The Vehicle Road Load Problem - An Approach by Non-Linear Modeling

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ABSTRACT

When vehicle exhaust emission tests or vehicle fuel consumption measurements are performed on a chassis dynamometer, the dynamometer is usually adjusted to simulate the road experience of the vehicle. Therefore, a method to accurately characterize the road experience of the vehicle is a prerequisite for accurate dynamometer testing of this nature. Commonly used methods for determining the road characteristics of the vehicle are manifold vacuum measurements, drive line torque measurements, and the coast down technique.

In the usual coast down technique, the vehicle road load force is calculated from a numerical derivative of the speed versus time measurements. This study significantly improves the coast down technique by eliminating the differentiation of the speed-time data while maintaining the ability to treat the forces on the vehicle as a general quadratic function of vehicle speed. A model equation for the forces on a freely decelerating vehicle is constructed; this equation is then analytically transformed into an expression for the speed of the freely decelerating vehicle as a function of time. This intrinsically non-linear equation is then computer fitted to the speed versus time data recorded from the vehicle.

The results of road load measurements by this method are compared with road load measurements on the same vehicle by the drive shaft torque method.

Introduction

Vehicle exhaust emission measurments are usually performed on a dynamometer so that the vehicle engine may be exercised in the laboratory at diverse power outputs. The power demand of the engine is usually determined by first adjusting the dynamometer power absorber to simulate the power versus speed or force versus speed characteristics of the vehicle; and then the vehicle is operated at the desired simulated speeds. A method to accurately characterize the road experience of the vehicle is therefore a prerequisite for accurate dynamometer testing of this nature.

Common methods for determining the road characteristics of a vehicle are manifold pressure measurements, drive line torque measurements and the coast down technique. Manifold pressure measurements are the easiest to conduct, but various engine emission control techniques, notably exhaust gas recirculation and fuel injection have made this approach unsatisfactory for many vehicles. Drive line torque measurements are quite satisfactory, but require placing strain gages or torque meters in the drive line of each test vehicle. The deceleration or coast down technique is well suited for measurements on diverse vehicles since only instrumentation to record the vehicle speed as a function of time is required.

The Deceleration Technique

The deceleration technique is perhaps the oldest method of determining the forces acting on an automobile as a function of the vehicle speed.[1] The concept is to determine the rate of deceleration of the vehicle and then, knowing the mass of the vehicle, the road load force may be calculated by Newton's second law.

Experimentally it is not practical to measure the deceleration as a function of velocity directly, however, it is easy to record speed as a function of time. It is possible to differentiate the recorded velocity versus time data to obtain acceleration versus time, and then to use the velocity versus time data to map the acceleration as a function of velocity. This is the most common approach but it is theoretically undesirable for two reasons. The non-analytical differentiation process is inherently noise sensitive. This can be a problem when attempting a least squares fit to the differentiated data, and can lead to a falsely large linear term in the least squares fitted curve. Also since the acceleration must be derived from the velocity, the initially random errors may no longer have a normal distribution. This makes many of the usual statistical tests invalid or of questionable validity.

Theoretically it is a better approach to assume a model or form of the acceleration versus speed equation and then perform analytical operations on this equation to convert it to the form of a speed versus time equation. This expression may then be directly fitted to the velocity versus time data. This approach has been used to investigate road load force equations of the usual constant plus a velocity squared term form.[2] However, tire data[3] indicate that terms linear in speed should be included in the force equation and this method may be generalized to include a linear term.

The System Energy

An expression for the system energy is constructed to develop a generalized expression for the force acting on the vehicle. The total energy of the decelerating vehicle is the sum of the translational kinetic energy and the rotational kinetic energy of any vehicle components in rotational motion. For all mechanical components of the wheels and drive train, the rotational velocity is proportional to the vehicle velocity, and the energy of the system is

$$E = 1/2\ mv^2 + 1/2\ v^2 \sum I_i\ \alpha i \tag{1}$$

where:

m = the vehicle mass
v = the vehicle speed
I_i = the rotational inertia of the i^{th} rotating component
α_i = the proportionality constant between the rotational velocity of the i^{th} rotating component and the vehicle speed

Differentiating equation (1) with respect to time; and comparing the resulting power expression with the similar time derivative of a purely translational system; the generalized force on the system may be expressed as:

$$f = (m + \Delta m_{eq}) \frac{dv}{dt} \tag{2}$$

The term Δm_{eq} is equal to $\sum I_i \alpha_i^2$ and is the mass equivalence of the inertia of the rotating components.

The Model Equation for the Vehicle Deceleration

For ideal wind free conditions the acceleration of a freely coasting vehicle is assumed to be a polynomial function of the velocity:

$$\frac{dv}{dt} = A(v) = -(a_0 + a_1 v + a_2 v^2) \qquad (3)$$

The a_0 term is usually identified as representing[4] tire losses, however, both tire losses and mechanical drive train losses probably appear in both the a_0 and $a_1 v$ term. The $a_2 v^2$ term of (3) is identified as the aerodynamic term and the velocity in this term must be the vehicle air speed. Therefore, if any ambient wind exists, equation (3) must be written as:

$$A = [a_0 + a_1 v + a_2 (v - \mathbf{w} \cdot \mathbf{s})^2] \qquad (4)$$

where:

$\mathbf{s}$ = a unit vector in the direction of vehicle travel

$\mathbf{w}$ = the wind velocity vector.

This analysis only considers the effect of the component of wind in the direction of vehicle travel. Also equation

(4) is only valid when $(v - \mathbf{w}\cdot\mathbf{s}) \geq 0$. If the component of wind speed in the direction of vehicle travel exceeds the vehicle speed, then the aerodynamic drag term must be replaced by an aerodynamic driving term by changing the sign of a_2. The effect of cross winds can be treated in this analysis if sufficient aerodynamic information is available about the vehicle to allow the cross wind effect to be approximated as a simple polynomial expression of the vehicle speed.

A term must be added to equation (4) to describe the effects of grade. The grade term is equal to $g \operatorname{Sin}\gamma$ where γ is the angle between the test surface and the horizontal and g is the gravitational acceleration. Since γ is a very small angle the approximation of using the grade is quite accurate. Inserting this grade term, equation (4) becomes:

$$A = -[a_0 + \mathbf{g}\cdot\mathbf{s} + a_1 v + a_2(v - \mathbf{w}\cdot\mathbf{s})^2] \qquad (5)$$

where:

$\mathbf{g}$ = a vector in the track direction of increasing grade with a magnitude equal to the product of the test surface grade and the gravitational acceleration.

Expanding and regrouping, equation (5) becomes:

$$A = -\left\{ [a_0 + a_2(\mathbf{w}\cdot\mathbf{s})^2 + \mathbf{g}\cdot\mathbf{s}] + [a_1 - 2a_2(\mathbf{w}\cdot\mathbf{s})]v + a_2 v^2 \right\} \tag{6}$$

Integration and Inversion

Using $A = dv/dt$ and separating variables of equation (6):

$$-dt = \frac{dv}{[a_0 + a_2(\mathbf{w}\cdot\mathbf{s})^2 + \mathbf{g}\cdot\mathbf{s}] + [a_1 - 2a_2(\mathbf{w}\cdot\mathbf{s})]v + a_2 v^2} \tag{7}$$

Equation (7) can be integrated in closed form.

The integrals of equation (2) will depend on the relative magnitude and sign of a_0, a_1, a_2, $\mathbf{w}\cdot\mathbf{s}$ and $\mathbf{g}\cdot\mathbf{s}$. The appropriate choice for the form of the integrals is more easily seen if the variable transformation:

$$v = \frac{2u - [a_1 - 2a_2(\mathbf{w}\cdot\mathbf{s})]}{2a_2} \tag{8}$$

and

$$B^2 = \frac{4a_2[\, a_0 + a_2(\mathbf{w}\cdot\mathbf{s})^2 + (\mathbf{g}\cdot\mathbf{s})] - [a_1 - 2a_2(\mathbf{w}\cdot\mathbf{s})]^2}{4} \tag{9}$$

are applied to equation (7). This yields:

$$dt = du/(B^2 + u^2) \tag{10}$$

Because of the constraint $[v - (\mathbf{w}\cdot\mathbf{s})] \geq 0$, u will always be real and non-negative. The B is not so strongly restrained, B^2 will always be real but it may be either negative, positive or zero.

Case A; $B^2 < 0$

If B^2 is negative the transformation:

$$B^2 = -D^2 \tag{11}$$

yields:

$$dt = -du/(u^2 - D^2) \tag{12}$$

Now both u^2 and D^2 are real and positive, hence both u and D are real.

The integral of equation D-12 can take either of two forms:

$$t = \frac{1}{D}[\tanh^{-1}(u/D)] + C_1 \tag{13}$$

or

$$t = \frac{1}{D}[\coth^{-1}(u/D)] + C_2 \tag{14}$$

The terms C_1 and C_2 are constants of integration, determined by the initial conditions.

The choice of equation D-13 or D-14 to represent the coast down data must depend on the physical possibilities of the motion of the system. If equation D-13 is used, the magnitude of u cannot exceed the magnitude of D since the argument of the $\tanh^{-1}$ function is bounded between +1 and -1. But, if u is less than D, then equation D-12 shows dt/du is positive. Hence, considering the transformations 8 and 9, dv/dt is positive and the vehicle must be accelerating. Consequently equation D-14 describes a "coast-up" which can only occur if there is a grade or wind driving term of sufficient magnitude that the vehicle freely accelerates from its initial speed to some higher steady state speed.

If equation 14 is to be used to describe the coast down data, u must be greater than D since the magnitude of the argument of the coth $^{-1}$ function must exceed 1. The physical interpretation of this can be seen by re-writing equation D-12 as an expression for du/dt.

$$du/dt = -(u^2 - D^2) \qquad (15)$$

If the vehicle is initially at some velocity where $u > D$, the rate of change of u is negative hence the vehicle will decelerate until $u = 0$ or $u = D$. If the $u = D$ condition occurs, du/dt vanishes and the vehicle will remain at the velocity where $u = D$. Physically this means there must be a grade or wind effect acting as a driving term. The vehicle will decelerate until the drag forces are balanced by this driving term, and the vehicle will remain in motion at a constant speed.

Case B; $B = 0$

The case $B = 0$ is a special case that can theoretically occur, however in practice it will probably never be observed because of experimental error. It is presented since it is the transition between two classes of motion, and numerical analysis difficulties may occur in this area.

If: $B = 0$

$$dt = -du/u^2 \tag{16}$$

and:

$$t = 1/u + C_3 \tag{17}$$

where C_3 is the constant of integration to be determined by the initial conditions of the system.

Case C; $B^2 > 0$

In the case $B^2 > 0$ equation D-9 may be integrated in the form:

$$-t = (1/B)[\tan^{-1}(u/B)] + C_4 \tag{18}$$

or:

$$t = (1/B)[\cot^{-1}(u/B)] + C_5 \tag{19}$$

Again C_4 and C_5 are constants of integration.

Equations 18 and 19 are equivalent. This may be shown by inverting equation 18 to yield:

$$u = B \tan [-B(C_4 + t)] \quad (20)$$

Using the trignometric identity relating the tangent and cotangent functions, equation 20 may be written as:

$$u = B \cot (Bt + \pi/2 + BC_4) \quad (21)$$

Inverting equation 19 yields:

$$u = B \cot [B (t-C_5)] \quad (22)$$

Comparing equations 21 and 22 these equations differ only by the sign of the constants of integration, and by the phase angle of $\pi/2$. Either equation may be used to describe the vehicle coast down. However, since the $B^2 < 0$ case requires the coth $^{-1}$ solution, equation 22 will be used for the case $B^2 > 0$ because of the convenient parallel nature of the cot and coth functions.

Applying the original transformations 8 to 22 and solving for v yields:

$$v = \frac{1}{a_2}[B \cot (Bt - C_5) - a_1/2 + a_2(\mathbf{w} \cdot \mathbf{s})] \quad (23)$$

where:

$$C_5 = - \cot^{-1} [(2a_2v_0 + a_1)/(2B)]$$

The velocity v_0 is the initial vehicle speed at $t = 0$.

Applying the same operations to equation D-13 yields:

$$v = \frac{1}{a_2} [D \coth (Dt - C_2) - a_1/2 + a_2(\mathbf{w \cdot s})] \quad (24)$$

and

$$C_2 = - \coth^{-1} [(2a_2v_0 + a_1)/2D]$$

Either equation 23 or 24 must be fitted to the speed time data by the method of least squares. The values of the coefficients a_0, a_1, and a_2 will determine which expression, 23 or 24 is the appropriate form to describe the data.

The Fitting Technique

The least squares method requires the sum of squares:

$$S = \sum_i [v_i - v(t_i)]^2 \quad (25)$$

be minimized with respect to the fitted parameters. To simplify the fitting process, the change of variables:

$$\begin{aligned} b_0 &= a_0 + a_2(\mathbf{w}\cdot\mathbf{s})^2 + \mathbf{g}\cdot\mathbf{s} \\ b_1 &= a_1 - 2a_2(\mathbf{w}\cdot\mathbf{s}) \\ b_2 &= a_2 \\ b_3 &= \text{the constant of integration} \end{aligned} \tag{26}$$

is made in equations 23 and 24. The equation for the vehicle velocity may now be written as

$$v = v(t,\mathbf{b}) \tag{27}$$

where $\mathbf{b}$ designates the four parameters of the fitting process, b_0, b_1, b_2, and b_3.

Equations 23 and 24 are transcendental and intrinsically non-linear in $\mathbf{b}$; hence, an iterative approximation method must be used somewhere in the fitting process. The equations expressing the necessary conditions for a minimum of equation 25 may be constructed analytically, but then this set of equations would have to be solved numerically by some iterative approximation method. An alternate approach, which will be used, is to approximate

$v(t_i,\mathbf{b})$ by a Taylor series in $\mathbf{b}$, about some trial point, and then iterate the entire fitting process until the system converges on a value for $\mathbf{b}$.

Expanding v about the trial point $\mathbf{b}^0$; and retaining only the terms through the first derivative:

$$v(t,\mathbf{b}) = v(t,\mathbf{b}^0) + \sum_{k=0}^{3} (b_k - b_k^0) \left.\frac{\partial v(t,\mathbf{b})}{\partial b_k}\right|_{\mathbf{b}^0} \qquad (28)$$

Substituting equation 28 into equation 25

$$S(\mathbf{b}) = \sum_i [v_i - v(t_i,\mathbf{b}^0) - \sum_{k=0}^{3} (b_k - b_k^0) \left.\frac{\partial v(t_i,\mathbf{b})}{\partial b_k}\right|_{\mathbf{b}^0}]^2 \qquad (29)$$

If S is to be a minimum, the partial derivative of S with respect to each of the fitting parameter must vanish. Applying this condition to equation 29;

$$\sum_i \left\{ [v_i - v(t_i,\mathbf{b}^0) - \sum_{k=0}^{3} (b_k - b_k^0) \left.\frac{\partial v(t_i,\mathbf{b})}{\partial b_k}\right|_{\mathbf{b}^0}] \left.\frac{\partial v(t_i,\mathbf{b})}{\partial b_j}\right|_{\mathbf{b}^0} \right\} = 0 \qquad (30)$$

Equation 30 represents a system of 4 equations linear in the four unknowns $\mathbf{b}$. This system can be solved for the $\mathbf{b}$, which are then used as a new trial $\mathbf{b}$ and the process repeated until convergence is obtained.

In the analysis of vehicle coast down data, the non-linear cotangent and hyperbolic cotangent models gave rise to a system which exhibited convergence difficulties. Convergence sometimes occured slowly, requiring many iterations, each one decreasing S(**b**), the sum of squares, but the solution vector would not stabilize. Sometimes the system would oscillate, each new correction complementing the correction of the previous step, while S(**b**) both increased and decreased. Finally, divergence sometimes occurred with S(**b**) increasing with each iteration.

Several techniques have been devised to circumvent these problems. A "relaxation technique,"which amends the correction vector by halving it whenever the correction increases S(**b**) and doubling the correction vector when it reduces S(**b**) is well known[5]. Another technique[6] computes a weighted average of the correction vector found from the linearization technique and the correction vector found from the steepest descent. Reparameterization of the model is yet another technique[7] which often helps, if a convenient reparameterization can be found. Unfortunately, however, there is as yet no general convenient way to determine a priori whether a proposed reparameterization will improve the situation. Although any of these techniques may provide the additional help needed to make a balky system converge, if they fail to help, no further insight about the problem or its possible solution is provided.

The approach used in this analysis attacks one of the fundamental problems and provides insight into the fitting of non-linear models. Since the theoretical convergence of these systems can be proven, failure to converge in practice must partially arise from a deficiency in the tools used to compute the solution. Specifically, computers use only a small subset of the rational numbers, and this must be considered when any algorithm-computer combination is used to approximate the solution of a difficult mathematical problem.

In virtually all cases where convergence difficulties are encountered, the linear system of equations which must be solved at each step in the iteration is "poorly" conditioned. Simply stated, a poorly conditioned linear system is one in which propagated round-off error dominates the computed solution.

A precise definition of the condition of a system of linear equations and an explanation of how this directly affects the convergence of the system can be expressed in terms of the norms of the system vectors and matrices. The norm of a vector is defined as

$$\|\mathbf{z}\| = \sqrt[p]{\sum_i z_i^p} \qquad (31)$$

for p = 2 this norm is merely the length in the Euclidian sense, when $p = \infty$ equation 31 designates the infinity norm, which is equivalent to the magnitude of the largest component of the vector. Analogous to the norm of a vector, the norm of a matrix is defined as:

$$\|\mathbf{A}\| = \max \|\mathbf{A}\,\mathbf{x}\|$$

where (32)

$$\|\mathbf{x}\| = 1$$

This norm can be viewed as the longest image of the unit sphere under the mapping associated with A. Norms defined in this manner have the properties.

$$\|c\,\mathbf{M}\| = |c| \cdot \|\mathbf{M}\|$$

$$\|\mathbf{M} \cdot \mathbf{N}\| \leq \|\mathbf{M}\| \cdot \|\mathbf{N}\| \qquad (33)$$

$$\|\mathbf{M}+\mathbf{N}\| \leq \|\mathbf{M}\| + \|\mathbf{N}\|$$

Consider a system of linear equations represented by $\mathbf{Ax} = \mathbf{r}$ where $\mathbf{A}$ is a non-singular matrix. If the components of $\mathbf{r}$ are not known precisely, we would like to know the effect of such uncertainty on the solution vector $\mathbf{x}$. Let $\delta\mathbf{r}$ represent the uncertainty in $\mathbf{r}$. Since;

$$\mathbf{A}\,\mathbf{x} = \mathbf{r} \qquad (34)$$

then:

$$\mathbf{A}\,(\mathbf{x} + \delta\mathbf{x}) = \mathbf{r} + \delta\mathbf{r} \qquad (35)$$

and consequently:

$$\delta\mathbf{x} = \mathbf{A}^{-1}\delta\mathbf{r} \qquad (36)$$

To establish a bound of the relative errors $\delta\mathbf{x}/\mathbf{x}$, the norms of $\delta\mathbf{x}$ and $\mathbf{r}$ are constructed from equations 34, 35, 36, and the property of norms 33. After rearranging terms:

$$\frac{\|\delta\mathbf{x}\|}{\|\mathbf{x}\|} \leq \|\mathbf{A}\|\cdot\|\mathbf{A}^{-1}\|\,\frac{\|\delta\mathbf{r}\|}{\|\mathbf{r}\|} \qquad (37)$$

The quantity $\|\mathbf{A}\|\cdot\|\mathbf{A}^{-1}\|$ is defined as the condition of the matrix $\mathbf{A}$ and is designated by COND($\mathbf{A}$).

A system of equations with a large condition number for the coefficient matrix is said to be poorly conditioned.

Such poorly conditioned systems are difficult to solve. The propagation of round-off error in the many multiplications and additions involved in computing the solution vector can lead to large inaccuracies. These problems are minimized by using Gaussian elimination with pivoting and scaling [8], augmented by iterative improvement.[9]

These algorithms are capable of solving systems of linear equations to machine precision accuracy. However, equation 37 demonstrates such techniques are not sufficient to produce accurate solutions if the right hand side vector of the system is uncertain.

Considering equation 30, the effect of the system condition on the analysis of the coast down data may be directly understood by defining:

$$
\begin{aligned}
r_j &= \sum_i [v_i - v(t_i \mathbf{b}^0)] \left.\frac{\partial v(t_i \mathbf{b})}{\partial b_j}\right|_{\mathbf{b}^0} \\
x_k &= b_k - b_k^0 \\
a_{jk} &= \left.\frac{\partial v(t_i, \mathbf{b})}{\partial b_k}\right|_{\mathbf{b}^0} \left.\frac{\partial v(t_i, \mathbf{b})}{\partial b_j}\right|_{\mathbf{b}^0}
\end{aligned}
\qquad (38)
$$

the system of equations depicted by 30 may then be written in the same form as equation 34:

$$\mathbf{A}\,\mathbf{x} = \mathbf{r} \qquad (39)$$

where the components of $\mathbf{r}, \mathbf{x}$ and $\mathbf{A}$ are the r_j x_k and a_{jk} respectively. As the iterative solution proceeds, the components of $\mathbf{r}$ tend to vanish since $\mathbf{x}$ must tend to zero if the system is converging. However the individual elements which comprise the sums of each component of $\mathbf{r}$ remain about the same size. Thus the precision of each component r_j is given by the accuracy of the largest term of the sum. If the calculations are performed on a t-digit machine using a base number system N, the precision of r_j may be expressed as:

$$\delta r_j \cong N^{-t} \max_i \left| [v_i - v(t_i,\mathbf{b}^0)] \frac{\partial v(t_i,\mathbf{b})}{\partial b_j}\Big|_{\mathbf{b}^0} \right| \tag{40}$$

$$\leq N^{-t} \max_i \left| v_i - v(t_i,\mathbf{b}^0) \right| \max_i \left| \frac{\partial v(t_i \mathbf{b})}{\partial b_j}\Big|_{\mathbf{b}^0} \right|$$

Using the infinity norm notation:

$$\delta r_j \leq N^{-t} \left\| v_i - v(t_i, \mathbf{b}^0) \right\| D_j \tag{41}$$

Where D_j is the jth component of the vector $\mathbf{D}$ whose elements are:

$$D_j = \max_i \left| \frac{\partial v(t_i,\mathbf{b})}{\partial b_j}\Big|_{\mathbf{b}^0} \right| \tag{42}$$

The bound of the precision of $\mathbf{r}$ is then:

$$\|\delta\mathbf{r}\| \leq N^{-t} \left\| v_i - v(t_i,\mathbf{b}^0) \right\| \cdot \|\mathbf{D}\| \tag{43}$$

Substituting equation 43 into equation 37, and using the definition of condition, the bound of the error of the correction vector is:

$$\frac{\|\delta\mathbf{x}\|}{\|\mathbf{x}\|} \leq \frac{\text{COND }(\mathbf{A})\ N^{-t} \|v_i - v(t_i,\mathbf{b}^0)\| \cdot \|\mathbf{D}\|}{\|\mathbf{r}\|} \tag{44}$$

Since $\|\mathbf{r}\|$ approaches zero as the system converges, the error bound for the relative error in the computed corrections increases without bound. If the condition number of matrix $\mathbf{A}$ is sufficiently large the error in the corrections can exceed the magnitude of the corrections before the system converges. This explains the phenomenon of a system which makes excellent initial progress toward convergence, then suddenly slows down and ceases to make additional progress in reducing the sum of the squares of the errors.

Considering the right side of equation 41; the condition of $\mathbf{A}$ is controlled by the choice of the model and its parameterization, as is $\|\mathbf{D}\|$. The "noise" of the data is represented by $\|v_i - v(t_i, \mathbf{b})\|$ and can only be reduced by improving the data collection instrumentation. The only computational hope for improving convergence is to decrease N^{-t} by increasing the precision in computing $\mathbf{r}$.

In analyzing the vehicle coast down data obtained from a Ford station wagon, the system of equations resulted in a coefficient matrix with a condition number of the order 10^{+6}. This poor condition caused severe convergence difficulties. Since the solution has been programmed in FORTRAN on an IBM 370/168 computer the precision of $\mathbf{r}$ could be increased by using double precision variables in the computation. This immediately improved the percentage of data sets which converged, however, results were still unsatisfactory. This computer had an additional mode of arithmetic, called extended precision, which was then used to compute $\mathbf{r}$ to a precision of 28 hexadecimal digits. This resulted in a dramatic improvement, and successful convergence was obtained for almost all data sets.

Data

A 1971 Ford station wagon was used as the test vehicle. This vehicle was equipped with an incremental digital magnetic tape recorder to record vehicle speed, drive shaft torque and time, all at one second intervals. The vehicle speed signal was generated by a rotary shaft encoder in the vehicle speedometer cable. The torque transducer was a rotary transformer torque meter installed in the vehicle drive shaft. The time signal was generated by a quartz crystal clock.

The skid pad of the Transportation Research Center of Ohio, in East Liberty Ohio, USA, was used for the test track. This facility is a multilane concrete straight track with large turnaround loops at each end. Approximately 1 kilometer of this straight track has a constant grade of 0.5% and this section was used for all measurements.

The test vehicle and tires were warmed up for 30 minutes at a steady speed of 80 km/hr. Steady state speed and torque data were then collected for 1 kilometer in each direction of travel on the track at approximately 9, 13, 18, 22 and 26 m/sec. Following these measurements, speed versus time data were recorded for ten coast downs, five in each direction of the track. The initial speed of the coast down was approximately 26 m/sec., and the terminal speed was 9 m/sec. The vehicle was then weighed with all instrumentation and test personnel still on board.

Wind velocity and direction were recorded manually during the test period. A mean value for the component of wind velocity in the direction of the test track was calculated for the time the steady state torque data were collected and for the time required for the coast downs. The mean values were 0.64 m/sec. during the steady state torque measurements and 0.85 m/sec. during the coast downs.

Each coast down was analyzed by the non-linear curve fitting process to yield the four **b** coefficients of equations 26 . An estimate of the standard error of each coefficient was also calculated. Equations 26 were then used to calculate the **a** coefficients from the **b** , the mean wind velocity and test track grade. A weighted mean [10] was then calculated for a_0, a_1 and a_2 using the reciprocal of the square of the estimate of the standard error as a weighting factor. The resulting set of acceleration coefficients were then multiplied by the total vehicle mass to yield a set of coefficients for predicting the force on the vehicle by the equation:

$$f = f_0 + f_1 v + f_2 v^2 \qquad (45)$$

The force coefficients calculated by the coast down method were:

$$f_0 = 4.4 \times 10^2 \quad \text{nt}$$
$$f_1 = -9.3 \quad \text{nt/(m/sec.)}$$
$$f_2 = 9.9 \times 10^{-1} \quad \text{nt/(m/sec.)}^2$$

The data from each set of steady state torque measurements were converted to vehicle driving force measurements using the rear axle ratio of the vehicle and the measured rolling radius of the tire. A regression analysis was then used to remove any acceleration effects and to yield an improved estimate of the vehicle driving force at the mean speed of each data set. The estimate of the mean force and the mean speeds were then fitted to the equation:

$$f = f_0 + \mathbf{h}\cdot\mathbf{s} + f_1 v + f_2 (v - \mathbf{w}\cdot\mathbf{s}) \qquad (46)$$

where

$\mathbf{h}$ = a vector in the track direction of increasing grade with a magnitude equal to the product of the test track grade, the gravitational acceleration, and the mass of the vehicle

Equation 46 is analogous to equation 5 . The f coefficients describe the vehicle road load on a level track with no wind. The values of these coefficients calculated from the drive shaft torque meter data are:

$$f_0 = 3.6 \times 10^2 \text{ nt}$$
$$f_1 = -2.0 \quad \text{nt/ (m/sec.)}$$
$$f_2 = 7.5 \times 10^{-1} \text{ nt/(m/sec.)}^2$$

The force versus speed, curves using the coefficients calculated by each method are plotted in figure 1.

Conclusions

The drive shaft torque measurements indicated slightly higher calculated road load forces, than did the coast down method, although the shape of the curves are in very good agreement. The difference in the predicted road load forces near the center of the measured speed range is less than 4%. There are several reasons why the road load calculated by the steady state drive shaft torque method might be expected to be slightly higher than the road load calculated by the coast down method. For example, the energy dissipated in tires transmitting power is greater than energy dissipation in freely rolling tires. Because of fuel consumption, the measured vehicle mass used in the coast down calculation was slightly less than the true mass during each coast down. Also the vehicle mass during the drive shaft torque measurements would be greater than the vehicle mass during the coast downs. However, the standard estimate of error for the coefficients calculated by each method would indicate that the 4% difference is not statistically significant.

The coast down technique can yield road load results very similar to drive shaft torque meter measurements when the added flexibility of a linear term is included. The results cannot be expected to be identical since the deceleration technique is intrinsically a transient experiment, however good precision may be obtained with either method. Since coast down measurements can be conducted with less instrumentation of the test vehicle than drive shaft torque meter measurements, the coast down technique is particularly useful for measurements on a diverse class of vehicles.

VEHICLE ROAD LOAD
1971 FORD STATION WAGON

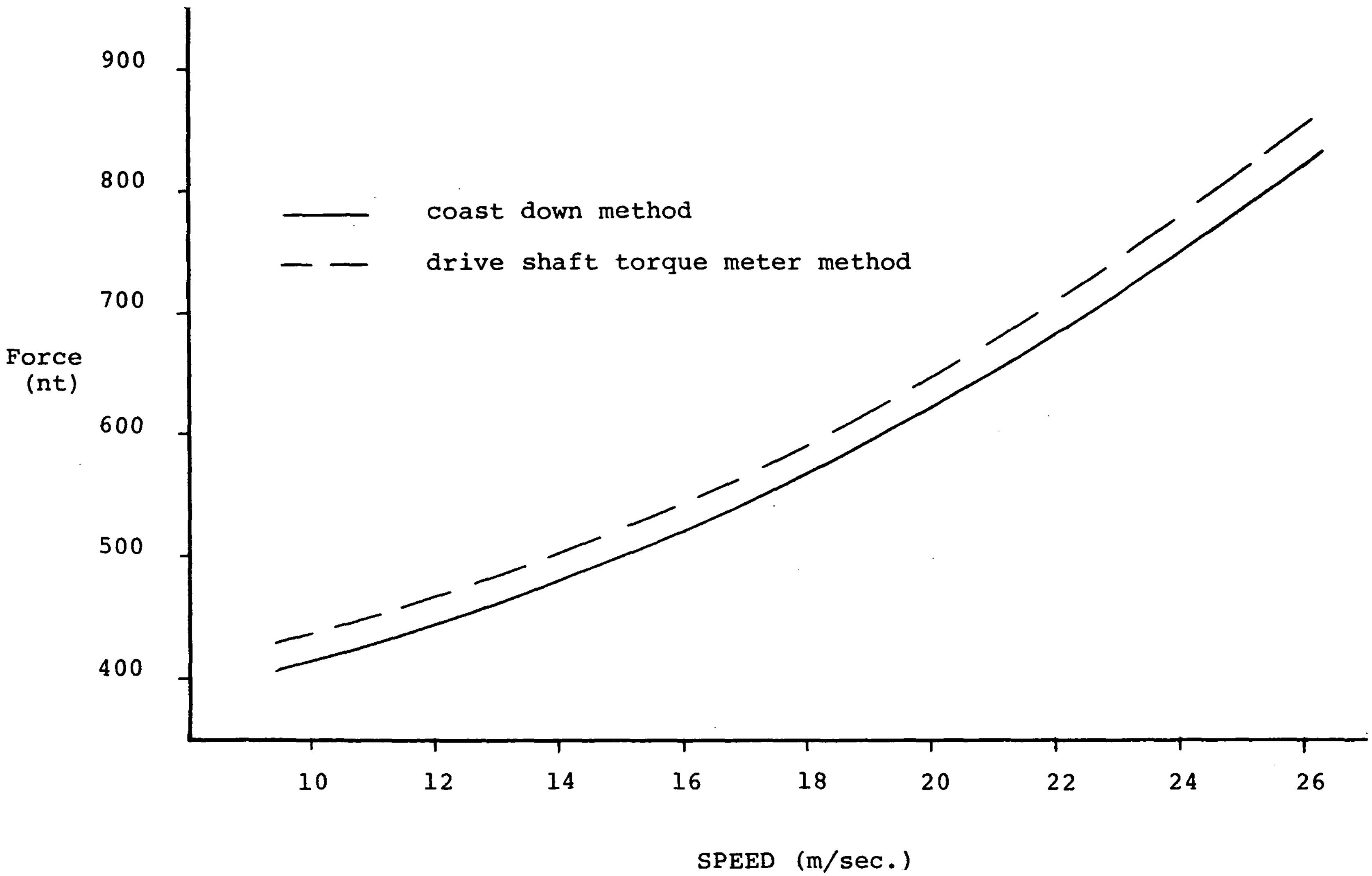

FIGURE 1

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