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Individual Manufacturer Procedures to Establish Fuel Economy Adjustment Factors

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INDIVIDUAL MANUFACTURER PROCEDURES TO ESTABLISH FUEL ECONOMY ADJUSTMENT FACTORS

by

Falcon Research & Development Co.

One American Drive
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"INDIVIDUAL MANUFACTURER PROCEDURES
TO
ESTABLISH FUEL ECONOMY ADJUSTMENT FACTORS"

Falcon R&D Report 3520-4/BUF-42
Final Report

February 1981

Page 64: Last equation should read:

$$v_k = u_k/U$$

Page 64: Second last line should read:

"The v_k represent ..."

Prepared: February 24, 1981

By: S. Kaufman

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Report 3520-4/BUF-42 Final Report

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ENVIRONMENTAL PROTECTION AGENCY

ANN ARBOR, MI 48105

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TABLE OF CONTENTS

Section	Title	Page
1	INTRODUCTION	1
	1.1 Background	1
	1.2 Scope of Work	2
2	SUMMARY	4
3	FUEL ECONOMY ADJUSTMENT BASED ON VEHICLE DESIGN PARAMETER VARIATIONS	6
	3.1 Fuel Economy Mathematical Model	8
	3.2 Estimation of Sensitivity Coefficient Functions from a Fuel Economy Data Set	10
	3.2.1 Derivation of Sensitivity Coefficient Data Sets	10
	3.2.2 Functional Estimation	13
	3.2.3 Results for a Specific Data Set	15
	3.3 Basic Considerations for Individual Manufacturer Procedure	17
	3.3.1 Accuracy	17
	3.3.2 Representativeness	24
	3.4 Draft Procedure for Individual Manufacturer Coefficients	25
	3.4.1 Data Requirements	26
	3.4.1.1 Minimal Set	26
	3.4.1.2 Additional Vehicle Tests	26
	3.4.1.3 Definitions	27
	3.4.2 Design Parameter Sensitivity Coefficient Estimation	28
	3.4.2.1 Test Weight	28
	3.4.2.2 Axle Ratio	35
	3.4.2.3 Road Load Horsepower	36

Table of Contents (Continued)

Section	Title	Page
3	FUEL ECONOMY ADJUSTMENT BASED ON VEHICLE DESIGN PARAMETER VARIATIONS (Continued)	
	3.5 Modified Draft Procedure if Sensitivity Coefficients are Assumed to be Constant (Parameter-Independent)	36
	3.5.1 Data Requirements	36
	3.5.2 Design Parameter Sensitivity Coefficient Estimation	36
4	FUEL ECONOMY ADJUSTMENT TO REFLECT IN-USE EXPERIENCE	39
	4.1 Road Adjustment Factor Estimation from In-Use Surveys	40
	4.2 Treatment of Sample Space Heterogeneity	48
	4.3 Confidence Intervals for Medians	52
	4.4 Basic Considerations for Individual Manufacturer Procedure	55
	4.4.1 Accuracy	55
	4.4.2 Representativeness	58
	4.5 Draft Procedure for Individual Manufacturer Road Adjustment Factors	59
	4.5.1 Data Requirements	59
	4.5.2 Adjustment Factor Estimation	61
5	ADDITIONAL ISSUES	63
	5.1 Computation of Design Parameter Adjusted Fuel Economy	63
	5.2 Regional/Seasonal Adjustment Factors	65
	5.3 Test and Parameter Adjustment Strategies	67
	REFERENCES	70

Table of Contents (Continued)

Section	Title	Page
Appendix A	VARIABILITY OF DYNAMOMETER FUEL ECONOMY MEASUREMENTS	71
Appendix B	FUEL ECONOMY LABEL ERROR DUE TO NORMAL ERROR BEFORE ROUND-OFF	77

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1. INTRODUCTION

This report is submitted as a deliverable on Task Order No. 4, "Fuel Economy Adjustment Factors," of Contract 68-03-2835 with the Environmental Protection Agency (EPA).

1.1 Background

The EPA has issued an Advance Notice of Proposed Rulemaking (Federal Register, Vol. 45, No. 190, September 29, 1980, pp. 64540-64544) with the objective of improving "the usefulness of vehicle fuel economy labels and the accuracy and completeness of the data used for determining corporate average fuel economy (CAFE) levels for new passenger vehicles and light trucks." Two of the ten regulatory options noted as being considered for this purpose are Design Factor Labeling and Shortfall Factor Labeling. The first would apply specific adjustment factors to normally available laboratory measured fuel economy test results in order to more closely estimate the fuel economy of (untested) design variations. The second option would apply an adjustment factor to each label value to account for the average industry difference (or "shortfall") between in-use experience and laboratory-measured fuel economy.

In connection with these two potential regulations, the EPA is considering developing procedures for manufacturer-specific adjustment factors for EPA fuel economy labeling: both vehicle design adjustment factors as well as in-use road adjustment factors. These procedures could be followed by a manufacturer if it feels that the adjustment factors provided by EPA regulations are not appropriate for its own vehicles.

EPA entered into a Task Order Agreement with Falcon Research and Development Company under which Falcon would perform the necessary engineering and statistical analyses to develop such procedures. The scope of work for this task follows.

1.2 Scope of Work

The proposed work will entail developing a method whereby an automotive manufacturer may develop fuel economy adjustment factors, for both vehicle design and road use, based upon the manufacturer's analysis of test data representative of his vehicles. The vehicle design adjustment factors will address the following technical parameters:

(a) Axle ratio;
(b) Road load horsepower;
(c) Estimated test weight.

The contractor shall develop a method for determining the quantity and nature of fuel economy data required to constitute a representative and statistically valid sampling of a manufacturer's vehicle fleet for purposes of design parameter sensitivity specification. The data analysis methods to be used by manufacturers shall also be specified by the contractor.

The above noted parameter effects on fuel economy shall be noted for both the EPA City and Highway cycles.

In addition, the Contractor shall review the EPA 404 Report (draft copy) for familiarization with the effects of road use on vehicle fuel economy. After reviewing the EPA 404 report, the contractor shall define the quantity and nature of in-use road fuel economy data, and analysis techniques, required in manufacturer development of road adjustment factors.

2. SUMMARY

This report addresses the question of automotive vehicle fuel economy as influenced by selected vehicle design parameters and by conditions which differentiate the on-road environment from the test environment. The central thrust of the report is the formulation of applicable fuel-economy adjustment factors in the context of a specific manufacturer's product line. The design parameter fuel economy adjustment problem is treated in Section 3. The on-road fuel economy adjustment problem is treated in Section 4. Section 5 considers a number of additional relevant issues.

The background discussion in Section 3 develops a fuel economy mathematical model in which the derivation and role of design parameter sensitivity coefficients is clarified. A general procedure for estimation of sensitivity coefficients from fuel economy data is presented. A significant issue raised is whether the sensitivity coefficient for each of the parameters (test weight, RLHP, axle ratio) should be expressed as a linear function of the parameter value at which it is to be applied or can be adequately represented by a constant value. The advantage of the constant value form lies in the simplicity and relative precision of estimation, but counterbalancing is the potential loss of accuracy of adjustment. Analysis of 1980 General Label File data suggests that non-zero slopes of the estimated sensitivity coefficient lines may have only marginal statistical significance. It is recommended that EPA carefully review its entire data set in order to decide this question.

The rationale and requirements for a procedure to estimate manufacturer-specific sensitivity coefficients is next presented. The section concludes with a draft procedure which covers data requirements and data analysis. Two alternatives are considered. The first is based on the assumption that each sensitivity coefficient is a linear function of its parameter. This is the assumption on which the EPA protocol is based. The second employs the simpler assumption that the sensitivity coefficients are not dependent on design parameters and can thus be treated as constants.

The background discussion in Section 4 reviews the concept of a numerical factor, derived from in-use vehicle surveys, which when multiplied into fuel economy label values brings these more into line with actually achieved fuel economy. The statistical objective is to achieve a match with the median in-use fuel economy. The present EPA method for estimating such factors for FTP (city) and HFET (highway) conditions is reviewed and an alternative method developed which more completely utilizes survey response data.

A critical issue in this problem is the heterogeneity of the sample space--due to variable environmental factors which greatly influence in-use fuel economy. A stratification procedure is recommended to ensure representativeness of the survey data sets to be used for road adjustment factor estimation. This leads to the employment of weighted median estimations.

The survey requirements for use in a procedure to estimate manufacturer-specific road adjustment factors is next presented. The section concludes with a draft procedure which covers survey design and data analysis. Provision is made for estimation of the factors by the present EPA approach or by a new method developed earlier in the section.

Issues discussed in Section 5 include: (1) Procedures for applying sensitivity coefficients to predict fuel economy of untested subconfigurations; (2) A public information program to enable individuals to make their own regional and seasonal adjustments for on-road fuel economy; and (3) The alternative strategies available to a manufacturer of establishing a revised mpg value for a subconfiguration by direct test vs. estimation of manufacturer-specific sensitivity coefficients.

3. FUEL ECONOMY ADJUSTMENT BASED ON VEHICLE DESIGN PARAMETER VARIATIONS

The present method of vehicle fuel economy* labeling by EPA averages test results over diverse configurations which, because of design/test parameter differences, are really not expected to have the same fuel economy. In the interest of achieving more accurate labeling, EPA is currently developing fuel economy adjustment factors that would explicitly account for variations in three significant vehicle design/test parameters: vehicle test weight (inertia setting of the test dynamometer), road load horsepower (dynamometer setting at 50 mph test speed), and axle ratio.

The EPA adjustment procedure is intended to be uniformly applicable to all manufacturers. However, a particular manufacturer could conceivably argue that its own vehicles are distinctly different as a class. Therefore, manufacturers should have the option of substituting alternative adjustment factors applicable to their own vehicles, so long as these factors satisfy appropriate criteria.

Selection of these criteria demands a careful enunciation of the intent of any protocol dealing with manufacturer-specific sensitivity coefficients as opposed to those promulgated by EPA. The view taken in this report is that whether the manufacturer-estimated coefficients are significantly different from the EPA-promulgated values is not an issue, nor does it need to be. Rather than considering manufacturer-specific coefficients in a hypothesis-testing context, one simply requires that the coefficients be estimated to some specified level of precision consistent with the aims of the fuel-economy labeling program.

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* Throughout this section the term "fuel economy" will be construed to mean FTP and/or HFET fuel economy as measured by chassis dynamometer testing.

This section develops two alternative recommended procedures to be followed by individual manufacturers who wish to challenge the EPA standard adjustment factors. (The choice between the two alternatives hinges on the specific form of the standard adjustment factors.) To lay the groundwork for these procedures, a background discussion of adjustment factor methodology is presented, along with some implications from available data.

3.1 Fuel Economy Mathematical Model

The measured FTP or HFET fuel economy of a vehicle is viewed as determined by the following factors:

(1) Basic Engine
(2) Engine Code
(3) Transmission Class
(4) Transmission Configuration
(5) Test Weight
(6) Road Load Horsepower
(7) Axle Ratio*
(8) Error Factor**

A unique combination of factors (1) through (7) is denoted by EPA as a vehicle subconfiguration,and all vehicles having this combination are essentially (though not precisely)*** identical design copies. The term "subconfiguration" is used because "configuration," as defined in the EPA regulations, refers to a unique combination of only factors (1) through (4), (7), and inertia weight class (which is close to but generally not equivalent to (5), test weight).

---

* The vehicle's N/V (engine rpm to vehicle speed (mph) ratio in highest gear) is probably the more fundamental parameter. However, axle ratio is a more accessible design parameter and, for a given transmission class/configuration and assumed fixed tire size, axle ratio determines N/V. The extent of variation in tire size among vehicles with same factors (1) through (6) is believed to be small.

** Includes both measurement errors and vehicle-to-vehicle variability within a subconfiguration.

*** For example, individual vehicle alternatives with curb weights differing by as much as 250 lbs. because of body differences could have the same test weight.

We shall find it convenient in this exposition to define a different aggregation of vehicles, namely, into unique combinations of factors (1) through (4), which we denote "design families." Note that "design family" fixes all design characteristics of a vehicle except the three parameters for which fuel economy adjustment factors are to be determined.

Our basic assumption is that within any design family the remaining three factors combine multiplicatively according to the following model:

$$E = K_i \cdot W(w) \cdot R(r) \cdot A(a) \cdot (1 + \varepsilon).$$

In this equation E is the measured fuel economy of a vehicle sampled from the specific subconfiguration defined by: design family i, test weight w, $RLHP_{50}$ r, and axle ratio a. Included in the model is a random error ε whose expected value is zero. If we drop the error factor $(1 + \varepsilon)$, then

$$E_0 = K_i \cdot W(w) \cdot R(r) \cdot A(a)$$

represents the true mean fuel economy within subconfiguration (i, w, r, a). Taking logarithms of both sides, we obtain

$$\ln E_0 = \ln K_i + \ln W(w) + \ln R(r) + \ln A(a).$$

A convenient way of representing the structural relationship between E_0 and design parameters w, r, a is to take the total differential of $\ln E_0$ with respect to w, r, a, and write

$$\frac{\Delta E_0}{E_0} = \frac{\frac{dW}{dw}}{W} \cdot \Delta w + \frac{\frac{dR}{dr}}{R} \cdot \Delta r + \frac{\frac{dA}{da}}{A} \cdot \Delta a$$

Then, defining sensitivity coefficients:

$$S_w = \frac{w}{E_0}\frac{\partial E_0}{\partial w} = \frac{w}{W}\frac{dW}{dw}$$

$$S_r = \frac{r}{E_0}\frac{\partial E_0}{\partial r} = \frac{r}{R}\frac{dR}{dr}$$

$$S_a = \frac{a}{E_0}\frac{\partial E_0}{\partial a} = \frac{a}{A}\frac{dA}{da}$$

one can write:

$$\frac{\Delta E_0}{E_0} = S_w \cdot \frac{\Delta w}{w} + S_r \cdot \frac{\Delta r}{r} + S_a \cdot \frac{\Delta a}{a}$$

Each sensitivity coefficient expresses the percentage change in resulting fuel economy per unit percentage change in design parameter value. Knowledge of S_w, S_r, S_a therefore permits estimation of the percentage change in E_0 due to any combination of small design perturbations in test weight, RLHP, and axle ratio. As seen from the assumed model, S_w, S_r, S_a are functions of w, r, a, respectively. We describe next the construction of a reasonable approach to estimation of these sensitivity coefficient functions.

3.2 Estimation of Sensitivity Coefficient Functions from a Fuel Economy Data Set

3.2.1 Derivation of Sensitivity Coefficient Data Sets

Each test record includes the tested vehicle's design family designation, design parameters (w, r, a), and measured fuel economy E. Generally, there will be both an FTP and HFET measurement, but the method of analysis is the same for each. Partition these records into groups containing only identical subconfigurations except possibly for test weight. Within each group, collapse all tests on vehicles with same test weight (hence, same subconfiguration) into a single composite test by calculating a mean fuel economy. Next, within each group containing at least two different test weights, order according to increasing test weight, i.e., (E_1, w_1, m_1), (E_2, w_2, m_2), ..., (E_k, w_k, m_k), where $w_1 < w_2 < \ldots, < w_k$ and m_j is the test multiplicity (from the above collapsing procedure) associated with test weight w_j. Define $k - 1$ estimates of S_w and associated fractional weight differences Δw as follows:

$$\Delta w_j = \frac{w_{j+1} - w_j}{\frac{1}{2}(w_{j+1} + w_j)}$$

$$\hat{S}_{w,j} = \frac{\dfrac{E_{j+1} - E_j}{\frac{1}{2}(E_{j+1} + E_j)}}{\Delta w_j}$$

It is important at this point to determine the relative precision of the computed $\hat{S}_{w,j}$. Our model assumes that errors arise only from measured fuel economy E, which has a fixed coefficient of variation σ_0 with respect to the mean fuel economy of the subconfiguration to which the vehicle belongs. (Both measurement errors and vehicle-to-vehicle differences contribute to this variability.) Then, the variances of $w_{j+1}/\frac{1}{2}(w_{j+1} + w_j)$ and $w_j/\frac{1}{2}(w_{j+1} + w_j)$ are approximately σ_0^2/m_{j+1} and σ_0^2/m_j, respectively. It can then be shown that the variance of $\hat{S}_{w,j}$ is given approximately by

$$\sigma_j^2 = \frac{\sigma_0^2}{\left(\dfrac{m_{j+1} m_j}{m_{j+1} + m_j}\right)(\Delta w_j)^2}$$

Hence, we define variance reduction factor u_j given by

$$u_j = \left(\frac{m_{j+1} m_j}{m_{j+1} + m_j} \right) (\Delta w_j)^2$$

This result depends on the assumption of test-to-test error independence, which is reasonable if all collapsed individual test results are from different vehicles.

After completing the above operations for all comparable configuration groups, pool the results from all of the groups to provide a derived data set

$$\{\hat{S}_{w,i}, w_i, u_i\} \qquad i = 1, \ldots, n$$

Repeat the above process for the other two design parameters, road load horsepower and axle ratio, substituting r and a, respectively, in place of w at each step in the procedure. This results in

$$\{\hat{S}_{r,i}, r_i, u_i'\} \qquad i = 1, \ldots, n'$$

$$\{\hat{S}_{a,i}, a_i, u_i''\} \qquad i = 1, \ldots, n''$$

3.2.2 Functional Estimation

At this point, it is appropriate to analyze the data qualitatively in order to assess the likely form of the regressed functional dependence of each sensitivity coefficient on its respective design parameter. If such analysis supports the possibility of a linear relationship in each case, or is at least not inconsistent with that assumption, then quantitative linear regressions may be performed to estimate regression lines which are of the form

$$S_w = \alpha + \beta w$$

$$S_r = \gamma + \delta r$$

$$S_a = \eta + \theta a$$

Given the heteroscedasticity of the data sets, i.e., nonuniform variance of the sensitivity coefficient estimates, an appropriate regression procedure would be weighted least squares regression for the three sensitivity coefficient lines, with weights $\{u_i\}$, $\{u'_i\}$, and $\{u''_i\}$, respectively. A detailed exposition of this procedure is provided in Section 3. .2.1.4 as part of the recommended protocol for individual manufacturers and will not be repeated here.

The above estimated sensitivity coefficient lines imply a general fuel economy model of the form

$$E_0 = K_i w^\alpha r^\gamma a^\eta \exp(\beta w + \delta r + \theta a)$$

Even if the data support straight-line sensitivity coefficients, these should be used essentially for interpolated estimation within the range of parameter values in the data. Extrapolation appreciably outside this range would be highly speculative.

Since S_w, S_r, and S_a are, by basic physical principles, expected to be non-positive,* should any of the lines cross the horizontal axis into the positive value region (while still within the range of parameter values in the data) it may be prudent to replace these positive values by zero for general application.

If qualitative data analyses suggest no significant relationship between each sensitivity coefficient and its respective design parameter, or if estimates of the linear slopes, β, δ, θ, are found to be not significantly different from zero, then one need only estimate a mean sensitivity coefficient in each case. Again, a weighted procedure is most appropriate and takes the form:

$$\hat{S}_w = \frac{\Sigma\, u_i S_{w,i}}{\Sigma\, u_i}$$

Similarly for $\hat{S}_r$ and $\hat{S}_a$. In this case the mathematical fuel economy model simplifies to:

$$E_0 = K_i w^{\hat{S}_w} \cdot r^{\hat{S}_r} \cdot a^{\hat{S}_a}$$

---

* It is understood that in some special cases this expectation has been shown to be incorrect, perhaps due to some unusual engine map characteristics.

3.2.3 Results for a Specific Data Set

Fuel economy test results from the 1980 General Label data base were provided by EPA. The data contained records on 1673 tested subconfigurations which included an unspecified number of duplications that were eliminated in the course of the processing. Search for groups of comparable subconfigurations and, then, estimation of sensitivity coefficients by pairing along adjacent increasing parameter values led to the calculation of 53 $\hat{S}_w$, 43 $\hat{S}_r$ and 56 $\hat{S}_a$ points. Weighted least squares linear regressions were carried out following the procedures described in Section 3.4.2.1.4. The results obtained are shown in Table 1.

Examination of Table 1 leads to a number of observations. The negativity of $\overline{S}$, the weighted mean of all sensitivity coefficient estimates (and also the estimated coefficient at the mean design parameter location) is generally confirmed. On the other hand, the existence of non-zero slopes is in some cases not established with statistical significance and, in other cases, only marginally so. Predicted FTP axle ratio sensitivity coefficients at the lower limit of the axle ratio range are slightly positive; this is the only instance of interpolated positive coefficient prediction. The estimated values of σ_0, the coefficient of variation in subconfiguration fuel economy measurement are quite consistent and in reasonable agreement with other estimates for σ_0 (see Appendix A).

The marginal statistical validity of non-zero slopes in the above numerical exercise is a situation that could conceivably also occur when EPA estimates its standard (industry-wide) sensitivity coefficient regression lines. It is recommended that careful attention be given to this matter. If parameter dependence of sensitivity coefficients is not confirmed at a suitable level of significance, then it would be prudent to choose the simpler mode of constant (parameter-independent) sensitivity coefficients.

Table 1. WEIGHTED REGRESSION OF SENSITIVITY COEFFICIENTS COMPUTED FROM 1980 GENERAL LABEL FILE

MODEL: $\hat{S} = \overline{S} + \hat{b}\,(P - \overline{P})$

ERROR SOURCE: Fuel Economy Measurement with Coefficient of Variation = σ_0

DESIGN PARAMETER, P	DRIVING SCHEDULE	n	$\overline{P}$	$\overline{S}$	$\hat{\sigma}_{\overline{S}}$	$\hat{b}$	$\hat{\sigma}_{\hat{b}}$	$\hat{\sigma}_0$	F*	α*
Test Weight, w	FTP	53	4226 lbs	- 0.266	0.084	$0.081/10^3$ lb	$0.099/10^3$ lb	0.026	0.68	> 0.25
Range = [2312, 5375] lbs	HFET	53	4226 lbs	- 0.153	0.083	$0.176/10^3$ lb	$0.097/10^3$ lb	0.025	3.28	~ 0.08
Road Load HP, r	FTP	43	9.8 HP	- 0.127	0.055	- 0.037/HP	0.022/HP	0.028	2.73	~ 0.11
Range = [7.2, 18.3] HP	HFET	43	9.8 HP	- 0.361	0.060	0.004/HP	0.024/HP	0.030	0.02	> 0.75
Axle Ratio, a Range = [2.35, 3.72]	FTP	56	3.06	- 0.234	0.050	- 0.369/ Unit Ratio	0.202/ Unit Ratio	0.035	3.34	~ 0.08
	HFET	56	3.06	- 0.579	0.063	- 0.562/ Unit Ratio	0.254 Unit Ratio	0.044	4.90	~ 0.035

* Analysis of variance F-ratio and associated significance probability α for slope b = 0.

3.3 Basic Considerations for Individual Manufacturer Procedure

EPA plans to promulgate standard sensitivity coefficients based on subconfiguration test data covering all light duty automotive manufacturers. There will be six such coefficients or coefficient lines--applicable to each of FTP or HFET fuel economy for each of three possible design parameter variations: test weight, RLHP, or axle ratio. The fact that preliminary examination of these results have revealed no consistent patterns among individual manufacturers,[1] is the basis for adopting single (industry-wide) standards. However, no special efforts were made by EPA to establish this conclusion with high confidence. An individual manufacturer may have good reason to believe that one or more of the EPA standards do not apply to its own vehicles. If that is the case, it can propose to use alternative sensitivity coefficients derived strictly from data on its own vehicles. Alternative coefficients (or coefficient lines) may be proposed for any number of the six EPA standards. Each proposed alternative should be considered independently. In order to be accepted by EPA, such manufacturer-specific sensitivity coefficients must meet certain accuracy and representativeness requirements. The procedures that will be presented have been formulated to ensure that that happens. It is appropriate at this point to consider first how the accuracy and representativeness requirements were developed.

3.3.1 Accuracy

The process of fuel economy adjustment based on design parameter sensitivity coefficients takes a tested vehicle subconfiguration (having fuel economy E) as a starting point and calculates the fuel economy E' of a second subconfiguration which is identical to the first subconfiguration in all respects except for a different value of single parameter P.* The relationships used for this calculation are:

---

* P refers to test weight, RLHP, or axle ratio. It is also possible for adjustments to be made for two or three simultaneous design parameter variations, but this possibility is disregarded.

$$E' = E + \Delta E$$

$$\Delta E = S \cdot E \cdot \Delta P$$

where S is the applicable sensitivity coefficient value, ΔP is the fractional change in P when going from the tested to the untested subconfiguration, and ΔE is the absolute adjustment in fuel economy (in mpg).

To begin with, it is recognized that the tested subconfiguration fuel economy measurement E will generally be in error relative to the true mean fuel economy for that subconfiguration, which error is carried directly into the estimate E' for the untested subconfiguration. A useful statistical characterization of this error is its coefficient of variation, i.e., the ratio of its standard deviation to the true mean fuel economy. (The latter is adequately approximated by E for small enough coefficients of variation.) Designate this coefficient of variation by σ_0. A review of available literature on fuel economy measurement errors together with additional analysis of recent EPA test data is described in Appendix A. It is concluded therein that a reasonable estimate for σ_0 is 0.04 (4%), apparently applicable to both FTP and HFET fuel economy.

Suppose, now, that the estimated sensitivity coefficient S is also in error relative to true value, that error being characterized by a variance σ_S^2. On the reasonable assumption of a statistical independence of the errors, we may then express the squared coefficient of variation of E' by

$$\sigma_1^2 = \sigma_0^2 + (\Delta P)^2 \sigma_S^2$$

In order to preclude any substantial increase in σ_1 relative to σ_0, it is recommended that $\Delta P \cdot \sigma_S$ be required not to exceed 0.02. The effect of that requirement would be to keep σ_1 to within about 12% of σ_0, as indicated by the following calculation:

$$\sqrt{(0.04)^2 + (0.02)^2} = 0.0447 \cong 0.04(1.12).$$

Any more stringent requirement would lead to only slight additional improvement in overall accuracy. On the other hand, degradation of overall accuracy becomes increasingly more rapid with relaxation of the 0.02 criterion.

It is of interest to look further into the implications of the requirement

$$\Delta P \cdot \sigma_S \leq 0.02$$

on the actual labeling process. Let E_L' be the label value assigned to the untested subconfiguration based on the adjusted calculation E'. E_L' will therefore be just E' rounded off to the nearest whole number in mpg. Let E'' and E_L'' be the corresponding quantities on the supposition that true S were known precisely and the correct design adjustment were made. Then

$$\varepsilon = E' - E''$$

$$\varepsilon_L = E_L' - E_L''$$

are errors (in mpg) due <u>only</u> to sensitivity coefficient error. ε is the unrounded error and would have standard deviation (in view of the above requirement)

$$\sigma_\varepsilon \leq 0.02\ E$$

which, for example, takes on limiting values of 0.25 mpg, 0.5 mpg, and 1 mpg at $E = 12.5$ mpg, 25 mpg, and 50 mpg, respectively. The second error quantity ε_L is the difference in label values achieved under actual and ideal (error-free) design parameter adjustment and therefore represents the ultimate impact of sensitivity coefficient uncertainty. ε_L is also a random value, but takes on only integer values.

On the reasonable assumptions that ε is normally distributed (about zero with variance σ_ε^2) and that the decimal component of any fuel economy measurement is uniformly distributed between 0 and 1, probability distributions for ε_L have been calculated for various σ_ε. These are shown in Table 2. The details of the calculation are given in Appendix B. Thus, the

Table 2. PROBABILITIES OF LABEL MPG ERROR FOR VARIOUS ERROR STANDARD DEVIATIONS IN CALCULATED MPG

σ_ε (mpg)	LABEL MPG ERROR ε_L (mpg)			
	0	±1	±2	±3
0.25	0.80	0.20	< 0.001	--
0.50	0.61	0.38	0.01	--
1.00	0.37	0.48	0.13	0.02

requirement $\Delta P \cdot \sigma_S \leq 0.02$ implies for $E = 12.5$ mpg cars at least 80% probability that there will be no label error due to the adjustment process. Moreover, if an error does occur, that error will rarely exceed ± 1 mpg. For $E = 25$ mpg cars these probabilities shift somewhat to at least 61% no error and no more than 38% ± 1 mpg error. In the case of 50 mpg cars (for which the requirement implies $\sigma_\varepsilon \leq 1$ mpg) there may be appreciable probabilities of ± 1 mpg and even ± 2 mpg errors. It should be noted however that for such cars ± 2 mpg is a relatively small change compared to absolute fuel economy; furthermore, in this case $\sigma_0 = 0.04$ implies that the error contribution due to fuel economy measurementsin the tested subconfiguration has a standard deviation of 2 mpg, which still dominates the $\leq$ 1 mpg σ_ε.

The above analysis tends further to support the reasonableness of the requirement

$$\Delta P \cdot \sigma_S \leq 0.02$$

The next question to be raised then is how does this translate into operational requirements for the manufacturer? It will be shown subsequently (Section 3.4.2.1.4) that, after the manufacturer has obtained and processed the appropriate data, an estimate for the maximum variance of the computed sensitivity coefficient within the range of parameter values in the data set* is

$$\sigma^2_{max} = \frac{2\,\sigma^2_0\, f^2}{n(\Delta P)^2_{rms}}$$

where

n is the number of individual sensitivity coefficient data points**

σ_0 is the underlying fuel economy coefficient of variation in the manufacturer's data set

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* As a consequence of the representativeness requirement, it is expected that the parameter range will encompass most, if not all of the respective parameter values over the entire set of manufacturers vehicles. Hence, fuel economy adjustments will involve interpolated rather than extrapolated estimates of sensitivity coefficient.

** n is not to be confused with the number of subconfiguration test results. As previously described, each data point derives from a pair of comparable subconfigurations from which a sensitivity coefficient has been made.

$(\Delta P)_{rms}$ is a root-mean-squared average of the fractional parameter differences between comparable pairs of subconfigurations used to calculate individual sensitivity coefficients

f is a parameter distribution shape factor which is defined in Section 3.4.2.1.4. If only mean sensitivity coefficients rather than sensitivity coefficient lines are to be estimated, then $f = 1$.

One may then derive a requirement on n as follows:

$$n = \frac{2\,\sigma_0^2 f^2}{(\Delta P)^2_{rms}\,\sigma^2_{max}} \geq 2\left(\frac{\Delta P}{\Delta P_{rms}} \cdot \frac{\sigma_0}{0.02} \cdot f\right)^2$$

An immediate simplification recommended is the identification of ΔP with ΔP_{rms}. This means that the accuracy requirement on the ΔE adjustment is to be imposed in the context of an "average" value for design parameter difference between comparable subconfigurations. Admittedly, adjustments will be made for $\Delta P > \Delta P_{rms}$ with correspondingly larger errors, but they will also be made for $\Delta P < \Delta P_{rms}$ as well which tends, overall, to balance out on a probabilistic basis. If this recommendation is acceptable, then the requirement on n reduces to

$$n \geq 2\left(\frac{\sigma_0}{0.02} \cdot f\right)^2$$

The difficulty with this expression is that neither σ_0 nor f is known in advance but must be calculated after the data set has been assembled.

The solution offered is to provide for a two-stage procedure. _A priori_ reasonable estimates are known for both σ_0 and f. On the basis of these, determine a required data set size n_1. Assemble a data set of size $n \geq n_1$ and carry out the estimation procedure including determination $\hat{\sigma}_0$ of σ_0 and f. Compute a revised n_2 using $\hat{\sigma}_0$ and f. If $n_2 \leq n$, then no additional data are required. If $n_2 > n$, then $n_2 - n$ additional data points must be introduced and the estimation procedure repeated with the augmented data set to yield final estimates.

We proceed now to derive the first stage data set size requirement n_1. As previously indicated, a reasonable _a priori_ estimate of σ_0 is 0.04. Consider, first, that EPA has selected the alternative of expressing sensitivity coefficients as linear functions of the design parameter. As implied by the definition of shape factor f, if the design parameter values of the individual data points are fairly uniformly distributed over their range, then $f \cong 2$. It is reasonable to expect that the requirement for representativeness of the data to all of the manufacturer's vehicles will tend to prevent peaked or polarized distributions from occurring.

In the preceding paragraph f was evaluated in the context of estimated sensitivity coefficient lines. If, on the other hand, EPA decides that estimation of mean sensitivity coefficients, independent of parameter values, is adequate, then $f = 1$.

We therefore arrive at the following recommended _first stage_ requirements on data set size:

$$n_1 \geq \begin{cases} 32 & \text{Linear Sensitivity Coefficients} \\ 8 & \text{Constant Sensitivity Coefficients} \end{cases}$$

Clearly, a number of arbitrary judgments were made along the way in arriving at this recommendation. For example, suppose there were additional compelling arguments for keeping the probability of any label error arising from the adjustment process to below 40% even with 50 mpg cars. Then a σ_ε of 0.5 mpg is called for at 50 mpg, and this fact imposes the more stringent requirement of $\Delta P \cdot \sigma_S \leq 0.01$. All n requirements would then be increased by a factor of four. Such a modification may well be feasible in the context of a constant sensitivity coefficient model. On the other hand, if sensitivity coefficients have to be represented as linear functions of parameter value, the test burden would probably effectively deny to all manufacturers the option of challenging EPA standards.

3.3.2 Representativeness

By requiring the manufacturer to use, as a minimum, all existing test results from emission data, fuel economy label and fuel economy data vehicles, we take advantage of an in-place structure which has as its objective the accumulation of a (sales-weighted) representative data set. This minimum requirement covers the forthcoming model year as well as the previous two model years, but, for the latter, excludes discontinued basic engine-transmission class combinations. If enough comparable subconfigurations are not found in this minimal set to meet the n_1 requirement, then the introduction of additional subconfiguration test data must be suitably spread over different Base Levels, with emphasis in some proportion to projected sales. The draft procedure defines specific rules by which this is to be accomplished. Base Levels, by definition, separate by inertia weight class; consequently, the derived data set could be expected to span all test weights having significant sales. Because of the moderate degree of correlation of RLHP with inertia weight class, a broad span of RLHP settings could also be expected. There is perhaps less assurance of obtaining a full span of axle ratio values, since this parameter is not defined at the Base Level. However, specific vehicle configurations are designated for inclusion in emission and fuel economy test fleets based on projected sales at the configuration level, and it is speculated that a broad span of values will naturally be achieved even in this case. It should also be noted that the manufacturer will have an incentive to achieve a broad spread of parameter values (in the linear sensitivity coefficient model) in order to minimize the value of distribution shape factor f which enters into the second stage n requirement.

3.4 Draft Procedure for Individual Manufacturer Coefficients

This section presents a draft procedure for individual manufacturers who wish to take exception to any or all of the six standard design parameter sensitivity coefficients (or coefficient lines) promulgated by EPA. It includes data requirements, first stage estimation of sensitivity coefficients, statistical test of need for additional data, and, if required, final estimation of sensitivity coefficients. The six cases are categorized as FTP or HFET fuel economy sensitivities to each of: test weight, RLHP, or axle ratio design parameters. Although each of the six cases may be considered independently, the procedure as structured presents parallel treatment of FTP and HFET sensitivities for each of the three design parameters, in recognition of the fact that most sub-configuration test results will provide both FTP and HFET fuel economies. The procedure is presented in full detail with respect to test weight design parameter. Application to the other two design parameters is by reference.

3.4.1 Data Requirements

3.4.1.1 Minimal Set

Fuel economy tests (FTP and HFET) conducted by the manufacturer or by EPA on all of the manufacturer's emission data vehicles, fuel economy label vehicles, and fuel economy data vehicles for the forthcoming model year are to be utilized. Each fuel economy test result is associated with a unique vehicle subconfiguration as specified in 3.4.1.3. Similar test results from the preceding two model years, with the exception of discontinued basic engine-transmission combinations, are to be included.

3.4.1.2 Additional Vehicle Tests

Utilize the procedure described in Sections 3.4.2.1.1 through 3.4.2.1.3 to estimate the number of sensitivity coefficient data points that can be generated from the minimal data set of Section 3.4.1.1. If the estimate is less than n_1^*, then additional subconfiguration FTP and/or HFET fuel economy test data need to be introduced to reach this requirement. Subconfigurations may be selected with some discretion by the manufacturer so as to match already-tested subconfigurations and thereby generate comparable subconfiguration groups as defined in 3.4.2.1.1. However, their distribution among Base Levels must reasonably match projected sales as provided for by the following rules:

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* In the final procedure n_1 will be replaced by a specific number.

(1) Base Levels with less than 1% or less than 5000 projected sales are excluded.

(2) Add each new subconfiguration test successively from different Base Levels starting with that with highest projected sales and working downward.

(3) There should be no more than one new subconfiguration from each Base Level until all eligible base Levels have contributed.

(4) Beyond this point, each additional subconfiguration added must be from a Base Level for which the ratio of number of already-added subconfigurations to projected (base Level) sales is smallest.

3.4.1.3 Definitions

The variables which uniquely define a vehicle's subconfiguration are:

Basic Engine Family (E)
Engine Code (E_C)*
Transmission Class (T)
Transmission Configuration (T_C)
Equivalent Test Weight (W)
Inertia Weight Setting (W_I)
Axle Ratio (A)
Road Load Horsepower Setting (R)

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* Practical considerations may lead to deletion of E_C as a defining variable, that is, EPA is considering the possibility of permitting aggregation over engine code in those instances where more than one code is compatible with a specified basic engine and transmission.

The Base Level of a given subconfiguration is uniquely defined by (E, T, W_I). Fuel economy test results (in mpg) are denoted by:

C (FTP Fuel Economy)

H (HFET Fuel Economy)

3.4.2 Design Parameter Sensitivity Coefficient Estimation

The three design parameters for which standard sensitivity coefficients are promulgated by EPA are: test weight (W), axle ratio (A), and road load horsepower (R). The procedure for estimating manufacturer's alternative coefficients may be carried out independently for any or all of these parameters.

3.4.2.1 Test Weight

3.4.2.1.1 Comparable Subconfiguration Groups

Partition the total set of tested vehicle subconfigurations (3.4.1.1 and 3.4.1.2) into groups within each of which members differ only in test weight (W) and inertia weight (W_I), i.e., all vehicles in a given group have identical E, E_c, T, T_c, A, and R, but different W.* Within each such group containing more than a single member, order according to increasing W, i.e., $W_1 < W_2 < \ldots < W_k$.

3.4.2.1.2 Weight Sensitivity Coefficients

For each group with $k > 1$ comparable subconfigurations, define k-1 estimates of weight sensitivity coefficients and related variables

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* If p such tested vehicles have equal W, they must be regarded as p samples of the same subconfiguration (even if the W_I differ). Accordingly, they are collapsed into a single test entry by taking the mean of their C and H test results.

as follows:

Average Fuel Economies:

$$\overline{C}_j = \frac{C_{j+1} + C_j}{2} \qquad \overline{H}_j = \frac{H_{j+1} + H_j}{2}$$

Fractional Changes in Fuel Economy:

$$\Delta C_j = \frac{C_{j+1} - C_j}{\overline{C}_j} \qquad \Delta H_j = \frac{H_{j+1} - H_j}{\overline{H}_j}$$

Average Weight:

$$\overline{W}_j = \frac{W_{j+1} + W_j}{2}$$

Fractional Change in Weight:

$$\Delta W_j = \frac{W_{j+1} - W_j}{\overline{W}_j}$$

Weight Sensitivity Coefficients:

$$SWC_j = \frac{\Delta C_j}{\Delta W_j} \qquad SWH_j = \frac{\Delta H_j}{\Delta W_j}$$

$j = 1, 2, \ldots, k-1$

Further, if C_j is the sample mean of p_j FTP tests on different vehicles of the same subconfiguration, compute the variance reduction factor

$$u_j = \left(\frac{P_{j+1}P_j}{P_{j+1} + P_j}\right) (\Delta W_j)^2$$

The variance of SWC_j is then

$$\sigma^2_{SWC_j} = \frac{\sigma_0^{\ 2}}{u_j} \qquad j = 1, \ldots, k - 1$$

Note that in most instances $P_{j+1} = P_j = 1$; hence $u_j = (\Delta W_j)^2/2$. Note also that if all FTP and HFET tests are paired, then u_j applies equally to SWH_j; otherwise different variance reduction factors need to be determined for the latter.

3.4.2.1.3 Pooling of Data

Pool the estimated sensitivity coefficients from all of the comparable groups into aggregated data sets as follows (replacing symbol $\overline{W}$ by W):

$$\{ SWC_i, W_i, u_i \} \qquad i = 1, 2, \ldots, n$$

$$\{ SWH_i, W_i, u_i \}$$

Thus, in the first data set there are aggregated n different determinations of FTP fuel economy weight sensitivity coefficient, SWC_i, each estimated at an average test weight W_i, and each with a variance reduction factor u_i relative to basic variance in subconfiguration FTP fuel economy measurement. Similarly for the second data set.

3.4.2.1.4 Weighted Least Squares Linear Regression

Assume a linear model for dependence of SWC on W, i.e.,

$$SWC_i = a + b\,(w_i - \overline{W}) + \varepsilon_i$$

where a and b are unknown coefficients to be estimated, $\overline{W}$ is a weighted sample mean of the W_i to be defined shortly and ε_i is an additive (unbiased) error with variance:

$$\sigma^2_{\varepsilon_i} = \frac{\sigma_0{}^2}{u_i}$$

Estimation of a and b is to be performed by weighted least squares linear regression. Define the normalized weights

$$v_i = u_i/\Sigma u_j = u_i/U^*$$

Define also:

$$\begin{aligned}
\overline{SWC} &= \Sigma v_i\ SWC_i \\
\overline{(SWC)^2} &= \Sigma v_i\ (SWC_i)^2 \\
\overline{W} &= \Sigma v_i\ w_i \\
\overline{W^2} &= \Sigma v_i\ w_i{}^2 \\
\overline{SWC \cdot W} &= \Sigma v_i\ SWC_i W_i
\end{aligned}$$

---

* In the special case of $u_i = \frac{1}{2}(\Delta W_i)^2$ for all i, which applies when none of the subconfiguration fuel economy tests are replicated, U may be expressed as $U = \frac{n}{2} \cdot \frac{1}{n}\,\Sigma(\Delta W_i)^2 = \frac{n}{2}\,(\Delta W_{rms})^2$

where ΔW_{rms} is a root-mean-square average of the ΔW_i.

Then unbiased estimates for a and b are:

$$\hat{a} = \overline{SWC}$$

$$\hat{b} = \frac{\overline{SWC \cdot W} - \overline{SWC} \cdot \overline{W}}{\overline{W^2} - (\overline{W})^2}$$

Variance estimates for $\hat{a}$ and $\hat{b}$ require also an estimate of $\sigma_0{}^2$, the squared coefficient of variation of subconfiguration FTP fuel economy determination. Such an unbiased estimate as given by:

$$\hat{\sigma}_0^2 = \left[\overline{(SWC)^2} - (\overline{SWC})^2 - \hat{b}^2 \cdot (\overline{W^2} - (\overline{W})^2) \right] U/(n-2)$$

The variances of $\hat{a}$ and $\hat{b}$ are then estimated by:

$$\hat{\sigma}_{\hat{a}}^2 = \hat{\sigma}_0^2 / U$$

$$\hat{\sigma}_{\hat{b}}^2 = \hat{\sigma}_0^2 \Big/ U(\overline{W^2} - (\overline{W})^2)$$

The covariance of a and b is zero by virtue of the centering of the data around $\overline{W}$ in the linear model. This implies that for an arbitrary test weight W, the estimated SWC at W

$$S\hat{W}C = \hat{a} + \hat{b}\ (W-\overline{W})$$

has variance given by

$$\hat{\sigma}^2_{S\hat{W}C} = \frac{\hat{\sigma}_0^2}{U} \left(1 + \frac{(W-\overline{W})^2}{\overline{W^2} - (\overline{W})^2} \right)$$

The maximum variance of $\hat{SWC}$ over the range of test weights in the data set is therefore:

$$\sigma^2_{max} = \frac{\hat{\sigma}^2_0}{U} \cdot \max_{W_L \le W \le W_U} \left\{ 1 + \frac{(W - \overline{W})^2}{\overline{W^2} - (\overline{W})^2} \right\}$$

$$= \frac{\hat{\sigma}^2_0 f^2}{U}$$

$$= \frac{2}{n} \frac{\hat{\sigma}^2_0 f^2}{(\Delta W_{rms})^2} \quad \text{(see earlier footnote)}$$

The HFET (highway) sensitivity coefficient estimates are obtained in the same manner by repeating the above regression procedure with SWH_i in place of SWC_i.

3.4.2.1.5 Accuracy Check and Second Stage Estimators

Define

$$f = \max_{W_L \le W \le W_U} \left\{ \sqrt{1 + \frac{(W - \overline{W})^2}{\overline{W^2} - (\overline{W})}} \right\}$$

where W_L and W_U are, respectively, the smallest and largest W_i values in the data set. Compute f and use it together with the previously computed estimate for basic fuel economy coefficient of variation, $\hat{\sigma}_0$, and requirement σ_E** to check whether n satisfies the inequality:

$$n \geq 2\left(\frac{\hat{\sigma}_0}{\sigma_E}\right)^2 f^2$$

If it does, then the procedure is terminated. If it does not then compute

$$\Delta n = \left[2\left(\frac{\hat{\sigma}_0}{\sigma_E}\right)^2 f^2 - n\right] + 1$$

as the additional number of data points required. (The notation $[x]$ denotes greatest integer less than x.) Generate the additional Δn data points by introduction of a suitable number of new tested sub-configurations in accordance with Section 3.4.1.2, continuing from the point reached in the first stage procedure. Repeat the procedures in Sections 3.4.2.1.1 through 3.4.2.1.4 with the augmented set of $n + \Delta n$ data points and discard the original (first-stage) estimates.

The above may be repeated for HFET weight sensitivity coefficients by use of corresponding values for f and $\hat{\sigma}_0$.*

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* If both FTP and HFET data are provided in all tested subconfigurations, then f will be identical for FTP and HFET cases. However, $\hat{\sigma}_0^2$ estimates will generally differ. If both FTP and HFET weight sensitivity coefficients are being estimated, then it would be wise to test adequancy of n for both cases together and then to generate new data points, as required, to meet the largest deficiency (if any).

** The recommended value for σ_E (required on bound coefficient variation of design parameter adjusted fuel economy) is 0.02. However, EPA may decide to set a more or less stringent requirement.

The final outputs of the estimation procedure are:

$$\hat{a}_c, \hat{b}_c, \overline{W}, \hat{\sigma}^2_{a_e}, \hat{\sigma}^2_{b_e}, \hat{\sigma}^2_{0_e} \qquad \text{(FTP)}$$

$$\hat{a}_H, \hat{b}_H, \overline{W}, \hat{\sigma}^2_{a_H}, \hat{\sigma}^2_{b_H}, \hat{\sigma}^2_{0_H} \qquad \text{(HFET)}$$

The manufacturer's estimated weight sensitivity coefficients are then represented by

$$S\hat{W}C = \hat{a}_c + \hat{b}_c\ (W-\overline{W})$$

$$S\hat{W}H = \hat{a}_H + \hat{b}_H\ (W-\overline{W})$$

with variances:

$$\hat{\sigma}^2_{S\hat{W}C} = \hat{\sigma}^2_{\hat{a}_c} + (W-\overline{W})^2\ \hat{\sigma}^2_{\hat{b}_c}$$

$$\hat{\sigma}^2_{S\hat{W}H} = \hat{\sigma}^2_{\hat{a}_H} + (W-\overline{W})^2\ \hat{\sigma}^2_{\hat{b}_H}$$

3.4.2.2 Axle Ratio

Repeat the procedure described in Sections 3.4.2.1.1 through 3.4.2.1.5, substituting axle ratio for test weight. Thus, comparable subconfiguration groups are formed on the basis of members differing only in axle ratio; pooled axle ratio sensitivity coefficient data sets are formed; and weighted least squares linear regression is carried out to estimate linear fit parameters and their variances.

3.4.2.3 Road Load Horsepower

Repeat the procedure described in Sections 3.4.2.1.1 through 3.4.2.1.4 substituting road load horsepower for test weight.

3.5 Modified Draft Procedure if Sensitivity Coefficients are Assumed to be Constant (Parameter-Independent)

As previously discussed in Section 3.2.3 the assumed linear dependence of true design parameter sensitivity coefficient on parameter value has not received definitive statistical confirmation. If the alternative assumption of no dependence is made, then the whole procedure of sensitivity coefficient estimation, by EPA as well as by an individual manufacturer, would be much simplified. This section presents the modifications that could then be made to the draft procedure described in section 3.4.

3.5.1 Data Requirements

No procedural changes are indicated. However, the data set size requirement, n_i, will be smaller (See Section 3.3.1).

3.5.2 Design Parameter Sensitivity Coefficient Estimation

The manufacturer continues to have the option of challenging any design parameter sensitivity coefficient for FTP or HFET fuel economy, independently of the others. The modified estimation procedure is presented in terms of test weight design parameter, paralleling Sections 3.4.2.1 through 3.4.2.5. However, exactly the same modifications apply to the other two design parameters, axle ratio, and RLHP.

No changes are indicated in Sections 3.4.2.1 through 3.4.2.3 except that the data sets need no longer include parameter values, i.e., they are of the form:

$$\{ SWC_i, \;\; u_i \} \qquad\qquad i = 1, \ldots, n$$

$$\{ SWH_i, \;\; u_i \}$$

In Section 3.4.2.4, the model for SWC values is

$$SWC_i = \mu_{SWC} + \varepsilon_i$$

where μ_{SWC} is the unknown constant sensitivity coefficient with respect to weight (city) and, as before, ε_i is an additive error with zero mean and with variance

$$\sigma^2_{\varepsilon_i} = \frac{\sigma_0^{\,2}}{u_i}$$

Estimation of μ_{SWC} is performed by weighted averaging. As in the regression case, define

$$v_i = v_i/\Sigma u_i = u_i/U$$

$$\overline{SWC} = \Sigma v_i \; SWC_i$$

$$\overline{SWC^2} = \Sigma v_i (SWC_i)^2$$

Then $\overline{SWC}$ provides an unbiased estimate of μ_{SWC}. An unbiased estimate of $\sigma_0^{\,2}$ for this case is given by

$$\hat{\sigma}_0^2 = \left[\overline{SWC^2} - (\overline{SWC})^2\right] U/(n-1)$$

The variance of the estimate $\overline{SWC}$ is then estimated by

$$\hat{\sigma}^2_{\overline{SWC}} = \hat{\sigma}_0^2/U$$

An analogous set of values is obtained from grouping the corresponding HFET results. Specifically, the weighted average $\overline{SWH}$ is obtained, which provides an unbiased estimate of μ_{SWH}, the sensitivity coefficient with respect to weight (highway). The variance of this estimate is itself estimated by the quantity $\hat{\sigma}^2_{\overline{SWH}}$.

In Section 3.4.2.5 the test for adequacy of n is revised to:

$$n \geq 2\left(\frac{\hat{\sigma}_0}{\sigma_E}\right)^2$$

since $f \equiv 1$ in this alternative estimation context. Otherwise, the procedure for determining Δn and second stage estimation is unchanged.

The final outputs of the estimation procedure are:

$$\overline{SWC},\quad \hat{\sigma}^2_{\overline{SWC}},\quad \hat{\sigma}^2_{0_C}$$

$$\overline{SWH},\quad \hat{\sigma}^2_{\overline{SWH}},\quad \hat{\sigma}^2_{0_H}$$

4. FUEL ECONOMY ADJUSTMENT TO REFLECT IN-USE EXPERIENCE

Following fuel economy label adjustment for vehicle design parameter differences, as described in Section 3, the adjusted values still represent chassis dynamometer fuel economies (FTP and HFET). In order to achieve label values that are more meaningful to the public, EPA is developing a transformation of dynamometer-based fuel economy values to correspond, on the average,* to road, i.e., actually realized in-use, fuel economies. Two factors, α_C and α_H are envisioned which multiply FTP and HFET fuel economies, respectively, to yield finally adjusted label values of "city" and "highway" mpg for each vehicle configuration. In view of the demonstrated shortfall of in-use mpg relative to EPA (dynamometer) mpg, on the average, both factors are expected to be smaller than 1.

In its initial implementation phase EPA plans that α_C and α_H would be two fixed numbers uniformly applied to all light duty vehicle configurations and *a fortiori* to all manufacturers. The possibility that dynamometer-to-in-use mpg scaling is substantially different over major vehicle design categories is also under investigation, and a possibility for the future is that sets of distinct (α_C, α_H) factors may be developed based on: diesel vs. spark ignition, front vs. rear wheel drive, trucks vs. cars, manual vs. automatic transmission, and/or other groupings shown to significantly affect the factors.

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* The meaning of "average" adopted by EPA[2] is the median. The rationale is to insure that equal numbers of in-use vehicles perform above and below their adjusted label fuel economies regardless of asymmetries in the distribution of road mpg. If distributions are symmetric, then the arithmetic mean and median are identical.

Another possibility is that a particular manufacturer could argue that its own vehicles are distinctly different as a class. Therefore, manufacturers should have the option of substituting alternative adjustment factors applicable to their own vehicles, so long as they are able to demonstrate that such factors satisfy appropriate criteria.

Selection of these criteria demands a careful enunciation of the intent of any protocol dealing with manufacturer-specific adjustment factors as opposed to those promulgated by EPA. The view taken in this report is that whether the manufacturer-estimated factors are significantly different from the EPA-promulgated values is not an issue, nor does it need to be. Rather than considering manufacturer-specific factors in a hypothesis-testing context, one simply requires that the factors be estimated to some specified level of precision consistent with the aims of the fuel-economy labeling program.

First, some general methodology for determining road adjustment factors based on in-use surveys is presented. This includes consideration of various environmental influences on in-use fuel economy and stratification methods to reflect these influences. The general structure of a manufacturer-specific data set together with criteria to be met and procedures to be followed is then described.

4.1 Road Adjustment Factor Estimation from In-Use Surveys

By some implemented survey mechanism, responses are received relative to the in-use experience of individual vehicles over limited driving intervals. These responses may provide all or some of the following information:

- Location of driving
- Time of year

- Vehicle identification (which enables determination of EPA fuel economy label values)
- Sequence of fuel purchases (gallons) (needed to calculate in-use fuel economy)
- Corresponding odometer readings
- Purchase dates
- Estimated number and lengths of trips
- Estimated percent split between urban and non-urban driving.

Because of the strong dependence of in-use fuel economy on the split between urban and non-urban driving, as well as the decision to compute separate adjustment factors for these two modes, it is essential to have some measure, either direct or indirect, of their relative proportions. We therefore assume that the following data are available for each sampled vehicle:

- EPA city fuel economy, C mpg [$c = 1/C$ gpm]
- EPA highway fuel economy, H mpg [$h = 1/H$ gpm]
- In-use fuel economy, R mpg [$r = 1/R$ gpm]
- Urban (city) fraction of total driving, u; $0 \leqq u \leqq 1$

As previously stated, u might be directly estimated by the respondent, derived from data on trip length or miles per day or computed as a weighted average of several such estimates.

Environmental factors, notably ambient temperature, wind speed, road grade, road surface condition, and degree of traffic congestion, can have an appreciable influence on in-use fuel economy.[1] Since knowledge of the location and time of year of each individual return modifies the distribution of environmental factors impinging on the reported driving, the statistical analysis should, strictly speaking, account for this hetereogeneity in the data. As a first approach, we make the simplifying assumption that the survey sample design gives equal probability to each vehicle in the U.S. fleet and each time of year. Then the sample median of individually calculated adjustment factors provides a reasonable estimate of the median road adjustment factor over all U.S. vehicles and all seasons. Recall, as previously remarked, that if the distribution of road adjustment factors is symmetric, then the median and mean parameters are identical. Inasmuch as the actual sample is not likely to conform to the above equal probability assumption, we shall need to address the issue of heterogeneity in the sample space. This will be done in Section 4.2.

We now consider two alternative methods of estimating average road adjustment factors for both city and highway driving.

The first method, which is that described in EPA draft documents,[2,3] extracts two extreme subsets of "nearly pure" highway driving and city driving respectively from the totality of responses. This is done by requiring $u \leq U_0$ and $u \geq U_1$, respectively, where U_0 is close to 0 and U_1 is close to 1. Specific cut-off values initially selected were $U_0 = 0.2$ and $U_1 = 0.9$.[2] Within subset $\{u_i \leq U_0\}$, the ratio $\alpha_{H_i} = R_i/H_i$ is computed for each response and the median $\tilde{\alpha}_H$ is designated the road adjustment factor for highway driving. Similarly, within subset $\{u_i \geq u_1\}$, the ratio $\alpha_{C_i} = R_i/C_i$ is computed for each response and the median $\tilde{\alpha}_C$ is designated the road adjustment factor for city driving.

An objection to the method just described is that it fails to use most of the survey responses. Ideally, u_0 should be very close to 0 and u_1 very close to 1 in order to generate subsets of reasonably pure highway driving and city driving, respectively. On the other hand, the closer these ideal cut-off limits are approached, the fewer are the responses actually utilized.*

We propose the following model which permits an alternative method of estimating α_H and α_C based on all of the data. Let S_i and T_i designate the (unknown) city and highway in-use fuel economies (in mpg) for the i^{th} response (a specific car at a specific time of year) in the survey data set. Let s_i and t_i designate the corresponding reciprocal fuel consumptions (in gpm). Then

$$r_i = u_i s_i + (1 - u_i) t_i$$

assuming r_i and u_i are accurately reported. Actual (r_i, u_i) data will, of course, introduce an error term. We can also write

$$\alpha_{H_i} = \frac{T_i}{H_i} = \frac{h_i}{t_i} = \alpha_H + \Delta\alpha_{H_i}$$

$$\alpha_{C_i} = \frac{S_i}{C_i} = \frac{c_i}{s_i} = \alpha_C + \Delta\alpha_{C_i}$$

where α_H and α_C are the average (median) road adjustment factors which we wish to estimate. Note, therefore, that

$$\operatorname*{med}_i (\Delta\alpha_{H_i}) = \operatorname*{med}_i (\Delta\alpha_{C_i}) = 0$$

---

* It is, of course, possible to use the intermediate u (mixed driving) responses as some kind of check on the estimates derived. There is no indication that this was done, nor is it clear how one would adjust the estimates in the light of inconsistencies found.

It follows that

$$t_i = \frac{h_i}{\alpha_H + \Delta\alpha_{H_i}} = \frac{h_i}{\alpha_H} + \Delta t_i$$

$$s_i = \frac{c_i}{\alpha_C + \alpha_{C_i}} = \frac{c_i}{\alpha_C} + \Delta s_i$$

where $\operatorname*{med}_i (\Delta t_i) = \operatorname*{med}_i (\Delta s_i) = 0$. Then, by substitution,

$$\frac{r_i}{c_i u_i + h_i(1 - u_i)} = \frac{c_i u_i(\frac{1}{\alpha_C}) + h_i(1 - u_i)(\frac{1}{\alpha_H}) + u_i\Delta s_i + (1 - u_i)\Delta t_i}{c_i u_i + h_i(1 - u_i)}$$

Define the derived data quantities

$$\beta_i = \frac{r_i}{c_i u_i + h_i(1 - u_i)}$$

$$\gamma_i = \frac{c_i u_i}{c_i u_i + h_i(1 - u_i)}$$

and the additive error term

$$\varepsilon_i = \frac{u_i\Delta s_i + (1 - u_i)\Delta t_i}{c_i u_i + h_i(1 - u_i)}$$

This leads to the simplified linear form:

$$\beta_i = \gamma_i\left(\frac{1}{\alpha_C}\right) + (1 - \gamma_i)\left(\frac{1}{\alpha_H}\right) + \varepsilon_i$$

$$= \frac{1}{\alpha_H} + \left(\frac{1}{\alpha_C} - \frac{1}{\alpha_H}\right)\gamma_i + \varepsilon_i; \quad 0 \leq i \leq 1$$

The result suggests that the quantities α_H and α_C may be estimated by linear regression of β_i on γ_i. However, one must be cautious about applying conventional least squares since that method assumes the mean error to be zero, and the mean behavior of ε_i is not known.

A reasonable assumption to make about the component deviations Δt_i and Δs_i is that they are highly positively correlated in sign. For example, an in-use highway fuel consumption above that predicted by the median highway road adjustment factor would seem to imply (for the same car in the same environment) an in-use city fuel consumption also above that predicted by the median road adjustment factor. If this assumption holds, it follows that

$$\operatorname*{med}_i (\varepsilon_i) \cong 0.$$

Some form of median linear regression of β_i on γ_i would then be appropriate. The iterative method described by Mood[4] is suggested.

We outline the method, but refer the reader to the reference for additional details. (See Figure 1 for an illustrative application.) Compute $\tilde{\gamma} = \operatorname*{med}_i (\gamma_i)$. Partition the data set into subsets S_- and S_+

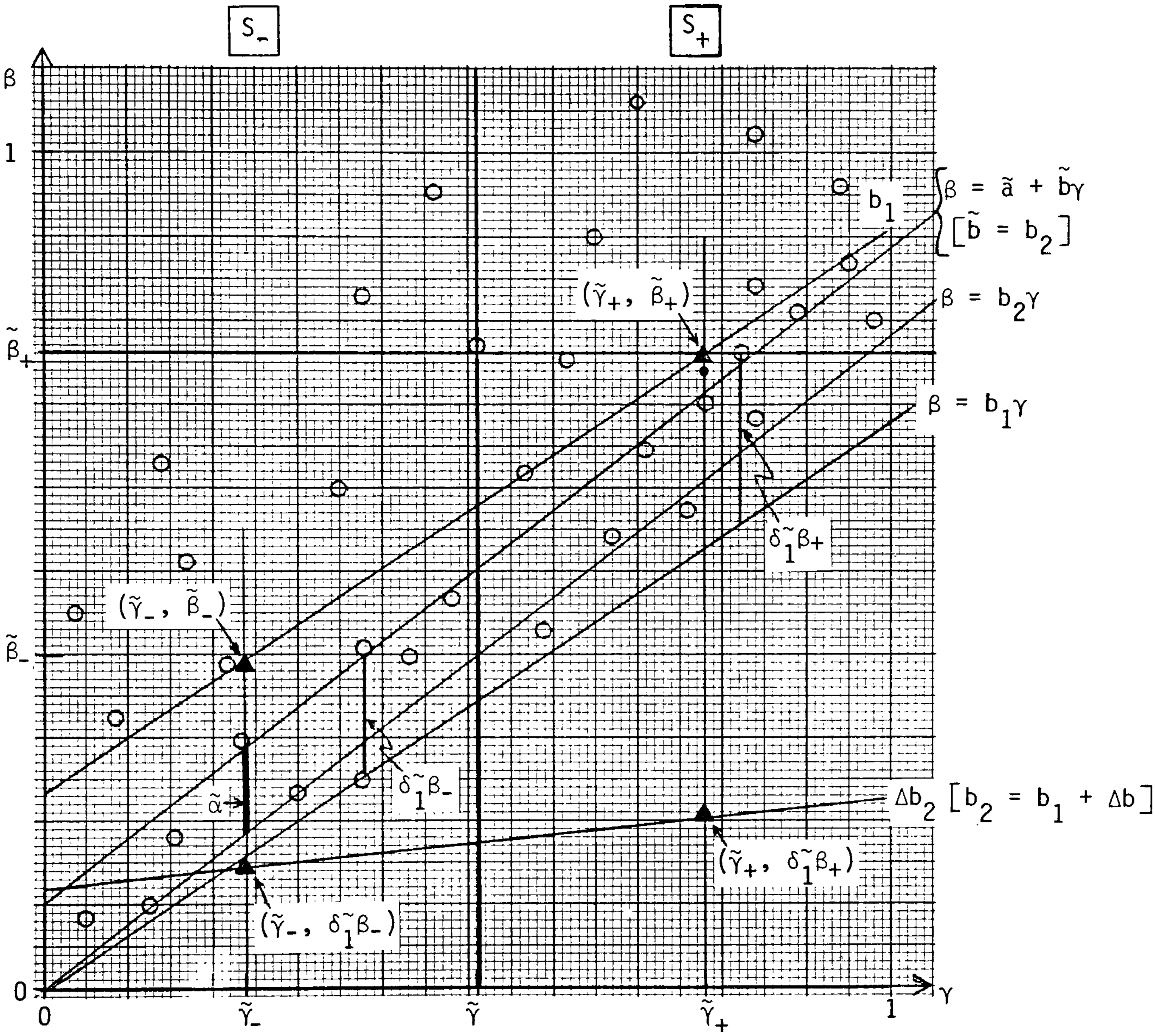

FIGURE 1. Illustrative Median Regression Procedure

to the left and right of $\tilde{\gamma}$, respectively. That is, $\gamma_i < \tilde{\gamma}$ implies (γ_i, β_i) is in S_-, etc. Determine the median of γ and β in each of these subsets: $(\tilde{\gamma}_-, \tilde{\beta}_-)$, $(\tilde{\gamma}_+, \tilde{\beta}_+)$. Compute the slope b_1 of the line joining these two points. Compute the deviations $\delta_1\beta_1$ from the line $\beta = b_1\gamma$. Compute the left and right medians of these deviations $\delta_1\tilde{\beta}_-$, $\delta_1\tilde{\beta}_+$. Compute the slope of the line joining $(\tilde{\gamma}_-, \delta_1\tilde{\beta}_-)$ to $(\tilde{\gamma}_+, \delta_1\tilde{\beta}_+)$. Add this slope to b_1 to yield a second approximation to the desired slope estimate b_2. Compute the deviations $\delta_2\beta_i$ from the line $\beta = b_2\gamma$. Proceed as before to compute a third slope estimate b_3. Continue this iteration to the desired degree of accuracy. Denote the final slope estimate by $\tilde{b}$. (In the illustration in Figure 1, the iteration stops at b_2.) The estimate for the intercept $\tilde{a}$ is then the median of the final total set of deviations. The final estimates achieved will have the property that

$$\operatorname*{med}_{i\epsilon S_-} (\beta_i - \tilde{a} - \tilde{b}\gamma_i) \cong \operatorname*{med}_{i\epsilon S_+} (\beta_i - \tilde{a} - \tilde{b}\gamma_i) = 0$$

which is a necessary condition for the true median line.

Finally, we estimate our desired median road adjustment factors by:

$$\tilde{\alpha}_H = \frac{1}{\tilde{a}}$$

$$\tilde{\alpha}_C = \frac{1}{\tilde{a} + \tilde{b}}$$

Several variations can be introduced as may be deemed appropriate from a preliminary analysis of the data. For one, if the data are partitioned into a moderate number of γ interval subsets, i.e., $[0, \gamma_1]$, $[\gamma_1, \gamma_2]$, ..., $[\gamma_{k-1}, \gamma_k]$, it is possible that the subset medians may show a considerably better fit to some nonlinear form than to a straight line. This would suggest the use of nonlinear regression. Another possibility is that relatively more scatter may be apparent at intermediate γ (away from 0 or 1). This may be due to inherently larger errors in estimation of urban fraction u by respondents who did substantial amounts of both urban and highway driving in contrast to respondents who did mostly one or the other. If such a phenomenon is evident, then one can consider weighted median linear regression which gives more weight to the median points closer to $\gamma = 0$ and $\gamma = 1$. The concept of the weighted median is defined in the next section.

Finally, it should be observed, as previously noted, that a symmetric error distribution implies identical mean and median regression lines. Furthermore, if this distribution is close to normal, then ordinary least squares regression would be preferred since it would produce efficient, i.e., minimum variance, estimates.

4.2 Treatment of Sample Space Heterogeneity

A practical way of accounting for the effects of sample space heterogeneity on a nonequal probability sample is through stratification and relative weighting of responses within each stratum.

Suppose that the important environmental parameters which affect road fuel economy are those previously listed, viz., ambient temperature T, wind speed W, road grade G, road surface conditions S, and degree of traffic congestion C. Now, it is presumed that a detailed sensitivity analysis, e.g., as described in the draft 404 report,[1] has led to a stratification of each environmental parameter into a manageably small number of intervals such that each interval can be viewed as approximately homogeneous. Denote this stratification by the parameter intervals

$$[T_0, T_1], [T_1, T_2], \ldots, [T_{I_T-1}, T_{I_T}]$$

$$[W_0, W_1], [W_1, W_2], \ldots, [W_{I_W-1}, W_{I_W}]$$

$$[G_0, G_1], [G_1, G_2], \ldots, [G_{I_G-1}, G_{I_G}]$$

$$[S_0, S_1], [S_1, S_2], \ldots, [S_{I_S-1}, S_{I_S}]$$

$$[C_0, C_1], [C_1, C_2], \ldots, [C_{I_C-1}, C_{I_C}]$$

The total number of product strata, corresponding to all possible combinations of the five parameters, is of course given by $I = I_T \cdot I_W \cdot I_G \cdot I_S \cdot I_C$. This suggests that the interval numbers $I_T, \ldots, I_C$ need to be as small as possible consistent with the requirements for reasonable homogeneity within strata.

For ease of exposition, assume that the I strata are indexed by i, $1 \leq i \leq I$, in some specified order. Suppose, further, that given the geographic locale x and time of year (say, month) τ of a response, one can quantify the most probable stratum (or alternatively the stratum containing the mean value for each environmental parameter) associated with the reported driving. Denote this stratum by $i(x, \tau)$. Thus, a response from (x, τ) falls into environmental stratum $i(x, \tau)$. If the total number of survey responses is N, this is partitioned by the function $i(x, \tau)$ into N_i responses associated with strata i, $i = 1, \ldots, I$, and $\Sigma N_i = N$. Define $n_i = N_i/N$.

A separate analysis of the actual distribution of registered vehicles over all U.S. locales would yield the fractional distribution over the I environmental strata, P_i, $i = 1, \ldots, I$. That is, the proportion of cars in use throughout the U.S. over the course of a year that are in stratum i is P_i. One should be careful to note that this analysis must take account of the fact that, whereas vehicle registrations are associated essentially with location, any given location can move through a number of different strata with time of year.

If the survey sample design gave equal probability to each car and time of year, then we should find a very close correspondence between P_i and n_i (any differences being a consequence of the random sampling process). In general, we would expect to find substantial discrepancies between P_i and n_i. To compensate for such a biased sample, we associate with every return from stratum i a relative weight $w_i = P_i/n_i$. Note that the w_i will always sum to N over the total set of returns.* Note further, that it is important for all strata to be occupied; in fact, for good performance of this weighting procedure it is desirable that a required minimum number or minimum fraction of returns from each stratum be achieved, alternatively that each w_i be smaller than a preselected bound.

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* The summation here is <u>not</u> over index i (which runs from 1 to I) but over individual response index j. Thus,

$$\sum_{j=1}^{N} w_{i(j)} = \sum_{i=1}^{I} N_i w_i = N \sum_{i=1}^{I} n_i w_i = N \sum_{i=1}^{I} P_i = N.$$

What do these relative weights mean in terms of the procedure for calculating road adjustment factors, which are based on median estimates? The appropriate modification is to calculate weighted medians. Suppose we have a set of N observations and associated weights which sum to N. After ordering according to increasing value of observation, denote these by $x_1 \leq x_2 \leq \ldots, \leq x_N$ with associated weights $w_1, w_2, \ldots, w_N$. Find k such that

$$\sum_{j=1}^{k-1} w_j \leq \frac{N}{2} < \sum_{j=1}^{k} w_j$$

Then the weighted median equals x_k. If strict equality holds on the left, then $\frac{1}{2}(x_{k-1} + x_k)$ is selected, in analogy with the unweighted case.

To recapitulate the results of this section:

(1) The total sample space (all U.S. cars x all times of year) is partitioned into I strata, each representing relatively homogeneous combinations of the significant environmental parameters. The appropriate stratum for each survey response is determinable from its locale and time of year.

(2) The relative frequency of survey responses within each stratum is n_i, $i=1, \ldots, I$.

(3) The (true) relative proportion of the population within each stratum is P_i, $i = 1, \ldots, I$ (determined by separate analysis of vehicle registrations by locale).

(4) Compute $w_i = P_i/n_i$, and use these relative weights (associated with the I strata) to estimate median road adjustment factors $\tilde{a}_H$ and $\tilde{a}_C$ according to one or the other of the two alternative procedures described: (a) weighted median estimation on "pure" highway-driving and "pure" city-driving respondents or (b) weighted median regressions on the reexpressed survey data, $\{\beta_j, \gamma_j\}$, $j = 1, \ldots, N$.

4.3 Confidence Intervals for Medians

Suppose that the road adjustment factors are estimated directly as medians of separate univariate samples for city and highway driving, viz., $\{\alpha_{C_i}\}$ and $\{\alpha_{H_i}\}$. Under the assumption of large sample size N, the probability that k observations fall below the true median is approximately normal with mean N/2 and standard deviation $\sqrt{N}/2$. Hence, a one-sided p-confidence interval is obtained by counting $z_{1-p} \cdot \sqrt{N}/2$ indices up (or down) from N/2 and noting the observation value at that index within the ordered list of observations. (z_{1-p} is the standard normal variate with tail probability = 1-p). Thus, for example, a 90% lower confidence bound ($z_{.1} = 1.28$) on the true median city road adjustment factor in a sample of 1000 would be given by the $[500 - (1.28)(31.62)/2]^{th} = 480^{th}$ ordered value, i.e., by $\alpha_{C_{(480)}}$. Even if weighted medians are estimated, as previously described, to compensate for biased sample space heterogeneity, the above confidence bound estimation procedure is generally still applicable. However, if the heterogeneity and sample bias are so large as to cause the median to be estimated by an order statistic $\alpha_{(j)}$ where $j < 0.2\ N$ or $j > 0.8\ N$, the confidence interval problem would have to be investigated more carefully.

Alternatively, if median linear regression on the total survey data set is performed, as described above, then we need to develop intermediate confidence bounds on a, the median line intercept (at $\gamma = 0$), and on $a + b$, the median line value at $\gamma = 1$. By merely taking reciprocals, we would then obtain corresponding confidence bounds on α_H and α_C, respectively. An approximate procedure for a one-sided bound suggested by Mood's discussion of the confidence interval problem[4] is as follows. The estimated median regression line, as schematically shown in the figure, partitions the total set of N points into four approximately equal subsets.

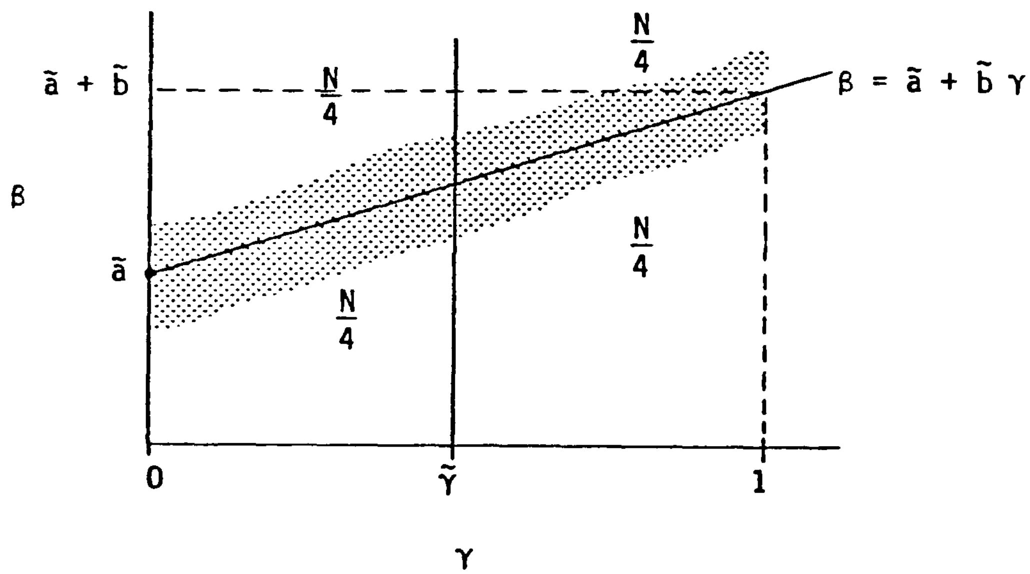

FIGURE 2

Suppose we desire to estimate a one-sided upper p-confidence bound on intercept $\tilde{a}$ (the value of β at $\gamma = 0$). Rotate the line clockwise about its $\tilde{\gamma}$ point until $N/4 - Z_{1-p} \cdot \sqrt{N}/2$ data points remain in the upper left region. Now translate the whole line in the upward vertical direction until $Z_{1-p} \cdot \sqrt{N}/2$ additional data points have crossed from the two upper into the two lower regions. The new intercept is taken to be $a_{U,p}$ the upper p-confidence bound on a. An analogous procedure involving first, counter-clockwise rotation of the line followed by upward translation would establish $(a + b)_{U,p}$ the upper p-confidence bound on $a + b$ (the value of β at $\gamma = 1$). If it is believed by symmetry considerations ($\tilde{\gamma} \cong \frac{1}{2}$ and comparable dispersion on left and right sides of $\tilde{\gamma}$) that the magnitude of the increase from $\tilde{a}$ to $a_{U,p}$ should equal the magnitude of the increase from $\tilde{a} + \tilde{b}$ to $(a + b)_{U,p}$, then these two differences could be averaged to provide symmetrically estimated upper confidence bounds.

The procedure described above is believed to provide a conservative estimate of confidence intervals/bounds. An exact method is not known.

As previously noted, we can now assert that

$$(\alpha_H)_{L,p} = \frac{1}{a_{U,p}}$$

$$(\alpha_C)_{L,p} = \frac{1}{(a + b)_{U,p}}$$

That is, the lower p-confidence bound on the median highway road adjustment factor $\tilde{\alpha}_H$ is given by the upper p-confidence bound on the $\gamma = 0$ intercept a of the median regression line. Similarly for $\tilde{\alpha}_C$. These regressions follow from the fact that

$$p = \Pr\{a_{U,p} \geq a\} = \Pr\left\{\frac{1}{a_{U,p}} \leq \frac{1}{a}\right\} = \Pr\left\{\frac{1}{a_{U,p}} \leq \alpha_H\right\},$$

and similarly for $(a + b)_{U,p}$.

4.4 Basic Considerations for Individual Manufacturer Procedure

EPA plans to promulgate two standard road adjustment factors (for FTP and HFET fuel economy, respectively) based on in-use experience of a representative survey sample of vehicles covering all light-duty automotive manufacturers. In the absence of evidence to the contrary, it is assumed that there are no substantial statistical differences among manufacturers with respect to the relationship of dynamometer to in-use fuel economy. This is the basis for adopting single (industry-wide) factors. An individual manufacturer may have reason to believe that the EPA factors do not apply to its own vehicles. In that case, it can propose to use alternative adjustment factors derived strictly from survey of in-use experience of its own vehicles to replace either one or both of the EPA factors. In order to be accepted by EPA, such manufacturer-specific road adjustment factors must meet certain accuracy and representativeness requirements. The procedures that will be presented have been formulated to ensure that that happens. It is appropriate at this point to consider first how the accuracy and representativeness requirements were developed.

4.4.1 Accuracy

One might argue that there is a fundamental limitation in the accuracy with which road adjustment factors can be determined because of the impossibility of defining truly objective classes of urban (city) and highway driving conditions. An alternative point of view which has much merit is that what a respondent reports as his mpg and his mix of driving conditions represents the reality to which the adjustment factors should relate. Thus, if the respondent says he did 90% urban driving and his average mpg (derived from his numbers) was 18.2, we should accept these numbers at face value. It may nevertheless be desirable, even in this context, to provide each respondent with a simple qualitative definition of "urban driving." Such an approach would reduce the chances of gross misinterpretation while still accommodating individual perceptions.

Adopting the above position, we see that the problem of accuracy is associated only with the sampling process. Let x be the (city or highway) road adjustment factor for an individual vehicle, and let x have distribution $F(x)$ over the total vehicle population. The median $F(x)$, denoted by α, is the quantity we wish to determine. If a truly random sample of N vehicles is obtained (i.e., every vehicle having equal probability of being selected) yielding individual factors x_1, x_2, ..., x_N, and we estimate α by the sample median $\tilde{\alpha}$, then what can be said about the accuracy of this estimate? It is known that for large sample sizes, $\tilde{\alpha}$ is approximately normally distributed about α with variance[5]

$$\sigma^2_{\tilde{\alpha}} = \frac{1}{4Np^2(\alpha)}$$

where $p(x) = F'(x)$ is the density function of the population distribution. From published data on in-use to EPA fuel economy ratios, it appears that x is centrally distributed mostly within the range between 0.5 and 1, and not greatly skewed. From this we estimate that a conservative lower bound on $p(\alpha)$ is 2. It follows that

$$\sigma^2_{\tilde{\alpha}} \leq \frac{1}{16N}$$

A requirement on minimal sample size can now be established directly in terms of desired level of precision in estimating the true population median road adjustment factor. Observe that $\sigma_{\tilde{\alpha}} \leq 0.02$ is equivalent to a 2% or smaller contribution to the coefficient of variation in the finally adjusted fuel economy value. It can be argued, in the same way as was done in connection with design parameter adjustment factors, that 2% added (by sum of squares) to the basic 4% coefficient of variation in fuel economy measurement results in very little increase in total error. It is therefore recommended that the requirement

$$\sigma_{\tilde{\alpha}} \leq 0.02$$

be adopted. This implies the following requirement on sample size (which applies individually to the sets of "pure" urban driving responses and "pure" highway driving responses),

$$N \geq 156.$$

If the median linear regression procedure is adopted in order to be able to utilize all survey responses regardless of reported urban percentage, then an f factor analogous to that developed in Section 3 needs to be applied. On the basis of survey data already accumulated[2,3] it is reasonable to assume a fairly flat distribution of responses over "percent urban driving." The applicable value of f is 2 and the modified recommended requirement for N is

$$N \geq \frac{f^2}{\sigma^2_{\tilde{\alpha}}} = 625.$$

Note that the requirement of a total survey response of 625 is probably less stringent than the requirements of 156 each at the urban and highway driving extremes.

4.4.2 Representativeness

In the preceding section it was assumed that the sample is randomly selected. As discussed earlier, known environmental factors very substantially influence in-use fuel economy and it is deemed necessary to stratify by these factors (as determined by locale and season) to make it possible to correct for any nonuniformities in the sample. Unfortunately, practicality dictates a moderate number of strata, say no more than 20, and, within any single stratum, environmental variations may still be large enough to have an appreciable differential effect. On the other hand, it is believed that biased sampling is much less likely to occur (intentionally or spuriously) within individual strata. Establishment of quantitative procedural requirements that would limit the extent of sampling bias to some prescribed level is not possible without additional details of environmental distributions and effects. What can be done at this point in time is to ensure a reasonably uniform probability of representation among strata by placing an upper bound on the relative weights w_i. Ideally, $w_i = 1$ for all i. It is recommended that none be permitted to exceed relative weight 2 when calculated on the basis of minimum N requirement.

That is,

$$w_i = \frac{P_i}{\frac{N_i}{N_0}} < 2$$

where N_0 is the minimum requirement (not the actual number of responses, which may be greater). Thus,

$$N_i > \frac{P_i N_0}{2}$$

For example, if $N_0 = 625$ and $P_i = 0.05$, then at least 16 returns should come from stratum i. If that number has not been achieved, then additional returns are required. This increases N, but the requirement that $N_i > 16$ remains unchanged.

4.5 Draft Procedure for Individual Manufacturer Road Adjustment Factors

This section presents a draft procedure for individual manufacturers who wish to take exception to the standard road adjustment factors promulgated by EPA. It includes data requirements and estimation of road adjustment factors.

4.5.1 Data Requirements

(1) All of the manufacturer's vehicle classes are to be covered in the survey. That is, the manufacturer is not permitted to deliberately exclude particular configurations or model types. Generally, the survey conducted during a given calendar year will be restricted to the most recent model year cars in order to limit the range of odometer mileage. However, surveyed vehicles should have accumulated at least 2000 miles.

(2) Each return should include as a minimum:

(a) Information which enables precise determination of vehicle fuel economy label values.

(b) Location of driving (zip code may be adequate).

(c) Time of year (month may be adequate).

(d) Three successive fuel purchases in gallons (motorist instructed to "top off" tank each time, and to wait until tank is at least half-empty on 2nd and 3rd fill-up).

(e) Corresponding odometer readings.

(f) Corresponding dates of purchases.

(g) Estimate of percentage urban driving.

(3) The sampling plan should be designed to make a reasonable effort at fair representation of all regions of the U.S. and all seasons.

(4) (a) If highway road adjustment factor is to be estimated from "pure" highway driving responses (i.e., $u \leq U_0$), then the total number of such valid responses received shall equal at least N_0.*

(b) If city road adjustment factor is to be estimated from "pure" city driving responses (i.e., $u \geq U_1$), then the total number of such valid responses received shall equal at least N_0.*

(c) (Alternative to (a) and (b)). If highway and/or city road adjustment factors are to be estimated by the median line regression procedure, then the total number of valid responses received shall equal at least N_0'.*

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* The recommended values of N_0 and N_0' based on a precision requirement of 2% are 156 and 625, respectively.

4.5.2 Adjustment Factor Estimation

(1) The responses are to be classified in accordance with the set of I environmental strata defined by EPA, based on the functional relationship of stratum to location and time of year of driving to be supplied by EPA.

(2) The relative frequencies of responses from the I strata $n_i = N_i/N$, $i = 1, \ldots, I$, are calculated and, using EPA supplied data on true proportions, P_i, the relative weights $w_i = P_i/n_i$ are calculated.

(3) In order to preclude excessive deviation from equal probability sampling which could raise questions about the validity of the relative weighting procedure, for each stratum, N_i is required to exceed $\frac{1}{2} P_i N_0$ (effectively a relative weight less than 2). If N_i is insufficient, then additional returns must be obtained from stratum i until the requirement is met.

(4) (a) Compute individual highway road adjustment factors $\alpha_{Hj} = R_j/H_j$ for those responses with $u < u_0$, where R_j is in-use average mpg* and H is HFET fuel economy of the vehicle subconfiguration. Compute the weighted median $\tilde{\alpha}_H$ using the relative weight for each response based on the stratum in which it is classified. The manufacturer-specific road adjustment factor for HFET (highway driving) fuel economy is set equal to $\tilde{\alpha}_H$.

* R_j may need to be adjusted for variable mileage accumulation effects by use of a standard formula.

(b) Compute individual city road adjustment factors α_{C_j} = R_j/C_j for those responses with $u \geq u_i$, where R_j is in-use average mpg* and C_j is FTP fuel economy of the vehicle subconfiguration. Compute weighted median $\tilde{\alpha}_C$ road adjustment factor for FTP fuel economy in a manner analogous to (a) above.

(c) (Alternative to (a) and (b)). Compute derived data, quantities β_j and γ_j and perform a weighted median linear regression as described in Sections 4.1 and 4.2. The outputs are estimates $\tilde{\alpha}_H$ and $\tilde{\alpha}_C$, HFET and FTP road adjustment factors, respectively. If this method is used, then both EPA standard factors are replaced by manufacturer-specific factors.

(5) If a manufacturer establishes its own specific road adjustment factor(s), then annual resurveys meeting the data requirements of Section 4.5.1 are required to update the factor(s) in succeeding years. Data for the three (or fewer) most recent annual surveys are pooled and the estimation procedures as described above are repeated. Failure to conduct a proper survey would cause denial of the manufacturer's petition for specific alternative factor(s) and reversion to EPA standard factors.

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* R_j may need to be adjusted for variable mileage accumulation effects by use of a standard formula.

5. ADDITIONAL ISSUES

5.1 Computation of Design Parameter Adjusted Fuel Economy

If only a single tested subconfiguration is comparable to a given untested subconfiguration, then the procedure for estimating its fuel economy based on design parameter adjustment is quite clear. The total differential formula in Section 3.1 leads to:

$$E' = E\ (1 + S_w\Delta w + S_r\Delta r + S_a\Delta a)$$

where E is the tested subconfiguration fuel economy

E' is the adjusted fuel economy for the untested subconfigurations

S_w, S_r, S_a are the applicable sensitivity coefficient values

Δw, Δr, Δa are the fractional design parameter increments from the tested to the untested subconfiguration.

If the two subconfigurations differ only in test weight, for example, then $\Delta r = \Delta a = 0$ and the formula is accordingly simplified.

Suppose, however, that we have K tested subconfigurations all comparable to a given untested subconfiguration. What is the best way to proceed? One suggestion that has been made is to adjust only from the "closest" test result. Aside from the problem of how to measure "closeness" in three-dimensional parameter space, it seems that such an approach simply ignores valuable information. The approach recommended here is that all K adjustments should be made and the final result computed as a weighted mean of the K individual estimates.

Let

$$E'_k = E_k\,(1 + S_w \Delta w_k + S_r \Delta r_k + S_a \Delta r_a) \qquad k = 1, \ldots, K$$

be the kth adjusted fuel economy estimate for the untested subconfiguration. Note that the sensitivity coefficients (assumed to be linear functions of their parameters) are evaluated at the untested subconfiguration point in parameter space. This is an adequate approximation to the mean parameter values for small increments. Assuming independence of error contributions, we may then estimate the variance of E'_k/E_k (equal approximately to the squared coefficient of variation of E'_k) as:

$$\sigma_k^2 = \sigma_0^2\left(\frac{1}{m_k} + (\Delta w_k)^2\,\sigma^2_{S_w} + (\Delta r_k)^2\,\sigma^2_{S_r} + (\Delta a_k)^2\,\sigma^2_{S_a}\right)$$

where σ_0 is the coefficient of variation in subconfiguration fuel economy measurement, m_k is the multiplicity of the test results that were averaged to estimate E_k, the kth subconfiguration fuel economy. The sensitivity coefficient variances are determined from the data used to estimate the coefficients (see Section 3.5.2.1.4). As was done in Section 3, define

$$u_k = \sigma_0{}^2/\sigma_k{}^2, \qquad U = \Sigma u_k$$

and

$$v_k\ \cancel{u'_k} = u_k/U$$

The v_k $\cancel{u'_k}$ represent appropriate weighting coefficients to use for estimating the untested subconfiguration fuel economy with maximum precision*,

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* This property depends on the k estimates being uncorrelated. In general, there can be substantial correlation due to the presence of the same sensitivity coefficients in different E_k estimates. An exact calculation of the proper weights could be made, but the improved precision is not believed to justify the added complexity entailed.

viz.,

$$E' = \Sigma\, v_k\, E'_k$$

The coefficient of variation of E' is

$$\sigma_0{}' = \sigma_0/\sqrt{U}$$

5.2 Regional/Seasonal Adjustment Factors

EPA is placing a great deal of emphasis on the achievement of realistic fuel economy numbers which people can associate with automotive vehicles. Implementation of design parameter and road adjustment factors for fuel economy labeling will constitute a major advance toward this objective. However, these alone cannot be sufficient because the final adjusted values will still represent an average for the whole country (and over all times of the year). When a person buys a car he specifies a particular subconfiguration. That is where the label value plays a crucial role. But he also restricts (and particularizes) the range of numerous influential environmental factors by virtue of his specific location. The national/yearly average of 22 mpg city label value for car X doesn't apply to that car driven in Phoenix AZ during the summer, nor to another copy in use in Duluth MN in January.

An obvious solution to this problem is to provide the general public with a set of regional/seasonal adjustment factors. Given a selected location and time of year, one looks up the indicated factor which is then multiplied into the label value of a car to yield an adjusted fuel economy that represents the conditional median value for the subconfiguration at the selected location and time of year. It is, of course, recognized that even after such adjustment, much uncertainty still remains--associated with such factors as small scale environmental detail, trip characteristics, and driver aggressiveness. Nevertheless, major environmental factors do have a very substantial influence and are at the same time strongly correlated with location and time of year.

Major practical questions relative to implementation of the concept are: how to compute such regional/seasonal factors and to what level of spatio-temporal detail?

With respect to the first question, one could attempt detailed analyses of environmental factor effects on fuel economy as, for example, presented in Reference 1 and couple the results with data on the distributions of these environmental conditions. The results could be presented in the form of U.S. maps of constant adjustment factor contours for different months or seasons of the year. Difficulties with this approach stems from the complexity of the analyses required, lack of knowledge regarding interaction of effects, and data requirements for environmental factor distributions.

An alternative approach which circumvents much of the above analyses yet leads to direct and meaningful results would make use of the I strata already developed for the in-use fuel economy survey design. Essentially all that need be done is to estimate separate median road adjustment factors for each stratum, i.e., $(\tilde{\alpha}_{C_i}, \tilde{\alpha}_{H_i})$, $i = 1, 2, \ldots, I$. These overall factors are then normalized by the overall road adjustment factors to yield correction factors per stratum:

$$\begin{aligned} f_{C_i} &= \tilde{\alpha}_{Ci} \big/ \tilde{\alpha}_C \\ f_{H_i} &= \tilde{\alpha}_{Hi} \big/ \tilde{\alpha}_H \end{aligned} \qquad i = 1, \ldots, I$$

Identification of the appropriate stratum for a given place and time of year could be provided by a series of seasonal maps or by a suitable tabulation.

The direct empirical validity of the f correction factors so derived should be clear. On the other hand, it is also recognized that one can no longer view the f's as consequences of just environmental influences. If, for example, drivers in one particular stratum just so happen to be very aggressive, on the average, in comparison to other strata, then that fact will be reflected in reduced f factor values.

In the final analysis, stratification of the in-use survey sample space is a means of discriminating systematically different locales and times of year. Judicious choice of strata boundaries or definitions can maximize the differences among the strata and minimize the spread of fuel economy variations within each stratum. By giving the public access to the differences so determined, EPA would be taking another significant step toward the provision of realistic fuel economy numbers which are as specifically applicable as possible.

5.3 Test and Parameter Adjustment Strategies

If a manufacturer is not satisfied with the label values derived for some of its untested subconfigurations by application of the EPA standard design parameter sensitivity coefficients, it has two alternatives:

(1) Estimation of manufacturer-specific sensitivity coefficients by procedure described in Section 3.

(2) Direct FTP/HFET fuel economy tests of the subconfiguration(s) in question.

Alternative (2) should always be available to the manufacturer inasmuch as it is fully consistent with the planned new labeling procedures. Under these procedures mandated* test subconfigurations would have their label values determined directly by the test results. If the number of untested subconfigurations which the manufacturer believes to be under-rated by EPA sensitivity coefficient adjustment is small, then the least costly strategy would likely be to test these subconfigurations directly. It is, of course, possible that a large manufacturer may have enough of its own comparable mandated test subconfigurations to carry out the manufacturer-specific sensitivity coefficient estimation procedure without having to introduce additional test data. This is even more likely if EPA adopts the constant sensitivity coefficient model. In that case the manufacturer would most certainly check out his own sensitivity coefficient estimates first. If he doesn't like their implications for some of his untested subconfigurations there would seem to be no way to prevent him from selectively testing those subconfigurations for the purpose of establishing direct test fuel economy label values.

It would appear, then, that manufacturers will have considerable flexibility in establishing fuel economy label values if they are willing to pay the price. They could, in effect try out each of the three available alternatives for their untested subconfigurations and choose the largest value. Inasmuch as there are random errors in all of the alternatives, this would be tantamount to introducing a positive bias in fuel economy labels of the non-mandated test subconfigurations.

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* As presently required for emission certification or fuel economy label and CAFE determinations.

It is recommended that EPA carefully review the process by which manufacturers would be permitted to take exception to proposed EPA standard design parameter sensitivity coefficients in order to preclude biasing of fuel economy label values. This review should also consider the question of whether to maintain the present specificity of mandated vehicle tests at the configuration level or to raise it to the subconfiguration level.

REFERENCES

[1] EPA, "Passenger Car Fuel Economy: EPA and Road," Draft Report in response to Section 404 of PL95-619, January 1980.

[2] EPA Informal Memo, "ECTD Deliverables--Fuel Economy Information Rulemaking," K. H. Hellman, dated 7/2/80 on Cover Sheet.

[3] EPA Memo, "Extremes Analyses of Ford In-Use Data Base," K. H. Hellman, 9/3/80.

[4] A. M. Mood, Introduction to the Theory of Statistics, McGraw-Hill, New York, 1950, pages 406-408.

[5] E. J. Gumbel, Statistics of Extremes, Columbia University Press, New York, 1958.

APPENDIX A

VARIABILITY OF DYNAMOMETER FUEL ECONOMY MEASUREMENTS

The fuel economy label given to a car is generally based on unreplicated dynamometer tests and hence subject to potentially significant errors. These are decomposable into: (1) test repeatability errors within a fixed test cell (dynamometer + driver + CVS apparatus), (2) differences among test cells, (3) differences among vehicles of same configuration, and (4) heterogeneity of the vehicle configurations which are aggregated into a single fuel economy label value. In this note we focus on (1) through (3). Components (1) and (2) are often loosely lumped together as "measurement variability." Some published estimates for fuel economy error standard deviations are shown in Table A-1. There seems to be considerable disparity among investigators, with no clear historical trend. Within-test-cell σ's under the carbon balance method range from 1.2% to 4.8%. ASTM estimates that metered mpg determination can reduce this error to 0.75%.

The correlation study by Sheth and Rice on five dynamometer test cells suggests that the additional contribution due to between-test-cell differences is reasonably limited, amounting to a standard deviation of about 2%. This is consistent with some earlier (1974) data from the Repca I correlation study,[7] involving several EPA and manufacturer test laboratories, from which a between-lab standard deviation in CO_2 measurement of about 2.5% was deduced.

The one precise datum in Table A-1 (Juneja[2]) that includes between-vehicle variability is based on a single 1975 model (unique subconfiguration),

Table A-1.

SOURCE	σ (%)*	ERROR COMPONENTS INCLUDED		
		Within Test Cell	Between Test Cell	Between Vehicle
Simpson (1975)[1]	2.4 2.8**	X X		
Juneja, et al (1977)[2]	1.2 2.8	X X		 X
Schurmann, et al (1978)[3]	4.8	X		
Sheth and Rice (1974)[4]	2.7 2.9*** 3.3 3.5***	X X X X	 X X	
ASTM (1980)[5]	1.9 0.75**	X X		
NHTSA (1979)[6]	2-3.5			X

Results shown are for FTP fuel economy tests unless otherwise stated.

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* The consensus is that σ is roughly proportional to true value; hence, it is generally reported in terms of coefficient of variation (as a percent of mean fuel economy).

** Volumetric or gravimetric procedure.

*** HFET (Highway Fuel Economy Test).

with six nominally identical vehicles multiply-tested at 5000, 10,000 and 15,000 miles. The reported results imply a between-vehicle component standard deviation of about 2.5%.

In order to be able to estimate the overall accuracy of vehicle subconfiguration fuel economy determinations with reasonable confidence, it would be helpful to have additional corroboration of the above reported results, particularly with respect to the between-vehicle component of error which is inferred from tests on only a single vehicle subconfiguration.

A recently acquired EPA report[8] on replicated and multiple-vehicle testing of a number of different 1977 models provide limited data for such supporting analysis.

Nominally, three low-mileage (3,000 to 9,000 miles) cars drawn from each of eleven models (representing subcompact fuel economy leaders) each received three replicate FTP and HFET fuel economy tests. On closer inspection of the data, however, it was determined that: 18 different subconfigurations were represented (due to variations in transmission, axle ratio, etc.); the number of vehicles per subconfiguration ranged from one to three; and the number of replications per vehicle ranged from two to four.

A two-fold hierarchical linear model was assumed for a components of variance analysis, viz.,[9]

$$Y_{ijk} = \mu + a_i + b_{ij} + e_{ijk}$$

where i = model (subconfiguration) index

j = vehicle index

k = replication index

$\mu + a_i$ = mean fuel economy of i^{th} model

b_{ij} = random perturbation due to $(i,j)^{th}$ vehicle

$$E\left[b_{ij}\right] = 0, \quad Var\left[b_{ij}\right] = \sigma_b^{\,2}$$

e_{ijk} = random error due to $(i,j,k)^{th}$ test

$$E\left[e_{ijk}\right] = 0, \quad Var\left[e_{ijk}\right] = \sigma_e^{\,2}$$

Furthermore, all the $\{b_{ij}\}$ and $\{e_{ijk}\}$ were assumed to be uncorrelated. In this representation σ_b is the (assumed common) standard deviation of between-vehicle differences for all models and σ_e is the (assumed common) measurement error standard deviation. σ_e includes the within-test-cell component plus an indeterminate fraction of the between-test-cell component due to partial test cell variation.

The analysis is somewhat complicated by the unbalanced design (unequal numbers) but the formulae are still straightforward[9] and were applied to the data in the EPA report to estimate σ_b and σ_e for both FTP and HFET tests. An alternate analysis was also performed based on the assumption of common coefficients of variation (COV) rather than common standard deviations. This was accomplished through normalization of all fuel economies by the appropriate estimated model mean. The resulting estimates (and estimated standard errors) are:

MODEL ASSUMPTION:	COMMON STANDARD DEVIATIONS			COMMON COEFFICIENTS OF VARIANCE*	
	$\hat{\sigma}_b$ (mpg)	$\hat{\sigma}_e$ (mpg)	$\hat{\mu}$ (mpg)	$\hat{COV}_b$ (%)	$\hat{COV}_e$ (%)
FTP Fuel Economy	0.93±0.21	0.42±0.04	28.3	3.3±0.73	1.5±0.13
HFET Fuel Economy	0.88±0.25	0.73±0.06	37.5	2.2±0.65	1.9±0.17

Note that the within-cell plus partial between-cell measurement error magnitudes of 1.5-1.9% represent test cell performance which is consistent with, though somewhat better than, most entries in the previous table. Finally, we observe that the inferred between-vehicle variabilities of 2.2 to 3.3% tend to corroborate the previously inferred value of 2.5%.

In conclusion, the review and analysis conducted suggest that vehicle variability, test cell variability, and test replication error all make fairly comparable contributions to the total error in vehicle subconfiguration fuel economy measurement, but with relative strengths in the order indicated. Furthermore a reasonably conservative estimate for total error coefficient of variation is 4%.

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* $\hat{COV}$ notation is used here to avoid confusion between the two model assumptions; however it is the COV representation which corresponds to the σ's of the previous table.

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1. B. R. Simpson, "Improving the Measurement of Chassis Dynamometer Fuel Economy," SAE Paper 750002, February 1975.

2. W. K. Juneja, D. D. Horchler, H. M. Haskew, "A Treatise on Exhaust Emission Test Variability," SAE Paper 770136, February 1977.

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4. N. S. Seth and T. I. Rice, "Identification, Quantification, and Reduction of Sources of Variability in Vehicle Emissions and Fuel Economy Measurements," SAE Paper 790232, February 1979.

5. L. J. Painter, "Review of Statistical Aspects of 'EPA Recommended Practice for Evaluating...Engine Oils'," Chevron Research Company Memorandum (No Number), March 1980.

6. NHTSA, "Review of Procedures for Determining Corporate Average Fuel Economy," Report Nos. DOT-HS-805396 and -805397, July 1979.

7. R. E. Lowery, "Emission Laboratory Correlation Study Between EPA and the MVMA," EPA Report (No Number), September 24, 1974.

8. F. P. Hutchins and J. Kranig, "An Evaluation of the Fuel Economy Performance of Thirty-One 1977 Production Vehicles Relative to Their Certification Counterparts," EPA Report 77-18 FPH (Technology Assessment and Evaluation Branch), January 1978.

9. O. Kempthorne, The Design and Analyses of Experiments, John Wiley, pages 103-110, 1952.

APPENDIX B

FUEL ECONOMY LABEL ERROR DUE TO NORMAL ERROR BEFORE ROUND-OFF

Let E and E' be true and estimated fuel economy (in mpg) with $x = E' - E$ (the error), normally distributed with mean zero and variance σ^2. Let

$$E_L = [E + 0.5]$$
$$E'_L = [E' + 0.5]$$

be the corresponding label values, i.e., round-off to nearest whole number. Then

$$y = E'_L - E_L$$

is the fuel economy label error (in mpg) due to x. In contrast to x, y can only take on integer values. It is of interest to determine the probability distribution of y for different values of σ.

A basic assumption that permits this determination to be made fairly straightforwardly is that the decimal portion of E is uniformly distributed on $[0, 1]$ and independent of x. Then the conditional probabilities for y given x may be expressed as:

$$P_0|x = \Pr\{y = 0|x\} = \begin{cases} 1 - |x|; & 0 \leq |x| \leq 1 \\ 0 \quad ; & |x| > 1 \end{cases}$$

$$P_1|x = Pr\{|y| = 1|x\} = \begin{cases} |x| & ;\ 0 \leq |x| \leq 1 \\ 2 - |x| & ;\ 1 \leq |x| \leq 2 \\ 0 & ;\ \quad |x| > 2 \end{cases}$$

$$\vdots$$

$$P_k|x = Pr\{|y| = k|x\} = \begin{cases} |x|-(k - 1) & ;\ k - 1 \leq |x| \leq k \\ k + 1 - |x| & ;\ k \leq |x| \leq k + 1 \\ 0 & ;\ \quad \text{elsewhere} \end{cases}$$

Invoking a basic relationship in conditional probabilities:

$$P_k = Pr\{|y| = k\} = \int_{-\infty}^{\infty} (P_k|x) \cdot f(x)dx$$

we determine:

$$P_0 = \int_0^1 (1 - x) \cdot \frac{2}{\sigma\sqrt{2\pi}} \ \exp\ (-x^2/2\sigma^2)dx$$

$$P_k = \int_{k-1}^{k} (x - (k-1)) \ \frac{2}{\sigma\sqrt{2\pi}} \exp\ (-x^2/2\sigma^2)dx$$

$$+ \int_k^{k+1} (k + 1 - x) \ \frac{2}{\sigma\sqrt{2\pi}} \exp\ (-x^2/2\sigma^2)dx;\ k > 0$$

Rewrite P_0 as:

$$P_0 = \int_{-1}^{0} (x - (-1)) \frac{1}{\sigma \sqrt{2\pi}} \exp(-x^2/2\sigma^2) dx$$

$$+ \int_{0}^{1} (1 - x) \cdot \frac{1}{\sigma \sqrt{2\pi}} \exp(-x^2/2\sigma^2) dx$$

Under transformation of variables $z = x/\sigma$ and $u = x^2/2\sigma^2$ and definition

$$\delta_k = \begin{cases} 1; & k = 0 \\ 2; & k = 1, 2, \ldots \end{cases}$$

we obtain (for $k = 0, 1, 2, \ldots$):

$$P_k = \delta_k \left[- (k - 1) \int_{\frac{k-1}{\sigma}}^{\frac{k}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) dz + (k + 1) \int_{\frac{k}{\sigma}}^{\frac{k+1}{\sigma}} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) dz \right.$$

$$\left. + \frac{\sigma}{\sqrt{2\pi}} \int_{\frac{(k-1)^2}{2\sigma^2}}^{\frac{k^2}{2\sigma^2}} e^{-u} du - \frac{\sigma}{\sqrt{2\pi}} \int_{\frac{k^2}{2\sigma^2}}^{\frac{(k+1)^2}{2\sigma^2}} e^{-u} du \right]$$

$$P_k = \delta_k \Big[(k+1)\left(\Phi\left(\frac{k+1}{\sigma}\right) - \Phi\left(\frac{k}{\sigma}\right)\right) - (k-1)\left(\Phi\left(\frac{k}{\sigma}\right) - \Phi\left(\frac{k-1}{\sigma}\right)\right)$$

$$+ \frac{\sigma}{\sqrt{2\pi}} \left(\exp\left(-(k+1)^2/2\sigma^2\right) - 2\exp\left(-k^2/2\sigma^2\right) + \exp\left(-(k-1)^2/2\sigma^2\right)\right)\Big]$$

$$= \delta_k \Big[(k+1)\,\Phi\left(\frac{k+1}{\sigma}\right) - 2k\Phi\left(\frac{k}{\sigma}\right) + (k-1)\,\Phi\left(\frac{k-1}{\sigma}\right)$$

$$+ \sigma\sqrt{\frac{2}{\pi}}\exp\left(-k^2/2\sigma^2\right)\left[\exp\left(-1/2\sigma^2\right)\cosh\left(\frac{k}{\sigma^2}\right) - 1\right]\Big]$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

Evaluation of P_k, for selected σ is given below:

	PROBABILITY OF A ± k MPG ERROR IN FUEL ECONOMY LABEL VALUE, P_k			
σ (MPG)	k = 0	k = 1	k = 2	k = 3
0.25	0.80	0.20	< 0.001	-
0.50	0.61	0.38	0.01	-
1.00	0.37	0.48	0.13	0.02