

Technical Report

February 1979

Computer Simulation of Tire Slip On  
A Clayton Twin Roll Dynamometer

by

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Office of Air and Waste Management  
U.S. Environmental Protection Agency

Abstract

Due to the occurrence of tire slip on a Clayton twin roll dynamometer, there is a difference between the velocities of the front and rear rolls of the dynamometer. This slip can be modeled by the method described in the following report. The results of this theoretical modeling show that, over the LA-4 and the HWFET driving schedules the velocity of the rear roll (which is currently used to determine the vehicle speed) exceeds the velocity of the front roll (which determines the power absorbed) by an average of approximately 1%. From this difference in velocities of the two rolls, a computation of the total energy effect, over transient driving cycles, can be obtained. Approximately a 2% to 5% increase in total energy dissipated over the city (LA-4) or highway driving schedules is predicted from the use of the front roll velocity to determine the vehicle speed, as compared to the currently used velocity of the rear roll.

## I. Introduction

When a vehicle is operated on a twin roll dynamometer the velocities of the front and rear rolls differ. This difference, or tire slip, is important because the tractive load imposed on the vehicle is dependent on the velocity and acceleration of the front roll, while the velocity of the rear roll is used to determine the vehicle speed. This report discusses a theoretical model of this tire slip phenomenon and investigates its effect on the EPA measurements of fuel economy and emissions.

## II. Discussion

Historically, the term tire slip has been used to describe several physical phenomena and hence is somewhat ambiguous. For the purpose of this report, which only considers the twin roll dynamometer, slip is defined as the difference between the velocities of the front and rear rolls of the dynamometer.

That is:

$$\text{Slip} = V - U \quad (1)$$

V = velocity of the rear roll

U = velocity of the front roll

This slip has been observed in an EPA tire test program which showed that at steady-state 50 MPH, the front roll consistently travels at a slower speed than the rear roll (1). This was confirmed for several different tires at five different power absorber settings.

This phenomenon is significant, since the tractive load imposed on the vehicle is dependent on the velocity and acceleration of the front dynamometer roll, however, the vehicle is driven over a speed versus time schedule based on the velocity of the rear roll. Consequently, the difference in the roll velocities may cause the vehicle to be underloaded with respect to the loading which would be imposed if the same speed schedule were followed on the road. Under steady state conditions, tests have shown that this under loading may be more than 1/3 HP (1). (FY78 program plans include an investigation into the relationship between the occurrence of slip on the twin roll dynamometer and the actual road experience of the vehicle.)

It is also important to consider the effect of tire slip under transient conditions. Since it is difficult to monitor slip during a transient cycle, a theoretical model was proposed as the most appropriate method of investigation. The subsequent subsections of the discussion consider the development of the tire slip model, a

comparison of some empirical slip results with the predicted results, a computation of the energy effects, and an estimate of the effect on fuel economy and emissions testing.

#### A. Theoretical Model

In order to theoretically model the tire slip that occurs on a twin roll dynamometer, a model equation which couples the slip to a dynamometer or cycle parameter must be chosen. Steady-state tests show that the difference between front and rear roll velocities increased as the dynamometer power setting increased. Consequently, it is assumed that tire slip is directly proportional to the force across the tire roll interface.

$$\text{Slip} = sF \quad (2)$$

or

$$V - U = sF \quad (3)$$

where

s = coefficient of slip

F = force at the tire/roll interface

The basic model equation 2, assumes that tire/roll slip is caused by a tire deformation resulting from the tangential forces that exist at the tire/front-roll interface.

Slip, during transient driving, can be calculated from the model equation (3) if the force across the interface is known. The force at the tire/front roll interface includes the dynamometer force and the inertial force.

$$F = \text{Dyno Force} + \text{Inertia Force} \quad (4)$$

The dynamometer force is assumed to be proportional to the square of the vehicle velocity, that is:

$$\text{Dyno Force} = b u^2 \quad (5)$$

where

b = proportionality constant dependent on the total dynamometer absorbed power.

This neglects the constant characteristic of the dynamometer bearing force, however, this is not critical since the velocities squared characteristics of the power absorber dominate over most of the speed regions. Hence minor variations in the assumed dyna-

momenter curve shape should not significantly affect the accuracy of the model results.

The inertia force is the mass effect of the vehicle.

$$\begin{aligned} \text{inertia force} &= m a \\ &= m (du/dT) \end{aligned} \quad (6)$$

where:

$m$  = dynamometer simulated inertia of the vehicle.

$a$  =  $du/dt$  = acceleration

Substituting equations 5 and 6 into equation 4.

$$F = b u^2 + m (du/dT) \quad (7)$$

using the model equation 3 this becomes:

$$v - u = s(b u^2 + m (du/dT)) \quad (8)$$

A linear approximation is used to calculate  $du/dt$  for each one second interval of the given speed versus time schedule. That is;

$$du/dt = u_2 - u_1 \quad (9)$$

where:

$T = 1$  second.

This acceleration is assumed to be the acceleration at the midpoint of the time interval. Therefore the midpoint velocities are used in the model equation:

$$v = (v_2 + v_1)/2 \quad (10)$$

$$u = (u_2 + u_1)/2 \quad (11)$$

Substituting 5, 6 and 7 equation 8 becomes:

$$(v_2 + v_1)/2 - (u_2 + u_1)/2 = s(b((u_2 + u_1)/2)^2 + m(u_2 - u_1)) \quad (12)$$

Expanding and regrouping terms in order to use the quadratic formula:

$$(sb/2) u_2^2 + (sbu_1 + 2 sm + 1)u_2 + (-(v_2 + v_1) + (sbu_1/2 + 2 sm + 1)u_1) \quad (13)$$

Using the quadratic formula to solve for  $u_2$

$$u_2 = (-B + (B^2 - r AC)^{1/2})/2A \quad (14)$$

Where:

$$A = sb/2 \quad (15)$$

$$B = sbu_1 + 2 sm + 1 \quad (16)$$

$$C = (sbu_1/2 - 2 sm + 1)u_1 - (v_1 + v_2) \quad (17)$$

In order to utilize this model for computer simulation  $u_1$  and  $v_1$  are initialized to zero, since the cycle begins with an idle.  $v_2$  is entered as the next speed point from the given driving schedule (see Appendix A for computer program "slip"). Each time a new  $u_2$  is calculated,  $v_2$  and  $u_2$  are replaced by  $v_1$  and  $u_1$  respectively and the next  $v_2$  is entered. This iterative process is continued for the entire driving schedule.

#### B. Theoretical vs. Experimental Results

The model was used to compute the dynamometer slip, using input parameters estimated to reflect the vehicle parameters of a 1978 Mercury Zephyr. The Zephyr was chosen since this vehicle was used in a recent test program on a Clayton dynamometer in which the difference between the front and rear roll velocities was recorded over an LA-4 driving cycle. Therefore, this allows a direct comparison between the theoretical tire slip model and experimentally measured dynamometer slip.

A mean coefficient of slip was calculated from the data acquired from steady-state tire tests (1).

$$s = 0.0008 \text{ sec/kg}$$

This program also requires the vehicle frontal area as an input parameter to calculate the appropriate force coefficient. In this case, the actual dynamometer power absorber setting used to test the Mercury Zephyr, was used to back calculate for the appropriate frontal area, according to the equation given in the Federal Register for non-fastback vehicles (2).

$$\text{HP at 50 mph} = 0.50 (\text{Frontal Area}) \quad (18)$$

Where:

$$\text{Actual Dynamometer HP at 50 mph} = 9.7 \text{ HP}$$

therefore;

Frontal Area = 19.4 square feet.

The dynamometer power at 50 mph is then used to calculate the proportionality constant (b) using equation 5.

$$\text{Dyno Force} = bv^2$$

$$\text{Dyno Power} = bv^3$$

therefore

$$b = (\text{Dyno power @ 50 mph}) / (50 \text{ mph})^3$$

$$50 \text{ mph} = 22.35 \text{ m/s}$$

$$745.7 \text{ watts} = 1 \text{ hp}$$

$$b = 0.648 \text{ kg/m}$$

The input parameter for the vehicle inertia (m) was the same as that used in the dynamometer tests with the Mercury Zephyr.

$$m = 1600 \text{ kg}$$

With these parameters the theoretical slip was then computed for the entire LA-4 and Highway Driving cycles.

The results of the tire slip modeling showed that the rear roll travelled, on the average, 0.09% faster than the front roll over the LA-4 driving schedule. Over the highway cycle the rear roll averaged 1.4% greater speed than the front roll.

These results can be compared to the results of a test conducted on a Clayton twin roll dynamometer with the Mercury Zephyr (Figure 1 shows both the actual and theoretical slip for the first 130 seconds of the LA-4 cycle).

There is a very close comparison between the theoretically modeled slip and the experimentally measured slip. The regions of positive slip correspond to accelerations or steady-state modes in the driving schedule, while the regions of negative slip correspond to decelerations. A minor discrepancy in the magnitudes of the modeled slip and the actual slip is mainly due to the use of a mean value for the slip coefficient, which appears to be slightly low for this particular vehicle. There are a few minor discrepancies in peak locations which are probably caused by transmission shift points. Also, a very slight shift in the data is observed which may be due to a slight miscalibration of the chart recorder.

Since the purpose of this report is to estimate the total fuel economy and emissions effects, these slight inaccuracies are not a major concern. During rapid decelerations an actual mechanical

slippage has also been observed. This phenomenon may result in a significant discrepancy when comparing actual slip to theoretical slip. However, since mechanical slip (actual sliding of the roll surface across the tire surface) primarily occurs during closed throttle modes it will not have a significant effect on fuel economy results. Thus the theoretical model may be used to accurately predict the effect of slip on EPA fuel economy and emissions testing.

### C. Computation of Energy Effects

In order to estimate the energy effect of the tire slip over a transient cycle, a second computer program is developed. This model utilizes the total road load equation of the vehicle on the dynamometer (see appendix B for program "LA4FORCE").

$$f = rr + b v^2 + m(dv/dT) \quad (19)$$

Where:

$rr$  = rolling resistance (tire force).

$b v^2$  = dynamometer force.

$m(dv/dT)$  = inertial force

The input velocity ( $v$ ) can be either the front roll velocity or the rear roll velocity, for any given driving cycle.

The power dissipated ( $P$ ) is then calculated

$$P = F * V \quad (20)$$

Finally the power is summed each second over the entire driving schedule to yield the total energy dissipated. However, if the power is negative the vehicle is assumed to be braking and this is summed as a separate quantity referred to as "Brake Energy".

This program may then be used to calculate the total energy that is assumed to be dissipated, over a driving cycle, using the rear roll velocity. Simultaneously, it may be used to calculate the actual energy that the vehicle has dissipated, by inputting the front roll velocity, that has been calculated by the tire slip model, for the same cycle.

This program was utilized in conjunction with the tire slip program in order to calculate the energy effects of this tire slip phenomenon. Once again, the input parameters were those of the Mercury Zephyr. In addition, a rolling resistance coefficient is



required in order to calculate the tire energy dissipated. A mean rolling resistance coefficient was used, typical of that particular tire type and size on the Mercury Zephyr(3).

$$rr_{\text{coef}} = 0.010$$

The total energy dissipated over the LA-4 cycle was calculated using the velocity of the rear roll, and then the computation was repeated using the velocity of the front roll (as computed from the tire slip program). The results showed that 4.3% more energy was calculated using the velocity of the rear roll as compared to that calculated from velocity of the front roll (see Table 1). The same procedure was repeated using the highway driving cycle and the results showed a 4.1% greater total energy absorbed by the dynamometer was calculated using the velocity of the rear roll as compared to using that of the front roll.

### III. Conclusions

There is a difference in the velocities of the front and rear rolls of a Clayton twin roll dynamometer due to tire slip. This slip is directly proportional to the tangential force at the front roll/tire interface. It accounts for approximately a 1% greater velocity of rear roll than the front roll, during transient driving cycles. This difference in velocities can be modeled approximately by the method as described in this report. A computation of the total energy dissipated, by a vehicle over an LA-4 or HWFET driving schedule indicates that approximately a 2% to 5% increase in energy will result from using the front roll velocity to determine vehicle speed rather than the currently used rear roll velocity.

### IV. Recommendation

It is important to determine if this occurrence of slip on the twin roll has a significant effect on fuel economy or emissions testing. It is most important to compare this phenomenon to the actual road experience of the vehicle. If such studies show that slip is an undesirable factor during vehicle testing, some possible alternatives are: to couple the front and rear rolls of the dynamometer; use the front roll to determine vehicle speed, or look to the possible use of flatbed dynamometers.

References

1. Richard Burgeson, Myriam Torres, "Tire Slip on the Clayton Dynamometer," EPA Technical Support Report, LDTP 78-02, March 1978.
2. "Control of Air Pollution from New Motor Vehicle Engines," Federal Register, Vol. 42, No. 176, September 12, 1977.
3. Richard N. Burgeson, "Clayton Dynamometer-to-Road Tire Rolling Resistance Relationship," Technical Report, LDTP 78-09.
4. Glenn D. Thompson, "Investigation of the Requested Alternate Dynamometer Power Absorption for the Ford Mercury Marquis," EPA Technical Report, LDTP 78-06.

Figure 1

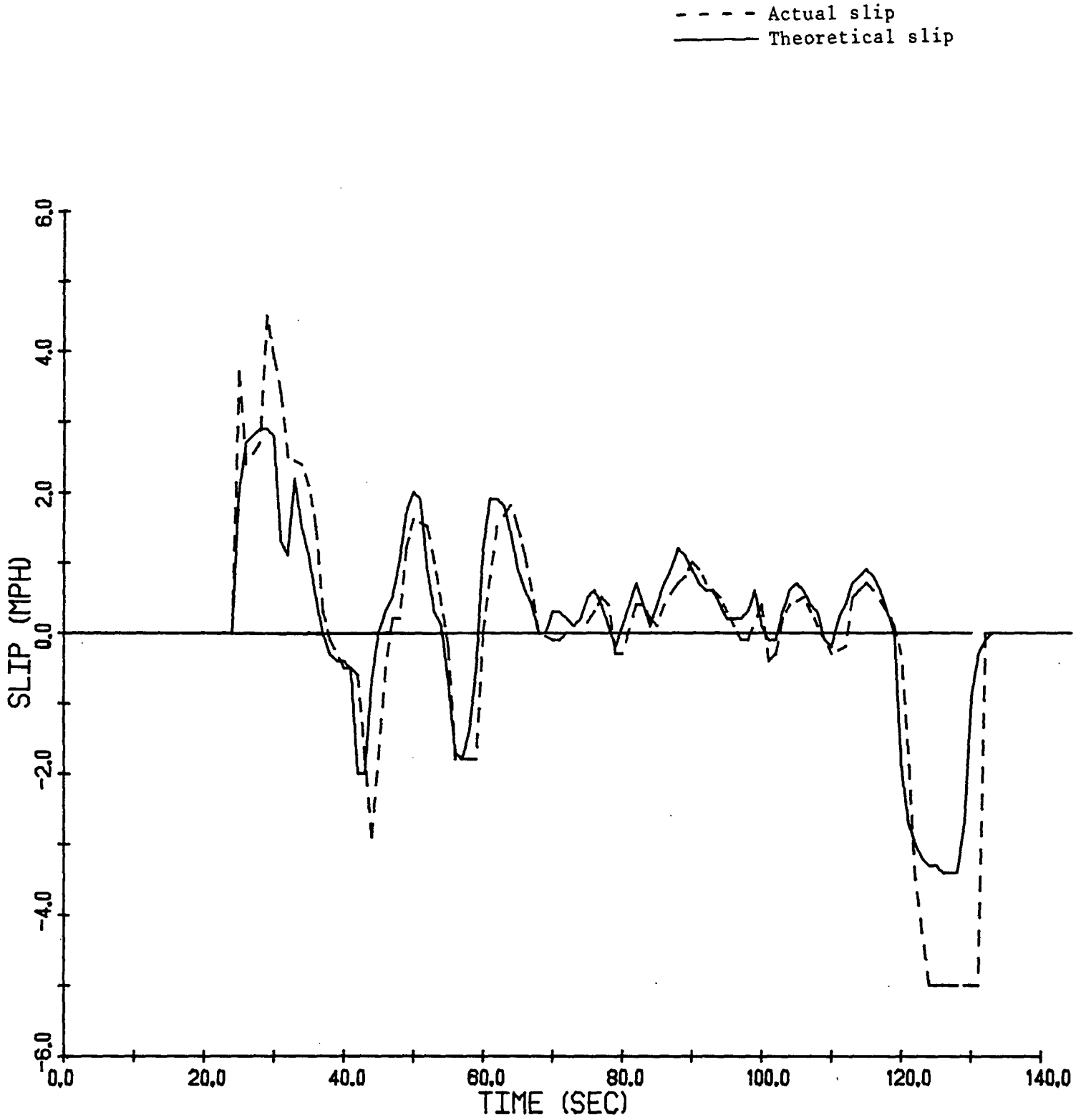


Table 1

LA-4 Cycle	Total Distance (meters)	Avg. Velocity (m/sec)	Total Energy (Joules) $\times 10^4$	Tire Energy (Joules) $\times 10^4$	Aerodynamic Energy (Joules) $\times 10^4$	Brake Energy (Joules) $\times 10^4$
(1) Rear Roll	11987.5	8.74	504.0	153.0	170.0	-181.0
(2) Front Roll	11877.4	8.65	483.0	151.0	163.0	-169.0
(1-2) Difference	110.1	0.09	21.0	2.0	7.0	- 12.0
(1-2)/2 % Diff.	0.93%	1.00%	4.3%	1.30%	4.30%	7.10%
Highway Cycle						
(1) Rear Roll	16496.2	21.5	807.0	210.0	552.0	-448.0
(2) Front Roll	16254.3	21.2	775.0	207.0	527.0	-400.0
(1-2) Difference	242.2	0.3	32.0	3.0	25.0	48.0
(1-2)/2 % Diff.	1.47%	1.42%	4.10%	1.45%	4.70%	12.00%

Appendix A

A-1

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C PROGRAM TO CALCULATE THE TOTAL FORCES, POWER, AND ENERGY DISSIPATED
C OVER THE LA4 SPEED
C VS. TIME SCHEDULE
C
C WRITTEN BY JOHN YURKO
C
C THE FOLLOWING DATA MUST BE SUPPLIED: VEHICLE MASS (KG), FRONTAL AREA
C (FT**2), ROLLING RESISTANCE
C COEFFICIENT (DIMENSIONLESS)
C
C FOR PROGRAMMING THE LABELS MAY NOT MATCH THE TERMS IN THIS REPORT:
C
C FAREA.....FRONTAL AREA
C PAUCOF.....COEFFICIENT OF AERODYNAMIC DRAG (B)
C VMASS.....VEHICLE MASS (M)
C PRCOEF.....ROLLING RESISTANCE COEFFICIENT
C
C
C
C
C DIMENSION V(10000),TT(10000)
C READ(5,100) VMASS,FAREA,PRCOEF
100  FORMAT(2F10.1,F10.3)
C WRITE(6,600)VMASS,FAREA,PRCOEF
600  FORMAT(1X,2F10.1,F10.3,/)
C
C CONVERT FRONTAL AREA TO A DYNAMOMETER LOAD COEFFICIENT USING THE
C FEDERAL REGISTER EQUATION
C FOR NON-FASTTRACK VEHICLES
C
C HP50 = 0.50*FAREA
C PWATTS =HP50 *745.7
C D=0.0
C PAUCOF = PWATTS/(22.35**3)
C
C READ IN VELOCITY, TIME, AND CALCULATE DV/DT
C
C
C DO 20 I=1,1372
C READ(5,200,END=999)IT(I),V(I)
20  CONTINUE
999  N=I-1
C
C
C 200  FORMAT(I4,F5.1)
C WRITE(6,300)
300  FORMAT('1',T6,'TIME',T12,'VELOCITY',T22,'ACCELR.',T33,
1  'TIRFFORCE',T44,'AFROFORCE',T55,'INERTIAL',T67,
2  'TOTAL',T79,'POWER',//1X,T5,'(SEC)',T12,'(M/SEC)',T23,
3  '(DV/DT)',T33,'(NEWTONS)',T44,'(NEWTONS)',T56,
4  'FORCE',T67,'FORCE',T78,'(WATTS)',//)
```

A-2

```
C
C
C      CALCULATE POLLING RESISTANCE (TIREF), AERODYNAMIC DRAG (AEROF),
C      INERTIAL FORCE (FINER),
C      AND TOTAL FORCE
C
C      TIREF = VMASS*0.8*RRCOEF
C      AEROE = 0.0
C      INERE = 0.0
C
C      TIRENG = 0.0
C      ENERGY = 0.0
C      DO 9 I=1,N
C
C      CONVERT VELOCITY FROM MPH TO METERS/SEC
C
C      V(I) = .447*V(I)
C      J=I-1
C      IF (T.EQ.1) GO TO 5
C      DVDT = V(I)-V(J)
C      GO TO 6
5     DVDT = V(I)
6     VMEAN = (V(I)+V(J))/2.0
C      AEROF = PAUCOF*VMEAN**2
C      FINER = VMASS*DVDT
C      FORCE = TIREF+AEROF+FINER
C
C      CALCULATE POWER AND ENERGY DISSIPATED
C
C      POWER = FORCE*VMEAN
C      TIREP = TIREF*VMEAN
C      PINER = FINER*VMEAN
C      AEROP = AEROF*VMEAN
C      TIRENG = TIRENG+TIREP
C      FINER = FINER+PINER
C      AEROE = AEROE+AEROP
C      CALCULATE THE TOTAL DISTANCE TRAVELED BY THE ROLL
C      D=D+VMEAN
C      IF (FORCE.LE.0.0) GO TO 7
C      ENERGY = ENERGY+POWER
C      GO TO 8
C
C      "BRAKE" REPRESENTS VEHICLE DECELERATING
C
7     BRAKE =BRAKE+POWER
C
8     WRITE(6,400) I,V(I),DVDT,TIREF,AEROF,FINER,FORCE,POWER
400  FORMAT('0',4X,'I4',F10.2,F10.2,F11.2,F12.2,F11.2,F10.2,F13.2)
C      CONTINUE
C      WRITE(6,500) TIRENG,EINER,AEROE,BRAKE,ENERGY
500  FORMAT('0',4X,'TIRE ENERGY = ',F10.3,/4X,'INERTIAL ENERGY = ',
5     F11.3,/4X,'AERODYNAMIC ENERGY = ',E12.3,/4X,'BRAKE ENERGY = ',
6     F12.3,/4X,'TOTAL ENERGY = ',E13.3)
C      WRITE(6,700) D
700  FORMAT(5X,'DISTANCE = ',F10.1)
C      STOP
C      END
```

Appendix B

B-1

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C PROGRAM TO CALCULATE THE VELOCITY(U2) OF THE FRONT ROLL FOR A TWIN
C POLL DYNAMOMETER.
C GIVEN THE REAR ROLL VELOCITY(V2): THAT IS THE VELOCITY FROM THE
C LA4 SCHEDULE
C WRITTEN BY JOHN YURKO
C
C THE FOLLOWING DATA MUST BE SUPPLIED: VEHICLE MASS (VM) IN KG,
C FRONTAL AREA (FAAREA)
C IN FT**2, ROLLING RESISTANCE COEFFICIENT (RR), VELOCITY (V2) IN MPH,
C AND SLIP
C COEFFICIENT (S)
C
C CALCULATE COEFFICIENTS B AND M FOR ROAD LOAD EQUATION:
C (F=A+R*V**2+M*DV/DT)
C CONSTANT "A" REPRESENTS ROLLING RESISTANCE AND DOES NOT
C CONTRIBUTE TO SLIP
C THEREFORE THIS TERM IS IGNORED
C
C DIMENSION V2(10000),IT(10000)
C READ(5,100)VM,FAAREA,RR,S
C DO 20 I=1,132
C READ(5,200,END=999)IT(I),V2(I)
200 FORMAT(I4,F6.1)
20 CONTINUE
999 N=I-1
C
C 100 FORMAT(2F10.1,F10.3,E14.3)
C WRITE(6,100)VM,FAAREA,RR,S
C R=0.50*FAAREA*745.7/22.35**3
C C=VM
C A=9.8*C*RR
C V1=0.0
C U1=0.0
C
C CALCULATE REAR ROLL VELOCITY
C
C DO 9 I=1,N
C
C CONVERT V2 FROM MPH TO M/SEC
C
C V2(I)=V2(I)*.447
C QA=S*R/2.0
C QB=S*(R+U1)+2.0*C)+1.0
C QC=(R*S*U1/2.0-2.0*S*C+1.0)*U1-V2(I)-V1
C U2=(-QB+SQRT(QB**2-4.0*QA*QC))/(2.0*QA)
C IF(U2.GE.0.0) GO TO 2
C U2=0.0
2 U1=U2
C V1=V2(I)

```

```
C
C   CALCULATE TOTAL DISTANCE TRAVELED ON EACH ROLL
C
DB=DB+V1
DF=DF+U1
U2=U2/.447
V2(I)=V2(I)/.447
TSLIP=V2(I)-U2
GO TO 21
IF WRITE(6,300)IT(I),U2,V2(I),TSLIP

300 FORMAT(I4,F6.1,F10.1,F10.1)
21 WRITE(6,700)IT(I),TSLIP
700 FORMAT(I5,F5.1)
  CONTINUE

C
C   THIS PROGRAM OUTPUTS THE FRONT ROLL VELOCITY IN (MPH) IN A
C   FORMAT THAT
C   CAN BE DIRECTLY READ INTO THE L4FORCE PROGRAM TO CALCULATE
C   THE FORCES
C

STOP
END
```