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Analysis of Multiple Cell Mechanical Draft Cooling Towers

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ANALYSIS OF MULTIPLE CELL MECHANICAL DRAFT COOLING TOWERS

by

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ABSTRACT

This report presents the development of a mathematical model designed to calculate the rise and dilution of plumes from multiple cell mechanical draft cooling towers. The model uses integral methods and includes the initial development zone, the individual single plume zone, and the zone of merging multiple plumes.

Although the governing equations for moist plumes are presented, the final working equations are for dry plumes only. Techniques are used that allow for a gradual merging of plumes without a discontinuity in the calculation of plume properties. Entrainment techniques that include the interference of unmerged plumes and the reduction of entrainment surfaces after merging are presented. The entrainment expression includes coefficients that need to be determined by tuning the model with experimental data.

This report was submitted by the Pacific Northwest Environmental Research Laboratory of the Environmental Protection Agency. Work was completed as of May 1975.

CONTENTS

SECTION I

CONCLUSIONS

This report presents a mathematical model capable of calculating plume rise and dilution from multiple cell mechanical draft cooling towers with the wind normal to the tower line. The model includes calculation techniques for each mode of plume development: 1) the zone of flow establishment, 2) the zone of fully developed single plumes, 3) the zone of merging multiple plumes, and 4) the zone of completely merged plumes. However, end effects have been left for future work. The model is particularly significant because calculations proceed smoothly from one zone to another without a discontinuity in plume properties. The entrainment functions presented include the effects of plume interference and variable entrainment surfaces on merging. In order to tune the model, however, the coefficients in the entrainment function need to be determined from suitable field or laboratory data. Although the present version is for dry plumes, the report includes the equations and simple modifications required to convert the model for moist plumes.

SECTION II

INTRODUCTION

A mechanical draft cooling tower for a modern power plant consists of clusters of tower cells arranged in rows. Each cell has its own discharge port and fan. The plumes resulting from each row of cells (i.e., tower) consist of a row of individual plumes that gradually merge together forming a long, oblong plume cross-section.

Most multiple cell plumes interfere with each other before rising very far into the environment. Interference affects individual plumes by reducing entrainment and changing the shape of the plume and distribution of its properties. Entrainment...pumping environmental fluid into and thereby diluting the plume...is caused by fluid shear at the boundaries of the plume where it contacts the environment and by interaction with the wind. Even before the plumes merge, neighboring plumes compete with one another by trying to entrain the same fluid that is between them, thus reducing the amount of dilution that occurs. Merging further reduces entrainment since only a portion of each plume subsequently contacts the environment.

Handling multiple port discharges mathematically is very difficult due to the non-symmetrical nature of the plume. For single plumes, one practical analytical method has been an integral analysis based on assumed symmetrical velocity, liquid, vapor, and temperature profiles. Entrainment is calculated from the plume size, relative velocity between plume and wind, and an entrainment coefficient. The conservation equations are integrated and solved stepwise for the desired properties along the plumes's trajectory.

Koh and Fan[1] made one of the first attempts to solve the multiple plume problem, assuming single round plumes up to a selected point of transition and then a two-dimensional slot plume after that. This approach was used to generate the nomograms for multiple port discharge in the "Workbook on Thermal Plume Prediction, Volume I"[2]. The problem with this approach is that continuity of plume centerline properties and conservation of mass, momentum, and energy cannot be obtained through the transition region. As a result, a sudden drop in plume centerline temperature was predicted using this method.

Another approach, suggested by Jirka and Harleman[3], ignores transition and assumes an equivalent slot discharge all the way from the source. The size of the equivalent slot is one having the same mass flux and momentum as the multi-port system. However, this approach overestimates dilution except for plumes that initially are very close together.

Briggs[4,5] modified his single plume equation by an "enhancement factor" to account for multiple sources. His equations are good if you want a quick, approximate answer.

Meyer, et al.[6] also used a modified version of the Briggs formula to predict plume rise and dispersion from a multiple cell tower. They found their equations fairly accurate in predicting visible plume length, but less accurate for trajectory predictions. Although they are working on a more rigorous mathematical model that includes the effects of merging, the details of that model presently are unknown.

Several existing single cell plume models provide insight into plume characteristics. A few are those by Slawson and Csanady[7,8], Csanady[9], Wigley and Slawson[10], Lee[11], Weil[12], Hirst[13], Hoult, et al.[14], and Hanna[15].

Data from multiple port discharges are scarce. One study by Carpenter, et al.[16] at TVA gives field information on plume characteristics for discharge from as many as nine stacks and as few as one in operation. Meyer, et al.[6] have been collecting data from the tower at Potomac Electric Power Company's Benning Road plant. Those data cover discharges from a tower with up to eight cells in operation.

In an ongoing study, Kannberg and Davis[17] at Oregon State University in Corvallis are conducting laboratory experiments on multiple port discharges. In this study, the characteristics of cooling tower plumes as well as those of submerged multi-port diffusers are being investigated by discharging hot water into cool water from selected discharge configurations. Since the study is interested in discharges from diffusers as well as from multiple cell cooling towers, discharge angles other than 90° have also been investigated. The parameters varied are the discharge densimetric Froude number, discharge port spacing, discharge angle, and ambient-to-discharge velocity ratio. The ambient has been neutrally stratified in all cases. Experiments to date indicate that dispersion of individual plumes from a row of multiple ports is significantly less than single port discharges of the same diameter and Froude number when the ports are spaced less then ten diameters apart. A complete report giving the results of this study is forthcoming. Figure 1 is an example of the results.

The following sections present a mathematical model for multiple cell cooling towers. The model, based on an integral analysis, includes

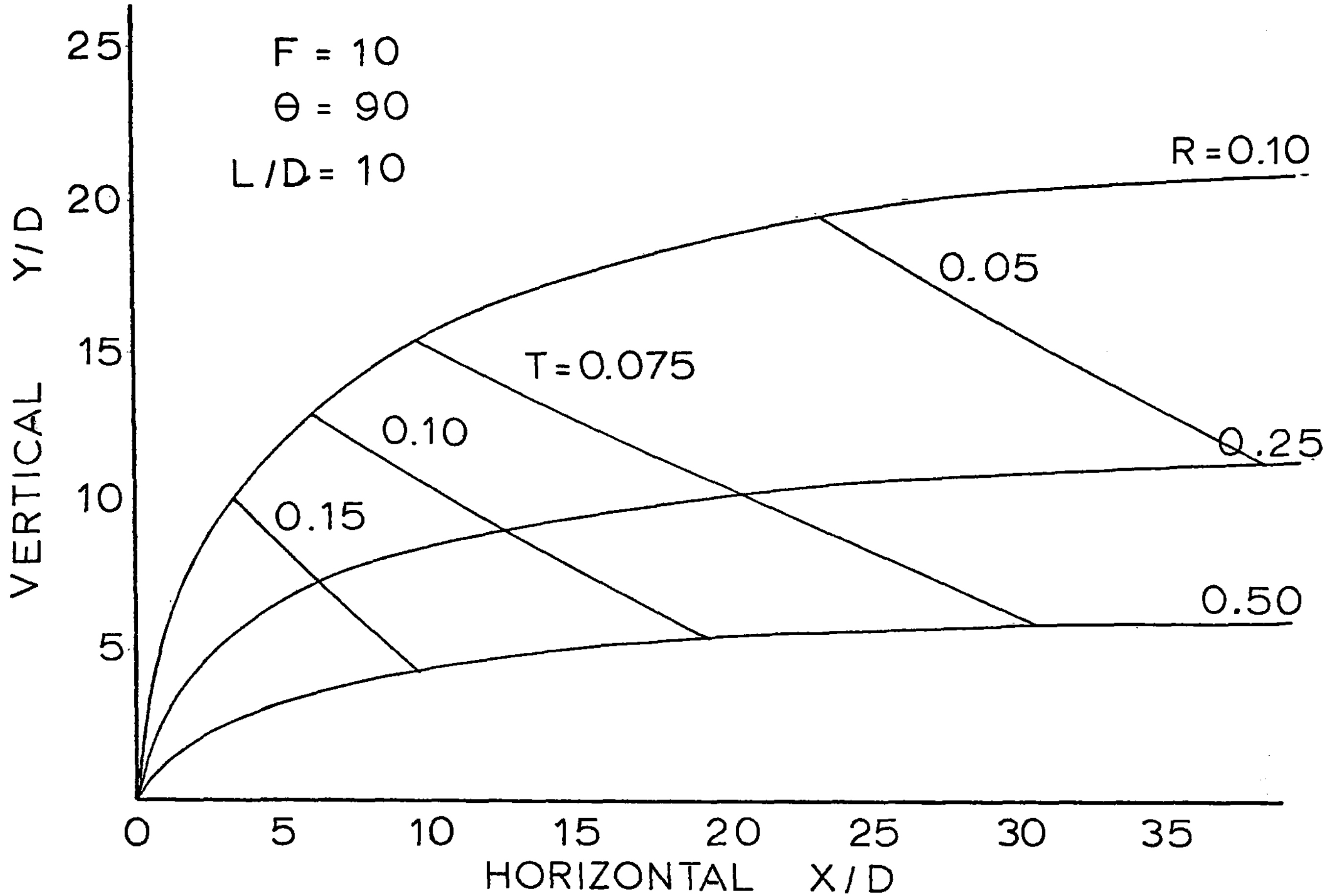

Fig. 1 Temperature - trajectory plot for multiport discharge with different current velocities.

the development and merging zones. The report presents the basic equations for moist plumes (including the effects of condensation) but only the final working equations for dry plumes. The experimental data of Kannberg and Davis and available field data will be used to tune the model in subsequent work.

SECTION III

MULTIPLE PLUME ANALYSIS - GENERAL

In order to analyze the plumes merging from a multiple cell cooling tower, the plume is divided into four major regions: 1) The zone of flow establishment, 2) the zone of fully developed single plumes, 3) the merging zone, and 4) the zone of completely merged plumes.

In the zone of flow establishment, the velocity, temperature, density, and moisture profiles change from those at the tower exit to fully developed profiles at some point downstream. In the fully developed single plume zone, the profiles within the plume retain their characteristic shapes, changing only in magnitude. The length of this zone largely depends on how close the discharge ports are. For cooling tower plumes with cells close together, this zone will probably only be a few diameters in length.

Long before they touch, the plumes actually interfere with one another by attempting to entrain the same fluid between them. The most dramatic effects of merging, however, are after the plumes touch and begin to diffuse into one another. Both the shape of the property profiles within the plumes and the surface area available for entrainment changes during this process. Merging continues until the properties at the mid-point between plumes equal those at the plume centerlines. Beyond this point, the plumes behave essentially like the discharge from a long slot.

The present analysis assumes that knowledge of the properties in the central plumes is desired since they differ the greatest from ambient conditions. It is further assumed that these central plumes are affected only slightly by the total number of cells in the towers. For this reason, end effects are ignored and left for future work.

Figure 2 gives the coordinate system and defines the angles used. The integral form of the governing conservation equations can be written as Hirst[13] and Weil[12]:

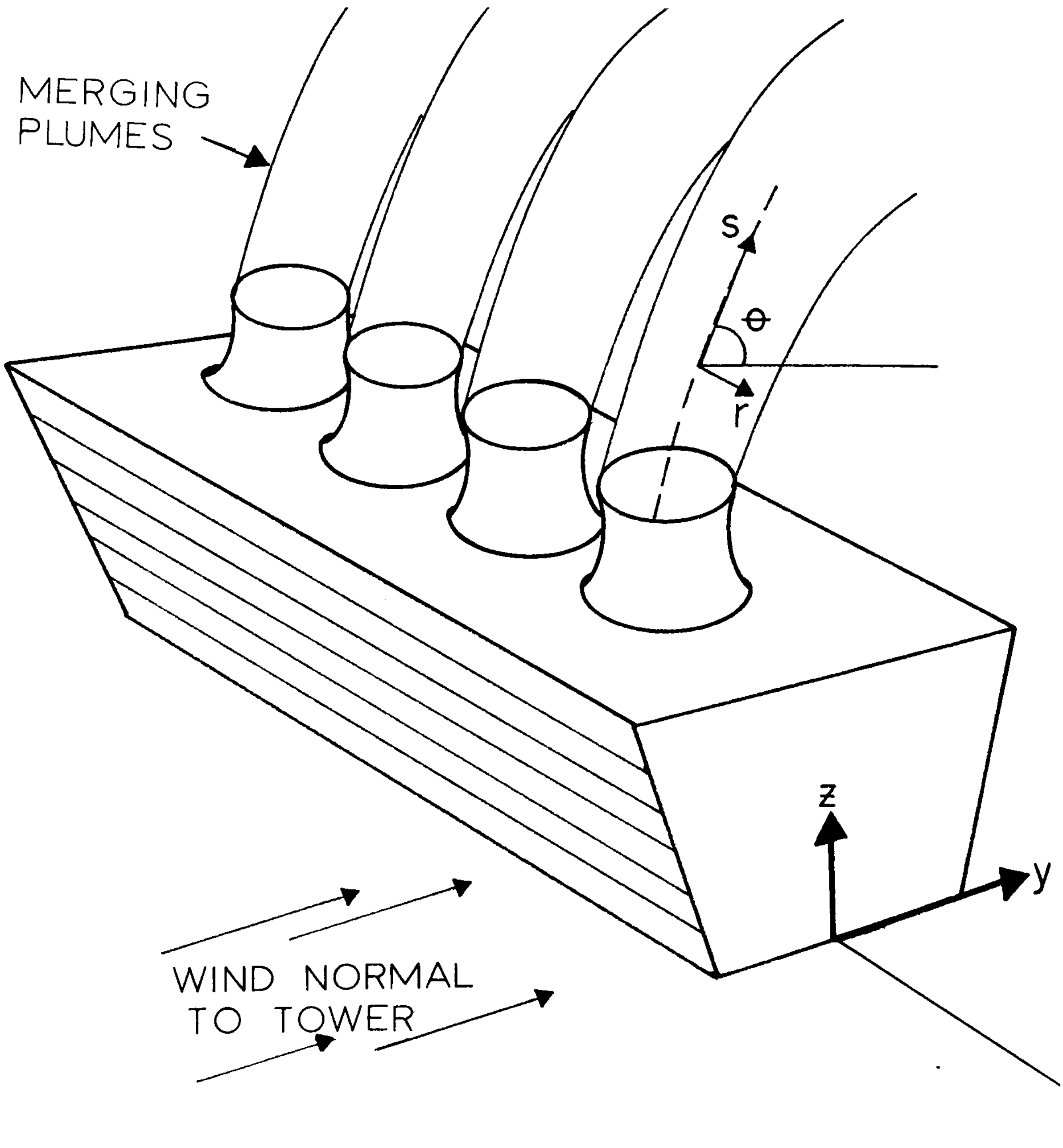

Fig. 2 Sketch of cooling tower with definitions of coordinates and plume angle.

Continuity equation

$$\frac{d}{ds}\int_0^{\infty} U dA = E \tag{1}$$

where E is the entrainment.

Energy equation

$$\frac{d}{ds}\int_0^{\infty} U(T-T_\infty)dA = -\left[\frac{dT_\infty}{ds} + \Gamma \sin\theta\right]\int_0^{\infty} UdA + \int_0^{\infty} \frac{LR_c}{\rho c_p} dA \tag{2}$$

where Γ is the abiabatic lapse rate, L is latent heat of vaporization and R_c is the condensation rate.

Conservation of Vapor (q)

$$\frac{d}{ds}\int_0^{\infty} U(q-q_\infty) dA = -\frac{dq_\infty}{ds}\int_0^{\infty} UdA - \int_0^{\infty} \frac{R_c}{\rho_1} dA \tag{3}$$

Conservation of moisture (σ)

$$\frac{d}{ds}\int_0^{\infty} U(\sigma-\sigma_o)dA = -\frac{d\sigma_\infty}{ds}\int_0^{\infty} UdA + \int_0^{\infty} \frac{R_c}{\rho_1} dA \tag{4}$$

Combining equations (3) and (4) with the energy equation (2) yields

$$\frac{d}{ds}\int_0^{\infty}\left[U(T-T_\infty) + \frac{L}{C_p}(q-q_\infty)\right] dA = -\left[\frac{dT_\infty}{ds} + \frac{dq_\infty}{ds} + \Gamma \sin\theta\right]\int_0^{\infty} UdA \tag{5}$$

Combining just (3) and (4) yields

$$\frac{d}{ds} \int_0^\infty U\left[(q-q_\infty) + (\sigma-\sigma_\infty)\right] dA = -\left[\frac{dq_\infty}{ds} + \frac{d\sigma_\infty}{ds}\right] \int_0^\infty U dA \qquad (6)$$

S - momentum equation

$$\frac{d}{ds} \int_0^\infty U^2 \, dA = E \, U_\infty \cos\theta + \int_0^\infty \frac{(\rho_\infty-\rho)}{\rho} g \sin\theta \, dA \qquad (7)$$

Curvature equation (combined r and s momentum)

$$\frac{d\theta}{ds} = \frac{\cos\theta \int_0^\infty \frac{(\rho_\infty-\rho)}{\rho} g dA - E \, U_\infty \operatorname{Sin}\theta}{\int_0^\infty U^2 dA - E^2/4} \qquad (8)$$

If thermodynamic equilibrium is assumed in the plume, the Clausius - Clapeyron relation can be used to yield a relation for the plume centerline vapor content.

$$(q_c - q_\infty) = (1-\phi)q_{\infty s} + \frac{q_{\infty s1} \, L(T_c-T_\infty)}{R_v T_1^2} \qquad (9)$$

where

ϕ is the local ambient relative humidity at height z.

$q_{\infty s}$ is the ambient saturation humidity at height z.

$q_{\infty s1}$ is the ambient saturation humidity at the top of the tower.

R_v is the vapor gas constant.

T_1 is the ambient temperature at the top of the tower.

$q_{\infty s}$ is given by

$$q_{\infty s} = q_{\infty s1} \exp\left[\frac{L}{R_v T_1^{\,2}} \quad (T_\infty - T_1) \quad + \frac{gz}{RT_1}\right] \tag{10}$$

Moisture effects on plume trajectory are only secondary[10]. For this reason the above equations first will be solved for merging dry plumes and later expanded to wet plumes.

SECTION IV

INTEGRAL ANALYSIS OF MULTIPLE DRY PLUMES

ZONE OF FLOW ESTABLISHMENT

Because of the fan and its hub, the distribution of velocity at the tower exit usually is shaped as shown in Figure 3.

To represent the actual profiles at discharge, this analysis uses approximate top-hat profiles determined by mean discharge values. This assumes that these profiles change to the fully developed, bell-shaped profiles at the end of the zone of flow establishment as shown on Figure 3.

Profiles assumed within the zone of flow establishment are:

$$U = U_o \quad , \quad r \leq r_u$$

$$= (U_o - U_\infty \cos\theta)\left[1 - \left(\frac{r-r_u}{b}\right)^{3/2}\right]^2 + U_\infty \cos\theta, \; r \geq r_u \tag{11}$$

$$T - T_\infty = T_o - T_\infty \; , \; r \leq r_t$$

$$= (T_o - T_\infty)\left[1 - \left(\frac{r-r_t}{b}\right)^{3/2}\right]^2, \; r \geq r_t \tag{12}$$

$$\rho_\infty - \rho = \beta(T - T_\infty) \tag{13}$$

where r_u and r_t are the radii to the edge of the velocity central core and to the temperature core, respectively, and U_∞ is the free ambient velocity. This expression for velocity and temperature profiles was selected over exponential or gaussian profiles because of the definite

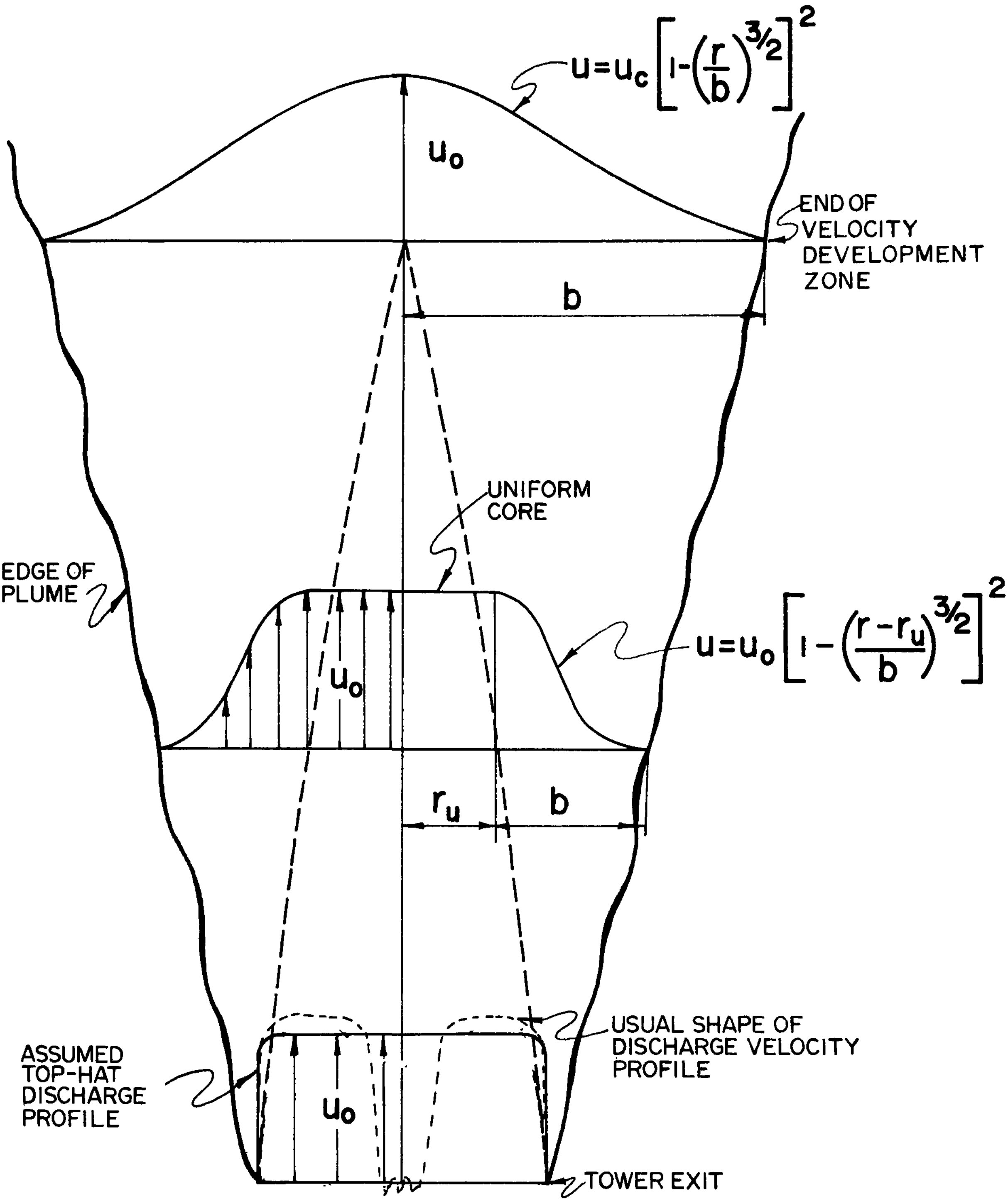

Fig. 3 Sketch of assumed velocity profiles within the development zone.

plume edge definition inherent in this expression. This profile property will become important later when the plumes begin to merge and will allow a smooth transition. With these assumptions, equations (1), (2), (7) and (8) can be integrated to yield:

Continuity

$$\frac{d}{ds}\left[\frac{U_o r_u^2}{2} + d_1 b r_u + d_2 \frac{b^2}{2}\right] = E \tag{14}$$

Energy

$$\frac{d}{ds}\left[U_o \, \Delta T_o \frac{r_t^2}{2} + \Delta T_o r_t \, b d_3 + \Delta T_o \frac{b^2}{2} d_4\right]$$
$$= -\left[\frac{dT_\infty}{ds} + \Gamma \sin\theta\right]\left[\frac{U_o r_u^2}{2} + d_1 b r_u + \frac{d_2 b^2}{2}\right] \tag{15}$$

Momentum

$$\frac{d}{ds}\left[\frac{U_o^2 r_u^2}{2} + d_5 r_u b + d_6 \frac{b^2}{2}\right] \tag{16}$$

$$= E\, U_\infty \cos\theta + g\, I_5 \sin\theta$$

Curvature equation

$$\frac{d\theta}{ds} = \frac{g\, I_5 \cos\theta - E\, U_\infty \sin\theta}{\frac{U_o^2 r_u^2}{2} + d_5 r_u b + d_6 \frac{b^2}{2} - E^2/4} \tag{17}$$

where

$$d_1 = .45U_o + .55U_\infty \cos\theta$$
$$d_2 = .2571U_o + .7429U_\infty \cos\theta$$
$$d_3 = .31558U_o + .13442U_\infty \cos\theta$$
$$d_4 = .13352U_o + .12362U_\infty \cos\theta$$
$$d_5 = .31558U_o^2 + .26885U_oU_\infty \cos\theta + .41558U_\infty^2 \cos^2\theta$$
$$d_6 = .13352U_o^2 + .24724U_oU_\infty \cos\theta + .61924U_\infty^2 \cos^2\theta$$

and

$$I_5 = \int_0^\infty \frac{\rho_\infty\ \rho}{\rho}\, dA = \beta\Delta T_o\left[\frac{r_t^2}{2} + .12857b^2 + .45r_tb\right] \qquad (18)$$

$\beta = -\frac{1}{\rho_o}\left(\frac{\partial\rho}{\partial t}\right)$, the coefficient of thermal expansion. For an ideal gas $\beta = \frac{1}{T_\infty}$.

Differentiating equations (14), (15) and (16) yields:

Continuity

$$U_or_u + d_1b)\ r_u' + (d_1r_u + d_2b)\ b' = E \qquad (19)$$

Energy

$$(U_or_t + bd_3)\ r_t' + (r_td_3 + bd_4)\ b' =$$

$$\frac{1}{\Delta T_o}\left[\Gamma \sin\theta + \frac{dT_\infty}{ds}\right]\left[\frac{U_o}{2}\ (r_t^2 - r_u^2) + b(r_td_3 - r_ud_1) + \frac{b^2}{2}(d_4 - d_2)\right] \qquad (20)$$

Momentum

$$(U_o^2 r_u + bd_5)\, r_u' + (r_u d_5 + bd_6)\, b' = E\, U_\infty \cos\theta + g\, I_5 \sin\theta \qquad (21)$$

Equations (17), (19), (20), and (21) can be solved for the four unknowns, r_u', r_t', b', and θ'. These variables can be integrated step-wise to yield r_u, r_t, b and θ in the development zone.

Since the change in r_t depends on Γ and $\frac{dT_\infty}{ds}$, it may disappear before or after r_u disappears. If r_t goes to zero first $r_t = 0$ and $r_u \geq 0$, then ΔT_c will change as r_u continues to decrease. In this case, equations (18) and (20) become:

$$I_5 = \beta \Delta T_c\, (0.12857\, b^2) \qquad (22)$$

$$\frac{b^2}{2} d_4\, \Delta T_c' + \Delta T_c\, bd_4 b' = -\left[\frac{dT_\infty}{ds} + \Gamma \sin\theta\right]\left[\frac{U_o r_u^2}{2} + d_1 b r_u + \frac{d_2 b^2}{2}\right] \qquad (23)$$

Using these two equations, (17), (19), and (21) yield b', r_u', θ_2' and $\Delta T_c'$.

Should r_u go to zero first $r_u = 0$ and $r_t \geq 0$, $\Delta U_c = U_c - U_\infty \cos\theta$ will change as r_t continues to decrease. In this case the equations become:

Continuity

$$.12855 b^2 \Delta U_c' + \left[.2571 \Delta U_c b + .4858 U_\infty \cos\theta\, b\right] b' = E \qquad (24)$$

Momentum

$$\left[.13352b^2\Delta U_c + .25714b^2 U_\infty \cos\theta \right] \Delta U_c'$$

$$+ \left[.13352b\Delta U_c^2 + .51824b\Delta U_c U_\infty \cos\theta + bU_\infty^2 \cos^2\theta \right] b'$$

$$= EU_\infty \cos\theta + g\, I_5 \sin\theta \tag{25}$$

Energy

$$\left[(\Delta U_c + U_\infty \cos\theta)\, r_t + d_7 b \right] r_t' + \left[\frac{r_t^2}{2} + .31588 r_t b + .13552\, \frac{b^2}{2} \right] \Delta U_c'$$

$$+ \left[d_7 r_t + d_8 b \right] b' = \frac{1}{\Delta T_o} \left[\frac{dT_\infty}{ds} + \Gamma \sin\theta \right] \left[- \frac{b^2}{2} (.2571\Delta U_c + .4858 U_\infty \cos\theta) \right.$$

$$\left. + (\Delta U_c + U_\infty \cos\theta) \frac{r_t^2}{2} + r_t b d_7 + \frac{b^2}{2} d_8 \right] \tag{26}$$

where $\Delta U_c = U_c - U_\infty \cos\theta$

$$d_7 = .31558\Delta U_c + .45 U_\infty \cos\theta$$

$$d_8 = .13352\Delta U_c + .25714\ U_\infty \cos\theta$$

$$\frac{d\theta}{ds} = \frac{g I_5 \cos\theta - EU_\infty \sin\theta}{\frac{b^2}{2} (.13352\Delta U_c^2 + .51428\Delta U_c U_\infty \cos\theta + U_\infty^2 \cos^2\theta) - E^2/4} \tag{27}$$

The integral I_5 is given by equation (18).

Equations (24), (25), (26), and (27) are sufficient to solve for r_t', $\Delta U_c'$, b', and θ'.

All of the above equations depend on the entrainment, E. This variable can be approximated using an expression similar to that used by Hirst[13], modified to account for the different profiles used in this work.

FULLY DEVELOPED SINGLE PLUME

After r_t and r_u both go to zero and before neighboring plumes merge, each individual plume will resemble a single fully developed plume. In this region assumptions are:

$$U = \Delta U + U_\infty \cos\theta \tag{28}$$

$$\text{where } \Delta U = \Delta U_c \left[1 - \left(\frac{r}{b}\right)^{3/2} \right]^2$$

$$\text{and } \Delta T = \Delta T_c \left[1 - \left(\frac{r}{b}\right)^{3/2} \right]^2 \tag{29}$$

Using this type of profile and assuming that velocity and temperature profiles are the same allows the merging process to be calculated smoothly without a discontinuity in either the conservation equation or the plume temperature, velocity, or width. Figure 5 compares this profile to the more popular gaussian profile given by $\Delta U = \Delta U_c \, e^{-\left(\frac{r}{b_1}\right)^2}$ with $b_1 = .529b$ ($\frac{\Delta T}{\Delta T_o} = 0.5$ for both).

With these assumptions the governing equations for the fully developed dry single plume become:

Continuity

$$\frac{d}{ds} \left[\Delta U_c b^2 I_1 + \frac{b^2}{2} U_\infty \cos\theta \right] = E \tag{30}$$

Momentum

$$\frac{d}{ds} \left[\Delta U_c{}^2 b^2 I_2 + 2\Delta U \, b^2 U_\infty \cos\theta \, I_1 + \frac{b^2}{2} U_\infty{}^2 \cos^2\theta \right]$$

$$= -\beta \Delta T_c b^2 g \sin\theta \, I_1 + E U_\infty \cos\theta \tag{31}$$

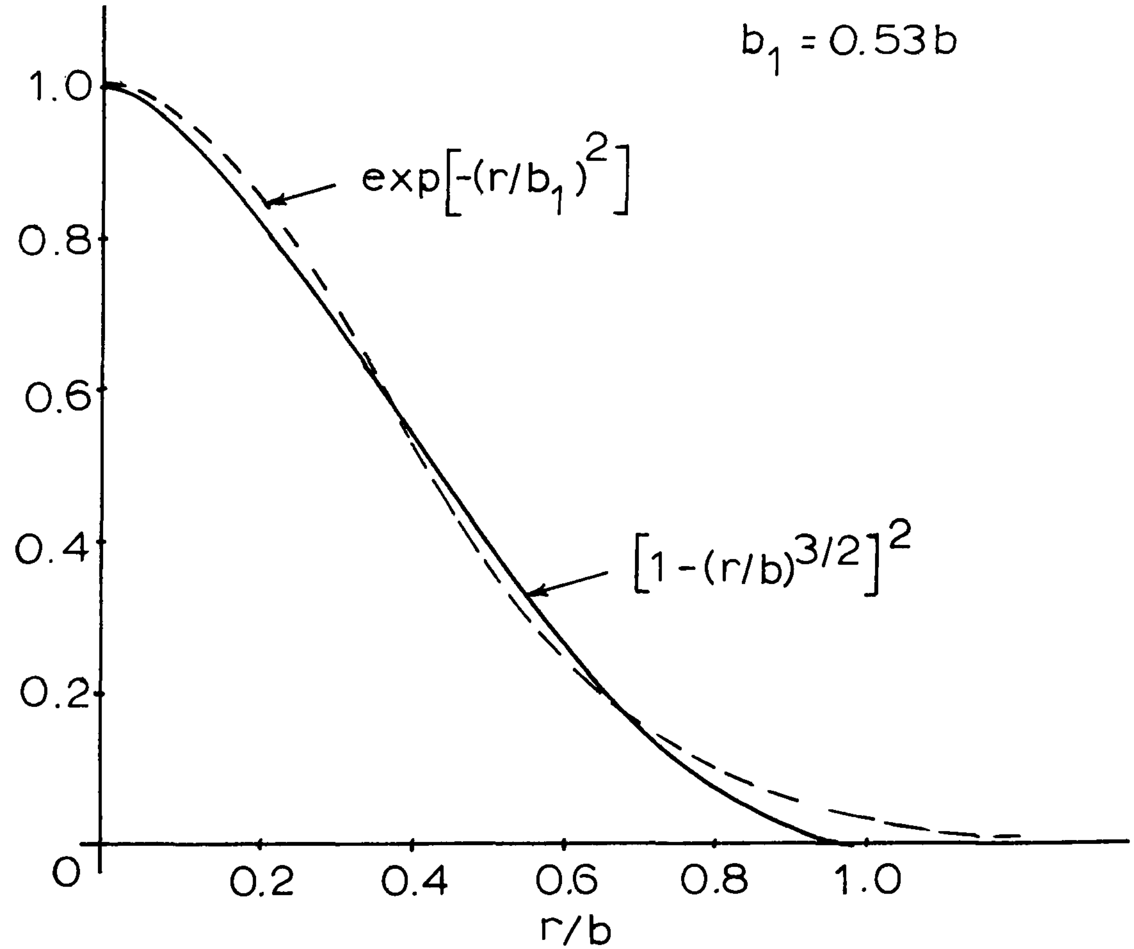

Fig. 5 Comparison of assumed to gaussian profile.

Energy

$$\frac{d}{ds}\left[\Delta T_c \Delta U_c b^2 I_2 + \Delta T_c b^2 U_\infty \cos\theta\, I_1\right]$$

$$= -\left[\frac{dT_\infty}{ds} + \Gamma \sin\theta\right]\left[\Delta U_c b^2 I_1 + \frac{b^2}{2} U_\infty \cos\theta\right] \qquad (32)$$

Curvature

$$\frac{d\theta}{ds} = \frac{\cos\theta\ \beta\Delta T_c g I_1 - E U_\infty \sin\theta}{\Delta U_c{}^2 b^2 I_2 + 2\Delta U_c b^2 I_1 U_\infty \cos\theta + \frac{b^2}{2} U_\infty{}^2 \cos^2\theta - E^2/4} \qquad (33)$$

where

$$I_1 = \int_0^1 (1-\eta^{3/2})^2\, \eta d\eta = 0.12857 \qquad (34)$$

and

$$I_2 = \int_0^1 (1-\eta^{3/2})^4\, \eta d\eta = 0.06676 \qquad (35)$$

The solutions to equations (30-33) yield ΔU_c, ΔT_c, b, and θ, assuming an appropriate entrainment function E. From θ and ds, the trajectory can be determined since:

$$dz = ds \sin\theta \qquad (36)$$

$$dy = ds \cos\theta \qquad (37)$$

MERGING PLUMES

To this point the present analysis has been essentially the same as Hirst's with a different profile and the addition of adiabatic expansion effects of a compressible fluid. Once the plumes start to merge, however, symmetrical profile no longer can be assumed.

Figure 6 shows a cross-section of merging plumes, and figure 7 shows the approximate shape of the temperature distribution along the line connecting the center of each individual plume.

In order to continue the integral analysis through this merging zone, profiles in the merging plumes must be known. As seen on Figure 7, the temperature at the mid-point between two plumes does not drop to the ambient temperature. In fact, limited data[17] indicate that the excess temperature at the mid-point between plumes is approximately twice the excess temperature at an equal distance from a plume centerline in a direction normal to the plume connecting line. If this is assumed true, the profiles must satisfy the following: Along the connecting line the profiles must be smooth curves with zero derivative at the mid-point and plume centerline with a value of ΔT twice the single plume value at the same distance from the center. In the limit as the plumes just start to merge, the profiles must be the same as the single plume.

Referring to Figure 6 for the definitions of the η and r direction and the terms c, b and L, the profile assumed in this analysis that satisfies the above condition is:

$$\Delta T_r = \Delta T_c \left[1-\left(\frac{r}{b}\right)^{3/2} \right]^2 \qquad , \quad 0 \leq r \leq L - b$$

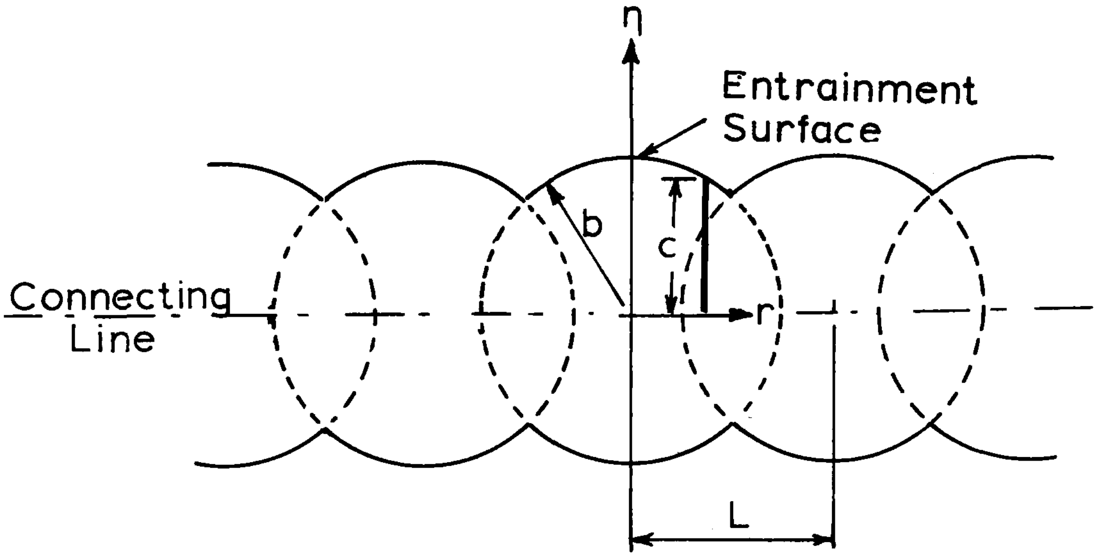

Fig. 6 Cross-section of merging plumes.

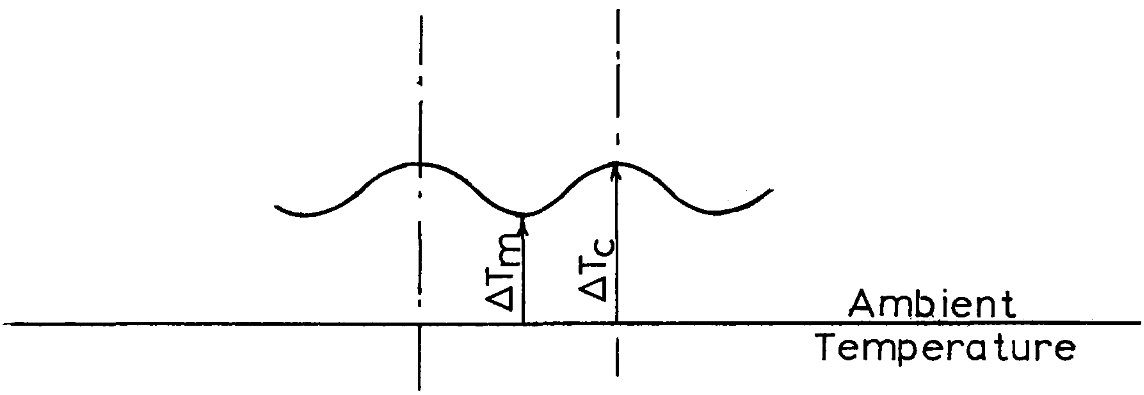

Fig. 7 Temperature profile along connecting line of merging plumes.

$$\Delta T_r = \Delta T_c \left\{ \left[1-(\frac{r}{b})^{3/2} \right]^2 + \left[1-(\frac{L-r}{b})^{3/2} \right]^2 \right\}, L-b \leq r \leq \frac{L}{2} \quad (38)$$

After $\Delta T_r \simeq \Delta T_c$ at $r = \frac{L}{2}$, it is assumed that $\Delta T_r = \Delta T_c$. In the η direction it is assumed that:

$$\Delta T_\eta = \Delta T_r \left[1-(\frac{\eta}{c})^{3/2} \right]^2 \quad (39)$$

where $c = \sqrt{b^2 - r^2}$

Making the same assumption for velocity yields:

$$\Delta U_\eta = \Delta U_r \left[1-(\frac{\eta}{c})^{3/2} \right]^2 + U_\infty \cos\theta \quad (40)$$

where

$$\Delta U_r = \Delta U_c \left[1-(\frac{r}{b})^{3/2} \right]^2, \quad 0 \leq r \leq L-b$$

$$= \Delta U_c \left\{ \left[1-(\frac{r}{b})^{3/2} \right]^2 + \left[1-(\frac{L-r}{b})^{3/2} \right]^2 \right\}, \quad L-b \leq r \leq \frac{L}{2} \quad (41)$$

and $\Delta U_r = \Delta U_c$ after $\Delta U_r \simeq \Delta U_c$ at $r = \frac{L}{2}$

MERGING PLUMES - INTEGRAL EQUATIONS

The continuity equation is still given by equation (1), but the integral must include the distorted profile. Therefore, the integral in equation (1) (called F_1) is:

$$\frac{1}{2\pi} \int UdA = \frac{2}{\pi} \int_0^{L/2} dr \int_0^{c} \Delta U_\eta d\eta = F_1 \tag{42}$$

In the single plume calculation, the integral $\int urdr$ was evaluated which is $\frac{1}{2\pi} \int UdA$. Substituting (40) and (41) into (42), then integrating yields:

$$F_1 = \frac{2}{\pi} b^2 \Delta U_c I_3 f_1(A) + \frac{b^2}{\pi} U_\infty \cos\theta \, f_3(A) \tag{43}$$

where $A = L/b$,

$$f_1(A) = \int_0^{\frac{A}{2}} \sqrt{1-x^2}\,(1-x^{3/2})^2\, dx + \int_{A-1}^{\frac{A}{2}} \sqrt{1-x^2} \left[1-(A-x)^{3/2}\right]^2 dx \tag{44}$$

$$f_3(A) = 2\int_0^{\frac{A}{2}} \sqrt{1-x^2}\, dx = \frac{A}{2}\sqrt{1-\left(\frac{A}{2}\right)^2} + \sin^{-1}\left(\frac{A}{2}\right) \tag{45}$$

and

$$I_3 = \int_0^1 \left[1-\eta^{3/2}\right]^2 d\eta = 0.45 \tag{46}$$

After $\Delta U_r = \Delta U_c$

$$F_1 = \frac{1}{2\pi} \int UdA = \frac{b^2}{\pi} \left[\Delta U_c I_3 + U_\infty \cos\theta \right] \left[\frac{A}{2} \; 1-(\frac{A}{2})^2 + \sin^{-1}(\frac{A}{2}) \right] \quad (47)$$

The variable A, represents the degree of merging. For A = 2 the plumes are just starting to merge. For A = 1 the plumes have nearly merged together.

In a similar manner, the value of

$$\frac{1}{2\pi} \int U^2 dA = F_2 \quad (48)$$

can be found

$$F_2 = \frac{2}{\pi} b^2 \Delta U_c{}^2 I_4 f_2(A) + \frac{4}{\pi} b^2 \Delta U_c I_3 f_1(A) U_\infty \cos\theta + \frac{b^2 U_\infty{}^2 \cos^2\theta}{\pi} f_3(A) \quad (49)$$

where

$$f_2(A) = \int_0^{A/2} \sqrt{1-x^2} \left(1-x^{3/2}\right)^4 dx + \int_{A-1}^{A/2} \sqrt{1-x^2} \left\{ (1-x^{3/2})^2 \left[1-(A-x)^{3/2} \right]^2 + \left[1-(A-x)^{3/2} \right]^4 \right\} dx \quad (50)$$

and

$$I_4 = \int_0^1 \left[1-x^{3/2} \right]^4 dx = 0.31558 \quad (51)$$

After $\Delta U_r = \Delta U_c$

$$F_2 = \frac{b^2}{\pi}\left[\Delta U_c{}^2 I_4 + 2\Delta U_c I_3 U_\infty \cos\theta_2 + U_\infty{}^2 \cos^2\theta_2\right] f_3(A) \tag{52}$$

For the energy equation

$$\frac{1}{2\pi}\int U\Delta T\, dA = \frac{2}{\pi}\int_0^{L/2} dr \int_0^{c} \Delta U_\eta \Delta T_\eta d\eta = F_3 \tag{53}$$

which when integrated becomes

$$F_3 = \frac{2}{\pi} b^2 \Delta T_c \Delta U_c I_4 f_2(A) + \frac{2}{\pi} b^2 \Delta T_c I_3 U_\infty \cos\theta \;\; f_1(A) \tag{54}$$

After $\Delta T_r = \Delta T_c$ and $\Delta U_r = \Delta T_c$ (which because of similar profile assumptions, occur at the same place):

$$F_3 = \frac{b^2}{\pi} \Delta T_c \left[\Delta U_c I_4 + I_3 U_\infty \cos\theta\right] f_3(A) \tag{55}$$

The density integral also must be evaluated.

$$\frac{1}{2\pi}\int \frac{\rho_\infty - \rho}{\rho}\, dA = F_4 \tag{56}$$

Since $\frac{\rho_\infty - \rho}{\rho} = \beta\Delta T$ this integral can be evaluated and becomes:

$$F_4 = \frac{2}{\pi} b^2 I_3 \beta \Delta T_c f_1(A) \tag{57}$$

After $\Delta T_r = \Delta T_c$ this becomes:

$$F_4 = \frac{b^2}{\pi} I_3 \beta \Delta T_c f_3(A) \qquad (58)$$

The values of $f_1(A)$, $f_2(A)$ and $f_3(A)$ can be tabulated as a function of A.

With these integrals evaluated, the conservation equation can be written as:

Continuity

$$\frac{d}{ds} F_1 = E \qquad (59)$$

Momentum

$$\frac{d}{ds} F_2 = EU_\infty \cos\theta + F_4 g \sin\theta \qquad (60)$$

Energy

$$\frac{d}{ds} F_3 = \left[\Gamma \sin\theta + \frac{dT_\infty}{ds}\right] F_1 \qquad (61)$$

and Curvature

$$\frac{d\theta}{ds} = \frac{F_4 g \cos\theta - EU_\infty \text{Sin}\,\theta}{F_2 - E^2/4} \qquad (62)$$

Equations (59)-(62) are sufficient to solve for ΔU_c, ΔT_c, b and θ and thus the trajectory when an appropriate entrainment function is introduced.

SECTION V

ENTRAINMENT

In order to achieve closure, the analysis presented in the previous sections requires an expression for the entrained ambient fluid. As far as this analysis is concerned, any realistic expression for entrainment could be used; however, the accuracy of the solutions depends to a large extent on how accurately the entrainment is calculated. The following expressions, developed according to the traditional line of Hirst[13], have been modified to include the effect of neighboring plumes and profile definitions.

For the development zone, the suggested expression is:

$$E/r_oU_o = \left[.0204 + .0144b/r_o\right]\left[\left|1.0 - R\cos\theta\right| \quad (1- \frac{A_1 r_o}{L}) + A_2 R\sin\theta\right]\left[1.0 + \frac{A_3}{Fr}\right] \tag{63}$$

Where R is the velocity ratio U_∞/U_o, A_1, A_2, and A_3 are empirical coefficients included to account for the effect of cell spacing, L/r_o, ambient cross current, R sin θ and local densimetric Froude Number, Fr.

For the fully developed single plumes, the suggested expression is:

$$E = (C_1 + \frac{C_2}{Fr})\left[b\left|U_c - U_\infty\cos\theta\right| \quad (1- \frac{C_3 b}{L}) + C_4 U_\infty b\sin\theta\right] \tag{64}$$

The values of C_1, C_2 and C_4 as suggested by Hirst are 0.0289, 0.492, and 4.56, respectively, when adjusted to account for the different definition of plume size, b, in this study. However, these values may have to be adjusted for multiple plumes to give best results. The value of C_3 needs to be determined from multiple cell discharge data.

Once the plumes start to merge, i.e., when $b=\frac{L}{2}$, the entrainment surface exposed to ambient fluid decreases with the degree of merging (see Figure 6). This arc length exposed to the ambient on each side of the plume is approximated by $A_c = b\pi - 2\cos^{-1}(\frac{L}{2}b)$. Therefore, an entrainment function of the following form is suggested after the plumes start to merge:

$$E = (C_1 + \frac{C_2}{Fr}) \Big[b \left| U_c - U_\infty \cos\theta \right| (1 - \frac{C_3}{2}) (1 - \frac{2}{\pi} \cos^{-1}\frac{L}{2b}) + C_4 U_\infty \frac{L}{2} \sin\theta \Big] \tag{65}$$

The above expressions seem reasonable since they include the parameters that possibly could affect entrainment. The values of the coefficients, however, would have to be determined from selected data so the effects of individual parameters could be isolated.

Wind directions other than normal to the line of towers and the drag induced on the plume by the tower wake have not been included in the model. How large a deviation in wind direction can be tolerated in the analysis remains to be determined. The drag induced by the tower wake on the plume could easily be included in the momentum equations when more information becomes available as to the magnitude of an appropriate drag coefficient for a particulate tower structure.

SECTION VI

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SECTION VII

NOMENCLATURE

A	=	Local aspect ratio, L/b, also used in integral equations as area and appears as $\cdot dA$.
b	=	characteristic size of plume.
c	=	$\sqrt{b^2 - r^2}$
c_p	=	specific heat
D	=	discharge port diameter
$d_1, d_2 \ldots$	=	coefficients defined in text
E	=	entrainment function
Fr	=	Froude Number = $U_o/(\Delta\rho/\rho gD)^{1/2}$
g	=	gravitational constant
$I_1, I_2 \ldots$	=	integral constants defined in text
L	=	distance between discharge ports, also used as latent heat of vaporization.
q	=	vapor concentration
r	=	plume coordinate normal to centerline
r_u, r_t	=	size of velocity and temperature central core in development zone
R_c	=	condensation rate
R_v	=	gas constant of vapor
s	=	distance along plume centerline
U	=	velocity
y, z	=	horizontal and vertical distance from discharge
β	=	coefficient of thermal expansion
Γ	=	adiabatic lapse rate
η	=	plume coordinate normal to tower connecting line

θ	=	plume angle relative to horizontal
ρ	=	density
σ	=	moisture content
ϕ	=	relative humidity

Subscripts

c	=	at plume centerline
o	=	discharge conditions
r	=	in r direction
η	=	in η direction
s	=	saturation conditions
∞	=	free stream conditions
1	=	at discharge level

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16. ABSTRACT

This report presents the development of a mathematical model designed to calculate the rise and dilution of plumes from multiple cell mechanical draft cooling towers. The model uses integral methods and includes the initial development zone, the individual single plume zone, and the zone of merging multiple plumes.

Although the governing equations for moist plumes are presented, the final working equations are for dry plumes only. Techniques are used that allow for a gradual merging of plumes without a discontinuity in the calculation of plume properties. Entrainment techniques that include the interference of unmerged plumes and the reduction of entrainment surfaces after merging are presented. The entrainment expression includes coefficients that need to be determined by tuning the model with experimental data.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
*Cooling Towers *Plumes *Mechanical Draft Merging Thermal Pollution Cooling Water	Mathematical Model	13B

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