

Final Report

THE EXEX METHOD: INCORPORATING  
VARIABILITY IN SULFUR DIOXIDE EMISSIONS  
INTO POWER PLANT IMPACT ASSESSMENT

Contract No. 68-01-3957

SAI No. 84-EF80-80R2

23 July 1980

PREPARED BY

**SYSTEMS APPLICATIONS, INC.**

[40802]

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Prepared for

Office of Regional Programs  
Office of Air Quality Planning and Standards  
Environmental Protection Agency  
Research Triangle Park, North Carolina 27711

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## DISCLAIMER

This report has been reviewed by the Office of Air Quality Planning and Standards, U.S. Environmental Protection Agency, and approved for publication. Approval does not signify that the contents necessarily reflect the views and policies of the U.S. Environmental Protection Agency, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.

## EXECUTIVE SUMMARY

The principal topic addressed in this report is the assessment methodology used to estimate near-source ambient air quality effects of contaminant emissions from coal-fired steam boilers. Attention is focussed specifically on a technical description of a new and practical development in this methodology that explicitly accounts for both the variability inherent in sulfur dioxide emissions from coal-fired boilers and in meteorological conditions when assessing the (new or modified) source's attainment of ambient standards. Policy questions associated with this new development are not discussed.

As currently practiced, air quality assessments entail the use of an appropriate mathematical model to calculate ground-level contaminant concentrations and the number of times per year those concentrations equal or exceed the applicable ambient air quality standard. If the computed second highest concentration equals or exceeds an ambient standard more than twice per year, emissions from the source are judged to be too great. Among the critical factors affecting this judgement, particularly for isolated sources located in flat terrain, is the selection of the meteorological conditions and contaminant emissions rates. If the emissions rate never varied, it would be possible to use such a model, employing a year's meteorological data to estimate (within the uncertainties of the model and its input data) the number of times an ambient standard is equalled or exceeded.

Named the ExEx Method, the new procedure, which utilizes the same (appropriate) air quality dispersion model mentioned previously, recognizes the problem associated with the use of a constant (usually maximum) emissions rate for the assessment, and specifically accounts for the low probability that maximum emissions and worst-case meteorology can occur simultaneously. The technique does not presume that these two events cannot occur together, but rather calculates the probability of their coincidence.

The technique by which emissions and meteorological fluctuations are combined in the ExEx Method is a Monte Carlo simulation. In this technique, random variations in emissions are simulated by first randomly selecting emissions values from an appropriate probability distribution. Such emissions values are then combined with air quality model estimates of atmospheric dispersion, and when the results of many samples are accumulated, an estimate is

obtained of the expected (average) number of times that an ambient standard is equalled or exceeded in a year. The ExEx Method also provides estimates of the probability that concentrations will exceed the standard two or more times in a year. An important result of the ExEx Method is that the determinism usually associated with air quality models in impact assessments is eliminated, since it cannot be stated that the second highest concentration would be, say,  $400 \mu\text{g SO}_2/\text{m}^3$ , but rather that the probability of occurrence of two  $\text{SO}_2$  concentrations greater than  $400 \mu\text{g}/\text{m}^3$  in the course of a year would be, say, 0.1 (10 percent, or once in ten years).

The majority of the report is devoted to a detailed description of the formulation of the ExEx Method and to documentation of a computer code that is applicable to isolated, coal-fired boilers located in flat terrain. Example calculations of this initial version of the ExEx Method are presented for two hypothetical 1000 MWe power plants--one with scrubbed and the other with unscrubbed  $\text{SO}_2$  emissions--using ground-level dispersion estimates obtained by exercising the EPA CRSTER dispersion model. As currently implemented, the ExEx Method utilizes several years of meteorological data and a distribution of  $\text{SO}_2$  emissions; the latter may be obtained from knowledge about the distribution of ("as-burned") sulfur in the coal, stack monitors, or plant design data. To date, practical experience in applying the current version of the ExEx Method indicates costs that are comparable to, or less than, those involved in the application of the EPA CRSTER model. Exercise of the ExEx computer code involves use of the output of the CRSTER model for each year of meteorological data, together with the distribution of  $\text{SO}_2$  emissions. Judgments about the acceptability of a source's  $\text{SO}_2$  emissions are based on consideration of:

- > The expected number of times an ambient standard is exceeded.
- > The probability of violating a standard (i.e., that the standard is exceeded two or more times in a year).

Both measures of acceptability are provided for the user, since at this time neither the suitability of a specific value of any measure has been established, nor has any decision been made as to which measure is preferable.

The effort reported herein is believed to represent a first step toward improving decisions associated with environmental protection and air quality management by logically relating ambient and emissions standards in the impact assessment, and by providing an analysis framework that is capable of explicitly accounting for the principal elements of uncertainty in the assessment decision. There are, however, many elements involved in an impact assessment decision that have not been incorporated into the current version of the ExEx procedure. Those worthy of further consideration are discussed in the final chapter of the report and include: dispersion model uncertainties, variations in background pollutant concentration, control equipment efficiency variations, load variations, and the treatment of multiple sources.

## AUTHORS' NOTE

This report has been reviewed extensively, inside and outside the EPA, for both technical content and clarity of exposition. In revising the report prior to its publication in final form, we considered all the comments we received. We would like to express our thanks to the reviewers; their careful efforts have without doubt improved our report.

However, we have not responded to certain reviewers' comments, a situation that derives from one of three causes: first, we take exception to the comment; second, we concur, but the comment covers work to be carried out in the future; or third, we consider the subject of the comment outside the scope of the report.

To clarify the third reason, we briefly discuss here what we consider to be the scope of this report (what it is and is not intended to encompass). We believe the ExEx methodology, whose development is described herein, is a step in the direction of a more realistic assessment of the effects of  $\text{SO}_2$  emissions from point sources on their surroundings. In this report we describe the ExEx method both in words (chapter 2) and by means of mathematical equations (chapter 3). We also describe some results for hypothetical sources (chapter 4) and suggest possible directions for future work (chapter 5). In an appendix we list a computer program we developed to implement the method.

What we do not discuss are those issues concerned with whether the method is adopted, or the important details associated with its possible implementation. This report is premised on the view that such matters are properly addressed by those responsible for policy within the EPA. In addition, we do not deal with issues such as the adequacy of existing air quality dispersion models or submodels. Issues associated with the inherent limitations of existing models or model components--like the assumptions of steady-state conditions and spatial homogeneity in Gaussian-based models, or suspected deficiencies in currently used dispersion coefficients and plume rise algorithms--are excluded. It is our view that the best, most acceptable (to the EPA) air quality models can always be used. For the purpose of introducing, illustrating, and performing preliminary evaluations of the method, any

model would meet our needs. However, if implemented, the model that is used is an important factor to be considered.\*

Our overall purpose here is to develop and evaluate analysis tools whose ultimate purpose is to improve decisions (by reducing uncertainties as to the consequences of decisions) associated with environmental protection and air quality management through ambient standards and emissions limits. The effort reported herein represents a small step toward achieving this goal by providing an analytical framework that can explicitly account for major areas of uncertainty in assessing near-source (< 100 km) SO<sub>2</sub> impacts of isolated coal-fired boilers. Consideration of all sources of uncertainty remains to be explicitly incorporated into the ExEx methodology; thus it is a topic for further work.

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\* It may be of interest here to note that the ExEx method, as currently implemented for an isolated source, could also use measured estimates of atmospheric dispersal strengths; therefore, the method need not rely on models.

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Hoff Stauffer and Ken Schweers of ICF, Incorporated, participated in many helpful discussions.

## I INTRODUCTION

When fossil fuels are burned to generate electricity, an unavoidable consequence is the introduction of contaminants into the atmosphere. The combustion of fuels containing sulfur impurities results in the emission of sulfur dioxide ( $\text{SO}_2$ ) through the plant's stack. A principal difference between oil or gas and coal combustion is the variability of their  $\text{SO}_2$  emissions. Because of the greater homogeneity of oil,  $\text{SO}_2$  emissions from an oil-burning point source are relatively constant for a particular fuel. Coal, on the other hand, exhibits short-term variability in composition; the result is that  $\text{SO}_2$  emissions from coal burning show stochastic variation from one time period to another. In this report, we describe the formulation of a methodology to assess the ground-level  $\text{SO}_2$  impacts of coal burning sources, taking into explicit account the statistical variability of the sulfur content of coal. We also describe the coding and use of a computer program to implement the methodology.

Variability in the sulfur content of coal derives from many causes. The basic source of variability is within the coal itself as it is mined. Within a mine, coal varies in its sulfur content from seam to seam and also within individual seams. This variability is changed by handling between the mine and the plant, and by subsequent treatment to prepare the coal for combustion: movement from one stockpile to another, transportation, cleaning, and crushing. The size of the combustion facility also has an effect on variability because of the averaging effect of large coal sample sizes. Finally, the technology and performance characteristics of emissions control systems installed at the plant depend on the coal composition as well as other factors. Thus, the control system affects the variability of  $\text{SO}_2$  emissions from the stack.

In assessing the impact of coal-fired facilities, current practice involves the use of a mathematical simulation model of  $\text{SO}_2$  emissions, transport, and dispersion. In contrast to appropriate physical modeling approaches, such models, in principle, enable calculations to be made of ground-level concentrations, given plant emissions and meteorological data. However, use of such a mathematical model introduces uncertainties into the impact assessment as a result of the model's deficiencies. These should be taken into account. In the work described in this report we have used the EPA's single-source (CRSTER) model (EPA, 1977), which is a single-source

Gaussian model. This model is recommended by the EPA for assessing SO<sub>2</sub> impacts from isolated point sources in flat terrain situations.

The EPA, as part of its legal responsibility to manage air quality, has established National Ambient Air Quality Standards (NAAQS) and Prevention of Significant Deterioration (PSD) increments.\* Both the short term NAAQS and PSD increments for SO<sub>2</sub> are defined in terms of concentration levels not to be exceeded more than once per year. Sources are not permitted to cause or contribute to the exceedance of these standards more than once per year. The numerical values of these standards are shown in table 1.

TABLE 1. APPLICABLE FEDERAL AIR QUALITY STANDARDS FOR SO<sub>2</sub>

Standard	SO <sub>2</sub> Concentrations ( $\mu\text{g}/\text{m}^3$ )	
	3-Hour Average	24-Hour Average
PSD Class I	25	5
PSD Class II	512	91
PSD Class III	700	182
NAAQS	1300	365

If SO<sub>2</sub> emissions never varied and a perfect air quality model were available, it would be possible to use that model, employing a year's meteorological data, to compute the second highest concentration at any point around the plant and to judge whether this value exceeded the federal standard. The problem with this approach in the context of stochastically varying emissions is choosing an appropriate emissions rate to use in applying the model. The second highest concentration computed using an average emissions rate requires further adjustment if it is to represent the anticipated second highest concentration (since about 50 percent of the time emissions will be above the average). On the other hand, if an upper-extreme emissions rate is used, it can be argued that combining an unusually high emissions rate with unusually poor meteorological conditions (the second worst in the year) is too stringent to use for an assessment of attainment. What is evidently needed is a methodology by which the stochastic variations in emissions rates are combined with the day-to-day variations in meteorology to give a picture of the like-

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\* With the passage of the 1977 Clean Air Act Amendments, the PSD increments originally established by the EPA were codified into law.

likelihood of violating the standard in a given year.\* Note that including the uncertainty in emissions rates eliminates determinism in the model results, so that we would not be able to state that the second highest concentration would be, say,  $400 \mu\text{g SO}_2/\text{m}^3$ , but rather that the probability of having two  $\text{SO}_2$  concentrations greater than  $400 \mu\text{g}/\text{m}^3$  in the course of a year would be, say, 0.1 (or 10 percent). Stating the result in this way modifies the current practice of judging acceptability, because an allowable probability of more than one exceedance† of the standard must be specified (since this probability cannot be reduced to zero). Thus, the method explicitly recognizes that, regardless of the extent to which emissions are reduced, there always remains a finite, however small, probability of exceeding the air quality standard (or increment) during periods of unfavorable meteorological conditions.

The technique by which we combine the uncertainty in emissions rates with the results of dispersion model computations is a Monte Carlo simulation. This technique simulates the effects of random variations in a quantity by randomly selecting values from an appropriate probability distribution and accumulating results over many samples to obtain a picture of the form that a long-term series of values should take.

The methodology we have developed utilizes a probability distribution of 3-hour or 24-hour average  $\text{SO}_2$  emissions. This distribution can be obtained from data collected on emissions estimates or coal sulfur content, or as estimates based on plant design or operating characteristics. Meteorological variability is treated by utilizing a multiyear (currently 5- to 10-year) meteorological record of wind speed, wind direction, and atmospheric stability. Using these meteorological data and the EPA CRSTER dispersion model, 3-hour or 24-hour average estimates of atmospheric dispersion are calculated

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\* This procedure would take into account both the meteorological and emissions fluctuations. It should be noted that the fluctuations in meteorology, as estimated by changes in ambient concentration levels for a fixed emissions rate, are significantly greater than typical fluctuations in emissions about the mean. Some may argue that, because of this, emissions fluctuations are unimportant. However, though emissions fluctuations may be relatively unimportant in determining mean ground-level concentrations over, say, a period of a year, such emissions fluctuations will be very important in determining maximum ground-level concentrations.

† We recognize that the word "exceedance" is a coinage. However, we believe that its use is sufficiently common in air quality data analysis to convey clear meaning. Therefore, throughout this report we have used "exceedance" to denote the observation or computation of a ground-level concentration exceeding the level set in the applicable standard, and "violation" to denote the occurrence of two or more exceedances in a single year.

for the entire meteorological record for a preselected array of critical receptors, usually 180.

For each 3-hour or 24-hour average dispersion estimate in the meteorological record and the receptor array, an estimate of the appropriately averaged  $\text{SO}_2$  emissions is randomly drawn from the emissions distribution and combined with that dispersion estimate to produce a corresponding estimate of the ground-level  $\text{SO}_2$  concentration. This 3-hour or 24-hour average  $\text{SO}_2$  concentration is compared with the applicable ambient standard, and a notation is made if that concentration exceeds the standard. The same procedure is followed to give sets of concentrations at each receptor for each time period throughout the year. The number of exceedances of the standard, and whether a standard violation (two or more exceedances) occurred at any receptor during the year, is recorded. This simulation of a year's concentrations is then repeated many (currently 1000) times to give an estimate of the long-run rate of occurrence of standard exceedances and violations. This methodology thus leads to estimates of both the expected number of exceedances of the applicable standard at each receptor and the probability that a violation can be expected to occur. In addition, we derive an estimate of the probability that a violation will occur somewhere in the network of receptors in the vicinity of the source. All of these estimates are provided for each meteorological year and over all meteorological years.

In this chapter we have provided a brief introduction to our newly developed methodology. In chapters 2 and 3 we present more detailed descriptions of the method and its theoretical development. Chapter 2 is pictorial in its presentation; chapter 3 requires the reader to have some acquaintance with elementary probability theory, though the main structure of the method can be understood without such prior knowledge. Reference to texts such as Parzen (1960) can supply any needed background. In chapter 4 we give details of using the methodology to evaluate two hypothetical power plants. The two plants are identical except that one emits a scrubbed plume and the other an unscrubbed plume. Differences in impacts are, therefore, the result of differences in exit temperature and velocity (and thus plume rise) rather than in  $\text{SO}_2$  emissions rates. In chapter 5 we list a series of concerns relevant to the general problem of assessing exceedances and violation probabilities. We point out those items that we have not covered in the present work and suggest that they be considered for future study. The report concludes with an appendix that gives a complete description (constituting a user's manual) for the computer program that we developed to implement the methodology. The program is written in standard FORTRAN and should be implementable on most computers.



## II PICTORIAL OVERVIEW OF THE EXEX METHODOLOGY

In the ExEx method a Monte Carlo simulation is the technique employed to combine the variability in emissions rates with the results of dispersion model computations. Stated simply, this methodology simulates the effects of random variation in a quantity by randomly selecting values from an appropriate probability distribution and accumulating results over many samples. An estimate of how an actually realized long-term series of values should look is thereby obtained.

All of the essential components of the ExEx methodology are depicted in figure 1:

- > A stochastic treatment of point source emissions
- > A suitable atmospheric diffusion model
- > A (Monte Carlo) simulation model.

Figure 1 shows the data and input requirements for each of these components, as well as a summary of the results of the methodology and the use of these results. The output of the simulation model is expressed in terms of exceedances and violations:

- > Expected (or average) number of exceedances of the ambient standard(s) for each receptor point specified in the diffusion model for each meteorological year and for all meteorological years.
- > Probability of violating the ambient standard. Three probabilities are provided: The first, provided for each receptor and year, is the probability that two or more exceedances will occur in that meteorological year. The second is the average of the foregoing probabilities (receptor violation probabilities) over all meteorological years. The third is the probability that there will be two or more exceedances at one or more receptors (in the array of receptors) over the meteorological period. This last quantity can be viewed as the probability that a vio-

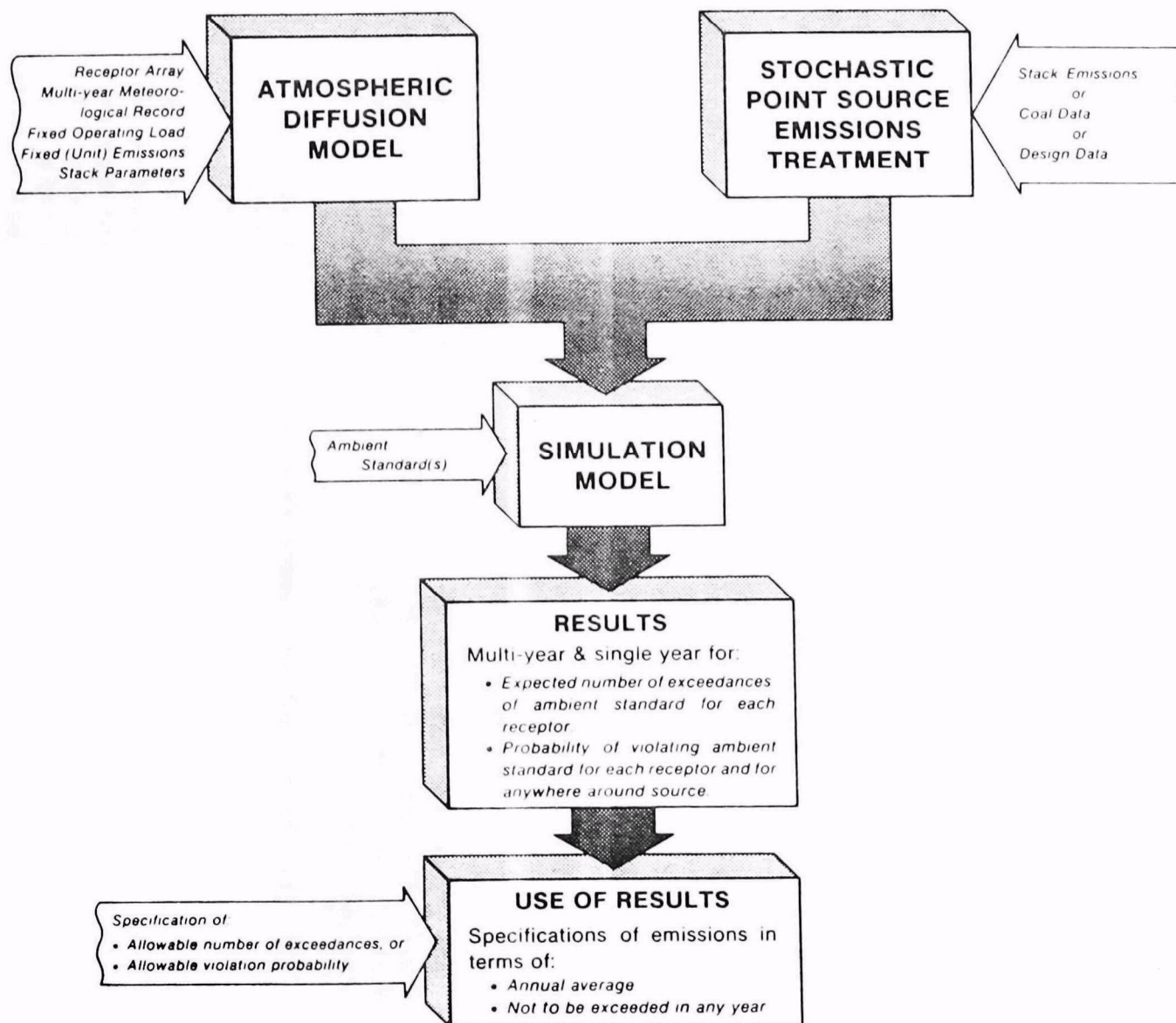


FIGURE 1. ELEMENTS OF THE SIMPLIFIED EXEX METHOD

lation of the ambient standard will occur anywhere (or somewhere) in the vicinity of the source.

These results can be used to determine allowable annual average emissions and an emissions limit not to be exceeded in any year in the life of the facility. Each of the principal elements of the methodology will be discussed briefly.

Figure 2 illustrates the input and output of the atmospheric dispersion model. Use of the model follows EPA-recommended procedure (EPA, 1977). It is convenient to run the model with unit emissions (i.e., 1.0 lb SO<sub>2</sub>/MMBtu). It is also convenient to view the output of the model as matrices (or tables) consisting of pairs of 24-hour-average and 3-hour-average normalized ground-level concentrations for each year in the meteorological record. For five years of meteorological data there would be 10 such matrices--five tables of 3-hour averages and five tables of 24-hour averages. Each matrix contains a normalized concentration for each receptor and each time period. Thus, for a 24-hour time period, assuming the model is CRSTER, which utilizes 180 receptors, there are 365 (or 366) x 180 entries in the matrix. For the 3-hour time period, there are 2920 (or 2928) x 180 entries in the matrix.

Figure 3 depicts the input requirements and output options of the stochastic treatment of source emissions. As noted in this figure, either stack emissions, coal, or engineering design data may be used to generate a frequency distribution of SO<sub>2</sub> emissions. This distribution can take the form of either an analytical expression, currently assumed to be lognormal, or of a table of values that specifies the number of occurrences for each SO<sub>2</sub> emissions value. The averaging period for the SO<sub>2</sub> emissions data should match that used in the dispersion model, i.e., 3-hour emissions data for 3-hour ambient standards, and 24-hour emissions data for 24-hour ambient standards. For the purposes of this report, we will assume that the 3-hour and 24-hour SO<sub>2</sub> emissions distributions will be identical. For even larger sources and coal sulfur variability that is not too large, it is logically conceivable that 24-hour emissions could equal the annual average emissions. We are unaware, however, of any data that support this possibility. In other situations, additional uncertainties can be introduced if the 3-hour and 24-hour distributions are assumed to be identical, but at this time the magnitude of such uncertainties is unknown.

A key feature of the methodology is the derivation, from the pollutant emissions distribution and meteorological conditions, of the probability with which the ambient standard will be exceeded in any averaging period. The calculation proceeds as follows (for more detail see chapter 3): Suppose that, for a particular period, the average ground-level concentration computed using the CRSTER model is one-third the level of the standard. Then, because of the linear relationship between emissions and concentrations included in CRSTER,

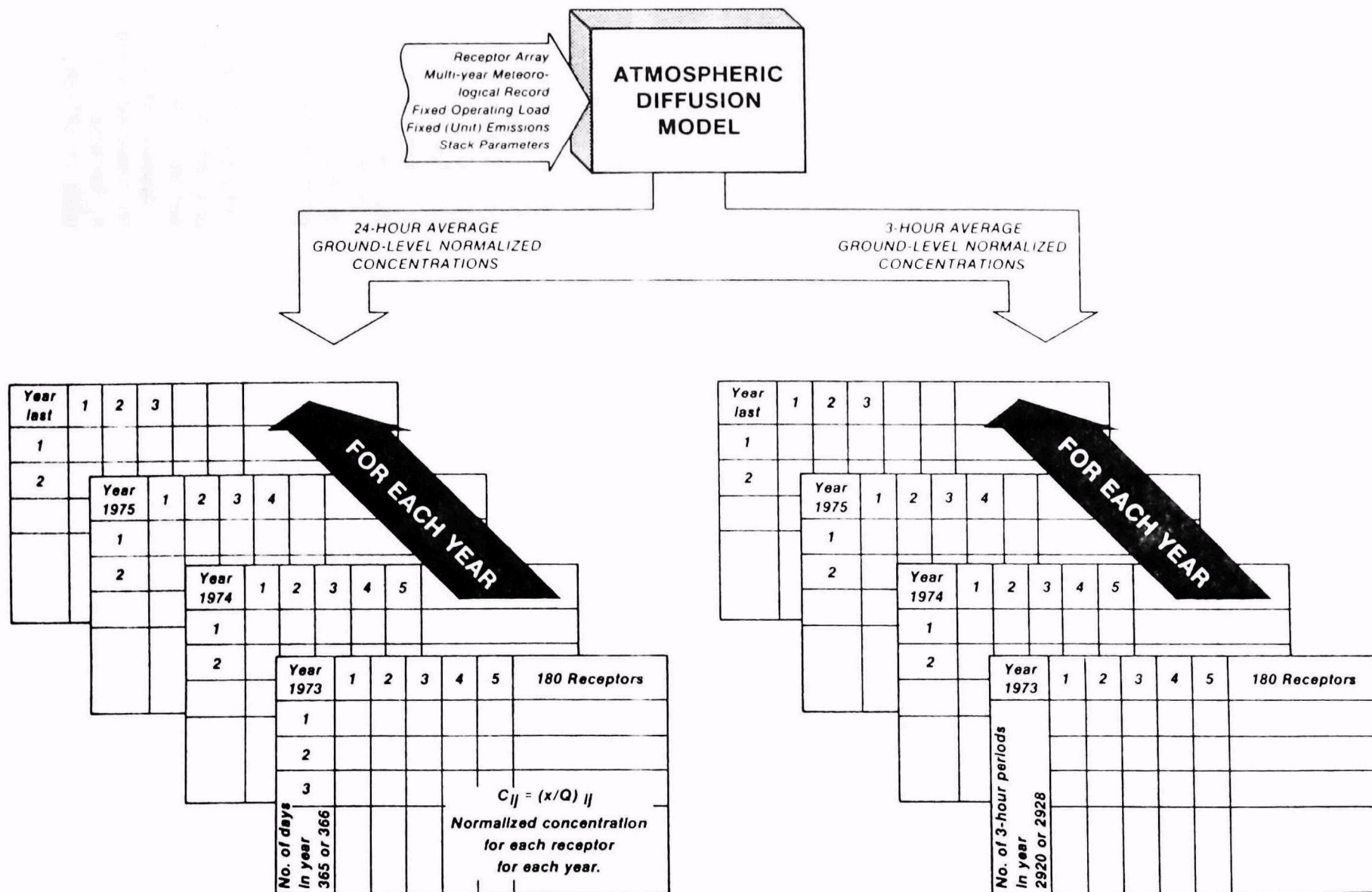


FIGURE 2. ATMOSPHERIC DIFFUSION MODEL

FIGURE 3. STOCHASTIC POINT SOURCE EMISSIONS TREATMENT

it can be seen that, if the emissions rate used as input had been three times higher, a concentration equal to the standard would have been computed. Thus, for this averaging period, the probability of exceeding the standard is equal to the probability of exceeding three times the emissions rate used for the CRSTER calculations. This probability can be derived from the (known or assumed) probability distribution of emissions rates. Use of the CRSTER model makes this computation particularly easy because of the linear scaling of emissions to concentrations, but, in principle, any model could be used so long as there is a one-to-one correspondence of emissions and concentrations (assuming this relationship is known).

Figure 4 illustrates how the results from both the dispersion model and the stochastic  $\text{SO}_2$  emissions treatment are used in the simulation model. The example in figure 4 is based on comparison with a 24-hour-average ambient standard. The upper left-hand portion of figure 4 shows the matrices of 24-hour-average ground-level concentrations normalized to unit  $\text{SO}_2$  emissions. Each normalized concentration is designated by the symbol  $C_{ij}$ , with the index  $i$  denoting the day of the year and the index  $j$  denoting the receptor location. The upper right-hand portion of figure 4 illustrates that 1000 vectors are generated from the  $\text{SO}_2$  emissions distributions, each vector containing 365 (or 366) randomly selected 24-hour-average  $\text{SO}_2$  emissions. Each emissions vector is designated  $Q_k$ , with the index  $k$  specifying the particular sample. For all receptors on each day in the meteorological year, the emissions value corresponding to the appropriate day is used to scale the normalized concentrations,  $C_{ij}$ . This results, for all receptors on that day, in a set of 24-hour-average ground-level  $\text{SO}_2$  concentrations, assuming that the randomly selected  $\text{SO}_2$  emissions occurred. The set consists of 365 (or 366) x 180 24-hour  $\text{SO}_2$  concentrations. This scaling process is repeated 1000 times for the meteorological year and generates a three dimensional matrix consisting of 365 (or 366) x 180 x 1000 values. This three-dimensional matrix is depicted in the lower portion of figure 4. For each year of meteorological data, one such matrix (box) is generated. Thus, for a five year meteorological record there are 5 such matrices, so that a total of 5 x 365 (or 366) x 180 x 1000 values are generated. Note that for the 3-hour ambient standard and a five year record, 5 x 2920 (or 2928) x 180 x 1000 values are generated. The program implementing the methodology incorporates a screening technique to reduce the number of operations so that only meteorological conditions that can possibly lead to exceedances are considered. In a typical computer run, only 10 to 1000 values per meteorological year would be considered in the calculations.

The elements of these boxes (see figure 5) are used to provide estimates of the expected number of exceedances of the ambient standard and of the probability of violating the ambient standard. For each year, each receptor,



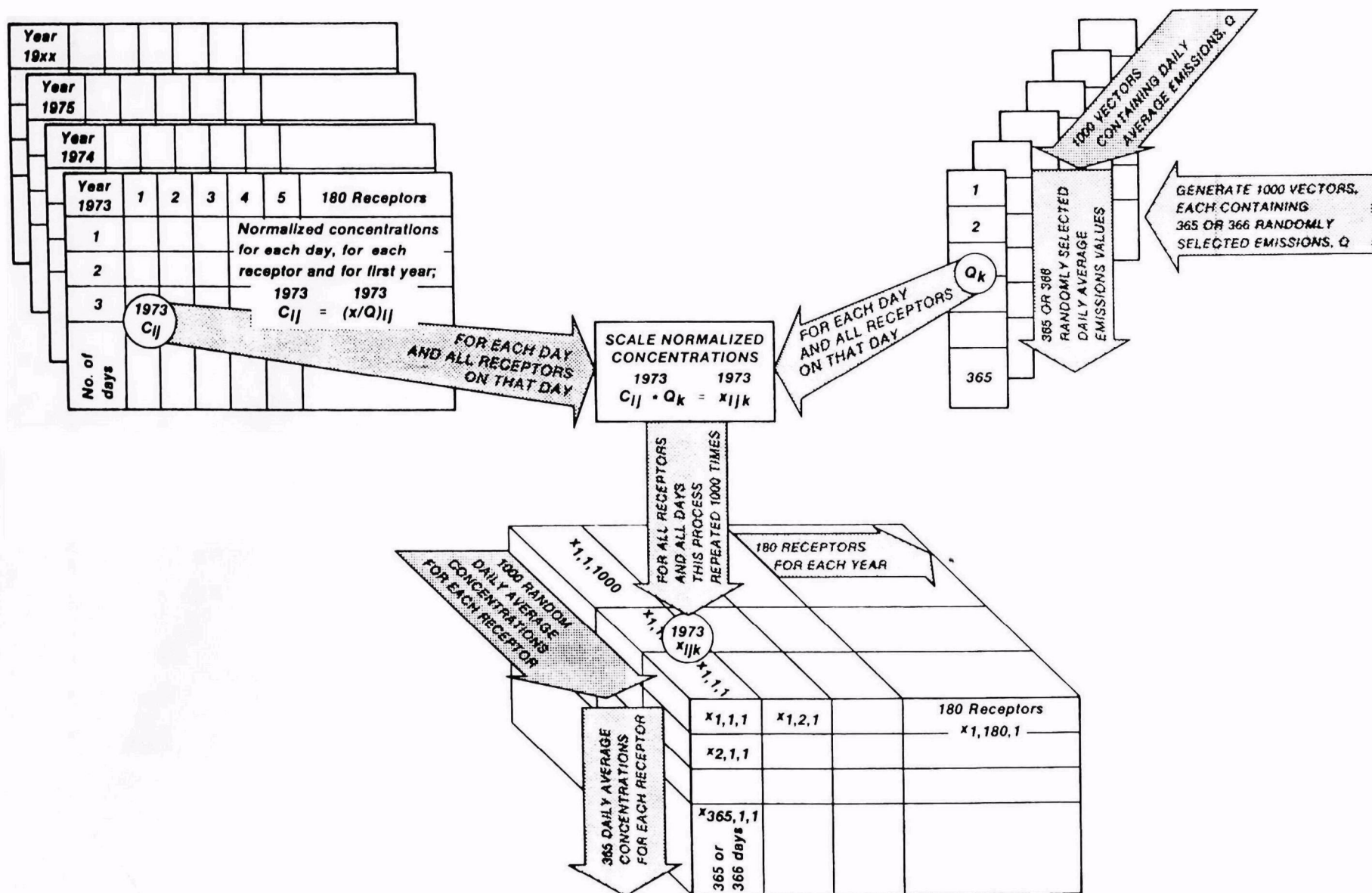


FIGURE 4. SIMULATION MODEL

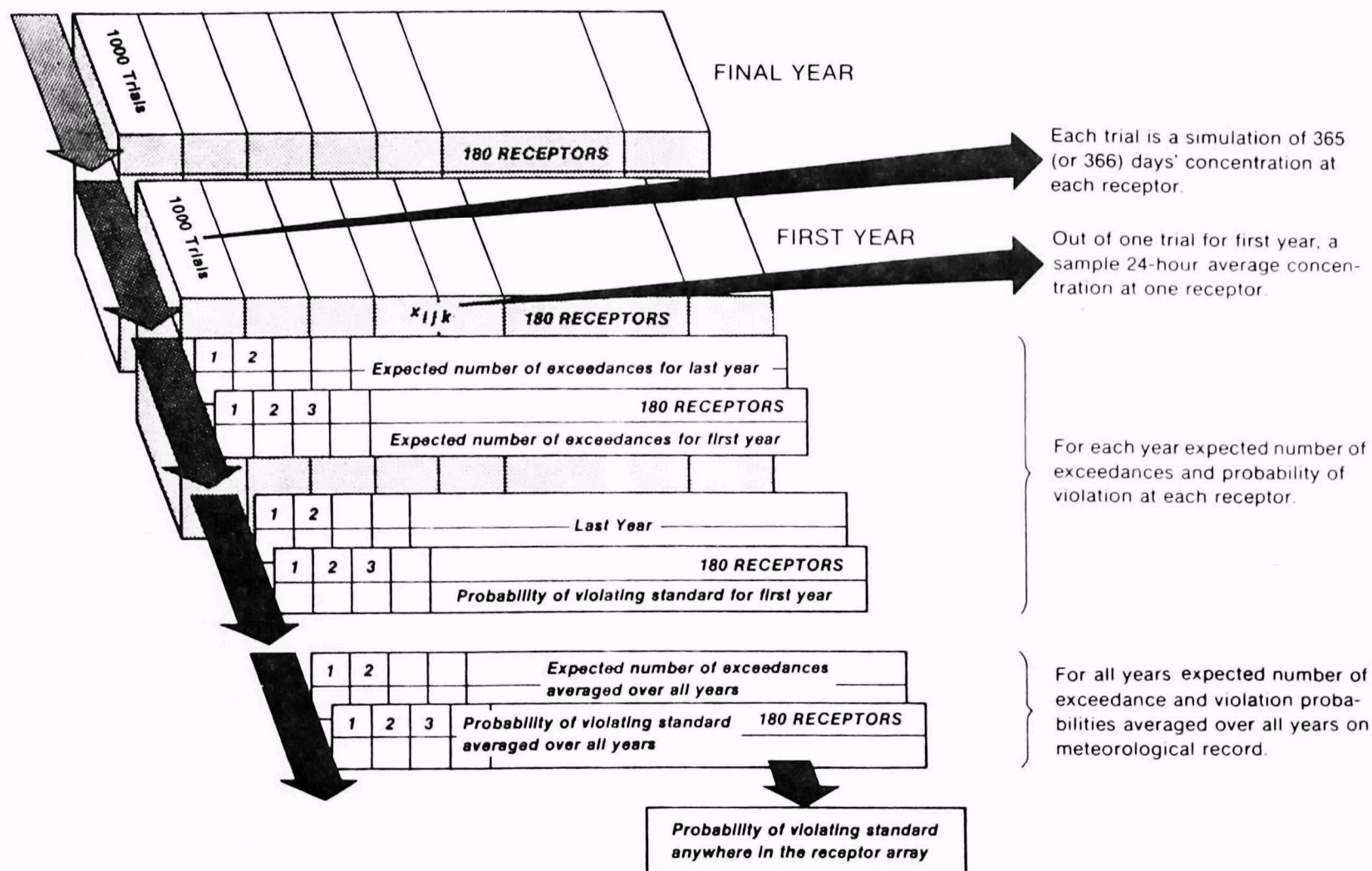


FIGURE 5. ANALYSIS OF EXCEEDANCES AND VIOLATIONS FROM EXEX METHODOLOGY



and each trial,<sup>\*</sup> the expected number of exceedances is obtained. The expected number of exceedances is the total number of exceedances of the ambient standard, after correcting for background,<sup>†</sup> divided by 1000. The probability of violating the ambient standard is also noted. For each receptor the probability of a violation is defined as the total number of occurrences of two or more exceedances in a year for each trial, divided by the number of trials.

The expected number of exceedances for each receptor and the probability of a violation for each receptor are also estimated for the entire meteorological period (record). This is done by averaging the expected number of exceedances and violation probabilities over all years in the period. The derivation of these average results is also shown in figure 5. In addition, an estimate is made of the probability of violating an ambient standard anywhere in the array of receptors; this probability is defined as the total number of trials (over all years in the meteorological period) for which there are two or more exceedances at one or more receptors in the receptor array.

The method is applicable in principle to any point source of a nonreactive pollutant (or any pollutant that can be treated as nonreactive). Although the current version of the method utilizes the CRSTER model, this is not a requirement for the application of the statistical methodology. Any model that can be used to give dispersion estimates for each period in a meteorological record could, in principle, be used to obtain the inputs to the statistical model. In fact, if experimental dispersion estimates are available, these can also be used for input.

Availability of a computer program makes the application of the methodology practical. Current experience shows that costs of applications are comparable to, or less than, those of the CRSTER model.

The methodology can also be used to evaluate different regulatory scenarios. Currently, emissions limits are not uniform across states: Both averaging time and allowed frequency of exceedance vary. The effect of different regulatory environments can be evaluated, provided meteorological and emissions information is available at an appropriate level of temporal averaging.

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<sup>\*</sup> A trial is defined as a simulation of 365 (or 366) days' concentrations at each receptor.

<sup>†</sup> Here "background" is used to represent the ambient concentration that would exist in the absence of the source under consideration.

### III THEORETICAL MATTERS

In this chapter we describe the theoretical development of a statistical methodology for evaluating the impact of SO<sub>2</sub>-emitting point sources. This method has become known as the ExEx, or expected exceedances, method. The name is appropriate because, instead of deterministic calculation of predicted ground-level SO<sub>2</sub> concentrations using a constant emissions rate, the method employs a Monte Carlo technique to take explicit account of the stochastic variation inherent in coal sulfur contents and the concomitant variation in emissions rates.

The development of the ideas in this chapter is, as implied in the title, theoretical. We believe that it is necessary to detail the development of the method in the body of the report rather than in an appendix, since the requirement that a Monte Carlo approach be used derives directly from the theory. The reader who wishes to know how the method works, and not how it is derived, could skim this chapter and rely on chapter 2 for a description of the method (or skip directly to chapter 4).

To obtain the necessary meteorological input for application of the method, we have used EPA's single-source (CRSTER) dispersion model (EPA, 1977). We point out here, however, that use of this particular model is in no way required for the application of the statistical technique. Any method, whether it be modeling or measurement, that will yield the required meteorological inputs can be used. Thus, the methodology described is flexible in its accommodation of current and future dispersion models.

The development of the ExEx method is accomplished in three stages. First, we develop expressions for calculating the number of exceedances of the standard and the probability of a violation, assuming that meteorological conditions remain constant throughout a year. This assumption enables a simple development of the theory, illustrating many of the salient points of the ExEx method. In the second section, we extend the development to include consideration of meteorological variation. In the third section, we show the

development of the ExEx method as it is implemented; particularly we show why use of a Monte Carlo technique is necessary for the method.\*

In the fourth section we discuss briefly some other topics that were studied in the course of our development of the ExEx method. These topics are not part of the method's development, but they are important in the context of point source impact assessment. First, we discuss the form of the probability distribution of coal sulfur contents and sulfur dioxide emissions rates. Next, we examine briefly the statistical consequences of some alternative forms of emissions limits. This subject is relevant, since the evaluation of the point source under consideration is obviously critically dependent on the form of the selected emissions limit. The principles used for this analysis, however, can be used to analyze any regulation once its form is stated. Finally, we consider an extension of the main problem considered in this report. Although our main concern in the report is evaluation of the impact of single sources on their surroundings, in many situations several sources contribute to pollutant concentrations at a given receptor point. We employ a well-known statistical inequality to derive a lower bound to the probability that emissions from multiple sources will not exceed a given value.

#### A. EXPECTED EXCEEDANCES AND VIOLATION PROBABILITY WITH CONSTANT METEOROLOGY

The ExEx method is based on the computation of the numbers of exceedances of the applicable SO<sub>2</sub> ambient air quality standard (referred to subsequently in this report as "the standard"). The number of exceedances at a given receptor point over a specified time period is not a fixed number. It varies from day to day, month to month, and year to year, depending on, among other factors, the meteorological conditions and the variation in the sulfur content of the fuel (coal) as it is burned. For any specified time period, we could not calculate the exact ground-level concentration that will be observed, even if we had available a perfectly precise air quality simulation model and very precise input data for that model. This uncertainty can be thought of as arising from either the imperfect state of our knowledge of atmospheric processes, or the inherently statistical nature of those processes. However, we can state the number of exceedances that will occur, in probabilistic terms, as an expectation.

The concept of expected value is well known in the field of probability. Formally, the expected value of a random variable  $X$  is denoted by  $E(X)$  and is given by

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\* Use of the Monte Carlo analysis is unnecessary if only results from individual receptors are of interest; in this case the analytical expressions developed in Section B are directly applicable.

$$E(x) = \begin{cases} \sum_{\text{all } x} x \Pr(X = x) = \sum_{\text{all } x} x p_X(x) & \text{if } X \text{ is a discrete} \\ & \text{random variable} \\ \int_{-\infty}^{\infty} t f_X(t) dt & \text{if } X \text{ is a continuous} \\ & \text{random variable} \end{cases} \quad (1)$$

where  $\Pr(X = x)$  is the probability that  $X$  takes the value  $x$ , and  $p_X(x)$  or  $f_X(t)$  is the probability density function of the random variable  $X$ . If many samples of the random variable are taken and the mean  $\bar{x}$  is computed,  $\bar{x}$  tends to the expectation of random variable  $X$  as the number of samples becomes large. That is,

$$\lim_{n \rightarrow \infty} \left( \frac{\sum_{i=1}^n x_i}{n} \right) = E(X) \quad (2)$$

Returning to our consideration of  $\text{SO}_2$  distributions, for a specified probability law describing the distribution of fuel (coal) sulfur contents, we may infer that the distribution of  $\text{SO}_2$  emissions has a corresponding distribution.\* If meteorology were fixed and constant for every day, the ground-level concentrations would also have a corresponding distribution.

Considering just one day, let us assume that the probability of observing an  $\text{SO}_2$  concentration above the standard is  $p$ . Then the formula for expectation given in eq. (1) can be used to compute the expected number of exceedances for that day as follows: The random variable appropriate to this situation is a binomial (i.e., two-valued) variable that takes the value 1 if an exceedance occurs, and 0 if no exceedance occurs. Thus the random variable takes the value 1 with probability  $p$  and the value 0 with probability  $(1 - p)$ , and its expectation is given by

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\* The correspondence between these two distributions can be quantified by making assumptions about control technology effectiveness and reliability. Given these assumptions and information about the distribution of coal sulfur contents, the distribution of emissions can in principle be derived.

$$E(X) = \sum x p_X(x) \quad (3)$$

$$1 \cdot p + 0 \cdot (1-p) \quad ,$$

or

$$E(X) = p \quad . \quad (4)$$

Thus, the expected number of exceedances for the day under consideration is  $p$ .<sup>\*</sup> If the probability of observing an exceedance were equal to  $p$  for all 365 days in a year (this could only occur if meteorology were fixed and constant), the expected number of exceedances for the year would be  $365 p$  (or  $366 p$  for a leap year).

We will now estimate the probability that a violation of the standard will occur during the year (i.e., the probability that more than the allowable number of exceedances will occur). For  $SO_2$ , a violation is currently defined as two or more exceedances during the year. For this definition of a violation, it is not sufficient to compare  $365 p$  to 2, since  $365 p$  is merely the expected number of exceedances in a year. The actual number of exceedances during a given year is a random variable (whose expectation is  $365 p$ ).

Thus, the occurrence of a violation will be evaluated as a probability. Considering all 365 days in the year, and taking the probability of an exceedance to be  $p$  on each day, we can estimate the probability of a violation as

$$\begin{aligned} \text{Pr(violation)} &= \text{Pr}(2 \text{ exceedances}) + \text{Pr}(3 \text{ exceedances}) \\ &\quad + \dots + \text{Pr}(365 \text{ exceedances}) \\ &= \sum_{i=2}^{365} \text{Pr}(i \text{ exceedances}) \quad . \end{aligned}$$

The individual probabilities are given by the properties of the binomial distribution. The following equation gives the probability of observing  $i$  "successes" (exceedances) in  $n$  independent trials (days for a 24-hour standard), assuming consecutive rather than running averages, with a constant probability of success  $p$  at each trial:

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<sup>\*</sup> Note that  $p$  is less than 1, that is, the expected number of exceedances for the one day is less than 1, though the actual number observed must be either 0 or 1. The situation is analogous to die rolling, where the expectation at each roll is 3.5, though this is not a realizable outcome.

$$p(i;n) = \binom{n}{i} p^i (1-p)^{n-i} \quad , \quad (5)$$

where  $\binom{n}{i}$  is the binomial coefficient and is equal to  $\frac{n!}{i!(n-i)!}$ . Thus, the probability of observing a violation of the standard during the year, assuming a constant of probability of exceedance  $p$  every day, is given by

$$\text{Pr(violation)} = \sum_{i=2}^{365} \binom{365}{i} p^i (1-p)^{365-i} \quad . \quad (6)$$

This series has 364 terms and, though the terms generally can be assumed to decay fairly rapidly, it is not worthwhile to evaluate eq. (6), since a simple analytic expression is available. To derive this expression we first note that the probability of a violation can be restated as

$$\begin{aligned} \text{Pr(violation)} &= 1 - \text{Pr(no violation)} \\ &= 1 - \text{Pr(0 exceedances)} - \text{Pr(1 exceedance)} \quad . \quad (7) \end{aligned}$$

From eq. (5), we have

$$\begin{aligned} \text{Pr(0 exceedances)} &= \binom{n}{0} p^0 (1-p)^n \\ &= (1-p)^n \end{aligned} \quad (8)$$

and

$$\begin{aligned} \text{Pr(1 exceedance)} &= \binom{n}{1} p^1 (1-p)^{n-1} \\ &= np(1-p)^{n-1} \quad . \end{aligned} \quad (9)$$

Combining eqq. (7), (8), and (9), we have

$$\begin{aligned} \text{Pr(violation)} &= 1 - (1-p)^n - np(1-p)^{n-1} \\ &= 1 - (1-p + np)(1-p)^{n-1} \\ &= 1 - [1 + (n-1)p](1-p)^{n-1} \end{aligned} \quad (10)$$

For  $n = 365$ , eq. (10) becomes

$$\text{Pr}(\text{violation}) = 1 - (1 + 364 p)(1 - p)^{364} \quad . \quad (11)$$

Figure 6 shows the probability of a violation given in eq. (11) as a function of  $p$ . This figure reveals, for this simplified but instructive example, that when the probability of exceeding the standard approaches 8/365 (i.e., 8 exceedances expected in one year), a violation of the standard becomes virtually certain. Note, however, that the analysis to this point does not represent a sufficiently realistic situation from which to draw conclusions about the probability of violations, since we have assumed that the probability of an exceedance is the same for every day during the year. In fact, of course, this probability is not constant, but varies because of changes in meteorological conditions. This additional factor will be discussed in the following section of this chapter.

There is one further point to make with the simple example illustrated in figure 6. For this case with constant daily exceedance probability, the probability corresponding to two expected exceedances in the year is 2/365, or 0.00548. This probability, when substituted in eq. (11), gives a probability of violation of 0.595 (cf. figure 6). Thus, assuming an expected exceedance of 2--which naive consideration might intuitively suggest to be virtually certain to correspond to a violation--a probability of violation of only about 60 percent is computed. From this result we may infer further that, if the daily probability of an exceedance is such that in the long run 2 exceedances per year are expected, in 40 percent of those years no violation will occur.

#### B. EXPECTED EXCEEDANCES AND VIOLATION PROBABILITY WITH VARIABLE METEOROLOGY

We now extend our analysis to the more realistic situation of variable meteorology and a time-varying probability of exceeding and violating the standard. First, we see that the expected number of exceedances for the whole year is simply the sum of the expected exceedances for all time periods during the year. That is, for a daily (i.e., 24-hour) time period,

$$E \sum_{i=1}^{365} p_i \quad , \quad (12)$$

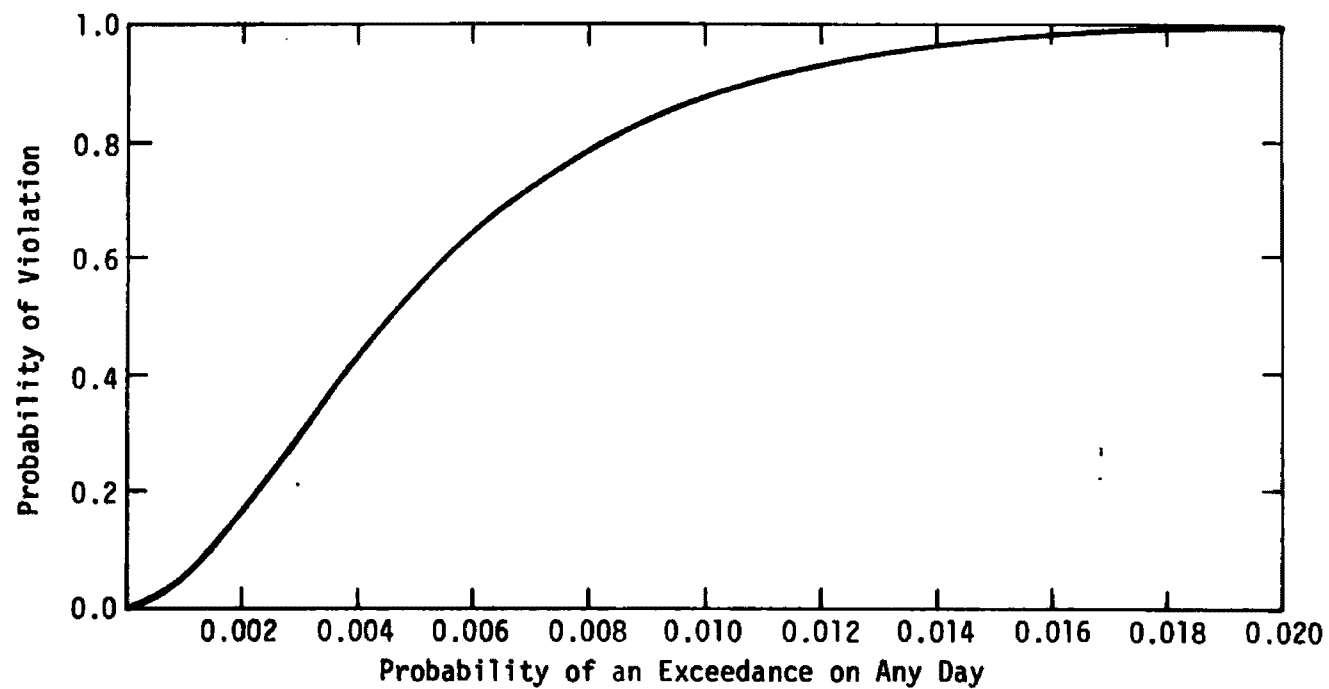


FIGURE 6. PROBABILITY OF A VIOLATION OF THE STANDARD AS A FUNCTION OF DAILY EXCEEDANCE PROBABILITY, ASSUMING CONSTANT METEOROLOGY



where  $E$  is the expected number of exceedances for the year and  $p_i$  is the probability of an exceedance on day  $i$ .

To calculate the probability of a violation, we first note that eq. (7) still applies, that is,

$$\Pr(\text{violation}) = 1 - \Pr(0 \text{ exceedances}) - \Pr(1 \text{ exceedance}) \quad .$$

As above, let  $p_i$  be the probability of an exceedance on day  $i$ , so that  $1 - p_i$  is the probability of not observing an exceedance on the same day. Then the probability of observing zero exceedances throughout the year is the product of the individual daily probabilities, assuming that the probability of an exceedance on any day is statistically independent of the probabilities on any other day:

$$\Pr(0 \text{ exceedances}) = (1 - p_1)(1 - p_2) \dots (1 - p_{365}) \quad (13)$$

$$= \prod_{i=1}^{365} (1 - p_i) \quad .$$

Further, the probability that exactly one exceedance will be observed during the year is the sum of the probabilities of observing an exceedance on day  $j$  and observing no exceedances on any other day. This sum is given by

$$\Pr(1 \text{ exceedance}) = \sum_{j=1}^{365} p_j \left[ \prod_{\substack{i=1 \\ i \neq j}}^{365} (1 - p_i) \right] \quad . \quad (14)$$

Equation 14 can be written more conveniently, provided  $p_j \neq 1$ , so that

$$\Pr(1 \text{ exceedance}) = \sum_{j=1}^{365} \frac{p_j}{1 - p_j} \left[ \prod_{i=1}^{365} (1 - p_i) \right] \quad , \quad (15a)$$

provided  $p_j \neq 1$  for all  $j$ , or, referring back to eq. (13),

$$\Pr(1 \text{ exceedance}) = \sum_{j=1}^{365} \frac{p_j}{1 - p_j} \Pr(0 \text{ exceedances}) \quad , \quad (15b)$$

Thus, combining eqs. (7), (13), and (15), we have

$$\begin{aligned} \Pr(\text{violation}) &= 1 - \prod_{i=1}^{365} (1 - p_i) - \sum_{j=1}^{365} \frac{p_j}{1 - p_j} \left[ \prod_{i=1}^{365} (1 - p_i) \right] \\ &= 1 - \prod_{i=1}^{365} (1 - p_i) \left( 1 + \sum_{j=1}^{365} \frac{p_j}{1 - p_j} \right) \quad , \quad (16a) \end{aligned}$$

provided  $p_j \neq 1$  for all  $j$ .

Equation (16a) can be modified to make calculation more convenient if the  $p_j$ 's are supplied in tabulated form. If there are  $n_1, n_2, \dots, n_k$  time periods with exceedance probabilities  $p_1, p_2, \dots, p_k$ , where

$$\sum_{i=1}^k n_i = 365 \quad ,$$

eq. (16a) can be rewritten

$$\Pr(\text{violation}) = 1 - \prod_{i=1}^k (1 - p_i)^{n_i} \left( 1 + \sum_{j=1}^k \frac{n_j p_j}{1 - p_j} \right) \quad , \quad (16b)$$

provided  $p_j \neq 1$  for all  $j$ .

If some  $p_j$ 's are equal to 1, the equation takes the less convenient form

$$\text{Pr(violation)} = 1 - \prod_{i=1}^{365} (1 - p_i) - \sum_{j=1}^{365} p_j \left[ \prod_{\substack{i=1 \\ i \neq j}}^{365} (1 - p_i) \right] ,$$

which reduces to

$$\text{Pr(violation)} = 1 - \sum_{j=1}^{365} p_j \left[ \prod_{\substack{i=1 \\ i \neq j}}^{365} (1 - p_i) \right] , \quad (16c)$$

since the second term must be zero if any  $p_i = 1$  (the probability of having no exceedances during the year must be zero if there are some days when an exceedance is certain).

These expressions for the probability of a violation (eqq. 16a, b, and c), it must be emphasized, all apply only when it can be assumed that emissions rates for any time period are statistically independent of those for any other time period. This assumption is also included in the current implementation of the ExEx method, but other assumptions could be substituted.

Having derived expressions for the annual expected exceedances and violation probability in terms of the daily exceedance probabilities, we now derive these probabilities from the results obtained from an atmospheric dispersion model combined with a knowledge (known or assumed) of the probability distribution of sulfur dioxide emissions or coal sulfur contents.

Atmospheric dispersion models are used to relate pollutant emissions to ground-level concentrations. They take into account emissions rate, atmospheric transport and dispersion, and, in some cases, other processes such as chemical reactions and wet and dry deposition. For an inert pollutant (and, in the present context, we consider  $\text{SO}_2$  to be inert), which is emitted from an elevated point source, the Gaussian dispersion model is commonly used. This formulation assumes a continuous emissions source, a steady-state downwind plume, and a Gaussian distribution of pollutant concentrations in both the crosswind and vertical directions. The ground-level concentration is given by

$$x = \frac{Q}{\pi \sigma_y \sigma_z u} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{y}{\sigma_y} \right)^2 + \left( \frac{H}{\sigma_z} \right)^2 \right] \right\} , \quad (17)$$

where the wind is advecting the plume at a speed  $u$  along the  $x$ -axis; plume dispersal in the crosswind and vertical directions is parameterized by the coefficients  $\sigma_y$  and  $\sigma_z$ , respectively. The pollutant emission from the source is at a uniform rate  $\bar{Q}$  and is assumed to be released at an effective stack height  $H$  (which depends on the velocity and temperature of the emissions).

The type of dispersion model around which the ExEx method is designed produces time-averaged concentrations at a number of points around the source (called receptors), for an extended period (one year or more). This is achieved by taking a formula such as eq. (17), substituting continuously observed meteorological data, and averaging the computed concentrations for the required time (typically, for  $SO_2$ , 1 hour, 3 hours, or 24 hours). Thus, concentrations at various receptors fluctuate according to the wind direction and other meteorological factors. As noted earlier, the model we have used exclusively in the present work is the single-source (CRSTER) model (EPA, 1977).

Using a dispersion model such as the single-source (CRSTER) model provides a set of ground-level concentrations for an array of receptors around a point source for a specified averaging period. However, a constant emissions rate is assumed for this application;<sup>†</sup> thus the computed concentrations are simply values for atmospheric dispersion (averaged over the period of interest)  $(x/Q)_{ij}$ ,<sup>§</sup> multiplied by the nominal emissions rate  $Q^*$  used in the calculations. That is, the computed concentration in period  $i$  at receptor  $j$  is

$$x_{ij} = \left( \frac{x}{Q} \right)_{ij} Q^* , \quad (18a)$$

or, alternatively, the dispersion in period  $i$  is given by

$$\left( \frac{x}{Q} \right)_{ij} = \frac{x_{ij}}{Q^*} . \quad (18b)$$

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<sup>†</sup> In using these models for this application, it is convenient to exercise them with a unit emissions rate, since concentrations computed by CRSTER are linearly related to emissions rate [see eq. (17)].

<sup>§</sup> Note that  $(x/Q)$  here represents a variable rather than a mathematical expression.

The emissions rate corresponding to a concentration  $x$  is given by

$$Q = \left( \frac{x}{\frac{x}{Q}} \right) \quad (19)$$

The particular emissions rate of interest in these calculations is that rate which will produce an  $SO_2$  concentration just equal to the standard,  $x_{st}$ . Allowing for a background concentration  $B_i$  in period  $i$ , we obtain from eq. (19) the emissions rate that will result in a computed concentration in period  $i$  at receptor  $j$  which is just equal to  $x_{st}$ :

$$Q_{ij} = \frac{x_{st} - B_i}{\left( \frac{x}{Q} \right)_{ij}} \quad (20)$$

Combining eqq. (18) and (20) yields

$$Q_{ij} = \left( \frac{x_{st} - B_i}{x_{ij}} \right) Q^* \quad (21)$$

where

$Q_{ij}$  = the  $SO_2$  emissions rate in period  $i$  that will result in a concentration just equal to the applicable standard  $x_{st}$  at the  $j$ -th receptor.

$B_i$  - the background concentration during time period  $i$  (in all applications to date  $B_i$  has been assumed to be constant<sup>†</sup> both spatially and temporally, but this assumption is not necessary to the development of our analysis),

$x_{ij}$  = the concentration computed by the dispersion model for time period  $i$  at receptor  $j$ ,

$Q^*$  = the nominal emissions rate used in the dispersion calculations.

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<sup>†</sup> Note that if  $B_i$  is not constant, the transformation described by eq. (21) is not necessarily monotonic, that is,  $x_{kj} > x_{lj}$  does not necessarily imply that  $Q_{kj} < Q_{lj}$  ( $k \neq l$ ).

The probability of exceeding the standard during the time period  $i$  at receptor  $j$  is just the probability that the emissions rate in that period is greater than  $Q_{ij}$ . This probability is obtained from the distribution of  $SO_2$  emissions rates,  $f(q)$ , as

$$p_{ij} = \int_{Q_{ij}}^{\infty} f(q) dq \quad . \quad (22)$$

Expected exceedances for the year and the probability of a violation may then be obtained by substituting the  $p_{ij}$ 's calculated from eq. (22) into eqs. (12) and (16), respectively. Carrying out this substitution yields the following expression for the expected number of exceedances in a year for the averaging period of interest:

$$\bar{N}_j = \sum_{i=1}^{365} p_{ij} = \sum_{i=1}^{365} \int_{Q_{ij}}^{\infty} f(q) dq \quad . \quad (23)$$

### C. FORMULATION OF THE EXEX METHOD

As outlined in the previous section, once the daily exceedance probabilities are known at a specific receptor, the expected number of exceedances per year and the probability of a standard violation for that ( $j$ -th) receptor may be calculated. However knowing the individual probabilities of violations occurring at 180 receptors separately does not seem to yield a reasonable basis for evaluating the  $SO_2$  impact of a power plant. For instance, is it better to have a moderate probability of a violation at one receptor (say 0.4) and zero probability everywhere else, or to have a small probability of violation (say 0.05) at 8 or 10 receptors and a 0.001 probability everywhere else? What appears more appropriate is some simpler, overall measure of the plant's impact. Specifically, a simpler measure is the probability that in any year, or over several years, a violation that is attributable to the facility will occur somewhere in the vicinity of the facility. This measure seems to provide a satisfactory resolution of the problem in that attention is shifted from individual receptors to the total area around the plant that experiences  $SO_2$  impacts as a result of the plant.

We now turn to the calculation of the probability that a violation will occur somewhere in the vicinity of the plant, given the 180 individual violation probabilities (derived from results of a Gaussian model such as CRSTER). First we restrict the problem by noting that we can only calculate an overall probability based on the specific receptor locations available, that is, we can compute the probability of a violation somewhere in the network of receptors. However, this restriction is only theoretical, since we may assume without loss of generality that we can locate receptors at all points of substantial interest.\*

To develop an expression for the probability that a violation will occur somewhere in the network, we first recall some results from probability theory (see, for example, Parzen, 1960). If we denote a series of events as  $E_1, E_2, \dots$  and the probability of occurrence of these events as  $\Pr(E_1), \Pr(E_2) \dots$ , the following result is obtained from the axiomatic definition of the probability function:  $\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2)$  if  $E_1 \cap E_2 = \phi$ . That is, the probability of the union of two mutually exclusive events is the sum of their probabilities. However, this definition is not useful in the present case since the events of an exceedance at different receptors are not necessarily mutually exclusive--obviously exceedances can occur at two receptors simultaneously if they are located appropriately (i.e., sufficiently close to one another). We therefore use the formula

$$\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2) - \Pr(E_1 \cap E_2) \quad , \quad (24)$$

which applies without restriction to two events,  $E_1$  and  $E_2$ , defined on the same probability space. This expression can be extended to a large number of events as follows:

$$\begin{aligned} \Pr(E_1 \cup E_2 \cup \dots \cup E_n) &= \sum_{i=1}^n \Pr(E_i) - \sum_{i < j} \Pr(E_i \cap E_j) \\ &\quad + \sum_{i < j < k} \Pr(E_i \cap E_j \cap E_k) - \dots \\ &= 1 - \Pr(E_1^c \cap E_2^c \cap \dots \cap E_n^c) \quad \dots \quad , \end{aligned} \quad (25)$$

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\* The question of appropriately locating receptors when using dispersion models is discussed in the EPA's Guideline on Air Quality Models (EPA, 1978).

where  $E_i^C$  is the complement of the event  $E_i$ . The probability given in eq. (25) can be evaluated if the events  $E_i$  are statistically independent:

$$\Pr(E_1^C \cap E_2^C \cap \dots \cap E_n^C) = \prod_{i=1}^n \Pr(E_i^C) \quad . \quad (26)$$

Combining eqq. (25) and (26), and recalling that

$$P(E^C) = 1 - P(E) \quad ,$$

we have

$$P(E_1 \cup E_2 \cup \dots \cup E_n) = 1 - \prod_{i=1}^n [1 - P(E_i)] \quad . \quad (27)$$

Unfortunately, the events of interest for the present application are not necessarily independent. Moreover, the degree of nonindependence of these events is dependent on the characteristics of each individual problem studied--in particular, the density of the receptor array and the meteorology. This relationship can be illustrated by a simple set of examples.

Consider two receptors, each with a violation probability of 0.3. Application of eq. (27) yields an estimate of 0.51 [= 1-(1-0.3)(1-0.3)] for the probability that a violation will occur at one or the other, assuming statistical independence (we will label this case A). Next, consider the receptors to be colocated, so that when a violation occurs at one, a violation always occurs at the other. (In probability terms, the conditional probability of a violation at receptor 2, given a violation at receptor 1,  $\Pr(E_2|E_1)$ , is 1.0, and vice versa.) Obviously, in this case  $\Pr(E_1 \cup E_2) = 0.3$  (label this case B). Conversely, suppose that the receptors are located such that violations never occur at both simultaneously, that is,  $\Pr(E_2|E_1) = 0$ . In this case it is obvious that  $\Pr(E_1 \cup E_2) = 0.6$  (case C). Thus, we obtain different answers for the probability of a violation occurring at one or the other receptor, depending on the nature of the dependence between the two receptors.



These three examples can be clarified if the joint distribution of violations at the two receptors is considered. In case A (in which eq. [27] is applicable), the occurrence of a violation at each receptor was assumed to be statistically independent of events at the other. Thus, by definition, the conditional probabilities are given by

$$\Pr(E_1|E_2) = \Pr(E_1)$$

and

$$\Pr(E_2|E_1) = \Pr(E_2).$$

That is, knowledge of whether a violation has occurred at one receptor yields no information on the likelihood of a violation occurring at the other. This is illustrated by the following tabulation of exceedance probabilities:

TABULATION OF VIOLATION PROBABILITIES  
FOR CASE A: THE INDEPENDENT CASE

		Receptor 1		Total
		Violation	No Violation	
Receptor 2	Violation	0.09	0.21	0.3
	No Violation	0.21	0.49	0.7
	Total	0.3	0.7	

In this tabulation, the probability of the simultaneous occurrence of violations at receptors 1 and 2 is 0.09 (the product of the two marginal probabilities,  $0.3 \times 0.3 = 0.09$ ). Symbolically,

$$\Pr(E_1 \cap E_2) = \Pr(E_1) \Pr(E_2) \quad ,$$

which is required by the definition of statistical independence. The probability of a violation at receptor 1, given that a violation occurs at receptor 2, is  $0.09/0.3 = 0.3$ . That is,

$$\Pr(E_2|E_1) = \frac{\Pr(E_1 \cap E_2)}{\Pr(E_1)} \quad . \quad (28)$$

From the tabulation, it can be seen that the probability of a violation at one receptor or the other is  $0.09 + 0.21 + 0.21 = 0.51$ , as before.

Turning to case B in which the receptors are colocated, the tabulation now becomes:

TABULATION OF VIOLATION PROBABILITIES FOR  
CASE B: CONDITIONAL PROBABILITY = 1

		Receptor 1		Total
		Violation	No Violation	
Receptor 2	Violation	0.3	0.0	0.3
	No Violation	0.0	0.7	0.7
	Total	0.3	0.7	

The tabulation shows that the probability of a violation at receptor 1, given the occurrence of a violation at receptor 2, is  $0.3/0.3 = 1.0$ , and the probability of a violation at one receptor or the other is  $0.3 + 0.0 + 0.0 = 0.3$ .

Case C results in the tabulation:

TABULATION OF VIOLATION PROBABILITIES FOR  
CASE C: CONDITIONAL PROBABILITY = 0

		Receptor 1		Total
		Violation	No Violation	
Receptor 2	Violation	0.0	0.3	0.3
	No Violation	0.3	0.4	0.7
	Total	0.3	0.7	

Here the conditional probability  $\Pr(E_1|E_2) = 0.0/0.3 = 0.0$ , and the probability of a violation at one receptor or the other is  $0.3 + 0.3 + 0.0 = 0.6$ .

To recapitulate these examples, in the first we considered the events at each receptor to be statistically independent of one another. This is an unrealistic assumption, since the receptors are dependent via their relative locations and the prevailing meteorological conditions. In the second and third examples, dependencies were assumed to be known, and the desired probability of a violation at either or both receptors could be calculated. These two examples, with conditional probabilities of 1.0 and 0.0, are special cases, and, in general, the conditional probabilities can have any value between 0.0 and 1.0. The desired probability of violation can be evaluated, however, for any conditional probability, so long as it is known. For instance, suppose that, as before,  $\Pr(E_1) = \Pr(E_2) = 0.3$ , and  $\Pr(E_1|E_2) = 0.5$ . That is, given a violation at receptor 2, there is an even chance of a violation also occurring at receptor 1. The tabulation for this case is:

TABULATION OF VIOLATION PROBABILITIES:  
CONDITIONAL PROBABILITY = 0.5

		Receptor 1		
		Violation	No Violation	Total
Receptor 2	Violation	0.15	0.15	0.3
	No Violation	0.15	0.55	0.7
	Total	0.3	0.7	

From the tabulation, it can be seen that in this case the probability of a violation at receptor 1 or receptor 2 is  $0.15 + 0.15 + 0.15 = 0.45$ .

An equation for the probability of a violation at one or both of two receptors can be derived from eqq. (24) and (28):

$$\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2) - \Pr(E_1|E_2) \Pr(E_2) \quad (29)$$

However, we cannot generalize eq. (29) to evaluate the probability of a violation somewhere in the receptor network because we do not know the conditional probabilities. As stated earlier, these conditional probabilities depend on relative receptor locations and meteorological conditions. Therefore, we use the results of the CRSTER calculations to represent the day-to-day dependencies of concentrations throughout the receptor array, and we use a Monte Carlo technique to evaluate the effect of these dependencies on the probability of a violation occurring somewhere in the vicinity of the source.

The Monte Carlo calculation is carried out by creating a large number (currently 1000) of samples of a year's emissions by making random draws from the probability distribution of emissions. (The appropriate form of the emissions distribution is discussed elsewhere in this report.) The (randomly selected) values for daily emissions are then combined with estimates of atmospheric dispersion obtained from the single source (CRSTER) model to yield ground-level concentrations at each receptor location. The number of exceedances of the standard, and whether a violation of the standard (two or more exceedances) occurred at any receptor during the year, are noted. This procedure yields estimates of the numbers of exceedances and occurrences of violations at each receptor for the 1000 samples from the emissions distributions. Expected exceedances are obtained by summing over all samples and dividing by 1000. Numbers of samples for which violations were recorded are noted for each receptor individually and for the network as a whole.

## D. OTHER TOPICS

### 1. The Probability Distribution of SO<sub>2</sub> Emissions

As eq. (22) shows, calculation of the probability that the air quality standard will be exceeded involves integrating a frequency (probability) distribution of SO<sub>2</sub> emissions rates. This operation raises the following question: What is an appropriate distribution to use? If specific information for the source under study can be obtained, the best course is to use the actual distribution. But if such specific information is not available, or if a hypothetical case is being investigated, some emissions distribution must be assumed.

Previous work (Rockwell, 1975; PEDCo, 1977; Foster, 1979) has centered on determining distributions for sulfur contents of coal, which should be directly related to SO<sub>2</sub> emissions distributions. Three distributions have been studied: the normal, the lognormal, and the inverted gamma. In the latest study, none of these distributions gave consistently better statistical fits to data, but the lognormal was the closest of the three.

Consideration of the characteristics of coal burned at power plants leads one to expect that, except in the rare case where a power plant utilizes coal purchased from a single supplier (mine), no one standard distribution is likely to adequately describe the emissions from a single plant. Purchases will be made with the objective of meeting an applicable emissions limit. Thus, purchases of relatively high sulfur coal from one source will be balanced by purchases of low sulfur coal from another source. The resulting distribution of emissions could very well be bimodal and not adequately represented by any standard distribution.

In the work described in chapter 3, we have used lognormal emissions distributions of emissions from two hypothetical power plants. This choice is not based on our own data analysis or on any theoretical foundation; rather it is one of convenience and custom. We believe that the results obtained are reasonable and are applicable in a wider context.

### 2. A Brief Consideration of Emissions Limits

SO<sub>2</sub> emissions from point sources are regulated by imposition of emissions limits. However, the effect on recorded emissions from a particular facility of a prescribed emissions limit is dependent not only on the numerical value of the limit, but also on the degrees of flexibility available to the regulator with respect to emissions limits. These include the averaging time over

which emissions are defined and the allowable frequency for exceeding the limit. Since emissions are a stochastically varying quantity, no matter how tightly the emissions from a plant are controlled, there will always remain a finite probability of exceeding the imposed limits. This probability can be made arbitrarily small if coal with a low enough sulfur content is burned, or if scrubbing efficiency is extremely good. The probability of exceeding the emissions limit can be made to approach zero by manipulation of the high emissions end of the distribution (e.g., by "early warning" information systems), or by rejecting noncompliance fuel. Our purpose in this section is to point out how one can calculate emissions limit exceedance probabilities for selected types of regulations. We use 24-hour averaging of emissions throughout, but the same procedures apply to any averaging time, with appropriate changes.

If the emissions limit is defined as a value not to be exceeded more than once during a year, this requirement may be interpreted as implying that the limit must only be exceeded with a probability of 1/365. Assuming the emissions rates to be distributed lognormally, and following Larsen (1971), it can be shown that this requirement further implies that the geometric mean (GM) of the emissions distribution must be less than

$$m = \frac{EL}{(GSD)^{2.94}} ,$$

where EL = emissions limit and GSD = the geometric standard deviation of the SO<sub>2</sub> emissions distribution. Thus, the operator of this facility should buy coal and operate a scrubber such that the geometric mean of the distribution of the plant's SO<sub>2</sub> emissions is less than m.

Another possible definition of the same emissions limit would be to specify the mean of 24-hour values. In this case, the plant operator would arrange coal purchases such that the long-term mean of the emissions distribution was equal to or less than the emissions limit. In actuality, such a regulation might result in emissions that were slightly lower than in the previous example, since facility operators might be inclined to institute a margin of safety by purchasing coal that would result in mean emissions of, say, 80 percent of the limit.

Another possible regulation would be to set the emissions limit as a 30-day average and allow no more than two exceedances per month. If we let the daily probability of an exceedance be constant and equal to p, and assume that emissions for each day are independent of those for other days, we can write (assuming a 30-day month):

Pr(more than 2 exceedances)

$$\begin{aligned}
 &= 1 - \text{Pr}(0, 1, \text{ or } 2 \text{ exceedances per month}) \\
 &= 1 - (1 - p)^{30} - 30p(1 - p)^{29} - \binom{30}{2} p^2(1 - p)^{28} \\
 &= 1 - (1 - p)^{28}[(1 - p)^2 + 30p(1 - p) + 435p^2] \\
 &\quad 1 - (1 - p)^{28}(1 - 2p + p^2 + 30p - 30p^2 + 435p^2) \\
 &= 1 - (1 - p)^{28}(1 + 28p + 406p^2) \quad . \quad (30)
 \end{aligned}$$

We can use eq. (30) to calculate the annual expected exceedances corresponding to any desired probability of observing more than two exceedances per month. For instance, if we set the probability of more than two exceedances at 1/13, i.e., an expectation that two exceedances will occur in a given month just less than once per year, we find that this probability corresponds to 12.4 expected exceedances per year. This result is obtained by setting the left hand side of eq. (30) to 1/13 and solving for the daily probability of an exceedance; this operation yields  $p = 0.034$ . Thus the annual expected exceedances are  $365 \times 0.034 = 12.4$ . Conversely, we calculate that if only one exceedance per year is to be allowed, the corresponding probability of two exceedances per month is only 0.00008. If running averages are used for the regulation, the problem is more complicated because of the lack of independence between successive exceedance probabilities.

This brief discussion is intended to indicate a need for careful examination of the consequences of any contemplated emissions limit regulatory strategy. Because of the variability inherent in a plant's emissions, the effect of different forms of emissions limits is not always predictable without a probabilistic analysis.

### 3. The Multiple Source Problem

One major problem in applying the methods described above to real situations is that of multiple sources. Whereas all of the development so far has been restricted to consideration of a single source (or colocated multiple sources; CRSTER is limited to a single plant, though that plant may have many stacks, all considered to be colocated), many potential applications could involve multiple sources. This extension of the method remains to be studied, but we discuss here some simplified considerations that serve to describe the nature of the problem.

Suppose  $n$  point sources exist, all emitting  $\text{SO}_2$  at identical rates and with identical frequency distributions. Suppose further that the emissions

are independently and lognormally distributed, with each plant having a mean emissions rate  $\mu$  and variance  $\sigma^2$ . We wish to calculate the probability that the total of emissions from the plants is greater than  $\beta n\mu$ , i.e., some factor  $\beta$  greater than the total mean emissions rate from all plants.

We first state Chebyshev's inequality:

$$\Pr[|x - E(X)| < \lambda S] > 1 - \frac{1}{\lambda^2} \quad , \quad (31)$$

where  $E(X)$  is the expected value of the random variable  $X$ ,  $\lambda$  is an arbitrary constant, and  $S$  is the standard deviation of  $X$ . This inequality gives a lower bound on the probability that the absolute value of the difference between a random variable and its mean will exceed an arbitrary multiple of its standard deviation. In this example, we put  $X = \sum x_i$ , where the  $x_i$  are the emissions from each individual plant, and  $S^2 = n\sigma^2$ , since the standard deviations of all plant emissions rates are assumed identical and equal to  $\sigma$ . Further, we wish to calculate  $\beta$  such that the above probability exceeds 0.95. Thus, we may state the problem to be solved as:

Find  $\beta$  such that

$$\Pr\left(\sum x_i < \beta n\mu\right) > 0.95 \quad . \quad (32)$$

We then manipulate this problem until its form corresponds to eq. (31). First, subtract  $n\mu$  from both sides of the probability expression:

$$\Pr\left[\sum_i x_i - n\mu < (\beta - 1)n\mu\right] > 0.95 \quad . \quad (33a)$$

Then note that if the distribution of  $\sum_i x_i - n\mu$  is symmetrical, eq. (33a) becomes

$$\Pr\left[\left|\sum_i x_i - n\mu\right| < (\beta - 1)n\mu\right] > 0.90 \quad . \quad (33b)$$

Equation (33b) is equivalent to eq. (31) if

$$(\beta - 1)n\mu = \lambda(n\sigma^2)^{1/2} \quad (34)$$



and

$$1 - \frac{1}{\lambda^2} = 0.90 \quad ,$$

or

$$\lambda = \sqrt{10} \quad .$$

Substituting  $\lambda = \sqrt{10}$  into eq. (34), we obtain

$$(\beta - 1)n\mu = \sqrt{10} (n\sigma^2)^{1/2} \quad ,$$

or

$$(\beta - 1) = \frac{\sqrt{10}}{\sqrt{n}} \left( \frac{\sigma}{\mu} \right)$$

$$\beta = 1 + \frac{\sqrt{10}}{\sqrt{n}} \left( \frac{\sigma}{\mu} \right) \quad . \quad (35)$$

Thus, we can conclude that if there are 10 sources contributing to concentrations at a particular location, and all 10 sources have identical emissions distributions, there is at least a 95 percent probability that at any given time the total emissions will be less than  $1 + \sigma/\mu$ . The term  $\sigma/\mu$  is the coefficient of variation of the individual emissions distributions (sometimes called the relative standard deviation). Since typical values of this quantity are about 0.2, we could state that there is at least a 95 percent probability that total emissions from all 10 plants will not be more than 20 percent greater than the total mean emissions from the plants. This calculation illustrates the result of the averaging effect of many plants, where if one plant is emitting a large amount of  $\text{SO}_2$ , some other plant will be emitting less to compensate. This considers only the variability of sulfur in the fuel; the operating load is assumed constant in the above analysis.

A further refinement can be made by using the restricted form of Chebyshev's inequality (Cramer, [1946]; Mallows, [1956]):

$$\text{Pr}[|x - M(X)| < \lambda S] > 1 - \frac{4}{9\lambda^2} \quad . \quad (36)$$

This form of the inequality holds only where an assumption of unimodality for the distribution of  $X$  can be made, and the difference is measured from the mode,  $M(X)$ , instead of from the expected value. Working through the same calculation as above, we find that  $\lambda = 2.11$  and

$$\beta = 1 + \frac{2.11}{\sqrt{n}} \left( \frac{\sigma}{\mu} \right) . \quad (37)$$

Thus, for  $n = 10$ ,

$$\beta = 1 + 0.67 \left( \frac{\sigma}{\mu} \right) . \quad (38)$$

Thus, by invoking an additional assumption of unimodality for the emissions distribution, we can state that there is a 95 percent probability that the total emissions from 10 plants will not be more than 13.4 percent greater than the total mean emissions rate (assuming  $\sigma/\mu = 0.2$ ).

## IV RESULTS

In this chapter we detail calculations made using the methodology developed in chapters 2 and 3. (The computer program is described in an appendix, along with required input and sample outputs.) The calculations described in this chapter were carried out for two hypothetical power plants. One of these plants has a scrubber and the other does not; the presence of the scrubber reduces the stack exit temperature and stack exit velocity. The characteristics of each plant are shown in table 2. Note that the same emissions rate was used in both sets of calculations; thus differences in computed  $X/Q$  values are the result of different plume rise characteristics for the two plants. Each plant is, in effect, utilizing fuels (coal) having different sulfur contents. For the unscrubbed plant to have the same emissions rate as the scrubbed plant, it must be burning higher sulfur coal.

Calculations of  $x/Q$  were made using the single source (CRSTER) model. By setting  $Q = 1 \text{ lb}/10^6 \text{ Btu}$  for the model runs, the calculated concentrations of  $x$  can be considered equal to  $x/Q$ . Concentrations for any other emissions rate,  $Q'$ , are then given by

$$\frac{x}{Q}_{\text{calc}} \cdot Q' = .$$

The CRSTER preprocessor runs were made using meteorological data from St. Louis for the years 1973 through 1977 (Lee, 1979). These computed  $X/Q$  values were then used as input to the ExEx method, and the exceedances and violations of the selected standard concentrations were noted. Because of the linear concentration scaling with emissions of CRSTER results, runs made with different standard concentrations for comparison could also be thought of as runs at the same standard concentration with coals having different mean sulfur values: For instance, a run using a standard concentration of, say,  $100 \text{ } \mu\text{g}/\text{m}^3$  and a coal producing mean emissions of  $1.2 \text{ lbs } \text{SO}_2/10^6 \text{ Btu}$  would produce exactly the same results as a run with a standard of  $200 \text{ } \mu\text{g}/\text{m}^3$  and mean emissions of  $2.4 \text{ lbs } \text{SO}_2/10^6 \text{ Btu}$ . That is, doubling all emissions rates results in doubling all computed concentrations and, thus, it produces exactly the same number of exceedances and violations of a standard set at double the original. Because of this property, the graphs, shown in this chapter, of probabilities of violation of a particular standard as a function of mean sulfur content can easily be converted into graphs of probabilities of viola-

TABLE 2. CHARACTERISTICS OF HYPOTHETICAL POWER PLANTS

	Parameter
Size	1000 Mw (Two 500 Mw units)
Stack height	500 ft (typically good engineering practice)
SO <sub>2</sub> emissions	Lognormally distributed
Stack exit temperature	175°F (scrubbed plant) 285°F (unscrubbed plant)
Stack exit velocity	13.14 m s <sup>-1</sup> (scrubbed plant) 15.42 m s <sup>-1</sup> (unscrubbed plant)
Emissions rate corresponding to 1 lb SO <sub>2</sub> /MMBtu	567 g/sec for each stack

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Other Characteristics

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St. Louis meteorology for period from 1973-1977  
Flat terrain assumed

tion for a given coal evaluated at various standard levels.

The cases studied were (for the scrubbed plume) emissions distributions with GSD's of 1.1, 1.2, and 1.4, and (for the unscrubbed plume) emissions distributions with GSD's of 1.05, 1.15, and 1.25. The rationale for these choices follows. The plant with the scrubbed plume was considered to be representative of the emissions ceiling for new plants meeting the New Source Performance Standard (NSPS) of 1.2 lbs SO<sub>2</sub>/MMBtu. Geometric standard deviations of emissions distributions ranging from 1.1 to 2.3 have been calculated for coal-fired power plants (ICF Incorporated, 1979); the great majority of values fall in the range 1.1 to 1.4. Therefore, we chose this range of values for our study of scrubbed plumes. The plant with the unscrubbed plume was presumed to be representative of existing power plants with higher average emissions not subject to NSPS emissions limitations. For a selected period in the past, coal burned at the Labadie power plant had been found to have a mean emissions rate of 4.8 lbs SO<sub>2</sub>/MMBtu and a GSD of 1.15 (Morrison, 1979). Moreover, the ICF work cited above had found GSD's of 1.04 - 1.18 for SO<sub>2</sub> emissions distributions at the inlet to a scrubber. (Emissions at the inlet to a scrubber should be representative of emissions from an unscrubbed plant; at that point no SO<sub>2</sub> has been removed from the effluent gases.) Thus, 1.05 to 1.25 was chosen as a range covering the GSD of emissions distributions for an unscrubbed plume.

The scrubbed power plant was studied using two concentration standards: 50 µg/m<sup>3</sup> and 91 µg/m<sup>3</sup>. The latter is the PSD increment for Class II areas (24-hour averaging time), and the former represents a case where only part (45%) of this PSD increment is available for the plant's emissions. The unscrubbed case was studied using concentration standards of 250 µg/m<sup>3</sup> and 325 µg/m<sup>3</sup>. These represent comparisons with the 24-hour NAAQS (365 µg/m<sup>3</sup>) with constant background concentrations of 115 µg/m<sup>3</sup> and 40 µg/m<sup>3</sup>, respectively. As we pointed out above, however, by suitable rescaling the results can be transformed to apply to any desired standard.

#### A. CRSTER RESULTS

Highest second high concentrations computed using CRSTER for each year's meteorology are shown in table 3. These concentrations were calculated assuming an SO<sub>2</sub> emissions rate of 1 lb SO<sub>2</sub>/MMBtu; to compute a concentration corresponding to any other rate, we merely multiple these numbers by that rate. Thus, we can calculate that, ignoring other sources, in 1973 the scrubbed plant would have violated the 24-hour average PSD Class II increment whenever its emissions rate exceeded 1.95 lbs SO<sub>2</sub>/MMBtu (= 91/46.76), and that the unscrubbed plume would have violated the 24-hour NAAQS whenever its emissions exceeded 13.97 lbs SO<sub>2</sub>/MMBtu (365/26.13).

TABLE 3. CRSTER RESULTS FOR THE HYPOTHETICAL  
POWER PLANT SIMULATIONS

(Concentrations in  $\mu\text{g}/\text{m}^3$  corresponding to  
a unit emissions rate of 1 lb  $\text{SO}_2$ /MMBtu)

<u>Year of Meteorology</u>	<u>Highest Second High Concentration In Receptor Network</u>	
	<u>Scrubbed Plume</u>	<u>Unscrubbed Plume</u>
1973	46.76	26.13
1974	39.85	24.97
1975	59.01	29.43
1976	45.24	25.66
1977	39.19	27.21

It may seem paradoxical that the scrubbed plume should yield higher computed concentrations than the unscrubbed plume. However, since all CRSTER computations were carried out using the same  $\text{SO}_2$  emissions rate (1 lb/MMBtu), the observed differences are all caused by different exit temperatures, and thus different calculated plume rise (see table 2). The unscrubbed plume, with a 110°F higher exit temperature, gives greater calculated plume rise, and thus the plume is carried farther from the plant before its effect is observed at ground level. Over this distance the plume has diffused more, and thus the highest estimated concentrations are lower. Since an unscrubbed plant's emissions can be 5 2/3 times those of a similarly sized scrubbed plant (assuming 85 percent scrubbing efficiency), the ratio between actual computed 1973 concentrations using real emissions rates can be 3.17 times higher for the unscrubbed plant ( $= 5.67 \times 26.13/46.76$ ). Higher scrubber efficiencies would result in even greater differences. (Of course, it is unlikely that scrubbed and unscrubbed plants would burn coal with the same sulfur content.)

## B. EXEX RESULTS

As outlined above, the CRSTER results computed using unit emissions were used to represent five years of 24-hour-averaged dispersion  $X/Q$  values. The yearly sequences were combined with randomly selected sequences of emissions rates using the program described in appendix A. Expected exceedances and the number of violations per 1000 sample emissions distributions were computed for each of the 180 receptors in the CRSTER network. In addition, the number of sample emissions distributions for which a violation was computed anywhere in the receptor network was recorded, and this provided the probability of a violation occurring anywhere around the plant. As discussed in chapters 2 and 3, this is intended to give a realistic assessment of the overall impact of the power plant  $\text{SO}_2$  emissions.

In carrying out the computations, we kept the sulfur emissions distribution constant and varied the standard against which computed concentrations were compared. As discussed above, this is entirely equivalent to calculations carried out for a given standard with the appropriate mean sulfur dioxide emissions rates. We present a summary of the results of these computations in tables 4 and 5. In these tables, we show both the expected exceedances at the worst site and the probability of a violation occurring somewhere in the receptor network, averaged over the five-year meteorological record. The data are plotted in figures 7 and 8, where the relationship of expected exceedances at the worst receptor site and violation probabilities are presented as a function of emissions rate.

The mean emissions rates quoted are the arithmetic means of the distributions and are given by  $\exp(\mu + \sigma^2/2)$ , where  $\mu = \ln$  (geometric mean) and  $\sigma$

**BLE 4. EXPECTED EXCEEDANCES AT THE WORST SITE AND VIOLATION PROBABILITY FOR THE SCRUBBED PLUME**

- (a) Calculations based on SO<sub>2</sub> emissions rate distribution parameters: geometric mean--1.2 lbs/MMBtu, geometric standard deviation--1.1

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
45	1.61	0.992
50	0.98	0.758
65	0.44	0.205
70	0.31	0.168
80	0.20	0.024
91	0.20	0.0
105	0.17	0.0
120	0.06	0.0

- (b) Calculations based on SO<sub>2</sub> emissions rate distribution parameters: geometric mean--1.2 lbs/MMBtu, geometric standard deviation--1.2

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
50	1.11	0.912
60	0.64	0.383
70	0.34	0.195
80	0.20	0.078
91	0.18	0.011
100	0.16	0.002
120	0.08	0.0
140	0.02	0.0



TABLE 4 (Concluded)

(c) Calculations based on SO<sub>2</sub> emissions rate distribution  
 parameters: geometric mean--1.2 lbs/MMBtu, geometric  
 standard deviation--1.4

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
55	1.11	0.963
70	0.49	0.448
80	0.25	0.225
91	0.18	0.096
105	0.12	0.024
120	0.09	0.006
140	0.05	0.001
160	0.03	0.0

TABLE 5. EXPECTED EXCEEDANCES AT THE WORST SITE AND VIOLATION PROBABILITY FOR THE UNSCRUBBED PLUME

(a) Calculations based on SO<sub>2</sub> emissions rate distribution  
parameters: geometric mean--4.8 lbs/MMBtu, geometric  
standard deviation--1.05

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
100	1.54	1.0
112	1.22	0.999
125	0.77	0.586
135	0.45	0.210
150	0.31	0.020
175	0.20	0.0
200	0.20	0.0
225	0.10	0.0

(b) Calculations based on SO<sub>2</sub> emissions rate distribution  
parameters: geometric mean--4.8 lbs/MMBtu, geometric  
standard deviation--1.15

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
100	1.78	1.0
118	1.01	0.919
135	0.58	0.414
150	0.36	0.127
175	0.19	0.008
200	0.16	0.0
250	0.04	0.0
300	0.00	0.0

TABLE 5 (Concluded)

(c) Calculations based on SO<sub>2</sub> emissions rate distribution  
 parameters: geometric mean--4.8 lbs/MMBtu, geometric  
 standard deviation--1.25

<u>Standard (<math>\mu\text{g}/\text{m}^3</math>)</u>	<u>Expected Exceedances</u>	<u>Violation Probability</u>
100	1.70	1.0
125	0.94	0.886
137	0.66	0.608
150	0.45	0.318
200	0.14	0.008
225	0.10	0.001
250	0.06	0.0
300	0.02	0.0

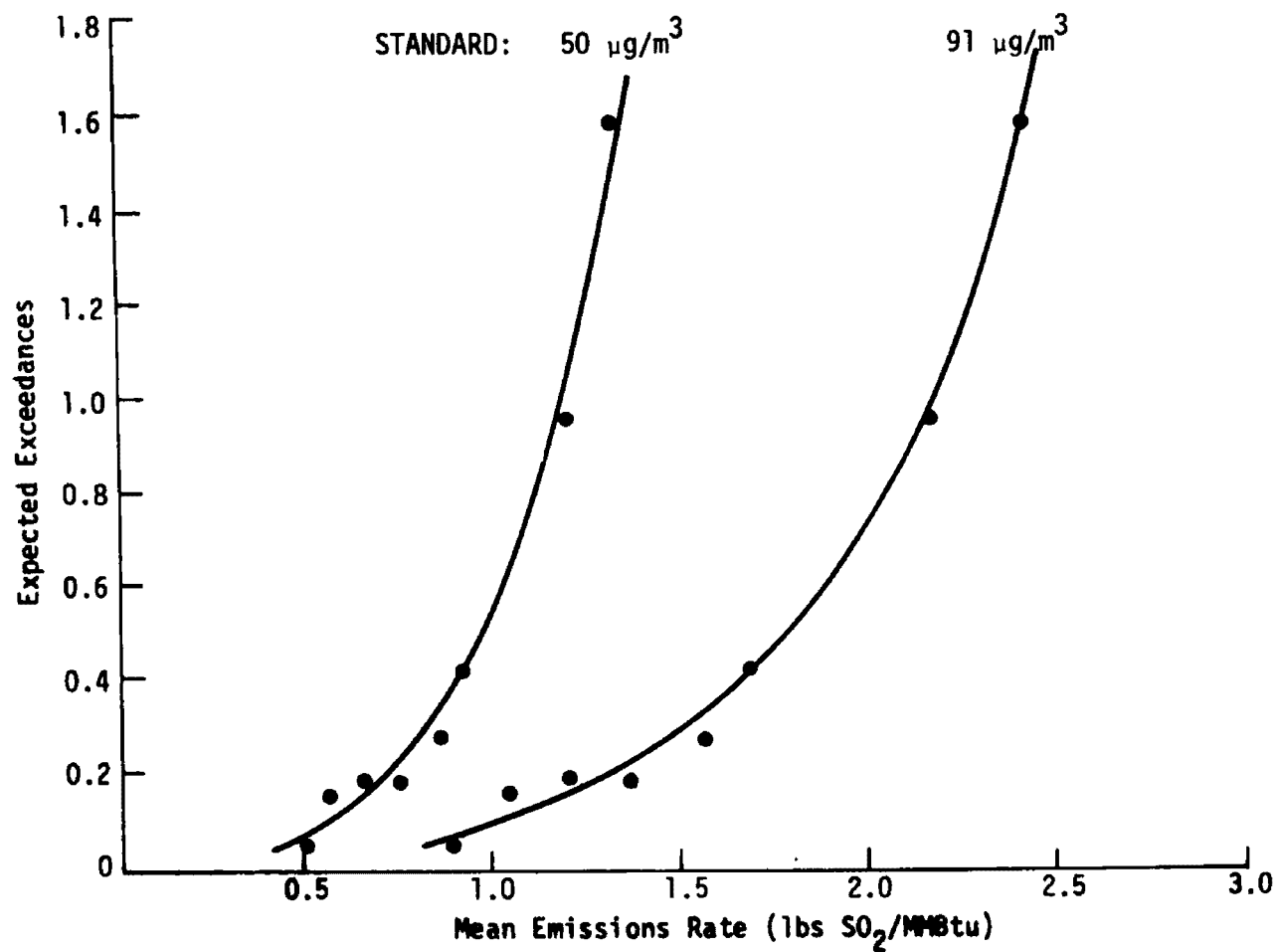
$\ln$  (geometric standard deviation). The arithmetic mean is a measure of long term (for instance, annual) average emissions rates, and thus is a relative measure of the total  $\text{SO}_2$  discharged to the atmosphere by a facility.

A few comments on the computations are in order.

First, for a given concentration level, the receptor site with the highest expected exceedances is not necessarily the site with the highest violation probability. Second, if expected exceedances are plotted against violation probabilities for many examples, the points do not fall along a smooth curve; rather, there is considerable scatter. Thus, there is no simple universal relationship between these two measures that holds for all cases. (See figures 9 and 10 for examples of these plots.) These effects result from the upper tails of the meteorology distributions having large discontinuities if one or more days (periods) during the year have significantly worse dispersion than all others. In an extreme case, one particularly bad day in an otherwise "good" year (one with no more very bad dispersion values) can result in one "guaranteed" exceedance every sample year but never in a violation, because the next worst dispersion value cannot cause an exceedance even with the highest possible emissions rate.

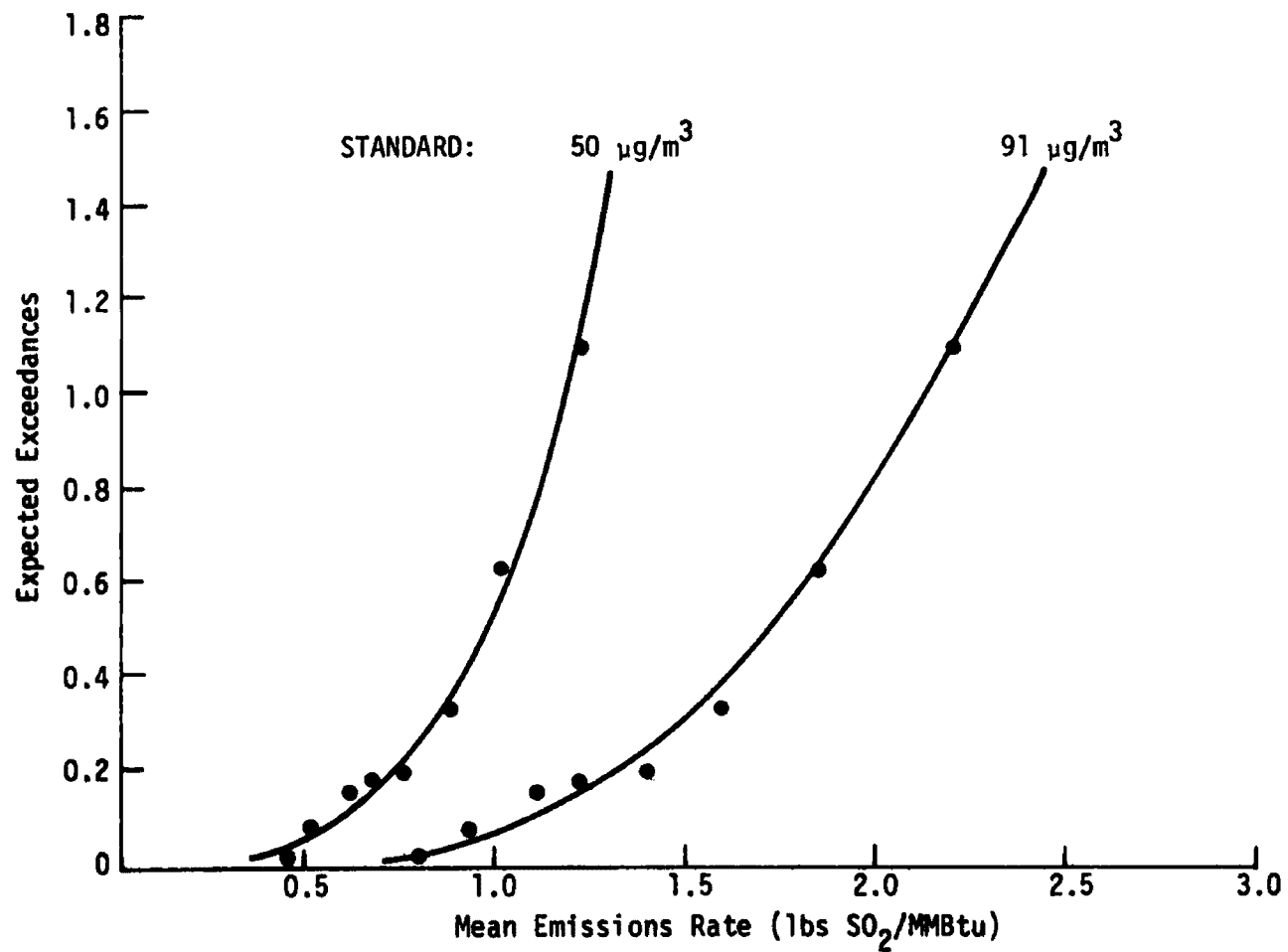
A further effect noted in the results is that when the five-year-average violation probabilities are plotted against mean emissions rates (e.g., figure 8a), a smooth relationship is not always found; "knees" in the curves for the scrubbed plume case were often observed at about 0.2 on the probability scale. The apparent reason for this effect is that, in the CRSTER calculations for the scrubbed plume, one year (1975) was markedly worse than the other four (see table 3), and thus the portion of the violation probability curves at low emissions rates is largely derived from 1975 results.

Figures 7 and 8 are self-explanatory, but some examples may clarify the uses to which they may be put. Suppose we arbitrarily select an ("acceptable") violation probability of 0.1 for the Class II PSD increment of  $91 \mu\text{g}/\text{m}^3$ ; that is, we accept a facility whose expected violation frequency anywhere in the vicinity of the plant is once every 10 years. Then from figure 8b, if a coal with an emissions rate GSD of 1.2 is burned in a scrubbed power plant, the mean emissions rate must be less than 1.45 lbs  $\text{SO}_2/\text{MMBtu}$  in order not to exceed this probability. Referring to figure 7b, this apparently corresponds to expected exceedances of 0.28 at the worst site in the receptor network, or to approximately one exceedance of the Class II PSD increment every four years. Furthermore, using the results in table 2, and using the mean emissions rate of 1.45, a highest second high concentration in the worst year (1975) of  $86 \mu\text{g}/\text{m}^3$  ( $= 1.45 \times 59.01$ ) would be computed using the CRSTER model. The emissions rate expected to occur with a frequency of once per year is  $2.44 \text{ lbs } \text{SO}_2/\text{MMBtu} - \left\{ \exp \left[ \ln 1.45 - \frac{(\ln 1.2)^2}{2} \right] \cdot 1.2^{2.94} \right\}$ , which



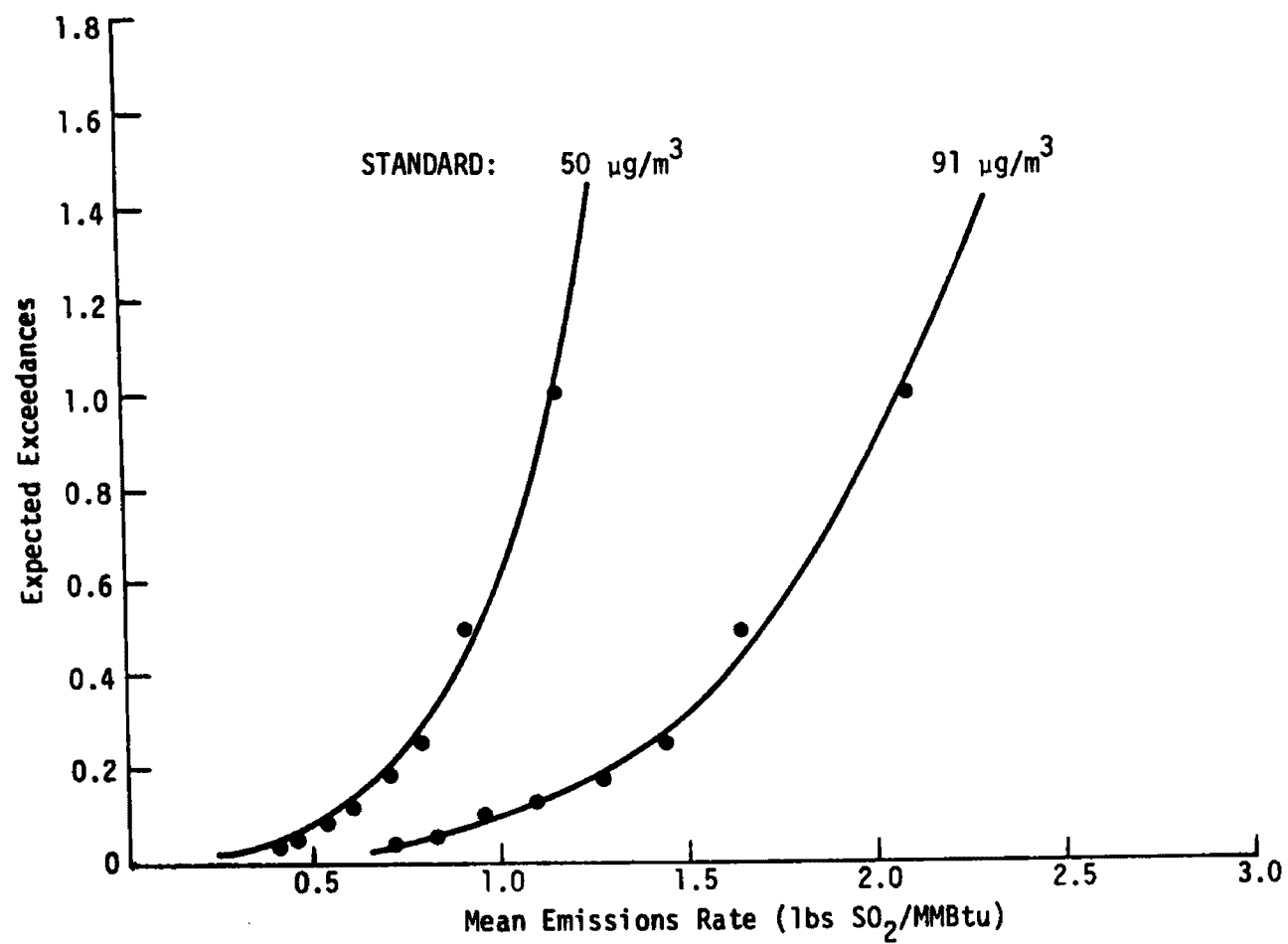
(a) GSD of SO<sub>2</sub> Emissions Distribution: 1.1--Scrubbed Plant

FIGURE 7. EXPECTED EXCEEDANCES



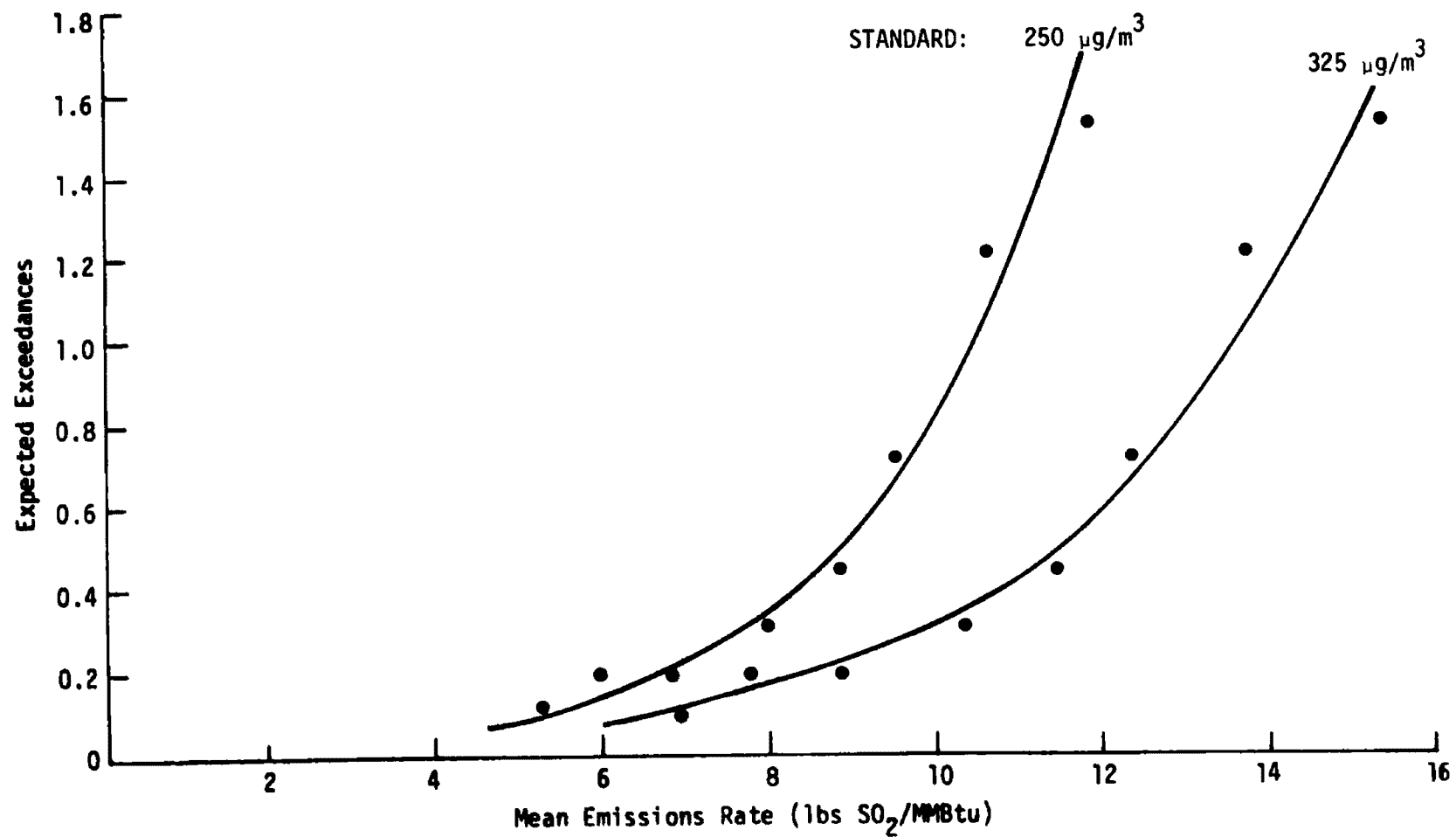
(b) GSD of SO<sub>2</sub> Emissions Distribution: 1.2--Scrubbed Plant

FIGURE 7 (Continued)



(c) GSD of SO<sub>2</sub> Emissions Distribution: 1.4--Scrubbed Plant

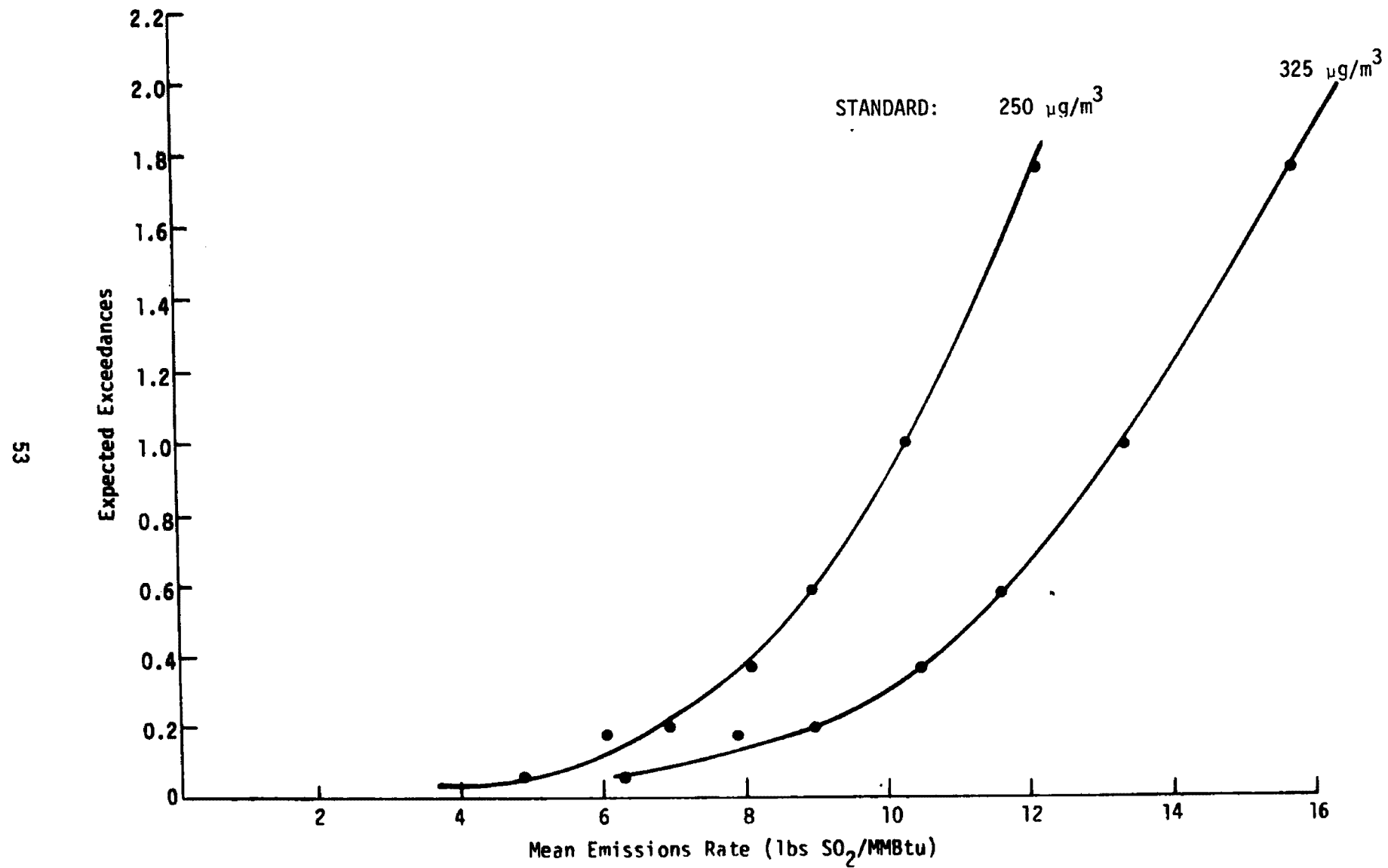
FIGURE 7 (Continued)



(d) GSD of SO<sub>2</sub> Emissions Distribution: 1.05--Unscrubbed Plant

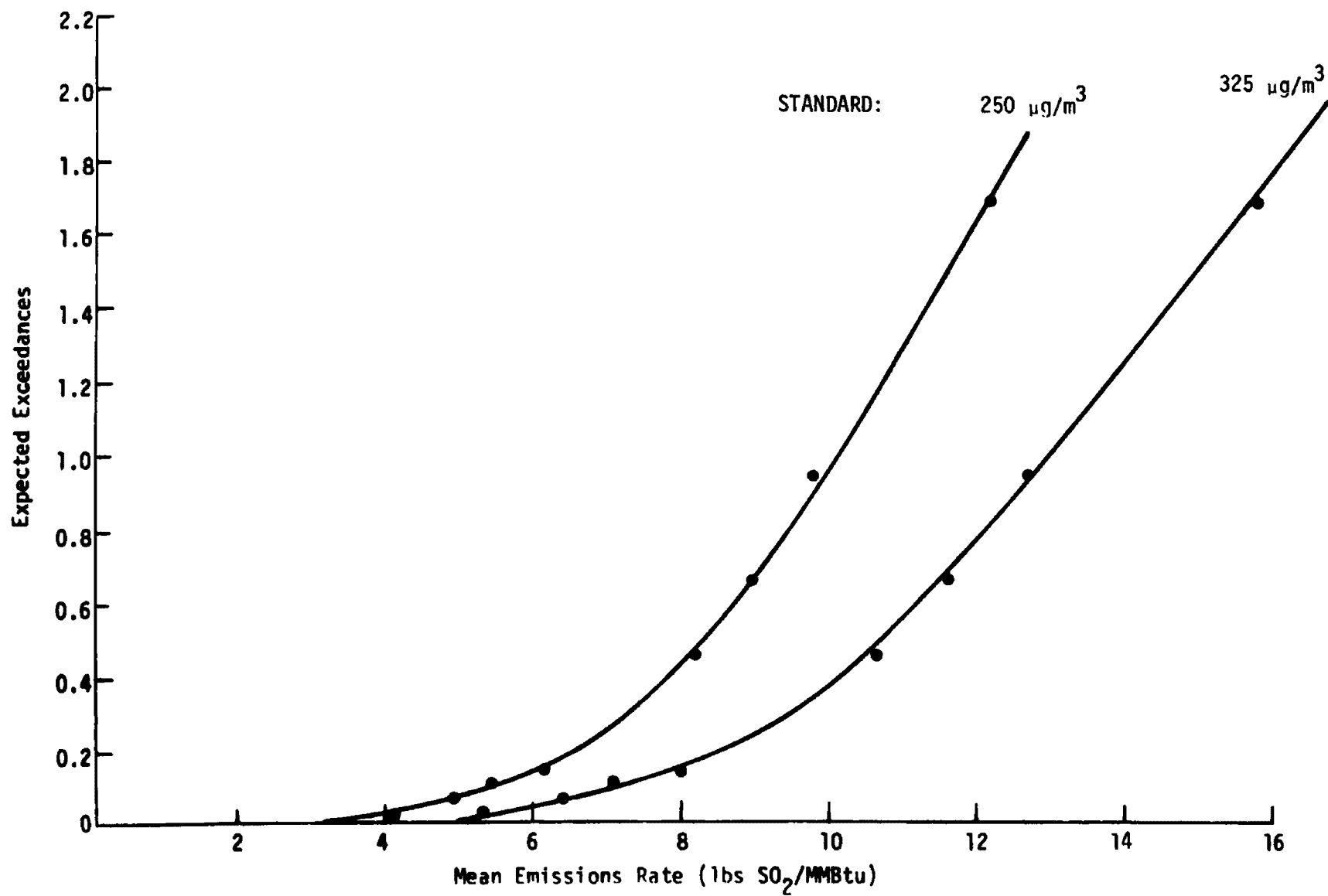
FIGURE 7 (Continued)





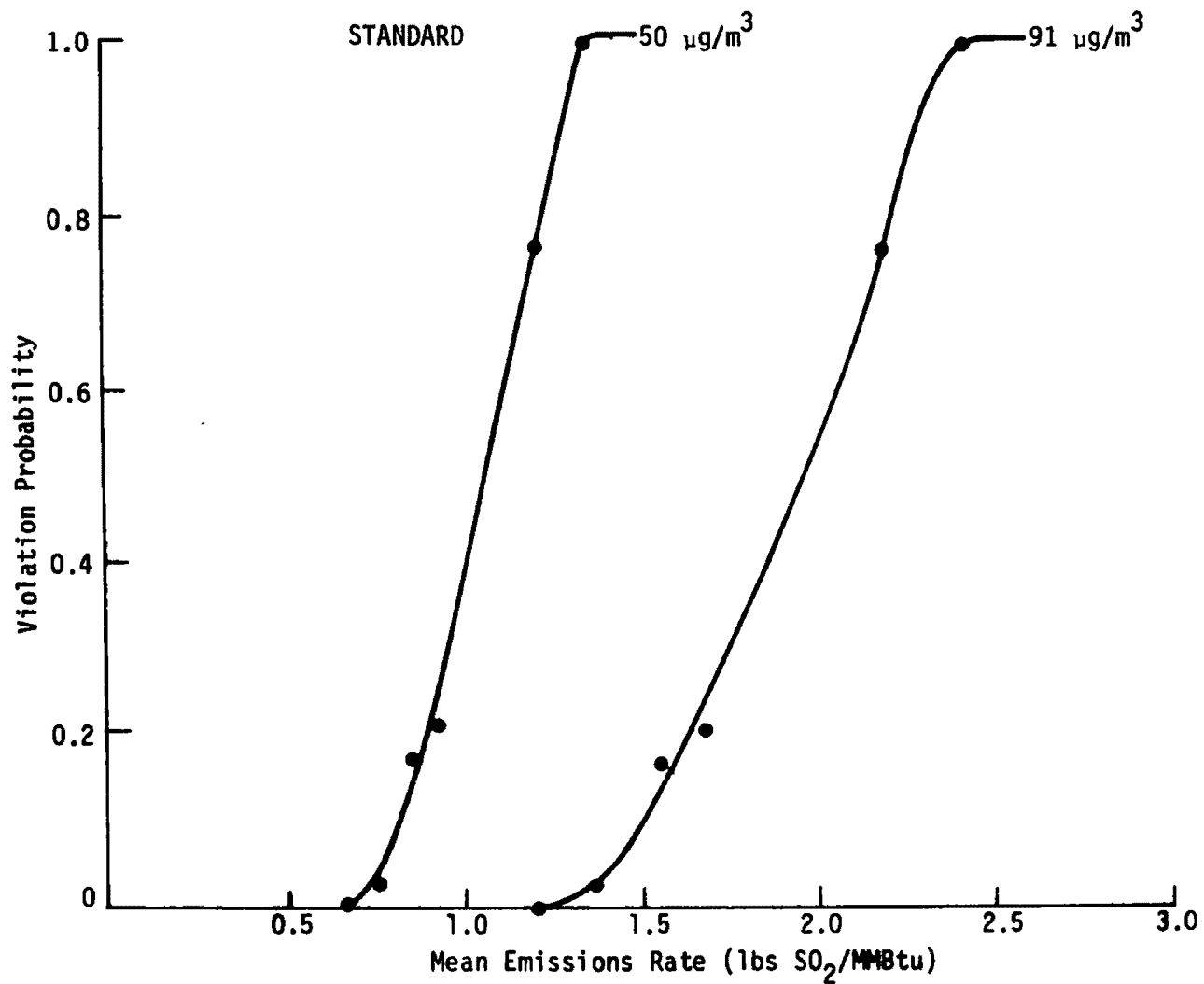
(e) GSD of SO<sub>2</sub> Emissions Distribution: 1.15--Unscrubbed Plant

FIGURE 7 (Continued)



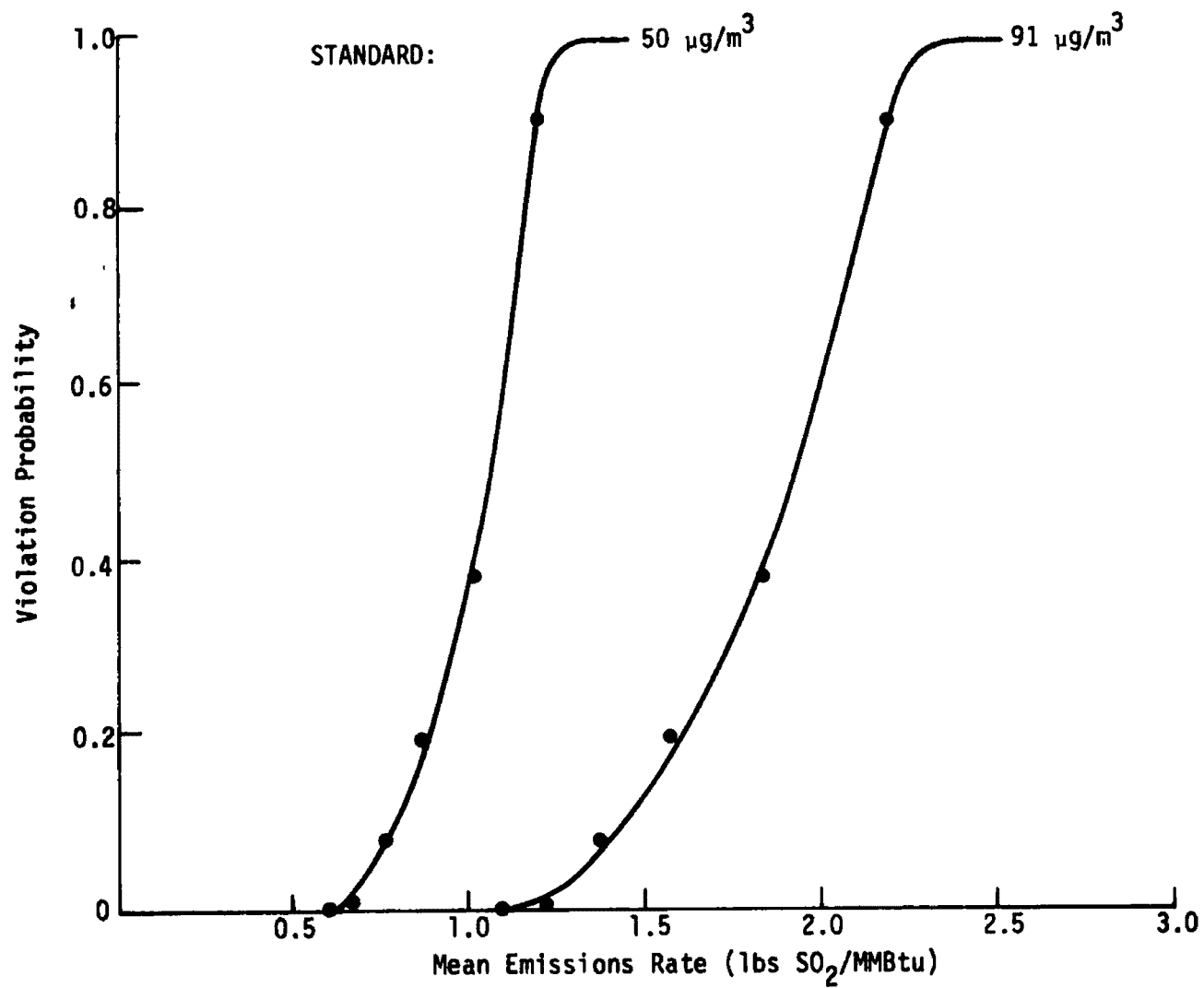
(f) GSD of SO<sub>2</sub> Emissions Distribution: 1.25--Unscrubbed Plant

FIGURE 7 (Concluded)

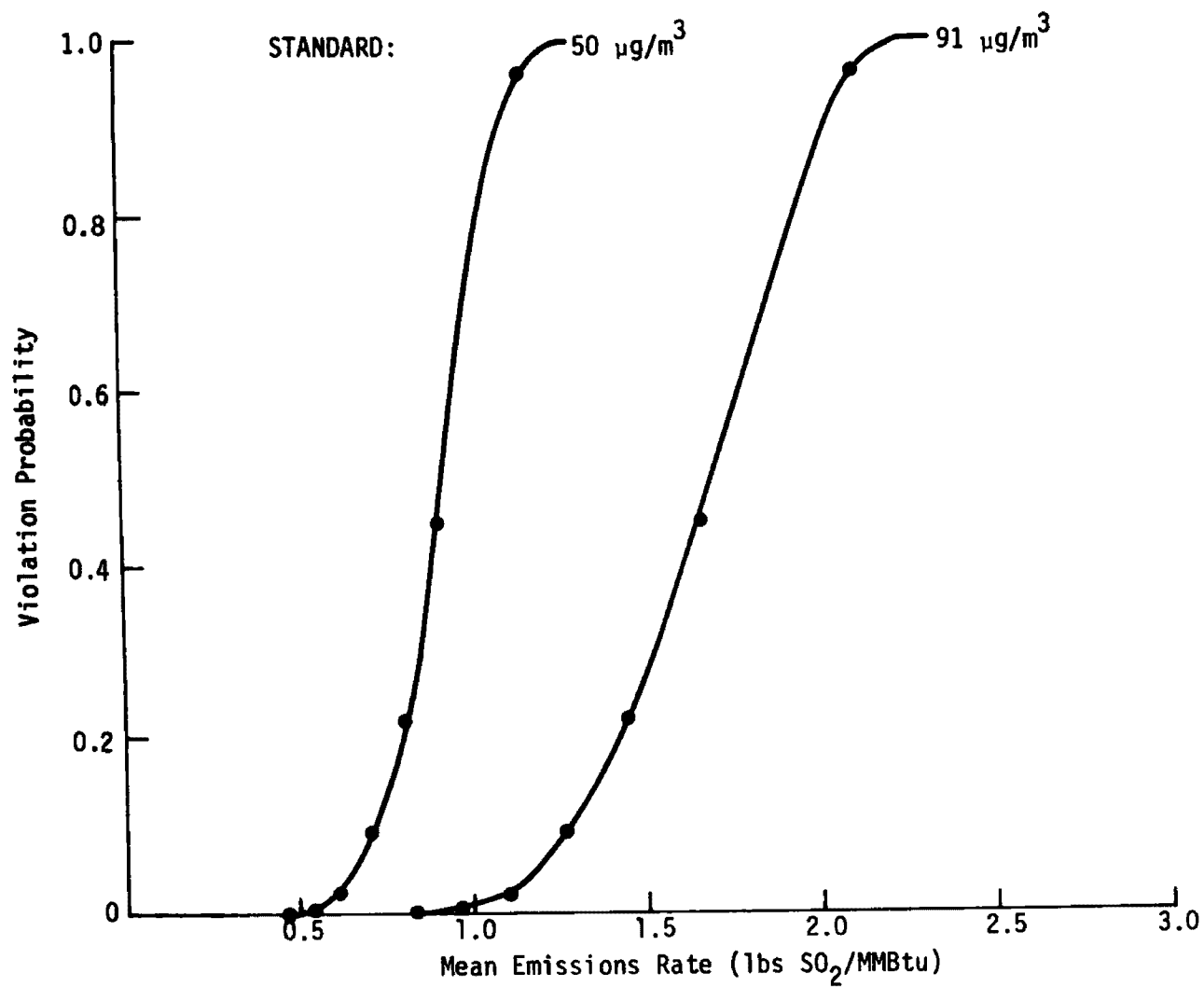


(a) GSD of SO<sub>2</sub> Emissions Distribution: 1.1--Scrubbed Plant

FIGURE 8. VIOLATION PROBABILITY

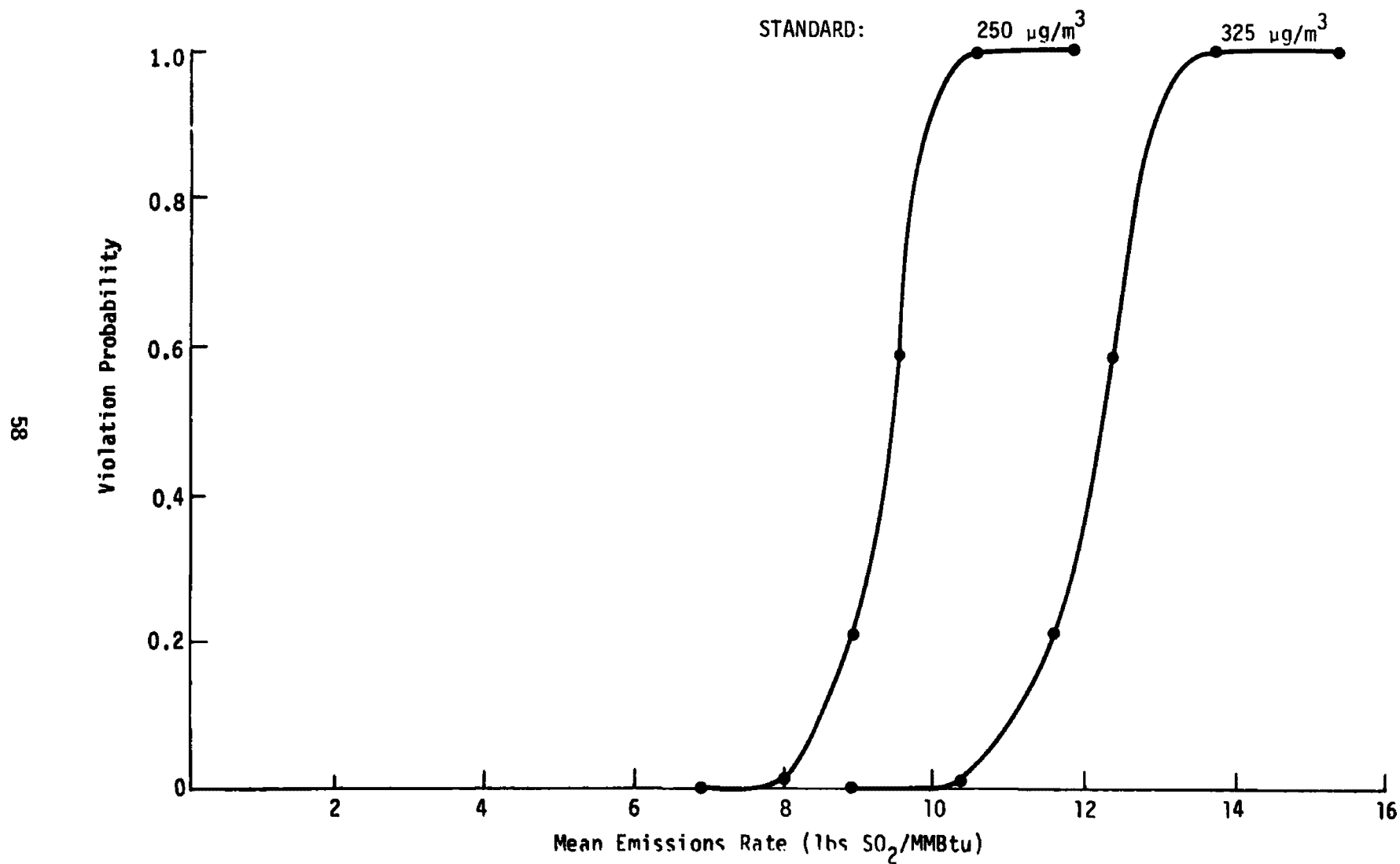


(b) GSD of  $\text{SO}_2$  Emissions Distribution: 1.2--Scrubbed Plant

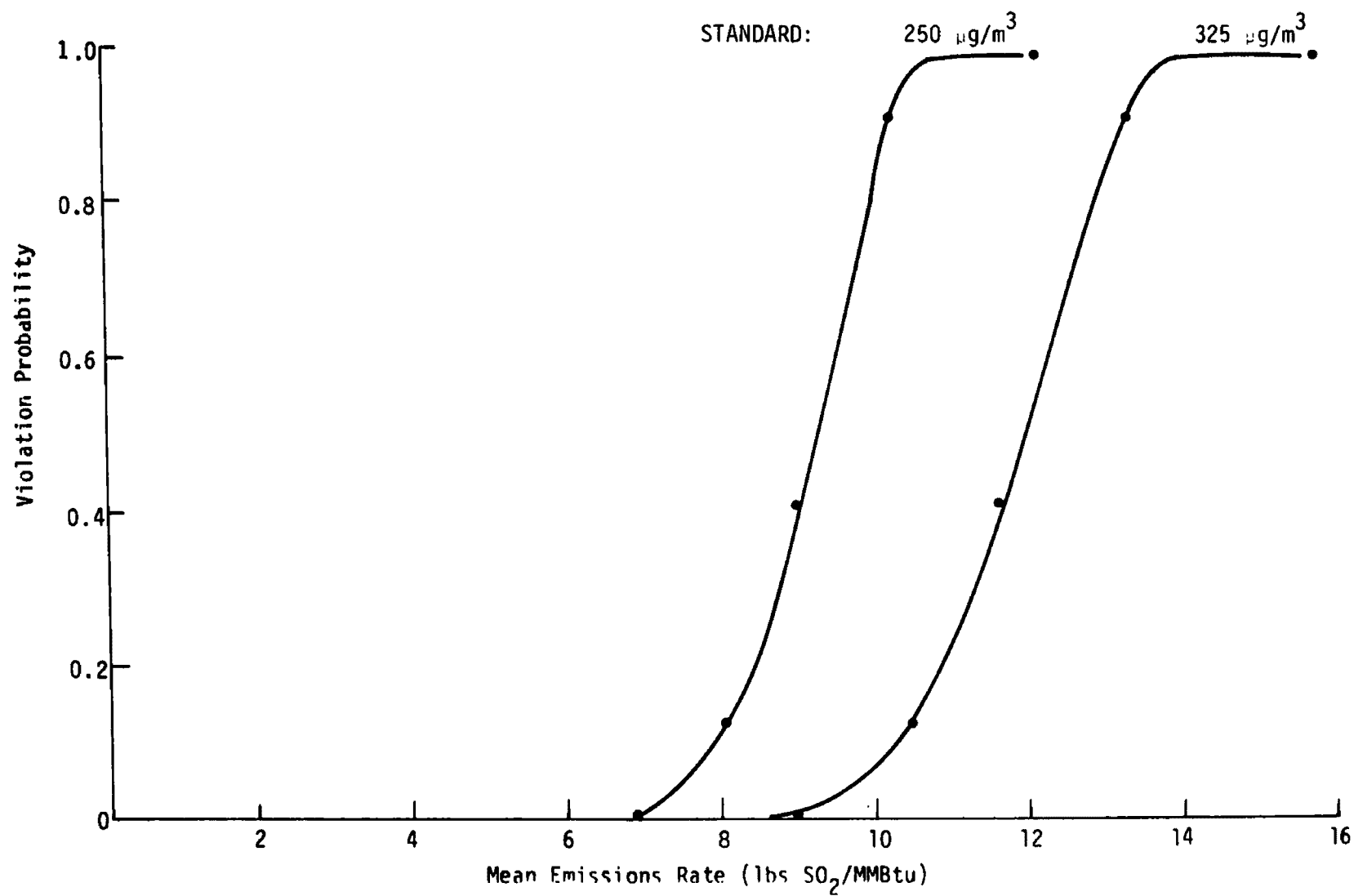


(c) GSD of SO<sub>2</sub> Emissions Distribution: 1.4--Scrubbed Plant

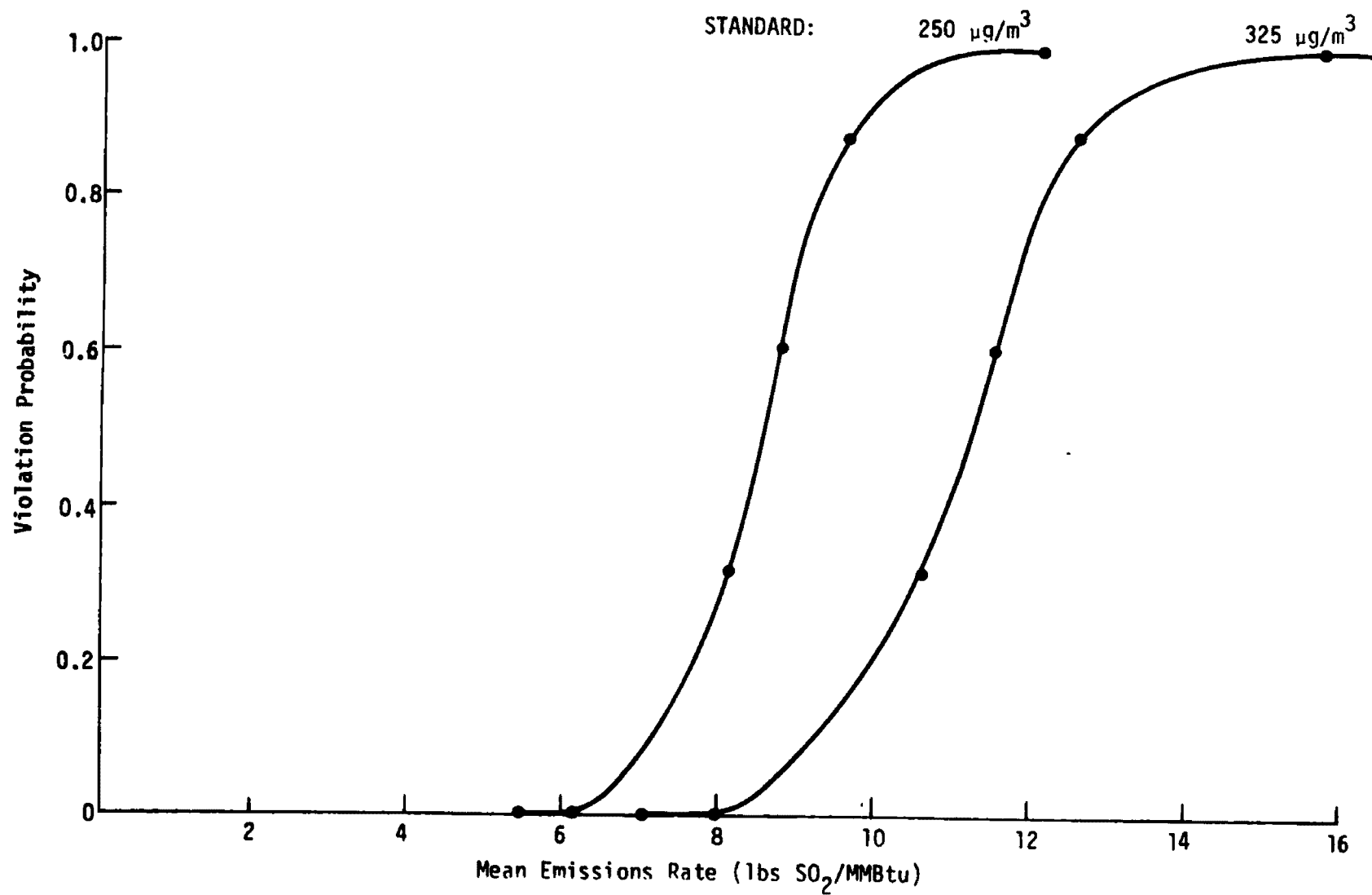
FIGURE 8 (Continued)



(d) GSD of  $\text{SO}_2$  Emissions Distribution: 1.05--Unscrubbed Plant



(e) GSD of SO<sub>2</sub> Emissions Distribution: 1.15--Unscrubbed Plant



(f) GSD of SO<sub>2</sub> Emissions Distribution: 1.25--Unscrubbed Plant



corresponds to a 1975 high second high of  $144 \mu\text{g}/\text{m}^3$ . That is, if the emissions rate that occurs with probability  $1/365$  were to occur on the day with the second worst dispersion during the year, the resulting highest ground-level  $\text{SO}_2$  concentration around the plant is computed as  $144 \mu\text{g}/\text{m}^3$ .

If a similar set of calculations is carried out for a GSD of 1.4, the mean emissions rate to ensure compliance with the  $91 \mu\text{g}/\text{m}^3$  increment (at the 10 percent violation probability) is 1.27 lbs  $\text{SO}_2/\text{MMBtu}$ . The expected number of exceedances at the worst site is 0.18 (i.e., the increment can be expected to be exceeded approximately once in six years). The mean expected highest second high concentration is  $75 \mu\text{g}/\text{m}^3$ , and the highest second high concentration estimated with the extreme ( $1/365$ ) emissions is  $190 \mu\text{g}/\text{m}^3$ .

Comparing the results for the 1.2 and 1.4 GSD values reveals that for the 17 percent increase in GSD (i.e., greater variance in emissions), a 12 percent decrease in the annual average emissions rate is required to comply with the  $91 \mu\text{g}/\text{m}^3$  increment at the 10 percent violation probability level. In addition, this 17 percent higher GSD involves a 32 percent higher worst-case (i.e.,  $1/365$ ) emissions and worst-case\* highest second high 24-hour-average ground-level  $\text{SO}_2$  concentration ( $144 \mu\text{g}/\text{m}^3$  to  $190 \mu\text{g}/\text{m}^3$ ).

Calculations such as these can be used to assess the effect of imposing different emissions limits on a plant. We would emphasize, however, that the results in this chapter apply only to the particular hypothetical plants and terrain situation modeled, for the particular five years of meteorological data. The results cannot be generalized to other circumstances, but they can be used to determine the direction and order of magnitude of effects resulting from changes in emissions distributions or other related matters. We believe that the results given here will be useful in outlining the general relationship between expected exceedances and violation probabilities. Figures 9 and 10 illustrate relationships for the expected number of exceedances at the worst receptor (over a five year period) and the average probability of violating the standard ( $91 \mu\text{g}/\text{m}^3$  PSD Class II increment) at that receptor, and anywhere in the network, respectively. In figure 9 it is apparent that, for these five years of St. Louis meteorology, these hypothetical power plants, these ranges of emissions GSD values, and an expected number of exceedances of 1.0 at the worst site, the corresponding probability of violating the standard is approximately 25 percent. Thus, over a long term period, we can expect approximately one violation of  $91 \mu\text{g}/\text{m}^3$  PSD Class II increment every four years. The results of figure 9 also indicate that for a violation probability of 10 percent at the worst receptor (i.e., one violation in 10 years at the worst receptor), the corresponding expected number of exceedances is approxi-

---

\* This worst-case condition is defined as the highest emissions rate and poorest dispersion occurring on the same day.

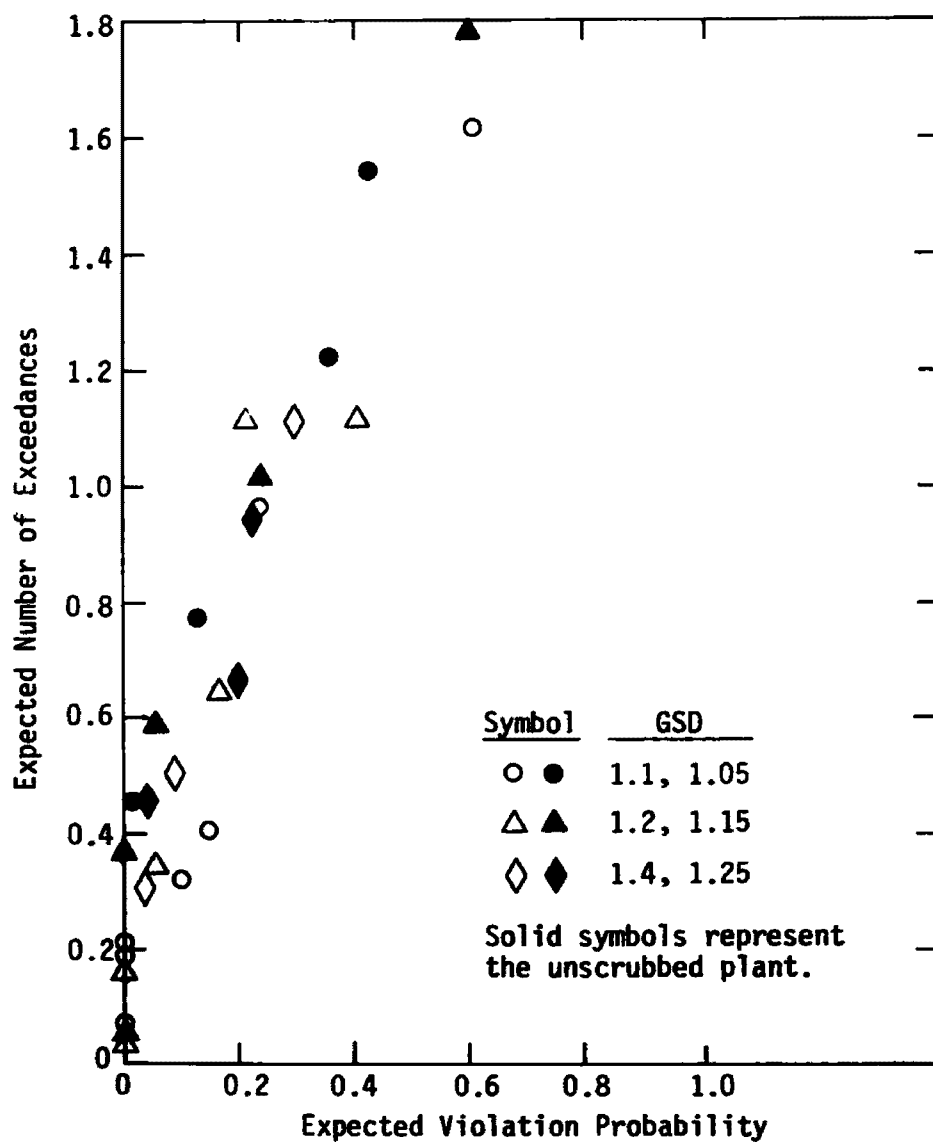


FIGURE 9. EXPECTED EXCEEDANCES VERSUS VIOLATION PROBABILITY OVER FIVE YEARS AT THE WORST SITE

mately 0.6 (the standard is expected to be exceeded approximately once every two years).

The results in figure 10 indicate that, for an expected number of exceedances of 1.0 at the worst receptor site, it is virtually certain that a violation will be predicted by the model at at least one receptor. However, for a predicted 10 percent probability of violations at one or more receptors in the network, the expected number of exceedances at the worst receptor is approximately 0.25, i.e., approximately once in every four years an exceedance of the PSD Class II increment ( $91 \mu\text{g}/\text{m}^3$ ) can be expected at that receptor.

The probability of violating an ambient standard is sensitive to changes in either the mean emissions rate or variability in emissions rates. Perusal of figures 7 and 8 reveals interesting features of this sensitivity. It is generally observed that:

- > As the annual mean emissions rate varies by 25 percent for emissions ranging from 0.75 to 5 lbs  $\text{SO}_2$ /MMBtu, the probability of violating an ambient standard (from 50 to  $91 \mu\text{g}/\text{m}^3$ ) increases from approximately 0.1 to 0.3, i.e., from one violation in 10 years to one in approximately 3 years. For annual mean emissions rates between 5 and 10 lbs  $\text{SO}_2$ /MMBtu, a 10 percent increase in the emissions rate leads to a similar expected increase in the violation probability (i.e., from 0.1 to 0.3).
- > As the variability of emissions rates increases, the probability of violating an ambient standard increases somewhat for a range in variability from approximately 1.1 to 1.4 (as measured by the GSD).
- > The general shapes of the probability of violations vs. annual emissions graphs for different GSD values were similar, each showing an S-shaped curve between a probability of violation of zero and one, over a range of mean emissions rates which depended on GSD.

### C. SUMMARY

In this chapter we have detailed results obtained using a Monte Carlo technique for simulating stochastically varying  $\text{SO}_2$  emissions from two power plants. The results presented enable an assessment of the relationships among results from a single source Gaussian model, expected numbers of exceedances of an air quality standard, and the probability of violating that standard (i.e., observing two or more exceedances) in any given year.

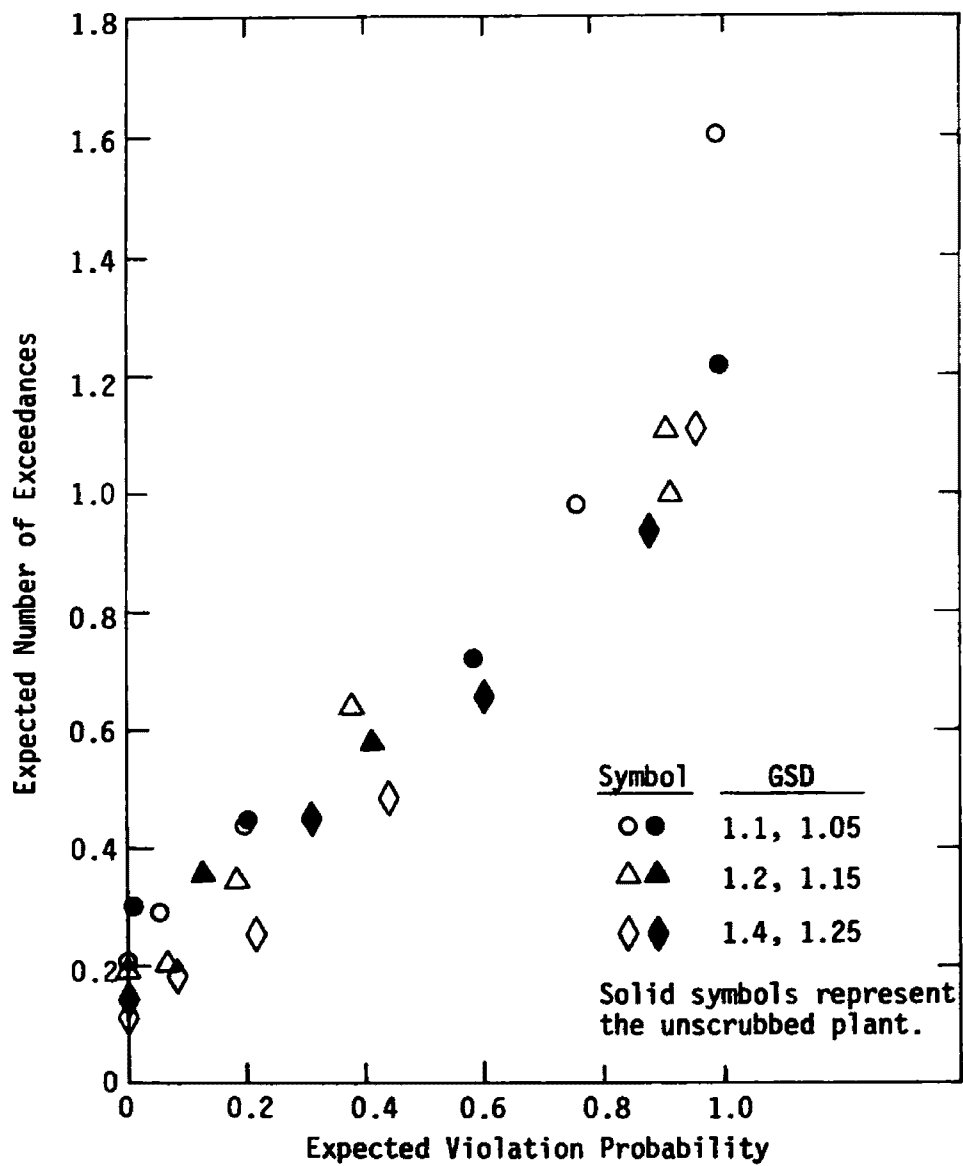


FIGURE 10. EXPECTED EXCEEDANCES AT THE WORST SITE VERSUS VIOLATION PROBABILITY FOR THE COMPLETE RECEPTOR NETWORK OVER A FIVE-YEAR PERIOD

## V SUGGESTIONS FOR FURTHER WORK

The work described in this report represents a new development in which the variability inherent in sulfur dioxide emissions from coal-burning facilities is explicitly accounted for in assessing their impact. This method recognizes the problems with using a constant, maximum emissions rate for this assessment and specifically accounts for the admittedly low probability that maximum emissions and worst-case meteorology can occur simultaneously. At the same time, the technique does not presume that these two events cannot occur together, but rather assigns a realistic probability to their coincidence.

That we treated only the variability in  $\text{SO}_2$  emissions from isolated power plants should not be misinterpreted; two important points must be remembered:

- > Emissions of  $\text{SO}_2$  are not the only, or major, cause of variability in ground-level  $\text{SO}_2$  concentrations.
- >  $\text{SO}_2$  emissions from other sources, and pollutants other than  $\text{SO}_2$ , may merit such stochastic treatment.

We limited the scope of this report to a technical description of a relatively simple version of the ExEx methodology and the presentation of example calculations for hypothetical power plant situations. This does not mean that we are unaware of the policy implications associated with incorporating into impact assessments factors that explicitly contribute to variability in ground-level concentrations. The issues that can, should, and are being raised are so numerous, and in some ways so fundamental, that we had to defer their consideration because of their scope.

However, because of the attention that has been given to the potential impact of implementing the ExEx methodology, we feel compelled to mention what appears to be the greatest concern: The apparent increase in  $\text{SO}_2$  emissions over those allowed by current regulations and modeling (assessment) procedures that could result, and the consequent increase in atmospheric loadings. We wish to point out that both the direction (i.e., increase or decrease) and magnitude of any changes in  $\text{SO}_2$  emissions and atmospheric loadings are inextricably linked to the policies and methods of implementation. As a consequence, control of  $\text{SO}_2$  emissions can be instituted, both in principle and in practice, so as to increase or decrease  $\text{SO}_2$  emissions or maintain them at the

current rate. Having mentioned this issue briefly, we will proceed to address additional technical topics that merit further investigation.

As noted previously, there are many other sources of uncertainty in the evaluation of SO<sub>2</sub> impacts of coal burning facilities. Several such important sources are discussed briefly:

- > The atmospheric dispersion model used. Whether an impact assessment is carried out as at present, using a constant SO<sub>2</sub> emissions rate, or by the ExEx method with variable emissions, the atmospheric dispersion models used yield results that are uncertain to some extent because of approximations inherent in their formulation or implementation, and uncertainties attributed to model inputs. Such model uncertainties introduce a corresponding uncertainty into impact assessments. It is recommended that the magnitude of the effect should be investigated in further studies.
- > Background pollutant concentrations. In most real situations the background concentrations of a pollutant are not constant at a given location, but fluctuate both from statistically random atmospheric effects and because of random influences of other nearby sources. These kinds of effects could be readily incorporated into the ExEx method.
- > Scrubber or other control equipment efficiency. The scrubber installed at a plant does not operate at constant efficiency. Study results involving the evaluation of variations in efficiency should be included in the model to account for their effect in contributing to emissions variability.
- > Load variations. The emissions from a power plant depend on the load. Load does not normally remain constant, but rather fluctuates somewhat to correspond with demand. Moreover, variations in load are, to a degree, autocorrelated, and they exhibit seasonal and diurnal trends. The effects of these variations on SO<sub>2</sub> impacts should be studied.
- > Consecutive versus running average. In all of the work described in this report, consecutive 3-hour or 24-hour averages have been used, with the periods starting at midnight on 1 January. However, compliance with the air

quality standard is currently evaluated in terms of running averages--a new averaging period is started in every hour of the year. Since the use of running averages can result in higher maximum concentrations, the effect on violation probabilities should be studied.

- > Form of the SO<sub>2</sub> emissions distribution. As discussed in chapter 3, no clear choice can be made as to a broadly applicable form of emissions distribution. Additional data are needed to establish suitable forms for the SO<sub>2</sub> emissions distribution. Incorporating different standard distributions into the ExEx method could enable its sensitivity to the form of SO<sub>2</sub> emissions distribution to be evaluated; this would also enable the user to select an alternative most suited to his application.
- > Multiple sources. The current version of the ExEx is applicable to isolated point sources only. Impact assessments involving interacting sources in either urban or rural environments are frequently required. Methods for treating these situations should be examined.
- > Random number generation. Implementation of a methodology containing a machine-dependent process such as random number generation can cause difficulties. Attention needs to be given to procedures for ensuring consistency in impact assessments carried out on different computers.

In addition to the technical issues listed above, there are other sources of uncertainty in impact assessments, particularly those relating to the formulation of regulations. For instance, the averaging period over which emissions are defined, the number of exceedances allowed in a given period (e.g., a month or a year) and the probability of violating any imposed standard are all matters that fall to the regulators to consider. The methods described in this report can be used to investigate different types of regulatory alternatives and practices. Such studies are recommended with the expanded capability of ExEx.

**APPENDIX**  
**THE COMPUTER PROGRAM**



## APPENDIX

### THE COMPUTER PROGRAM

The computer program to implement the ExEx method is very simple: annual emissions distributions are simulated, these distributions are combined with dispersion values calculated by the Single Source (CRSTER) Model to give ground-level SO<sub>2</sub> concentrations, these concentrations are compared against a standard, and the number of standard exceedances is counted. However, a few subtleties must be noted to ensure that the program carries out the computations efficiently and accurately.

First, it is efficient to avoid storing even one year of concentration outputs from CRSTER; for 3-hour averages in a leap year there are 366 x 8 x 180 = 527,040 individual values. Efficiency is achieved by storing only those values that can result in exceedances when combined with possible emissions values. This is done by computing a quantity called ZERO:

$$\text{ZERO} = \frac{\text{STANDARD CONCENTRATION}}{\text{MEAN SO}_2 \cdot (\text{GSD})^6} \quad (\text{A-1})$$

and eliminating those CRSTER concentrations that are smaller than ZERO. As the formula implies, ZERO is that x/Q value that results in a concentration just equal to the standard value, when it is combined with an emissions rate six standard deviations higher than the mean. Since such an emissions rate occurs with probability  $\sim 10^{-8}$ , this procedure eliminates potential exceedances with an upper bound of this probability. If all 527,040 3-hour-average values were as large as ZERO, the total probability of missing an exceedance by this procedure would be  $527,040 \times 10^{-8}$ , or 0.00527, or around 0.5 percent. In actual fact, of course, the probability is much less; most concentrations are several orders of magnitude less than the computed ZERO and, thus, the actual probability of missing an exceedance is much less than 0.5 percent. However, the savings in computer time is spectacular; generally we have observed that very few concentrations pass this screening procedure--the number is usually less than a few hundred per year unless the standard value is exceeded by a wide margin by the maximum concentration.

The second feature we point out is designed to make the program simulate a lognormal distribution of emissions more accurately. The usual method for

generating normally distributed random variables is to sum at least 12 uniformly distributed random variables. However, this method is known to be inaccurate in simulating the tails of the distribution outside the  $3\sigma$  limits (Naylor, et al., 1966), or those values that occur with a frequency of 1/1000 or less. However, we are particularly interested in these extreme events, which are the most likely to produce exceedances of the standard. Therefore, we chose a method for generating normally distributed variables developed by Box and Muller (1958), which is known to be more accurate in the distribution tails. The uniform random number generator shown in the program, RANF(X), is a congruential generator available as a system routine on the CDC7600 computer.

We present a listing of the program in exhibit A-1, with a sample set of input data shown at the end. In our own version of the program, we store the CRSTER output files in a series of disc files that are sequentially accessed by the ExEx program, but this portion of the program can be adjusted to suit the user. The CRSTER model, as supplied by the EPA, will generate files of 24-hour-average concentrations directly. To generate 3-hour-average concentrations, the user should insert an appropriate WRITE statement into CRSTER or write a small program to average three 1-hour-average concentrations and write these out to a file.

If the user wishes to study a particular power plant, and a lognormal distribution of emissions is not appropriate, code can be substituted to sample from any distribution, even one that is empirically derived. To do this, we have set up a vector of length 10,000, with the statement

DIMENSION DSTRB (10000)

inserted after line 17. (In exhibit A-1, all code necessary to accommodate a user-supplied distribution has been inserted as comments and preceded by C\*\*\*\*.) This vector is filled out with the emissions values as follows. Suppose the distribution to have 0.5 percent frequency of emissions of 2.1 lbs  $\text{SO}_2/\text{MMBtu}$ , 1.3 percent with 2.2 lbs/MMBtu, 5.6 percent with 2.5 lbs/MMBtu, and so on. Then we insert a DATA statement after line 30, as follows:

DATA DSTRB /50 \* 2.1, 130 \* 2.2, 560 \* 2.5 . . . . /

The way in which ZERO is calculated is also changed. Instead of the standard code at line 67, insert

ZERO = (STD \* 1.E-6) / DSTRB (10000)

Then, in place of lines 190 to 195, substitute the following code:

C        EXEX. PROGRAM

C        EXEX. PROGRAM

C\*\*\*\*\*

C        PROGRAM TO IMPLEMENT THE EXEX METHOD

C        WRITTEN BY M.J.HILLYER  
C                SYSTEMS APPLICATIONS INC  
C                950 NORTHGATE DRIVE  
C                SAN RAFAEL  
C                CALIFORNIA 94903

C\*\*\*\*\*

C        CODE TO ENABLE SUBSTITUTION OF AN ARBITRARY DISTRIBUTION  
C        IS NOTED BY C\*\*\*\*

C  
C        DIMENSION NEXCD(180), NVIOL(180,9), CONCS(180)  
C        DIMENSION IEXCD(180,9), EXPEXC(180,9), IVIOL(9)  
C        DIMENSION NVIOL5(180), IEXCD5(180), EXPEX5(180)  
C        DIMENSION Q(2928), INCR(9), CNCHR(10000), INDEX(10000)  
C        DIMENSION MAX(9)  
C        DIMENSION TITLE(20)  
C\*\*\*\* DIMENSION DSTRB(10000)

C  
C        LOGICAL LViol, THREE

C  
C        DATA P12 /6.283185308/  
C\*\*\*\* DATA DSTRB / 50\*2.1, 130\*2.2, 560\*2.5, .... /

C  
C        DEFINITIONS OF INPUT VARIABLES

C        SMEAN -- GEOMETRIC MEAN OF SO2 EMISSIONS DISTRIBUTION  
C        GSD    -- GEOMETRIC STANDARD DEVIATION OF EMISSIONS DISTRIBUTION  
C        STD    -- CONCENTRATION AGAINST WHICH TO DEFINE EXCEEDANCES  
C        NSIM   -- NUMBER OF SAMPLES FOR MONTE CARLO PROCEDURE  
C                ( GENERALLY SET TO 1000)  
C        ISTART-- FIRST METEOROLOGICAL YEAR (E.G., 1973, ETC)  
C        NYRS   -- NUMBER OF YEARS OF METEOROLOGY AVAILABLE IN CRSTER OUTPUT  
C        INCR   -- STARTING WITH 1 FOR THE FIRST MET. YEAR, THE INCREMENT  
C                DEFINING EACH SUBSEQUENT YEAR, EG, FOR FIVE SUCCESSIVE  
C                YEARS, ENTER 1 2 3 4 5  
C        THREE -- LOGICAL VARIABLE USED TO DEFINE THE AVERAGING PERIOD  
C                WHEN THREE IS 'TRUE', 3-HR AVERAGES ARE USED,  
C                WHEN THREE IS 'FALSE', 24-HR AVERAGES ARE USED

C  
C        \*\*\*\*\* READ DATA CARDS FOR RUN \*\*\*\*\*

C        READ IN FREE FORMAT

EXHIBIT A-1. EXEX PROGRAM LISTING WITH SAMPLE INPUT DATA

```

C      EXEX. PROGRAM

      READ (5,1000) TITLE
      READ (5,*) STD, SMEAN, GSD, NSIM, ISTART, NYRS
      READ (5,*) (INCR(I), I=1, NYRS)
      READ (5,*) THREE

C
C
C      ***  INITIALISE SOME VARIABLES FOR LATER USE  ***
C
      SSTD  ALOG (GSD)
      ZERO = (STD * 1.E-6 ) / ( SMEAN * EXP(6*SSTD) )
C****  ZERO = (STD * 1.E-6 ) / DSTRB(10000)
      IF (THREE) GO TO 20
      NOLEAP = 365
      LEAPN = 366
      GO TO 25
20  NOLEAP = 2920
      LEAPN = 2928
25  CONTINUE
      DO 49 K = 1,180
          NVIOL5(K) = 0
49  CONTINUE
      DO 50 K = 1,180
          IEXCD5(K) = 0
50  CONTINUE
      IVIOL5 = 0

C
C
C      HERE STARTS THE MAIN LOOP, THROUGH THE MET YEARS
C
C      THE VARIOUS LOOP INDICES ARE DEFINED AS FOLLOWS:
C
C      L      YEAR OF METEOROLOGICAL RECORD ( 1...5 OR SO )
C      K      RECEPTOR NUMBER ( 1... 180 )
C      I      MONTE CARLO SAMPLE NUMBER ( 1...1000 )
C      J      - INDEX OF 'SIGNIFICANT' CONCENTRATIONS (THESE
C              ARE DEFINED BELOW) ( 1..... )
C      IPERD  TIME PERIOD NUMBER ( 1...365,366,2920,OR 2928 )
C      INDEX  INDEX WHICH IS A COMPOSITE OF IPERD AND K, WHICH
C              CAN BE DECODED TO RECOVER THE TIME PERIOD AND
C              RECEPTOR NUMBER FOR A GIVEN CONCENTRATION
C
      DO 600 L = 1, NYRS
          LL = L + 10
          IYEAR = ISTART + INCR(L) - 1
          NPERDS  NOLEAP

C
C      CHECK FOR LEAP YEAR
C
      IF ( MOD(IYEAR,4).EQ.0 ) NPERDS = LEAPN

C
C      INITIALISE IN-LOOP VARIABLES
C
      DO 87 K = 1,180
          NEXCD(K) = 0
87  CONTINUE

```

EXHIBIT A-1 (Continued)



```

C      EXEX. PROGRAM

C
C      IF NO EXCEEDANCES, SKIP TO OUTPUT FOR THIS YEAR
C
C      IF ( MAXNO.EQ.0 ) GO TO 550
C
C      EACH ITERATION OF THIS LOOP
C      SIMULATES A YEAR'S OPERATION
C
C      DO 500 I      1, NSIM
C
C      GENERATE A SET OF RANDOM EMISSIONS RATES.
C      THE METHOD OF BOX AND MULLER IS USED TO
C      GENERATE NORMALLY DISTRIBUTED RANDOM VARIABLES.
C
C      RANF(X) IS A RANDOM NUMBER GENERATOR
C      WHICH GENERATES NUMBERS UNIFORMLY DISTRIBUTED
C      ON THE INTERVAL (0,1)
C
C      DO 200 IPERD = 1, NPERDS
C          R1 = RANF(X)
C          R2 = RANF(X)
C          RANDN = SQRT (-2.*ALOG(R1)) * COS (PI2 * R2)
C          Q(IPERD) = SMEAN * ( EXP (SSTD * RANDN) )
200      CONTINUE
C
C      DO 200 IPERD = 1, NPERDS
C          R1 = RANF(X)
C          IRAND = IFIX ( 10000*R1 ) + 1
C          Q(IPERD) = DSTRB(IRAND)
C*200      CONTINUE
C
C      CALCULATE CONCENTRATION, CHECK FOR AN EXCEEDANCE AND,
C      IF ONE IS FOUND, INCREMENT THE EXCEEDANCE COUNTER
C      NEXCD FOR THE APPROPRIATE RECEPTOR.
C
C      DO 400 J      1, MAXNO
C          IPERD = INDEX(J) / 181 + 1
C          CHI = CNCHR(J) * Q(IPERD)
C          IF ( CHI.LT.STD ) GO TO 400
C          K = MOD ( INDEX(J), 181 )
C          NEXCD(K) = NEXCD(K) + 1
400      CONTINUE
C
C      FOR EACH RECEPTOR SHOWING TWO OR MORE EXCEEDANCES,
C      SET THE LOGICAL SWITCH WHICH INDICATES THAT A VIOLATION
C      HAS BEEN DETECTED, AND INCREMENT THE VIOLATION COUNTER
C      FOR THAT METEOROLOGICAL YEAR.
C
C      DO 420 K      1, 180

```

EXHIBIT A-1 (Continued)

C        EXEX. PROGRAM

```

      IF ( NEXCD(K).LT.2 ) GO TO 420
      NVIOL(K,L) = NVIOL(K,L) + 1
      LVIOL = .TRUE.
420   CONTINUE
C
C      IF ( LVIOL ) IVIOL(L) = IVIOL(L) + 1
      LVIOL = .FALSE.
C
C      DO 450 K = 1,180
      IEXCD(K,L) = IEXCD(K,L) + NEXCD(K)
      NEXCD(K) = 0
450   CONTINUE
C
500   CONTINUE
C
C      DO 520 K = 1,180
      EXPEXC(K,L) = FLOAT ( IEXCD(K,L) ) / FLOAT (NSIM)
520   CONTINUE
C
C      WRITE OUT RESULTS FOR YEAR BEING SIMULATED
C
550   KSTD = IFIX(STD)
      WRITE (6,2000) TITLE
      WRITE (6,2100) IYEAR, SMEAN, GSD, KSTD
      WRITE (6,2200) (EXPEXC(K,L),K=1,180)
      WRITE (6,2300) NSIM, (NVIOL(K,L),K=1,180)
      WRITE (6,2400) IYEAR, NSIM, IVIOL(L)
600   CONTINUE
C
C      ADD UP RESULTS FOR ALL YEARS
C
      NSIM5 = NSIM * NYRS
      DO 700 L = 1,NYRS
      DO 690 K = 1,180
      NVIOL5(K) = NVIOL5(K) + NVIOL(K,L)
      IEXCD5(K) = IEXCD5(K) + IEXCD(K,L)
      EXPEX5(K) = FLOAT ( IEXCD5(K) ) / FLOAT (NSIM5)
690   CONTINUE
C
      IVIOL5 = IVIOL5 + IVIOL(L)
700   CONTINUE
C
C      PROB = FLOAT ( IVIOL5 ) / FLOAT (NSIM5) * 100.0
C
C      WRITE OUT SUMMARY RESULTS

```

EXHIBIT A-1 (Continued)

```

C      EXEX. PROGRAM

C
C
C      WRITE (6,2000) TITLE
      WRITE (6,2500) SMEAN, CSD, KSTD
      WRITE (6,2200) (EXPEX5(K),K=1,180)
      WRITE (6,2300) NSIMS, (NVIOL5(K),K=1,180)
      WRITE (6,2600) NYRS, NSIMS, IVIOL5, PROB
      WRITE (6,2700) ( L,MAX(L),L=1,NYRS )

C
C
C      TERMINATE RUN
      STOP

C
C
C      ***** FORMAT STATEMENTS *****

C
C      1000 FORMAT(20A4)
      2000 FORMAT(1H1,20A4)
C
      2100 FORMAT (1H-,10X,14,12H METEOROLOGY,
      $          4X,F5.2,18H LB SO2/MMBTU COAL,
      $          4X,6HCSD = ,F4.2,13X,14,
      $          26H UC/M3 COMPARISON STANDARD)
C
      2200 FORMAT(1H-,10X,20HEXPECTED EXCEEDANCES,/,
      $          12X,6HRING 1,/,3(19X,12(F6.2,2X),/),
      $          12X,6HRING 2,/,3(19X,12(F6.2,2X),/),
      $          12X,6HRING 3,/,3(19X,12(F6.2,2X),/),
      $          12X,6HRING 4,/,3(19X,12(F6.2,2X),/),
      $          12X,6HRING 5,/,3(19X,12(F6.2,2X),/))
C
      2300 FORMAT(1H0,10X,26HNUMBER OF SAMPLES (OUT OF ,14,
      $          51H) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES),
      $          17H AT EACH RECEPTOR,/,
      $          12X,6HRING 1,/,3(19X,12(16,2X),/),
      $          12X,6HRING 2,/,3(19X,12(16,2X),/),
      $          12X,6HRING 3,/,3(19X,12(16,2X),/),
      $          12X,6HRING 4,/,3(19X,12(16,2X),/),
      $          12X,6HRING 5,/,3(19X,12(16,2X),/))
C
      2400 FORMAT(1H-,/, 12X,5HUSING, 15, 27H METEOROLOGY, THE NUMBER OF,
      $          21H SAMPLE YEARS (OUT OF, 15, 1H),/,12X,15HRESULTING IN A ,
      $          38HVIOLATION AT AT LEAST ONE RECEPTOR WAS, 15)
C
      2500 FORMAT(1H-,39H RESULTS FOR ALL 5 METEOROLOGICAL YEARS,
      $          4X,F5.2,18H LB SO2/MMBTU COAL,
      $          4X,6HCSD = ,F4.2,13X,14,
      $          26H UC/M3 COMPARISON STANDARD)
C
      2600 FORMAT(1H-,/,10H USING ALL,12,22H YEARS OF METEOROLOGY,,
      $          35H THE NUMBER OF SAMPLE YEARS (OUT OF,15,1H),/,

```

EXHIBIT A-1 (Continued)



C        EXEX. PROGRAM

```

      $      50H RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR.
      $      4H WAS, 15, 111., /, 28H THUS, THE PROBABILITY OF A ,
      $      29H VIOLATION AT SOME RECEPTOR IS, F6.1, 4H PCT)
C
2700 FORMAT(1H1, 39H IN CASE YOU'RE INTERESTED, THE NUMBER ,
      $      17H OF CONCENTRATIONS, /, 28H LOOKED AT FOR EACH YEAR WAS, /,
      $      5(5H YR , 15, 110, /) )
      END
```

\*\*\*\*\*  
\*\*\*\*\*

#### EXAMPLE DATA INPUT

```
1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC
118. 4.8 1.15 2 1973 5
1 2 3 4 5
0
```

\*\*\*\*\*  
\*\*\*\*\*

EXHIBIT A-1 (Concluded)

```

      DO 200 IPERD = 1, NPERDS
        R1 = RANF(X)
        IRAND = IFIX (10000 * R1) + 1
        Q(IPERD) = DSTRB (IRAND)
200    CONTINUE

```

This will result in an emissions rate that is randomly selected from the entire distribution.

A flow diagram for the program is shown in figure A-1. As pointed out above, the program is very simple and self-explanatory.

The input data are shown at the bottom of exhibit A-1. It consists of four cards:

Card 1: Title (up to 80 characters).

Card 2: STD--the standard concentration for making concentration comparisons.

SMEAN--the geometric mean of the emissions distribution (not needed if an empirical distribution is being used--see above).

GSD--Geometric standard deviation of emissions distribution (not needed if an empirical distribution is being used).

NSIM--the number of sample years to use in the Monte Carlo analysis--generally set to 1000.

ISTART--the first year of meteorological data (CRSTER output).

NYRS--the number of years of meteorological data (CRSTER output) available.

Card 3: INCR--A set of increments that are used to increase ISTART to equal the numbers of subsequent years. For instance, if five successive years are available, INCR would be entered as 1 2 3 4 5.

Card 4: THREE--A logical variable that indicates whether 3- or 24-hour average concentrations are being used.

A sample output is shown in exhibit A-2. Again, this material is self-explanatory. We have run this program on a CDC 7600 computer. However, we believe it can be run on any computer with a standard FORTRAN compiler.

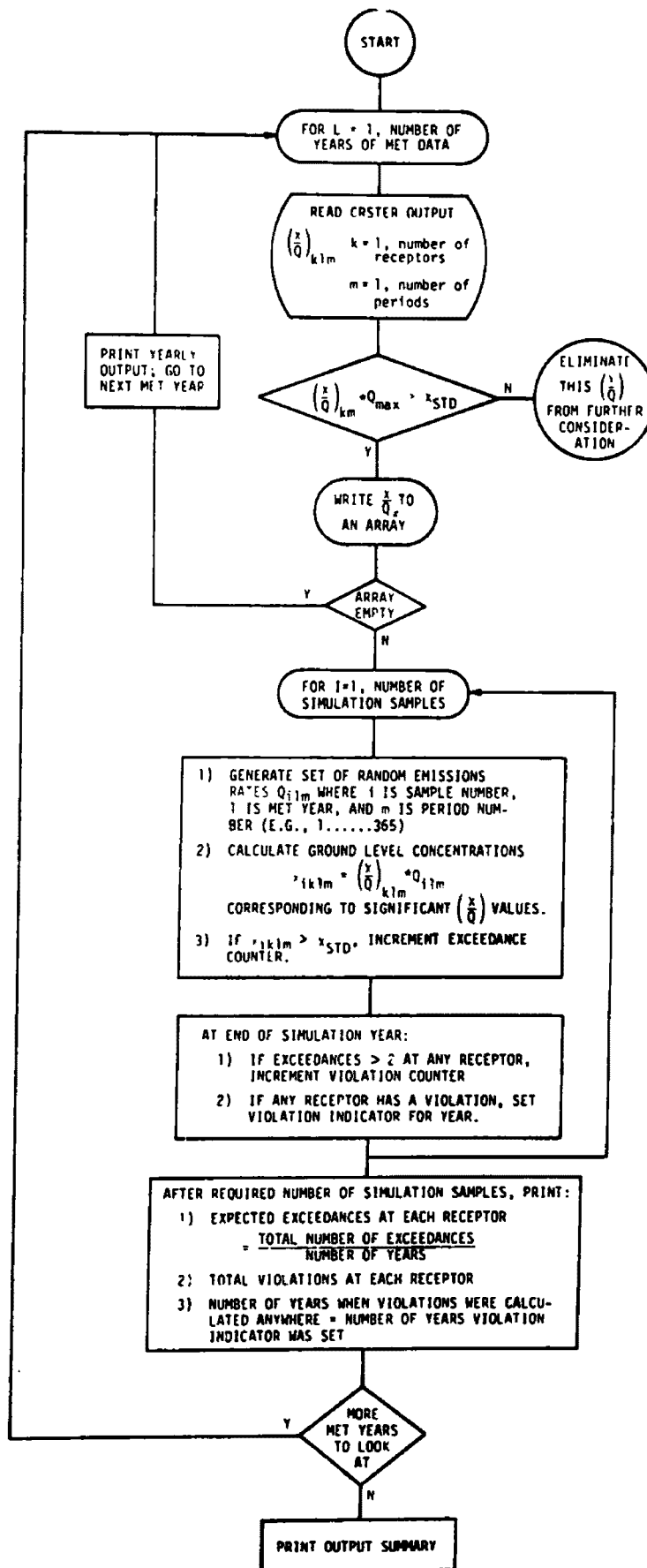


FIGURE A-1. FLOW DIAGRAM FOR EXEX PROGRAM

1000 PM UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

1973 METEOROLOGY 4 80 LB SO2/MMBTU COAL GSD = 1 15 118 UG/M3 COMPARISON STANDARD

EXPECTED EXCEEDANCES

RING 1	0 96	0 78	0 00	0 00	0 00	0 01	0 00	0 00	0 00	0 00	0 00	0 00
	0 00	0 00	0 00	0 17	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 03
	0 00	0 00	0 00	0 00	0 00	0 02	0 03	0 00	0 00	0 00	0 00	0 13
RING 2	0 01	0 11	0 21	0 04	0 31	0 98	0 96	0 16	0 15	0 13	0 22	0 36
	1 11	0 93	0 00	0 07	0 19	0 92	0 61	0 04	0 00	0 00	0 00	0 04
	0 00	0 04	0 12	0 04	0 00	0 39	0 84	0 03	0 00	0 00	0 01	1 64
RING 3	0 00	0 01	0 00	0 00	0 00	0 00	0 47	0 00	0 00	0 03	0 01	0 00
	0 00	0 12	0 00	0 00	0 00	0 03	0 00	0 00	0 00	0 00	0 00	0 07
	0 00	0 03	0 34	0 00	0 00	0 03	0 10	0 00	0 03	0 00	0 00	0 31
RING 4	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 01
RING 5	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00
	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00	0 00

NUMBER OF SAMPLES (OUT OF 1000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES) AT EACH RECEPTOR

RING 1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1
RING 2	0	1	16	0	71	5	1	8	0	4	2	2
	245	191	0	0	5	0	0	0	0	0	0	0
	0	0	0	0	0	9	128	0	0	0	0	656
RING 3	0	0	1	0	0	0	34	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	15
RING 4	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 5	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

USING 1973 METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 1000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 838

1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

1974 METEOROLOGY 4.80 LB SO2/MMBTU COAL OSD = 1.15 118 UG/M3 COMPARISON STANDARD

EXPECTED EXCEEDANCES												
RING 1	0.82	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.01	0.02	0.10	0.00	0.91	0.96	1.02	1.24	0.63	0.43	0.02	0.06
	0.98	0.35	0.09	0.37	0.14	0.35	0.05	0.01	0.00	0.01	0.01	0.18
RING 2	0.03	0.00	0.01	0.03	0.60	0.92	0.63	0.00	0.10	0.00	0.32	0.11
	0.09	0.34	0.05	0.00	0.56	0.28	0.14	0.34	0.70	0.37	0.18	0.68
	0.40	1.43	1.12	0.47	0.01	0.12	0.01	0.06	0.92	0.00	0.19	0.01
RING 3	0.00	0.00	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.03	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.00	0.05	0.00
	0.00	0.04	1.27	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
RING 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.03	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
RING 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NUMBER OF SAMPLES (OUT OF 1000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES) AT EACH RECEPTOR												
RING 1	0	0	0	0	0	0	0	0	0	0	0	0
	0	3	0	0	1	136	58	326	3	0	0	0
	224	3	3	3	0	0	0	0	0	0	0	5
RING 2	0	0	0	0	27	90	105	0	0	0	21	5
	0	1	0	0	4	15	3	30	11	0	0	77
	15	48	115	8	0	2	0	0	123	0	2	0
RING 3	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	400	0	0	0	0	0	0	0	0	0
RING 4	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 5	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

USING 1974 METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 1000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 897

1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

1975 METEOROLOGY 4 80 LB 802/MMBTU COAL GSD = 1.15 118 UG/M3 COMPARISON STANDARD

EXPECTED EXCEEDANCES

RING 1	0.03	0.55	0.37	0.00	0.00	0.00	0.00	0.01	0.70	0.08	0.00	0.01
	0.00	0.08	1.00	1.78	1.30	1.13	0.46	0.25	0.04	0.00	0.01	0.33
	1.00	1.03	0.58	0.25	1.17	0.74	0.00	0.02	0.25	0.00	0.52	0.72
RING 2	1.21	0.69	0.13	0.20	0.60	0.01	0.00	0.41	0.71	1.01	0.77	1.04
	1.24	0.34	2.63	0.94	0.80	0.96	0.00	0.06	0.01	0.00	0.01	0.05
	1.11	1.04	0.75	0.67	0.03	0.05	0.86	0.02	0.22	0.14	0.76	0.37
RING 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00
	0.39	0.00	2.91	0.00	2.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	0.44	0.00	0.00	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.27	0.00
RING 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.00
RING 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NUMBER OF SAMPLES (OUT OF 1000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES) AT EACH RECEPTOR

RING 1	0	52	26	0	0	0	0	0	103	0	0	0
	0	0	3	640	318	185	1	15	0	0	0	0
	0	70	2	1	247	0	0	0	0	0	0	0
RING 2	273	138	1	3	67	0	0	28	6	16	41	64
	252	31	964	169	0	224	0	0	0	0	0	0
	201	258	39	4	1	0	18	0	1	0	16	41
RING 3	0	0	0	0	0	0	0	0	0	2	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 4	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 5	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

USING 1975 METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 1000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 1000

1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

1976 METEOROLOGY 4 80 LB SO<sub>2</sub>/MMBTU COAL GSD = 1.15 118 UG/M<sup>3</sup> COMPARISON STANDARD

EXPECTED EXCEEDANCE

RING 1	0.02	0.29	0.18	0.00	0.00	0.00	0.00	0.00	0.00	1.11	1.45	1.04	0.96
	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.94	1.00	0.98	0.00
	0.33	0.02	0.86	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.60
RING 2	0.00	0.04	0.00	0.23	0.13	0.38	0.20	0.69	0.06	1.65	0.55	0.46	
	0.94	0.00	0.00	0.01	0.03	0.00	0.00	0.01	0.02	0.95	0.07	0.02	
	0.04	0.66	1.66	0.31	0.32	0.66	0.10	0.10	1.02	0.03	0.44	0.38	
RING 3	0.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	0.69	0.02	0.00	0.01	0.00	0.00	0.12	0.00	0.01	0.00	
RING 4	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
RING 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

NUMBER OF SAMPLES (OUT OF 1000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES) AT EACH RECEPTOR

RING 1	0	2	9	0	0	0	0	0	136	396	194	0	
	272	0	0	0	0	0	0	0	0	0	0	0	
	0	0	77	0	0	0	0	0	0	0	0	0	
RING 2	0	0	0	0	2	22	2	47	1	368	46	48	
	179	0	0	0	0	0	0	0	1	145	1	0	
	0	0	247	0	0	37	5	0	25	0	23	5	
RING 3	0	0	0	0	0	0	0	0	0	0	2	0	
	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	1	0	0	0	0	0	0	0	0	0	
RING 4	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	
RING 5	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	0	0	

USING 1976 METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 1000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 957

1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

1977 METEOROLOGY 480 LB SO<sub>2</sub>/MMBTU COAL OSD = 1.15 118 UG/M<sup>3</sup> COMPARISON STANDARD

EXPECTED EXCEEDANCES

RING 1	0.09	0.01	0.36	0.00	0.61	0.91	0.15	0.12	0.11	0.28	0.49	0.00
	0.00	0.00	0.02	0.93	0.90	0.33	0.00	0.00	0.00	0.00	0.00	0.34
	0.01	0.08	0.67	0.00	0.60	0.52	0.00	0.01	0.39	0.01	0.00	0.01
RING 2	0.58	0.01	1.18	0.62	1.00	1.51	0.64	0.13	0.15	0.15	0.23	0.33
	0.00	0.02	0.09	0.00	0.35	0.00	0.00	0.38	1.04	1.24	0.92	0.05
	0.02	0.90	1.43	0.36	0.02	0.58	0.09	0.02	0.01	0.00	0.00	0.00
RING 3	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.12	0.67	0.01	0.09	0.00
	0.00	0.10	0.43	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RING 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RING 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NUMBER OF SAMPLES (OUT OF 1000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES, AT EACH RECEPTOR

RING 1	0	0	0	0	5	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 2	5	0	322	51	217	566	113	1	8	3	15	19
	0	0	1	0	0	0	0	0	43	342	8	0
	0	20	460	36	0	33	2	0	0	0	0	0
RING 3	0	0	0	0	0	2	0	0	0	0	0	2
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	2	0	0	0	0	0	0	0	0	0
RING 4	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
RING 5	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0

USING 1977 METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 1000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 933



1000 MW UNSCRUBBED POWER PLANT - ST LOUIS MET DATA - 24 HOUR AVG CONC

RESULTS FOR ALL 5 METEOROLOGICAL YEARS

4 80 LB SQ/MMBTU COAL

GSD = 1.15

118 UG/M3 COMPARISON STAN

EXPECTED EXCEEDANCES

RING 1

0.38	0.33	0.18	0.00	0.13	0.18	0.03	0.03	0.38	0.36	0.31	0.19
0.24	0.02	0.23	0.38	0.64	0.48	0.34	0.30	0.32	0.29	0.20	0.19
0.40	0.28	0.44	0.11	0.38	0.34	0.02	0.01	0.43	0.00	0.11	0.34

RING 2

0.37	0.17	0.31	0.23	0.57	0.80	0.49	0.28	0.23	0.59	0.42	0.46
0.68	0.37	0.55	0.21	0.39	0.43	0.15	0.16	0.35	0.51	0.24	0.17
0.31	0.82	1.02	0.37	0.12	0.36	0.38	0.04	0.43	0.04	0.28	0.48

RING 3

0.01	0.05	0.02	0.30	0.00	0.02	0.14	0.50	0.40	0.07	0.03	0.03
0.08	0.02	0.00	0.00	0.00	0.02	0.00	0.03	0.14	0.00	0.03	0.01
0.09	0.03	0.55	0.30	0.00	0.01	0.02	0.00	0.03	0.00	0.06	0.06

RING 4

0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

RING 5

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NUMBER OF SAMPLES (OUT OF 5000) RESULTING IN VIOLATIONS (TWO OR MORE EXCEEDANCES) AT EACH RECEPTOR

RING 1

0	54	35	0	6	0	0	0	239	396	194	0
272	0	3	640	319	341	59	341	3	0	0	0
224	78	82	4	247	0	0	0	0	0	0	7

RING 2

278	139	339	57	384	643	221	84	15	591	125	138
676	223	965	159	9	239	3	30	35	1487	9	77
216	772	1261	74	1	77	153	0	149	0	41	702

RING 3

0	0	1	0	0	2	34	0	0	2	2	2
2	0	0	0	0	0	0	0	0	0	0	0
0	0	403	0	0	0	0	0	0	0	0	15

RING 4

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

RING 5

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

USING ALL 5 YEARS OF METEOROLOGY, THE NUMBER OF SAMPLE YEARS (OUT OF 5000) RESULTING IN A VIOLATION AT AT LEAST ONE RECEPTOR WAS 462. THIS THE PROBABILITY OF A VIOLATION AT SOME RECEPTOR IS 9.25 PCT

IN CASE YOU'RE INTERESTED, THE NUMBER OF CONCENTRATIONS LOOKED AT FOR EACH YEAR WAS

YR	1	430
YR	2	518
YR	3	605
YR	4	454
YR	5	463

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