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# FUEL ECONOMY MEASUREMENT
# CARBON BALANCE METHOD

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## 1. Introduction

This paper gives the equations for determining fuel economy by the carbon balance method for gasoline, diesel fuel, alcohols and blends of the above. Derivations of the fuel economy equation constants and several sample calculations are given. Fuel economy calculation using compresses natural gas and method for stoichiometric A/F determinations are included in Appendix 2.

Comparisons of carbon balance and volumetric fuel economy measurement were made in an earlier study.[1/]

## 2. Summary

The carbon balance equation for determining fuel economy is:

$$F.E. = \frac{x}{y\ HC + 0.429\ CO + 0.273\ CO_2 + z\ TP}$$

Where HC, CO, $CO_2$, and TP are in grams per mile and x and y are given in the following table for some typical fuels. A more complete table, including A/F, is given in Appendix 1.

| Fuel | x | y |
|---|---|---|

| | | |
|---|---|---|
| Indolene | 2421 | .865 |
| Diesel #2 | 2778 | .865 |
| Methanol (MOH) | 1124 | .375 |
| Ethanol (ETOH) | 1557 | .521 |
| 90% IND/10% ETOH | 2335 | .829 |

TP is the particulate emission level in grams carbon per mile. It is neglible for computing fuel economy for gasoline fueled vehicles. For light duty diesel vehicles it is of the same order of magnitude as the HC contribution (which is very small). In this paper z is assumed to be 0.85 however available data ranges from .75 to .95. Note that the "official" fuel economy equation in the Federal Register does not include particulate emissions.

## 3. Discussion

Fuel economy by the carbon balance method is accurate when used under conditions where certain assumptions are valid. These assumptions are:

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1/ Evaporative and Exhaust Emissions of Two Automobiles Fueled With

Volatility adjusted Gasohol, David Lawrence, D. Niemczak,

EPA-AA-TEB-81-12

1 - All carbon in the exhaust comes from carbon in the fuel. Corrections for carbon in the exhaust from sources other than the fuel are made (such as background air corrections).

2 - The HC composition of the exhaust is the same as that of the fuel.

3 - Emissions of HC, CO, $CO_2$ and total particulate ($^{g}/mi$) are measured accurately. This includes proper accounting for interferences such as water vapor.

4 - The vehicle exhaust system does not have any leaks.

5 - The weight fraction carbon (WFc) and specific gravity (SG) are known. Ideally, they should be accurately determined for each batch of gasoline or diesel fuel.

6 - For vehicles with particulate traps or trap oxidizers the carbon trapped and emissions during purge are properly accounted for. The first four assumptions are valid for gasoline and diesel fueled vehicles tested in accordence with FR 40 CFR 86. The $WF_c$ and SG must be accurately determined. This is not difficult for pure fuels, such as the alcohols.

For gasoline and diesel fuels the SG is easily determined from the API gravity.[2/] SG for the alcohols is available from various handbooks.

$WF_c$ is not easily determined for gasoline and diesel fuels and can vary by several percent from batch to batch. For fuel economy comparisons between a base gasoline or diesel fuel and a blend of alcohol with that same base fuel the effect of the uncertainty of $WF_c$ of the base fuel cancels. Thus, fuel economy comparisons for fuel blends can be done accurately using the carbon balance method. and the same base fuel. However, batch to batch fuel economy comparisons of a single fuel type (e.g. Indolene HO) can induce an error on the order of up to 2% if the $WF_c$ and SG are assumed rather than measured. Such a situation exists under current fuel economy regulations (40 CFR part 600) where an assumed value of 2421 grams carbon per gallon of gasoline and 2778 grams carbon per gallon of diesel fuel are used.

Particulate emissions from diesel fueled vehicles are not included in the "official" EPA equation given in 40 CFR 600. However, at the current particulate standard of 0.6 $^{g}/mi$ the impact of excluding particulate emissions will cause overstatement of fuel economy, especially for higher fuel economy vehicles. For example, assuming that a diesel fueled vehicle particulate consisting of 85% carbon the impact of excluding the particulate emission will be to overstate fuel economy by:

| TP = 0.6 g/mi | TP = 0.2 g/mi | |
|---|---|---|
| 0.07 MPG | 0.02 MPG | (20 MPG vehicle) |
| 0.44 MPG | 0.14 MPG | (50 MPG vehicle) |
| 1.80 MPG | 0.61 MPG | (100 MPG vehicle) |

For vehicles with other particulate emission rates the impact of excluding these emissions from the carbon balance equation will change proportionately.

## 4. Calculations

Carbon balance fuel economy is given by:

$$\text{F.E.} = \frac{\text{grams carbon / gal fuel}}{\text{grams carbon in exhaust / mile}} = \text{MPG}$$

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2/ $$SG = \frac{141.5}{131.5 + \text{deg API}}$$

A. The numerator "N" of equation 2 is determined by:

N = Grams carbon/gal fuel

$N = 3785 \times SG \times WF_c$

Where: 3785 = density of water (grams per gallon)

SG = Specific gravity of fuel ($g_c/g_w$)

$WF_c$ = weight fraction of carbon in the fuel = $MW_c/MW_f$

$MW_c$ = molecular weight of carbon per fuel molecule

$MW_f$ = molecular weight of fuel.

Example 1: pure ethanol: C2H6O

SG = .789

$MW_c = 2 \times 12.011 = 24.022$

$MW_f = 2 \times 12.011 + 6 \times 1.008 + 16.0 = 46.070$

$WF_c = 24.02/46.07 = .5214$

$N = 3785 \times .789 \times .5214$

N = 1557 grams carbon/gal fuel

Example 2: Gasoline: $CH_{1.86}$ (typical value)

Note that the gasoline is reduced to the $C_1$ value to simplify calculations.

SG = .740

$MW_c = 1 \times 12.01 = 12.011$

$MW_f = 1 \times 12.011 + 1.86 \times 1.008 = 13.88$

$WF_c = 12.011/13.886 = .865$

$N = 3785 \times .740 \times .865$

$N = 2421$ grams carbon/gal fuel

Example 3: Diesel Fuel: $CH_{1.86}$ (Typical value)

$SG = 0.8475$

$MW_c = 1 \times 12.011 = 12.011$

$MW_f = 1 \times 12.011 + 1.86 \times 1.008 = 13.886$

$MF_c = 12.011/13.886 = .865$

$N = 3785 \times 0.8485 \times 0.865$

$N = 2778$ grams carbon/gal fuel

Example 4: a mixture of 10% ethanol and 90% gasoline:

Calculate components individually and then weight by volume fraction.

$N = .1 \times 1557 + .9 \times 2421$

$N = 2334$ grams carbon/gal fuel

Where: $1557 = g_c$/gal ETOH

$2421 = g_c$/gal Gasoline

B. The Denominator "D" of equation 2 is determined by:

$$D = WF_c \times HC\ {}^{g}/mi + .429 \times CO\ {}^{g}/mi + .273 \times CO_2\ {}^{g}/mi + 0.85 \times TP\ {}^{g}/mi$$

HC, CO, $CO_2$ $^{g}/mi$ are obtained from the emissions test.

TP is obtained from the emission test for diesel pueled vehicles and is assumed equal to zero for other vehicles with low particulate emission rates.

0.429 is the weight fraction of carbon in CO:

$$MW_c/MW_{co} = 12.011/(12.011 + 16.0) = 0.429$$

0.273 is the weight fraction of carbon in $CO_2$:

$$MW_c/MW_{co_2} = 12.011/(12.011 + 2 \times 16.0)$$

The weight fraction carbon of diesel particulate is assumed to be 0.85

$WF_c$ for single component fuels is determined as in the calculations for the numerator.

For fuel blends:

$$WF_c = \frac{(VF_i \; WF_i \; SG_i)}{(VF_i \; SG_i)}$$

Example 5 a mixture of 10% ethanol and 90% gasoline:

$$WF_c = \frac{.1 \times .5214 \times .789 + .9 \times .865 \times .739}{.1 \times .789 + .9 \times .739} = \frac{.6164}{.7440}$$

$WF_c$ = .829 grams carbon/gram fuel

$$D = .829\ HC + .429\ CO + .273\ CO_2$$

C. Carbon balance equation:

Combining above information (from ex 4 and 5) for a mixture of 10% ethanol and 90% gasoline in eq 1 gives:

$$FE,\ MPG = \frac{2334 \quad g_c/\text{gal fuel}}{(.829\ HC + .429\ CO + .273\ CO_2) \quad g_c/\text{mile}}$$

The carbon balance equation for one batch of "Anafuel" is shown to demonstrate the method for combining three fuel components (Appendix, Table 2).

APPENDIX 1

TABLE 1

Fuel Properties - Alcohols, Gasoline, Diesel Fuels

| (1) Fuel | (2) Formula | (3) MW | (4) SG | (5) $g_{fuel}$/GAL (3785 x SG)1/ | (6) $WF_{carbon}$ ($MW_c/MW_{fuel}$) | (7) $g_c$/GAL (5 x 6) | | (8) A/F (STOICH) |
|---|---|---|---|---|---|---|---|---|
| Indolene | $CH_{1.86}$ | 13.89 | .740 | 2801 | .865 | 2423 | [Note 2] | 14.5 |
| #2 Diesel | $CH_{1.86}$ | 13.89 | .848 | 3210 | .865 | 2776 | [Note 3] | 14.5 |
| Methanol | $CH_4O$ | 32.04 | .792 | 2997.7 | .3749 | 1124 | | 6.5 |
| Ethanol | $C_2H_6O$ | 46.07 | .789 | 2986.4 | .5214 | 1557 | | 9.0 |
| N-Propanol | $C_3H_8O$ | 60.09 | .804 | 3043.1 | .5997 | 1825 | | 10.3 |
| N-Butanol | $C_4H_{10}O$ | 74.12 | .810 | 3065.9 | .6482 | 1987 | | 11.2 |
| "Gasohol" | 10% ETOH in gasoline | | | | .8294 | 2335 | | 14.0 |
| | 10% MOH in gasoline | | | | .8138 | 2179 | | 13.7 |

$$\text{F.E., MPG} = \frac{(7)\ g_c/\text{gal}}{6 \times HC + .429\ CO + .273\ CO_2\ g_c/\text{mi} + z \times TP}$$

Where: HC, CO, $CO_2$, and TP are in g/mi

And: Z is the weight fraction carbon in the particulate of diesel fueled vehicles. Assume Z = .85 if no other information is available. 40 CFR 600 does not include TP in fuel economy calculation.

1/ 1 gal = 3785 cc; 1 cc = 1 g water at 4°C.

2/ 40 CFR 600 uses 2421 $g_c$/gal Indolene.

TABLE 2

Fuel Properties - "Anafuel"

| Fuel | FORMULA | $MW_c$ | SG | g/GAL (3785 x SG)1/ | $WF_c$ | $g_c$/GAL | | A/F |
|---|---|---|---|---|---|---|---|---|
| Indolene | $CH_{1.86}$ | 13.89 | .740 | 2801 | .865 | 2421 | [F.R. Value] | 14.5 |
| Gasoline Portion of Anafuel | $CH_{1.71}$ | 13.73 | .7696 2/ | 2913 | .8748 | 2548 | | |
| MOH | $CH_4O$ | 32.04 | .792 | 2998 | .3749 | 1124 | | 6.5 |
| BOH | $C_4H_{10}O$ | 74.12 | .810 | 3066 | .6482 | 1987 | | 12.1 |
| Anafuel 7/81<br>9.8% MOH<br>2.7% BOH<br>87.5% Gasolene | | | | | .819 4/ | 2393 3/ | | 13.7 |

$$\text{F.F., MPG} = \frac{2393\ g_c/\text{gal}}{[.819\ HC + .429\ CO \mp .273\ CO_2]\ g_c/\text{mi}}$$

1/ 1 gal = 3785 cc; 1 cc = 1 g water

2/ Calculated

3/ .875 (2548) + .098 (1124) + .027 (1987) = 2393 gc/gal

4/ (Vf) (Wf) (SG)

$$\frac{(.875 \times .875 \times .7696) + (.098 \times .3749 \times .792) + (.027 \times .6482 \times .810)}{(.875 \times .7696) + (.098 \times .792) + (.027 \times .810)} = \frac{.6328}{.7729} = \underline{.819}$$

(Vf) (SG)

Doc 5?3?B

# I. Method for Calculation of Fuel Economy of Compressed Natural Gas (CNG) 1/

A. An accurate analysis of the fuel giving mole fraction data is require[d]. Below is given a typical CNG analysis.

Gas Analysis in Mole %

| | |
|---|---|
| Nitrogen ($N_2$) | 4.24 |
| $CO_2$ | 1.23 |
| He | .12 |
| $CH_4$ | 90.52 |
| $C_2H_6$ | 3.22 |
| $C_3H_8$ | .45 |
| i-$C_4H_{10}$ | .06 |
| n-$C_4H_{10}$ | .07 |
| i-$C_5H_{12}$ | .02 |
| n-$C_5H_{12}$ | .02 |
| $C_6H_{14}$ | .02 |
| $C_7H_{16}$ | .01 |
| $C_8H_{18}$ | .01 |
| $C_9H_{20}$ | .00 |
| $C_{10}H_{22}$ | .00 |
| $C_{11}H_{24}$ | .00 |
| $C_{12}H_{26}$ | .00 |
| $C_{13}H_{28}$ | .00 |
| $C_{14}H_{30}$ | .00 |
| $H_2$ | .00 |

Higher Heating Value = 976 BTU/SCF

Specific Gravity = .607

B. Calculation of Carbon and Hydrogen Weight Fractions.

1. The weight of Carbon per constituent is:

   # of carbon Atoms x (the weight of a carbon Atom = 12.01115) x mole fractio[n]

   The sum of the carbon weight fractions will be the carbon weight fractio[n] for the fuel.

2. The molecular weight of the fuel is found by:

   (Molecular weight of constituent) x (Mole fraction)

   The sum of the weight fractions will be molecular weight for the fuel.

3. The weight of the hydrogen per constituent is:

   (# of Hydrogen Atoms) x (the weight of a Hydrogen Atom = 1.008) x (H[2] mole fraction)

   The sum of the hydrogen weight fractions will be the hydrogen weight fraction of the fuel.

1/ PREPARED BY THOMAS PENNINGA -

4. Because the $CO_2$ in the fuel will simply pass through the engine (assumed the carbon fraction of the fuel not counting the $CO_2$ in also needed.

A sample calculation is given below:

| Component | Mole Fraction | Molecular Weight of Constituent | Weight Carbon | Weight Hydrogen | Molecular Weight |
|---|---|---|---|---|---|
| $N_2$ | 0.0450 | 28.0134 | 0 | 0 | 1.26060 |
| $CO_2$ | 0.0043 | 44.00995 | 0.05165 | 0 | 0.18924 |
| He | 0.0012 | 4.0026 | 0 | 0 | 0.00480 |
| $CH_4$90 | 0.9076 | 16.0430 | 10.90132 | 3.65933 | 14.56065 |
| $C_2H_6$ | 0.0362 | 30.0700 | 0.86961 | 0.21893 | 1.08854 |
| $C_3H_8$ | 0.0039 | 44.0972 | 0.14053 | 0.03145 | 0.17198 |
| i-$C_4H_{10}$ | 0.0005 | 58.1243 | 0.02402 | 0.00504 | 0.02906 |
| n-$C_4H_{10}$ | 0.0006 | 58.1243 | 0.02883 | 0.00605 | 0.03487 |
| i-$C_5H_{12}$ | 0.0002 | 72.1513 | 0.01201 | 0.00242 | 0.01443 |
| n-$C_5H_{12}$ | 0.0001 | 72.1513 | 0.00601 | 0.00121 | 0.00722 |
| $C_6H_{14}$ | 0.0002 | 86.1784 | 0.01441 | 0.00282 | 0.01724 |
| $C_7H_{16}$ | 0.0001 | 100.2055 | 0.00841 | 0.00161 | 0.01002 |
| $C_8H_{18}$ | 0.0001 | 114.2327 | 0.00961 | 0.00181 | 0.01142 |
| TOTALS | 1.0000 | | 12.06641 | 3.93068 | 17.40007 |

$$\text{Carbon weight fraction for fuel} = \frac{\text{weight Carbon}}{\text{molecular weight of fuel}} = \frac{12.06641}{17.40007} = .693$$

$$\text{Carbon weight fraction for fuel not counting } CO_2 = \frac{\text{weight Carbon} - \text{weight } CO_2}{\text{molecular weight of fuel}} = \frac{12.06641 - .0516}{17.40007}$$

.691

$$\text{Hydrogen weight of fraction of fuel} = \frac{\text{weight of Hydrogen}}{\text{molecular weight of fuel}} = \frac{3.93068}{17.40007} = .226$$

C. Carbon Balance Method of Fuel Economy Calculation for CNG.

1. The weight fraction of carbon in CO is:

$$\frac{12.01115}{12.01115 + 15.9994} = .429$$

2. The weight fraction of carbon in $CO_2$ is:

$$\frac{12.01115}{12.01115 + 2(15.9994)} = .273$$

3. The weight fraction of HC in the vehicle exhaust is assumed to be equal t that in the fuel not counting $CO_2$.

The gram/mile of carbon in the exhaust is then:

$$.429\ (CO) + .273\ (CO_2) + \left(\begin{array}{c}\text{weight fraction of carbon not} \\ \text{counting } CO_2 \text{ in the fuel}\end{array}\right) (HC)$$

Where CO, $CO_2$, and HC are in grams/mile from the exhaust analysis.

4. The density of the fuel is calculated as follows:

$$= \frac{\text{Pressure x Mass Air}}{\text{R x T}} (\text{S.G.})$$

Where R = Universal Gas Constant = 1545.33 ft - lbf/lbm - °R
T = Temperature in °R
S.G.= Specific gravity compared to air given in the fuel analysis.

example = At atmospheric pressure, 60°F the

$$= \frac{(14.767 \text{ psi})(144 \text{ in}^2/\text{ft}^2)(28.967 \text{ lbm/lb-mole Air})(453.592 \text{ gms/l}}{(1545.33 \text{ ft-lbf/lbm-°R})(520\text{°R})} \underline{(\text{S.G.})}$$

$= (34.77 \text{ gms/ft}^3)(\text{S.G.})$

for the analysis given previously S.G. = .607

example = $(34.77 \text{ gms/ft}^3)(.607) = 21.11 \text{ gms/ft}^3 = 2111$ gms/100 SCF

5. Fuel Economy Calculations

The fuel economy calculations are found by

$$\frac{(\text{Gms/100 SCF})(\text{Carbon weight fraction for the fuel})}{.429\ (CO) + .273(CO_2) + \genfrac{}{}{0pt}{}{(\text{Carbon weight fraction for})}{(\text{the fuel not counting } CO_2)}(HC)} = \text{miles/100 SCF}$$

example: for the analysis given above and HC = 1.0 gms/mile, CO = 7.0 gms/m
$CO_2$ = 400 gms/mile, = 21.11 gms/$Ft^3$ = 2111 gms/100 SCF

$$\frac{(2111 \text{ gms/100SCF})(.693)}{(.429)(7.0) + (.273)(400) + (.691)(1.0)} = 12.96 \text{ miles/100 SCF}$$

6. Equivalent gasoline MPG Calculations

Using the higher heating value from the CN4 Analysis for 100SCF the lov heating value must be calculated.

This is because we will need to compare the lower heating values of CNG ε Gasoline. The lower heating value is calculated as follows:

Grams of Hydrogen/100 SCF = (grams of fuel/100SCF)(Weight fraction Hydrogen)
(in the fuel)

The $H_2O$ produced per 100 SCF

$$= (\text{Grams of Hydrogen/100 SCF})\frac{[(2)(1.00797) + 15.9994)]}{(2)(1.00797)}$$

The heating value of $H_2O$ at 60°F is:

$$\frac{(H2O\ \text{produced}/100\ SCF)}{453.592\ \text{gms/lbm}} \times 1059.9\ \text{BTU/lbm}$$

Where 1059.9 BTU/lbm is the energy required to change 1 lbm of $H_2O$ fro liquid to steam.

The lower heating value = the higher heating value - the heating value o $H_2O$

Ex. using the same set of example data

Grams of $H_2$/100 SCF = (2111 gm/100 SCF)(.226) = 477.1 gms $H_2$/10 SCF

$$H_2O\ \text{produced}/100\ SCF = 477.1\ \frac{[(2)(1.00797) + 15.9994)]}{(2)(1.00797)} = 4263.58\ \text{gms}\ H_2O/10$$

The heating value of the $H_2O$ is:

$$\frac{4263.58\ \text{gms}\ H2O/100\ SCF}{453.592\ \text{gms/lbm}} \times 1059.9\ \text{BTU/lbm} = 9962.6\ \text{BTU}/100\ SCF$$

The lower heating value = 97600 - 9962.6 = 87637.4 BTU/100 SCF

The number of SCF of CNG to have an equivelent BTU content of one gallon is given by

$$\frac{\text{Lower heating value 1 gallon of gasoline (100)}}{\text{Lower heating value of 100 SCF of CNG}} = \frac{\text{No. of SCF}}{\text{Gallon of Gasoline}}$$

The mile per gallon gasoline equivelent is given by

$$\frac{\text{miles}}{100\ SCF} \times \frac{\text{No. of SCF}}{\text{Gallon of Gasoline}} \times \frac{1}{100} = \text{MPG gasoline equivelent.}$$

ex. Using same example with BTU/gallon of gasoline = 118,000 BTU/gal.

$$\frac{\text{No. of SCF}}{\text{Gallon of Gasoline}} = \frac{118{,}000}{87637.4} \times 100 = 134.65\ SCF/\text{gallon.}$$

$$\text{MPG gasoline equivelent} = 12.96\ \frac{\text{miles}}{100\ SCF} \times \frac{134.65}{100}\ SCF/\text{Gallon} = 17.45\ \text{MPG equi}$$

## II. Calculation of Air/Fuel Ratio at Stochiometric A/R.

1. For any hydrocarbon fuel $C_xH_yO_z$

2. The Equation is:

$$C_xH_yO_z + (x + y/4 - z/2)\ O_2 + 79/21\ (x + y/4 - z/2)\ N_2$$
$$x\ CO_2 + y/2\ H_2O + 79/21\ (x + y/4 - z/2)\ N_2$$

$$A/F_{STOICH} = \frac{(x + y/4 - z/2)(32) + 79/21\ (x + y/4 - z/2) \times (28)}{12.011\ (x) + 1.008\ (y) + 16.0\ (z)}$$

$$A/F_{STOICH} = \frac{137.333\ (x + y/4 - z/2)}{12.011\ (x) + 1.008\ (y) + 16.0\ (z)}$$