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USER'S GUIDE AND DOCUMENTATION FOR OUTFALL PLUME MODEL

CLEAN WATER

ENVIRONMENTAL
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REGION X

PACIFIC NORTHWEST
WATER LABORATORY

CORVALLIS, OREGON

ERRATA: "User's Guide and Documentation for Outfall Plume Model," by D. J. Baumgartner, D. S. Trent, and K. V. Byram. Working Paper #80, EPA, Pacific Northwest Water Laboratory. May 1971.

A program modification has been made to the above publication which improves its response for plumes which exhibit both low initial densimetric Froude numbers and small but non-zero angles of inclination. Under this modification, the angular orientation of the plume axis at the end of the zone of flow establishment is determined by a linear interpolation process. This replaces the assumption in the original program that the plume axis maintains the port orientation angle throughout the zone of flow establishment for all angles other than zero. The effects of this modification are represented by a small decrease in the predicted dilution over that of the original program, fostered, of course, by the shorter travel distances associated with the improved specification of the angle of the plume.

The revised program is useful for all densimetric Froude numbers between 10^{-1} and 10^{+4} and for all angular orientation between and including horizontal and vertical. The program changes are included on revised pages 25 and 26.

W. F. Rittall

D. J. Baumgartner

June 1972

```
C
C  PROGRAM PLUME, VERSION OF 4/6/72
C
      COMMON G,FK,FN,COSTH,SINTH,COSTHE,DS,C1,C2,E13,FLAG,GRAV
      DIMENSION ZD(50),DG(50),RHO(50),DP(50)
      LOGICAL NDC,TRAPPD,FLAG,CHGDEN,METERS
      NOCASE=0
   20 READ(5,73,END=74)NDC,METERS,NPTS,ANGLE,DIA,DEPTH,RHOJ,RFD,Q,FN,PS
   73 FORMAT(2L1,I3,F5.0,7F10.0)
      FD=ABS(RFD)
      DO 102 I=1,NPTS
      READ(5,75)DP(I),RHO(I),SAL
      IF(SAL.EQ.0.)GO TO 102
      RHO(I)=1.+.001*SIGMAT(SAL,RHO(I))
  102 CONTINUE
      IF(NPTS.NE.1)GO TO 76
      NPTS=2
      DP(2)=DEPTH
      RHO(2)=RHO(1)
   76 NOCASE=NOCASE+1
   75 FORMAT(8F10.0)
      GRAV=32.172
      IF(METERS)GRAV=9.80665
      DO 55 I=1,NPTS
      IF(DP(I).GE.DEPTH)GO TO 56
   55 CONTINUE
      WRITE(6,59)NOCASE
   59 FORMAT('-NO DENSITY INFORMATION FOR JET LEVEL.  EXECUTION FOR',
     *  ' CASE NO.',I2,' DELETED.')
      GO TO 20
   56 NP=I
      NM=I-1
      RHOB=(DEPTH-DP(NM))*(RHO(NP)-RHO(NM))/(DP(NP)-DP(NM))+RHO(NM)
      DISP=RHOJ-RHOB
      DO 54 I=1,NM
      J=NP-I
      ZD(I)=(DEPTH-DP(J))/DIA
   54 DG(I)=(RHO(J+1)-RHO(J))*DIA/(DISP*(DP(J+1)-DP(J)))
      IF(NDC)GO TO 58
      U0=Q/(FN*.7853982*DIA*DIA)
      RFD=U0*U0*RHOJ/(-DISP*DIA*GRAV)
      FD=ABS(RFD)
   58 IF(FD.LE.4.01.OR.FD.GE.9.99)GO TO 61
      S=.113*FD+4.
      GO TO 62
   61 IF(FD.LE.4.01)S=2.6*FD**.333333
      IF(FD.GT.10.)S=5.6*FD/SQRT(FD*FD+18.)
   62 TEMP=ATAN(1.416667*S/FD)
      THETA0=.0111*ANGLE*(1.5708-TEMP)+TEMP
      COSTHE=COS(THETA0)
      SINTHE=SIN(THETA0)
      DSI=DEPTH/(177.*DIA)
```

```
      IF(DG(1).EQ.0.)GO TO 77
      DGTEMP=.01/DG(1)
      IF(DGTEMP.LE.0.)DGTEMP=-DGTEMP
      DSI=.12*1.6**(ALOG10(FU/10.))-2.**(ALOG10(DGTEMP))
   77 NPO=IFIX(PS/(DSI*DIA)+.5)
      N=0
      Z=S*SINTHE
      X=S*COSTHE
      E=(4./S)**3
      R=.25
      C1=E**.6666667
      C2=.75/RFD
      IPTS=1
      G=DG(IPTS)
      ZLIM=ZD(IPTS)
      CHGDEN=.FALSE.
      FLAG=.FALSE.
      TRAPPD=.FALSE.
      XP=X*DIA
      ZP=DEPTH-Z*DIA
      DSIP=DSI*DIA
      SP=S*DIA
C**
C  WRITE INITIAL CONDITIONS
C**
      WRITE(6,23)NOCASE
   23 FORMAT('1-******A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA****
     ******'/'-CASE NO.',I2,' INITIAL CONDITIONS ......')
      IF(NDC)WRITE(6,104)
  104 FORMAT('0     UNITS: NON DIMENSIONAL')
      IF(.NOT.NDC.AND.METERS)WRITE(6,105)
  105 FORMAT('0     UNITS: MKS')
      IF(.NOT.NDC.AND..NOT.METERS)WRITE(6,106)
  106 FORMAT('0     UNITS: FPS')
      WRITE(6,108)ANGLE,RFD,SP,DSIP,PS,XP,ZP,RHOJ,DEPTH
  108 FORMAT(
     *40H-      PORT ANGLE . . . . . . . . . . . . . . .,F7.1/
     *40H0      FROUDE NUMBER  . . . . . . . . . . . . .,F7.1/
     *40H       LENGTH FOR FLOW ESTABLISHMENT  . . .,F8.2/
     *40H       INTEGRATION STEP LENGTH  . . . . . . .,F9.3/
     *40H       PRINTOUT INTERVAL  . . . . . . . . . .,F8.2/
     *40H       X0 . . . . . . . . . . . . . . . . . .,F8.2/
     *40H       Z0 . . . . . . . . . . . . . . . . . .,F8.2/
     *40H       DISCHARGE DENSITY  . . . . . . . . . .,F11.5/
     *40H       PORT DEPTH . . . . . . . . . . . . . .,F8.2)
      IF(.NOT.NDC)WRITE(6,60)Q,FN,U0,DIA
   60 FORMAT(
     *40H       FLOWRATE . . . . . . . . . . . . . . .,E15.5/
     *40H       NUMBER OF PORTS  . . . . . . . . . . .,F5.0/
     *40H       DISCHARGE VELOCITY . . . . . . . . . .,F8.2/
```

USER'S GUIDE AND DOCUMENTATION FOR OUTFALL PLUME MODEL

Prepared by

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Working Paper #80

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INTRODUCTION

Many EPA offices have need for a computational program for analysis of pipeline discharges into lakes, reservoirs, estuaries, or the ocean. The enclosed plume program, P L U M E, is offered as a standard procedure for analysis of industrial waste, thermal, and sewage streams, incorporating the most recent knowledge of jets and plumes generally applicable.

DESCRIPTION OF PROGRAM P L U M E

P L U M E is designed to solve for the geometric and dynamic behavior of a buoyant round plume issuing from a port in stagnant, density stratified surroundings. The port may be oriented at any angle from 0° (horizontal) to 90° (vertical). The governing differential equations used are based on similarity principles and are similar to those presented by Baumgartner and Trent (1, pp. 64-71). Solution is carried out by a fourth-order Runge-Kutta technique. Additional information concerning the exact equations used and detailed theory will be given by Trent, Baumgartner, and Byram (2).

Calculation of the potential core length is based on Abraham's method and is an integral part of the results. Computed results include coordinates for the plume centerline, centerline dilution as a function of position, and the maximum height of rise if the receiving water is stratified. Calculation of centerline dilution is terminated should the centerline buoyancy become neutral. The reason for this is that similarity assumptions become invalid above this point and dilution results may be largely inaccurate. The ℄ dilution at the point of neutral buoyancy may be used as a conservative (minimum) estimate of the trap level ℄ dilution.

As in the case with all plume similarity solutions, the program should not be applied to cases where the receiving water is very shallow (water should be at least 10 port diameters deep). Initial densimetric Froude numbers may range from values smaller than 1 to greater than 10,000. Calculations may be carried out in either MKS or FPS units. Flow rate inputs must be specified for the entire diffuser section, along with the number of ports and a constant port diameter. Hence, the program assumes equal flow conditions at each diffuser port.*

* No difficulty is encountered if actual conditions are not consistent with this assumption. Flow distribution between unequal diameter ports may be approximated (see also Rawn, Bowerman, and Brooks (4)) and P L U M E employed for each port.

INPUT DESCRIPTION (IBM 360)

Data input for program P L U M E consists of the following records (see Figure 1):

Initial Conditions

Format: (2L1, I3, F5.0, 7F10.0)

Field	Column	Description
1	1	Leave blank.
2	2	Logical: T (true)=MKS units; Blank=FPS units.
3	3-5	Number of ambient density points to be entered, $\leq$ 50, right justify (do not use a decimal point).
4	6-10	Angle of port orientation from horizontal, degrees.
5	11-20	Port diameter.
6	21-30	Vertical distance between water surface and outfall port centerline (port depth).
7	31-40	Density of effluent in grams per cubic centimeter.
8	41-50	Leave blank.
9	51-60	Total volumetric flow rate.
10	61-70	Number of ports.
11	71-80	Desired data printout interval along plume centerline.

Density (s) Profile Cards

The program needs a profile of either density or temperature and salinity. If density is input, the units must be grams per cc.

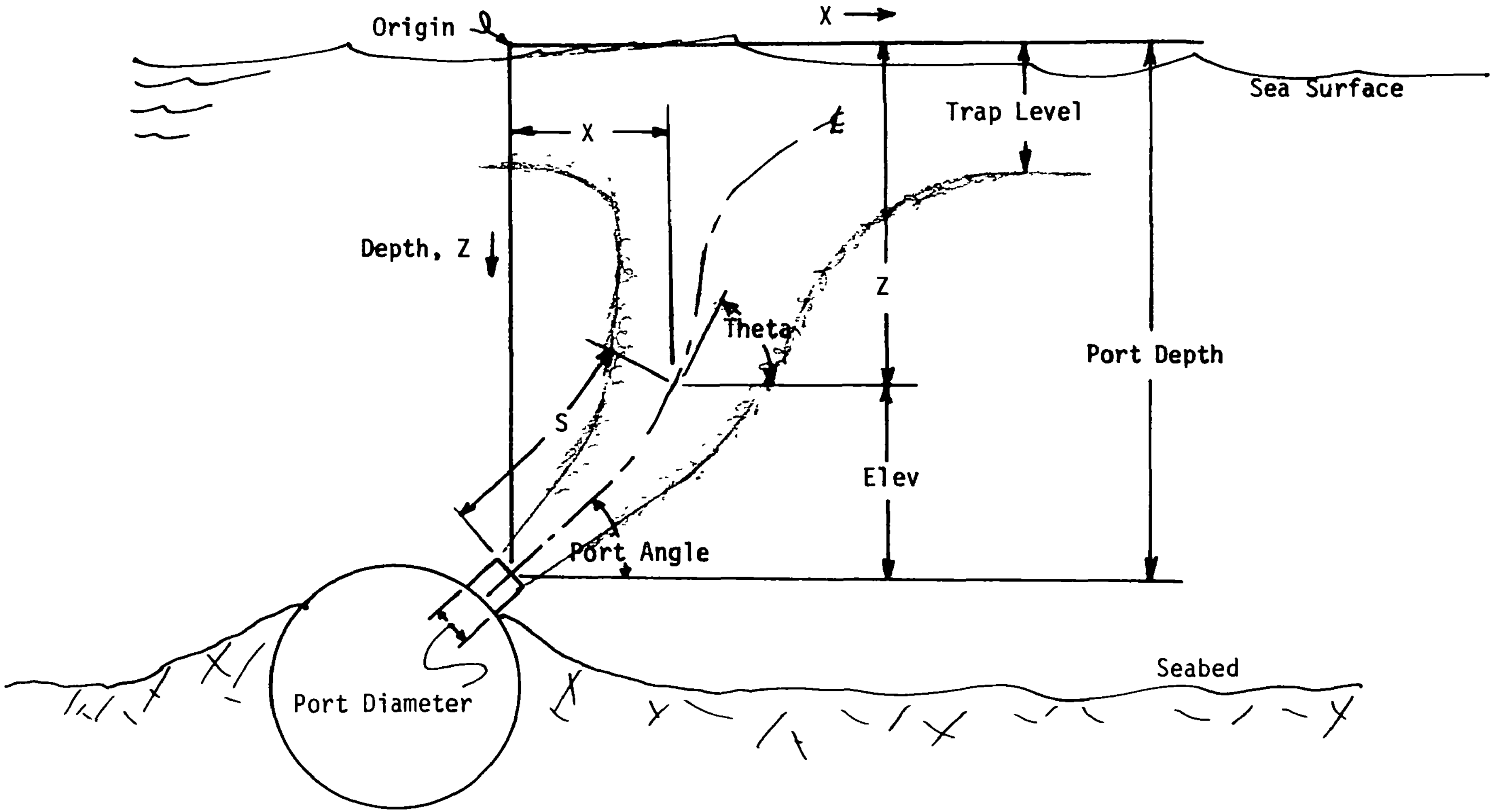

FIGURE 1. Definition Sketch for Input Specification and Output Interpretation.

One set of paired depth and density values is entered on each card, starting at the surface. If ambient density is constant with depth, use only one card.

Format: (3F10.0)

Field	Column	Description
1	1-10	Depth (distance measured from surface=0)
2	11-20	Density (or temperature, in degrees Celsius)
3	21-30	Salinity (in parts per thousand) but entered only if temperature is entered in columns 11-20. Otherwise, blank. You must use a non-zero salinity if temperature is used.

Any number of additional cases may be run by providing additional sets of input data cards as described above.

OUTPUT DESCRIPTION

Example Format


```
*********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA*********

CASE NO. n  INITIAL CONDITIONS ......

     UNITS: aaa

    PORT ANGLE . . . . . . . . . . . . . . .          nnn

    FROUDE NUMBER  . . . . . . . . . . . .            nnn
    LENGTH FOR FLOW ESTABLISHMENT  . . .              nnn
    INTEGRATION STEP LENGTH  . . . . . .              nnn
    PRINTOUT INTERVAL  . . . . . . . . .              nnn
    XO . . . . . . . . . . . . . . . . .              nnn
    ZO . . . . . . . . . . . . . . . . .              nnn
    DISCHARGE DENSITY  . . . . . . . . .              nnn
    PORT DEPTH . . . . . . . . . . . . .              nnn
    FLOWRATE . . . . . . . . . . . . . .              nnn
    NUMBER OF PORTS  . . . . . . . . . .              nnn
    DISCHARGE VELOCITY . . . . . . . . .              nnn
    PORT DIAMETER. . . . . . . . . . . .              nnn

DENSITY STRATIFICATION DEPTH     RHO

                          nnn      nnn
                          nnn      nnn
                          nnn      nnn

        S        X        Z        ELEV     THETA     DILN
       nnn      nnn      nnn       nnn      nnn       nnn
       nnn      nnn      nnn       nnn      nnn       nnn
       nnn      nnn      nnn       nnn      nnn       nnn
PLUME HITS SURFACE
       nnn      nnn      nnn       nnn      nnn

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL IS        nnn WITH DILUTION OF     nnn
```

Explanation

The "FROUDE NUMBER" is an initial densimetric Froude number based on velocity squared.

The "LENGTH FOR FLOW ESTABLISHMENT" is the length of the plume potential core.

"INTEGRATION STEP LENGTH" is the length of the increment along the plume centerline used for numerical calculation. If there is no stratification, the increment is 1/177 of the depth. If there is stratification the increment is computed as a function of the Froude number and the density gradient at the outfall port. This function was arrived at arbitrarily based on operating experience with the model.

"XO" is the horizontal coordinate of the point for flow establishment measured from the origin at the port centerline.

"ZO" is the depth of the point for flow establishment.

"DENSITY STRATIFICATION" is a read out of the density distribution input. If salinity and temperature data are input, the output will be given in terms of density.

Each line of the final tabular listing above corresponds to a point along the plume centerline defined by coordinates X and Z.

Heading	Description
S	Distance along plume centerline
X	Horizontal distance to point on plume centerline
Z	Depth of point on plume centerline

ELEV	Distance above outfall port to point on plume centerline (i.e., the difference between port depth and Z)
THETA	Inclination of plume trajectory above horizontal at point.
DILN	Dilution at point on the plume centerline.

No dilution values (DILN) are printed after the plume first reaches neutral buoyancy.

Four plume configurations are described by the model (see Figure 2):

Case a

If the plume reaches the surface with residual buoyancy the messages PLUME HITS SURFACE and TRAPPING LEVEL NOT REACHED will be printed along with a message announcing that the last line in the table describes conditions at the surface.

Case b

If a heavy fluid is discharged upward the plume may proceed toward the surface for some distance before falling back toward the bottom. In this case the computations cease as soon as the centerline angle, THETA, becomes negative, and the last data printout will describe the dilution at the highest level of the plume centerline. The message TRAPPING LEVEL NOT REACHED will be printed to advise that the fluid retains residual negative buoyancy.

Case c

If fluid near the region of the plume centerline becomes sufficiently diluted so that it is neutrally buoyant, the message

TRAP LEVEL IS NNN WITH DILUTION OF NNN

will be printed to serve as an estimate of the lower level of the horizontally spreading lens of diluted waste.

Case d

The message PLUME HITS SURFACE will be printed when the case c condition is met and if the fluid has sufficient momentum to reach the surface.

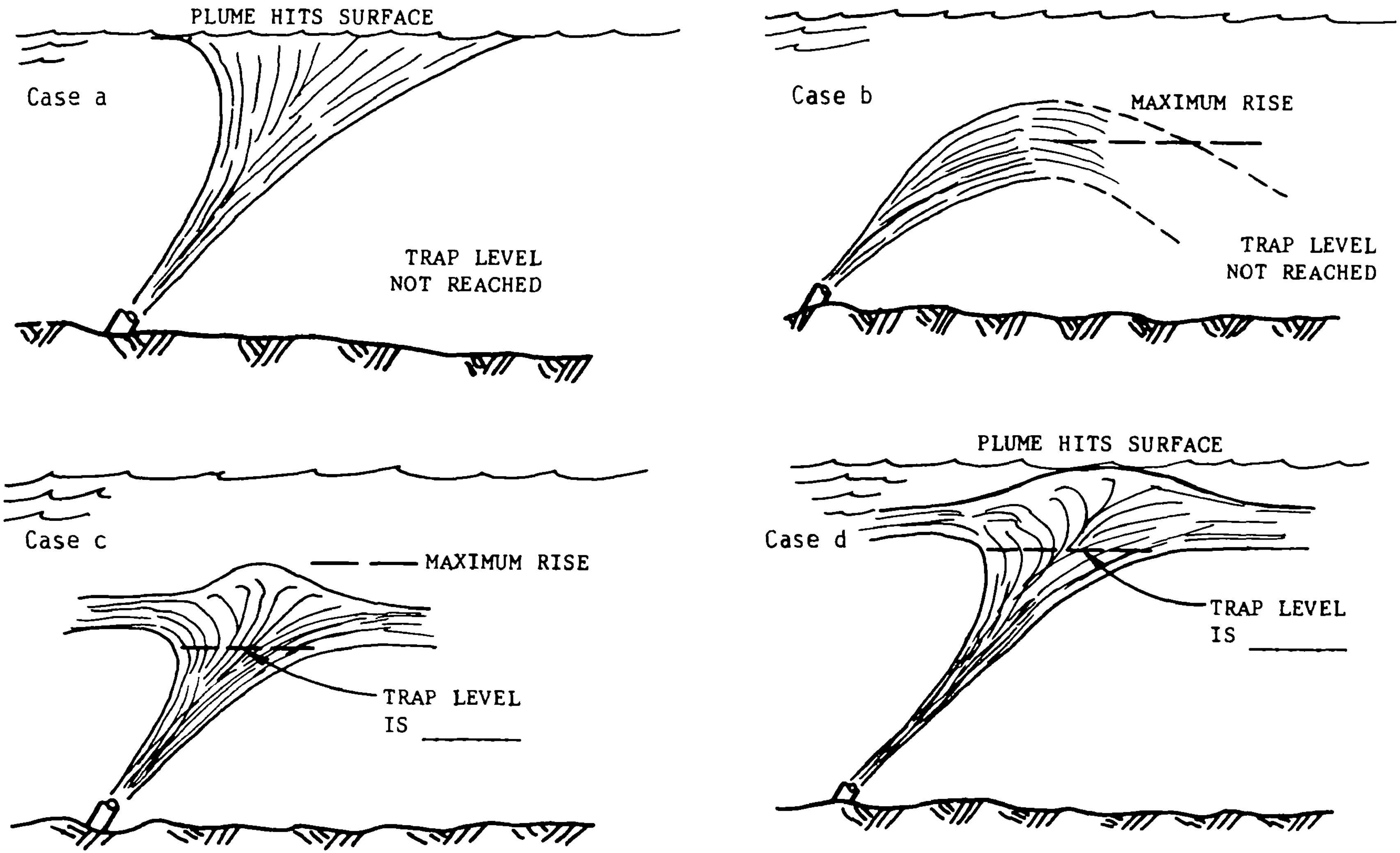

FIGURE 2. Four Possible Plume Configurations Analyzed by Program P L U M E

EXAMPLES

Case Number One

An industrial waste consisting of 0.7 cfs of heated, essentially fresh process water (specific gravity 0.9933) is discharged through a single 4" horizontal port 36.7 feet below the surface of the ocean. A pycnocline occurs between 8.6 feet and 14.1 feet and a slight increase in density occurs with depth below the pycnocline.

It is desired to determine if the plume will reach the surface or not, and what minimum dilution can be expected. Results are desired at 4 feet increments along the plume path. The density gradient data are:

Depth	Density
0	1.02272
8.6	1.02272
14.1	1.02452
36.7	1.02500

Results are shown on the computer printout. Neutral buoyancy was first reached between the 8th and 9th increments (trapping level at Z=13.4 feet). This is a "case c" example.

Case Number Two

Same conditions as example one except specific gravity is 1.040 and port angle is 30°. Anticipating that the plume will not rise very far, calculations are requested for every foot of travel.

********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA********

CASE NO. 1 INITIAL CONDITIONS ......

UNITS: FPS

PORT ANGLE	0
FROUDE NUMBER	197.7
LENGTH FOR FLOW ESTABLISHMENT . . .	1.85
INTEGRATION STEP LENGTH	.229
PRINTOUT INTERVAL	4.00
XO	1.85
ZO	36.63
DISCHARGE DENSITY	.99330
PORT DEPTH	36.70
FLOWRATE	7.00000E-01
NUMBER OF PORTS	1
DISCHARGE VELOCITY	8.18
PORT DIAMETER.	3.30000E-01

DENSITY STRATIFICATION

DEPTH	RHO
0	1.02272
8.60	1.02272
14.10	1.02452
36.70	1.02500

S	X	Z	ELEV	THETA	DILN
5.52	5.48	36.16	.54	14.2	2.9710
9.42	9.02	34.58	2.12	34.4	5.4971
13.32	11.84	31.90	4.80	51.5	8.9473
17.22	13.94	28.63	8.07	62.1	13.3521
21.12	15.55	25.08	11.62	68.5	18.5041
25.02	16.85	21.40	15.30	72.5	24.1900
28.91	17.93	17.65	19.05	75.1	30.1967
32.81	18.87	13.88	22.82	76.7	36.2566
36.71	19.85	10.10	26.60	71.8	
39.07	21.14	8.28	28.42	0	

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL IS 13.5 WITH DILUTION OF 36.8181

Results (following) show that the plume rose 4.16 feet before turning downward due to a residual negative buoyancy in the fluid. This program does not compute the additional dilution achieved in the downward travel, because the models available to describe this portion of the flow field are less reliable and are still under development. This is a "case b" example.

Case Number Three

A nuclear power plant intends to use ocean water for cooling and discharge 2500 cfs of heated water (specific gravity 1.02081) through 93 horizontal ports 1.5 feet in diameter. The water depth at the proposed discharge site is 30 feet. Determine centerline coordinates and dilutions at two-foot intervals when the ambient salinity is 33 parts per thousand and the temperature distribution is as follows:

Depth	Temperature
0	20°C
5	19.7
10	19.4
15	18.9
20	18.7
25	18.6
30	18.3

Results (following) show that plume spreads below the surface after achieving a minimum dilution of 5.1. This also is a "case c" example.

********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA********

CASE NO. 2 INITIAL CONDITIONS ......

UNITS: FPS

PORT ANGLE 30.0

FROUDE NUMBER . . . . -437.4
LENGTH FOR FLOW ESTABLISHMENT . . . 1.83
INTEGRATION STEP LENGTH 215
PRINTOUT INTERVAL 1.00
XO 1.59
ZO 35.78
DISCHARGE DENSITY 1.04000
PORT DEPTH 36.70
FLOWRATE 7.00000E-01
NUMBER OF PORTS 1
DISCHARGE VELOCITY 8.18
PORT DIAMETER. 3.30000E-01

DENSITY STRATIFICATION DEPTH RHO

DEPTH	RHO
0	1.02272
8.60	1.02272
14.10	1.02452
36.70	1.02500

S	X	Z	ELEV	THETA	DILN
2.69	2.33	35.36	1.34	29.3	1.4362
3.77	3.28	34.84	1.86	28.1	1.9984
4.85	4.24	34.35	2.35	26.3	2.5492
5.92	5.21	33.89	2.81	24.1	3.0864
7.00	6.20	33.47	3.23	21.1	3.6091
8.08	7.22	33.12	3.58	17.5	4.1183
9.15	8.26	32.83	3.87	13.1	4.6183
10.23	9.31	32.63	4.07	7.8	5.1178
11.31	10.39	32.54	4.16	1.7	5.6313
11.52	10.60	32.54	4.16	0	5.7370

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL NOT REACHED

********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA********

CASE NO. 3 INITIAL CONDITIONS ......

UNITS: FPS

PORT ANGLE	0
FROUDE NUMBER	1698.0
LENGTH FOR FLOW ESTABLISHMENT . . .	8.40
INTEGRATION STEP LENGTH	.555
PRINTOUT INTERVAL	2.00
XO	8.40
ZO	29.96
DISCHARGE DENSITY	1.02081
PORT DEPTH	30.00
FLOWRATE	2.50000E 03
NUMBER OF PORTS	93
DISCHARGE VELOCITY	15.21
PORT DIAMETER.	1.50000E 00

DENSITY STRATIFICATION DEPTH RHO

DEPTH	RHO
0	1.02326
5.00	1.02334
10.00	1.02342
15.00	1.02354
20.00	1.02359
25.00	1.02362
30.00	1.02369

S	X	Z	ELEV	THETA	DILN
10.07	10.07	29.95	.05	.3	1.1743
12.29	12.29	29.94	.06	.5	1.4334
14.51	14.51	29.92	.08	.6	1.6925
16.73	16.73	29.89	.11	.8	1.9517
18.95	18.95	29.86	.14	1.0	2.2108
21.17	21.17	29.81	.19	1.1	2.4700
23.39	23.39	29.77	.23	1.3	2.7292
25.61	25.61	29.71	.29	1.5	2.9884
27.83	27.83	29.65	.35	1.7	3.2477
30.05	30.05	29.58	.42	1.9	3.5071
32.27	32.27	29.50	.50	2.1	3.7664
34.49	34.49	29.41	.59	2.3	4.0258
36.71	36.71	29.32	.68	2.4	4.2853
38.94	38.92	29.22	.78	2.5	4.5447

41.16	41.14	29.12	.88	2.6	4.8040
43.38	43.36	29.02	.98	2.7	5.0633
45.60	45.58	28.92	1.08	2.7	
47.82	47.80	28.82	1.18	2.6	
50.04	50.02	28.72	1.28	2.5	
52.26	52.23	28.63	1.37	2.3	
54.48	54.45	28.54	1.46	2.0	
56.70	56.67	28.48	1.52	1.6	
58.92	58.89	28.42	1.58	1.1	
61.14	61.11	28.39	1.61	.5	
62.81	62.78	28.39	1.61	0	

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL IS 29.0 WITH DILUTION OF 5.1487

Case Number Four

A municipality discharges 0.126 m^3/sec of raw sewage (s.g.=1.000) through a single 0.76 m diameter port oriented at 15° in 27 meters of water off Florida's coast. Calculate dilutions and coordinates every three meters. The ambient salinity is a constant 36 °/₀₀ over the depth. The temperature is constant at 85°F from zero to 12 meters and 83°F below 13 meters.

Results (following) show the plume spreading at the surface with residual buoyancy and a minimum dilution of 52. This is a "case a" example.

If the temperature data are entered erroneously in °F rather than the °C units as specified in the INPUT DESCRIPTION (page 3), the computed ambient densities would be unrealistically low, and the computed plume results would be erroneous.

Case Number Five

A pulp mill discharges 10.8 cfs of process water (s.g.=1.000) through 26 horizontal 3" ports in 40 feet of water. Surface density is 1.0245 grams/cc while density at 40 feet is 1.0250. Plume calculations are desired at an interval of 3.5 feet.

Results (following) show that the plume hits the surface and also that a trapping level is reached 8.7 feet below the surface. The minimum dilution expected is 64. This is a "case d" example.

The horizontally spreading layer may be exposed at the surface or spread submerged. This program does not include this information

********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA********

CASE NO. 4 INITIAL CONDITIONS ......

UNITS: MKS

PORT ANGLE	15.0
FROUDE NUMBER	.4
LENGTH FOR FLOW ESTABLISHMENT . . .	1.56
INTEGRATION STEP LENGTH	.153
PRINTOUT INTERVAL	3.00
XO	1.50
ZO	26.60
DISCHARGE DENSITY	1.00000
PORT DEPTH	27.00
FLOWRATE	1.26000E-01
NUMBER OF PORTS	1
DISCHARGE VELOCITY	.28
PORT DIAMETER.	7.60000E-01

DENSITY STRATIFICATION DEPTH RHO

DEPTH	RHO
0	1.02267
12.00	1.02267
13.00	1.02307
27.00	1.02307

S	X	Z	ELEV	THETA	DILN
4.45	4.08	25.33	1.67	39.4	3.1353
7.51	5.98	22.96	4.04	60.8	6.6508
10.56	7.19	20.16	6.84	71.3	11.5495
13.61	8.01	17.23	9.77	76.8	17.6167
16.66	8.62	14.24	12.76	80.0	24.7170
19.54	9.07	11.40	15.60	81.3	31.1603
22.59	9.53	8.38	18.62	81.6	36.4967
25.64	9.97	5.36	21.64	81.8	42.0214
28.69	10.40	2.34	24.66	82.1	47.7505
PLUME HITS SURFACE					
31.05	10.72	.00	27.00	82.2	52.3366

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL NOT REACHED

*********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA*********

CASE NO. 5 INITIAL CONDITIONS ......

UNITS: FPS

PORT ANGLE	0
FROUDE NUMBER	356.1
LENGTH FOR FLOW ESTABLISHMENT . . .	1.40
INTEGRATION STEP LENGTH	.233
PRINTOUT INTERVAL	3.50
XO	1.40
ZO	39.97
DISCHARGE DENSITY	1.00000
PORT DEPTH	40.00
FLOWRATE	1.08000E 01
NUMBER OF PORTS	26
DISCHARGE VELOCITY	8.46
PORT DIAMETER.	2.50000E-01

DENSITY STRATIFICATION DEPTH RHO

DEPTH	RHO
0	1.02450
40.00	1.02500

S	X	Z	ELEV	THETA	DILN
4.66	4.64	39.70	.30	9.8	3.2838
8.15	7.95	38.63	1.37	26.7	6.0331
11.64	10.78	36.61	3.39	44.0	9.6010
15.13	12.99	33.92	6.08	56.3	14.2168
18.62	14.70	30.88	9.12	64.2	19.7461
22.11	16.07	27.67	12.33	69.2	25.9852
25.60	17.21	24.37	15.63	72.6	32.7390
29.09	18.18	21.02	18.98	74.8	39.8085
32.58	19.05	17.64	22.36	76.4	46.9702
36.07	19.84	14.24	25.76	77.4	53.9555
39.56	20.59	10.83	29.17	77.9	60.4244
43.05	21.31	7.42	32.58	77.9	
46.54	22.06	4.01	35.99	77.4	
50.03	22.85	.61	39.39	75.9	

PLUME HITS SURFACE

50.66	23.01	-0.00	40.00	75.5

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL IS 8.7 WITH DILUTION OF 64.0537

because the mechanics of mixing in the transition region are less well understood, and the models which exist are not sufficiently well varied.

Case Number Six

Same case as number five except the density at the surface is increase to 1.02499.

Results (following) show that the centerline dilution calculation is very sensitive to the value of the density gradient. In this case - essentially no gradient - the dilution is increased to 107, and no trapping level is reached. Since the plume reaches the surface with residual buoyancy, the spreading lens is considerably thinner, but no estimate of thickness is generally available. This is a "case a" example.

********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA********

CASE NO. J INITIAL CONDITIONS ......

UNITS: FPS

PORT ANGLE 0

FROUDE NUMBER 356.1
LENGTH FOR FLOW ESTABLISHMENT . . . 1.40
INTEGRATION STEP LENGTH 755
PRINTOUT INTERVAL 3.50
XO 1.40
ZO 39.97
DISCHARGE DENSITY 1.00000
PORT DEPTH 40.00
FLOWRATE 1.08000E 01
NUMBER OF PORTS 26
DISCHARGE VELOCITY 8.46
PORT DIAMETER. 2.50000E-01

DENSITY STRATIFICATION DEPTH RHO

DEPTH	RHO
0	1.02499
40.00	1.02500

S	X	Z	ELEV	THETA	DILN
4.42	4.41	39.74	.26	8.8	3.1131
8.20	8.00	38.62	1.38	27.1	6.0814
11.98	11.01	36.36	3.64	45.9	10.0493
15.75	13.28	33.36	6.64	58.9	15.3324
19.53	14.98	29.99	10.01	66.9	21.8086
23.31	16.30	26.45	13.55	72.0	29.3221
27.08	17.35	22.82	17.18	75.4	37.7670
30.86	18.22	19.15	20.85	77.8	47.0715
34.64	18.96	15.44	24.56	79.6	57.1827
38.41	19.59	11.72	28.28	81.0	68.0580
42.19	20.14	7.99	32.01	82.1	79.6614
45.97	20.63	4.24	35.76	83.0	91.9613
49.74	21.07	.49	39.51	83.7	104.9285
PLUME HITS SURFACE					
50.24	21.12	.00	40.00	83.8	106.6648

LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.

TRAPPING LEVEL NOT REACHED

REFERENCES

1. Baumgartner, D. J. and D. S. Trent, "Ocean Outfall Design: Part I, Literature Review and Theoretical Development." FWPCA, April 1970.

2. Trent, D. S., D. J. Baumgartner, and K. V. Byram, "Forced Plumes in Stratified, Quiescent Media." To be published.

3. Abraham, G., "Jet Diffusion in Stagnant Ambient Fluid." Delft Hydraulics Publication. November 29, 1963.

4. Rawn, A. M., F. R. Bowerman, and Norman H. Brooks, "Diffusers for Disposal of Sewage in Sea Water." Proceedings of the American Society for Civil Engineers, Journal of the Sanitary Engineering Division, Volume 86, 1960, pp. 65-105.

APPENDIX I

Program Listing

```
C
C  PROGRAM PLUME, VERSION OF 1/6/71
C
      COMMON G,FK,FM,COSTH,SINTH,COSTHE,DS,C1,C2,F13,FLAG,GRAV
      DIMENSION ZO(50),DG(50),RHO(50),DP(50)
      LOGICAL NDC,TRAPPD,FLAG,CHGDEN,METERS
      NOCASE=1
   20 READ(5,73,END=74)NDC,METERS,NPTS,ANGLE,DIA,DEPTH,RHOJ,RFD,Q,FN,PS
   73 FORMAT(2L1,I3,F5.0,7F10.0)
      FD=ABS(RFD)
      DO 102 I=1,NPTS
      READ(5,75)DP(I),RHO(I),SAL
      IF(SAL.EQ.0.)GO TO 102
      RHO(I)=1.+.001*SIGMAT(SAL,RHO(I))
  102 CONTINUE
      IF(NPTS.NE.1)GO TO 76
      NPTS=2
      DP(2)=DEPTH
      RHO(2)=RHO(1)
   76 NOCASE=NOCASE+1
   75 FORMAT(3F10.0)
      GRAV=32.172
      IF(METERS)GRAV=9.80665
      DO 55 I=1,NPTS
      IF(DP(I).GE.DEPTH)GO TO 56
   55 CONTINUE
      WRITE(6,59)NOCASE
   59 FORMAT(≠-NO DENSITY INFORMATION FOR JET LEVEL.  EXECUTION FOR≠,
     *  ≠ CASE NO.≠,I2,≠ DELETED.≠)
      GO TO 20
   56 NP=I
      NM=I-1
      RHOB=(DEPTH-DP(NM))*(RHO(NP)-RHO(NM))/(DP(NP)-DP(NM))+RHO(NM)
      DISP=RHOJ-RHOB
      DO 54 I=1,NM
      J=NP-I
      ZO(I)=(DEPTH-DP(J))/DIA
   54 DG(I)=(RHO(J+1)-RHO(J))*DIA/(DISP*(DP(J+1)-DP(J)))
      IF(NDC)GO TO 58
      U0=Q/(FN*.7853982*DIA*DIA)
      RFD=U0*U0*RHOJ/(-DISP*DIA*GRAV)
      FD=ABS(RFD)
   58 IF(ANGLE.NE.0.)GO TO 66
      IF(FD.LE.4.01.OR.FD.GE.9.99)GO TO 61
      S=.113*FD+4.
      GO TO 62
   61 IF(FD.LT.4.01)S=2.8*FD**.333333
      IF(FD.GT.10.)S=5.6*FD/SQRT(FD*FD+18.)
   62 THETA0=ATAN(1.416667*S/FD)
      SINTHE=0.
      GO TO 63
   66 NIT=0
      S=2.
   64 IF(NIT.GE.100)STOP
      SO=S
      S=5.6*SQRT(FD/(1.42*S+FD))
```

```
      NIT=NIT+1
      IF(ABS(S-S0).GT..0001)GO TO 64
      THETA0=ANGLE/57.29578
   63 COSTHE=COS(THETA0)
      SINTHE=SIN(THETA0)
      DSI=DEPTH/(177.*DIA)
      IF(DG(1).EQ.0.)GO TO 77
      DGTEMP=.01/DG(1)
      IF(DGTEMP.LE.0.)DGTEMP=-DGTEMP
      DSI=.12*1.6**(ALOG10(FD/10.))*2.**(ALOG10(DGTEMP))
   77 NPO=IFIX(PS/(DSI*DIA)+.5)
      N=0
      Z=S*SINTHE
      X=S*COSTHE
      E=(4./S)**3
      R=.25
      C1=E**.6666667
      C2=.75/RFD
      IPTS=1
      G=DG(IPTS)
      ZLIM=ZD(IPTS)
      CHGDEN=.FALSE.
      FLAG=.FALSE.
      TRAPPD=.FALSE.
      XP=X*DIA
      ZP=DEPTH-Z*DIA
      DSIP=DSI*DIA
      SP=S*DIA
C**
C  WRITE INITIAL CONDITIONS
C**
      WRITE(6,23)NOCASE
   23 FORMAT(≠1*********A BUOYANT PLUME IN A DENSITY STRATIFIED MEDIA****
     ******≠/≠-CASE NO.≠,I2,≠ INITIAL CONDITIONS ......≠)
      IF(NDC)WRITE(6,104)
  104 FORMAT(≠0     UNITS: NON DIMENSIONAL≠)
      IF(.NOT.NDC.AND.METERS)WRITE(6,105)
  105 FORMAT(≠0     UNITS: MKS≠)
      IF(.NOT.NDC.AND..NOT.METERS)WRITE(6,106)
  106 FORMAT(≠0     UNITS: FPS≠)
      WRITE(6,108)ANGLE,RFD,SP,DSIP,PS,XP,ZP,RHOJ,DEPTH
  108 FORMAT(
     *40H-    PORT ANGLE . . . . . . . . . . . . . .,F7.1/
     *40H0    FROUDE NUMBER  . . . . . . . . . . . .,F7.1/
     *40H     LENGTH FOR FLOW ESTABLISHMENT  . . . .,F8.2/
     *40H     INTEGRATION STEP LENGTH  . . . . . . .,F9.3/
     *40H     PRINTOUT INTERVAL  . . . . . . . . . .,F8.2/
     *40H     X0 . . . . . . . . . . . . . . . . . .,F8.2/
     *40H     Z0 . . . . . . . . . . . . . . . . . .,F8.2/
     *40H     DISCHARGE DENSITY  . . . . . . . . . .,F11.5/
     *40H     PORT DEPTH . . . . . . . . . . . . . .,F8.2)
      IF(.NOT.NDC)WRITE(6,60)Q,FN,U0,DIA
   60 FORMAT(
     *40H     FLOWRATE . . . . . . . . . . . . . . .,E15.5/
     *40H     NUMBER OF PORTS  . . . . . . . . . . .,F5.0/
     *40H     DISCHARGE VELOCITY . . . . . . . . . .,F8.2/
```

```
     *40H    PORT DIAMETER. . . . . . . . . . . . .,F15.5)
      WRITE(6,24)((DP(I),RHO(I)),I=1,NPTS)
   24 FORMAT(≠-DENSITY STRATIFICATION DEPTH   RHO≠//10(24XF6.2,F11.5/))
C**
C  SET INITIAL CONDITIONS
C**
      DS=DSI
      WRITE(6,109)
  109 FORMAT(≠1      S         X          Z         ELEV     THETA≠
     *   ,≠      DILN≠)
      GO TO 16
   11 DS=DSI
   45 DELX=COSTH*DS
      DELZ=SINTH*DS
      DELE=FK
      DELT=FM
      SPDS2=S+DS/2.
      DO 10 I=1,2
      CALL SDERIV(SPDS2,E+.5*FK,R+.5*FM)
      DELX=DELX+2.*COSTH*DS
      DELZ=DELZ+2.*SINTH*DS
      DELE=DELE+2.*FK
      DELT=DELT+2.*FM
   10 CONTINUE
      CALL SDERIV(S+DS,E+FK,R+FM)
      ZLAST=Z
      ZINCR=(DELZ+SINTH*DS)/6.
      Z=Z+ZINCR
      IF(CHGDEN)GO TO 41
      IF(Z.GT.ZLIM)GO TO 40
   43 X=X+(DELX+COSTH*DS)/6.
      E=E+(DELE+FK)/6.
      R=R+(DELT+FM)/6.
      S=S+DS
      IF(E.LE.0.)FLAG=.TRUE.
C
C  THIS STOPPING CRITERIA IS BASED ON VELOCITY GOING TO ZERO
C
      IF(FLAG)GO TO 13
   16 CALL SDERIV(S,E,R)
      IF(TRAPPD)GO TO 14
      IF(R.GT.0.)GO TO 15
      RAT=R/(R0-R)
      E13TRP=E13+(E13-E130)
      ZTRAP=DEPTH-(Z+ZINCR*RAT)*DIA
      SMTRAP=.245*(S+DS*RAT)*E13TRP
      TRAPPD=.TRUE.
      GO TO 14
   15 R0=R
      E130=E13
   14 N=N+1
      IF(N-(N/NPO)*NPO.NE.0)GO TO 11
   13 XP=X*DIA
      ELEV=Z*DIA
      ZP=DEPTH-ELEV
      SP=S*DIA
```

```
      DILN=.245*S*E13
      THETA=ARCOS(COSTH)*57.2958
      IF(.NOT.TRAPPD)GO TO 72
      WRITE(6,101)SP,XP,ZP,ELEV,THETA
      GO TO 71
   72 WRITE(6,101)SP,XP,ZP,ELEV,THETA,DILN
  101 FORMAT(4F10.2,F10.1,F10.4)
   71 IF(.NOT.FLAG)GO TO 11
      WRITE(6,65)
   65 FORMAT(*-LAST LINE ABOVE IS FOR MAXIMUM HEIGHT OF RISE.*)
   67 FORMAT(*0TRAPPING LEVEL IS*F10.1,* WITH DILUTION OF*F10.4)
      IF(TRAPPD)WRITE(6,67)ZTRAP,SMTRAP
      IF(.NOT.TRAPPD)WRITE(6,68)
   68 FORMAT(*0TRAPPING LEVEL NOT REACHED*)
      GO TO 21
C
C  FIND NEXT STRATIFICATION AND RECOMPUTE LAST STEP IF NECESS)
C
   40 DS=DS*(ZLIM-ZLAST)/(Z-ZLAST)
      CALL SDERIV(S,E,R)
      CHGDEN=.TRUE.
      Z=ZLAST
      GO TO 45
   41 CHGDEN=.FALSE.
      IPTS=IPTS+1
      IF(IPTS.GT.NM)GO TO 42
      G=DS(IPTS)
      ZLIM=ZD(IPTS)
      GO TO 43
   42 WRITE(6,44)
   44 FORMAT(*0PLUME HITS SURFACE*)
      FLAG=.TRUE.
      GO TO 43
   74 STOP
      END
      SUBROUTINE SDERIV(S,E,R)
      COMMON G,FK,FM,COSTH,SINTH,COSTHE,DS,C1,C2,F13,FLAG,GRAV
      LOGICAL FLAG
      E13=0.
      IF(E.LE.0.)GO TO 3
      E13=E**.3333333
      COSTH=COSTHE*C1/(E13*E13)
      IF(COSTH.LT.1.)GO TO 1
C
C  THIS STOPPING CRITERIA IS BASED ON PLUME BECOMING HORIZONTAL
C  AGAIN
C
      FLAG=.TRUE.
      SINTH=0.
      COSTH=1.
      GO TO 3
    1 SINTH=SQRT(1.-COSTH*COSTH)
    3 FK=C2*S*R*SINTH*DS
      FM=.109*S*E13*G*DS
      RETURN
      END
```

```
FUNCTION SIGMAT(SAL,T)
SIG0     = (((6.8E-6*SAL)-4.82E-4)*SAL+.8149)*SAL-.093
B        = 1.E-6*T*((.01667*T-.8164)*T+18.03)
A        = .001*T*((.0010843*T-.09818)*T+4.7867)
SUMT     = (T-3.98)*(T-3.98)*(T+283.)/(503.57*(T+67.26))
SIGMAT   = (SIG0+.1324)*(1.-A+B*(SIG0-.1324))-SUMT
RETURN
END
```