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EMISSION CORRECTION FACTORS:

A STUDY TO DEVELOP AN IMPROVED MATHEMATICAL APPROACH

Prepared for:

Environmental Protection Agency

Office of Air and Waste Management

Mobile Source Air Pollution Control

Under:

Purchase Order No. CD-8-0145-A

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A SUBSIDIARY OF Whittaker

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I. INTRODUCTION

This report is submitted as deliverable item in fulfillment of work performed under Purchase Order No. CD-8-0145-A for the Environmental Protection Agency (EPA), Office of Air and Waste Management (OAWM), Mobile Source Air Pollution Control (MSAPC). The objective of the program was to examine the mathematical formulation of existing emission correction factors and to suggest a new mathematical approach which is simple to use, simple to update when new data are available, and which considers the dependencies among various factors.

The scope of work was aimed at isolating a set of prospective statistical techniques and discussing the advantages/disadvantages of each technique. Evaluation criteria were to include but not be limited to, the following:

- Ease in application (manually and computer).
- Ease in updating when additional data are available.
- Required input data sample.
- Ability to relate to engineering concepts.
- Ability to assess and express correction factor uncertainty.
- Required test procedure changes to obtain needed data.

This report summarizes findings and offers recommendations for future correction factor development. Because of the broad scope of the effort, these recommendations are necessarily exemplary rather than definitive. Their purpose is to serve as an outline or prospectus for further development of correction factor methodology.

II. SUMMARY

This report is developed as a series of three working papers developed in chronological sequence. Each of these papers touches on one or more issues pertinent to the formulation of emission correction factors.

The formulation and application of emission correction factors represents an attempt to deal with the multiplicity of variables which influence vehicle emissions. Mathematically the problem can be represented as a function

$$y_p = f(x_1, x_2, \ldots, x_k) \tag{1}$$

where y_p represents emission of a particular pollutant in grams per mile and $x_1, x_2, \ldots, x_k$ are variables such as speed, ambient temperature and vehicle variables known to affect emissions. Equation (1) is often referred to as a response relation and can be represented as a hypersurface (response surface) in $(k + 1)$-dimensional space. The subspace consisting of the variables $x_1, x_2, \ldots, x_k$ is often referred to as the "treatment space" or the "x-space."*

To explore the emission response space in sufficient detail to allow formulation of the response relation (1) is the crux of the correction-factor problem. The standard emission test as formulated in the Federal Test Procedure (FTP) constitutes only a single point in the treatment space and consequently provides no information pertinent to emissions under any conditions other than those specified in the FTP. Moreover, because of the large number of variables which can affect emissions and the prospect of interaction among these variables, it is quite costly to explore the x-space with a sufficient number of treatments and emission tests to represent all possible driving and use scenarios.

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* The term "treatment" is a vestige of the fact that the science of experiment design originated in agricultural research, in which various fertilizer treatments were applied to plots of land to determine the effect of these treatments on crop yield.

Efforts of the EPA to provide correction factors applicable to a wide range of scenarios provide an approximation to the ideal response-surface approach. Because of cost constraints and other considerations, data-acquisition methods were sometimes less than optimum and it was necessary to draw heavily on engineering knowledge and judgment in order to fill in data lacunae. For example, it is well known that if two variables x_1 and x_2 are subject to interaction in the sense of experiment design, then it is necessary to explore the (x_1, x_2)-plane with a set of treatments which are distributed areally in that plane. Under realistic circumstances, however, it may be possible to sample x_1 only at some fixed value of x_2, and to sample x_2 only at some fixed value of x_1. Such a sampling design can not provide the information needed to assess the interaction between x_1 and x_2, and it thus becomes necessary to supplement such data with engineering judgment.

The working papers which constitute the bulk of this report provide a critique of past methodology and offer suggestions for future correction factor development. They address certain general considerations in the philosophy of correction-factor formulation as well as certain specific issues involved in implementing that philosophy.

Working Paper No. 1 well exemplifies these two aspects of the present study. Though the paper is concerned primarily with the representation of preliminary speed correction factors, it also raises such basic and general issues as the degree of commonality of correction-factor functions for various groups of vehicles and the most parsimonious representation of functional relations. In this connection it was shown that principal component analysis provides a means for structuring the information content of correction factors in a concise way, as well as a means for identifying areas of commonality among various makes and models or other homogeneous groups of vehicles. Specifically, it was shown that the functional relation between correction factor and average speed tends to have a common shape for various groups of vehicles and that among-group variation can be accommodated by one or two group-specific parameters rather than the four or five parameters assumed in the exponential and polynomial regression relations.

Working Paper No. 2 extends the analysis of preliminary speed correction factors to their incorporation in the more general "R-factors," which take into account, in addition to average speed, a number of other factors affecting emissions. These include mileage accumulation, the prevailing ambient temperature, and the fraction of vehicle operation performed in the cold transient, hot transient, and stabilized modes. In addition, the paper makes generalizations pertaining to the identification and definition of variables affecting emissions and the important role played by nondimensionalization in the formulation of correction factors.

One of the difficulties encountered in the use and interpretation of the R-factors arises from the notion that the fraction of miles driven in the cold transient, stabilized and hot transient conditions can be driven at arbitrary speeds, whereas the FTP definition of these conditions imply associated speeds of 26, 16 and 26 mph, respectively. When average speed and mode of operation are incorporated in a common correction factor, therefore, this combination impacts on trip lengths and the proportion of trips originated in the cold-start mode. To reconcile all the constraints it is necessary to invoke certain assumptions about either the lengths or time durations of trips and the portions thereof spent in a Bag 1, Bag 2 or Bag 3 condition of operation. Otherwise, the implied warm-up times may exceed the time allowable under the FTP test. Reformulation of the R-factors, therefore, is indicated to be a fruitful area for refinement of correction factors.

In the general area of methodology, Working Paper No. 2 provides a good example of how correction factors may be simplified by an adroit definition of variables affecting emissions. In particular, it is shown that it may be advantageous to represent the effect of speed on emissions in terms of grams per unit time rather than in terms of grams per unit distance. This observation arises from a dimensional analysis type of argument and is shown empirically to lead to considerable simplification of the speed correction factor function. It is further shown that difficulties arising from numerical constraints imposed by choice of units can often be avoided by nondimensionalization of the independent as well as the dependent variables in a functional expression.

Working Paper No. 3 delves further into the problem of transient versus stabilized operation. In particular, it suggests that the thermal state of operation of a vehicle comprises a continuum and that it may be more advantageous to view the operating history of a vehicle in this light rather than in the light of discrete cold transient, hot transient and stabilized states. This type of treatment would circumvent the need to define and estimate "warm-up times" under various ambient and use conditions. An accretion-depletion model of thermal transient effects is outlined and it is proposed that any state of operation of a vehicle can be represented as a linearly weighted combination of initial and final emission values as represented by cold transient and stabilized operation, respectively.

In summary, several directions for possible refinement of correction factors have been identified. In most instances, these directions have been made explicit by an example based on actual data.

3. RECOMMENDATIONS

Recommendations aimed at an optimal approach for future correction factor development are evolved primarily in Working Paper No. 2. These recommendations are based on the incontrovertible fact that correction factors represent an attempt to express the functional dependence of emissions on a host of variables and that a designed experiment is best suited for defining this function. The experiment should be based on the best available engineering estimates and experience pertinent to the complexity of the functional relation, the interaction of variables, and the relative importance of these variables in the emission-generation process. The thrust of the emission-factor formulation should be to separate vehicle or vehicle-class dependent aspects of emissions from incremental effects more or less common to all vehicles.

Specific recommendations, therefore, are as follows:

- Systematic analysis of the degree of commonality of the effects of emission-related variables should precede attempts to develop a correction-factor response function.

- Correction factors are best determined through a designed experiment in which the allocation of "treatments" (that is, combinations of levels of the variables $x_1, x_2, \ldots, x_p$) is made according to the anticipated degree of nonlinearity within variables and degree of interaction among variables.

- Mathematical representation of correction factors should be approached with due regard to the magnitude and engineering importance of variables perceived to be important. The prospect of combined or derived variables should not be overlooked nor should the fact that variables originally identified may be highly covariant.

- Difficulties arising from numerical constraints imposed by choice of units can often be avoided by nondimensionalization of independent or predictor variables as well as the dependent variable.

It is recognized that any proposed program aimed at refinement of correction factors must be balanced against associated costs. Though such considerations are beyond the scope of the current effort, a cost-benefit examination of correction-factor refinement is considered to be an essential prelude to future correction factor development.

4. WORKING PAPERS

4.1 WORKING PAPER NO. 1:

A FACTOR-ANALYTICAL APPROACH TO EMISSION CORRECTION FACTORS

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Working Paper No. 1
Project No. 8411
Environmental Protection Agency
April 5, 1978
H. T. McAdams

A FACTOR-ANALYTIC APPROACH TO EMISSION CORRECTION FACTORS

1. INTRODUCTION

Vehicle exhaust emissions are functions of a large number of variables. Some of these variables are vehicle related, others pertain to the operating environment, and still others are the consequence of use priorities dictated by the needs of a mobile society. Consequently, one can envision an infinite variety of emission scenarios, each having its own peculiar impact on air quality.

Because emissions are either continuous or discrete functions of a large number of variables it is impractical if not impossible to measure emissions for every scenario of interest. The only alternative is to attempt to derive a modeling or scaling procedure by means of which a limited number of measurements can be employed to predict emissions over the entire domain of the multivariate emission function.

Perhaps the most "vehicle-related" variable is the vehicle itself. Different vehicles exhibit different emission characteristics, and these differences, of course, stem from many design variables which differentiate one vehicle from another. If all these variables are pooled, however, one can think of a "vehicle variable" which changes value discretely as one goes from one make or model year to another. In short, vehicles can be considered as comprising a state space. Viewed as a process, pollutant generation assumes successively different states as attention is directed successively to different automobiles or different classes of automobiles. Within a given state (i.e., a particular automobile or automobile group) the emission process is responsive to other, continuous, process variables, such as operating speed, ambient temperature, and the like.

It is important to distinguish between data analysis applicable to a given state of the process and that applicable to some aggregation of states. In short, it is important to know to what extent the effect of the process variables are common to all states and to what extent these effects must be particularized to a given state. Factor analytic methods--in particular, principal component analysis--provide an approach to this problem and is indicated to have important bearing on the parsimonious formulation of emission correction factors. The method is demonstrated in relation to speed correction factors but is evidently applicable to other variables in different contexts.

2. SPEED CORRECTION FACTORS

Speed correction factors for 18 groups of vehicles are given in the attached Tables II.12, II.13, II.14 from "Supplement 8 Light Duty Vehicle Correction Factors," Memo *Janet Becker to J. Hidinger/J. Horowitz (3/7/77).* The vehicle groups play the role of states in the process, whereas speed plays the role of a continuous variable affecting emissions within a given state. Consider, for example, the formulation of CO correction factors as

$$\ell n \text{ CF} = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 + A_5 s^5 \qquad (1)$$

or

$$\text{CF} = \exp (A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 + A_5 s^5) \qquad (2)$$

The correction factors are normalized to yield CF = 1.0 at s = 19.6 mi/hr. Note that separate coefficients (and separate correction factors as a function of speed) are given for each of the 18 vehicle groups. One might well ask if some "commonality" might not exist among the groups so that a single functional form might unify all groups except for a state "scaling factor" peculiar to each vehicle group.

Table II.12

Group Definitions

Group Number	Group Definition
Group 1	Denver pre-controlled
Group 2	All low altitude cities pre-controlled
Group 3	1966-1967 California
Group 4	1968 low altitude cities
Group 5	1969 low altitude cities
Group 6	1970 low altitude cities
Group 7	1971 low altitude cities
Group 8	1968 Denver
Group 9	1969 Denver
Group 10	1970 Denver
Group 11	1971 Denver
Group 12	1972 Denver
Group 13	1972 Los Angeles
Group 14	1972 low altitude cities
Group 15	1973-1974 Denver
Group 16	1973-1974 Los Angeles
Group 17	1973-1974 low altitude cities
Group 18	1975 low altitude cities

Table II.13

Speed Correction Factors

Normalized Equations

$$\ln HC = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 + A_5 s^5$$

$$\ln CO = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 + A_5 s^5$$

$$NOx = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4$$

```
GROUP= 1
HC=     0.22461E+01   -0.29097E+00    0.15889E-01   -0.47249E-03    0.69408E-05   -0.39200E-07
CO=     0.18198E+01   -0.25466E+00    0.15235E-01   -0.48740E-03    0.75821E-05   -0.44951E-07
NOX=    0.24442E+01   -0.25011E+00    0.13829E-01   -0.28703E-03    0.20758E-05    0.0

GROUP= 2
HC=     0.23103E+01   -0.28957E+00    0.15299E-01   -0.44669E-03    0.64818E-05   -0.36346E-07
CO=     0.23399E+01   -0.29698E+00    0.16007E-01   -0.47740E-03    0.70675E-05   -0.40398E-07
NOX=    0.16863E+01   -0.11830E+00    0.65497E-02   -0.13714E-03    0.10085E-05    0.0

GROUP= 3
HC=     0.21656E+01   -0.26999E+00    0.14422E-01   -0.43364E-03    0.65074E-05   -0.37810E-07
CO=     0.24415E+01   -0.29147E+00    0.14293E-01   -0.38785E-03    0.52978E-05   -0.28244E-07
NOX=    0.11265E+01   -0.39340E-01    0.26864E-02   -0.50802E-04    0.47729E-06    0.0
```

(Table II.13 con't)

```
GROUP= 4
HC=     0.23973E+01   -0.29998E+00   0.16135E-01   -0.48749E-03   0.72909E-05   -0.41977E-07
CO=     0.24655E+01   -0.30502E+00   0.16950E-01   -0.47397E-03   0.69908E-05   -0.39976E-07
NOX=    0.12268E+01   -0.44498E-01   0.26248E-02   -0.56715E-04   0.43429E-06    0.0

GROUP= 5
HC=     0.24087E+01   -0.30819E+00   0.16817E-01   -0.50684E-03   0.75385E-05   -0.43160E-07
CO=     0.27780E+01   -0.31913E+00   0.15318E-01   -0.42233E-03   0.58495E-05   -0.31497E-07
NOX=    0.10174E+01   -0.11896E-01   0.91437E-03   -0.21574E-04   0.18230E-06    0.0

GROUP= 6
HC=     0.22322E+01   -0.28499E+00   0.15383E-01   -0.45674E-03   0.67349E-05   -0.38380E-07
CO=     0.27890E+01   -0.32711E+00   0.16294E-01   -0.46757E-03   0.67191E-05   -0.37440E-07
NOX=    0.98760E+00   -0.19567E-01   0.16964E-02   -0.40400E-04   0.32800E-06    0.0

GROUP= 7
HC=     0.22522E+01   -0.28772E+00   0.15682E-01   -0.47318E-03   0.70795E-05   -0.40846E-07
CO=     0.27074E+01   -0.33131E+00   0.17618E-01   -0.53856E-03   0.81740E-05   -0.47780E-07
NOX=    0.11592E+01   -0.44454E-01   0.29643E-02   -0.66899E-04   0.52236E-06    0.0

GROUP= 8
HC=     0.20278E+01   -0.27305E+00   0.15358E-01   -0.46030E-03   0.67853E-05   -0.38488E-07
CO=     0.18692E+01   -0.27662E+00   0.17233E-01   -0.55828E-03   0.87168E-05   -0.51698E-07
NOX=    0.18866E+01   -0.16129E+00   0.90499E-02   -0.18561E-03   0.13256E-05    0.0

GROUP= 9
HC=     0.21506E+01   -0.28362E+00   0.15384E-01   -0.44214E-03   0.62873E-05   -0.34631E-07
CO=     0.18213E+01   -0.27205E+00   0.17030E-01   -0.55202E-03   0.86254E-05   -0.51144E-07
NOX=    0.15578E+01   -0.11393E+00   0.67183E-02   -0.14341E-03   0.10608E-05    0.0

GROUP=10
HC=     0.22302E+01   -0.29365E+00   0.16236E-01   -0.48415E-03   0.71159E-05   -0.40286E-07
CO=     0.20142E+01   -0.29519E+00   0.19635E-01   -0.62161E-03   0.99366E-05   -0.59978E-07
NOX=    0.20452E+01   -0.19401E+00   0.11074E-01   -0.23175E-03   0.16637E-05    0.0
```

(Table II.13 con't)

```
GROUP=11
HC=      0.21223E+01   -0.29107E+00    0.16909E-01   -0.52615E-03    0.80271E-05   -0.47012E-07
CO=      0.20453E+01   -0.31062E+00    0.20495E-01   -0.70853E-03    0.11621E-04   -0.71569E-07
NOX=     0.14326E+01   -0.12136E+00    0.70302E-02   -0.14629E-03    0.10614E-05    0.0
FE=      0.35076E-01    0.87843E-01   -0.27727E-02    0.47466E-04   -0.33220E-06    0.0

GROUP=12
HC=      0.21536E+01   -0.28345E+00    0.15695E-01   -0.46976E-03    0.69383E-05   -0.39471E-07
CO=      0.23187E+01   -0.34115E+00    0.20945E-01   -0.66589E-03    0.10223E-04   -0.59827E-07
NOX=     0.14482E+01   -0.12244E+00    0.79502E-02   -0.17108E-03    0.12578E-05    0.0


GROUP=13
HC=      0.20735E+01   -0.28935E+00    0.17304E-01   -0.55471E-03    0.86420E-05   -0.51311E-07
CO=      0.25752E+01   -0.32889E+00    0.18975E-01   -0.62826E-03    0.10092E-04   -0.61273E-07
NOX=     0.24597E+00    0.84195E-01   -0.34084E-02    0.62988E-04   -0.41397E-06    0.0


GROUP=14
HC=      0.23495E+01   -0.30496E+00    0.16842E-01   -0.50962E-03    0.75952E-05   -0.43496E-07
CO=      0.26845E+01   -0.33282E+00    0.17628E-01   -0.52412E-03    0.77222E-05   -0.43702E-07
NOX=     0.12817E+01   -0.80487E-01    0.53574E-02   -0.11889E-03    0.90106E-06    0.0


GROUP=15
HC=      0.21134E+01   -0.28568E+00    0.16318E-01   -0.50079E-03    0.75507E-05   -0.43719E-07
CO=      0.21549E+01   -0.32912E+00    0.21011E-01   -0.68906E-03    0.10839E-04   -0.64712E-07
NOX=     0.15345E+01   -0.12567E+00    0.78592E-02   -0.16943E-03    0.12549E-05    0.0


GROUP=16
HC=      0.21194E+01   -0.29863E+00    0.18447E-01   -0.61654E-03    0.99206E-05   -0.60402E-07
CO=      0.25456E+01   -0.36295E+00    0.23277E-01   -0.81504E-03    0.13623E-04   -0.85591E-07
NOX=     0.70481E+00    0.38153E-01   -0.17391E-02    0.32614E-04   -0.20385E-06    0.0


GROUP=17
HC=      0.26838E+01   -0.34463E+00    0.19542E-01   -0.62572E-03    0.97844E-05   -0.58337E-07
CO=      0.28393E+01   -0.36876E+00    0.21078E-01   -0.67644E-03    0.10627E-04   -0.63641E-07
NOX=     0.78384E+00    0.32855E-03    0.10603E-02   -0.31935E-04    0.29039E-06    0.0


GROUP=18
HC=      0.23954E+01   -0.33578E+00    0.21161E-01   -0.73155E-03    0.12072E-04   -0.74857E-07
CO=      0.24275E+01   -0.39156E+00    0.27072E-01   -0.97618E-03    0.16527E-04   -0.10432E-06
NOX=     0.94213E+00   -0.42324E-01    0.38625E-02   -0.93985E-04    0.75308E-06    0.0
```

Table II.14

Selected Speed Correction Factors - Warm Operation

Hydrocarbon

Average Speed

GROUP	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	55.000	60.000
1	3.107	1.679	1.201	0.987	0.858	0.761	0.684	0.629	0.597	0.585	0.571	0.516
2	3.297	1.749	1.224	0.986	0.844	0.740 ✓	0.659	0.600	0.565	0.547	0.530	0.482
3	3.083	1.708	1.219	0.986	0.841	0.733	0.650	0.592	0.556	0.534	0.503	0.429
4	3.470	1.308	1.246	0.984	0.821	0.700	0.606	0.538	0.497	0.472	0.445	0.381
5	3.419	1.773	1.231	0.985	0.834	0.720 ✓	0.630	0.565	0.526	0.504	0.479	0.414
6	3.123	1.695	1.208	0.987	0.853	0.754	0.677	0.622	0.591	0.576	0.557	0.695
7	3.160	1.739	1.215	0.986	0.845	0.740	0.658	0.600	0.567	0.551	0.529	0.460
8	2.700	1.548	1.160	0.990	0.889	0.811	0.748	0.703	0.681	0.678	0.672	0.616
9	2.903	1.600	1.170	0.990	0.891	0.819	0.762	0.720	0.699	0.694	0.691	0.649
10	3.039	1.649	1.190	0.988	0.867	0.775	0.702	0.649	0.619	0.609	0.596	0.538
11	2.798	1.571	1.168	0.989	0.877	0.788	0.716	0.667	0.644	0.638	0.617	0.525
12	2.928	1.623	1.184	0.988	0.871	0.781	0.709	0.659	0.632	0.624	0.612	0.553
13	2.705	1.548	1.165	0.989	0.871	0.773	0.694	0.640	0.616	0.610	0.589	0.493
14	3.276	1.726	1.216	0.986	0.844	0.736	0.650	0.589	0.554	0.538	0.519	0.456
15	2.815	1.582	1.172	0.989	0.876	0.788	0.717	0.669	0.646	0.641	0.628	0.553
16	2.763	1.575	1.181	0.987	0.848	0.729	0.635	0.573	0.543	0.531	0.497	0.383
17	3.963	1.931	1.285	0.981	0.784	0.635	0.523	0.446	0.401	0.373	0.337	0.258
18	3.194	1.708	1.228	0.984	0.803	0.653	0.540	0.468	0.432	0.414	0.273	0.260

(Table II.14 con't)

Carbon Monoxide

Ave. e Speed

GROUP	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	55.000	60.000
1	2.389	1.463	1.142	0.991	0.889	0.803	0.733	0.686	0.665	0.663	0.647	0.558
2	3.319	1.751	1.225	0.986	0.841	0.734	0.650	0.592	0.557	0.539	0.518	0.454
3	3.656	1.856	1.251	0.985	0.837	0.738	0.663	0.608	0.572	0.554	0.542	0.515
4	3.621	1.844	1.253	0.984	0.823	0.707	0.619	0.556	0.517	0.493	0.465	0.399
5	4.554	2.120	1.329	0.979	0.721	0.644	0.543	0.469	0.418	0.384	0.358	0.323
6	4.511	2.103	1.326	0.979	0.778	0.637	0.533	0.457	0.407	0.374	0.345	0.299
7	4.174	2.003	1.299	0.975	0.776	0.633	0.526	0.453	0.406	0.375	0.341	0.273
8	2.345	1.418	1.121	0.992	0.905	0.827	0.760	0.717	0.703	0.711	0.701	0.600
9	2.277	1.395	1.113	0.993	0.913	0.841	0.780	0.743	0.737	0.755	0.756	0.662
10	2.541	1.488	1.149	0.990	0.873	0.770	0.684	0.629	0.608	0.608	0.589	0.478
11	2.516	1.474	1.148	0.989	0.863	0.746	0.651	0.591	0.568	0.566	0.534	0.397
12	2.885	1.540	1.149	0.991	0.891	0.804	0.729	0.677	0.657	0.658	0.643	0.538
13	3.791	1.916	1.291	0.980	0.771	0.612	0.496	0.420	0.378	0.355	0.324	0.246
14	4.056	1.950	1.281	0.982	0.804	0.675	0.577	0.510	0.470	0.451	0.436	0.391
15	2.599	1.459	1.127	0.992	0.900	0.814	0.739	0.691	0.675	0.681	0.661	0.530
16	3.384	1.744	1.237	0.983	0.795	0.641	0.527	0.458	0.427	0.410	0.360	0.227
17	4.239	1.980	1.293	0.981	0.762	0.634	0.525	0.454	0.415	0.395	0.364	0.278
18	2.988	1.580	1.183	0.986	0.821	0.671	0.557	0.493	0.475	0.478	0.434	0.265

Nitric Oxide

Average Speed

GROUP	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	55.000	60.000
1	1.505	1.060	0.941	1.010	1.161	1.319	1.440	1.511	1.551	1.608	1.764	2.129
2	1.242	1.031	0.974	1.004	1.074	1.146	1.203	1.239	1.265	1.306	1.404	1.615
3	0.990	0.946	0.960	1.004	1.058	1.109	1.150	1.182	1.213	1.258	1.340	1.489
4	1.063	0.992	0.980	1.002	1.038	1.075	1.105	1.129	1.152	1.189	1.257	1.384
5	0.978	0.970	0.981	1.002	1.026	1.049	1.070	1.091	1.115	1.151	1.208	1.298
6	0.927	0.924	0.956	1.004	1.056	1.102	1.141	1.173	1.206	1.250	1.323	1.445
7	1.003	0.949	0.960	1.004	1.059	1.110	1.150	1.180	1.207	1.250	1.331	1.483
8	1.284	1.006	0.944	1.008	1.128	1.255	1.359	1.429	1.477	1.531	1.641	1.877
9	1.143	0.966	0.944	1.007	1.105	1.200	1.275	1.323	1.358	1.406	1.511	1.733
10	1.324	0.997	0.930	1.010	1.152	1.297	1.410	1.480	1.524	1.582	1.721	2.031
11	1.181	0.981	0.946	1.007	1.109	1.214	1.300	1.361	1.407	1.462	1.570	1.786
12	1.014	0.860	0.887	1.012	1.174	1.330	1.454	1.542	1.606	1.678	1.810	2.070
13	0.589	0.806	0.934	1.004	1.043	1.070	1.097	1.132	1.175	1.221	1.258	1.268
14	0.999	0.903	0.924	1.008	1.112	1.208	1.282	1.332	1.369	1.421	1.526	1.736
15	1.082	0.907	0.909	1.010	1.148	1.280	1.382	1.452	1.501	1.564	1.691	1.955
16	0.856	0.943	0.986	1.001	1.002	1.000	1.002	1.014	1.036	1.067	1.103	1.136
17	0.808	0.864	0.934	1.006	1.069	1.121	1.161	1.193	1.226	1.274	1.353	1.486
18	0.816	0.819	0.897	1.009	1.124	1.222	1.294	1.344	1.386	1.446	1.560	1.777

Evidence of undue complexity in the formulation of speed correction factors is found in the fact that the coefficients in (1) or (2) appear to be highly correlated, as shown in Figure 1, in which A_0 has been plotted against A_1 for CO for the 18 groups. The correlation coefficient for the plot is 0.69. Similarly, one can compute correlations for all pairs of coefficients to obtain a 6×6 correlation matrix, as shown in Table I. Note that many of the correlation coefficients approach either +1 or -1. The result is that the correlation matrix is effectively of less than full rank. Indeed, if one computes the eigenvalues of the matrix only two are found to be of appreciable magnitude. This fact suggests that the coefficients do not vary independently from group to group but are highly covariant. Consequently, a simpler expression with fewer terms should suffice to represent the speed vs. emission relations for the various vehicle groups. Similar conclusions can be drawn from Table II for hydrocarbon emissions (HC) and from Table III for nitrogen oxide emissions (NO_x). Note, particularly, that the correlation matrix for NO_x has only one eigenvalue of appreciable magnitude and is consequently of rank 1.

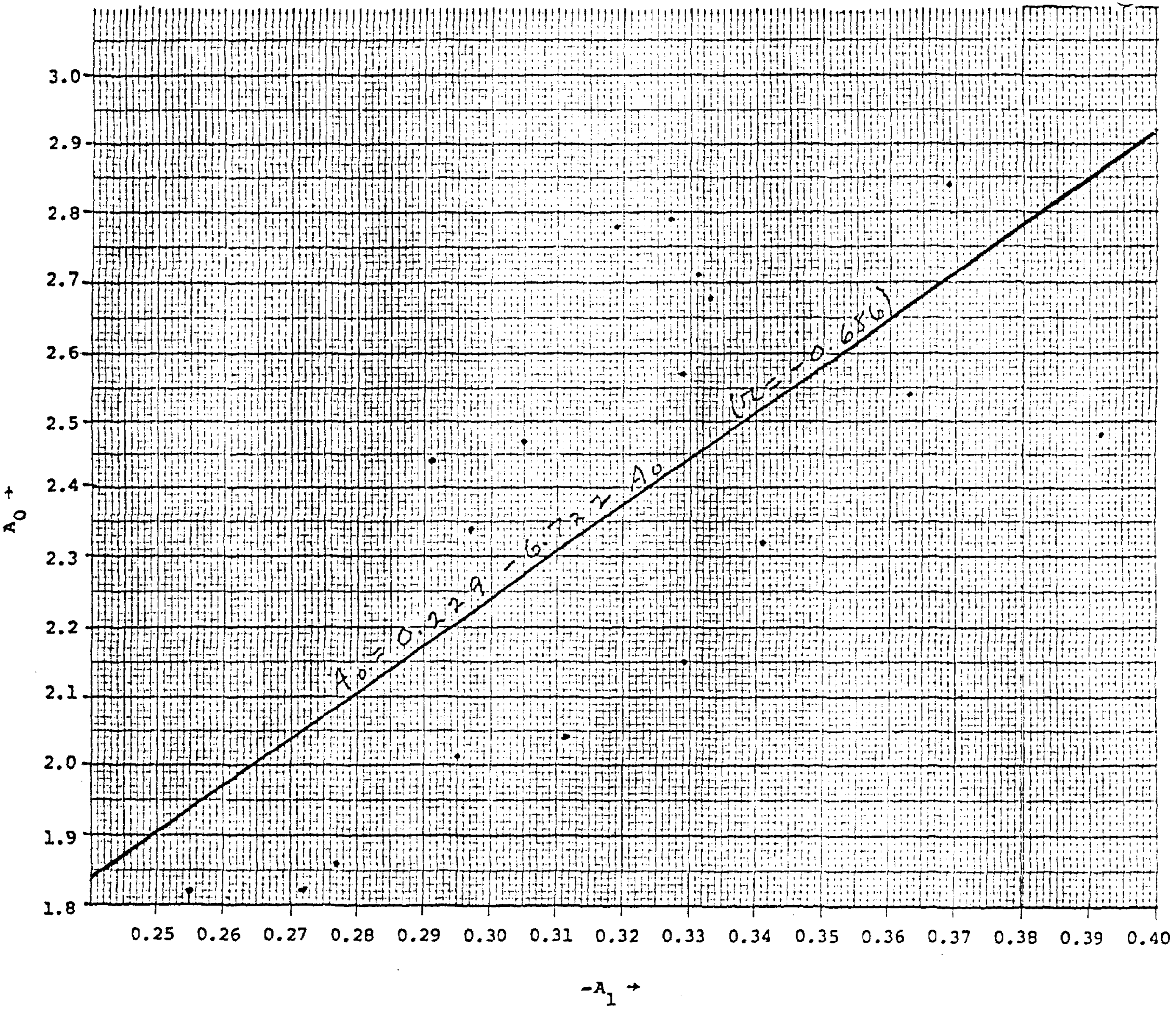


FIGURE 1 - Relation Between Coefficients in Speed Correction Factor Equation

TABLE I

CORRELATION MATRIX FOR COEFFICIENTS IN CORRECTION FACTOR vs SPEED EQUATION FOR CARBON MONOXIDE (CO)

	A_0	A_1	A_2	A_3	A_4	A_5
A_0	1.0000	-0.6871	0.0933	0.0330	-0.0705	0.0851
A_1	-0.6871	1.0000	-0.7825	0.6860	-0.6489	0.6309
A_2	0.0933	-0.7825	1.0000	-0.9868	0.9741	-0.9655
A_3	0.0330	0.6860	-0.9868	1.0000	-0.9977	0.9943
A_4	-0.0705	-0.6489	0.9741	-0.9977	1.0000	-0.9992
A_5	0.0851	0.6309	-0.9655	0.9943	-0.9992	1.0000

EIGENVALUES: 4.5115
1.4534
0.0350
0.0002
0.0000
0.0000

TRACE = 6.0001 = SUM OF EIGENVALUES

TABLE II

CORRELATION MATRIX FOR COEFFICIENTS IN CORRECTION FACTOR vs SPEED EQUATION FOR HYDROCARBONS (HC)

	A_0	A_1	A_2	A_3	A_4	A_5
A_0	1.0000	-0.8320	0.4854	-0.3922	0.3568	-0.3375
A_1	-0.8320	1.0000	-0.8821	-0.8148	-0.7823	-0.7622
A_2	0.4854	-0.8821	1.0000	-0.9883	0.9752	-0.9650
A_3	-0.3922	0.8148	-0.9883	1.0000	-0.9975	0.9935
A_4	0.3568	-0.7823	0.9752	-0.9975	1.0000	-0.9991
A_5	-0.3375	0.7622	-0.9650	0.9935	-0.9991	1.0000

EIGENVALUES: 4.9589
1.0004
0.0405
0.0001
0.0000
0.0000

TRACE = 5.9999 = SUM OF EIGENVALUES

TABLE III

CORRELATION MATRIX FOR COEFFICIENTS IN CORRECTION FACTOR vs SPEED EQUATION FOR NITROGEN OXIDES (NO_x)

	A_0	A_1	A_2	A_3	A_4
A_0	1.0000	-0.9868	0.9677	-0.9584	0.9525
A_1	-0.9868	1.0000	-0.9957	0.9918	-0.9886
A_2	0.9677	-0.9957	1.0000	-0.9993	0.9979
A_3	-0.9584	0.9918	-0.9993	1.0000	-0.9996
A_4	0.9525	-0.9886	0.9979	-0.9996	1.0000

EIGENVALUES: 4.9356
0.0637
0.0008
0.0000
0.0000

TRACE = 5.0001 = SUM OF EIGENVALUES

The correlations among coefficients of the emission versus speed equations suggest that attention be directed to Table II.4, in which the equations have been used to compute emissions for 5-mph increments of speed. One can compute the covariance (or correlation) between correction factors for 5 mph and 10 mph, 5 mph and 15 mph, ..., 55 mph and 60 mph to produce a 12 x 12 matrix of covariances or correlations for each of the three pollutants. For the purpose of this analysis the covariance matrix is preferred, because it retains the scale aspect of the relation between correction factors for various speeds. It can be shown that the number of linearly independent functions needed in a regression equation to represent emissions as a function of speed can be deduced from the eigenvalues of the covariance matrix and that the form of these "basis functions" can be deduced from the corresponding eigenvectors.[1]

First, let us direct attention to the eigenvalues of the 12 x 12 covariance matrices (see Table 4). Only eigenvalues significant to the fourth decimal place are tabulated. Note that the first eigenvalue constitutes 96.5% of the trace for CO, and 90.6% and 89.2% of the trace for HC and NO_x respectively. Thus it appears that "most" of the particularization for vehicle group could be achieved by a single basis function. If further refinement is required, a second basis function could be used, but this second function would serve only to "trim" the effect of the first function and provide a second-order refinement.

TABLE 4

EIGENVALUES OF COVARIANCE MATRICES
FOR SPEED CORRECTION FACTORS
EVALUATED AT 5-mph SPEED INTERVALS

CO	HC	NO_x
0.7126	0.1364	0.2499
0.0250	0.0141	0.0298
0.0009	0.0002	0.0005
	0.0001	

---

[1] H. T. McAdams, "A Factor Analytic Approach to the Identification of Manufacturing Systems," Proc. of the CIRP Seminars on Manufacturing Systems, Vol. 1, No. 2 (1972), pp. 79-97

The eigenvectors resulting from the covariance analysis consist of 12 components corresponding to 5 mph, 10 mph, ..., 60 mph. These components are plotted versus speed in Figure 2. for the eigenvectors corresponding to the eigenvalues of greatest magnitude. Note that the eigenvectors for CO and HC are essentially monotonic for increasing speed, whereas the eigenvector for NO_x peaks at about 15 mph. The eigenvectors corresponding to the second largest eigenvalues are plotted in Figure 3. Note that certain similarities are apparent in the first eigenvectors for CO and HC and the second eigenvector for NO_x. It is known that factors affecting CO and HC emissions often tend to affect NO_x emissions inversely, and it is hypothesized that this tendency is reflected in the emission eigenvectors.

It is informative to compare the eigenvectors in Figure 2 with the plots in Figure 4, which represents the actual relations between correction factors and speed for Group 2. Except for sign (sign is arbitrary!) corresponding curves in Figure 2 and Figure 4 have quite similar shapes.

The implications of the analysis can now be expressed as follows. Let f(x) denote the correction factor for a given pollutant at speed x when averaged over all 18 groups. For the present it is assumed that the domain of the function f(x) is

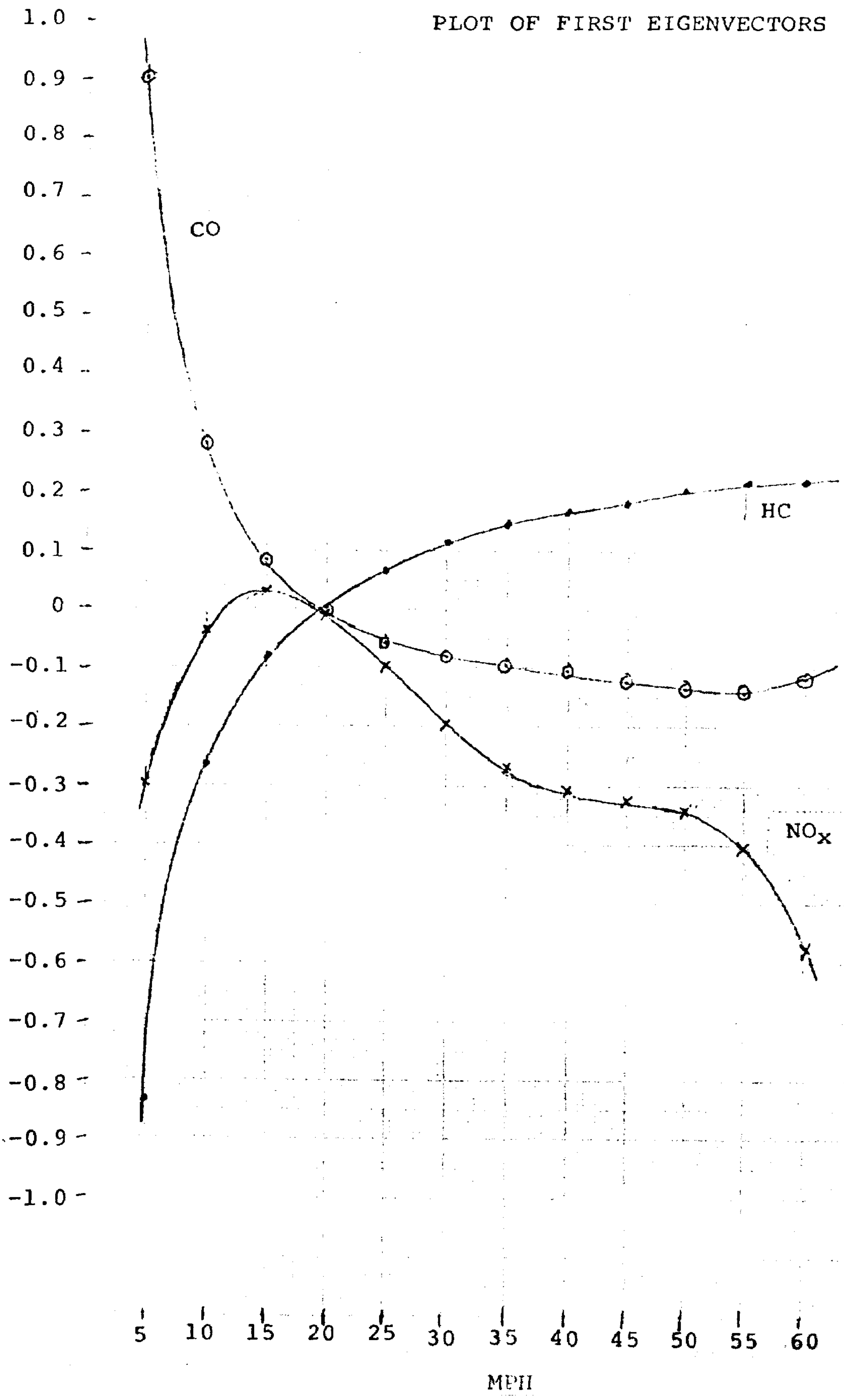
$$D\left[f(x)\right] = \left\{5 \text{ mph}, 10 \text{ mph}, \ldots, 60 \text{ mph}\right\}$$

but the matter of extension to a continuous domain will be considered later (see Section 3, Summary and Conclusions.) Let $v_1(x)$ denote the first eigenvector and let $v_2(x)$ denote the second eigenvector for that pollutant. Considering $v_1(x)$ and $v_2(x)$ as functions defined on the same domain as f(x), one can then write

$$g_i(x) = f(x) + b_{1i}\, v_1(x) + b_{2i}\, v_2(x)$$

where b_{1i} and b_{2i} are regression coefficients for the ith group and $g_i(x)$ is an approximation to the observed correction factor vs speed relation for the ith group. The coefficients b_{1i} and b_{2i} can be determined very simply by virtue of the fact that the vectors $v_1(x)$ and $v_2(x)$ are orthogonal.

FIGURE 2


PLOT OF FIRST EIGENVECTORS
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
CO
HC
NO_x
5
10
15
20
25
30
35
40
45
50
55
60
MPH

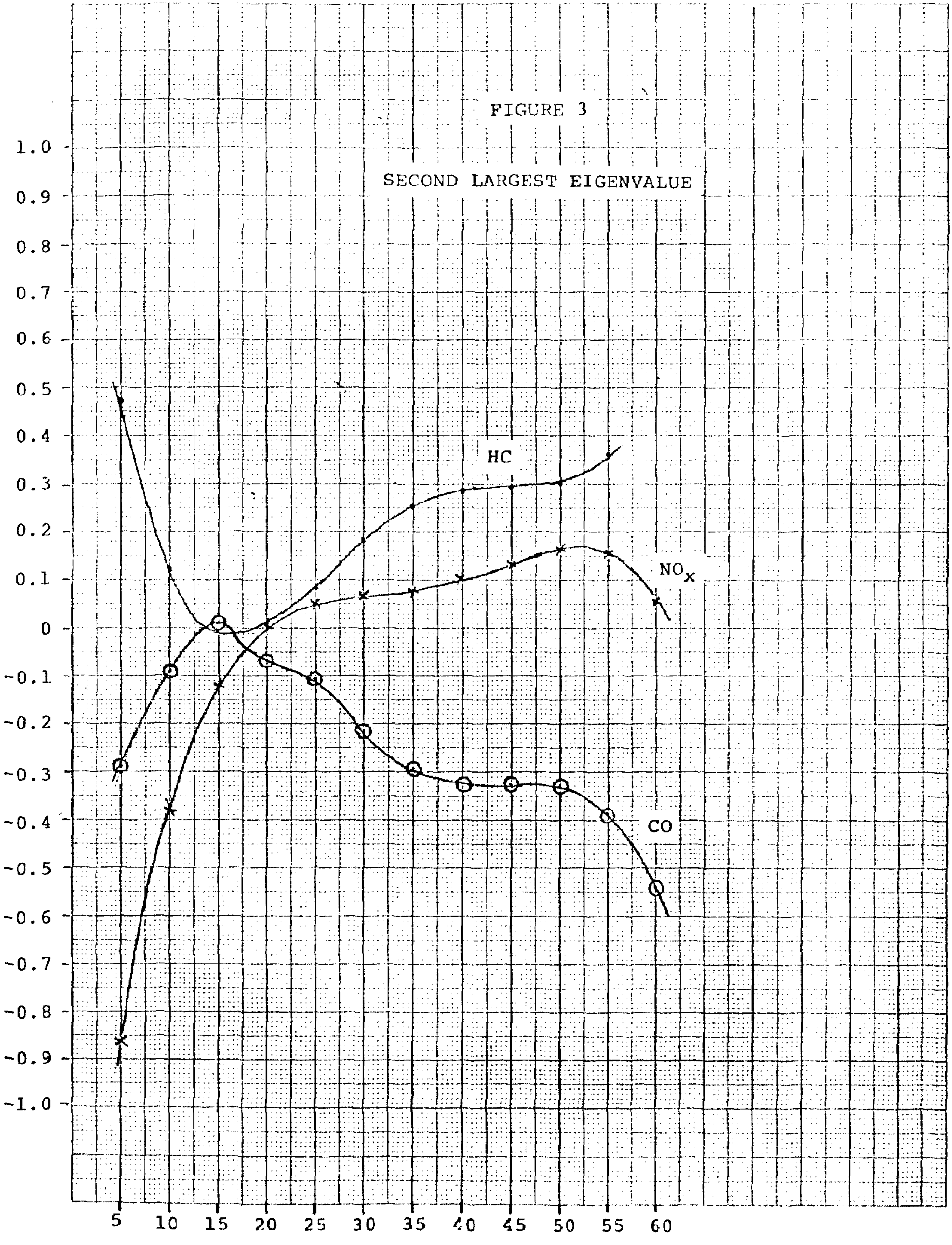
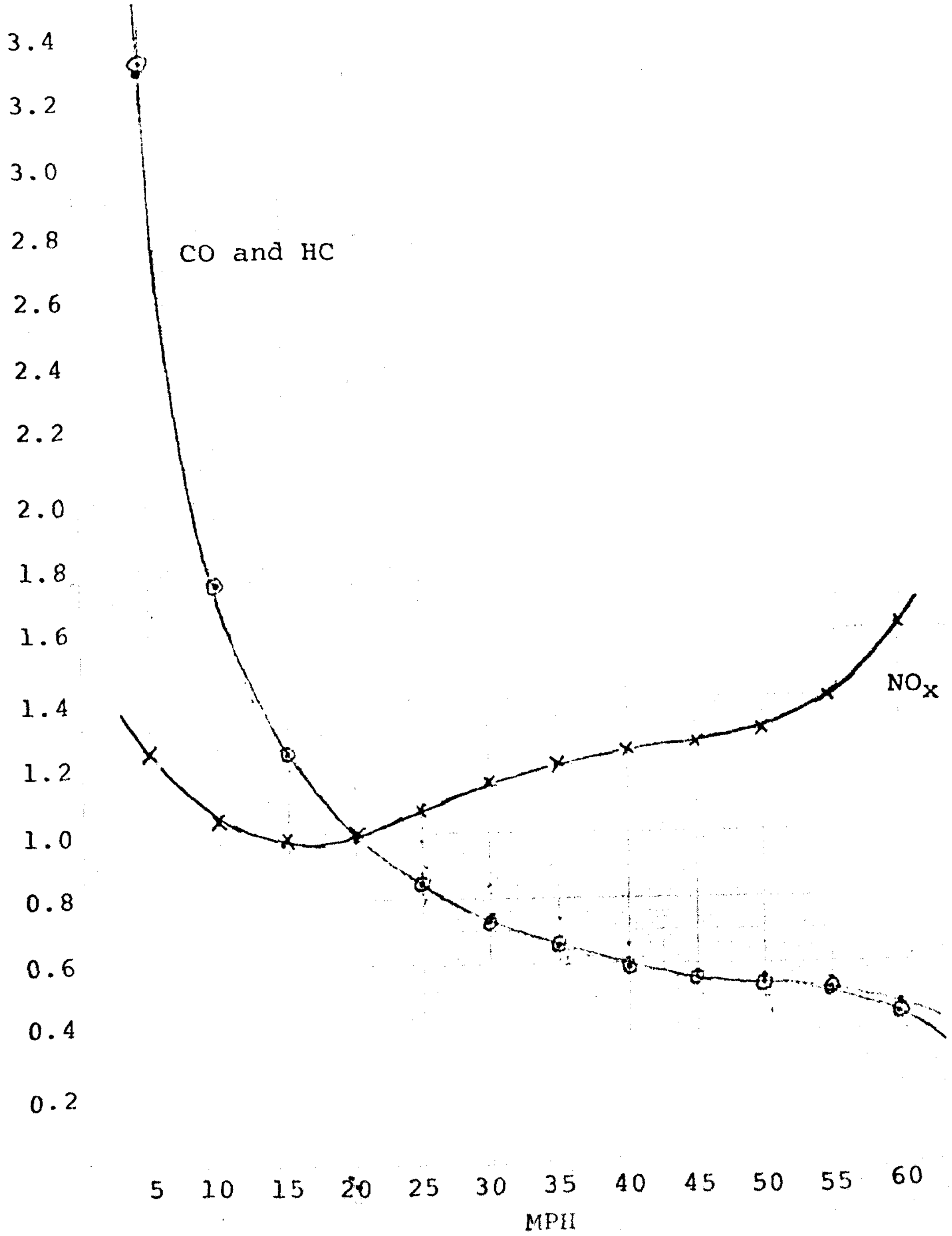


FIGURE 3

SECOND LARGEST EIGENVALUE

FIGURE 4

PLOT OF GROUP 2 CORRECTION FACTORS



Also, by virtue of orthogonality, the last term in the equation can be deleted if it makes only a small contribution to $g_i(x)$ without the need to recompute the coefficient b_{1i}.

An illustration is informative. Consider the correction factors for CO as displayed in Table II.14. The mean computed across all 18 groups gives rise to the following "correction-factor vector" or "correction-factor function" (see Table 5).

TABLE 5

MEAN CO CORRECTION FACTOR FOR
18 VEHICLE GROUPS AT 5 mph INCREMENTS

Speed (mph)	Mean CO Correction Factor
5	3.325
10	1.727
15	1.217
20	0.985
25	0.836
30	0.718
35	0.627
40	0.567
45	0.536
50	0.525
55	0.501
60	0.413

Each of the vehicle groups deviates to some extent from these correction factors, and it is the intent of the principal component analysis to allow for this correction in the most parsimonious way.

Consider Group 2, for example. Its deviations from the mean correction-factor relation are tabulated in Table 6.

TABLE 6

GROUP 2 DEVIATIONS FROM MEAN CO CORRECTION FACTORS AT 5 mph INCREMENTS

SPEED (mph)	DEVIATION FROM MEAN CORRECTION FACTOR
5	-0.0057
10	0.0214
15	0.0074
20	0.0006
25	0.0053
30	0.0156
35	0.0227
40	0.0251
45	0.0206
50	0.0140
55	0.0170
60	0.0411

The object is to express these deviations as a linear combination of the significant eigenvectors $v_1(x)$ and $v_2(x)$ for CO as derived from the principal-component analysis. In short, we want to solve the following equation for b_1 and b_2

$$b_1 \begin{bmatrix} 0.9015 \\ 0.2881 \\ 0.0867 \\ -0.0059 \\ -0.0541 \\ -0.0793 \\ -0.0937 \\ -0.1059 \\ -0.1206 \\ -0.1374 \\ -0.1436 \\ -0.1159 \end{bmatrix} + b_2 \begin{bmatrix} -0.2909 \\ -0.0923 \\ 0.0149 \\ -0.0067 \\ -0.1061 \\ -0.2179 \\ -0.2942 \\ -0.3246 \\ -0.3239 \\ -0.3263 \\ -0.3887 \\ -0.5417 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \\ e_{11} \\ e_{12} \end{bmatrix} = \begin{bmatrix} -0.0057 \\ 0.0241 \\ 0.0074 \\ 0.0006 \\ 0.0053 \\ 0.0156 \\ 0.0227 \\ 0.0251 \\ 0.0206 \\ 0.0140 \\ 0.0170 \\ 0.0411 \end{bmatrix} \qquad (3)$$

in such a way as to minimize the sum of squared errors

$$\sum_{i=1}^{12} e_i^2 \quad .$$

Equation (3) thus falls in the framework of a linear model

$$X\,\underline{b} + \underline{\varepsilon} = \underline{y} \qquad (4)$$

and one can solve for $\underline{b}$ by means of the least-squares normal equations

$$X'X\,\underline{b} = X'\,\underline{y} \qquad (5)$$

from which

$$\underline{b} = (X'X)^{-1}\,X'\,\underline{y} \qquad (6)$$

It should be noted, however, that the column vectors of the matrix X are orthogonal, so that (6) becomes simply

$$\underline{b} = X'\,\underline{y} \qquad (7)$$

Solution of (7) for the specific case of (3) gives

$$\underline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -0.0154784 \\ -0.0593595 \end{bmatrix} \qquad (8)$$

With the computed values of b_1 and b_2 one can then compute an estimated correction factor vector for Group 2 and can compare this vector with the observed correction factors. Thus,

$$-0.0155 \begin{bmatrix} 0.9015 \\ 0.2881 \\ 0.0867 \\ -0.0059 \\ -0.0541 \\ -0.0793 \\ -0.0937 \\ -0.1059 \\ -0.1206 \\ -0.1374 \\ -0.1436 \\ -0.1159 \end{bmatrix} -0.0594 \begin{bmatrix} -0.2902 \\ -0.0923 \\ 0.0149 \\ -0.0067 \\ -0.1061 \\ -0.2179 \\ -0.2942 \\ -0.3246 \\ -0.3239 \\ -0.3263 \\ -0.3887 \\ -0.5417 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.0033 \\ 0.0010 \\ -0.0022 \\ 0.0005 \\ 0.0071 \\ 0.0142 \\ 0.0189 \\ 0.0209 \\ 0.0211 \\ 0.0215 \\ 0.0253 \\ 0.0340 \end{bmatrix}}_{\text{Computed Deviations from Mean}} \leftrightarrow \underbrace{\begin{bmatrix} -0.0057 \\ 0.0214 \\ 0.0074 \\ 0.0006 \\ 0.0053 \\ 0.0156 \\ 0.0227 \\ 0.0251 \\ 0.0206 \\ 0.0140 \\ 0.0170 \\ 0.0411 \end{bmatrix}}_{\text{Observed Deviations from Mean}}$$

The error vector can now be computed as the difference between the observed and the computed vectors according to

$$\underline{\varepsilon} = \underline{y} - X\,\underline{b}$$

as shown below.

$$\begin{bmatrix} -0.0057 \\ 0.0214 \\ 0.0074 \\ 0.0006 \\ 0.0053 \\ 0.0156 \\ 0.0227 \\ 0.0251 \\ 0.0206 \\ 0.0140 \\ 0.0170 \\ 0.0411 \end{bmatrix} - \begin{bmatrix} 0.0033 \\ 0.0010 \\ -0.0022 \\ 0.0005 \\ 0.0071 \\ 0.0142 \\ 0.0189 \\ 0.0209 \\ 0.0211 \\ 0.0215 \\ 0.0253 \\ 0.0340 \end{bmatrix} = \begin{bmatrix} -0.0090 \\ 0.0204 \\ 0.0096 \\ 0.0001 \\ -0.0018 \\ 0.0014 \\ 0.0038 \\ 0.0042 \\ -0.0005 \\ -0.0075 \\ -0.0083 \\ 0.0072 \end{bmatrix}$$

Note that the maximum error is 0.02, at 10 mph, where the actual value of the correction factor as tabulated in Table II.14 is 1.751. Thus the maximum error is only slightly over 1%.

Similar analysis can be performed for other vehicle groups and for other pollutants, but such analysis is not within the scope of this paper, which is intended to be exemplary only. The inputs required for additional analysis are, however provided in the Appendix.

3. SUMMARY AND CONCLUSIONS

It has been demonstrated, in connection with emission speed correction factors, that principal component analysis provides a useful tool for consolidating correction factor data. By virtue of the fact that the speed correction factor curves for various groups of vehicles tends to have a common 'shape," this common (mean) curve can be expressed as the basic input to speed correction. Refinement to this curve can be achieved by means of a linear model having no more than two correction terms, the coefficients of which are vehicle-group specific. Thus, if $g_i(x)$ is the mean speed-correction function for the ith vehicle group and $f(x)$ is the mean speed-correction function for all groups, then

$$g_i(x) = f(x) + b_{1i}\ v_1(x) + b_{2i}\ v_2(x) \qquad (10)$$

where $v_1(x)$ and $v_2(x)$ are determined from principal component analysis. Thus to characterize the data of Table II.14 it is necessary to know only:

$f(x)$, the mean curve	(12 values)
$v_1(x)$, the first eigenvector	(12 values)
$v_2(x)$, the second eigenvector	(12 values)
b_{1i}, $i = 1, 2, \ldots, 18$	(18 values)
b_{2i}, $i = 1, 2, \ldots, 18$	(18 values)

Thus a total of 72 (and possibly as few as 42) key parameters retains the correction factor information for each pollutant, as opposed to $18 \times 12 = 216$ in the complete tabulation. If viewed in the form of Table II.13, employing logarithmically transformed exponential functions, the correction-factor milieu required $18 \times 5 = 90$ coefficients for CO and HC and $18 \times 4 = 72$ coefficients for NO_x, in addition to the assumption of specific forms of equations for the correction factor vs speed relations.

Though it is recognized that in (10) each of the functions $g_i(x)$, $f(x)$, $v_1(x)$ and $v_2(x)$ are considered as being defined on a discrete domain of vehicle speeds, extension to a continuous interval of speeds is straightforward. Indeed, if $f(x)$, $v_1(x)$ and $v_2(x)$ are expressed as functional forms, each of these functions can be represented by much fewer than the 12 values which emerge as components of the eigenvectors. For example, each of these characteristic functions could be expressed as an approximating polynomial of relatively small degree. Note that the coefficients in these polynomials need be defined <u>only</u> <u>once</u>, <u>not</u> for each vehicle group, since the group-specific part of the representation is contained entirely in the coefficients b_{1i} and b_{2i}.

In summary, principal component analysis provides a means for structuring the information content of correction factors in a concise way, as well as identifying the areas of commonality among various vehicle makes and models or other homogeneous groups. The approach is applicable to other multivariate aspects of correction factors.

APPENDIX

TABLE A-1

CO CORRECTION FACTORS AT 5 mph INCREMENTS

Speed (mph)

5	10	15	20	25	30	35	40	45	50	55	60
Raw Data Column Means											
3.3247	1.7269	1.2176	.9854	.8357	.7184	.6273	.5669	.5364	.5250	.5010	.4129
Raw Data Column Sigmas											
.7624	.2447	.0739	.0053	.0489	.0755	.0919	.1032	.1141	.1272	.1361	.1308
Transformed Data (Deviation Scores)											
- .9357	-.2639	-.0756	.0056	.0533	.0846	.1057	.1191	.1286	.1380	.1460	.1451
- .0057	.0241	.0074	.0006	.0053	.0156	.0227	.0251	.0206	.0140	.0170	.0411
.3313	.1291	.0334	-.0004	.0013	.0196	.0357	.0411	.0356	.0290	.0410	.1021
.2963	.1171	.0354	-.0014	-.0127	-.0114	-.0083	-.0109	-.0194	-.0320	-.0360	-.0139
1.2293	.3931	.1114	-.0064	-.0547	-.0744	-.0843	-.0979	-.1184	-.1410	-.1430	-.0899
1.1863	.3761	.1084	-.0064	-.0577	-.0814	-.0943	-.1099	-.1294	-.1510	-.1560	-.1139
.8493	.2761	.0814	-.0104	-.0597	-.0854	-.1013	-.1139	-.1304	-.1500	-.1600	-.1399
- .9797	-.3089	-.0966	.0066	.0693	.1086	.1327	.1501	.1666	.1860	.2000	.1871
-1.0477	-.3319	-.1046	.0076	.0773	.1226	.1527	.1761	.2006	.2300	.2550	.2491
- .7837	-.2389	-.0686	.0046	.0373	.0516	.0567	.0621	.0716	.0830	.0880	.0651
- .8087	-.2529	-.0696	.0036	.0273	.0276	.0237	.0241	.0316	.0410	.0330	-.0159
- .4397	-.1869	-.0686	.0056	.0553	.0856	.1017	.1101	.1206	.1330	.1420	.1251
.4663	.1891	.0734	-.0054	-.0647	-.1064	-.1313	-.1469	-.1584	-.1700	-.1770	-.1669
.7313	.2231	.0634	-.0034	-.0317	-.0434	-.0503	-.0569	-.0664	-.0740	-.0650	-.0219
- .7257	-.2679	-.0906	.0066	.0643	.0956	.1117	.1241	.1386	.1560	.1600	.1171
.0593	.0171	.0194	-.0024	-.0407	-.0774	-.1003	-.1089	-.1094	-.1150	-.1410	-.1859
.9143	.2531	.0754	-.0044	-.0537	-.0844	-.1023	-.1129	-.1214	-.1300	-.1370	-.1349
-.3367	-.1469	-.0346	.0006	-.0147	-.0474	-.0703	-.0739	-.0614	-.0470	-.0670	-.1479

TABLE A-1 (Continued)

Covariance Matrix

.5813	.1856	.0555	-.0038	-.0339	-.0494	-.0580	-.0657	-.0751	-.0858	-.0894	-.0706
.1856	.0599	.0180	-.0012	-.0109	-.0158	-.0185	-.0210	-.0241	-.0276	-.0287	-.0223
.0555	.0180	.0055	-.0004	-.0034	-.0050	-.0059	-.0067	-.0076	-.0087	-.0091	-.0072
-.0038	-.0012	-.0004	.0000	.0002	.0004	.0004	.0005	.0006	.0006	.0007	.0006
-.0339	-.0109	-.0034	.0002	.0024	.0037	.0044	.0050	.0055	.0062	.0066	.0058
-.0494	-.0158	-.0050	.0004	.0037	.0057	.0069	.0078	.0086	.0096	.0102	.0095
-.0580	-.0185	-.0059	.0004	.0044	.0069	.0084	.0095	.0104	.0116	.0124	.0117
-.0657	-.0210	-.0067	.0005	.0050	.0078	.0095	.0106	.0117	.0130	.0140	.0131
-.0751	-.0241	-.0076	.0006	.0055	.0086	.0104	.0117	.0130	.0145	.0155	.0143
-.0858	-.0276	-.0087	.0006	.0062	.0096	.0116	.0130	.0145	.0162	.0173	.0157
-.0894	-.0287	-.0091	.0007	.0066	.0102	.0124	.0140	.0155	.0173	.0185	.0171
-.0706	-.0223	-.0072	.0006	.0058	.0095	.0117	.0131	.0143	.0157	.0171	.0171

Eigenvalues

.7126	.0250	.0009	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Eigenvectors

.9015	-.2902	-.2991	.0498
.2881	-.0923	-.7316	-.2374
.0867	.0149	-.3209	.0096
-.0059	-.0067	.0183	.0038
-.0541	-.1061	.1256	-.1809
-.0793	-.2179	.0798	-.3750
-.0937	-.2942	.0132	-.4617
-.1059	-.3246	.0126	-.3906
-.1206	-.3239	.1046	-.1901
-.1394	-.3263	.2200	.1212
-.1436	-.3887	.1410	.3908
-.1159	-.5417	-.4127	.4453

(Other eigenvectors make insignificant contributions)

TABLE A-1 (Continued)

Transformed Data (Standard Scores)

-1.2273	-1.0785	-1.0230	1.0536	1.0904	1.1204	1.1497	1.1544	1.1275	1.0852	1.0730	1.1092
- .0075	.0985	.1000	.1054	.1080	.2067	.2466	.2434	.1807	.1101	.1249	.3139
.4345	.5277	.4518	- .0843	.0262	.2597	.3881	.3984	.3122	.2280	.3013	.7804
.3886	.4786	.4788	- .2739	- .2604	- .1508	- .0907	- .1055	- .1700	- .2516	- .2646	- .1066
1.6123	1.6067	1.5071	-1.2222	-1.1200	- .9851	- .9176	- .9487	-1.0378	-1.1088	-1.0509	- .6877
1.5559	1.5372	1.4665	-1.2222	-1.1814	-1.0778	-1.0264	-1.0650	-1.1343	-1.1874	-1.1464	- .8713
1.1139	1.1285	1.1012	-1.9808	-1.2233	-1.1307	-1.1025	-1.1037	-1.1430	-1.1795	-1.1758	- .0701
-1.2650	-1.2624	-1.3071	1.2433	1.4179	1.4382	1.4435	1.4548	1.4606	1.4626	1.4698	1.4303
-1.3742	-1.3564	-1.4154	1.4329	1.5816	1.6236	1.6611	1.7068	1.7586	1.8086	1.2740	1.9044
-1.0279	- .9764	- .9283	.8640	.7629	.6834	.6165	.6019	.6278	.6527	.6467	.4974
-1.0607	-1.0336	- .9418	.6743	.5583	.3656	.2575	.2337	.2771	.3224	.2425	- .1219
- .5767	- .7638	- .9233	1.0536	1.1318	1.1337	1.1062	1.0671	1.0573	1.0459	1.0436	.9562
.6116	.7729	.9929	-1.0326	-1.3246	-1.4088	-1.4289	-1.4236	-1.3885	-1.3368	-1.3008	-1.2765
.9591	.9119	.6577	- .6532	- .8492	- .5746	- .5476	- .5513	- .5820	- .5819	- .4777	- .1678
- .9518	-1.0949	-1.2260	1.2433	1.3155	1.2661	1.2150	1.2028	1.2151	1.2267	1.1753	.8951
.0777	.0699	.2623	- .4636	- .8334	-1.0248	-1.0917	-1.0553	- .9589	- .9043	-1.0362	-1.4218
1.1991	1.0345	1.0200	- .8429	-1.0995	-1.1175	-1.1134	-1.0941	-1.0641	-1.0223	-1.0868	-1.0318
- .4416	- .6003	- .4683	.1054	- .3013	- .6275	- .7652	- .7161	- .5382	- .3696	- .4524	-1.1312

Correlation Matrix

1.0000	.9947	.9853	- .9349	- .9108	- .8573	- .8280	- .8345	- .8637	- .8854	- .8616	- .7085
.9947	1.0000	.9947	- .9422	- .9146	- .8565	- .8245	- .8311	- .8628	- .8874	- .8619	- .6968
.9853	.9947	1.0000	- .9590	- .9469	- .8977	- .8691	- .8744	- .9021	- .9232	- .9010	- .7499
- .9349	- .9422	- .9590	1.0000	.9683	.9380	.9191	.9224	.9408	.9538	.9408	.8335
- .9108	- .9146	- .9469	.9683	1.0000	.9912	.9807	.9820	.9906	.9945	.9881	.9150
- .8593	- .8565	- .8977	.9380	.9912	1.0000	.9979	.9979	.9982	.9946	.9949	.9572
- .8280	- .8245	- .8691	.9191	.9807	.9979	1.0000	.9999	.9963	.9893	.9928	.9723
- .8345	- .8311	- .8744	.9224	.9820	.9979	.9997	1.0000	.9978	.9917	.9948	.9716
- .8637	- .8628	- .9021	.9408	.9906	.9982	.9963	.9978	1.0000	.9980	.9982	.9583
- .8854	- .8874	- .9232	.9538	.9945	.9946	.9893	.9917	.9980	1.0000	.9981	.9466
- .8616	- .8619	- .9010	.9408	.9881	.9949	.9988	.9948	.9982	.9981	1.0000	.9609
- .7085	- .6968	- .7499	.8335	.9150	.9572	.9723	.9716	.9583	.9436	.9609	1.0000

TABLE A-1 (Continued)

Eigenvalues

11.2318	.6865	.0456	.0225	.0110	.0023	.0002	.0000	.0000	.0000	.0000	.0000

Eigenvectors

-.2738	.4599	.3528	-.5005	.3833	.0979	-.3596
-.2738	.4772	.1535	-.0164	-.1524	-.0182	.3743
-.2826	.3798	.0941	.3291	-.3008	-.2482	.3324
.2892	-.1802	.8977	.2482	-.0855	-.0428	-.0516
.2979	.0020	.0685	-.3409	-.1294	.4432	.4267
.2954	.1574	-.0220	-.2652	-.3200	.2544	.0195
.2926	.2294	-.0597	-.1612	-.3767	-.0566	-.1038
.2933	.2172	-.0791	-.0971	-.2364	-.3444	-.2477
.2960	.1469	-.0532	-.0873	.0471	-.4713	-.1036
.2973	.0851	-.0197	-.0709	.3772	-.3753	.2402
.2957	.1484	-.0310	.0751	.4964	.0308	.4392
.2747	.4567	-.1341	.5823	.1554	.4240	-.3187

(Other eigenvectors make insignificant contribution)

TABLE A-2

HC CORRECTION FACTORS AT 5 mph INCREMENTS

Speed (mph)

5	10	15	20	25	30	35	40	45	50	55	60
Raw Data (Column Means)											
3.0969	1.6768	1.2035	.9868	.8509	.7464	.6644	.6072	.5759	.5622	.5414	.4701
Raw Data (Column Sigmas)											
.3110	.0987	.0321	.0023	.0276	.0475	.0615	.0704	.0765	.0829	.0918	.1022
Transformed Data (Deviation Scores)											
.0102	.0022	-.0025	.0002	.0071	.0146	.0196	.0218	.0211	.0228	.0296	.0459
.2002	.0722	.0205	-.0008	-.0069	-.0064	-.0054	-.0072	-.0109	-.0152	-.0144	.0119
-.0138	.0312	.0155	-.0008	-.0099	-.0134	-.0144	-.0152	-.0199	-.0282	-.0384	-.0411
.3732	.1312	.0425	-.0028	-.0299	-.0464	-.0584	-.0692	-.0789	-.0902	-.0964	-.0891
.3222	.0962	.0275	-.0018	-.0169	-.0264	-.0344	-.0422	-.0499	-.0582	-.0624	-.0561
.0262	.0162	.0045	.0002	.0021	.0076	.0126	.0148	.0151	.0138	.0156	.0248
.0632	.0322	.0115	-.0008	-.0059	-.0064	-.0064	-.0072	-.0089	-.0122	-.0124	-.0101
-.3968	-.1288	-.0435	.0032	.0381	.0646	.0836	.0958	.1051	.1158	.1306	.1459
-.1938	-.0768	-.0335	.0032	.0401	.0726	.0976	.1128	.1231	.1318	.1496	.1789
-.0578	-.0278	-.0135	.0012	.0161	.0286	.0376	.0418	.0431	.0468	.0546	.0679
-.2988	-.1058	-.0355	.0022	.0261	.0416	.0516	.0598	.0681	.0758	.0756	.0549
-.1688	-.0538	-.0195	.0012	.0201	.0346	.0466	.0518	.0561	.0618	.0706	.0829
-.3918	-.1288	-.0385	.0022	.0201	.0266	.0296	.0328	.0401	.0478	.0476	.0229
.1792	.0492	.0125	-.0008	-.0069	-.0104	-.0144	-.0182	-.0219	-.0242	-.0224	-.0141
-.2818	-.0948	-.0315	.0022	.0251	.0416	.0526	.0618	.0701	.0788	.0866	.0829
-.3338	-.1018	-.0225	.0002	-.0029	-.0174	-.0294	-.0342	-.0329	-.0312	-.0444	-.0871
.8662	.2542	.0815	-.0058	-.0669	-.1114	-.1414	-.1612	-.1749	-.1892	-.2044	-.2121
.0972	.0312	.0245	-.0028	-.0479	-.0934	-.1244	-.1392	-.1439	-.1482	-.1684	-.2101

TABLE A-2 (Continued)

Covariance Matrix

.0967	.0304	.0097	-.0006	-.0073	-.0117	-.0147	-.0169	-.0188	-.0209	-.0223	-.0214
.0304	.0097	.0031	-.0002	-.0023	-.0038	-.0047	-.0055	-.0061	-.0068	-.0072	-.0069
.0097	.0031	.0010	-.0001	-.0008	-.0014	-.0017	-.0020	-.0022	-.0024	-.0026	-.0026
-.0006	-.0002	-.0001	.0000	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0002
-.0073	-.0023	-.0008	.0001	.0008	.0013	.0017	.0019	.0021	.0023	.0025	.0027
-.0117	-.0038	-.0014	.0001	.0013	.0023	.0029	.0033	.0036	.0039	.0043	.0047
-.0147	-.0047	-.0017	.0001	.0017	.0029	.0038	.0043	.0047	.0051	.0056	.0062
-.0169	-.0055	-.0020	.0002	.0019	.0033	.0043	.0050	.0054	.0058	.0064	.0071
-.0188	-.0061	-.0022	.0002	.0021	.0036	.0047	.0054	.0058	.0063	.0070	.0077
-.0209	-.0068	-.0024	.0002	.0023	.0039	.0051	.0058	.0063	.0069	.0076	.0083
-.0223	-.0072	-.0026	.0002	.0025	.0043	.0056	.0064	.0070	.0076	.0084	.0092
-.0214	-.0069	-.0026	.0002	.0027	.0047	.0062	.0071	.0077	.0083	.0092	.0104

Eigenvalues

.1364	.0141	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Eigenvectors

-.8282	.4684	.2934	-.0457
-.2627	.1224	-.8847	-.0793
-.0862	-.0038	-.3290	.0286
.0060	.0043	.0111	-.0014
.0694	.0843	.0698	-.1288
.1141	.1822	-.0305	-.2645
.1450	.2538	-.1212	-.3175
.1666	.2869	-.1314	-.3262
.1840	.2937	-.0201	-.3124
.2024	.3005	.1488	-.2027
.2191	.3630	.2013	.0860
.2196	.5199	-.1568	.7423

(Other eigenvectors make insignificant contribution)

TABLE A-2 (Continued)

Transformed Data (Standard Scores)

.0327	.0225	- .0778	.0977	.2359	.3067	.3177	.3100	.2761	.2754	.3227	.4497
.6437	.7318	.6378	- .3420	- .2519	- .1358	- .0885	- .1018	- .1424	- .1829	- .1241	.1169
- .0445	.3164	.4822	- .3420	- .3607	- .2832	- .2347	- .2154	-. 2602	- .3397	- .4184	- .4019
1.2001	1.3296	1.3222	-1.2213	-1.0862	- .9785	- .9496	- .9822	-1.0319	-1.0875	-1.0604	- .8718
1.0061	.9750	.8555	- .7817	- .6147	- .5571	- .5597	- .5988	- .6526	- .7016	- .6799	- .5487
.0841	.1846	.1400	.0977	.0746	.1592	.2040	.2106	.1977	.1668	.1701	.2442
.2031	.3265	.3572	- .3420	- .2156	- .1358	- .1047	- .1018	- .1163	- .1347	- .1350	- .0984
-1.2762	-1.3048	-1.3533	1.4167	1.3805	1.3600	1.3577	1.3609	1.3749	1.3971	1.4234	1.4286
- .6233	- .7779	-1.0422	1.4167	1.4530	1.5286	1.5851	1.6023	1.6103	1.5901	1.6365	1.7517
- .1860	- .2815	- .4200	.5374	.5824	.6016	.6102	.5940	.5639	.5649	.5952	.6651
- .9610	-1.0718	-1.1044	.9771	.9452	.8755	.8377	.8497	.8909	.9146	.8240	.5378
- .5430	- .5449	- .6066	.5374	.7275	.7280	.7240	.7360	.7339	.7458	.7695	.8119
-1.2601	-1.3048	-1.1977	.9771	.7275	.5595	.4802	.4662	.5247	.5769	.5189	.2246
.5762	.4987	.3889	- .3420	- .2519	- .2200	- .2347	- .2580	- .2863	- .2915	- .2440	- .1376
- .9063	- .9603	- .9800	.9771	.9089	.8755	.8539	.8781	.9171	.9508	.9439	.6119
-1.0736	-1.0313	- .7000	.0977	- .1068	- .3675	- .4784	- .4852	- .4302	- .3759	- .4837	- .6522
2.7855	2.5759	2.5354	-2.5404	-2.4284	-2.3479	-2.2983	-2.2886	-2.2876	-2.2816	-2.2274	-2.0758
.3125	.3164	.7622	-1.2212	-1.7392	-1.9687	-2.0220	-1.9762	-1.8821	-1.7871	-1.8351	-2.0562

Correlation Matrix

1.0000	.9920	.9702	- .9132	- .8465	- .7918	- .7681	- .7730	- .7926	- .8094	- .7828	- .6728
.9920	1.0000	.9863	- .9274	- .8634	- .8045	- .7797	- .7845	- .8057	- .8250	- .8005	- .6892
.9702	.9863	1.0000	- .9727	- .9313	- .8860	- .8658	- .8692	- .8862	- .9018	- .8835	- .7919
- .9132	- .9274	- .9727	1.0000	.9851	.9632	.9525	.9548	.9648	.9727	.9625	.9044
- .8485	- .8634	- .9313	.9851	1.0000	.9939	.9881	.9886	.9926	.9952	.9908	.9557
- .7918	- .8045	- .8860	.9632	.9939	1.0000	.9989	.9987	.9987	.9973	.9969	.9795
- .7681	- .7797	- .8658	.9525	.9881	.9989	1.0000	.9998	.9984	.9955	.9967	.9862
- .7730	- .7845	- .8692	.9548	.9886	.9987	.9998	1.0000	.9991	.9966	.9974	.9857
- .7926	- .8057	- .8862	.9648	.9926	.9987	.9984	.9991	1.0000	.9991	.9986	.9806
- .8094	- .8250	- .9018	.9727	.9952	.9973	.9955	.9966	.9991	1.0000	.9987	.9752
- .7828	- .8005	- .8835	.9625	.9908	.9969	.9967	.9974	.9986	.9987	1.0000	.9845
- .6728	- .6892	- .7919	.9044	.9557	.9795	.9862	.9857	.9806	.9752	.9845	1.0000

TABLE A-2 (Continued)

Eigenvalues

11.1529	.8122	.0189	.0085	.0043	.0030	.0001	.0000	.0000	.0000	.0000	.0000

Eigenvectors

-.2607	-.5349	-.6983	-.2229	-.0409	-.0537	-.2138
-.2645	-.5191	.1984	-.0848	-.1730	.1608	.3254
-.2827	-.3571	.4992	.1653	-.2503	-.0351	.0334
.2970	.1215	-.8421	.1419	-.8186	.3742	.0678
.2989	-.0435	-.0768	-.3910	.2240	.3041	.0276
.2960	-.1608	.1306	-.3741	.1280	.2465	.3547
.2940	-.2053	.2159	-.2514	-.0158	.1114	-.1177
.2945	-.1974	.2163	-.1358	-.0942	-.1446	-.4720
.2962	-.1603	.0964	-.0511	-.1808	-.3862	-.2388
.2974	-.1250	-.0745	.0539	-.0866	-.5113	.1332
.2956	-.1719	-.1810	.2499	.0968	-.2972	.5941
.2834	-.3498	-.0453	.6733	.3323	.3802	-.2276

(Other eigenvectors make insignificant contribution)

TABLE A-3

NO_X CORRECTION FACTORS AT 5 mph INCREMENTS

Speed (mph)

5	10	15	20	25	30	35	40	45	50	55	60
Raw Data (Column Means)											
1.0477	.9402	.9437	1.0062	1.0910	1.1726	1.2375	1.2837	1.3208	1.3702	1.4651	1.6499
Raw Data (Column Sigmas)											
.2100	.0683	.0274	.0032	.0492	.0965	.1325	.1528	.1618	.1712	.2021	.2868
Transformed Data (Deviation Scores)											
.4603	.1198	-.0027	.0038	.0700	.1464	.2025	.2273	.2302	.2378	.2989	.4791
.1973	.0908	.0303	-.0022	-.0170	-.0266	-.0345	-.0447	-.0558	-.0642	-.0611	-.0349
-.0547	.0058	.0163	-.0022	-.0330	-.0636	-.0875	-.1017	-.1078	-.1122	-.1251	-.1609
.0183	.0518	.0363	-.0042	-.0530	-.0976	-.1325	-.1547	-.1688	-.1812	-.2081	-.2659
-.0667	.0298	.0373	-.0042	-.0650	-.1236	-.1675	-.1927	-.2058	-.2192	-.2571	-.3519
-.1177	-.0162	.0123	-.0022	-.0350	-.0706	-.0965	-.1107	-.1148	-.1202	-.1421	-.2049
-.0417	.0088	.0163	-.0022	-.0320	-.0626	-.0875	-.1037	-.1138	-.1202	-.1431	-.1669
.2393	.0658	.0003	.0018	.0370	.0824	.1215	.1453	.1562	.1608	.1759	.2271
.0983	.0258	.0003	.0008	.0140	.0274	.0375	.0393	.0372	.0358	.0459	.0831
.2793	.0568	-.0137	.0038	.0610	.1244	.1725	.1963	.2032	.2118	.2559	.3811
.1363	.0408	.0023	.0008	.0180	.0414	.0625	.0773	.0862	.0918	.1049	.1361
-.0307	-.0802	-.0567	.0058	.0830	.1574	.2165	.2583	.2852	.3078	.3349	.4201
-.4557	-.1342	-.0097	-.0022	-.0480	-.1026	-.1405	-.1517	-.1458	-.1492	-.2071	-.3819
-.1457	-.0372	-.0197	.0018	.0210	.0354	.0455	.0483	.0482	.0508	.0609	.0861
.0373	-.0332	-.0347	.0038	.0570	.1074	.1455	.1683	.1802	.1938	.2259	.3051
-.1887	.0028	.0423	-.0052	-.0890	-.1726	-.2355	-.2697	-.2848	-.3032	-.3621	-.5139
-.2367	-.0762	-.0097	-.0002	-.0220	-.0516	-.0765	-.0907	-.0948	-.0962	-.1121	-.1639
-.2287	-.1212	-.0467	.0028	.0330	.0494	.0565	.0603	.0652	.0758	.0949	.1271

TABLE A-3 (Continued)

Covariance Matrix

.0441	.0123	.0004	.0003	.0058	.0124	.0175	.0198	.0202	.0207	.0257	.0408
.0123	.0047	.0011	-.0000	.0002	.0008	.0013	.0014	.0012	.0010	.0017	.0042
.0004	.0011	.0007	-.0001	-.0011	-.0020	-.0026	-.0031	-.0033	-.0036	-.0041	-.0053
.0003	-.0000	-.0001	.0000	.0002	.0003	.0004	.0005	.0005	.0005	.0006	.0009
.0058	.0002	-.0011	.0002	.0024	.0047	.0065	.0075	.0079	.0084	.0099	.0139
.0124	.0008	-.0020	.0003	.0047	.0093	.0128	.0147	.0156	.0165	.0195	.0275
.0175	.0013	-.0026	.0004	.0065	.0128	.0176	.0202	.0214	.0226	.0268	.0377
.0198	.0014	-.0031	.0005	.0075	.0147	.0202	.0233	.0247	.0261	.0309	.0434
.0202	.0012	-.0033	.0005	.0079	.0156	.0214	.0247	.0262	.0277	.0326	.0457
.0207	.0010	-.0036	.0005	.0084	.0165	.0226	.0261	.0277	.0293	.0345	.0482
.0257	.0017	-.0041	.0006	.0099	.0195	.0268	.0309	.0326	.0345	.0408	.0574
.0408	.0042	-.0053	.0009	.0139	.0275	.0377	.0434	.0457	.0482	.0574	.0818

Eigenvalues

.2499	.0298	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Eigenvectors

-.2964	-.8616	-.0563
-.0341	-.3819	-.1699
.0358	-.1188	-.1082
-.0059	.0063	.0071
-.0966	.0531	.0779
-.1915	.0686	.0515
-.2636	.0796	-.0682
-.3033	.1037	-.2478
-.3195	.1372	-.4121
-.0371	.1654	-.4179
-.4007	.1541	-.1155
-.5709	.0561	.7236

(Other eigenvectors make insignificant contribution)

TABLE A-3 (Continued)

Transformed Data (Standard Scores)

2.1922	1.7548	- .0995	1.1976	1.4231	1.5171	1.5277	1.4875	1.4233	1.3889	1.4793	1.6747
.9397	1.3300	1.1066	- .7045	- .3456	- .2758	- .2603	- .2927	- .3448	- .3751	- .3021	- .1220
- .2603	.0846	.5949	- .7045	- .6709	- .6592	- .6601	- .6657	- .6663	- .6555	- .6168	- .5624
.0872	.7586	1.3259	-1.3385	-1.0775	-1.0116	- .9996	-1.0126	-1.0434	-1.0586	-1.0295	- .9294
- .3175	.4363	1.3624	-1.3385	-1.3215	-1.2810	-1.2637	-1.2613	-1.2722	-1.2805	-1.2720	-1.2300
- .5603	- .2377	.4487	- .7045	- .7116	- .7318	- .7280	- .7246	- .7096	- .7023	- .7029	- .7162
- .1984	.1286	.5949	- .7045	- .6506	- .6489	- .6601	- .6788	- .7034	- .7023	- .6634	- .5884
1.1397	.9637	.0102	.5636	.7522	.8538	.9166	.9508	.9658	.9392	.8706	.7939
.4683	.3777	.0102	.2466	.2846	.2838	.2829	.2571	.2201	.2090	.2273	.2905
1.3302	.8318	- .5015	1.1976	1.2402	1.2891	1.3014	1.2846	1.2564	1.2371	1.2665	1.3322
.0492	.5974	.0832	.2466	.3660	.4289	.4715	.5058	.5331	.5361	.5193	.4785
- .1460	-1.1753	-2.0731	1.8316	1.6874	1.6311	1.6334	1.6904	1.7633	1.7978	1.7069	1.4685
-2.1700	-1.9665	- .3553	- .7045	- .9759	-1.0634	-1.0600	- .9930	- .9012	- .8717	-1.0246	-1.3249
- .2175	- .5453	- .7208	.5626	.4269	.3667	.3357	.3160	.2981	.2966	.3016	.3010
.1778	- .4867	-1.2690	1.1976	1.1588	1.1129	1.0902	1.1013	1.1142	1.1319	1.1180	1.0665
- .8985	.0407	1.5452	-1.6555	-1.8094	-1.7888	-1.7767	-1.7653	-1.7606	-1.7712	-1.7916	-1.7963
-1.1270	-1.1167	- .3553	- .0704	- .4473	.5349	- .5771	- .5937	- .5859	- .5621	- .5545	- .5789
-1.0890	-1.7760	-1.7076	.0806	.6709	.5118	.4263	.3945	.4032	.4426	.4698	.4448

Correlation Matrix

1.0000	.8615	.0700	.4144	.5603	.6130	.6283	.6174	.5933	.5767	.6061	.6796
.8615	1.0000	.5668	- .1027	.0627	.1284	.1488	.1369	.1087	.0883	.1216	.2135
.0700	.5668	1.0000	- .8740	- .7862	- .7431	- .7280	- .7342	- .7506	- .7638	- .7457	- .6834
.4144	- .1027	- .8740	1.0000	.9825	.9691	.9632	.9642	.9682	.9720	.9684	.9449
.5603	.0627	- .7862	.9825	1.0000	.9976	.9953	.9946	.9943	.9949	.9965	.9875
.6130	.1284	- .7431	.9691	.9976	1.0000	.9995	.9987	.9970	.9962	.9991	.9947
.6283	.1488	- .7280	.9632	.9953	.9995	1.0000	.9996	.9976	.9967	.9992	.9949
.6174	.1369	- .7342	.9642	.9946	.9987	.9996	1.0000	.9998	.9984	.9994	.9919
.5933	.1087	- .7506	.9682	.9943	.9970	.9978	.9993	1.0000	.9998	.9988	.9868
.5767	.0883	- .7538	.9720	.9949	.9962	.9967	.9984	.9998	1.0000	.9984	.9845
.6061	.1216	- .7457	.9684	.9965	.9991	.9992	.9994	.9988	.9994	1.0000	.9926
.6796	.2135	- .6834	.9449	.9875	.9947	.9949	.9919	.9868	.9845	.9926	1.0000

TABLE A-3 (Continued)

Eigenvalues

9.8535	2.1174	.0215	.0064	.0011	.0001	.0000	.0000	.0000	.0000	.0000	.0000

Eigenvectors

-.1911	.5490	.2833	-.0780	.1775	.0326
-.0359	.6828	.0335	.0793	.0297	.4294
.2399	.4507	-.3083	.3065	-.2173	-.6857
-.3099	-.1472	.3962	.8396	-.1400	.0279
-.3130	-.0344	.2010	-.1497	.3286	-.5231
-.3185	.0109	.0626	-.0923	.2805	-.0811
-.3183	.0252	-.0909	-.0187	.2733	-.0906
-.3182	.0170	-.2737	.0439	.1861	.0385
-.3179	-.0023	-.4386	.0923	.0301	-.0820
-.3179	-.0164	-.4449	.0516	-.1790	.1721
-.3184	.0064	-.1582	-.1150	-.4631	.0261
-.3163	.0704	.3518	-.3672	-.5978	-.1531

(Other eigenvectors make insignificant contribution)

TABLE A-4

REGRESSION COEFFICIENTS FOR CO CORRECTION FACTORS

PRINCIPAL COMPONENTS PROGRAM

Covariance Matrix

.11180E + 00	-.78435E - 02	.98686E - 04	.15818E - 05	-.64443E - 07	.52888E - 09
	.11655E - 02	-.84526E - 04	.33546E - 05	-.60594E - 07	.40040E - 09
		.10011E - 04	-.44718E - 06	.84291E - 08	-.56788E - 10
			.20514E - 07	-.39083E - 09	.26477E - 11
				.74804E - 11	-.50808E - 13
					.34561E - 15

Eigenvalues

.11235E + 00	.62202E - 03	.13417E - 06	.46185E - 12	.10209E - 12	.30625E - 15

Eigenvectors

.99752E + 00	.11940E - 01	.79114E - 02	-.26439E - 08	.82568E - 05	-.12681E - 07
-.70371E - 01	-.98962E + 00	.12518E + 00	.50308E - 02	-.16204E - 03	-.23231E - 06
.92923E - 03	.12541E + 00	.98851E + 00	-.04350E - 01	-.31904E - 02	-.22265E - 05
.11940E - 04	-.56053E - 01	-.84351E - 01	.99453E + 00	-.61425E - 01	.13586E - 03
-.53415E - 06	.10535E - 03	.20110E - 02	.61471E - 01	-.99800E + 00	.14408E - 01
.44445E - 08	-.70796E - 06	-.15285E - 04	-.75079E - 03	.14389E - 01	.99990E + 00

TABLE A-5

REGRESSION COEFFICIENTS FOR HC CORRECTION FACTORS

PRINCIPAL COMPONENTS PROGRAM

Covariance Matrix

.23837E - 01	-.23725E - 02	.12123E - 03	-.45466E - 05	.80177E - 07	-.51954E - 09
	.34111E - 03	-.26353E - 04	.11299E - 05	-.21030E - 07	.14038E - 09
		.26167E - 05	-.12003E - 06	.22962E - 08	-.15567E - 10
			.56372E - 08	-.10901E - 09	.74382E - 12
				.21187E - 11	-.14501E - 13
					.99441E - 16

Eigenvalues

.24074E - 01	.10587E - 03	.55098E - 07	.58084E - 13	.11181E - 17	.24840E - 21

Eigenvectors

.99503E + 00	-.99251E - 01	.83972E - 02	-.26351E - 03	-.95207E - 05	.12889E - 06
-.99474E - 01	-.98583E + 00	.13497E + 00	-.50465E - 02	-.18636E - 03	.25240E - 05
.51201E - 02	.13508E + 00	.98722E + 00	-.84279E - 01	-.36026E - 02	.51042E - 04
-.19261E - 03	-.64121E - 02	-.84239E - 01	-.99421E + 00	-.66324E - 01	.11165E - 02
.34012E - 05	.12360E - 03	.20101E - 02	.66396E - 01	-.99736E + 00	.29268E - 01
-.22057E - 07	-.83998E - 06	-.15353E - 04	-.82734E - 03	.29278E - 01	.99957E + 00

TABLE A-6

REGRESSION COEFFICIENTS FOR NOX CORRECTION FACTORS

PRINCIPAL COMPONENTS PROGRAM

Covariance Matrix

.26107E + 00	-.40772E - 01	.21252E - 02	-.42892E - 04	.30230E - 06
	.65390E - 02	-.34608E - 03	.70242E - 05	-.49658E - 07
		.18475E - 04	-.37619E - 06	.26644E - 08
			.76712E - 08	-.54385E - 10
				.38584E - 12

Eigenvalues

.26746E + 00	.16858E - 03	.13350E - 08	.82688E - 15	.25399E - 18

Eigenvectors

-.98798E + 00	-.15453E + 00	.47653E - 02	.18743E - 03	.40029E - 05
.15439E + 00	-.98456E + 00	.82412E - 01	.36301E - 02	.79084E - 04
-.80505E - 02	.82215E + 00	.99434E + 00	.66750E - 01	.16147E - 02
.16251E - 03	-.18897E - 02	-.66836E - 01	.99710E + 00	.36236E - 01
-.11454E - 05	.14214E - 04	.81028E - 03	-.36263E - 01	.99934E + 00

4.2 WORKING PAPER NO. 2:

SPEED-TEMPERATURE-HOT/COLD CORRECTION FACTORS: A CRITIQUE AND PROSPECTUS

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Project No. 8411
Environmental Protection Agency
May 12, 1978
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SPEED-TEMPERATURE-HOT/COLD CORRECTION FACTORS:

A CRITIQUE AND PROSPECTUS

1. INTRODUCTION

One of the most important components of the corrections applied to emission factors is the correction factor denoted R_{ipstwx} and sometimes referred to as the speed-temperature-hot/cold correction factor. To understand the omnibus effect of this factor one must examine the influence of each of the variables which enter into its makeup. The R-factor is, of course, specific for vehicle group i and pollutant p, but the items of interest here are the average speed s, the ambient temperature t, and the hot/cold partitioning specified by the quantities w and x.

2. AVERAGE SPEED CONSIDERATIONS

Speed correction has been previously discussed in Working Paper No. 1.(1) The speed correction factors treated in that document, however, are only the "preliminary" inputs to the R-factors. It is appropriate, therefore, to examine the assumption whereby these preliminary speed-correction factors were incorporated into the more inclusive R-factors.

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(1) H. T. McAdams, "A Factor-Analytic Approach to Emission Correction Factors," Working Paper No. 1, Project 8411, Falcon Research and Development Company, Buffalo, N. Y. (April 5, 1978).

The preliminary speed-correction factors were generated by application of the Modal Emission Model (MEM). It should be noted, however, that this model applies only to warmed-up vehicles and to vehicles operating in a standard ambient of 75°F. A further complication is introduced by the fact that the MEM takes into consideration the actual speed vs time profile prevailing in a driving sequence, whereas any correction factor based on average speed does not. In reality there is an unlimited number of speed-time profiles which could map into the same average speed, but a concession made in the R-factor is that this many-to-one mapping is permissible.

The preliminary speed correction factors, as derived by application of the MEM, were found to be nonlinear functions of average speed. Whether the complexity of the functional forms assumed for these relations is justifiable is subject to question, however. For example, the correction factor vs speed relations for HC and CO show steep gradients only at low speed, and it is possible that a simpler functional form would be capable of retaining "most" of the information in the exponential fifth-order polynomials. Some further observations pertinent to this point are given in Appendix I.

Whatever the form of the preliminary speed correction factors, one needs now a way to adjust these correction factors for the operating condition of the engine, whether cold start, hot start or stabilized. In other words one must make the speed correction factors for warmed-up engines bag specific.

The method employed for this purpose, as described by Becker [2], is based on two factors which can affect bag emissions. One is that the average speeds for the several bags are different and different from the average speed of 19.6 mpg over the FTP driving cycle. The other stems from the state of warm-up of the vehicle as that state affects the actual generation of pollutants. In other words, even if the average speed in Bag 1 (cold start) were the same as for the total FTP driving cycle, there would still be a difference between cold-start emissions and emissions from vehicles in the warmed-up state.

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(2) "Supplement 8 Light Duty Vehicle Correction Factors," Memo Janet Becker to J. Hidinger/J. Horowitz (3/7/77).

It is at this point that difficulties in terminology seem to arise as well as difficulties in assumptions. To speak of measuring emissions over the FTP driving cycle at an arbitrary speed seems contradictory, because the FTP in a strict sense *implies* an average speed, namely 19.6 mph. Similarly, to speak of measuring Bag i emissions at an arbitrary speed also seems incorrect, since with each bag there is an *associated* specific average speed. Further challenging of these concepts will be held in abeyance for the present, however, in order to set forth what was actually done in the generation of the bag-specific speed correction factors.

First consider Bag 1. This is the cold-start bag and has an associated average speed of 26 mph. For Bag 1, the final speed correction factor for a specific pollutant is given as

$$V_{g, s_1} = \frac{V_{2, s_1}}{V_{2, 26}}$$

where the subscript g refers to vehicle group, and the subscript 2 identifies Group 2, low-altitude pre-controlled vehicles. This definition is based on the assumption that "emission dependency on speed during cold operation is ... similar for all model year vehicles and ... is equal to the dependency of pre-controlled vehicles during warmed-up operation." (3) In short, it is assumed that it is necessary only to normalize the Group 2 correction factor at an arbitrary speed s_1 by dividing by the Group 2 correction factor at the bag speed of 26 mph (see Table 1). This adjustment has the effect of re-referencing the correction factor curves to the Bag 1 average speed of 26 mph rather than to the FTP average speed of 19.6 mph.

Adjustment of Bag 2 and Bag 3 preliminary correction factors were performed in a somewhat different manner. Here the re-referencing for bag average speed is done on a group-wise basis.

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(3) Becker, *op. cit.*

TABLE 1

DERIVATION OF BAG 1 SPEED CORRECTION FACTORS

Speed (mph)	5	10	15	20	25	26	30	35	40	45	50	55	60
Hydrocarbons													
Preliminary Correction Factor	3.297	1.749	1.224	.986	.844	.821	.740	.659	.600	.565	.547	.530	.482
Final Correction Factor	4.016	2.130	1.491	1.201	1.028	1.000	.901	.803	.731	.688	.666	.646	.587
Carbon Monoxide													
Preliminary Correction Factor	3.319	1.751	1.225	.986	.841	.817	.734	.650	.592	.557	.539	.518	.454
Final Correction Factor	4.062	2.143	1.499	1.207	1.029	1.000	.898	.796	.725	.682	.660	.634	.556
Nitrogen Oxides													
Preliminary Correction Factor	1.242	1.031	.974	1.004	1.074	1.089	1.146	1.203	1.239	1.265	1.306	1.404	1.615
Final Correction Factor	1.140	.947	.894	.922	.986	1.000	1.052	1.105	1.138	1.162	1.199	1.289	1.483

Thus,

$$V_{g,\ s_2} = \frac{V_{g,\ s_2}}{V_{g,\ 16}} \qquad \text{for Bag 2}$$

and

$$V_{g,\ s_3} = \frac{V_{g,\ s_3}}{V_{g,\ 26}} \qquad \text{for Bag 3.}$$

The assumption in all cases is that "emissions as measured over the cold (hot) (sic) bag driving cycles and over the stabilized driving cycle at a given average speed are equal to emissions as measured over the FTP driving cycle at that speed." (4)

Though understandable because of data limitations, the assumptions on which the speed correction factors are based seem tenuous, and certain aspects of the procedure raise questions. For example, it is asserted that in the Monte Carlo implementation of the Modal Emission Model "all cycles were transient; no steady state cycles were generated." (5) Yet to obtain an average speed of--say--60 mph, much of the time would have to be spent, it would seem, in a high-speed cruise mode. A second consideration applies to the bag-specific average speeds. It might be considered to be "very unlikely" that a vehicle operating in the cold-start mode would do so at a high average speed; in short, there is an upper bound of speed beyond which the correction factor for cold start is essentially irrelevant. Finally, there is an anomaly in the notion of "FTP driving cycle" or "Bag i driving cycle" at anything other than the design average speeds.

(4, 5) Becker, op. cit

3. TEMPERATURE-DETUNING-AGE DETERIORATION CONSIDERATIONS

According to Becker (*op. cit.*) ambient temperature was found to affect only Bag 1 emissions for only HC and CO. Results obtained for 1975 vehicles showed an exponential relation between these emissions and ambient temperature.

Pre-1975 vehicles were assigned the relations:

$$\text{Bag 1 CO} = e^{5.6548 - .015965\ t}$$

$$\text{Bag 1 HC} = e^{2.9310 - .014779\ t}$$

where the response is in units of gms/mi.

When $t = 75^{\circ}$, one has:

$$\text{Bag 1 CO} = e^{4.457425} = 86.266 \text{ gms/mi.}$$

$$\text{Bag 1 HC} = e^{1.822575} = 6.188 \text{ gms/mi.}$$

It might seem reasonable to divide the general Bag 1 expressions by their corresponding standard emissions at 75° to obtain a "Bag 1 temperature correction factor." In the case of carbon monoxide, this would have led to

$$\text{CO Bag 1 CF} = \frac{1}{86.266}\ e^{5.6548}\ e^{-.015965\ t}$$

$$= \frac{286.015}{86.266}\ e^{-.015965\ t}$$

$$= 3.318\ e^{-.015965\ t}$$

Note that when $t = 75^{\circ}$,

$$e^{-.015965\ t} = e^{-1.197375} = .302$$

and the correction factor is (3.318)(.302) = 1.00.

Further insight would be gained by differentiating the correction factor with respect to temperature to obtain

$$\frac{d\ (\text{CO Bag 1 CF})}{dt} = (3.318)(-.015965)\, e^{-.019565\ t}$$

$$= -.015965\ (\text{CO Bag 1 CF})$$

Thus a 1° change from a specified temperature would change the correction factor by about 1½% of its value at the initial temperature. A similar analysis for HC would indicate the range of the correction factor and its rate of change at any given temperature.

Though the above type of approach sheds light on the specific effect of ambient temperature on Bag 1 emissions, this was <u>not</u> the approach used in developing the current correction factors. Instead, terms were added to account for the "detuning" of vehicles in use and for mileage-accumulation "deterioration." In these contributions distinction is made between pre-1968 vehicles and 1968-74 vehicles, whereas no distinction is made in the exponential parts of the Bag 1 expressions (see Table 2).

Table 2

DETUNING AND DETERIORATION CONTRIBUTIONS TO

CORRECTION FACTOR EXPRESSIONS (A = age in years - 1)

	Detuning	Deterioration
	HYDROCARBONS	
Pre-1968	0.673	0.569A
1968-74	-2.410	0.863A
	CARBON MONOXIDE	
Pre-1968	-14.74	9.62A
1968-74	-33.89	9.77A

Moreover, the correction factors incorporating temperature, age and detuning effects are normalized in terms of the overall FTP emissions rather than in terms of the Bag 1 emissions.

Let us examine the implications of this approach for HC. As given in Table II.1 of Becker (*op. cit.*), the relevant part of the general expression for the R-factor is:

$$\frac{e^{2.9310 - .014779\ t} + .673 + .569A}{5.67 + .47A} \quad \text{for pre-1968 vehicles}$$

and

$$\frac{e^{2.9310 - .014779\ t} - 2.41 + 863A}{2.8 + .64A} \quad \text{for 1978-74 vehicles.}$$

When $t = 75^{\circ}$ and $A = 0$, these expressions reduce to:

$$\frac{6.188 + .673}{5.67} = \frac{6.861}{5.67} = 1.21 \quad \text{for pre-1968 vehicles}$$

$$\frac{6.188 - 2.41}{2.8} = \frac{3.778}{2.8} = 1.35 \quad \text{for 1968-74 vehicles}$$

These results imply that hydrocarbon emissions in Bag 1 are 21% higher than emissions over the FTP for pre-1968 vehicles and 35% higher for 1968-74 vehicles. *A much stronger conclusion than this, however, can be made*: these factors are independent of age so long as temperature is held at 75°. For:

$$\frac{6.861 + .569A}{5.67 + .47A} \equiv 1.21$$

and

$$\frac{3.778 + .863A}{2.8 + .64A} \equiv 1.35.$$

Similar conclusions can be drawn for Bag 2 and Bag 3 for pre-1968 and for 1968-74 vehicles. For, in Table II.1 one has for HC:

$$\text{For Bag 2}\quad \begin{cases} \dfrac{5.69 + .471A}{5.67 + .47A} \equiv 1.002 \text{ for pre-1968 vehicles} \\[2ex] \dfrac{2.61 + .597A}{2.8 + .64A} \equiv 0.932 \text{ for 1968-74 vehicles} \end{cases}$$

$$\text{For Bag 3}\quad \begin{cases} \dfrac{4.75 + .393A}{5.67 + .47A} \equiv 0.837 \text{ for pre-1968 vehicles} \\[2ex] \dfrac{2.43 + .555A}{2.8 + .64A} \equiv 0.867 \text{ for 1968-74 vehicles} \end{cases}$$

Thus for pre-1975 vehicles the age-effect term is a "fictional" or "dummy" correction. Note, however, that this is not the case for 1975 vehicles.

In summing up the considerations of this section several points can be made. In Table 2, the effects of detuning are tabulated. In three of the four entries the detuning terms are negative. This fact suggests that as-received, in-use vehicles give lower emissions than tuned-up vehicles. Though it is not inconceivable that such could be the case, it seems more likely that the results may be an artifact of the sampling involved in obtaining the data on which the corrections are based. This possibility further suggests that a program aimed at assessing deteriorations in use should exercise as much control as possible and practical over the sampling procedure. Finally, there may be advantage in normalizing emission correction factors on a bag-by-bag basis rather than on the FTP basis and to report emission factors bagwise rather than (or in addition to) FTP-wise. More on this point will follow in the next section, which deals with the hot start/cold start aspect of the R-factor.

4. HOT START/COLD START CONSIDERATIONS

The three bags in the FTP test are combined to give the overall emission factor. The weightings, under standard circumstances, are as follows:

w = cold start fraction = 20.58% (Bag 1)
$1-w-x$ = stabilized fraction = 52.13% (Bag 2)
x = hot start fraction = 27.28% (Bag 3)

The average speeds for these bags under standard conditions are respectively 26 mph, 16 mph and 26 mph.

In the overall expressions (Tables II.1, II.2, II.3 of Becker, <u>op. cit.</u>) the factors w, $1-w-x$, and x are used as simple weighting factors. When these factors assume their standard values and when both temperatures and speeds are at their standard values, then the R-factor should compute to unity. That this is true can be noted by returning to the previous section and considering weighted combinations of the bag-specific correction factors for HC for pre-1968 vehicles and for 1968-1974 vehicles. For pre-1968 vehicles, one has

$$\underbrace{1.21\ (.2058)}_{\text{(Bag 1)}} + \underbrace{1.002\ (.5213)}_{\text{(Bag 2)}} + \underbrace{0.837\ (.2728)}_{\text{(Bag 3)}} = 1.00$$

and, for 1968-74 vehicles,

$$1.35\ (.2058) + 0.932\ (.5213) + 0.867\ (.2728) = 1.00$$

It is when the values of w and x depart from their standard values of $w = 0.2058$ and $x = 0.2728$ that the correction factors become useful. However, that is also when any difficulties or errors in the correction-factor expressions will be manifested. A particularly troublesome point, it would seem, arises when both the fraction of occupancy time as well as the prevailing speeds in the three bags depart from their nominal values. Indeed, as was mentioned earlier, such departures are "coincidentia oppositorum" if the usual concepts of "Bag i" emissions are to hold in a strict sense. What is really implied, when one postulates a Bag 1 speed of--say--10 mph, is a driving cycle quite different from the normal Bag 1 cycle, but still one in which the vehicle is operating in the cold-start condition. Apparently the assumption relied upon to resolve this apparent contradiction is that any scaling of emissions for speed effects, whether in Bags 1, 2 or 3, can be performed "as if" the vehicle were operating over some portion of the FTP consistent with that average speed.

In applying the weighting fractions w, x and 1-w-x to the three bags certain questions of a sample-space nature arise. According to definition, these weighting factors relate to the fraction of total miles driven in the cold start, hot start and stabilized conditions, respectively. Note, however, that the fraction of miles driven in a given mode is not, in general, the same as the fraction of the number of vehicles operating in that mode at a given point in time nor to the fraction of time during which a vehicle operates in that mode. Indeed, when corrections are made simultaneously for mode weight fraction and for average speed in mode, the prospect of interaction of the two should not be overlooked.

To investigate this possibility let us return to the basis by which the standard weightings

$$
\begin{aligned}
w &= 20.58\% \\
1-w-x &= 52.13\% \\
x &= 27.28\%
\end{aligned}
$$

arise. Consider the following.

Bag 1. First 505 seconds (0.1403 hr.)
Mileage = 3.59 miles
Average speed = 25.6 mph

Bag 2. Next 870 seconds (0.2417 hr.)
Mileage = 3.91 miles
Average speed = 16 mph

Bag 3. Repeat first 505 seconds
Mileage = 3.59 miles
Average speed = 25.6 mph.

If it is assumed that 43% of the vehicle trips begin in the cold start mode and 57% in the hot start mode, then a "typical" or "expected" vehicle trip would be represented as

$$
\text{FTP GMS/MI} = \left[0.43 \text{ Bag 1 Gms.} + \text{Bag 2 GMS.} + 0.57 \text{ Bag 3 Gms.}\right] \Big/ 7.5 \text{ mi.}
$$

and

$$\text{FTP mi.} = 0.43 \text{ Bag 1 mi.} + \text{Bag 2 mi.} + 0.57 \text{ Bag 3 mi.}$$

$$= (0.43)(3.59) + 3.91 + (0.57)(3.59)$$

$$= 1.5437 + 3.91 + 2.0463$$

$$= 7.5 \text{ mi.}$$

Then

$$\frac{1.5437}{7.5} = .2058 = \text{fraction of miles in cold start mode}$$

$$\frac{3.91}{7.5} = .5213 = \text{fraction of miles in stabilized mode}$$

and

$$\frac{2.0463}{7.5} = .2728 = \text{fraction of miles in hot start mode}$$

Now it is presumed that the FTP weightings were devised with a view toward "representative" operation of vehicles in cold-start, stabilized and hot-start modes. The gms/mi. emissions are derived on the basis that the driving cycle consists of a transient part 3.59 miles long and a stabilized part 3.91 miles long and that the total trip length is 7.5 miles. When average speeds in the three stages of operation are taken into account, the assumptions of the FTP translate into certain fractions of "vehicle-hours" spent in cold-start, stabilized and hot-start modes, given a trip length of 7.5 miles.

$$\frac{(0.43)(3.59 \text{ mi.})}{25.6 \text{ mi/hr}} = 0.0603 \text{ hrs. (3.6 min.) in "Bag 1"}$$

$$\frac{3.91 \text{ mi.}}{16 \text{ mi/hr}} = 0.2444 \text{ hrs. (14.7 min) in "Bag 2"}$$

$$\frac{(0.57)(3.59 \text{ mi.})}{25.6 \text{ mi/hr}} = 0.0799 \text{ hrs. (4.8 min) in "Bag 3"}$$

Thus the total "trip time" is

$$0.0603 + 0.2444 + 0.0799 = 0.3846 \text{ hrs.} = 23 \text{ minutes}$$

and the fractions of time in each "bag" are:

$$\frac{.0603}{.3846} = 0.1568 \quad \text{for cold start} \quad \text{(Bag 1)}$$

$$\frac{0.2444}{0.3846} = 0.6355 \quad \text{for stabilized (Bag 2)}$$

$$\frac{0.0799}{0.3846} = 0.2077 \quad \text{for hot start (Bag 3).}$$

The point to be made here is that the FTP driving sequence taken as the reference to which other driving scenarios are compared, implies not only certain average speeds and fractions of miles in each mode but also a "typical" or average trip length and certain fractions of total vehicle operating times in each of the three modes of operation. It is to be understood, of course, that either the vehicle executes the trip from a cold start or from a hot start. Thus the time for "warm-up" in the cold start, when it occurs, is (3.59 mi.)/(25.6 mi. hr.) = 0.14 hr. = 8.4 minutes, not the "expected" value of 3.6 minutes as calculated above.

In view of the above considerations it seems logical to consider their implications when one envisions a scenario departing from the reference conditions. For example, consider one of the cases given in Table II.5 of Becker (op. cit.):

% cold, % stable, % hot start = 40, 30, 30

Ave. speed Bag 1, 2, 3 = 10, 10, 10

The fraction of trips originating "cold" is given by

$$\frac{.40}{.40 + .30} = \frac{0.40}{0.70} = 0.5714 \text{ (i.e., 57\%)}$$

and the fraction of trips originating "hot" is given by

$$\frac{.30}{.40 + .30} = \frac{0.30}{0.40} = 0.4286 \quad (\text{i.e., } 43\%)$$

To reconcile all the constraints one must make certain assumptions about either trip lengths or trip times and the portions thereof spent in each bag.

For example, suppose that the trip length is assumed to be 7.5 miles, as in the FTP, and that the disposition of this length is 3.59 miles in either cold or hot transient and 3.91 miles in stabilized operation. Then

$$\frac{3.59 \text{ miles}}{10 \text{ mi/hr}} = 0.359 \text{ hr.} = 21.5 \text{ min.}$$

which is too long for the vehicle to operate without being in stabilized mode, whether the start is cold or hot. Suppose, instead, that one assumes the warm-up time of 8.4 minutes (0.14 hr.) as in the FTP. The corresponding distance at 10 mi/hr. is (0.14 hr.) x (10 mi/hr.) = 1.4 mi. If a trip length of 7.5 miles is assumed, then 7.5 - 1.4 = 6.1 miles would have to be in stabilized operation. Note that

$$(.57)(1.4) + 6.1 + (.43)(1.4) = 7.5$$

satisfies the trip length requirement but violates the original assumption of 40%/30%/30% mileage split in Bag 1/Bag 2/Bag 3. For,

$$\frac{(0.57)(1.4 \text{ mi.})}{7.5 \text{ mi.}} = .1064 \neq 0.40$$

$$\frac{6.1 \text{ mi.}}{7.5 \text{ mi.}} = .8133 \neq 0.30$$

$$\frac{(0.43)(1.4 \text{ mi.})}{7.5 \text{ mi.}} = .0803 \neq 0.30$$

Evidently, then, trip length must be much shorter than in the FTP, a fact made evident by the consideration that average speed over every segment is no greater than about half the average speed of the FTP (19.6 mph). A consistent solution, based on adjusted trip length, would be found by solving

$$\frac{x}{x + 1.4} = 0.3$$

where x is the distance covered during the stabilized portion of the trip. The solution is $x = 0.6$ miles, for a total trip length of $1.4 + 0.6 = 2$ miles. Then

$$(0.57)(1.4) + 0.6 + (0.43)(1.4) = 2 \text{ miles}$$

and

$$\frac{(0.57)(1.4)}{2} = 0.4$$

$$\frac{0.6}{2} = 0.3$$

$$\frac{(0.43)(1.4)}{2} = 0.3$$

and it is seen that the desired split of cold start, stabilized and hot start driving is preserved but only if an average trip length of 2 miles is postulated.

To summarize, cold/stabilized/hot fractions, average speeds, trip lengths and fractions of trips initiated in cold start or hot start--all are interrelated. Correction factors designed to accomodate both average speed and mode of operation must, therefore, be developed and used with care.

Specific points to consider are the fact that the fraction of miles driven in cold start is different from the fraction of time spent in the cold start mode and is also different from the fraction of trips which originated from a cold start.

Moreover, for a given fraction of miles driven in cold start conditions, the average cold-start speed acts as a constraint on trip length. Though rate of warm-up may vary with driving speed, it is likely that vehicles tend to stabilize after some fixed length of time rather than after some fixed distance driven. Further study to define warm-up time as a function of speed, ambient temperature and other factors would help resolve this question. Any survey to define local use patterns for the purpose of defining the impact of vehicle emissions on air quality should aim at consistency among the various elements involved.

5. SUMMARY AND PROSPECTUS

Correction factors for ambient temperature, average speed, percent hot start/percent cold start operation and other factors are incorporated in a quantity R_{ipstwx}. Assumptions on which this factor is based have been critically examined, certain difficulties and possible anomalies indicated, and some suggestions made for simplification and/or revision. It is realized that the approach taken in the formulation of the R-factor is to a certain extent expedient in that definitive data pertinent to variables affecting the correction factor were not always available. This fact necessitated the use of cogent estimation based on engineering judgment and reasonable assumptions. Without calling into question the efficacy of such an approach, which certainly was reasonable under the circumstances, it is nonetheless appropriate to consider alternatives, given the resources to supplement data sources. This prerogative is exercised in response to the charge given in the Scope of Work to offer a recommendation "as to the optimal approach for future correction factor development." A comprehensive pronouncement on this point is premature at this time, but certain pertinent observations deriving from the examination of the R-factor will be advanced. These observations deal generally with the assumptions and the data on which the R-factor is based, as well as the mathematical form into which the factor is cast.

At the outset it is observed that correction factors represent an attempt to express the functional dependence of emissions on a host of variables known to influence them. Mathematically, therefore, one hopes for an expression or response relation of the form

$$\text{Emissions (gms./mi.)} = f\ (x_1,\ x_2,\ \ldots\ x_p) \qquad (1)$$

where $x_1, x_2, \ldots, x_p$ refer to such quantities as ambient temperature, vehicle average speed, and the like. Even to identify the appropriate "variables" is problematical, as evidenced by concern previously expressed with regard to the adequacy of the concept of average speed. Moreover, the equation (1) is vehicle specific, and to develop such an equation for every vehicle or class of vehicle would clearly entail exorbitant effort. The hope of the correction factor concept is that the vehicle or vehicle-class dependent aspect of equation (1) can be extracted as a scalar multiplier and that the equation can thereby be reduced to nondimensional form. In short, it is assumed that the relative effect on emissions of incremental changes $\Delta x_1, \Delta x_2, \ldots, \Delta x_p$ is unaffected by the absolute level of emissions factored out of the response relation.

Viewed in the above light, equation (1) becomes

$$\text{Emissions (gms/mi.)} = \text{Emission factor (gms/mi.)} \cdot CF(x_1,\ x_2,\ \ldots,\ x_p) \qquad (2\ )$$

where the first quantity on the right-hand side is vehicle or group dependent and the second quantity is a non-dimensional function of the variables $x_1, x_2, \ldots, x_p$ and is considered to be common to all vehicles in the class of interest. Thus the very existence of $CF(x_1, x_2, \ldots, x_p)$ rests on an implicit assumption which can be either too strong or too weak, depending on the complexity of the response relation and the degree of commonality of that function as one goes from one vehicle or class of vehicles to another. Means for analytically evaluating the degree of commonality reside in factor analytic methods such as principal component analysis, as illustrated in Working Paper No. 1. The correction factor function, viewed as a response "surface" (hypersurface) does not necessarily have the same "shape" for all cases of interest and may need to be resolved into two or more component surfaces, each of which has to be scaled by a multiplier much as in the original assumption of single separable scalar and non-dimensional functional parts.

A first recommendation, therefore, is that:

- Systematic analysis of the degree of commonality of the effects of emission-related variables should precede attempts to develop a correction-factor response function.

It is believed that such an approach might better structure the correction process, which should be viewed not necessarily as a factor but as a general mathematical transformation until its form has been delineated by systematic analysis.

Once the form of $CF(x_1, x_2, \ldots, x_p)$ has been postulated, either by mathematical or engineering analysis, data requirements for its definition can be specified. The function can be structured as a "linear model"

$$CF(x_1, x_2, \ldots, x_p) = b_1 f_1(x_1, x_2, \ldots, x_p) + \ldots$$
$$+ b_k f_k(x_1, x_2, \ldots, x_p)$$

in which $f_1, f_2, \ldots f_k$ are linearly independent "basis functions" of arbitrary (and often nonlinear) form. It follows, therefore, that the effects of $x_1, x_2, \ldots, x_p$ on the correction factor may be both nonlinear and interactive--that is, the effect of x_i on emissions may depend on the level at which x_j is set $(i \neq j)$. Accordingly, an experiment designed to evaluate the correction-factor surface should take such nonlinearities and interactions into account. On the other hand the experiment should not be "overstructured." For example, if the effect of speed on emissions were thought to be quadratic, it would be wasteful to structure an experiment at--say--ten speed levels.

A second recommendation thus arises:

- Correction factors are best determined through a designed experiment in which the allocation of "treatments" (that is, combinations of levels of the variables $x_1, x_2, \ldots, x_p$) is made according to the anticipated degree of nonlinearity within variables and degree of interaction among variables.

One notes that the above recommendations are idealized and break with historical precedent, in which it is often necessary to gain information by a "piggy-back" process. In other words, data made available from an experiment designed for a purpose other than correction factor development must be adapted to that purpose and, in the process, becomes subject to uncertainties over which the investigator has little control.

So much for the assumptions regarding emission factor correction and the nature of the data-acquisition process. Let us now move to the question of mathematical representation of the correction-factor response relation.

There is evidence that, in some instances, the dependence of emissions on certain variables--e.g., average speed--is less complicated than is implied by available expressions in current use. Excess complication in functional representation can arise from: (1) incorporation of refinements having only minor engineering significance; (2) less than optimum choice of independent variable; (3) covariance of variables influencing emissions; and (4) inopportune choice of units.

Appreciation of the engineering importance of variables influencing emissions can be obtained by evaluating the differential effects of variables. For example, it would be informative to know the incremental change in grams per mile for a 1-mile-per-hour increment of average speed or a 1°F change in ambient temperature. It is realized, of course, that if interaction is present these incremental changes are not constant, but the lack of constancy can be evaluated by means of mixed partial derivatives. Further, if the response of emissions to a given variable is substantially linear, then incorporation of additional terms may add little to the precision of estimation of emissions even though the added terms can be shown to be statistically significant. In such cases it is possible that the simplification of representation more than offsets the small gain in precision, expecially in view of the uncertainties that may exist in the estimation of levels of the independent variable. For example, if average speed must be estimated by a rough sampling process, it is doubtful that a fourth or fifth order polynomial or exponential is justified in representing the relation between speed and emissions.

What is referred to as less than optimum choice of independent variable is exemplified by the observation that emissions expressed in grams per unit time may plot as a simpler function of average speed than when expressed in grams per unit distance. In short, the effect of speed tends to be "unified" by the distance-to-time transformation. The fact that certain variables are perceived as having important effects on emissions is to a certain extent an accident of human perception--that is, we are accustomed to think in terms of speed, temperature and the like when other, derived or even "contrived" variables may lead to simpler relations or scaling laws.

One of the ways to originate variables which seem artificial but which are in reality quite meaningful is to note the covariance or interdependence of two or more variables. Such covariance can be imposed by real physical constraints, such as those influencing temperature, absolute humidity and relative humidity. A cogent example, also, is afforded by the fact that high speeds are not likely to be associated with short trip lengths or with cold transient operation. The mutual interdependence of cold start/hot start ratio, average speeds and trip length is another case in point. If added complexity of representation serves no other purpose than to extend the domain of functional representation to such unlikely or impossible combinations, then it is clearly not justified. In the event that two or more variables are so interrelated that this interrelation can be expressed mathematically, then it may be possible to combine several variables into a single, "combined" variable having the same effect on emissions as the original variables within their allowable ranges of variation under the constraints.

Another recommendation, therefore, naturally arises:

- Mathematical representation of correction factors should be approached with due regard to the magnitude and engineering importance of variables perceived to be important. The prospect of combined or derived variables should not be overlooked nor should the fact that variables originally identified may be highly covariant.

Finally, a word is in order regarding choice of units. The issue can be illustrated by currently used expressions to define emissions as a function of average speed. For example, NO_x emission correction factors are related to average speed through a fourth-degree polynomial:

$$CF = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4$$

Since speed s is in miles per hour, s^2 has units of $(mph)^2$, s^3 has units of $(mph)^3$ and s^4 has units of $(mph)^4$. In reality however, each term in the sum must be dimensionless. Therefore, a_1 must units which are the reciprocal of mph, a_2 must have units of $(mph)^{-2}$, and so on up to a_4, which must have units of $(mph)^{-4}$. The result is that when $s = 60$ mph, $s^4 = 12{,}960{,}000$ $mi.^4/hr^4$. To compensate for such a numerically large quantity in the fourth-degree term, the magnitude of a_4 must be very small. Under such conditions computational precision becomes critical and it is difficult to appreciate the actual importance of the higher powers of speed. It would be preferable to express speed in nondimensional form before developing the regression relation. Such nondimensionalization can be achieved by expressing speed as a dimensionless multiple of some nominal or "standard" speed, such as the 19.6 mph average speed of the FTP. In the case of exponential expressions such a transformation facilitates a Taylor-series expansion which may be capable of capturing "most" of the effect of the variable in a linear or quadratic expression.

Consideration of units and dimensional homogeneity therefore suggests that:

- Difficulties arising from numerical constraints imposed by choice of units can often be avoided by nondimensionalization of independent or predictor variables as well as the dependent variable.

In conclusion, examination of the R-factor and its basis of derivation suggests the desirability of an experimental program specifically aimed at developing a data base for the formulation of improved correction factors.

APPENDIX I

SOME CONSIDERATIONS PERTINENT TO SPEED CORRECTION FACTORS

The speed correction factors for CO and HC, as developed in Mobile Source Emission Factors, Final Document (January 1978) and "Supplement 8 Light Duty Vehicle Correction Factors"* take the functional form

$$F = e^{A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 + A_5 s^5} \qquad \text{(I-1)}$$

Differentiating (I-1) with respect to s one obtains

$$\dot{F} = \frac{dF}{ds} = (A_1 + 2A_2 s + 3A_3 s^2 + 4A_4 s^3 + 5A_5 s^4) F \qquad \text{(I-2)}$$

Thus the fractional change in correction factor per mph is

$$\frac{\dot{F}}{F} = A_1 + 2A_2 s + 3A_3 s^2 + 4A_4 s^3 + 5A_5 s^4 \qquad \text{(I-3)}$$

Similarly, for NO_x the correction factor relation takes the form

$$F = A_0 + A_1 s + A_2 s^2 + A_3 s^3 + A_4 s^4 \qquad \text{(I-4)}$$

---

* Memo Janet Becker to Jack Hidinger/Joel Horowitz (March 7, 1977)

and

$$\dot{F} = \frac{dF}{ds} = A_1 + 2A_2\ s + 3A_3\ s^2 + 4A_4\ s^3 \qquad (I-5)$$

or

$$\frac{\dot{F}}{F} = \frac{A_1 + 2A_2\ s + 3A_3\ s^2 + 4A_4\ s^3}{A_0 + A_1\ s + A_2\ s^2 + A_3\ s^3 + A_4\ s^4} \qquad (I-6)$$

An appreciation of these relations can be obtained by comparing correction factors computed at 1-mph increments in Table II.1e of Becker (op. cit.). In Table I-1 below these incremental changes in correction factors are tabulated at 5 mph to 6 mph, 10 mph to 11 mph, ..., 60 mph to 61 mph for HC, CO and NO_x for Group 2 vehicles. Values are expressed both as incremental changes in correction factors and as fractions of the correction factors prevailing at 5, 10, ..., 60 mph respectively. It is noted that although the correction factors for CO and HC range from 3.3 to less than 0.5, most of the sensitivity to speed is in the range below the average speed of 19.6 mph prevailing in the FTP. This fact could possibly be of value in attempts to simplify the correction factor vs speed relations.

The shape of the correction factor curves for CO and HC suggest a hyperbola of the form xy = constant in cartesian coordinates. Quite clearly it would be fortuitous if such a simple function applied. Note, however, that dimensionally this notion has an interesting aspect. Recalling that the correction factors are simply multiples of grams/mile emissions at standard conditions (19.6 mph), one has

$$\text{gms/mi} \cdot \text{mi/hr.} = \text{gms/hr}$$

or emissions per unit time rather than emissions per unit distance traveled. It is quite possible that this change of basis, though not reducing to a constant, could give a much simpler relation and one having a certain amount of engineering credibility.

To examine this suggestion, consider Table I-2. In this table the correction factors for Group 2 hydrocarbons are tabulated at 5 mph increments together with the product of correction factor and speed. In the last column there is

Table I-1

DIFFERENTIAL ANALYSIS OF CORRECTION FACTORS

Group 2

SPEED (mph)	5-6	10-11	15-16	20-21	25-26	30-31	35-36	40-41	45-46	50-51	55-56	60-61
HYDROCARBONS												
	3.297	1.749	1.224	.986	.844	.740	.659	.600	.565	.547	.530	.482
	2.816	1.601	1.163	.953	.821	.722	.645	.591	.560	.544	.525	.464
Delta	-.481	-.148	-.061	-.033	-.023	-.018	-.014	-.009	-.005	-.003	-.005	-.018
Δ/Initial	-.146	-.085	-.049	-.033	-.027	-.024	-.021	-.015	-.009	-.005	-.009	-.037
CARBON MONOXIDE												
	3.319	1.751	1.225	.986	.841	.734	.650	.592	.557	.539	.518	.454
	2.829	1.602	1.164	.952	.817	.715	.637	.583	.552	.536	.510	.433
Delta	-.490	-.149	-.061	-.034	-.024	-.017	-.013	-.009	-.005	-.003	-.008	-.021
Δ/Initial	-.148	-.085	-.050	-.034	-.029	-.023	-.020	-.015	-.009	-.006	-.015	-.046
NITROGEN OXIDES												
	1.242	1.031	.974	1.004	1.074	1.146	1.203	1.239	1.265	1.306	1.404	1.615
	1.184	1.010	.975	1.017	1.089	1.159	1.211	1.244	1.271	1.320	1.436	1.677
Delta	-.058	-.021	.001	.013	.015	.013	.008	.005	.006	.014	.032	.062
Δ/Initial	-.047	-.020	.001	.013	.014	.011	.007	.004	.005	.011	.023	.038

TABLE I-2

GROUP 2 CORRECTION FACTOR ANALYSIS

HYDROCARBONS

SPEED (mph) s	CORRECTION FACTOR F	PRODUCT s F	SUCCESSIVE RATIOS
5	3.297	16.485	
10	1.749	17.490	1.061
15	1.224	18.360	1.050
20	.986	19.720	1.074
25	.844	21.100	1.070
30	.740	22.200	1.052
35	.659	23.065	1.034
40	.600	24.000	1.041
45	.565	25.425	1.059
50	.547	27.350	1.076
55	.530	29.150	1.066
60	.482	28.920	0.992

TABLE I-3

GROUP 2 CORRECTION FACTOR ANALYSIS

NITROGEN OXIDES

SPEED (mph) s	CORRECTION FACTOR F	PRODUCT s F	SUCCESSIVE RATIOS
5	1.242	6.210	
10	1.031	10.310	1.660
15	.974	14.610	1.417
20	1.004	20.080	1.374
25	1.074	26.850	1.337
30	1.146	34.380	1.415
35	1.203	42.105	1.225
40	1.239	49.560	1.177
45	1.265	59.925	1.209
50	1.306	65.300	1.090
55	1.404	77.220	1.182
60	1.615	96.900	1.255

tabulated the ratio of successive products. The fact that these ratios are relatively constant suggests that an exponential relation applies:

$$s\ F = e^{a + b\ s}$$

or

$$F = 1/s\ e^{a + b\ s}$$

where a and b are constants. Further,

$$\log_e (s\ F) = a + b\ s$$

and the product $s\ F$ should plot as a linear function of speed on semilog coordinates. The validity of this hypothesis is shown in Figure I-1. It is further evident that CO would exhibit similar behavior because the correction factors for CO and HC are closely related.

Let us now examine the multiplicative relation for NO_x, as shown in Table I-3. Though a simple exponential relation is not evident, the product $s\ F$ is a monotonic increasing function of speed, and when plotted on semilog coordinates appears capable of being represented by perhaps a quadratic function of s (see Figure I-2).

In conclusion, it appears that it may be advantageous to represent the effect of speed on emissions in terms of grams per unit time rather than in terms of grams per unit distance.

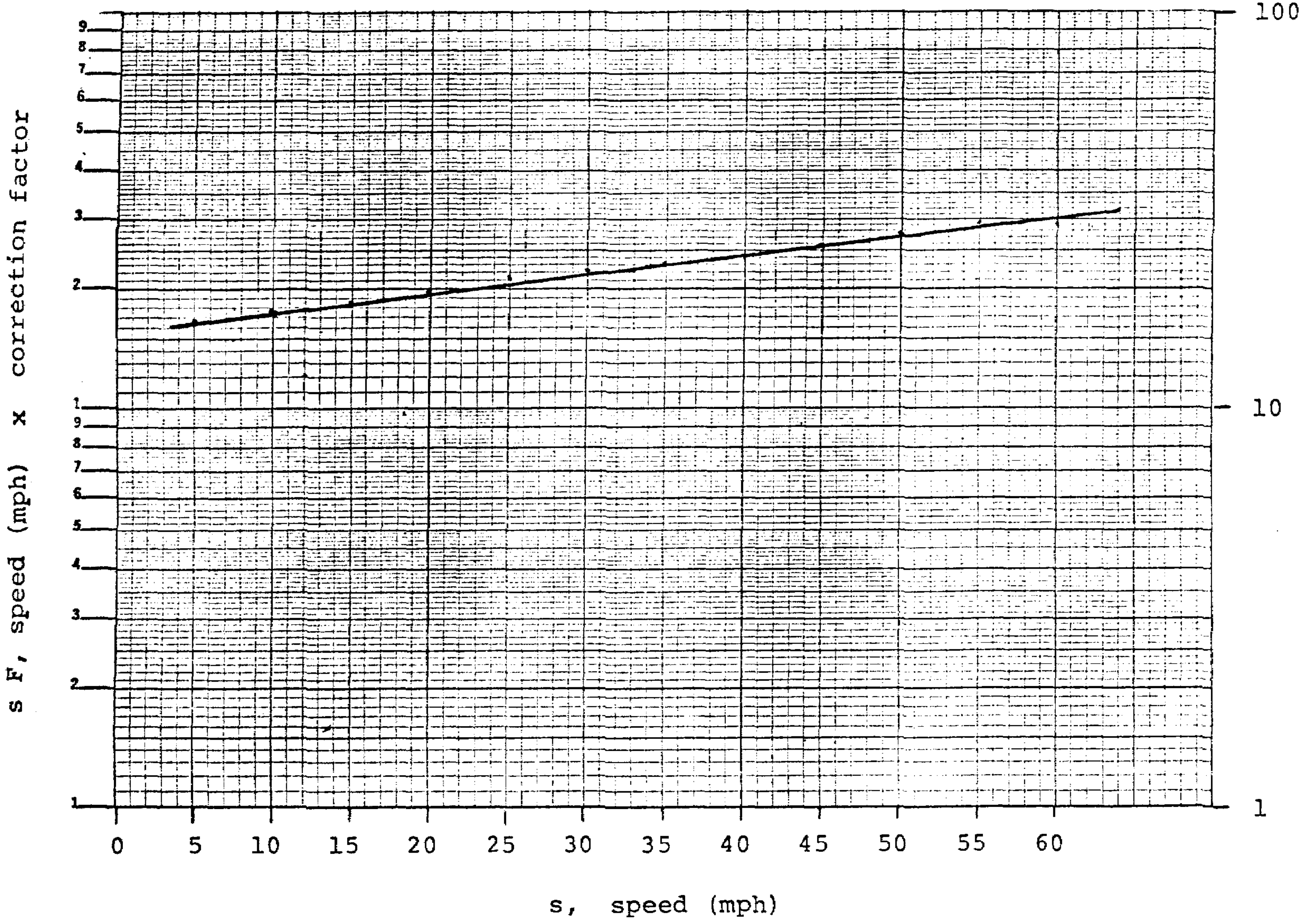

FIGURE I-1 TRANSFORMED PLOT OF CORRECTION FACTOR AS A FUNCTION OF SPEED FOR GROUP 2 HYDROCARBONS

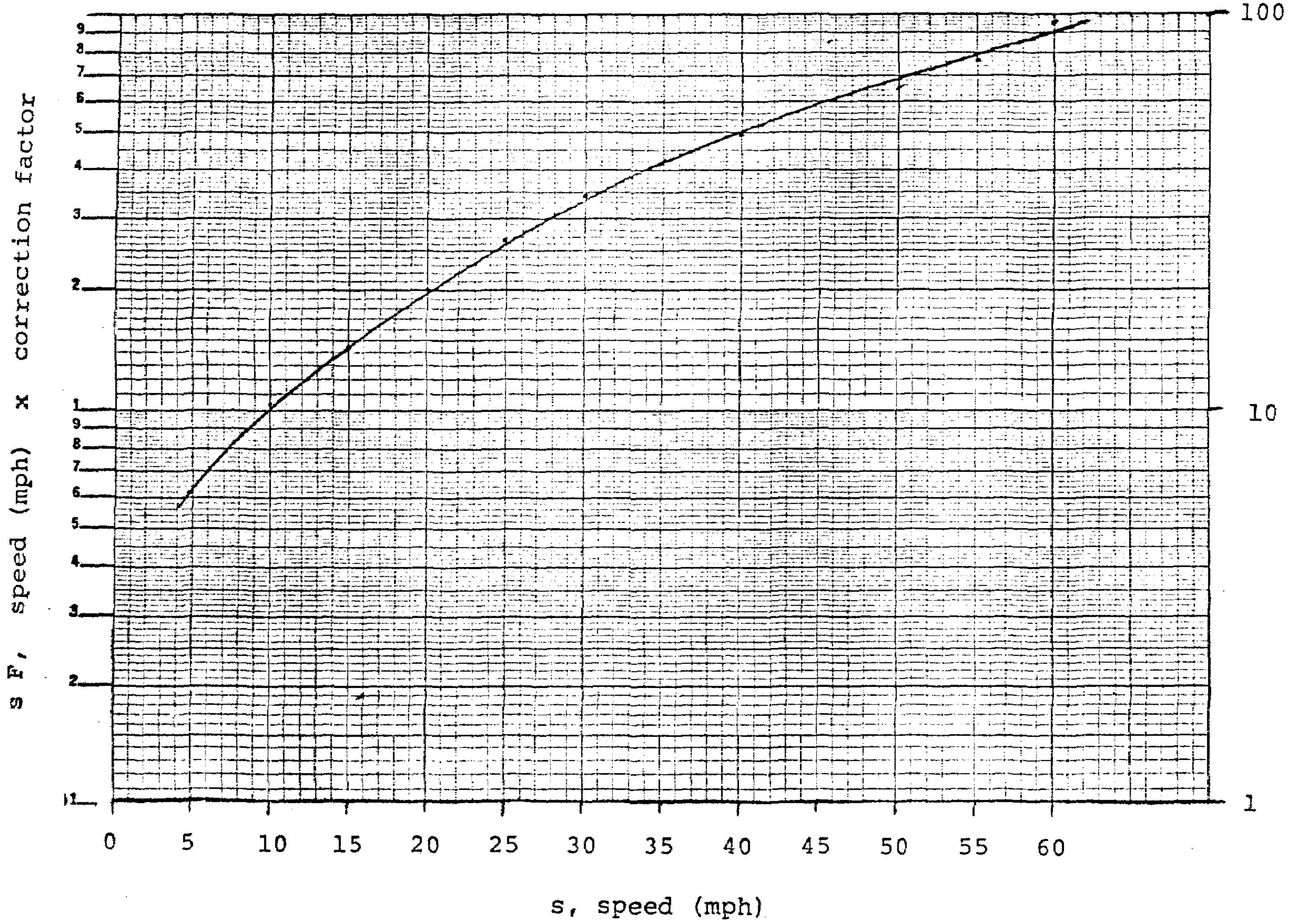

FIGURE I-2 TRANSFORMED PLOT OF CORRECTION FACTOR AS A FUNCTION OF SPEED FOR GROUP 2 OXIDES OF NITROGEN

4.3 WORKING PAPER NO. 3:

HOT/COLD/STABILIZED VEHICLE OPERATION: A CRITIQUE AND CANDIDATE APPROACH

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HOT/COLD/STABILIZED VEHICLE OPERATION: A CRITIQUE AND CANDIDATE APPROACH

1. INTRODUCTION

The concepts of cold-start, stabilized and hot-start operation of automotive vehicles represents an attempt to address the effects of "state of warm-up" on emissions and fuel economy and were developed in recognition of the importance of correcting for these effects. Though one is tempted to refer to "vehicle temperature," it is recognized that to do so would be simplistic, because (1) temperature takes on different meanings according to where in the vehicle it is measured, and (2) the mechanism by which such temperatures translate into emissions is complicated and not well understood.

Presently used correction factors based on fraction of miles driven in the cold-start, stabilized and hot-start modes represent a compromise with reality. The conditions under which a vehicle operates vary continuously rather than discretely, and it is safe to say that it is seldom, if ever, that a vehicle can be said to be ideally in the cold-start, stabilized or hot-start state as defined in the FTP. For example, few indeed must be the times when a vehicle is restarted exactly 10 minutes after completing a trip which is the exact equivalent of the FTP driving cycle. Consequently, some protocol must be adopted by which operation can be classified into one of the three states on the basis that a given set of circumstances is sufficiently "like" one of the states to justify its inclusion in that state. As will be suggested later, however, there may be an advantage in abandoning such a compartmentalized approach in favor of a methodology which views heating and cooling of a vehicle as a continuous process and modifies emission rates accordingly. In the continuum, warm-up is considered as a matter of degree rather than as a "go" or "no-go" affair.

2. PRESENT PROCEDURES: A REVIEW

The present method for dealing with hot and cold starts is to view them as processes which are switched on or off according to cold soak time (that is, how long the vehicle has been standing unused) and length of time the vehicle has been running since start-up. It is recognized, of course, that the nature and severity of the driving cycle can modify the effects of soak times and run times on emissions, as can also ambient conditions.

According to an EPA "Emission Factor User Information Sheet" (see Appendix I), the break between hot-start and cold-start operation can be defined in terms of a threshold for engine-off time. For example, in the case of a catalyst-equipped vehicle, the threshold is based on the maximum engine-off time "that can occur without causing the catalyst to cool down sufficiently" so that upon engine restart the catalyst is still operational. Any such threshold time will, as noted above, be affected by ambient temperature. The definition given in the cited document is:

> "Following an engine-off period, vehicle operation is said to be hot transient (hot start) if the engine-off time is less than 30 minutes and the temperature is 75° F or greater. If the temperature is 20° F or less, the allowable engine-off time drops to 10 minutes. Interpolation can be used between the two temperature levels."

In the previous EPA emission factor document, AP-42, Supplement 5, cold operation was defined as 505 seconds of operation following a 4 hour engine-off period for noncatalyst vehicles and a 1 hour engine-off period for catalyst vehicles.

Even though engine-off time can be used, albeit arbitrarily, to differentiate between hot transient and cold transient operation, it is clear that the ensuing transient period "remembers" the past operating history of the vehicle. It seems reasonable to believe that a vehicle restarted after only a 2-hour soak would exhibit a different transient response than one not restarted until after a 12-hour soak. Thus a complete model of transient phenomena should take into account both heating and cooling cycles.

In the "Emission Factor User Information Sheet" of Appendix I, it is proposed that the time to reach stabilized emissions can be defined by an equation of the type

$$t = 2.51\ s^{0.36} \qquad (1)$$

where t is the time, in minutes, required to reach stabilized emissions, and s is the soak time in hours. Evaluation of this formula for various soak times is shown in Table 1.

TABLE 1

Relation Between Stabilization and Soak Times
$t = 2.51\ s^{0.36}$, Temperature = 75° F

s soak time (hrs)	t stabilization time (min.)
0	0
1	2.51
2	3.22
4	4.14
16	6.81
32	9.09
64	11.22

Now it is clear that, according to the table, the time required for stabilization after a 16-hour soak is only 6.81 minutes whereas the transient portion of the FTP is based on 505 seconds (8.4 minutes) of operation.

The equation can be adjusted so as to constrain the time to a value of 8.4 minutes at a soak time of 16 hours. The revised equation is

$$t = 3.11 \; s^{0.36} \qquad (2)$$

and is evaluated for various soak times in Table 2.

TABLE 2

Constrained Relation Between Stabilization and Soak Times
$t = 3.11 \; s^{0.36}$, Temperature = 75° F

s soak time (hrs)	t stabilization time (min.)
0	0
1	3.11
2	3.99
4	5.12
16	8.44
32	11.26
64	13.90

An interpretation of Table 2 is that it gives the times required to bring the vehicle to a state of operation comparable to that which occurs at the end of the Bag 1 sequence in a standard FTP test (following the prescribed 16-hour soak). Inasmuch as the vehicle stabilized, according to Equation (1), in less than the allotted 8.4 minutes, however, it might be said to have spent 8.4 - 6.8 = 1.6 minutes in stable operation. This time increment, expressed as a percent of the <u>actual</u> warm-up time of 6.8 minutes, is

$$\frac{1.6}{6.8} \times 100\% = 24\%$$

Note that by expressing the ratio in terms of the actual times, as predicted by Equation (1), one is able to apply the same argument to soak times other than the standard 16-hour period. For example, when $s = 4$ hours, one has

$$\frac{5.12 - 4.14}{4.14} \times 100\% = \frac{0.98}{4.14} \times 108\% = 24\%$$

Thus the shift is a proportional one and is shown graphically in Figure 1.

A similar approach applied to tests conducted at 20° F suggested (see Appendix I) that a constant difference of 1.27 minutes between 20° F ambient and 75° F ambient applies regardless of engine-off time. Thus the time to reach stabilized emissions becomes

$$t = 2.51\, s^{0.36} + 1.27 \qquad (3)$$

or, upon renormalization,

$$t = 2.61\, s^{0.36} + 1.32 \qquad (4)$$

and it is proposed that linear interpolation/extrapolation be used between 0° F and 100° F.

Several comments regarding the suggested approach can now be made. First, the notion of a power law to represent the relation between stabilization times and soak times does not seem compatible with physical reality. According to Equations (1) - (4), the time required for stabilization continues to increase as a monotonic function of soak time, rather than approaching some limiting value. Second, units seem difficult to reconcile. Of course it can always be argued that the equations are only for descriptive purposes and are not to be applied outside a specified range of soak times. On the other hand, their use is not in keeping with criteria of the Purchase Order, one of which is "ability to relate to engineering concepts."

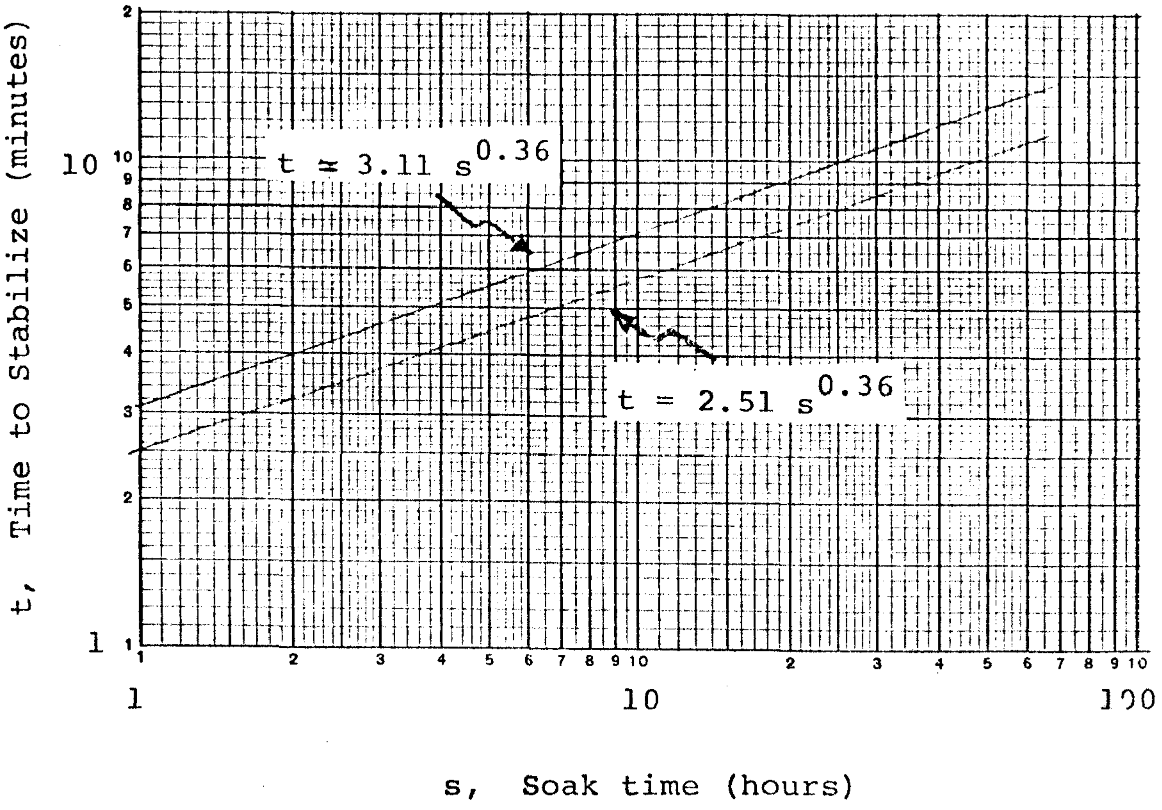

FIGURE 1

Time to Stabilize as a Function of Soak time

Perhaps the most serious drawback of an approach based on definition of a specific time required for stabilization is the difficulty in selecting a criterion for defining when vehicle operation is "warm or stabilized." Temperature levels, such as catalyst temperature, oil temperature, water temperature or air-intake temperature can be considered as bases, but these do not necessarily track each other over time and are not readily translated into effects on emissions. If emission levels are taken as indicators, it may well be that each pollutant has its own "stabilization time" so that it would not be possible to specify a unique time required to reach a stabilized state.

3. AN ACCRETION-DEPLETION VIEW OF THERMAL TRANSIENT EFFECTS

Insight pertinent to the development of a predictive equation for engine warm-up is afforded by an examination of the physical processes involved. An internal combustion engine is both a heat source and a heat reservoir. Much of the heat generated is converted to mechanical energy to drive the vehicle, but a certain amount goes to raise the temperature of elements of the engine itself, and excess heat is dissipated by the cooling system. When the engine is turned off, the heat stored in the engine and associated elements such as the catalytic converter is lost to the surrounding atmosphere. It is conjectured that heat is transfered <u>mostly</u> through a slow process of radiation. Thus it might be expected that heat is lost by the engine much more slowly than it is gained, and that the effect of the ambient temperature would be to increase or decrease the temperature differential between engine and atmosphere and hence influence the rate of heat loss accordingly. Also for any <u>given</u> ambient temperature, it is reasonable to expect that as the engine cools and approaches ambient, the rate of heat dissipation decreases so that the cooling cycle might be expected to be essentially an exponential process. By similar reasoning it might be conjectured that heat accretion is also exponential, but with a much shorter time constant caused by the much larger temperature differential driving the process.

In short, it appears that the accretion and depletion of heat might be analogous to the charging and discharging of an electrical condenser in a simple resistance-capacitance circuit, as developed in Appendix II. Because some of the processes involved (such as the action of the choke and the activation of the analytic converter) are discontinuous or only quasi-continuous, the simple exponential process postulated may be subject to step-function perturbations which may need to be modeled by non-linear circuit elements. In any event, however, such a view has the conceptual advantage that the "heat budget" of the vehicle, or its effects on emissions, can be visualized as a continuous function of time.

A schematic view of how the accretion and depletion of heat might be manifested is shown in Figure 2. At what point in the vehicle this temperature is measured is not particularly germane to the argument, but it is assumed to be related in some way to emission performance. When the vehicle is first started, designated by the notation "ON" at Time = 0, the temperature rises, presumably exponentially, as the vehicle accumulates heat from the combustion process. Because of the cooling system, however, a maximum operating temperature is approached as time continues. As the difference is narrowed between the limiting temperature and the temperature at time t, the rate of temperature rise tends to become smaller. When the vehicle is stopped, designated by the notation "OFF" at Time = a, the temperature falls, again presumably exponentially, as temperature is lost to the surround. Again, as the difference is narrowed between ambient temperature and vehicle temperature, the rate of cooling tends to decrease with time.

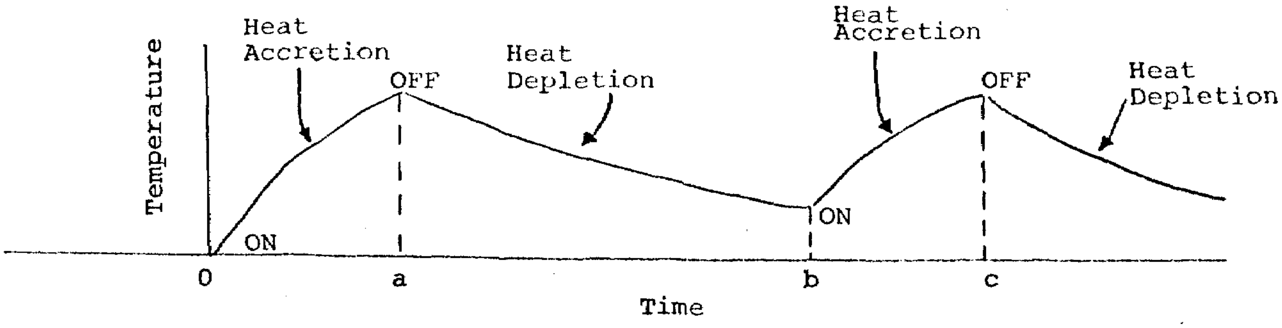

FIGURE 2

Figure 2 does not suggest any mechanism by which vehicle temperature is translated into emissions. Let us suppose, however, that there exists a critical, minimum temperature level requisite to "stable operation"--for example, the operating temperature of the catalytic converter. As shown in Figure 3, this critical level would be reached sooner after a relatively short "OFF" time than after a quasi-infinite time (compare t_1 and t_2), as was assumed to be the case at Time = 0. Because of the mechanism by which temperature is translated into

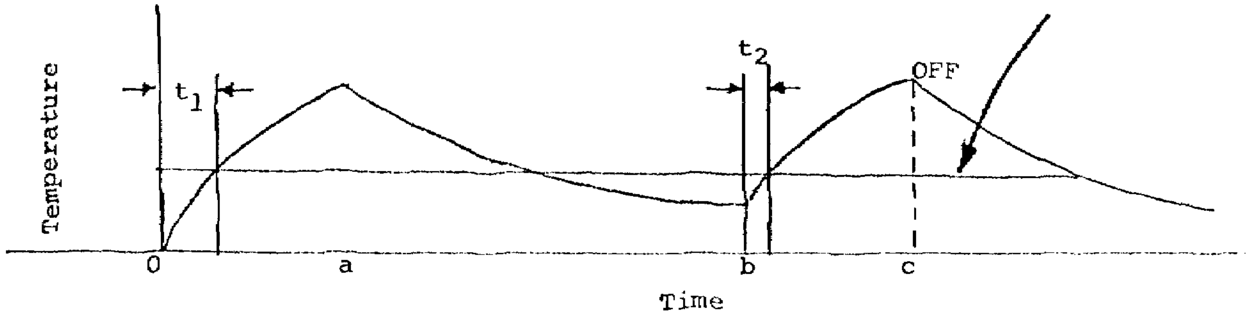

FIGURE 3

emissions, emission rates as functions of time are not necessarily exponential though driven by an exponential thermal process. On the other hand, an exponential decay of the choke effect has been postulated as part of a computer simulation of emissions and fuel economy.(1)

In the event that emissions are directly expressible as an exponential function of time, the accretion-depletion model could be implemented by determining the time constants of the heating and cooling cycles. An attempt to evaluate the required time constants was undertaken, using data from an EPA-funded program on vehicle soak and run times and their effects on emissions(2) (see Appendix III). For reasons cited above, the attempt had only limited success, but it is suggestive for further, less simplistic analysis. The implications of the approach are also exploited in the discussion of the following section, in which transient emission rates are regarded as linear combinations of two limiting emission rates associated, respectively, with cold transient and hot transient operation.

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(1) W. K. Juneja, W. J. Kelly and R. W. Valentine, "Computer Simulations of Emissions and Fuel Economy," SAE Paper No. 780287, Society of Automotive Engineers (1978)

(2) R. L. Srubar, "Emission and Fuel Economy Sensitivity to Changes in Light Duty Vehicle Test Procedures," Final Report of Task No. 10, EPA Contract 68-03-2196, Southwest Research Institute (May 1977)

4. A CONTINUUM APPROACH TO TRANSIENT OPERATION

The 1975 Federal Test Procedure employs three types of driving: a cold transient phase (representing vehicle start-up after a long engine-off period); a hot transient phase (representing vehicle start-up after a short engine-off period); and a stabilized phase (representing warmed-up vehicle operation). Emissions measured during these three phases are combined as a weighted sum, the weighting factors being 0.20, 0.27, and 0.53, respectively. It is presumed that the FTP is run at a standard temperature of 75° F, but a range from 68° to 86° is allowed.

The basis for quoting FTP emission results suggest that the measure is a sort of "hybrid" quantity partaking of the properties of both cold-start and hot-start operation. By an extension of this argument it is not a great step to propose that a vehicle restarted after some period of soak time can similarly be characterized as a "hybrid" in the sense that it represents neither a cold-start nor a hot-start situation, but some weighted combination of the two. Viewed in this way, adjustment of emissions for various combinations of soak and run times can be achieved by the device of variable weighting factors rather than by the device of predicting warm-up times.

An example of how this approach might be used is shown in Table 3 below. The data are taken from R. L. Srubar (op. cit.) for a 1976 Chevrolet Impala. Emission tests were run after

TABLE 3

Soak Time	HC (gms/mi) First 505 sec. of FTP	P_{HC} Percent Cold Start
10 min	0.39*	0.0
20 min	0.48	11.2
30 min	0.53	17.5
1 hr	0.69	37.5
2 hr	0.64	31.2
4 hr	0.85	57.5
8 hr	1.03	80.0
16 hr	1.19*	100.0
36 hr	1.27	110.0

* Reference values (see text)

various soak times as indicated. The starred values are of especial significance. The results shown for a 10-minute soak are essentially Bag 3 results, that is, emissions representing hot transient conditions. The results shown for a 16-hour soak are essentially Bag 1 results, that is, emissions representing cold transient conditions.

A procedure is now proposed whereby results for soak times other than 10 minutes and 16 hours can be interpreted as linear combinations of hot transient and cold transient contributions. For example, consider the emissions for the 1-hour soak as consisting of a fraction P of cold-start emissions and a fraction $1-P$ of hot-start emissions. Then

$$P(1.19 \text{ gms/mi}) + (1-P)(0.39 \text{ gms/mi}) = 0.69 \text{ gms/mi}$$

Solving this equation gives $P = 0.375$. Thus the 1-hour test acts "as if" it were a composite of 37.5% cold transient and 62.5% hot transient operation. The weightings should not be construed as fractions of either the total time to execute the test (8.4 minutes) or the total distance covered (3.59 miles). Rather, their only purpose is to generate, from available Bag 1 and Bag 3 emissions, a quantity which is equivalent to the results observed for any given soak time. Note that this approach would have the advantage that the required computation could be performed in terms of emission results readily available as components of the standard FTP test, provided that the fraction P is known as a function of soak time and run time.

Tables 3 through 7 provide further analysis for the five vehicles studied by Srubar (op. cit.). Results are given for HC and CO emissions and for fuel economy. No attempt was made to analyze the results for NO_x, since that pollutant does not seem to be very sensitive to the cold-start phenomenon.

Two anomalies may be noted in these tables. One is that negative values of P sometimes occur. The other is that emissions after a 36-hour soak may be greater than emissions after a 16-hour soak. This fact implies values of P greater than 1.0 and negative values of $1-P$.

TABLE 4

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Chevrolet Impala Over the First 505 Seconds of the FTP

(Data from Table A-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel Economy (F.E.) (mi/gal)	$P_{F.E.}$
10 min	0.53	0	7.13	0	13.47	0
20 min	0.61	6.45	6.08	-4.3	13.20	12.05
30 min	0.88	28.20	6.87	-1.1	13.08	17.41
1 hr	1.09	45.20	7.95	3.3	12.61	38.39
2 hr	1.38	68.5	10.10	12.0	12.14	59.38
4 hr	1.34	65.3	14.18	28.5	11.88	70.98
8 hr	1.27	59.7	16.52	38.0	11.28	97.77
16 hr	1.77	100.0	31.86	100.0	11.23	100.00
36 hr	1.56	83.0	28.70	87.2	11.06	107.6

TABLE 5

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1977 Ford LTD Over the First 505 Seconds of the FTP

(Data from Table B-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel Economy (F.E.) (mi/gal)	$P_{F.E.}$
10 min	0.39	0.0	3.91	0.0	15.98	0.0
20 min	0.72	37.50	2.51	-5.49	15.10	21.10
30 min	0.93	61.36	3.67	-0.94	15.52	11.03
1 hr	1.03	72.73	3.38	-2.08	15.29	16.54
2 hr	1.08	78.41	3.56	-1.37	14.84	27.34
4 hr	0.78	44.32	3.96	0.20	14.15	43.88
8 hr	1.23	95.45	12.48	33.63	12.40	85.85
16 hr	1.27	100.00	29.39	100.00	11.81	100.00
36 hr	2.06	189.80	37.63	132.33	12.46	84.41

TABLE 6

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Plymouth Fury Over the First 505 Seconds of the FTP

(Data from Table C-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel Economy (F.E.) (mi/gal)	$P_{F.E.}$
10 min	0.52	0.0	1.71	0.0	16.87	0.0
20 min	0.70	13.7	1.88	0.8	15.82	26.05
30 min	0.72	15.3	2.70	4.4	15.43	35.73
1 hr	0.87	26.7	3.18	6.5	15.42	35.98
2 hr	1.40	67.2	12.91	49.7	14.67	54.59
4 hr	1.42	68.7	16.95	67.6	13.88	74.19
8 hr	1.64	85.5	20.84	92.4	13.18	91.56
16 hr	1.83	100.0	24.24	100.0	12.84	100.00
36 hr	1.89	104.6	28.37	118.3	12.18	116.40

TABLE 7

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Chevrolet Vega Over the First 505 Seconds of the FTP

(Data from Table D-2, Srubar, *op. cit.*)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel Economy (F.E.) (mi/gal)	$P_{F.E.}$
10 min	0.39	0.00	1.01	0.00	22.71	0.00
20 min	0.48	11.25	2.09	8.05	22.84	-3.05
30 min	0.53	17.50	1.89	6.56	23.20	-11.50
1 hr	0.69	37.50	2.24	9.16	22.19	12.21
2 hr	0.64	31.25	3.68	19.89	20.97	40.84
4 hr	0.85	57.50	5.36	32.41	19.70	70.66
8 hr	1.03	80.00	14.63	101.49	19.41	77.46
16 hr	1.19	100.00	14.43	100.00	18.45	100.00
36 hr	1.27	110.00	19.90	140.80	18.45	100.00

TABLE 8

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Honda Civic CVCC Over the First 505 Seconds of the FTP

(Data from Table E-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel Economy (F.E.) (mi/gal)	$P_{F.E.}$
10 min	0.85	0.00	4.83	0.00	31.53	0.00
20 min	0.97	8.39	4.39	-14.33	31.08	9.49
30 min	1.00	10.49	4.17	-21.50	31.58	-1.05
1 hr	0.80	-3.50	4.38	-14.66	30.75	16.45
2 hr	1.00	10.49	4.89	1.95	28.72	59.28
4 hr	1.24	27.27	6.32	48.53	27.84	77.85
8 hr	2.03	82.52	7.82	97.39	25.03	137.13
16 hr	2.28	100.00	7.90	100.00	26.79	100.00
36 hr	2.65	125.90	9.48	151.46	24.13	156.10

These anomalies, however, are not considered serious nor obstructive to the formulation of a weighting-factor approach. Part of the difficulty, it is believed, arises from errors of emission measurement and the manner in which these errors propagate in the computation of P. In general,

$$P = \frac{E_t - E_{HS}}{E_{CS} - E_{HS}}$$

where E_t = emissions after soak time t, E_{HS} = hot start emissions, and E_{CS} = cold start emissions. Thus errors in the determination of E_{HS} and E_{CS} could induce appreciable error in P. In general application of the method, however, it is likely that P would be determined on an aggregated basis (i.e., for groups of vehicles) and errors would tend to be averaged out. It is also possible, of course, that soak times somewhat greater than 10 minutes could produce emissions slightly lower than those associated with the classical hot transient by virtue of evaporative losses and other minor effects of relatively short duration. It is believed, however, that such phenomena would have such a small effect that they could be ignored. Finally, the tendency for the 36-hour soak to exhibit higher emissions than the 16-hour soak suggests that a reference other than the 16-hour soak might be appropriate. On the other hand, such long soak times would rarely occur in practice and for that reason could likewise be ignored.

Implementation of the variable-weighting approach requires further development in order to be practical. For example, it would be necessary to explore the question of vehicle-to-vehicle commonality of P, expressed as a function of soak and run times, and to develop the necessary functional representations. Moreover, it would be necessary to develop a strategy by which a local or area survey of vehicle use patterns could be converted into either a distribution of P values or into some form of appropriate average P value. Finally, it might well be that cold-start and hot-start transient emissions do not represent the best choice of limiting emissions on which to base the continuum approach; perhaps cold-start and stabilized emission levels might be better (see Appendix 4). At any rate it is suggested that two emission levels, representing respectively the most favorable and least favorable operating conditions could form the basis for a weighted combination reflecting different scenarios of soak and run times. The proposed approach is believed to offer a viable alternative to the determination of time to stabilize and is considered worthy of further evaluation.

5. SUMMARY AND PROSPECTUS

One of the most important scenario variables affecting emissions is the "thermal operating history" of the vehicle. The term in quotation marks, admittedly a coined one, is meant to signify the state of warm-up of the vehicle as a function of time. It suggests that the vehicle can, at certain times in its operating history, be in a fully warmed up condition but at other times can fall short of this condition to varying degrees.

The present approach to the thermal history problem is to postulate three "states" of operation: cold transient, stabilized, hot transient. Because these three states do not represent the whole spectrum of degrees of "warmed-upness," it becomes necessary to implement a methodology by which an arbitrary driving scenario can be decomposed into the three discrete states. An important aspect of this methodology is to develop a criterion for determining the length of time required in order to declare a vehicle "stabilized."

Equations previously developed by EPA for this purpose are of such a nature that they predict stabilization times which increase without limit as soak times increase. It seems more reasonable to believe, however, that there exists an upper limit for stabilization time and that this limit would be approached asymtotically with increasing length of soak time. The lower bound for stabilization time is, of course, zero and occurs at zero soak time. Thus stabilization times have "floor" and "ceiling values," though it is understood that the range between these two values can vary with ambient conditions and with vehicle characteristics and the nature of the emission-control system.

A hypothesis advanced in this working paper is that emission rates, expressed in terms of grams/mile, also have "floor" and "ceiling" values. The floor value, mathematically speaking, must be zero, but in practice the emission rates will tend to be of some finite value even when the vehicle is operating under the most ideal conditions. As a practical assumption, these "most ideal conditions" can be taken as "stabilized operation." The ceiling value is not so clearly limited, but it would have to be of finite, albeit very large, magnitude even if all the fuel burned were converted to pollutant. In practice, one can visualize a "worst possible" condition which is seldom if ever exceeded. For the purposes of the arguments in this paper it is not particularly damaging that the proposed ceiling is occasionally exceeded if these exceedances are very rare or tend to occur under use scenarios (e.g., very long soak times) which practically never occur.

It is now proposed that the determination of realistic floor and ceiling emission rates may suffice to define emissions over the entire spectrum of thermal operating histories, once the envelope of histories has been "calibrated" for various combinations of soak and run times. Note that this approach eschews completely the notion of "warm-up time" or "time to stabilization." Under the assumption that stabilization is approached asymtotically, time to stabilization can be defined only arbitrarily. Moreover, if a means is provided for computing emissions for all scenarios of interest, the question of stabilization time is irrelevant.

Data required to implement the proposed approach are acquired readily with little, if any, modification of present testing practice. If cold-start emissions (Bag 1) are taken as the ceiling and stabilized emissions (Bag 2) are taken as the floor, the spectrum of all values between these limits can be represented as weighted sums of the two. The presently used hot-transient phase (Bag 3) could provide a check point for such weighting and could be augmented by one or more additional check points representing soak times intermediate between 10 minutes and 16 hours. In this way the standard method of reporting emissions according to the FTP would not need to be disturbed, but the information provided by the FTP could be augmented and used in a more effective way. One difficulty, however, does remain. Since cold-start (Bag 1) emissions and stabilized emissions (Bag 2) are measured over different driving cycles, it might be preferable to normalize the two results to the same average speed. The necessary calibration for this purpose could be provided by a series of tests in which the first 505 seconds of the FTP is followed by a repetition of the first 505 seconds without any vehicle off time. In this way the Bag 2 results could be referenced to a "Bag 1 result obtained under stabilized conditions."

It is true, of course, that to implement the methodology for purposes of regional air-quality assessment one must have available a distribution of soak and run times. It is believed, however, that the required data could be obtained by sample survey methods and that average or "effective" soak and run times characteristic of the scenario can be derived. These effective times would have to be defined in such a way that they reflect the impact of the joint distribution of soak and run times, rather than as a simple average, but statistical approaches to such problems are well known.

In summary, an alternative to the three-bag method of adjusting emissions for cold-start/stabilized/hot-start fractions is proposed. It is believed that the method could be implemented with only minor modification of present test procedures and that it has the advantage of circumventing the need to determine effective warm-up times for various use scenarios. Further evaluation of the approach is recommended. An essential part of this evaluation would be the development of a data base to define how the weighting factors vary as a function of soak and run times. As noted above, the required data base could be developed as an addendum to existing emission-testing programs, such as certification and in-use surveillance.

APPENDIX I

EMISSION FACTOR USER INFORMATION SHEET: HOT/COLD/ STABILIZED OPERATION

Emission Factor User Information Sheet:
Hot/Cold/Stabilized Operation

Problem Identification:

The recent Revised Emission Factors Document contains a single correction factor for hot/cold weighting, average speed, and ambient temperature. In the previous EPA emission factor document, AP-42, Supp 5, cold operation was defined as 505 seconds of operation following a 4 hour engine-off period for non-catalyst vehicles and a 1 hour engine-off period for catalyst vehicles. The recent emission factor document did not provide a definition of cold or hot transient operation. This information guide will provide users with the appropriate definitions and some methodologies which can be used to collect the necessary input data.

Definitional Constraints:

A correction factor for vehicle temperature would ideally relate emissions to a series of variables including time since vehicle start-up, time vehicle was turned-off prior to start-up, severity of cycle over which vehicle has been driven since start-up, vehicle identifying information and ambient conditions. This relationship would be a predictive regression relationship and would be normalized to equal one if the vehicle were completely warmed-up. This relationship would then be applied on a second by second basis to the EPA modal emission model. Since vehicle temperature changes on a second by second basis, it would not be absolutely correct to apply a single correction factor to an entire cycle.

The data are not available to develop a functional relationship of the type just described; the development of such a relationship would require extensive amounts of second by second emission data as a function of all of the variables of interest. An equally important limitation is that Federal Test Procedure emission data collected in the large studies of in-use vehicles have divided the data into three distinct operational categories. A single emission value is available for the first 505 seconds of operation following a 16 hour engine off period. A second emission value is available for an 870 second period of stabilized (warmed-up) operation. Finally, a third emission value is available for 505 seconds of operation following a ten minute engine off period. These three pieces of data can be used to develop average correction factors for cold start, stabilized, and hot start operation. In each case, the correction factor is an average of the effect of vehicle temperature over a fairly long time period during which, vehicle temperature and vehicle emissions are changing.

The cold start correction factor presented in AP-42 will underestimate the emissions during the first minute after a 16 hour soak and overestimate the emissions during the seventh minute after a 16 hour soak. Therefore, the correct application of the factor requires a knowledge of the number of vehicles which are operating within a 505 second period after a 16 hour soak. It is assumed that the distribution of vehicles is equally distributed throughout the 505 second period. For example, if 16 percent of the vehicles are operating in the first 505 seconds since start-up, it is assumed that 2 percent are operating in the first minute, 2 percent in the second minute, ... and 2 percent in the eighth minute.

From an emission standpoint, cold operation can occur when a vehicle soaks (engine off condition) for less than 16 hours. Limited data are available to determine the length of time it takes before emissions stabilize as a function of engine-off time and time since engine start-up. If emissions averaged over start-up periods which are less than 505 seconds following a less than 16 hour engine off period can be shown to be equivalent to emissions averaged over 505 seconds following a 16 hour engine off period, then all such equivalent operation should be defined as cold start operation and the AP-42 correction factor should be applicable. Again, the assumption of equal vehicle distribution throughout the time period is required.

The differentiation between cold start operation and hot start operation is strictly dependant upon the length of the engine off period. A hot start condition attempts to simulate a case where the length of engine off time is sufficiently short so that vehicle engine temperatures/ emission control systems do not cool down significantly; for example,a hot start situation would not activate the vehicle choke. Emissions are increased during a hot start condition due to the dumping of excess fuel which is stored in the carburetor, evaporative canisters, etc. Thus, emissions following a hot start quickly return to their normal stabilized levels. Again, as in the case of the cold start, the AP-42 correction factor averages these emissions over 505 seconds and assumes that vehicles are equally distributed throughout the time period.

Thus, the break between hot start and cold start operation is dependant upon engine-off time. It is that engine-off time that differentiates between a vehicle where the engine/emission control system is still in a warmed-up condition and a vehicle where the engine/emission control system has cooled down. In the case of a catalyst vehicle, it is the engine-off time that can occur without causing the catalyst to cool down sufficiently so that during a hot start the catalyst is still operational. EPA does not have data to define this point. Clearly, the time period could be expected to be a function of ambient temperature. At this time, the recommended definition is given below.

Following an engine-off period, vehicle operation is said to be hot transient (hot start) if the engine-off time is less than 30 minutes and the temperature is 75° F or greater. if the temperature is 20° F or less, the allowable engine-off time drops to 10 minutes. Interpolation can be used between the two temperature levels.

Once engine-off time is used to differentiate between hot transient and cold transient operation, a definition is needed for transient operation as a function of engine-off time and ambient temperature.

Several studies have been performed by EPA to address the transient operation definition. These studies are.

1. Bureau of Mines, "Ambient Temperature and Vehicle Emissions", EPA 460/3/74-028, December 1974; 26 vehicles, 4 different ambient temperatures, one soak condition, emission readings at 2, 5.5, 8.4, 15.6, and 22.9 minutes.

2. In house work on five vehichles, January, 1977 (unpublished); 2 different ambient conditions, five different soak conditions, emission readings at 2, 5.5, 7.1, 8.4, 12.8, 17.1, and 22.9 minutes.

3. Ongoing contract work on five vehicles; one ambient condition, nine different soak conditions, emission readings at 1, 2, 3, 4, 5, 6, 7, and 8 minutes.

At the present time, detailed analyses have not been performed. However, the necessary stages of detailed analysis can be outlined. First, it is necessary to define when operation is "warm or stabilized". Two criteria are possible for this assessment; emission levels or temperature levels. Temperature levels can be catalyst temperature, oil temperature, water temperature, or intake air temperature. While emission levels have been recorded in discrete bag samples, temperature levels are normally recorded continuously. Given the difference in measurement/recording techniques, there may be less variability in using a temperature definition. However, since the bottom line item is emissions, extra variability is introduced with the undefined link between temperature and emissions. Future analyses will pursue both approaches.

The first stage of analysis will be to develop a curve of time to reach stabilized emissions as a function of engine-off time for ambient temperatures in the range of the FTP. Data sources 2 and 3 can be used for this assessment. If possible, separate curves should be developed for pre-1975 models, 1975 and later catalyst equipped models, and 1975 and later non-catalyst modes.

The seond stage of analysis is to factor in ambient temperature effects. Data sources 1 and 2 can be applied and two approaches can be used. The approach used with data source 2 would be a straightforward graphical application of the data. Data source 1 has the largest amount of information regarding ambient temperature effects. However, all data were collected at one soak time, an overnight soak taken to be 16 hours. By defining the 16 hour point and making the assumption that temperature and soak time effects are independent, a parallel set of curves can be drawn for a range of ambient temperatures.

Finally, the definition of transient operation must be related to the definition used in the development of emission correction factors. In the most recent AP-42 work, cold and hot transient emissions were defined as emissions over the first 505 seconds where the first 505 seconds contain some stabilized operation. In order to appropriately apply the AP-42 factors, the warm-up time/engine-off time curves need to be shifted. The cold correction factors assume a sixteen hour soak period and 505 seconds (8.4 minutes) of transient operation while the hot transient correction factors assume a 10 minute engine-off period and 505 seconds of transient operation. Thus, the curves should be shifted so that the 16 hour and 10 minute soak periods respectively are equivalent to 8.4 minutes of operation. The shift is a percentage shift. That is, if the time for the 16 hour soak period is shifted up by 25%, the time for the 2 hour soak period is also increased by 25%. This approach allows the same percentage mix of cold/stabilized or hot/stabilized operational time in the estimate of the cold or hot transient correction factors.

Definitions:

The data sources referenced above were analyzed on a preliminary basis. At 75° F, the time to reach stabilized emission operations can be defined as

$$t = 2.51\ s^{.36}\ ,\ r = .86$$

where t is the time to reach stabilized emissions (in minutes) and s is the engine-off time (in hours).

To get the AP-42 definition for cold operation, the equation must be adjusted so that a 16 hour engine-off period results in 505 seconds of cold operation. This is accomplished by including 24% stabilized operation in the definition. Thus,

A vehicle is operating in a cold transient condition if:
1) The ambient temperature is 75°F
2) The engine-off period is 30 minutes or greater
3) The vehicle has been operating for t minutes or less where $t = 3.11\ s^{.36}$ and s is the engine-off time in hours.

To get the AP-42 definition of hot operation, the equation must be adjusted so that a 10 minute engine-off period results in 505 seconds of hot operation. This is accomplished by including 538% stabilized operation in the definition. Thus,

A vehicle is operating in a hot transient condition if:
1) The ambient temperature is 75°F
2) The engine-off period is 30 minutes or less
3) The vehicle has been operating for t minutes or less where $t = 16.01\ s^{.36}$ and s is the engine-off time in hours.

Based on very limited data, it appears that a fixed difference in time to reach stabilized emissions exists between t at 75°F and t at 20°F, regardless of engine-off time. The fixed difference was obtained from two catalyst vehicles in data source 2; the fixed difference is 1.27 minutes. Thus, at 20°F, the time to reach stabilized emissions is defined as

$$t = 2.51\ s^{.36} + 1.27$$

where t and s are given earlier. Using the same normalization schemes discussed above, the following definitions hold

A vehicle is operating in a cold transient condition if:
1) The ambient temperature is 20°F
2) The engine-off period is 10 minutes or longer
3) The vehicle has been operating for t minutes or less where $t = 2.61\ s^{.36} + 1.32$.

At temperatures different from 20°F or 75°F, linear interpolation/extrapolation can be used between 0°F and 100°F by computing a t/°F rate and then normalizing. The t/°F rate is .023 minutes/°F. Thus, at 30°F, the time constant is 1.04 rather than 1.27, the normalizing multiplier is 1.07, and the final equation would be $t = 2.69\ s^{.36} + 1.11$.

A vehicle is operating in a hot transient condition if:
1) The ambient temperature is 20°F
2) The engine-off period is 10 minutes or less
3) The vehicle has been operating for t minutes or less where $t = 8.15\ s^{.36} + 4.12$

At temperatures different from 20°F or 75°F, linear interpolation/extrapolation can be used between 0°F and 100°F by computing a t/°F rate and then normalizing. The t/°F rate is .023 minutes/°F. Thus, at 30°F, the time constant is 1.04 rather than 1.27, the normalizing multiplier is 3.56, and the final equation would be $t = 8.94\ s^{.36} + 3.70$.

Sources of Data:

Various methods are available to obtain data on the percentage of vehicles which are operating in cold transient, hot transient, or stabilized emission scenarios. These are discussed below.

1. Origin - Destination Studies - On a regional basis, these data banks can be analyzed to determine the number of trips which begin after various engine-off times. By adding information on average trip length and average trip speed, the percentage of miles of each type of operation can be readily determined.

   On a local basis, O-D studies can be used to determine the distribution of vehicles at a given location with respect to operating time and engine-off time prior to start-up. Average link speed may have to be added to the data base to perform the analysis. This methodology was recently used by GCA under contract to EPA. The report is titled Characterization of Cold Mode Operation and is available from Mr. Jim Wilson, EPA, Research Triangle Park, NC 27711.

2. Survey data - Clearly, only two pieces of information are needed in order to determine whether a vehicle is in a transient or a stabilized emission condition. A quick roadside survey can be designed to ask motorists how long ago they started their engine (time and/or miles) and how long the engine was off prior to start-up. In many cases, a rough estimate is entirely adequate. Depending upon the design of the survey, localized or regional percentage values can be determined.

3. Direct Measurement Data - Direct measurement techniques exist which can determine whether a vehicle is in a stabilized emission configuration. These techniques can be difficult to implement due to the need to perform extensive calibration. Some measurement methods require the vehicles to stop for several minutes while other techniques require only a 30 second stop or no stop. Broadly, the measurement techniques can be divided into two types; those which use a thermocouple measurement technique and those which use an infrared measurement technique. Temperature measurements from a number of areas of

the vehicle can be fairly well correlated with emissions in a gross sense (that is, emissions stabilized or emissions not stabilized). These areas include: inlet to vehicle radiator, oil pan, catalyst skin, exhaust gas, and difference between vehicle hood and vehicle trunk. The EPA is in the process of implementing this type of study. Results and documentation of the methodology should be available by July, 1978.

APPENDIX II

A CAPACITIVE MODEL OF SOAK AND RUN TIME EFFECTS ON VEHICLE EMISSIONS AND FUEL ECONOMY

Experimental data collected on vehicles during varying periods of run and soak (vehicle on and off) indicate that temperature, as well as emissions and fuel economy, may respond in time as voltage on a capacitor responds in time to periods of charge and discharge. The accumulation of electric charge in the capacitor is analogous to the accumulation of heat in the vehicle when it is in the "run" condition. The discharge of the capacitor is analogous to the cooling phase when the vehicle is in the "soak" condition.

Consider an electric circuit represented by Figure II-1 below:

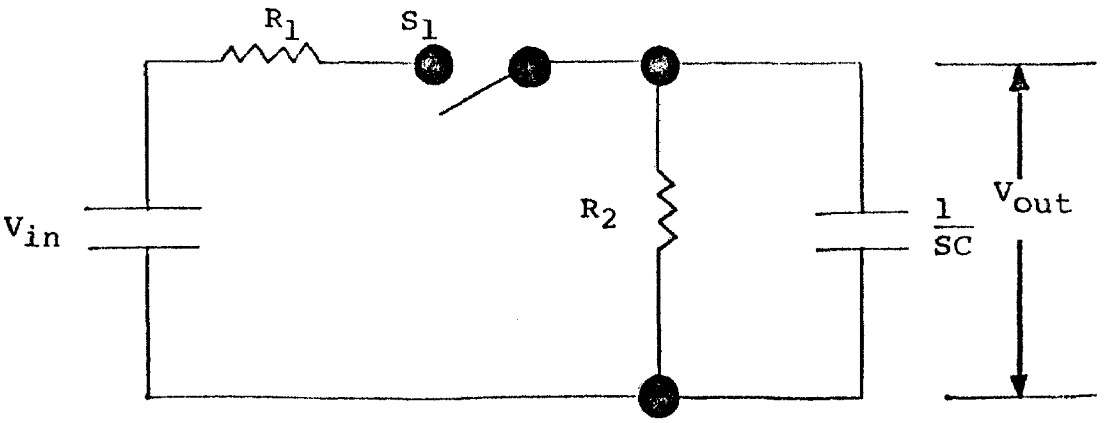

FIGURE II-1

In this figure, $R_2 >> R_1$ and most of the current passes through R_1 when switch S_1 is closed. This is equivalent to the "on" cycle when the car is started. The rate at which the capacitor is charged (or the car heats up) is a function of the time constant,

$$\left(\frac{R_1R_2}{R_1 + R_2}\right) \quad C.$$

R_2 represents a condition in the circuit allowing some leakage from the capacitor during charging, and is analogous to the cooling effect of the ambient conditions which act in opposition to the accumulation of heat when the vehicle is started. For hot ambient, R_2 is much larger than R_1 and the time constant is approximately R_1C. For cold ambient, R_2 is of smaller magnitude than for hot ambient and can provide an appreciable leakage for the capacitor. In short, a longer time would be required for the capacitor to charge, just as a longer time would be required for the vehicle to heat up. When switch S_1 is opened, the battery and R_1 are eliminated from the circuit and the capacitor is discharged through the resistance R_2 (the engine is cooled by heat transfer to ambient with a time constant R_2C).

When charging the capacitor the voltage time response is

$$V_{out}(t) = \left(1 - e^{-t/RC}\right) \quad \text{where} \quad R \quad \text{is} \quad \frac{R_1R_2}{R_1 + R_2}$$

However, when discharging the capacitor, the voltage response is

$$V_{out}(t) = -V_0\left(e^{-t/R_2C}\right) \quad \text{where}$$

V_0 is the voltage attained during charging for a time of $t = a$. Thus

$$V_0 = \left(1 - e^{-a/RC}\right) \quad \text{and}$$

$$V_{out}(t) = -\left(1 - e^{-a/RC}\right)\left(e^{-t/R_2C}\right) \quad \text{for} \quad t > a$$

If a is "long enough" to approximately charge the capacitor, then the magnitude of $V_{out}(t)$ is approximately

$$e^{-t/R_2C}$$

By means of Laplace transform methods, $V_{out}(S)$ may be described in terms of $V_{in}(S)$ in the frequency domain (S-space) by the relation:

$$\frac{V_{out}(S)}{V_{in}(S)} = \frac{1}{1 + SCR_1 + R_1/R_2}$$

$$\approx \frac{1}{1 + SCR_1} \quad \text{for} \quad R_2 >> R_1 \quad \text{for charging}$$

and

$$\frac{V_{out}(S)}{V_{in}(S)} = \frac{1}{1 + SCR_2} = \frac{1}{1 + SCR_2} \quad \text{for discharging.}$$

The charging-discharging sequence can represent one cycle each part of which can have a different charging and discharging time constant. Several continuous cycles, each of different duration may be as demonstrated in Figure II-2 below, where the input is represented by a step function of unity (engine on) or zero (engine off).

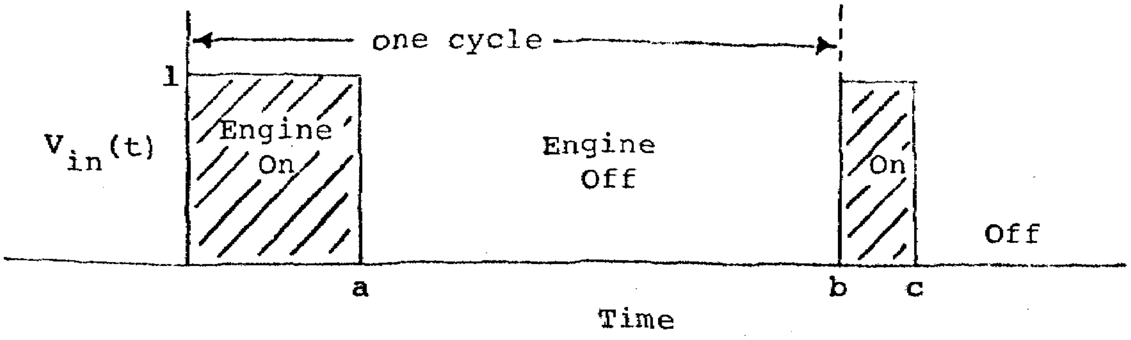

FIGURE II-2

The output may be represented by Figure II-3 which follows:

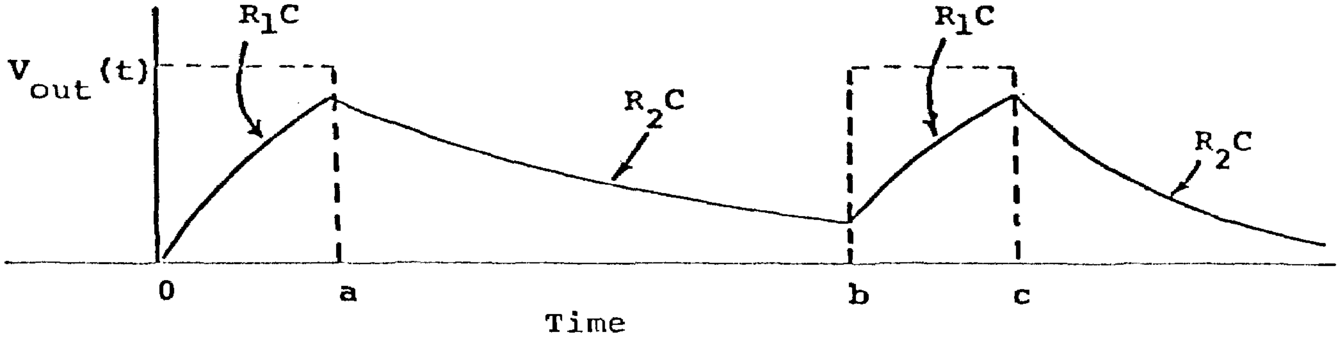

FIGURE II-3

The transform of $\frac{V_{out}(S)}{V_{in}(S)}$ is represented by $L^{-1}\left[\frac{V_{out}(S)}{V_{in}(S)}\right]$

and

$$V_{out}(t) = \begin{cases} \left(1 - e^{-t/RC}\right) & 0 < t < a \\ -\left(1 - e^{-a/RC}\right) e^{-t/R_2C} & a < t < b \\ \left[\left(1 - e^{-a/RC}\right) e^{-b/R_2C}\right]\left(1 - e^{-t/RC}\right) & b < t < c \\ \left[\left(1 - e^{-a/RC}\right) e^{-b/R_2C}\left(1 - e^{-C/RC}\right)\right] & \\ \quad \text{times} \quad e^{-t/R_2C} & \text{for } t > c \end{cases}$$

This treatment of a capacitive model in frequency space is easily generalized to include inputs which may be characterized by functions other then step functions.

APPENDIX III

TIME-CONSTANT CONSIDERATIONS FOR A CAPACITIVE MODEL OF EMISSIONS AND FUEL ECONOMY

For the capacitive model to be applicable to emission and fuel economy of automobiles, it is necessary to be able to evaluate the time constants corresponding to the heating and cooling cycles of a vehicle. These cycles correspond to vehicle on (run time) and vehicle off (soak time), respectively.

Data relevant to time-constant evaluation are afforded by R. L. Srubar in a report on soak-time and run-time effects.* Five vehicles were tested for emissions and fuel economy after various soak times. The only way in which the effects of run time were monitored, however, was by bagging emissions for various periods of "time into the driving cycle." Though the bagged results do, indeed, reflect warm-up effects, these effects are confounded with changes in the driving cycle from one bag to another. Accordingly, an estimation of time constants must be by an indirect process, but this fact does not preclude the possibility of designing an experiment specifically for the purpose of time constant estimation.

Consider the data tabulated in Table III-1 for a 1977 Ford LTD. Each of the bags corresponds to a 63-second period of operation, but the distances covered, average speeds and driving sequence severity are different for the various bags. If it is assumed that the 10-minute soak represents essentially stabilized conditions (actually it probably exhibits a short transient), then one can compute for each bag a "correction factor" reflecting only the effects of bag differences in driving sequence.

The average fuel consumption for the 505-second run is 15.35 ℓ/100 km. By dividing the fuel consumption in the first bag by this number one obtains 24.99/15.34 = 1.63. Thus one can assume that the first 63 seconds of operation is 1.63

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* R. L. Srubar, Emission and Fuel Economy Sensitivity to Changes in Light Duty Vehicle Test Procedures, Final Report of Task No. 10, Contract 68-03-2196, Southwest Research Institute (May 1977).

TABLE III-1

FUEL CONSUMPTION FOR A 1977 FORD LTD AS A FUNCTION OF RUN TIME

Bag No.	Time (sec.)	Distance km.	Fuel Consumption 10-minute Soak* (A)	(ℓ/100 km) 16-hour Soak** (B)	Ratio B/A
1	0-63	0.35	24.99 (1.63)***	59.22	2.37
2	64-126	0.72	11.90 (0.78)	16.37	1.38
3	127-189	0.24	32.31 (2.11)	37.63	1.16
4	190-252	1.37	15.39 (1.00)	19.22	1.25
5	253-315	1.40	9.20 (0.60)	11.14	1.21
6	316-378	0.55	20.00 (1.30)	19.87	0.99
7	379-441	0.42	19.02 (1.24)	20.59	1.08
8	442-505	0.73	14.60 (0.95)	16.18	1.11
Combined		5.78	15.38		

* Average of two tests

** Average of three tests

*** Numbers in parentheses are fuel consumptions divided by overall fuel consumption of 15.34 ℓ/100 km times "as severe" as the composite 505 seconds of operation. One can now adjust the fuel consumption figures for the 16-hour soak by dividing each of the tabulated fuel consumption values by the corresponding values in parentheses. These results are the same as are obtained by dividing the 16-hour soak results by the 10-minute soak results on a bag-by-bag basis.

As shown in Figure III-1, the ratios fall off rapidly with increasing time into cycle and tend to approach as asymtote of unity. It appears that about 6 minutes of running time from a cold start is sufficient to reduce fuel consumption to within about 10% of the quasi-stabilized values represented by the hot start condition.

Whereas the duration of the warm-up cycle is of the order of minutes, the duration of the cool-down cycle is of the order of several hours. This effect can be seen in Table III-2, in which average fuel consumption rates are tabulated for the 1977 Ford LTD as determined for results bagged over the first 505 seconds of the FTP driving cycle.

TABLE III-2

FUEL CONSUMPTION FOR A 1977 FORD LTD
AS A FUNCTION OF SOAK TIME

Soak length	Fuel Consumption ℓ/100 km	Ratio
10 min	14.72	1.00
20 min	15.58	1.04
30 min	15.16	1.03
1 hr	15.39	1.05
2 hr	15.85	1.08
4 hr	16.62	1.13
8 hr	18.97	1.29
16 hr	19.92	1.35
36 hr	18.89	1.28

Taking the 10-minute soak time as reference, one sees that it takes approximately 2 hours of cooling to degrade fuel economy to the extent that about 10% more fuel is consumed over the 505-second cycle. This time is compared wihtout about 6 minutes in the warm-up cycle.

The "time constants" estimated by the above procedures can not be interpreted in the strict exponential sense because of the mechanisms involved in translating temperature into fuel economy (or emissions). In reality, fuel economy is a composite function

$$F.E. = g \left[f \left(t_{run}, t_{soak} \right) \right] .$$

Although f (t_{run}, t_{soak}) can be regarded as temperature and although temperature may vary exponentially as the run and soak times t_{run} and t_{soak}, fuel economy may take a somewhat different form from exponential. Similar comments can be made for HC, CO and NO_x emissions. Nevertheless, the prospect of a "heat budget" or time-history approach to emissions as a function of soak and run times is attractive from the standpoint of physical understanding of the process.

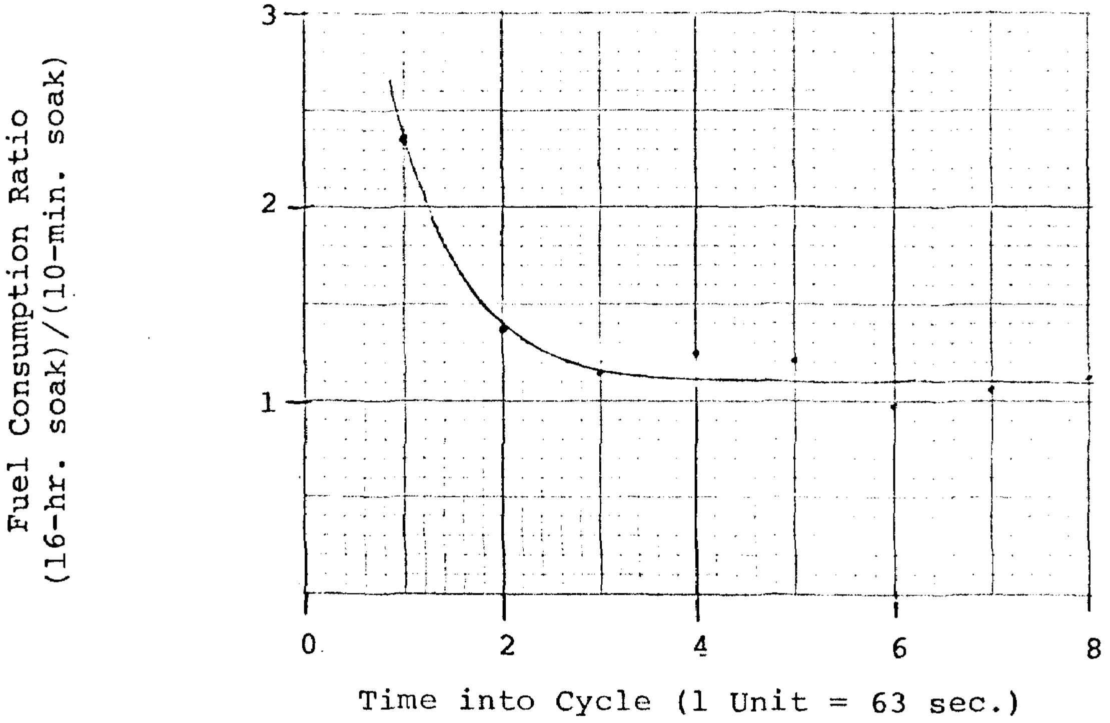

FIGURE III-1 FUEL CONSUMPTION FOR A 1977 FORD LTD AS A FUNCTION OF RUN TIME

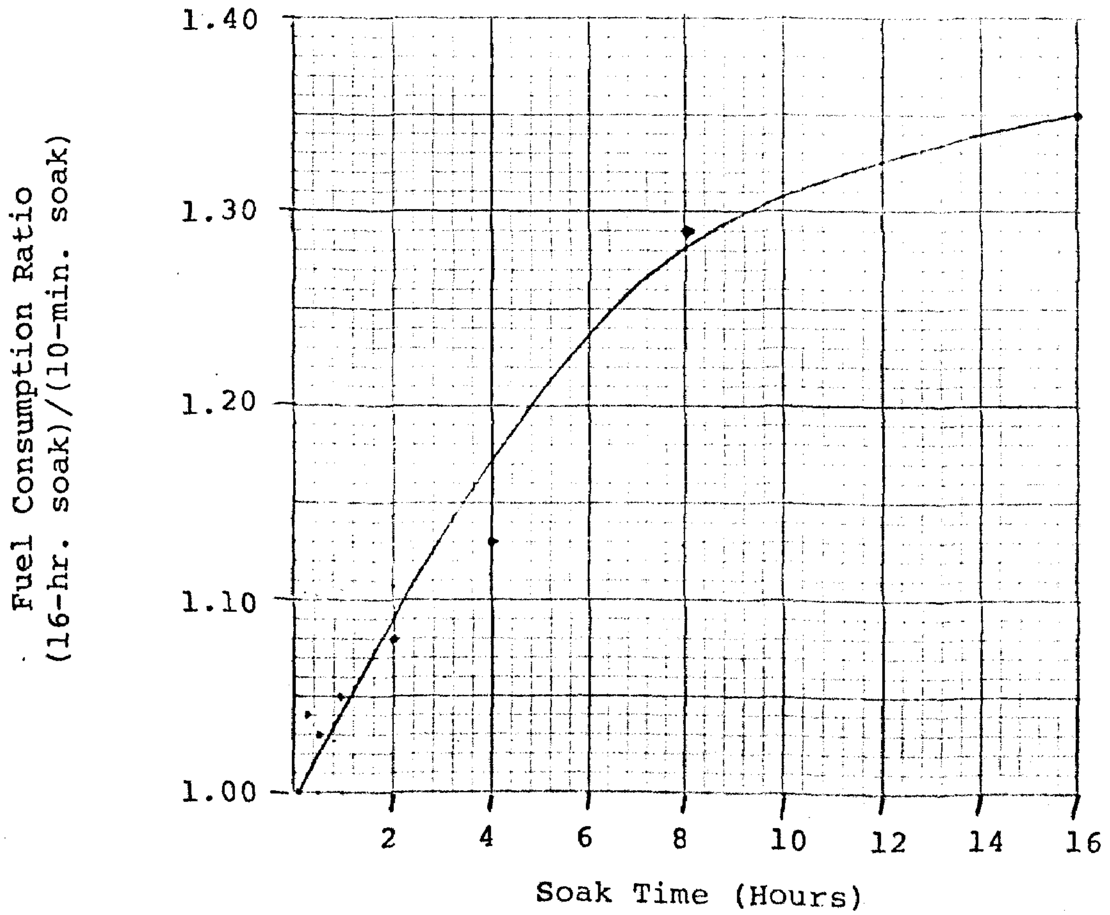

FIGURE III-2 FUEL CONSUMPTION FOR A 1977 FORD LTD AS A FUNCTION OF SOAK TIME

APPENDIX IV

WEIGHTING FACTORS FOR ADJUSTING EMISSIONS FOR VEHICLE THERMAL OPERATING HISTORY

An approach is proposed for adjusting emissions according to the thermal operating history of a vehicle, as expressed by its profile of soak and run times. The approach assumes a "floor" value representing the lowest level of emissions which might ever be expected and a "ceiling" value representing the highest level anticipated. It is then postulated that emission levels for all combinations of soak and run times can be expressed as weighted combinations of these two extreme values.

Reasonable definitions of the floor and ceiling values might be based on the hot stabilized (HST) and cold transient (CTR) values of the FTP respectively. Then one can assume that

$$E_t = P\ E_{CTR} + (1-P)\ E_{HST} \qquad (IV-1)$$

or

$$P = \frac{E_t - E_{HST}}{E_{CTR} - E_{HST}} \qquad (IV-2)$$

where E_t denotes emissions during the first 505 seconds of the FTP after a soak time t.

Values of P computed by equation (IV-2) are given in Tables IV-1 through IV-5. The data are taken from Srubar (<u>op</u>. <u>cit</u>.) and represent five vehicles which were subjected to tests after various soak times. One notes that the 10 minute soak times exhibit P values which are generally of the order of 10% to 20%. Since the 10-minute soak times represent conditions which are essentially hot transient, the results suggest that this condition is only slightly worse than stabilized operation but is "slightly contaminated" with cold-start behavior.

A word of caution is in order concerning literal interpretation of the results in the tables, however. The first 505 seconds of the FTP and the next 870 seconds represent different driving sequences, different average speeds, and hence different levels of severity as far as both emissions and fuel economy are concerned. It is for this reason that the fuel-economy calculations show a rather severe anomaly of negative P values for short soak times. Clearly an improved basis for the calculation of weighting factors would be obtained if either (1) driving cycles for the end-points were the same, or (2) adjustments were made for speed differences or differences in severity for the two driving sequences. In that sense the results presented here are only suggestive of further refinements of approach.

TABLE IV-1

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Chevrolet Impala Over the First 505 seconds of the FTP

(Data from Table A-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel (gal/mi)	$P_{F.E.}$
10 min	0.53	22.98	7.13	16.14	13.47	-128.57
20 min	0.61	27.95	6.08	12.58	13.20	-101.02
30 min	0.88	44.70	6.87	15.26	13.08	-88.77
1 hr	1.09	57.76	7.95	18.92	12.61	-40.82
2 hr	1.38	75.78	10.10	26.21	12.14	7.14
4 hr	1.34	73.29	14.18	40.04	11.88	33.67
8 hr	1.27	68.94	16.52	47.98	11.28	94.89
16 hr	1.77	100.00	31.86	100.00	11.23	100.00
36 hr	1.56	86.96	28.70	89.28	11.06	117.3
Hot Stabilized	0.16		2.37		12.21	

TABLE IV- 2

Percent Cold Start Operation as a Function of Soak Time

for HC, CO and Fuel Economy for a 1977 Ford LTD

Over the First 505 seconds of the FTP

(Data from Table B-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel (gal/mi)	$P_{F.E.}$
10 min	0.39	10.20	3.91	12.53	15.98	-66.89
20 min	0.72	43.88	2.51	7.72	15.10	-28.51
30 min	0.93	65.31	3.67	11.71	15.52	-44.92
1 hr	1.03	75.51	3.38	10.71	15.29	-35.94
2 hr	1.08	80.61	3.56	11.33	14.84	-18.36
4 hr	0.78	50.00	3.96	12.70	14.15	8.59
8 hr	1.23	95.92	12.48	41.95	12.40	76.95
16 hr	1.27	100.00	29.39	100.00	11.81	100.00
36 hr	2.06	180.60	37.63	128.30	12.46	74.61
Hot Stabilized	0.29		0.26		14.37	

TABLE IV- 3

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Plymouth Fury Over the First 505 seconds of the FTP

(Data from Table C-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel (gal/mi)	$P_{F.E.}$
10 min	0.52	21.56	1.71	5.53	16.87	-110.00
20 min	0.70	32.33	1.88	6.25	15.82	-56.02
30 min	0.72	33.53	2.70	9.68	15.43	-35.60
1 hr	0.87	42.51	3.18	11.70	15.42	-35.08
2 hr	1.40	74.25	12.91	52.49	14.67	4.19
4 hr	1.42	75.45	16.95	69.43	13.88	45.55
8 hr	1.64	88.62	20.84	85.57	13.18	82.20
16 hr	1.83	100.00	24.24	100.00	12.84	100.00
36 hr	1.89	103.60	28.37	117.30	12.18	134.50
Hot Stabilized	0.16		0.39		14.75	

TABLE IV- 4

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Chevrolet Vega Over the First 505 seconds of the FTP

(Data from Table D-2, Srubar, <u>op</u>. <u>cit</u>.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel (gal/mi)	$P_{F.E.}$
10 min	0.39	26.60	1.01	-0.90	22.71	-8.67
20 min	0.48	34.86	2.09	7.22	22.84	-11.99
30 min	0.53	39.45	1.89	5.71	23.20	-21.17
1 hr	0.69	54.13	2.24	8.34	22.19	4.59
2 hr	0.64	49.54	3.68	19.17	20.97	35.71
4 hr	0.85	68.81	5.36	31.80	19.70	68.11
8 hr	1.03	85.32	14.63	101.50	19.41	75.51
16 hr	1.19	100.00	14.43	100.00	18.45	100.00
36 hr	1.27	107.3	19.90	141.13	18.45	100.00
Hot Stabilized	0.10		1.13		22.37	

TABLE IV-5

Percent Cold Start Operation as a Function of Soak Time for HC, CO and Fuel Economy for a 1976 Honda Civic CVCC Over the First 505 seconds of the FTP

(Data from Table E-2, Srubar, op. cit.)

Soak Time	HC (gms/mi)	P_{HC} (%)	CO (gms/mi)	P_{CO} (%)	Fuel (gal/mi)	$P_{F.E.}$
10 min	0.85	16.86	4.83	3.15	31.53	-138.20
20 min	0.97	23.84	4.39	-10.72	31.08	-115.60
30 min	1.00	25.58	4.17	-17.66	31.58	-140.70
1 hr	0.80	13.95	4.38	-11.04	30.75	-99.00
2 hr	1.00	25.58	4.89	5.05	28.72	3.01
4 hr	1.24	39.53	6.32	50.16	27.84	47.24
8 hr	2.03	85.46	7.82	97.48	25.03	188.40
16 hr	2.28	100.00	7.90	100.00	26.79	100.00
36 hr	2.65	121.50	9.48	149.80	24.13	233.60
Hot Stabilized	0.56		4.73		28.78	