

MANAGEMENT AND MEASUREMENT OF "DO" IN IMPOUNDMENTS

James M. Symons, William H. Irwin, Robert M. Clark, and Gordon G. Robeck

U. S. Department of the Interior  
Federal Water Pollution Control Administration  
Cincinnati, Ohio 45226  
September 1966

## MANAGEMENT AND MEASUREMENT OF "DO" IN IMPOUNDMENTS

James M. Symons, William H. Irwin, Robert M. Clark, and Gordon G. Robeck

This paper develops a basis for calculating the hydraulic efficiency of any mechanical destratification device by the use of the stability concept and a mathematical model, including a solution, for studying the DO budget. Also, the influence of mixing on reduced substances and DO resources in stratified impoundments is demonstrated.

### INFORMATION ABSTRACT

This paper recommends a basis for the comparison of various mechanical devices used for artificially destratifying impoundments by calculating their hydraulic efficiency based on impoundment stability. Three problems with this calculation are discussed to avoid misinterpretation, influence of size and geometry on stability, natural changes in stability with time, and the basic hydraulic inefficiency of spring mixing if cold water is raised. Also, a mathematical model is developed for studying the DO budget in impoundments. A solution to the model is presented, using specially designed DO probe equipment. Finally, data is presented on the destratification of a 2930-ac-ft impoundment in northern Kentucky by pumping. The hydraulic efficiency of the pump is calculated and the influence of the mixing on the reduced substances and the DO resources in the impoundment is shown. The most favorable time for destratification is also discussed. A bibliography on artificial destratification is included.

### KEY WORDS

Impoundments, Stratification, Destratification, Water Quality, Stability, Mechanical Pumping, DO Resources

# MANAGEMENT AND MEASUREMENT OF "DO" IN IMPOUNDMENTS<sup>a</sup>

James M. Symons<sup>1</sup>, M.ASCE, William H. Irwin<sup>2</sup>, Robert M. Clark<sup>3</sup>, A.M. ASCE  
and Gordon G. Robeck<sup>4</sup>, F. ASCE

## INTRODUCTION

The problem of thermal stratification in storage reservoirs is widespread throughout the United States. Considerable attention has been given to developing engineering methods that will prevent downstream problems from the discharge of poor quality bottom water from impoundments, particularly during peaking power operations. Several possible design features that might be incorporated into dams and outlet works to prevent the escape of poor quality water from impoundments have been extensively reviewed by Kittrell<sup>5</sup>. One of these techniques, artificial destratification, has been researched during the past two years by staff members of the Cincinnati Water Research Laboratory. The first

a - Presented at the ASCE National Symposium on Quality Standards for Natural Waters, Ann Arbor, Michigan, July 19-22, 1966.

Submitted for publication in the Proceedings of the Symposium and in the Journal Sanitary Engineering Division of the American Society of Civil Engineers.

1. In Charge, Impoundment Behavior Studies, Engineering Activities, Research and Development, Cincinnati Water Research Laboratory, FWPCA Cincinnati, Ohio
2. Aquatic Biologist, Impoundment Behavior Studies, Engineering Activities, Research and Development, Cincinnati Water Research Laboratory, FWPCA Cincinnati, Ohio.
3. Applied Mathematician, Research and Development, Cincinnati Water Research Laboratory, FWPCA, Cincinnati, Ohio.
4. Chief, Engineering Activities, Research and Development, Cincinnati Water Research Laboratory, FWPCA, Cincinnati, Ohio

This work was performed while the authors were members of the Division of Water Supply and Pollution Control, Public Health Service, Department of Health, Education, and Welfare.

publication on this subject<sup>6</sup>, discussed the theoretical aspects of artificially breaking the thermal stratification of an impoundment, and presented early experimental results. The second paper<sup>7</sup> dealt with a larger field trial, and extensive data were presented to demonstrate the improvement in water quality that could be affected by pumping cold water from the bottom of a stratified impoundment and discharging it in the surface, thereby creating an overturn in the body of water.

Although to date (1966) the method has only been tried on relatively small bodies of water, 3,000-4,000 ac-ft, artificial destratification is an effective method for water quality improvement. This technique is doubly attractive since it improves the water quality both in the reservoir itself as well as protecting downstream quality. This can be contrasted to some of the techniques discussed by Kittrell<sup>5</sup> that only prevent poor quality water from escaping an impoundment, and do not prevent the formation of poor quality water in the impoundment itself.

Since research on this process will continue, one purpose of this article, the third in the series, is to establish a firm theoretical basis for comparing the hydraulic efficiencies of various pieces of equipment that might be used for artificially destratifying a body of water. Secondly, although dissolved oxygen (DO) is not the only quality parameter in natural waters, it certainly is an important one. Therefore, the other purpose of this article is to examine the influence that artificial destratification has on the DO resources in an impoundment

from four aspects; a) the elimination of reduced substances, b) the addition and redistribution of DO, c) when artificial destratification should be used for quality control during a summer season, and d) a description of a method for evaluating DO resources in impoundments. In addition, a complete bibliography on the subject of artificial destratification is included.

## THEORETICAL CONSIDERATIONS

### Management of DO in Impoundments

#### Hydraulic Efficiency

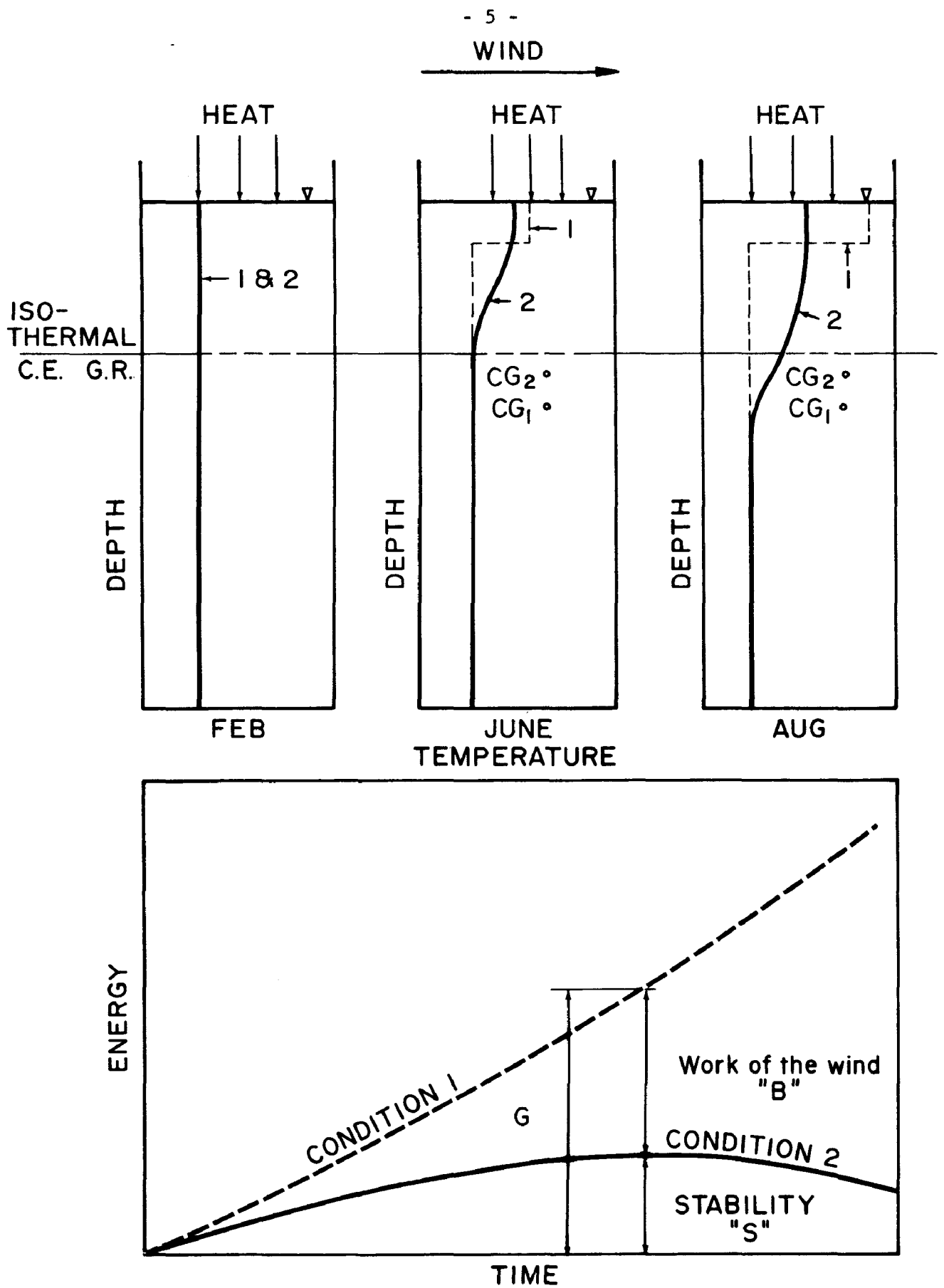
There are four major methods currently (1966) available for artificial destratification, mechanical pumping<sup>6,7</sup>, the Air-Aqua<sup>\*</sup> system<sup>8</sup>, compressed air,<sup>9</sup> and the Aero-Hydraulics Gun<sup>\*\*10</sup>. If these four systems are to be compared, their hydraulic performance must be judged on an equal basis, through the calculation of their hydraulic efficiency. The hydraulic efficiency of any destratification device is evaluated by comparing the actual energy input necessary to cause overturn in a given body of water, to the theoretical minimum energy required to overturn that body of water. The theoretical minimum energy required for artificial destratification is defined as the stability of the body of water.

The stability of stratified impoundments has been discussed by Koberg and Ford<sup>9</sup> and by Hutchinson<sup>11</sup>, and can be calculated according to the following definition - the work or the energy required to lift the weight of the entire body of water the vertical distance between the center of gravity of the water mass when isothermal and the mass center of gravity when the impoundment is thermally stratified. For the purpose of this paper the center of gravity is taken as a point anywhere on a horizontal plane that passes through the true mass center of gravity.

Figure 1 schematically illustrates the stability concept. In the upper portion the temperature profile in a column of water, taken as a rectangular slice from a wedge-shaped body of water, is shown at three different months of a year. In the first column, the water is

<sup>\*</sup>Product of Hinde Engineering Company, Highland Park, Illinois.

<sup>\*\*</sup>Product of Aero-Hydraulics Corporation, Montreal, Canada.



**Figure 1. Schematic diagram to demonstrate stability and work of the wind.**

isothermal, and the center of gravity along the vertical axis is approximately  $1/3$  the depth of the water column. The isothermal center of gravity does not change throughout the summer season. In the subsequent discussion movement of the position of the center of gravity is considered along the vertical axis only.

As the water column is heated from the surface, two different temperature profiles, labeled "1" and "2", are shown for June and August. Profile "1" in June, dotted, would occur if there were no wind blowing across the surface of the water. In this condition, most of the incident heat would be stored in the upper layers of the water, making this water lighter, relative to the remainder of the water column. This decrease in weight would lower the center of gravity of the stratified water column to the position shown by  $CG_1$ . As heating continues into August, thermal stratification becomes more intense, and the center of gravity moves down into the water column even farther. Since the actual center of gravity continues to lower throughout the heating season, the energy required to mix the water column, which is evaluated by multiplying the weight of the water column by the distance between the isothermal center of gravity and  $CG_1$  at any given time, continually increases with time. This is shown in the lower portion of Figure 1, by the curve labeled "condition 1."

In reality, wind is always present in any natural situation. The influence of wind blowing across the surface of the water column changes the temperature profile from the condition labeled "1" to the condition labeled "2". The surface of the water column does not become as warm, and the warm water extends farther down into the water column. At any given time, therefore, the center of gravity of the stratified



water column does not move down to the level of  $CG_1$ , but only descends to the level shown in Figure 1 as  $CG_2$ . Since wind has the effect of preventing stratification from being as intense as without wind, the actual energy necessary to mix the water column is considerably reduced because of wind action. This is shown in the lower portion of Figure 1, where the "condition 2" curve is considerably below that of the "condition 1," curve. The distance between these two curves has been defined as "work of the wind."<sup>11</sup>

In summary, at any given time, the energy that is represented by the distance "G" in Figure 1, is the energy that would have been required to mix the stratified water column in the absence of any wind. Since the wind has put into the water the quantity of work indicated by dimension "B", only the remainder, the stability, "S", must be supplied by any mechanical device, to make the water column isothermal. In any natural situation, the stability can be arithmetically calculated<sup>9</sup>. This theoretical minimum energy requirement for destratification, when compared with the energy input of a given mechanical device that caused destratification, defines the hydraulic efficiency of the mechanical equipment.

The stability curve, "S" in the lower portion of Figure 1 also shows that stability reaches a maximum in an impoundment during the middle of the heating season, usually in late July, and then begins to decline. The maximum stability in a body of water occurs when the warm water has penetrated to the depth of the isothermal center of gravity along the vertical axis. As the water warms below this depth, and the water body approaches an isothermally warm condition, the

distance decreases between the isothermal and the stratified centers of gravity, and the stability declines, reaching zero when the water body is again isothermal, but warm.

#### Problems with the Stability Concept

While stability is a useful method of calculating the hydraulic efficiency of an artificial destratification device, it must be used carefully to avoid misinterpretation. Three problems will be discussed. The first is the effect of size and geometry on the calculation of stability. Table 1 summarizes data from three lakes near Cincinnati, Ohio, which are under study. Column 2 through 6 lists the pertinent physical data for these lakes. The stability of each lake, column 7, was calculated assuming the same temperature profile in each lake, 20°C for the upper 5 feet of water, and remainder of the water body at 4°C. As seen in column 7, the stability of these three lakes is different.

Hutchinson<sup>11</sup> suggested that stability should be calculated per unit of lake surface, to eliminate differences of size and geometry. Column 8 shows, however, this is not a reliable method of calculating a comparable parameter. At least for these three lakes, the stability per unit volume, column 9, is a better method of obtaining a parameter that can be compared from lake to lake. Table 1 demonstrates that the stability calculated for one body of water cannot be assumed to exist in other lakes and reservoirs in the same general geographic location. To properly evaluate the hydraulic efficiency of a mixing

Table 1

COMPARISON OF STABILITY IN LAKES WITH  
THE SAME ASSUMED TEMPERATURE PROFILE BUT DIFFERENT SIZE AND GEOMETRY

Lake	Surface ac	Greatest Depth ft	Avg. Depth ft	Vertical Isothermal Center of Gravity ft	Volume ac-ft	Stability kw-hr	<u>Stability</u> area w-hr ac	<u>Stability</u> vol w-hr ac-ft
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Falmouth	225	42	20.5	14.232	4605	21.8	96.9	4.7
Bullock Pen	142	44	22.8	15.059	3237	15.5	109.2	4.8
Boltz	93	62	31.5	20.867	2930	15.0	161.3	5.2

operation in a certain impoundment, the stability for that particular impoundment must be calculated. Table 1 also illustrates the difficulty in using a second nearby impoundment as a control for comparison purposes in an artificial destratification study. Differences in size and geometry between the two impoundments may prevent the second from being used as an "exact" control.

Second, the continuing natural change that the stability of a given body of water undergoes throughout a summer season is another difficulty with its use for the calculation of hydraulic efficiency. As shown in the lower portion of Figure 1, as stratification begins, the stability rises from zero, reaches a maximum in late July, and declines towards zero again, because of continued wind-mixing and nighttime cooling in August and September. Therefore, if the stability in a given impoundment were calculated when a mixing operation began, the natural change in stability occurring during the time of the mixing operation, must be included in any proper evaluation of hydraulic efficiency.

If a destratification operation should take place in the spring, and should require a month to complete, the actual energy input during mixing should be compared with the calculated stability at the start of the operation, plus the natural increase in stability that would have taken place during the month. During a fall mixing operation, the natural decline in stability with time should be subtracted from the initial stability, in order to properly evaluate the hydraulic efficiency of the overturn operation.

Third, the hydraulic inefficiency of a spring destratification operation, in which cold water is pumped from the bottom to the surface, must be recognized. This is illustrated by the data in Table 2. Here, temperature profiles were assumed for four different months during the year and the stability calculated for each of the profiles, (see Column 3). The energy that must be added to the lake for mixing is low in the spring and fall, and high in mid-summer.

If destratification is to be carried out by bringing cold water from the bottom as a heat sink to absorb heat from the upper layers and thereby make the lake isothermal, the volume that must be moved is the volume of cold water that is present at the start of the operation. For this example, these volumes are shown in Column 4, Table 2.

Although the volume of cold water that must be pumped in the spring is high, the work requirements are not, since the elevation head through which the water must be lifted, based on density difference<sup>12</sup>, is much smaller in the spring than in the summer or fall. Practically, however, mixing equipment cannot take advantage of this low elevation head in the spring and so the actual energy input at any time is directly proportional to the volume of water that must be moved, Column 4, Table 2. Therefore, although theoretical energy requirements are low in the spring,<sup>an</sup> hydraulically inefficient operation results because of practical considerations. The improvement in hydraulic efficiency as the summer season progresses is again shown in Column 5, Table 2 where the volume required to mix the lake is calculated per unit of original stability.

Table 2

VARIATION IN PUMPING REQUIREMENTS TO DESTRATIFY BULLOCK PEN LAKE AT  
DIFFERENT SEASONS

Month	Assumed Temperature Profile	Stability kw-hr	Cold Water Volume Required to mix Lake ac-ft	Cold Water Volume Required to mix the Lake per Unit Stability ac-ft/kw-hr
(1)	(2)	(3)	(4)	(5)
April	0-5 ft - 20°C	13.8	2560	180
	5-45 ft - 9°C			
June	0-15 ft - 23°C	32.9	1440	43
	15-45 ft - 9°C			
August	0-25 ft - 25°C	28.2	640	23
	25-45 ft - 10°C			
September	0-40 ft - 25°C	2.5	35	15
	40-45 ft - 10°C			

---

Note: Data calculated for Bullock Pen Lake, see Table 1. for physical data.

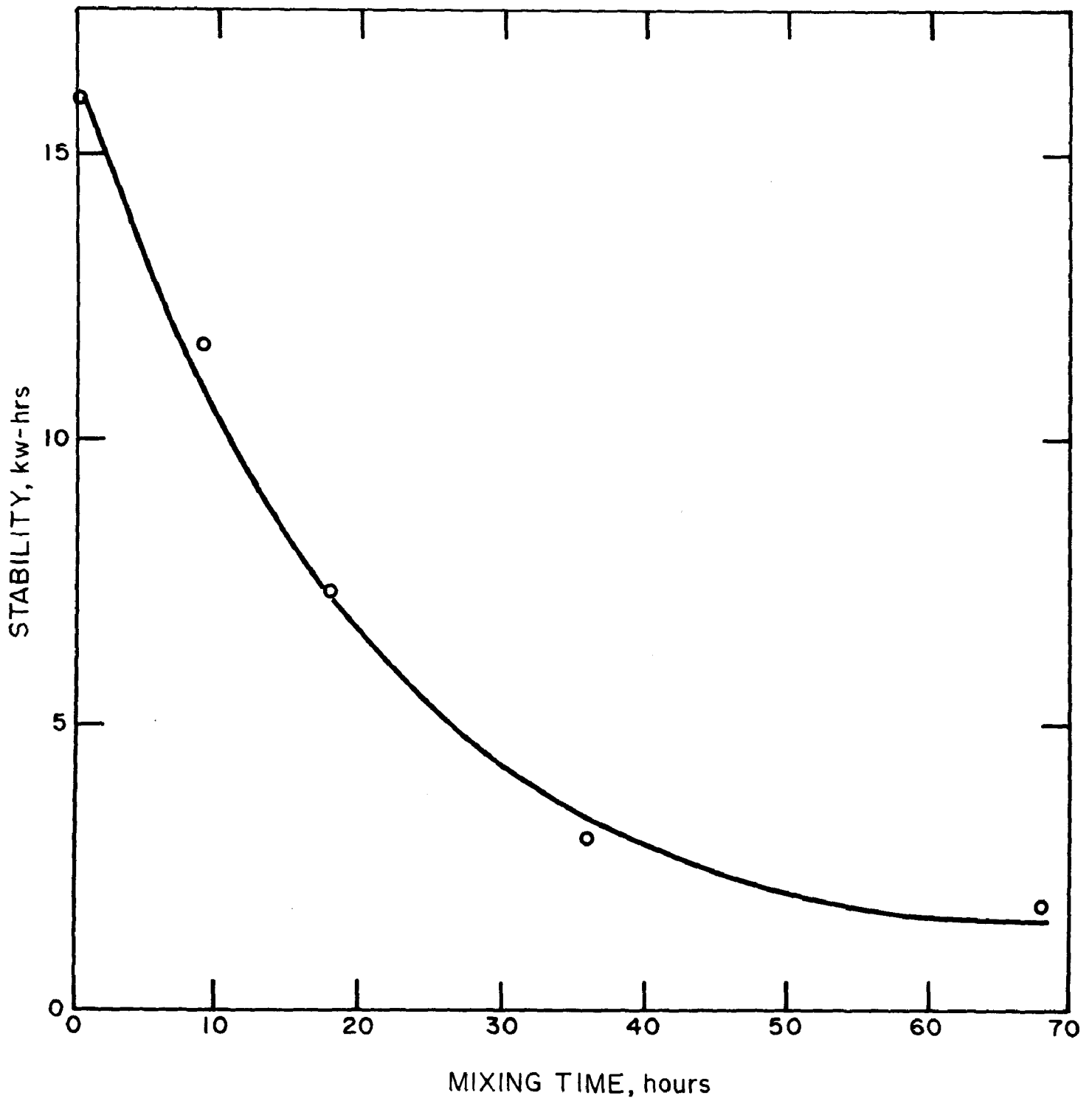
The practical inefficiency of destratification in the spring could be overcome by pumping the smaller volume of warm water down into the colder layers. The theoretical energy requirements would be the same, since pumping cold water up involves moving a large volume through a small elevation head while pumping warm water down would result in moving a small volume through a larger elevation head. As noted before, however, most mechanical equipment cannot take advantage of the elevation head differences, therefore the actual input energy is directly proportional to the water volume moved. This would make pumping warm water down in the spring a hydraulically more efficient operation.

This variation in pumping requirements with changing seasons does not eliminate the usefulness of the stability concept for calculating hydraulic efficiency, but the discussion above should indicate that a low calculated hydraulic efficiency may occur during a spring destratification operation and should also indicate that different season comparisons are unfair.

Another practical consideration for any mixing operation is the change in hydraulic efficiency as the mixing nears completion. In Figure 2 from the work of Koberg and Ford<sup>9</sup>, the rate of change in stability per unit of time decreases as the stability approaches zero. There are at least three reasons for this. First, since so much of the water in most lakes is in the upper portions, a very slight temperature gradient can produce some stability. Practically, complete elimination of stability, particularly in the spring may be impossible. Second, as mixing nears completion the density of the body of water is nearly uniform. Under these conditions some of the water that has been raised to the surface may sink to the bottom and be repumped. This would lead to excessive energy inputs to complete the job.

Finally, as a body of water is cooled to near its final temperature, there is a smaller decrease in stability per unit volume of water pumped. This is caused by the non-linear change in water density with changing temperature.<sup>12</sup> In the example shown in Table 2, for the April data, the first 155 ac-ft of bottom water pumped up decreases the stability 2.3 kw-hrs, while the last 155 ac-ft of water mixed only eliminates 0.8 kw-hrs of stability. The higher the average (final) temperature of the water, the less pronounced this effect would be, as the density change of water tends to be linear over small temperature ranges. Keeping a continuous record of stability during a destratification operation is recommended to avoid excessive energy inputs in the late stages of the mixing.





**Figure 2. Decrease in stability with application of compressed air (Reference 9).**

## Measurement of DO Budget in Impoundments

### Mathematical Model

One common mathematical model used to describe the change of DO concentrations with time in the euphotic zone of impoundments, where sedimentation is not an important factor, is shown in Equation (1), (see O'Connell and Thomas<sup>13</sup>.)

$$\frac{dD}{dt} = k_1(L_a - y) - k_2(D) + (R - P) \quad (1)$$

where D = DO deficit at any time

t = time

$k_1$  = Deoxygenation rate constant

$L_a$  = Total organic biochemical oxygen demand

y = Biochemical oxygen demand satisfied at any time

$$\{y = L_a [1 - \exp(-k_1 t)]\}$$

$k_2$  = Atmospheric reoxygenation rate constant

R = Rate of oxygen demand by the algal population

P = Rate of oxygen produced by the algal population

Integration yields Equation (2).

$$D = \frac{k_1(L_a)}{k_2 - k_1} [\exp(-k_1 t) - \exp(-k_2 t)] + \frac{(R - P)}{k_2} [1 - \exp(-k_2 t)] + D_a [\exp(-k_2 t)] \quad (2)$$

where  $D_a$  = Initial DO deficit.

Although Hull's<sup>14</sup> suggestion of measuring all the DO sources and sinks in a given impoundment to evaluate the DO budget is valid, this is often time-consuming and evaluates the budget only under one set of environmental conditions. If a reliable model can be developed, the changes in the constants, as related to changing sunlight, wind, algal counts, and nutrient level, will permit general conclusions to be drawn, permit predictions to be made, and permit comparisons of one impoundment to another.

Through the years, research efforts have attempted to devise methods to evaluate and understand the physical meaning of the five constants ( $k_1$ ,  $k_2$ ,  $L_a$ ,  $R$ , and  $P$ ) in Equation (2). While excessive concern over the precise meaning of these constants has, with some justification, been criticized by Hull<sup>14</sup> and Ettinger<sup>15</sup>, this does not negate the utility of the overall model. The important feature of any mathematical model is whether or not its results accurately reproduce what is occurring in nature. The model shown in Equation (2) may not be an exact representation of the complete DO budget in an impoundment, but, as will be shown in Figure 10, the change in "D" with time derived from the solution of the model, and the change in the dissolved oxygen deficit with time in the natural environment are similar. Therefore, the model is acceptable for further use.

#### Evaluation of the Model by the DO Probe Method

There are many different methods of studying the DO budget in natural waters, for example, Odum<sup>16</sup> - diurnal cycle method, Hull<sup>17,18</sup> - "light" and "dark" - bottle method, Goldman and Carter<sup>19</sup> - carbon-14 method, and Verduin<sup>20</sup> - changes in  $CO_2$  concentration in

unconfined water method. Each method has advantages and drawbacks. The method developed in this study, called the DO probe method, using special equipment has three distinct advantages; (a) the inaccuracies in bottle experiments are minimized, (b) DO is measured directly and (c) the model, Equation (2), may be solved from data gathered. Future study will undoubtedly reveal further improvements in this approach, but this is a significant advance.

Details of the design and operation of this equipment, and a discussion of its improvements over the bottle technique are contained in reference 21, and shall be noted only briefly here. The apparatus consists of three DO probes that will record the changes in DO concentration with time under three different conditions. One probe is placed in a black container that prevents the admission of either sunlight or air. Under these conditions the constants " $k_2$ ", and " $P$ " are zero in Equation (1). Integrating Equation (1) under these conditions yields Equation (3).

$$D = L_a [1 - \exp(-k_1 t)] + R(t) + D_a \quad (3)$$

A second DO probe is in a clear plastic container that admits sunlight, but not air. Under these conditions, Equation (1) can be integrated with " $k_2$ " = 0. This integration yields Equation (4).

$$D = L_a [1 - \exp(-k_1 t)] + (R-P)(t) + D_a \quad (4)$$

The third DO probe is unconfined, therefore the change in DO deficit with time is represented by Equation (1), which when integrated yields Equation (2).

Evaluation of the five constants in the model is accomplished by solving Equation (3) for " $k_1$ ", " $R$ ", and " $L_a$ " using DO vs. time data obtained from the DO probe in the dark container. These constants are then entered in Equation (4) and " $P$ " is evaluated through the use of the DO vs. time data taken from the light container DO probe. Finally, Equation (2) is solved from the DO data taken with the probe that is unconfined, and the previously developed constants.

For those interested in the details of the solution, an example is included in the Appendix. The application of this method, using data from the equipment developed for this purpose<sup>21</sup>, will be demonstrated in the results section of this paper.

## EQUIPMENT AND METHODS

### Management of DO in Impoundments

The equipment and methods used in this study have been explained in detail in references 7 and 21 and shall be briefly summarized here. Artificial destratification was studied from August 6, to September 10, 1965 in Boltz Lake, a 2,930 ac-ft impoundment near Cincinnati, Ohio. Destratification was accomplished by pumping cold water from within 2 feet of the bottom and discharging it in the surface with a 13 ac-ft/day capacity, raft-mounted pump, driven by a 15.5 kw gasoline engine. Analysis of temperature, and DO, sulfide, manganese, and total COD concentrations at various depths, weekly from May 14 to October 29, 1965 demonstrated the changes in stability and DO resources that occurred in this impoundment before, during, and after the pumping operation. A nearby, unmixed lake, Bullock Pen, served as the control. Physical details of both lakes are summarized in Table 1.

### Measurement of DO Budget in Impoundments

The ability of the special DO probe equipment to produce data necessary for the solution of Equations (2), (3), and (4) was demonstrated by making studies in a 750-gal. tank filled with artificial impoundment water. The simulated impoundment water was created by mixing some settled sewage, nitrogen, phosphorus, sodium bicarbonate, and algal seed with Cincinnati, Ohio tap water. This mixture was stirred 15 minutes each hour and illuminated 14 hours each day until an observable algal growth developed. The day of the test, some sewage was added as a fresh source of organic material. The three DO probes, with

open trap doors on the containers were lowered into the tank, the triggers were tripped, and the test begun. The DO concentration data was obtained for an 8-hour period, under illumination, using the recording and timing circuitry detailed in reference 21. Before the test began, the DO probes were calibrated and adjusted against the Winkler method for DO analysis. The temperature of the test was fairly constant at 20°C.

## RESULTS

### Management of DO in Impoundments

#### Hydraulic Efficiency

The approximate magnitude of the work of the wind in the control lake, Bullock Pen, during the 1965 season is shown in Figure 3. The "G" energy was approximated by assuming that all of the heat incident to the lake was concentrated in the upper 5 feet of the water. The "G" energy is much higher than the stability, the lower curve, indicating that, at least for this impoundment, the wind has applied much more energy for mixing than would mechanical equipment.

Figure 4 compares the stability pattern for the test and the control lakes. The control lake, Bullock Pen, shows the typical stability pattern, reaching a maximum stability of 45 kw-hrs in mid-July. At this time the calculated vertical distance between the isothermal and actual centers of gravity was 0.17 inches. The test lake, Boltz Lake, also reaches its maximum stability about the same time, but the calculated stability for Boltz Lake is somewhat above that for the control throughout the unmixed period because of differences in size and geometry. In spite of this slight inconsistency, Bullock Pen Lake was assumed to be a proper control for this operation.

Artificial destratification of Boltz Lake was started on August 6, 1965 when the pumping equipment became available. As shown in Figure 4, through the 5 weeks of pumping, the stability in Boltz Lake dropped more sharply than the natural decline of the control. After the 5 weeks of pumping, the calculated stability in the test



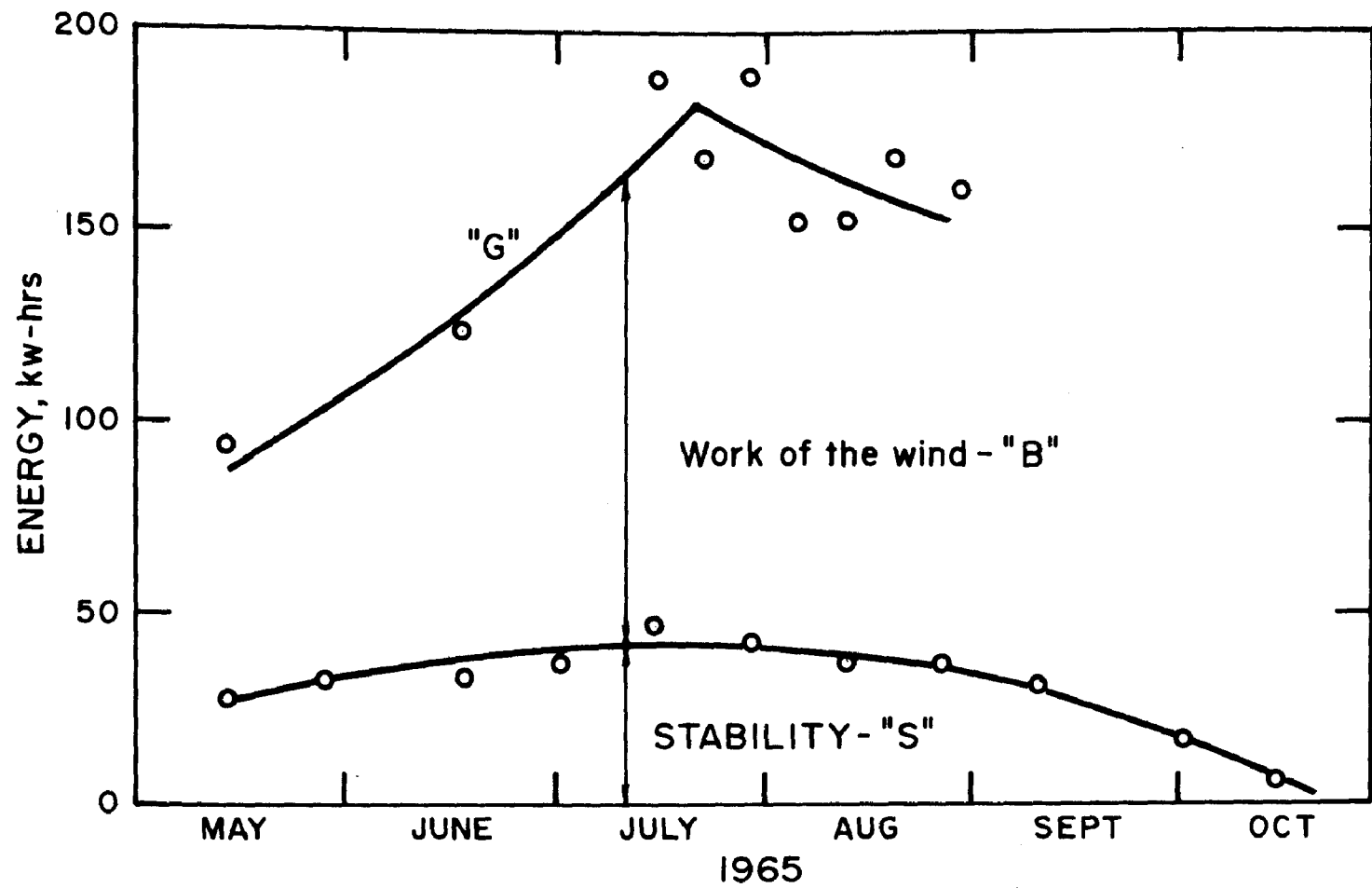


Figure 3. Estimation of the work of the wind in Bullock Pen Lake (Control lake).

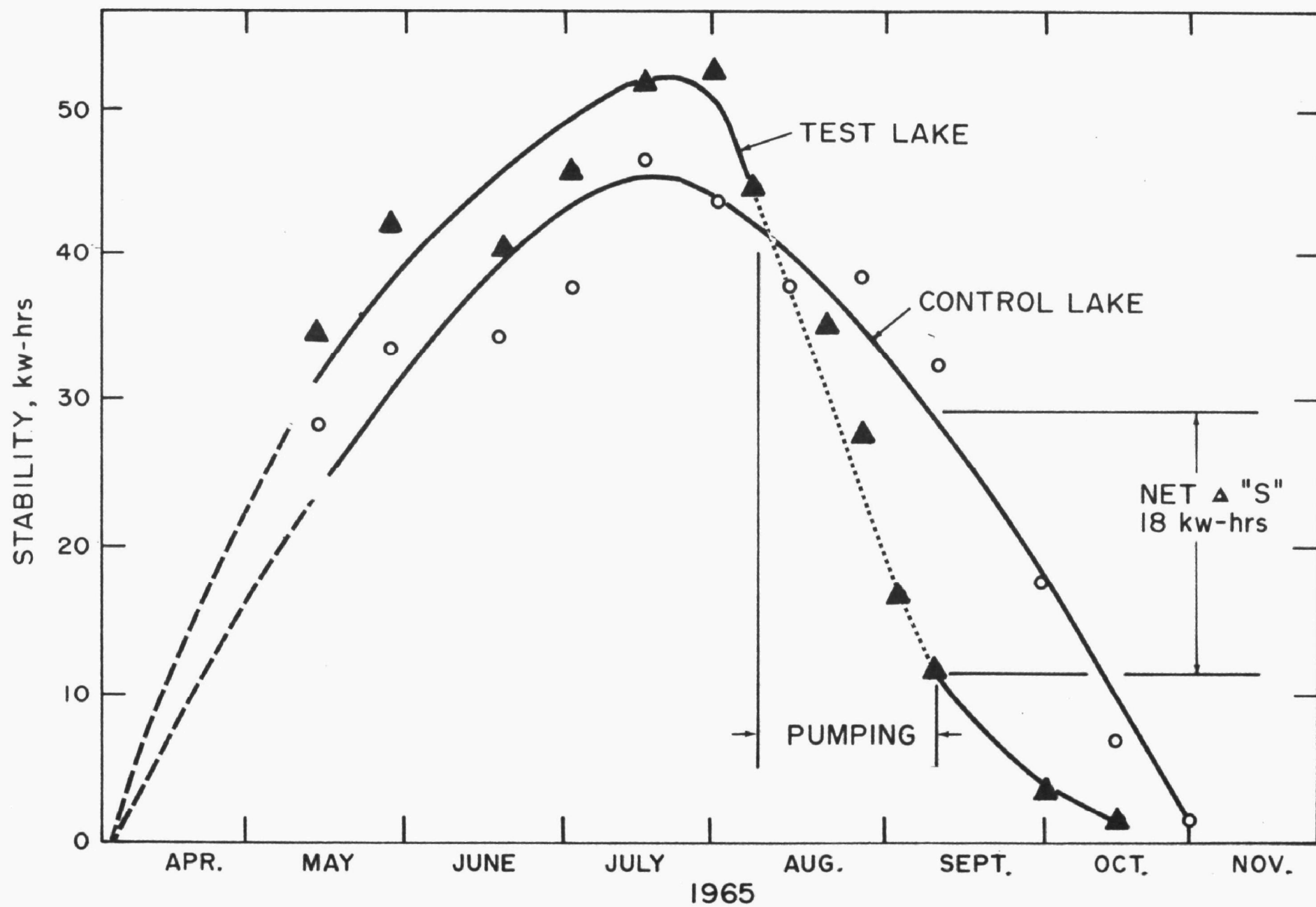


Figure 4. Stability in test and control lakes.

lake was 18 kw-hr lower than the control at the same time. Therefore, the hydraulic efficiency of this particular piece of mixing equipment, calculated according to Equation (5) was a rather low  $18/14,300 = 0.13\%$ .

$$\frac{\text{Net change in stability}}{\text{Total energy input}} = \text{Hydraulic efficiency} \quad (5)$$

#### Elimination of Reduced Substances

In Figure 5, the total quantity of sulfide, in the control lake, calculated as the equivalent oxygen demand rises throughout the summer, reaching a peak in mid-September. After this time, the natural overturn of the lake provides DO throughout the depth, thus oxidizing and eliminating the sulfides. The test lake rises similarly to the control prior to the artificial destratification operation. As pumping provided DO throughout the depth of the test lake, the sulfides were oxidized and their quantity lowered, as shown in the dotted portion of the test lake curve. Assuming that the test lake would have risen to a level approximately equal to that of the control lake on September 10, the net change in the sulfide content caused by pumping was 13,000 pounds, as the oxygen equivalent. The oxygen equivalent of the sulfide was calculated as twice its concentration, since two moles of oxygen are required to oxidize one mole of sulfide.

A similar, but smaller, change is shown in the manganese oxygen demand in Figure 6. For this analysis, all of the manganese measured in both lakes was taken to be soluble, divalent manganese and the oxygen equivalent of the manganese was calculated as 0.29 of its concentration since one-half mole of oxygen is required to oxidize one mole of manganous manganese. The net change in the manganese oxygen equivalent

between the control and test lakes at the end of the pumping operation, September 10, was 1,300 pounds of oxygen equivalent. As with the sulfides, the oxidation of the manganese in the test lake was accomplished by the destratification providing DO to the lower level of the lake.

The total COD content of the test and control lake was also calculated for each week during the study period. These data were more scattered than those shown in Figures 5 and 6, however, and no significant reduction in the total COD content in the test lake could be noted during the pumping period. This is probably because the degradable portion of the total COD was small, so even though the addition of DO to the lake enhanced the biodegradation of this portion of the organic material, the total COD measurement probably was too insensitive to detect any biodegradation.

In summary, the artificial destratification operations on Boltz Lake, from August 6 through September 10, 1965 supplied a total of 14,300 pounds of DO, which satisfied an equivalent quantity of inorganic oxygen demand.

#### Addition and Redistribution of DO

The decline in the total quantity of DO in both lakes through the early part of the summer season, because of continuing oxygen demand in the water, is shown in Figure 7. When pumping began in the test lake, the quantity of DO rose, but at this same time, because of continued wind-mixing, additional DO was appearing in the control lake.

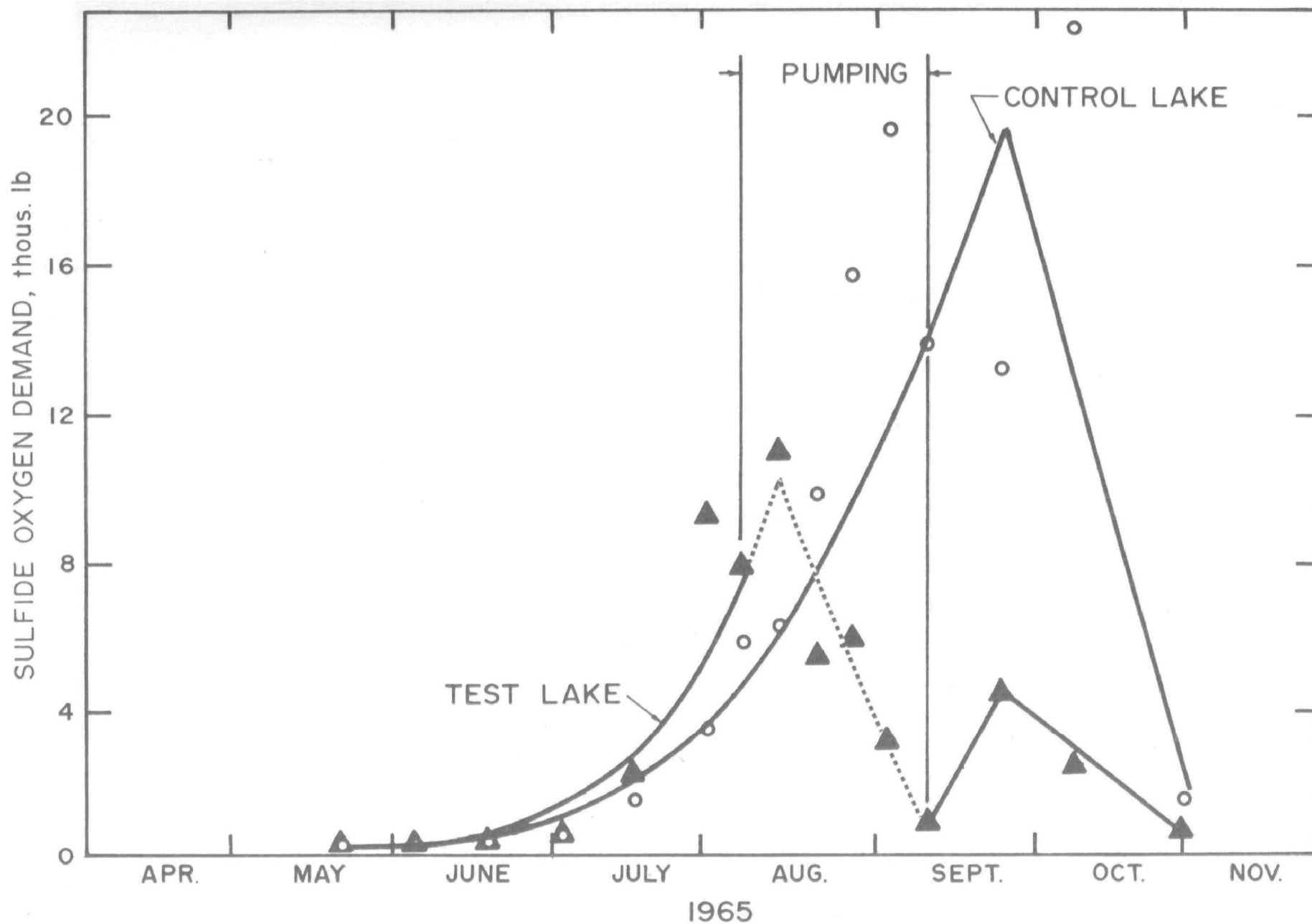


Figure 5. Total sulfides as oxygen equivalent in test and control lakes.

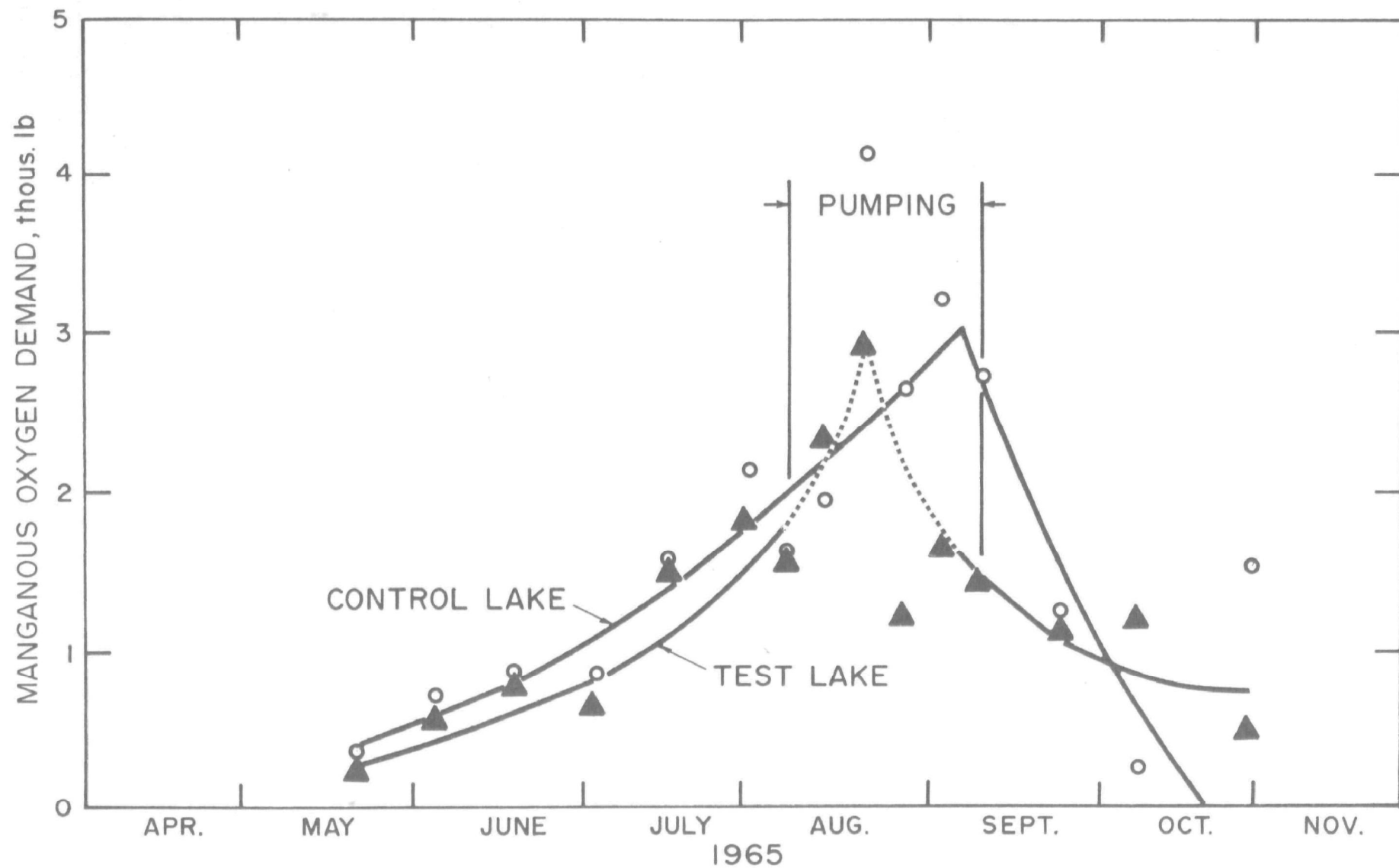


Figure 6. Total manganese as oxygen equivalent in test and control lakes.

Based on these data, although 14,300 pounds of DO were supplied to the test lake (Figures 5 and 6) the pumping capacity of the equipment was not sufficient to add the desired quantity of DO above the inorganic oxygen demand. The oxygen demand caused by the natural fall overturn in late September is also shown in Figure 7. This occurred even in the test lake, which restratified slightly after the pumping operation stopped.

Although the desired quantities of DO beyond the inorganic oxygen demand were not added to Boltz Lake, Figure 8 shows the redistribution of the existing DO resources caused by the pumping operation. Here, DO, absent below 20 feet before pumping began, was present, although in low concentrations, at all depths after pumping. Since the goal of any destratification operation is to provide reasonably high levels of DO at all depths, Figure 8 indicates further the undercapacity of this particular piece of equipment. It does show in general, however, that a favorable redistribution of DO resources is accomplished by artificial destratification.

#### Optimum Time for Destratification

Figures 5, 6, and 7 show a general deterioration in water quality from May to August. The total quantity of DO is lowered in the lakes, and the quantity of sulfide and manganese rises. While the mixing operation improved the water quality markedly in August, the goal of managing the DO resources in an impoundment through artificial destratification is to maintain quality throughout the summer season. The Boltz Lake studies were planned to begin in spring, in an effort to prevent any water quality deterioration, but the lake was actually mixed in August because the mixing equipment was not ready for operation. Early season mixing is currently (1966) under study.

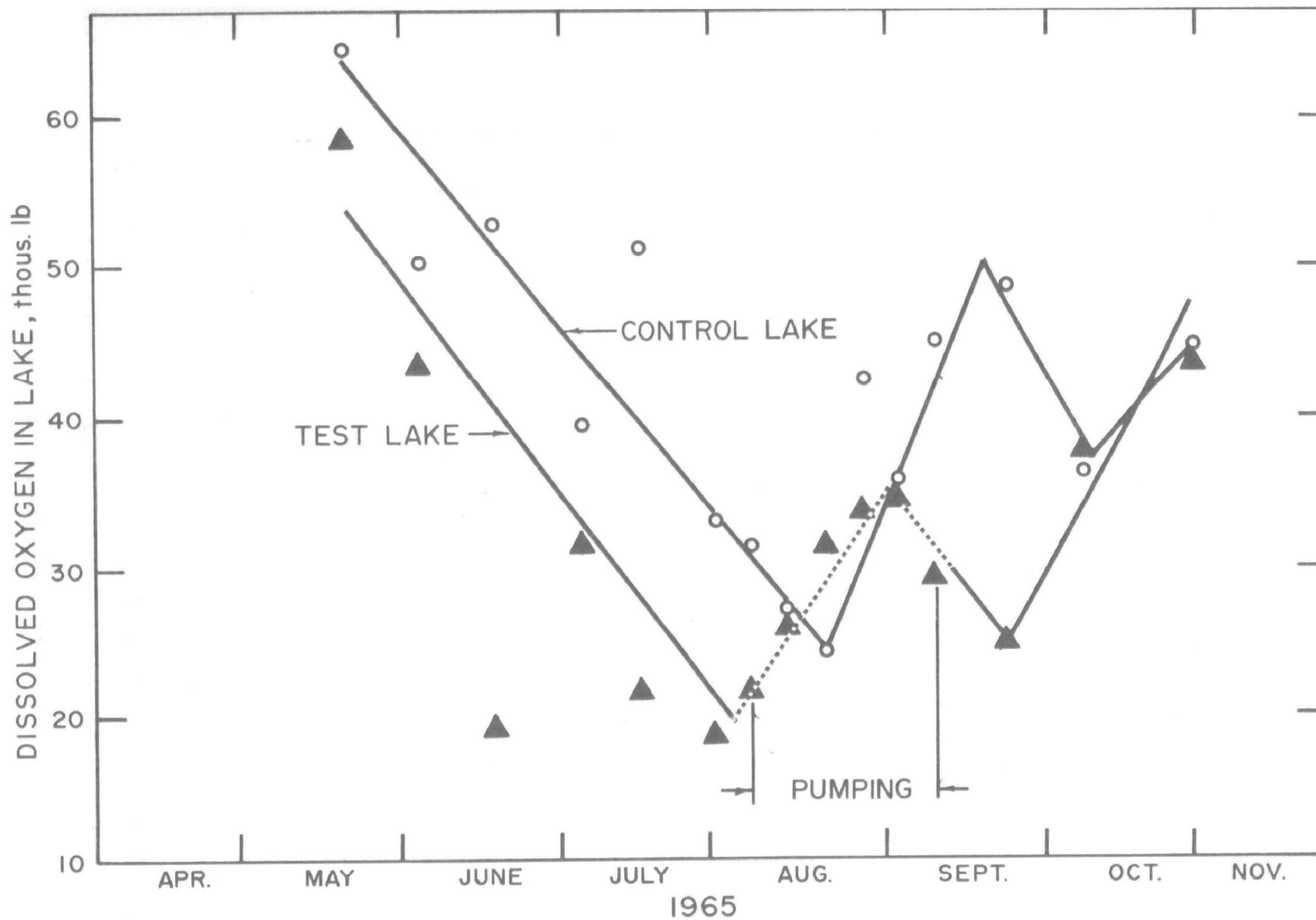
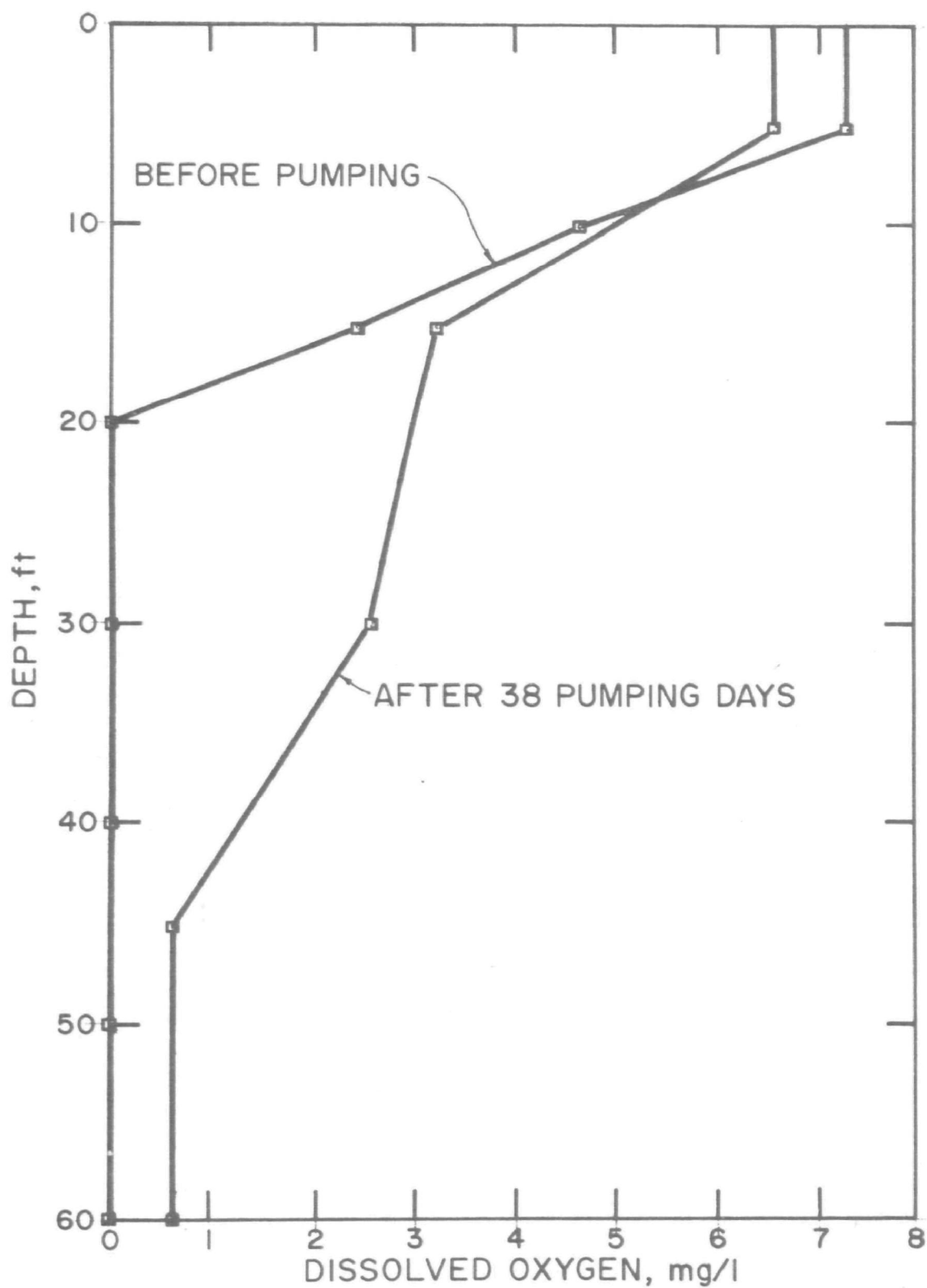


Figure 7. Total quantity of dissolved oxygen in test and control lakes.





**Figure 8. Redistribution of dissolved oxygen resources in Boltz Lake during artificial destratification.**

Spring mixing has some hydraulic drawbacks: (a) the practical inefficiency of raising cold water when the volume of cold water is high (see Table 2) and (b) a naturally increasing stability with time at this season of the year, (see the lower portion of Figure 1 and Figure 4). Quality protection, not hydraulic efficiency, is the goal of this technique, however, and so some hydraulic sacrifices should be made in an attempt to prevent the quality deterioration that naturally occurs from the beginning of the heating season through the time of maximum stability in late July. Some of this inefficiency could be overcome by pumping the warm water layer down into the water mass in the spring.

#### Measurement of DO Budget by the DO Probe Method

The data plotted in Figure 9 was obtained with the special equipment previously described.<sup>21</sup> Some important water quality parameters of the artificial impoundment water used for the test are noted on Figure 9. This figure shows the excellent data that may be obtained using this equipment, note the extremely expanded ordinate scale. Approximate curves showing the trends have been placed by eye through the data points.

The solutions to Equations (2), (3), and (4) are taken from the deficit data taken from these curves. Table 3 summarizes the data used for the solution of the equations.

Using these data and the method in Appendix I the solution for Equation (2) is:

$$D = \frac{(0.15)(0.91)}{0-0.15} \{ \exp[(-0.15)(t)] - \exp[(-0)(t)] \} + \frac{0.002}{0} \{ 1 - \exp[(-0)(t)] \} + Da \{ \exp[(-0)(t)] \} \quad (6)$$

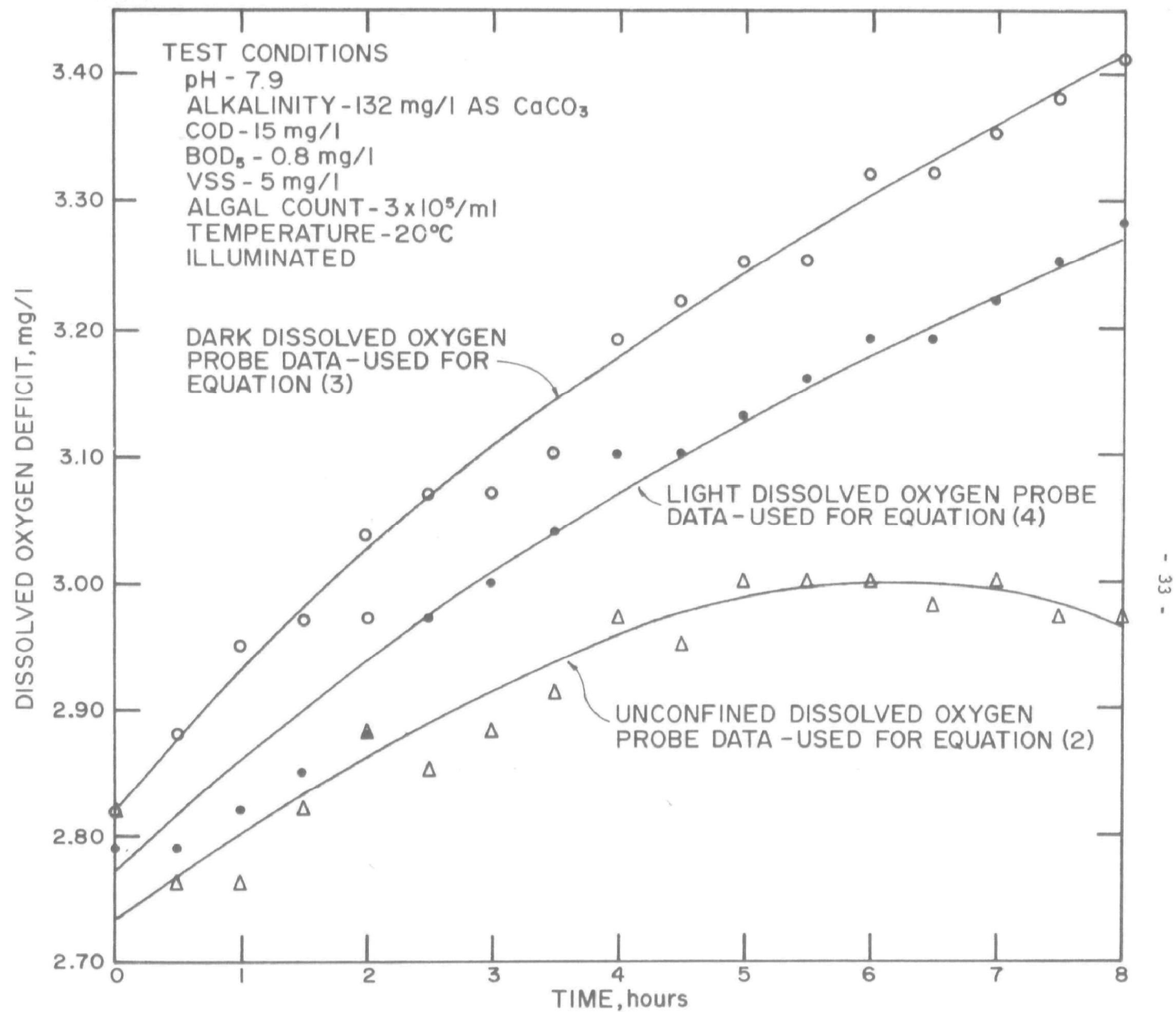


Figure 9. Data from special dissolved oxygen probe equipment.

TABLE 3

DATA FROM FIGURE 9 FOR THE SOLUTION OF THE MATHEMATICAL MODEL

DO Deficit - mg/l

Time Hours	Dark Probe	Light Probe	Unconfined Probe
0	2.82	2.77	2.74
0.5	2.88	2.82	2.78
1.0	2.94	2.86	2.81
1.5	2.99	2.90	2.84
2.0	3.03	2.94	2.87
2.5	3.07	2.98	2.90
3.0	3.11	3.01	2.92
3.5	3.14	3.04	2.95
4.0	3.18	3.08	2.97
4.5	3.21	3.10	2.99
5.0	3.24	3.13	3.00
5.5	3.27	3.16	3.01
6.0	3.30	3.19	3.01
6.5	3.33	3.21	3.01
7.0	3.36	3.23	3.00
7.5	3.39	3.25	2.99
8.0	3.41	3.27	2.98

Figure 10 compares the line of best fit from Equation (6) with the deficit data obtained with the unconfined DO probe. The fairly close comparison of the data generated from the mathematical model and the data taken with the DO probe shows the general reliability of the model. The low values of  $k_2$  and (R-P) are in agreement with the low turbulence and low algal population in the test media. These data show that the "DO probe method," using this special equipment, and the mathematical model represented by Equation (2), is a useful method for studying the DO budget in impoundments.

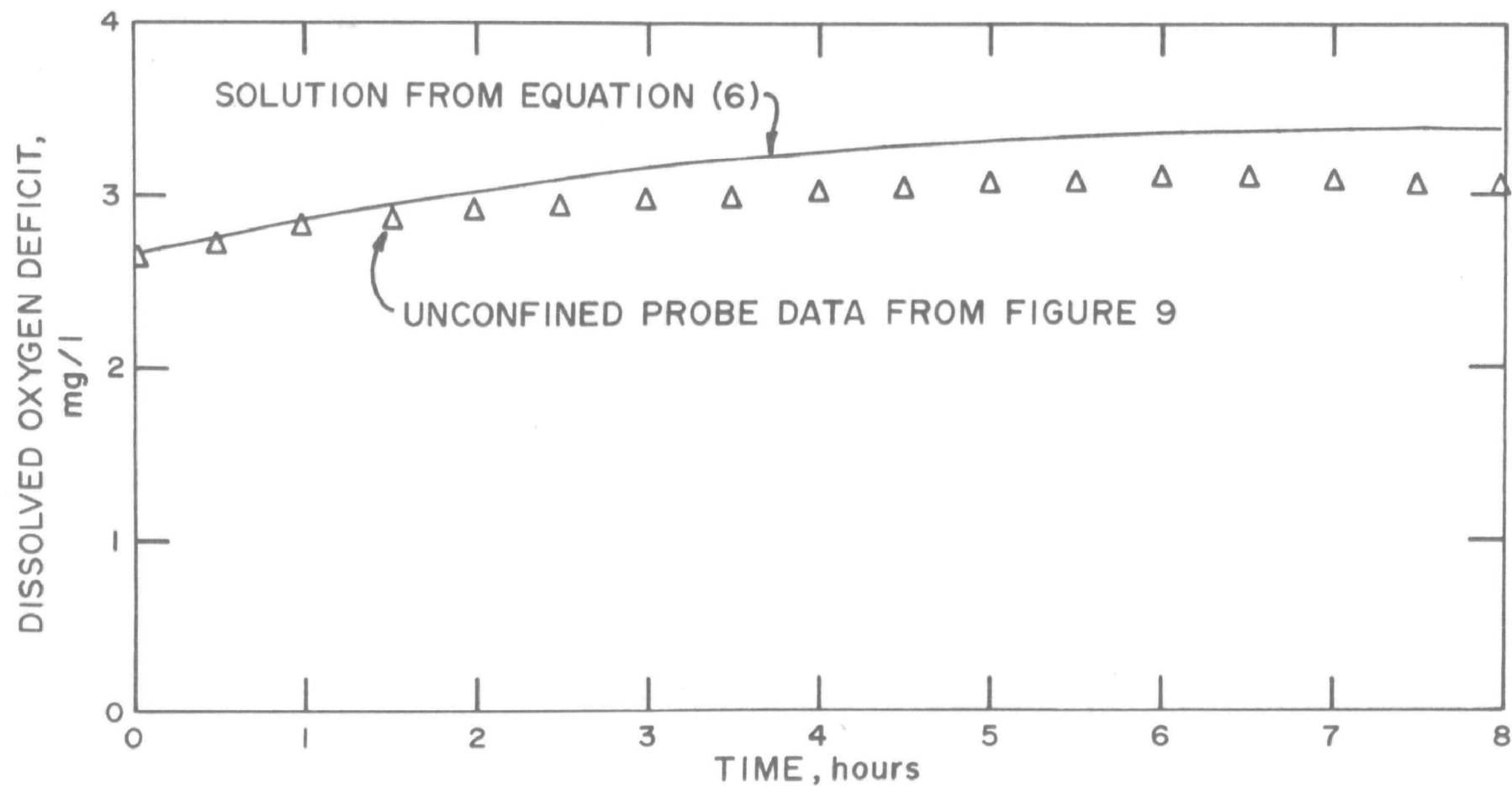


Figure 10. Comparison of Equation (6) with the unconfined probe data.

## FUTURE PLANS

Continued study and research are necessary to completely define the potential of artificial destratification for water quality control. Comparison of different destratification methods, spring mixing, and the mixing of larger bodies of water, must be accomplished. The first two items are currently (1966) under study, a diffused air system was used, starting in the spring of 1966, for the destratification of Boltz Lake. This system has the disadvantage that it cannot be adapted to mixing warm water down in the spring. Eventually, other destratification methods must be tried, and water quality control by this technique must be demonstrated in a large storage reservoir. Finally, the DO probe equipment must be used in the field to gain experience with the influence of various natural environmental factors on the constants in the mathematical model, and to determine whether good correlations between the model and data taken may be obtained under a variety of conditions.

## SUMMARY AND CONCLUSIONS

### Management of DO in Impoundments

#### Hydraulic Efficiency

The data presented here, and in reference 6 and 7 have indicated, that artificial destratification is an effective method of impoundment water quality improvement, although to date (1966) it has only been researched on a relatively small scale. Any piece of mechanical artificial destratification equipment should be as hydraulically efficient as possible, and this can be evaluated by calculating the stability of the impoundment to be mixed. The calculated hydraulic efficiency, "the net change in stability/the total energy input," is a good method of comparing one piece of destratification equipment with another, or evaluating various designs of a given piece of equipment. The equipment used in this study had a relatively poor hydraulic efficiency, 0.13%, probably because of poor hydraulic design.

#### Oxygenation Efficiency

Effective water quality improvement is even more important than a high hydraulic efficiency, and although the hydraulic efficiency of this pumping equipment was relatively low, the pump did supply 14,300 pounds of oxygen to the test impoundment with an expenditure of 14,300 kw-hr of input energy. The calculated oxygenation efficiency of this equipment was 1 pound of oxygen/kw-hr. This is an acceptable value, but this equipment was still undersized for Boltz Lake, as only enough oxygen was provided to satisfy the inorganic oxygen demand, and very little DO was actually added to the water. In spite of this, the DO resources in the impoundment were favorably redistributed, and at least some DO was present at all levels of the



lake after the mixing operation.

#### Best Time for Destratification

The major advantage of artificial destratification over the devices for preventing escape of poor quality water from impoundments<sup>5</sup>, is that mixing creates a two-fold benefit, protecting downstream users as well as improving the quality of the water in the impoundment itself. In order to obtain these benefits, an impoundment should be destratified in the spring, to prevent the formation of bad quality water, rather than in mid-summer to improve poor quality water. Spring mixing is necessary, in spite of the practical hydraulic inefficiency of raising cold water at this season of the year, and the necessity to occasionally remix during the summer season. These hydraulic sacrifices must be made in order to maintain good quality water throughout the heating season.

#### Measurement of DO Budget

To properly evaluate the DO budget in impoundments, an adequate mathematical model and a method of solving the model are needed.

The model presented in Equation (2),

$$D = \frac{k_1(L_a)}{k_2 - k_1} [\exp(-k_1 t) - \exp(-k_2 t)] + \frac{(R-P)}{k_2} [1 - \exp(-k_2 t)] + D_a [\exp(-k_2 t)] \quad (2)$$

is adequate, and the special three DO probe system<sup>21</sup> is effective for determining the five constants. Use of this tool will permit further study of the DO budget and will permit comparison of the DO budget from one impoundment to another.

## Conclusions

Although research on this project is continuing, the conclusions that may be drawn at this time are:

1. Engineering methods are available that will prevent the escape of poor quality bottom water from impoundments.
2. Artificial destratification is a better method of impoundment water quality control since it has the double benefit of preventing downstream damage and improving water quality in the impoundment itself.
3. The proper use of the stability concept for the calculation of the hydraulic efficiency of a mechanical mixing device is a useful method of comparing various pieces of equipment and improving equipment design.
4. In this study, reduced materials, sulfides and manganese, were eliminated from the test lake by the artificial destratification operation.
5. The equipment used in this study was insufficient in capacity to add the desired dissolved oxygen beyond the inorganic oxygen demand, but the available DO resources in the test lake were favorably redistributed by the pumping so that DO was present at all levels.
6. In spite of the practical hydraulic inefficiency of raising large volumes of cold water, spring mixing is better than mid-summer destratification because early mixing will prevent any bottom water quality deterioration.

7. Equation (2)

$$D = \frac{k_1(L_a)}{k_2 - k_1} [\exp(-k_1 t) - \exp(-k_2 t)] + \frac{(R-P)}{k_2} [1 - \exp(-k_2 t)] + D_a [\exp(-k_2 t)] \quad (2)$$

is a satisfactory mathematical model for representing the DO budget in impounded waters.

8. The special three DO probe equipment designed for this study is capable of supplying the data necessary for solving the DO model.

#### ACKNOWLEDGMENTS

The authors wish to thank Lawrence Kamphake, George Holtzer, and Richard Shibiya for the laboratory chemical analyses, Glenn R. Gruber for help with the field work, and Thomas A. Entzminger of the Ohio River Basin Project who supplied the computer program for the solution of the Table 3 data, yielding Equation (6). In addition, acknowledgment is made of the cooperation of the personnel of the State of Kentucky Department of Fish and Wildlife Resources.

REFERENCES

5. Kittrell, F.W., "Effects of Impoundments on Dissolved Oxygen Resources," Sewage and Industrial Wastes, Vol. 31, 1959, p. 1065-1078.
6. Irwin, W.H., Symons, J.M., and Robeck, G.G., "Impoundment Destratification by Mechanical Pumping," Presented at ASCE National Symposium on Sanitary Engineering Research, Development, and Design at University Park, Pa., July 27-30, 1965. In Press.
7. Symons, J.M., Irwin, W.H., and Robeck, G.G., "Impoundment Water Quality Changes Caused by Mixing," Presented at the ASCE Water Resources Engineering Conference, Denver, Colorado, May 16-20, 1966. In Press.
8. Ogborn, C.M., "Aeration System Keeps Water Tasting Fresh," Public Works Magazine, Vol. 97, April 1966, p. 84-86.
9. Koberg, G.E. and Ford, M.E., Jr., "Elimination of Thermal Stratification in Reservoirs and the Resulting Benefits," Geological Survey Water Supply Paper 1809-M, U.S. Government Printing Office, Washington, D.C., 1965, 28 pp.
10. Bryan, J.G., "Physical Control of Water Quality," The Journal, British Waterworks Association, Vol. XLVI, No. 395, August 1964, p. 546.
11. Hutchinson, G.E., A Treatise on Limnology, John Wiley & Sons, Inc., New York, New York, 1957, Vol. 1.
12. Symons, J.M., Weibel, S.R., and Robeck, G.G., "Impoundment Influences on Water Quality," Journal American Water Works Association, Vol. 57, 1, Jan. 1965, pp. 51-75.
13. O'Connell, R.L., and Thomas, N.A., "Effect of Benthic Algae on Stream Dissolved Oxygen," Journal of the Sanitary Engineering Division, ASCE, Vol. 91, SA3, Proc. Paper 4345, June 1965, pp. 1-16.
14. Hull, C.H.J., Discussion of "Effect of Benthic Algae on Stream Dissolved Oxygen" by Richard L. O'Connell and Nelson A. Thomas, Journal of the Sanitary Engineering Division, ASCE, Vol. 92, SA1, Proc. Paper 4637, Feb. 1966, pp. 306-313.
15. Ettinger, M.B., "How to Plan an Inconsequential Research Project," Journal of the Sanitary Engineering Division, ASCE, Vol. 91, SA4, Proc. Paper 4437, August, 1965, pp. 19-22.
16. Odum, H.T., "Primary Production in Flowing Waters," Limnology and Oceanography, Vol. 1, 1956, pp. 102-117.
17. Hull, C.H.J., "Oxygenation of Baltimore Harbor by Planktonic Algae," Journal Water Pollution Control Federation, Vol. 35, 1963, pp. 587-606.

18. Hull, C.H.J., "Photosynthetic Oxygenation of a Polluted Estuary," *Advances in Water Pollution Research, Proceedings of the 1st International Conference, held in London, United Kingdom, in September, 1962*, Pergamon Press, London, U.K., 1964, Vol. 3, pp. 347-374.
19. Goldman, C.R. and Carter, R.A., "An Investigation by Rapid Carbon-14 Bioassay of Factors Affecting the Cultural Eutrophication of Lake Tahoe, California-Nevada," Journal Water Pollution Control Federation, Vol. 37, 1965, pp. 1044-1059.
20. Verduin, J., "Energy Fixation and Utilization by Natural Communities in Western Lake Erie," Ecology, Vol. 37, 1956, pp. 40-50.
21. Symons, J.M., Discussion of "Effect of Benthic Algae on Stream Dissolved Oxygen," By Richard L. O'Connell and Nelson A. Thomas, Journal of the Sanitary Engineering Division, ASCE, Vol. 92, SA1, Proc. Paper 4637, Feb. 1966, pp. 301-306.
22. Willers, F.A., Practical Analysis, Dover Publications, New York, New York (1948).
23. Whittaker, E.T. and Robinson, G., The Calculus of Observations, D. Van Nostrand Company, Princeton, New Jersey (1926).

# APPENDIX I

## Solution of Mathematical Model with Assumed Data

The assumed constants for this example are:  $k_1 = 0.10/\text{hr}$ ,  $k_2 = 0.05/\text{hr}$ ,  $R = 0.10/\text{hr}$ ,  $P = 0.30/\text{hr}$ ,  $L_a = 3.00 \text{ mg/l}$ ,  $D_a = 0.47 \text{ mg/l}$ . Using the appropriate constants and solving Equations (2), (3), and (4) yields the deficit data shown in Table 4, and plotted in Figure 11. For simplicity, the units of the constants will not be used.

Several approaches to the solution of this problem were attempted. Various forms of exponential approximation or Prony's method<sup>22</sup>, and a modification of Newton Raphson's method<sup>23</sup> were tried without success. Much of the difficulty in using these techniques was caused by an insufficient number of significant figures and error caused by rounding off. The most productive technique was simple iteration, which will be developed using the data in Table 4. The solution presented was developed using a Honeywell 400 computer.

Considering the simulated dark probe data and solving Equation (3) at  $t = 7.5$  and  $t = 8.0$  yields,

$$2.81 = L_a \{ 1 - \exp [-k_1 (7.5)] \} + R (7.5) + 0.47 \quad (7)$$

$$2.92 = L_a \{ 1 - \exp [-k_1 (8.0)] \} + R (8.0) + 0.47 \quad (8)$$

Solving Equations (7) and (8) for  $R$  and  $L_a$  yields,

$$R = \frac{2.45 \{ 1 - \exp [-k_1 (7.5)] \} - 2.34 \{ 1 - \exp [-k_1 (8.0)] \}}{8.0 \{ 1 - \exp [-k_1 (7.5)] \} - 7.5 \{ 1 - \exp [-k_1 (7.5)] \}} \quad (9)$$

$$L_a = \frac{8.0 (2.34) - 7.5 (2.45)}{8.0 \{ 1 - \exp [-k_1 (7.5)] \} - 7.5 \{ 1 - \exp [-k_1 (8.0)] \}} \quad (10)$$

TABLE 4

DO DEFICIT DATA FROM ASSUMED CONSTANTS

DO Deficit - mg/l

$$D_a = 0.47 \text{ mg/l}$$

Time hrs (1)	Eq. (3) Simulated Dark Probe (2)	Eq. (4) Simulated Light Probe (3)	Eq. (2) Simulated Unconfined Probe (4)
0	0.47	0.47	0.47
0.5	0.67	0.52	0.52
1.0	0.87	0.57	0.56
1.5	1.04	0.59	0.59
2.0	1.21	0.61	0.61
2.5	1.38	0.63	0.63
3.0	1.55	0.65	0.65
3.5	1.72	0.67	0.66
4.0	1.86	0.66	0.66
4.5	2.00	0.65	0.66
5.0	2.14	0.64	0.65
5.5	2.28	0.63	0.64
6.0	2.42	0.62	0.62
6.5	2.56	0.61	0.60
7.0	2.67	0.57	0.58
7.5	2.81	0.56	0.55
8.0	2.92	0.52	0.52

$k_1 = 0.10, k_2 = 0$	$k_1 = 0.10, k_2 = 0$	$k_1 = 0.10, k_2 = 0.05$
$P = 0, R = 0.10$	$P = 0.30, R = 0.10$	$P = 0.30, R = 0.10$
$L_a = 3.00$	$L_a = 3.00$	$L_a = 3.00$



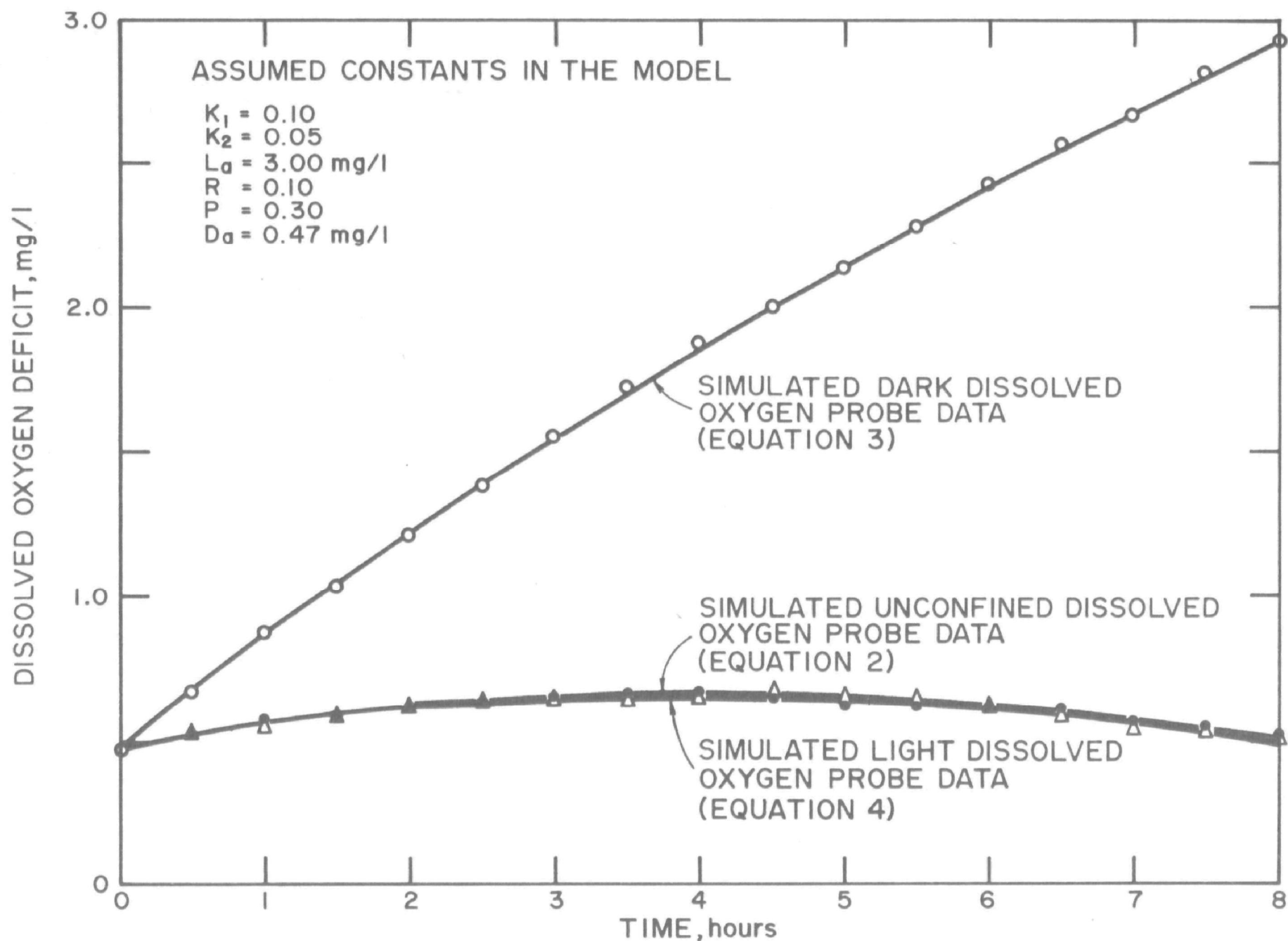


Figure 11. Assumed dissolved oxygen data to verify solution of dissolved oxygen model.

To solve for  $k_1$ , Equation (3) can be reformulated as follows:

$$D = L_a [1 - \exp(-k_1 t)] + R(t) + D_a \quad (3)$$

$$D - D_a - R(t) = L_a [1 - \exp(-k_1 t)] \quad (11)$$

$$\frac{D - D_a - R(t)}{L_a} = 1 - \exp(-k_1 t) \quad (12)$$

$$1 - \left[ \frac{D - D_a - R(t)}{L_a} \right] = \exp(-k_1 t) \quad (13)$$

$$\ln \left\{ 1 - \left[ \frac{D - D_a - R(t)}{L_a} \right] \right\} = -k_1 t \quad (14)$$

$$\ln \left\{ 1 - \left[ \frac{D - D_a - R(t)}{L_a(t)} \right] \right\} = -k_1 \quad (15)$$

Using  $\left[ \frac{D - D_a - R(t)}{L_a} \right]$  as K, Equation (15) may be written

$$\ln \left( \frac{1 - K}{t} \right) = -k_1 \quad (16a)$$

Now, a high value of  $k_1$  is assumed, for example  $k_1^1 = 0.50$  (the superscript is the iteration number) and Equations (9) and (10) solved for  $L_a$  and  $R$ , values of  $L_a^1 = 0.76799$  and  $R^1 = 0.21201$  are obtained. Using this  $L_a^1$  and  $R^1$ , a value may be obtained for  $k_1$  at each set of values for  $D$  and  $t$  using the dark probe data of Table 4 and Equation (16). The average value for  $k_1$  is the new value,  $k_1^2$ . Then Equations (9) and (10) are resolved for  $L_a^2$  and  $R^2$ . The process is repeated until  $k_1^i = k_1^{i+1}$  and the solution has converged. A tolerance of 0.005 is placed on  $k_1$  for convergence, that is when  $k_1^{i+1}$  is within 0.005 of  $k_1^i$  the iteration stops.

Rearranging Equation (3) into  $\frac{(D-D_a)}{1-\exp(-k_1 t)} = L_a + R \left[ \frac{1}{1-\exp(-k_1 t)} \right]$  (16b)

a least squares technique gives the best value of  $L_a$  and  $R$ , using the final value of  $k_1$ .

To illustrate, starting with  $k_1^1 = 0.50$  and  $L_a^1 = 0.76799$  and  $R^1 = 0.21201$  from Equations (9) and (10), the solution for  $k_1^2$  would be as shown in Table 5. Discarding any values for which  $1-K$  is zero or negative,  $k_1^2 = 0.33426$ . Continuing in this manner until  $k_1^i = k_1^{i+1}$ ,  $k_1^{13} = 0.11$ . The least squares solution of Equation (16b) with  $k_1 = 0.11$ , yields  $R = 0.10$  and  $L_a = 2.72$ .

In the next step, using  $k_1 = 0.11$  and  $L_a = 2.72$  from the previous solution, Equation (4) is solved for  $(R-P)$  at each time except  $t=0$  using the light probe data in Table 4. This yields 16 values for  $(R-P)$  that are averaged to yield the final value of  $(R-P)$ . For these data  $(R-P) = -0.19$ .  $P$  can then be evaluated by difference, if required.

In solving Equation (2) a slightly different approach is used. Here, equally spaced abscissa values are used, then Equation (2) is reformulated as follows,

$$D = C_1 [\exp(-k_1 t)] + C_2 [\exp(-k_2 t)] + Q \quad (17)$$

where  $C_1 = \frac{k_1(L_a)}{k_2 - k_1} \quad (18)$

$$C_2 = \frac{-k_1(L_a)}{k_2 - k_1} - \frac{(R-P)}{k_2} + D_a \quad (19)$$

$$Q = \frac{(R-P)}{k_2} \quad (20)$$

TABLE 5

DATA FOR SOLUTION OF EQUATION (16)

t	R(t)	D-D <sub>a</sub>	$\left[ \frac{D-D_a - R(t)}{L_a} \right] = K$	$\ln\left(\frac{1-K}{t}\right) = -k_1$
0.0	0.0	0.00	0.0000	-
0.5	0.105	0.20	0.12239	-0.26111
1.0	0.212	0.40	0.24478	-0.28075
1.5	0.315	0.57	0.32811	-0.26511
2.0	0.424	0.74	0.41144	-0.26504
2.5	0.525	0.91	0.49477	-0.27309
3.0	0.630	1.08	0.57810	-0.28766
3.5	0.735	1.25	0.66142	-0.30943
4.0	0.840	1.39	0.70569	-0.30578
4.5	0.945	1.53	0.74995	-0.30802
5.0	1.050	1.67	0.79422	-0.31619
5.5	1.155	1.81	0.83848	-0.33148
6.0	1.260	1.95	0.88275	-0.35724
6.5	1.365	2.09	0.92701	-0.40269
7.0	1.470	2.20	0.93222	-0.38449
7.5	1.575	2.34	0.97648	-0.50000
8.0	1.680	2.45	0.98168	-0.50000

Solving Equation (17) at  $(t_1, D_1)$  for  $C_2[\exp(-k_2 t_1)]$  yields,

$$C_2[\exp(-k_2 t_1)] = D_1 - C_1 [\exp(-k_1 t_1)] - Q \quad (21)$$

The solution for  $C_2[\exp(-k_2 t_2)]$  at  $(t_2, D_2)$  is

$$C_2[\exp(-k_2 t_2)] = D_2 - C_1 [\exp(-k_1 t_2)] - Q \quad (22)$$

Dividing Equation (21) by Equation (22) yields,

$$\exp[-k_2(t_1 - t_2)] = \frac{D_1 - C_1 [\exp(-k_1 t_1)] - Q}{D_2 - C_1 [\exp(-k_1 t_2)] - Q} \quad (23)$$

Repeating this process at  $(t_2, D_2)$  and  $(t_3, D_3)$  yields,

$$\exp[-k_2(t_2 - t_3)] = \frac{D_2 - C_1 [\exp(-k_1 t_2)] - Q}{D_3 - C_1 [\exp(-k_1 t_3)] - Q} \quad (24)$$

Since equally spaced abscissa values are used,  $(t_1 - t_2) = (t_2 - t_3)$ ,

the right hand side of Equations (23) and (24) may be equated,

yielding

$$\frac{D_1 - C_1 [\exp(-k_1 t_1)] - Q}{D_2 - C_1 [\exp(-k_1 t_2)] - Q} = \frac{D_2 - C_1 [\exp(-k_1 t_2)] - Q}{D_3 - C_1 [\exp(-k_1 t_3)] - Q} \quad (25)$$

Forming the cross products and substituting for  $C_1$  and  $Q$  yields,

the quadratic in  $k_2$ .

$$(Z_4)(k_2)^2 + [k_1(Z_4) - k_1(L_a)(Z_2) + (R-P)(Z_1)][k_2] + k_1[L_a(R-P)(Z_3) - (R-P)(Z_3)] = 0 \quad (26)$$

$$\text{where } Z_1 = [D_1 + D_3 - 2(D_2)] \quad (27)$$

$$Z_2 = [D_3[\exp(-k_1 t_1)] + D_1[\exp(-k_1 t_3)] - 2[D_2][\exp(k_1 t_2)]] \quad (28)$$

$$Z_3 = [\exp(-k_1 t_1) + \exp(-k_1 t_3) - 2[\exp(-k_1 t_2)]] \quad (29)$$

$$\text{and } Z_4 = [D_1(D_3) - (D_2)^2] \quad (30)$$

Using values of  $k_1$ ,  $L_a$ , and (R-P) derived from the solution of Equations (3) and (4), the quadratic Equation (26) may be solved for its positive root, as follows.

The coefficients for  $k_2$  and the constant in Equation (26) may be determined from each three successive data points, for example ( $t=0.5, 1.0, 1.5$ ), ( $t=1.0, 1.5, 2.0$ ) and so on. This yields 14 polynomial equations in the form of Equation (26). The fourteen sets of coefficients and constants are then averaged to yield an "average" Equation (26) which is then solved for  $k_2$  as the only unknown, which in this example = 0.09. A summary of all of the solutions for the constants in Equation (2) is shown in Table 6.

TABLE 6  
COMPARISON OF SOLUTIONS FOR CONSTANTS IN EQUATION (2)

	$k_1$	$k_2$	R	(R-P)	$L_a$
Assumed	0.10	0.05	0.10	-0.20	3.00
Dark Probe	0.11	-	0.10	-	2.72
Light Probe	-	-	-	-0.19	-
Unconfined Probe	-	0.09	-	-	-

To demonstrate the adequacy of the model, Equation (2), and the method of solution, deficit data was generated using the constants of Table 6, in Equation (2). This curve is plotted in Figure 12 and is compared to the simulated unconfined DO data from Column 4 in Table 4. The fit curve is low as time progresses, probably because  $k_2$  is too large. Fit curves calculated for the dark- and light-probe systems are much better, as  $k_2$  is not included.

The solution is very sensitive to the final value of  $k_1$ . The computational procedure was repeated using a tolerance of 0.001 as a measure of convergence for  $k_1$ . Under these circumstances  $k_1$  is reduced 0.03 to 0.08. This slight reduction has a large impact on the remaining calculations. One method of reducing this instability might be to evaluate Equations (9) and (10) at each group of two times (0.5, 1.0; 1.0, 1.5; and so forth) instead of just at  $t = 7.5$  and  $8.0$  to obtain an "average"  $L_a^1$  and  $R^1$  before calculating  $k_1^2$ . In spite of these minor difficulties the usefulness of this equipment and method for studying the DO budget in impoundments has been demonstrated.

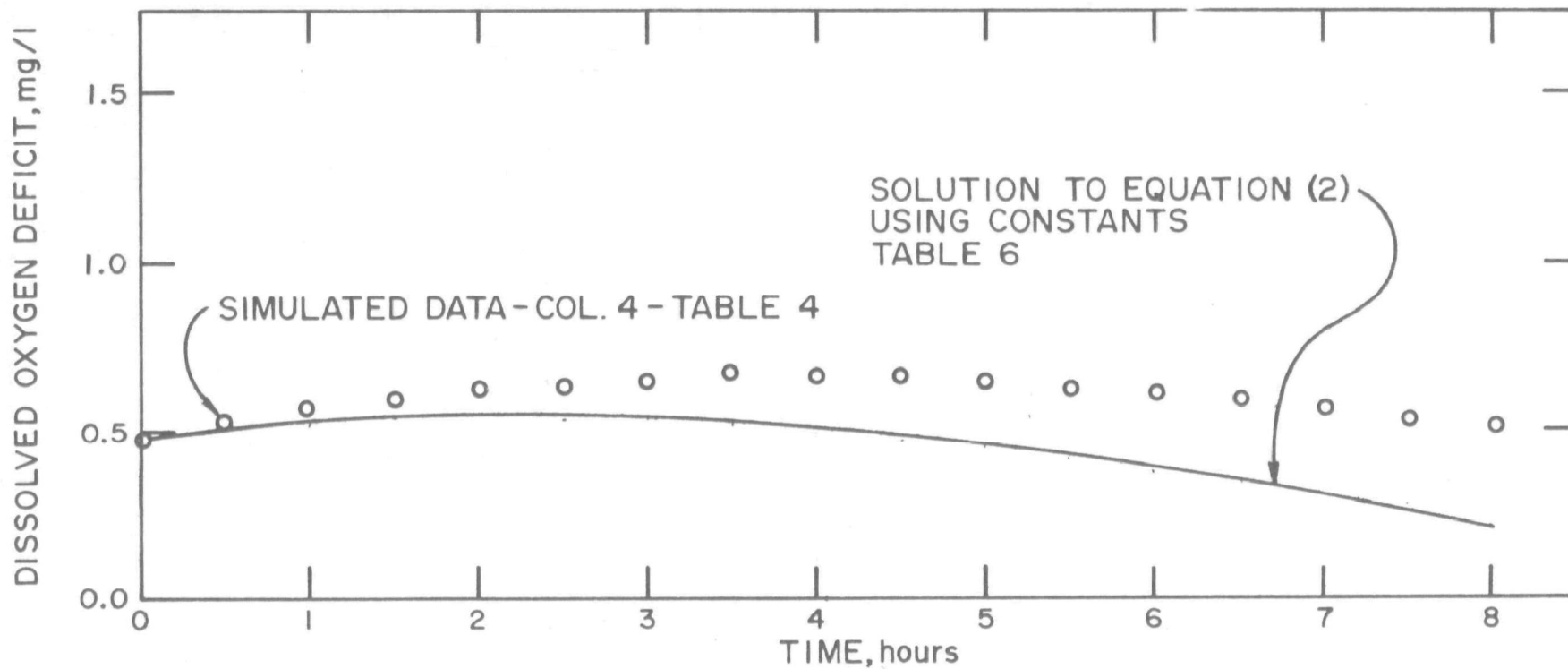


Figure 12. Comparison of the solution of Equation (2) with the simulated data.



APPENDIX II

BIBLIOGRAPHY

1. Hooper, F.F., Ball, R.C. and Tanner, H.A., "An Experiment in the Artificial Circulation of a Small Michigan Lake," Transactions of the American Fisheries Society, Vol. 82, 1952, p. 222-240.
2. Grim, J., "Ein See Wird Umgepflugt," Allg. Fisch-Ztg., Jahrg., Vol. 77, 1952, p. 281-283.
3. Mercier, P. and Gay, S., "Effects de L'aeration artificielle sous-lacustre au lac de Bret," Revue Suisse D'Hydrologic, Vol. XVI, Fasc. 2, 1954.
4. Cooley, P. and Harris, S.L., "Prevention of Stratification in Reservoirs," Journal of the Institution of Water Engineers, Vol. 8, 1954, p. 517-537.
5. Streiff, A., "Compressed Air vs. Drought," Compressed Air Magazine, Vol. 60, August 1955, p. 232.
6. Derby, R.L., "Chlorination of Deep Reservoirs for Taste and Odor Control," Journal American Water Works Association, Vol. 48, 1956, p. 775-780.
7. Mercier, P., "L'aeration Naturelle et Artificielle des Lacs," Rev. suisse d'hydrol., Vol. 19, 1957, p. 613.
8. Schmitz, W.R. and Hasler, A.D., "Artificially Induced Circulation of Lakes by Means of Compressed Air," Science, Vol. 128, October 1958, p. 1088-1089.
9. Riddick, T.M., "Forced Circulation of Reservoir Waters," Water and Sewage Works, Vol. 104, 1957, p. 231-237. "Forced Circulation of Large Bodies of Water," Proceedings of the American Society of Civil Engineers, Journal Sanitary Engineering Division, Vol. 84, Paper 1703, SA-4, July 1958, 21 pp.
10. Heath, W.A., "Compressed Air Revives Polluted Swedish Lakes," Water and Sewage Works, Vol. 108, 1961, p. 200.
11. Nickerson, H.D., "Gloucester-Forced Circulation of Babson Reservoir," Sanitalk, Vol. 9, Summer 1961, p. 1-10 & 27.
12. Patriarche, M.H., "Air-Induced Winter Circulation of Two Shallow Michigan Lakes," Journal of Wildlife Management, Vol. 25, 1961, p. 282-289.
13. Laurie, A.H., "The Application of the 'Bubble-Gun' Low Lift Pump," Water and Waste Treatment, Vol. 8, 1961, p. 363.
14. Anonymous, "'Bubble-Gun' for Destratification," Pneumatics Breakwaters Ltd., Water and Water Engineering, Vol. 65, 1961, p. 7071.

15. Yount, J.L., Biologist, Florida State Board of Health, Mixed 300 Acre Lake in 1962 with Compressed Air, Personal Communication.
16. Anonymous, "Improvement in the Water Quality of Reservoir Discharge Through Reservoir Mixing and Aeration," Water and Water Engineering, Vol. 66, 1962, p. 112.
17. Ford, M.E., Jr., "Air Injection for Control of Reservoir Limnology," Journal American Water Works Association, Vol. 55, 1963, p. 267.
18. Karlgren, L. and Lindgren, O., "Aeration Studies in Lake Trask," Vattenhygien, Vol. 19, 1963, p. 67.
19. Bernhardt, H., "Erste Ergebnisse über die Belüftungsversuche an der Wahnachtalsperre," Vom Wasser, Vol. XXX, 1963, p. 11.
20. Bryan, J.G., "Physical Control of Water Quality," The Journal, British Waterworks Association, Vol. XLVI, No. 395, August 1964, p. 546.
21. Ridley, J.E., "Thermal Stratification and Thermocline Control in Storage Reservoirs," Proceedings of the Society for Water Treatment and Examination, Vol. 13, 1964, p. 275.
22. Forty-first Report on the Results of the Bacterial, Chemical, and Biological Examination of the London Waters for the Years 1963-1964, "Thermal Stratification and the Possibility of Thermocline Control of Storage Reservoirs," p. 104.
23. Kobert, G.E., and Ford, M.E., Jr., "Elimination of Thermal Stratification in Reservoirs and the Resulting Benefits," Geological Survey Water Supply Paper 1809-M, U.S. Government Printing Office, Washington, D.C., 1965, 28 pp.
24. Bryan, J.G., "Improvement in the Quality of Reservoir Discharges Through Reservoir Mixing and Aeration," Symposium on Streamflow Regulation for Quality Control, PHS Publ. No. 999-WP-30, Cincinnati, Ohio, June 1965, 420 pp.
25. Anonymous, New Zealand Outdoors, May 1965.
26. Irwin, W.H., Symons, J.M. and Robeck, G.G., "Impoundment Destratification by Mechanical Pumping," Presented at ASCE National Symposium on Sanitary Engineering Research, Development, and Design at University Park, Pa., July 27-30, 1965, In Press.
27. Ogborn, C.M., "Aeration System Keeps Water Tasting Fresh," Public Works Magazine, Vol. 97, April 1966, p. 84-86.
28. Symons, J.M., Irwin, W.H., and Robeck, G.G., "Impoundment Water Quality Changes Caused by Mixing," Presented at the ASCE Water Resources Engineering Conference, Denver, Colorado, May 16-20, 1966. In Press.

29. Burns, J.M., Jr., "Reservoir 'Turn-over' Improves Water Quality," Water and Wastes Engineering, Vol. 3, No. 5, May 1966, p. 81.

30. Anonymous, "Water Treatment with Compressed Air," Industrial Water Engineering, Vol. 3, No. 6, June 1966, p. 40-41.

31. Symons, J.M., DeMarco, J., Irwin, W.H. and Robeck, G.G., "Enhancing Biodegradation of Synthetic Organics in Stratified Impoundments by Artificial Modification of the Thermal Profile," To be Presented at the International Association of Scientific Hydrology's meeting "Symposium on Lakes and Reservoirs" at Lake Garda, Italy, October 10-15, 1966. In Press.

32. Fast, A., reported by Roach, R., "Reservoir Aeration Reduces Evaporation, Improves Water," Palo Alto Times, August 4, 1966, p. 1.