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COLLABORATIVE STUDY OF METHOD FOR DETERMINATION OF STACK GAS VELOCITY AND VOLUMETRIC FLOW RATE IN CONJUNCTION WITH EPA METHOD 5

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COLLABORATIVE STUDY OF METHOD FOR DETERMINATION OF STACK GAS VELOCITY AND VOLUMETRIC FLOW RATE IN CONJUNCTION WITH EPA METHOD 5

by

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SUMMARY AND CONCLUSIONS

This report presents the results of a statistical analysis on data collected in the use of EPA Method 2 in conjunction with collaborative testing of Method 5 (Particulate Emissions). Method 2 is for the determination of stack gas velocity and volumetric flow rate and specifies that the stack gas velocity be determined from the gas density and from measurement of the velocity head using a Type S Pitot tube.

The collaborative tests of Method 5 were conducted at three sites: a Portland cement plant, a coal-fired power plant, and a municipal incinerator. There were 15, 16 and 12 sampling runs, respectively, at the three sites and four collaborating laboratories at each. The data from one laboratory at the power plant site were not used, and some determinations were not made due to equipment failure during the sampling run. This resulted in a total of 150 separate determinations of both velocity and flow rate being used in the analyses.

The runs at each site were grouped into blocks based upon the velocity heads. The precision components, within-laboratory, between-laboratory and laboratory bias, are shown to be proportional to the mean of the determinations and are expressed as percentages of the true mean, denoted by δ. The results are summarized below for each factor.

Velocity—The between-laboratory standard deviation estimate is 5.0% of δ with 8 degrees of freedom. The within-laboratory standard deviation estimate is 3.9% of δ with 113 degrees of freedom. From these, a laboratory bias standard deviation of 3.2% of δ may be estimated.

Volumetric Flow Rate—The estimated between-laboratory standard deviation is 5.6% of δ with 8 degrees of freedom. The estimated within-laboratory standard deviation is 5.5% of δ with 113 degrees of freedom. These give a laboratory bias standard deviation of 1.1% of δ.

The emission rate, denoted by r, is defined in the Federal Register as the product of the volumetric flow rate and the pollutant concentration. Using the estimates for the precision of the flow rate determination and estimates for the precision of Methods 5, 6, and 7, the precision of r is estimated for each Method.

Based upon the results obtained, the precision of the volumetric flow rate seems adequate for use with other test methods in determining the emission rate. The precision of r depends primarily upon the precision of the test method used, which is the desirable result.

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I. INTRODUCTION

This report describes the work performed on Contracts 68-02-0623 and 68-02-0626, and the results obtained on Southwest Research Institute Project 01-3462-008, Contract 68-02-0626, which includes collaborative testing of the method for determination of stack gas velocity and volumetric flow rate with use of Method 5 for particulate emissions as given in "Standards of Performance for New Stationary Sources"(1).

This report describes the statistical analysis of data from collaborative tests conducted in a Portland cement plant,(2) a coal-fired power plant,(3) and a municipal incinerator.(4)

The collaborative tests of the method for determination of stack gas velocity and volumetric flow rate were not run as separate tests of Method 2 but as this method is used in conjunction with Method 5 for particulate emissions.(1)

The results of the data analyses are given in this report.

II. COLLABORATIVE TESTING

A. Collaborative Test Sites

The site of the Portland cement plant test was the Lone Star Industries Portland Cement Plant in Houston, Texas. This plant utilizes the wet feed process and operates three kilns. The flue gas from each kiln passes through a separate electrostatic precipitator. The flue gases are then combined and fed into a 300-foot-high stack.(2) Samples were taken at the sample ports located on the stack 150 feet above grade. Inside diameter of the stack at the sample ports is 13 feet.* The cross-sectional area of the stack at the sample ports is 132.73 ft^2.* The average stack gas velocity ranged from about 50 to 60 ft/sec* during the test period. A typical velocity profile is shown in Figure 1. The typical volumetric flow rate was about 12×10^6 ft^3/hr,* dry gas basis at 70°F and 1 atmosphere.

The site of the coal-fired power plant was the Allen King Power Plant, The Northern States Power Company, near St. Paul, Minnesota. The exhaust gas from the combustion chamber passes through the heat exchanger and splits into two identical streams upstream of twin electrostatic precipitators. The twin emission gas streams are fed into an 800-foot-high stack through two horizontal ducts.(3) The sample ports were located in the south horizontal duct upstream of the entrance to the stack. The inside duct dimensions are 12 feet wide by 27 feet high. The duct cross sectional area is 324 ft^2. The average gas velocity was about 50 ft/sec. A typical velocity profile is shown in Figure 2. The typical total volumetric flow rate (flow rate in the duct times 2) was about 70×10^6 ft^3/hr.

The site for the municipal incinerator test was the Holmes Road Incinerator, City of Houston, Houston, Texas. The facility consists of two independent parallel furnace trains. Refuse feeds continuously onto traveling grate stokers in the furnaces. Gases leaving the furnaces are cooled in water spray chambers and then enter the flue gas scrubbers to remove particulates. The gases are then drawn through induced draft fans and exhaust into the 148-foot-high stacks. Samples were taken from the sample ports located on the stacks 102 feet above grade. The inside diameter of both stacks is 6.5 ft. The cross-sectional area of each stack is 33.18 ft^2. The typical stack gas velocity for both stacks was about 50 ft/sec (Fig. 3). The typical volumetric flow rate for either unit was about 3.5×10^6 ft^3/hr. Determinations were made on both stacks during the test. Only one furnace train was operating at any time during the test.

B. Collaborators and Test Personnel

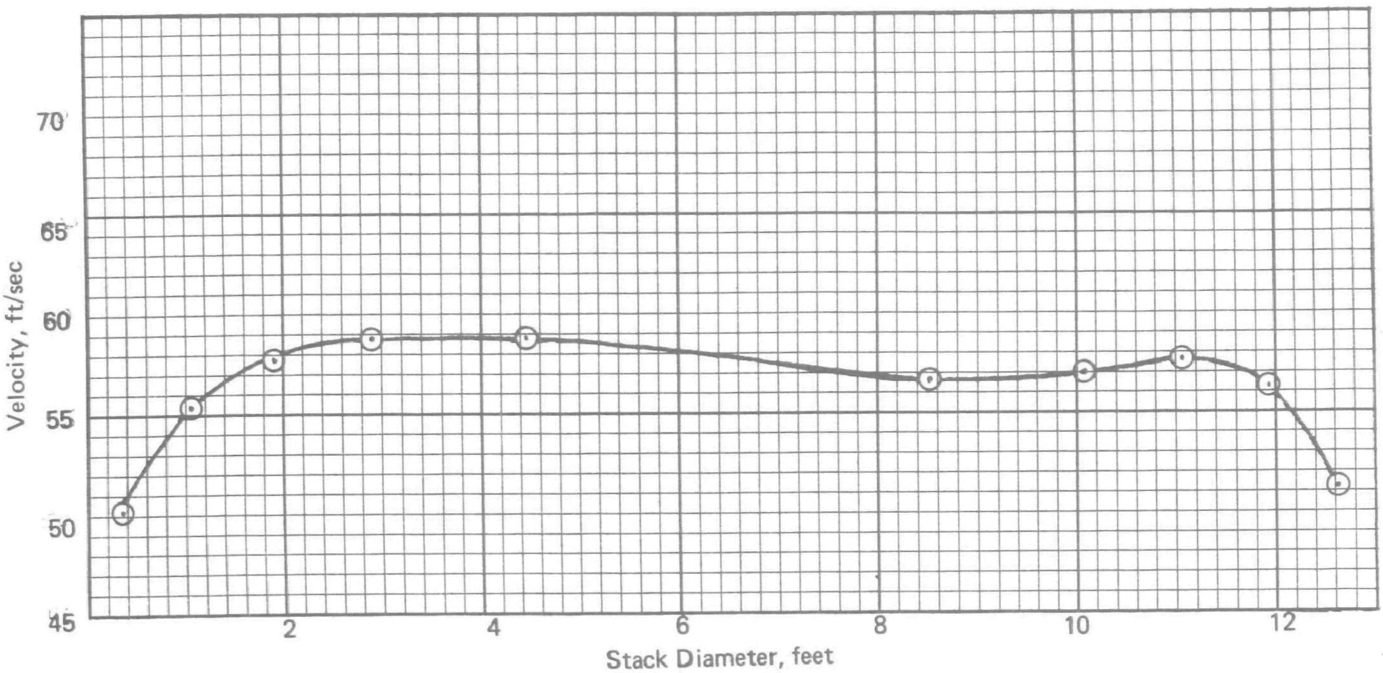
The collaborators for the Lone Star Industries Portland Cement Plant test were Mr. Charles Rodriguez and Mr. Nollie Swynnerton of Southwest Research Institute, San Antonio Laboratory, San Antonio, Texas; Mr. Mike Taylor and Mr. Ron Hawkins of Southwest Research Institute, Houston Laboratory, Houston, Texas; Mr. Quirino Wong, Mr. Randy Creighton, and Mr. Vito Pacheco, Department of Public Health, City of Houston, Houston, Texas; and Mr. Royce Alford, Mr. Ken Drummond, and Mr. Lynn Cochran of Southwestern Laboratories, Austin, Texas.

The collaborators for the Allen King Power Plant test were Mr. Mike Taylor and Mr. Hubert Thompson of Southwest Research Institute, Houston Laboratory, Houston, Texas; Mr. Charles Rodriguez and Mr. Ron Hawkins of Southwest Research Institute, San Antonio Laboratory, San Antonio, Texas; Mr. Gilmore Sem, Mr. Vern Goetsch, and Mr. Jerry Brazelli of Thermo-Systems, Inc, St. Paul, Minn.; and Mr. Roger Johnson and Mr. Harry Patel of Environmental Research Corporation, St. Paul, Minn.

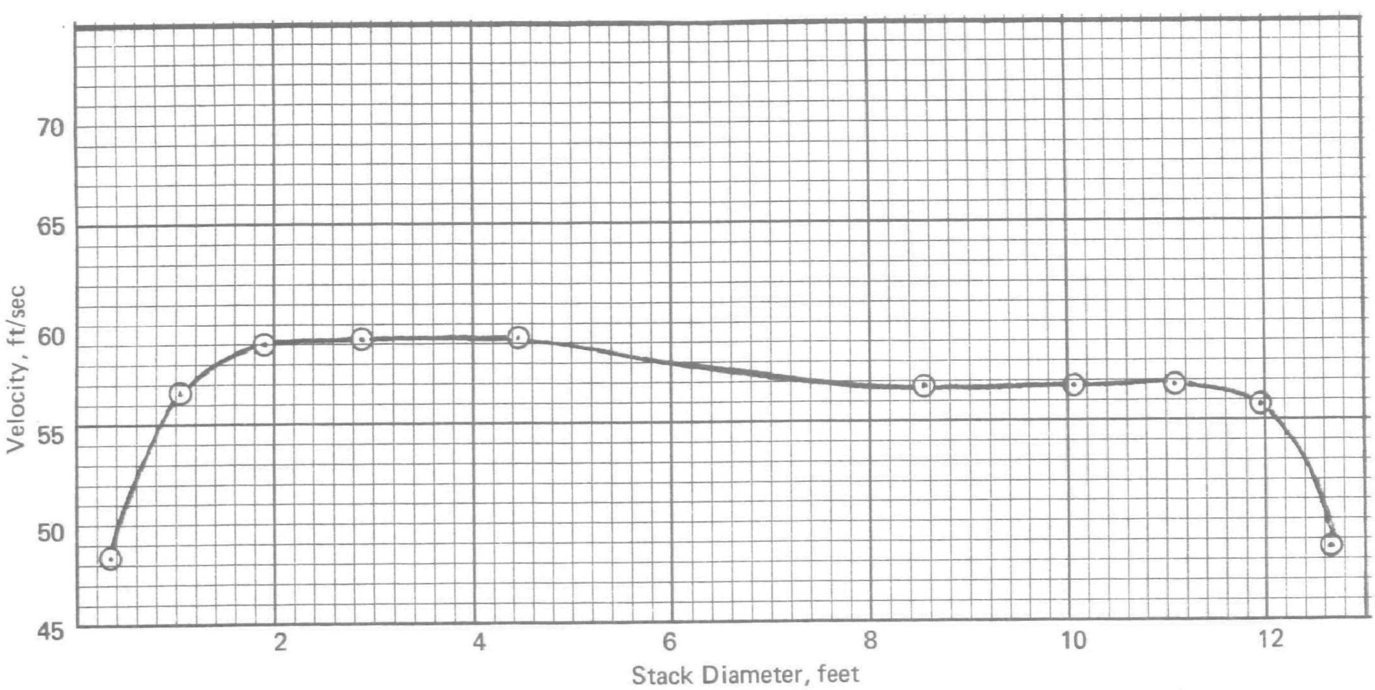
The collaborators for the Holmes Road Incinerator test were Mr. Mike Taylor and Mr. Rick Hohmann of Southwest Research Institute, Houston Laboratory, Houston, Texas; Mr. Charles Rodriguez and Mr. Ron Hawkins of

*EPA policy is to express all measurements in Agency documents in metric units. When implementing this practice will result in undue cost or difficulty in clarity, NERC/RTP is providing conversion factors for the particular non-metric units used in the document. For this report, the factors are:

1 ft = 0.3048 meters	1.0 ft^2 = 0.0929 $meters^2$
1 ft/sec = 0.3048 meters/sec	1 ft^3/hr = 0.0283 $meters^3$/hr

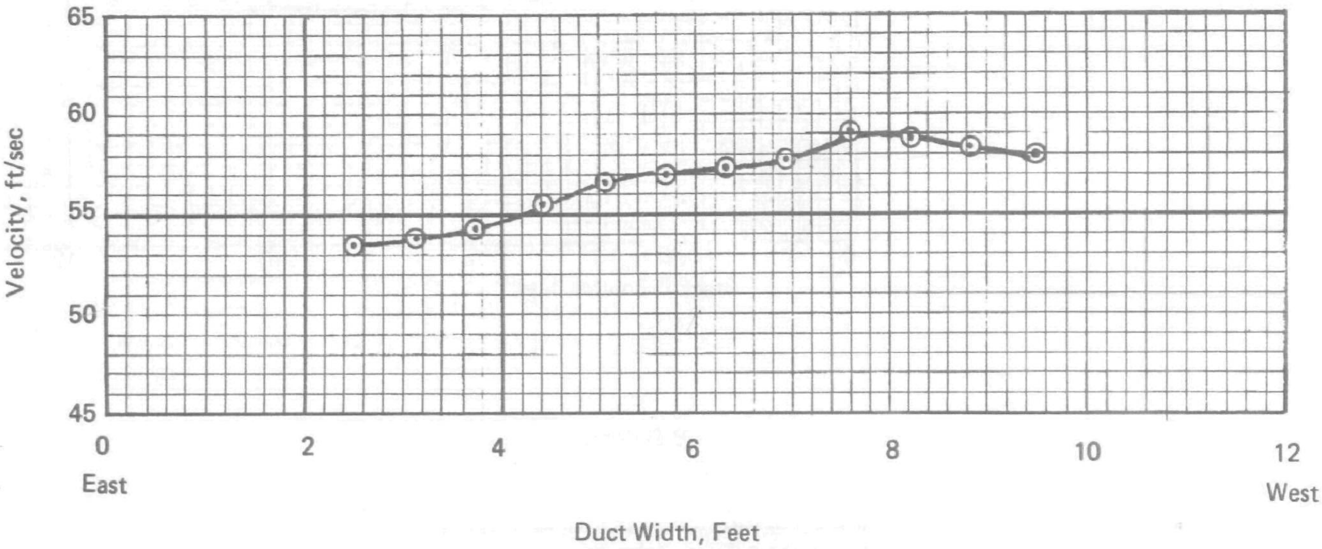


B-D Ports

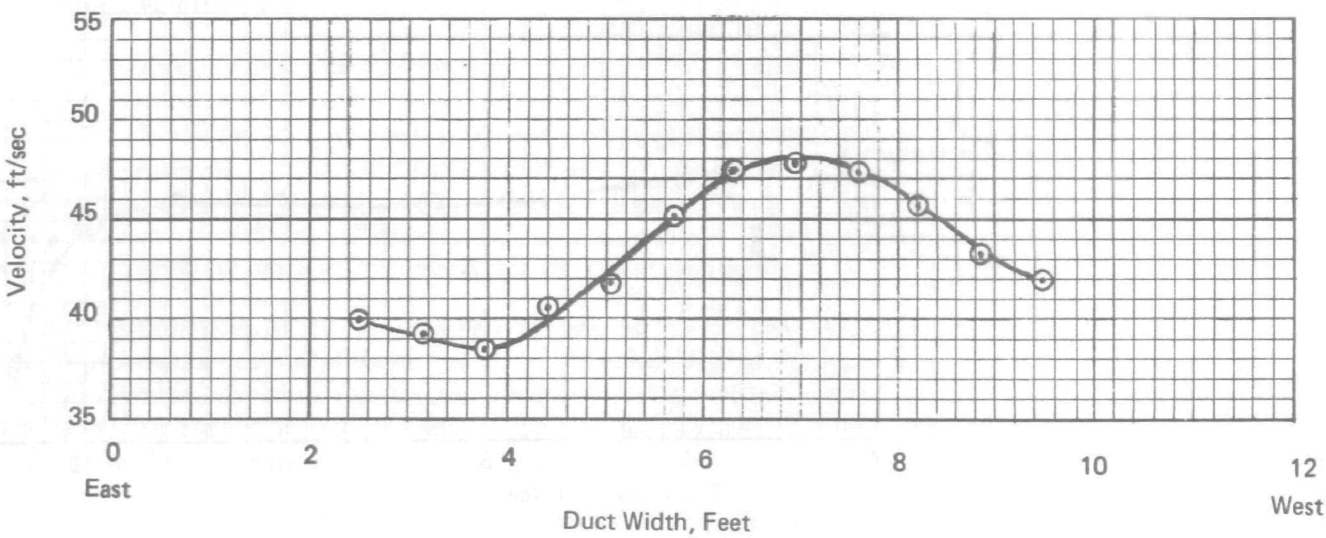


A-C Ports

FIGURE 1. TYPICAL VELOCITY PROFILES,
LONE STAR PORTLAND CEMENT PLANT

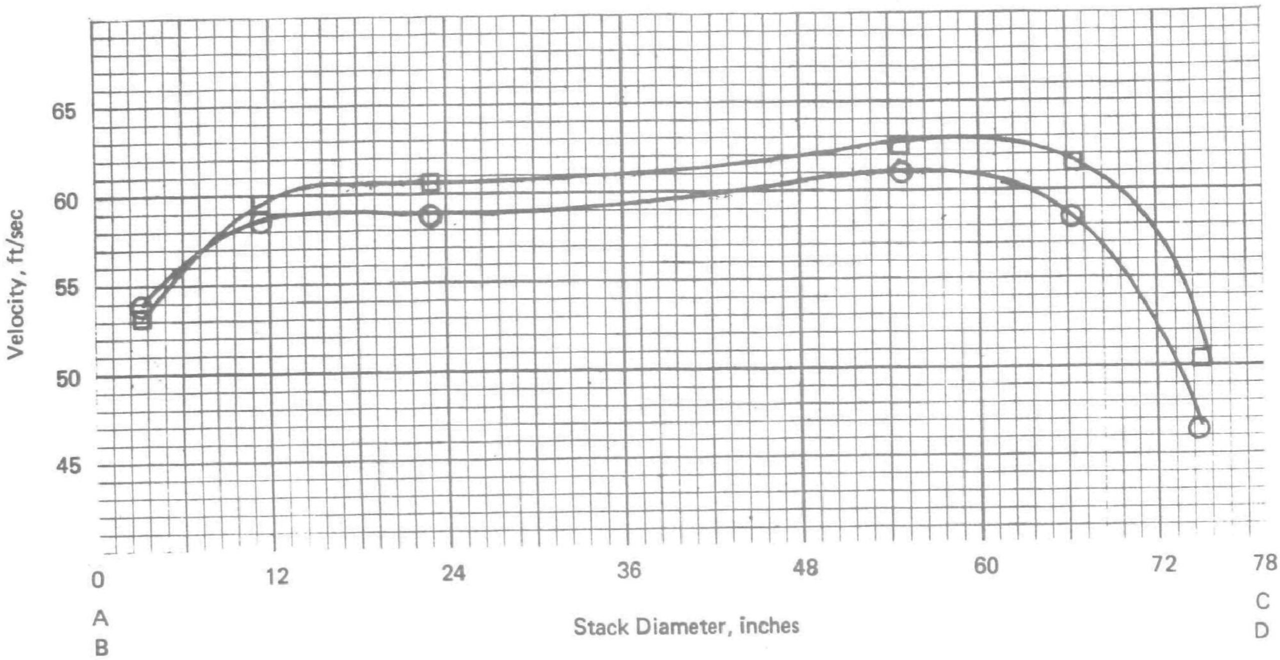


Profile Across Upper Ports

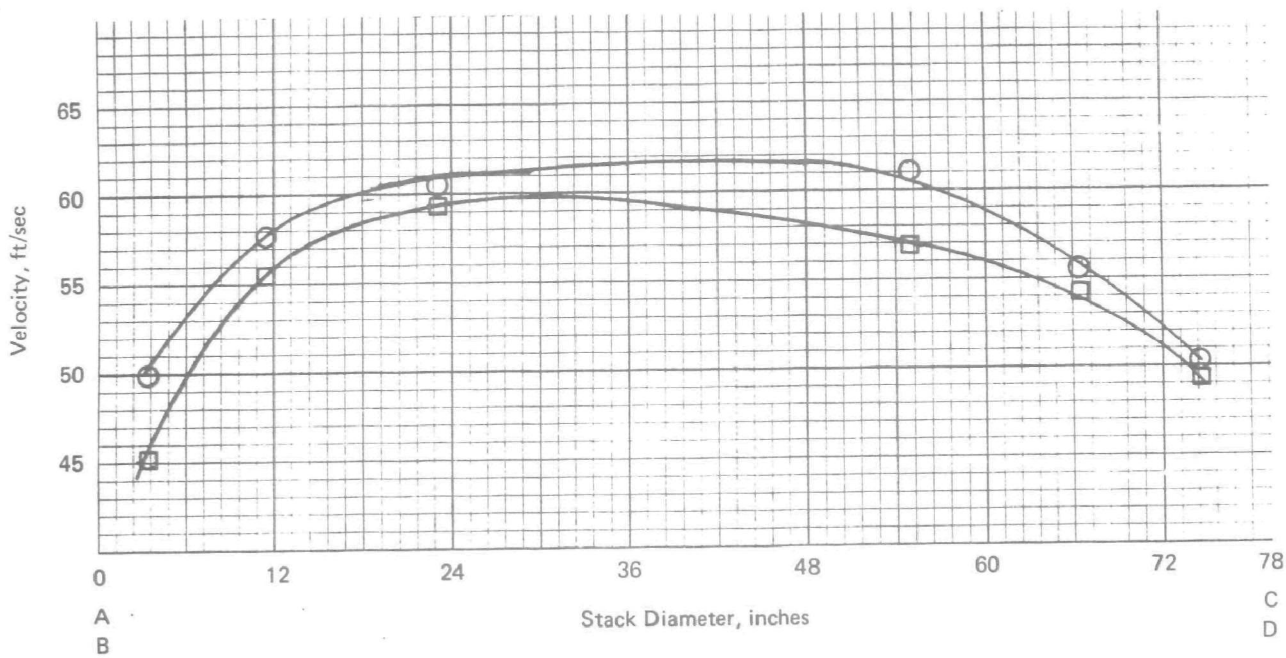


Profile Across Lower Ports

FIGURE 2. TYPICAL VELOCITY PROFILES, ALLEN KING POWER PLANT



Profile, East Stack



Profile, West Stack

□ Axis through ports A, C
○ Axis through ports B, D

FIGURE 3. TYPICAL VELOCITY PROFILES, HOLMES ROAD INCINERATOR

Southwest Research Institute, San Antonio Laboratory, San Antonio, Texas; Mr. Quirino Wong, Mr. Randy Creighton, and Mr. Steve Byrd, City of Houston, Department of Public Health; Mr. John Key, Mr. James Draper, Mr. Tom McMickle, Mr. Tom Palmer, Mr. Michael Lee, and Mr. Charles Goerner, Air Pollution Control Services, Texas State Department of Health.*

The Portland cement plant test was conducted under the supervision of Dr. Henry Hamil, and the power plant and municipal incinerator tests were conducted under the supervision of Mr. Nollie Swynnerton, both of Southwest Research Institute.

Collaborators for all three tests were selected by Dr. Hamil.

*Throughout the remainder of this report, the collaborative laboratories are referred to by randomly assigned code numbers. For the cement plant test, code numbers 101, 102, 103, and 104 are used. For the power plant test, code numbers 201, 202, 203, and 204 are used. For the cement plant test, code numbers 301, 302, 303, and 304 are used. These numbers do not correspond to the above ordered listing of laboratories, and may differ from the code numbers assigned in the previous reports.[2,3,4]

III. STATISTICAL DESIGN AND ANALYSIS

A. Statistical Terminology

To facilitate the understanding of this report and the utilization of its findings, this section explains the statistical terms used in this report. The procedures for obtaining estimates of the pertinent values are developed and justified in the subsequent sections.

We say that an *estimator,* $\hat{\theta}$, *is unbiased for a parameter* θ if the expected value of $\hat{\theta}$ is θ, or expressed in notational form, $E(\hat{\theta}) = \theta$. From a population of method determinations made at the same true level, μ, let $x_1, \ldots, x_n$ be a sample of n replicates. Then we define:

(1) $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ as the *sample mean,* an unbiased estimate of the *true determination mean,* δ, the center of the distribution of the determinations. For an accurate method, δ is equal to μ, the true level.

(2) $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ as the *sample variance,* an unbiased estimate of the *true variance,* σ^2. This term gives a measure of the dispersion in the distribution of the determinations around δ.

(3) $s = \sqrt{s^2}$ as the sample standard deviation, an alternative measure of dispersion, which estimates σ, *the true standard deviation.*

The sample standard deviation, s, however, is not unbiased for σ,[5] so a *correction factor* needs to be applied. The correction factor for a sample of size n is α_n, and the product of α_n and s is unbiased for σ. That is, $E(\alpha_n s) = \sigma$. As n increases, the value of α_n decreases, going for example from $\alpha_3 = 1.1284, \alpha_4 = 1.0854$ to $\alpha_{10} = 1.0281$. The formula for α_n is given in Appendix B.3.

We define

$$\beta = \frac{\sigma}{\delta}$$

as the *true coefficient of variation* for a given distribution. To estimate this parameter, we use a *sample coefficient of variation,* $\hat{\beta}$, defined by

$$\hat{\beta} = \frac{\alpha_n s}{\bar{x}}$$

where $\hat{\beta}$ is the ratio of the unbiased estimates of σ and δ. The coefficient of variation measures the percentage scatter in the observations about the mean and thus is a readily understandable way to express the precision of the observations.

There were a total of 43 sampling *runs* for the three tests. Since the actual velocity, and hence the flow rate, fluctuates, one can in general expect different true levels for each run. To permit a complete statistical analysis, the individual runs are grouped into *blocks,* where each block has approximately the same true level.

We can apply the statistical terms of the preceding paragraphs both to the collaborators' values during a given run and to each collaborator's values in a given block. In this report, statistical results from the first situation are referred to as *run results.* Those from the second situation are referred to as *collaborator-block results.*

For example, a run mean is the average of all the determinations made in a run as obtained by Method 2. A collaborator-block coefficient of variation is the ratio of the unbiased standard deviation to the sample mean for all the collaborator's runs grouped in the block.

The variability associated with a Method 2 determination is estimated in terms of the *within-laboratory* and the *between-laboratory precision components.* In addition, a *laboratory bias component* can be estimated. The following definitions of these terms are given with respect to a *true level,* μ.

- *Within-laboratory*—The within-laboratory standard deviation, σ, measures the *dispersion in replicate single determinations* made using Method 2 by one laboratory team (same field operators, laboratory analyst, and equipment) sampling the same true level, μ. The value of σ is estimated from within each collaborator-block combination.

- *Between-laboratory*—The between-laboratory standard deviation, σ_b, measures the *total variability in a determination* due to simultaneous Method 2 determinations by different laboratories sampling the same true stack level, μ. The between laboratory variance, σ_b^2, may be expressed as

$$\sigma_b^2 = \sigma_L^2 + \sigma^2$$

  and consists of a within-laboratory variance plus a *laboratory bias variance,* σ_L^2. The between-laboratory standard deviation is estimated using the run results.

- *Laboratory bias*—The laboratory bias standard deviation, $\sigma_L = \sqrt{\sigma_b^2 - \sigma^2}$, is that portion of the total variability that can be ascribed to differences in the field operators, analysts and instrumentation, and due to different manners of performance of procedural details left unspecified in the method. This term measures that part of the total variability in a determination which results from the use of the method by different laboratories, as well as from modifications in usage by a single laboratory over a period of time. The laboratory bias standard deviation is estimated from the within- and between-laboratory estimates previously obtained.

B. Test Data

This study is based upon velocities and volumetric flow rates obtained in the use of Method 5. The average velocity, $(V_s)_{avg}$, is calculated as

$$(V_s)_{avg} = K_p C_p (\sqrt{\Delta_p})_{avg} \sqrt{\frac{(T_s)_{avg}}{P_s M_s}} \text{ ft/sec}$$

where

K_p = 85.48 for the units used,

C_p — the pitot tube coefficient

$(\sqrt{\Delta_p})_{avg}$ — the average square root of the velocity head of the stack, inches H_2O

$(T_s)_{avg}$ — the average absolute stack gas temperature, °R

P_s — the absolute stack gas pressure, inches Hg

and

M_s — the molecular weight of the stack gas (wet basis), lb/lb-mole.

The data used in the calculation of the velocities and flow rates were obtained during the sampling runs and not from the preliminary velocity traverses. These, then, represent 2-hour average velocities and flow rates across the stack. The volumetric flow rate, Q_s, is calculated as

$$Q_s = (3600)(1 - B_{wo})V_sA \left(\frac{T_{\text{std}}}{(T_s)_{\text{avg}}}\right)\left(\frac{P_s}{P_{\text{std}}}\right) \text{ft}^3/\text{hr}$$

where A is the cross-sectional area of the stack, and B_{wo} is the volume fraction of water vapor in the gas stream.

In conjunction with the testing of Method 5, the collaborators calculated average stack velocities but not volumetric flow rates, since the concentration determinations in the previous studies were the final results used in the analysis. The velocities were recalculated to ensure their accuracy, and the flow rates calculated using these velocities and the other test data.

The results obtained by Lab 201 were excluded from the analysis. In a study on moisture fraction determination, Lab 201 was eliminated due to the probable development of leakage during some runs and filter contamination due to use of a low-melting ground-joint lubricant. Since this would adversely affect the volume of liquid collected due to the introduction of ambient air into the train, their moisture fractions were not usable. Since the moisture fraction is involved directly in the Q_s determination and indirectly, through M_s, in the $(V_s)_{\text{avg}}$ determination, these data were not judged acceptable.

C. Test Design and Analysis

The data were arranged in blocks where the true velocity was assumed to be essentially constant. The velocity determination has been shown[6] to be principally dependent upon the value of $(\sqrt{\Delta_p})_{\text{avg}}$. Thus, this provides a valid means of determining when there was a change in the stack gas velocity. The actual reading of the velocity head is a function of the particular pitot tube that is used, but by comparing the values of all collaborators, the increases and decreases in velocities can be determined. The average $\sqrt{\Delta_p}$'s are shown in Table 1, along with the blocks to which the runs were assigned. The determinations used in the analyses are shown arranged in blocks in Table 2 and 3.

To determine the best method of analyzing the data, Bartlett's test for homogeneity of variance was used to determine the appropriateness of an analysis of variance approach. The data were then transformed using the logarithmic transformation and retested by Bartlett's test. The details are contained in Appendices B.1 and B.2.

For the velocity data, significance levels under both transformations indicate suitability. If a logarithmic transformation is accepted, the conclusion is that there is a proportional relationship between the true mean and true standard deviation. If a linear transformation is accepted, then the indication is that the variance is independent of the mean. An investigation of the proportional relationship was conducted on an empirical basis to determine which of the two models should be used.

The correlation between the sample means and standard deviations is determined for both the run data and the collaborator-block data. The model chosen is a no-intercept model, meaning that when the sample mean is zero, the sample standard deviation must also be zero. The coefficient of determination, r^2, is the measure of the appropriateness of the model. For the run data, the value of r^2 was 0.80, which gives a correlation coefficient, $r = \sqrt{r^2}$, of 0.89 based on 43 pairs. This value is significant at the 5 percent level. For the collaborator-block data, the value of r^2 was 0.75, and the correlation coefficient, r, was 0.86, based on 37 pairs. This value also exceeds the value for the 5 percent significance level.

Thus, there is evidence of a proportional relationship between the mean and the standard deviation for the velocity data. This is equivalent to saying that the standard deviations, σ_b and σ, change as the mean, δ, changes. That is,

TABLE 1. AVERAGE $\sqrt{\Delta p}$'s AND BLOCK DESIGNATIONS.

Run	Labs				Block
Site 1					
	101	102	103	104	
1	0.83	0.82	–*	0.83	1
2	0.83	0.83	0.91	0.83	1
3	0.83	0.84	1.02	0.83	1
4	0.75	0.75	0.77	0.79	2
5	0.75	0.78	0.76	0.75	2
6	0.78	0.75	0.76	0.79	2
7	0.68	0.69	0.69	0.74	4
8	0.72	0.76	0.73	0.72	3
9	0.74	0.74	0.73	0.77	2
10	0.74	0.73	0.74	0.73	3
11	0.72	0.72	0.72	0.75	3
12	0.73	0.76	0.73	0.76	3
13	0.72	0.73	0.71	0.75	3
14	–*	0.69	0.67	0.70	4
15	0.67	0.70	0.69	0.70	4
Site 2					
	202	203	204		
1	0.75	0.80	0.76		1
2	0.74	0.75	0.76		2
3	0.70	0.79	0.77		1
4	0.78	0.80	0.78		1
5	0.73	0.75	0.73		2
6	0.74	0.79	0.77		1
7	0.69	0.76	0.74		2
8	0.79	0.80	0.76		1
9	0.68	0.71	0.74		3
10	0.73	0.74	0.77		2
11	0.72	0.75	0.75		2
12	0.71	0.73	0.67		3
13	0.76	0.75	0.75		2
14	0.76	–*	0.74		2
15	0.71	0.73	0.75		2
16	0.75	0.80	0.72		2
Site 3					
	301	302	303	304	
1	0.79	0.81	0.78	–*	3
2	0.77	0.91	0.73	0.88	2
3	0.78	1.00	0.79	0.78	1
4	0.78	0.73	0.73	0.86	3
5	0.83	0.86	0.78	0.83	2
6	0.82	0.90	0.74	0.81	2
7	0.79	0.86	0.75	0.84	3
8	0.83	0.89	0.70	0.74	3
9	0.68	0.75	0.67	0.72	4
10	0.84	0.84	0.78	0.87	1
11	0.77	0.83	0.77	0.84	3
12	0.82	0.87	0.76	0.94	1

*Run not made due to equipment failure.

TABLE 2. STACK GAS VELOCITY DATA, ARRANGED BY BLOCK.

Velocity, ft/sec

Block	Run	Labs			
Site 1					
		101	102	103	104
1	1	62.4	62.3	–*	58.2
	2	61.1	63.2	59.0	60.7
	3	61.7	63.1	66.1	63.3
2	4	56.5	56.5	57.3	60.1
	5	55.9	60.3	56.3	57.6
	6	57.6	56.0	56.4	60.4
	9	51.9	52.8	52.3	55.6
3	8	49.5	53.9	51.7	51.8
	10	52.6	53.4	53.0	53.8
	11	51.2	54.1	52.4	55.9
	12	51.9	55.3	53.3	57.0
	13	51.7	53.2	51.9	55.3
4	7	46.9	48.7	49.0	51.9
	14	–*	52.0	48.6	52.9
	15	52.0	51.5	49.9	52.2
Site 2					
		202	203	204	
1	1	51.2	52.3	47.6	
	3	47.3	51.8	51.3	
	4	53.1	52.5	49.6	
	6	50.2	52.0	49.3	
	8	53.7	51.3	47.5	
2	2	50.1	48.7	49.9	
	5	49.8	49.3	48.1	
	7	46.5	48.8	45.4	
	10	48.8	48.3	44.8	
	11	48.1	46.6	48.8	
	13	51.9	47.6	48.8	
	14	51.3	–*	49.8	
	15	48.2	46.8	48.6	
	16	50.6	51.7	47.0	
3	9	46.0	45.0	49.5	
	12	48.6	46.5	43.0	
Site 3					
		301	302	303	304
1	3	46.9	52.3	57.9	47.7
	10	50.9	51.8	53.5	53.0
	12	50.5	50.2	55.5	57.6
2	2	48.0	48.2	58.4	54.2
	5	51.7	51.6	55.6	53.2
	6	51.1	48.9	57.5	50.2
3	1	48.6	51.0	55.9	–*
	4	48.0	47.9	56.7	54.3
	7	48.8	49.6	55.3	52.0
	8	51.6	46.1	56.5	46.0
	11	47.4	51.0	53.4	51.5
4	9	41.7	44.8	48.7	48.5

*Run not made due to equipment failure.

TABLE 3. VOLUMETRIC FLOW RATE DATA, ARRANGED BY BLOCK

Volumetric Flow Rate, $ft^3/hr \times 10^{-4}$

Block	Run	Labs			
		Site 1			
		101	102	103	104
1	1	1267.6	1176.5	–*	1254.2
	2	1244.2	1243.7	1384.0	1251.4
	3	1238.5	1220.3	1496.7	1410.4
	4	1101.3	1141.6	1250.2	1274.7
	5	1123.3	1065.6	1238.4	1158.1
	6	1183.7	1110.5	1248.5	1220.9
	9	1257.4	1256.7	1403.8	1330.8
	8	1243.9	1335.0	1337.1	1320.1
	10	1240.1	1245.9	1341.2	1255.2
	11	1241.7	1167.2	1255.8	1198.7
	12	1208.4	1198.1	1257.8	1192.6
	13	1184.5	1160.7	1266.2	1240.5
	7	1078.2	1261.9	1270.0	1431.5
	14	–*	1086.1	1152.5	1114.7
	15	1136.8	1118.8	1205.0	1135.8
		Site 2			
		202	203	204	
1	1	7170.8	7145.4	6947.2	
	3	6617.2	7110.8	7240.3	
	4	7324.7	7162.2	6855.4	
	6	6995.2	7011.5	6880.2	
	8	7445.7	7199.8	6683.3	
2	2	6895.3	6606.4	6958.7	
	5	6935.4	6741.9	6823.1	
	7	6555.7	6850.2	6551.8	
	10	6835.6	6861.3	6364.2	
	11	7364.3	6534.0	6839.9	
	13	7173.4	6887.8	7015.9	
	14	7119.1	–*	6928.9	
	15	6694.1	6586.4	6830.0	
	16	7093.1	7177.6	6602.6	
3	9	5819.7	6582.5	7101.8	
	12	6835.7	6730.8	6107.2	
		Site 3			
		301	302	303	304
1	3	335.8	371.7	333.5	311.8
	10	354.1	369.1	317.3	353.9
	12	319.8	367.6	342.2	374.5
	2	311.3	355.5	334.6	345.1
	5	261.3	350.9	310.5	284.5
	6	335.4	349.5	325.6	320.2
	1	309.3	368.3	357.1	–*
	4	301.4	329.5	316.6	317.8
	7	308.4	339.4	325.4	326.2
	8	323.8	323.4	352.0	281.1
	11	312.6	348.7	303.7	329.0
	9	273.6	311.7	276.7	289.5

*Run not made due to equipment failure.

$$\sigma_b = \beta_b \delta$$

and

$$\sigma = \beta\delta$$

where β_b and β are the true coefficients of variation for between-laboratory and within-laboratory, respectively. The standard deviations are estimated, then, as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta}\delta$$

where $\hat{\beta}_b$ and $\hat{\beta}$ are the estimated coefficients of variation.

For the volumetric flow rates obtained, a similar investigation is done. For these values, the only acceptable transformation is the logarithmic, which implies, on a theoretical basis, an underlying proportional relationship between the population mean and the population standard deviation for both the run data and the collaborator-block data.

To establish this empirically, the paired sample means and standard deviations are fit to a no-intercept regression model. The run data give an r^2 of 0.73 and a correlation coefficient of 0.85, based on 43 pairs. The collaborator-block r^2 is 0.63, with $r = 0.79$ based on 37 pairs. Both r values are significant at the 5 percent significance level.

As a result, the volumetric flow rate within-laboratory and between-laboratory standard deviations can be said to be proportional to the mean level. The estimates of these standard deviations will be expressed using coefficients of variation times an unknown mean in the same manner as the velocity data.

At each site, there were occasional missing values due to equipment malfunctions and varying block sizes, so that not all coefficients of variation are based on the same number of observations. To account for this, for each site the individual beta estimates are weighted so that a greater contribution to the final estimate is made by those values based on larger samples. The weighting technique is based upon the number of values in each run or block and is discussed in detail in Appendix B.4. The beta values from all three sites form a composite estimate of the coefficients of variation for both velocity and flow rate.

IV. VELOCITY DETERMINATION PRECISION ESTIMATES

The between-laboratory standard deviation, σ_b, and the within-laboratory standard deviation, σ, for (V_s) avg are estimated as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta}\delta.$$

In Appendix B.5, the data from the three sites are used to obtain estimates of these terms using a linear combination of the individual values.

The between-laboratory coefficient of variation is $\hat{\beta}_b = (0.050)$. This gives a between-laboratory standard deviation of

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

$$= (0.050)\delta$$

or 5.0% of the mean. This estimate has 8 degrees of freedom associated with it.

The within-laboratory coefficient of variation is estimated as $\hat{\beta} = (0.039)$. This gives an estimated within-laboratory standard deviation of

$$\hat{\sigma} = \hat{\beta}\delta$$

$$= (0.039)\delta$$

or 3.9% of the mean. There are 113 degrees of freedom associated with this term.

From the formula in Section IIIA, the laboratory bias standard deviation, σ_L, is given by

$$\sigma_L = \sqrt{\sigma_b^2 - \sigma^2}.$$

Substituting the estimates above into this formula gives

$$\hat{\sigma}_L = \sqrt{\hat{\sigma}_b^2 - \hat{\sigma}^2}$$

$$= \sqrt{(0.050)^2 \delta^2 - (0.039)^2 \delta^2}$$

$$= \sqrt{[(0.050)^2 - (0.039)^2]\,\delta^2}$$

$$= \sqrt{(0.001)\delta^2}$$

$$= (0.032)\delta$$

or 3.2% of the mean level.

V. VOLUMETRIC FLOW RATE PRECISION ESTIMATES

The between-laboratory standard deviation, σ_b, and the within-laboratory standard deviation, δ, for Q_s are estimated as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta}\delta$$

where $\hat{\beta}_b$ and $\hat{\beta}$ are the estimated coefficients of variation. In Appendix B.6 the individual beta estimates are combined to obtain estimates of these from the run data and collaborator-block data, respectively.

The estimated between-laboratory coefficient of variation is $\hat{\beta}_b = (0.056)$. This gives an estimated between-laboratory standard deviation of

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$
$$= (0.056)\delta$$

or 5.6% of the mean. This estimate has 8 degrees of freedom associated with it.

The within-laboratory coefficient of variation is estimated by $\hat{\beta} = (0.055)$. This gives an estimated within-laboratory standard deviation of

$$\hat{\sigma} = \hat{\beta}\delta$$
$$= (0.055)\delta$$

or 5.5% of the mean. There are 113 degrees of freedom associated with this estimate.

The laboratory bias standard deviation is defined as

$$\sigma_L = \sqrt{\sigma_b^2 - \sigma^2}.$$

Substituting $\hat{\sigma}_b$ and $\hat{\sigma}$ into this formula gives

$$\hat{\sigma}_L = \sqrt{\hat{\sigma}_b^2 - \hat{\sigma}^2}$$
$$= \sqrt{(0.056)^2\delta^2 - (0.055)^2\delta^2}$$
$$= \sqrt{[(0.056)^2 - (0.055)^2]\delta^2}$$
$$= (0.011)\delta$$

or 1.1% of the mean level.

VI. EMISSION RATE VARIATION

The standards of performance[1] for certain sites (e.g., power plants, nitric acid plants, Portland cement plants) specify that the product of the volumetric flow rate and the emission concentration obtained by the appropriate method be used in determining compliance with the regulations. The rate is denoted in this study by r, where

$$r = Q_s \cdot c$$

It is of interest to determine the precision of this product based upon the precision of the individual components.

In Appendix B.7, the formula is developed for estimating a precision component for this product when both the flow rate and the concentration determination follow the coefficient of variation hypothesis. The formulas for the within-laboratory and between-laboratory variances are given by

$$\sigma_b^2 = [\beta_b^2(Q_s)\beta_b^2(c) + \beta_b^2(c) + \beta_b^2(Q_s)]\,\delta_r^2$$

and

$$\sigma^2 = [\beta^2(Q_s)\beta^2(c) + \beta^2(c) + \beta^2(Q_s)]\,\delta_r^2$$

where $\beta_b(Q_s)$ and $\beta(Q_s)$ are the between and within-laboratory coefficients of variation for flow rate, $\beta_b(c)$ and $\beta(c)$ are the coefficients of variation for emission concentrations, and δ_r is the mean emission rate.

TABLE 4. PRECISION ESTIMATES FOR EMISSION CONCENTRATIONS

Method	$\hat{\beta}(c)$	$\hat{\beta}_b(c)$
5[4]	0.253	0.387
6[7]	0.040	0.058
7[8]	0.066	0.095

In Table 4 are listed values of $\hat{\beta}_b(c)$ and $\hat{\beta}(c)$ for Methods 5, 6 and 7 based upon previous collaborative studies. Using these and the coefficients of variation for Q_s developed in this study, estimates can be made of the precision associated with r.

Method 5—The between-laboratory standard deviation estimate is 39.2% of δ_r, and the within-laboratory standard deviation estimate is 25.9% of δ_r. This gives a laboratory bias term of 29.4% of δ_r.

Method 6—The estimated between-laboratory standard deviation is 8.1% of δ_r, with an estimated within-laboratory standard deviation of 6.8% of δ_r. From these, the laboratory bias standard deviation is estimated as 4.5% of δ_r.

Method 7—The estimated between-laboratory standard deviation is 11.0% of δ_r. The estimated within-laboratory standard deviation is 8.6% of δ_r. Using these, the laboratory bias standard deviation is estimated as 7.1% of δ_r.

As can be seen from these results, the precision in r depends primarily upon the precision of the emission concentration determination, and little variation is introduced by the volumetric flow rate determination.

APPENDIX A

METHOD 2. DETERMINATION OF STACK GAS VELOCITY AND VOLUMETRIC FLOW RATE (TYPE S PITOT TUBE)

METHOD 2—DETERMINATION OF STACK GAS VELOCITY AND VOLUMETRIC FLOW RATE (TYPE S PITOT TUBE)

1. *Principle and applicability.*

1.1 Principle. Stack gas velocity is determined from the gas density and from measurement of the velocity head using a Type S (Stauscheibe or reverse type) pitot tube.

1.2 Applicability. This method should be applied only when specified by the test procedures for determining compliance with the New Source Performance Standards.

2. *Apparatus.*

2.1 Pitot tube—Type S (Figure 2-1), or equivalent, with a coefficient within ±5% over the working range.

2.2 Differential pressure gauge—Inclined manometer, or equivalent, to measure velocity head to within 10% of the minimum value.

2.3 Temperature gauge—Thermocouple or equivalent attached to the pitot tube to measure stack temperature to within 1.5% of the minimum absolute stack temperature.

2.4 Pressure gauge—Mercury-filled U-tube manometer, or equivalent, to measure stack pressure to within 0.1 in. Hg.

2.5 Barometer—To measure atmospheric pressure to within 0.1 in. Hg.

2.6 Gas analyzer—To analyze gas composition for determining molecular weight.

2.7 Pitot tube—Standard type, to calibrate Type S pitot tube.

3. *Procedure.*

3.1 Set up the apparatus as shown in Figure 2-1. Make sure all connections are tight and leak free. Measure the velocity head and temperature at the traverse points specified by Method 1.

3.2 Measure the static pressure in the stack.

3.3 Determine the stack gas molecular weight by gas analysis and appropriate calculations as indicated in Method 3.

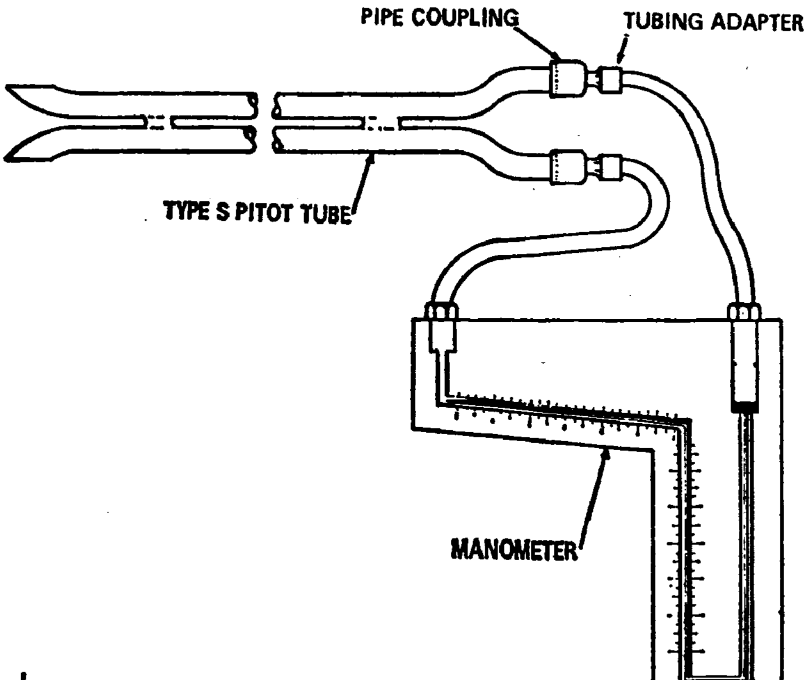


Figure 2-1. Pitot tube-manometer assembly.

4. *Calibration.*

4.1 To calibrate the pitot tube, measure the velocity head at some point in a flowing gas stream with both a Type S pitot tube and a standard type pitot tube with known coefficient. Calibration should be done in the laboratory and the velocity of the flowing gas stream should be varied over the normal working range. It is recommended that the calibration be repeated after use at each field site.

4.2 Calculate the pitot tube coefficient using equation 2-1.

$$C_{p_{test}} = C_{p_{std}} \sqrt{\frac{\Delta p_{std}}{\Delta p_{test}}} \quad \text{equation 2-1}$$

where:

$C_{p_{test}}$ = Pitot tube coefficient of Type S pitot tube.

$C_{p_{std}}$ = Pitot tube coefficient of standard type pitot tube (if unknown, use 0.99).

Δp_{std} = Velocity head measured by standard type pitot tube.

Δp_{test} = Velocity head measured by Type S pitot tube.

4.3 Compare the coefficients of the Type S pitot tube determined first with one leg and then the other pointed downstream. Use the pitot tube only if the two coefficients differ by no more than 0.01.

5. *Calculations.*

Use equation 2-2 to calculate the stack gas velocity.

$$(V_s)_{avg.} = K_p C_p (\sqrt{\Delta p})_{avg.} \sqrt{\frac{(T_s)_{avg.}}{P_s M_s}} \quad \text{Equation 2-2}$$

where:

$(V_s)_{avg.}$ = Stack gas velocity, feet per second (f.p.s.).

$K_p = 85.48 \frac{ft.}{sec.} \left(\frac{lb.}{lb.\ mole-°R}\right)^{1/2}$ when these units are used.

C_p = Pitot tube coefficient, dimensionless.

$(T_s)_{avg.}$ = Average absolute stack gas temperature, °R.

$(\sqrt{\Delta p})_{avg.}$ = Average velocity head of stack gas, inches H_2O (see Fig. 2-2).

P_s = Absolute stack gas pressure, inches Hg.

M_s = Molecular weight of stack gas (wet basis), lb./lb.-mole.

$M_d(1-B_{wo})+18B_{wo}$

M_d = Dry molecular weight of stack gas (from Method 3).

B_{wo} = Proportion by volume of water vapor in the gas stream (from Method 4).

Figure 2-2 shows a sample recording sheet for velocity traverse data. Use the averages in the last two columns of Figure 2-2 to determine the average stack gas velocity from Equation 2-2.

Use Equation 2-3 to calculate the stack gas volumetric flow rate.

$$Q_s = 3600\ (1-B_{wo}) V_s A \left(\frac{T_{std}}{(T_s)_{avg.}}\right) \left(\frac{P_s}{P_{std}}\right) \quad \text{Equation 2-3}$$

where:

Q_s = Volumetric flow rate, dry basis, standard conditions, ft.³/hr.

A = Cross-sectional area of stack, ft.²

T_{std} = Absolute temperature at standard conditions, 530° R.

P_{std} = Absolute pressure at standard conditions, 29.92 inches Hg.

6. *References.*

Mark, L. S., Mechanical Engineers' Handbook, McGraw-Hill Book Co., Inc., New York, N.Y., 1951.

Perry, J. H., Chemical Engineers' Handbook, McGraw-Hill Book Co., Inc., New York, N.Y., 1960.

Shigehara, R. T., W. F. Todd, and W. S. Smith, Significance of Errors in Stack Sampling Measurements. Paper presented at the Annual Meeting of the Air Pollution Control Association, St. Louis, Mo., June 14–19, 1970.

Standard Method for Sampling Stacks for Particulate Matter, In: 1971 Book of ASTM Standards, Part 23, Philadelphia, Pa., 1971, ASTM Designation D-2928-71.

Vennard, J. K., Elementary Fluid Mechanics, John Wiley & Sons, Inc., New York, N.Y., 1947.

PLANT____________________

DATE____________________

RUN NO.____________________

STACK DIAMETER, in.____________________

BAROMETRIC PRESSURE, in. Hg.____________________

STATIC PRESSURE IN STACK (P_g), in. Hg.__________

OPERATORS____________________

SCHEMATIC OF STACK CROSS SECTION

Traverse point number	Velocity head, in. H_2O	$\sqrt{\Delta_p}$	Stack Temperature (T_S), °F
	AVERAGE:		

Figure 2-2. Velocity traverse data.

APPENDIX B

STATISTICAL METHODS

APPENDIX B. STATISTICAL METHODS

This appendix consists of various sections which contain detailed statistical procedures carried out in the analyses of the Method 2 data. Reference to these sections has been made at various junctures in the body of this report. Each Appendix B section is an independent ad hoc statistical analysis pertinent to a particular problem addressed in the body of the report.

B.1 Proportional Relationship Between Mean and Standard Deviation in the Velocity Determinations

The velocities shown in Table 2 are tested to determine if the variance is independent of the mean level in their original form (linear) and after having undergone a logarithmic transformation. Bartlett's test for homogeneity of variance[9] is used to determine the suitability of each transformation. The obtained values of the statistic with degrees of freedom and significance levels are shown in Table B-1. The significance levels are obtained from a chi-square distribution with the degrees of freedom shown.

TABLE B-1. VELOCITY TRANSFORMATION RESULTS

Data	Transformation	Test Statistic	DF	Significance Level
Run	Linear	44.391	42	0.371
	Logarithmic	46.219	42	0.302
Collaborator-Block	Linear	47.932	36	0.088
	Logarithmic	48.084	36	0.086

For both the run data and the collaborator-block data, either form is acceptable. The acceptance of the linear form of the data implies that the variance is independent of the mean, that is, constant regardless of the mean value. The acceptance of the logarithmic transformation implies a proportionality between the population mean and the population standard deviation, or that as the mean level rises (falls), the standard deviation rises (falls) in a proportional manner.

Both transformations are acceptable at nearly equal significance levels. To determine if there is further evidence of a proportional relationship between the mean and standard deviation, a regression model is fit to the data. The model chosen is a no-intercept model,

$$y = bx$$

so that a sample mean of zero implies a sample standard deviation of zero. Define

x_{ijk} as the determination by collaborator i on run k in block j.

$\bar{x}_{\cdot jk} = \frac{1}{p} \sum_{i=1}^{p} x_{ijk}$ as the mean of run k in block j for p collaborators

and

$s_{jk} = \sqrt{\frac{1}{p-1} \sum_{i=1}^{p} (x_{ijk} - \bar{x}_{\cdot jk})^2}$ as the run standard deviation.

The paired means and standard deviations, $(\bar{x}_{\cdot jk}, s_{jk})$, shown in Table B-2 are fit to the model, and the degree of fit determined by the coefficient of determination, r^2. For this model, r^2 is calculated as[10]

$$r^2 = \frac{[\Sigma x_i y_i]^2}{\Sigma x_i^2 \, \Sigma y_i^2} = \frac{[\Sigma \bar{x}_{\cdot jk} s_{jk}]^2}{\Sigma \bar{x}_{\cdot jk}^2 \, \Sigma s_{jk}^2}$$

TABLE B-2. RUN MEANS AND STANDARD DEVIATIONS (Velocity, ft/sec)

Run	Mean Velocity	Standard Deviation
	Site 1	
1	61.0	2.4
2	61.0	1.7
3	63.5	1.8
4	57.6	1.7
5	57.5	2.0
6	57.6	2.0
9	53.1	1.7
8	51.7	1.8
10	53.2	0.5
11	53.4	2.0
12	54.4	2.2
13	53.0	1.7
7	49.1	2.1
14	51.2	2.3
15	51.4	1.0
	Site 2	
1	50.4	2.5
3	50.1	2.5
4	51.7	1.9
6	50.5	1.4
8	50.8	3.1
2	49.6	0.8
5	49.1	0.9
7	46.9	1.7
10	47.3	2.2
11	47.8	1.1
13	49.4	2.2
14	50.5	1.1
15	47.9	0.9
16	49.8	2.5
9	46.8	2.4
12	46.0	2.8
	Site 3	
3	51.2	5.1
10	52.3	1.2
12	53.4	3.7
2	52.2	5.0
5	53.0	1.9
6	51.9	3.8
1	51.8	3.7
4	51.7	4.5
7	51.4	2.9
8	50.0	5.0
11	50.8	2.5
9	45.9	3.3

For the run data, $r^2 = 0.80$, which indicates that 80 percent of the variation in the magnitude of the standard deviation is attributed to variation in the magnitude of the mean. The correlation coefficient, $r = \sqrt{r^2}$, is 0.89 based on 43 pairs of observations, which is significant at the 5 percent level.

For the collaborator-block data, we define

$\bar{x}_{ij.} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} x_{ijk}$ as the mean of collaborator-block ij, for n_{ij} determinations.

and

$s_{ij} = \sqrt{\frac{1}{n_{ij}-1} \sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij.})^2}$ as the collaborator-block standard deviation.

The values obtained are shown in Table B-3. Fitting these to a no-intercept model, we have a coefficient of determination of $r^2 = 0.75$ and a correlation coefficient of 0.86. This value is also significant at the 0.05 level, based upon 37 pairs of observations.

Thus, we have that on a theoretical basis, from the acceptability of the logarithmic transformation, and an empirical basis, from the regression model, there is strong evidence that a proportional relationship exists between the mean and standard deviation for the velocity data. This is equivalent to saying that the coefficients of variation for both between- and within-laboratory components remain constant. This gives the equations

$$\sigma_b = \beta_b \delta$$

and

$$\sigma = \beta \delta.$$

Then we estimate the standard deviations by estimating the coefficients of variation and defining new estimators $\hat{\sigma}_b$ and $\hat{\sigma}$,

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta} \delta$$

where $\hat{\beta}_b$ and $\hat{\beta}$ are the estimated coefficients of variation for between-laboratory and within-laboratory, respectively. Thus, the standard deviations are estimated as percentages of an unknown mean, δ.

B.2 Proportional Relationship Between Mean and Standard Deviation in the Flow Rate Determination

The calculated volumetric flow rates in Table 3 are tested for equality of variance in two forms: their original form (linear) and after having been passed through a logarithmic transformation. Bartlett's test[9] for homogeneity of variance is used to determine the adequacy of each transformation, and the test statistic is compared to a chi-square

TABLE B-3. COLLABORATOR-BLOCK MEANS AND STANDARD DEVIATIONS
(Velocity, ft/sec)

Block	Collaborator	Mean Velocity	Standard Deviation
Site 1			
1	Lab 101	61.73	0.65
	Lab 102	62.87	0.49
	Lab 103	62.55	5.02
	Lab 104	60.73	2.55
2	Lab 101	55.47	2.49
	Lab 102	56.40	3.07
	Lab 103	55.57	2.23
	Lab 104	58.42	2.26
3	Lab 101	51.38	1.16
	Lab 102	53.98	0.82
	Lab 103	52.46	0.69
	Lab 104	54.76	2.02
4	Lab 101	49.45	3.61
	Lab 102	50.73	1.78
	Lab 103	49.17	0.67
	Lab 104	52.33	0.51
Site 2			
1	Lab 202	51.10	2.55
	Lab 203	51.98	0.47
	Lab 204	49.06	1.58
2	Lab 202	49.48	1.72
	Lab 203	48.47	1.62
	Lab 204	47.91	1.82
3	Lab 202	47.30	1.84
	Lab 203	45.75	1.06
	Lab 204	46.25	4.60
Site 3			
1	Lab 301	49.43	2.20
	Lab 302	51.43	1.10
	Lab 303	55.63	2.20
	Lab 304	52.77	4.95
2	Lab 301	50.27	1.99
	Lab 302	49.57	1.80
	Lab 303	57.17	1.43
	Lab 304	52.53	2.08
3	Lab 301	48.88	1.62
	Lab 302	49.12	2.12
	Lab 303	55.56	1.33
	Lab 304	50.95	3.52
4*	Lab 301	41.70	–
	Lab 302	44.80	–
	Lab 303	48.70	–
	Lab 304	48.50	–

*No standard deviations since block contains only one run.

TABLE B-4. FLOW RATE TRANSFORMATION RESULTS

Data	Transformation	Test Statistic	DF	Significance Level
Run	Linear	192.451	42	0.000
	Logarithmic	48.401	42	0.230
Collaborator-Block	Linear	192.416	36	0.000
	Logarithmic	62.844	36	0.004

distribution with the appropriate degrees of freedom. The values for both the run and collaborator-block data are shown in Table B-4.

Clearly, for the run data the logarithmic transformation is acceptable, while the linear form of the data is not. The reason for this is apparent from the formula for Q_s. The principal factor upon which Q_s depends is $(V_s)_{avg}$, but the use of the multipliers of (3600) and the cross-sectional area of the stack increases the magnitude of the velocity variation. However, the relative variation, as expressed by the coefficient of variation, tends to remain constant from site to site. For the collaborator-block data, the logarithmic transformation would not be considered acceptable but is an improvement over the linear form. The acceptance of the logarithmic transformation implies, on a theoretical basis, a proportional relationship between the mean and standard deviation of the distribution.

To further investigate the proportional relationship, a least squares model is fit to the paired sample means and sample standard deviations. For the run data, define

x_{ijk} – the flow rate determined by collaborator i on run k in block j.

$\bar{x}_{.jk} = \frac{1}{p} \sum_{i=1}^{p} x_{ijk}$ as the run mean, where p is the number of collaborators

and

$s_{jk} = \sqrt{\frac{1}{p-1} \sum_{i=1}^{p} (x_{ijk} - \bar{x}_{.jk})^2}$ as the run standard deviation.

A no-intercept model is fit to the pairs $(\bar{x}_{.jk}, s_{jk})$, since a mean of zero automatically implies a standard deviation of zero. The paired means and standard deviations are shown in Table B-5. The fit to this model

TABLE B-5. RUN MEANS AND STANDARD DEVIATIONS (Volumetric Flow Rate, $ft^3/hr \times 10^{-4}$)

Run	Mean Flow Rate	Standard Deviation
Site 1		
1	1232.8	49.2
2	1280.8	68.9
3	1341.5	134.3
4	1191.9	83.7
5	1146.3	72.3
6	1190.9	59.8
9	1312.2	70.3
8	1309.0	44.1
10	1270.6	47.5
11	1215.8	40.5
12	1214.2	29.8
13	1213.0	48.8
7	1260.4	144.4
14	1117.8	33.3
15	1149.1	38.2
Site 2		
1	7087.8	122.4
3	6989.4	328.8
4	7114.1	238.3
6	6962.3	71.6
8	7109.6	389.1
2	6820.1	187.8
5	6833.5	97.2
7	6652.6	171.2
10	6687.0	279.9
11	6912.7	419.9
13	7025.7	143.1
14	7024.0	134.5
15	6703.5	122.1
16	6957.8	310.5
9	6501.3	644.9
12	6557.9	393.8
Site 3		
3	338.2	24.8
10	348.6	22.0
12	351.0	25.0
2	336.6	18.9
5	301.8	38.4
6	332.7	12.9
1	344.9	31.3
4	316.3	11.5
7	324.8	12.7
8	320.1	29.2
11	323.5	19.8
9	287.9	17.3

is measured by the coefficient of determination, r^2. For the no-intercept model, r^2 is calculated as[10]

$$r^2 = \frac{[\Sigma x_i y_i]^2}{\Sigma x_i^2 \, \Sigma y_i^2} = \frac{[\Sigma \bar{x}_{.jk} s_{jk}]^2}{\Sigma \bar{x}_{.jk}^2 \, \Sigma s_{jk}^2}.$$

For the run data, $r^2 = 0.73$ based on 43 pairs of observations. This indicates that 73% of the variation in the magnitude of the standard deviation is attributed to variation in the magnitude of the mean. The correlation coefficient, $r = \sqrt{r^2}$, is 0.85 which is a significant value at the 5 percent level.

Similarly, for the collaborator block data define

$\bar{x}_{ij.} = \frac{1}{n_{ij}} \sum_{k=1}^{n_{ij}} x_{ijk}$ as the collaborator-block mean for collaborator i in block j, and n_{ij} determinations in the collaborator-block

and

$s_{ij} = \sqrt{\frac{1}{n_{ij}-1} \sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij.})^2}$ as the collaborator-block standard deviation.

The paired values $(\bar{x}_{ij.}, s_{ij})$ are shown in Table B-6. For the no-intercept model, $r^2 = 0.63$ and the correlation coefficient, r, is 0.79 based on 37 pairs. As in the case of the transformations, the proportional relationship does not appear as strong for the collaborator-block data. However, a correlation coefficient of 0.79 is significant at the 5 percent level.

As a result, then, we have the model for the between-laboratory and within-laboratory standard deviations of

$$\sigma_b = \beta_b \delta$$

and

$$\sigma = \beta \delta$$

where β_b and β are the true between-laboratory and within-laboratory coefficients of variation, and δ is an unknown mean. The coefficients of variation remain constant, and the standard deviation may be expressed as a percentage of the mean value. Thus, the standard deviations are estimated by obtaining estimates of the coefficients of variation, $\hat{\beta}_b$ and $\hat{\beta}$, and expressing the estimators as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta} \delta.$$

TABLE B-6. COLLABORATOR-BLOCK MEANS AND STANDARD DEVIATIONS
(Volumetric Flow Rate, ft^3/hr X 10^{-4})

Block	Collaborator	Mean Flow Rate	Standard Deviation
Site 1			
1	Lab 101	1250.10	15.42
	Lab 102	1213.50	34.11
	Lab 103	1440.35	79.69
	Lab 104	1305.33	91.00
2	Lab 101	1166.42	69.94
	Lab 102	1143.57	81.63
	Lab 103	1285.22	79.22
	Lab 104	1246.12	73.87
3	Lab 101	1223.72	26.32
	Lab 102	1221.38	71.89
	Lab 103	1291.62	43.59
	Lab 104	1241.42	51.45
4	Lab 101	1107.50	41.44
	Lab 102	1155.60	93.50
	Lab 103	1209.17	58.86
	Lab 104	1227.33	177.13
Site 2			
1	Lab 202	7110.72	323.46
	Lab 203	7125.94	71.54
	Lab 204	6921.28	203.17
2	Lab 202	6962.89	251.59
	Lab 203	6780.70	210.72
	Lab 204	6768.34	215.90
3	Lab 202	6327.70	718.42
	Lab 203	6656.65	104.86
	Lab 204	6604.50	703.29
Site 3			
1	Lab 301	336.57	17.16
	Lab 302	369.47	2.07
	Lab 303	331.00	12.64
	Lab 304	346.73	31.96
2	Lab 301	302.67	37.80
	Lab 302	351.97	3.14
	Lab 303	323.57	12.18
	Lab 304	316.60	30.46
3	Lab 301	311.10	8.19
	Lab 302	341.86	17.65
	Lab 303	330.96	22.95
	Lab 304	313.52	22.13
4*	Lab 301	273.60	–
	Lab 302	311.70	–
	Lab 303	276.70	–
	Lab 304	289.50	–

*No standard deviations since block contains only one run.

B.3 Unbiased Estimation of Standard Deviation Components

In Appendices B.1 and B.2, the theoretical and empirical arguments from the collaborator-block data indicate that a suitable model for the within-lab standad deviations of both variables is

$$\hat{\sigma} = \beta\delta$$

To estimate this standard deviation, we use the relationship

$$s_{ij} = C\bar{x}_{ij}.$$

where C is a constant, representing the proportionality. As previously discussed, s_{ij} is a biased estimator for the true standard deviation, σ. The correction factor for removing the bias is dependent on the sample size n, and is given by Ziegler[5] as

$$\alpha_n = \sqrt{\frac{2}{3}}\,\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)},$$

where Γ represents the standard gamma function. Thus, we can say that

$$E(\alpha_n s_{ij}) = \sigma$$

or

$$\begin{aligned}\sigma &= \alpha_n E(s_{ij}) \\ &= \alpha_n E(C\bar{x}_{ij.}) \\ &= \alpha_n C E(\bar{x}_{ij.}) \\ &= \alpha_n C\delta \\ &= \beta\delta\end{aligned}$$

so that in obtaining an unbiased estimate of β, we can obtain an unbiased estimate of σ as well. Thus, we *define an estimator* for σ, $\hat{\sigma}$, where

$$\hat{\sigma} = \hat{\beta}\delta.$$

From Appendices B.1 and B.2, we determine that a suitable model for the run data from both variables is

$$\sigma_b = \beta_b\delta$$

where $\sigma_b = \sqrt{\sigma_L^2 + \sigma^2}$ is the *between-lab standard deviation.* Empirically, we have

$$s_{jk} = C_b \bar{x}_{.jk}$$

and s_{jk} is a biased estimator for σ_b. Thus, for p collaborators,

$$E(\alpha_p s_{jk}) = \sigma$$

and we have

$$\begin{aligned} \sigma &= E(\alpha_p s_{jk}) \\ &= \alpha_p E(C_b \bar{x}_{.jk}) \\ &= \alpha_p C_b E(\bar{x}_{.jk}) \\ &= \alpha_p C_b \delta \\ &= \beta_b \delta . \end{aligned}$$

Obtaining an estimate of β_b, we have a *new estimator,* $\hat{\sigma}_b$, of σ_b given by

$$\hat{\sigma}_b = \hat{\beta}_b \delta .$$

But $\sigma_b = \sqrt{\sigma_L^2 + \sigma^2}$ implies

$$\begin{aligned} \sigma_b^2 &= \sigma_L^2 + \sigma^2 \\ \sigma_L^2 &= \sigma_b^2 - \sigma^2 \\ \sigma_L &= \sqrt{\sigma_b^2 - \sigma^2} \end{aligned}$$

and substituting our estimates of σ_b and σ, we have

$$\begin{aligned} \hat{\sigma}_L &= \sqrt{\hat{\beta}_b^2 \delta^2 - \hat{\beta}^2 \delta^2} \\ &= \sqrt{\hat{\beta}_b^2 - \hat{\beta}^2}\ \delta , \end{aligned}$$

so that the *laboratory bias standard deviation* may be estimated as a percentage of the mean as well.

B.4 Weighted Coefficient of Variation Estimates

The technique used for obtaining estimates of the coefficients of variation of interest is to use a linear combination of the individual beta values obtained. The linear combination used will be of the form

$$\frac{1}{k} \sum_{j=1}^{k} w_j \hat{\beta}_j$$

where $\hat{\beta}_j$ is the jth coefficient of variation estimate, k is the total number of estimates, and w_j is a weight applied to the jth estimate.

As previsouly discussed, the individual estimate of β is obtained as

$$\hat{\beta} = \frac{\alpha_n s}{\bar{x}}$$

for a sample of size n. This estimator is shown in B.5 to be unbiased for the true coefficient of variation. However, since we are dealing with small samples to obtain our individual estimates, weighting is more desirable in that it provides for more contribution from those values derived from larger samples. There is more variability in the beta values obtained from the smaller samples, as can be seen by inspecting the variance of the estimator. We have that

$$\begin{aligned}\mathrm{Var}(\hat{\beta}) &= \mathrm{Var}\left(\frac{\alpha_n s}{x}\right)\\ &= \alpha_n^2\,\mathrm{Var}\left(\frac{s}{\bar{x}}\right)\\ &= \alpha_n^2\left[\frac{\beta^2}{2n}(1+2\beta^2)\right]\end{aligned}$$

for normally distributed samples,[11] and true coefficient of variation, β. Rewriting this expression, we have

$$\mathrm{Var}(\hat{\beta}) = \frac{\alpha_n^2}{n}\left[\frac{\beta^2}{2}(1+2\beta^2)\right]$$

and all terms are constant except for α_n^2 and n. Thus, the magnitude of the variance changes with respect to the factor α_n^2/n. Now, since α_n decreases as n increases, the factor α_n^2/n must decrease as n increases, and the variance is reduced.

The weights, w_j, are determined according to the technique used in weighted least squares analysis[6], which gives a minimum variance estimate of the parameter. The individual weight, w_i, is computed as the inverse of the variance of the estimate, $\hat{\beta}_i$, and then standardized. Weights are said to be standardized when

$$\frac{1}{k}\sum_{j=1}^{k} w_j = 1$$

To standardize, the weights are divided by the average of the inverse variances for all the estimates. Thus, we can write

$$w_i = \frac{u_i}{\bar{u}}$$

where

$$u_i = \frac{1}{\mathrm{Var}(\hat{\beta}_i)}$$

and

$$\bar{u} = \frac{1}{k}\sum_{j=1}^{k}\frac{1}{\mathrm{Var}(\hat{\beta}_j)}$$

Now, from the above expressions we can determine u_i, $\bar{u}$ and w_i for the beta estimates. For any estimate, $\hat{\beta}_i$,

$$u_i = \frac{1}{\mathrm{Var}(\hat{\beta}_i)}$$

$$u_i = \frac{n_i}{\alpha_{n_i}^2}\left[\frac{2}{\beta^2\,(1+2\beta^2)}\right]$$

for sample size n_i, and

$$\bar{u} = \frac{1}{k}\sum_{j=1}^{k}\frac{1}{\mathrm{Var}(\hat{\beta}_j)}$$

$$= \frac{1}{k}\sum_{j=1}^{k}\frac{n_j}{\alpha_{n_j}^2}\left[\frac{2}{\beta^2\,(1+2\beta^2)}\right]$$

$$= \frac{1}{k}\left[\frac{2}{\beta^2\,(1+2\beta^2)}\right]\sum_{j=1}^{k}\frac{n_j}{\alpha_{n_j}^2}\,.$$

Thus, the ith weight, w_i, is

$$w_i = \frac{u_i}{\bar{u}}$$

$$= \frac{\dfrac{n_i}{\alpha_{n_i}^2}\left[\dfrac{2}{\beta^2\,(1+2\beta^2)}\right]}{\dfrac{1}{k}\left[\dfrac{2}{\beta^2\,(1+2\beta^2)}\right]\sum\dfrac{n_j}{\alpha_{n_j}^2}}$$

$$= \frac{\dfrac{n_i}{\alpha_{n_i}^2}}{\dfrac{1}{k}\displaystyle\sum_{i=1}^{k}\dfrac{n_j}{\alpha_{n_j}^2}}$$

$$= \frac{\dfrac{kn_i}{\alpha_{n_i}^2}}{\displaystyle\sum_{j=1}^{k}\dfrac{n_j}{\alpha_{n_j}^2}}\,.$$

The estimated coefficient of variation is

$$\hat{\beta} = \frac{1}{k}\sum_{j=1}^{k} w_j\hat{\beta}_j$$

$$= \frac{1}{k}\sum_{i=1}^{k}\frac{k n_i \alpha_{n_i}^2}{\displaystyle\sum_{j=1}^{k}\frac{n_j}{\alpha_{n_j}^2}}\,\hat{\beta}_i$$

$$\hat{\beta} = \left[\sum_{j=1}^{k} \frac{n_j}{\alpha_{n_j}^2}\right]^{-1} \sum_{i=1}^{k} \frac{n_i}{\alpha_{n_i}^2} \cdot \frac{\alpha_{n_i} s}{\bar{x}}$$

$$= \left[\sum_{j=1}^{k} \frac{n_j}{\alpha_{n_j}^2}\right]^{-1} \sum_{i=1}^{k} \frac{n_i s}{\alpha_{n_i} \bar{x}} .$$

B.5 Estimating Precision Components For Velocity Determination

In Appendix B.1 the models are given for the between-laboratory and within-laboratory standard deviations, σ_b and σ, respectively,

$$\sigma_b = \beta_b \delta$$

and

$$\sigma = \beta\delta$$

where β_b and β are the true coefficients of variation for between-laboratory and within-laboratory, respectively. The coefficients of variation remain constant for changing mean levels.

Estimates of σ_b and σ are obtained using the technique of Appendices B.3 and B.4. The coefficients of variation are estimated as a linear combination of beta values obtained from each run or collaborator-block. The estimator is of the form

$$\hat{\beta} = \frac{1}{k} \sum_{i=1}^{k} w_i \beta_i$$

where k is the number of individual estimates and w_i is the weight applied to the ith beta estimate.

From the run data, the estimated between-laboratory coefficient of variation is estimated as

$$\hat{\beta}_b = \frac{1}{43} \sum_{i=1}^{43} w_i \hat{\beta}_i .$$

The individual beta estimate and the weights applied are shown in Table B-7. Substituting these into the above formula gives

$$\hat{\beta}_b = (0.050).$$

The estimated between-laboratory standard deviation, then, is

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

$$= (0.050)\delta .$$

The degrees of freedom associated with this estimate are determined by taking the number of collaborators at a site less one, summed for all three sites. This gives (4 – 1) + (3 – 1) + (4 – 1) = 8 degrees of freedom for this estimate.

TABLE B-7. RUN BETA ESTIMATES AND WEIGHTS
(Velocity)

Run	Beta Hat	Weight
Site 1		
1	0.0444	0.723
2	0.0307	1.043
3	0.0315	1.043
4	0.0322	1.043
5	0.0375	1.043
6	0.0374	1.043
9	0.0342	1.043
8	0.0377	1.043
10	0.0105	1.043
11	0.0416	1.043
12	0.0477	1.043
13	0.0339	1.043
7	0.0457	1.043
14	0.0500	0.723
15	0.0220	1.043
Site 2		
1	0.0551	1.030
3	0.0555	1.030
4	0.0408	1.030
6	0.0307	1.030
8	0.0694	1.030
2	0.0172	1.030
5	0.0201	1.030
7	0.0417	1.030
10	0.0520	1.030
11	0.0265	1.030
13	0.0506	1.030
14	0.0263	0.556
15	0.0223	1.030
16	0.0557	1.030
9	0.0569	1.030
12	0.0693	1.030
Site 3		
3	0.1073	1.026
10	0.0244	1.026
12	0.0748	1.026
2	0.1047	1.026
5	0.0382	1.026
6	0.0800	1.026
1	0.0810	0.712
4	0.0938	1.026
7	0.0616	1.026
8	0.1092	1.026
11	0.0535	1.026
9	0.0789	1.026

The within-laboratory coefficient of variation, β, is estimated from the collaborator-block data as

$$\hat{\beta} = \frac{1}{37} \sum_{i=1}^{37} w_i \hat{\beta}_i .$$

The collaborator block beta estimates and their corresponding weights are shown in Table B-8. Substituting into the above equation gives

$$\hat{\beta} = 0.039$$

and the within-laboratory standard deviation estimate is

$$\hat{\sigma} = \hat{\beta}\delta$$

$$= (0.039)\delta .$$

There are are $(n_{ij} - 1)$ degrees of freedom for this estimate from each collaborator-block, where n_{ij} is the number of determinations in the collaborator block. Summing over the 37 blocks gives 113 degrees of freedom.

B.6 Estimating Precision Components For Volumetric Flow Rate

In Appendix B.2, the models are developed for the standard deviation components, σ_b and σ,

$$\sigma_b = \beta_b \delta$$

and

$$\sigma = \beta\delta ,$$

where β_b and β are the between-laboratory and within-laboratory coefficients of variation, and δ is the mean method determination. The coefficients of variation are shown to remain constant as the mean changes. To estimate σ_b and σ, then, estimators $\hat{\sigma}_b$ and $\hat{\sigma}$ are defined as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$

and

$$\hat{\sigma} = \hat{\beta}\delta$$

Estimated coefficients of variation are used to estimate the standard deviations as percentages of the mean value, δ.

The technique used in estimating β_b and β is discussed in Appendices B.3 and B.4. The estimator is a linear combination of beta values,

TABLE B-8. COLLABORATOR-BLOCK BETA ESTIMATES AND WEIGHTS (Velocity)

Block	Collaborator	Beta Hat	Weight
Site 1			
1	Lab 101	0.0119	0.786
	Lab 102	0.0089	0.786
	Lab 103	0.1006	0.425
	Lab 104	0.0474	0.786
2	Lab 101	0.0486	1.133
	Lab 102	0.0591	1.133
	Lab 103	0.0435	1.133
	Lab 104	0.0420	1.133
3	Lab 101	0.0241	1.475
	Lab 102	0.0162	1.475
	Lab 103	0.0139	1.475
	Lab 104	0.0392	1.475
4	Lab 101	0.0914	0.425
	Lab 102	0.0396	0.786
	Lab 103	0.0153	0.786
	Lab 104	0.0111	0.786
Site 2			
1	Lab 202	0.0531	0.960
	Lab 203	0.0095	0.960
	Lab 204	0.0342	0.960
2	Lab 202	0.0360	1.837
	Lab 203	0.0346	1.618
	Lab 204	0.0391	1.837
3	Lab 202	0.0487	0.277
	Lab 203	0.0291	0.277
	Lab 204	0.1245	0.277
Site 3			
1	Lab 301	0.0503	0.796
	Lab 302	0.0241	0.796
	Lab 303	0.0447	0.796
	Lab 304	0.1059	0.796
2	Lab 301	0.0446	0.796
	Lab 302	0.0409	0.796
	Lab 303	0.0282	0.796
	Lab 304	0.0447	0.796
3	Lab 301	0.0352	1.494
	Lab 302	0.0458	1.494
	Lab 303	0.0254	1.494
	Lab 304	0.0749	1.148
4*	Lab 301	–	–
	Lab 302	–	–
	Lab 303	–	–
	Lab 304	–	–

*No estimates possible since this block contains only one run.

$$\hat{\beta} = \frac{1}{k} \sum_{i=1}^{k} w_i \hat{\beta}_i$$

where $\hat{\beta}_i$ is the ith beta estimate from a run or collaborator-block, w_i is the weight assigned, and k is the number of estimates.

From the run data, there are 43 separate estimates of β_b, which gives

$$\hat{\beta}_b = \frac{1}{43} \sum_{i=1}^{43} w_i \hat{\beta}_i.$$

The values and their weights are shown in Table B-9. Substituting these into the above formula gives

$$\hat{\beta}_b \doteq 0.056.$$

The between-laboratory standard deviation is estimated as

$$\hat{\sigma}_b = \hat{\beta}_b \delta$$
$$= (0.056)\delta$$

or 5.6% of its mean value.

The degrees of freedom for this estimate are determined by taking the number of collaborators less one at each site, and summing over the three sites. This gives $(4 - 1) + (3 - 1) + (4 - 1) = 8$ degrees of freedom for this estimate.

The collaborator-block data gives an estimate of the within-laboratory precision components. The within-laboratory coefficient of variation is estimated as

$$\hat{\beta} = \frac{1}{37} \sum_{i=1}^{37} w_i \hat{\beta}_i.$$

The individual beta estimates and weights are shown in Table B-10. Substituting into this equation gives

$$\hat{\beta} = 0.055$$

and a standard deviation estimate of

$$\hat{\sigma} = \hat{\beta}\delta$$
$$= (0.055)\delta.$$

Thus, the within-laboratory standard deviation is estimated as 5.5% of the mean level. Letting n_{ij} be the number of determinations in a collaborator-block, there are $(n_{ij} - 1)$ degrees of freedom for this estimate from each. Summing, there are a total of 113 degrees of freedom from the 37 collaborator-blocks for this estimate.

TABLE B-9. RUN BETA ESTIMATES AND WEIGHTS (Volumetric Flow Rate)

Run	Beta Hat	Weight
Site 1		
1	0.0450	0.723
2	0.0584	1.043
3	0.1087	1.043
4	0.0762	1.043
5	0.0685	1.043
6	0.0545	1.043
9	0.0581	1.043
8	0.0365	1.043
10	0.0406	1.043
11	0.0362	1.043
12	0.0266	1.043
13	0.0436	1.043
7	0.1244	1.043
14	0.0336	0.723
15	0.0361	1.043
Site 2		
1	0.0195	1.030
3	0.0531	1.030
4	0.0378	1.030
6	0.0116	1.030
8	0.0618	1.030
2	0.0311	1.030
5	0.0160	1.030
7	0.0290	1.030
10	0.0472	1.030
11	0.0685	1.030
13	0.0230	1.030
14	0.0240	0.556
15	0.0205	1.030
16	0.0504	1.030
9	0.1119	1.030
12	0.0678	1.030
Site 3		
3	0.0796	1.026
10	0.0686	1.026
12	0.0774	1.026
2	0.0610	1.026
5	0.1381	1.026
6	0.0420	1.026
1	0.1025	0.712
4	0.0396	1.026
7	0.0425	1.026
8	0.0991	1.026
11	0.0664	1.026
9	0.0653	1.026

TABLE B-10. COLLABORATOR-BLOCK BETA ESTIMATES AND WEIGHTS (Volumetric Flow Rate)

Block	Collaborator	Beta Hat	Weight
Site 1			
1	Lab 101	0.0139	0.786
	Lab 102	0.0317	0.786
	Lab 103	0.0693	0.425
	Lab 104	0.0787	0.786
2	Lab 101	0.0651	1.133
	Lab 102	0.0775	1.133
	Lab 103	0.0669	1.133
	Lab 104	0.0643	1.133
3	Lab 101	0.0229	1.475
	Lab 102	0.0626	1.475
	Lab 103	0.0359	1.475
	Lab 104	0.0441	1.475
4	Lab 101	0.0469	0.425
	Lab 102	0.0913	0.786
	Lab 103	0.0549	0.786
	Lab 104	0.1628	0.786
Site 2			
1	Lab 202	0.0484	0.960
	Lab 203	0.0107	0.960
	Lab 204	0.0312	0.960
2	Lab 202	0.0373	1.837
	Lab 203	0.0322	1.618
	Lab 204	0.0329	1.837
3	Lab 202	0.1423	0.277
	Lab 203	0.0197	0.277
	Lab 204	0.1335	0.277
Site 3			
1	Lab 301	0.0575	0.796
	Lab 302	0.0063	0.796
	Lab 303	0.0431	0.796
	Lab 304	0.1040	0.796
2	Lab 301	0.1409	0.796
	Lab 302	0.0101	0.796
	Lab 303	0.0425	0.796
	Lab 304	0.1086	0.796
3	Lab 301	0.0280	1.494
	Lab 302	0.0549	1.494
	Lab 303	0.0738	1.494
	Lab 304	0.0766	1.148
4*	Lab 301	–	–
	Lab 302	–	–
	Lab 303	–	–
	Lab 304	–	–

*No estimates possible since block contains only one run.

B.7 Emission Rate Variability

The emission rate, r, is given by

$$r = Q_S \cdot c$$

where

Q_S is the volumetric flow rate of the stack, ft^3/hr

and

c is the concentration of pollutant, determined by the applicable test method, appropriate weight units/scf.

The flow rate calculation does not involve the mass or volume of pollutant obtained, and the concentration of pollutant is determined separately from the velocity calculation. Thus, it is reasonable to say that Q_S and c are independent variables. Under this assumption, we can estimate a variance component for r, $V(r)$, from the estimated terms for Q_S and c.

In this section, these relationships will be used.

[1] The variance of any random variable, x, is defined as

$$V(x) = E(x^2) - [E(x)]^2$$

[2] For independent variables x and y(12)

$$E(x \cdot y) = E(x) \cdot E(y)$$

and

$$E[f_1(x) \cdot f_2(y)] = E[f_1(x)] \cdot E[f_2(y)]$$

for any two functions f_1 and f_2.

[3] For any variables x and y

$$V(x) \cdot V(y) = E(x^2)E(y^2) - E(x^2)[E(y)]^2 - E(y^2)[E(x)]^2 + [E(x)]^2[E(y)]^2$$

This can be derived easily from [1].

For the variable r, from [1]

$$V(r) = E(r^2) - [E(r)]^2$$

$$= E(Q_S^2 \cdot c^2) - [E(Q_S \cdot c)]^2.$$

From [2] with Q_S and c taken to be independent variables

$$V(r) = E(Q_S{}^2)E(c^2) - [E(Q_S)]^2[E(c)]^2$$

From [3], solving for $E(x^2)E(y^2)$ gives

$$V(r) = \left\{V(Q_S) \cdot V(c) + E(Q_S{}^2)[E(c)]^2 + E(c^2)[E(Q_S)]^2 - [(E(Q_S)]^2[E(c)]^2\right\} - [E(Q_S)]^2[E(c)]^2$$

Rearranging terms gives

$$V(r) = V(Q_s)V(c) + \left\{E(Q_s^2)[E(c)]^2 - [E(Q_s)]^2[E(c)]^2\right\} + \left\{E(c^2)[E(Q_s)]^2 - [E(c)]^2[E(Q_s)]^2\right\}$$

$$= V(Q_s)V(c) + \left\{E(Q_s^2) - [E(Q_s)]^2\right\}[E(c)]^2 + \left\{E(c^2) - [E(c)]^2\right\}[E(Q_s)]^2$$

and from 1

$$V(r) = V(Q_s)V(c) + V(Q_s)[E(c)]^2 + V(c)E(Q_s)^2$$

$$= V(Q_s)V(c) + V(Q_s)\delta_c^2 + V(c)\delta_{Qs}^2$$

where

δ_c^2 — the mean pollutant concentration, and

δ_{Qs}^2 — the mean flow rate

When both the flow rate and pollutant concentration have constant coefficients of variation, $\beta(Q_s)$ and $\beta(c)$, respectively, the variances are written as

$$V(Q_s) = \beta^2(Q_s)\delta^2{}_{Q_s}$$

$$V(c) = \beta^2(c)\delta_c^2$$

and substituting these into the equation for $V(r)$ gives

$$V(r) = [\beta^2(Q_s)\delta^2{}_{Qs}]\,[\beta^2(c)\delta_c^2] + \beta^2(Q_s)\delta_{Qs}^2\delta_c^2 + \beta^2(c)\delta_c^2\delta^2{}_{Qs}$$

$$= [\beta^2(Q_s)\beta^2(c) + \beta^2(Q_s) + \beta^2(c)]\;\delta_c^2\delta^2{}_{Qs}$$

$$= [\beta^2(Q_s)\beta^2(c) + \beta^2(Q_s) + \beta^2(c)]\delta_r^2$$

where

δ_r is the mean emission rate, $\delta_r = \delta_c\delta_{Qs}$.

LIST OF REFERENCES

1. Environmental Protection Agency "Standards of Performance for New Stationary Sources," *Federal Register*, Vol. 36, No. 247, December 23, 1971, pp 24876-24893.

2. Hamil, H.F. and Camann, D.E., "Collaborative Study of Method for the Determination of Particulate Matter Emissions from Stationary Sources (Portland Cement Plants)," Southwest Research Institute report for Environmental Protection Agency, May, 1974.

3. Hamil, H.F. and Thomas, R.E., "Collaborative Study of Method for the Determination of Particulate Matter Emissions from Stationary Sources (Fossil Fuel-Fired Steam Generators)," Southwest Research Institute report for Environmental Protection Agency, June 30, 1974.

4. Hamil, H.F. and Thomas, R.E., "Collaborative Study of Method for the Determination of Particulate Matter Emissions from Stationary Sources," (Municipal Incinerators). Southwest Research Institute report for Environmental Protection Agency, July 1, 1974.

5. Ziegler, R.K., "Estimators of Coefficients of Variation Using k Samples," *Technometrics*, Vol. 15, No. 2, May 1973, pp 409-414.

6. Hamil, H.F., "Laboratory and Field Evaluations of EPA Methods 2, 6 and 7." Southwest Research Institute report for Environmental Protection Agency, October, 1973.

7. Hamil, H.F. and Camann, D.E., "Collaborative Study of Method for the Determination of Nitrogen Oxide Emissions from Stationary Sources," (Fossil Fuel-Fired Steam Generators) Southwest Research Institute report for Environmental Protection Agency, October 5, 1973.

8. Hamil, H.F. and Camann, D.E., "Collaborative Study of Method for the Determination of Sulfur Dioxide Emissions From Stationary Sources (Fossil Fuel-Fired Steam Generators)." Southwest Research Institute report for Environmental Protection Agency, December 10, 1973.

9. Dixon, W.J. and Massey, F.J., Jr., **Introduction to Statistical Analysis**, 3rd Edition. McGraw-Hill, New York, 1969.

10. Searle, S.R., **Linear Models**. Wiley, New York, 1971.

11. Cramér, H., **Mathematical Methods of Statistics**. Princeton University Press, New Jersey, 1946.

12. Hogg, R.V. and Craig, A.T., **Introduction to Mathematical Statistics**, 3rd Edition. The Macmillan Company, New York, 1970.

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16. ABSTRACT

This study concerns itself with the determination of stack gas velocity and volumetric flow rate (EPA Method 2) as used with EPA Method 5 (Particulates). The determinations were calculated from data obtained in collaborative testing of Method 5 at three sites: a Portland cement plant, a coal-fired power plant and a municipal incinerator. These data were submitted to statistical analysis to obtain estimates of the precision that can be expected with the use of Method 2. The standard deviations for both velocity and flow rate are shown to be proportional to the mean value.

The between-laboratory standard deviations are estimated as 5.0% of the mean and 5.6% of the mean for velocity and volumetric flow rate, respectively. The volumetric flow rate is used to calculate the emission rate for compliance testing. The precision of the emission rate is shown to be primarily a function of the precision of the pollutant test method used. The conclusion, then, is that the volumetric flow rate determination is sufficiently precise as it appears in Method 2.

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