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Comparisons of Models Predicting Ambient Lake Phosphorus Concentrations

Working Paper 704

COMPARISONS OF MODELS PREDICTING

AMBIENT LAKE PHOSPHORUS CONCENTRATIONS

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COMPARISONS OF MODELS PREDICTING AMBIENT LAKE PHOSPHORUS CONCENTRATIONS

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NATIONAL EUTROPHICATION SURVEY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY

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FOREWORD

The National Eutrophication Survey was initiated in 1972 in response to an Administration commitment to investigate the nationwide threat of accelerated eutrophication to freshwater lakes and reservoirs. The Survey was designed to develop, in conjunction with State environmental agencies, information on nutrient sources, concentrations, and impact on selected freshwater lakes as a basis for formulating comprehensive and coordinated national, regional, and State management practices relating to point source discharge reduction and nonpoint source pollution abatement in lake watersheds.

The Survey collected physical, chemical, and biological data from 815 lakes and reservoirs throughout the contiguous United States. To date, the Survey has yielded more than two million data points. In-depth analyses are being made to advance the rationale and data base for refinement of nutrient water quality criteria for the Nation's freshwater lakes.

ABSTRACT

The Vollenweider, Dillon, and Larsen/Mercier models for predicting ambient lake phosphorus concentrations and classifying lakes by trophic state are compared in this report. The Dillon and Larsen/Mercier models gave comparable results in ranking 39 lakes relative to known ambient phosphorus concentrations. The Vollenweider model, which does not include a phosphorus retention capacity component, was unable to achieve the high rank correlations found with the other models.

Trophic state predictions from the phosphorus loading models are compared with National Eutrophication Survey lake report designations. Disagreements of 14, 18, and 25 percent, respectively, were found with the Dillon, Larsen/Mercier, and Vollenweider concepts.

COMPARISONS OF MODELS PREDICTING
AMBIENT LAKE PHOSPHORUS CONCENTRATIONS

The phosphorus loading - mean depth - relationship formulated by Vollenweider (1968) has been widely accepted and used to indicate the degree of eutrophy of lakes and evaluate the level of phosphorus loading to lakes. Dillon (1975) pointed out that there has been too little thought and criticism given to the limitations of the model by people using it. Subsequently, modifications of the basic mass balance equation have been derived to predict mean ambient lake phosphorus concentrations at equilibrium. Dillon (1975) utilizes phosphorus areal loading (L), the retention coefficient for phosphorus (R), the hydraulic flushing rate (P), and mean depth (Z) in a plot of the form

$$\frac{L\,(1-R)}{P} \quad \text{versus} \quad Z$$

to estimate trophic state. Vollenweider (1975) revised his original formula to include T , hydraulic residence time, so that areal phosphorus loading (L) is plotted against mean depth (Z) divided by T . Larsen and Mercier (1976) provide an alternative (to the prior loading concepts) which avoids the criticism of Edmondson (1970) that the effect of an increasing phosphorus load upon a lake depends, in part, upon whether that increase results from increases in influent flows, concentrations, or both. The Larsen/Mercier formula plots mean tributary phosphorus concentration against phosphorus retention coefficient, called R experimental, computed in the same way as Dillon's R, i.e.,

$$R = 1 - \frac{\text{Total phosphorus leaving}}{\text{Total phosphorus entering}}$$

It is interesting to note that in applying the above-mentioned formulas, each of the authors have selected to use levels of 10 and 20 μg/l of ambient lake phosphorus to divide lakes into the three standard trophic classifications -- oligotrophic, mesotrophic, and eutrophic.

To compare the three models, we selected 39 lakes sampled during 1973 by the National Eutrophication Survey (U.S. Environmental Protection Agency, 1975) which represented the entire range of water transparency as measured by Secchi disk. Table 1 demonstrates the variety of lakes selected.

TABLE 1. THE NUMERICAL AVERAGE AND RANGE OF MEAN CHEMICAL, PHYSICAL AND BIOLOGICAL CHARACTERISTICS OF 39 LAKES, SELECTED FROM THOSE SAMPLED DURING 1973 BY THE NATIONAL EUTROPHICATION SURVEY

PARAMETER	MEAN	RANGE		
Surface Area (km^2)	40.75	0.23	-	263.05
Drainage Area (km^2)	3502.6	4.3	-	38850.0
Mean Depth (m)	7.5	0.9	-	21.0
Maximum Depth (m)	21.6	1.5	-	57.8
Volume ($m^3 \times 10^6$)	379.28	0.45	-	2608.00
Hydraulic Retention Time (days)	251	1	-	2446
Secchi Disk (cm)	177.8	15.2	-	563.9
Total Phosphorus (μg/liter)	147	5	-	1120
Chlorophyll a (μg/liter)	53.9	1.4	-	456.6

The data used in this report are from the various individual National Eutrophication Survey (NES) lake reports for the lakes listed in Table 2, e.g., Report on Lake Lulu (EPA, 1976). Similarly, NES lake reports provide data for Lake Mead (EPA, 1977a) and Flaming Gorge Reservoir (EPA, 1977b) included as examples of the application of the formulas.

Solution analyses based on 20 μg/l were employed to place the three formulas on an equivalent basis by dividing the appropriate theoretical minimum eutrophic "loading" rate for a given lake (i.e., that which would produce an ambient lake concentration of 20 μg/l) into the actual "loading" rate determined for that lake. Hereafter, this ratio will be referred to as the "trophic ratio". Trophic ratios which exceed or equal 1.0 represent eutrophic loadings, whereas trophic ratios extending from 0.5 to less than 1.0 represent mesotrophic loadings and trophic ratios below 0.5 represent oligotrophic loadings, regardless of the formula employed.

Table 2 lists the 39 lakes used in this study, ranked in descending order by total phosphorus concentration, and gives the mean total phosphorus concentration and mean Secchi disk value for each lake. In addition, the trophic states given in the individual NES lake reports are listed along with the trophic ratios calculated for the Larsen/Mercier, Dillon, and Vollenweider models and the trophic states predicted by the ratios. The trophic states indicated in the NES reports were based largely upon lake mean total phosphorus, chlorophyll a, Secchi depth, hypolimnetic dissolved oxygen values and phytoplankton data (Allum et al., 1977).

Spearman rank correlation coefficients (rs) were calculated for each trophic ratio against measured mean ambient phosphorus concentrations with the following results: Larsen/Mercier (.94), Dillon (.92), and Vollenweider (.82). The Larsen/Mercier model provided the best estimation of the relative rank of the lakes based on ambient phosphorus concentrations. It was followed closely by the Dillon model. Both of these models take into consideration the phosphorus retention capacity of lakes. These models are

TABLE 2. COMPARISON OF LAKE AMBIENT PHOSPHORUS PREDICTION MODELS AND TROPHIC CLASSIFICATIONS WITH ACTUAL AMBIENT LAKE CONDITIONS. THE 39 LAKES COMPARED ARE RANKED IN DESCENDING ORDER BY MEAN SUMMER AMBIENT PHOSPHORUS CONCENTRATION

RANK	LAKE NAME (STATE)	MEAN AMBIENT TOTAL-P (μg/l)	MEAN AMBIENT SECCHI DEPTH (cm)	NES TROPHIC STATE*	LARSEN/MERCIER		DILLON		VOLLENWEIDER	
					TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*
1	Lake Lulu (Fla.)	1120	23	E	259.90	E	Δ		75.62	E
2	Slocum Lake (Ill.)	882	20	E	61.77	E	74.00	E	29.42	E
3	Lake Hancock (Fla.)	608	30	E	79.90	E	Δ		Δ	
4	Alligator Lake (Fla.)	429	58	E	38.92	E	Δ		Δ	
5	Fox Lake (Ill.)	322	23	E	10.81	E	3.40	E	12.66	E
6	Highland (Silver) Lake (Ill.)	258	20	E	14.61	E	15.38	E	5.43	E
7	Horseshoe Lake (Ill.)	256	25	E	11.81	E	12.00	E	1.96	E
8	Killen Pond (Del.)	216	66	E	5.58	E	5.67	E	9.25	E
9	Lake Loramie (Ohio)	204	15	E	18.57	E	18.67	E	5.02	E
10	Crab Orchard Lake (Ill.)	184	58	E	7.40	E	7.33	E	7.42	E
11	Duhernal Lake (N.J.)	179	61	E	3.17	E	3.33	E	18.92	E

* E = eutrophic, M = mesotrophic, O = oligotrophic and Δ = insufficient data to make estimate of trophic ratio.

(Continued)

TABLE 2. COMPARISON OF LAKE AMBIENT PHOSPHORUS PREDICTION MODELS AND TROPHIC CLASSIFICATIONS WITH ACTUAL AMBIENT LAKE CONDITIONS. THE 39 LAKES COMPARED ARE RANKED IN DESCENDING ORDER BY MEAN SUMMER AMBIENT PHOSPHORUS CONCENTRATION (Continued)

RANK	LAKE NAME (STATE)	MEAN AMBIENT TOTAL-P (µg/l)	MEAN AMBIENT SECCHI DEPTH (cm)	NES TROPHIC STATE*	LARSEN/MERCIER		DILLON		VOLLENWEIDER	
					TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*
12	Lake Charleston (Ill.)	164	23	E	8.57	E	7.50	E	15.91	E
13	Lake Apopka (Fla.)	161	29	E	19.17	E	22.00	E	3.94	E
14	Marsh Lake (Ind.)	115	127	E	5.80	E	5.92	E	4.54	E
15	Saluda Lake (S.C.)	73	68	M	1.84	E	1.88	E	5.07	E
16	Arkabutla Reservoir (Miss.)	58	64	E	11.28	E	11.39	E	2.58	E
17	Barren River Reservoir (Ky.)	49	123	E	3.35	E	2.32	E	2.10	E
18	Lake Chesdin (Va.)	40	120	E	2.20	E	2.21	E	2.24	E
19	Lay Lake (Ala.)	39	104	E	4.84	E	4.20	E	5.72	E
20	Cherokee Lake (Tenn.)	37	141	E	2.28	E	2.43	E	3.61	E
21	Hickory Lake (N.C.)	34	114	E	2.04	E	1.74	E	3.13	E
22	Walter F. George Reservoir (Ga.)	30	110	E	3.12	E	3.25	E	3.44	E

* E = eutrophic, M = mesotrophic, O = oligotrophic and Δ = insufficient data to make estimate of trophic ratio.

(Continued)

TABLE 2. COMPARISON OF LAKE AMBIENT PHOSPHORUS PREDICTION MODELS AND TROPHIC CLASSIFICATIONS WITH ACTUAL AMBIENT LAKE CONDITIONS. THE 39 LAKES COMPARED ARE RANKED IN DESCENDING ORDER BY MEAN SUMMER AMBIENT PHOSPHORUS CONCENTRATION (Continued)

					LARSEN/MERCIER		DILLON		VOLLENWEIDER	
RANK	LAKE NAME (STATE)	MEAN AMBIENT TOTAL-P (µg/l)	MEAN AMBIENT SECCHI DEPTH (cm)	NES TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*	TROPHIC RATIO*	TROPHIC STATE*
23	Moultrie Lake (S.C.)	25	134	E	1.51	E	1.55	E	1.70	E
24	Lake Hopatcong (N.J.)	25	231	E	1.32	E	1.00	E	0.34	0
25	Tims Ford Reservoir (Tenn.)	25	240	E	2.22	E	2.70	E	1.11	E
26	Lake Minnehaha (Fla.)	22	140	E	2.11	E	Δ		Δ	
27	Murray Lake (S.C.)	20	218	E	1.56	E	1.56	E	1.49	E
28	Chatuge Lake (Ga.)	17	310	M	1.06	E	1.10	E	0.56	M
29	Liberty Reservoir (Md.)	15	381	M	0.71	M	0.76	M	1.80	E
30	Wanaque Reservoir (N.J.)	14	467	M	1.31	E	0.96	M	0.48	0
31	Maxinkuckee Lake (Ind.)	14	221	M	1.20	E	1.33	E	0.75	M
32	Dale Hollow Reservoir (Ky.)	13	318	M	0.49	0	0.47	0	0.49	0

* E = eutrophic, M = mesotrophic, 0 = oligotrophic and Δ = insufficient data to make estimate of trophic ratio.

(Continued)

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RANK	LAKE NAME (STATE)	MEAN AMBIENT TOTAL-P (μg/l)	MEAN AMBIENT SECCHI DEPTH (cm)	NES TROPHIC STATE*	LARSEN/MERCIER TROPHIC RATIO*	LARSEN/MERCIER TROPHIC STATE*	DILLON TROPHIC RATIO*	DILLON TROPHIC STATE*	VOLLENWEIDER TROPHIC RATIO*	VOLLENWEIDER TROPHIC STATE*
33	Lake Wallenpaupack (Penn.)	13	434	M	1.10	E	1.12	E	0.50	M
34	Martin Lake (Ala.)	13	230	M	1.04	E	0.96	M	1.35	E
35	John W. Flannagan Reservoir (Va.)	11	366	M	0.66	M	0.69	M	1.92	E
36	Deep Creek Lake (Md.)	11	366	M	0.39	O	0.38	O	0.34	O
37	Summersville (W.Va.)	10	549	M	0.83	M	0.83	M	1.28	E
38	Harveys Lake (Penn.)	9	564	M	0.55	M	0.59	M	0.54	M
39	Tygart Reservoir (W.Va.)	5	320	M	0.95	M	0.94	M	2.02	E

* E = eutrophic, M = mesotrophic, O = oligotrophic and Δ = insufficient data to make estimate of trophic ratio.

very similar, as areal phosphorus loading (L) divided by mean depth (Z) approximates mean tributary concentration. The major difference between the models is the flushing rate (P) employed by Dillon.

The Vollenweider model, which does not contain a phosphorus retention capacity element, produced the poorest estimate of the ambient phosphorus concentration.

Comparison of NES trophic state assignments to various phosphorus model predictions revealed a 14, 18, and 25 percent disagreement, respectively, for the Dillon, Larsen/Mercier, and Vollenweider concepts. The Dillon and Larsen/Mercier models predicted the same trophic state in 33 of 35 lakes. The only exceptions were Martin Lake and Wanaque Reservoir where the trophic ratios, although very close, lay on opposite sides of the somewhat arbitrary borderline. Nine of the 35 Vollenweider trophic state predictions differed from the Dillon model predictions, while 8 differed from the Larsen/Mercier model estimates. Generally, the Dillon and Larsen/Mercier models not only predicted the same trophic state but their respective trophic ratios were quite similar. By comparison, the Vollenweider model porvided less consistent trophic state results and greater variation in trophic ratios.

Wanaque Reservoir was classified as eutrophic, mesotrophic, and oligotrophic by the models. However, the Dillon and Larsen/Mercier trophic ratios were actually very close (0.96 and 1.13, respectively).

Even though all of the models produced relatively high Spearman rank correlation coefficients, these models must be used with caution as the comparisons given in Table 3 illustrate. For the two western reservoirs, the Vollenweider model predicted a much higher eutrophic loading rate than the other models. Both of these reservoirs have high phosphorus retention capacities (associated with high suspended sediment deposition) -- Lake Mead (0.93), and Flaming Gorge Reservoir (0.82). These examples reinforce the concept that the Dillon and the Larsen/Mercier models give comparable results. However, the Larsen/Mercier model requires less information [(P) flushing rate and (Z) mean depth are not required] and produced the highest Spearman rank correlation coefficient (rs). Also, the Dillon model required mean depth information which is neither uniformily available nor necessarily accurate, without an extensive bathymetric survey. The mean depth of the lake is the only independent variable which sets the level of the theoretical eutrophic "loading" rate in the Dillon model. In the Larsen/Mercier model the independent variable which controls the theoretical eutrophic loading rate is the phosphorus retention capacity.

TABLE 3. COMPARISON OF THREE MODELS TO PREDICT MEAN AMBIENT LAKE PHOSPHORUS CONCENTRATIONS AT EQUILIBRIUM IN TWO RESERVOIRS

	CALCULATED MODEL VALUE	TROPHIC STATE LEVELS		TROPHIC RATIO	PREDICTED TROPHIC STATE
		EUTROPHIC	OLIGOTROPHIC		
Lake Mead, Nev./Ariz.					
Vollenweider	6.23 g/m^2/yr	0.78 g/m^2/yr	0.39 g/m^2/yr	7.98	Eutrophic
Larsen/Mercier	372 µg/l	298.5 µg/l	149.5 µg/l	1.29	Eutrophic
Dillon	1.47 g/m^2	1.18 g/m^2	0.59 g/m^2	1.24	Eutrophic
Flaming Gorge, Wyo./Utah					
Vollenweider	1.35 g/m^2/yr	0.76 g/m^2/yr	0.38 g/m^2/yr	1.77	Eutrophic
Larsen/Mercier	93.5 µg/l	152.6 µg/l	76.9 µg/l	0.60	Mesotrophic
Dillon	0.41 g/m^2	0.68 g/m^2	0.34 g/m^2	0.60	Mesotrophic

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