BACKGROUND DOCUMENT

RESOURCE CONSERVATION AND RECOVERY ACT
SUBTITLE C-HAZARDOUS WASTE MANAGEMENT

SECTION 3004 - [STANDARDS APPLICABLE TO OWNERS AND OPERATORS OF HAZARDOUS WASTE TREATMENT, STORAGE AND DISPOSAL FACILITIES]

SECTION 250.43 - STATISTICAL METHOD

U.S. ENVIRONMENTAL PROTECTION AGENCY
OFFICE OF SOLID WASTE

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PREFACE

This document provides background information for §250.43-8(c)(4) of EPA's proposed section 3004 hazardous waste treatment, storage, and disposal regulations under the Resource Conservation and Recovery Act of 1976 (RCRA). It is being made available as a draft document to support the reproposed portion of §250.43-8(c)(4) which was first published on December 18, 1978 (43 FR 59005). As new information is obtained, changes may be made in the background information which will be used as support for the regulations when promulgated.

This document will address the following areas:

(1) Comments as they relate to the statistical methodology proposed in §250.43-8(c)(4)

(2) Rationale for selecting the Mann-Whitney U Test

(3) Instruction for computing and interpreting the Mann-Whitney U Test

Comments in writing may be made to:

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STATISTICAL METHOD
BACKGROUND DOCUMENT

1.1 Introduction

The regulation as proposed on December 18, 1978 (43 FR 59005) called for the use of the Student's t Test to detect statistically significant differences in the concentration of leachates found in upgradient and downgradient ground water samples. The Student's t Test was initially choosen because it would provide a relatively sensitive (powerful) test for detecting statistically significant differences in groundwater leachate concentration using small sample sizes (i.e., sample sizes of seven (7) observations per sample of well water). Based on comments received in response to the proposed use of the Student's t Test, EPA has changed the statistical methodology to the Mann-Whitney U Test. The rationale for this change is documented in the remainder of this paper.

1.2 Comments as they relate to the Statistical Methodology

Comments relating to the statistical methodology proposed in the December 18, 1978 publication (43 FR 59005) can be classified into two general areas: (1) violations of the mathematical assumptions underlying the statistical methodology; and (2) statistical sensitivity (power) of the test.

Violations of the underlying distribution assumptions of the Statistical Model

The mathematical model underlying the Student's t Test assumes that the sample observations (data) have been drawn from a population (all possible observations) in which the measured observations of leachate concentration are independent and normally distributed. Further, the model assumes that all sample measurements were taken at the same point in time and represent a random sample of observations from populations with a constant mean and variance.

Concern was expressed that the above assumptions would not be met by the sampling methodology presented in the proposed regulations. EPA agrees that these concerns would be valid under certain circumstances. In response to those concerns EPA has changed the statistical methodology and will use the Mann-Whitney U Test instead of the Student's t Test. The Mann-Whitney U Test is a nonparametric test, (i.e., a statistical test which makes no assumptions regarding the nature of the underlying population distribution), and as such is not dependent on samples drawn from populations with known distributions.

Rationale for Use of the Mann-Whitney U Test

Based on the comments received the EPA is proposing to substitute the nonparametric Mann-Whitney U Test for the more stringent, (i.e., more assumptions underlying the mathematical model) Student's t Test. The Mann-Whitney U Test is nonparametric and only requires that the sample be drawn from a population of independent and continuous measurements. The Student t Test assumes the underlying population to be normally distributed in addition to the measurements being independent and continuous. The assumption of normality inherent in the Student's t Test is what caused concern in applying that methodology to detect significant differences in leachate concentration. The observations (seven measurements/well water sample) in each sample are subject to measurement error as well as laboratory bias; therefore it is possible that the underlying error distribution is not normal and the sample of seven measurements represents some other unknown population.1/ Given this situation, the nonparametric Mann-Whitney U Test is better suited as a decision model than the more "stringent" Student's t Test.

1/ See Tai, Larry S.L., Statistical Methods for Determining the Measurement Precision of Drinking Water Contaminants, EPA, TSC-PD-A223-1, Final Report Task 1 Contract No. 68-01-5086, July 1979, for a discussion of non-normal error distribution for samples

Nonparametric statistical tests are uniformly less powerful than their parametric analogs when all assumptions of the parametric model are met; however, when the underlying assumptions are not met, one nonparametric technique can provide greater sensitivity than the parametric analog.

The power of the Mann-Whitney U Test compares favorably with the Student's t Test; when the two populations sampled are assumed to differ only in location (i.e., mean or median) the Mann-Whitney U Test is almost equal in power to the Student's t Test.[2/]

In summary, the Mann-Whitney U Test was selected because of its computational simplicity and broad range of applicability in situations where the more stringent assumptions of parametric techniques would be violated.

1.3 Protocol for using the Mann-Whitney U-test

The Agency is considering requiring a minimum of seven observations in each of the samples to be compared. This should provide adequate power for detecting meaningful differences in leachate concentrations. In this example, the seven observations for the experimental sample will be compared to the seven observations in the control sample using the following procedure.

(1) Combine the two data sets in a single list, arranged from lowest to highest values. For example, assume we obtained the following sets of observations for the control (C) and the experimental (E) wells, measured in mg/l:

C 3.1, 3.2, 3.3, 3.4. 4.2, 4.5, 5.0

E 4.0, 4.3, 4.8, 5.2, 5.5, 5.6, 5.8

These fourteen data points would be reordered as follows:

3.1	3.2	3.3	3.4	4.0	4.2	4.3
C	C	C	C	E	C	E
4.5	4.8	5.0	5.2	5.5	5.6	5.8
C	E	C	E	E	E	E

2/ See Gibbons, Non-Parametric Statistical Inference, McGraw-Hill, 1971, pp. 148-149, for a discussion of the relative efficiency of the Mann-Whitney U and Student's statistics.

(2) For each control value (C) count the number of experimental (E) values which precede it. The process is shown below:

C	C	C	C	E	C	E	C	E	C	E	E	E	E
0	0	0	0		1		2		3				

That is to say, each of the first four control values has no experimental value preceding it, the fifth control value has one, the sixth two, and the last has three values preceeding it. In the case of ties list the control value before the experimental value.

(3) Sum the counts obtained for the control values. The Mann-Whitney U statistics is the sum of these counts, that is:

$$U = 0 + 0 + 0 + 0 + 1 + 2 + 3 = 6$$

(4) Determine if there is a statistically significant difference between the control and experimental samples. If the calculated value of U is less than eleven, there is a statistically significant difference in the control and experimental samples at the 95% confidence level.

<u>REFERENCES</u>

1. Mary Gibbons Natrella, <u>Experimental Statistics</u>, (Washington, D.C. - U.S. Government Printing Office, 1966) p. 1-14; pp. 2-12 to 2-15.

2. Sidney Siegel, <u>Nonparametric Statistics</u>, (New York: McGraw-Hill Book Co., 1956) pp. 116-126.