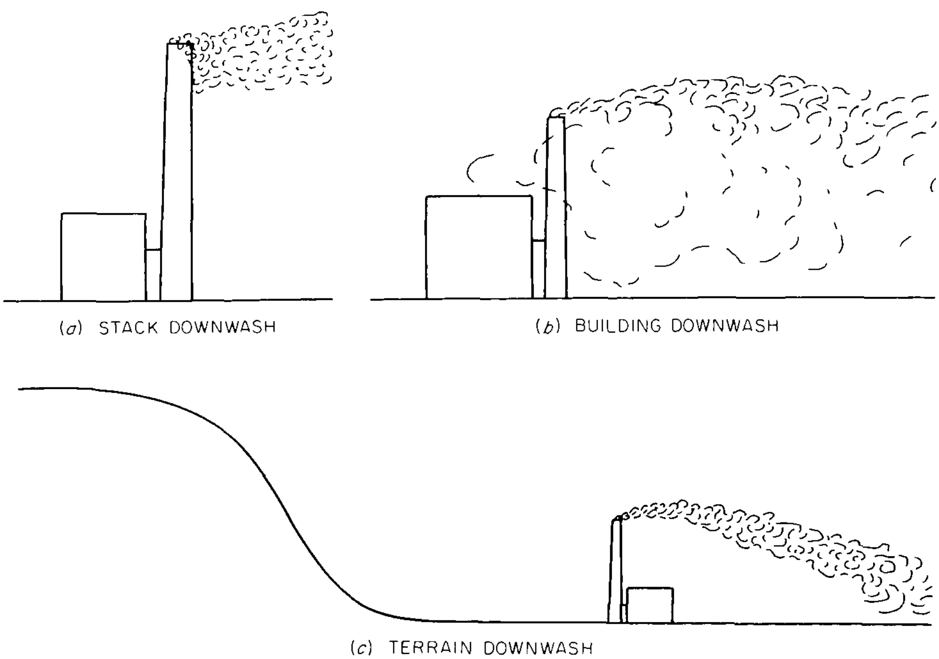

Fig. 2.1 Undesirable aerodynamic effects.

to the ground and even inside the buildings. A similar effect can occur in the wake of a terrain feature. These three effects are illustrated in Fig. 2.1.

The wind-tunnel studies of Sherlock and Stalker[34] indicate that downwash is slight as long as $w_0 > 1.5u$, where w_0 is the efflux velocity of gases discharging from the stack and u is the average wind speed at the top of the stack. These results are consistent with elementary theoretical considerations: when $w_0 > 1.8u$, the upward momentum of the stack gases should overcome the downward pressure gradient produced by the wind blowing around the stack on the basis of the pressure distribution around an infinite circular cylinder in a crosswind given by Goldstein;[35] when $w_0 < 0.8u$, the smoke can be sucked into the lower pressure region across the entire back of the chimney. If the plume is very buoyant, i.e., if the efflux Froude number, Fr, is 1.0 or less, the buoyancy forces are sufficient to counteract some of the adverse pressure forces, and the preceding criterion for w_0 could be relaxed. This factor probably abates downwash at the Tallawarra plant, cited in Table 5.1, where

$$\mathrm{Fr} = \frac{w_0^2}{g(\Delta T/T)D} = 0.5$$

Experiments are still needed to determine quantitatively the effect of the efflux Froude number on the abatement of downwash, unfortunately, the experiments of

Sherlock and Stalker involved only high values of Fr, and thus buoyancy was not a significant factor.

Nonhebel[21] recommends that w_0 be at least 20 to 25 ft/sec for small plants (heat emission less than 10^6 cal/sec) and that w_0 be in the neighborhood of 50 to 60 ft/sec for a large plant (e.g., with a heat emission greater than 10^7 cal/sec). Larger efflux velocities are not necessary since such high winds occur very rarely; in fact, much higher velocities may be detrimental to the rise of a buoyant plume because they are accompanied by more rapid entrainment of ambient air into the plume. Scorer[36] reports that, when efflux velocity must be low, placing a horizontal disk that is about one stack diameter in breadth about the rim of the chimney top will prevent downwash.

One of the most enduring rules of thumb for stack design was a recommendation[37] made in 1932 that stacks be built at least 2.5 times the height of surrounding buildings, as illustrated in Fig. 2.2. If such a stack is designed with sufficient efflux velocity to avoid downwash, the plume is normally carried above the region of downflow in the wake of the building. If the stack height or efflux velocity is slightly

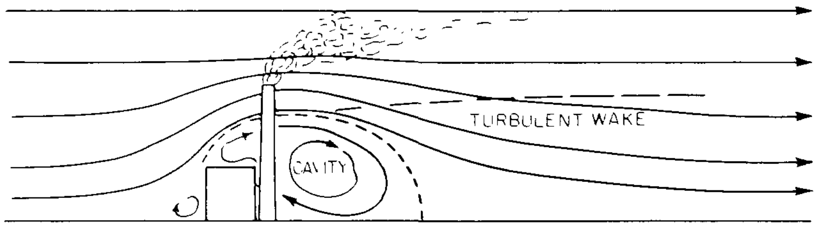

Fig. 2.2 Flow past a typical power plant.

lower, in high winds the plume will get caught in the downflow and be efficiently mixed to the ground by the increased turbulence in the wake of the building. If the stack is less than twice the building height, at least part of the plume is likely to be caught in the cavity of air circulating in the lee of the building; this can bring high concentrations of effluent to the ground near the building and even into the building. The streamlines in Fig. 2.2 also illustrate the advantage of constructing a chimney on the side of the building facing the prevailing wind, where the air is still rising.

Still, this is only a rough rule, because the air-flow pattern around a building depends on the particular shape of the building and on the wind direction. Details on these effects are given by Halitsky.[38] Also, for sources with very small emissions, the rule for stacks 2.5 times higher than nearby buildings may be impracticable. Lucas[39] suggested a correction factor for smaller stacks, and this has been incorporated into the British *Memorandum on Chimney Heights.*[22] The correction factor is also reported by Ireland[40] and Nonhebel.[23] The behavior of effluents from very short stacks has been discussed by Barry,[41] Culkowski,[42] and Davies and Moore.[43] For such sources plume rise is probably negligible.

It is much more difficult to give any rules about the effect of terrain features, partly because of the great variety of possibilities. Fortunately the general effect of terrain and buildings on a plume can be fairly well modeled in a wind tunnel, such as the one at New York University or at the D.S.I.R. (National Physics Laboratory, England). Stumke[44,45] gives a method for correcting effective stack height for a simple step in the terrain, but only streamline flow is considered.

A curious aerodynamic effect sometimes observed is bifurcation, in which the plume splits into two plumes near the source. This is discussed by Scorer,[36] and a good photograph of the phenomenon appears in Ref. 46. Bifurcation arises from the double-vortex nature of a plume in a crosswind, but it is not clear under what conditions the two vortices can separate. However, bifurcation is rare and appears to occur only in light winds.

Scriven[47] discusses the breakdown of plumes into puffs due to turbulent fluctuations in the atmosphere. Scorer[46] discusses the breakdown into puffs of buoyant plumes with low exit velocity and includes a photograph. The process appears to be associated with a low efflux Froude number, but a similar phenomenon could be initiated through an organ-pipe effect, e.g., if the vortex-shedding frequency of the stack corresponds to a harmonic mode of the column of gas inside the stack.

PLUME RISE

Although quantitative aspects of plume rise are the concern of the bulk of this report, only the qualitative behavior is discussed in this section. More detailed discussions can be found in a paper by Batchelor[48] and a book by Scorer.[46] It is assumed that the plume is not affected by the adverse aerodynamic effects discussed in the previous section since these effects can be effectively prevented.

The gases are turbulent as they leave the stack, and this turbulence causes mixing with the ambient air; further mechanical turbulence is then generated because of the velocity shear between the stack gases and the air. This mixing, called entrainment, has a critical effect on plume rise since both the upward momentum of the plume and its buoyancy are greatly diluted by this process. The initial vertical velocity of the plume is soon greatly reduced, and in a crosswind the plume acquires horizontal momentum from the entrained air and soon bends over.

Once the plume bends over, it moves horizontally at nearly the mean wind speed of the air it has entrained; however, the plume continues to rise relative to the ambient air, and the resulting vertical velocity shear continues to produce turbulence and entrainment. Measurements of the mean velocity distribution in a cross section of a bent-over plume show the plume to be a double vortex, as shown in Fig. 2.3. Naturally the greatest vertical velocity and buoyancy occur near the center of the plume, where the least mixing takes place. As the gases encounter ambient air above the plume, vigorous mixing occurs all across the top of the plume. This mixing causes the plume diameter to grow approximately linearly with height as it rises.

If the plume is hot or is of lower mean molecular weight than air, it is less dense than air and is therefore buoyant. If the heat is not lost and the atmosphere is well mixed, the total buoyant force in a given segment of the moving plume remains constant. This causes the total vertical momentum of that segment to increase at a constant rate, although its vertical velocity may decrease owing to dilution of the momentum through entrainment.

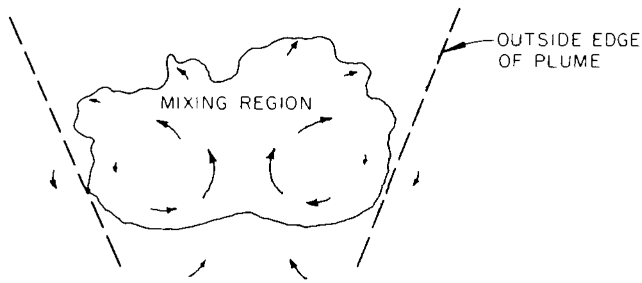

Fig. 2.3 Cross section of mean velocity distribution in a bent-over plume.

At some point downwind of the stack, the turbulence and vertical temperature gradient of the atmosphere begin to affect plume rise significantly. If the atmosphere is well mixed because of vigorous turbulent mixing, it is said to be neutral or adiabatic. In such an atmosphere the temperature decreases at the rate of 5.4°F per 1000 ft. This rate of decrease, which is called the adiabatic lapse rate (Γ), is the rate at which air lifted adiabatically cools owing to expansion as the ambient atmospheric pressure decreases. If the temperature lapse of the atmosphere is less than the adiabatic lapse rate, the air is said to be stable or stably stratified. Air lifted adiabatically in such an environment becomes cooler than the surrounding air and thus tends to sink back. If the temperature actually increases with height, the air is quite stable. Such a layer of air is called an inversion. If the temperature lapse of the atmosphere is greater than the adiabatic lapse rate, the air is said to be unstable or unstably stratified. Air lifted adiabatically in such an environment becomes warmer than the surrounding air, and thus all vertical motions tend to amplify.

The potential temperature, θ, is defined as the temperature that a sample of air would acquire if it were compressed adiabatically to some standard pressure (usually 1000 millibars). The potential temperature is a convenient measure of atmospheric stability since

$$\frac{\delta\theta}{\delta z} = \frac{\delta T}{\delta z} + \Gamma \tag{2.1}$$

where Γ = 5.4°F/1000 ft = 9.8°C/km. Thus the potential temperature gradient is positive for stable air, zero for neutral air, and negative for unstable air.

If the ambient air is stable, i.e., if $\delta\theta/\delta z > 0$, the buoyancy of the plume decays as it rises since the plume entrains air from below and carries it upward into regions of warmer ambient air. If the air is stable throughout the layer of plume rise, the plume eventually becomes negatively buoyant and settles back to a height where it has zero buoyancy relative to the ambient air. The plume may maintain this height for a distance of 20 miles or more from the source. In stable air atmospheric turbulence is suppressed and has little effect on plume rise.

If the atmosphere is neutral, i.e., if $\delta\theta/\delta z = 0$, the buoyancy of the plume remains constant in a given segment of the plume provided the buoyancy is a conservative property. This assumes no significant radiation or absorption of heat by the plume or loss of heavy particles. Since a neutral atmosphere usually comes about through vigorous mechanical mixing, a neutral atmosphere is normally turbulent. Atmospheric turbulence then increases the rate of entrainment; i.e., it helps dilute the buoyancy and vertical momentum of the plume through mixing.

If the atmosphere is unstable, i.e., if $\delta\theta/\delta z < 0$, the buoyancy of the plume grows as it rises. Increased entrainment due to convective turbulence may counteract this somewhat, but the net effect on plume rise is not well known. The few usable data for unstable situations seem to indicate slightly higher plume rise than in comparable neutral situations. On warm, unstable afternoons with light wind, plumes from large sources rise thousands of feet and even initiate cumulus clouds.

Measurements are made difficult by fluctuations in plume rise induced by unsteady atmospheric conditions. On very unstable days there are large vertical velocity fluctuations due to convective eddies that may cause a plume to loop, as shown in Fig. 2.5d. Figure 2.4 illustrates the large variations in plume rise at a fixed distance downwind during unstable conditions. On neutral, windy days the plume trajectory at any one moment appears more regular, but there still may be large fluctuations in plume rise due to lulls and peaks in the horizontal wind speed. Since the wind is responsible for the horizontal stretching of plume buoyancy and momentum, the wind strongly affects plume rise. In stable conditions there is very little turbulence, and plume rise is also less sensitive to wind-speed fluctuations. This can be seen in Fig. 2.4. In this case the plume leveled off in stable air, and its rise increased in a smooth fashion as the air became less and less stable owing to insolation at the ground.

One might ask whether plume rise is affected by the addition of latent heat that would occur if any water vapor in the stack gases were to condense. This is an important question because there may be as much latent heat as there is sensible heat present in a plume from a conventional power plant. It is true that some water vapor may condense as the plume entrains cooler air, but calculations show that in most conditions the plume quickly entrains enough air to cause the water to evaporate again. Exceptions occur on very cold days, when the air has very little capacity for water vapor, and in layers of air nearly saturated with water vapor, as when the plume rises through fog. Observations by Serpolay[49] indicate that on days when cumulus clouds are present condensation of water from entrained air may increase the buoyancy of the plume and enhance its ability to penetrate elevated stable layers.

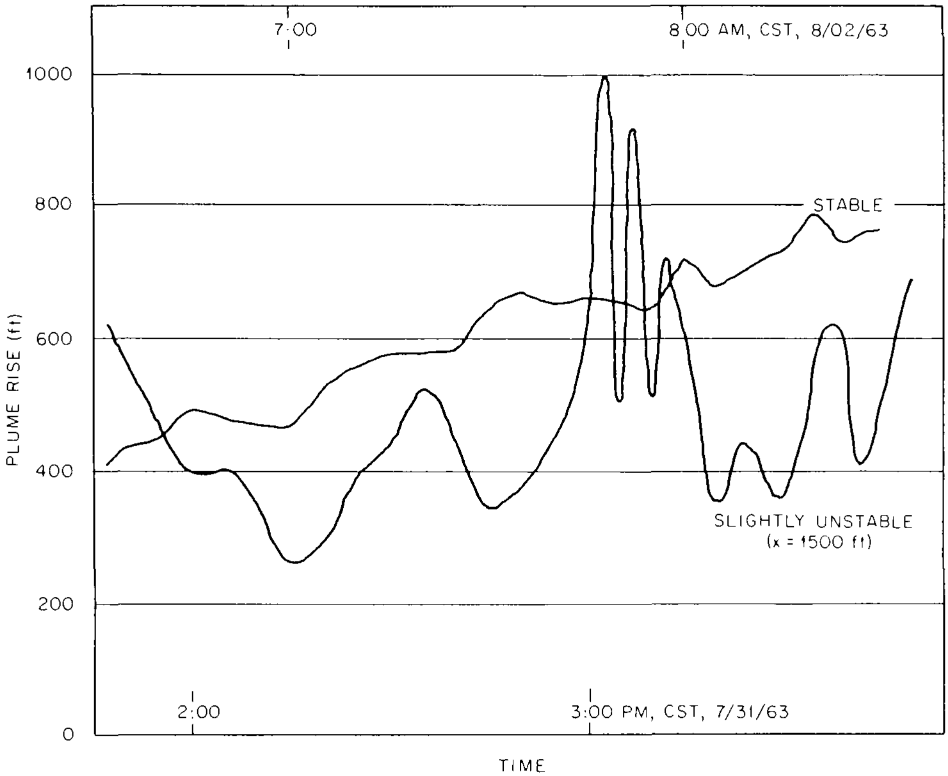

Fig. 2.4 Fluctuations of plume rise with time (Gallatin Plant, Tennessee Valley Authority).

Ordinarily only the sensible heat of the plume should be used in calculations.

One might also ask whether thermal radiation can significantly alter the heat content of a plume, i.e., its buoyancy. Not much is known about the radiative properties of smoke plumes, but crude calculations show that radiation is potentially important only for very opaque plumes some thousands of feet downwind and should have little effect on clean plumes from modern power plants or on plumes from air-cooled reactors. Plumes from TVA plants have been observed to maintain a constant height for 20 miles downwind in the early morning; thus there appears to be negligible heat loss due to radiation.

DIFFUSION

Detailed diffusion calculations are beyond the scope of this review, but the main types of diffusion situations should be discussed with regard to plume rise. On a clear

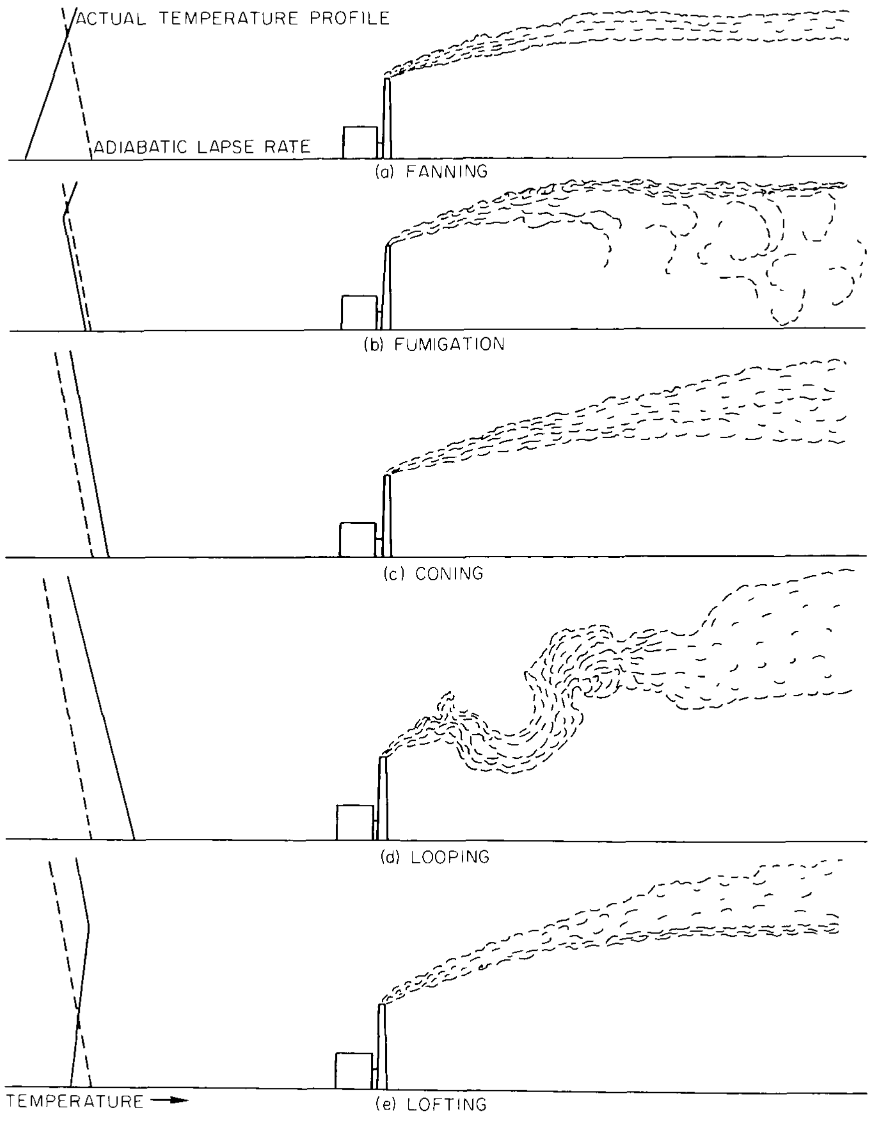

Fig. 2.5 Effect of temperature profile on plume rise and diffusion.

night the ground radiates heat, most of which passes out into space. In this process the air near the ground is cooled, and an inversion is formed. The stable layer may be several thousand feet deep; so most plumes rising through it lose all their buoyancy and level off. This behavior is called fanning and is pictured in Fig. 2.5a. When the sun comes up, convective eddies develop and penetrate higher and higher as the ground warms up. As the eddies reach the height at which the plume has leveled off, they rapidly mix the smoke toward the ground while the inversion aloft prevents upward diffusion. This phenomenon, called fumigation, can bring heavy concentrations of effluent to the ground (Fig. 2.5b). Just after an inversion has been broken down by convective eddies or in cloudy, windy conditions, the atmosphere is well mixed and nearly neutral. Then the plume rises and diffuses in a smooth fashion known as coning (Fig. 2.5c). As the heating of the ground intensifies, large convective eddies may develop and twist and fragment the plume in a looping manner (Fig. 2.5d). Diffusion is then more rapid than in a neutral atmosphere. The convection dies out as the sun gets lower, and an inversion again starts to build from the ground up. This ground inversion is weak enough at first that the plume can penetrate it, and the plume diffuses upward but is prevented by the stability below from diffusing downward. This lofting period (Fig. 2.5e) is the most ideal time to release harmful effluents since they are then least likely to reach ground.

The meteorological conditions that should be considered in stack design depend on the size of the source, the climatology of the region, and the topography. In reasonably flat terrain, high wind with neutral stratification usually causes the highest ground concentrations since there is the least plume rise in these conditions. The mean concentration of the effluent in the plume is reasonably well described by a Gaussian distribution, for which the maximum ground concentration is given by

$$\chi = \frac{\sigma_z}{\sigma_y} \frac{2Q}{\pi e u h^2} = 0.164 \frac{Q}{uh^2} \tag{2.2}$$

where Q is the rate at which pollutant is emitted, u is the mean wind speed at the source height, and h is the effective stack height (defined as the sum of the actual stack height, h_s, and the plume rise, Δh); σ_z/σ_y is the ratio of the vertical dispersion to the horizontal dispersion and is equal to about 0.7 in a neutral atmosphere for an averaging period of 30 min.[25] Variation with distance has been neglected in deriving Eq. 2.2. This equation is valid only when the atmosphere is neutral from the ground up to at least twice the effective stack height. Inversions may exist below this height even in windy conditions. A diffusion model for this case is given by Smith and Singer.[50] If the plume reaches the height of the inversion and penetrates it, as can be predicted by Eq. 4.30,† none of the effluent reaches the ground. If the plume does not penetrate, the inversion acts as an invisible ceiling and prevents upward diffusion.

A good measure of the efficiency of the diffusion process on a given occasion is

†See "Basic Theory Simplified" in Chapter 4.

Q/χ, the effective ventilation, which has the dimensions of volumetric flow rate (l^3/t). For the case just described,

$$\frac{Q}{\chi} = 6.1uh^2 = 6.1u(h_s + \Delta h)^2 \qquad (2.3)$$

Naturally the effective ventilation is large for extremely high wind speeds, but it is also large at low values of u because of very high plume rise. It is at some intermediate wind speed that Q/χ attains a minimum, i.e., χ attains a maximum; this wind speed is called the critical wind speed. If the dependence of Δh on u is known, Eq. 2.3 can be differentiated and set equal to zero to find the critical wind speed. The result can be substituted into the plume-rise equation and into Eq. 2.2 to find the highest expected ground concentration for the neutral, windy case, χ_{max}.

There is evidence that fumigation during calm conditions may lead to the highest ground-level concentrations at large power plants. This type of fumigation can occur near the center of large slow-moving high-pressure areas in so-called "stagnation" conditions. Such high-pressure systems usually originate as outbreaks of cold, relatively dense air, and, as these air masses slow down, they spread out much in the manner of cake batter poured into a pan. Since the air underneath the upper surface of these air masses is appreciably colder than the air above it, a subsidence inversion forms and presents a formidable barrier to upward mixing; such an inversion normally occurs 1500 to 4000 ft above the ground.[51] In combination with a near-zero wind speed, a subsidence inversion severely limits atmospheric ventilation, and the little ventilation that occurs is due to convective mixing from the ground up to the inversion.

Fortunately such circumstances are rare except in certain geographical areas. The southeastern United States, one such region, averages 5 to 15 stagnation days a year with the higher figure occurring in the Carolinas and Georgia.[51] Nevertheless, there is only one outstanding case of fumigation during stagnation in all the years of monitoring SO_2 around TVA power plants. In this instance ground concentration near an isolated plant was 50% higher than the maximum observed in windy, neutral conditions, and this condition continued for most of one afternoon. The wind speed was 0 to 1 mph, and the effective ventilation, as defined above, was 1.5×10^8 cu ft/sec (4.3×10^6 m^3/sec). This value is adequate for a small plant but too small for a large plant. There is not much hope of improving the effective ventilation in this rare condition, for a stack would have to be thousands of feet high to ensure that the plume could penetrate a subsidence inversion. The only way to reduce ground concentrations in this case seems to be to reduce the emission of pollutants; accordingly, TVA stockpiles low-sulfur coal for use when the Weather Bureau predicts stagnation conditions.

Similar conditions occur under marine inversions, such as are found along the Pacific coast of the United States. The inversions there are sometimes less than 1000 ft above the ground,[51] and plumes from high stacks can often penetrate them. Such penetration can be predicted by equations presented in later chapters.

Fumigation associated with inversion breakdown may be serious when topography is prominent. If the plume does not rise out of a deep valley during the period of the nighttime inversion, the pollutant will mix fairly uniformly down to the ground during fumigation; therefore concentration is given by

$$\chi = \frac{Q}{uhW} \tag{2.4}$$

where u is the average velocity of the along-valley drainage flow at night, h is the effective stack height at night, and W is the average width of the valley up to height h.

An elevated plateau can also be subjected to intensified fumigation if during an inversion the plume rises slightly higher than the plateau and drifts over it. This has occurred at a plant on the Tennessee River where the river cuts a 1000-ft-deep gorge through the Cumberland Plateau.[17] Careful consideration should be given to this possibility at such a site. Topographic effects are discussed by Hewson, Bierly, and Gill.[52]

3 OBSERVATIONS OF PLUME RISE

Dozens of plume-rise observations have been made, and each is unique in terms of type of source and technique of measurement. Observations have been made in the atmosphere, in wind tunnels, in towing channels, and in tanks. Brief descriptions of these experiments are given in this chapter.

MODELING STUDIES

Plume rise is a phenomenon of turbulent fluid mechanics and, as such, can be modeled; i.e., it can be simulated on some scale other than the prototype. There are obvious advantages to modeling plume rise. For example, the model plume can be measured much less expensively than the real plume since it is not necessary to probe high above the ground, and the variables can be controlled at will. The main difficulty is in ensuring that the behavior of the model plume essentially duplicates that of a real smoke plume. The most obvious requirements are that all lengths be scaled down by the same factor and that the wind speed and efflux velocity be scaled down by identical factors. For exact similarity the Reynolds number has to be the same in model and in prototype. The Reynolds number is defined by

$$\mathrm{Re} = \frac{\mathrm{v}l}{\nu} \tag{3.1}$$

where v is a characteristic velocity, l is a characteristic length, and ν is the kinematic viscosity of air or the fluid in which the model is measured. Exact similarity is seldom possible in modeling since Re is of the order of 10^6 for a real plume. Fortunately fully turbulent flow is not very dependent on Reynolds number so long as it is sufficiently high. In most experiments Re is at least 10^3 on the basis of efflux velocity and stack diameter, but the adequacy of this value is not certain.

For buoyant plumes the Froude number must be the same in model and in prototype. Since we are unable to scale down gravity, which is a prerequisite for the existence of buoyancy, the basic requirement is that

$$\left(\frac{v^2}{l}\right)_{\text{model}} = \left(\frac{v^2}{l}\right)_{\text{prototype}} \tag{3.2}$$

provided the temperature or density ratios are kept unchanged.

Numerous measurements have been made on the simple circular jet.[53,54] Schmidt[55] first investigated the heated plume with zero wind. Yih[56] studied the transition from laminar to turbulent flow in a heated plume. Later, Rouse, Yih, and Humphreys[57] studied the detailed distribution of vertical velocity and temperature in a fully turbulent hot plume from a gas flame near the floor of an airtight, high-ceilinged room. They measured temperature with a thermocouple and velocity with a 1¼-in. vane on jeweled bearings. The important result of all these investigations is that both jets and hot plumes are cone shaped in calm, unstratified air. The half-cone angle is smaller for the heated plume than for a jet, and the decreases of temperature and velocity with distance above the source are consistent with heat and momentum conservation principles. Also, the cross-sectional distributions of vertical velocity and temperature excess are approximately Gaussian except close to the source. The characteristic radius describing the temperature distribution in a heated plume is 16% greater than that for the velocity distribution.

Several modeling studies have been made on heated plumes rising through a stable environment. Morton, Taylor, and Turner[58] confirmed predictions by using measured releases of dyed methylated spirits in a 1-m-deep tank of stratified salt solution. Crawford and Leonard[59] ran a similar experiment with a small electric heater to generate a plume on the floor of an ice rink. The invisible plume was observed with the Schlieren technique, and convection thermocouples were used to measure the temperature profile of the air above the ice. Their results are, in fact, in good agreement with those of Ref. 58, although they miscalculated the constant in the equation of Ref. 58 by a factor of 2¼. Vadot[60] conducted experiments with an inverted plume of heavier fluid in a tank of salt solution. His inversions were quite sharp in contrast to the smooth density gradients used in the preceding studies.

A number of wind-tunnel investigations of jets in a crosswind have been made. The early study of Rupp and his associates[61] has been used as the basis for a momentum contribution to plume rise by several investigators. Callaghan and Ruggeri[62] measured the temperature profile of heated jets in experiments in which the efflux velocities were of the order of the speed of sound. Keffer and Baines[63] measured rise for only four stack diameters downwind and obtained some velocity and turbulence intensity measurements within the jets. Halitsky[64] and Patrick[65] summarized the work of previous investigators. In addition, Patrick presented new measurements to about 20 stack diameters downstream, including detailed profiles of velocity and concentration of a tracer (nitrous oxide).

The effect of buoyancy on plume rise near the stack was studied by Bryant and Cowdrey[66,67] in low-speed wind in a tunnel. Vadot[60] made a study of buoyancy effects in a towing channel with both stratified and unstratified fluids. This study was unusual in that the ambient fluid was at rest and the effect of crosswind was incorporated by towing the source at a constant speed down the channel. This is a valid experimental technique since motion is only relative. However, Vadot's source was a downward-directed stream of dense fluid. There is some question whether a bent-over plume from such a source behaves as a mirror image of a bent-over plume from an upward-directed stream of light fluid. Subtle changes in the entrainment mechanism could take place owing to centrifugal forces acting on the more dense fluid inside the plume. The recent treatment by Hoult, Fay, and Forney[68] of past modeling experiments tends to confirm this. The bent-over portion of a hot plume behaves much like a line thermal, which was modeled for both dense and light plumes by Richards,[69] who found that the width of the thermals increased linearly with vertical displacement from their virtual origins, just as had been observed for jets and plumes that were not bent over. The line thermal was also modeled numerically by Lilly.[70] Lilly did not have enough grid points to reach the shape-preserving stage found in laboratory thermals, but, as larger computers are developed, numerical modeling should be quite feasible. Extensive experiments made recently by Fan[71] in a modeling channel included plume rise both into a uniform crossflow and into a calm stream with a constant density gradient. In the latter case most of the plumes were inclined; i.e., the stacks were not vertical. Although the buoyancy of these plumes was varied, they were momentum dominated for the most part. The behavior of plumes with negative buoyancy in a crosswind was modeled by Bodurtha.[72]

ATMOSPHERIC STUDIES

The first full-scale plume-rise data were given in an appendix to the Bosanquet, Carey, and Halton paper[19] of 1950. The center lines of plumes from four chimneys were traced from visual observation onto a Perspex screen. The observations were carried only as far as 800 ft downwind of the stacks, where apparently the visibility of the plumes was lost. These observations also appear in a paper by Priestley.[73] Holland[6] published some of the details of the observations that he used in deriving the Oak Ridge formula, but the distance of observation was not mentioned. According to Hawkins and Nonhebel[7] the plume heights were measured at only two or three stack heights downwind and were obtained from photographs. Holland found only a small correlation between plume rise and the temperature gradient, which was measured near the ground. However, the plume is affected only by the temperature gradient of the air through which it is rising, and the gradient near the ground is not a good measure of the gradient higher up. Stewart, Gale, and Crooks[74,75] published a survey of plume rise and diffusion parameters at the Harwell pile. Vertical surveys of the invisible plume were conducted by mounting up to ten Geiger counter units on the cable of a mobile barrage balloon. The stack was a steady, known source of radioactive argon (^{41}Ar), and the Geiger units were arranged to measure the disintegration of beta particles, which have a maximum range of only 3 m in air. Again, the temperature gradient was measured well below plume level except for a few runs that were made in neutral conditions. Most of the wind-speed measurements were also made at a height well below the plume height. Since wind speed generally increases with height, the reported wind speeds are probably too low for such runs.

Ball[76] made measurements on very small plumes from lard-pail-type oil burners. The heights were estimated at 30 and 60 ft downwind by visual comparison with 10-ft poles and were averaged over 2 or 3 min. There was some tendency for the burning rate to increase with wind speed. Moses and Strom[77] ran experiments on a source with about the same heat emission, but here the effluent was fed into an 111-ft experimental stack with a blower. Plume-rise data at 30 and 60 m downwind were obtained photographically and averaged over 4 min. Wind speed was interpolated at plume level from measurements from a nearly 150-ft meteorological tower. The temperature gradient was measured between the 144- and 5-ft levels of the tower. This provided only a fair measure of the actual gradient at plume level since the gradient above 111 ft may be quite different from that near the ground. In only 2 of the 36 runs, the plume appeared to level off owing to stable conditions. These data tend to be dominated by momentum rise.

Danovich and Zeyger[78] published some plume-rise data obtained from photographs. However, the effective rise was assumed to occur when the plumes were still inclined at 10 to 15° above horizontal, and plumes have been observed to rise many times the height at this point. Some interesting data were obtained from exhaust plumes of rocket motors by van Vleck and Boone,[79] including some runs with complete temperature profiles furnished. The sources ranged up to 1000 Mw, which is about ten times the heat-emission rate of a large power plant stack. However, they were not true continuous sources since burning times varied from 3 to 60 sec.

Extensive plume photography was carried out at two moderate-size power plants in Germany by Rauch.[80] Plume center lines were determined for 385 runs at Duisburg and for 43 runs at Darmstadt. Each determination consisted of two or three time exposures of about 1 min each, together with five instantaneous pictures taken at set time intervals. The horizontal speed of the plume was calculated by following irregular features of the plume from one negative to the next. This method should provide a good measure of the wind speed experienced by the plume. In most of the photographs, the plume center line could not be determined for a distance downwind of more than 1000 ft, although a few could be determined out to 3000 ft. The accuracy of the temperature-gradient measurements was such that only general stability classifications could be made. In practice no measurements in very unstable conditions were made because of looping, and no measurements in stable conditions were made far enough downwind to show the plume leveling off. In fact, not one of the 428 plume center lines leveled off. It would therefore seem that Rauch's extrapolation of these center lines to a final rise is rather speculative.

Much more extensive observations, consisting of about 70 experiments on more than 30 smoke stacks in Sweden, were recently made by Bringfelt,[81] and some of the preliminary data have been reported by Högström.[82] Each experiment consisted in taking about one photograph a minute for 30 to 60 min. The center lines were measured up to 9000 ft downwind, and wind speed and temperature gradient were measured at the plume level.

Some observations of plume rise at a small plant were reported by Sakuraba and his associates.[83] The best fit to the data was given by $\Delta h \propto u^{-3/2}$, but downwash was likely at the higher wind speeds since the wind speeds exceeded the efflux velocity. The temperature gradient and distance downwind were not given. More observations were carried out by the Central Research Institute of Electric Power Industry, Japan,[84] in which temperature and wind profiles were measured, as well as the vertical profile of SO_2 concentration at 1 km downwind.

Several groups have shown continuing interest in plume-rise measurements. The Meteorology Group at Brookhaven National Laboratory has conducted several programs by burning rocket fuel on the ground near their well-instrumented 420-ft meteorological tower. Limited data[85] were published in 1964 from tests in which there was some difficulty in obtaining a constant rate of heat release. This problem has been overcome, and more detailed data are available.[86]

Csanady published plume-rise observations[87] from the Tallawarra power station in New South Wales in 1961. Plume rise was measured photographically, and wind speed was determined from displacement of plume features in a succession of photographs. Csanady has been conducting a continuing program of plume-rise and dust-deposition research at the University of Waterloo in Ontario since 1963. More-elaborate photographic measurements of plume rise made at the Lakeview Generating Station were published by Slawson and Csanady.[5,88] Tank, wind-tunnel, and small-scale outdoor studies are now in progress.[89,90]

The Central Electricity Research Laboratory in England has been conducting plume-rise studies for some time. In 1963 they published results from the Earley and Castle Donington power stations.[91] The measurements were unique in that the plumes were traced a long distance downwind by injecting balloons into the base of the chimney.[92] The balloons were observed to stay within the plumes when the plumes were purposely made visible, but there may have been systematic errors due to the relative inertia and buoyancy of the balloons. Although some of the balloons continued to rise beyond 2 miles downwind, the reported rises were in the range 3600 to 6000 ft downwind. The motion of the balloons provided a convenient measure of wind speed. More recently measurements were made by Hamilton[93] at the Northfleet Power Station by using lidar to detect the plume. Lidar is an optical radar that uses a pulsed ruby laser. It measures the range and concentration of light-reflecting particles and can detect smoke plumes even when they become invisible to the eye.[94,95] Some searchlight determinations of the height of the Tilbury plant plume are also given in Refs. 93 and 96.

The Tennessee Valley Authority has also conducted plume-rise measurements over many years. The plume-rise and dispersion results[97,98] published in 1964 were based on helicopter probes of SO_2 concentrations in the plume. The helicopter also measured the temperature gradient up to the top of the plume. Plumes in inversions were observed to become level and maintain a nearly constant elevation as far as 9 miles downwind. Much more detailed studies at six TVA plants have recently been completed.[99] Heat emissions ranged from 20 to 100 Mw per stack with one to nine stacks operating. Complete temperature profiles were obtained by helicopter, and wind profiles were obtained from pibal releases about twice an hour. Such intermittent sampling of wind speed does not provide a good average value, however, and may account for some of the scatter in the results. After several different techniques were tried, with good agreement among them, infrared photography was used to detect the plume center line. Complete plume trajectories as far as 2 miles downwind were obtained from the photographs.

There have been a few atmospheric studies concerned particularly with plume rise in stable air. Vehrencamp, Ambrosio, and Romie[100] conducted tests on the Mojave Desert, where very steep surface inversions occur in the early morning. The heat sources were shallow depressions, 2.5, 5, 10, and 20 ft in diameter, containing ignited diesel oil. Temperature profiles were measured with a thermocouple attached to a balloon, and the dense black plumes were easily photographed. Davies[101] reported a 10,000-ft-high plume rise from an oil fire at a refinery in Long Beach, Calif. The heat release was estimated to be of the order of 10,000 Mw;[102] i.e., about 100 times the heat emission from a large power plant stack. Observations of plume rise into multiple inversions over New York City were presented recently by Simon and Proudfit.[103] The plumes were located with a fast-response SO_2 analyzer borne by helicopter, and temperature profiles were also obtained by helicopter.

4 FORMULAS FOR CALCULATING PLUME RISE

There are over 30 plume-rise formulas in the literature, and new ones appear at the rate of about 2 a year. All require empirical determination of one or more constants, and some formulas are totally empirical. Yet the rises predicted by various formulas may differ by a factor greater than 10. This comes about because the type of analysis and the selection and weighting of data differ greatly among various investigators.

Emphasis is given here on how the formulas were derived and on the main features of each. Complicated formulations are omitted since readers may check the original references. For convenience all symbols are defined in Appendix B.

EMPIRICAL FORMULAS

Formulas for Buoyant Plumes

Of the purely empirical plume-rise formulas, the first to be widely used was that suggested by Holland[6] on the basis of photographs taken at three steam plants in the vicinity of Oak Ridge, Tenn. The observed scatter was large, but the rise appeared to be roughly proportional to the reciprocal of wind speed. Holland used the wind-tunnel results of Rupp and his associates[61] for the momentum-induced part of the rise and, by assuming a linear combination of momentum and buoyancy rises, found the best fit to the data with

$$\Delta h = 1.5 \left(\frac{w_0}{u}\right) D + 4.4 \times 10^{-4} \left[\frac{\text{ft-ft/sec}}{\text{cal/sec}}\right] \frac{Q_H}{u} \tag{4.1}$$

The dimensions of constants are given in brackets. Thomas[15] found that a buoyancy term twice as large as that in Eq. 4.1 gave a better fit to observations at the TVA Johnsonville plant, and Stumke[104] recommended a rise nearly three times that given by Eq. 4.1 on the basis of comparisons with many sets of observations.

Another early empirical formula was suggested by Davidson[105] in 1949 on the basis of Bryant's[66] wind-tunnel data:

$$\Delta h = \left(\frac{w_0}{u}\right)^{1.4} D \left(1 + \frac{\Delta T}{T_s}\right) \qquad (4.2)$$

Equation 4.2, although a dimensionally homogeneous formula, is physically oversimplified in that the buoyancy term ($\Delta T/T_s$) does not take into consideration the total heat emission or the effect of gravity, without which buoyancy does not exist. The main weakness of Eq. 4.2 is that it is based on data obtained at only seven stack diameters downwind and often greatly underestimates observed rises.

Berlyand, Genikhovich, and Onikul[106] suggested

$$\Delta h = 1.9 \left(\frac{w_0}{u}\right) D + 5.0 \frac{F}{u^3} \qquad (4.3)$$

where F is a quantity that is proportional to the rate of buoyancy emission from the stack. This formula is dimensionally consistent, but few details are given about the observations on which it is based. The constant in the buoyancy term, 5.0, is curiously almost two orders of magnitude smaller than the constants recommended by Csanady,[87] by Briggs,[11,107] and by the new ASME manual.[27]

On the basis of data from four stacks, namely, the Harwell stack,[74,75] Moses and Strom's experimental stack,[77] and the two stacks reported by Rauch,[80] Stumke[108] derived the formula

$$\Delta h = 1.5 \left(\frac{w_0}{u}\right) D + 118 \left[\frac{m^{1/2}}{sec}\right] D^{3/2} \left(1 + \frac{\Delta T}{T_s}\right)^{1/4} u^{-1} \qquad (4.4)$$

The argument for omitting emission velocity from the buoyancy term is not clear. The constants and exponents for the various terms resulted from applying the method of least squares to the observed and calculated rises.

Lucas, Moore, and Spurr[91] fitted observed plume rises at two of their plants with

$$\Delta h = 258 \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{1/4}}\right] \frac{Q_H^{1/4}}{u} \qquad (4.5)$$

The heat emissions varied from 4 to 67 Mw, and the plumes were traced to about a mile downwind by releasing balloons in the stacks (see "Atmospheric Studies" in Chap. 3). The formula is based on a simplification of Priestley's theoretical plume-rise model.[73] The best values for the constant in Eq. 4.5 differed by 25% at the two plants, and further variations have been observed at other plants.[93] Lucas[109] noted some correlation with stack height and suggested a modification of Eq. 4.5:

$$\Delta h = (134 + 0.3\ [\mathrm{ft}^{-1}]\ h_s) \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{1/4}}\right] \frac{Q_H^{1/4}}{u} \tag{4.6}$$

Recently a CONCAWE working group[25,26] developed a regression formula based on the assumption that plume rise depends mainly on some power of heat emission and some power of wind speed. The least-squares fit to the logarithms of the calculated-to-observed plume-rise ratios was

$$\Delta h = 1.40 \left[\frac{\text{ft-(ft/sec)}^{3/4}}{(\text{cal/sec})^{1/2}}\right] \frac{Q_H^{1/2}}{u^{3/4}} \tag{4.7}$$

Data from eight stacks were used, but over 75% of the runs came from Rauch's[80] observations at Duisburg, i.e., from just one stack. Most of these data fall into a small range of Q_H and of u, and therefore it is difficult to establish any power-law relation with confidence.

Even more recently Moses and Carson[110] developed a formula of the same type as Eq. 4.7 with data for ten different stacks, but again the Duisburg observations were heavily weighted. A momentum term of the type that appears in the formulas of Holland,[6] Berlyand and his associates,[106] and Stümke[108] was included, but the optimum value of the constant turned out to be very small. The least-squares fit was given by

$$\Delta h = 1.81 \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{1/2}}\right] \frac{Q_H^{1/2}}{u} \tag{4.8}$$

Actually, changing the exponent of Q_H to ⅓ or ¾ increased the standard error very little. This insensitivity is due partly to the small range of Q_H into which the bulk of the data fell. Another shortcoming of this analysis, as well as of the analysis by Stümke, is that absolute values of the error in predicted rises were employed. This tended to weight the analysis in favor of situations with high plume rise; cases with high wind speed counted very little since both the predicted and the observed rises, and hence their differences, were small. Relative or percentage error, such as used by CONCAWE by means of logarithms, results in more even weighting of the data.

Formulas for Jets

One of the first empirical relations for the rise of pure jets was given by Rupp et al.[61] This relation was determined from photographs of a plume in a wind tunnel. The investigators found the height of the jet center line at

$$\Delta h = 1.5 \left(\frac{w_o}{u}\right) D \tag{4.9}$$

the point at which the plume became substantially horizontal, i.e., when its inclination was only 5 to 8°

Subsequent investigators have all given empirical relations for the jet center line as a function of downwind distance. The results are summarized in Table 4.1 for the case in which the density of the jet is the same as that of air. A theoretical formula to be given later in the chapter is included for comparison.

Table 4.1

COMPARISON OF EMPIRICAL RESULTS FOR JET CENTER LINES AS A FUNCTION OF DOWNWIND DISTANCE

Investigator	Range of $R = (w_0/u)$	Maximum x/D	$\Delta h/D$	$\Delta h/D$ at 5.7° Inclination
Eq. 4.33			$1.44\ R^{0.67}(x/D)^{0.33}$	$3.2\ R^{1.00}$
Rupp et al.[61]	2 to 31	47		$>1.5\ R^{1.00}$
Callaghan and Ruggeri[62]		81	$1.91\ R^{0.61}(x/D)^{0.30}$	$4.0\ R^{0.87}$
Gordier (by Patrick[65])			$1.31\ R^{0.74}(x/D)^{0.37}$	$3.3\ R^{1.17}$
Shandorov (by Abramovich[111])	2 to 22		$0.84\ R^{0.78}(x/D)^{0.39}$	$1.8\ R^{1.28}$
Patrick[65]				
Concentration	6 to 45	22	$1.00\ R^{0.85}(x/D)^{0.34}$	$1.9\ R^{1.29}$
Velocity	8 to 54	34	$1.00\ R^{0.85}(x/D)^{0.38}$	$2.3\ R^{1.37}$

The early Callaghan and Ruggeri[62] experiments involved heated, supersonic jets in a very narrow wind tunnel; so application of their results to free, subsonic jets is questionable. Since the penetration was determined as the highest point at which the temperature was 1°F above the free-stream temperature, the rises given represent the very top of the plume and are noticeably higher than in other experiments. The Gordier formula was obtained from total-head traverses in a water tunnel as reported by Patrick.[65] The formula attributed to Shandorov by Abramovich[111] was based on experiments that included various angles of discharge and density ratios. The Patrick[65] formulas were based both on the height or maximum concentration of nitrous oxide tracer and on the height of maximum velocity as determined by a pitot tube.

THEORETICAL FORMULAS

There are many theoretical approaches to the problem of plume rise, and some of them are quite complex. To reproduce them all here would be tedious and of little help to most readers. Instead, the various theories are compared with a relatively simple basic plume-rise theory based on assumptions common to most of the theories.

It will be shown later that this basic theory in its simplest form gives good agreement with observations.

Basic Theory

In most plume-rise theories, buoyancy is assumed to be conserved; i.e., the motion is considered to be adiabatic. This means that the potential temperature of each element of gas remains constant. It is also assumed that pressure forces are small and have little net effect on the motion, that they merely redistribute some of the momentum within the plume. Molecular viscosity is also negligible because the plume Reynolds number is very high, and local density changes are neglected. These assumptions lead to three conservation equations:

$$\nabla \quad \rho_p \vec{v}_p = 0 \qquad \text{(continuity of mass)} \tag{4.10}$$

$$\frac{d\theta_p}{dt} = 0 \qquad \text{(buoyancy)} \tag{4.11}$$

$$\frac{d\vec{v}_p}{dt} = \frac{g}{T}\,\theta'\vec{k} \qquad \text{(momentum)} \tag{4.12}$$

where $\vec{v}_p$ = the local velocity of the gas in the plume
ρ_p = the local gas density
θ_p = the local potential temperature
$\theta' = \theta_p - \theta$ = the departure of the potential temperature from the temperature of the environment at the same height
$\vec{k}$ = the unit vector in the vertical direction (buoyancy acts vertically)

Equations 4.10, 4.11, and 4.12 can be transformed to describe the mean motion of a plume by integrating them over some plane that intersects the plume. It is most convenient to integrate over a horizontal plane because then the mean ambient values of potential temperature (θ), density (ρ), and velocity ($\vec{v}_e$) can be considered constant over the plane of integration and are assumed to be functions of height only. Furthermore, if $\vec{v}_e$ is assumed to be horizontal, the vertical component of $\vec{v}_p$, denoted by w', is due entirely to the presence of the plume. Thus w' is a convenient variable with which to identify the plume.

A further simplification results from assuming that the vertical velocity and the buoyancy are everywhere proportional to each other in a horizontal section of the plume since it is then unnecessary to assume any specific distribution of either. This assumption is approximately true for measured cross sections of vertical plumes.[57] Admittedly it does not hold near the height of final rise in a stable atmosphere, because buoyancy decays more rapidly than vertical velocity in such a situation.

A steady state is assumed. To obtain Eq. 4.13, we combine Eq. 4.10 times θ' with Eq. 4.11 times ρ_p and integrate the resulting equation over a horizontal plane, assuming that the vertical velocity and the buoyance are everywhere proportional to each other. Similarly, to obtain Eq. 4.14, we combine Eq. 4.10 times $\vec{v}_p$ with Eq. 4.12 times ρ_p and integrate the resulting equation over the same horizontal plane. The plane of integration must completely intersect the plume so that $\theta' \doteq 0$ around the perimeter of the plane. The resulting equations for the net buoyancy flux and momentum flux in a plume are

$$\frac{dF_z}{dz} = -sV \tag{4.13}$$

$$\frac{d(\vec{v}V)}{dz} = \frac{F_z}{w}\vec{k} + \vec{v}_e\frac{dV}{dz} + \text{drag} \tag{4.14}$$

where

$$V = \frac{\iint \rho_p w' \, dx \, dy}{\pi\rho} \quad \text{(vertical volume flux)} \tag{4.15}$$

$$s = \frac{g}{T}\frac{\partial\theta}{\partial z} \quad \text{(stability parameter)} \tag{4.16}$$

$$F_z = \frac{\iint (g/T)\theta' \rho_p w' \, dx \, dy}{\pi\rho} \quad \text{(buoyancy flux)} \tag{4.17}$$

$$\vec{v}V = \frac{\iint \vec{v}_p \rho_p w' \, dx \, dy}{\pi\rho} \quad \text{(momentum flux)} \tag{4.18}$$

The vertical volume flux of the plume, as defined in Eq. 4.15, is the total vertical mass flux divided by $\pi\rho$, where ρ is the environmental density. The stability parameter, s, can be interpreted as the restoring acceleration per unit vertical displacement for adiabatic motion in a stratified atmosphere (either stable or unstable); in an unstable atmosphere, s is negative; F_z is the vertical flux of the buoyant force divided by $\pi\rho$; $\vec{v}$ is an average plume velocity at a given height, as defined by the total velocity field at that height weighted by the normalized vertical mass flux; w is the vertical component of $\vec{v}$ and is the velocity of plume rise at any given height. The drag term in Eq. 4.14 is not written out since it will be dropped later, but it can be interpreted as the net horizontal advection of momentum deficit across the boundary of the plane of integration.

The initial conditions are

$$\vec{v} = w_0\vec{k} \tag{4.19a}$$

$$\vec{v}V = \frac{\rho_0}{\rho} w_0^2 r_0^2 \, \vec{k} \equiv F_m\vec{k} \tag{4.19b}$$

and

$$F_z = \left(1 - \frac{\rho_0}{\rho}\right) g w_0 r_0^2 \equiv F \tag{4.19c}$$

For a hot source

$$\begin{aligned} F &= \frac{g Q_H}{\pi c_p \rho T} \\ &\doteq 4.3 \times 10^{-3} \left[\frac{\text{ft}^4/\text{sec}^3}{\text{cal/sec}}\right] Q_H \\ &\doteq 3.7 \times 10^{-5} \left[\frac{\text{m}^4/\text{sec}^3}{\text{cal/sec}}\right] Q_H \end{aligned} \tag{4.20}$$

where c_p is the specific heat of air at constant pressure.

Equations 4.13 and 4.14 can be solved for the mean motion of a plume through any atmosphere, including one with stability varying with height and wind shear. However, the equations cannot be solved until some specific assumption is made about the growth of volume flux with the height (dV/dz). This assumption, called an entrainment assumption, is necessary to describe the bulk effect of turbulence in diffusing momentum and buoyancy in a plume.

Basic Theory Simplified

It is desirable to reduce the basic theory to the simplest form that works. To be more specific, we would like to derive from the basic theory simple formulas that agree with data. To do this, we must make the simplest workable entrainment and drag-force assumptions, assume simple approximations for the atmosphere, treat the stack as a point source, and treat the plume as being either nearly vertical or nearly horizontal, i.e., ignore the complicated bending-over stage.

When the wind speed is sufficiently low, a plume rises almost vertically, and the drag force and mechanically produced atmospheric turbulence are negligible. The turbulence that causes entrainment of ambient air is generated within the plume by the shear between the vertical plume motion and the almost stationary environment. The simplest workable entrainment hypothesis for this case is that the entrainment velocity, or the average rate at which outside air enters the plume surface, is proportional to the characteristic vertical velocity (w) at any given height. This assumption, based on dimensional analysis, will be called the Taylor entrainment assumption after the author[112] who suggested it in 1945. If $(V/w)^{1/2}$ is defined as a characteristic plume radius, the rate at which the volume flux grows in a given increment of height is then $2\pi(V/w)^{1/2}\alpha w$, where α is called the entrainment constant and is dimensionless. The complete set of equations governing the vertical plume are then

$$\frac{dF_z}{dz} = -sV$$

which was given as Eq. 4.13,

$$\frac{d(wV)}{dz} = \frac{F_z}{w} \tag{4.21}$$

and

$$\frac{dV}{dz} = 2\alpha(wV)^{1/2} \tag{4.22}$$

This set of equations is equivalent to the relations given by Taylor[112] in 1945 and further developed in 1956 in a classic paper[58] by Morton, Taylor, and Turner, who found that a value of 0.093 for the entrainment constant gave the best fit to observed profiles of heated plumes. Briggs[113] found that $\alpha = 0.075$ gives the best predictions of the height of the top of stratified plumes in stable air, based on the height at which the buoyancy flux decays to zero. The latter value is used here. The direct empirical determination of entrainment in jets by Ricou and Spalding[114] yields a comparable value of 0.080.

The case of a bent-over plume, in which the vertical velocity of the plume is much smaller than the horizontal velocity, is simpler. Both the total plume velocity and its horizontal component are then very close to the ambient wind speed, u, which is assumed constant; wind shear is neglected. It is more reasonable in this case to integrate Eqs. 4.10 to 4.12 over a vertical plane intersecting the plume since a vertical plane is more nearly perpendicular to the plume axis. When this is done, the resulting equations are identical to Eqs. 4.13 and 4.21, provided that s is constant over the plane of integration, that F_z, V, and wV are defined as fluxes of plume quantities through a vertical plane, and that the drag term is zero. Measurements by Richards[69] of the mean streamlines near horizontal thermals suggest that the drag term is zero provided the chosen plane of integration is large enough. This is also intuitively evident since one would not expect a vertically rising plume to leave a very extensive wake underneath it.

In the initial stage of rise of a bent-over plume, the self-induced turbulence dominates the mixing process, and the Taylor entrainment hypothesis can be used again. The main difference from a vertical plume is that in this case the velocity shear is nearly perpendicular to the plume axis, rather than parallel to it. This apparently results in more efficient turbulent mixing since the entrainment constant for a bent-over plume is about 5 times as large as that for a vertical plume. With a characteristic plume radius defined as $(V/u)^{1/2}$, the rate at which the volume flux grows in a given increment of axial distance is $2\pi(V/u)^{1/2}\ \gamma w$, where γ is the entrainment constant for a bent-over plume. If this is transformed to vertical coordinates, the plume rise is governed by Eqs. 4.13, 4.21, and

$$\frac{dV}{dz} = 2\gamma(uV)^{1/2} \tag{4.23}$$

which is comparable to Eq. 4.22. Since u is a constant, Eq. 4.23 can readily be integrated. For a point source this yields a characteristic radius equal to γz. The relation is confirmed by modeling experiments of Richards[69] and by photographs of full-scale plumes made by TVA[99] (see Fig. 4.1). On the basis of these photographic plume diameters, $\gamma \doteq 0.5$.

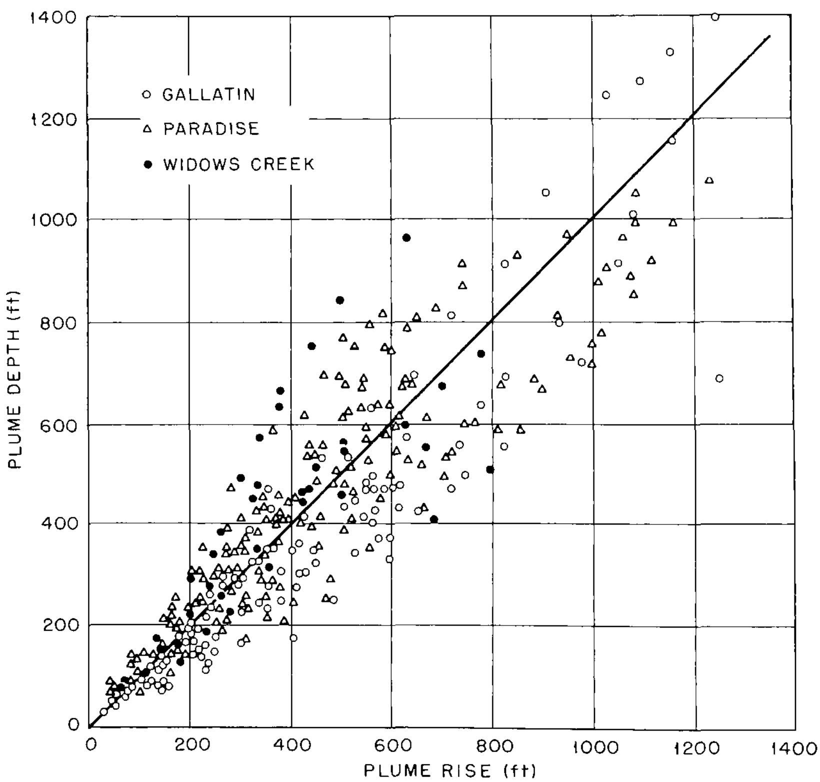

Fig. 4.1 Photographic plume depth (top to bottom) vs. plume rise (center line) at TVA plants.

Atmospheric turbulence is small in a stable environment and can be neglected, in which case Eq. 4.23 is valid up to the point where a bent-over plume reaches its maximum rise. However, in a neutral or unstable atmosphere, turbulence is vigorous enough to eventually dominate the entrainment process. This occurs some distance downwind of the stack when the vertical velocity of the plume becomes small compared with ambient turbulent velocities. The simplest measure of the effective intensity of atmospheric turbulence is the eddy energy dissipation, ϵ, because it adequately describes the part of the turbulence spectrum that is most effective at diffusing the plume relative to its axis, i.e., the inertial subrange. The characteristic radius of the plume, $(V/u)^{1/2}$, determines the range of eddy sizes that most efficiently diffuse the plume. If these two terms are adequate enough to characterize entrainment, the effective entrainment velocity must be given by $\beta\epsilon^{1/3}(V/u)^{1/6}$, where β is a dimensionless constant; the exponents of the terms result from dimensional considerations. Since the entrainment velocity in the initial stage of plume rise is γw, for the simplest model of a bent-over plume an abrupt transition to an entrainment velocity of $\beta\epsilon^{1/3}(V/u)^{1/6}$ is assumed to occur when $\gamma w = \beta\epsilon^{1/3}(V/u)^{1/6}$

The solution for the bending-over stage of a plume in a crosswind is less certain because both shear parallel to the plume axis and shear perpendicular to the axis are present. Both mechanisms operate at once to cause turbulent entrainment. Drag force could contribute to the bending over of the plume since there could be an extensive wake downwind of the plume in this case, but the drag force will have to be neglected at present owing to insufficient knowledge. In the early stage of bending over, the vertical-plume model is applicable except that there is a perpendicular shear velocity nearly equal to u. If the two contributions to entrainment can be summed in the manner of vectors, the resultant entrainment velocity becomes $(\alpha^2 w^2 + \gamma^2 u^2)^{1/2}$, and the plume center line is given by Eqs. 4.13, 4.21, and

$$\frac{dV}{dz} = 2\left(\frac{V}{w}\right)^{1/2} (\alpha^2 w^2 + \gamma^2 u^2)^{1/2} \qquad (4.24)$$

Before applying models of the vertical plume and bent-over plume to specific cases, some approximations about the source can be made. Usually it is reasonable to assume that either the initial vertical momentum or the buoyancy dominates the rise. In the former case the plume is called a jet, and we set F equal to 0. Unheated plumes composed mostly of air are in this category. Most hot plumes are dominated by buoyancy, and we can neglect the initial vertical momentum flux, F_m. At a sufficient distance from the stack, e.g., beyond 20 stack diameters downwind, we can neglect the finite size of the source and treat the stack merely as a point source of momentum flux or buoyancy flux.

Some of the approximations that come out of the simplified theory are given in Eqs. 4.25 to 4.34. Vertical plumes are indicated by the term "calm" and bent-over plumes by "wind." For rise into stable air in which s is constant, we have

$$\Delta h = 5.0 F^{1/4} s^{-3/8} \qquad \text{(buoyant, calm)} \qquad (4.25)$$

$$\Delta h = 2.4 \left(\frac{F}{us}\right)^{1/3} \qquad \text{(buoyant, wind)} \qquad (4.26)$$

$$\Delta h = 4\dagger \left(\frac{F_m}{s}\right)^{1/4} \qquad \text{(jet, calm)} \tag{4.27}$$

$$\Delta h = 1.5 \left(\frac{F_m}{u}\right)^{1/3} s^{-1/6} \qquad \text{(jet, wind)} \tag{4.28}$$

In the calm case, Eq. 4.25 gives the height at which the buoyancy goes to zero. In the windy cases for a bent-over plume, the equations are integrated to the point where $w = 0$, and the plume is assumed to fall back to the level at which the buoyancy is zero with no further mixing. More details are given by Briggs.[113] The plume will penetrate a ground-based inversion or stable layer if the preceding formulas predict a rise higher than the top of the stable air. If the air is neutrally stratified above this level, a buoyant plume will continue to rise since it still has some buoyancy. A jet will fall back and level off near the top of the stable air because it acquires negative buoyancy as it rises.

The model predicts penetration of a sharp, elevated inversion of height z_i through which the temperature increases by ΔT_i if

$$z_i \leqslant 7.3 F^{0.4} b_i^{-0.6} \qquad \text{(buoyant, calm)} \tag{4.29}$$

$$z_i \leqslant 2.0 \left(\frac{F}{ub_i}\right)^{1/2} \qquad \text{(buoyant, wind)} \tag{4.30}$$

$$z_i \leqslant 1.6\dagger \left(\frac{F_m}{b_i}\right)^{1/3} \qquad \text{(jet, calm)} \tag{4.31}$$

where $b_i = g\ \Delta T_i/T$. The buoyant plume is assumed to penetrate if its characteristic temperature excess, given by $(T/g)F_z/V$, exceeds ΔT_i at the height of the inversion.

For the first stage of rise, the bent-over model predicts plume center lines given by

$$\Delta h = 1.8 F^{1/3} u^{-1} x^{2/3} \qquad \text{(buoyant, wind)} \tag{4.32}$$

$$\Delta h = 2.3 F_m^{1/3} u^{-2/3} x^{1/3} \qquad \text{(jet, wind)} \tag{4.33}$$

For the general case where s is positive and constant, Eqs. 4.13 and 4.21 can be combined with the transform $dz = (w/u)\, dx$ to give

$$\frac{d^2(wV)}{dx^2} = -\frac{s}{u^2}(wV)$$

This is the equation of a simple harmonic oscillator. Since V always increases, the plume center line behaves like a damped harmonic oscillator (the author has observed

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†Empirical; numerical value difficult to determine from present model.

such behavior at a plant west of Toronto in the early morning). Since $V \simeq u\gamma^2 z^2$, the preceding expression can be integrated and satisfies the initial conditions when

$$[(\gamma^2/3)us^{1/2}]\ \Delta h^3 = F_m \sin(xs^{1/2}/u) + Fs^{-1/2}[1 - \cos(xs^{1/2}/u)]$$

This equation is valid only up to the point of maximum rise because beyond this point a negative entrainment velocity would be implied. According to this equation a jet ($F = 0$) reaches its maximum height at $x = (\pi/2)\ us^{-1/2}$ and a buoyant plume ($F_m = 0$) reaches its maximum height at $x = \pi us^{-1/2}$. At much smaller distances the plume center line is approximated by

$$\Delta h = 2.3 F_m^{1/3} u^{-2/3} x^{1/3} \left(1 + \frac{Fx}{2F_m u}\right)^{1/3}$$

From this equation it is seen that the ratio $Fx/F_m u$ is a general criterion of whether a bent-over plume is dominated by buoyancy or by momentum at a given distance downwind. It, in fact, represents the ratio of buoyancy-induced vertical momentum to initial vertical momentum.

For the buoyant bent-over plume in neutral conditions, the first stage of rise is given by Eq. 4.32 up to the distance at which atmospheric turbulence dominates the entrainment. The complete plume center line is given by Eq. 4.32 when $x < x^*$ and by

$$\Delta h = 1.8 F^{1/3} u^{-1} x^{*2/3} \left[\frac{2}{5} + \frac{16}{25}\frac{x}{x^*} + \frac{11}{5}\left(\frac{x}{x^*}\right)^2\right]\left(1 + \frac{4}{5}\frac{x}{x^*}\right)^{-2} \qquad (4.34)$$

when $x > x^*$, where x^* is the distance at which atmospheric turbulence begins to dominate entrainment. This distance is given by

$$x^* = 0.43 F^{2/5} (\beta^{-3} u \epsilon^{-1})^{3/5}$$

Results from puff and cluster diffusion data and from measurements of eddy energy dissipation rates, given in Appendix A, show that $\beta = 1$ is acceptable as a somewhat conservative approximation. In the surface layer of the atmosphere defined by constant stress, e.g., the lowest 50 ft or so, it is well established[115] that $\epsilon = u^{*3}/0.4\bar{z}$, where $\bar{z}$ is the height above the ground and u^* is the friction velocity. If we approximate $\bar{z}$ by $\bar{z} = \Delta h$, the final plume rise given by Eq. 4.34 is $\Delta h = 4.5\ F/uu^{*2}$; since $u \propto u^*$ and changes only gradually with height in the neutral surface layer, this result is similar to those of earlier theories[36,46,107] that predict $\Delta h \propto F/u^3$ Unfortunately, this clear relation between ϵ and u^* breaks down at heights more typical of smoke plumes. In Appendix A, data from 50 to 4000 ft above the ground give more support to the empirical relation

$$\epsilon \simeq 0.73 \left[\frac{ft^2}{sec^2}\right] \frac{u}{\bar{z}}$$

up to $\bar{z} \simeq 1000$ ft, then becoming constant with height. If we conservatively approximate $\bar{z}$ with the stack height, the resulting estimate for x^* becomes

$$x^* = 0.52 \left[\frac{\sec^{6/5}}{\mathrm{ft}^{6/5}}\right] F^{2/5} h_s^{3/5} \qquad (h_s < 1000 \text{ ft})$$

$$x^* = 33 \left[\frac{\sec^{6/5}}{\mathrm{ft}^{3/5}}\right] F^{2/5} \qquad (h_s > 1000 \text{ ft}) \tag{4.35}$$

Other Theories

There is such a variety of plume-rise theories in the literature that only the briefest discussion of each must suffice. One can only be amazed, and perhaps perplexed, at the number of different approaches to the solution of this fascinating fluid-dynamics problem. The theories will be discussed chronologically, first for the calm case and then for the crosswind case.

The first theoretical treatment was of a jet in calm surroundings and was given by Tollmien[116] in 1926. Rather than making an entrainment assumption, he used the Prandtl mixing-length hypothesis to derive a specific velocity-profile law that agrees quite well with data. A similar approach was used for heated plumes in calm air by Schimdt[55] in 1941. Rouse, Yih, and Humphreys[57] treated the same problem by assuming eddy viscosity diffusion of the buoyancy and momentum by a process analogous to molecular diffusion. They determined experimentally that the mean temperature and velocity profiles are approximately Gaussian with the characteristic plume radius growing linearly with height. Yih[56] also considered the case of a laminar plume, which does not apply to full-scale plumes.

Batchelor[48] considered the same problem in 1954 by dimensional analysis. He included the case of a stratified environment and found power-law expressions for the mean plume velocity and temperature as functions of height in an unstable atmosphere whose potential temperature gradient is also approximated by a power law. The first theoretical model for a vertical plume rising through any type of stratification was given by Priestley and Ball[117] in 1955. Their equations are similar to the preceding equations for the vertical plume except that the entrainment assumption, Eq. 4.22, is replaced by an energy equation involving an assumption about the magnitude and distribution of the turbulent stress. Vehrencamp, Ambrosio, and Romie[100] were the first to apply the results from an entrainment model to final rise in stable air by using the Taylor entrainment assumption. A general model involving this assumption and complete with experimental verification was put forth by Morton, Taylor, and Turner[58] in 1956. This model is called the M,T,&T model in the discussion that follows. The M,T,&T model is virtually identical to the vertical-plume model presented in the section "Basic Theory Simplified" of this chapter and differs from the Priestley and Ball[117] model mainly by predicting a wider half-cone angle for jets than for buoyant plumes. This is actually observed in the laboratory. Both the M,T,&T model and the Priestley and Ball model predict a linear increase of radius with height in the unstratified case and give similar results for the final plume height but disagree somewhat on the values of the numerical constants. Estoque[118] further compares these two theories.

Morton[119] extended the numerical integrations of the M,T,&T model to the case of a buoyant plume with nonnegligible initial momentum and concluded that increasing the efflux velocity can actually lessen rise in stable conditions because of increased entrainment near the stack level. In another paper,[120] he extended the theory to include augmented buoyancy due to the condensation of moisture of the entrained air. Hino[121,122] made further calculations with the M,T,&T model, including the effects of a finite source radius. Turner[123] coupled the M,T,&T model with a vortex ring model to predict the speed of rise for a starting plume in neutral surroundings. Okubo[124] expanded the M,T,&T model to the case of a plume rising through a salinity gradient in water.

A generalized theory for steady-state convective flow incorporating several of these solutions was given by Vasil'chenko.[125] Recently Telford[126] proposed another type of entrainment assumption in which the entrainment velocity is proportional to the magnitude of turbulent fluctuations in the plume as calculated from a turbulent kinetic energy equation. Telford's results are similar to those of the M,T,&T model for a buoyant plume, except near the stack, but his model predicts too-rapid growth for a jet. This happens because the model is, in effect, based on the assumption that the scale of the energy-containing turbulent eddies is proportional to the plume radius, but this is not true for a jet, because most of the turbulent energy is generated while the jet radius is relatively small. Morton[127] has further criticized Telford's model in a recent note.

Lee[128] developed a model for a turbulent swirling plume. He used the Prandtl mixing-length hypothesis. Still another problem was explored by Fan,[71] who extended the M,T,&T theory to the case of nonvertical emissions and tested the result in a modeling tank with linear density stratification.

One of the earliest theories for a bent-over buoyant plume was given by Bryant[66] in 1949. A drag-force assumption was included, and the entrainment assumption was in the form of a fairly complicated hypothesis about how the plume radius grows with distance from the source along its center line. Eventually the radius in this model becomes proportional to $x^{1/3}$, which is too small a growth rate compared with subsequent observations.

In 1950 Bosanquet, Carey, and Halton[19] published a well-known theory that was later revised by Bosanquet.[20] The entrainment assumptions were similar to those made in the simplified theory here except that the same entrainment constant was applied to both the vertical and the bent-over stages of plume development, i.e., $\gamma = \alpha$. In addition, a contribution to the entrainment velocity due to environmental turbulence was assumed that was proportional to the wind speed. This assumption eventually led to a linear growth of plume radius with distance downwind and resulted in a final height for a bent-over jet and rise proportional to log x for a buoyant plume. The theory tends to underestimate rise at large distances downwind (see Fig. 5.3 in the next chapter).

About the same time, Sutton[129] developed a simple theory for a buoyant plume in a crosswind which was based on Schmidt's[55] result for a vertical plume, i.e., $w \propto (F/z)^{1/3}$. Sutton replaced z in this relation with the distance along the plume center line and took the horizontal speed of the plume to be equal to u. The expression is dimensionally correct and, at large distances, approaches the form given by Eq. 4.32.

Priestley[73] adapted his and Ball's vertical-plume model to the bent-over case. The average radius of a horizontal section was assumed to grow linearly with height, and the entrainment constant was modified by a factor proportional to $u^{1/2}$. Thus the equations of rise were identical to those for a vertical plume except for the entrainment constant modification. Priestley coupled this first-phase theory with a second phase in which atmospheric turbulence dominates the mixing. This latter phase is complicated and yields some unrealistic results, as was mentioned by Csanady.[87] The first phase leads to an asymptotic formula identical to Eq. 4.32 times a factor proportional to $(F/x)^{-1/12}$; namely,

$$\Delta h = 2.7 \left[\left(\frac{\text{ft}}{\text{sec}}\right)^{1/4}\right] F^{1/4} u^{-1} x^{3/4} \tag{4.36}$$

Lucas, Moore, and Spurr[91] were able to simplify Priestley's theory considerably. For the first stage of rise, they obtained a plume rise 15% greater than that given by Eq. 4.36, and, for the atmospheric-turbulence-dominated stage, they obtained

$$\Delta h = 258 \left[\frac{\text{ft-ft/sec}}{(\text{cal/sec})^{1/4}}\right] \frac{Q_H^{1/4}}{u} \left[1 - \left(0.6 + 0.2\,\frac{x - x_1}{1000\ \text{ft}}\right) \exp\left(-\frac{x - x_1}{1000\ \text{ft}}\right)\right] \tag{4.37}$$

where x_1 is the distance of transition to the second stage. It was estimated that x_1 = 660 ft, in contrast to the transition distance x* given by Eq. 4.35, which depends on both the source strength and the height in the atmosphere.

Scorer[36,130] introduced a simple plume-rise model for which he assumed that the plume radius grows linearly with height (see Fig. 4.1). The constant governing the growth rate depends on whether the plume is nearly vertical or bent over and also on whether it is dominated by momentum or by buoyancy in a given stage. Scorer considered all the separate possibilities and then matched them at the bend-over point to get a complete set of formulas for rise in neutral conditions. The predictions for transitional rise, the plume center line before final height is reached, are similar to those given by Eqs. 4.32 and 4.33. In addition, he postulated that the active rise terminates when the vertical velocity of the plume reduces to the level of atmospheric turbulence velocities, which he took to be some fraction of the wind speed. This led to the prediction that $\Delta h \propto F/u^3$ for a very buoyant plume. This type of formula has been given by many authors, but the leveling off of the plume in neutral conditions has not yet actually been observed. Furthermore, it now appears that atmospheric turbulence velocities are less strongly related to wind speed at typical plume heights.[131]

A great variety of work has been done in the last 6 years. Lilly[70] constructed a numerical model of the two-dimensional vortex pair seen in a vertical cross section of a bent-over buoyant plume. Keffer and Baines[63] presented a model for the bending-over stage of a jet with an entrainment assumption similar to the one in the bending-over model given in this review except that only the horizontal shear was included. Danovich and Zeyger[78] developed a theory along the lines of the Priestley theory for the first stage but with the second-stage dynamics determined by the diffusion of buoyancy by atmospheric turbulence. The type of diffusion assumed was essentially the same as that observed for total diffusion of gases in a passive plume. However, total diffusion includes the meandering of the plume axis caused by shifts in wind direction, whereas the action of buoyancy on the plume is affected only by the diffusion of buoyancy relative to the plume axis. Only relative diffusion should be used. The same criticism applies to a theory developed by Schmidt,[132] which is based on the assumption that the spread of material equals that given by the total diffusion of a passive plume. There is also the criticism that the diffusion of a rising plume, especially in its early stages, is not the same as for a passive plume, because the rising plume generates its own turbulence in addition to the ambient turbulence. These problems were also pointed out by Moore.[133]

Equations 4.25 through 4.28 and Eqs. 4.32 and 4.33 were proposed by Briggs[107] on the basis of rather elementary dimensional analysis as an extension of Batchelor's[48] and Scorer's[46] approaches. Briggs[113] recently considered in some detail the penetration of inversions by plumes of all types by using a model based on the simplified theory given here. Gifford[134] extended this type of model to the case of a bent-over plume whose total buoyancy flux increases linearly with time as it moves away from the source, again using the Taylor entrainment assumption. Modeling experiments of Turner[135] with thermals of increasing buoyancy support this assumption.

A model by Csanady[136] for the bent-over buoyant plume included the effect of eddy-energy dissipation and of inertial subrange turbulence in the relative diffusion of plume buoyancy. In a later paper by Slawson and Csanady,[5] a three-stage model was proposed. In the first stage, self-generated turbulence dominates, and the governing equations are in fact the same as those given in the bent-over plume model here. The second stage is dominated by inertial subrange atmospheric turbulence, and, in the third stage, the plume is supposed to be large enough for the eddy diffusivity to be essentially constant, as is the case for molecular diffusion. This model yields a radius proportional to $x^{1/2}$ and a constant rate of rise in the final stage rather than any limiting height of rise.

Very recently a model along the lines of the basic theory presented here was developed by Hoult, Fay, and Forney,[137] in which entrainment velocity depends on the longitudinal and transverse shear velocities. This theory is more elaborate than the simplified theory presented here, in that γ may be a function of w_0/u and the Froude number at the stack but does not take into account the effect of atmospheric turbulence.

5 COMPARISONS OF CALCULATED AND OBSERVED PLUME BEHAVIOR

NEUTRAL CONDITIONS

Buoyant Plumes in Neutral Conditions

Some previous comparisons of plume-rise formulas with data for the case of hot bent-over plumes in near-neutral conditions were reviewed by Moses, Strom, and Carson[138] and are only summarized here. Moses and Strom[139] compared a number of formulas with data from their experimental stack. However, there was much scatter in the data, and only the absolute differences between observed and calculated values were used in the analysis, rather than their ratios. The results of the comparisons were rather inconclusive. Rauch[80] made a brief comparison of the Holland[6] formula and that of Lucas, Moore, and Spurr[91] with his own data and found the latter formula, multiplied by a factor of 0.35, to be a better fit. Stümke[104] made more extensive comparisons between 8 different formulas and the data of Bosanquet, Carey, and Halton,[19] of Stewart, Gale, and Crooks,[75] and of Rauch.[80] By computing the ratios of calculated to observed rises, Stümke concluded that the Holland formula, multiplied by a factor of 2.92, works best.

Since these comparisons were made, a number of new formulas have appeared, including those by Stümke,[108] Moses and Carson,[110] CONCAWE,[25,26] the modified Lucas formula,[109] and Eq. 4.34, published for the first time here. In addition, more data are now available, especially the data from three Central Electricity plants and six TVA plants; therefore comparisons can now be mad· over a much wider range of conditions.

First, a simple wind-speed relation would be convenient since this would allow some reduction of a large amount of data that covers a wide range of wind speeds, source strengths, and measuring distances. Many formulas, both empirical and theoretical, suggest that plume rise is inversely proportional to wind speed, at least at a fixed point downwind. In Fig. 5.1, data from a large number of sources tend to confirm this. In each graph the plume rise at one or more fixed distances is plotted against wind speed on logarithmic coordinates so that $\Delta h \propto u^{-1}$ is represented by a straight line with a slope of -1; such lines are indicated for reference.

For most of the sources, $\Delta h \propto u^{-1}$ is the best elementary relation. It would be difficult to make a case for $\Delta h \propto u^{-3/4}$, as appears in the CONCAWE[25,26] formula. A better fit would result only for the Duisburg data, upon which the CONCAWE formula is very largely based. A few of the sources, in particular Shawnee and Widows Creek, show a greater decrease of Δh with increasing u, which probably indicates some form of downwash at higher wind speeds. However, the Davidson–Bryant[105] prediction that Δh is proportional to $u^{-1.4}$ would not fit most of the data.

With the inverse wind-speed law reasonably well established for neutral conditions, we can now average the product of plume rise and wind speed for all wind speeds to greatly reduce the volume of data. Such a presentation was first employed by Holland.[6] In Fig. 5.2, $u\,\Delta h$ is plotted as a function of x for all available data sources. The average heat efflux per stack, in units of 10^6 cal/sec, is given in parentheses following each identification code, along with the number of stacks if more than one. The key to the code is given in Table 5.1. In system A at Harwell (HA), wind measurements were at a height of 27 m, whereas in system B (HB) the measurements were at 152 m, which is much closer to the height of the plume. A considerable amount of data is presented in Fig. 5.2. A general criterion was that each point plotted should represent at least three periods of 30 to 120 min duration each and that each period should be represented by at least five samples of plume rise or some equivalent amount of data.

The outstanding feature of Fig. 5.2 is that all the plume center lines continued to rise as far as measurements were made; there is no evidence of leveling off. In general, the plume center lines approximate a 2/3 slope, as predicted by the "2/3 law" in Eq. 4.32. This means that the final rise has not definitely been measured in neutral conditions, and therefore we will have to find some other way of defining effective stack height.

The same data as in Fig. 5.2, along with the data of Ball,[76] are plotted in Fig. 5.3. Both the rise and the distance downwind are made nondimensional by means of the length $L = F/u^3$ The result is a somewhat entangled family of curves that lie between 1.0 and 3.0 times $F^{1/3}u^{-1}x^{2/3}$. Rise for a buoyant plume according to the Bosanquet theory[20] and the asymptotic plume rises according to Csanady[87] in 1961 and to Briggs[107] in 1965 are shown. They all underestimate rise at large values of x/L.

The Bosanquet[20] formulation underestimates plume rise when $x/L > 10^3$ The CONCAWE[25,26] relation that Δh is proportional to $u^{-3/4}$ and the Davidson–Bryant[105] relation that Δh is proportional to $u^{-1.4}$ are not valid for most data sources. Formulas of the type $\Delta h \propto L = F/u^3$ are difficult to test because they apply only to final rise in

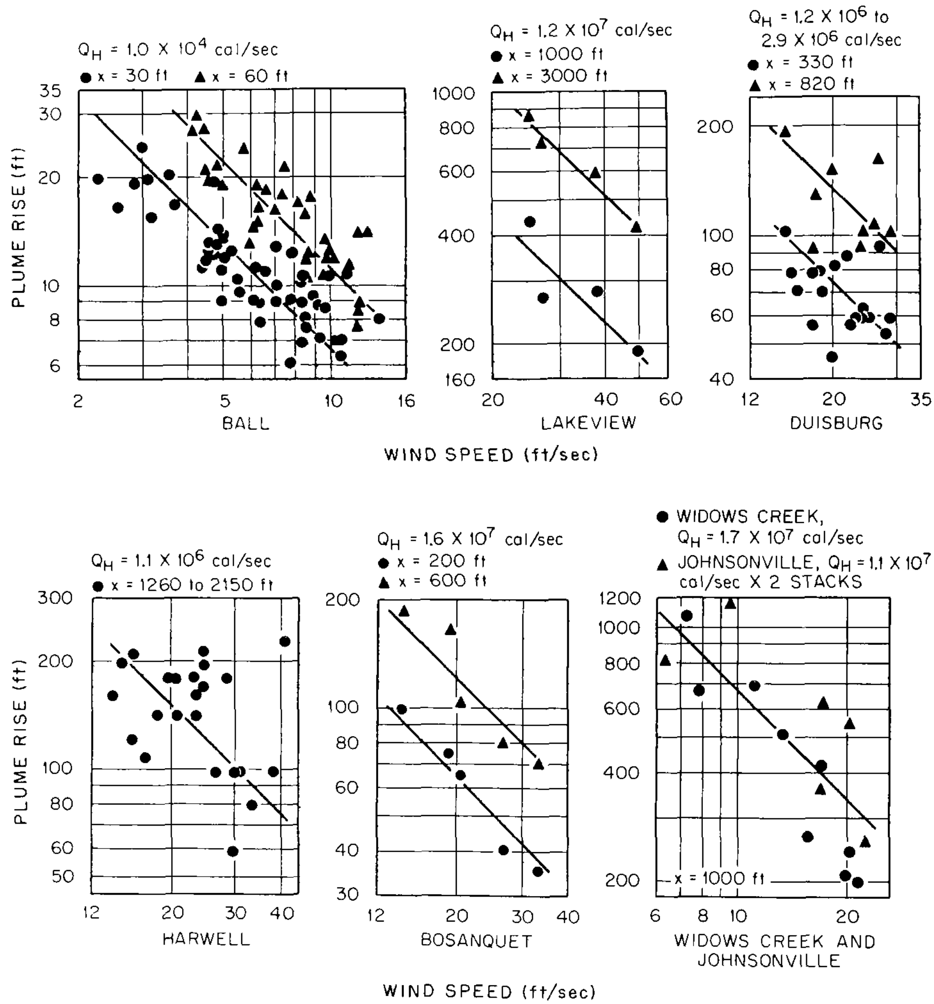
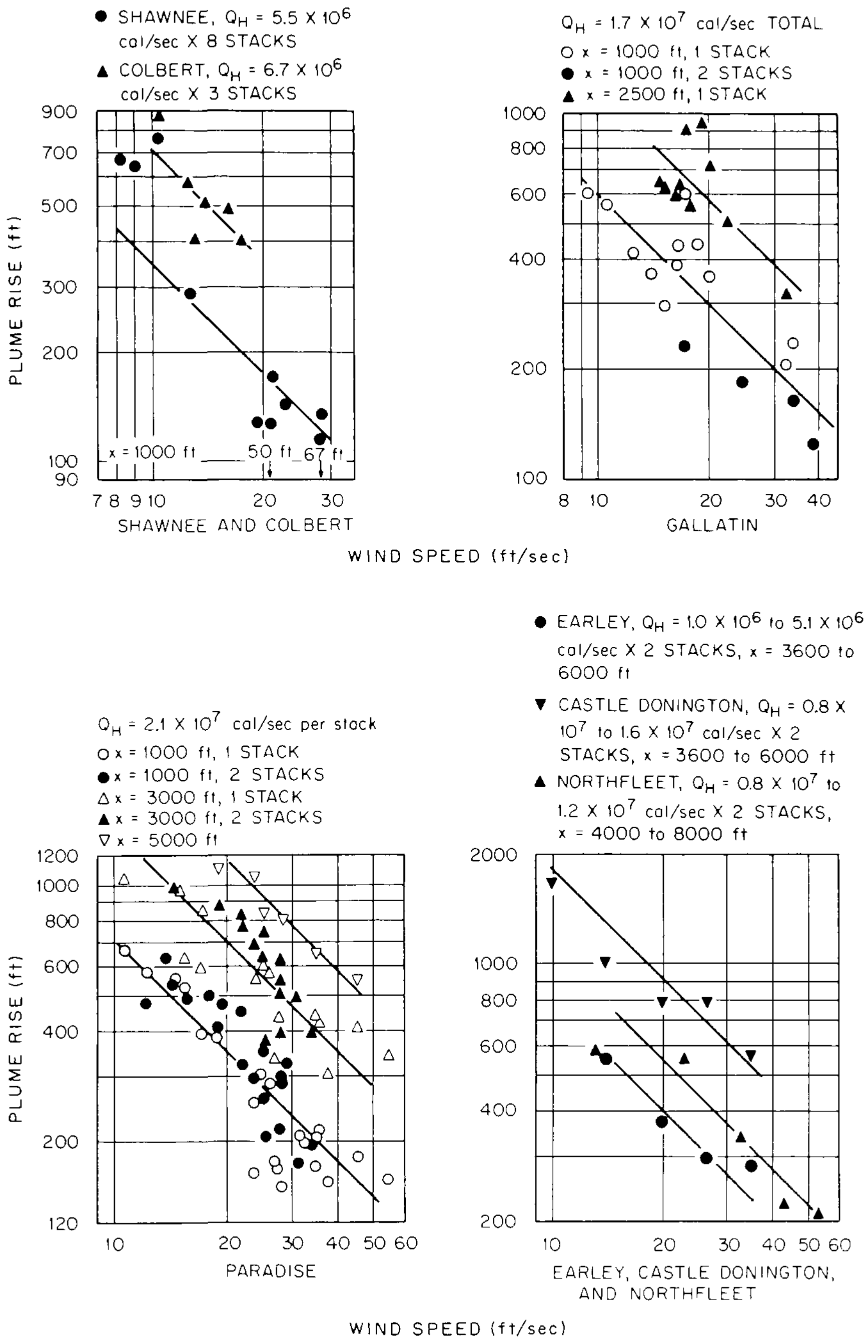


Fig. 5.1 Plume rise vs. wind speed in near-neutral conditions.


● SHAWNEE, Q_H = 5.5 X 10^6 cal/sec X 8 STACKS
▲ COLBERT, Q_H = 6.7 X 10^6 cal/sec X 3 STACKS
x = 1000 ft
50 ft
67 ft
PLUME RISE (ft)
SHAWNEE AND COLBERT
Q_H = 1.7 X 10^7 cal/sec TOTAL
○ x = 1000 ft, 1 STACK
● x = 1000 ft, 2 STACKS
▲ x = 2500 ft, 1 STACK
GALLATIN
WIND SPEED (ft/sec)
Q_H = 2.1 X 10^7 cal/sec per stack
○ x = 1000 ft, 1 STACK
● x = 1000 ft, 2 STACKS
△ x = 3000 ft, 1 STACK
▲ x = 3000 ft, 2 STACKS
▽ x = 5000 ft
PLUME RISE (ft)
PARADISE
● EARLEY, Q_H = 1.0 X 10^6 to 5.1 X 10^6 cal/sec X 2 STACKS, x = 3600 to 6000 ft
▼ CASTLE DONINGTON, Q_H = 0.8 X 10^7 to 1.6 X 10^7 cal/sec X 2 STACKS, x = 3600 to 6000 ft
▲ NORTHFLEET, Q_H = 0.8 X 10^7 to 1.2 X 10^7 cal/sec X 2 STACKS, x = 4000 to 8000 ft
EARLEY, CASTLE DONINGTON, AND NORTHFLEET
WIND SPEED (ft/sec)

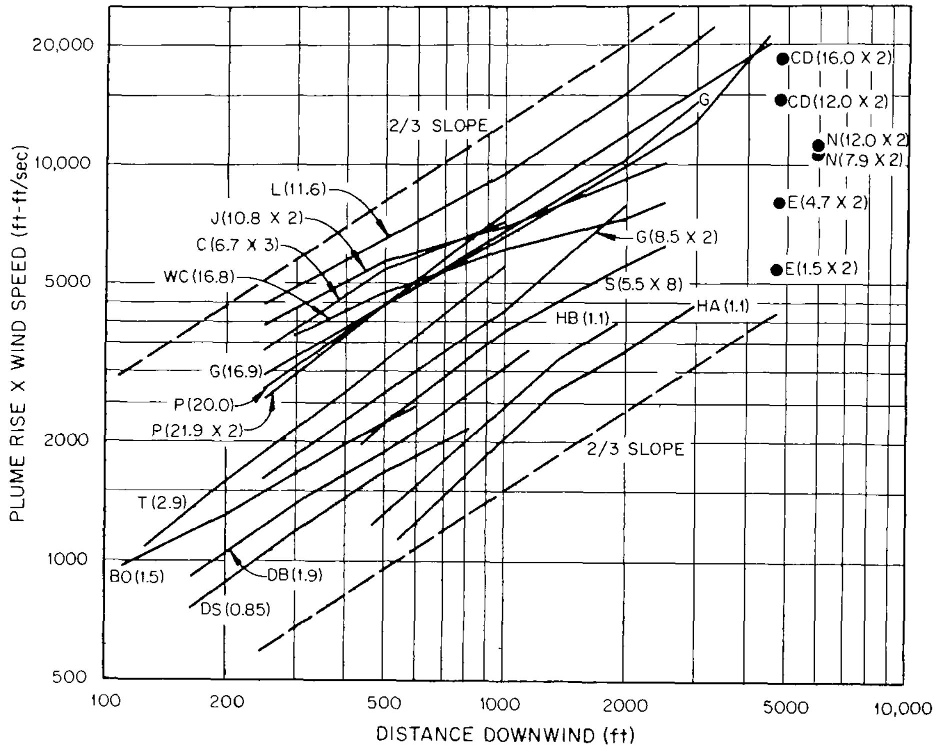


Fig. 5.2 Plume rise times wind speed vs. downwind distance in near-neutral conditions. The average heat efflux per stack, in units of 10^6 cal/sec, and the number of stacks, if more than one, are given in parentheses. See Table 5.1 for identification of sources and for additional data.

neutral conditions, which has not yet been clearly observed. Therefore only relations of the type $\Delta h \propto \bar{u}^{-1}$ have been chosen for the comparison shown in Table 5.1. Data are given for the plume rises at the maximum distance downwind for which there was sufficient information to meet the data criterion set up for Fig. 5.2. The ratio of calculated to observed plume rises times wind speed was computed for each source and each formula, and the results were analyzed on a one-source one-vote basis. The exceptions to this rule were plants that were run both with one stack and with two stacks emitting (Paradise and Gallatin) and plants at which there were substantial amounts of data for different rates of heat emission (Earley, Castle Donington, Northfleet). The median value of the ratio was also computed for each plume-rise formula, along with the average percentage deviation from the median. The same computation was repeated for a selected set of data that excluded the following data sources: Ball, source very small; Harwell A, wind speed measured much below plume and obviously lower than that measured with system B; Bosanquet, no stack heights indicated and length of runs uncertain; Darmstadt, low efflux velocity and insufficient

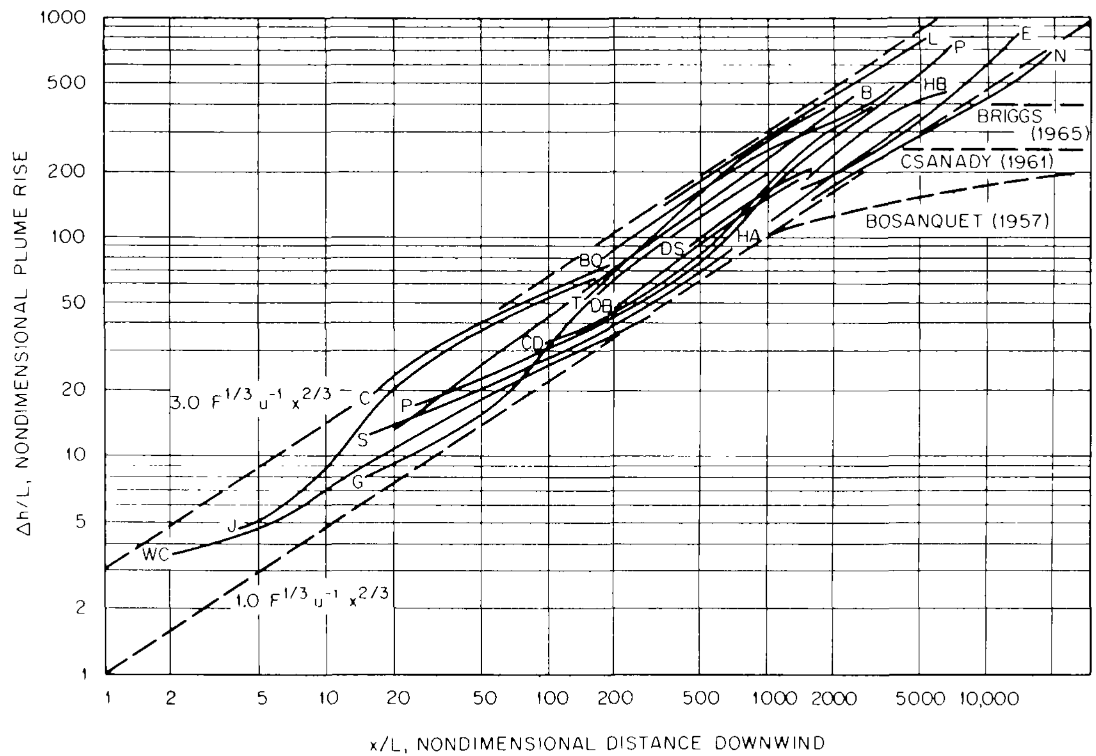


Fig. 5.3 Nondimensional plume rise vs. nondimensional distance downwind in near-neutral conditions. Rise for a buoyant plume according to Bosanquet[20] and the asymptotic plume rises according to Csanady[87] and Briggs[107] are shown. See Table 5.1 for identification of other curves and for additional data.

Table 5.1

COMPARISON OF CALCULATED VALUES WITH OBSERVATIONS FOR NEUTRAL CONDITIONS

Code	Source	Reference	Number of stacks	h_s, ft	D, ft	w_0, ft/sec	Range of u, ft/sec	Q_H/stack, 10^6 cal/sec	x*,†, ft	x, ft	u Δh, ft-ft/sec
B	Ball‡	76					2 to 14	0.0096	14§	60	112
HA	Harwell A‡	75	1	200	11.3	32.6	14 to 30	1.10	370	2950	4,430
HB	Harwell B	75	1	200	11.3	32.6	17 to 38	1.10	370	1900	3,980
BO	Bosanquet‡	73	1		6.5	31.9	14 to 33	1.54	485¶	600	2,450
DS	Darmstadt‡	80	1	246	7.5	15.7	16 to 25	0.855	380	820	2,150
DB	Duisburg	80	1	410	11.5	28.0	15 to 29	1.88	705	1150	3,400
T	Tallawarra‡	87	1	288	20.5	12.0	20 to 23	2.93	680	1000	5,500
L	Lakeview‡	88	1	493	19.5	65.0	25 to 49	11.6	1630	3250	22,100
	CEGB plants										
E	Earley	91	2	250	12.0	18.3	14 to 35	1.54	485	4800	5,580
E	Earley	91	2	250	12.0	56.0	14 to 35	4.72	760	4800	8,150
CD	Castle Donington	91	2	425	23.0	40.9	10 to 26	11.95	1510	4800	14,800
CD	Castle Donington	91	2	425	23.0	54.7	10 to 35	16.0	1700	4800	18,600
N	Northfleet‡	93	2	492	19.7	46.3	13 to 52	7.9	1400	5900	10,900
N	Northfleet‡	93	2	492	19.7	70.0	13 to 52	11.95	1660	5900	11,150
	TVA plants	99									
S	Shawnee‡		8	250	14.0	48.7	8 to 29	5.45	805	2500	6,210
C	Colbert‡		3	300	16.5	42.9	10 to 17	6.74	975	1000	7,200
J	Johnsonville		2	400	14.0	94.8	6 to 22	10.8	1400	2500	10,100
WC	Widows Creek‡		1	500	20.8	71.5	8 to 21	16.8	1910	2500	8,000
G	Gallatin		1	500	25.0	52.4	7 to 34	16.9	1920	3000	14,250
G	Gallatin		2	500	25.0	23.7	5 to 39	8.55	1460	2000	7,850
P	Paradise		1	600	26.0	51.3	6 to 55	20.2	2300	4500	21,200
P	Paradise		2	600	26.0	57.2	12 to 34	21.9	2380	4500	20,000

†Calculated from Eq. 4.35.
‡Not included in selected data.
§Height chosen for computing x* = 20 ft.
¶Height chosen for computing x* = 250 ft.

Table 5.1 (Continued)

Code	Source	Reference	Ratio of calculated to observed values of u Δh: Moses and Carson[110]	Stümke[108]	Holland[6]	Priestley[73,87] (first phase)	Lucas, Moore, and Spurr[91]	Lucas[109]	Eq. 4.32 ("2/3 law")	Eq. 4.34
B	Ball	76	1.59		0.04	1.31	1.51	0.78	0.86	0.72
HA	Harwell A	75	0.43	0.74	0.23	2.00	1.70	1.27	1.40	0.95
HB	Harwell B	75	0.48	0.83	0.25	1.60	1.59	1.19	1.17	0.93
BO	Bosanquet	73	0.92	0.75	0.44	1.19	1.38	1.12	0.98	0.98
DS	Darmstadt	80	0.78	1.04	0.25	1.49	1.69	1.36	1.13	1.09
DB	Duisburg	80	0.74	1.12	0.38	1.47	1.62	1.60	1.17	1.16
T	Tallawarra	87	0.57	1.53	0.30	0.91	1.02	0.87	0.76	0.75
L	Lakeview	88	0.28	0.41	0.31	0.78	0.62	0.68	0.66	0.64
	CEGB plants									
E	Earley	91	0.40	0.72	0.18	2.49	1.59	1.30	1.73	1.05
E	Earley	91	0.48	0.57	0.37	2.26	1.44	1.16	1.71	1.25
CD	Castle Donington	91	0.43	0.74	0.44	1.57	1.01	1.01	1.29	1.17
CD	Castle Donington	91	0.39	0.62	0.47	1.34	0.86	0.86	1.13	1.05
N	Northfleet	93	0.47	0.79	0.44	2.24	1.24	1.35	1.75	1.46
N	Northfleet	93	0.56	0.84	0.65	2.43	1.35	1.47	1.96	1.73
	TVA plants	99								
S	Shawnee		0.68	0.90	0.54	1.88	1.69	1.36	1.53	1.40
C	Colbert		0.66	0.96	0.55	0.86	0.96	0.82	0.77	0.77
J	Johnsonville		0.59	0.66	0.66	1.37	1.24	1.21	1.19	1.17
WC	Widows Creek		0.94	1.32	1.18	1.94	1.76	1.92	1.73	1.72
G	Gallatin		0.53	0.90	0.64	1.35	1.05	1.15	1.10	1.09
G	Gallatin		0.68	1.53	0.58	1.41	1.37	1.50	1.21	1.20
P	Paradise		0.39	0.65	0.51	1.19	0.80	0.96	1.03	1.00
P	Paradise		0.42	0.70	0.58	1.28	0.85	1.03	1.11	1.09
	Median for all data		0.54 ± 34%	0.79 ± 27%	0.44 ± 37%	1.44 ± 26%	1.36 ± 21%	1.18 ± 20%	1.17 ± 23%	1.09 ± 19%
	Median for selected data		0.48 ± 19%	0.72 ± 24%	0.47 ± 26%	1.41 ± 18%	1.24 ± 22%	1.16 ± 14%	1.17 ± 12%	1.09 ± 7%

data; Tallawara and Lakeview, much higher rise than comparable sources in Fig. 5.2, possibly due to lakeshore effect; Widows Creek, downwash, possibly due to a 1000-ft plateau nearby, shown in Figs. 5.1 and 5.2; Northfleet, terrain downwash reported by Hamilton[93] and rise much lower than at Castle Donington at same emission; Colbert and Shawnee, many stacks. The results in Table 5.1 help justify the exclusion of these data, since with the selected data the average deviation from the median is considerably reduced for seven of the eight formulas.

The first three formulas tested in Table 5.1 are completely empirical and do not allow for the effect of distance of measurement on plume rise as the remaining five formulas do; consequently, these three formulas give poorer agreement with data. The Holland[6] formula (Eq. 4.1) in particular shows a high percentage of scatter. The formula of Stümke[108] (Eq. 4.4) is perhaps slightly preferable to that of Moses and Carson[110] (Eq. 4.8), although the latter shows less scatter in comparison with the selected data. All three of these formulas underestimate plume rise, but this shortcoming can be corrected by multiplying the formulas by a constant that optimizes the agreement.

The next three formulas are based on the Priestley[73] theory. The first is the asymptotic formula for the first-phase theory[87] (Eq. 4.36), which predicts a rise proportional to $x^{3/4}$. Even though this is a transitional-rise formula, which does not apply to a leveling off stage of plume rise, it shows less scatter compared with observations than the three empirical final-rise formulas. The next formula (Eq. 4.37), by Lucas, Moore, and Spurr,[91] includes both a transitional- and a final-rise stage and gives a little better agreement with data. When Eq. 4.37 is multiplied by the empirical stack-height factor suggested by Lucas,[109] i.e., $0.52 + 0.00116\ h_s$, the agreement is considerably better. However, one should be cautious about applying this formula to plants with heat emission less than 10 Mw, because it predicts continued plume rise to almost 1 km downwind regardless of source size. For instance, for the very small source used by Ball,[76] the predicted final rise is 12 times the rise measured at 60 ft downwind; it seems unlikely that such a weakly buoyant plume so close to the ground, where turbulence is stronger, will continue to rise over such a long distance.

The last two formulas are based on the simplified theory given in the section, "Basic Theory Simplified" in Chapter 4. The "2/3 law" (Eq. 4.32), another transitional-rise formula, agrees about as well with these data as the Lucas[109] formula just discussed. Equation 4.34, which includes both a transitional-rise and a final-rise stage, gives both improved numerical agreement and much less percentage of scatter. Clearly it is the best of the eight formulas tested in Table 5.1 and is the one recommended for buoyant plumes in neutral conditions (for optimized fit it should be divided by 1.09).

Eq. 4.34 should not be applied beyond $x = 5x^*$, because so few data go beyond this distance. In some cases the maximum ground concentration occurs closer to the source than this, and Eq. 4.34 applied at the distance of the maximum gives the best measure of effective stack height. (Beyond this distance plumes diffuse upward, and the interaction of diffusion with plume rise cannot be neglected.) One conservative approach is to set $x = 10\ h_s$, which is about the minimum distance downwind at which maximum ground concentration occurs. For the fossil-fuel plants of the Central

Electricity Generating Board (CEGB) and TVA in Table 5.1, at full load this distance turns out to be in the range $2.5 < (x/x^*) < 3.3$. At $x/x^* = 3.3$, Eq. 4.34 gives a plume rise only 10% lower than Eq. 4.32, but at twice this distance the plume rise is increased by only 27%. This suggests a rule of thumb that Eq. 4.34 can be approximated by Eq. 4.32, the "2/3 law," up to a distance of 10 stack heights, beyond which further plume rise is neglected, i.e.,

$$\begin{aligned} \Delta h &= 1.8\, F^{1/3} u^{-1} x^{2/3} \qquad (x < 10\, h_s) \\ \Delta h &= 1.8\, F^{1/3} u^{-1} (10\, h_s)^{2/3} \quad (x > 10\, h_s) \end{aligned} \tag{5.1}$$

For other sources a conservative approximation to Eq. 4.34 is to use Eq. 4.32 up to a distance of $x = 3x^*$ and then to consider the rise at this distance to be the final rise. Surprisingly, Eq. 5.1 compares even better with the data in Table 5.1 than the recommended Eq. 4.34. Excluding Ball's data, which were for a ground source, the median ratio of calculated to observed plume rises is about 1.13, and the average deviations are ±17% for all data and ±4% for the selected data. Because of the nature of the approximation used in Eq. 5.1 and the scarcity of data beyond $x = 5x^*$, Eq. 5.1 is recommended as an alternative to Eq. 4.34 only for fossil-fuel plants with a heat emission of at least 20 Mw at full load.

For multiple stacks the data show little or no enhancement of plume rise over that from comparable single stacks in neutral conditions. Observations at the Paradise Steam Plant were about equally split between one-stack operation and two-stack operation with about the same heat emission from the second stack. In Fig. 5.1 the plume rises in these two conditions can be seen to be virtually indistinguishable. However, the same figure shows a clear loss in plume rise at Gallatin for the cases in which the same heat emission was split between two stacks. In Table 5.1 average plume rises for plants with two stacks are somewhat less than those for plants with one stack, at least in comparison with Eq. 4.34. Colbert, with three stacks, seems to have an enhanced rise, but Shawnee, with eight or nine stacks operating, has a lower rise than would be expected for a single stack. This may be due to downwash, as noted in the discussion of Fig. 5.1. In summary, the observations do not clearly support any additional allowance for plume rise when more than one stack is operating. It is beneficial to combine as much of the effluent as possible into one stack to get the maximum heat emission and the maximum thermal plume rise. This has been the trend for large power plants both in England and in the United States.

Few data are available to evaluate plume rise in unstable conditions. Slawson[88] found a just slightly higher average rise in unstable than in neutral conditions, as well as more scatter, as might be expected owing to convective turbulence. The same general features are evident in the TVA data. The buoyancy flux of the plume increases as it rises in unstable air, but there is also increased atmospheric turbulence; it is not clear which influence has the greater effect on the plume. However, because of lack of empirical evidence, it is possible only to recommend for unstable conditions the same formulas that apply in neutral conditions, specifically Eq. 5.2.

Jets in Neutral Conditions

Most data for jets in a crosswind do not extend very far downwind; so in Fig. 5.4 they are compared with the bending-over plume model in "Basic Theory Simplified," Chapter 4; $\Delta h/D$ is plotted as a function of $R = w_0/u$ for two different distances

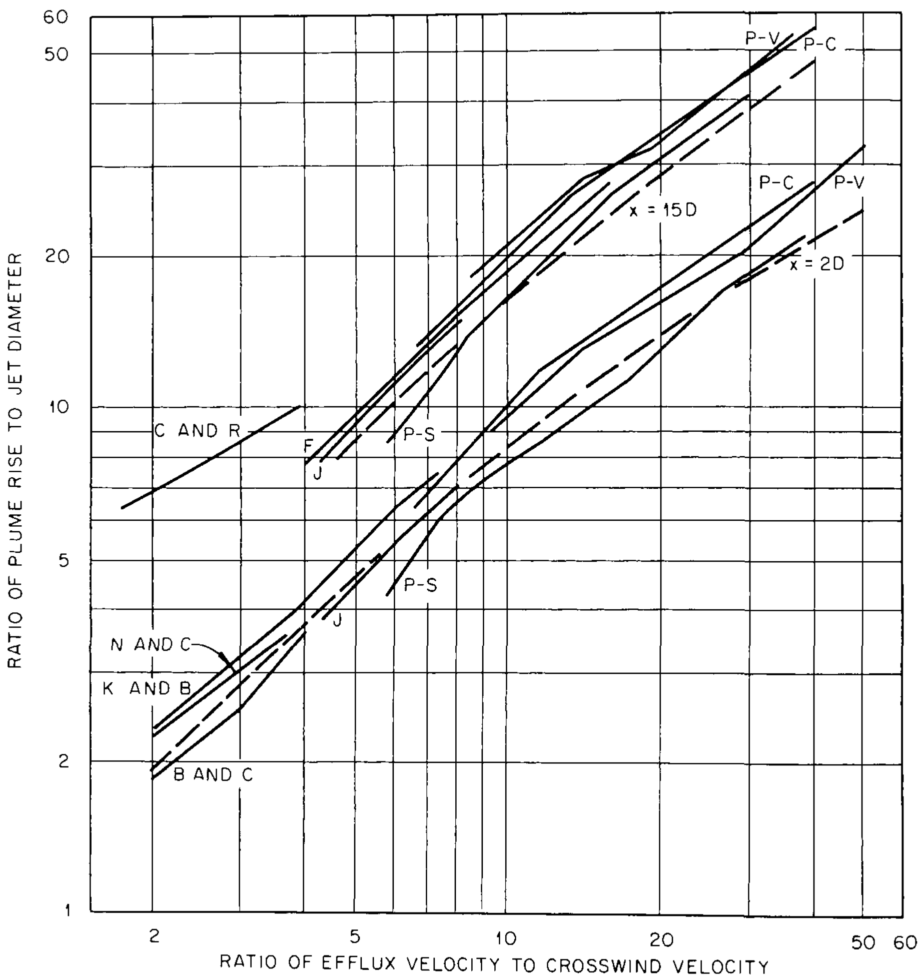


Fig. 5.4 Plume rise of jets in crosswind compared with values for bending-over plume model.

B and C, Bryant and Cowdry[67]
C and R, Callaghan and Ruggeri[62]
F, Fan[71]
J, Jordinson[65]
K and B, Keffer and Baines[63]
N and C, Norster and Chapman[65]
P–C, concentration profiles, Patrick[65]
P–S, Schlieren photographs, Patrick[65]
P–V, velocity profiles, Patrick[65]

downwind, x = 2D and x = 15D. The two families of curves group together rather well, considering the variety of experiments and measurement techniques, which include the photographic center lines by Bryant and Cowdry[67] (B and C), the temperature survey by Norster and Chapman[65] (N and C), the velocity survey by Keffer and Baines[63] (K and B), the total pressure measurements by Jordinson[65] (J), the top of the temperature profile measured by Callaghan and Ruggeri[62] (C and R), the photographic measurements by Fan[71] (F), and the three different sets of measurements made by Patrick,[65] i.e., concentration profiles (P-C), velocity profiles (P-V), and Schlieren photographs (P-S). The data are fit rather well by the dashed line that represents the formula given by the bending-over plume model (Eqs. 4.14 and 4.24); the resultant formula is probably not of practical value since it applies only near the source and, being unwieldy, is not written out. This is just a test of the entrainment assumption. Only the Callaghan and Ruggeri data do not fit the pattern. A number of reasons are possible, one being that the jet velocities were near supersonic and another being that this jet was more nearly horizontal, the distance downwind being about twice the rise. The main reason this curve is higher is probably that it represents the top of the jet rather than the center line.

A comparison of values from Eq. 4.33 with the few sets of data that go as far as 100 or 200 stack diameters downwind is shown in Fig. 5.5. Equation 4.33 does fairly

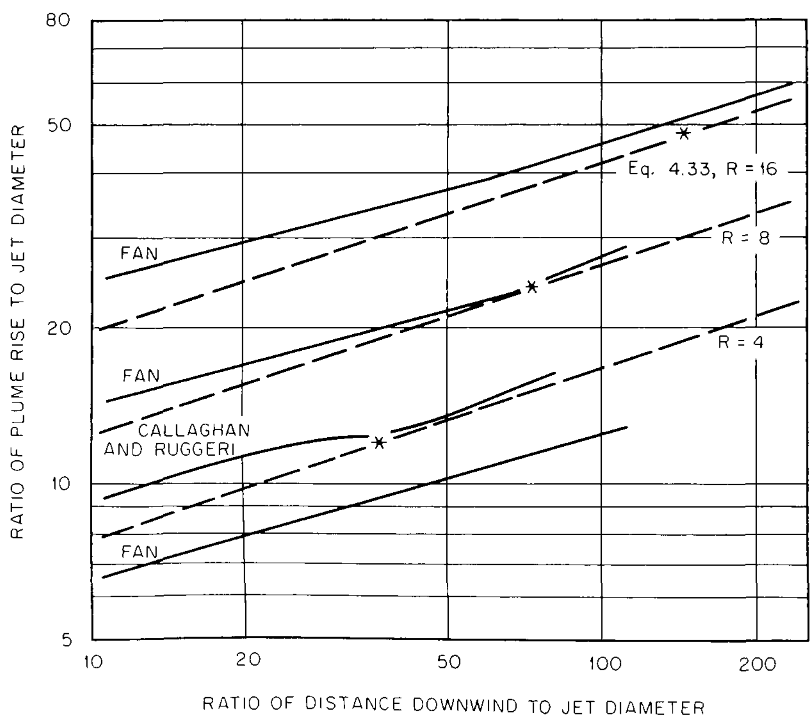


Fig. 5.5 Plume rise of jets in crosswind compared with values from Eq. 4.33. ($R = w_0/u$; asterisks denote $\Delta h/D = 3.0\ R$)

well even when the plume is more vertical than horizontal ($\Delta h > x$) and works quite well when the plume is more horizontal. The exception is that it overestimates the rise measured by Fan at the lower value of $R = w_0/u$, specifically at $R = 4$. This lends some credence to the suggestion made by Hoult, Fay, and Forney[68] that the entrainment constant γ may be a function of R although the particular function that they suggest works poorly in the present model. It should be noted that Fan's plumes were partially buoyant, but these effects are minimized by rejecting data for which $Fx/F_m u$, the ratio of buoyancy-induced momentum flux to initial momentum flux, is greater than 0.5.

As for the final rise of a jet, again it appears that none has been measured, but the asterisks in Fig. 5.5 at $\Delta h/D = 3.0\,R$ (see Table 4.1) indicate a reasonable value for maximum observed rise; i.e.,

$$\Delta h = 3 \frac{w_0}{u} D \tag{5.2}$$

This is twice the value given by Eq. 4.9, the often-cited formula of Rupp and his associates.[61]

STABLE CONDITIONS

Penetration of Elevated Inversions

A hot plume will penetrate an inversion and continue to rise if at that elevation the plume is warmer than the air above the inversion, i.e., if its temperature excess exceeds ΔT_i. A jet, on the other hand, must have enough momentum to force its way through an inversion, and then it must eventually subside back to the level of the inversion since it is cooler than the air above. For the case of no wind, the simplified vertical model with boundary conditions implies that penetration ability is a function of b_i, z_i, F_m, and F Then conventional dimensional analysis predicts penetration when

$$z_i b_i^{0.6} F^{-0.4} \leqslant f(F_m b_i^{0.8} F^{-1.2}) \tag{5.3}$$

where f is a function to be determined empirically. Relevant empirical data, collected in a modeling experiment of Vadot,[60] are plotted in Fig. 5.6 to confirm Eq. 5.3. The two data points on the left side of Fig. 5.6 provide some support for Eq. 4.29 for penetration by a buoyant plume. As F_m is increased, the penetration ability actually decreases because a jet expands faster than a buoyant plume and therefore any heat content is diluted more rapidly. For very large values of the nondimensional momentum flux, the height of possible penetration becomes proportional to $(F_m/b_i)^{1/4}$ Three data points on the right side of Fig. 5.6 suggest that the constant of

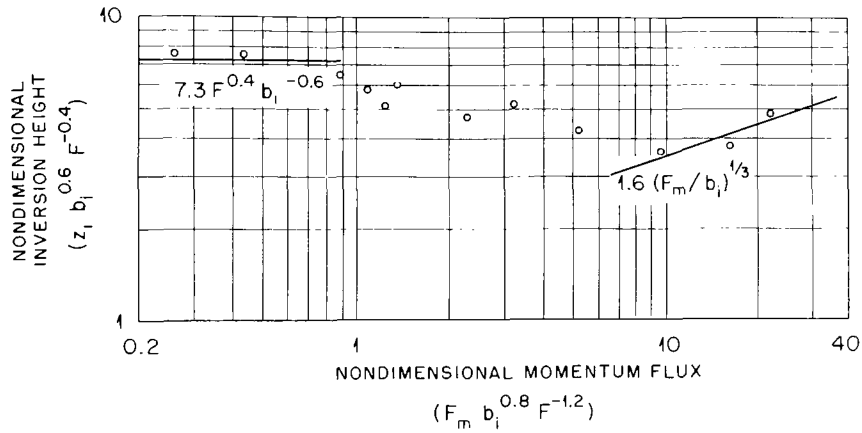


Fig. 5.6 Maximum nondimensional inversion height for penetration by plume vs. nondimensional momentum flux (based on data from Vadot[60]).

proportionality is roughly 1.6, as given in Eq. 4.31. As a simple, conservative criterion for a vertical plume, Vadot's experiments suggest penetration when

$$z_i \leqslant 4F^{0.4}b_i^{-0.6} \tag{5.4}$$

A bent-over buoyant plume rising through neutrally stratified air should penetrate an inversion at height z_i if, as expressed by Eq. 4.30,

$$z_i \leqslant 2.0\left(\frac{F}{ub_i}\right)^{1/2}$$

This equation (Eq. 4.30) was derived from the simplified bent-over plume model, which gives a characteristic temperature excess of the plume of

$$\theta' = 4.0\frac{T}{g}\frac{F_z}{uz^2} \tag{5.5}$$

for a plume rising through neutral air. Eq. 5.5 is easier to apply to cases where there are two or more inversions separated by neutral stratification. Initially $F_z = F$, and θ' decreases with the inverse square of the height above the source until the plume reaches the first inversion. As the plume rises through the inversion, its potential temperature is unaffected, but the potential temperature of the ambient air increases by ΔT_i; thus θ' is reduced by ΔT_i. If θ' remains positive, the plume is buoyant and continues to rise with θ' proportional to z^{-2} until it reaches the height of the next inversion. The same procedure is repeated until the plume reaches an inversion it cannot penetrate, i.e., until $\theta' < \Delta T_i$.

The results obtained by applying this procedure to the data of Simon and Proudfit[103] from the Ravenswood plume in New York City, which include plume penetrations of multiple inversions, are shown in Table 5.2, along with the temperature excesses of the plume relative to the air above the inversion as calculated by subtracting ΔT_i from Eq. 5.5 applied at the top of the inversion. It can be seen that every one of the eight nonpenetrations is predicted by a negative calculated θ'. In one case penetration is questionable because the plume center line ascended only 10 m higher than the inversion; so the lower part of the plume was undoubtedly below the inversion. Only one of the five penetrations was not predicted, and that was with a negative θ' of only 0.2°C, near the limits of the accuracy of temperature measurements. The procedure given in the discussion following Eq. 5.5 appears to be a good predictor but, perhaps, just slightly conservative.

Rise Through Uniform Temperature Gradient

Also of particular interest is the case in which the plume rises through air with a fairly uniform temperature gradient. In this case we can approximate s as a constant. For the calm case the simple vertical model predicts that the buoyancy of a hot plume decays to zero according to Eq. 4.25. This formula was derived by M,T,&T[58] from virtually the same model, and a similar formula was derived by Priestley and Ball.[117] The ability of Eq. 4.25 to predict the final height of the tops of plumes is shown in Fig. 5.7. Data are plotted from the modeling experiment in stratified salt solution by M,T,&T,[58] from the modeling experiment in air near the floor of an ice rink of Crawford and Leonard,[59] from the experiments of Vehrencamp, Ambrosio, and Romie[100] on the Mojave Desert, and from the observation by Davies[101,102] of the plume from a large oil fire. Equation 4.25 correctly approximates the top of the massive smoke plume that billowed out of the Surtsey volcano in 1963.[140] The rate of thermal emission was estimated to be of the order of 100,000 Mw,[141] or about a thousand times greater than the heat emission from a large stack. For the average lapse rate observed in the troposphere (6.5°C/km), Eq. 4.25 gives a rise of 5 km, or about 16,000 ft; the observed cloud top ranged from 3 to 8 km.

As the nondimensional momentum flux is increased, Morton's[119] numerical solution indicates lessened plume rise, just as inversion penetration ability was seen to decline in Fig. 5.6. There are no data to show this, but three experiments with vertical plumes by Fan[71] indicate gradual enhancement of rise over that given by Eq. 4.25 when $F_m s^{1/2}/F > 1.8$. Dimensional analysis of the vertical model indicates that

$$\Delta h = C F_m^{1/4} s^{-1/4} \tag{5.6}$$

for a pure jet, where C is a constant. The values of C that correctly describe Fan's plumes, which were momentum dominated but not pure jets, are 4.53, 4.43, and 4.18. A value of C = 4 is suggested as an approximation, as in Eq. 4.27.

Table 5.2

INVERSION PENETRATION AT THE RAVENSWOOD PLANT†

Date	Time	$\overline{Q}_H$, 10^7 cal/sec	$\bar{u}$, m/sec	Plume height, m	Inversion height, m: Bottom	Inversion height, m: Top	ΔT_i, °C	Calculated θ', °C	Penetration
May 25	1825	1.97	9.0	295	145	180	0.2	15	Yes
					325	475	0.7	−0.5	No
July 20	0552–0559	0.98	10.5	350	255	275	0.3	0.05	Yes
					365	395	2.0	−2.0	No
	0617–0820	1.11	7.3	360	540	580	1.9	−1.9	No
July 21	0600–0724	1.13	4.3	360	410	450	0.6	−0.45	No
	0828	1.64	2.7	510	240	280	0.6	1.7	Yes
					360	410	0.4	0.0	Yes
September 8	0648–0930	1.66	7.5	410	360	400	0.8	−0.6	?
	1000–1020	1.77	5.4	560	620	650	0.4	−0.3	No
September 9	0640–0705	1.20	9.6	350	360	400	2.1	−2.0	No
	0747–0850	1.54	9.1	370	260	300	0.7	−0.2	Yes
					370	410	1.6	−1.6	No
	0930–1000	2.13	9.6	390	420	530	1.8	−1.7	No

†Stack height, 155 m.

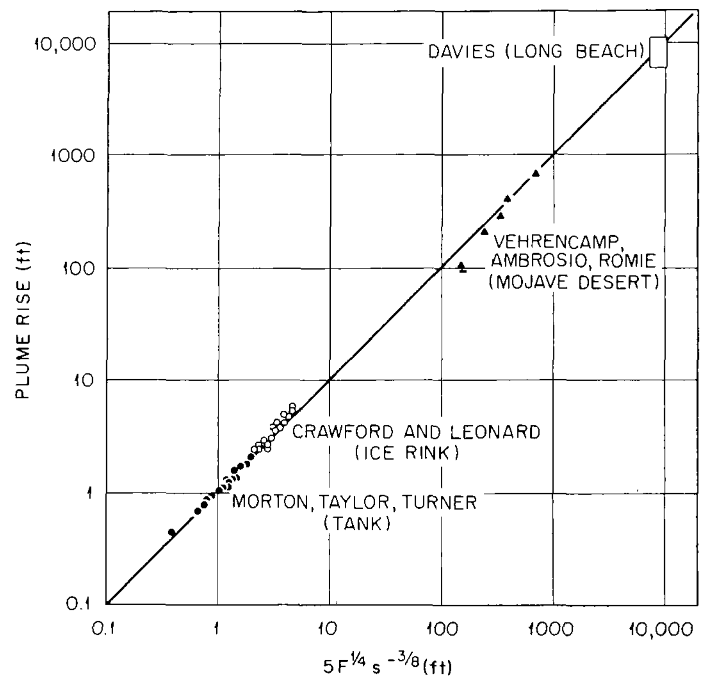


Fig. 5.7 Rise of buoyant plumes in calm, stable air.

For the case of a bent-over plume rising through stable air with constant s, the quasi-horizontal model can be applied both to a buoyant plume and to a jet to yield Eqs. 4.26 and 4.28, respectively. There are no data to test Eq. 4.28, but Eq. 4.26 and several other formulas can be compared with data from buoyant plumes released in stable air. These data include nine runs made at Brookhaven[86] with 15-sec ignitions of rocket fuel, six runs by TVA[99] with large single stacks, and seven runs by Van Vleck and Boone[79] with 60-sec firings of horizontal rocket motors. Admittedly the plumes were not continuous in two of these experiments, and the plume rises were defined somewhat differently in each case. In each case the ratios of the calculated to observed rises were computed. The resulting median values of this ratio and mean deviation from the median are

Holland[6]	0.44 ± 131%
Priestley[73]	0.42 ± 43%
Bosanquet[20]	1.22 ± 26%
Briggs, Eq. 4.26	0.82 ± 13%

Holland[6] suggested that Eq. 4.1 be reduced by 20% to predict rise in stable conditions, but this may be seen to work poorly. The Priestley[73] and Bosanquet[20] theoretical formulations are both complex; so they were simplified to the case for a buoyant

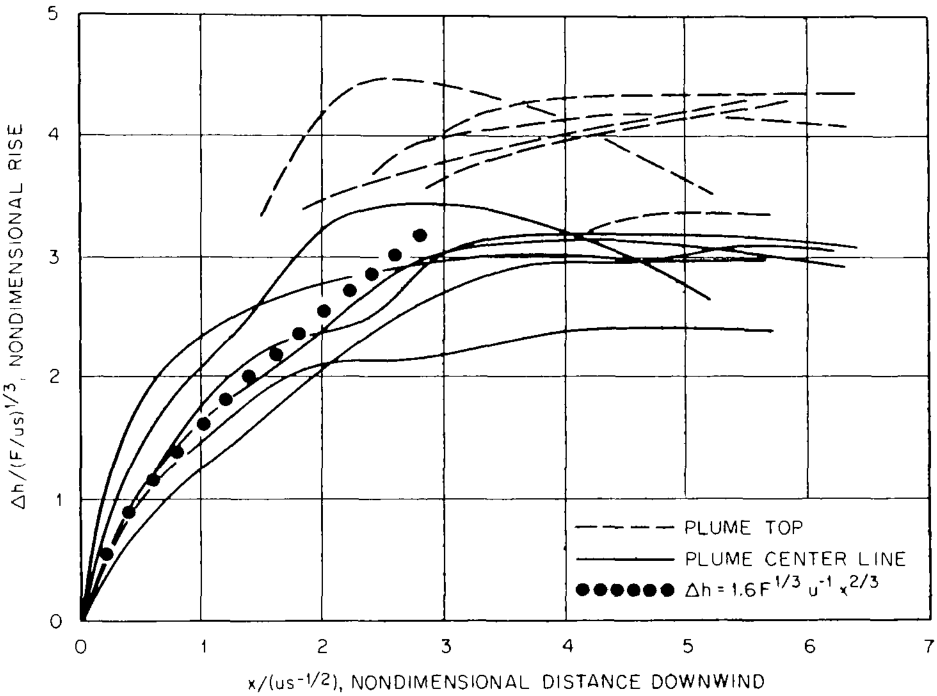


Fig. 5.8 Rise of buoyant plumes in stable air in crosswind at the TVA Paradise and Gallatin plants.

point source. Clearly Eq. 4.26 gives the most consistent agreement, and on the average it slightly underestimates rise. A constant of 2.4/0.82 = 2.9 works best, i.e.,

$$\Delta h = 2.9\left(\frac{F}{us}\right)^{1/3} \tag{5.7}$$

A further test of the simplified theory for bent-over plumes is shown in Fig. 5.8 for six periods of TVA data, which include the complete trajectories of the plume center lines and plume tops in stable air. The center lines follow the "2/3 law" in the first stage of rise with a fairly typical amount of scatter and reach a maximum in the neighborhood of $x = \pi\, us^{-1/2}$ as is predicted by theory. There is less scatter in the final-rise stage, where four of the six trajectories almost coincide. The actual final heights range from 450 to 1500 ft. The plume tops level out at

$$\Delta h = 4.0\left(\frac{F}{us}\right)^{1/3} \tag{5.8}$$

When two or three stacks were operating at the TVA plants, there was some evidence of enhanced final rise in stable conditions. The maximum enhancement that could be expected according to Eq. 5.7 would be 26 and 44% for two and three stacks, respectively, if the total heat emission could simply be lumped together in computing F. The averaged observed enhancement relative to Eq. 5.7 was +20% with two stacks operating and +30% with three stacks operating except that when the wind was blowing along the line of three stacks at Colbert the enhancement was +40%. Enhancement also depends on stack spacing since the plumes can hardly be expected to interact with each other if they are too far apart, especially if the wind is perpendicular to the line of stacks. In the preceding cases the stacks were spaced less than $0.9(F/us)^{1/3}$, or about one-fourth of the plume rise apart.

6 CONCLUSIONS AND RECOMMENDATIONS

There is no lack of plume-rise formulas in the literature, and selection is complicated by the fact that no one formula applies to all conditions. For a given situation many different predictions emerge, as is shown in Table 5.1. The variety of theoretical predictions follows from the great variety of assumptions used in the models; the disagreement among empirical formulas is due to the different weighting of data used in their formulations and to variability among the data. Another factor is the frequent disregard of the dependence of plume rise on distance downwind of the stack. In the formulas recommended in the following paragraphs, all symbols are given in Appendix B, and the constants in the formulas are optimized for the best fit to data covered by this survey. Readjustment of the constants in previously cited equations is indicated by primes on the equation numbers.

An important result of this study is that buoyant plumes are found to follow the "2/3 law" for transitional rise for a considerable distance downwind when there is a wind, regardless of stratification; i.e.,

$$\Delta h = 1.6 F^{1/3} u^{-1} x^{2/3} \tag{4.32'}$$

The bulk of plume-rise data are fit by this formula.

In neutral stratification Eq. 4.32′ is valid up to the distance $x/x^* = 1$, beyond which the plume center line is the most accurately described by

$$\Delta h = 1.6\, F^{1/3} u^{-1} x^{*2/3} \left[\frac{2}{5} + \frac{16}{25}\,\frac{x}{x^*} + \frac{11}{5}\left(\frac{x}{x^*}\right)^2\right]\left(1 + \frac{4}{5}\,\frac{x}{x^*}\right)^{-2} \tag{4.34'}$$

where

$$x^* = 0.52 \left[\frac{\sec^{6/5}}{\mathrm{ft}^{6/5}}\right] F^{2/5} h_s^{3/5} \qquad (h_s < 1000\ \mathrm{ft})$$
$$x^* = 33 \left[\frac{\sec^{6/5}}{\mathrm{ft}^{3/5}}\right] F^{2/5} \qquad (h_s > 1000\ \mathrm{ft}) \tag{4.35}$$

Equation 4.35 is the best approximation of x^* at present for sources 50 ft or more above the ground; for ground sources an estimated plume height can be used in place of h_s. Equation 4.34′ applies to any distance such that $x/x^* > 1$, but owing to lack of data at great distances downwind $x/x^* = 5$ is suggested as the maximum distance at which it be applied at present. Even though Eq. 4.34′ is the best of the dozen or so formulas considered, the average plume rise at a given plant may deviate from the value given by Eq. 4.34′ by ±10% if the site is flat and uniform and by ±40% if a substantial terrain step or a large body of water is nearby. Furthermore, normal variations in the intensity of turbulence at plume heights at a typical site cause x^* to vary by about ±20% on the average, with corresponding variations in Δh. For fossil-fuel plants with a heat emission of 20 Mw or more, a good working approximation to Eq. 4.34′ is given by

$$\Delta h = 1.6\, F^{1/3} u^{-1} x^{2/3} \qquad (x < 10\, h_s)$$
$$\Delta h = 1.6\, F^{1/3} u^{-1} (10\, h_s)^{2/3} \qquad (x > 10\, h_s) \tag{5.1′}$$

For other sources, a conservative approximation to Eq. 4.34′ is to use Eq. 4.32′ up to a distance of $x = 3x^*$, then to consider the rise at this distance to be the final rise.

Equations 4.34′ and 5.1′ are also recommended for the mean rise in unstable conditions although larger fluctuations about the mean should be expected (see Fig. 2.4).

In stable stratification Eq. 4.32′ holds approximately to a distance $x = 2.4us^{-1/2}$, beyond which the plume levels off at about

$$\Delta h = 2.9 \left(\frac{F}{us}\right)^{1/3} \tag{5.7}$$

as illustrated in Fig. 5.8. The top of the stratified plume is about 38% higher than that predicted by Eq. 5.7, which describes the plume center line. Although no significant increase in transitional rise is found when more than one stack is operating, some enhancement of the final rise in stable conditions is observed provided the stacks are close enough. If the wind is so light that the plume rises vertically, the final rise is given accurately by

$$\Delta h = 5.0\, F^{1/4} s^{-3/8} \tag{4.25}$$

In computing s for Eqs. 4.25 and 5.7, an average potential temperature gradient is calculated for the stable layer or for the layer expected to be traversed by the plume.

A buoyant plume will penetrate a ground inversion if both Eq. 5.7 and Eq. 4.25 give a height higher than the top of the inversion. The plume will penetrate an elevated inversion if the top of the inversion lies below both Eq. 5.4 and Eq. 4.30, i.e.,

$$z_i \leqslant 4F^{0.4} b_i^{-0.6} \qquad \text{(calm)} \tag{5.4}$$

$$z_i \leqslant 2.0 \left(\frac{F}{ub_i}\right)^{1/2} \qquad \text{(wind)} \tag{4.30}$$

All the preceding formulas apply to buoyant plumes, which include most plumes from industrial sources, and they are fairly well confirmed by observations. Because of a relative lack of data, it is more difficult to make firm recommendations of formulas for jets. It appears that in neutral, windy conditions the jet center line is given by

$$\Delta h/D = 1.44 \left(\frac{w_0}{u}\right)^{2/3} \left(\frac{x}{D}\right)^{1/3} \tag{4.33}$$

at least up to the point that

$$\Delta h = 3 \frac{w_0}{u} D \tag{5.2}$$

as long as $w_0/u \geqslant 4$. It can be only tentatively stated that in windless conditions the jet rises to

$$\Delta h = 4 \left(\frac{F_m}{s}\right)^{1/4} \tag{5.6}$$

where 4 is used as the value of C. This is on the basis of only three experiments. If there is some wind and the air is stable, the minimum expected theoretical rise is

$$\Delta h = 1.5 \left(\frac{F_m}{u}\right)^{1/3} s^{-1/6} \tag{4.28}$$

Unfortunately there are no published data for this case, and it would be presumptuous to recommend any formula without testing it. However, since Eq. 4.28 is based on the same model, we should not use Eq. 5.6 or Eq. 5.2 if it gives a higher rise than Eq. 4.28 does. The most conservative of the three formulas is the one that best applies to a given situation. The same can be said of Eqs. 4.34′, 5.7, and 4.25 for a buoyant plume.

Obviously more experiments are needed to complete our basic understanding of plume rise. In particular they are needed for jets at large distances downwind for all stability conditions and for buoyant plumes at distances greater than ten stack heights downwind in neutral conditions. Once the fundamental results are complete, it will be worthwhile to study in detail the effect of the finite source diameter, the bending-over stage of plume rise, the effect of wind shear and arbitrary temperature profiles, the interaction of plumes from more than one stack, and the interaction of plume-rise dynamics with diffusion processes.

APPENDIX A: EFFECT OF ATMOSPHERIC TURBULENCE ON PLUME RISE

As discussed in "Basic Theory Simplified," in Chapter 4, entrainment of ambient air into the plume by atmospheric turbulence is due mostly to eddies in the inertial subrange; so, for a bent-over plume or a puff in a neutral atmosphere, the entrainment velocity, or velocity of growth, is given by

$$dr/dt = \beta \epsilon^{1/3} r^{1/3} \qquad (A.1)$$

where β is a dimensionless entrainment constant, ϵ is the eddy energy dissipation rate, and r is a characteristic radius defined as $(V/u)^{1/2}$ for a bent-over plume. To apply this entrainment assumption, some simple method of estimating ϵ at plume heights is needed, and β must be determined.

Ideally ϵ would be related in some simple way to wind speed (u) and height above the ground ($\bar{z}$). In the neutral surface layer, e.g., the lowest 50 ft or so, such a relation is well described by the expression[115] $\epsilon = u_*^3/0.4\bar{z}$, where u_* is the friction velocity and is proportional to the wind speed at some fixed height. Unfortunately, at typical plume heights no such simple relation is found to exist. The turbulence becomes more intermittent and is affected more by departures from neutral stability and by terrain irregularities over a wide area. Still, enough data exist to estimate mean values of ϵ along with the amount of variability that should be expected.

Recent estimates of ϵ were made by Hanna,[142] who used vertical-velocity spectra measured in a variety of experiments, and by Pasquill,[143] who used high-frequency standard deviations of wind inclination measured with a lightweight vane mounted on captive balloons at Cardington, England. Hanna used data from towers at Round Hill, Mass.,[144] and Cedar Hill, Tex.,[145] from aircraft measurements made over a great variety of terrain by the Boeing Company,[146] and from several low-level installations (below 50 ft). These values of ϵ are used in Table A.1 to test the relation $\epsilon^{1/3} \propto u^m$ by computing the median value of $\epsilon^{1/3} u^{-m}$ and the average deviation from the median value for m = 0, 1/3, 2/3, and 1 at each height of each experiment. Because ϵ is sensitive to atmospheric stability, only runs in which $-1.0 < \mathrm{Ri} < 0.15$ were used from the Round Hill and Cedar Hill data, where Ri is the local Richardson number; the Boeing runs during very stable conditions and Pasquill's measurements above inversions were omitted. Also omitted were the few runs made during very low wind speeds, i.e., less than 2m/sec.

Table A.1 shows that the excellence of the fit is rather insensitive to increasing the value of m, especially at Round Hill and Cedar Hill. The best overall fit is with m = 1/3; the average percentage deviation from the median is lowest with m = 1/3 for four of the eight sets of data and, on the average, is only 9% greater than the minimum value of percentage deviation (indicated by † in Table A.1). This is fortunate because the expression for x^*, the distance at which atmospheric turbulence begins to dominate entrainment, turns out to be independent of wind speed when $\epsilon^{1/3} \propto u^{1/3}$ (see Eq. 4.35 and the preceding discussion in Chap. 4). It is therefore very desirable to adopt this approximation, keeping in mind the scatter about the median values shown in the table.

It is evident in Table A.1 that $\epsilon^{1/3}/u^{1/3}$ decreases with height. With a power law relation of $\epsilon^{1/3}/u^{1/3} \propto \bar{z}^{-n}$, the optimum value of n depends on which data are used. The best least-squares fit to $\log \epsilon^{1/3}/u^{1/3} = \text{constant} - n \log \bar{z}$ is n = 0.29 for all the data but n = 0.37 if the Pasquill data at 4000 ft are omitted. At Round Hill n = 0.31 between 50 ft and 300 ft, and at Cedar Hill n = 0.39 between 150 ft and 450 ft, but in Pasquill's data n is only 0.15 between 1000 ft and 4000 ft. These values are roughly consistent with the following three published conclusions: (1) Hanna[142,147] confirmed the relation $\epsilon^{1/3} = 1.5\, \sigma_w\, \lambda_m^{-1/3}$ for a wide variety of data, where σ_w is the variance of vertical velocity and λ_m is the wavelength of maximum specific energy in the vertical-velocity spectra; (2) data compiled in a note by Moore[131] indicate almost no dependence of σ_w on height from about 100 to 4000 ft except for very high wind speeds ($u > 10$ m/sec); (3) Busch and Panofsky[148] conclude that $\lambda_m \propto \bar{z}$ near the ground and reaches a maximum or a constant value somewhere above $\bar{z} = 200$ m. The simplest expression consistent with all of the preceding evidence is $\epsilon^{1/3}/u^{1/3} \propto \bar{z}^{-1/3}$ up to a height of the order of 1000 ft and then becomes constant with height. In the last column of Table A.1, an expression of this type is compared with the data. The best estimate of energy dissipation appears to be

$$
\begin{aligned}
\epsilon^{1/3} &= 0.9\ [\mathrm{ft}^{2/3}/\mathrm{sec}^{2/3}]\, u^{1/3}\, \bar{z}^{-1/3} \qquad (\bar{z} < 1000\ \mathrm{ft}) \\
\epsilon^{1/3} &= 0.09\ [\mathrm{ft}^{1/3}/\mathrm{sec}^{2/3}]\, u^{1/3} \qquad (\bar{z} > 1000\ \mathrm{ft})
\end{aligned}
\qquad \text{(A.2)}
$$

There remains the problem of how to determine the value of the dimensionless constant β, particularly when no observations of plume, puff, or cluster growth include

Table A.1

ENERGY DISSIPATION VS. WIND SPEED AND HEIGHT

Source	Height, ft	Number of runs	$\epsilon^{1/3}$, $ft^{2/3}/sec$	$\epsilon^{1/3}/u^{1/3}$, $ft^{1/3}/sec^{2/3}$	$\epsilon^{1/3}/u^{2/3}$, $sec^{-1/3}$	$\epsilon^{1/3}/u$, $ft^{-1/3}$	$(\epsilon\bar{z}/u)^{1/3}$, $ft^{2/3}/sec^{2/3}$
Round Hill	50	8	0.636 ± 17%	0.266 ± 14%†	0.103 ± 16%	0.042 ± 18%	0.98
Round Hill	150	11	0.495 ± 11%	0.177 ± 10%	0.063 ± 8%†	0.022 ± 10%	0.94
Cedar Hill	150	9	0.457 ± 20%	0.159 ± 18%	0.057 ± 16%†	0.020 ±19%	0.84
Round Hill	300	4	0.470 ± 11%	0.151 ± 7%†	0.049 ± 7%†	0.017 ± 7%†	1.01
Cedar Hill	450	6	0.331 ± 9%†	0.104 ± 9%†	0.034 ± 11%	0.010 ± 17%	0.80
Boeing	750	22	0.256 ± 20%†	0.083 ± 24%	0.028 ± 34%	0.009 ± 53%	0.75
Pasquill	1000	31	0.269 ± 38%†	0.097 ± 44%	0.042 ± 46%	0.018 ± 53%	0.97
Pasquill	4000	10	0.172 ± 49%	0.079 ± 42%†	0.030 ± 47%	0.011 ± 59%	0.79‡

†Minimum value of percentage deviation.
‡$\bar{z}$ = 1000 ft.

simultaneous, independent measurements of ϵ. The approach used in this review is to assume the validity of Eq. A.2 at the time and place of diffusion experiments and to compare the results with Eq. A.1.

Frenkiel and Katz[149] used two motion-picture cameras to photograph smoke puffs released above an island in the Chesapeake Bay. The puffs were produced by small detonations of gunpowder from an apparatus on the cable of a tethered balloon. The radii of the puffs were calculated from their visible areas at 1-sec intervals. The values of $\beta\epsilon^{1/3}$ shown in Table A.2 were calculated from the first 2 sec of puff growth by using Eq. A.1 as a finite difference equation, i.e., by setting $dr/dt = \Delta r/\Delta t$. Smith and Hay[150] published some data from several experiments on the expansion of clusters of particles. In their short-range experiments, Lycopodium spores were released at a height of 2 m and were collected on adhesive cylinders lined up perpendicular to the wind at 100 m downwind, yielding a lateral standard deviation of particle distribution (σ_y). In their medium-range experiments, fluorescent particles were released from an airplane at heights of 1500 to 2500 ft several miles upwind of a sampling apparatus mounted on the cable of a captive balloon, yielding a vertical standard deviation of particle distribution (σ_z). The values of $\beta\epsilon^{1/3}$ shown in Table A.2 for the Smith and Hay experiments were calculated from the integral of Eq. A.1 for a point source, namely,

$$\frac{3}{2} r^{2/3} = \beta\epsilon^{1/3} t = \beta\epsilon^{1/3} \frac{x}{u}$$

Interpreting the effective radius of a rising plume in terms of σ_y or σ_z is difficult, but in this case it was assumed that $\sigma_z = \sigma_y$ and that $r = 2^{1/2} \sigma_y$, as is true in the "top hat" model equivalent to a Gaussian plume in the Morton, Taylor, and Turner[58] theory.

The last column of Table A.2 shows the value of $\beta\epsilon^{1/3}$ inferred from the diffusion data divided by the value of $\epsilon^{1/3}$ calculated from Eq. A.2. The values of β inferred from this calculation range from 0.62 to 0.82, a remarkably small range considering the

Table A.2

GROWTH RATE OF PUFFS AND PARTICLE CLUSTERS

Source	Number of runs	$\bar{z}$, ft	$\bar{u}$, ft/sec	$\beta\epsilon^{1/3}$, ft$^{2/3}$/sec	$\beta\epsilon^{1/3}/\epsilon^{1/3}_{\text{calculated}}$
Smith and Hay					
Runs 1–5	5	$14 = \sigma_y$	18	0.60 ± 7%	0.62
Runs 7–10	4	$13 = \sigma_y$	30	0.96 ± 18%	0.82
Frenkiel and Katz					
$\bar{z}$ = 15 to 22 m	6	58	19	0.40 ± 7%	0.64
$\bar{z}$ = 39 to 61 m	7	164	52	0.48 ± 23%	0.78
Smith and Hay					
(May 7, 1959)	4	2500	16	0.17 ± 17%	0.74

indirectness of this approach and the wide range of variables involved. Note that the short-range experiments of Smith and Hay were probably carried out within the surface layer, where Eq. A.2 is not actually valid; nevertheless, the error in estimating ϵ is not large for moderate wind speeds at these heights. Table A.2 suggests that $\beta \simeq 0.7$, but, considering the small number of data and the indirectness of this analysis, the more conservative value of $\beta = 1.0$ is recommended.

It should be cautioned that the characteristic plume radius, r, that appears in Eq. A.1 is not necessarily the same as the visible radius or other measures of size of a passive puff or plume, and so the evaluation of β made in Appendix A is not directly applicable to diffusion problems other than plume rise.

B APPENDIX: NOMENCLATURE

Dimensions of each term are given in brackets: l = length, t = time, τ = temperature, m = mass.

b_i	Inversion parameter = $g\ \Delta T_i/T$ [l/t^2]
C_D	Drag coefficient [dimensionless]
D	Internal stack diameter [l]
F	Buoyancy flux parameter [l^4/t^3] ; see Eqs. 4.19c and 4.20
F_m	Momentum flux parameter [l^4/t^2] ; see Eq. 4.19b
F_z	Vertical flux of buoyant force in plume divided by $\pi\rho$ [l^4/t^3] ; see Eq. 4.17
Fr	Froude number = $w_0^2/[g(\Delta T/T)D]$ [dimensionless]
g	Gravitational acceleration [l/t^2]
h	Effective stack height = $h_s + \Delta h$ [l]
h_s	Stack height [l]
Δh	Plume rise above top of stack [l]
$\vec{k}$	Unit vector in the vertical direction [dimensionless]
L	Characteristic length for buoyant plume in crosswind = F/u^3 [l]
Q	Emission rate of a gaseous effluent [m/t]
Q_H	Heat emission due to efflux of stack gases [ml^2/t^3]
R	Ratio of efflux velocity to average windspeed = w_0/u [dimensionless]
r	Characteristic radius of plume or puff, defined as $(V/u)^{1/2}$ for a bent-over plume [l]
r_0	Internal stack radius [l]

s	Restoring acceleration per unit vertical displacement for adiabatic motion in atmosphere [t^{-2}]; see Eq. 4.16
T	Average absolute temperature of ambient air [τ]
T_s	Average absolute temperature of gases emitted from stack [τ]
ΔT	Temperature excess of stack gases = $T_s - T$ [τ]
ΔT_i	Temperature difference between top and bottom of an elevated inversion [τ]
$\partial T/\partial z$	Vertical temperature gradient of atmosphere [τ/l]
t	Time [t]
u	Average wind speed at stack level [l/t]
u^*	Friction velocity in neutral surface layer [l/t]; see Ref. 115
V	Vertical volume flux of plume divided by π [l^3/t]; see Eq. 4.15
$\vec{v}$	Average velocity of plume gases [l/t]; see Eq. 4.18
$\vec{v}'$	Velocity excess of plume gases = $\vec{v}_p - \vec{v}_e$ [l/t]
$\vec{v}_e$	Average velocity of ambient air [l/t]
$\vec{v}_p$	Average local velocity of gases in plume [l/t]
w	Vertical component of $\vec{v} = \vec{k} \cdot \vec{v}$ [l/t]
w'	Vertical component of $\vec{v}_p = \vec{k} \cdot \vec{v}_p$ [l/t]
w_0	Efflux speed of gases from stack [l/t]
x	Horizontal distance downwind of stack [l]
x^*	Distance at which atmospheric turbulence begins to dominate entrainment [l]; see Eq. 4.34.
y	Horizontal distance crosswind of stack [l]
z	Vertical distance above stack [l]
$\bar{z}$	Height above the ground [l]
z_i	Height of penetratable elevated inversion above stack [l]
α	Entrainment constant for vertical plume [dimensionless]; see Eq. 4.22
β	Entrainment constant for mixing by atmospheric turbulence [dimensionless]; see Eq. A.2
Γ	Adiabatic lapse rate of atmosphere = 5.4°F/1000 ft [τ/l]
γ	Entrainment constant for bent-over plume [dimensionless]; see Eq. 4.23
ϵ	Eddy energy dissipation rate for atmospheric turbulence [l^2/t^3]; see Ref. 115
θ	Average potential temperature of ambient air [τ]
θ'	Potential temperature excess of plume gases = $\theta_p - \theta$ [τ]
θ_p	Average potential temperature of gases in plume [τ]
$\partial\theta/\partial z$	Vertical potential temperature gradient of atmosphere [τ/l]; see Eq. 2.1
ρ	Average density of ambient air [m/l^3]
ρ_0	Density of gases emitted from stack [m/l^3]
ρ_p	Average density of gases in plume [m/l^3]
σ_z/σ_y	Ratio of vertical dispersion to horizontal dispersion [dimensionless]
χ	Concentration of a gaseous effluent [m/l^3]

C APPENDIX: GLOSSARY OF TERMS

Adiabatic lapse rate The rate at which air lifted adiabatically cools owing to the drop of pressure with increasing height, 5.4°F/1000 ft in the earth's atmosphere.

Advection The transport of a fluid property by the mean velocity field of the fluid.

Buoyant plume A plume initially of lower density than the ambient fluid after the pressure is adiabatically brought to equilibrium. Usually, the term "buoyant plume" refers to a plume in which the effect of the initial momentum is small, and the term "forced plume" refers to a plume with buoyancy in which the effect of the initial momentum is also important.

Convection Mixing motions in a fluid arising from the conversion of potential energy of hydrostatic instability into kinetic energy. It is more precise to term this motion "free convection" to distinguish it from "forced convection," which arises from external forces.

Critical wind speed In the context of this critical review, the wind speed at the height of an elevated plume for which the maximum ground concentration is highest in neutral conditions.

Diffusion The mixing of a fluid property by turbulent and molecular motions within the fluid.

Downwash The downward motion of part or all of a plume due to the lower pressure in the wake of the stack or building or due to a downward step of the terrain.

Effective stack height Variously defined. The three most common definitions are: (1) the height at which a plume levels off, which has been observed only in stable conditions; (2) the height of a plume above the point of maximum ground concentration; (3) the virtual height of plume origin based on the diffusion pattern at large distances downwind of the stack. Definition 1 is the easiest to apply in stable conditions; definition 2 is the most practical in neutral and unstable conditions; definition 3 is comprehensive but difficult to apply.

Efflux velocity The mean speed of exiting stack gases.

Entrainment The dilution of plume properties due to mixing with the ambient fluid.

Final rise The total plume rise after leveling off, if this occurs, especially as opposed to the term "transitional rise."

Froude number The ratio of pressure forces to buoyant forces. The efflux Froude number of a stack may be defined as $w_0^2/[g(\Delta T/T)D]$.

Fumigation The downward diffusion of pollutants due to convective mixing underneath an inversion that prevents upward diffusion.

Inversion A layer of air in which temperature increases with height. Such a layer is also stable.

Jet A nonbuoyant plume.

Lapse rate The rate at which temperature drops with increasing altitude; the negative of the vertical temperature gradient.

Neutral In hydrostatic equilibrium. A neutral atmosphere is characterized by an adiabatic lapse rate, i.e., by potential temperature constant with height.

Plume rise The rise of a plume center line or center of mass above its point of origin due to initial vertical momentum or buoyancy, or both.

Potential temperature The temperature that a gas would obtain if it were adiabatically compressed to some standard pressure, usually 1000 mb in meteorological literature.

Stable Possessing hydrostatic stability. A stable atmosphere has a positive potential temperature gradient.

Stratification The variation of potential temperature with height. Usually the term "stratified fluid" refers to a fluid possessing hydrostatic stability, as does the atmosphere when the potential temperature gradient is positive.

Temperature gradient In meteorology, usually the vertical gradient of mean temperature.

Transitional rise The rise of a plume under the influence of the mean wind and the properties of the plume itself; i.e., the rise before atmospheric turbulence or stratification has a significant effect.

Turbulence Three-dimensional diffusive motions in a fluid on a macroscopic scale. According to Lumley and Panofsky,[115] turbulence is also rotational, dissipative, nonlinear, and stochastic.

Unstable Possessing hydrostatic instability. An unstable atmosphere has a negative potential temperature gradient.

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AUTHOR INDEX

SUBJECT INDEX

NUCLEAR SAFETY INFORMATION CENTER

Plume Rise was originally prepared for the Nuclear Safety Information Center, one of the U. S. Atomic Energy Commission's specialized information analysis centers. Established in 1963 at the Oak Ridge National Laboratory, the Nuclear Safety Information Center serves as a focal point for the collection, storage, evaluation, and dissemination of nuclear safety information. The subject coverage, which is comprehensive in the nuclear safety field, includes such primary subject areas as

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Plant Safety Features	Reactor Operating Experiences
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C

EFFECTIVE STACK HEIGHT PROBLEMS

SET ONE

Submit all calculations leading to and involving final answer(s) to each problem.

1. A power plant with an 80-meter stack 3.5 meters in diameter, emits effluent gases at 93°C with an exit velocity of 15 meters/sec. What is the effective stack height when the wind speed is 4 meters/sec, using the Bryant-Davidson stack rise equation? Assume air temperature is 20°C.

$$\Delta H \quad d\left(\frac{V_s}{u}\right)^{1.4}\left(1+\frac{\Delta T}{T_s}\right)$$

$$\Delta H = 3.5\left(\frac{15}{4}\right)^{1.4}\left(1+\frac{73^\circ}{366^\circ}\right) =$$

$$3.5(6.3)(1.2) = 26.7\ m.$$

effective stack Height = 80 m + 26.7 m = 106.7 m.

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You should refer to Part B, "Effective Stack Height", of this package while solving these problems.

USE THIS SIDE FOR CALCULATIONS BEFORE USING ADDITIONAL SHEETS

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Problem Set One (C)

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2. Using the above conditions and atmospheric pressure of 1010 mb, what is the effective stack height calculated from the Holland equation for neutral stability?

$$\Delta H = \frac{v_s d}{4}\left(1.5 + 2.68\times10^{-3}\, p\, \frac{\Delta T}{T_s}\, d\right)$$

$$= \frac{15\times3.5}{4}\left(1.5 + 2.68\times10^{-3}(1010)\frac{13}{366}\times3.5\right)$$

$$= 44.5\,m$$

$$H = h + \Delta H = 80.0\,m + 44.5\,m = 124.5\,m$$

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1

USE THIS SIDE FOR CALCULATIONS BEFORE USING ADDITIONAL SHEETS

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3. Briggs has published generalized plume rise equations which EPA is incorporating into dispersion calculations involving elevated emission sources. A simplified working equation is given by:

$$\Delta h = 1.6\ F^{1/3} u^{-1}\ x^{2/3}; \quad (x < 10\ h_s)$$

$$\Delta h = 1.6\ F^{1/3} u^{-1}\ (10\ h_s)^{2/3}; \quad (x > 10\ h_s)$$

Where:

Δh = plume rise (m)

F = buoyancy flux = $\frac{\Delta T}{T_s} g v_s r^2$ (m^4/sec^3)

= $\frac{73}{366}$ (9.8 m/sec²) 15 m/sec (1.75 m)² = 89.8

u = wind speed (m/sec) - 4 m/sec

x = downwind distance (m)

h_s = physical stack height (m) - 80 m

v_s = exit velocity (m/sec) 15 m/sec

r = stack radius (m) 3.5 m/2 1.75 m

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For the power plant in Problems #1 and #2, assuming ambient air temperature of 20°C, what is the plume rise at: a) 350 m and b) 1750 m downwind?

a) where $(x < 10 h_s)$ $x = 350$ m

$\Delta h = 1.6\ (898)^{1/3} (4)^{-1}\ (350)^{2/3} =$

$= 1.6\ (4.48)(.25)(49.66) - 88.95$ m

b) where $(x > 10 h_s)$ $x - 1750$ m

$\Delta h = 1.6\ (898)^{1/3} (4)^{-1} (800\ m)^{2/3} = 154.4$ m

1

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Problem Set One (C)

Discuss very briefly whether or not this simplification of the Briggs equation should be used for this power plant.

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1

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F

EFFECTIVE STACK HEIGHT PROBLEMS

SET TWO

Submit all calculations leading to and including final answer(s) to each problem.

Using the Colbert power plant data in Table 5.1, p. 44, of Plume Rise by Briggs, calculate the expected plume rise under the following stability conditions:

1. Neutral and Unstable $u = 5$ m/s, Eq. 4.20 for F, Eq. 5.1' for Δh

   a. Δh at 800 m

   b. Δh at 8000 m

Colbert Data

$h_s = 300 \text{ ft} = 91.43 \text{ m}$

$d = 16.5 \text{ ft} = 5.03 \text{ m}$

$w_0 = 42.9 \text{ ft/sec} = 13.08 \text{ m}$

$Q_H = 6.74 \times 10^6 \text{ cal/sec}$

$$F = 3.7 \times 10^{-5} \left[\frac{m^4/sec^3}{cal/sec}\right] Q_H$$

$$= 3.7 \times 10^{-5} (6.74 \times 10^6) = 249.4 \ m^4/sec^3$$

a) where $x < 10 h_s (914.3 m)$ & $u = 5 m/s$

$$\Delta h = 1.6 F^{1/3} u^{-1} x^{2/3}$$

$$\Delta h = 1.6 (249.4 \ m^4/sec^3)^{1/3} (5 \ m/s)^{-1} (800 \ m)^{2/3}$$

$$= 173.6 \ m$$

b) where $x > 10 h_s (914.3 m)$ & $u = 5 m/sec$

$$\Delta h = 1.6 F^{1/3} u^{-1} (10 h_s)^{2/3}$$

$$= 1.6 (249.4 \ m^4/sec^3)^{1/3} (5 \ m/s)^{-1} (914.3 \ m)^{2/3}$$

$$= 189.7 \ m$$

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Problem Set Two (F)

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2. Stable $u = 2$ m/s, $T = 280°K$, $\frac{\partial\Theta}{\partial Z} = 2 \times 10^{-2}$

a. At what distance is $x = 2.4\ us^{-1/2}$; why is this calculation important?

b. Δh at 800 m

a. $x = 2.4\ (2\ m/sec)\ s^{-1/2}$ where $s = \frac{g}{T}\frac{\partial\Theta}{\partial Z}$ $\frac{9.8}{280°}(2\times10^{-2})$

$x = 2.4(2m/sec)(7.0\times10^{-4})^{-1/2}$ $s = 7.0 \times 10^{-4}$

$= 181.4$ m

up to this point ~~[illegible]~~ in stable stratification with wind the Equation ($\Delta h = 1.6 F^{1/3} u^{-1} x^{2/3}$) held. ~~[illegible]~~ Beyond this point the plume levels off at $\Delta h = 2.4 \frac{F}{us}^{1/3}$

b. Δh @ 800 m

$\Delta h = 2.4\left(\frac{F}{us}\right)$ $2.4\left[\frac{249.4}{2.0 \times 7.0\times10^{-4}}\right]^{1/3}$

$\Delta h = 135.0$ m

2

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3. "No wind" Δh from Eq. 4.25; Assume top of surface-based inversion at 500 m.

$$\Delta h = 5.0\, F^{1/4}\, s^{-3/8} \qquad F = 249.4 \qquad s = 7.0 \times 10^{-4}$$

$$= 5.0\,(249.4)^{1/4}\,(7.0\times10^{-4})^{-3/8}$$

$$= 302.89 \text{ m}$$

plume will not penetrate inversion @ 500 m so will not rise above 500 m.

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I

EFFECTIVE STACK HEIGHT PROBLEMS

SET THREE

Submit all calculations leading to and including final answer(s) to each problem.

Refer to Part H of this package, which reflects current usage of the Briggs' equations in the Meteorology Laboratory, EPA.

Under Unstable or Neutral Conditions:

1. Using the Colbert plant data, what is x*?

h_s 91.43 m

d = 5.03 m

V_s = 13.08 m/sec

$$V_f = \frac{\pi}{4} V_s d^2$$

$$V_f = 0.785 \, (13.08 \text{ m/sec})(5.03 \text{ m})^2$$

$$V_f = 259.78 \text{ m}^3/\text{sec}$$

$$F = \frac{g}{\pi} V_f \left(\frac{T_s - T}{T_s}\right) = \frac{9.8}{3.14}(259.78)\left(\frac{93 \quad 20}{366}\right)$$

$$= 161.6 \qquad F > 55$$

$$x^* = 34(161.6)^{2/5} = 259.95 \text{ m}$$

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Problem Set Three (I)

Under Unstable or Neutral Conditions:

2. What is the distance of the final plume rise, x_f?

$$x_f = 3.5\, x^* = 3.5(2599.5) = 9098 \text{ m}$$

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ALL Problem Sheets and ANY additional calculations must be returned to APTI to receive credit.

Problem Set Three (I)

Under Unstable or Neutral Conditions:

3. What is the plume rise Δh, that can be expected a mile (1500 m) from the plant if the wind speed is 5 m/sec?

This page is perforated in the margin for easy removal

Name__________

Address__________

State__________ Zip__________

USE THIS SIDE FOR CALCULATIONS BEFORE USING ADDITIONAL SHEETS

EFFECTIVE STACK HEIGHT/PLUME RISE
STUDENT CRITIQUE

PLEASE CIRCLE A RESPONSE.

1. How many hours did you spend completing this package?

   <2   2-4   4-6   6-8   8-10   >10

2. In terms of coverage of the topic, how would you rate this package?

Too Narrow	1	2	3	4	5	6	7	Too Broad
Too Elementary	1	2	3	4	5	6	7	Too Advanced
Too Little Material	1	2	3	4	5	6	7	Too Much Material

3. How would you rate this package in terms of overall value to you?

Not Worth the Time	1	2	3	4	5	6	7	Significantly Improve My Work

4. What responsibility do you have for calculating or reviewing effective stack height estimates?

   None at Present   Currently Involved   Will Assume Shortly   None Planned (Within next two years)

5. Additional Comments:

   A. Package Contents

   B. Administrative Aspects (grading, Certificate, etc.)

   C. General Suggestions

Name __________

Address __________

State __________ Zip __________

A CRITIQUE MUST BE SUBMITTED BEFORE A CERTIFICATE CAN BE AWARDED BY THE AIR POLLUTION TRAINING INSTITUTE.