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LAKE PHOSPHORUS LOADING GRAPHS:

AN ALTERNATIVE

by

D. P. Larsen and H. T. Mercier

Eutrophication & Lake Restoration Branch

Working Paper No. 174

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ABSTRACT

As an alternative to loading graphs, a graph of mean influent phosphorus concentration versus phosphorus retention capacity is proposed to express the relationship between phosphorus supply and hydraulic flow to, and resultant trophic state of, lakes. Lines of constant lake phosphorus concentration drawn on the proposed graph delineate predicted trophic states and changes caused by altering influent phosphorus concentrations and/or lake phosphorus retention capacity. The graph, derived from the steady state solution of a phosphorus mass balance model, expresses lake mean phosphorus concentration as a function of mean influent phosphorus concentration and phosphorus retention capacity. Because the mean influent phosphorus concentration is a potential lake concentration reduced by the lake phosphorus retention capacity, a determination of the latter is critical for predicting lake phosphorus concentrations. Several empirical expressions are developed for oligotrophic lakes relating phosphorus retention capacity to areal hydraulic load or hydraulic washout coefficient. These expressions can be used to predict average lake phosphorus concentrations from mean influent concentrations.

INTRODUCTION

Mass balance models have been developed to describe the relationship between phosphorus (P) concentrations in lakes and the supply of P to lakes (see review by Dillon, 1974; also, Imboden, 1973, 1974; Lerman, 1974; Snodgrass and O'Melia, 1974; Sonzogni, et al., 1974; Vollenweider, 1969, 1975). Since lake P concentrations often indicate trophic state (Sawyer, 1947; Vollenweider, 1968; Dillon, 1975), these models can be useful for predicting water quality and changes in water quality associated with changes in P supply. P loading graphs, which may or may not be developed from mass balance models, have also been used to summarize the effects of P supply on the trophic state of lakes and to provide an assessment of lake changes with changes in P supply (Vollenweider, 1968, 1975; Dillon, 1975). The loading graphs are based on the premise that increased P supply to lakes increases the productivity of the lakes and that the effects of supply can be modified by lake characteristics, such as mean depth and hydraulic retention time. Loading relationships suggest the supply of P (as g $P/m^2/yr$) as the critical factor in determining a lake's potential trophic state; however, the average influent P concentration may be more appropriate as a measure of this potential trophic state.

An alternative is proposed to the above class of P loading graphs. The average P concentration in a lake can be described as a relationship between the mean influent P concentration and a lake's ability to assimilate P. A graph using this expression can be constructed to predict trophic state and changes in trophic state caused by manipulating influent P concentrations and/or manipulating a lake's P assimilative capacity. This relationship derives directly from the steady state solution of a simple P mass balance model.

O'Melia (personal communication*) suggested that the important feature about the P supply to lakes is the P concentration of the influent, not the loading rate itself. Thus, the greater the influent P concentration, the greater the average concentration of P in the lake is likely to be--hence the higher the trophic state--regardless of the loading rates. In a well mixed system the steady state lake P concentration will be equivalent to the inflowing P concentration in the absence of P transport to the sediments. The lake P retention capacity--possibly a function of a particular lake's physical, chemical and biological properties--reduces the steady state P concentration from that of the influent.

Since the P retention capacity of lakes is critical in determining the steady state lake concentrations, lake features which control its magnitude were examined. Kirchner and Dillon (1975) suggested that the retention capacity was related to areal hydraulic flow. The equation which they developed was valid for additional lakes examined, however, several alternative expressions can be generated which are equally valid. These relate the P retention capacity to either areal hydraulic flow or to the hydraulic washout coefficient.

THE MASS BALANCE MODEL

A mass balance model for P in lakes can be written as (Vollenweider, 1969, 1975):

$$\frac{d[P]}{dt} = \frac{\Sigma\ v_j\ [p_j]}{V} - \rho_w[P] - \sigma_p\ [P] \qquad (1)$$

where $[P]$ = concentration of P in the lake

*Personal communication with Dr. Charles R. O'Melia. University of North Carolina, Chapel Hill, North Carolina.

ERRATA

1. Page 10. "The correlation between R_p determined by equation (14) and R_{exp} was 0.94." should read "The correlation between R_p determined by equation (14) and R_{exp} was 0.94 (Figure 5)."
2. Page 21, Table 1. "$[\bar{p}]$ mg/l" should read "$[\bar{p}]$ μg/l"
3. Page 22, Table 2. "$[\bar{p}]$ mg/l" should read "$[\bar{p}]$ μg/l"
4. Page 23, Lake #70 "Pelican/ME" should read "Pelican/MN"

v_j = flow rate of the j^{th} tributary

$[p_j]$ = P concentration in j^{th} tributary

V = lake volume

ρ_w = theoretical hydraulic washout coefficient

σ_p = P sedimentation coefficient

t = time.

Assumptions and rationale for adoption of such a mass balance model discussed by Vollenweider (1969, 1975), Dillon (1974), and Sonzogni, et al., (1974) will not be repeated here.

The steady state solution is:

$$[P]_\infty = \frac{\ell_p}{\rho_w+\sigma_p} \qquad (2)$$

where $\ell_p = \frac{\Sigma\, v_j\, [p_j]}{V}$ and is defined as the volumnar loading.

In order to predict $[P]_\infty$, an estimate of σ_p is necessary. Experimental determination of σ_p is difficult; therefore Dillon and Rigler (1974a) and Vollenweider (1975) defined a retention coefficient, R_p, which derives from the steady state solution as:

$$R_p = 1 - \frac{\rho_w}{\rho_w+\sigma_p} = \frac{\sigma_p}{\sigma_p+\rho_w}. \qquad (3)$$

Solving equation (3) for σ_p and substituting for σ_p in equation (2),

$$[P]_\infty = \frac{\ell_p \, (1-R_p)}{\rho_w} \, . \qquad (4)$$

Conceptually, the supply of P to lakes can be considered as originating through a single tributary with a flow weighted average P concentration equivalent to $\frac{\Sigma v_j[p_j]}{\Sigma v_j}$. Since $\ell_p = \frac{\Sigma v_j[p_j]}{V}$ and $\rho_w = \frac{\Sigma v_j}{V}$, the average concentration of P in the incoming tributaries, $[\bar{p}]$, is equivalent to $\frac{\ell_p}{\rho_w}$ under the steady state assumption. Thus equation (4) can be rewritten as:

$$[P]_\infty = [\bar{p}] \; (1 - R_p). \qquad (5)$$

The retention coefficient can be experimentally determined as (Dillon and Rigler, 1974a):

$$R_{exp} = 1 - \frac{v_o[p_o]}{\Sigma \; v_j[p_j]} \qquad (6)$$

where v_o and $[p_o]$ respectively are hydraulic outflow and P concentration in the outflow. Substituting R_{exp} for R_p, equation (5) becomes

$$[P]_\infty = [\bar{p}] \; (1-R_{exp}) \qquad (7)$$

or

$$[\bar{p}] = \frac{[P]_\infty}{(1-R_{exp})}$$

Equation 7 describes the relationship between steady state lake and mean influent P concentrations. This relationship can be summarized by graphing $[\bar{p}]$ vs. R_{exp} showing the modifying effect of a lake's P retention capacity upon the influent P concentration. Lines of constant lake P concentration can be drawn as solutions to the mass balance model for values of $[\bar{p}]$ and R_p. If particular lake mean P concentrations can be used to designate the trophic state of a lake, curves delineating trophic states can be drawn analogous to the method in which they have been drawn on loading graphs. This graph can be useful to determine the reduction in inflowing P concentration necessary to attain a desired trophic state, or the influent P concentration above which the receiving lake can be expected to exhibit noxious conditions.

$[\bar{p}]$ VS. R_{exp} GRAPH

Three sets of data have been used to map the position of lakes on $[\bar{p}]$ vs. R_{exp} graphs (Figure 1). The first two sets were those summarized by Dillon and Rigler (1974a) and Dillon (1975) and included estimates of L_p (areal P loading, g P/m^2/yr), ρ_w, $\bar{z}$ (mean depth, m), and R_{exp} for various lakes in North America and Europe. The quotient, $L_p/(\rho_w \cdot \bar{z})$ was used to obtain values for $[\bar{p}]$. Gakstatter (personal communication*) provided the third set which includes ρ_w and $\bar{z}$, as well as inflow and outflow P concentrations and loadings, for many lakes in northeastern and midwestern United States studied by the National Eutrophication Survey. The above loading estimates apparently included rainfall and direct discharges to the lakes but did not include internal loadings. Tables 1 and 2 summarize the data used.

The scientific community has not yet agreed upon objective criteria for identifying the trophic state of lakes, therefore the designations O (oligotrophic), M (mesotrophic), E (eutrophic) and HE (hypereutrophic) are qualitative, based on the assessments of limnologists who have investigated these lakes. In some cases, the identity of the trophic state was not available.

*Personal communication with Dr. Jack Gakstatter, Pacific Northwest Environmental Research Laboratory, Corvallis, Oregon.

The selection of lake P concentrations delineating trophic state is difficult. Sawyer (1947) and Vollenweider (1968) indicate that springtime concentrations of available P in excess of 10 μg/l are likely to produce noxious blooms of algae. Interpretation of Sakamoto's (1966) and Dillon and Rigler's (1974b) average summer chlorophyll vs. springtime total P concentration suggest that concentrations of total P in excess of 20 μg/l are likely to produce average summer chlorophyll a concentrations of about 10 μg/l or greater. Therefore, values of 10 and 20 μg P/l were selected to delineate oligotrophic, mesotrophic and eutrophic states. These values are used primarily for illustrative purposes.

The positions of the lakes on Figure 1 agree reasonably with their trophic states. Some lakes violate the pattern. Several reasons can be suggested for their departure from the model and a more detailed investigation would be necessary to determine the cause of this discrepancy. Reasons for the discrepancies might be:

1. The observed lakes might not be in or near a steady state and, consequently, recent changes in $[\bar{p}]$ may not yet be reflected by changes in lake characteristics.

2. The data base for the calculations might be inadequate for the present analysis which requires an adequate estimate of the mean concentration of P in inflowing and outflowing waters. The optimum method for obtaining P flux into or out of lakes has only recently been critically evaluated (Treunert, et al., 1974). Thus, the data reflect the notions of the principal investigators so sampling frequency may be neither consistent nor optimal.

3. The characteristics of some lakes may sufficiently disagree with model assumptions to invalidate a comparison. For example, vertical and horizontal inhomogeneities may be large enough to seriously affect

the model assumption of homogeneously and instantaneously distributed properties, or factors other than phosphorus may limit primary production.

4. The identity of trophic state is subjective so that designations are not likely to be universally acceptable and may not be specifically related to lake P content.

PHOSPHORUS RETENTION CAPACITY

Determining the P retention capacity is critical to the successful prediction of lake P concentrations. Empirically, R_p can be determined from the ratio of outflow loads or concentrations to inflow loads or concentrations. Therefore, experimental determination of R_p is only as good as measurements of imported and exported P. Furthermore, steady state values are required if R_{exp} is to be representative of a particular lake. Here again no attempts have been made to quantify the effect of deviation from steady state. Valid questions are: Can R_{exp} be related statistically to any lake properties such as mean depth and hydraulic retention time? Do these statistically derived relationships have theoretical foundation?

Kirchner and Dillon (1975) obtained a good relationship between R_{exp} and the areal hydraulic loading ($q_s = \rho_w \cdot \bar{z}$), but did not obtain an expected good relationship between R_{exp} and the hydraulic washout coefficient. Vollenweider (1975) obtained a statistical correlation between ln σ_p and ln $\bar{z}$ and suggested that σ_p could be estimated by $\frac{10}{\bar{z}}$. Substituting $\frac{10}{\bar{z}}$ for σ_p in $\frac{\sigma_p}{\sigma_p+\rho_w}$ provides:

$$R_p = \frac{10}{10+\rho_w \cdot \bar{z}}, \tag{8}$$

a relationship that also suggests a correlation between R_{exp} and q_s.

The data summarized in Table 2 were analyzed to determine whether similar relationships existed and whether alternative relationships were equally valid, particularly whether R_{exp} was related to ρ_w. Only lower mesotrophic or oligotrophic lakes were examined because increasing levels of productivity may decrease a lake's P retention capacity (Vollenweider, 1975) thus obscuring relationships between R_{exp} and other lake properties.

Initial examination revealed a high correlation between R_{exp} and $\ln q_s$, as summarized in Figure 2. However, considerable variability is evident. To try to reduce this variability by including only those lakes likely to be oligotrophic and to which P loadings probably had not undergone recent changes, the data from those lakes whose inflowing P concentration was $\leq$ 25 μg/l were analyzed further. Table 3 summarizes the correlations and the correlation coefficients for the 20 lakes selected.

For these lakes, the highest correlation existed between R_{exp} and $\ln q_s$ (Figure 3, r = -0.92). Note that slightly more than half the lakes used for these correlations were used by Kirchner and Dillon (1975). A least squares linear fit to the data provided the relationship:

$$R_{exp} = 0.854 - 0.142 \ln q_s \quad (9)$$

The observation that R_{exp} correlated better with areal hydraulic load than with volumetric hydraulic load (Kirchner and Dillon, 1975) is also supported by the results presented in Table 3. However, if values for Lakes Raven and Talbot are excluded as outliers, a high correlation is obtained between R_{exp} and $\ln \rho_w$ (Figure 4, r = -0.91). The best fit linear regression equation was

$$R_{exp} = 0.482 - 0.112 \ln \rho_w. \qquad (10)$$

Raven and Talbot are the shallowest lakes ($\bar{z}$ = 0.73 m and 0.85 m, respectively) in the data set so perhaps this relationship between R_{exp} and $\ln \rho_w$ is not valid for extremely shallow lakes. Dillon and Rigler (1974a), testing a simple phosphorus mass balance model, reported that predicted and observed P values differed markedly for Lakes Raven and Talbot.

Although equations (9) and (10) provide a good fit to observations, they predict R_p greater than 1 for low values of q_s (<0.36 m/yr) or ρ_w (<0.01 yr^{-1}) and R_p less than 0 for high values of q_s (>395 m/yr) and ρ_w (>74 yr^{-1}). For example, equation (10) predicts R_p = 1.22 for Lake Tahoe. Predictions of R_p < 0 or > 1 are contrary to limnological experience except in unusual circumstances, thus equations (9) and (10) are not entirely adequate.

Figure 4 also includes a graph of $R_p = \frac{\sigma_p}{\sigma_p+\rho_w}$ for $\sigma_p = 1.0$ yr^{-1} to compare the data spread with the shape of the theoretical R_p vs $\ln \rho_w$ curve (at $\sigma_p = 1.0$ yr^{-1}). For $\ln \rho > 0$ the curve evidently underestimates observed values of R_{exp} and overestimates observed values for $\ln \rho < 0$. Because σ_p and ρ_w seemed related, $\ln \sigma_p$ was regressed on $\ln \rho_w$. Direct measures of σ_p are not available for these lakes. Therefore, σ_p was determined as $\frac{\rho_w R_{exp}}{1-R_{exp}}$ (rearranging equation (3)). Data for Lakes Tahoe and Superior were included because they provided data points at extremely low values of ρ_w, but Lakes Raven and Talbot were excluded. The relationship

$$\ln \sigma_p = \ln 0.761 + 0.472 \ln \rho_w \ (r=0.84) \qquad (11)$$

or

$$\sigma_p = 0.761\ \rho_w^{.472}$$

resulted. Substituting into (3) and simplifying provided an equation of the form

$$R_p = \frac{1}{1+\alpha\rho_w^{\beta}} \qquad (13)$$

where α = 1.3 and β = 0.4.

The coefficients α and β were also estimated using a Gauss-Newton non-linear least squares algorithm available from the Oregon State University Computer Library. The best estimate for α was 1.12 and for β, 0.49. Thus, equation (13) can be approximated by the expression:

$$R_p = \frac{1}{1+\rho_w^{1/2}} \qquad (14)$$

The correlation between R_p determined by equation (14) and R_{exp} was 0.94. Further, the solutions of this expression are not theoretically impossible at extremely high and extremely low values of ρ_w. This expression is similar to that which develops from a relationship between σ_p and $\bar{z}$ (equation (8)). The correlation between R_p determined from equation (8) and R_{exp} was 0.93.

Table 4 summarizes the various expressions which relate the P retention capacity to lake properties. The expressions presented are all based upon hydraulic flow normalized to lake size as either surface area or volume, the lake attribute which seems to control the P retention coefficient to the greatest degree in oligotrophic lakes.

DISCUSSION

Vollenweider (1968) first summarized the notion that the trophic state of lakes could be related to the supply of P. He proposed a P loading graph relating the trophic state of lakes to the areal supply of P (g P/m^2/yr) modified by lake mean depth (L_p vs $\bar{z}$ graph). Although an important first step in stimulating limnologists to consider supplies of critical nutrients to lakes, the graph has been criticized because it does not distinguish between conditions of low flow-high influent P concentration or high flow-low influent P concentration (see review in Dillon, 1975). Vollenweider (1975) modified this P loading graph, dividing $\bar{z}$ by τ_w (L_p vs. $\bar{z}/\tau_w$ graph). Since $\bar{z}/\tau_w$ is q_s, the areal hydraulic load, this eliminates the effect of $\bar{z}$ on the resultant trophic state. The latter graph expresses roughly the same concepts as a graph of $[\bar{p}]$ vs. R_p if L_p and $[\bar{p}]$ are directly correlated and if R_p is related to q_s (Table 4).

Dillon (1975) proposed a graph of $L_p(1-R_p)/\rho_w$ vs. $\bar{z}$ modifying L_p to include the effects of hydraulic washout and P retention. This formulation can be derived directly from equation (4) since

$$\ell_p = \frac{\Sigma v_j[p_j]}{V} = \frac{L_p}{\bar{z}} .$$

Lines of constant lake P concentration can be drawn on the graph as the steady state solution to equation (1) for values of L_p, R_p, ρ_w, and $\bar{z}$. Dillon chose lines of constant [P] representing the division between oligotrophic and mesotrophic (10 μg/l) and between mesotrophic and eutrophic (20 μg/l) conditions. Since $L_p = \frac{\Sigma\, v_j[p_j]}{S}$ (S = lake surface area, m^2) and $\rho_w = \frac{\Sigma v_j}{V}$, $\frac{L_p}{\rho_w} = [\bar{p}] \cdot \bar{z}$. Thus this graph of $\frac{L_p(1-R_p)}{\rho_w}$ vs. $\bar{z}$ is equivalent to a graph of $[\bar{p}] \cdot \bar{z}\ (1-R_p)$ vs. $\bar{z}$.

A graph of $[\bar{p}]$ vs. R_p better represents the concepts expressed by the steady state equation. Importantly, $[\bar{p}]$ combines both P supply and hydraulic flow. It is the concentration attainable in a lake in the absence of P retention, i.e., a potential lake P concentration. (This differs from the potential concentration defined by Edmondson (1961) which is ℓ_p, the phosphorus volumnar loading.) Thus the steady state lake P concentration is a function of the magnitude of $[\bar{p}]$ and the ability of a particular lake to assimilate P, expressed as its P retention capacity. This assimilation occurs as P is incorporated into the lake sediments. The graph of $[\bar{p}]$ vs. R_p expresses these concepts and conveniently separates the potential lake P concentration from the lake assimilative capacity. It can be derived directly from the steady state solution of a simple mass balance model for phosphorus (equation (7)) and does not produce results contrary to theoretical expectation. If trophic state can be related to lake P, then this graph provides a method for quickly assessing the trophic state of a lake.

The importance of R_p as a factor in determining the steady state P concentration in lakes is evident from Figure 1 and equation (7). R_p, as defined by equation (3), is sensitive to the relative values of σ_p and ρ_w. If ρ_w is $>> \sigma_p$, such as may exist for lakes with high flushing rates, R_p approaches 0 and is nearly independent of σ_p. These lakes are particularly sensitive to $[\bar{p}]$ and changes in $[\bar{p}]$; steady state P concentrations would be only slightly lower than $[\bar{p}]$. Thus, in order to prevent these lakes from becoming eutrophic or to reduce the trophic state of these lakes, emphasis must be placed upon manipulating $[\bar{p}]$. When ρ_w is $<< \sigma_p$, R_p is nearly independent of the value of either variable and approaches 1. Large lakes such as Lakes Tahoe and Superior with slow water renewal may exhibit these characteristics. These lakes can absorb high $[\bar{p}]$, yet maintain good quality. However, these lakes are extremely sensitive to changes in R_p, and require large reductions in $[\bar{p}]$ to significantly reduce lake P concentrations.

A comparison of Lakes Superior and Zurichsee illustrates the relative importance of $[\bar{p}]$ and R_p. Lake Superior (R_{exp} = 0.9, $[\bar{p}]$ = 40 μg/l, mean lake P = 3-5 μg/l) is oligotrophic (Beeton and Chandler, 1966; Patalas, 1972; Dobson, et al., 1974), while Zurichsee (R_{exp} = 0.25, $[\bar{p}]$ = 40 μg/l, [P] = 32 μg/l at vernal circulation) is eutrophic (Vollenweider, 1968). Assuming that both lakes approximate a steady state, Lake Superior's ability to assimilate incoming P allows it to manifest oligotrophic characteristics while Zurichsee's much lower assimilatory ability dictates that the same $[\bar{p}]$ results in eutrophic conditions. Lake Superior probably could absorb a 40-50 μg/l increase in $[\bar{p}]$ and remain oligotrophic, while a similar increase in $[\bar{p}]$ for Zurichsee probably would aggravate an already eutrophic problem. Slight changes in Zurichsee's assimilatory capacity would only slightly affect the steady state P concentrations, while a slight reduction in Superior's retention capacity--for example, from 0.9 to 0.8--could double the steady state lake concentration. This sensitivity of lakes with high R_p to changes in R_p suggests that a careful determination of R_p is necessary if the results of influent P changes are to be reasonably predicted for these lakes.

The P retention capacity of oligotrophic lakes may be primarily controlled by ρ_w or q_s,--that is, hydraulic flow normalized to a measure of lake size (Table 4). The relationships summarized in Table 4, if generally valid, may provide an upper limit for R_p because it apparently decreases as productivity increases (Vollenweider, 1975). These relationships may be useful for predicting lake P content for known or hypothesized influent P levels. No attempt has been made to quantify this reduction in R_p due to increased productivity. Other factors such as lake morphometry, climate, or lake chemistry may influence R_p, but a more thorough investigation is necessary to refine these predictive estimates.

If loading and flow data are available, the expression of trophic characteristics in terms of $[\bar{p}]$ and R_p not only allows a rapid assessment of trophic state, but also provides a preliminary guide useful for management decisions. Effects of alternatives such as loading reduction or nutrient dilution can be expressed as changes in $[\bar{p}]$. Furthermore, the effect of manipulations affecting R_p, if quantifiable in terms of R_p, can be assessed. Thus the results of nutrient inactivation or hypolimnetic aeration might be determined as increases in R_p and, therefore, as reductions in steady state P levels. In addition, the effect of combined treatments can be predicted through their changes in $[\bar{p}]$ and R_p.

The $[\bar{p}]$ vs. R_p graph provides an alternative to the P loading graphs. It develops from a simple mass balance equation for P (equation (1)) and, therefore, is only as good as the mass balance model itself. Mass balance models of this nature are intended to describe general characteristics of phosphorus dynamics in lakes. Thus annual averages of the input variables and rate coefficients are used and results are compared with average conditions in lakes. This type of model has been used to explain existing conditions (Vollenweider 1969, 1975) or to predict changes as a result of reducing P supplies to lakes (Sonzogni, et al., 1974; Sonzogni and Lee, 1974; Megard, 1970). Slightly more complex mass balance models developed by Imboden (1973, 1974) and Snodgrass and O'Melia (1974) may provide a closer fit to average conditions and, more importantly, to seasonal changes in lake P. However, these models require further testing to ascertain their generality.

SUMMARY

An expression derived from the steady state solution of a phosphorus mass balance model was developed relating mean lake phosphorus concentration to mean influent phosphorus concentration and lake retention capacity. A graph of mean influent phosphorus concentration versus lake phosphorus retention capacity was proposed to summarize this relationship. Lines of constant lake phosphorus concentration drawn on the graph to delineate trophic states help predict changes in trophic state occurring when mean influent phosphorus concentration and/or phosphorus retention capacity are changed.

The mean phosphorus influent concentration is a potential lake phosphorus concentration. The assimilative capacity of lakes acts to reduce this potential to produce mean lake concentrations. Empirical expressions were developed to relate phosphorus assimilative capacity in oligotrophic lakes to either areal hydraulic load or hydraulic washout coefficient. These expressions can be used to predict the resultant mean lake phosphorus concentrations if mean influent concentrations are known.

LIST OF SYMBOLS

Symbol	Definition
L_p	Areal phosphorus loading (g $P/m^2/yr$).
ℓ_p	Volumnar phosphorus loading (g $P/m^3/yr$).
[P]	Phosphorus concentration in lake (μg/l).
$[\bar{p}]$	Mean inflowing phosphorus concentration (μg/l).
$[p_j]$	Phosphorus concentration of j^{th} tributary (μg/l).
$[p_o]$	Phosphorus concentration of outlet (μg/l).
q_s	Areal hydraulic load ($m^3/m^2 \cdot yr$ = m/yr).
R_{exp}	Experimentally determined phosphorus retention coefficient (dimensionless).
R_p	Theoretical phosphorus retention coefficient (dimensionless).
S	Lake surface area (m^2).
V	Lake volume (m^3).
v_j	Flow rate of j^{th} tributary (m^3/yr).
v_o	Flow rate of outlet (m^3/yr).
$\bar{z}$	Mean depth (m).

ρ_w Hydraulic washout coefficient (yr^{-1}).

σ_p Phosphorus deposition coefficient (yr^{-1}).

τ_w Hydraulic retention time = ρ_w^{-1} (yr).

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Table 1

Summary of lakes and characteristiscs used in Figure 1. Data for lakes 1-17 were obtained from Dillon (1975) and references therein and for lakes 18-37 from Gakstatter (personal communication).

No.	Lake Name	Trophic State	$[\bar{p}]$ mg/l	R_{exp}
1	Baldeggersee	E	230	0.61
2	Bodensee-Obersee	E	200	0.65
3	Erie	M-E	150	0.84
4	Greifensee	E	170	0.62
5	Hallwilersee	O	75	0.36
6	Kalamalka	E	610	0.90
7	Kegonsa	E	505	0.09
8	Menona	E	330	0.71
9	Michigan	O-M	85	0.9
10	Norrviken (1961-1962)	E	425	0.49
11	Okanagan	O	305	0.95
12	Pfaffikersee	E	195	0.77
13	Sebasticook	E	90	0.48
14	Skaha	M-E	95	0.65
15	Waubesa	E	620	0
16	Wood	E	2590	0.90
17	227 (ELA)	E	325	0.89
	Lake/State			
18	Beaverdam (Dodge County)/WI	E	265	0.0
19	Bemidji/MN	E	35	0.36
20	Blackduck/MN	E	135	0.55
21	Chemung/MI	E	105	0.63
22	Clearwater/MN	E	180	0.76
23	Cokato/MN	E	385	0.44
24	Delavan/WI	E	410	0.55
25	Fremont/MI	HE	550	0.47
26	Green/MN	M-E	55	0.67
27	Jordan/MI	E	130	0.17
28	Le Homme Dieu/MN	E	135	0.78
29	Nagawicka/WI	E	205	0.57
30	Nest/MN	E	90	0.56
31	Sebasticook/ME	E	60	0.29
32	Shawano/WI	E	35	0.23
33	Swan/WI	E	140	0.18
34	Townline/WI	E	255	0.52
35	Trout/MN	E	375	0.92
36	Wagonga/MN	E	4300	0.57
37	Wapogasset/WI	E	70	0.43

TABLE 2

Summary of lakes and characteristics used in Figure 1 and for an analysis of relationships between R_{exp} and lake characteristics. Data for lakes 38-45 were obtained from Dillon (1975) and references therein, for lakes 46-57 from Dillon and Rigler (1974a), and for lakes 58-73 from Gakstatter (personal communication). Asterisks (*) indicate lakes used for generating best empirical relationships between R_{exp} and lake characteristics.

No.	Lake Name	Trophic State	$[\bar{p}]$ mg/l	R_{exp}	ρ_w (yr^{-1})	$\bar{z}$ (m)	σ_p (yr^{-1})
38	Aegerisee	O	30	0.68	0.115	49	0.24
39	*Clear	O	25	0.80	0.13	12.5	0.52
40	Leman	M	55	0.20	0.083	155	0.02
41	Ontario	M	50	0.78	0.152	84	0.54
42	*Superior	O	40	0.90	0.0053	148	0.05
43	*Tahoe	O	100	0.93	0.0014	303	0.02
44	Turlersee	M	45	0.80	0.465	14	1.86
45	Zurichsee	E	40	0.25	0.680	50	0.23
46	*Beech		8	0.07	22.7	9.8	1.71
47	*Bob		24	0.71	0.37	18.0	0.91
48	*Cameron		16	0.30	18.9	7.1	8.1
49	*Eagle-Moose		9	0.36	2.03	12.8	1.14
50	Four Mile		45	0.82	0.26	9.3	1.18
51	*Oblong-Haliburton		20	0.72	0.32	17.7	0.82
52	*Halls		8	0.53	0.96	27.2	1.08
53	*Maple		9	0.26	8.0	11.6	2.81
54	*Pine		8	0.01	18.5	7.4	0.19
55	Raven		20	0.55	14.9	0.73	18.2
56	Talbot		24	0.69	4.9	0.85	10.9
57	*Twelve Mile--Boshkung		8	0.33	2.38	18.1	1.18
	Lake/State						
58	*Bay of Naples/ME	O	9	0.19	14.0	4.3	3.28
59	Canadaigua/NY	O	55	0.65	0.07	39.0	0.13
60	Carlos/MN	M	40	0.56	0.27	13.1	0.34
61	*Carry Falls/NY	O	14	0.28	9.61	5.4	3.74
62	Cass/MN	M	40	0.63	1.17	7.6	1.99
63	*Charlevoix/MI	O	24	0.63	0.31	16.8	0.53
64	Higgins/MI	O	28	0.70	.06	14.9	0.14
65	Houghton/MI	M	32	0.47	0.76	2.3	0.68
66	Long (Aroostook County)/ME	M	30	0.56	0.31	13.4	0.39

TABLE 2 (Continued)

67	*Long (Cumberland County)/ME	M	16	0.50	0.83	10.4	0.83
68	*Mattawamkegg/ME	M	20	0.31	8.69	3.7	3.90
69	*Moosehead/ME	O	15	0.46	0.33	16.4	0.28
70	Pelican/ME	M	77	0.46	0.30	2.7	0.25
71	*Rangeley/ME	O	18	0.49	0.36	14.3	0.35
72	*Sebago/ME	O	14	0.57	0.19	30.8	0.25
73	Winnipesaukee/NH	O	40	0.77	0.25	13.1	0.84

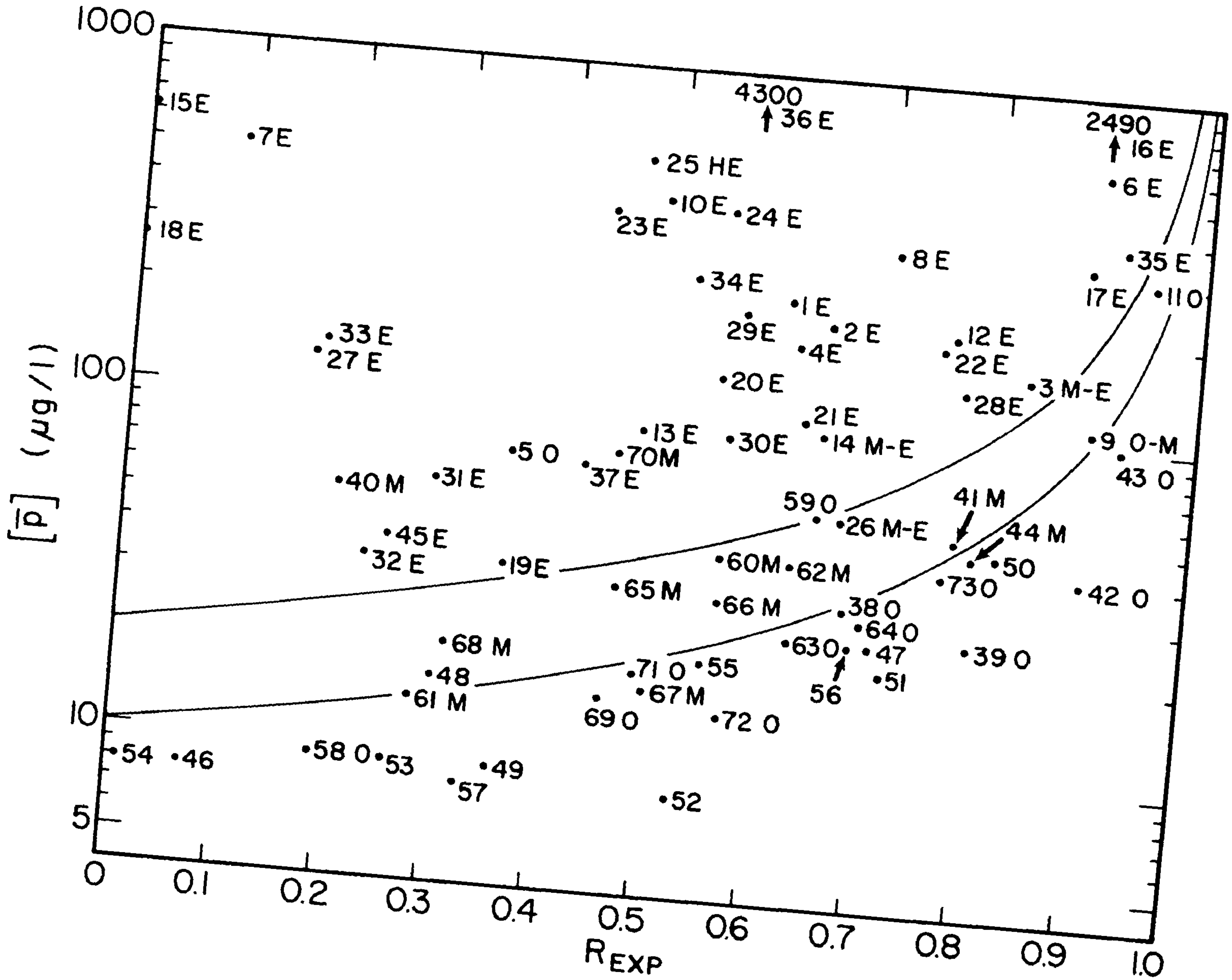
Table 3. Correlations between R_{exp} and various transformations and combinations of ρ_w and $\bar{z}$ for 20 lakes whose $[\bar{p}]$ $\leq$ 25 μg/l (Table 2).

Correlation		Correlation Coefficient	Correlation		Correlation Coefficient
R_{exp} vs.	ρ_w	-0.74	R_{exp} vs.	$\frac{\bar{z}}{\rho_w}$	0.57
	$\bar{z}$	0.33			
	$\frac{1}{\rho_w}$	0.68		$\frac{\rho_w}{\bar{z}}$	0.00
	$\frac{1}{\bar{z}}$	0.18			
	$\ln \rho_w$	-0.77		$\ln \frac{\bar{z}}{\rho_w}$	0.58
	$\ln \bar{z}$	0.09		$\bar{z}^2$	0.34
	$\rho_w \cdot \bar{z}$	-0.80		ρ^2_w	-0.69
	$\ln \rho_w \cdot \bar{z}$	-0.92		$\frac{1}{\bar{z}^2}$	0.25
	$\frac{1}{\rho_w \cdot \bar{z}}$	0.75		$\frac{1}{\rho^2_w}$	0.55

Table 4. Summary of various relationships between R_p and ρ_w or q_s for twenty selected lakes (indicated by (*) in Table 2). Correlation coefficients in parenthesis are values obtained when Lakes Superior and Tahoe are deleted.

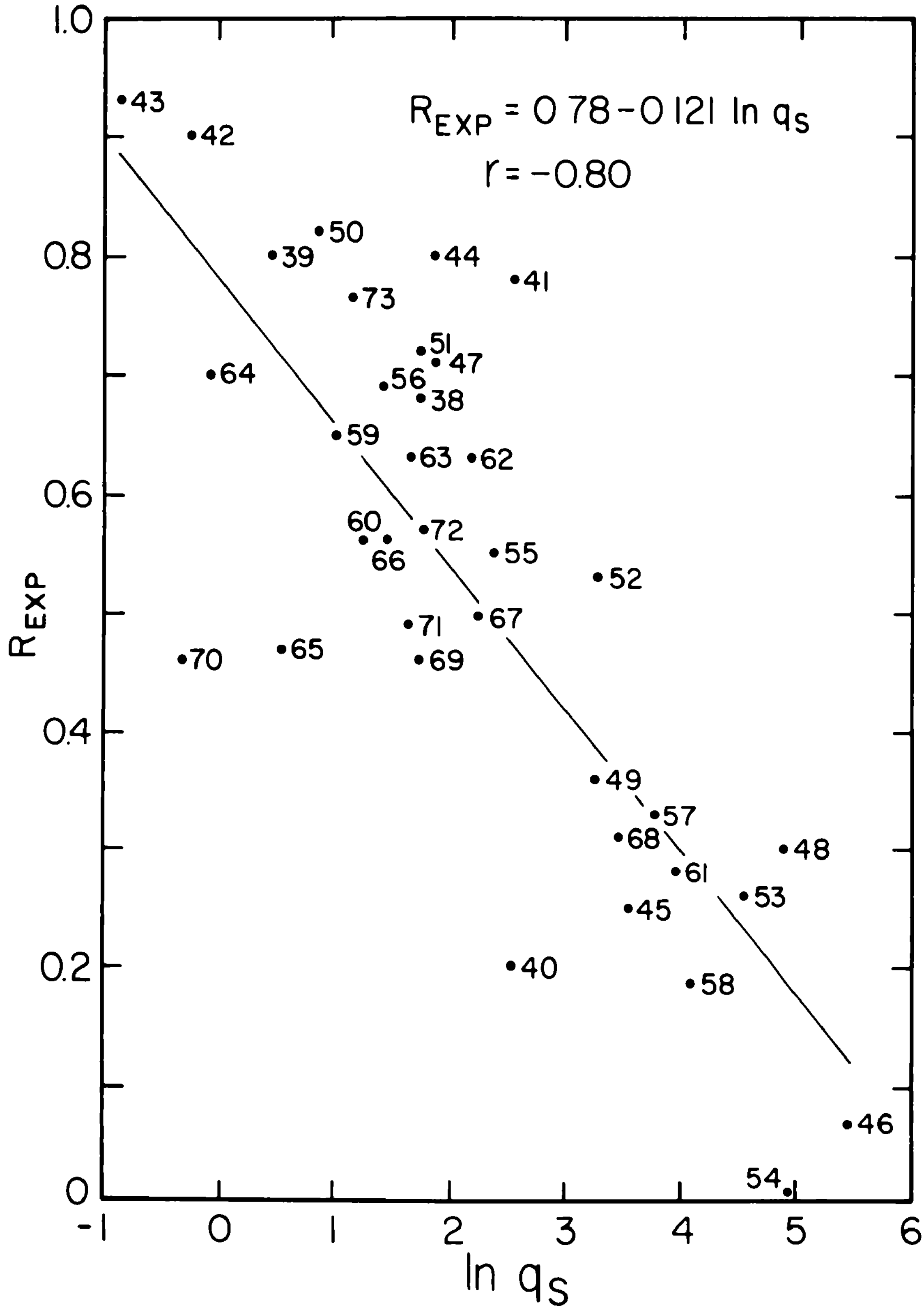
1.	$R_p = \dfrac{1}{1 + \rho_w^{1/2}}$	r = 0.94 (0.91)
2.	$R_p = 0.482 - 0.112 \ln \rho_w$	r = 0.93 (0.91)
3.	$R_p = \dfrac{10}{10 + q_s}$	r = 0.93 (0.89)
4.	$R_p = 0.426e^{(-0.271\ q_s)} + 0.574e^{(-0.00949\ q_s)}$	r = 0.94 (0.90)
5.	$R_p = 0.854 - 0.142 \ln q_s$	r = 0.94 (0.91)

LIST OF FIGURES



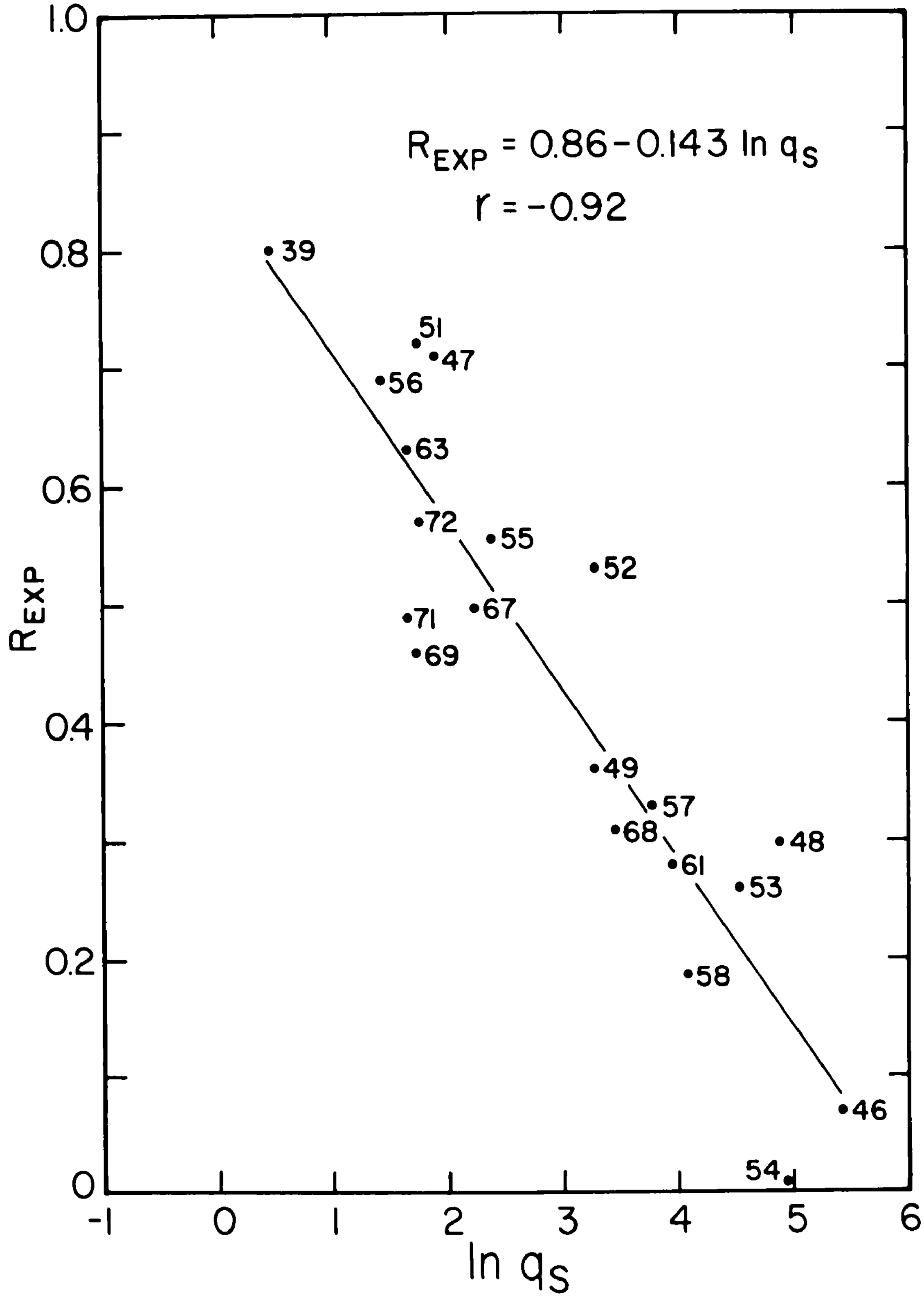
1000
100
10
5
[p̄] (μg/l)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
R EXP
15E
7E
18E
4300
36E
25 HE
10E
23E
24E
2490
16E
6E
8E
34E
35E
17E
110
1E
29E
2E
4E
12E
22E
33E
27E
20E
3 M-E
28E
21E
13E
5 0
70M
30E
14 M-E
9 0-M
43 0
40M
31E
37E
59 0
41 M
26 M-E
44 M
45E
32E
19E
60M
62M
50
65M
73 0
42 0
66 M
38 0
64 0
68 M
63 0
47
71 0
55
39 0
48
56
51
61 M
67M
69 0
72 0
54
46
58 0
53
49
57
52

FIGURE 1



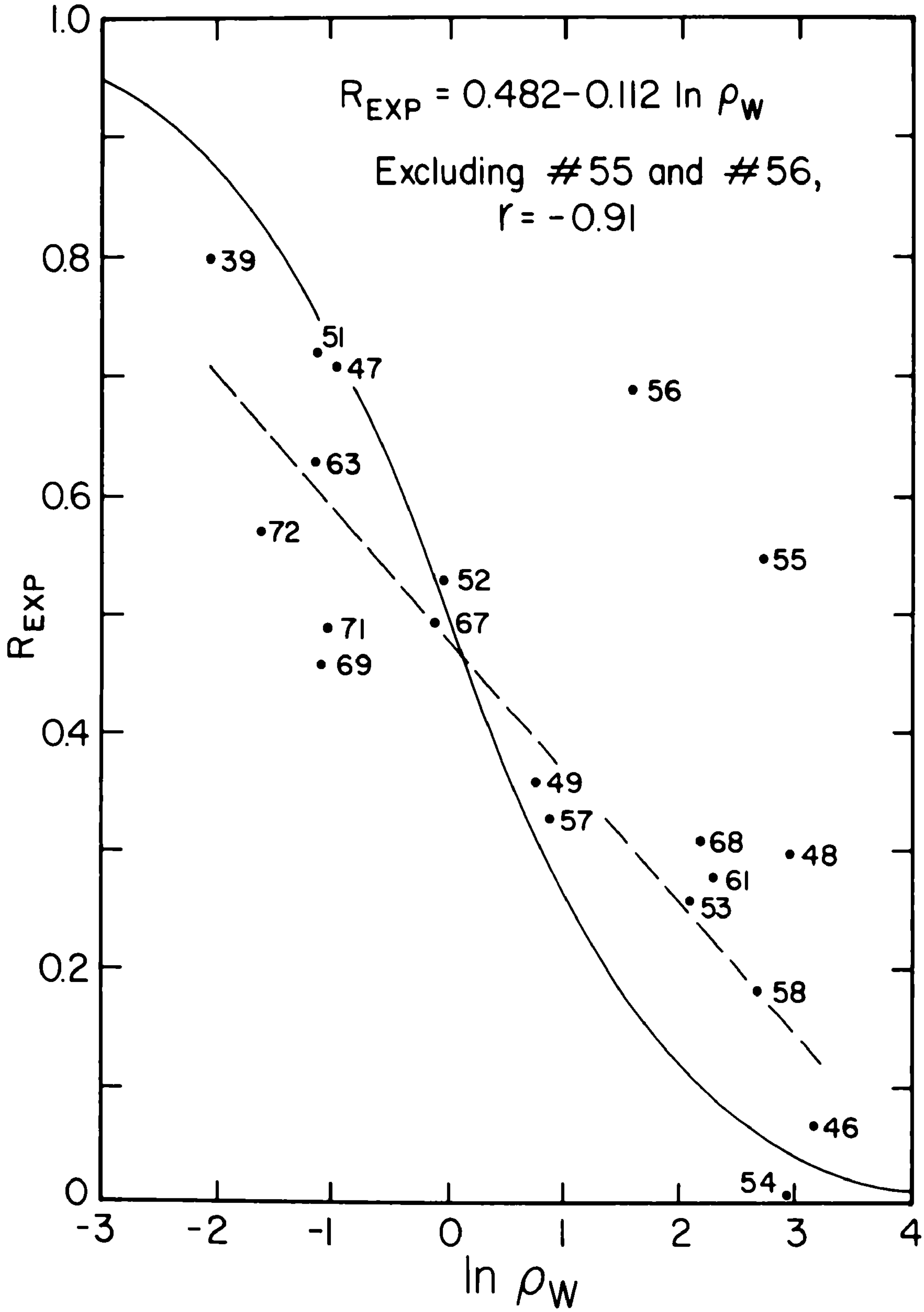
REXP = 0.78−0.121 ln qS
r = −0.80
REXP
ln qS
1.0
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
43
42
50
39
44
41
73
51
47
64
56
38
59
63
62
60
72
66
55
52
67
71
70
65
69
49
57
68
48
61
53
45
40
58
46
54

FIGURE 2



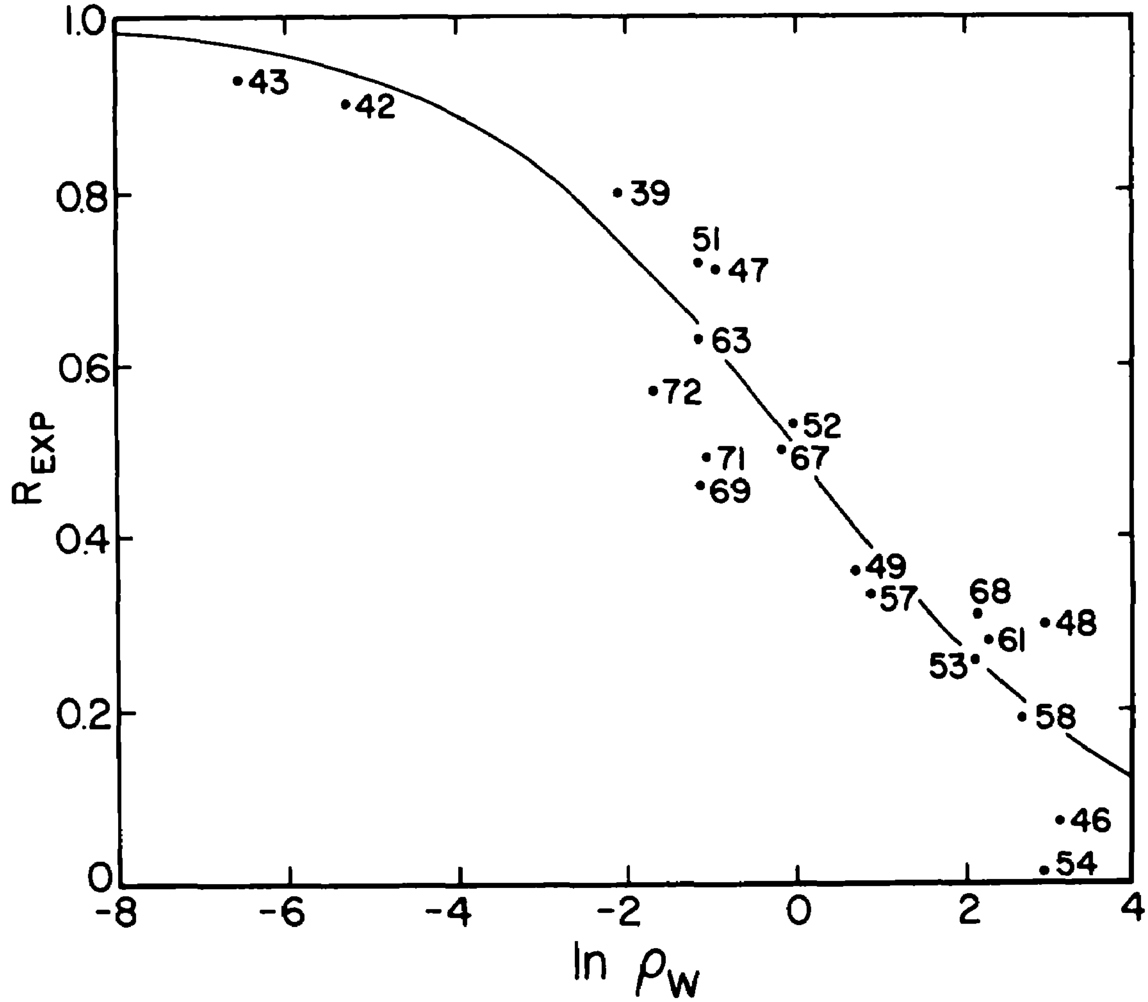
$R_{EXP} = 0.86 - 0.143 \ln q_S$
$r = -0.92$
R_{EXP}
$\ln q_S$
1.0
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
39
51
47
56
63
72
55
52
67
71
69
49
57
68
48
61
53
58
46
54

FIGURE 3



R_{EXP} = 0.482-0.112 ln ρ_W
Excluding #55 and #56,
r = -0.91
1.0
0.8
0.6
0.4
0.2
0
R_{EXP}
-3
-2
-1
0
1
2
3
4
ln ρ_W
39
51
47
56
63
72
55
52
71
67
69
49
57
68
48
61
53
58
46
54

FIGURE 4



1.0
0.8
0.6
0.4
0.2
0
R_{EXP}
-8
-6
-4
-2
0
2
4
ln ρ_W
43
42
39
51
47
63
72
52
71
67
69
49
57
68
48
61
53
58
46
54

FIGURE 5