

Technical Report  
Tire Test Variability

by

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March, 1978

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Standards Development and Support Branch  
Emission Control Technology Division  
Office of Mobile Source Air Pollution Control  
Office of Air and Waste Management  
U.S. Environmental Protection Agency

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## Tire Test Repeatability

### I. Introduction

In order to determine the effects of the twin small-roll dynamometer on tire rolling resistance, EPA has undertaken a program to measure tire rolling resistance on both a twin small-roll dynamometer and a single large-roll dynamometer. The results generated on the single large-roll dynamometer will then be corrected to a flat surface (the road) and a correlation between the twin small-roll dynamometer and the road established with regard to tire rolling resistance. In order to estimate the accuracy of these measurements, three (3) sets of tires (2 tires are a set), each set of different construction type (radial, bias belted, and bias ply), were tested repeatedly under the same conditions (vertical load, cold tire pressure, etc.) on the single large-roll dynamometer. The results of these tests are discussed in this report.

### II. Summary and Conclusions

To estimate the test-to-test variability of our tire rolling resistance test method, a set of tires representing each construction type (radial, bias belted, bias ply) were chosen for repeat testing. All the tires selected were of the "78" series, 15 inch nominal diameter and "H" load carrying capability. Prior to testing, each tire was set to a cold inflation pressure of 26 psig which was unregulated throughout the test (trapped air method). The vertical load was fixed as the rear axle weight of the test vehicle to include a full tank of fuel. Each tire test was preceded by a warm-up cycle which consisted of the vehicle being operated according to the current Federal Test Procedure (3-bag Urban Speed-Time Driving Cycle). The vehicle was then accelerated to and maintained at 50 mph for a period of 15 minutes, during which time data were collected (all tests were conducted on the large (48" diameter) single-roll dynamometer). The power absorbed by the tire was obtained by monitoring both the power transmitted from the vehicle and the power received by the dynamometer and computing the difference. The tire rolling resistance was then determined from this differential quantity.

From the data collected, it was found that the test-test variability is approximately 9.16% for the radial tire, 7.57% for the bias belted and 10.01% for the bias ply tire. Vehicle speed fluctuations, dynamometer residual friction, and the vehicle differential power losses are identified as the primary sources of this variability. An investigation into other possible sources failed to result in any significant conclusions.

### III. Technical Discussion

#### A. Program Objectives

To determine the variability associated with the tire and tire rolling resistance test procedure currently used by EPA.

## B. Program Design

Tire rolling resistance measurements were conducted on three (3) sets of tires, each set representing a popular construction type (radial, bias belted, bias). The initial cold inflation pressure was set at 26 psig and allowed to increase during testing. Each set of tires was mounted on an instrumented vehicle and the vehicle operated first, over the Federal Test Procedure (speed-time) driving schedule as a warm-up and second, at a velocity of 50 mph for 15 minutes. It was during the latter vehicle operation that tire rolling resistance measurements were conducted. Each set of tires was tested in this manner seven (7) times. An individual set of tires was not retested until at least a period of four (4) hours [1] had elapsed to permit the tires to return to ambient temperature. All tests were conducted on the large single-roll dynamometer.

## C. Equipment

### 1. Test Vehicle

A 1971 Ford stationwagon equipped with a driveshaft torque sensor and optical speed pick-up was utilized for this program. The combination of these two sensor outputs provide a measure of the power transmitted from the vehicle to the tire. By measuring the driveshaft torque and the driveshaft speed the power to the rear axle of the vehicle may be computed as follows:

$$P_{\text{Engine}} = T_{\text{Engine}} W_E \quad 1$$

where

$P_{\text{Engine}}$  = Power generated by the engine

$T_{\text{Engine}}$  = The torque measured at the driveshaft

$W_E$  = Angular velocity of the driveshaft.

In order to determine the power at the tire, however, the amount of power lost due to differential bearing friction and to brake drag must be subtracted from the power generated by the engine,  $P_{\text{Engine}}$ . In this study brake drag was minimized by disablement of the self-adjustors and readjustment of the brakes. The following equation assumes zero brake drag.

$$P_{\text{Tire}} = P_{\text{Engine}} - P_{\text{Diff}} \quad 2$$

where

$P_{\text{Diff}}$  = Power required to revolve the rear axle and associated bearings and gearing which compose the differential.

$P_{\text{Diff}}$  was previously determined by measuring the driveshaft torque and speed while the vehicle's rear wheels were raised off the ground. These data were collected by sampling these parameters while the vehicle was

operating at velocities from 10-60 mph. The vehicle velocity was increased and then decreased in 10 mph increments between 10 mph and 60 mph. Thirty seconds of data were collected at each velocity increment. A regression line defining  $P_{Diff}$  as a function of driveshaft speed was then computed.

#### 2.2.3. Dynamometer

A large (48" diameter) single-roll LABECO dynamometer was utilized for this experiment. The dynamometer roll was equipped with a magnetic proximity detector so that roll revolutions (roll RPM) could be monitored. In addition, the torque load cell sensor output was recorded simultaneously. The product of these two variables is the power received by the dynamometer through the tire. However, since the dynamometer is an electro-mechanical device, a certain amount of power is consumed by bearing friction. Therefore, the power at the tire/roll interface is as follows:

$$P_R = P_{LC} + P_{BL} \quad 3$$

where

$P_{LC}$  = Power measured as a function of the torque load cell sensor

$P_{BL}$  = Power consumed by bearing friction

The power consumed by bearing friction was determined by dynamometer coastdown on a daily basis after a 30 minute warm-up period at 50 mph.

Prior to this program, a study was conducted to determine the test-test variability of the dynamometer residual friction. After a warm-up period of 30 minutes at 50 mph, five (5) successive coastdowns of the dynamometer were performed. The dynamometer roll speed, load cell torque and real time were recorded during each coastdown.

#### D. Tires

Three (3) sets of tires (2 tires constitute a set) of various manufacture were chosen for this program. All tires were of the same size (15" nominal diameter), series ("78") and load carrying capacity ("H"). Each set of tires represented one of the common construction types (radial, bias belted, bias) and had at least 400 miles of tread wear.

#### E. Data Collection

During the 15 minute test period, data were collected and recorded at a frequency of once per second. Vehicle and dynamometer-roll speeds, vehicle and dynamometer torques, real time, test code, tire manufacturer code and a size code were recorded on 7-track magnetic tape utilizing a Kennedy 7-track tape transport and a Datum digital data acquisition system. Only those data recorded while the vehicle was operating at a velocity of 50 mph were utilized for this program (approximately 900 data points per test).

F. Analysis

The power absorbed by the tire was computed for each data point (each second) recorded during testing. This quantity was calculated according to the following equation:

$$P_{AT} = P_{Engine} - P_{Diff} - P_{LC} - P_{BL} \quad 4$$

where

$P_{AT}$  = Power absorbed by the tire

$P_{Engine}$  and  $P_{Diff}$  are as defined in 2,

and  $P_{LC}$  and  $P_{BL}$  are as defined in 3.

The force required to roll the tire was then derived from  $P_{AT}$ . In actuality,  $P_{AT}$  is the product of the torque at the tire/roll interface and the angular velocity of the tire.

$$P_{AT} = T_T W_T \quad 5$$

The torque at the tire/roll interface is defined as the product of the tire rolling force,  $F_R$  and the rolling radius of the tire,  $r$ ,

$$T_T = F_R \times r \quad 6$$

By substituting for  $T_T$  in equation 5, the following equation results;

$$P_{AT} = (F_R \times r) W_T \quad 7$$

Since the angular velocity of a rotating body is the ratio of the body's linear velocity to its radius of rotation,  $P_{AT}$  can be expressed as a function of the linear velocity of the tire,

$$P_{AT} = \frac{(F_R \times r) V_T}{r} = F_R V_T \quad 8$$

where  $V_T$  is the linear velocity of the tire and  $F_R$  and  $r$  are defined in 6, above. The linear velocity,  $V_T$ , is in actuality the ground or test surface velocity. Therefore, either the test vehicle speed or the dynamometer roll speed can be utilized for the determination of  $F_R$ . For this experiment the vehicle speed was utilized for this computation.

For each test, the following mean values were computed; vehicle speed,  $V_T$ , power absorbed by the tire,  $P_{AT}$ , rolling force,  $F_R$ , power absorbed by the dynamometer,  $P_{LC}$ , and the power out of the vehicle engine,  $P_{engine}$ . Each test mean value was then considered to represent one test point. For each tire type, a weighted mean,  $\bar{x}_p$ , and pooled standard deviation,  $s_p$ , were calculated from the test point mean values and then

a coefficient of variability, in percent, was derived. The  $\bar{x}_p$  and  $s_p$  values were computed according to the following equations:

$$\bar{x}_p = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + \dots + n_i \bar{x}_i}{N_T} \quad 9$$

$$s_p = \left( \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \dots + (n_i - 1)s_i^2}{n_1 + n_2 + \dots + n_i - i} \right)^{1/2} \quad 10$$

where;

$i$  = number of tests

$n_i$  = the number of observations in the  $i^{\text{th}}$  test

$N_T$  = total number of observations

$s_i^2$  = the variance of the  $i^{\text{th}}$  test

$\bar{x}_i$  = the mean value of the  $i^{\text{th}}$  test

The coefficient of variability was calculated as follows:

$$\text{coefficient of variability} = \frac{s_p}{\bar{x}_p} \times 100\% \quad 11$$

#### G. Test Procedure

Prior to testing each day, the vehicle and dynamometer were warmed-up for 30 minutes at a steady state 50 mph. The vehicle tires used for warm-up were then removed and a pair of test tires installed. A cold tire pressure of 26 psig was set upon installation of the test tires. The vehicle was then driven in accordance with the current Federal Urban driving (3-bag speed-time) schedule used for vehicle certification. Upon completion of this schedule the vehicle was then accelerated to and then maintained at 50 mph for 15 minutes. The performance of the Federal Driving Schedule was considered to be the tire warm-up period. At the completion of each test, the vehicle fuel tank was refilled and a different set of test tires were installed and the above process repeated. Once tested, a given set of tires was not retested unless a minimum of four (4) hours [1] had elapsed. A minimum of six (6) tests per set of tires were conducted in the above manner.

#### IV. Results

The values for rolling force,  $F_R$ , for all the tests conducted were analyzed to determine the variability of the current tire test procedure. The coefficient of variability was computed by tire type according to equation 11. These results are presented in Table 1.

Table 1

Coefficients of Variability for  
Tire Rolling Force,  $F_R$ , by Tire Type

<u>Tire Type</u>	<u>Weighted Mean <math>F_R</math> * (Newtons)</u>	<u>Pooled Standard Deviation</u>	<u>Coefficient of Variability (%)</u>
Radial	117.09	10.72	9.16
Bias Belted	167.78	12.70	7.57
Bias	167.97	16.81	10.01

\* Rolling force mean values for two (2) tires.

An Analysis of variance (ANOVA) was performed on the above data to determine if any significant differences could be detected with respect to tire type. The results of this analysis indicated that the mean rolling force displayed by the radial tire tested was significantly different from that displayed for the bias belted and bias ply tires. However, no significant difference was detected between bias belted and bias ply tire rolling force means. This result is not unusual and is due to the small sample and the magnitude of the variability displayed. Possible sources of this variability are instrumentation calibration errors, dynamometer repeatability, the frictional losses within the vehicle differential, and vehicle speed fluctuation. Each item is discussed separately below.

A. Instrumentation Calibration

Prior to the commencement of the program, the driveshaft torque transducer, dynamometer load cell and the vehicle and dynamometer-roll speed sensors were checked to establish linearity. It was found that all the sensors, including associated electronics, conformed with the precision specified by their respective manufacturers. Set points for each sensor were determined at that time and were checked during the calibration procedure. Since each set point calibration was recorded on magnetic tape, any errors which may have occurred during the procedure would have been detected during data review. It is assumed that the variability contributed by the instrumentation and associated sensors is no greater than the basic design precision and is negligible.

B. Dynamometer

A study was conducted to determine the test-test variability of the dynamometer residual friction. After a warm-up period of 30 minutes at 50 mph, five (5) successive coastdowns of the dynamometer were performed. The dynamometer roll speed, load cell torque and real time were recorded during each coastdown.

The bearing losses,  $P_{BL}$ , was computed for each coastdown (see Appendix B for a derivation of  $P_{BL}$ ). All  $P_{BL}$  and roll speed data were then combined and a regression analysis  $P_{BL}$  was performed. The value of  $P_{BL}$

at 50 mph was then calculated from the regression equation and a coefficient of variability computed as follows:

$$\text{coefficient of variability} = \frac{\text{Regression Standard Error}}{\text{the value of } P_{BL} \text{ at 50 mph}} \times 100\%$$

The result was a variability of 8.64% with a value of 367.003 watts (.49 Hp) for  $P_{BL}$  at 50 mph. Given this variability, the effect on  $F_R$  variability may be estimated. Those estimates are presented in Table 2 below.

Table 2

Effects of  $P_{BL}$  Variability on  
The Tire Rolling Force,  $F_R$

<u>Tire Type</u>	<u>Increase in <math>F_R</math> Variability</u>
Radial	1.21%
Bias Belted	0.85%
Bias	0.85%

C. Differential Losses

Initial data analysis indicated that the variability of the differential losses was higher than one would desire for an engineering experiment. To obtain a better estimate of these losses, the experiment explained in Section III.C.1 was repeated. Since the steady state speed utilized for this program was 50 mph, data were generated at 45 mph, 50 mph and 55 mph only.

A regression analysis was performed on all the  $P_{DIFF}$  data generated and a coefficient of variability was then computed using the same method as in Section IV.B. above. It was determined that  $P_{DIFF}$  had a variability of 5.38% with a value at 50 mph of 1184.52 watts (1.65 Hp). Estimates of the effect of this variability on  $F_R$  are as presented in Table 3.

Table 3

Effects of  $P_{DIFF}$  on the Tire Rolling Force,  $F_R$

<u>Tire Type</u>	<u>Increase in <math>F_R</math> Variability</u>
Radial	2.43%
Bias Belted	1.70%
Bias	1.70%

D. Vehicle Speed

Analysis of the vehicle speed data for each tire type indicated that the variability of this parameter was small. However, even small fluctuations produce large effects on  $F_R$  variability. From the "Analysis" portion of this report, it can be seen that the vehicle speed influences each variable comprising the computation of  $F_R$ . Statistically, it is almost impossible to estimate the exact magnitude of the effect on  $F_R$  of this variability. However, the coefficients of variability for the major parameters affected can be calculated using equation 11 and are presented below in Table 4. Also included is the coefficient of variability for the vehicle speed itself.

Table 4

Coefficients of Variability for Major Parameters in the Tire Rolling Force Computations

<u>Tire Type</u>	<u>Percent Variability</u>		
	<u>Vehicle Speed</u>	<u>P<sub>Engine</sub></u>	<u>P<sub>LC</sub></u>
Radial	0.92	4.43	2.70
Bias Belted	1.02	4.44	3.27
Bias	0.83	5.35	2.23

If all parameters except vehicle speed were assumed to remain constant, an estimate of the effect on  $F_R$  variability of the vehicle speed variability can be computed. These estimates are presented in Table 5.

Table 5

Effect of Vehicle Speed Variations on Tire Rolling Force

<u>Tire Type</u>	<u>Percent Increase in <math>F_R</math> Variability</u>
Radial	1.43
Bias Belted	1.41
Bias	1.17

The above estimates are based on the vehicle speed variabilities presented in Table 4. It must be noted, however, that the displayed variations in speed also affect the torque measurement variability due to the inertia of the system, i.e., torque spiking occurs on accelerations and decelerations. The above estimates also ignore the additional variability in the dynamometer portion of the tire rolling force computations which would also be affected by the vehicle speed fluctuations.

It is generally accepted that a given tire is repeatable to less than 1 percent and is a function of the accuracy of the instrumentation used to measure the force. By summing the variabilities identified thus far and subtracting these totals from the variabilities presented in Table 1, an estimate of the remaining unidentified variability can be determined. These results are shown in Table 6.

Table 6

## Unidentified Variability

<u>Tire Type</u>	<u>Percent Variability</u>
Radial	3.09%
Bias Belted	2.61%
Bias	5.29%

It is believed that if the variability associated with the interactive effects of vehicle speed variations on the vehicle-dynamometer power relationships could be quantified, the remainder of the test variability shown in Table 6 can be explained.

E. Other Parameters Investigated

In an effort to identify other possible sources of test variability, many of the parameters recorded during testing were analyzed for possible correlation with tire rolling force. The following is a list of the parameters investigated.

1. Vehicle Differential Temperature
2. Wet Bulb Temperature
3. Dry Bulb Temperature
4. Barometric Pressure
5. Test Date
6. Test Time-of-Day
7. Tire Test Sequence
8. Test Driver

The results of these analyses indicates that the above parameters investigated were not correlated with tire rolling force.

V. Conclusions/Recommendations

The above analyses indicates that the test-test variability of the present tire test procedure is larger than one might desire for an engineering experiment. The method of determining the tire rolling force via the subtraction of two large quantities (the power transmitted by the vehicle and the power received by the dynamometer) to obtain a relatively small quantity tends to create variable results. Small

deviations in the large quantities produce large deviations in the difference. The variability of the results is compounded further by uncontrollable parameters such as the losses in the vehicle differential and the bearing friction in the dynamometer.

In order to reduce the variability of future results several improvements to the current test procedure can be made. Since a driver is only capable of controlling the vehicle speed for a short period of time, the sample period of each tire test can be shortened to 30-60 seconds, instead of 15 minutes. An alternative to this method is to install an automatic speed controller with drivetrain feedback. This would permit a longer sampling period and reduce the variability due to speed fluctuations. The speed controller would also permit better estimates of the differential losses, again reducing the test variability. A moderately expensive method of variability reduction would be to eliminate the vehicle altogether. This could be accomplished by modifying the existing dynamometer. A shaft torque sensor could be installed between the D.C. motor and the rolls, in addition to the installation of low friction bearings. A test stand could then be constructed and used to support the test tire and vary vertical loading. The dynamometer could then be used to control the tire test according to whatever type of speed cycle is chosen. The ultimate method of reducing the test variability is to construct a tire test machine specifically designed to reduce frictional forces and to accurately control test speed. This machine would be equipped with precision sensors so that the tire rolling force could be measured as accurately as possible.

References

- [1] D.A. Glemming and P.A. Bowers, "Tire Testing for Rolling Resistance and Fuel Economy", Tire Science and Technology, TSTCA, Vol. 2, No. 4 November 1974, pp. 286-311.
- [2] D.J. Schuring, "The Energy Loss of Tires on Twin Rolls, Drum, and Flat Roadway - A Uniform Approach", SAE Paper 770875, Society of Automotive Engineers, New York, September 1977.

APPENDIX A

Tire Descriptions and Test Data

Table A-1

Tire Description

<u>Tire ID</u>	<u>Manufacturer</u>	<u>Size</u>	<u>Model/Type</u>
090	Goodyear	HR 78X15	Custom Polysteel Radial
13A	B.F. Goodrich	H 78X15	Custom Long Miler
15A	B.F. Goodrich	H 78X15	Silvertown Belted

Table A-2

Ambient Tire Test Data

<u>Tire ID</u>	<u>Diff. Temp.</u>	<u>Dry Bulb</u>	<u>Wet Bulb</u>	<u>Baro. Press.</u>	<u>Test Time</u>
090	139.80	73.5	63.5	29.10	1040
090	138.30	72.0	61.5	29.05	1310
090	135.10	73.0	64.0	29.11	930
090	139.15	72.5	63.5	29.02	1035
090	136.10	73.5	64.5	29.08	910
090	135.70	73.5	63.5	29.02	905
13A	140.75	74.0	65.0	29.05	1015
13A	139.15	73.0	62.0	29.10	1035
13A	139.45	72.5	61.0	29.11	1440
13A	136.95	75.5	65.0	29.11	1440
13A	137.55	73.5	63.0	29.06	1435
13A	135.80	74.0	64.0	29.10	925
13A	139.95	72.0	60.0	29.02	1440
15A	140.40	72.0	60.5	29.00	1500
15A	138.55	72.5	63.5	29.10	1040
15A	134.95	71.0	61.7	29.06	1430
15A		74.0	64.0	29.10	900
15A	138.25	73.5	62.5	29.10	1020
15A	138.40	72.5	63.5	29.00	920

Table A-3

Tire Test Data

Tire ID	N	$P_{\text{Engine}}$		$P_{\text{LC}}$		$P_{\text{DIFF}}$		$F_{\text{R}}$		Vehicle Speed	
		$\bar{x}$	$\sigma$	$\bar{x}$	$\sigma$	$\bar{x}$	$\sigma$	$\bar{x}$	$\sigma$	$\bar{x}$	$\sigma$
090	891	5422.186	219.745	1433.869	31.372	1072.485	7.369	113.149	9.706	50.350	.233
090	940	5431.267	267.323	1341.797	27.328	1036.208	8.241	121.210	12.533	49.193	.265
090	859	5501.703	255.473	1389.574	65.839	1066.361	30.823	116.343	9.272	50.105	.992
090	893	5524.109	215.412	1434.324	25.976	1069.518	5.767	115.847	9.344	50.256	.182
090	942	5545.504	157.695	1447.653	33.416	1062.383	8.737	116.682	7.216	50.030	.278
090	923	5432.466	310.642	1388.097	31.090	1068.828	9.895	119.014	14.493	50.234	.314
13A	917	6496.567	356.475	1398.335	31.282	1068.359	16.695	161.598	13.521	50.218	.534
13A	936	6733.672	534.912	1375.443	26.811	1070.710	7.601	177.212	24.208	50.294	.241
13A	914	6713.409	444.015	1467.492	41.738	1090.410	13.603	165.567	22.536	50.914	.431
13A	913	6384.386	217.222	1487.961	23.448	1076.499	17.944	148.863	10.322	50.475	.578
13A	922	6922.609	196.461	1464.225	24.731	1081.616	6.158	175.236	9.730	50.638	.194
13A	901	7032.563	331.529	1437.751	25.162	1070.353	6.561	184.850	14.791	50.283	.208
13A	928	6548.309	295.495	1456.060	44.696	1067.095	16.431	162.118	16.570	50.178	.522
15A	870	6569.348	252.440	1367.694	28.580	1068.868	7.700	170.425	11.151	50.235	.244
15A	887	6716.341	236.653	1365.728	24.890	1046.294	14.133	175.116	11.057	49.516	.460
15A	908	6551.850	208.066	1405.039	32.903	1076.227	10.449	166.492	10.849	50.468	.330
15A	927	6359.213	352.204	1399.000	26.208	1068.616	8.727	155.416	16.306	50.227	.277
15A	908	6724.486	434.141	1470.095	93.108	1070.293	32.104	170.301	15.413	50.274	1.017
15A	877	6594.408	183.688	1389.559	24.443	1056.749	6.853	169.530	9.603	49.851	.218

APPENDIX B

Dynamometer Bearing Loss Computations

Dynamometer Bearing Loss Computations

The power losses within the dynamometer due to bearing friction,  $P_{BL}$ , is the difference between the power at the dynamometer roll surface and that measured by the dynamometer load cell. This may be expressed as a function of torque as follows:

$$P_{BL} = (T_R - T_{LC})W_R \quad B-1$$

where

$T_R$  = the torque at the tire/roll interface

$T_{LC}$  = the torque measured by the dynamometer load cell

and  $W_R$  = the angular velocity of the dynamometer roll.

During a coastdown,  $T_R$  can be computed by knowing the amount of inertia,  $I$ , that is rotating and the time it takes this inertia to traverse from one velocity to another.

$$T_R = I\alpha = I \frac{dW_R}{dt} \approx I \frac{\Delta W_R}{\Delta t} \quad B-2$$

where  $\alpha$  = the angular acceleration of the inertia.

For this program, a  $\Delta t$  of 30 seconds was utilized. The velocity of the dynamometer roll at the center of this 30 second interval was then defined as  $W_R$  and substituted into equation B-1. The value of  $T_{LC}$  at the appropriate  $W_R$  was then subtracted from the computed  $T_R$  and a value for  $P_{BL}$  at  $W_R$  results from equation B-1. Each  $P_{BL}$  value was then regressed against the appropriate  $W_R$  and a defining equation derived.