METHODS FOR ANALYZING EXTREME EVENTS UNDER CLIMATE CHANGE

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SECTION 1

INTRODUCTION

This report summarizes work completed through a cooperative agreement between the National Center for Atmospheric Research (NCAR) and the Environmental Protection Agency (EPA) to study methods for analyzing extreme climate events. The overall goal of this project is to develop statistical models for extreme climate events that will be useful for the construction and application of scenarios of future climate. The results of this study will provide methods for determining how the likelihood of extreme climate events may change as other more general climate parameters (e.g., the mean or variance) change.

Research in the first year focused on the general problem of expressing climate change in terms of the likelihood of extreme events. A statistical "paradigm" for climate change was formulated, and theoretical properties of the relative sensitivity of extreme events were derived. During the second year, these theoretical results were extended to treat more realistic situations for climate variables (e.g., autocorrelation, finite samples). The interpretation of the theory in terms of a "spatial analogue" for climate change was also begun. Research in the third year concentrated on refinements that naturally arise in actual applications or case studies of extreme climate events. For instance, the accuracy of approximations based on extreme value theory was investigated for extreme maximum and minimum temperature events, and a specialized treatment for extreme precipitation events was devised. Finally, as another analogue for climate change, the so-called "heat island effect" was considered.

These accomplishments are summarized in greater detail in Section 2. Their possible extensions and their implications for the generation of scenarios of future climate are discussed in Section 3. A list of papers produced under this cooperative agreement is given in Appendix 1, and a reprint of an article that appeared in *Climatic Change* is included as Appendix 2.

SECTION 2

SUMMARY OF ACCOMPLISHMENTS

2.1. General theory

Appreciation of the need for a statistical paradigm for climate change arises when considering how the relative frequency of extreme events might change as more conventional statistics, such as the mean or standard deviation, change. A climate variable X is assumed to have a probability distribution with a *location parameter* μ and a *scale parameter* σ. If this distribution were the normal, then μ would be the mean and σ the standard deviation. Climate change is envisioned to involve a combination of two different statistical operations: (i) the distribution is shifted, producing a change in location (μ); and (ii) the distribution is rescaled, producing a change in scale (σ).

Figure 1 illustrates this concept for one hypothetical choice of distribution. The two forms of climate change are included: (i) a change in the location parameter μ to a new, in this case larger, value μ^*; and (ii) a change in the scale parameter σ to a new, in this case larger, value σ^*. Especially noteworthy is how much these distributions differ in the tails, the shape of which determines the probability of extreme events. Katz (1991) treated this statistical paradigm for climate change in more detail.

Some standard statistical theory for extremes can be applied to reveal some broad generalizations that can be made about the relative sensitivity of extreme events to the location and scale parameters. Attention is focused on two specific types of extreme events:

(i) the exceedance of a threshold [event $E_1 = \{X > c\}$, where the constant c denotes a threshold]; and

(ii) the maximum of a sequence of length n exceeding a threshold [event $E_2 = \{\max(X_1, X_2, ..., X_n) > c\}$].

The *sensitivity* of an extreme event to the location parameter μ or the scale parameter σ is defined to be the corresponding partial derivative of the probability of the event; that is, $\partial P(E)/\partial\mu$ or $\partial P(E)/\partial\sigma$. Because extreme events vary in their likelihood, it is reasonable to deal with the *relative sensitivity*, $[\partial P(E)/\partial\mu]/P(E)$ or $[\partial P(E)/\partial\sigma]/P(E)$, comparing the sensitivity of an event to its probability.

Katz and Brown (1992a) show that the relative sensitivity of an extreme event (either E_1 or E_2) to the scale parameter σ becomes proportionately greater than its relative sensitivity to the location parameter μ as the event becomes more extreme (i.e., the larger the threshold c). Moreover, in many instances, the relative sensitivity of an extreme event to both μ and σ increases as the event becomes more extreme. These theoretical properties are illustrated in the next subsection for an extreme temperature example.

Some of the theoretical results for extreme event E_2 (i.e., the maximum of a sequence exceeding a threshold) are based on large sample approximations that do not directly take into account certain prominent statistical features, like autocorrelation, of climate time series. Nevertheless, simulation studies show that these results are actually quite robust (Katz and Brown, 1992b). These results are also presented in the next subsection.

2.2. Extreme temperature example

Extreme high temperature events of a form known to be deleterious to the corn crop in the midwestern U.S. are considered (Mearns *et al.*, 1984). The July time series of daily maximum temperature at Des Moines, Iowa is utilized (mean $\mu \approx 30$ °C, standard deviation $\sigma \approx 3.9$ °C). Figure 2 shows plots of the relative sensitivity of extreme event E_1 to μ and σ as the threshold c increases (i.e., as the event becomes more extreme). In this application, the event E_1 corresponds to the temperature exceeding a threshold on a given day in July. These curves are based on the assumption of a normal distribution for daily maximum temperature. The relative sensitivity of E_1 to μ increases at an approximately linear rate, whereas the relative sensitivity to σ increases at an approximately quadratic rate, for large threshold c.

Figure 3 shows plots of the relative sensitivity of extreme event E_2 to μ and σ as the threshold c increases. In this application, the event E_2 corresponds to the temperature ever exceeding the threshold within the entire month of July (i.e., $n = 31$). These curves are based on the assumption of a Type I extreme value distribution for the maximum of a sequence [i.e., a distribution function of the form $G(x) = \exp(-e^{-x})$] (Katz and Brown, 1992a). The relative sensitivity of E_2 to μ is approximately constant, whereas the relative sensitivity to σ increases at an approximately linear rate, for large c.

To convert these results into more concrete terms, Table 1 gives the probability of event E_1 for a threshold of $c = 38$ °C, when μ and σ are changed by $\pm$ 0.5 °C. Relative to the current probability of 0.020, $P(E_1)$ changes by roughly twice as much for a change in σ as for the corresponding change in μ. Table 1 also includes the probability of event E_2 for the same threshold and changes in μ and σ. Again, the relative changes in $P(E_2)$ are roughly twice as large when σ is varied as when μ is varied. Unlike event E_1 which always remains rare for all of the values of μ and σ considered, event E_2 becomes quite likely when σ is increased and somewhat rare when σ is decreased.

As mentioned previously, the Type I extreme value distribution serves only as an approximation in determining $P(E_2)$. By means of a simulation study, the exact relative sensitivity of the maximum of a finite sequence of a normally distributed, autocorrelated time series can be determined. It is more convenient to actually perform the simulation in terms of the hazard rate of the exact distribution of the maximum (i.e., the hazard rate H for a distribution function F is $H(x) = F'(x)/[1 - F(x)]$). Katz and Brown (1992a) established that this hazard rate curve has the same shape as the relative sensitivity of the extreme event E_2 to the mean μ.

Figure 4 shows the simulated hazard rate for the exact distribution of the maximum of a sequence of length $n = 30$ with a first-order autocorrelation coefficient of $\phi = 0.5$ (for the Des Moines application, $n = 31$ and $\phi \approx 0.58$). For comparison sake, the hazard rate for the Type I extreme value distribution (i.e., equivalent to the dashed curve in Figure 3) and the hazard rate for the exact distribution of the maximum of a sequence of $n = 30$ normally distributed, independent observations (i.e., $\phi = 0$) are also included in Figure 4. It is evident that these curves are quite similar, with any discrepancies for the exact curve being in the direction of even more sensitivity than either the asymptotic theory or the exact theory under independence would predict. Katz and Brown (1992b) treated this issue in more detail.

2.3. Regional analysis/Spatial analogue

One approach to the interpretation of the assumptions on which the theoretical results presented in Section 2.1 are based involves the so-called "spatial analogue" for climate change. Actual differences in climate across space are substituted for hypothetical changes over future time horizons. This concept is similar to the "regional analysis" approach that is employed in hydrology, for instance, to estimate flood probabilities.

Time series of daily maximum temperature for July at 30 sites in the U.S. Midwest and of daily minimum temperature for January at 28 sites in the U.S. Southeast were subjected to such a regional analysis. Figure 5 gives a plot of the relative frequency of the maximum temperature on a given day in July exceeding a threshold of c = 35 °C (i.e., event E_1) versus the standardized threshold of $(c - \mu)/\sigma$ for each of the 30 stations in the Midwest. Here the location and scale parameters, μ and σ, were estimated using the sample means and standard deviations of the July daily maximum temperatures for the individual stations. The points fall remarkably close to a smooth decreasing curve, in agreement with our statistical paradigm for climate change (Section 2.1). Similar results were obtained for the analogous case of January minimum temperature in the Southeast (Brown and Katz, 1991).

As an additional check, Figure 6 gives a plot of the relative frequency of the temperature ever exceeding c = 35 °C during the entire month of July (i.e., event E_2) versus the same standardized threshold. The plot has a greater degree of scatter than Figure 5, in part because these relative frequencies are based on a much smaller sample (i.e., only one observation for each July instead of 31 for event E_1). Nevertheless, the indication of an underlying relationship is present. Again, similar results were obtained for minimum temperature (Brown and Katz, 1991).

These checks have served so far to help interpret our statistical paradigm for climate change. Additional conditions, however, were imposed in examining the relative sensitivity of extreme events. In particular, the relative sensitivity of extreme event E_2 (i.e., the maximum of a sequence exceeding a threshold) was derived through the Type I extreme value approximation (Figure 3). Further regional analysis of the same daily time series of July maximum temperature was performed to investigate whether this approximation is appropriate. When the two parameters of the Type I extreme value distribution are derived indirectly by assuming a normal distribution for daily maximum temperature, the approximation is quite inaccurate (Brown and Katz, 1992). Although this behavior indicates that the conventional assumptions about the statistical properties of time series of daily maximum temperature do not all hold, it does not necessarily conflict with our statistical paradigm for climate change.

In this regard, Figure 7 shows the results when the parameters are estimated directly from the monthly maxima. The event E_2 of the maximum exceeding a threshold of $c \approx 37.8$ °C is considered for each of the 30 midwestern sites. There is reasonably good agreement between the theoretical probabilities based on the Type I extreme value distribution and the observed relative frequencies. For minimum temperatures in the Southeast, the Type I approximation is not as accurate, with the Type III extreme value distribution (i.e., a distribution function of the form $G(x) = \exp[-(-x)^\alpha]$, $\alpha > 0$, $x < 0$) being a considerable improvement. Although no simple physical explanation for this disagreement exists, such behavior for daily minimum temperature extremes has been noted previously (e.g., Faragó and Katz, 1990). Because the

Type III distribution requires an additional shape parameter, our statistical paradigm for climate change might need to be made more complex to encompass this situation. Brown and Katz (1992) provided further details on these issues.

2.4. Extreme precipitation events

Katz and Garrido (1992) employed a somewhat more specialized approach to address the issue of how the frequency of extreme precipitation events might change with an overall change in climate. Such an approach is required, because the forms of distribution generally fit to precipitation totals do not satisfy the location and scale model on which our statistical paradigm for climate change is based (Section 2.1). Nevertheless, results that are qualitatively similar to those stated in Section 2.1 and illustrated in Section 2.2 can still be obtained in this case.

The climate variable, Y say, represents the precipitation totaled over a month or season. The extreme event of interest is the total precipitation exceeding a threshold c, say $E = \{Y > c\}$, referred to as the "right-hand tail event." Analogous to the approach followed in Section 2.1, the relative sensitivity of event E is defined with respect to the median (rather than the location parameter or mean) and to the scale parameter of the distribution of total precipitation Y. The median is adopted as a measure of central tendency, because the distribution of total precipitation has a substantial degree of positive skewness.

A technique based on a power transformation to normality is employed to account for this skewness. Here $X = Y^s$, for some s, $0 < s < 1$, is assumed to have a normal distribution with mean μ_X and variance σ_X^2 [written $N(\mu_X, \sigma_X^2)$] (Katz and Garrido, 1992). Let m_Y denote the median of the distribution of total precipitation Y [i.e., m_Y satisfies $F_Y(m_Y) = 1 - F_Y(m_Y) = 1/2$, where F_Y denotes the distribution function of Y]. This median m_Y is related to the mean μ_X of the normal distribution for the transformed variable X by $m_Y = \mu_X^r$, where $r = 1/s$. Analogous to the relative sensitivity to the location parameter previously treated (Section 2.1), the relative sensitivity of the right-hand tail event E to the median m_Y is $[\partial P(E)/\partial m_Y]/P(E)$.

The scale parameter, λ say, of the distribution of total precipitation Y is introduced by considering a new random variable $Y_\lambda = \lambda Y$, $0 < \lambda < \infty$, representing a multiplicative effect. The relative sensitivity of the right-hand tail event E to the scale parameter is defined as $\{[\partial P(E_\lambda)/\partial \lambda]/P(E_\lambda)\}|_{\lambda=1}$, where $E_\lambda = \{Y_\lambda > c\}$ is the analogous extreme event for the rescaled variable Y_λ. The partial derivative is evaluated at $\lambda = 1$, because this value corresponds to an instantaneous change from the current climate. In effect, an additional parameter λ has been included in the power transform distribution of total precipitation Y (Katz and Garrido, 1992).

Figure 8 shows the relative sensitivity of the right-hand tail event E to the median and to the scale parameter for summer total precipitation at Segovia, Spain. Both relative sensitivities increase as the event becomes more extreme (i.e., as the threshold c increases). Moreover, the relative sensitivity to the scale parameter is greater than that to the median for virtually all values of the threshold c, and it increases at a much faster rate as c increases. Similar results are obtained for the relative sensitivity of the left-hand tail event (i.e., $\{Y < c\}$) to the median and to the scale parameter (Katz and Garrido, 1992).

2.5. Heat island effect

Another analogue for climate change is the "urban heat island," in which real temporal climate changes associated with human activity have been produced inadvertently for local environments. As metropolitan areas develop, a warming can occur that is comparable in magnitude to that anticipated for the enhanced greenhouse effect (i.e., 2-3 °C according to Changnon, 1992). This heat island has been detected in cities across the globe, ranging from the tropics to high latitudes and, to a lesser extent, in relatively small communities.

Research on the heat island effect has dwelt on average temperatures, with little mention of any changes in variability or in the frequency of extreme events. One notable exception is the recent work by Balling *et al.* (1990). They examined the trend in the occurrence of extreme maximum and minimum temperatures at Phoenix, Arizona, an area that has experienced a marked heat island effect in recent decades. Among other things, the inadequacy of a statistical model for climate change in which simply the mean (or location parameter) is allowed to change (i.e., the variance or scale parameter is held constant) was established. Although changes in the mean are apparently sufficient to explain the trend in occurrence of extreme minimum temperatures, such a model overestimates the frequency of extreme maximum temperatures.

A reanalysis of the same Phoenix temperature data has been performed. The goal is to establish whether our statistical paradigm for climate change (Section 2.1), allowing for a change in variance or scale, could satisfactorily explain the observed trend in the occurrence of extreme maximum temperatures. Figure 9 shows the trend in summer (July-August) standard deviation of minimum and maximum daily temperatures. For minimum temperature, the slight apparent increase in standard deviation makes only a minor contribution to the decrease in the frequency of extreme low temperature events. On the other hand, the more substantial decrease in the standard deviation for maximum temperature does make a major contribution to the frequency of extreme high temperature events. In particular, it explains why the change in the mean alone results in too high a frequency of extreme high temperatures. Tarleton and Katz (1993) included further details on this example.

The urban heat island, thus, provides a real-world application in which changes in variability need to be taken into account to anticipate changes in the frequency of extreme events. Of course, as pointed out by Balling *et al.* (1990), the heat island effect is not necessarily analogous to the enhanced greenhouse effect. Further, being situated in a desert region, the heat island effect for Phoenix is not necessarily typical of that for other urban areas.

SECTION 3

EXTENSIONS AND IMPLICATIONS

A myriad of ways exist in which this study of extreme events and climate change could be extended. For instance, all of our work has concentrated on time series of a single climate variable treated in isolation. It would be natural to consider simultaneously the extremes of two or more variables, either different variables (such as temperature and precipitation) for the same site or the same variable at several locations (e.g., fields of temperature or precipitation). Other more specialized approaches could also be taken. For example, extreme precipitation events could be more systematically studied by developing an underlying stochastic model (i.e., on a daily or hourly time scale) for the precipitation process. It would be more physically meaningful to change these basic model parameters (e.g., the frequency or intensity of "storms"), using probabilistic methods to induce the effects on any extreme events of interest.

As it stands, this study has significant implications for scenarios of future climate. Stochastic weather generators that are convenient to employ for simulating climate variables, such as daily maximum and minimum temperature and precipitation amounts, do exist (e.g., Richardson, 1981). Such models were originally intended to be used to simulate time series for the present climate. However, these models have not been extensively validated with respect to their ability to reproduce the frequency of extreme events. In particular, our results concerning the lack of fit of the Type I extreme value distribution to extreme minimum temperature events in the U.S. Southeast (Section 2.3) would appear to cast doubt upon the use of Richardson's model when extremes are of importance. Specifically, this model essentially represents both time series of daily minimum and maximum temperature as first-order autoregressive processes having normal distributions. Under these assumptions, the Type I extreme value distribution is known to be a good approximation for the maximum or minimum of a sequence (i.e., as generated by Richardson's model). In other words, the Type III extreme value distribution that sometimes arises in practice as a better fit to extreme minimum temperature events could not be reproduced by Richardson's model.

Recently, it is becoming increasingly popular to utilize these same stochastic weather generators to simulate time series for a changed climate (Wilks, 1992). But it would be potentially misleading to apply such models by changing only those parameters (e.g., mean values) for which some information about future changes is currently available. For instance, Richardson's model requires the specification of the means and variances of the normal distributions for daily maximum and minimum temperature, as well as their contemporaneous cross correlation coefficient and individual first-order autocorrelation coefficients (technically, these parameters are conditional on whether or not precipitation occurs). As our results convincingly establish, attention needs to be devoted to how these other model parameters (especially variances) might change as well. Without this information, stochastic weather generators may produce scenarios of future climate whose frequency of extremes turns out to be far off the mark.

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TABLE 1

Probability of extreme events, E_1 and E_2, with threshold of c = 38 °C associated with changes in mean μ and standard deviation σ of July daily maximum temperatures at Des Moines, Iowa (current climate of μ = 30 °C and σ = 3.9 °C) [Source: Katz and Brown, 1992a].

Change in μ (°C)	Change in σ (°C)	$P(E_1)$ (Relative Change)	$P(E_2)$ (Relative Change)
0	0	0.020	0.492
+0.5	0	0.027 (+34.7%)	0.612 (+24.5%)
0	+0.5	0.034 (+70.8%)	0.713 (+44.9%)
-0.5	0	0.015 (-27.7%)	0.384 (-22.0%)
0	-0.5	0.009 (-54.0%)	0.264 (-46.2%)

Figure 1. Hypothetical distribution of a climate variable with location parameter μ and scale parameter σ (solid line); location parameter μ^* and scale parameter σ (dashed line); and location parameter μ and scale parameter σ^* (long and short dashed line) [Source: Katz and Brown, 1992a].

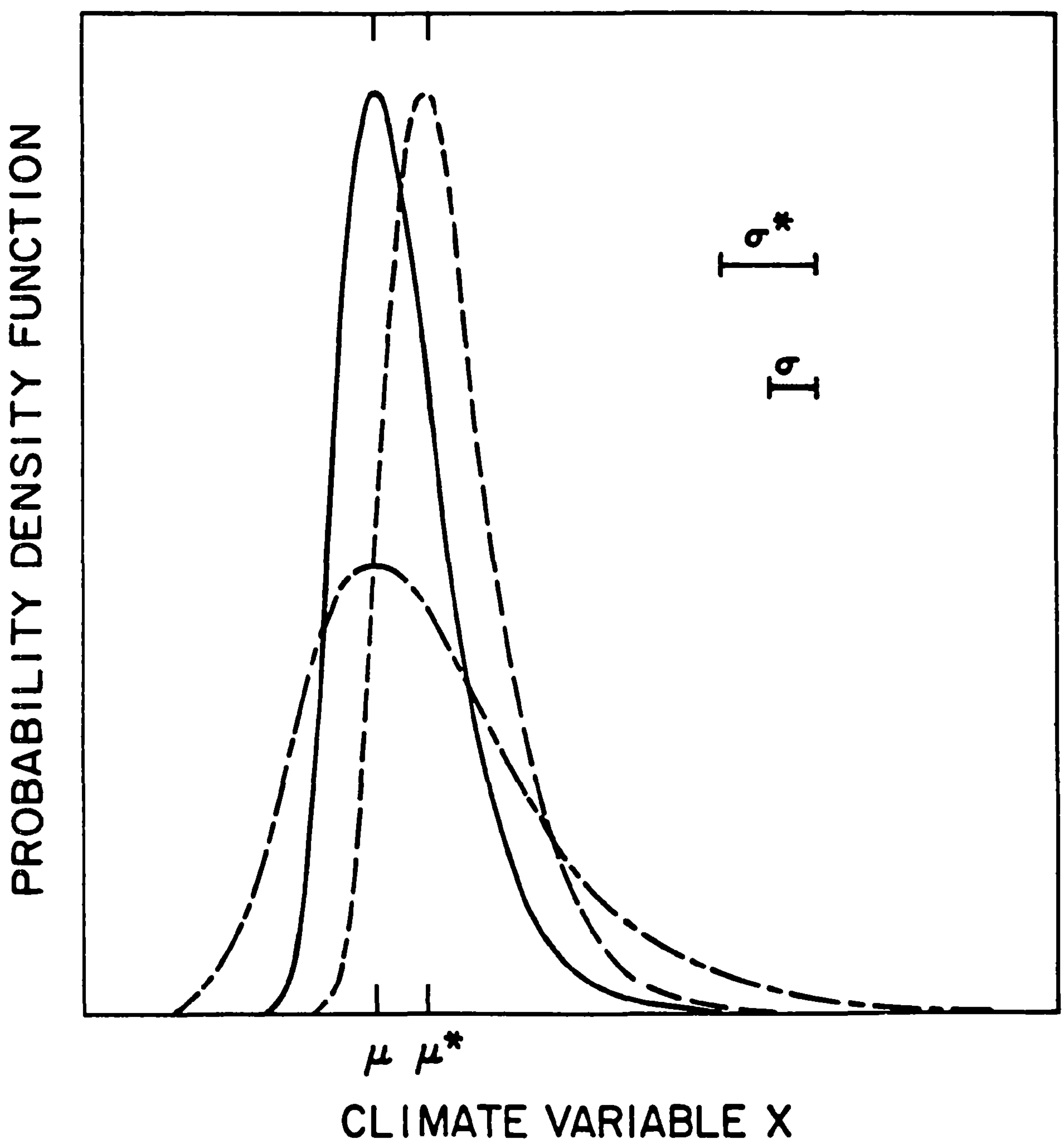


Figure 2. Relative sensitivity of extreme event E_1, temperature exceeding threshold on given day in July, to mean (dashed line) and standard deviation (solid line) [Source: Katz and Brown, 1992a].

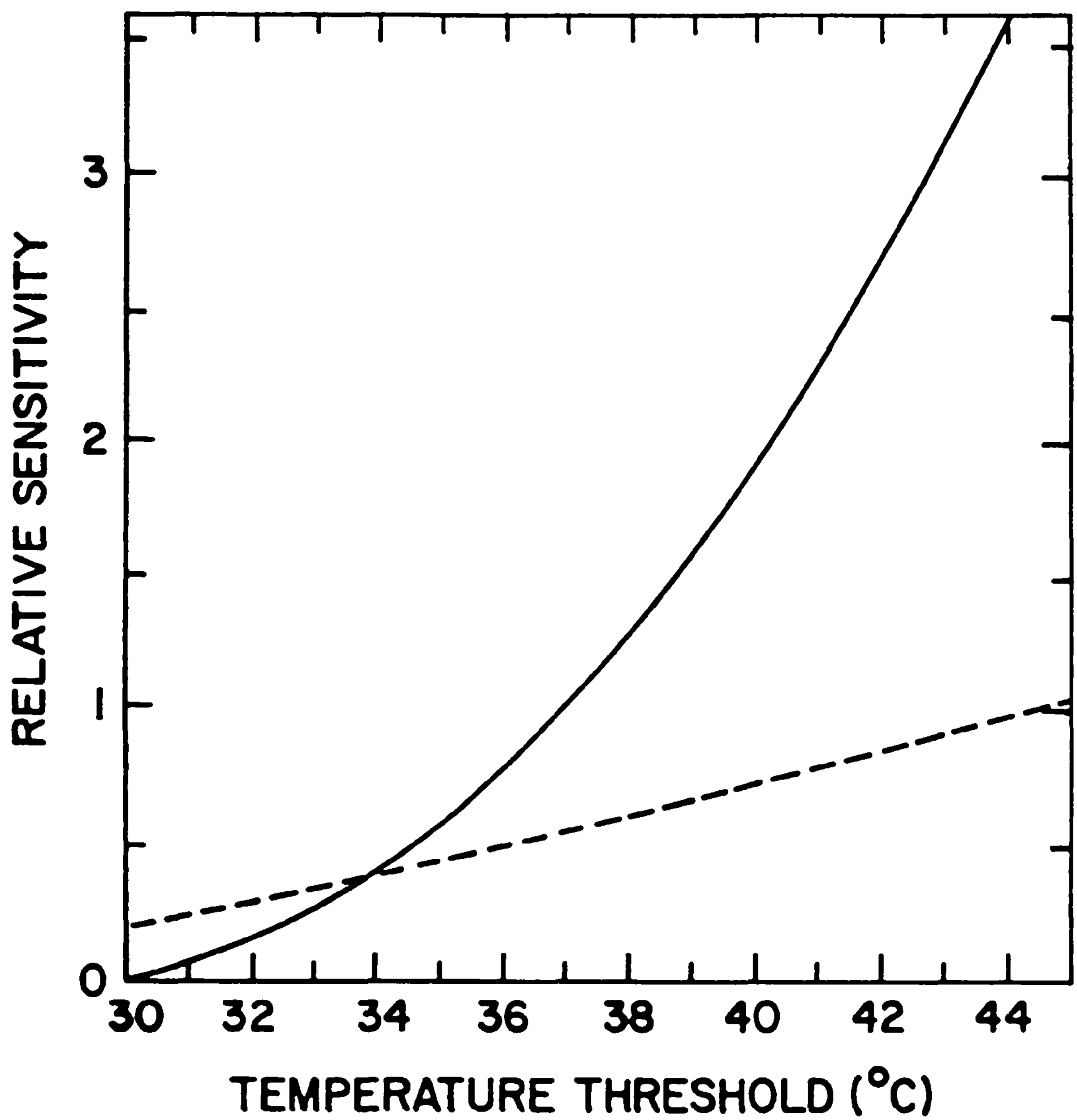


Figure 3. Relative sensitivity of extreme event E_2, temperature ever exceeding threshold during entire month of July, to mean (dashed line) and standard deviation (solid line) [Source: Katz and Brown, 1992a].

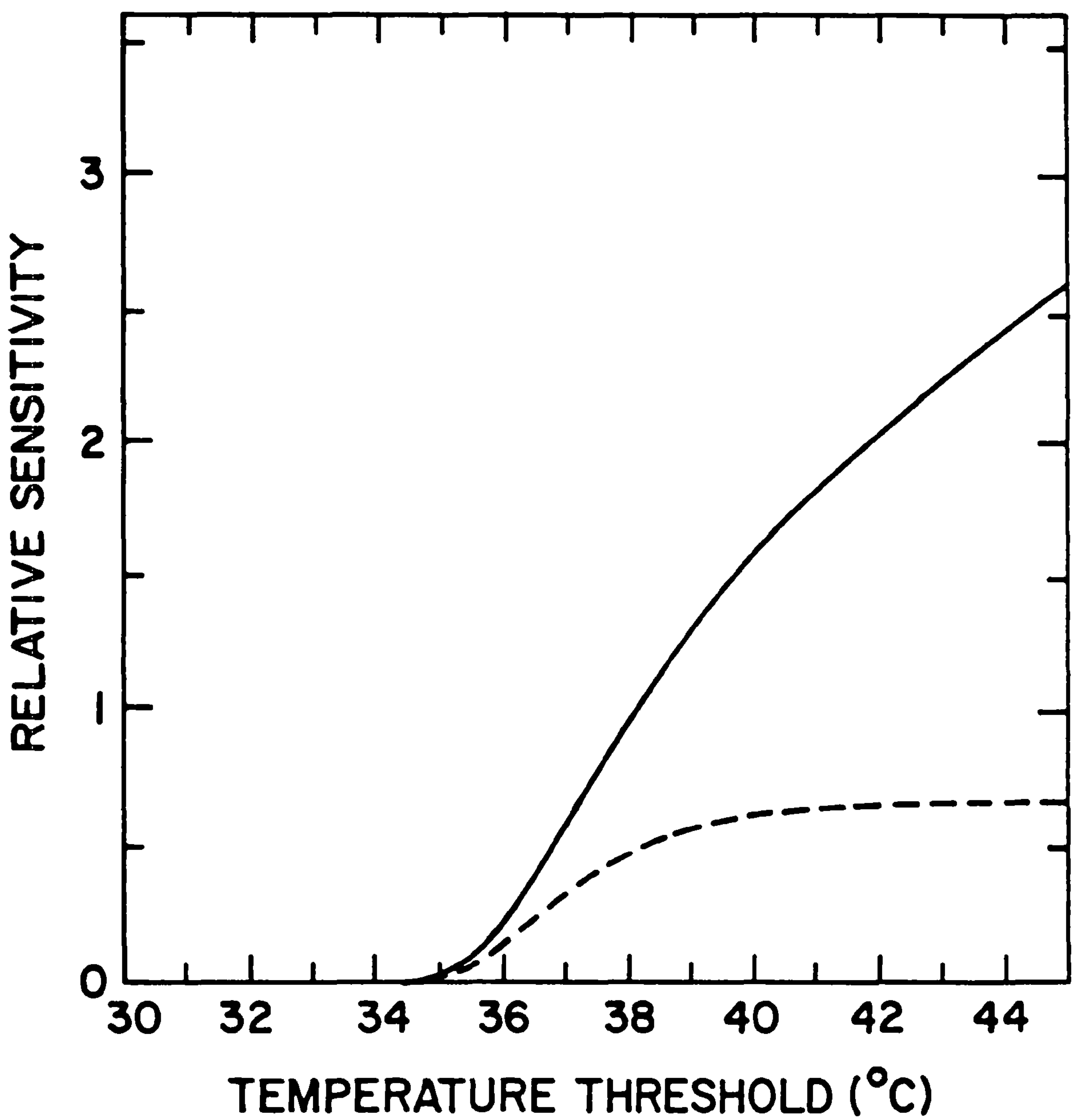


Figure 4. Simulated hazard rate for the exact distribution of the maximum (dotted curve) from a time series of length $n = 30$ with a first-order autocorrelation coefficient $\phi = 0.5$. Curves showing theoretical hazard rates for the exact distribution of the maximum under independence (dashed curve) and for the Type I extreme value distribution (solid curve) are included for comparison [Source: Katz and Brown, 1992b].

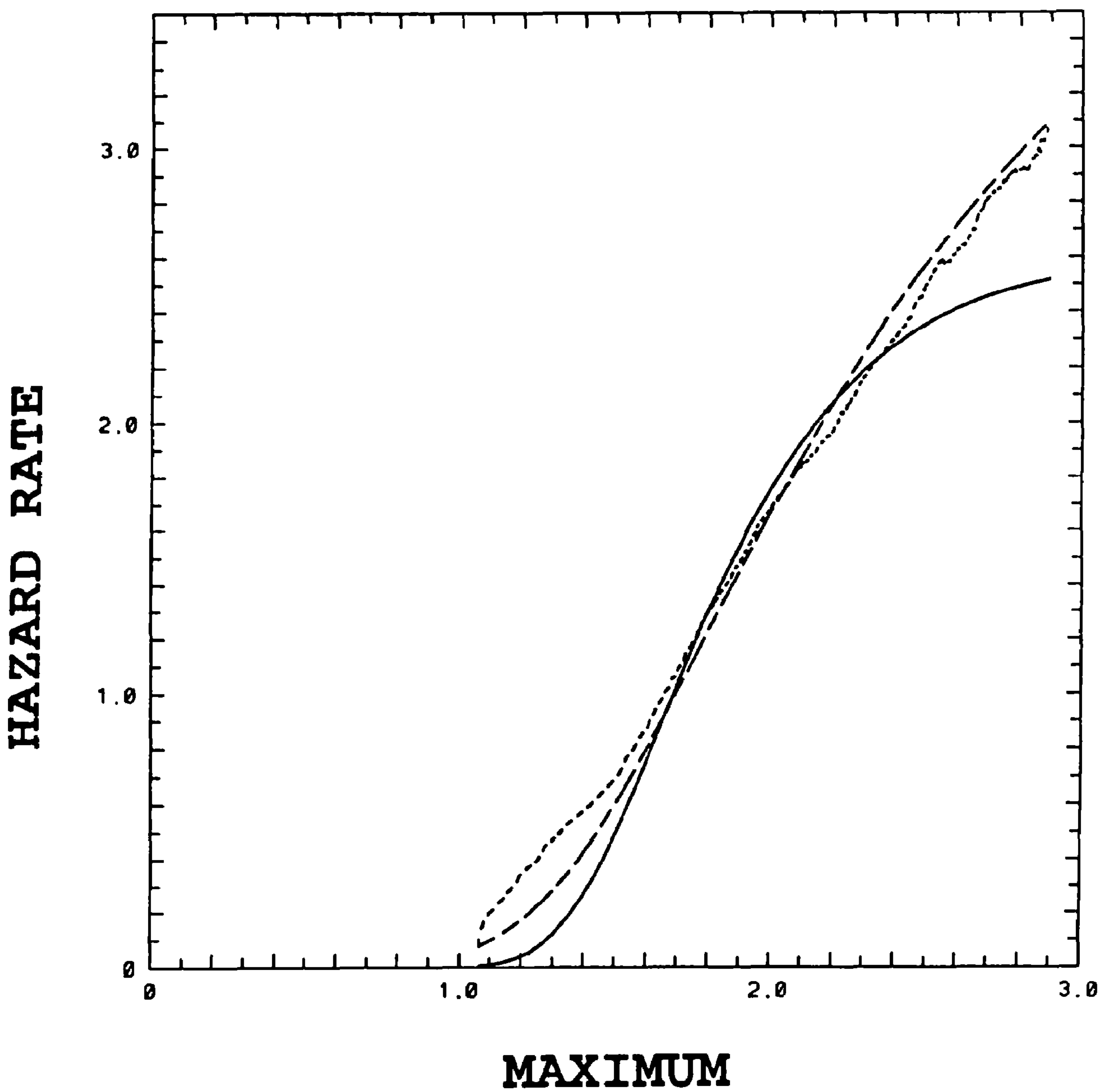


Figure 5. Relative frequency of extreme event E_1, temperature exceeding c = 35 °C on given day in July, versus standardized threshold $(c - \mu)/\sigma$ for 30 stations in the U.S. Midwest [Source: Katz and Brown, 1992a].

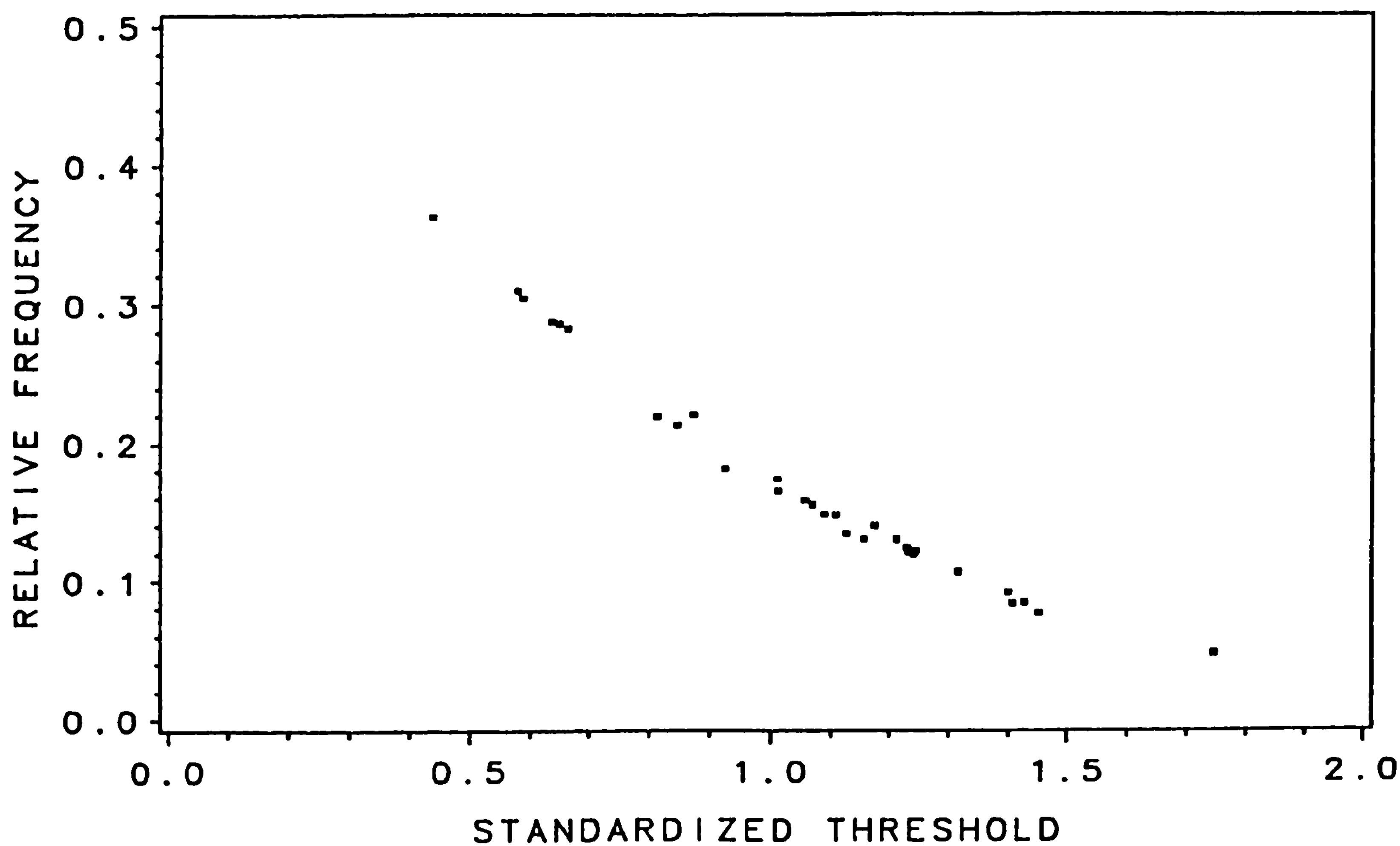


Figure 6. Relative frequency of extreme event E_2, temperature ever exceeding c = 35 °C during entire month of July, versus standardized threshold $(c - \mu)/\sigma$ for 30 stations in the U.S. Midwest [Source: Katz and Brown, 1992a].

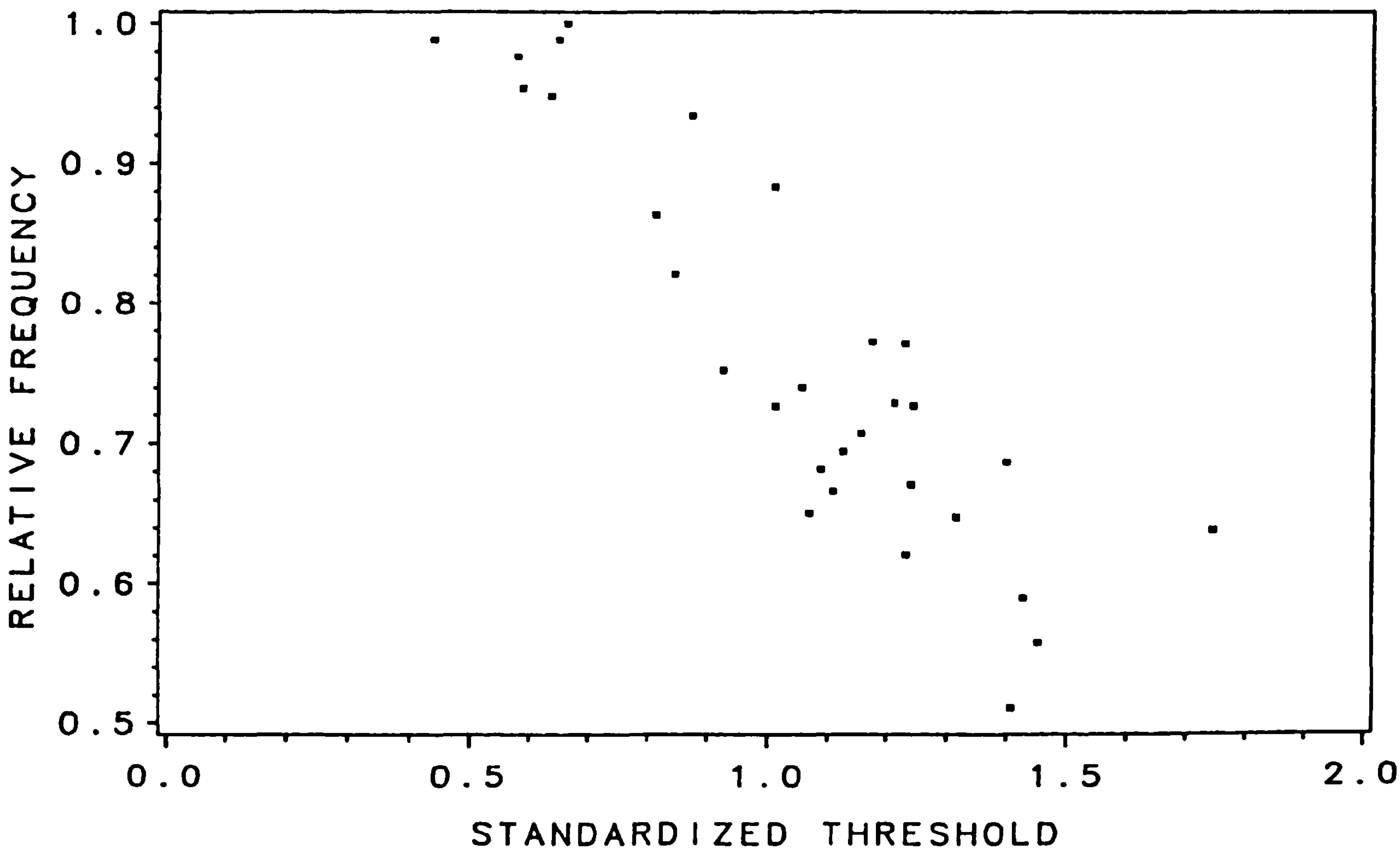


Figure 7. Observed relative frequencies of extreme event E_2, temperature in July ever exceeding $c \approx 37.8$ °C, versus probabilities estimated using Type I extreme value distribution with location and scale parameters fit directly to monthly maxima (for 30 stations in the U.S. Midwest) [Source: Brown and Katz, 1992].

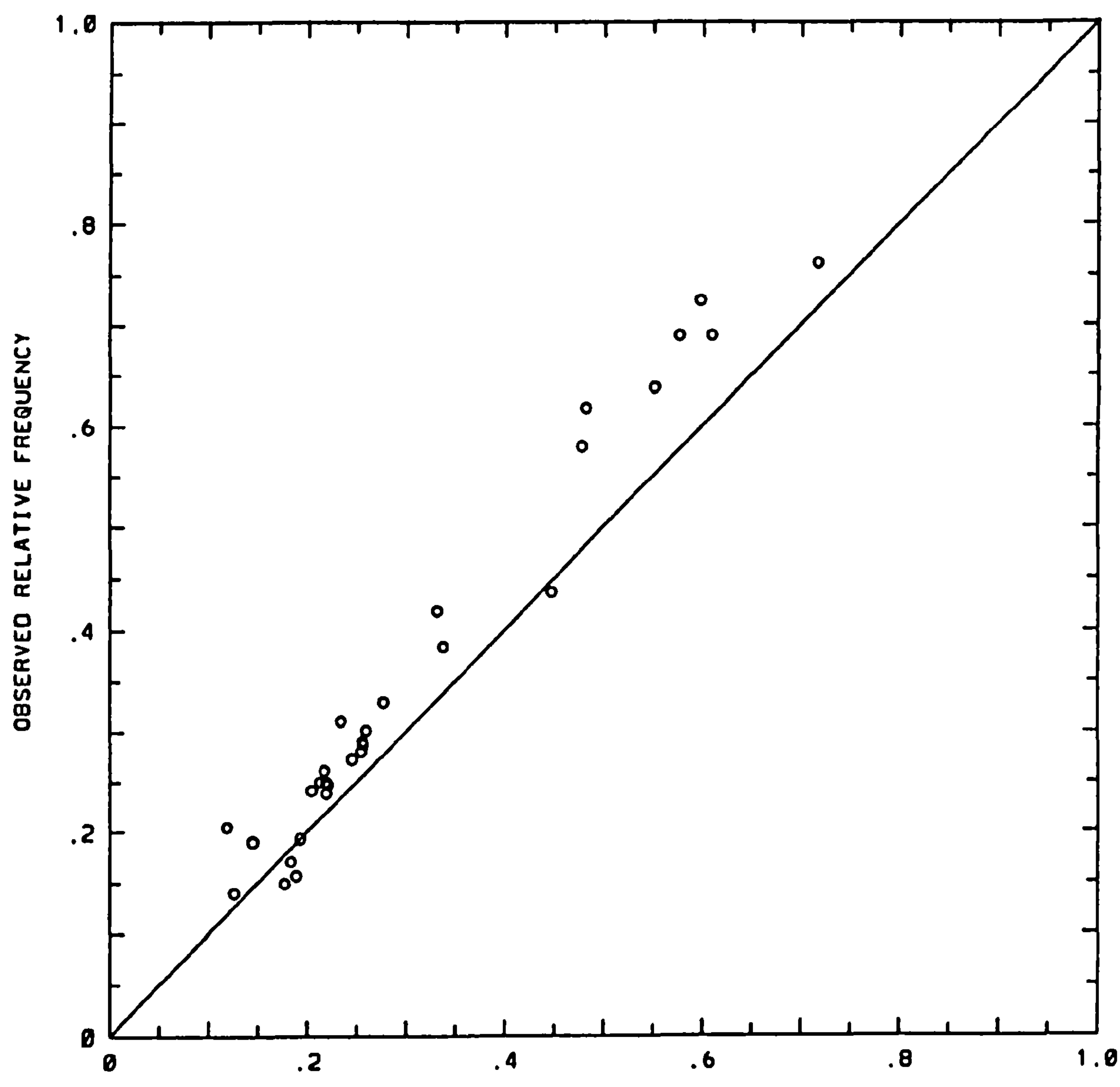


Figure 8. Relative sensitivity of right-hand tail event E (i.e., summer total precipitation exceeding threshold c) to the median (solid line) and to the scale (dashed line) as a function of c at Segovia, Spain [Source: Katz and Garrido, 1992].

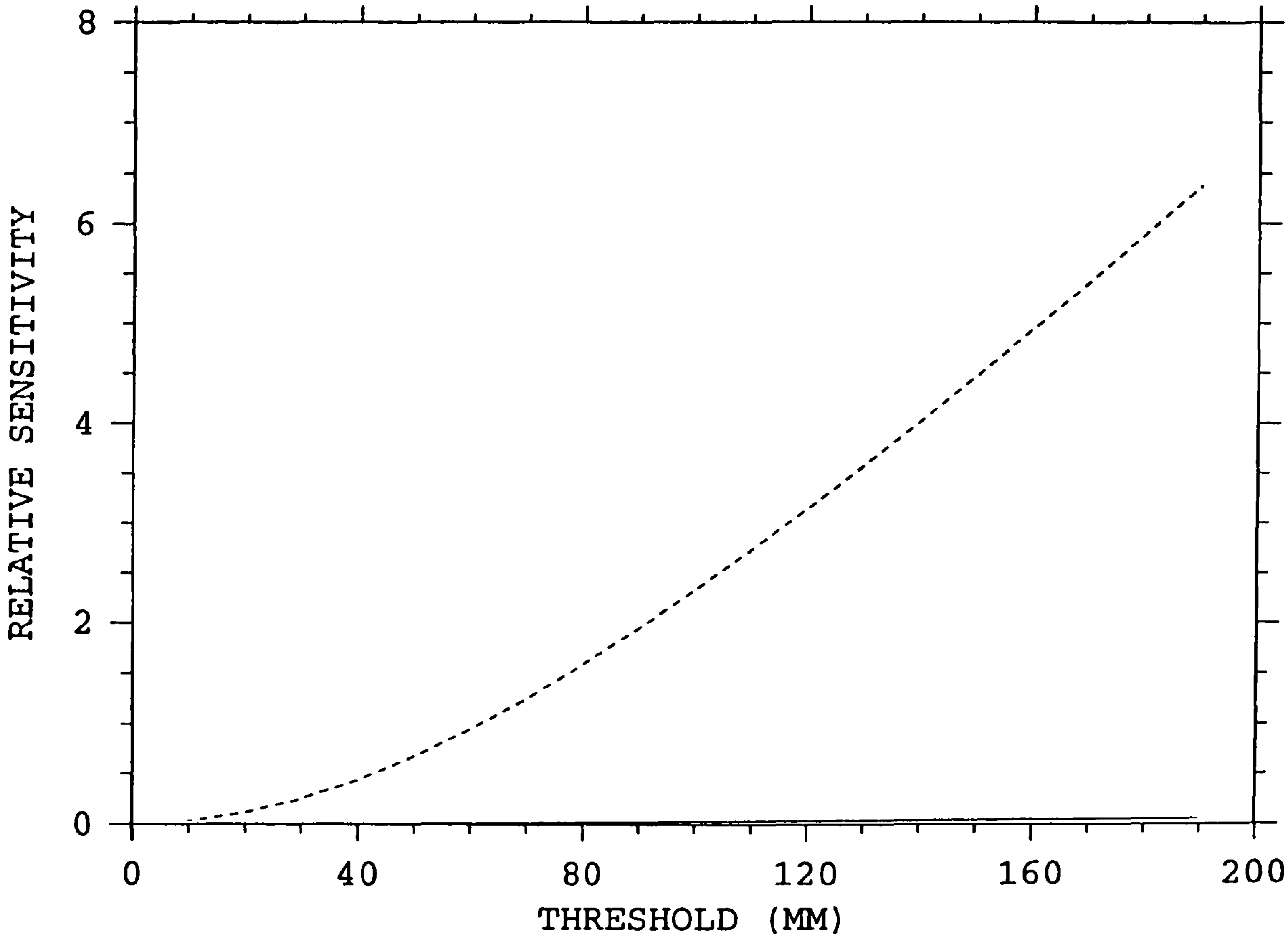
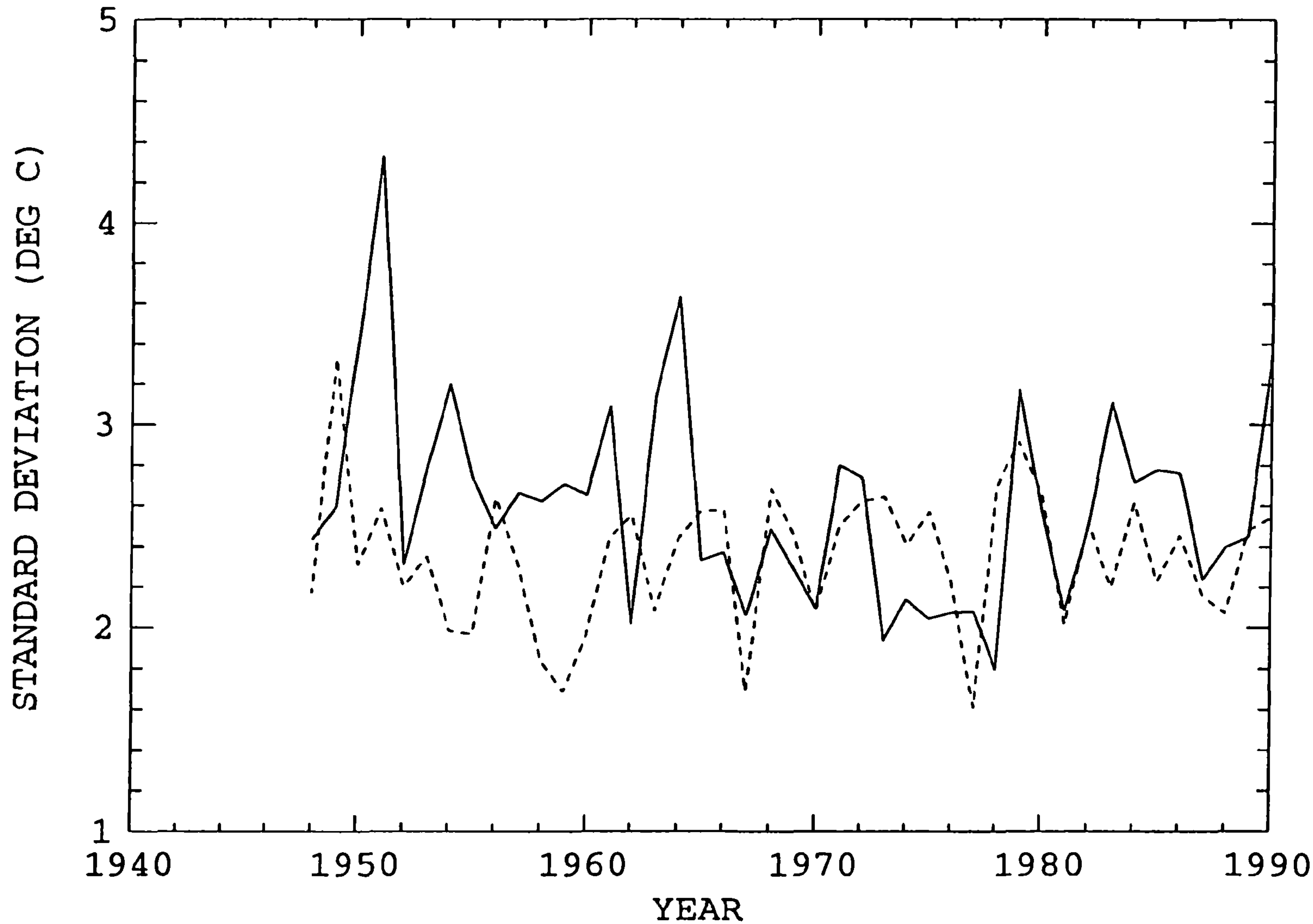


Figure 9. Time series of standard deviation of daily maximum (solid line) and minimum (dashed line) temperature for July-August at Phoenix, Arizona, for the period 1948-1990 [Source: Tarleton and Katz, 1993].



APPENDIX 1

Papers Based on Research Supported by Cooperative Agreement

Journal Articles

Katz, R.W., and B.G. Brown, 1992: Extreme events in a changing climate: Variability is more important than averages. *Climatic Change*, **21**, 289-302.

Katz, R.W., and J. Garrido, 1992: Sensitivity of extreme precipitation events to climate change. Submitted to *Water Resources Research*.

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Tarleton, L.F., and R.W. Katz, 1993: Effects of urban heat island on temperature variability and extremes. *Preprints, AMS Eighth Conference on Applied Climatology*, Anaheim, CA (in press).

APPENDIX 2

Reprint of "Extreme Events in a Changing Climate: Variability is More Important than Averages" [R.W. Katz and B.G. Brown, 1992: *Climatic Change*, **V. 21**, pp. 289-302]

EXTREME EVENTS IN A CHANGING CLIMATE: VARIABILITY IS MORE IMPORTANT THAN AVERAGES

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Abstract. Extreme events act as a catalyst for concern about whether the climate is changing Statistical theory for extremes is used to demonstrate that the frequency of such events is relatively more dependent on any changes in the variability (more generally, the scale parameter) than in the mean (more generally, the location parameter) of climate Moreover, this sensitivity is relatively greater the more extreme the event These results provide additional support for the conclusions that experiments using climate models need to be designed to detect changes in climate variability, and that policy analysis should not rely on scenarios of future climate involving only changes in means

1. Introduction

Recent hot spells and droughts, as well as evidence of a gradual warming trend in global mean temperature (Hansen and Lebedeff, 1987, 1988), have led to a heightened awareness of possible greenhouse gas-induced climate change. Although it is natural that society tends to notice the extremes and variability of weather, climate change experiments, based on the use of general circulation models (GCMs) of the atmosphere, have to date dwelt on potential changes in average climate (Schlesinger and Mitchell, 1987; Schneider, 1989) Likewise, efforts to establish the statistical significance of apparent changes in the observed climate have only been successful in detecting changes in average conditions (e g , Solow and Broadus, 1989)

Assessments of the economic impacts of global climate change also focus on averages, rather than on variability or extremes (Adams *et al.*, 1990). But the primary impacts of climate on society result from extreme events, a reflection of the fact that climate is inherently variable (Parry and Carter, 1985). A hot spell during the summer of 1983 in the midwestern U.S. resulted in a substantial decrease in corn yields (Mearns *et al*, 1984). Freezes during the winters of 1983 and 1985 killed a significant fraction of the citrus trees in the state of Florida (Miller and Glantz, 1988) Droughts are a frequent cause of adverse societal impacts, for instance, the recurrent episodes of famine in Africa (Glantz, 1987).

In spite of the need to examine how the frequency of extreme events might change as the mean climate changes (Wigley, 1985; Mitchell *et al*, 1990), attempts

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to quantify the nature of such relationships have been rare (Mearns *et al*, 1984; Wigley, 1988). Here some standard statistical theory for extremes is applied to reveal some amazingly broad generalizations that can be made about the relative sensitivity of extreme events to the mean and variability (more generally, the location and scale parameters) of climate. The extreme events considered include the exceedance of a threshold by a climate variable. (i) on a particular occasion; or (ii) on one or more occasions within a certain time period (i.e., by the maximum value of a sequence of observations). In essence, our results indicate that (i) extreme events are relatively more sensitive to the variability of climate than to its average, and (ii) this sensitivity is relatively greater the more extreme the event.

2. Statistical Model for Climate Change

Just as the need for a paradigm to monitor global climate change has been recognized (Wood, 1990), a prototype model is needed to define climate change in statistical terms. A given climate variable X has some probability distribution (with distribution function $F_X(x) = P\{X \leq x\}$), possessing a *location parameter* μ and a *scale parameter* σ; that is, the distribution of the standardized variable

$$Z = \frac{X - \mu}{\sigma} \tag{1}$$

is assumed to not depend on either μ or σ. In the special case of F_X being the normal distribution, the location parameter μ is simply the mean and the scale parameter σ is simply the standard deviation. For nonnormal distributions, it is more meaningful from a statistical perspective to deal with location and scale parameters, rather than with the mean and standard deviation.

The relation (1) implies that the distribution function F_X can be expressed in terms of the distribution function F_Z of the standardized variable Z as

$$F_X(x) = F_Z[(x - \mu)/\sigma]. \tag{2}$$

Differentiating (2), the probability density function of X, $F'_X(x)$, can be expressed as

$$F'_X(x) = (1/\sigma)F'_Z[(x - \mu)/\sigma]. \tag{3}$$

By use of (2) and (3), properties of location and scale parameter distributions can be obtained directly from those for the simpler standardized variable (1).

Climate change is envisioned to involve a combination of two different statistical operations: (i) the distribution function F_X is shifted, producing a change in location μ, and (ii) F_X is rescaled, producing a change in scale σ. Figure 1 illustrates this concept for one arbitrary choice of F_X that happens to be positively skewed. For

this particular distribution, the location parameter μ is the mode, rather than the mean, whereas the scale parameter σ is equivalent to the standard deviation (i.e., the same except for a constant multiple). The two hypothetical forms of climate change are also included in Figure 1: (i) a change in the location parameter μ to a new, in this case larger, value μ^*; and (ii) a change in the scale parameter σ to a new, in this case larger, value σ^*. These new values, μ^* and σ^*, have been intentionally selected so that the probability density function evaluated at the original location parameter value μ is identical for both changes. What is remarkable is how much these distributions differ in the tails, the shape of which determines the probability of extreme events. Katz (1991) treated this model for climate change in more detail.

Attention is focused on two specific types of extreme events:

(i) the exceedance of a threshold [event $E_1 = \{X > c\}$, where the constant c denotes a threshold],
(ii) the maximum of a sequence exceeding a threshold [event $E_2 = \{\max(X_1, X_2, \ldots, X_n) > c\}$]

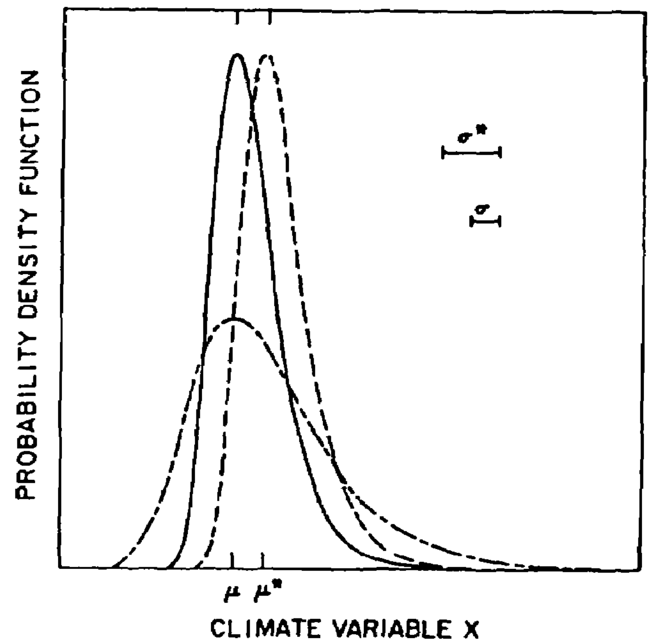

Fig. 1. Hypothetical distribution of a climate variable with location parameter μ and scale parameter σ (solid line); location parameter μ^* and scale parameter σ (dashed line), and location parameter μ and scale parameter σ^* (long and short dashed line)

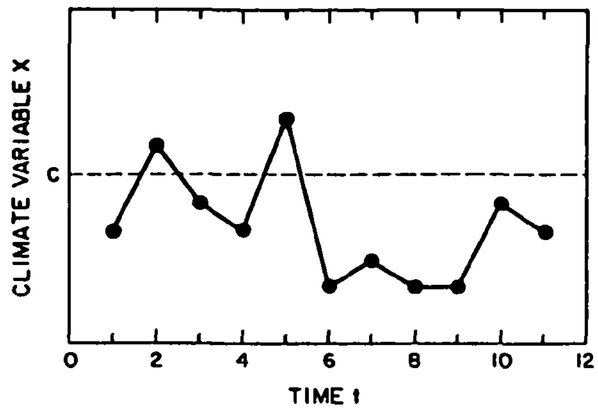

Fig. 2. Hypothetical climate time series (solid line) of length $n = 11$, along with threshold value c (dashed line) used to define extreme events

Figure 2 shows a hypothetical climate time series of length $n = 11$. Since $X_6 < c$, the event E_1 did not occur at the particular time $t = 6$. On the other hand, since the threshold was exceeded on at least one occasion (actually by both X_2 and X_5), the event E_2 did occur.

In the notation previously introduced, the probability of the extreme event E_1 is given by

$$P(E_1) = 1 - F_X(c). \tag{4}$$

If the sequence $\{X_1, X_2, \ldots, X_n\}$ consists of independent and identically distributed random variables, then the probability of the extreme event E_2 is given by

$$P(E_2) = 1 - [F_X(c)]^n. \tag{5}$$

Rather than rely on this exact expression (5), the theory of extreme values (Leadbetter *et al.*, 1983) can be invoked to produce an approximation that holds even when the sequence is autocorrelated, as is typical of climate variables. Specifically,

$$P(E_2) \approx 1 - G[a_n(c - b_n)], \tag{6}$$

for large sequence length n. Here G is one of three possible extreme value distributions:

(i) *Type I* (or Gumbel)

$$G(x) = \exp(-e^{-x}), \tag{7}$$

(ii) *Type II* (or Fréchet)

$$G(x) = \exp(-x^{-\alpha}), \quad \alpha, x > 0; \tag{8}$$

(iii) *Type III* (or Weibull)

$$G(x) = \exp[-(-x)^{\alpha}], \quad \alpha > 0, \quad x < 0. \tag{9}$$

In (6), $a_n > 0$ and b_n are normalizing constants that depend on the original distribution F_X.

For the most part, we will assume that the approximation (6) holds for G being the Type I extreme value distribution (7). This assumption is valid if F_X is any one of the distributions commonly fit to climate variables (e.g., exponential, gamma, lognormal, normal, squared normal, Weibull) (Essenwanger, 1976). Nevertheless, the Type II or III extreme value distributions have sometimes been found to provide a better fit in practice to the empirical distribution of the maximum of climate sequences (e.g., Faragó and Katz, 1990). Tiago de Oliveira (1986) and Buishand (1989) review the application of the theory of extreme values to climatology.

3. Sensitivity of Extreme Events

We are interested in how the probability $P(E)$ of an extreme event E would change as the location or scale parameters, μ or σ, change. The *sensitivity* of an extreme event to the location or to the scale is defined to be the corresponding partial derivative of the probability of the event, that is, $\partial P(E)/\partial\mu$ or $\partial P(E)/\partial\sigma$. Since extreme events vary in their likelihood, it is reasonable to deal with the *relative sensitivity*,

$$\left[\frac{\partial P(E)}{\partial \mu}\right] / P(E) \quad \text{or} \quad \left[\frac{\partial P(E)}{\partial \sigma}\right] / P(E), \tag{10}$$

comparing the sensitivity of an event to its probability.

3.1. *Extreme Event E_1*

Using (2)–(4), it follows that the two sensitivities can be expressed as

$$\frac{\partial P(E_1)}{\partial \mu} = F_X'(c), \tag{11}$$

$$\frac{\partial P(E_1)}{\partial \sigma} = \left(\frac{c - \mu}{\sigma}\right) F_X'(c) \tag{12}$$

Comparing (11) and (12),

$$\frac{\partial P(E_1)}{\partial \sigma} = \left(\frac{c-\mu}{\sigma}\right) \frac{\partial P(E_1)}{\partial \mu} \tag{13}$$

Equation (13) implies that the (relative) sensitivity of an extreme event to the scale becomes proportionately greater than its (relative) sensitivity to the location as the event becomes more extreme (i e, the larger the threshold c). Since the (relative) sensitivity to the scale is always greater than the (relative) sensitivity to the location, provided $(c-\mu)/\sigma > 1$, this condition could even be taken as an objective criterion for what constitutes an 'extreme' event.

Another important issue concerns whether the relative sensitivity of an extreme event to the location (or to the scale) increases or decreases as the event becomes more extreme. One way to attack this problem involves recognizing that the relative sensitivity of event E_1 to the location is mathematically equivalent to the so-called 'hazard rate' or 'failure rate' for distribution function F_X; namely,

$$H(x) = \frac{F_X'(x)}{1 - F_X(x)} \tag{14}$$

In fact, it follows directly from (11) that

$$\left[\frac{\partial P(E_1)}{\partial \mu}\right] / P(E_1) = H(c) \tag{15}$$

The hazard rate is a fundamental measure in engineering studies of system reliability (Hillier and Lieberman, 1986), representing the 'rate' of occurrence of the event $\{X = x\}$ given that $X \geq x$.

The question of whether the relative sensitivity of E_1 to μ increases as the threshold c increases is equivalent to asking whether F_X has a hazard rate $H(x)$ that is an increasing function for large x, a question that is commonly addressed in the statistical literature (Johnson and Kotz, 1970). For instance, since the normal distribution has a hazard rate that is an increasing function [in particular, $H(x)$ increases at an approximately linear rate for large x], the relative sensitivity of E_1 to μ increases as c increases in this case. However, other forms of distribution F_X have hazard rates that are not strictly increasing functions, implying that the relative sensitivity of E_1 to μ need not always increase For instance, the exponential distribution has a constant hazard rate, and the Weibull distribution has either a strictly increasing or strictly decreasing hazard rate, depending on the value of its shape parameter.

Equation (13) implies that, if the relative sensitivity of extreme event E_1 to the location either increases, remains constant, or decreases at slower than a linear rate as the event becomes more extreme, then the relative sensitivity to the scale must increase For instance, if F_X were the normal distribution, then the relative sensitivity of E_1 to σ would increase at an approximately quadratic rate for large c. If F_X were the exponential distribution, then the relative sensitivity to σ would increase at a linear rate.

3.2. *Extreme Event E_2*

If we make use of the large-sample approximation (6), then the same relation (13) between the sensitivity to the location and scale parameters, μ and σ, also holds for extreme event E_2, no matter which of the three types of extreme value distributions (7)–(9) arises. Moreover, analogous to (15), the relative sensitivity of E_2 to μ is the same as the hazard rate, not for F_X, but for the appropriate choice of extreme value distribution G. For the Type I extreme value distribution (7), the hazard rate is an increasing function [with $H(x)$ being approximately constant for large x]. Consequently, in many cases the relative sensitivity of E_2 to μ should increase as the threshold c increases. The Type II extreme value distribution (8) has a hazard rate $H(x)$ that is decreasing for large x. So it is possible in some circumstances that the relative sensitivity of E_2 to μ would decrease as c increases.

Equation (13) can again be employed to infer the relative sensitivity of E_2 to the scale parameter σ from that of E_2 to μ. If the Type I extreme value approximation (7) were employed, then the relative sensitivity of E_2 to σ would increase at an approximately linear rate as c increases. If the Type II extreme value approximation (8) were employed, then the relative sensitivity of E_2 to σ would gradually level off to a constant Katz and Brown (1989) presented a more technical discussion of the relative sensitivity of extreme event E_2.

3.3 *Other Extreme Events*

Although only two forms of extreme event, namely E_1 and E_2, have been explicitly treated here, the theoretical results concerning relative sensitivity apply much more generally. For instance, a perfectly analogous theory exists for the event of falling below a relatively small threshold, say $X < c$ (analogous to extreme event E_1), and for the event of the minimum value of a sequence falling below a threshold, say $\min(X_1, X_2, \ldots, X_n) < c$ (analogous to extreme event E_2) It is common to consider how often a threshold c is exceeded for a climate time series $X_1, X_2, \ldots, X_n$. Then $\tau_n = n[1 - F_X(c)]$ is the mean number of exceedances, and it is straightforward to show that the relative sensitivity of τ_n to μ and σ is the same as that for event E_1. Moreover, the number of exceedances has an approximate Poisson distribution Katz and Brown (1989) showed that somewhat analogous results can be obtained concerning how these Poisson probabilities change as a function of μ and σ.

4. Extreme High Temperature Events

We consider extreme high temperature events of a form known to be deleterious to the corn crop in the midwestern U S (Mearns *et al*, 1984) In this region the corn crop is subject to climate conditions close to its tolerance limits, especially high temperatures during the particularly sensitive phenological stage of tasseling (typically, during the month of July). The July time series of daily maximum temperature at Des Moines, Iowa is utilized (data previously analyzed by Mearns *et al*, 1984). It is assumed that F_X is the normal distribution, so that the location parameter μ is the mean and the scale parameter σ is the standard deviation. The 31 years of historical data indicate that μ is about 30 °C and σ is about 3.9 °C. Thresholds of $c = 35$ and 38 °C are of special interest.

Figure 3a shows plots of the relative sensitivity of extreme event E_1, the temperature exceeding a threshold c on a given day in July, to μ and σ as c increases (i.e., as the event becomes more extreme). As specified by (13), the two curves intersect when the threshold is one standard deviation above the mean. The separation between the two curves becomes greater as c increases, as anticipated, with the relative sensitivity to μ increasing at an approximately linear rate in contrast with the approximately quadratic rate for the relative sensitivity to σ. Strictly speaking, the relative sensitivity curves shown in Figure 3a apply only to infinitesimal changes in either μ or σ. To convert these results into more concrete terms, Table I gives the probability of event E_1 for a threshold of $c = 38$ °C when μ and σ are changed by ± 0.5 °C. Relative to the current probability of 0.020, $P(E_1)$ changes by roughly twice as much for a change in σ as for the corresponding change in μ.

Figure 3b shows plots of the relative sensitivity of extreme event E_2, the temperature ever exceeding c within the entire month of July (i.e., $n = 31$), to μ and σ as a function of c. These relative sensitivities are based on the use of the Type I extreme value approximation for the maximum (7), with the normalizing constants, a_n and b_n, for the case of F_X being the normal distribution (see Leadbetter *et al*, 1983, p. 14) Again, the two curves intersect at the point specified in (13) More

TABLE I Probability of extreme events, E_1 and E_2, with threshold of $c = 38$ °C associated with changes in mean μ and standard deviation σ of July daily maximum temperatures at Des Moines, Iowa

Change in μ (°C)	Change in σ (°C)	Probability of extreme event (relative change) $P(E_1)$	Probability of extreme event (relative change) $P(E_2)$
0*	0*	0.020	0.492
+0.5	0	0.027 (+34.7%)	0.612 (+24.5%)
0	+0.5	0.034 (+70.8%)	0.713 (+44.9%)
−0.5	0	0.015 (−27.7%)	0.384 (−22.0%)
0	−0.5	0.009 (−54.0%)	0.264 (−46.2%)

* Current climate of $\mu = 30$ °C and $\sigma = 3.9$ °C

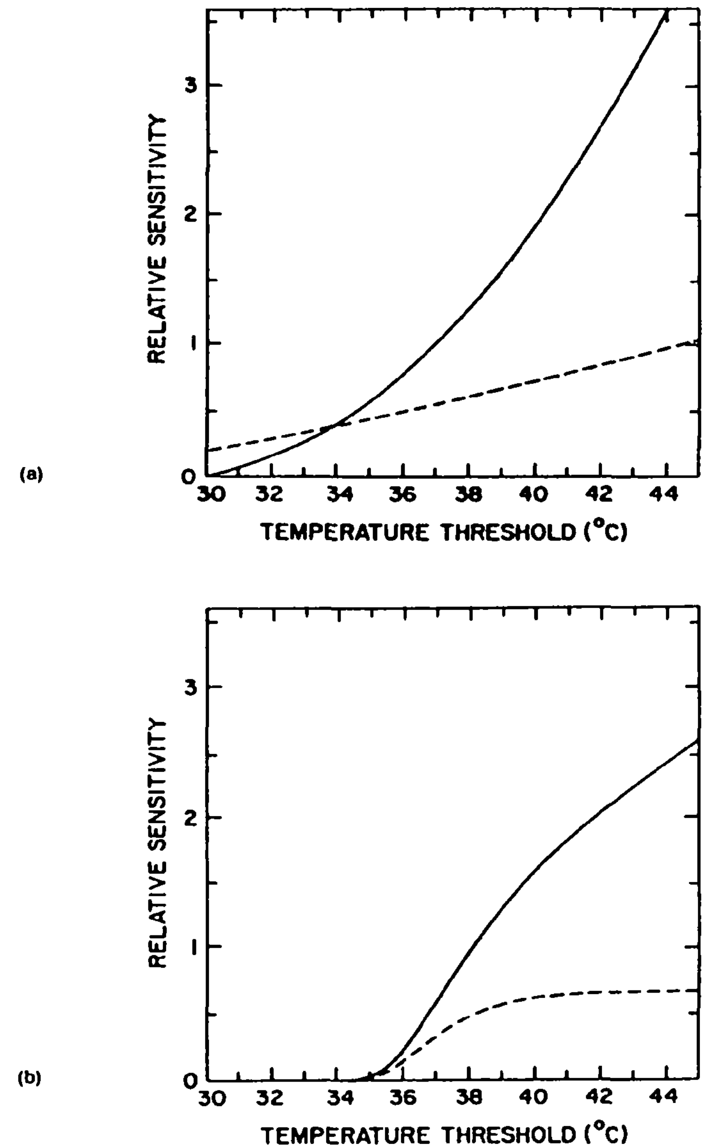

Fig 3 Relative sensitivity of extreme event to mean (dashed line) and standard deviation (solid line) versus threshold c (a) event E_1, temperature exceeding threshold on given day in July, (b) event E_2, temperature ever exceeding threshold during entire month of July

importantly, the two relative sensitivities rapidly separate as the event becomes more extreme. As the theory has established, the relative sensitivity to μ increases, but gradually levels off to a constant, whereas the relative sensitivity to σ increases at an approximately linear rate for large c. Table I also includes the probability of event E_2 for the same threshold and changes in μ and σ. Again, the relative changes in $P(E_2)$ are roughly twice as large when σ is varied as when μ is varied. Unlike event E_1 which always remains rare, event E_2 becomes either quite likely or somewhat rare depending on these seemingly small changes in σ.

Since the relative sensitivity curves shown in Figure 3b are based on an approximation derived in extreme value theory, the question arises as to the realism of the application to time series of daily maximum temperature. Using a first-order autoregressive process to represent daily maximum temperature (the same stochastic model assumed by Mearns *et al.*, 1984), both the effects of taking the maximum over a relatively small sample (i.e., $n = 31$ days) and the positive day-to-day autocorrelation of daily temperature were investigated in a simulation study. At least qualitatively, our theoretical results turn out to be quite robust. Moreover, any discrepancies that exist appear to be in the direction of even greater actual relative sensitivity than the theory predicts (Katz and Brown, 1992)

As a check on the plausibility of our assumptions about climate change (Section 2), the relative frequency of occurrence of these two extreme events was recorded at 30 sites within the midwestern U.S that possess relatively long records of daily maximum temperature (i.e., 40–90 years). Figure 4 gives plots of these relative frequencies versus the standardized threshold $(c - \mu)/\sigma$ for $c = 35$ °C. Because daily maximum temperature is known to have an approximately normal distribution (e.g., Mearns *et al.*, 1984), the location and scale parameters μ and σ were estimated by the sample mean and standard deviation of the July time series at the corresponding site. If a change in the future climate were analogous to a spatial relocation, then our model for climate change would imply that the scatter plot for event E_1 simply represents the right-hand tail of the distribution of daily maximum temperature $1 - F_X$ Since the points (Figure 4a) fall remarkably close to a smooth decreasing curve, these results serve as motivation for our statistical concept of climate change. The plot for event E_2 (Figure 4b) has a greater degree of scatter, in part because these relative frequencies were obtained from a much smaller sample (i.e., only one observation for each July instead of 31 for event E_1) Nevertheless, the indication of an underlying relationship is present Brown and Katz (1991) presented a more extensive validation of this climate change model.

5. Implications and Extensions

These theoretical results concerning the relative sensitivity of extreme events to the average and variability (or, more generally, location and scale parameters) of climate are quite compelling. Of course, our statistical model for climate change may be an oversimplification of the actual circumstances of future climate change

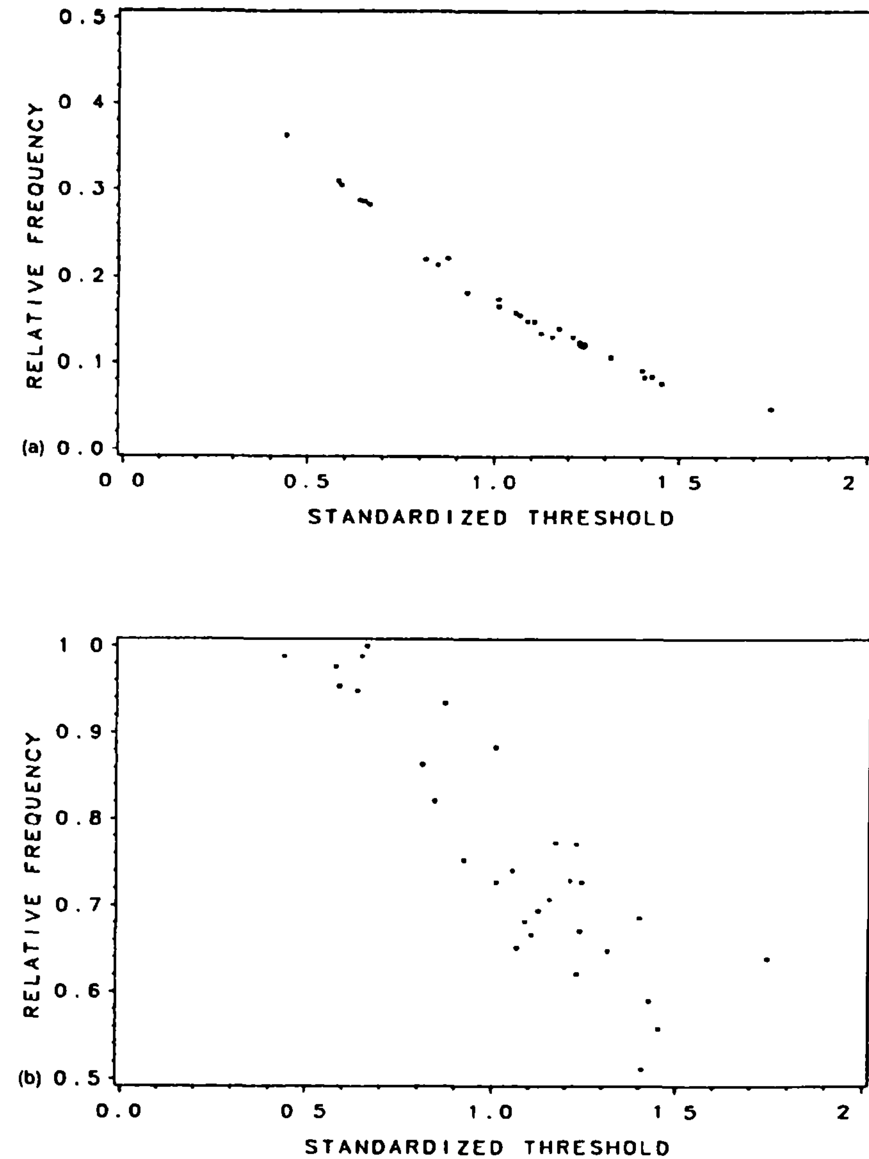

Fig 4 Relative frequency of extreme event versus standardized threshold $(c - \mu)/\sigma$ for 30 stations in the U.S Midwest (a) event E_1, temperature exceeding $c = 35$ °C on given day in July. (b) event E_2 temperature ever exceeding $c = 35$ °C during entire month of July

Moreover, since the theory only deals with sensitivities (i.e., partial derivatives), it does not eliminate the possibility that the magnitude of the change in the mean could be enough larger than that for the variance to still have a dominant effect on the actual change in the frequency of extreme events.

Still, the implications for climate change experiments that rely on GCMs are clear. More emphasis needs to be placed on the validation of such models in terms of their ability to reproduce climate variability, not just the mean climate. Experiments specifically designed to detect changes in the overall variability of climate also remain to be performed (Katz, 1988a; Rind *et al.*, 1989, Mearns *et al.*, 1990). Discussion of the generation of scenarios of future climate is common (Lamb, 1987; Katz, 1988b), but so far, the enhanced greenhouse effect in terms of changes in the overall variability of climate is not well known.

There are also some lessons for policy analysis that attempts to deal with the societal impacts of climate change. Assessments that rely on scenarios of future climate involving only changes in mean values or that infer changes in the frequency of extreme events from only changes in means (i.e., holding the variability of climate constant) are suspect. Although it is true that climate modelers are currently unable to provide policy analysts with much information on how either the variability of climate or the frequency of extreme events would change, these issues need to be addressed before impact assessments for greenhouse gas-induced climate change can be expected to gain much credibility.

The characterization of the sensitivity of extreme events to climate change could be extended to situations more representative of climate variables. Besides considering small samples and autocorrelation (as in work in progress mentioned in Section 4), results could be obtained that allow for diurnal or seasonal cycles. Further, extreme events involving climate variables with nonnormal distributions (e g., 'drought' or 'flood' events defined in terms of total precipitation) would constitute interesting case studies. Although some of the distributions treated do not fall within the framework of location and scale parameter families, quite analogous results have been derived (Garrido and Katz, 1992). Waggoner (1989) has attempted to study this issue for precipitation, but without making any explicit use of the theory of extreme values. Finally, more complex forms of extreme events such as ones that take into account the duration of an excursion above a threshold (e g., runs of consecutive hot days as in Mearns *et al*, 1984) or below (e.g, cold spells as in LeBoutillier and Waylen, 1988) could be investigated.

The issue of how to verify the applicability of this theory to actual climate change remains problematic. Nevertheless, several observational studies of climate could be performed that should at least produce some complementary results. For instance, the spatial analogue introduced in Section 4 would provide one technique for examining relationships between the probability of extreme events and statistics such as means and variances (see Brown and Katz, 1991). Another approach would involve the study of how the frequency of extreme events has changed for sites which are known to have already experienced a change in climate over time –

one such instance is the so-called 'heat island' effect associated with the growth of cities (Balling *et al.*, 1990).

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