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AN INVESTMENT DECISION MODEL FOR CONTROL TECHNOLOGY

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FOREWORD

Man and his environment must be protected from the adverse effects of pesticides, radiation, noise and other forms of pollution, and the unwise management of solid waste. Efforts to protect the environment require a focus that recognizes the interplay between the components of our physical environment--air, water and land. The multidisciplinary programs of the National Environmental Research Centers provide this focus as they engage in studies of the effects of environmental contaminants on man and the biosphere and in a search for ways to prevent contamination and recycle valuable resources.

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ABSTRACT

Investment decisions in control technology for environmental management are similar to those in other areas of public finance. These decisions, which may include the decision to construct a water or wastewater treatment system or an incinerator, depend on adequate financial support, which means not only availability of money in sufficient quantity, but also at the time when needed. A mathematical model, incorporating borrowing and lending variables, has been structured to provide an efficient method of studying the problem. The model formulation assumes that investment decisions for control technology can be separated into a total operating and capital cost decision and an investment cost decision. These costs are minimized in two stages. The first stage utilizes a fixed-charge algorithm and the second stage, a linear programming algorithm. A problem is solved utilizing the construction, operation and financing of an incinerator subject to capacity and monetary constraints for an example.

AN INVESTMENT DECISION MODEL FOR CONTROL TECHNOLOGY

Investment decisions in control technology for environmental management are similar to those in other areas of public finance. These decisions, which may include the decision to construct a water or wastewater treatment system or an incinerator, depend on adequate financial support, which means not only availability of money in sufficient quantity but also at the time when needed. To illustrate the interdependence of engineering design (and operation) of control technology systems and the timing of sufficient quantities of money, an investigation has been undertaken to isolate the critical variables involved. The problem has been structured in the form of a mathematical model to provide a more efficient method of defining the important variables. Basically, the model says nothing more than money comes in from taxes or user charges and money goes out to pay for control technology management, but the money coming in may not match the money going out in quantity or timing. For this reason, a method, such as a borrowing or lending device, is needed to bring the expenditures into phase with incoming funds. In addition, the model must be consistent with reality by conforming to reasonable limitations on such things as treatment capacities and user charges or tax burdens. The study might be characterized as one that studies the interface between engineering design and adequate financial support.

The manager of control technology facilities must plan a system that will meet the desired operational or functional requirements of the community at minimum cost. In such a system, development and enlargement of engineering facilities over time must be considered, as well as long-range financial planning. The manager is concerned with optimal financing and with timing the enlargement of facilities in accordance with growth requirements, borrowing and payback opportunities, service charge possibilities, etc. In view of the uncertainty associated with information required for an analysis of this problem (e.g., growth rates and market opportunities), it seems unlikely that any analytical solution would result in a conclusive optimal design in the sense that the answer is the best of all possible designs. Standard approaches to the evaluation of capital investment decisions are based on the present value or present worth concept [1]. These methods do not allow the consideration of limitations on fund availability. What is needed, however, is the ability to study a particular situation in detail and to assess a variety of possible conditions for which suboptima can be generated [2]. Techniques from mathematical programming are used here to suggest an approach to such a study.

The model formulation will assume that investment decisions for control technology can be separated into a total operating and capital cost decision and an investment cost decision.

A model will be developed that performs a two-stage minimization of these costs. The first stage minimizes the total operating and capital cost of a facility subject to operational constraints, and the second stage minimizes the cost of financing its construction. The facility will be assumed to have a characteristic capital and operating cost function and an investment cost dependent on the rate of return associated with the bonds used to finance the capital expenditures.

The operating and capital cost function will take the form $C = MK + C_0$, where:

C = total construction and operation costs;

M = slope;

K = capacity supplied; and,

C_0 = set-up or start-up cost.

The total operating and capital cost function may be composed of one or of many of these types of relationships and will be minimized subject to operational requirements. The solution to this problem will indicate the amount of capital investment that must be undertaken in each period to supply the required level of capacity. To meet requirements, for example, one facility might have to be constructed in period one and another in period three. We must, therefore, invest in capital equipment in each of these periods, equal to the fixed cost or set-up charge. The total cost of required capital investment in each period will be the sum of the construction costs required for that period. This problem is essentially a fixed charge problem and requires appropriate solution techniques.

After finding the "best" solution to our operational problem, we will determine the most efficient allocation of funds for capital investment. To examine the inflow and outflow of funds for timing and investment requirements and for investment alternatives, a linear programming approach will be applied to this problem.

It should be emphasized that this approach leads to a suboptimal solution. The "best" approach would be to minimize a total cost function including operating, capital, and financing costs. Unfortunately, the state-of-the-art in mathematical programming is such that there are no techniques available to solve a problem formulated in this manner. The approach suggested in this paper does, however, provide a systematic method for evaluating various alternatives for minimizing the cost of control technology facilities.

To rationally apply this approach to control technology investment decisions, it is important to understand something about control technology systems.

CONTROL TECHNOLOGY SYSTEMS

Control technology systems are those systems that assist in environmental management by abating and controlling pollution and/or protecting the public's health. These systems may be designed for water and wastewater treatment, air pollution control, solid waste management, and noise abatement and control.

Although the example that will be developed here is for investment, operating, and capital cost decisions in solid waste management, the general techniques apply equally well to other investment decision problems in control technology.

The most common methods presently used for treatment and disposal of solid waste are the sanitary landfill, the central incinerator, or combinations of both. Incineration is a volume reduction method and requires investment in an expensive fixed facility. Sanitary landfill requires acquisition of land area, which may be excavated; waste material is deposited in the excavation and covered with earth at the end of each day's operation.

Central incineration will be considered in this analysis as an example for investment in control technology. An incinerator may have a high initial cost and a low operating cost or a low initial cost and a high operating cost. The selection of an incinerator to be used in a specific community strongly depends on the availability of funds for investment and operation and on the amount of wastes to be handled. Decisions such as this must be made within a time frame or planning horizon.

FACILITY COST MINIMIZATION MODEL

A model for this problem must reflect the requirements of the situation, which include the planning period, the level and type of operation, and quantity of waste to be treated. The objective is to minimize the total cost of operation and equipment acquisition for the time periods under consideration, while respecting the requirements or restrictions imposed on the problem. An optimal solution to the problem will indicate: (a) the types of facilities to be constructed in each time period; and, (b) the types of facilities to be constructed for each increment of capacity.

To provide the required capacity for the period under consideration, we can choose from among any of several available incinerators. We can assume n different incinerators and j periods in which treatment capacity must be supplied. Thus, the capacity of treatment available must be sufficient to handle the quantity of waste Q_j. For the first period, this can be:

(1)
$$\sum_{i=1}^{n} x_{i1} \geq Q_1$$

in which x_{11} represents the increment of capacity of incinerator type 1 required in period one; x_{21}, the increment of capacity of incinerator type 2 required in period one, etc. Incremental additions to incinerator capacity in period j can be obtained from among the n types, x_{1j}, x_{2j} . . . x_{nj}, such that for any period j, (j = 2, 3, . . . m) the capacity is given by:

(2)
$$\sum_{i=1}^{n} x_{ij} \geq Q_j$$

where Q_j ($j = 2 . . . m$) is the incremental waste quantity handled in period j. This formula assumes that each increment directly augments capacity. To describe the situation in a realistic manner, we can introduce the idea of a fixed cost, f_i, which is associated with the use of activity x_i and is incurred with the first increment of x_i constructed.

The value for f_i represents the capital cost or acquisition cost of incinerator i; thus, if in period one, treatment capacity is required from incinerator 1, we would designate this capacity as x_{11}. This implies that a capital investment cost of f_1 will be incurred. If more capacity is needed, for example x_{12} in period two, the only additional cost will be the variable cost $c_1 x^*_{12}$, where x^*_{12} is the level of capacity for incinerator 1 constructed in period two [4].

In addition to the fixed cost of the initial facility construction, we can also allow for a fixed cost of facility expansion. For example, in incinerator 1, the expansion capacity is x_{12}, and the fixed cost associated with it would be f_{12}. We can formulate the model as follows:

(3a) Minimize; $z = \sum_{i=1}^{n} (f_i + c_i\ x_i) + \sum_{i=1}^{n} \sum_{j=2}^{m} (f_{ij} + 0\ x_{ij})$

Subject to;

$$
\begin{array}{llll}
 & x_{11}+x_{12}+\ldots+x_{1m} & & = x_1 \\
 & \quad +x_{21}+x_{22}+\ldots x_{2m} & & = x_2 \\
 & \quad\quad \ldots\ldots\ldots\ldots\ldots & & \\
\text{(3b)} & \quad\quad +x_{n1}+x_{n2}+\ldots+x_{nm} & & = x_n \\
 & x_{11} \quad +x_{21}+\ldots\ldots\ldots\ldots+x_{n1} & & \geq Q_1 \\
 & x_{12} \quad +x_{22}\ldots\ldots\ldots\ldots+x_{n2} & & \geq Q_2 \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \\
\text{(3c)} & x_{1m} \quad +x_{2m}\ldots\ldots\ldots\ldots+x_{nm} & & \geq Q_m \\
 & \quad\quad x_{ij} & & \geq 0
\end{array}
$$

Equation (3a), composed of the concave cost functions for the various investment alternatives, represents the function to be minimized. For example, consider the cost function for incinerator 1 represented by $f_1 + c_1 x_1$; here f_1 represents the purchase or fixed cost for acquiring the treatment facility, and c_1 is the variable cost associated with the level of activity, x_1, at which it is operated. The fixed cost of adding the incremental capacity, x_{12}, would be f_{12}. The (3b) group of equations gives the total capacity at which the facility is operated. Again, when discussing incinerator 1, x_1 is the total capacity needed by incinerator 1; x_{11} is the increment of capacity needed in period one; x_{12} is the increment of capacity needed in period two; etc., which can be written as:

$$x_1 = x_{11} + x_{12} + \cdot \cdot \cdot + x_{1m}$$

Equations (3c) are the constraints on capacity required in period one . . . m. In period one, the required capacity must be greater than Q_1. The options for capacity include the available capacity of incinerator 1 in period one, x_{11}, or incinerator 2, in period one, x_{21}, etc. This holds true for each period of interest.

If incineration capacity is not available in certain time periods, the capacity equals zero. Thus, if there will be no capacity available for incinerator x_2 in time period one, $x_{21} = 0$, and,

$$x_2 = x_{22} + \cdot \cdot \cdot + x_{2m}$$

and, $$x_{12} + x_{32} \cdot \cdot \cdot + x_{n2} \geq Q_2$$

This allows us to specify that some investment alternatives will not be available until later in the planning period. Any constraints on the size of capacity for each incinerator are given by the form:

$$x_{ij} \geq K_s$$

where x_{ij} is the ith type treatment in the jth period, and K_s is its constraint.

This model demonstrates the capital cost that must be incurred each period to maintain capacity subject to operational requirements. If the results of the optimization indicate that the first increment of capacity for incinerator 1 is to be constructed in period one, the first increment of capacity for incinerator 2 is to be constructed in period three, and no other incinerators are to be constructed, then the capital investment costs will be f_1 in period one and f_2 in period three. The following notation is used to designate the capital investment required in any period j; F_1 is the capital investment required in period one, F_2 the capital investment required in period two, etc. In our example, $F_1 = f_1$ and $F_3 = f_2$, with all other capital investment being zero.

BORROWING MODEL

The construction of solid waste facilities or of any types of control technology involve sizable sums of money obtained through some financing procedure, normally through general tax funds. We will assume, however, for this analysis that revenue bonds will be used to finance the project. If this financing method is unavailable to a particular community, the revenue and individual taxpayer cost must be analyzed to assess the economic validity of the level of service to be rendered. Where financing of control technology systems is entirely through taxation, a meaningful analysis requires the budget and the tax resources available to the particular project to be partitioned [2].

For any period, the funds required for construction of control technology facilities must be sufficient to cover the cost of construction of plant capacity in that period, and they must be available to pay interest and principal payments due in that period. At the beginning of a period, the funds available must represent the accumulated revenue collected in all preceding periods. In this analysis, the time value of money is considered, as well as differences between borrowing and lending rates, dependence of funds usages, capital rationing, and "imperfect markets."

To formulate equations for these concepts, we must be able to write explicit constraints for: usage opportunities of funds; requirements and timing of funds, e.g., interest payments, construction charges, etc.; and the interdependence of fund usages.

The variables in the capital allocation model include borrowings and amounts set aside for reserves. These are related, through expressions, to the funds available for the uses in each period and to the needs that the available funds must cover in each period [3].

The funds available during period one for construction of the facility must be greater than the obligations for that period, or:

$$(4) \quad F_o + \sum_k L_{k1} \geq \sum_k L_{k1}\rho_{k1} + \sum_k L_{k1}r_k + F_1 - \Sigma A_{k1}$$

In equality (4), the left side of the inequality consists of: cash on hand at the beginning of period one, F_o; plus the loan (or funds borrowed) of borrowing type k at the beginning of period one. The right side of the inequality consists of the funds employed to retire a portion, ρ_{k1}, of the outstanding debt, L_{k1}, or $\Sigma\, L_{k1}\rho_{k1}$; plus the money required for interest payments, $\sum_k L_{k1}r_k$; plus the capital investment required in period one, F_1; minus additional funds used to develop cash reserves (or sinking funds) to decrease the outstanding debt or any other special purposes, $\sum_k A_{k1}$.

We can now define a variable W_1 that is equal to:

$$(5) \quad W_1 = F_o + \sum_k L_{k1} - \sum_k L_{k1}\rho_{k1} - \sum_k L_{k1}r_k - F_1 + \sum_k A_{k1}$$

Equation (5) represents the accumulated funds remaining at the end of the first period.

During period two, the funds available will consist of the revenue collected at the end of period one, g_1P_1, where g_1 is the service charge, and P_1 is the population served during period one; plus the funds remaining from period one, W_1; plus funds borrowed at the beginning of period two, $\sum_k L_{k2}$. The sum of the funds available in period two must be greater than or equal to the funds employed to retire a portion, ρ_{k2}, of the outstanding debt, L_{k1}, or $\sum_k L_{k1}\rho_{k2}$; minus the interest due on the outstanding debt, L_{k1}, or $\sum_k L_{k1}r_k\ (1 - \rho_{k1})$; and, minus additional funds used to develop cash reserves, $\sum_k A_{k2}$; plus funds employed to retire a portion, ρ_{k2}, of the outstanding debt, L_{k2}, or $\sum_k L_{k2}\rho_{k2}$; plus interest due on the outstanding debt, $\sum_k L_{k2}r_k$; plus the capital investment required in period two, F_2. In summary, funds available at the beginning of period two can be stated as:

$$g_1P_1 + W_1 + \sum_k L_{k2} \geq {}_k\, L_{1k}\rho_{k2} + \sum_k L_{k1}r_k\ (1 - \rho_{k1}) - \sum_k A_{k2}$$
$$+ \sum_k L_{k2}\rho_{k2} + \sum_k L_{k2}r_k + F_2$$

W_2 is defined as the funds left over at the end of period two, or:

$$W_2 = g_1P_1 + W_1 + \sum_k L_{12} - \{\sum_k L_{k1}\rho_{k2} + \sum_k L_{k1}r_k\ (1 - \rho_{k1})$$
$$- \sum_k A_{k2} + \sum_k L_{k2}\rho_{k2} + \sum_k L_{k2}r_k + F_2\}.$$

For all succeeding periods, j, (j = 3, 4, 5 . . . n), the funds available at the beginning of period will be given by:

$$g_{j-1}P_{j-1} + W_{j-1} + \sum_k L_{kj} \geq \sum_k \sum_{b=1}^{j} L_{kb}\rho_{kj} + \sum_k \sum_{b=1}^{j-1} L_{kb}r_k$$
$$(1 - \sum_{s=1}^{j-b-1} \rho_{ks}) - \sum_k A_{kj} + \sum_k L_{kj}\rho_{kj} + \sum_k L_{kj}r_k + F_j$$

We can categorize the funds in three different ways:

1. Interest payments:

$$\sum_k \sum_{b=1}^{j} L_{kb}r_k\ \{(M_k + 1) - \sum_{s=1}^{M_k} \rho_k\ (M_k - s + 1)\}$$

2. Funds recovered for investment:

$$- \sum_k \sum_{b=1}^{j} A_{kb}\ (1 + i_b)^{j-b}$$

3. Revenue from service charges:

$$\sum_{b=1}^{j} g_b P_b$$

In these terms, the objective may be stated as:
Minimize total cost, C:

$$(6)\quad C = \sum_{k} \sum_{b=1}^{j} L_{kb} r_k \left\{ (M_k + 1) - \sum_{s=1}^{M_k} \rho_{ks} \, (M_k - s + 1) \right\} - \sum_{k} \sum_{b=1}^{j} A_{kb} \, (1 + i_b)^{j-b} + \sum_{b=1}^{j} g_b P_b$$

where M_k is the life of issue and i_b is the interest rate in period b. The objective is to be attained subject to the constraints detailed above on the variables involved. By combining this model with the facility selection model, we can achieve a more efficient allocation of funds for control technology investment decisions.

EXAMPLE

This technique is used to solve the following problem. Assume three basic incinerators designated by x_1, x_2, and x_3, respectively. The first type, x_1, has a capital investment cost, f_1, of \$3 million and a variable operating cost of \$2 per ton of solid waste processed; this means that \$3 million is needed to acquire the facility represented by treatment type x_1. We will also assume a zero fixed cost for capacity addition to the incinerators. After the facility is built, the operating cost will be \$2 for every ton of solid waste processed. Incinerator x_2 has a capital cost of \$2 million and an operating cost of \$3 per ton, and incinerator x_3 has a capital cost of \$1 million and an operating cost of \$4 per ton. Combinations of these incinerators can be used to satisfy the demand over four planning periods.

Assume that a municipality has an increasing production of solid waste over time; 5 million tons must be treated in period one, and the incremental capacity requirements in the following periods are 7 million in period two, 8 million in period three, and 9 million in period four. The constraint set for this problem is:

$$
\text{(7a)} \quad
\begin{array}{llll}
x_{11}+x_{12}+x_{13}+x_{14} & & & = x_1 \\
 & +x_{21}+x_{22}+x_{23}+x_{24} & & = x_2 \\
 & & +x_{31}+x_{32}+x_{33}+x_{34} & = x_3
\end{array}
$$

$$
\text{(7b)} \quad
\begin{array}{lllll}
x_{11} & x_{21} & x_{31} & \geq 5 \times 10^6 \\
+x_{12} & +x_{22} & +x_{32} & \geq 7 \times 10^6 \\
+x_{13} & +x_{23} & +x_{33} & \geq 8 \times 10^6 \\
+x_{14} & +x_{24} & +x_{34} & \geq 9 \times 10^6
\end{array}
$$

Equations (7a), the sum of which equals the total capacity for the treatment type, represent the capacity available over the four periods. That is, x_{11} is the capacity of incinerator 1 used in period one, x_{12} the capacity of incinerator 2 used in period two, etc.

Equations (7b) represent the capacity of the various treatment types available to fulfill the demand in each period, e.g., in period one we have x_{11}, the capacity of incinerator 1 in period one; plus x_{21}, the capacity of incinerator 2 in period one; plus x_{31}, the capacity of incinerator 3 in period one.

The following system could then be solved:

Minimize:

$$z = (3 \times 10^6 + 2x_1) + (2 \times 10^6 + 3x_2) + (1 \times 10^6 + 4x_3)$$

Subject to:

$$\begin{array}{llllllllllllllll}
x_1 & & & -x_{11} & -x_{12} & -x_{13} & -x_{14} & & & & & & & & & = 0 \\
 & x_2 & & & & & & -x_{21} & -x_{22} & -x_{23} & -x_{24} & & & & & = 0 \\
 & & x_3 & & & & & & & & & -x_{31} & -x_{32} & -x_{33} & -x_{34} & = 0 \\
 & & & x_{11} & & & & +x_{21} & & & & +x_{31} & & & & \geq 5 \times 10^6 \\
 & & & & x_{12} & & & & +x_{22} & & & & +x_{32} & & & \geq 7 \times 10^6 \\
 & & & & & x_{13} & & & & +x_{23} & & & & +x_{33} & & \geq 8 \times 10^6 \\
 & & & & & & x_{14} & & & & +x_{24} & & & & +x_{34} & \geq 9 \times 10^6
\end{array}$$

After solving the model with the use of Walker's algorithm, we find incinerator 1 provides the least expensive solution to handle capacity for all four periods [5]. Incinerator x_1 will be built in period one and will require a capital investment of \$3 million; therefore, $F_1 = 0$, $F_3 = 0$, $F_4 = 0$.

The conditions for the bond type, assuming one type of borrowing available only at the beginning of period one, are given as:

Bond Type (k)	Life of Issue (M_k) Periods	ρks % of Original Loan to be Retired in Each Successive 5-Year Period S			Borrowing Interest Rate Per Period % (r_k)
		1	2	3	
1	3	21	32	47	18

Assume, for simplicity, an $F_o = 0$, with no sinking funds, and at the beginning of each period, a population of 2,000, 3,000, and 4,000, respectively. A constraint placed on the user charges requires that they be equal in each period. The constraint set will be as follows:

(8a) $$0.61L_{11} \geq 3 \times 10^6$$

$$1{,}000g_1 + 0.15L_{11} \geq 3 \times 10^6$$

$$2{,}000g_1 + 3{,}000g_2 - 0.40L_{11} \geq 3 \times 10^6$$

$$2{,}000g_1 + 3{,}000g_2 + 4{,}000g_3 \geq 3 \times 10^6$$

(8b) $$g_1 = g_2 = g_3$$

Constraints (8a) are of the same form as shown in constraint (5), and (8b) reflects the equality conditions on the user charges. The objective function is obtained from equation (6) and has the form:

Minimize:

$$z = 0.41L_{11} + 2{,}000g_1 + 3{,}000g_2 + 4{,}000g_3$$

The solution to the above system is obtained with the use of a standard linear programming algorithm [3]. The values are:

$$L_{11} = 5.80 \times 10^6 \text{ dollars}$$

$$g_1 = 1{,}060 \text{ dollars/period}$$

$$g_2 = 1{,}060 \text{ dollars/period}$$

$$g_3 = 0{,}060 \text{ dollars/period}$$

Total cost of the system is:

$$C = 11.90 \times 10^6 \text{ dollars.}$$

SUMMARY AND CONCLUSION

With this approach, the most efficient allocation of funds for investment in control technology can be analyzed. Obviously, there is a great deal of flexibility in this method. The various demand patterns can be examined, as well as many different alternatives for investment. By using sensitivity analysis in conjunction with the linear programming portion of the model, the effects of changing the investment pattern on the borrowing costs can also be examined. The writer believes this technique can be beneficially applied to financing control technology systems.

12-00

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