COMBINED SEWER OVERFLOW CHARACTERISTICS
FROM
TREATMENT PLANT DATA

by

JAMES A. MUELLER
DOMINIC M. DI TORO

APRIL, 1981

MANHATTAN COLLEGE
ENVIRONMENTAL ENGINEERING AND SCIENCE PROGRAM
BRONX, NEW YORK 10471

DISCLAIMER

This report has been reviewed by the Municipal Environmental Research Lab, U.S. Environmental Protection Agency, and approved for publication. Approval does not signify that the contents necessarily reflect the views and policies of the U.S. Environmental Protection Agency, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.

ABSTRACT

This research was undertaken to evaluate the adequacy of using a mass balance technique with daily treatment plant data to determine combined sewer runoff and overflow characteristics.

An hourly simulator was utilized to generate known runoff and overflow concentrations as well as plant concentrations, similar to raw treatment plant data. The daily balance technique was used to analyze the simulated treatment plant data which provided comparisons of the calculated to the known runoff and overflow concentrations.

The bias and variability associated with the mass balance technique together with a theoretical analysis of the plant measurement error effects are presented. The unit loads and average concentrations from the NYC 26th Ward Treatment Plant area as well as the effect of rainfall characteristics on combined sewer runoff concentrations are also presented.

This report was submitted in fulfillment of Grant No. R 806519-01 by Manhattan College under the sponsorship of the U. S. Environmental Protection Agency and covers the project period June 1, 1979 to February 28, 1981.

TABLE OF CONTENTS

Abstract iii
Figures vi
Tables ix
Acknowledgement xi

1. Introduction 1
2. Summary and Conclusions 3
3. Recommendations 6
4. Computational Framework 7
   Hourly Simulator 7
   Daily Mass Balance Technique 11
      Flow Weighted Composite Sampling 12
      Equal Volume Composite Sampling 13
5. Evaluation of Daily Mass Balance Method 14
   Flow Weighted and Equal Volume Plant Sampling Analyses . 14
   Effect of Runoff Coefficient 15
   Diurnal Sewage Variation 15
6. Hourly Mass Balance Method 22
   Derivation of Estimating Equations 22
   Performance of Hourly Mass Balance Method 25
      Measurement Error 29
      Dry Weather Random Variability 33
      Wet Weather Variability 33
      Conclusion 38
7. Theoretical Analysis of Measurement Error 39
   Variability of Overflow Concentrations 39
   Variability of Runoff Concentrations 41
   Comparisons of Predicted and Simulated Effects of Measurement Errors 45
   Conclusion 52
8. Analysis of Rainfall-Runoff Relationships 53
   Simulator Data 53
   NYC 26th Ward Data 63
   Conclusion 63
9. Analysis of Hourly Sample Collection 67

Bibliography 72
Appendices 73

I. Estimation of $E\{1/RD^2\}$ 73
II. Theoretical Analysis of Dry Weather Sewage Variability 77
   $C_2 - C_0$ Analysis 77

TABLE OF CONTENTS (cont'd)

$C_2 - C_R$ Analysis 79
$Q_2 - C_0$ Analysis 80
$Q_2 - C_R$ Analysis 80
Verification 81
III. Analysis of 26th Ward Plant Data. 83
Flow Weighted and Equal Volume Analyses. 83
Effect of Tidegate Leakage on Mass Balance 85
Hourly Mass Balance Method 89
Rainfall-Runoff Relationships. 97
IV. Applicability 100
V. Definition of Symbols 108

FIGURES

Number		Page
1.	Hourly Simulator Schematic	10
2.	Effect of Minimum Rainfall Analyzed and Interceptor Capacity on Runoff and Overflow Values using Flow Weighted Analysis	16
3.	Effect of Minimum Rainfall Analyzed and Interceptor Capacity on Runoff and Overflow Values using Equal Volume Analysis	16
4.	Effect of Assumed Runoff Coefficient on Calculated Runoff and Overflow Loads - Equal Volume Analysis	17
5.	Effect of Assumed Runoff Coefficient on Calculated Runoff and Overflow Concentrations - Equal Volume Analysis . .	17
6.	Diurnal Dry Weather Sewage Values for 26th Ward Plant[4], Tuesday, Oct. 12-13, 1976.	18
7.	Bias of Runoff and Overflow Concentrations for Diurnal and Constant Sewage Characteristics.	19
8.	Variability of Runoff and Overflow Concentrations for Diurnal and Constant Sewage Characteristics.	19
9.	Runoff and Overflow Histograms for Constant and Diurnal Sewage Characteristics at α_{min} = 4 Hr.	21
10.	Runoff and Overflow Histograms for Diurnal Sewage Characteristics Using Hourly Constant C_o Analysis.	26
11.	Runoff and Overflow Histograms for Diurnal Sewage Characteristics Using Hourly Constant C_o Analysis for i_{min} = 0.03 in/hr	26
12.	Runoff and Overflow Histograms for Constant C_o Analysis with Measurement Error for i_{min} = 0.03 in/hr	31
13.	Runoff and Overflow Histograms for Constant C_R Analysis with Measurement Error for i_{min} = .03 in/hr.	31
14.	Runoff and Overflow Histograms for Variability on Dry Weather Sewage Concentration	34
15.	Runoff and Overflow Histograms for Variability on Dry Weather Sewage Flow Rate	34

FIGURES (cont'd)

Number		Page
16.	Effect of Number of Samples and Rainfall Intensity on Runoff and Overflow Variability Parameters	44
17.	Runoff, Variability Factors for Measurement Error (STD = 10 mg/ℓ).	46
18.	Daily Averaging Error for Constant Rainfall and Variable Sewage Characteristics During an Event	47
19.	Runoff Variability Factors for Combined Measurement (STD = 10 mg/ℓ) and Daily Averaging Errors	48
20.	Overflow Variability Factors for Combined Measurement (STD = 10 mg/ℓ) and Daily Averaging Errors	49
21.	Runoff Variability for Varying Runoff Characteristics. . .	50
22.	Overflow Variability for Varying Runoff Characteristics. .	51
23.	Runoff Concentration - Storm Interval Regressions for Strong Interval Effect	55
24.	Probability Distribution of Runoff Residuals for a 10 mg/ℓ Measurement Error for Strong Interval Effect	56
25.	Runoff Residuals Histograms for a 10 mg/ℓ Measurement Error for Strong Interval Effect	56
26.	Effect of Length of Record on Regression Parameters for a 15 mg/ℓ Measurement Error for Strong Interval Effect . .	57
27.	Effect of Length of Record on Regression Parameters for a 10 mg/ℓ Measurement Error for Weak Interval Effect . . .	58
28.	Effect of Measurement Error on Runoff Concentration - Interval Correlation Coefficients.	59
29.	Effect of Length of Record on Interval Regression Parameters for a Significant First Flush	61
30.	Effect on Length of Record on Duration Regression Parameters for a Significant First Flush	61
31.	Effect of Minimum Duration on Runoff Concentration - Duration Regression Parameters for a Significant First Flush .	62
32.	Effect of Minimum Duration on Runoff Concentration-Interval Regression Parameters for a Significant First Flush. .	62
33.	Effect of Minimum Duration Analyzed on Runoff-Duration Regression Parameters for NYC 26th Ward Data	64
34.	Duration Regression Analysis for 26th Ward Suspended Solids Data.	65

FIGURES (Cont'd)

Number		Page
35.	Interval Regression Analysis for 26th Ward Suspended Solids Data	65
36.	Effect of Minimum Storm Duration on Error Magnification Factors for Hourly Plant Sampling	70
37.	Duration Regression Parameters Using Hourly Plant Sampling Technique	71
38.	Interval Regression Parameters Using Hourly Plant Sampling Technique	71
A-1.	Effect of Minimum Storm Duration on Runoff Suspended Solids Histograms for 1957-26th Ward Data Using Equal Volume Analysis.	87
A-2.	Effect of Minimum Rainfall Intensities and Durations on Runoff Suspended Solids Histograms for 1969 - 26th Ward Hourly Analysis	90
A-3.	Yearly Runoff Suspended Solids Concentrations for the 26th Ward Data Using the Hourly Analysis.	94
A-4.	Yearly Runoff BOD Concentrations for the 26th Ward Data Using the Hourly Analysis	94
A-5.	Yearly Sewage Concentrations for the 26th Ward Data	95
A-6.	Percentage of Population in each State Served by Combined Sewers.	102
A-7.	Number of Urban Areas in each State Having More than 30% of the Population Served by Combined Sewers	102

TABLES

Number		Page
1.	Daily and Hourly Mass Balance Results for Diurnal Sewage Characteristics	28
2.	Variability of Laboratory Analyses	30
3.	Measurement Error Results	32
4.	Dry Weather Random Variability Results	35
5.	Runoff and Overflow Variabilities for Varying Runoff Characteristics	37
6.	Effect of Neglecting Short Storms on Runoff and Overflow Concentrations for Significant First Flush	38
7.	Comparison of Theoretical and Actual Statistical Parameter for Overflow Measurement Error	41
8.	Comparison of Theoretical and Actual Statistical Parameters for Runoff Measurement Error	43
A-1.	Duration and Sampling Probabilities	75
A-2.	Overall Probabilities for RD Values	76
A-3.	Effect of RD_{min} on $E\{\frac{1}{RD^2}\}$	76
A-4.	Runoff and Overflow Variability Due to Dry Weather Random Variability	82
A-5.	Flow Weighted and Equal Volume Analysis for 1957 - 26th Ward Data	84
A-6.	Equal Volume Analysis for 1957 - 26th Ward Data, Alpha Minimum = 0	86
A-7.	Effect of Tidegate Leakage on Volumes and Yearly Loads from Equal Volume Analysis of 26th Ward Data	86
A-8.	Effect of Tidegate Leakage on Runoff Concentrations from Equal Volume Analysis of 6 Years of 26th Ward Data	88
A-9.	Comparison of Runoff and Overflow Concentrations Using One and Two Rainfall Stations for 1969 - 26th Ward Data with Hourly Analysis	91
A-10.	Yearly Average Runoff and Overflow Concentrations Using the Hourly Analysis	92

TABLES (cont'd)

Number		Page
A-11.	Comparison of Average Yearly Unit Loads by Daily and Hourly Mass Balance Methods for 26th Ward Data.	96
A-12.	Weighted Average Concentrations for 26th Ward Data Over Total Study Using Hourly Analysis.	97
A-13.	Rainfall-Runoff Relationships for Four Parameters Analyzed at 26th Ward Using Separate Linear Regressions[1] for Duration and Interval	98
A-14.	Rainfall-Runoff Relationship for Four Parameters Analyzed at 26th Ward Using Multiple Linear Regression[1] for Duration and Interval	99

ACKNOWLEDGEMENTS

The capable participation of Edward J. Garland as Research Assistant in the study is acknowledged as well as the initial work of Charles R. Dabrusco. Special thanks are due to Mrs. Eileen Lutomski and Mrs. Margaret Cafarella for their typing of the report manuscript. The interest and cooperation of Richard Field, Chief Storm and Combined Sewer Section, and Douglas Ammon, Project Officer, are gratefully acknowledged.

SECTION 1

INTRODUCTION

The assessment of the magnitude and characteristics of urban runoff loads for specific regions is a difficult task due to the random nature of storm events. Techniques typically used to evaluate urban runoff inputs include (1) direct sampling of storm overflow concentrations and flows or (2) use of a stormwater quality model based on land use and rainfall characteristics. Due to the highly variable nature of rainfall and associated runoff phenomena, an extensive sampling program is generally necessary with the first technique in order to provide accurate estimates of overflow loads, a costly and time-consuming undertaking. The latter method, if based on default values incorporated in the models, may lead to significant errors. To obtain reliability in the latter approach, the models must be calibrated for specific areas, normally by direct sampling of stormwater overflows. A third possibility is to use existing data bases namely, treatment plant influent data, for determination of combined sewer overflow loads. This should provide municipalities with an alternate method to rapidly and economically assess the importance of their combined sewer overflows when formulating water quality management plans.

The objective of this research was to evaluate the adequacy of a mass balance technique using treatment plant influent data to determine the magnitude of combined sewer runoff and overflow loads. The initial concept of using treatment plant data to obtain these loads was developed [1-3] to evaluate the relative importance of urban runoff inputs to New York Bight. The mass balance method is a mathematical framework consisting of mass and flow balances for the sewer system and regulators over the total drainage area served by a treatment plant. Inputs to the sewer system include the dry weather sewage flow, runoff into the combined sewer system during storm events, and tidegate leakage. Outputs from the system include the wastewater flow to the treatment plant and the combined sewer overflow from the regulators to the receiving waters. During the overflow event, the quality of the overflow from the regulators is assumed equal to that of the treatment plant influent. Since daily treatment plant data is normally available, hourly mass and flow balance equations are integrated over the sampling day to provide estimates of the temporal and areal average daily overflow and runoff concentrations. The runoff concentration includes the contribution from both surface runoff and interceptor scour. The initial study utilized data from the 26th Ward plant in New York City. A large degree of variability in the daily runoff and overflow concentrations resulted, values over a number of years used to characterize loads from the drainage area. Partial verification was obtained by comparison to existing combined sewer sampling data from

portions of the area.

The present study was conducted to determine the bias and variability associated with the technique and evaluate modifications required to provide maximum accuracy for the available data base. The approach taken was to develop an hourly simulator in which all influent characteristics, both dry weather sewage and runoff, were known. The daily composite simulator output was analyzed by the mass balance technique and compared to the known inputs. This comparison served as the basis for modifying the computational technique. Two modifications were developed: one employing equal volume plant sampling similar to the N.Y.C. sampling technique, and the other employing real time which allows rainfall events to be correlated with dry weather sewage diurnal variability. The effects of errors in the estimation of dry weather sewage characteristics and runoff volumes were evaluated along with the effect of plant concentration measurement error. The ability of the technique to extract the effects of rainfall characteristics, interval between storms and storm duration, on runoff loads was studied for both the New York City sampling routine (every 4 hours skipping the 2 AM sample) and an hourly sampling routine.

The modified computational techniques were utilized on the existing 26th Ward data from N.Y.C. to evaluate the impact of the improved methodology on the runoff and overflow load estimates. A literature review and letter survey were also conducted to evaluate the nationwide applicability of the methodology.

SECTION 2

SUMMARY AND CONCLUSIONS

The ability of a mass balance technique using treatment plant influent data to accurately determine the overflow loads and runoff characteristics from combined sewers was evaluated using an hourly simulator to generate known runoff, overflow and plant influent concentrations. The plant influent data generated by the simulator were analyzed by the daily mass balance technique to determine concentrations which were compared to the true values generated above. This provided the basis for analyzing the bias and variability associated with the technique.

The initial results showed that a significant bias existed when interceptor capacity was greater than dry weather flow if a flow weighted analysis of influent data was used on plant composite samples collected in equal volumes. The bias was removed by modifying the technique to an equal volume analysis of the plant composite samples. Lastly, variability due to the averaging technique was minimized by using the hourly dry weather concentration coinciding with the time of the storm.

The variability in the calculated runoff and overflow concentrations due to plant measurement error is significant. A theoretical analysis of the error structure indicated that the variability of the runoff estimates was greater than the overflow estimates. The variability in the individual concentrations could be reduced by deleting low average storm intensities (<0.03 in/hr) and low storm durations which provided only one wet sample at the plant. However, excluding lower duration storms from the mass balance analysis reduced the capability of extracting first flush effects from the data. Random variability in hourly dry weather sewage concentrations using standard deviations of 10 and 20% on the hourly values was found to be significant but somewhat lower than that due to the measurement error. A summation of the variance of each of the individual errors provided an excellent estimate of the total variance of the estimated runoff and overflow concentrations.

The ability of the mass balance technique to analyze for the effect of rainfall characteristics on runoff concentrations when both averaging and measurement errors were present was evaluated for the New York City sampling mode by linear regression analysis. The actual effects of both interval and duration on the storm average runoff concentrations provided by the simulator were successfully obtained from an analysis of the daily plant data. Approximately 150 to 200 days of data are required to insure the confidence limits on the interval effect, as measured by the slope of the regression curve, are above zero when the runoff concentrations are significantly affected by a

first flush. The correlation coefficients obtained from these regressions are low, explaining only 3 to 14% of the observed variability. The remainder of the variability of these simulator data is due to the averaging and measurement errors inherent in the analysis and not random variability of runoff concentrations. Thus the mass balance technique is capable of accurately predicting effects of duration and interval on storm weighted average runoff concentrations.

In the simulated runoff data, the first flush effect was limited to the first hours of the storm events with background levels attained after three to four hours. Therefore, when short storms were neglected in the analysis, lower runoff concentrations resulted with regression parameters similarly reduced. Thus to properly evaluate the first flush effects on runoff characteristics, short duration storms had to be included in the analysis.

Collecting samples every hour instead of the NYC sampling routine of every 4 hours (skipping the 2 AM sample) caused a higher degree of variability in the results especially when short duration storms were analyzed. This is due to the fact that with durations of 1 or 2 hours, less than 10% of the collected samples reflect wet weather conditions. Analyzing durations only equal to or greater than 4 hours for the hourly sampling routine provided results similar to analysis of all plant data sampled by the NYC routine as long as a runoff event occurred during a plant sampling time. Thus a plant sampling routine based on hourly sampling reduces the capability of evaluating runoff and overflow characteristics from plant data.

The actual daily data from the 26th Ward Plant in New York City were then analyzed using the hourly mass balance technique. Unit loads were similar to those for the previous flow weighted daily mass balance analysis with the exception of the soluble BOD_5 data, which was significantly lower than previously estimated. For these estimates, the hourly variability in dry weather concentrations for all four parameters: (SS, VSS, BOD_5, soluble BOD_5) was taken from the BOD_5 variability. Interval and duration significantly affected runoff concentrations. For the 26th Ward data, similar first flush effects were obtained when both 1 and 2 hour minimum duration storms were analyzed, with a higher correlation coefficient for the latter. Plant data analysis using a minimum storm duration of two hours and minimum average intensity of 0.03 in/hr provided the best estimate of average runoff and overflow concentrations as well as the effects of storm characteristics on runoff concentration.

The following conclusions have been drawn from the study.

1. Average annual runoff and overflow loads and concentrations can be obtained using long term influent data from treatment plants with combined sewer systems by using a mass balance analysis.

2. To remove bias from the analysis, the original flow weighted mass balance technique must be modified to reflect the type of composite sampling being conducted at the plant. For the New York City plant sampling routine, an equation based on equal sample volumes is required

for the measured plant concentrations.

3. Individual estimates of daily runoff and overflow concentrations have a high degree of variability due to subtractions inherent in the mass balance technique. Therefore long data bases are required to provide good estimates of average loads.

4. An hourly mass balance technique using dry weather hourly sewage concentrations and flows with hourly rainfall intensities reduces averaging errors inherent in the daily analysis. However, for evaluation of long term BOD_5, suspended solids and volatile suspended solids loads for the the 26th Ward Plant in New York City, the extra complexity of the hourly analysis was not justified since average loads were not significantly different. This was not the case for soluble BOD_5 which was significantly lower for the hourly analysis. In the hourly analysis of the 26th Ward data, the diurnal variability of all dry weather constituents was assumed similar to that for BOD_5 for which data was available.

5. Measurement error associated with plant concentrations causes a major portion of the variability in estimated runoff and overflow concentrations using the hourly analysis. Other causes of this variability are hourly dry weather sewage concentration variability and within storm variable hourly runoff concentrations.

6. Runoff concentrations can be related to rainfall characteristics reliably if a sufficient length of record is analyzed, the hourly analysis providing greater reliability than the daily analysis. The manner of sample collection and compositing significantly affects the length of record required. For example, hourly sampling for the daily plant composite requires approximately 400 days of data while sampling at 4 hour intervals would require approximately 150 days data.

7. Use of the mass balance technique to obtain drainage area integrated runoff and overflow concentrations from plant influent data should provide significant costs saving when laboratory analytical costs are high as in the case of the toxics.

SECTION 3

RECOMMENDATIONS

With the ability to obtain accurate estimates of combined sewer runoff and overflow characteristics from plant data, plant influent sampling for the toxics should be initiated to define the combined sewer contributions of these loads to receiving waters and to treatment plants. The manner of sample collection, compositing and analysis should be optimized to minimize costs of laboratory analyses. Utilization of suspended solids with a toxics-suspended solids correlation may be the most cost effective approach.

SECTION 4

COMPUTATIONAL FRAMEWORK

Two mathematical models were used in the study. The first was an hourly simulator to develop daily composite treatment plant, runoff and overflow loads using hourly rainfall input data. The second was the daily mass balance technique which analyzed the above treatment plant data. This section describes the characteristics of both models along with the modifications to the daily balance technique to reduce bias and variability associated with the methodology.

HOURLY SIMULATOR

The adequacy of the daily balance technique used in the previous work on the 26th Ward treatment plant[1-3] was evaluated by comparing the estimates to measured average overflow characteristics from field studies of Jamaica Bay. The spread of the average field values from the two studies was large, a factor of approximately 3 to 1. Thus no absolute runoff and overflow concentrations existed to accurately evaluate the accuracy of the daily mass balance technique. The hourly simulator was developed to fill this gap.

To develop an efficient hourly simulator which did not require a significant amount of raw data handling, two modifications of the previous balance programs were utilized. The first was that tidal inflow was not included in the analysis so a chloride balance was not required. The second, and most time saving, was the calculation of hourly rainfall volumes using internally generated characteristics of storm average rainfall intensity, duration, and interval between storms. It has been found[5], that the intensity, i, duration, d, and time between storms, δ, are essentially independent, serially uncorrelated and exponentially distributed. Thus the probability density functions for these random variables are:

$$P_i(i) = \frac{1}{I} e^{-i/I} \qquad i \geq o$$

$$P_d(d) = \frac{1}{D} e^{-d/D} \qquad d \geq o$$

$$P_\delta(\delta) = \frac{1}{\Delta} e^{-\delta/\Delta} \qquad \delta \geq o$$

where I,D, and Δ, are the average intensity, duration, and time between storms, respectively. In order to generate exponentially distributed random variables, consider a uniformly distributed random variable, x:

$$P_x(x) = 1 \qquad o \leq x \leq 1$$

and the relationship, using rainfall intensity, for example,

$$i = - I \ln (x)$$

It can be shown using the relationships that given the probability density function of a random variable (5) that

$$P_i(i) = P_x (e^{-i/I}) \frac{d}{di} (e^{-i/I})$$

$$= \frac{1}{I} e^{-i/I}$$

Uniformly distributed random variables are directly available from internal subroutines that are part of most programming languages (the RND function), and the logarithmic transformation converts them to exponentially distributed random variables with the appropriate mean. Similiar equations are used for duration and time between storms. This provided storm average rainfall characteristics which were exponentially distributed, similiar to actual distributions. These are then converted to hourly sequences.

Hourly fluctuations of rainfall intensity within each event were obtained using a random number generator with a zero mean and a specified variance, so that

$$i_{jk} = i_j + \varepsilon_{ik}; \qquad \varepsilon_{ik} = N(0, \sigma_{\varepsilon_i}^{2})$$

where k and j refer to the hour and day, respectively. Fluctuations on the hourly intensities providing standard deviations (σ) of 0 and 100% of the hourly values were utilized with a constraint that the minimum value of i_{jk} = 0.01 in/hr. The hourly runoff flow rate (Q_{1jk}) was obtained from the hourly rainfall values using the rational method:

$$Q_{1jk} = Ci_{jk}$$

where C combines the runoff coefficient, drainage area and unit conversion factor. Using rainfall values in units of hundredths in/hr., a drainage area of 5,000 acres and a runoff coefficient of 0.7 similar to the 26th Ward data, a C value of 0.95 provides runoff flow rates in units of MG/hr.

The runoff concentration, C_1, was varied deterministically as a function of both interval between storms and storm duration as follows:

$$C_{1jk} = C_{10} f(d,\delta)$$

A linear increase of runoff concentration with interval between storms, δ, was utilized similar to the previous correlations obtained with the 26th Ward data[1]. An exponentially decreasing effect of storm duration, d, on runoff concentration provided a strong first flush effect.

Hourly values of dry weather sewage concentration and flow rate from 26th Ward plant data provided a significant diurnal fluctuation in the hourly sewage characteristics. In addition to the known diurnal fluctuation, random variability was assigned to the hourly values as follows:

$$C_{2jk} = C_{2k} + \varepsilon_{C_2k}; \quad \varepsilon_{C_2k} = N(0, \sigma_{C_2}{}^2)$$

$$Q_{2jk} = Q_{2k} + \varepsilon_{Q_2k}; \quad \varepsilon_{Q_2k} = N(0, \sigma_{Q_2}{}^2)$$

Fluctuations on the hourly values which provided standard deviations (σ) of 0, 10 and 20% of the hourly values were utilized.

The above parameters served as input data to a regulator which proportioned flow to the treatment plant influent and to the overflow as a function of the plant interceptor capacity, QI, as shown in Figure 1. A constant value of interceptor capacity typically equal to 2.5 times the average dry weather flow rate was used in the simulator. During an event, the overflow concentration was assumed equal to the plant concentration. The latter value was calculated each hour from a mass balance on runoff and sewage loads. No storage capacity of volume or load was assumed to exist in the interceptor and regulator.

The plant influent concentrations were then sampled according to a specified sampling routine and composited to provide the daily plant concentration. Two sampling routines with equal volume compositing were used in the analysis, the one historically used by NYC, every 4 hours but skipping the 2:00 A.M. sample as well as an hourly sampling routine. To account for plant measurement error, the daily plant concentration was varied randomly to provide the measured plant concentration as follows:

$$C_{Pj} = C_P + \varepsilon_{C_Pj}; \qquad \varepsilon_{C_Pj} = N(0, \sigma_{C_P}{}^2)$$

Standard deviations of measurement error in the range of 0 to 20% of the actual concentrations were utilized.

The daily loads and volumes at each location in the flow diagram were calculated by taking the sum of the hourly values over the day and the flow weighted average concentrations calculated as seen in Fig. 1. A summation of the total wet hours (α) occurring each day was also made. Output then consisted of the daily volumes and average flow weighted concentrations at each location, the measured plant concentration, the hourly runoff flow rates, along with the hours of rainfall and day of the year.

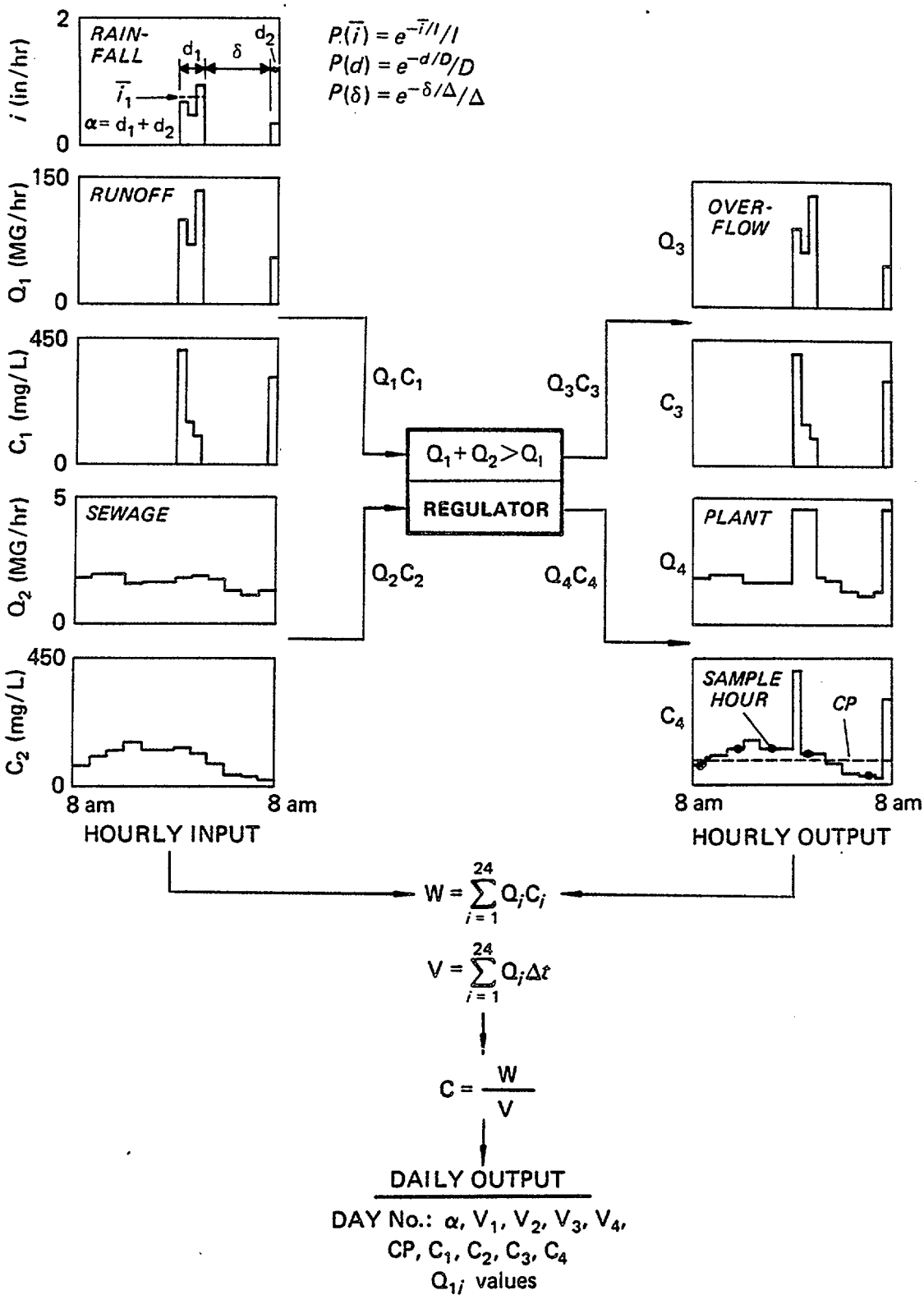

Figure 1. Hourly Simulator Schematic

All simulator runs were developed in BASIC and conducted on a TRS-80 microcomputer. The simulator studies were initially conducted with constant runoff and dry weather sewage characteristics. As the evaluation of the daily mass balance technique progressed, the degree of complexity of the input data was increased.

DAILY MASS BALANCE TECHNIQUE

The analysis of influent treatment plant data relies on flow and mass balance equations. The derivation is given in detail since they form the basis for all results in this report. The schematic is given in Fig. 1 which defines surface runoff flow, Q_1, and concentration, C_1; sewage flow, Q_2, and concentration, C_2; overflow flow Q_3, and concentration, C_3; and finally treatment plant flow, Q_4, and concentration, C_4. The time scale of the analysis is one day. Wet periods during the day (during rainfall) are subscripted by "w" and the length of rainfall is t_w. Dry periods, subscripted "d", have length t_d.

The flow balance equations are

Dry:

$$Q_{2d} = Q_{4d} \tag{1}$$

Wet:

$$Q_{1w} + Q_{2w} = Q_{4w} \qquad Q_{1w} + Q_{2w} \leq Q_I \tag{2}$$

$$Q_{1w} + Q_{2w} = Q_{3w} + Q_I \qquad Q_{1w} + Q_{2w} > Q_I \tag{3}$$

where Q_I is the interceptor capacity of the treatment plant. Overflow occurs if runoff plus sewage flow exceeds interceptor capacity.

The mass balance equations are similar:

Dry:

$$C_{2d}Q_{2d} = C_{4d}Q_{4d} \tag{4}$$

Wet:

$$C_{1w}Q_{1w} + C_{2w} + Q_{2w} = C_{4w}Q_{4w} \qquad Q_{1w} + Q_{2w} \leq Q_I \tag{5}$$

$$C_{1w}Q_{1w} + C_{2w}Q_{2w} = C_{3w}Q_{3w} + C_{4w}Q_{4w} \qquad Q_{1w} + Q_{2w} > Q_I \tag{6}$$

The critical assumption that allows the analysis to proceed is that the plant concentration, C_{4w}, is equal to the overflow concentration, C_{3w}, if one

occurs. Then eqs. (5) and (6) become:

$$C_{1w}Q_{1w} + C_{2w}Q_{2w} = C_{3w}(Q_{4w}) \qquad Q_{1w} + Q_{2w} \leq Q_I \tag{7}$$

$$= C_{3w}(Q_{3w} + Q_{4w}) \qquad Q_{1w} + Q_{2w} > Q_I \tag{8}$$

and using the flow balance eqs. (2) and (3) yields:

$$C_{1w}Q_{1w} + C_{2w}Q_{2w} = C_{3w}(Q_{1w} + Q_{2w}) \tag{9}$$

independent of whether the system is overflowing or not. This equation forms the basis of the daily average analysis. Two different models are obtained depending on the sampling routine followed at the treatment plant.

Flow Weighted Composite Sampling

If the treatment plant sample is a flow-weighted daily composite, then the reported plant concentration is:

$$C_P = \frac{Q_{4d}C_{4d}t_d + Q_{4w}C_{4w}t_w}{Q_{4d}t_d + Q_{4w}t_w} \tag{10}$$

Define wet and dry period volumes as, $V_{1w} = Q_{1w}t_w$, $V_{2d} = Q_{2d}t_d$, $V_{2w} = Q_{2w}t_w$, etc. Then using eqs. (1) and (4), and these definitions, eq. (10) becomes:

$$C_P = \frac{C_{2d}V_{2d} + C_{3w}V_{4w}}{V_{4w} + V_{4d}} \tag{11}$$

If the total daily volumes are defined as $V_2 = V_{2d} + V_{2w}$, and $V_4 = V_{4d} + V_{4w}$ then:

$$V_{2d} = V_2 - V_{2w} = V_2(1 - \alpha) \tag{12}$$

where $\alpha = V_{2w}/V_2$, the fraction of total sewage volume which corresponds to wet periods. For constant within day sewage flow α is the wet fraction of the day. Further,

$$\begin{aligned} V_{4w} &= V_4 - V_{4d} \\ &= V_4 - V_{2d} \\ &= V_4 - V_2(1 - \alpha) \end{aligned} \tag{13}$$

Hence the overflow concentration, C_0, is, from eq. (11):

$$C_0 = C_{3w} = \frac{C_P V_4 - (1-\alpha) V_2 C_{2d}}{V_4 - (1-\alpha) V_2} \tag{14}$$

The runoff concentration, C_R, follows from the wet weather mass balance eq. (9) applied over the wet period, t_w:

$$C_{1w} Q_{1w} t_w + C_{2w} Q_{2w} t_w = C_{3w} (Q_{1w} + Q_{2w}) t_w \tag{15}$$

$$C_{1w} V_{1w} + C_{2w} \alpha V_2 = C_{3w} (V_{1w} + \alpha V_2) \tag{16}$$

so that:

$$C_R = C_{1w} = C_{3w} + \frac{\alpha V_2}{V_{1w}} (C_{3w} - C_{2w}) \tag{17}$$

These equations (14 and 17) form the basis of the mass balance analysis reported previously (1,2).

Equal Volume Composite Sampling

If the treatment plant sampling is not flow weighted, then the defining equation for C_P is not equation (10), but:

$$C_P = (1 - \alpha) C_{2d} + \alpha C_{3w} \tag{18}$$

where α is the fraction of time corresponding to wet weather. Thus,

$$C_0 = C_{3w} = \frac{C_P - (1 - \alpha) C_{2d}}{\alpha} \tag{19}$$

gives the overflow concentration and equation (17), the runoff concentration as before.

SECTION 5

EVALUATION OF DAILY MASS BALANCE METHOD

FLOW WEIGHTED AND EQUAL VOLUME PLANT SAMPLING ANALYSES

The original flow weighted mass balance analysis was initially tested using the simulator output. All analyses were made with constant values of dry weather sewage flow and concentration (C_2 = 100 mg/ℓ, Q_2 = 2 MG/hr) and constant runoff concentration and volumetric runoff coefficient (C_1 = 50 mg/ℓ, C_V = 0.7). The New York City plant sampling procedure (10 AM, 2PM, 6PM, 10PM, 6AM) was also utilized.

Two comparisons were conducted: one with the interceptor capacity Q_I, equal to 5 MG/hr ($Q_I = 2.5Q_2$) and the other with a capacity of 2 MG/hr ($Q_I = Q_2$). For each comparison, the amount of wet weather data analyzed was varied by specifying α_{min}, and analyzing those data for which $\alpha \geq \alpha_{min}$. The results are summarized in Figure 2 which compares the ratio of the average runoff and overflow concentrations to the known concentrations. When the plant capacity is significantly greater than the dry weather flow, errors of 30 to 40% result when the majority of the rainfall is analyzed ($\alpha_{min} \leq$ 4 hours). This occurs due to the non-flow weighted plant sampling. During high intensity storms the concentration at the plant would be lower during the wet period than during the dry period while the flow is higher due to the larger interceptor capacity.

Since New York City composites samples according to equal volume increments independent of flow, too great a weight is proportioned to the dry weather conditions at the lower flows. When only these larger duration storms are analyzed this proportioning error is reduced with no error resulting for storms lasting 24 hours, since all concentrations are constant over the day. Of course only a small proportion of the total rainfall is analyzed at the larger α_{min} thus neglecting the major portion of the data.

When the interceptor capacity is equal to the dry weather flow, plant flow is constant for both wet and dry days and no flow proportioning error results. The overall error in the analysis is therefore significantly reduced to a maximum of about 12% at α_{min} of 8 to 12 hours. When the majority of the data is analyzed, at $\alpha_{min} \leq 4$ hours, the error is small, $\pm$ 3-5%.

Underestimation of the runoff and overflow loads at the lower storm duration (some negative values result) help to balance out the overestimation of the larger storms.

The results using the equal sample volume mass balance analysis to calculate the runoff and overflow concentrations are shown in Figure 3. The previous large error for the higher capacity plants has been removed with interceptor capacity having no effect on runoff concentration and only slight effect on overflow concentrations. This modification adds no increased complexity to the daily mass balance technique and was therefore used for the remainder of the analyses using the New York City plant sampling routine. Depending on the actual method of plant compositing, either technique can be employed to analyze plant data.

EFFECT OF RUNOFF COEFFICIENT

Using the equal sample volume mass balance technique, the effect of runoff coefficient, C_V, on runoff and overflow loads and concentrations was evaluated. The actual C_V value used in the hourly simulator was 0.7. It was varied from 0.3 to 0.9 in the daily mass balance analysis. The loads are seen to vary significantly with the C_V coefficient as indicated in Figure 4 while runoff and overflow concentrations (Figure 5) vary only slightly for C_V from 0.5 to 0.9. The lowest C_V coefficient of 0.3 resulted in numerous negative overflow volumes, an indication that the estimated runoff coefficient is unrealistically low for this test case. The above analysis confirms the results previously obtained using the 26th Ward data: the daily mass balance technique provides good estimates of runoff and overflow quality independent of the quantity estimation. The latter is still required for a good estimate of the load.

DIURNAL SEWAGE VARIATION

The hourly sewage flow and BOD_5 concentrations during dry weather from the 26th Ward Plant (Oct. 12 & 13, 1976, Figure 6) were used in the hourly simulator. Random rainfall was again utilized and average daily treatment plant sample concentrations were generated. To maintain consistency with the previous analysis using a constant concentration of 100 mg/ℓ, the concentrations were adjusted slightly (+5%) to yield a flow weighted average concentration over the day of 100 mg/ℓ. The daily runoff and overflow concentrations were calculated from the daily mass balance technique using a constant average sewage concentration and flow. Figure 7 indicates that no consistent bias in the runoff or overflow concentrations was introduced to the analysis as a function of rainfall duration analyzed α_{min}. Bias is the average difference for the year of record analyzed between the actual and estimated daily concentrations. For storms with durations $\geq$ 2 and 4 hours, the bias introduced in the daily mass balance analysis was greater than the bias using a constant sewage input but lower when all storms (α_{min} = 0) and

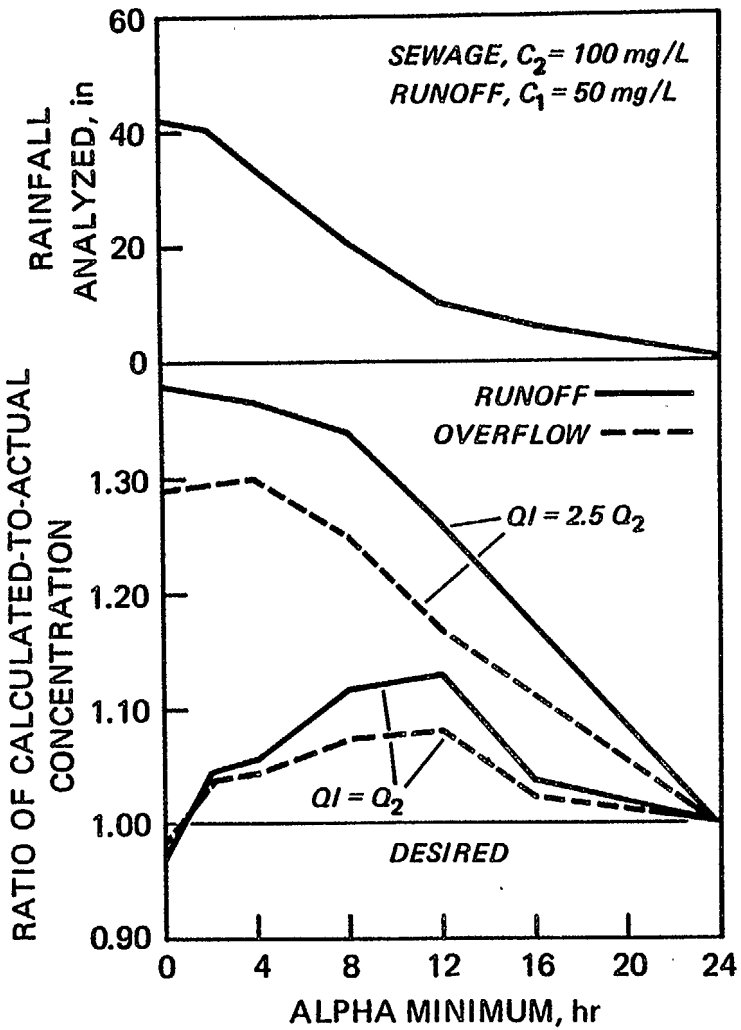

Figure 2. Effect of Minimum Rainfall Analyzed and Interceptor Capacity on Runoff and Overflow Values using Flow Weighted Analysis

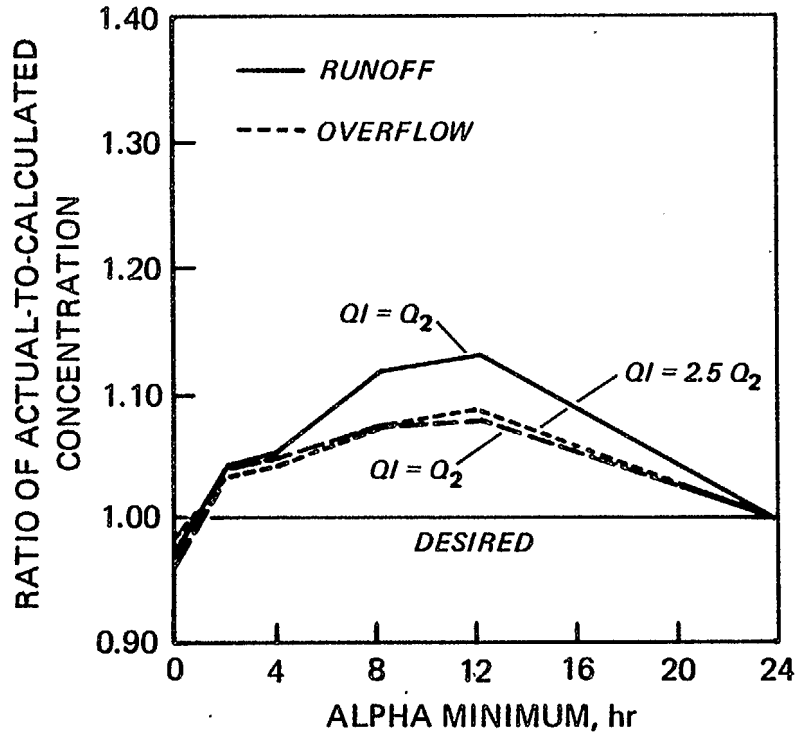

Figure 3. Effect of Minimum Rainfall Analyzed and Interceptor Capacity on Runoff and Overflow Values using Equal Volume Analysis

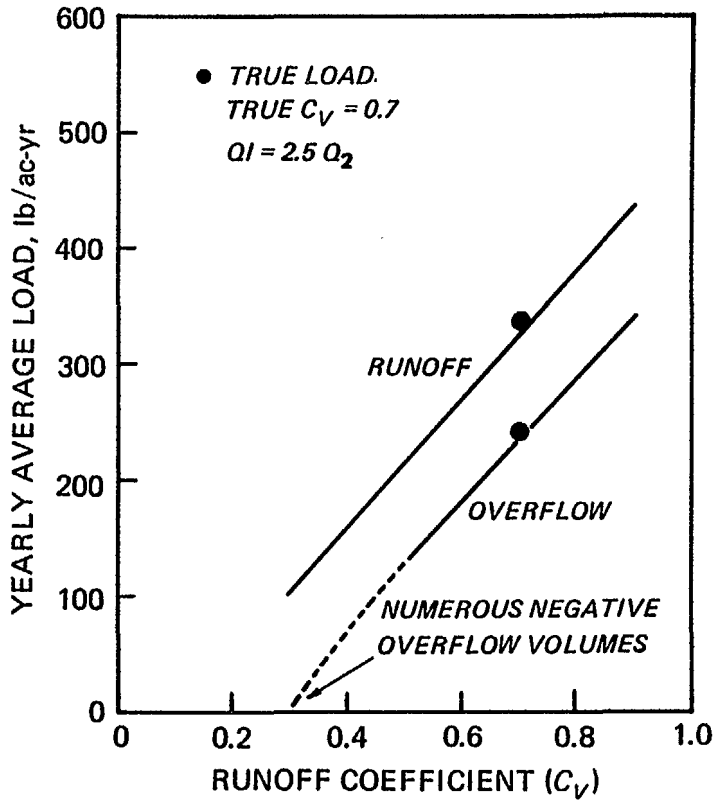

Figure 4. Effect of Assumed Runoff Coefficient on Calculated Runoff and Overflow Loads - Equal Volume Analysis

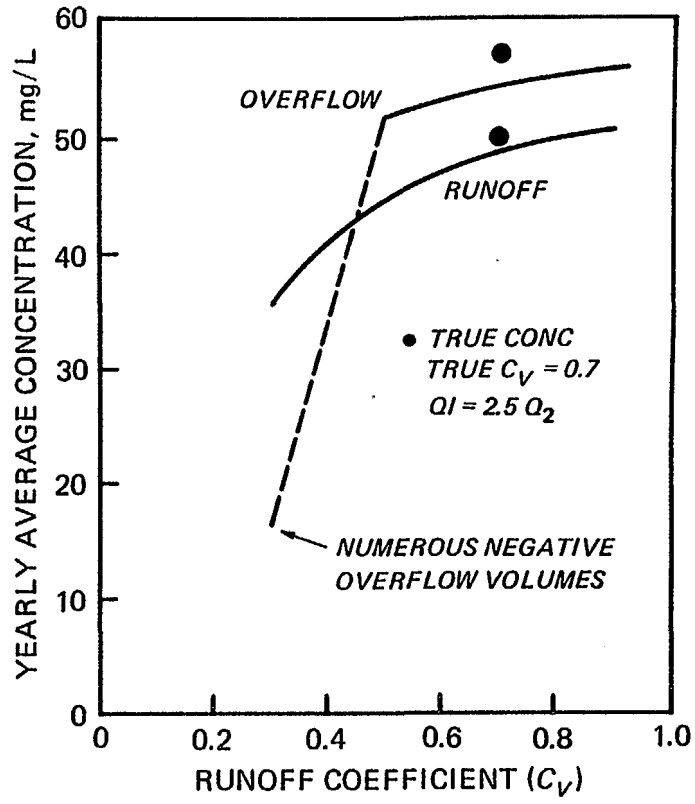

Figure 5. Effect of Assumed Runoff Coefficient on Calculated Runoff and Overflow Concentrations - Equal Volume Analysis

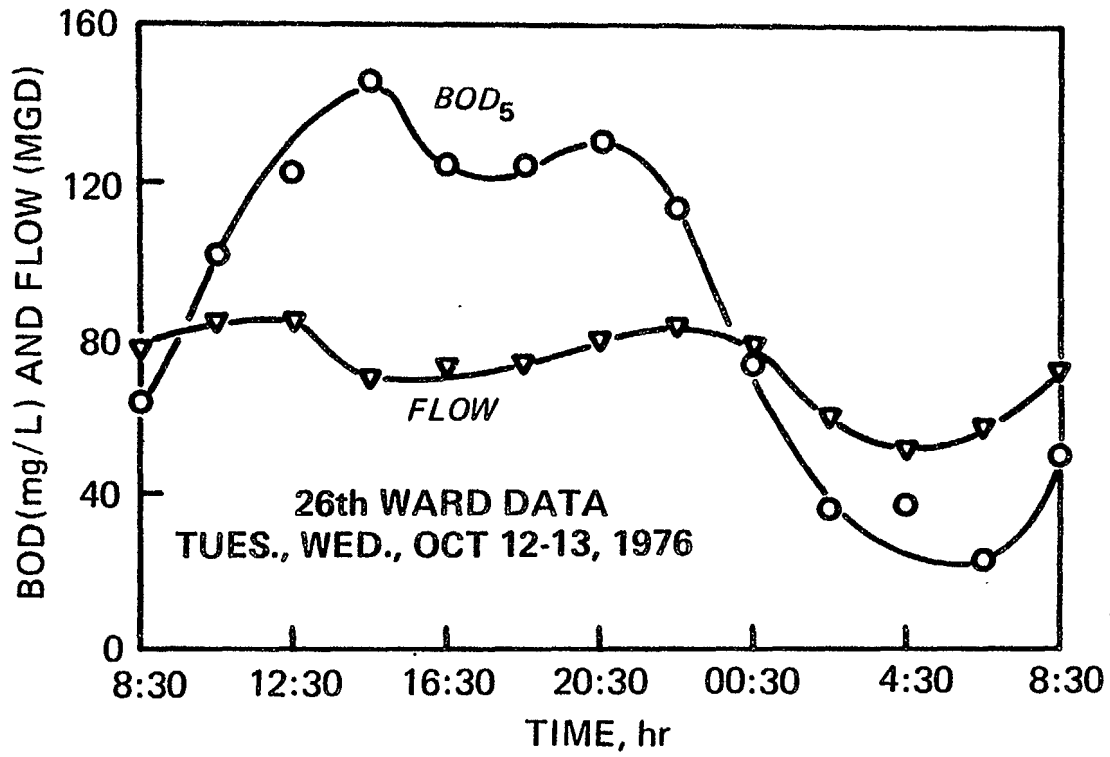

Figure 6. Diurnal Dry Weather Sewage Values for 26th Ward Plant[4], Tuesday, Oct. 12-13, 1976

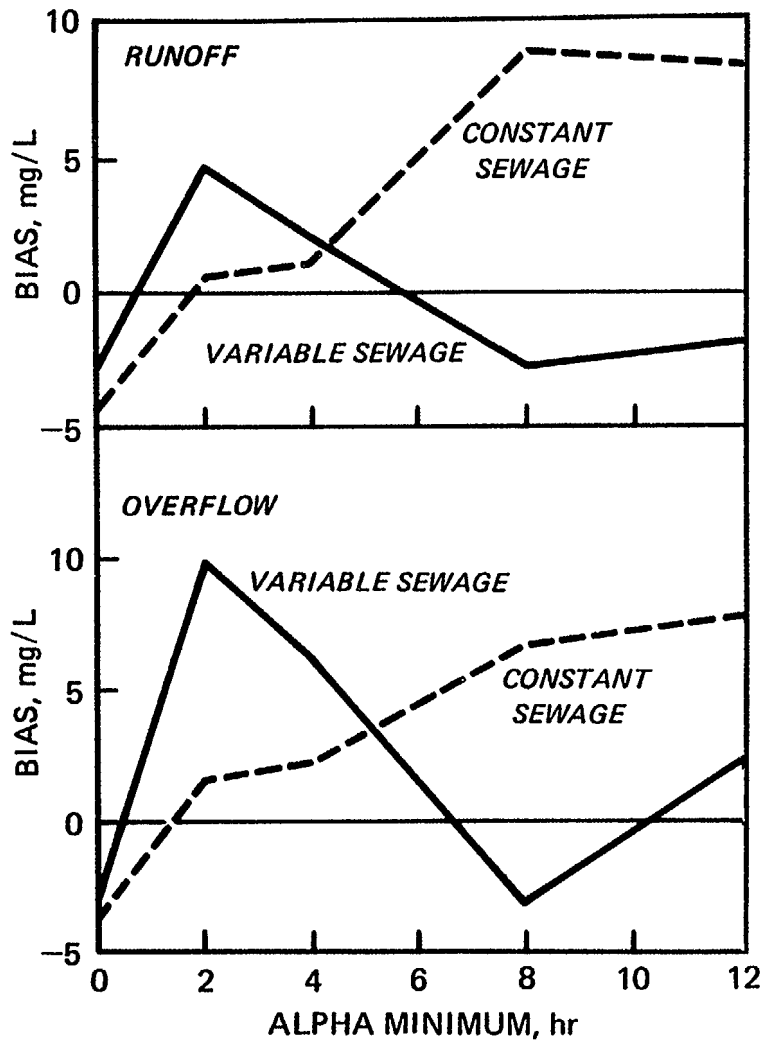

Figure 7. Bias of Runoff and Overflow Concentrations for Diurnal and Constant Sewage Characteristics

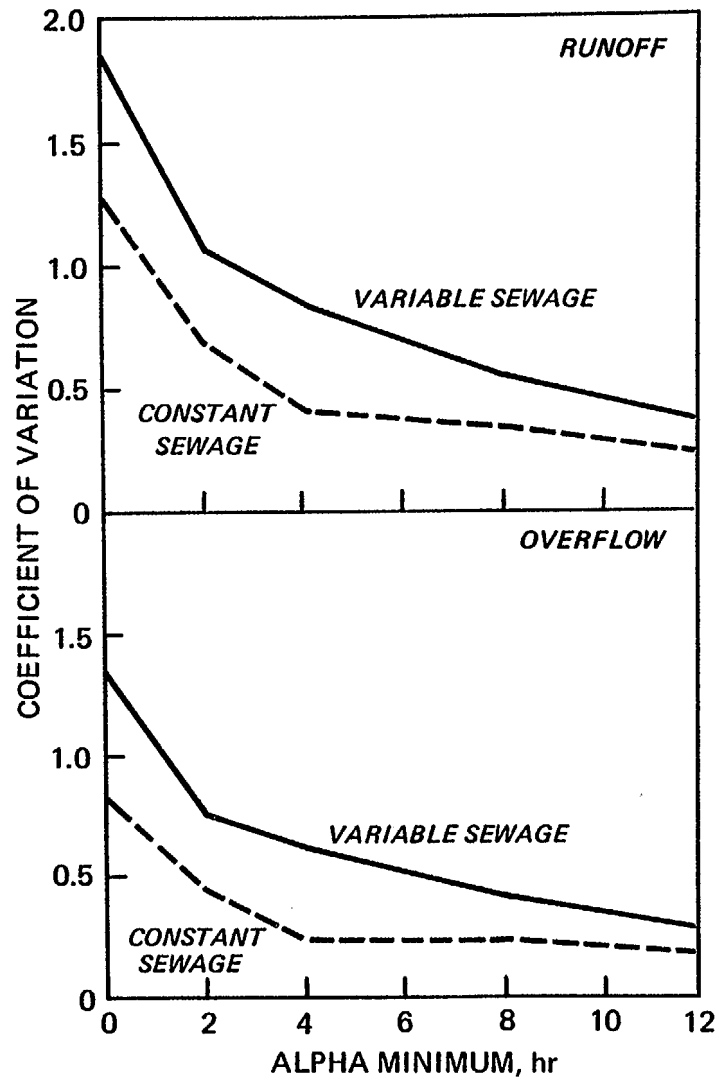

Figure 8. Variability of Runoff and Overflow Concentrations for Diurnal and Constant Sewage Characteristics

those with durations $\geq$ 8 hours were analyzed.

Although no significant bias was introduced using diurnal dry weather concentrations, the variability of the results was significantly increased as shown in Figure 8. For both runoff and overflow concentrations the diurnal sewage characteristics caused the coefficient of variation of calculated versus actual values to be 40 to 50% greater than those using constant sewage characteristics. The longer the storm duration analyzed, the lower the variability.

Figure 9 shows typical histograms for the above results at $\alpha_{min} = 4$ hours with the actual compared to the calculated values. The diurnal sewage histograms show a significantly greater number of errant values at both the negative and positive ends of the histogram than the constant sewage characteristics. Using greater rainfall intensities and durations with diurnal sewage characteristics gave similar variabilities as those above. This increased variability is inherent in an analysis that ignores the actual time of day and partitions the day into only wet and dry periods without regard to the actual times that it rained. Since this information is directly available from the rainfall record it can be incorporated within the mass balance framework at the expense of some complexity of the resulting formulas. As shown in the next chapter, this refinement significantly improves the performance of the mass balance analysis.

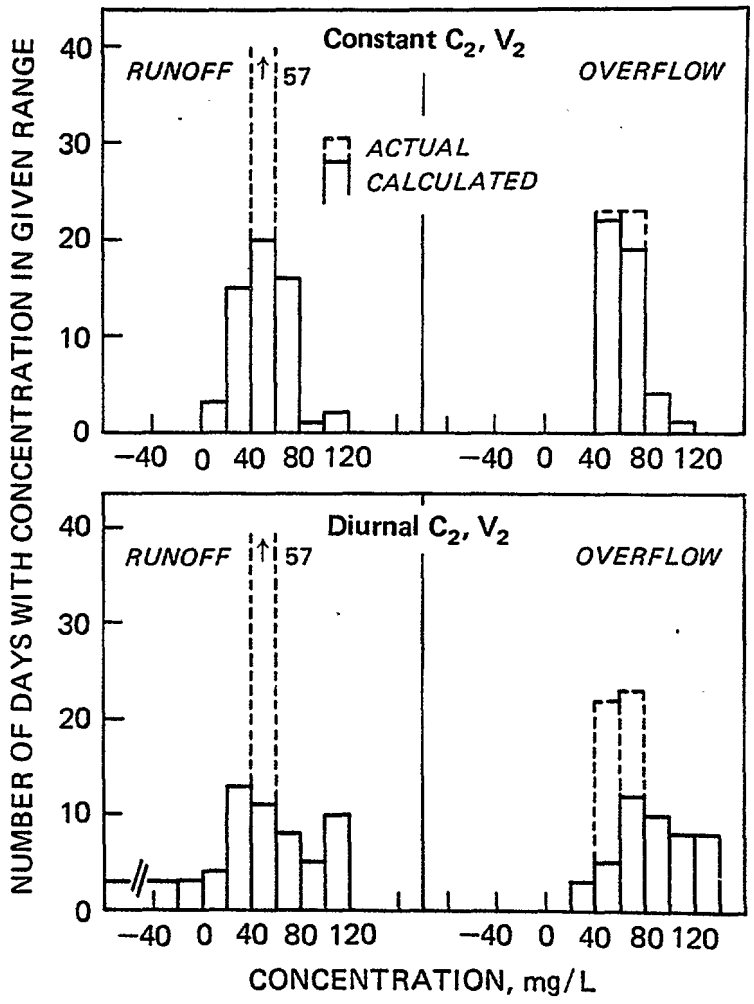

Figure 9. Runoff and Overflow Histograms for Constant and Diurnal Sewage Characteristics at α_{min} = 4 hr.

SECTION 6

HOURLY MASS BALANCE METHOD

DERIVATION OF ESTIMATING EQUATIONS

The estimating equations for runoff and overflow concentrations follow from flow and mass balance equations applied to each hour during the day. Letting i denote the hour of concern the flow balance and mass balance equations are:

Dry Hour:

$$Q_{2i} = Q_{4i} \tag{20}$$

$$C_{2i}Q_{2i} = C_{4i}Q_{4i} \tag{21}$$

Wet Hour:

$$Q_{1i} + Q_{2i} = Q_{4i} \qquad Q_{1i} + Q_{2i} \leq Q_I \tag{22}$$

$$C_{1i}Q_{1i} + C_{2i}Q_{2i} = C_{4i}Q_{4i} \qquad Q_{1i} + Q_{2i} \leq Q_I \tag{23}$$

$$Q_{1i} + Q_{2i} = Q_{3i} + Q_I \qquad Q_{1i} + Q_{2i} > Q_I \tag{24}$$

$$C_{1i}Q_{1i} + C_{2i}Q_{2i} = C_{3i}Q_{3i} + C_{4i}Q_I \qquad Q_{1i} + Q_{2i} > Q_I \tag{25}$$

The assumption: $C_{4i} = C_{3i}$ yields the wet hour mass balance equation, analogous to equation (9); that is, equation (25) becomes:

$$C_{1i}Q_{1i} + C_{2i}Q_{2i} = C_{3i}(Q_{1i} + Q_{2i}) \tag{26}$$

The dry hour mass balance equation (21) similarly becomes:

$$C_{2i}Q_{2i} = C_{3i}Q_{2i} \tag{27}$$

so that when Q_{1i} is zero, equation (26) is equivalent to equation (27).

Thus it applies to all hours, wet or dry.

Constant Runoff Approximation

Since the application of these methods is for New York City data, only equal volume treatment plant sampling will be investigated. Consider a

variable, s_i, which is equal to one if a sample is taken at the ith hour, and zero otherwise. The reported daily treatment plant concentration is:

$$C_P = \sum_i s_i C_{4i} / \sum_i s_i \tag{28}$$

For NYC sampling, $s_i = 1$ for the 5 hours that samples are taken (10 AM, 2 PM, 6 PM, 10 PM, 6 AM) and $\Sigma_i s_i = 5$.

Using the mass balance equation (26) yields:

$$C_P = \left(\sum_i \frac{s_i C_{1i} Q_{1i}}{Q_{1i}+Q_{2i}} + \sum_i \frac{s_i C_{2i} Q_{2i}}{Q_{1i}+Q_{2i}} \right) / \sum_i s_i \tag{29}$$

This equation cannot be solved for the hourly runoff concentrations since there are 24 unknowns, C_{1i}, and one equation. The assumption that suggests itself is that the runoff concentration is constant: $C_{1i} = C_1 = C_R$. For this situation, equation (29) can be solved for C_R:

$$C_R = \frac{C_P \sum_i s_i - \sum_i \frac{s_i Q_{2i} C_{2i}}{Q_{1i} + Q_{2i}}}{\sum_i \frac{s_i Q_{1i}}{Q_{1i}+Q_{2i}}} \tag{30}$$

The flow weighted overflow concentration is defined to be:

$$C_O = \frac{\sum_i o_i Q_3 C_{3i}}{\sum_i o_i Q_{3i}} \tag{31}$$

where $o_i = 1$ if $Q_{1i} + Q_{2i} > Q_I$ indicating an overflow occurs at hour i, and zero otherwise. The overflow concentrations follow from the mass balance:

$$C_{3i} = \frac{C_R Q_{1i} + C_{2i} Q_{2i}}{Q_{1i} + Q_{2i}} \tag{32}$$

and the flows from the flow balance:

$$Q_{3i} = Q_{1i} + Q_{2i} - Q_I \qquad Q_{1i} + Q_{2i} > Q_I \tag{33}$$

$$Q_{3i} = 0 \qquad \text{otherwise.}$$

The daily average interceptor capacity can be estimated from the difference of sewage plus runoff flow and the recorded volume of influent to the treatment plant. From equations (22) and (24), the wet weather flow balance equations;

$$\sum_i (Q_{1i} + Q_{2i} - Q_{3i}) = V_4 \tag{34}$$

where V_4 is the reported daily volume treated at the plant. Expressing this equation in terms of hours for which overflows occurred ($o_i = 1$) and did not occur ($o_i = 0$ and $1 - o_i = 1$) yields:

$$\sum_i (1 - o_i)(Q_{1i} + Q_{2i}) + \sum_i o_i(Q_{1i} + Q_{2i} - Q_{3i}) = V_4 \tag{35}$$

and using equation (33) for overflow hours yields:

$$\sum_i (1 - o_i)(Q_{1i} + Q_{2i}) + Q_I \sum_i o_i = V_4 \tag{36}$$

or:

$$Q_I = [V_4 - \sum_i (1 - o_i)(Q_{1i} + Q_{2i})]/\sum_i o_i \tag{37}$$

The solution technique for Q_I is iterative. An initial daily average interceptor capacity is chosen (say $Q_I = 0$). Then equation (33) is applied to each hour, which established Q_{3i} and $o_i = 1$ for $Q_{3i} > 0$ and $o_i = 0$ for $Q_{3i} = 0$. These are used in equation (37) to compute a new estimate of Q_I. The cycle is repeated until Q_I converges. A maximum of three iterations has been required when analyzing simulator output.

Constant Overflow Approximation

An alternative to the assumption that runoff concentration is constant, is that overflow concentration is constant: $C_{3i} = C_0$. The treatment plant sample concentration can be expressed as:

$$C_P = \sum_i^+ s_i C_{3i} + \sum_i^\circ s_i C_{2i} \quad / \sum s_i \tag{38}$$

where $\sum_i^+$ is the sum over all wet hours ($Q_{1i} > 0$) and $\sum_i^\circ$ is the sum over all

dry hours ($Q_{1i} = 0$). For the wet hours, the treatment plant concentration is the overflow concentration $C_{4i} = C_{3i}$; for the dry hours it is the sewage flow: $C_{4i} = C_{2i}$. Assuming that the overflow concentration is constant yields from equation (38):

$$C_0 = \frac{C_P \sum_i s_i - \sum_i^{\circ} s_i C_{2i}}{\sum_i^{+} s_i} \tag{39}$$

This formula is valid only if at least one sampling time corresponds to a wet hour and $\sum_i^{+} s_i > 0$.

Since runoff concentration is also required, it is convenient to compute the flow weighted average runoff concentration:

$$C_R = \sum_i^{+} Q_{1i} C_{1i} / \sum_i^{+} Q_{1i} \tag{40}$$

This is available from the mass balance equation (26). Summing it over the wet hours only yields:

$$\sum_i^{+} Q_{1i} C_{1i} = C_3 \sum_i^{+} (Q_{1i} + Q_{2i}) - \sum_i^{+} Q_{2i} C_{2i} \tag{41}$$

so that:

$$C_R = C_0 \left(1 + \frac{\sum_i^{+} Q_{2i}}{\sum_i^{+} Q_{1i}}\right) - \frac{\sum_i^{+} Q_{2i} C_{2i}}{\sum_i^{+} Q_{1i}} \tag{42}$$

These equations complete the specification of the hourly mass balance techniques.

PERFORMANCE OF HOURLY MASS BALANCE METHOD

The results of the hourly mass balance analysis which assumes constant overflow concentration, C_0 are shown in Figure 10 for the diurnal sewage characteristics. Comparing the results to those in Figure 9, it is seen that variability is significantly reduced by using the hourly analysis method.

The hourly mass balance analysis results indicated that the variability did not significantly decrease when only longer duration storms were analyzed. However, upon inspection of the individual daily values, it was determined that low rainfall intensities produced the greatest errant values. Figure 11

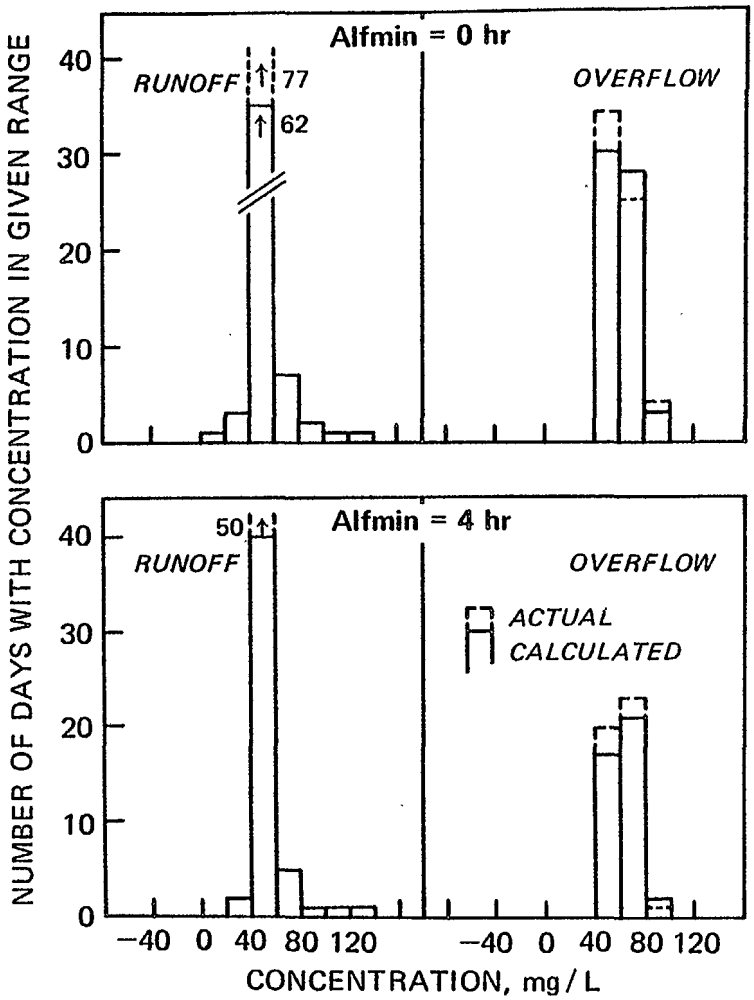

Figure 10. Runoff and Overflow Histograms for Diurnal Sewage Characteristics Using Hourly Constant C_o Analysis

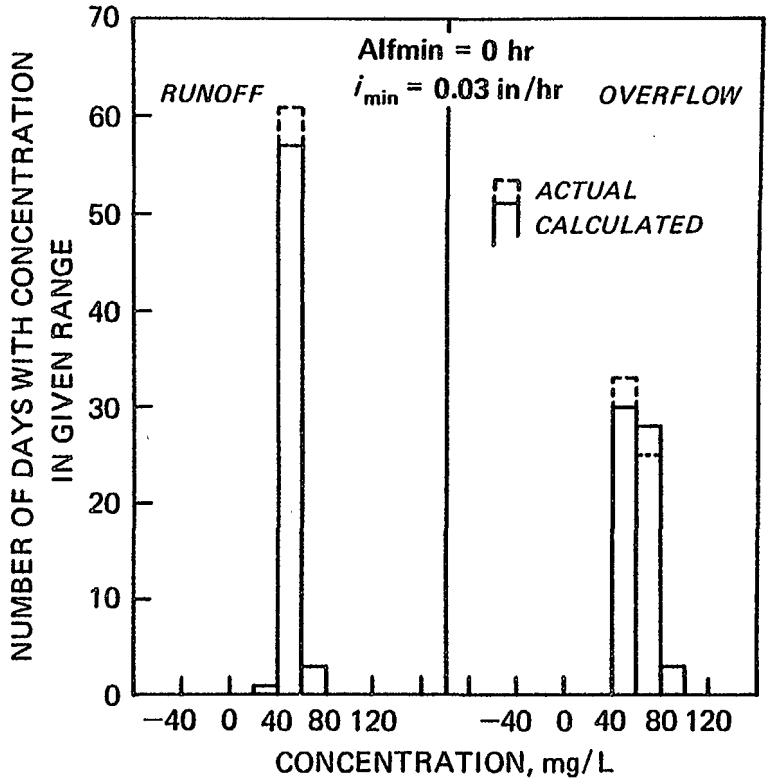

Figure 11. Runoff and Overflow Histograms for Diurnal Sewage Characteristics Using Hourly Constant C_o Analysis for i_{min} = 0.03 in/hr.

indicates that deleting rainfalls less than 0.02 in/hr significantly reduced the variability in runoff concentrations. No effect on overflow variability occurred since the low intensity storms did not overflow.

The low rainfall intensities that showed the greatest error in the above analysis were normally the longer rainfalls from 8 to 12 hr. duration which occurred over the early morning periods. Since NYC sampling was every 4 hours except for 2 AM which was skipped, only 1 or 2 wet period samples were obtained for these long duration storms giving too great a weight to the dry weather data.

Table 1 summarizes the results for both runoff and overflow concentrations in terms of bias and coefficient of variation. Define the error in each estimated daily flow weighted runoff and overflow concentration as:

$$\varepsilon_{rj} = C_{R\text{-true}(j)} - C_{R\text{-estimated}(j)} \tag{43}$$

$$\varepsilon_{oj} = C_{O\text{-true}(j)} - C_{O\text{-estimated}(j)} \tag{44}$$

where j is the jth day of record analyzed. Then the bias is defined as:

$$\bar{\varepsilon}_r = \frac{1}{N}\sum_{j=1}^{N} \varepsilon_{rj} \tag{45}$$

$$\bar{\varepsilon}_o = \frac{1}{N}\sum_{j=1}^{N} \varepsilon_{oj} \tag{46}$$

for N days of record analyzed. They are the average of the difference between true and estimated daily concentrations. The variability of the daily estimates are represented by the coefficients of variation of runoff and overflow which are the ratios of the error standard deviation to the true concentrations:

$$\nu_R = \sigma_{CR}/\bar{C}_R \tag{47}$$

$$\nu_O = \sigma_{CO}/\bar{C}_O \tag{48}$$

where $\bar{C}_R$ and $\bar{C}_O$ are averages of the true runoff and overflow concentrations and:

$$\sigma^2_{CR} = \frac{1}{N}\sum_{j=1}^{N} (\varepsilon_{rj} - \bar{\varepsilon}_r)^2 \tag{49}$$

TABLE 1. DAILY AND HOURLY MASS BALANCE RESULTS FOR DIURNAL SEWAGE CHARACTERISTICS

RAINFALL ANALYZED				RUNOFF		OVERFLOW	
Alpha Min.	Total Inches	Wet Days	Overflow Days	Bias mg/ℓ	Coeff. of Var.	Bias mg/ℓ	Coeff. of Var.
(a) Daily Mass Balance Method - Equal Sampling Volumes							
0	42.4	117	84	-2.9	1.85	-3.1	1.34
2	40.3	85	68	+4.7	1.06	+9.9	0.75
4	33.6	57	46	+2.0	0.83	+6.2	0.61
8	21.2	27	22	-2.8	0.55	-3.1	0.41
12	10.5	11	10	-1.9	0.36	+2.1	0.28
(b) Daily Mass Balance Method - Equal Sampling Volumes - Greater Intensity and duration							
0	85.5	126	95	+11.0	1.15	+12.0	0.94
2	83.5	101	76	+4.0	0.89	+7.4	0.72
(c) Hourly Mass Balance Method - Constant C_O							
0	37.6	77	62	+2.3	0.32	+1.6	0.14
4	29.5	50	42	+3.0	0.34	+2.0	0.17
8	20.2	25	21	+4.4	0.50	+4.2	0.21
(d) Hourly Mass Balance Method - Constant C_O - $i > 0.02$ in/hr							
0	36.1	61	61	+0.2	0.10	+0.7	0.10
(e) Hourly Mass Balance Method - Constant C_R							
0	38.6	82	63	0	0.0004	+0.4	0.0005

$$\sigma_{CO}^2 = \frac{1}{N}\sum_{j=1}^{N} (\varepsilon_{oj} - \bar{\varepsilon}_o)^2 \tag{50}$$

For the daily analysis methods, Table 1 (a) and (b), the bias in both the runoff and overflow concentration is small (< 10 mg/ℓ except in one case) but the coefficient of variation is substantial, especially for $\alpha_{min} < 4$ where it exceeds one, indicating that the standard deviation exceeds the mean concentration.

For the hourly analysis method with C_0 assumed constant, Table 1(c), the coefficient of variation for both runoff and overflow are significantly reduced and the effect of α_{min} is eliminated. This is a significant improvement since short duration storms can make a significant contribution to overflow and they contain the information from which first flush effects are extracted, as shown subsequently.

A significant reduction in variability (ν_R and $\nu_0 \sim 0.1$) can be achieved if low intensity rainfalls are ignored Table 1(d). This residual variation is due to the assumption of constant overflow concentration which is not the case if the sewage concentration has a diurnal variation, as it does for these simulations. Note, however, that the small bias (< 1.0 mg/ℓ) and coefficient of variation (0.1) indicate that this method of analysis is quite good and indeed extracts the runoff and overflow concentrations from the composited treatment plant sampling information.

The hourly analysis method with assumed constant C_R, performs exactly since it conforms to the assumptions of the simulation output used for this case (constant C_R) and it has exact knowledge of the diurnal sewage fluctuations. The small errors and variation are due to numerical roundoff. This result also serves as a check on the computer program implementation of the method.

Measurement Error

In analysis of the plant samples, a certain amount of variability exists in the laboratory technique. Table 2 summarizes the variability data from Standard Methods[7] for three of the parameters measured at 26th Ward.

TABLE 2. VARIABILITY OF LABORATORY ANALYSES[7]

Parameter	Average Concentration (mg/ℓ)	Standard Deviation (mg/ℓ)	Coefficient of Variation (%)
Suspended Solids	15	5.2	33
Suspended Solids	242	24	10
Volatile Susp. Solids	170	11	6.5
BOD_5	175	26	15

Since the mass balance analysis techniques depend upon differences of measured concentrations it was suspected that measurement error would increase significantly the variability in computed runoff and overflow concentrations. In order to simulate the effect of measurement error, the simulated treatment plant concentration was corrupted by adding Gaussian random variables with zero mean and standard deviation = STD corresponding to reasonable measurement precision.

A measurement error of 6 and 12% of the average plant concentration (85-88 mg/ℓ) was used in the analysis which is in the range of the volatile and suspended solids variabilities. This was equivalent to standard deviations of 5 and 10 mg/ℓ.

Figures 12 and 13 show that a significant variability was reintroduced into both runoff and overflow concentration estimates due to measurement error for both the constant overflow and constant runoff analyses. The smaller 5 mg/ℓ standard deviation perturbation produces one-half the variability of the 10 mg/ℓ perturbation.

Table 3 summarizes the measurement error results. An α_{min} = 1 hr was used for all runs. Due to the measurement errors in the plant concentrations, some bias was again introduced into the analysis for the approximately 60 days analyzed. Duplicate runs using different random numbers on the perturbations provided similar results with bias and variability slightly different. Greater variability resulted when the low rainfall intensities, 0.01 and 0.02 in/hr, were included in the constant C_R analysis.

The variability on the overflow is consistently lower than that of the runoff concentrations in both analysis schemes. This results since the overflow concentration is directly related to the plant concentration while the runoff concentration involves an additional subtraction due to the mass balance analysis.

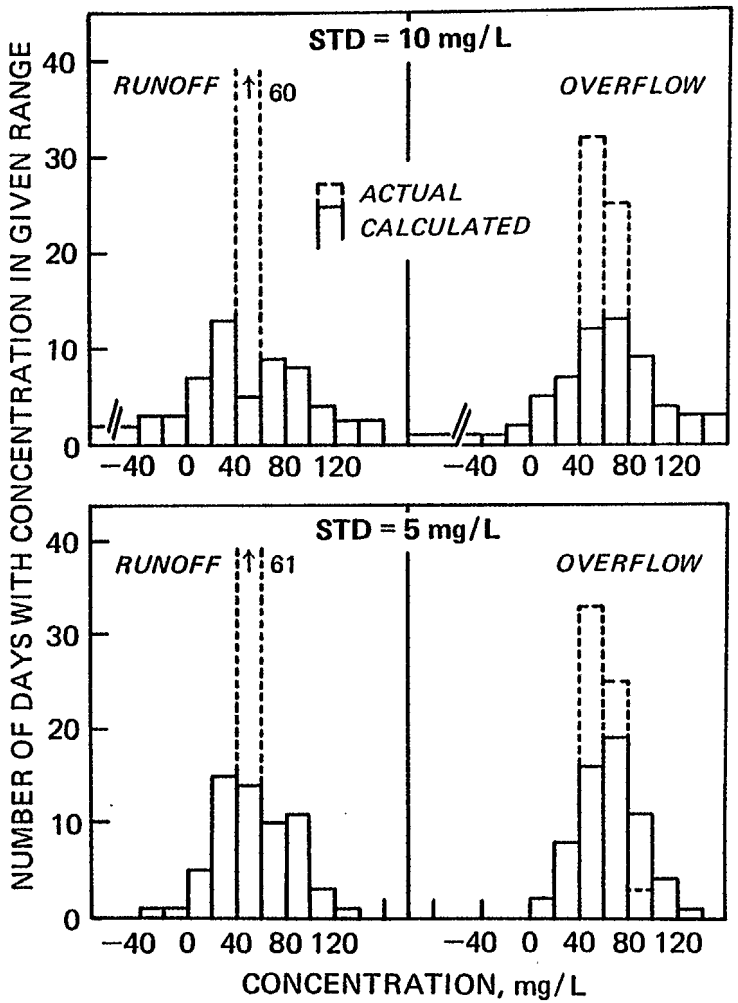

Fig. 12. Runoff and Overflow Histograms for Constant C_O Analysis with Measurement Error for i_{min} = 0.03 in/hr.

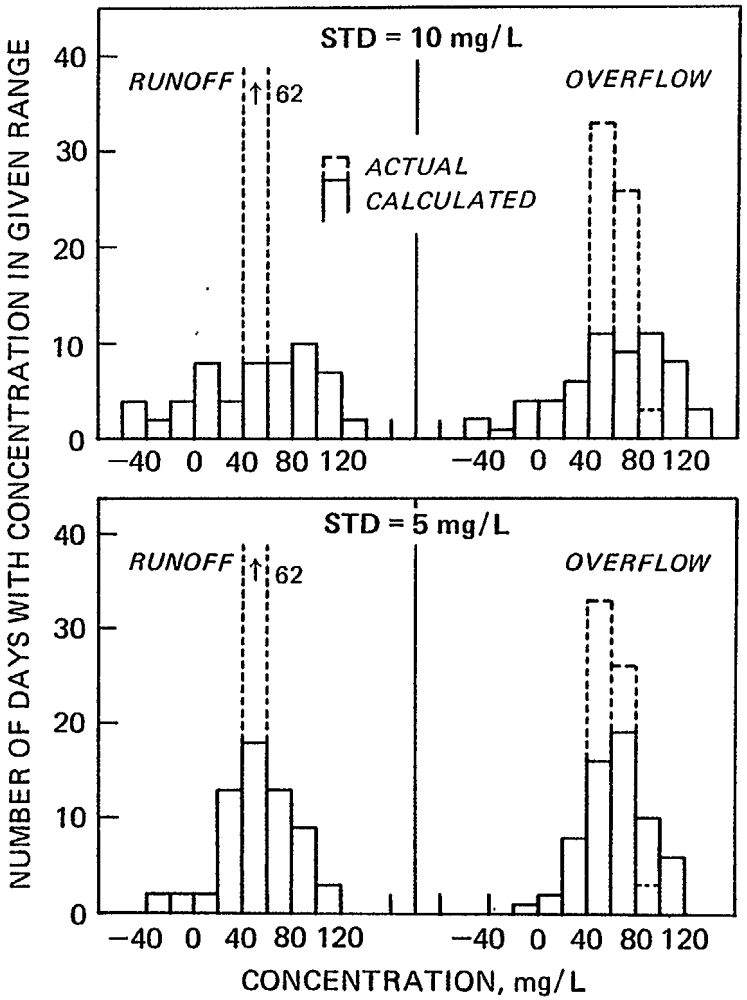

Fig. 13. Runoff and Overflow Histograms for Constant C_R Analysis with Measurement Error for i_{min} = .03 in/hr.

TABLE 3. MEASUREMENT ERROR RESULTS

S.D. of Plant Concentration Perturbation, mg/ℓ	RAINFALL ANALYZED			RUNOFF		OVERFLOW	
	Minimum Intensity in/hr	Total Inches	Wet Days	Bias mg/ℓ	Coeff. of Var.	Bias mg/ℓ	Coeff. of Var.
CONSTANT C_0 ANALYSIS							
10	0.03	35.4	60	+ 4.0	1.08	+ 3.3	0.71
10[a]	0.03	36.1	61	- 3.9	1.20	- 3.0	0.79
5	0.03	36.1	61	+ 4.4	0.62	+ 3.9	0.39
CONSTANT C_R ANALYSIS							
10	0	38.6	82[b]	+ 4.0	1.54	+ 1.5	0.89
10	0.03	36.6	62	+ 5.2	1.23	+ 4.6	0.79
10[a]	0.03	36.6	62	+13.2	1.22	+10.9	0.83
5	0.03	36.6	62	+ 2.6	0.61	+ 2.3	0.39

(a) same as above with different random numbers for measurement error

(b) 63 overflow days

Measurement errors of 6 and 12% completely swamp any previous differences in output between the two analysis techniques (constant C_R or constant C_0) with coefficients of variation of 40 to 120% reintroduced into the estimated overflow and runoff concentrations. This effect of the measurement error precludes use of any single estimated daily value for comparison to observed overflow concentrations. Rather an average value must be utilized in order to lessen the measurement error effect.

Dry Weather Random Variability

Errors in the estimation of dry weather sewage characteristics of both 10 and 20% on the hourly flows and concentrations were evaluated using the constant C_0 analysis.

Figure 14 shows that 10 and 20% perturbations on the dry weather concentrations causes significant variability in both runoff and overflow concentrations. Similar perturbations on the average sewage flows cause a much lower variability to result on the overflow and runoff values (Figure 15). This is to be expected since overflow concentration is a function of only the plant and dry weather concentrations, not flow. A summary of the dry weather random variability results is shown in Table 4. Once again bias is not significantly affected by the dry weather variability. The dry weather concentration perturbations of 10 and 20% cause coefficients of variation of 45 and 76% to result for the calculated runoff concentrations. Values of only 15 to 17% resulted in the runoff concentrations when the same perturbations were applied to the sewage flows. Much of this latter effect is due to the inherent model error ($\sim$ 10%) since there is very little difference in the results for the 10 and 20% perturbations. A number of runs have been made with the flow perturbations which verified the similarity of results for both the 10 and 20% perturbations.

Wet Weather Variability

In addition to dry weather variability, runoff concentration and flow variability within a storm are known to occur. Their effect was analyzed by varying runoff characteristics in the hourly simulator. Runoff concentration as a function of rainfall duration, similar to a first flush effect was specified as follows:

$$C_R(t) = C_{R_\infty} + (C_{R_{max}} - C_{R_\infty})\, e^{-\beta_R t}$$

The above expression provides the peak runoff concentration during the first hour of the storm. The concentration then exponentially decreases to a constant value, the time to attain the constant value a function of the rate coefficient, β_R. For the hourly simulation data the following parameters were used in the above equation:

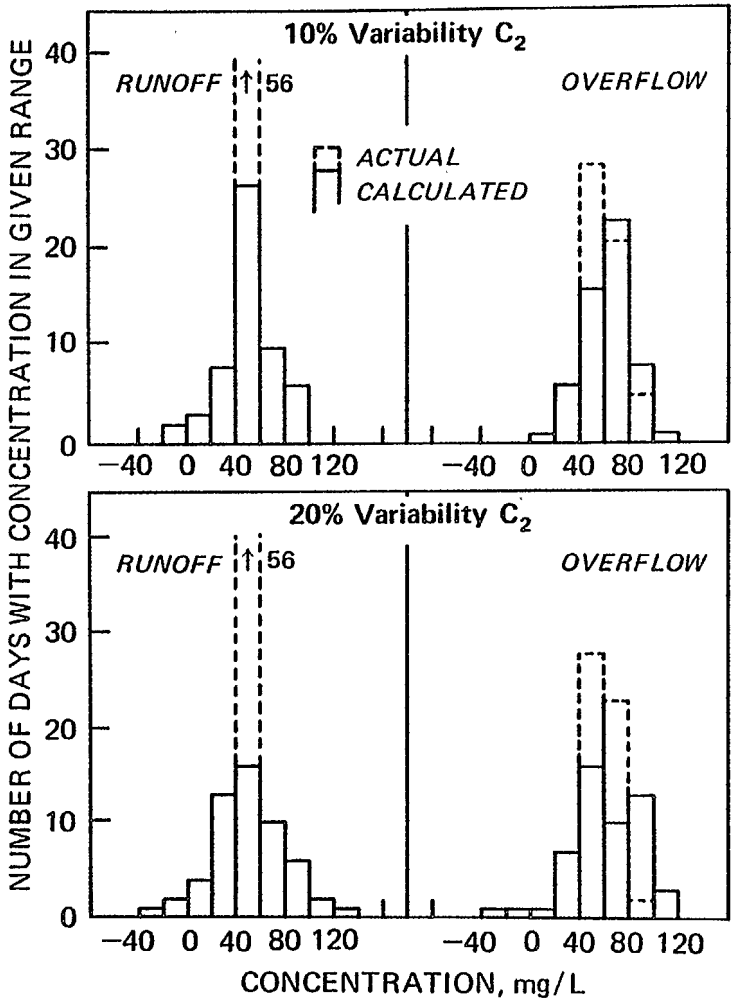

Figure 14. Runoff and Overflow Histograms for Variability on Dry Weather Sewage Concentration

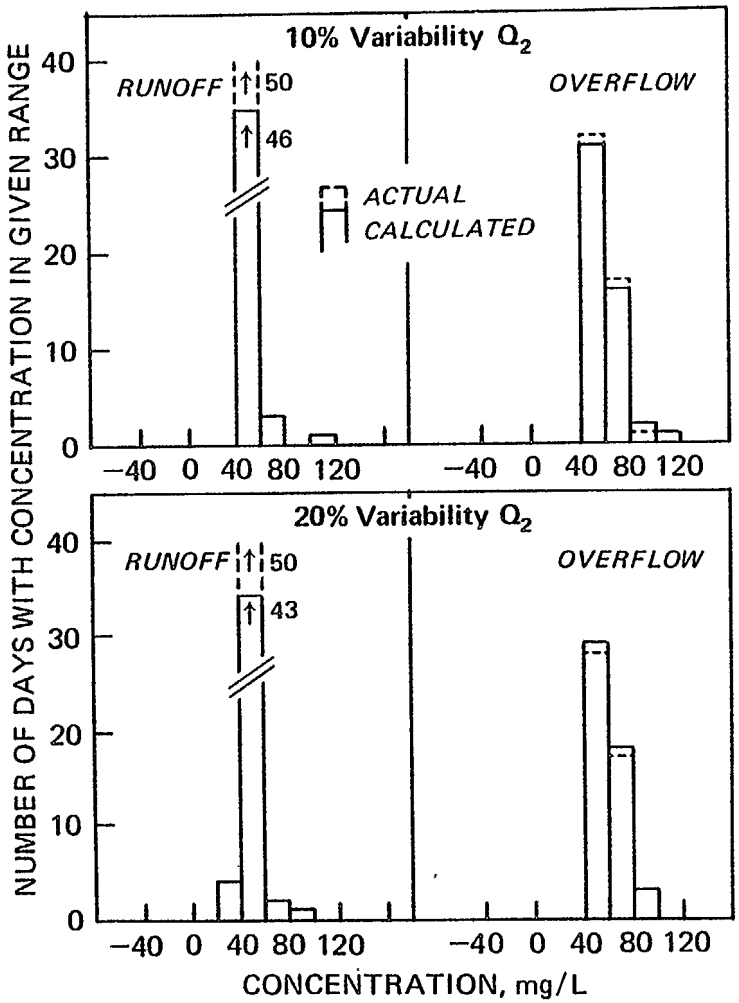

Figure 15. Runoff and Overflow Histograms for Variability on Dry Weather Sewage Flow Rate

TABLE 4. DRY WEATHER RANDOM VARIABILITY RESULTS

S.D. of Perturbation on Hourly Sewage Values, P %		RAINFALL ANALYZED			RUNOFF		OVERFLOW	
C_2	Q_2	i_{min} in/hr	Total inches	Wet Days	Bias mg/ℓ	Coeff. of Var.	Bias mg/ℓ	Coeff. of Var.
10	0	0.03	45.4	56	+ 0.4	0.45	+ 1.0	0.30
20	0	0.03	35.9	56	+ 4.0	0.76	+ 1.4	0.46
0	10	0.03	37.3	50	+ 1.6	0.17	+ 0.8	0.15
0	10[a]	0.03	36.6	52	+ 1.4	0.15	+ 1.7	0.16
0	10	0	38.3	63[b]	+ 0.7	0.29	+ 1.0	0.15
0	20	0.03	34.3	50	- 0.4	0.19	+ 0.4	0.16
0	20[a]	0.03	36.5	50	+ 1.4	0.17	+ 0.8	0.15
0	20[a]	0.03	36.6	52	+ 1.4	0.15	+ 1.3	0.16
20	20	0.03	37.2	59	+ 1.2	0.88	- 0.4	0.57

(a) different rainfall than above

(b) 51 overflow days

$C_{R_\infty} = 40$ mg/ℓ

$C_{R_{max}} = 1000$ mg/ℓ

$\beta_R = 2$/hr

which provided the following runoff concentrations during a storm:

time (hr)	$C_R(t)$ (mg/ℓ)
1	170
2	58
3	42
4	40
∞	40

To more closely simulate actual storm events, the rainfall intensity was varied randomly during an event similar to the random generation of measurement error and dry weather sewage variability. A high degree of variability was given to the hourly rainfall intensity by using a standard deviation of 100% of the mean intensity with the constraint of a minimum intensity of 0.01 in/hr.

Table 5 indicates that the runoff and overflow variabilities due to averaging errors were significantly increased from a previous standard deviation of 5 to 15 mg/ℓ for constant rainfall characteristics to 30 to 40 mg/ℓ for variable characteristics. The major portion of the variability was due to the varying runoff concentrations with varying hourly intensities providing a relatively slight effect. Using both the constant C_0 and constant C_R computational techniques provided similar variabilities due to averaging errors. Use of minimum rainfall intensities and minimum durations had negligible effect on averaging error.

With the strong first flush effect in the runoff concentrations, neglecting low duration rainfalls significantly reduces the actual runoff and overflow concentrations as seen in Table 6. As the minimum number of samples is increased, from 1 to 3, the storm duration is significantly increased and thus more dilute samples obtained. The minimum duration of 4 hours also has reduced runoff and overflow concentrations, but not as significantly as a two sample minimum. Thus the technique of minimizing variability by excluding shorter duration events is unacceptable when a significant first flush is present. Minimum rain intensities from 0.01 to 0.04 in/hr had no effect on average runoff concentrations.

TABLE 5. RUNOFF AND OVERFLOW VARIABILITIES FOR VARYING RUNOFF CHARACTERISTICS

WET DAYS	STD (mg/ℓ)	RD_{min} (# samples)	i_{min} (0.01 in/hr)	σ_{CR} (mg/ℓ)	σ_{CO} (mg/ℓ)
		C_1 VARIABILITY - CONSTANT C_0 ANALYSIS			
78	0	1	1	37.0	30.1
67	0	1	2	34.2	29.6
60	0	1	3	33.2	28.0
78	10	1	1	81.4	53.2
		C_1 VARIABILITY - CONSTANT C_R ANALYSIS			
78	0	1	1	33.9	32.5
		C_1 & Q1 VARIABILITY - CONSTANT C_0 ANALYSIS			
90	0	1	1	38.7	32.9
67	0	1	3	39.6	33.1
69	0	$1(\alpha_{min}=4)$	1	34.9	30.7
90	10	1	1	82.8	54.8
90	10	1	1	76.4	50.0
69	10	$1(\alpha_{min}=4)$	1	69.7	50.2
		C_1 & Q1 VARIABILITY - CONSTANT C_R ANALYSIS			
90	0	1	1	39.2	34.1

TABLE 6. EFFECT OF NEGLECTING SHORT STORMS ON RUNOFF AND OVERFLOW CONCENTRATIONS FOR SIGNIFICANT FIRST FLUSH

RD_{min} (# samples)	α_{min} (hr.)	Flow Weighted Average Concentration (mg/ℓ)	
		Runoff, C_1	Overflow, C_3
1	1	63.0	66.9
2	-	50.4	56.0
3	-	44.7	50.8
1	4	60.3	64.6

Conclusion

The conclusion is that measurement error causes significant variability in the daily estimates and that it is affected by the characteristics of the rainfall analyzed. Errors in dry weather sewage and wet weather concentrations also cause significant variability but not to the extent of measurement errors. These results suggest that a theoretical analysis of the relationships that produce these variations would be useful in understanding the results obtained using simulation techniques and may suggest strategies for mitigating their impact such as ignoring small rainfall intensities in the analysis.

SECTION 7

THEORETICAL ANALYSIS OF MEASUREMENT ERROR

To evaluate the effect of the important parameters on measurement error, a theoretical statistical analysis of the errors on overflow and runoff concentrations was conducted using the hourly-constant C_0 analysis formulas.

VARIABILITY OF OVERFLOW CONCENTRATIONS

The statistical error analysis seeks to compute the mean and standard deviation of the perturbations in overflow and runoff concentration due to the random measurement errors. Let ε_p be the random measurement error with zero mean and standard deviation, σ_p. The overflow concentration, computed using equation (39), is:

$$C_0 + \varepsilon_o = \frac{(C_p + \varepsilon_p)\ \Sigma_i s_i - \Sigma_i^{\circ} s_i C_{2i}}{\Sigma_i^{+} s_i} \tag{51}$$

where ε_o is the perturbation in the overflow concentration, C_0, produced by ε_p. Subtracting equation (39) from equation (51) yields:

$$\varepsilon_o = \frac{\varepsilon_p \Sigma_i s_i}{\Sigma_i^{+} s_i} \tag{52}$$

Let $\Sigma_i s_i = N_s$, the number of sampled hours (= 5 for NYC sampling). Let $\Sigma_i^{+} s_i$ = RD which is the number of wet hours sampled. Thus equation (52) becomes:

$$\varepsilon_o = \frac{N_s}{RD}\ \varepsilon_p \tag{53}$$

Note that RD is a random variable since its value depends on the timing of the rainfall. Hence the statistics of ε_o depend not only on the statistics of the measurement error, ε_p, but also on the rainfall characteristics as they affect RD.

The bias introduced by measurement error is the statistical average (i.e. the expected value) of ε_o:

$$E\{\varepsilon_o\} = N_s \; E\{\frac{\varepsilon_p}{RD}\} \tag{54}$$

since N_s is constant. And since measurement error is independent of rainfall:

$$E\{\varepsilon_o\} = N_s \; E\{\frac{1}{RD}\} \;\; E\{\varepsilon_p\} \tag{55}$$

If the measurement error has zero mean as used in the simulator then $E\{\varepsilon_o\} = 0$ or no bias is introduced by plant measurement error into overflow concentration. This agrees with the simulation results.

The variance of a random variable, x, is defined by the equation:

$$V\{x\} = E\{ \; (x-E\{x\})^2\} \tag{56}$$

i.e. the variance is the average of the square of the deviations between x and its average. By squaring and combining terms this equation becomes:

$$V\{x\} = E\{x^2\} - E^2\{x\} \tag{57}$$

Applying this to ε_o yields:

$$V\{\varepsilon_o\} = N_s^2 \; E\{\frac{\varepsilon_p^2}{RD^2}\} - E^2\{\varepsilon_o\} \tag{58}$$

but the second term is zero and, by independence of ε_p and RD, the result is:

$$V\{\varepsilon_o\} = N_s^2 \; \sigma_p^2 \; E\{\frac{1}{RD^2}\} \tag{59}$$

where $\sigma_p^2 = V\{\sigma_p^2\} = E\{\varepsilon_p^2\}$ since $E\{\varepsilon_p\} = 0$. Hence the variance of ε_o is linearly related to the expected value of $1/RD^2$.

In order to compute $E\{1/RD^2\}$ it is necessary to examine the possible values of RD and compute their probability. This is evaluated in Appendix I. Comparing the above techniques to results of three sets of simulator data shows that the theoretical value of $E\{1/RD^2\}$ to be comparable to the simulator results (Table 7).

TABLE 7. COMPARISON OF THEORETICAL AND ACTUAL STATISTICAL PARAMETER FOR OVERFLOW MEASUREMENT ERROR

Number of Days Analyzed	Expected Value of $1/RD^2$		
	Theoretical	Actual (=95% Confidence Limits)	
59	0.64	0.65	(0.106)
82	0.64	0.73	(0.086)
121	0.64	0.60	

The importance of $E\{1/RD^2\}$ can be seen from equation (59) which express the variance of the overflow estimation error in terms of the variance of the plant measurement error. For NYC sampling, $N_s = 5$, and the variance magnification factor is $N_s^2\ E\{1/RD^2\} = 25(0.6) = 15$ so that if the standard deviation of plant measurement error is 5 mg/ℓ, the standard deviation of the errors in overflow concentration estimates is predicted to be: $\sigma_{CO} = \sqrt{15} \times 5$ mg/ℓ = 19.4 mg/ℓ. As shown subsequently these theoretical predictions agree with simulator results.

VARIABILITY OF RUNOFF CONCENTRATIONS

A similar analysis is possible for runoff concentration. The runoff concentration is determined from the overflow concentration, dry weather concentrations and flow ratios:

$$C_R = C_O\ (1 + \frac{\Sigma_i^+ Q_{2i}}{\Sigma_i^+ Q_{1i}}) - \frac{\Sigma_i^+ Q_{2i} C_{2i}}{\Sigma_i^+ Q_{1i}} \tag{60}$$

For an overflow perturbation, ε_o, the resulting runoff perturbation, ε_r is:

$$\varepsilon_r = \varepsilon_o (1 + V_{2w}/V_1) \tag{61}$$

where $V_{2w} = \Sigma_i^+ Q_{2i}$, the total sewage volume during wet hours and V_1 is the total runoff flow during wet hours. The variance is found using equation (57):

$$V\{\varepsilon_r\} = E\{\varepsilon_o^2\ (1 + V_{2w}/V_1)^2\} - E^2\{\varepsilon_o (1 + V_{2w}/V_1)\}$$

$$= E\{\frac{N_s^2\ \varepsilon_p^2}{RD^2}(1 + V_{2w}/V_1)^2\} - E^2\{\frac{N_s \varepsilon_p}{RD^2}(1 + V_{2w}/V_1)\} \tag{62}$$

Since ε_p is independent of rainfall characteristics the second term of this equation is zero and the result is:

$$V\{\varepsilon_r\} = N_s^2\sigma_p^2\ E\{\frac{1}{RD^2}\ (1 + V_{2w}/V_1)^2\} \tag{63}$$

Consider the ratio: V_{2w}/V_1. For constant sewage flow, Q_{2i}, and runoff flow, Q_{1i} during a rainfall event of duration d:

$$\frac{V_{2w}}{V_1} = \frac{\Sigma_i^+ Q_{2i}}{\Sigma_i^+ Q_{1i}} = \frac{d\ Q_2}{d\ Q_1} = \frac{Q_2}{Q_1}$$

which is independent of duration and is only a function of rainfall intensity i, through the runoff flow $Q_1 = f\ C_V Ai = Ci$ for drainage area, A, units conversion factor, f, and runoff coefficient, C_V. Thus:

$$V\{\varepsilon_r\} = N_s\sigma_p^2\ E\{\frac{1}{RD^2}\}\ E\{(1 + Q_2/Q_1)^2\}$$

$$= V\{\varepsilon_o\}\ E\{(1 + Q_2/Q_1)^2\} \tag{64}$$

and the runoff variation due to measurement error is increased over the overflow variation by the expression $E\{(1 + Q_2/Q_1)^2\}$.

To compute this expectation, let:

$$\beta = Q_2/fC_VA = Q_2/C$$

so that the expectation $E\{(1 + Q_2/Q_1)^2\}$ becomes:

$$E\{(1 + \beta/i)^2\} = \int_{i_{min}}^{\infty} (1 + \beta/i)^2\ P_i(i)di / \int_{i_{min}}^{\infty} P_i(i)di \tag{65}$$

where $P_i(i) = \frac{1}{I}\ e^{-i/I}$, the probability density function of the intensity. The integral in the numerator can be evaluated numerically. For example,

$$\int_{i_{min}}^{\infty} (1 + \beta/i)^2 \, P_i(i)di = \sum_{n=1}^{\infty} \left(1 + \beta/i_{n+1/2} \right)^2 \left(e^{-i_{n+1}/I} - e^{-i_n/I} \right) \quad (66)$$

where $i_{n+1/2} = (i_n + i_{n+1})/2$ and i_n = 0, 0.015, 0.025, 0.035, etc. (in/hr).

For the following values used in the simulator: $\overline{Q}_2$ = 2 MG/hr, C_V = 0.7 and A = 5000 acres, f = 2.715 x 10^{-4} MG/acre-hundredths inch, then β = 2.105, and:

$$E\{1 + \beta/i)^2\} = 3.36$$

Two runs were used to compare the above theoretical values to the simulator results and close agreement was obtained as shown in Table 8.

TABLE 8. COMPARISON OF THEORETICAL AND ACTUAL STATISTICAL PARAMETERS FOR RUNOFF MEASUREMENT ERROR

No. of Days Analyzed	β			$E\{(1 + \beta/i)^2\}$		
	Theoretical	Actual	(95% C.L.)	Theoretical	Actual	(95% C.L.)
82	2.105	2.107	(0.058)	3.36	3.106	(0.632)
80	2.105	2.109	(0.059)	3.36	3.213	(0.624)

With the above theoretical framework, the effect of varying both the number of samples taken during a rain event, RD, and the rainfall constant intensity, i, can be evaluated.

Figure 16 indicates that significant reductions in the error magnification factors, $E\{1/RD^2\}$ and $E\{(1 + \beta/i)^2\}$, occur at i_{min} > 0.02 in/hr and $RD_{min} \sim 2$ samples. Again good agreement between theoretical and simulated values was obtained as shown. These results are important for two reasons. They confirm that the methods used to compute $E\{1/RD^2\}$ and $E\{(1 + \beta/i)^2\}$, as described above are correct. In addition they suggest that significant reductions in the variability of overflow and runoff concentration estimates can be achieved by using an $RD_{min} \sim 2$ and $i_{min} \sim 0.02$ (in/hr). These results provide the explanation of the large increases in estimated concentration variability that were observed to occur in the simulator investigations if short duration - low intensity storms were included in the analysis.

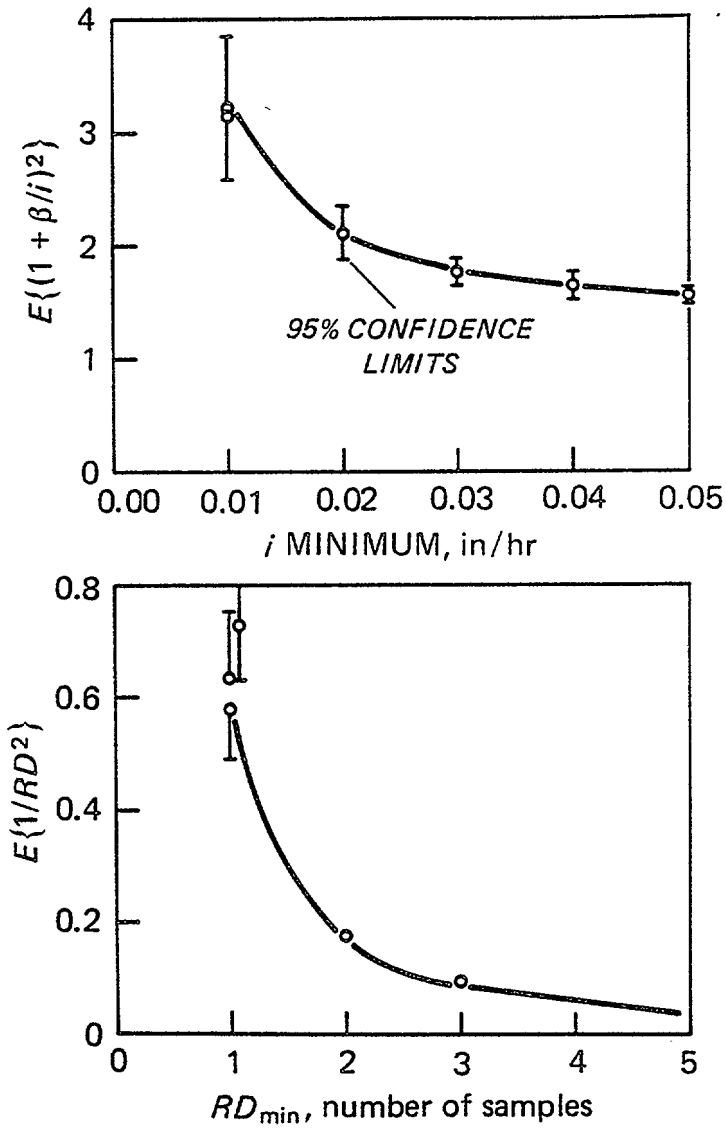

Figure 16. Effect of Number of Samples and Rainfall Intensity on Runoff and Overflow Variability Parameters

COMPARISONS OF PREDICTED AND SIMULATED EFFECTS OF MEASUREMENT ERRORS

The theoretical expressions for the effect of measurement errors on overflow, equation (59), and runoff, equation (63), concentration variances can be compared directly to simulation results.

The effect of measurement error on runoff variability is given in Figure 17. The only source of variability in this figure is measurement error since constant runoff and sewage characteristics were used. Good agreement between theoretical and observed values is obtained with a significant degree of scatter in the data. Again the major effect is the number of samples, RD_{min}, rather than rainfall intensity analyzed.

Incorporation of variable sewage characteristics introduces additional variability into the analysis as shown in Figure 18. Using a storm interval of 1 day caused additional variability since the number of multiple rain events occurring on one day was significantly increased over the three day interval data. With the presence of significant model error, due to the daily averaging of plant samples, as well as significant measurement errors, it is necessary to combine these effects theoretically in order to compare to simulator results. Assuming the measurement variance and averaging variance are independent, the total variability can be obtained by summing the variances.

$$V\{C_R\} = V_1\{C_R\} + V_2\{C_R\}$$

where $V_1\{C_R\}$ = Measurement Error Variance

$V_2\{C_R\}$ = Averaging Error Variance

$V\{C_R\}$ = Total Variance = σ^2_{CR}

For the constant runoff characteristics used in the analysis, the magnitude of the averaging error is small compared to the measurement error as seen in Figure 19. The data again show a significant amount of scatter but in good agreement with the predicted values. Figure 20 shows the predicted and observed variabilities on the overflow concentrations. As predicted, rainfall intensity has no effect on overflow variability. To obtain overflow values at low rainfall intensities, for this data the interceptor capacity was set at the maximum dry weather flow rate of 2.32 MG/hr. instead of 5 MG/hr. used previously.

When measurement errors are combined with wet weather concentration variation significantly higher variabilities were obtained. The effect of both averaging and model errors on the runoff and overflow variabilities is given in Figures 21 and 22. For runoff variability, the averaging error is still less than the measurement error using a STD = 10 mg/ℓ. However for overflow the situation is reversed when more than two samples are taken during a storm. Good agreement between predicted and observed variabilities is obtained with a relatively large degree of scatter in the data due to the one year data base (only 5 to 10 values exist for an RD_{min} of 3). The good

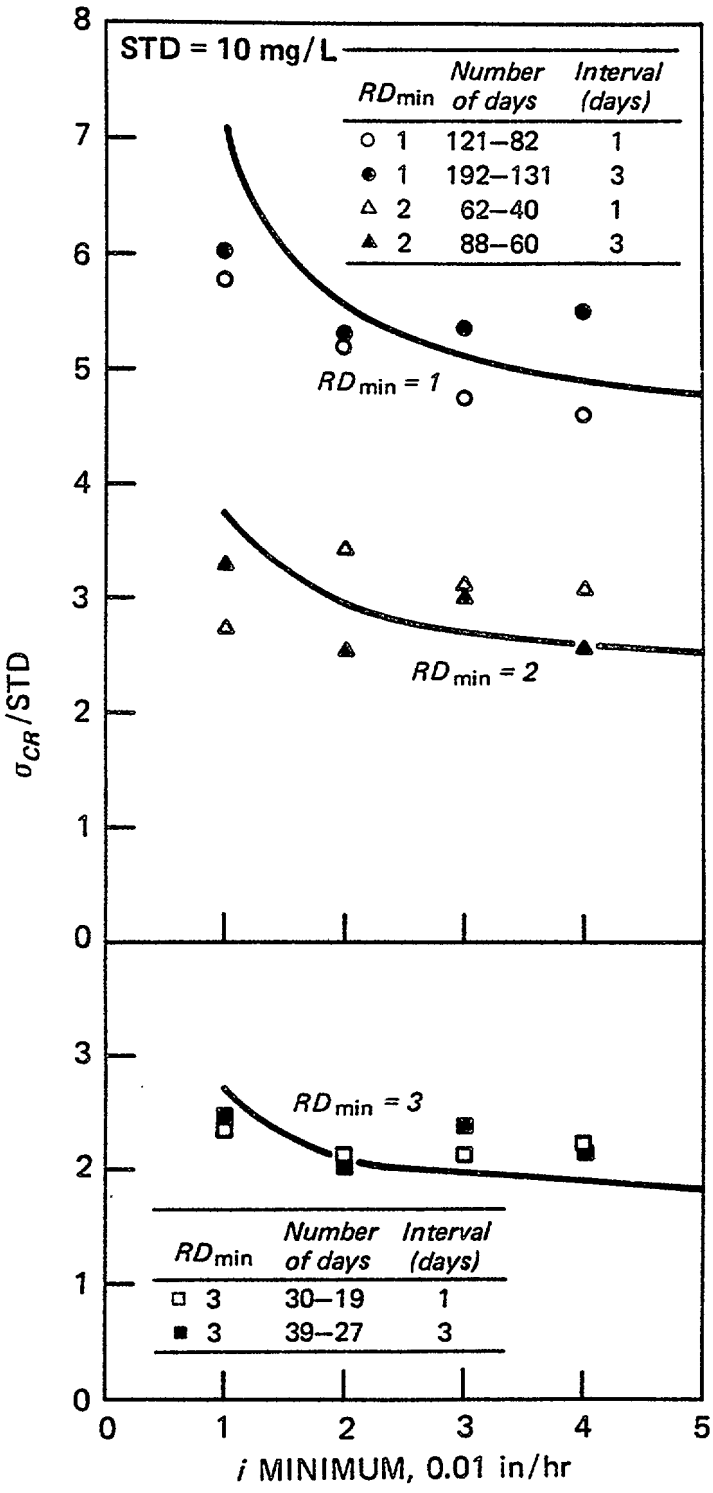

Figure 17. Runoff, Variability Factors for Measurement Error (STD = 10 mg/ℓ)

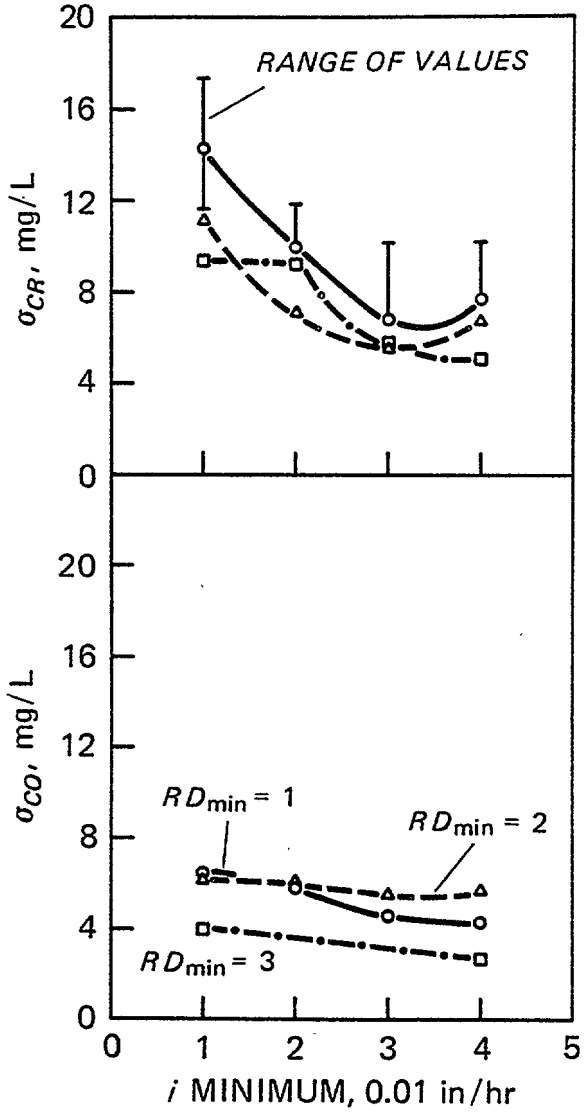

Figure 18. Daily Averaging Error for Constant Rainfall and Variable Sewage Characteristics During an Event

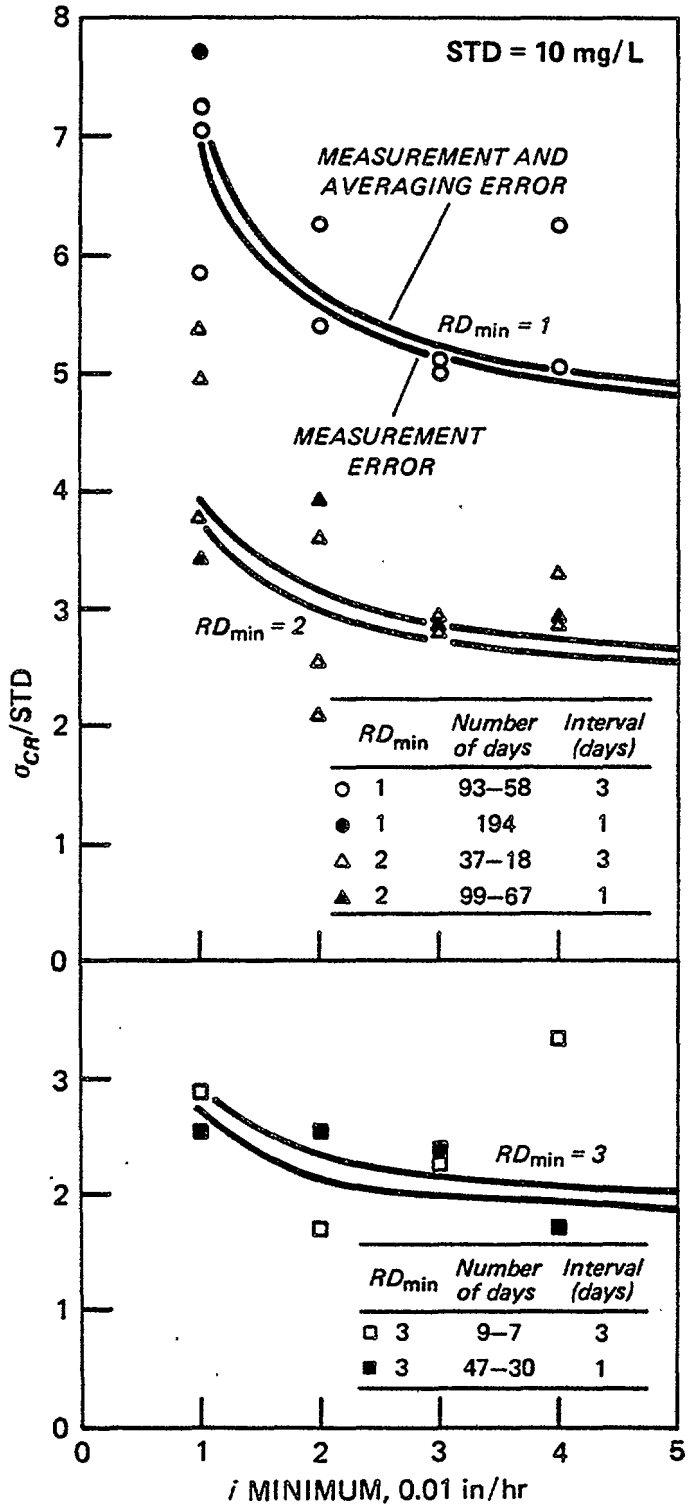

Figure 19. Runoff Variability Factors for Combined Measurement (STD = 10 mg/ℓ) and Daily Averaging Errors

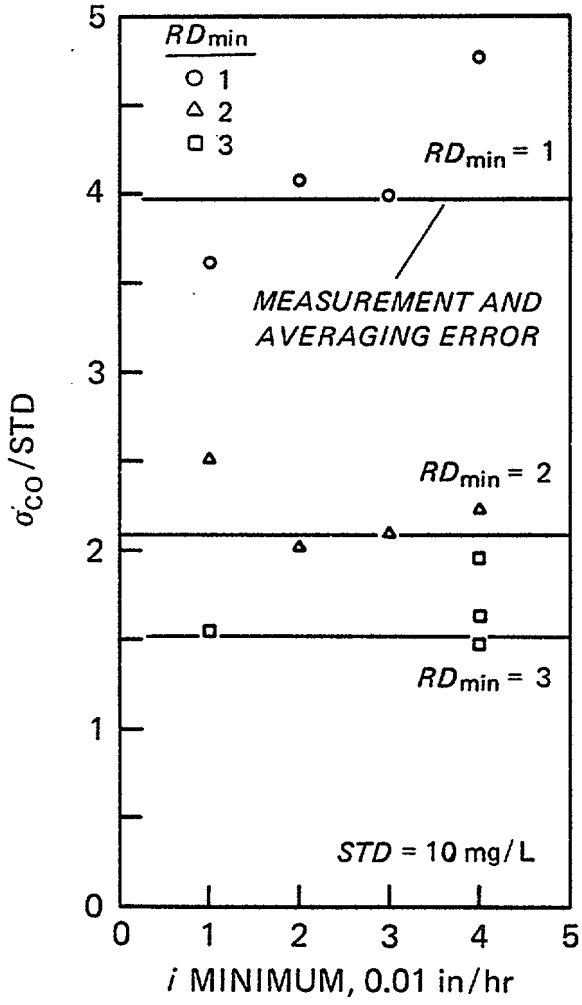

Figure 20. Overflow Variability Factors for Combined Measurement (STD = 10 mg/ℓ) and Daily Averaging Errors

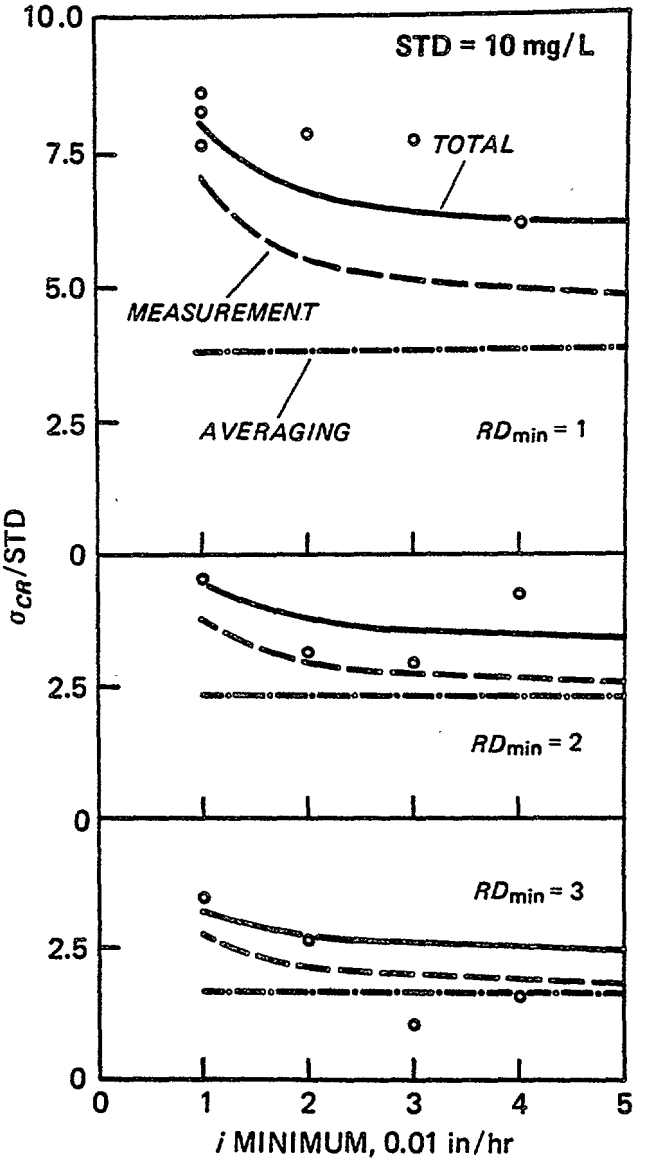

Figure 21. Runoff Variability for Varying Runoff Characteristics

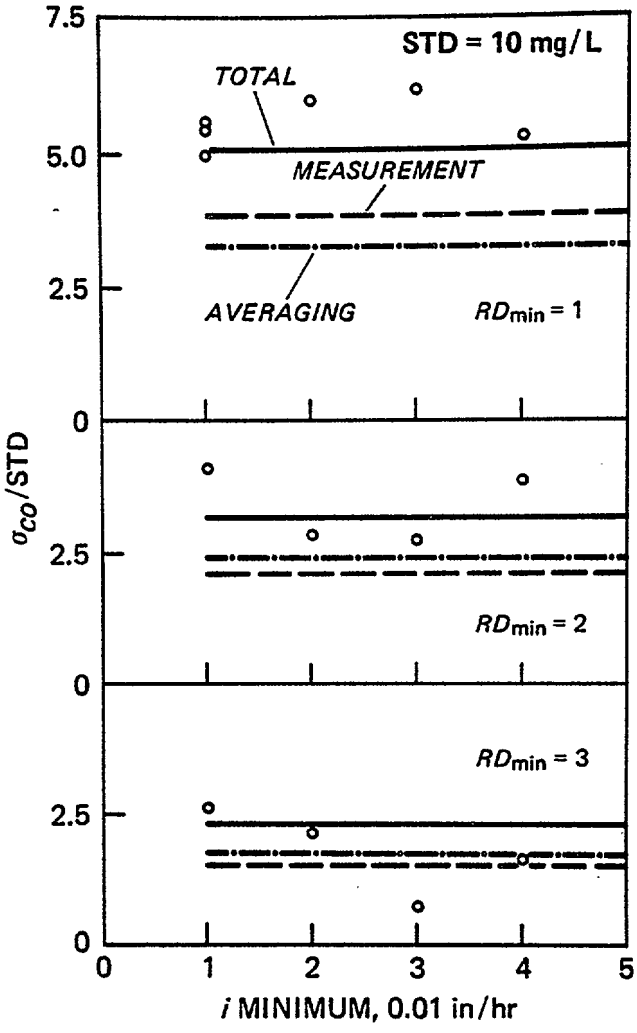

Figure 22. Overflow Variability for Varying Runoff Characteristics

agreement again verifies the technique of taking the sum of the variances of the individual errors to obtain the total variance on the runoff and overflow concentrations.

A theoretical analysis of the effect of dry weather random variability, similar to the measurement error analysis, is presented in Appendix II. The results of this analysis show that random variability on hourly dry weather concentrations can produce significant variability on overflow and runoff concentrations. To accurately assess the magnitude of this error, data on the hourly variability in dry weather sewage concentrations is required.

CONCLUSION

From the above analysis, it is seen that the variability due to treatment plant measurement error is magnified 4 to 7 times for estimates of overflow and runoff concentrations respectively by the mass balance technique. By analyzing longer duration storms in which two out of the five samples taken for the composite occur during the runoff event, the variability due to measurement error can be reduced to factors of about 2 and 3.5 to 4 respectively for overflow and runoff. The average rainfall intensity has no effect on overflow variability and some effect on runoff variability, especially for 0.01 and 0.02 in/hr. average intensities. The theoretical analysis substantiates these results and explains the source of the magnification of measurement errors at the plant.

These results suggest that the variability of the individual daily estimates of overflow and runoff concentrations are an inherent part of the mass balance method and are large, relative to measurement errors, because of the magnification factors. These are an unavoidable consequence of the method employed, which attempts to extract the runoff and overflow concentrations from differences of measured concentrations. However, the analysis also confirms that there are no biases present in the resulting estimates. This suggests that although the estimates are noisy they may be still useful for analyzing the properties of runoff concentrations obtained from an analysis of actual treatment plant data. This is investigated in the next section.

SECTION 8

ANALYSIS OF RAINFALL - RUNOFF RELATIONSHIPS

An important topic in the modeling and analysis of runoff generation mechanisms is the relationship between rainfall properties and resulting runoff concentrations. For example, if a strong first flush effect exists, then storm-averaged runoff concentrations should show a significant inverse relationship to storm duration. Also if dry deposition of pollutants is the principle mechanism by which they accumulate on the drainage basin, or if in combined sewers solids are accumulating during dry periods, then a positive correlation is expected between storm interval and runoff concentratration.

The purpose of this section is to investigate the degree to which the hourly mass balance methods, using daily (equal volume) composite treatment plant data, can be used to uncover these relationships. The methodology used involves building into the simulator a known relationship between rainfall properties and runoff concentrations. This simulator output is then sampled and composited as before, measurement error is introduced at the plant, and these observations are analyzed. Since runoff concentrations will be varying, the constant overflow method is employed to estimate the runoff for each day, C_R. These concentrations are then analyzed using regression analysis to estimate the relationships between runoff concentrations and rainfall properties.

SIMULATOR DATA

The hourly simulator was modified to incorporate the effect of interval between storms on runoff concentration. Two interval correlations were utilized. The first uses the same effect as that previously found in the 26th Ward data, Mueller and Anderson, 1979:

$$C_1 = 0.542 * \delta + 146$$

$$\delta = \text{interval between storms, hr}$$

where the second contains a weaker effect in order to obtain lower runoff concentrations, similar to those previously used in the simulator.

$$C_1 = 0.147 * \delta + 39.5$$

Both strong and weak interval effects on runoff concentration were

obtained by the daily balance analysis. Figure 23 shows the regression plots for the strong interval effect for the total length of record of 260 days. The data are plotted in groups of 20 to reduce some of the scatter. The degree of variability (r^2) explained by the interval effect is relatively low, 12% for the strong interval effect and only 2% for the weaker interval effect. The reason for these low correlation coefficients is the errors inherent in the daily balance analysis due to the averaging technique and measurement errors. As can be seen from Figure 23, the results are quite good. The estimated slope of the relationship (0.584) is very close to the actual slope (0.542) as are the intercepts: 136 and 146 mg/ℓ respectively. In order to make a quantitative statement of the goodness of the estimates it is necessary to know how close is close enough. This information is available from the regression analysis since the 95% confidence limits for slope and intercept are available using standard regression theory. However it is necessary to check that indeed the assumptions implicit in linear regression theory are met. Most important is that the residuals are normally distributed. A normal probability plot and histogram of the residuals are shown in Figs. 24 and 25. These confirm the assumption of normally distributed residuals and allow the use of the confidence intervals for slope and intercept as correct indications of the extent to which slope and intercept are known. These limits are then compared to the true values used in the simulator.

It is clear that the length of record analyzed, and therefore the number of C_R data used in the regression analysis, will affect the confidence limits associated with the slope and intercept. These indicate the quantity of data required in order to use the mass balance method for the investigation of rainfall-runoff relationships.

The effect of length of record on the slope and intercept of the regression analyses is given in Figs. 26 and 27. For the strong interval correlation, a nonzero intercept is ruled out (95% confidence) but not a nonzero slope for a record length of 65 days. A data base of 150 to 200 days provides tighter confidence limits. For the weak interval effect, the 95% confidence limits on the slope are relatively wide and the lower limit still approaches zero at a data base as high as 330 days. Thus the weaker the effect, the longer the data base required.

The effect of the magnitude of the measurement error on the percent of variability (r^2) explained by the interval correlation is shown in Fig. 28. The lower the measurement error, the greater the r^2. The stronger interval effects on runoff concentration have significantly greater r^2 values than the weaker effects. Averaging error, although relatively small since constant C_R values were used over an event, result in a maximum r^2 value of 29 and 78% respectively, for the weak and strong interval effects. Thus as measurement error increases, r^2 decreases since more of the total variability is due to the measurement error and less to the interval effects.

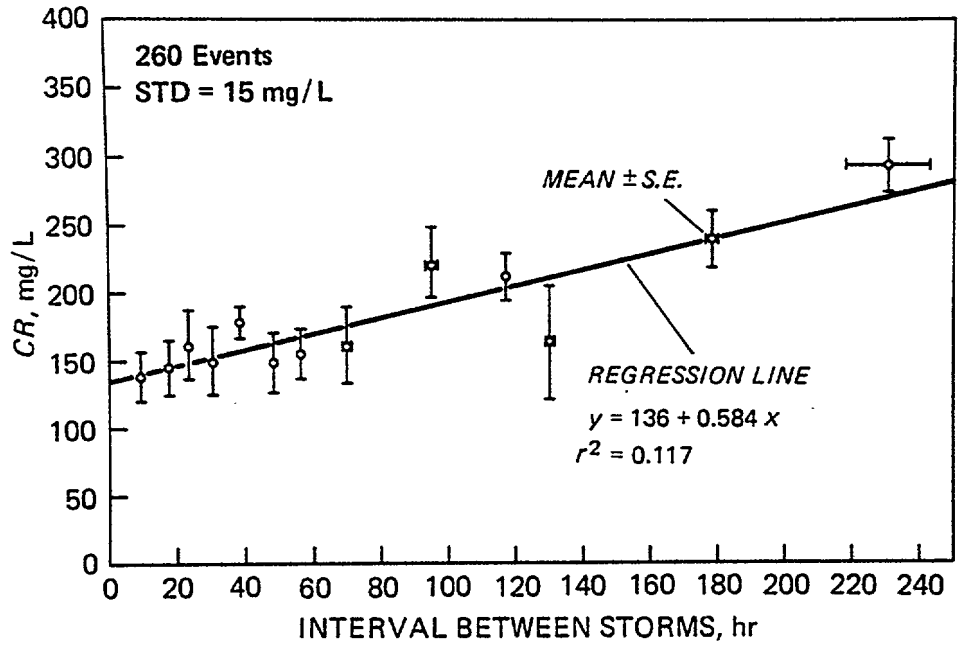

Figure 23. Runoff Concentration - Storm Interval Regressions for Strong Interval Effect

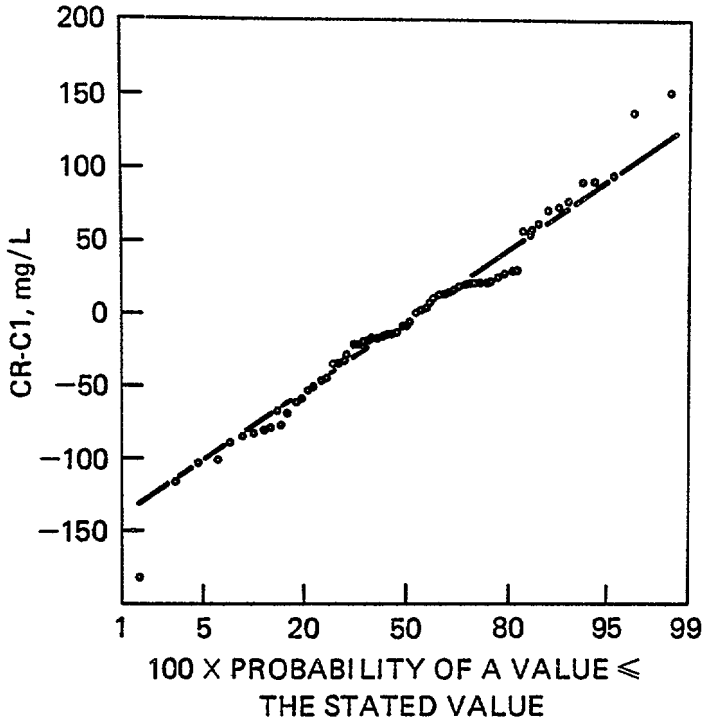

Figure 24. Probability Distribution of Runoff Residuals for a 10 mg/ℓ Measurement Error for Strong Interval Effect

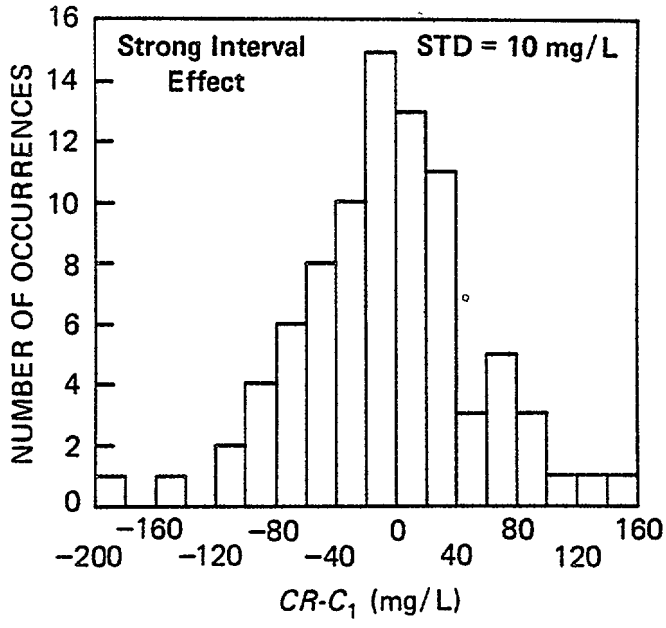

Figure 25. Runoff Residuals Histograms for a 10 mg/ℓ Measurement Error for Strong Interval Effect

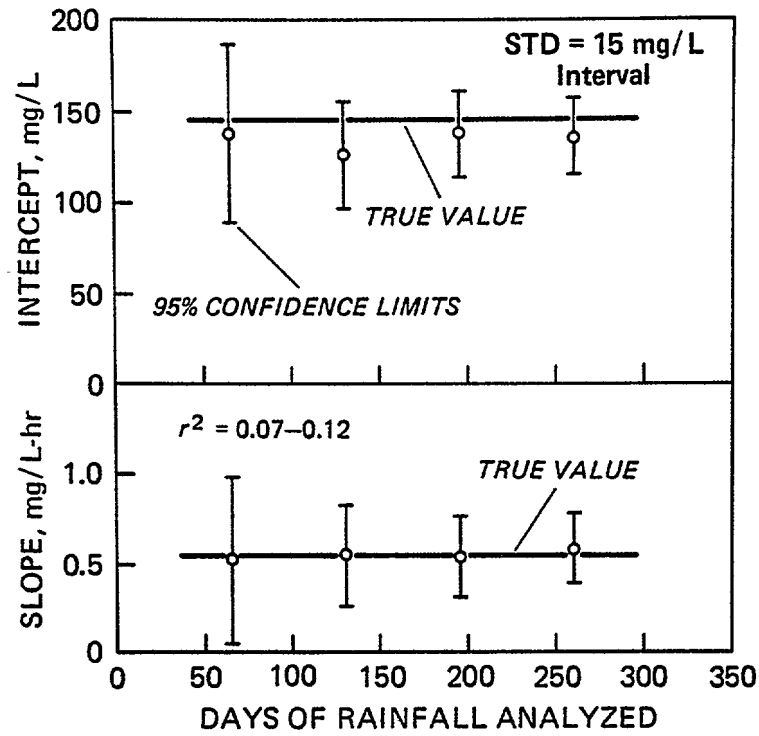

Figure 26. Effect on Length of Record on Regression Parameters for A 15 mg/ℓ Measurement Error for Strong Interval Effect

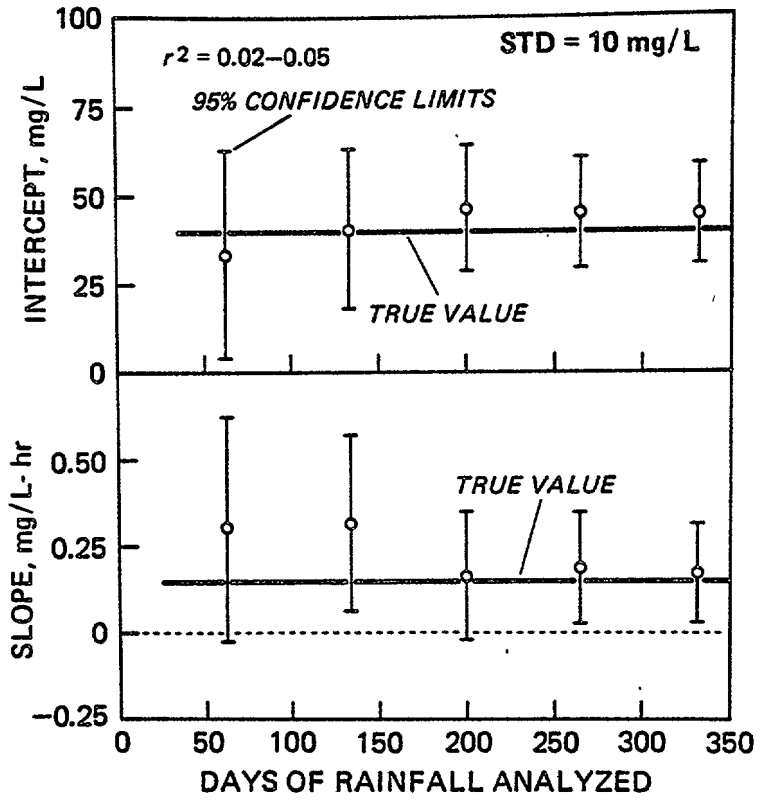

Figure 27. Effect of Length of Record on Regression Parameters for A 10 mg/ℓ Measurement Error for Weak Interval Effect

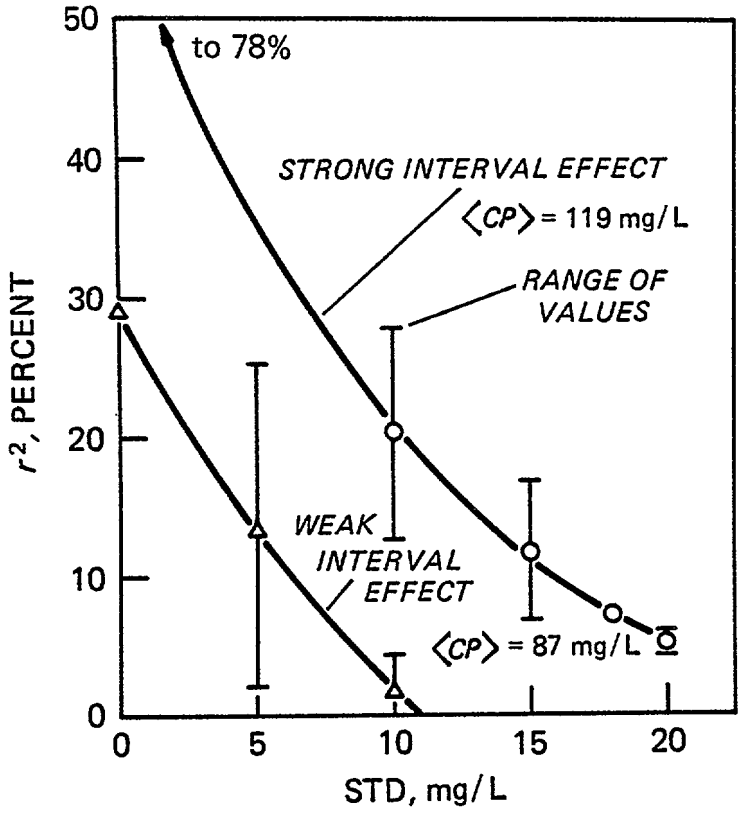

Figure 28. Effect of Measurement Error on Runoff Concentration-Interval Correlation Coefficients

The ability of the daily balance technique to determine the runoff properties when both first flush and dry interval between storms were incorporated in the model was next evaluated. The following strong interval and first flush effects were used for the analysis:

$$C_R(t) = [40 + (1000 - 40)e^{-2t(hrs)}][0.0067\ \delta + 1.8]$$

Typical hourly values for an interval of 68 hours (2.8 days) are as follows:

t hr	$C_R(t)$ mg/ℓ
1	383
2	130
3	96
4	91
5	90
6	90

The effect of first flush is negligible after 4 hours for this assumed runoff relationship. The concentrations are much higher than previously used due to the stronger interval effect.

Figures 29 and 30 show the slopes and intercepts of the regression equations as a function of length of record analyzed. For both interval and duration, close to the true values of both parameters are obtained. Between 150 and 200 days data are required to insure the 95% confidence limit on the slopes are different than zero in both cases.

If the significant first flush exists in the rainfall properties then analyzing data at greater minimum durations reduces the average runoff concentration significantly since the highest values occur in the first hour of the storm. This is shown in Figs. 31 and 32, using the full record length with 15 mg/ℓ measurement error. For the duration regressions the intercept decreases as the average concentration analyzed decreases, while the slope approaches zero. The amount of variance explained by the duration effect is also markedly reduced. As the duration effect becomes weakened, the interval effect becomes somewhat greater as seen from the increasing slope, but concentration is still reduced. The correlation coefficient also increases, but the large degree of measurement and averaging error inherent in the data base and daily analysis technique keeps the r^2 values below 10%. However in all cases the regression slopes and intercepts obtained from the calculated C_R data are remarkably consistent with the regression slopes and intercepts obtained from the true runoffs, C_1. Further, these analyses indicate that to obtain the true runoff effects, all duration data must be analyzed. Additional regressions conducted at an i_{min} of 0.03 in/hr had negligible effect on the duration and interval correlations.

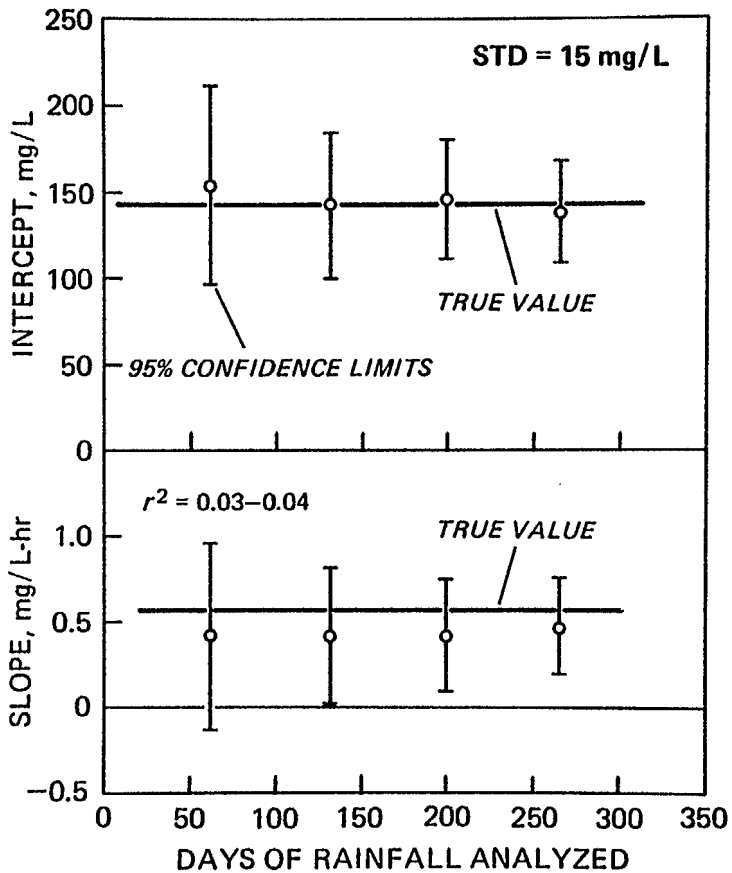

Figure 29. Effect of Length of Record on Interval Regression Parameters for a Significant First Flush

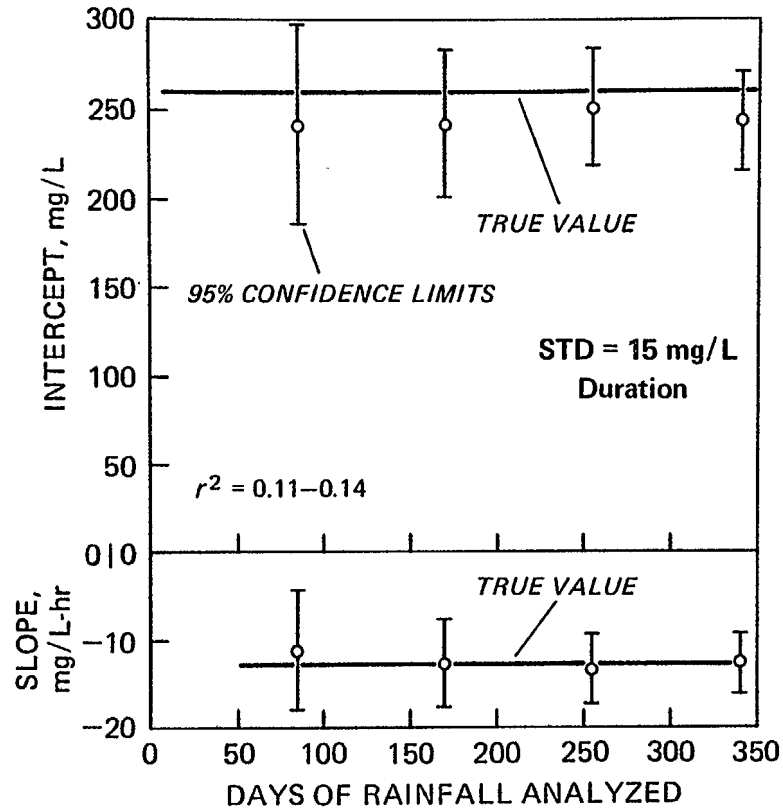

Figure 30. Effect on Length of Record on Duration Regression Parameters for a Significant First Flush

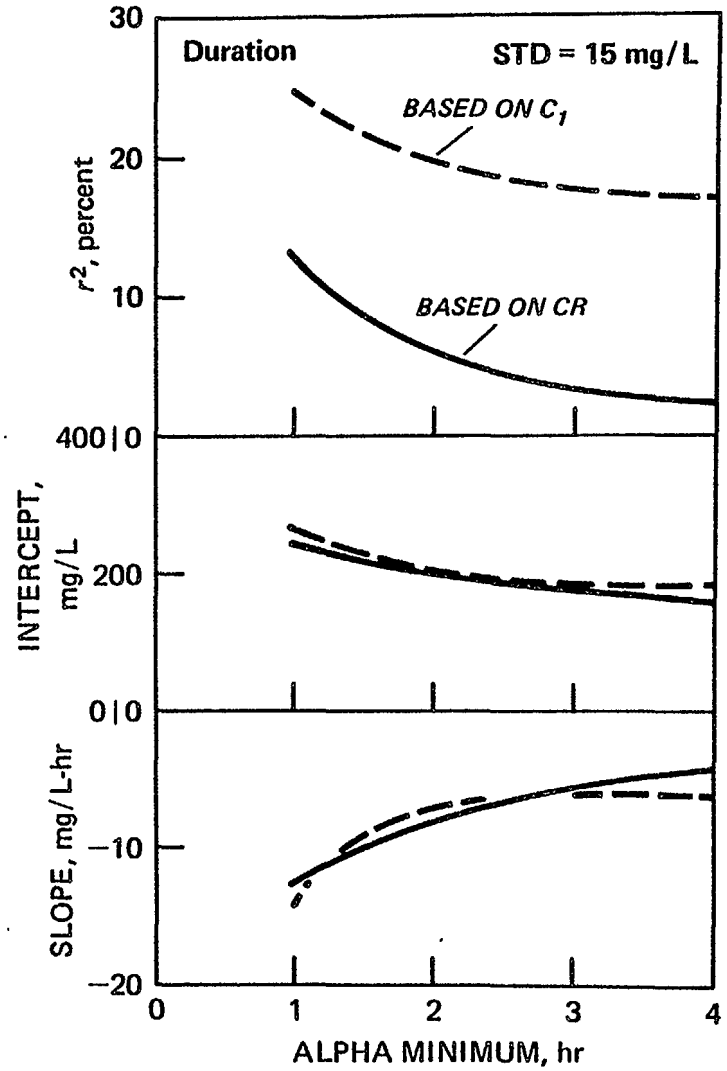

Figure 31. Effect of Minimum Duration on Runoff Concentration - Duration Regression Parameters for a Significant First Flush

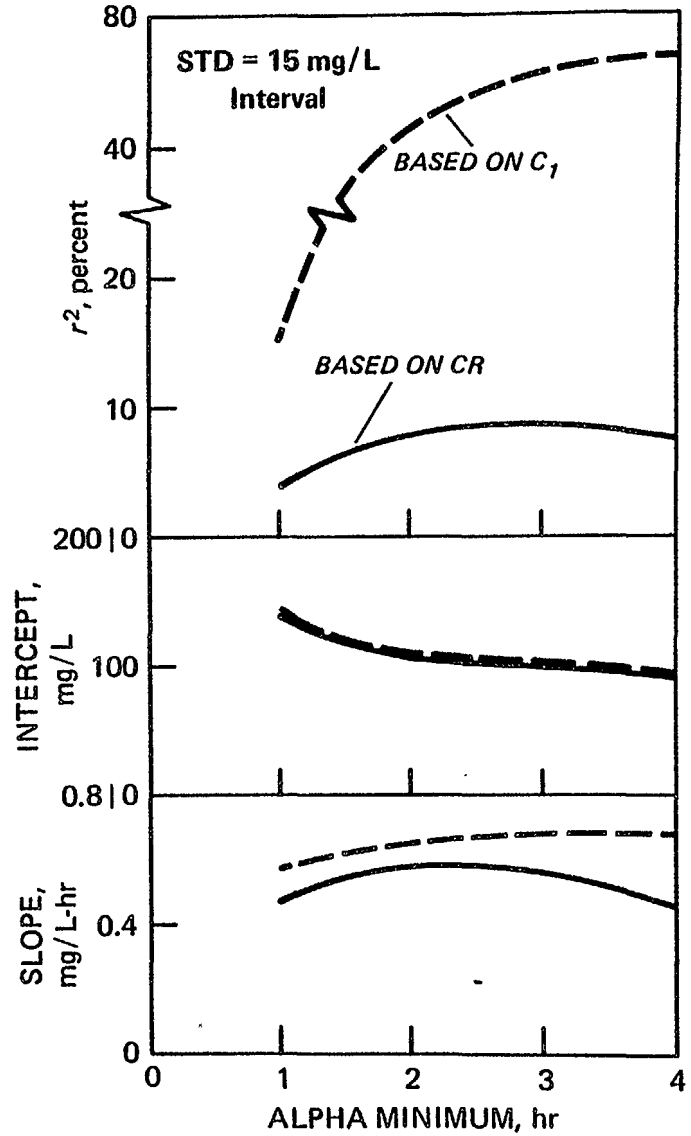

Figure 32. Effect of Minimum Duration on Runoff Concentration - Interval Regression Parameters for a Signifiant First Flush

NYC - 26th WARD DATA

Numerous analyses were conducted on the New York City 26th Ward data with the various model changes as described in detail in Appendix III. Analyses of rainfall-runoff characteristics were conducted with the hourly mass balance analysis at 0.03 in/hr. minimum average intensity to reduce the variability in the runoff estimates for the low rainfall storms. Also storms that lasted more than one day were combined into one event.

The regression parameters describing the effect of minimum duration analyzed are shown in Fig. 33 for concentration vs. duration. The 95% confidence limits on the slope and intercept show that the analysis can predict the effect of duration, even when small storms are included. The plot of slope vs. alpha minimum shows that a first flush does exist, and is most pronounced in storms of at least 2 hours in duration and 0.03 in/hr intensity. With the exception of the 1 hour rainfall, these results are similar to those from the hourly simulator with the first flush.

They also agree well with the NYC 208 results[7] which showed a significant first flush to exist in combined sewer overflows for BOD_5 and suspended solids over the first two hours of storm events. The linear regression plots for suspended solids versus storm duration and interval at α_{min} of 2 hours and a minimum rainfall intensity of 0.02 in/hr are given in Figs. 34 and 35.

The first flush effect as measured by the duration regression is significantly greater than that obtained previously ($y = 257 - 4.7x$, $r^2 = 0.033$) using the flow weighted balance technique at an α_{min} of 4 hours[1]. The interval regression has a similar slope but with a greater intercept and lower correlation coefficient than obtained previously ($y = 13x + 146$, $r^2 = 0.078$) as predicted by the simulator results (Fig. 32). Rainfall characteristics have less of an effect on the three remaining parameters analyzed at the 26th Ward plant as summarized in Appendix III.

CONCLUSION

These results strongly suggest that the mass balance estimates of runoff concentrations, although noisy, can be successfully used to obtain the relationship between runoff concentration and rainfall properties and that the proper method of analysis is linear regression. The confidence limits for slope and intercept decrease as record length increases as expected. The surprising result is that the regression estimates are quite close to the actual values even for the cases where the confidence limits are quite large. This suggests that the 95% confidence limits are a conservative estimate of the probable range of the true values.

The low value of r^2 obtained from the regression is not to be interpreted as an indication that the estimates of slope and intercept from the regression analysis are not useful. Their utility should be judged from

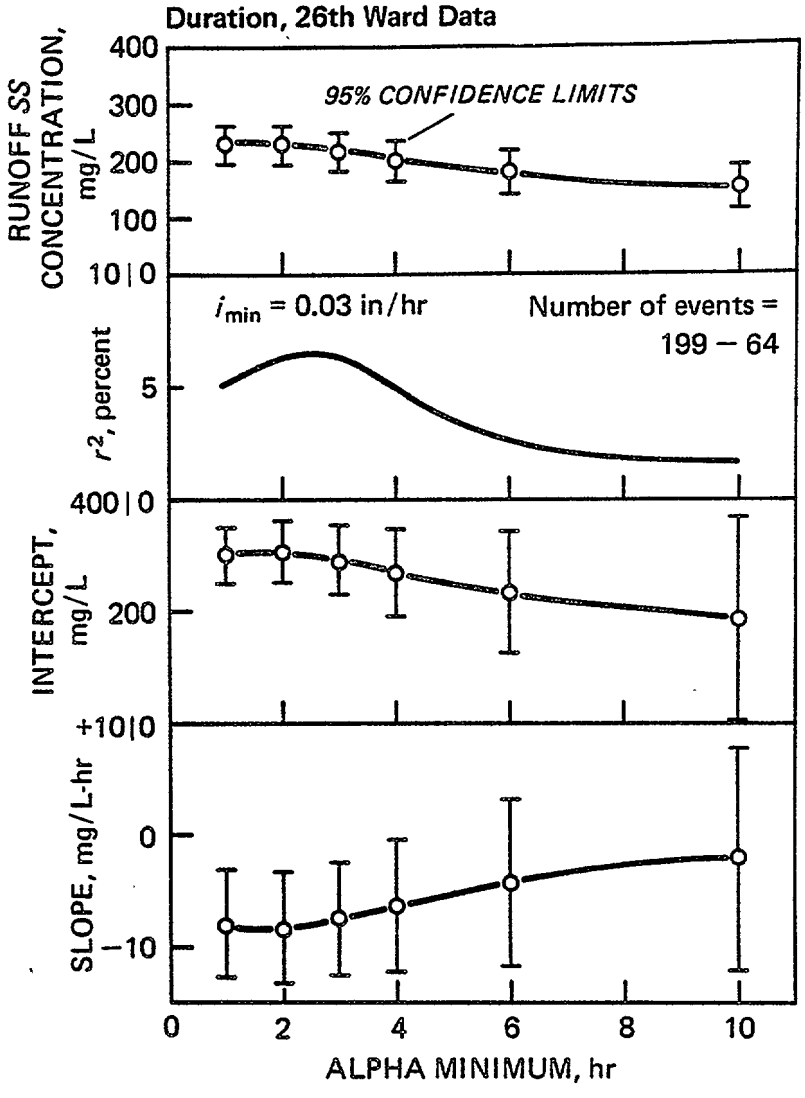

Figure 33. Effect of Minimum Duration Analyzed on Runoff-Duration Regression Parameters for NYC 26th Ward Data

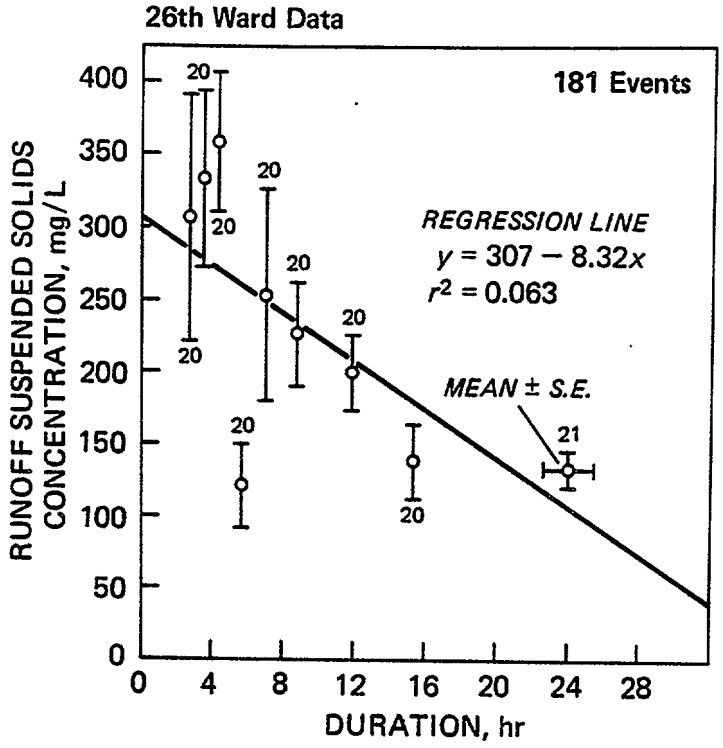

Figure 34. Duration Regression Analysis for 26th Ward Suspended Solids Data

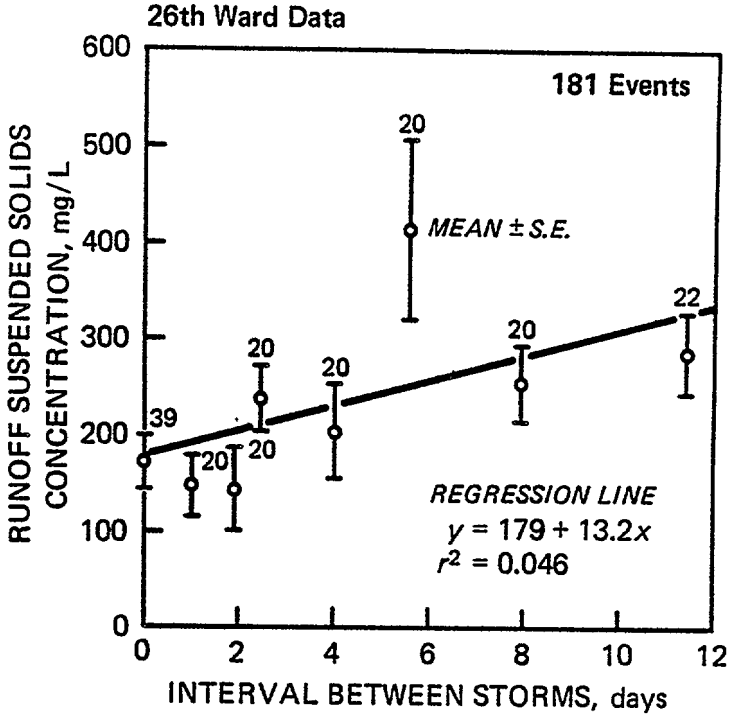

Figure 35. Interval Regression Analysis for 26th Ward Suspended Solids Data

their confidence limits. The small r^2 values are a result of the large variability in the estimates of C_R, which is inherent in the mass balance method. There is no reason to expect that a large fraction (r^2 close to one) of this variability, which is due to measurement error, should be explained by the rainfall property correlations. As shown above, r^2 decreases sharply as measurement error increases. However, the slope and intercept are still reasonably well estimated, as judged by the confidence limits.

Analysis of the actual 26th Ward data shows a significant first flush to exist at this location for suspended solids concentration. The proper magnitude of this effect could not be obtained when only storm durations greater than 4 hours were analyzed as required by the flow weighted balance technique used previously.

SECTION 9

ANALYSIS OF HOURLY SAMPLE COLLECTION

The impact of the sample collection procedure was investigated. The New York City regime composites five equal volume samples over the day and measures the concentration of the composite. An alternate method of collection is to obtain a sample at each hour and composite the resulting twenty-four samples. The composite sample is then analyzed. As shown in Appendix IV, a significant number of plants (24 out of 54 surveyed) composite samples at a one hour interval or less. This regime does not increase the quantity of measurements obtained, rather it samples the influent more frequently. It was expected that this regime may improve the behavior of the mass balance estimates.

For the constant overflow assumption, the treatment plant concentration is, (equation 38):

$$C_P = \left\{ \sum_i^+ C_{3i} + \sum_i^\circ C_{2i} \right\} / N_s$$

where now $s_i = 1$ for each hour since a sample is removed at each hour, so that $N_s = 24$.

The overflow concentration estimate is (equation 39):

$$C_O = \frac{N_s C_P - \sum_i^\circ C_{2i}}{\sum_i^+ s_i} = \frac{\sum_i^+ C_{3i} + \sum_i^\circ C_{3i} - \sum_i^\circ C_{2i}}{\sum_i^+ s_i}$$

$$= \frac{\sum_i C_{3i} - \sum_i^\circ C_{2i}}{\sum_i^+ s_i} = \frac{N_s C_P - \sum_i^\circ C_{2i}}{\alpha}$$

where $\alpha = \sum_i^+ s_i$ = numbers of hours of rainfall in the day.

The last term in the above equation is the dry weather contribution for the daily sample. The runoff concentration is calculated as previously discussed from a mass balance on the collection system.

Two data bases were used in the analysis, one with a weak interval effect and no first flush, the other with a strong interval effect with first flush.

When no measurement error was present, the linear regression analysis on runoff concentration versus interval successfully predicted the input data with r^2 values of 70 and 12% for no first flush and first flush, respectively. However, when measurement error was introduced into the analysis the degree of variability was greatly increased over that observed previously with the NYC sampling technique. This surprising result requires an explanation.

The theoretical analysis developed previously was applied to predict the variability of the results with the hourly sample collection. Applying a perturbation on the above equation for overflow concentration and subtracting the overflow concentration yields:

$$\varepsilon_o = \frac{N_s}{\alpha} \varepsilon_p$$

where: ε_p = the plant concentration perturbation, α = the hours of rainfall, and $N_s = 24$.

The variance of the overflow concentration due to measurement error is then:

$$V\{\varepsilon_o\} = E\{(\frac{N_s}{\alpha} \varepsilon_p)^2\} - E^2\{\frac{N_s}{\alpha} \varepsilon_p\}$$

Since $E\{\varepsilon_p\} = 0$, the last term in the above equation is zero leaving:

$$V\{\varepsilon_o\} = N_s^2 \; E\{\varepsilon_p^2\} = N_s^2 \; STD^2 \; E\{\frac{1}{\alpha^2}\}$$

The standard deviation of the overflow concentration is then:

$$\sigma_{CO} = N_s \; STD \; E^{1/2}\{\frac{1}{\alpha^2}\}$$

As previously determined for the NYC sampling routine, the standard deviation on the overflow concentration when all rainfall is analyzed, $i_{min} = 1$, is

$$\sigma_{CR} = 1.83 \; \sigma_{CO}$$

The expected value of $1/\alpha^2$ can be determined at any minimum duration, $RD_{min} = \alpha_{min}$, as:

$$E\{1/\alpha^2\} = \frac{\sum_{d=RD_{min}}^{\infty} P_d(1/\alpha^2)}{\sum_{d=RD_{min}}^{\infty} P_d}$$

The resulting error magnification factors are compared to those from the NYC sampling routine in Figure 36. Good agreement between theoretical and calculated values is again observed. The magnitude of the error magnification factor is much larger than that for the NYC sampling routine at a similar RD_{min}. A minimum of 4 wet samples provides magnification factors similar to those for 1 wet sample using the NYC routine. This is equivalent to 17% of the plant samples being wet for the hourly sampling routine compared to 20% for the NYC sampling routine. With the large measurement error at low rainfall durations, the ability to obtain information at these low durations is impaired. The reason is that the sample collected during the short storm is mixed with many samples collected during dry weather thus reducing the effect of the runoff contribution. A small measurement error effectively masks this small concentration impact.

Figure 37 indicates that the daily mass balance analysis can still determine the correct values of slope and intercept from a regression analysis of runoff concentration versus duration analyzing all storm durations for 476 days of data. However, the confidence limits are wide due to the large measurement error effect. Increasing the minimum duration analyzed reduces the confidence limits, however lower values of runoff concentration result due to missing the first flush effect. Figure 38 shows that the true values of the effect of interval on runoff concentration cannot be obtained with 388 days data at $RD_{min} = 1$. The predicted slope is lower and the intercept higher than the true values. The confidence limits on the slope are also large and show the slope to be not significantly different than zero. Analyzing storm durations greater than 4 hours, provides higher correlation coefficients than the lower durations, since the first flush effect is diminished. Again, this results in low estimated runoff concentrations due to missing the first flush.

The above analysis indicates that hourly sample collection at a treatment facility has significant drawbacks when analyzing for runoff and overflow characteristics. To obtain reliable estimates, greater duration storms have to be analyzed which miss a first flush effect if present. Therefore this type of sample collection procedure is not recommended. The New York City sampling collection method is superior since a larger proportion of the collected sample is affected by runoff. These results suggest that separate sampling during wet and dry periods may be beneficial.

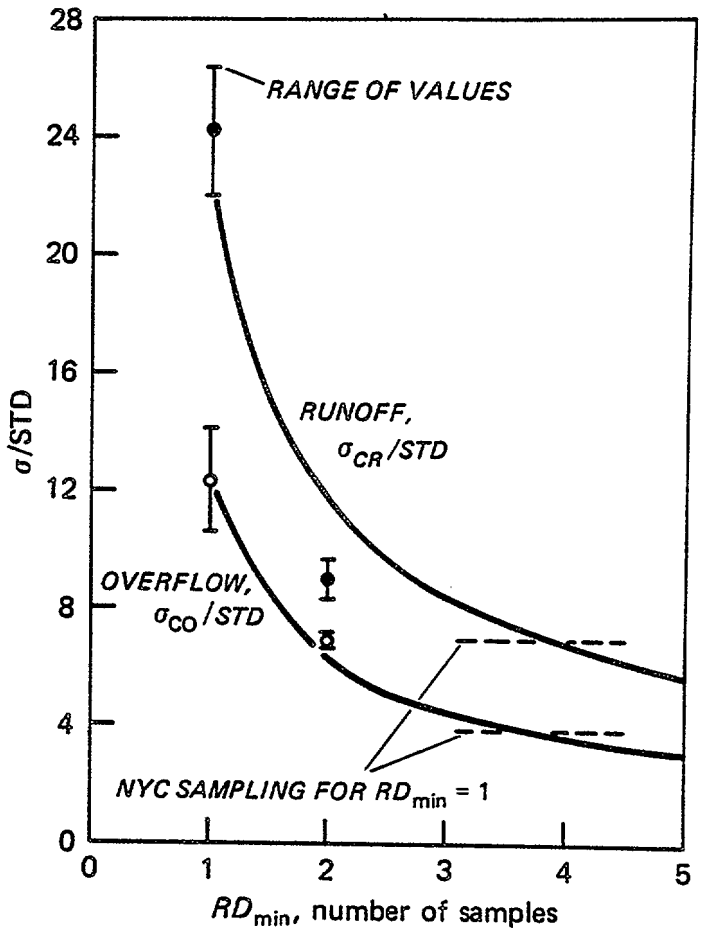

Figure 36. Effect of Minimum Storm Duration on Error Magnification Factors for Hourly Plant Sampling

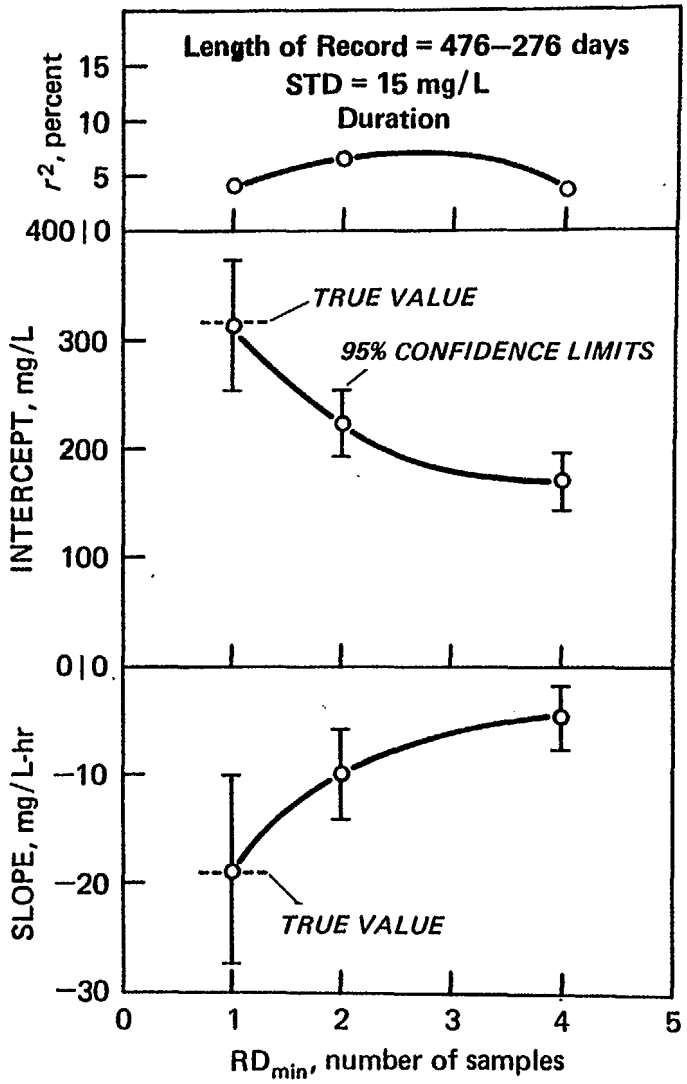

Figure 37. Duration Regression Parameters Using Hourly Plant Sampling Technique

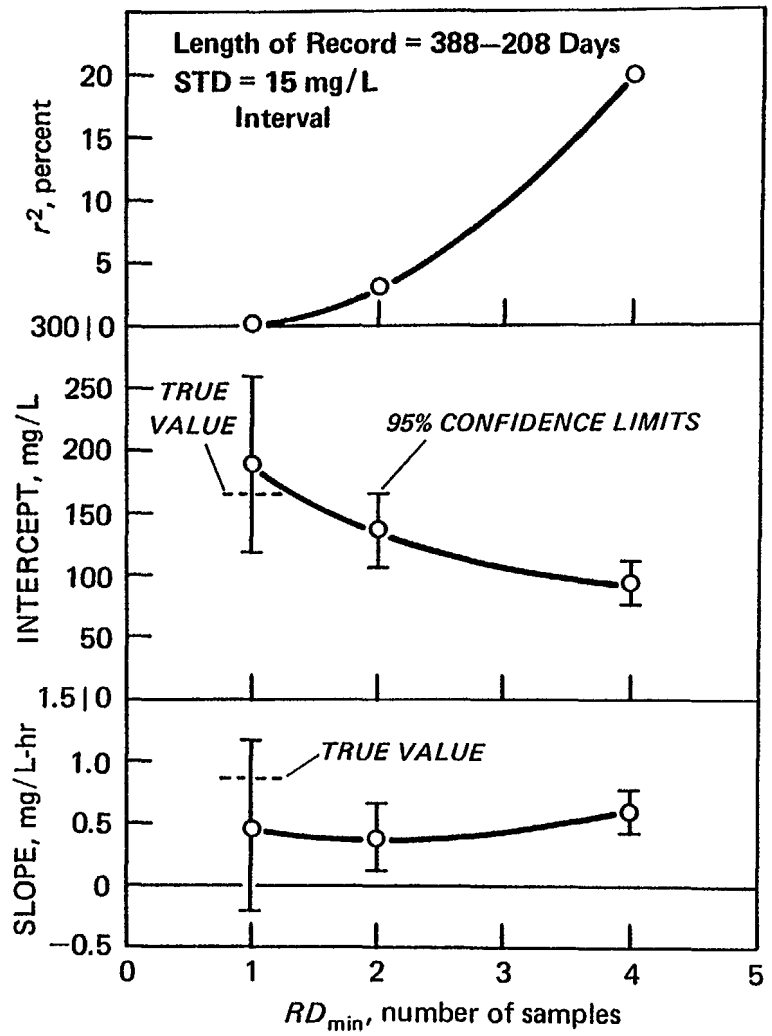

Figure 38. Interval Regression Parameters Using Hourly Plant Sampling Technique

BIBLIOGRAPHY

1. Mueller, J.A. and Anderson, A.R., "Contaminant Inputs to New York Bight, Phase II, Urban Runoff", NOAA, MESA Report. October 1977.

2. Mueller, J.A. and Anderson, A.R., "A Mass Balance Method for Estimating Combined Sewer Runoff and Overflow Quality from Sewage Treatment Plant Data", Prog. Wat. Tech. 10, pp. 727-739. 1978.

3. Mueller, J.A. and Anderson, A.R., "Combined Sewer Overflow Quality from Treatment Plant Data", JWPCF, 51, pp. 958-973. 1979.

4. Hydroscience, Inc., "Available Storm/CSO Sampling", NYC 208 Task Report, PCP TASK 222. March 1978.

5. Di Toro, D.M., and Small, M.J., "Stormwater Interception and Storage", Journal of the Environmental Engineering Division, ASCE, 105, pp. 43-54. 1979.

6. Davenport, W.B. and Root, W.L., "Random Signals and Noise", McGraw-Hill Book Co., Inc., New York, p. 34. 1958.

7. Hydroscience, Inc., "Rainfall, Runoff and Statistical Receiving Water Models", NYC 208 Task Report, PCP Task 225. March 1978.

APPENDIX I

ESTIMATION OF E $\{1/RD^2\}$

The number of samples obtained during storm events, RD, is a function of a number of factors; the time at which the storm begins, storm duration and interval between storms. The first two factors have a major effect since sampling at the treatment plant occurs at specified times with the greater the duration the greater the likelihood of obtaining samples during the wet period of the day. The following equation was used to describe this effect:

$$E\left\{\frac{1}{RD^2}\right\} = \sum_{RD_{min}}^{N_s} \left(\frac{1}{RD^2}\right) P_{RD} \Big/ \sum_{RD_{min}}^{N_s} P_{RD} \qquad \text{(A1)}$$

where P_{RD} = probability of RD occurring over all durations,

and N_s = 5 for the New York City sampling regime.

The overall probability of a sample being wet is a function of both the probability of a specified duration occurring during a day and the probability of that duration being wet as follows:

$$P_{RD} = \sum_{d=1}^{24} P_d \times P_{wd} \qquad \text{(A2)}$$

where P_d = probability of occurrence of a specified duration from 1 to 24 hours

$$= \frac{1}{D} \int_{d_1}^{d_2} e^{-d/D}\, dd = e^{-d_1/D} - e^{-d_2/D} \qquad \text{(A3)}$$

and P_{wd} = probability of duration, d, having RD wet samples.

The above equation ignores the effect of multiple events occurring in a day, the smaller the storm interval the greater the probability of two or more events occurring in 1 day. It also ignores the impact of starting time on spreading the total duration over 2 days instead of a single day. These two effects are somewhat self compensating in that the first effect tends to give longer storm durations in a day and the latter effect shorter durations. Storm durations greater than 24 hours which occur relatively infrequently (< 1.5% of the time), are neglected since these would necessarily occur over 2 sampling days.

Evaluation of the first probability, P_d, is straightforward using the average duration of the simulator, D = 6 hr, and increments of 0-1.5, 1.5-2.5, 2.5-3.5, etc.

To evaluate the latter probability, P_{wd}, an evaluation procedure was utilized to properly account for the New York City Sampling regime which skipped the 2 AM sample. A given duration continuous storm was assumed to start at a specified hour. The number of samples which would be taken over the storm duration were counted. The storm was then assumed to start on the next hour and the same procedure followed. This was continued until all possible 24 starting times were evaluated. For example, a 13 hour storm first appearing at the plant at 10 AM would be sampled at 10 AM, 2 PM, 6 PM, and 10 PM resulting in an RD of 4. However when started an hour later it would only be sampled at 2, 6, and 10 PM resulting in an RD of 3. Continuing this procedure over 24 hours provides 9 possibilities for RD = 2, 13 for RD = 3, and 2 for RD = 4. Dividing each of the total number of possible RD values by 24 yields P_{wd} for each RD, so P_{wd} = 0.375 for RD = 2, 0.542 for RD = 3, and 0.083 for RD = 4. The results for all durations and RD values is given in Table A-1.

TABLE A-1. DURATION AND SAMPLING PROBABILITIES

Storm Duration	Duration Probability Pd x 100	Probability of d having RD wet samples, P_{wd} x 100, %, for specified number of samples				
d,hr	%	RD				
		1	2	3	4	5
1	22.1	20.8	0	0	0	0
2	12.0	41.6	0	0	0	0
3	10.1	62.5	0	0	0	0
4	8.6	83.3	0	0	0	0
5	7.3	70.8	16.7	0	0	0
6	6.1	58.3	33.3	0	0	0
7	5.2	45.8	50.0	0	0	0
8	4.4	33.3	66.7	0	0	0
9	3.7	25.0	62.5	12.5	0	0
10	3.2	16.7	58.3	25.0	0	0
11	2.7	8.3	54.2	37.5	0	0
12	2.3	0	50.0	50.0	0	0
13	1.9	0	37.5	54.2	8.3	0
14	1.6	0	25.0	58.3	16.7	0
15	1.4	0	12.5	62.5	25.0	0
16	1.2	0	0	66.7	33.3	0
17	0.98	0	0	50.0	45.8	4.2
18	0.83	0	0	33.3	58.3	8.3
19	0.70	0	0	16.7	70.8	12.5
20	0.60	0	0	0	83.3	16.7
21	0.50	0	0	0	62.5	37.5
22	0.43	0	0	0	41.6	58.3
23	0.36	0	0	0	20.8	79.2
24	0.31	0	0	0	0	100.0

For each RD value in Table A-1, the summation of $P_d * P_{wd}$ is made over all probability values which are then combined with the $\frac{1}{RD^2}$ and RD values as shown in Table A-2. The last column in Table A-2 is used in the next section for determining E{RD}.

TABLE A-2. OVERALL POSSIBILITIES FOR RD VALUES

RD	Overall probability $P_{RD} \times 100$, %	$P_{RD} \times \frac{1}{RD^2} \times 100$, %	$P_{RD} \times RD \times 100$, %
1	37.3	37.3	37.3
2	16.8	4.2	33.6
3	7.88	0.88	23.6
4	3.65	0.23	14.6
5	1.33	0.05	6.7
Sum	66.96	42.7	115.8

The above evaluation accounts for only 66.96% of the number of wet samples since the effects of multiple storm events occurring in a day and a storm event being spread over 2 days were ignored. In using the above results, the assumption is implicit that the remaining 33.04% of the wet samples will be distributed in the same fashion as that given in Table A-2. Comparison of the above distributions to a number of computer simulations showed the results to be acceptable.

When all data are analyzed ($RD_{min} = 1$), then $E\{\frac{1}{RD}2\} = 42.7/66.96 = 0.638$. The summations from specified RD_{min} values can then be made to yield the $E\{\frac{1}{RD^2}\}$ as shown in Table A-3.

TABLE A-3. EFFECT OF RD_{min} ON $E\{\frac{1}{RD^2}\}$

RD_{min}	$\sum_{RD_{min}}^{5} P_{RD} \times \frac{1}{RD^2} \times 100$	$\sum_{RD_{min}}^{5} P_{RD} \times 100$	$E\{\frac{1}{RD^2}\}$
1	42.7	66.96	0.638
2	5.4	29.66	0.182
3	1.16	12.86	0.090
4	0.28	4.98	0.056
5	0.05	1.33	0.038

APPENDIX II

THEORETICAL ANALYSIS OF DRY WEATHER SEWAGE VARIABILITY

The constant C_0 equations for the hourly mass balance technique were used for the theoretical analysis of dry weather variability on the runoff and overflow variabilities.

$C_2 - C_0$ ANALYSIS

The overflow concentration with the sewage concentration perturbation (ε_i) is:

$$C_0 + \varepsilon_o = \frac{C_p \Sigma_i s_i \overset{\circ}{\Sigma}_i s_i (C_{2i} + \varepsilon_i)}{\overset{+}{\Sigma}_i s_i} \tag{A4}$$

Subtracting the overflow concentration from the above yields:

$$\varepsilon_o = \frac{\overset{\circ}{\Sigma}_i s_i \varepsilon_i}{\overset{+}{\Sigma}_i s_i} = \frac{1}{RD} \overset{\circ}{\Sigma}_i s_i \varepsilon_i \tag{A5}$$

Applying the definition of variance yields:

$$V\{\varepsilon_o\} = E\{(\frac{1}{RD} \overset{\circ}{\Sigma}_i s_i \varepsilon_i)^2\} - E^2\{(\frac{1}{RD} \overset{\circ}{\Sigma}_i s_i \varepsilon_i)\} \tag{A6}$$

The latter term is zero since $E\{\varepsilon_i\} = 0$, giving:

$$V\{\varepsilon_o\} = E\{\frac{1}{RD^2}\} \; E\{(\overset{\circ}{\Sigma}_i s_i \varepsilon_i)^2\} \tag{A7}$$

If one wet sample occurs during the day for the New York City sampling regime of 5 samples total over a day, then 4 dry samples result, 2 wet samples require 3 dry, etc. The latter term in equation A7 can thus be expressed as follows:

$$E\{(\sum_i^{\circ} s_i\varepsilon_i)^2\} = E\{(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4)^2\}Pr_{RD=1} + E\{(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)^2\}$$

$$Pr_{RD=2} + E\{(\varepsilon_1 + \varepsilon_2)^2 Pr_{RD=3} + E\{(\varepsilon_1)^2\}Pr_{RD=4} \qquad \text{(A8)}$$

where

$$Pr_{RD=1} = P_{RD=1}/\sum_1^5 P_{RD} = 37.3\%/66.96\% = 0.557 \text{ from Table A-2}$$

= Probability that RD wet sample occurs over a day.

Expanding the above and realizing that all cross products are equal to zero since $E\{\varepsilon_i\} = 0$, yields:

$$E\{(\sum_i^{\circ} s_i\varepsilon_i)^2\} = E\{(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2)\}Pr_{RD=1} + E\{(\varepsilon_1^2 + \varepsilon_2^2$$

$$+ \varepsilon_3^2)\}Pr_{RD=2} + E\{(\varepsilon_1^2 + \varepsilon_2^2)\}Pr_{RD=3} + E\{\varepsilon_1^2\}Pr_{RD=4} \qquad \text{(A9)}$$

An analysis of the different sampling intervals showed that $E\{\varepsilon^2\}$ was not a function of RD but a constant, thus Equation A9 becomes:

$$E\{(\sum_i^{\circ} s_i\varepsilon_i)^2\} = E\{\varepsilon_i^2\}(4\ Pr_{RD=1} + 3\ Pr_{RD=2} + 2\ Pr_{RD=3} + Pr_{RD=4}) \qquad \text{(A10)}$$

The latter term in the above can be simplified as follows:

$$4\ Pr_{RD=1} + 3\ Pr_{RD=2} + 2\ Pr_{RD=3} + Pr_{RD=4} = (5-1)Pr_{RD=1} + (5-2)Pr_{RD=2}$$

$$+ (5-3)Pr_{RD=3} + (5-4)Pr_{RD=4} + (5-5)Pr_{RD=5}$$

$$= 5\sum_1^5 Pr_{RD} - \sum_1^5 RD\ Pr_{RD} = 5 - E\{RD\} = N_s - E\{RD\} \qquad \text{(A11)}$$

For the hourly simulator, the dry weather sewage variability was input as a fraction (F) of the dry weather concentration as:

$$\sigma_{\varepsilon i} = F \times C_{2i}$$

thus:

$$E\{\varepsilon_i^2\} = V\{\varepsilon_i\} = \sigma_{\varepsilon i}^2 = F^2 \times \overline{C_{2s}^2} \tag{A12}$$

where: $\overline{C_{2s}^2} = \frac{1}{N_s} \sum_{i=1}^{N_s} C_{2i}^2$ = average of the dry sewage concentrations squared for the sampling hours.

Substituting Equations A-12, A-11 and A-10 into A-7, yields:

$$V\{\varepsilon_o\} = F^2 \overline{C_{2s}^2} E\{\frac{1}{RD^2}\} (N_s - E\{RD\}) \tag{A-13}$$

<u>$C_2 - C_R$ ANALYSIS</u>

The runoff perturbation due to sewage variability is given:

$$C_R + \varepsilon_R = (C_O + \varepsilon_o)(1 + \beta/i) - \frac{\sum_i^+ Q_{2i}(C_{2i}+\varepsilon_i)}{V_1} \tag{A-14}$$

Subtracting the runoff concentration yields:

$$\varepsilon_R = \varepsilon_o(1 + \beta/i) - \frac{1}{V_1} \sum_i^+ Q_{2i}\varepsilon_i$$

$$V\{\varepsilon_R\} = E\{(\varepsilon_o(1 + \beta/i) - \frac{1}{V_1} \sum_i^+ Q_{2i}\varepsilon_i)^2\} \tag{A-15}$$

No E^2 value results in the above since $E\{\varepsilon_o\} = 0$, $E\{\varepsilon_i\} = 0$. Equation A15 can be expanded and the cross product deleted for the above reason to yield:

$$V\{\varepsilon_R\} = V\{\varepsilon_o\} E\{(1 + \beta/i)^2\} + E\{\frac{1}{V_1^2}\} E\{(\sum_i^+ Q_{2i}\varepsilon_i)^2\} \tag{A-16}$$

when:

$$E\{(\sum_i^+ Q_{2i}\varepsilon_i)^2\} = \overline{Q_2^2} E(\sum_i^+ \varepsilon_i)^2$$

$$= \overline{Q_2^2} E\{\varepsilon_i^2\} E\{\sum_i^+\}$$

$$= D \overline{Q_2^2} V\{\varepsilon_i\} \tag{A-17}$$

since $D = E\{\sum_i^+\}$ = average storm duration.

The approach used to obtain Equation A-17 is similar to that used to obtain A-11. The $V\{\varepsilon_i\}$ in the above is somewhat different than Equation A-12 since it is taken over the total day not only over the sampled hours, thus:

$$V\{\varepsilon_i\} = F^2 \overline{C_2^{\,2}}$$

$$\text{where } \overline{C_2^{\,2}} = \frac{1}{24} \sum_{i=1}^{24} C_{2i}^{\,2} \tag{A18}$$

Letting V_1 = Cid where i is the mean storm intensity for the event, then:

$$E\{\frac{1}{V_1^{\,2}}\} = \frac{1}{C^2} E\{\frac{1}{d^2}\} E\{\frac{1}{i^2}\} \tag{A19}$$

Substituting A19-A17 into A16 yields:

$$V\{\varepsilon_R\} = V\{\varepsilon_o\} E\{(1 + \beta/i)^2\} + \frac{D\ P^2\ \overline{Q_2^{\,2}}\ \overline{C_2^{\,2}}}{C^2} E\{\frac{1}{d^2}\} E\{\frac{1}{i^2}\} \tag{A20}$$

Q_2 - C_0 ANALYSIS

Variability in dry weather sewage flow rate has no effect on overflow concentration since Q_2 does not appear in the equation.

Q_2 - C_R ANALYSIS

The runoff concentration is given by:

$$C_R + \varepsilon_R = C_0(1 + \frac{\sum_i^+ (Q_{2i} + \varepsilon_i)}{V_1}) - \frac{\sum_i^+ (Q_{2i} + \varepsilon_i) C_{2i}}{V_1} \tag{A21}$$

which after subtracting C_R yields:

$$\varepsilon_R = \frac{C_0}{V_1} \sum_i^+ \varepsilon_i - \frac{1}{V_1} \sum_i^+ C_{2i} \varepsilon_i \tag{A22}$$

and:

$$V\{\varepsilon_R\} = E\{(\frac{C_0 - \overline{C_2}}{V_1})^2\} \; E\{(\sum_i^+ \varepsilon_i)^2\} \tag{A23}$$

where $\overline{C_2}$ = average dry weather sewage concentration and no E^2 term exists since $E\{\varepsilon_i\} = 0$. From the results in Equations A17 and A19, Equation A23 becomes:

$$V\{\varepsilon_R\} = \frac{(C_0-\overline{C_2})^2}{C^2} \; E\{\frac{1}{d^2}\} \; E\{\frac{1}{i^2}\} \; D \; V\{\varepsilon_i\} \tag{A24}$$

since $V\{\varepsilon_i\} = F^2 \; \overline{Q_2^2}$ where $\overline{Q_2^2} = \frac{1}{24} \sum_{i=1}^{24} Q_{2i}^2$, the above equation becomes:

$$V\{\varepsilon_R\} = \frac{D \; F^2 \overline{Q_2^2} \; (C_0-\overline{C_2})^2}{C^2} \; E\{\frac{1}{d^2}\} \; E\{\frac{1}{i^2}\} \tag{A25}$$

VERIFICATION

Verification of the above equations for the dry weather effects are obtained from the hourly simulator results using the following values:

$$\overline{C_{2s}^2} = 67^2 + 128^2 + 131^2 + 137^2 + 37^2 = 11634 \; (mg/\ell)^2$$

$$E\{\frac{1}{RD^2}\} = 0.64 \text{ for } RD_{min} = 1, \text{ Table A3}$$

$$E\{RD\} = 115.8/66.96 = 1.729 \text{ for } RD_{min} = 1, \text{ Table A2}$$

i_{min} (0.01 in/hr.	$E\{(1+\beta/i)^2\}$	$E\{\frac{1}{i^2}\}$
0	3.36	0.218
3	1.77	0.0306

$$E\{\frac{1}{d^2}\} = 0.276$$

$$C = 0.950$$

$D = 6$ hr

$\overline{Q_2^2} = 4.09\ (MG/hr)^2$

$\overline{C_2^2} = 11020\ (mg/\ell)^2$

$\overline{C_2} = 100\ mg/\ell$

$C_0 = 58\ mg/\ell$ (average of 6 runs)

Table A4 shows excellent agreement between predicted and observed standard deviations on overflow and runoff concentrations. The effect of dry weather flow variability is relatively small, normally less than the averaging error associated with the daily data base. Neglecting small storm average rainfall intensities (<0.03 in/hr) significantly reduces the runoff variability similar to the effect of measurement error.

TABLE A-4. RUNOFF AND OVERFLOW VARIABILITY DUE TO DRY WEATHER RANDOM VARIABILITY

F, %		i_{min}	σ_{C_0}, mg/ℓ		σ_{CR}, mg/ℓ	
C_2	Q_2	in/hr	Predicted	Observed[1]	Predicted	Observed[1]
10	0	0	15.6		30.2	
10	0	0.03	15.6	17.8	21.1	21.9
20	0	0	32.5		60.4	
20	0	0.03	32.5	27.8	42.2	37.5
0	10	0	0	∿0[2]	5.4	∿0[2]
0	10	0.03	0	7.1[2]	2.0	6.2[2]
0	20	0	0		10.7	
0	20	0.03	0	7.1[2]	4.0	7.0[2]
20	20	0	32.5			
		0.03	32.5	35.1	43.5	43.7

[1] Variance of Averaging error removed from results

[2] Averaging error same magnitude or greater than sewage variability error

APPENDIX III

ANALYSIS OF 26th WARD PLANT DATA

FLOW WEIGHTED AND EQUAL VOLUME ANALYSES

The New York City 26th Ward data was analyzed initially with the daily mass balance technique using flow weighted composite sampling. A chlorides balance was used to include tidegate leakage effects, the equations and results given in previous publications[1-3]. To compare the results of the equal volume composite sampling analysis as developed in this study to the above flow weighted results required development of equations incorporating the chlorides balance into the equal volume technique. The resulting equation for overflow concentration is:

$$C_0 = C_{3W} = \frac{[V_4 - (1-\alpha)(V_2 + V_5)][\frac{CP}{\alpha} - (\frac{1-\alpha}{\alpha})(\frac{V_2C_2 + V_5C_5}{V_2 + V_5})] - \alpha V_5C_5}{V_4 - (1-\alpha)V_2 - V_5} \quad \text{(A26)}$$

which reduces to Equation 19 when $V_5 = 0$ and C_2 is constant over the day.

V_5, the tidegate leakage volume over the day, is given by:

$$V_5 = \frac{B \pm \sqrt{B^2 - 4C}}{2} \quad \text{(A27)}$$

where:

$B = D/F$, $C = E/F$

$$D = CC_P(2V_2(1-\alpha) - V_4) - CC_2V_2(1-\alpha)^2 + CC_3(V_4 - V_2(2-\alpha))$$

$$+ CC_5(V_4(1-\alpha) - V_2(1-2\alpha))$$

$$E = V_2(V_4 - V_2(1-\alpha))(CC_P - CC_2(1-\alpha) - \alpha CC_3)$$

$$F = \alpha CC_3 + (1-2\alpha)CC_5 - (1-\alpha)CC_P$$

and CC refers to the concentration of chlorides at the various locations.

Table A-5 compares the equal volume to the flow weighted analysis for an alpha minimum of 4 hours using the 1957 data. In both analyses, the dry

TABLE A-5. FLOW WEIGHTED AND EQUAL VOLUME ANALYSIS FOR 1957 - 26th WARD DATA

ALPHA MINIMUM = 4 HOURS

	FLOW WEIGHTED ANALYSIS				EQUAL VOLUME ANALYSIS			
	Suspended Solids	Volatile Suspended Solids	BOD_5	Soluble BOD_5	Suspended Solids	Volatile Suspended Solids	BOD_5	Soluble BOD_5
SEWAGE								
Avg Conc., mg/ℓ	140	113	122	51	SAME AS FLOW WEIGHTED ANALYSIS			
Coeff. Variation	0.26	0.26	0.17	0.29				
RUNOFF								
Avg Conc., mg/ℓ								
Yearly load/volume	167	105	103	43	173	95	93	35
Arithmetic mean	238	167	138	51	261	173	134	44
Coeff. Variation	1.04	1.05	0.88	0.79	1.22	1.30	1.08	1.03
OVERFLOW								
Avg. Conc., mg/ℓ								
Yearly load/volume	157	100	103	44	161	89	94	37
Arithmetic mean	205	145	128	49	220	147	123	43
Coeff. Variation	0.77	0.78	0.66	0.65	0.93	1.00	0.78	0.83
WET DAYS	25	25	24	21	25	25	24	21
RAINFALL, in.	16.9				16.9			

weather sewage data is analyzed similarly, thus no differences result. In the table, the average concentration is calculated by 2 techniques, (1) by dividing the total load over the year by the total volume and (2) by taking an arithmetic mean of the daily concentrations.

Since sewage flows are relatively constant, arithmetic means are approximately equal to the yearly load/volume. However both runoff and overflow concentrations have significant differences due to the large flow variations. Comparing average concentrations between flow weighted and equal volume analyses shows differences between 2 and 23% depending on the parameter analyzed. From the analysis conducted with the hourly simulator, this magnitude of difference would be expected for an interceptor capacity of 1.5 to 2.0 Q_2, typical for New York City. The variability of the equal volume daily concentrations about the arithmetic means is somewhat greater than the flow weighted values.

Table A-6 presents the results of the equal volume analysis when all rainfall durations were analyzed for the 1957 data. A significantly greater variability results as well as some negative arithmetic mean concentrations. Thus it is obvious that the individual concentrations from the short duration storms with the resulting low rainfall volumes cannot be analyzed with the daily mass balance technique. The overall load is not significantly affected due to the low rainfall volumes of the shorter duration storms.

Figure A-1, the histogram for the runoff suspended solids concentration for the 26th Ward data, indicates the large number of values existing at the low and high runoff concentrations when all data are analyzed using the equal volume analysis. These results are similar to those obtained previously with the flow weighted analysis, resulting in a minimum duration storm analyzed of 4 hours.

EFFECT OF TIDEGATE LEAKAGE ON MASS BALANCE

If the effect of tidegate leakage is the same in wet and dry periods, incorporating the tidegate leakage into dry weather sewage should result in negligible changes in wet weather concentrations. This effect on the mass balance calculation was studied by setting the tidegate volume equal to zero, thereby incorporating the effect of tidegate leakage into the dry weather sewage. Table A-7 indicates that the sewage volumes calculated without chlorides are higher, but the change in overflow volumes is negligible. The loads per unit area differ very little with a maximum difference of 3%. The mean runoff concentrations (Table A-8) calculated without tidegate leakage vary little from those calculated with tidegate leakage, and in most cases the standard deviation of those calculated without was less. This shows that the effect of tidegate leakage is approximately the same during wet and dry periods, and therefore, the tidegate leakage was incorporated into the dry weather sewage in the following analyses.

TABLE A-6. EQUAL VOLUME ANALYIS FOR 1957 - 26th WARD DATA, ALPHA MINIMUM = 0

	Suspended Solids	Volatile Suspended Solids	BOD_5	Soluble BOD_5
RUNOFF				
Avg Conc., mg/ℓ				
Yearly load/volume	169	98	109	51
Arithmetic mean	-231	-231	155	46
Coeff. Variation	-8.83	-9.15	7.54	18.5
OVERFLOW				
Avg Conc., mg/ℓ				
Yearly load/volume	159	99	104	56
Arithmetic mean	10	-0.7	148	62
Coeff. Variation	95.1	-1220	4.47	7.7
WET DAYS	77	77	76	65
RAINFALL, in.	24.0			

TABLE A-7. EFFECT OF TIDEGATE LEAKAGE ON VOLUMES AND YEARLY LOADS FROM EQUAL VOLUME ANALYSIS OF 26th WARD DATA

	RUNOFF		SEWAGE		OVERFLOW	
Parameter	with Tidegate	w/o Tidegate	with Tidegate	w/o Tidegate	with Tidegate	w/o Tidegate
Volume (MG)	2220	2220	8600	9230	1990	2000
LOAD (lb/acre)						
Susp. Solids	609	598	1971	1982	522	508
Volatile S.S.	383	375	1521	1526	341	332
BOD	241	239	1643	1646	230	227
Soluble BOD	32	33	517	518	43	44

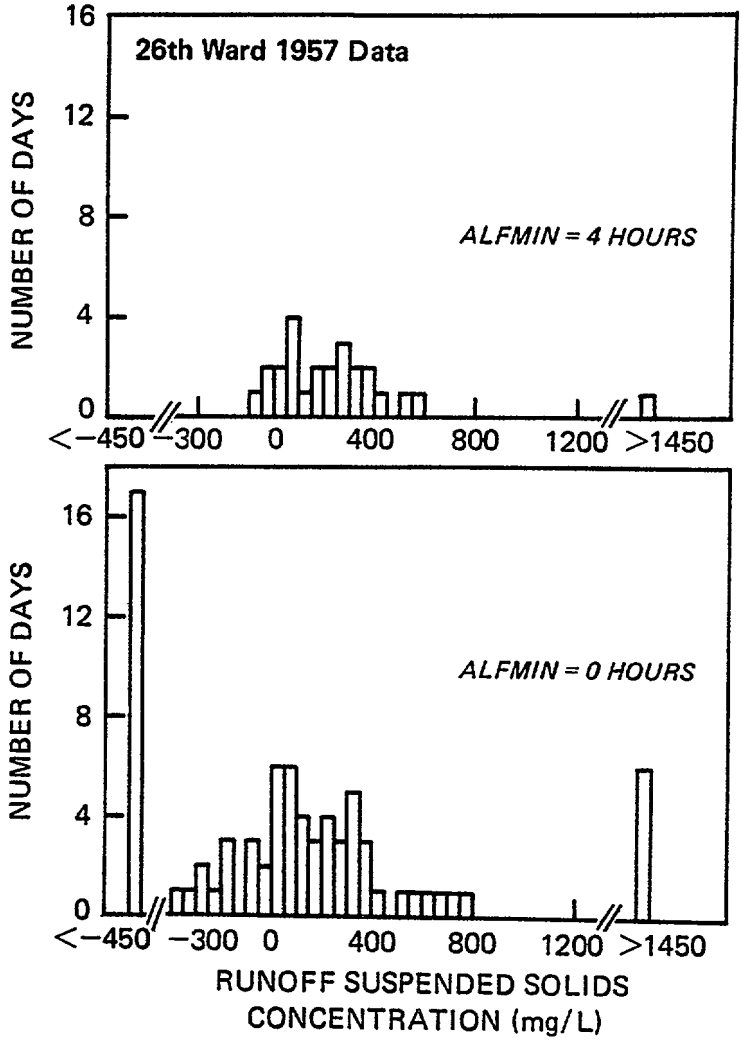

Figure A-1. Effect of Minimum Storm Duration on Runoff Suspended Solids Histograms for 1957 - 26th Ward Data Using Equal Volume Analysis

TABLE A-8. EFFECT OF TIDEGATE LEAKAGE ON RUNOFF CONCENTRATIONS FROM EQUAL VOLUME ANALYSIS OF 6 YEARS OF 26th WARD DATA

YEAR	PARAMETER	ARITHMETIC MEAN, mg/ℓ		STANDARD DEVIATION, mg/ℓ	
		TIDEGATE LEAKAGE	w/o TIDEGATE LEAKAGE	TIDEGATE LEAKAGE	w/o TIDEGATE LEAKAGE
	Suspended Solids	150	144	247	244
54	Volatile S.S.	72	69	181	177
	BOD	34	33	180	175
	Soluble BOD				
	Suspended Solids	261	259	319	315
57	Volatile S.S.	173	172	225	222
	BOD	134	134	146	144
	Soluble BOD	44	44	45	45
	Suspended Solids	145	141	213	209
60	Volatile S.S.	80	78	145	143
	BOD	55	53	143	141
	Soluble BOD	11	10	59	58
	Suspended Solids	273	271	283	272
63	Volatile S.S.	184	183	232	221
	BOD	171	174	427	412
	Soluble BOD	- 4	1	68	69
	Suspended Solids	221	213	158	151
66	Volatile S.S.	139	134	130	128
	BOD	164	160	140	139
	Soluble BOD	34	11	110	113
	Suspended Solids	162	158	162	165
69	Volatile S.S.	114	111	119	119
	BOD	93	90	92	89
	Soluble BOD	29	27	74	73

HOURLY MASS BALANCE METHOD

To analyze the 26th Ward WPCP data by the Hourly method the "Daily Balance" program was expanded from one to four parameters and changed to run on a Tecktronix 4051 computer. The hourly rainfall data used by the program was taken from the Avenue V, Brooklyn rainfall station, since this is the closest rainfall station with an adequate data base. The equal volume variation of the program, without chlorides, was run to obtain the wet weather data required by the hourly analysis program. The wet weather data required were: alpha (the wet fraction of the day); total daily rainfall; daily volumes for runoff, dry weather sewage, overflow, and the plant; as well as plant and dry weather sewage concentrations for each parameter.

The first year analyzed was 1969. Various runs were made to determine the sensitivity of the analysis to different minimum rainfall durations and intensities. Figure A-2 shows histograms of runoff suspended solids concentrations calculated by: analyzing all data; using a minimum duration of 1 hour and intensity 0.03 in/hr; and a minimum duration of 2 hours and intensity of 0.03 in/hr, respectively. Twenty-eight days were analyzed with a minimum duration of 2 hours and intensity of 0.03 in/hr, with a standard deviation of 131 mg/ℓ. By lowering the minimum duration to 1 hour, 2 more days were analyzed while the standard deviation increased to 170 mg/ℓ. If all the data were analyzed (minimum duration of 1 hour and intensity of 0.01 in/hr) concentrations are calculated for 41 days, with a standard deviation of 329 mg/ℓ. For these 3 analyses the range of the calculated mean runoff suspended solids concentration is 67 mg/ℓ, while the flow weighted average concentration varies little, with a range of 19 mg/ℓ. This is explained by the large variability in the runoff concentration resulting from short storms (<1 hr duration, <0.03 in/hr intensity) which have small volumes associated with them.

Since the Avenue V rainfall station is southwest of the drainage area, there was concern that the rainfall data recorded at Avenue V would not be the same as the rainfall occurring over the drainage area. Using hourly rainfall data from the La Guardia Airport weather station, a new hourly rainfall record was developed by combining the two rainfall records. This combined hourly data was used by the hourly analysis to test for the number of samples taken at the plant during wet periods of the day. Alpha (the wet fraction of the day) was still input from the Daily Balance program based on Avenue V, La Guardia, and Central Park rainfall data. As can be seen in Table A-9, there is little difference between the analyses, thus for all additional analyses, the Avenue V data was used.

Beginning in April 1959 the ten samples taken on Saturday and Sunday were analyzed in one composite, as if the weekend was one 48 hour day. Table A-10 shows the yearly average concentrations and standard deviations for runoff and overflow calculated with and without weekends. Since the calculated concentrations vary little, and the standard deviations are generally higher without weekends with less data analyzed, the analysis to follow included weekends.

The hourly analysis program obtains hourly plant and dry weather sewage

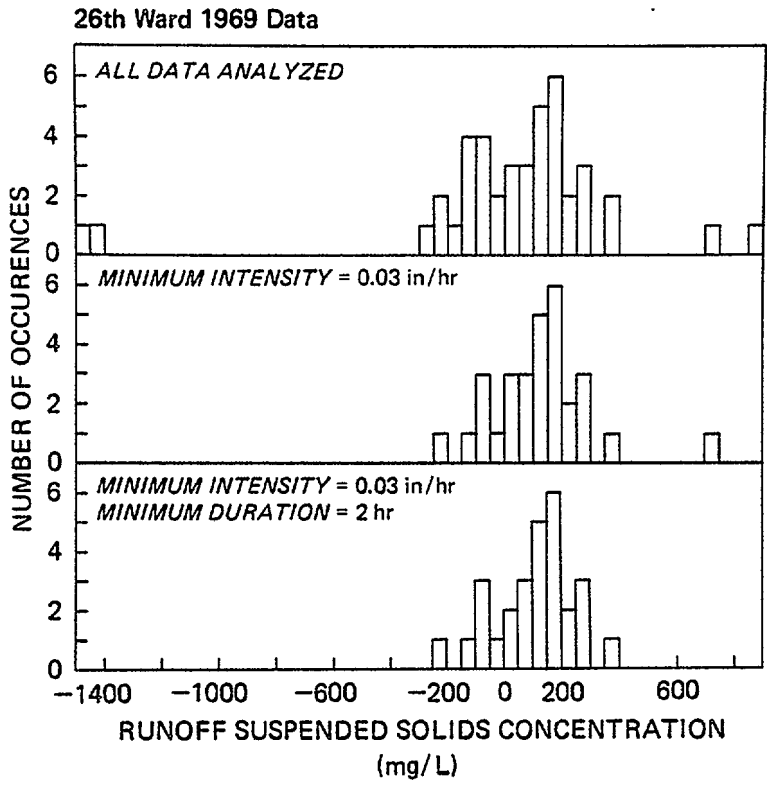

Figure A-2. Effect of Minimum Rainfall Intensities and Durations on Runoff Suspended Solids Histograms for 1969 - 26th Ward Data Using Hourly Analysis

TABLE A-9. COMPARISON OF RUNOFF AND OVERFLOW CONCENTRATIONS USING ONE AND TWO RAINFALL STATIONS FOR 1969 - 26th WARD DATA WITH HOURLY ANALYSIS[1]

	RUNOFF VALUES, mg/ℓ				OVERFLOW VALUES, mg/ℓ			
	ONE STATION		TWO STATIONS		ONE STATION		TWO STATIONS	
PARAMETER	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation
Suspended Solids	110	122	109	131	117	91	116	95
Volatile S.S.	76	85	74	86	81	63	80	64
BOD	39	122	44	142	56	94	61	103
Soluble BOD	-6	70	-7	63	6	54	5	51
Days Analyzed	23		25		23		25	

[1] α_{min} = 2 hr

i_{min} = 0.03 in/hr

TABLE A-10. YEARLY AVERAGE RUNOFF AND OVERFLOW CONCENTRATIONS USING THE HOURLY ANALYSIS[1]

		RUNOFF VALUES, mg/ℓ				OVERFLOW VALUES, mg/ℓ			
		WITH WEEKENDS		WITHOUT WEEKENDS		WITH WEEKENDS		WITHOUT WEEKENDS	
YEAR	PARAMETER	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation	Mean Conc.	Standard Deviation
1960	Suspended Solids	54	206	55	213	63	159	65	164
	Volatile SS	12	174	21	180	22	133	31	136
	BOD	-3	173	2	179	9	131	13	135
	Soluble BOD		56	3	62	3	42	4	45
	Days Analyzed	28		26					
1963	Suspended Solids	263	306	303	313	232	246	277	254
	Volatile SS	191	249	209	262	172	200	193	214
	BOD	33	404	63	439	35	233	58	250
	Soluble BOD	-29	74	-15	73	16	56	-4	54
	Days Analyzed	26		21					
1966	Suspended Solids	182	163	188	170	178	110	182	115
	Volatile SS	123	146	131	136	127	98	132	99
	BOD	79	150	68	130	79	95	74	95
	Soluble BOD	-3	79	-4	95	1	64	1	76
	Days Analyzed	20		14					
1969	Suspended Solids	109	146	110	122	116	93	117	91
	Volatile SS	82	102	76	85	86	73	81	63
	BOD	43	133	39	122	61	96	56	94
	Soluble BOD	-6	63	-6	70	5	49	6	54
	Days Analyzed	28		23					

[1] α_{min} = 2 hr

i_{min} = 0.03 in/hr

concentrations and volumes by assuming them to vary according to the variation of BOD_5 and flow obtained by the Hydroscience Inc. 208 study, Oct. 12-13, 1976 at the 26th Ward WPCP. When the concentrations calculated by the hourly analysis program (min. duration = 2 hr., min intensity = 0.03 in/hr) were compared to those calculated previously by the "Daily Balance" program (min duration = 4 hr) the hourly analysis results were significantly lower. The runoff suspended solids, for example, were more than 25% lower. To determine if the assumed variable sewage characteristics were the cause of the lower concentrations, the analysis was run for 1960 with constant sewage characteristics. The concentrations calculated with constant characteristics were an average of 50% higher than those calculated with variable characteristics. Assuming the sewage variation to be correct, the effects of the New York City sampling schedule were studied. New York City samples at 6 AM, 10 AM, 2 PM, 6 PM, and 10 PM, using equal volume composites. The dry weather sewage concentration, C_{Pd}, obtained from analysis of the dry weather data is greater than the arithmetic average concentration as follows:

$$5C_{Pd} = 0.99\langle C_2\rangle + 1.59\langle C_2\rangle + 1.36\langle C_2\rangle + 1.36\langle C_2\rangle + 0.25\langle C_2\rangle$$

$$C_{Pd} = 1.11\langle C_2\rangle$$

where C_{Pd} = REPORTED AVERAGE DRY WEATHER SEWAGE CONCENTRATION

$\langle C_2\rangle$ = ACTUAL AVERAGE DRY WEATHER SEWAGE CONCENTRATION

When this was incorporated into the Hourly Analysis the resulting runoff suspended solids concentrations were an average of 23% higher. The bias resulting from the NYC sampling scheme would also affect the equal volume concentrations, where the same dry weather sewage error exists, but the error in the plant concentration would be a function of the number of hours of rainfall.

A comparison of the average unit loads calculated by the flow weighted and equal volume variations of "Volbal" and by the hourly analysis is shown in Table A-11. All unit loads except for soluble BOD_5 are similar. The hourly analysis shows that runoff and overflow soluble BOD_5 are significantly lower than previously estimated. The above analyses assume that all parameters have diurnal fluctuations similar to BOD_5. Dry weather hourly data for all parameters would be required to verify this assumption.

The yearly runoff concentrations for all 4 parameters using the hourly analysis technique are shown in Figures A-3 and A-4 and in Figure A-5 for sewage concentrations. A significant degree of variability occurs from year to year while the values for soluble BOD_5 are close to zero. Table A-12 presents the weighted average concentrations for runoff, sewage and overflow from the hourly analysis results. The suspended solids concentrations are higher in the runoff and overflow than in the sewage while the other parameters are lower except for the volatile suspended solids which is similar for the three locations.

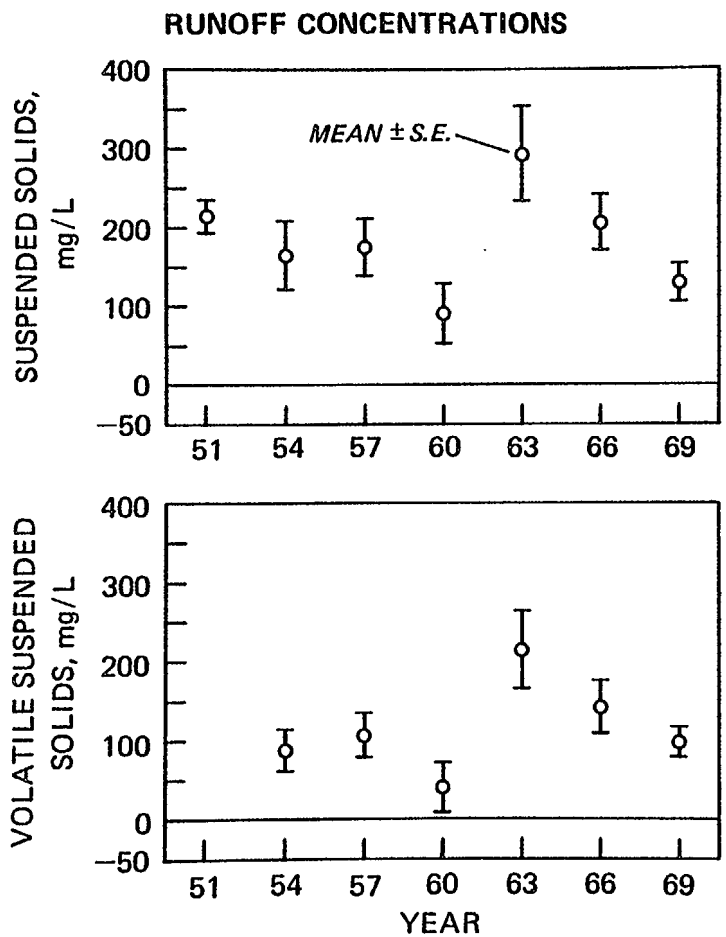

Figure A-3. Yearly Runoff Suspended Solids Concentrations for the 26th Ward Data Using the Hourly Analysis

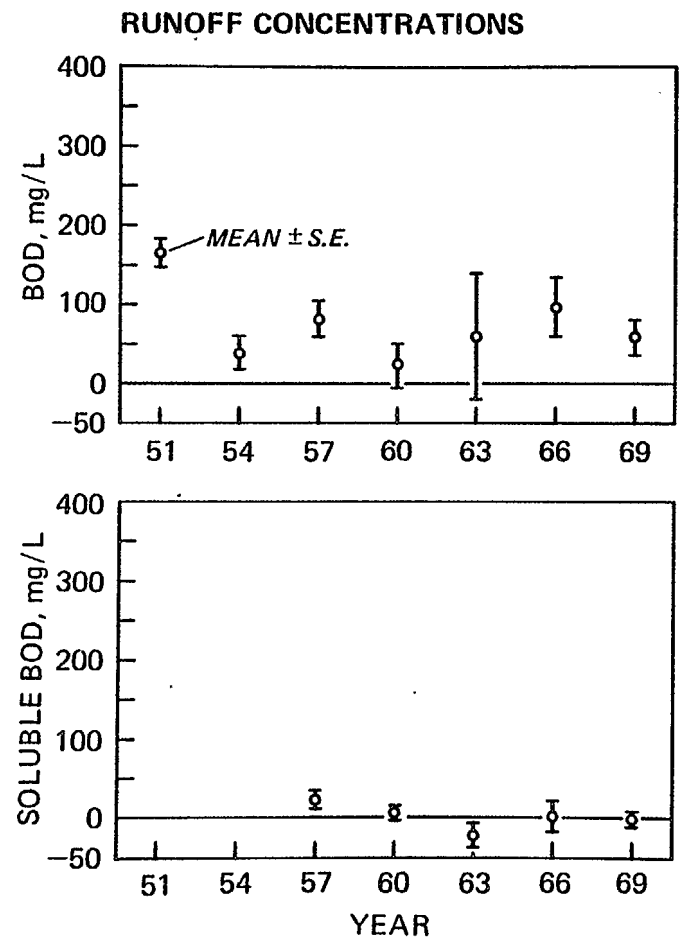

Figure A-4. Yearly Runoff BOD Concentrations for the 26th Ward Data Using the Hourly Analysis

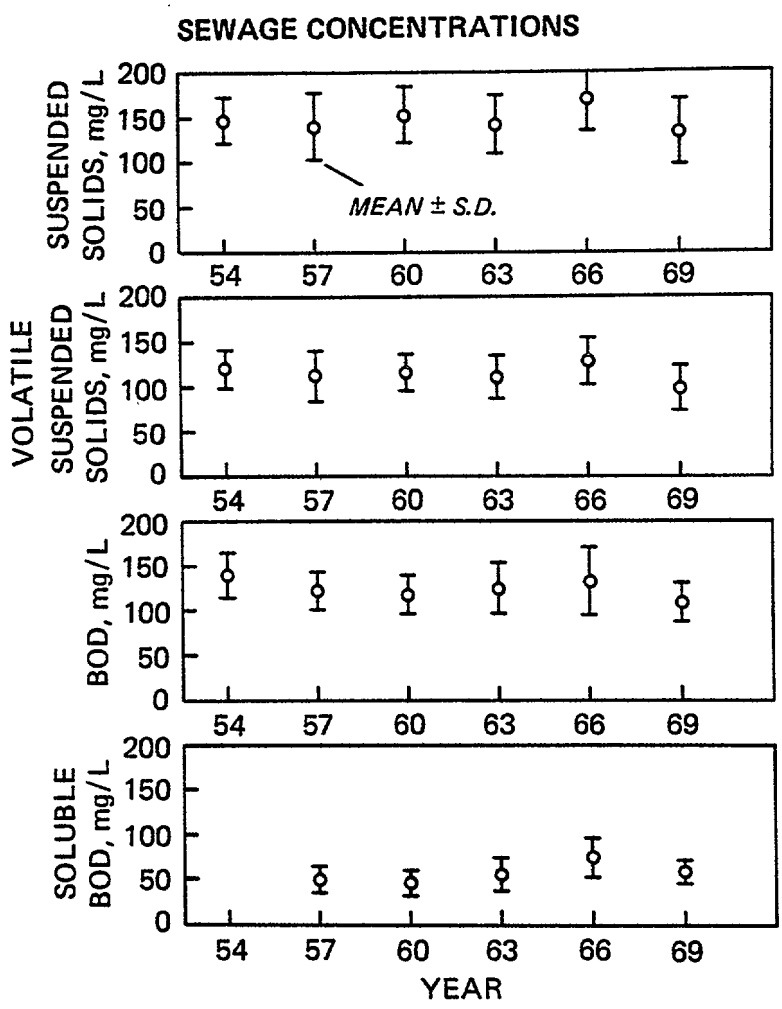

Figure A-5. Yearly Sewage Concentrations for the 26th Ward Data

TABLE A-11. COMPARISON OF AVERAGE YEARLY UNIT LOADS BY DAILY AND HOURLY MASS BALANCE METHODS FOR 26th WARD DATA

	RUNOFF LOAD (lb/ac-in)			OVERFLOW LOAD (lb/ac-in)		
	Daily Balance[1]		Hourly Analysis[2]	Daily Balance[1]		Hourly Analysis[2]
Parameter	Flow Weighted	Equal Volume		Flow Weighted	Equal Volume	
Suspended Solids	28.5	29.9	29.0	24.4	25.2	24.2
Volatile S.S.	18.2	17.7	17.4	16.1	13.2	15.0
BOD	13.0	14.6	12.0	12.1	15.7	12.9
Soluble BOD	2.47	2.41	0.42	2.80	3.21	1.23
Days Analyzed	221	221	226			

[1] α_{min} = 4 hr.

[2] Skip Intensities <0.03 in/hr, α_{min} = 2 hr.

TABLE A-12. WEIGHTED AVERAGE CONCENTRATIONS FOR 26th WARD DATA OVER TOTAL STUDY USING HOURLY ANALYSIS

Flow Point	Weighted Average Concentration (mg/ℓ)			
	SS	VSS	BOD_5	Soluble BOD_5
Runoff	183	109	82	3
Sewage	143	109	121	55
Overflow	174	107	87	9

RAINFALL - RUNOFF RELATIONSHIPS

Linear regression was performed on the runoff concentrations to determine the relationship between concentration and duration of storms, and between concentration and interval between storms. For storms that lasted more than one day the two days were combined into 1 event.

For both duration and interval the regression was performed on the results of three analyses: all data analyzed, alpha minimum of 1 hour and minimum intensity of 0.03 in/hr, and alpha minimum of 2 hours and minimum intensity of 0.03 in/hr. For both duration and interval the best correlation occurred with an alpha minimum of 2 hours and minimum intensity of 0.03 in/hr.

The regression plots for suspended solids and the effect of minimum duration analyzed have been shown previously in Section 8. The regression parameters for suspended solids, BOD_5, volatile suspended solids, and soluble BOD_5 are shown in Table A-13 for both duration and interval between events, obtained using an alpha minimum of 2 hr. and minimum intensity of 0.03 in/hr. Only the suspended solids data, duration data for BOD_5 and interval data for volatile suspended solids have slopes that are not zero within the 95% confidence limits. The regression parameters from a multiple regression of runoff concentration vs. duration of events and interval between events is shown in Table A-14. Higher correlation coefficients result when both parameters are analyzed together as anticipated.

TABLE A-13. RAINFALL-RUNOFF RELATIONSHIPS FOR FOUR PARAMETERS ANALYZED AT 26th WARD USING SEPARATE LINEAR REGRESSIONS[1] FOR DURATION AND INTERVAL

PARAMETER	SLOPE	Y-INTERCEPT, mg/ℓ	r^2, %	N, days
		PARAMETER VS. DURATION		
Suspended Solids	-8.34(± 4.8)	307.2(±56.0)	6.3	181
Volatile Suspended Solids	-1.44(± 1.8)	162.3(±36.4)	1.8[2]	138
BOD_5	-4.49(± 4.4)	156.4(±50.2)	2.3[2]	179
Soluble BOD_5	-0.50(± 2.9)	33.5(±33.0)	0.2[2]	74
		PARAMETER VS. INTERVAL		
Suspended Solids	13.15(± 9.0)	178.8(±48.4)	4.6	181
Volatile Suspended Solids	9.88(± 7.7)	108.1(±43.0)	4.6	138
BOD_5	4.23(± 8.1)	98.6(±43.7)	0.6[2]	179
Soluble BOD_5	2.25(± 4.9)	20.1(±20.0)	1.2[2]	74

[1] α_{min} = 2 hr., i_{min} = 0.03 in/hr.

( ) refers to 95% confidence limits

[2] r^2 not different than zero at 5% level of significance.

TABLE A-14. RAINFALL-RUNOFF RELATIONSHIP FOR FOUR PARAMETERS ANALYZED AT 26th WARD USING MULTIPLE LINEAR REGRESSION[1] FOR DURATION AND INTERVAL

PARAMETER	EQUATION	r^2, %	N, days
Suspended Solids	$y = 256-7.41\ x_1 + 11.0\ x_2$	9.4	181
Volatile S.S.	$y = 123-1.12\ x_1 + 9.16\ x_2$	5.7	138
BOD_5	$y = 142-4.24\ x_1 + 3.07\ x_2$	2.7[2]	179
Soluble BOD_5	$y = 22.6-0.25\ x_1 + 2.17\ x_2$	1.2[2]	74

where y = Concentration of Parameter (mg/ℓ)

x_1 = Duration of Storm (hours)

x_2 = Interval Between Storms (days)

[1] α_{min} = 2 hr, i_{min} = 0.03 in/hr.

[2] r^2 not different than zero at 5% level of significance.

APPENDIX IV

APPLICABILITY

The applicability of the mass balance technique throughout the U.S. was investigated. The first step taken was to determine the extent of combined sewers in the U.S. It was found that 37.6 million people living in urban areas in the U.S. are served by combined sewers (Sullivan, Richard H. et al., 1977)[1]. This represents 25.3% of the urban population.

Using the above data on population served by type of sewerage system, Figure A-6 was developed. This map of the U.S. shows the range of combined sewerage service in the U.S. Within each EPA region there is a wide range of percentage of population served by combined sewers. States with over 30% of the population served by combined sewers are mostly in the Northeast and Midwest, with Washington and Oregon also included.

Since the Balance technique is applicable to specific urban areas containing CSOs and treatment facilities, an analysis of these areas was conducted. Table A-15 lists 71 urban areas in 27 states with over 30% of the population served by combined sewers. This table contains 72% of the total population served by combined sewers and gives a much better picture of the extent of combined sewerage. For example, the state of Georgia has 21.3% of the population served by combined sewers, seemingly low, but Albany, Savannah and Augusta, Ga. have 100, 61, and 49% served, respectively. California, as a state, has only 9.2%, but San Francisco has 47% of the urban population served by combined sewers. This represents 84.8% of the combined sewered population in the state. In some states, rather than having a state-wide application, the "Balance" technique would be limited to one or two urban areas, but still include most of the combined sewerage in the state. This is the case for California, as well as Nebraska, Nevada, Oregon, Rhode Island, Texas, Virginia and Washington.

Three states, Maine, New Hampshire, and Vermont, are sewered exclusively by combined sewers. Some states, Illinois, Indiana, Michigan, Missouri, New York, and West Virginia, have several urban areas, each with high percentages of combined sewers, accounting for most of the combined sewerage in the state and a high percentage of the total sewerage in the state. The remaining 11 states from Table II, Arkansas, Connecticut, Georgia, Iowa, Kansas, Kentucky, Massachusetts, Ohio, Pennsylvania and Tennessee, have percentages of combined sewerage in specific urban areas ranging from 100-34%, but a small percentage of the total sewerage of the state.

---

[1] Sullivan, Richard H., et al. "Nationwide Evaluation of Combined Sewer Overflows and Urban Stormwater Discharges, Volume I: Executive Summary". EPA-600/2-77-064a. Sept. 1977.

Figure A-7 shows the number of urban areas in each state. It is clear that the concentration of combined sewerage is greatest in the Northeast and Midwest, with a few additional urban areas scattered in other parts of the country.

In reviewing Canadian literature Gore and Storrie Limited, 1978[2], determined that 20.4% of the population is served by combined sewers. Twenty urban areas in Ontario have more than 35% of their population served by combined sewers (Sullivan, Richard H., et al., 1978)[3]. Similar statistics might be expected in Quebec and Manitoba since the percentage of combined sewerage is higher in these provinces than in Ontario.

Once the extent of combined sewerage was determined, a questionnaire was prepared to be used in gathering information on the method of sampling used by treatment plants across the country. Initially, attempts were made to obtain information over the telephone, this however proved to be ineffective. Questionnaires were mailed to EPA regional offices with a request for addresses of state offices if the information was not available at the regional office. While some state offices were able to supply the requested information others forwarded the questionnaire to local governments. Twenty-six cities from eleven states returned the questionnaires with information from fifty-four sewage treatment plants. All plants reported using composite samples, 35 of which were equal volume, and 19 flow weighted. Of the 54 plants, 21 take samples for the composite four hours apart, 9 plants take samples every two hours, 12 plants take samples every hour, and 12 plants sample between every ten and twenty minutes. The mass balance hourly analysis would be adaptable to the above plants with greater variability expected from the 24 plants sampling hourly or relatively continuously.

---

[2] Gore and Storrie Limited. "Review of Canadian Municipal Urban Drainage Policies and Practices." Ontario Ministry of the Environment. 1978.

[3] Sullivan, Richard H., et al. "Evaluation of the Magnitude and Significance of Pollution Loadings from Urban Stormwater Runoff in Ontario." Ontario Ministry of the Environment. 1978.

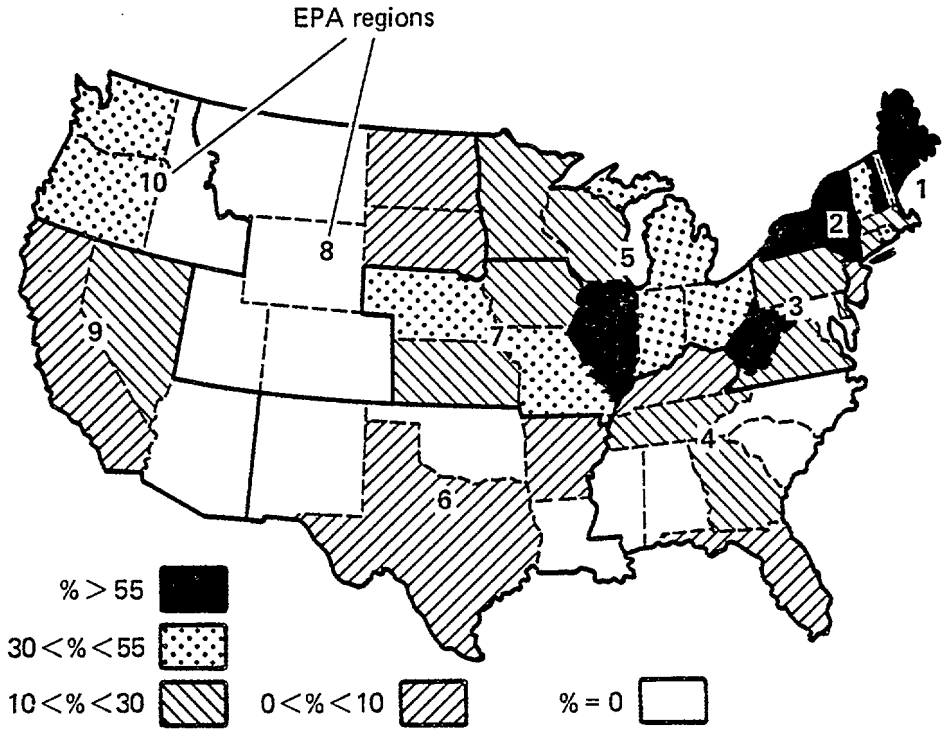

Figure A-6. Percentage of Population in each State Served by Combined Sewers

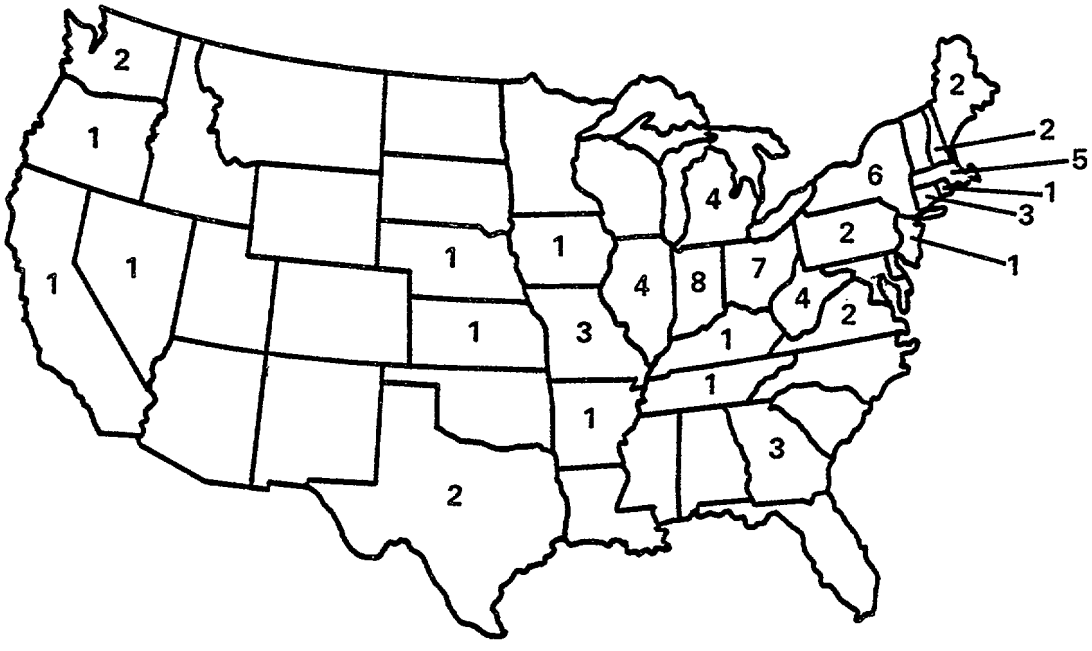

Figure A-7. Number of Urban Areas in Each State Having More Than 30% of the Population Served by Combined Sewers

SEWAGE TREATMENT PLANT DATA SURVEY

State New York

City New York City

Number of Plants in City 12

Plant Name Summary for City

Drainage Area _____

Population Served _____

Population Served by Combined Sewers _____

FLOW:

Peak Hourly Hydraulic Capacity 2x Plant Design Capacity if Primary ByPass Available - 1.5x Plant Design Capacity if Primary Bypass Not Available

Daily Plant Flow (Yearly Average) _____

SAMPLING:

Type of Sampling: Grab _____ or Composite X

If Composite

1) Number of Samples Comprising Composite 6

2) Times of Sampling for Composite 10AM, 2PM, 6PM, 10PM, 2AM, 6AM (Prior to ∿ 1975 no 2 AM sample)

3) Method of Composite:

a) Equal Volume X

b) Flow Weighted ____

Number of Composite Samples per Week 6 No Friday Samples

ANALYSIS

Number of years for which data is available Approximately 35 years

Parameters Analyzed	Frequency of Analysis Daily	Weekly	Monthly	Other
BOD	X			
Soluble BOD	X			
Suspended Solids	X			
Volatile S.S.	X			
NO_2-N				2/Month
NO_3-N				2/Month
NH_3-N				2/Month
Org-N				2/Month
Total P				2/Month
Total Coliform	X			
Fecal Coliform	X			
C_u			X	
C_R			X	
C_d			X	
N_i			X	
H_g			X	
P_b			X	
Z_N			X	

TABLE A-15. URBAN AREAS WITH HIGH PERCENTAGES OF POPULATION SERVED BY COMBINED SEWERS

Urban Area	State	% of Total Pop. Served by Combined Sewer	Pop. Served by Combined Sewers (1000 Persons)	Area Served by Combined Sewers (1000 Acres)	% of Total Area
Albany	Ga.	100	76	12.0	56.9
Anderson	In.	100	81	20.0	72.7
Evansville	In.	100	142	15.8	60.3
Huntington	WV	100	121	14.8	63.8
Lima	Oh.	100	70	10.1	58.4
Lynchberg	Va.	100	71	10.4	43.9
St. Joseph	Mo.	100	77	14.4	70.2
Steubenville	Oh.	100	45	4.2	53.2
Steubenville Metro	WV	100	40	6.6	38.6
Philadelphia Metro	NJ	99.5	201	20.3	64.6
Lafayette	In.	92.4	73	5.0	41.0
Binghampton	NY	86.8	145	15.2	45.6
Chicago	Il.	77.3	4416	204.9	32.7
Springfield	Oh.	76.6	72	7.3	45.6
New Bedford	Ma.	75.4	101	8.9	40.8
Manchester	NH	74.7	71	7.0	28.0
Lewiston	Me.	73.8	48	4.7	10.8
Owensboro	Ky.	73.6	39	3.5	45.5
Portland	Me.	72.6	77	10.3	28.8
Scranton	Pa.	72.5	148	17.1	27.3
Wheeling	WV	72.0	67	6.7	37.4
Spokane	Wa.	71.7	165	19.4	38.9
Cincinnati	Oh.	70.1	778	73.4	10.3
Saginaw	Mi.	69.6	103	11.5	40.8
Fall River	Ma.	66.2	92	7.2	26.2
Galveston	Tx.	64.5	40	2.5	17.0
New York City	NY	64.3	6764	108.3	44.5

TABLE A-15 (cont'd)

Urban Area	State	% of Total Pop. Served by Combined Sewer	Pop. Served by Combined Sewers (1000 Persons)	Area Served by Combined Sewers (1000 Acres)	% of Total Area
Detroit	Mi.	62.3	2475	166.2	29.8
Lawrence	Ma.	61.5	123	12.0	22.3
Savannah	Ga.	61.0	100	6.5	15.9
South Bend	In.	60.8	175	20.2	30.7
Omaha	Ne.	60.2	296	20.9	21.6
Buffalo	NY	59.1	624	38.3	28.0
Hartford	Ct.	59.1	275	20.8	24.8
Nashua	NH	59.0	36	4.9	22.5
St. Louis	Mo.	58.2	1096	112.7	38.2
Bay City	Mi.	57.7	45	5.2	31.3
Albany	NY	55.8	271	19.3	20.0
Indianapolis	In.	55.6	456	34.0	13.9
Decatur	Il.	54.0	54	6.5	27.4
Charleston	WV	53.2	84	6.8	17.1
Washington D.C.	--	52.8	400	12.7	32.3
New Haven	Ct.	51.4	179	14.9	21.8
Norwalk	Ct.	51.4	55	1.5	5.5
Fort Wayne	In.	50.7	114	8.1	18.3
Davenport Metro	Il.	50.0	56	5.7	25.3
Jackson	Mi.	50.0	39	4.4	19.1
Springfield	Ma.	49.4	254	32.9	21.6
Augusta	Ga.	49.0	73	9.0	24.3
Richmond	Va.	48.1	200	15.9	17.1
Muncie	In.	47.8	43	4.3	26.9
Lowell	Ma.	47.5	87	6.2	15.6
Peoria	Il.	47.4	117	14.8	51.9
San Francisco	Ca.	47.0	1410	54.1	12.4
Des Moines	Ia.	46.3	118	4.0	5.7

TABLE A-15 (cont'd)

Urban Area	State	% of Total Pop. Served by Combined Sewer	Pop. Served by Combined Sewers (1000 Persons)	Area Served by Combined Sewers (1000 Acres)	% of Total Area
Chicago Metro	In.	45.3	453	32.3	16.9
Youngstown	Oh.	43.5	172	13.7	16.6
Syracuse	NY	42.3	159	13.2	21.5
Providence	RI	41.9	333	21.0	13.4
Toledo	Oh.	41.8	204	15.9	15.0
Nashville	Tn.	40.2	180	14.7	6.7
Rochester	NY	39.9	240	14.3	15.3
Fort Smith	Ar.	39.5	30	4.3	11.2
Seattle	Wa.	39.0	483	37.9	14.3
Hamilton	Oh.	38.5	35	2.4	9.9
Portland	Or.	38.0	316	24.2	14.2
Kansas City	Mo.	37.2	308	21.0	8.1
Pittsburgh	Pa.	36.1	667	31.5	8.3
Reno	Nv.	35.0	35	2.4	9.9
Topeka	Ks.	34.1	45	3.7	10.9
Beaumont	Tx.	32.8	38	2.8	5.8
TOTAL			27,076	1,559.6	

SOURCE: Heany, James P. et al., "Nationwide Evaluation of Combined Sewer Overflows and Urban Stormwater Discharges Volume II: Cost Assessment and Impacts" EPA-600/2-77-064 March 1977.

APPENDIX V

DEFINITION OF SYMBOLS

Symbol	Subscript	Superscript	Description
A			drainage area
C			overall runoff coefficient = fC_vA
	1,2,3,4,5		actual concentrations at specified location: 1 = Runoff 2 = Dry weather sewage 3 = Overflow 4 = Plant 5 = Tidegate leakage
	i,k		concentration during specified hour
	j		concentration during specified day
		-	average concentration over day
	d,w		concentration during dry and wet portions of day, respectively
	O,P,R		estimated concentration at specified location: O = Overflow P = Plant R = Runoff
	R_{max}		peak hourly runoff concentration
	R_{∞}		runoff concentration at infinite time after beginning of event
	s	-	average concentration during sampling hours
	v		volumetric runoff coefficient
CC	2,3,4,5,P		chloride concentration at specified location (see "C")
d			duration of storm
	i		duration of storm during specified hour
D			mean duration of storm for total record
E{ }			expected value
f			units conversion factor for rainfall-runoff relationship

DEFINITION OF SYMBOLS (continued)

Symbol	Subscript	Superscript	Description
F			ratio of the hourly random sewage variability (standard deviation) to the hourly sewage concentration
i			average rainfall intensity for event
	j		average rainfall intensity for event on specified day
	k		rainfall intensity for specified hour
I			mean event rainfall intensity for total record
N			number of days analyzed
	s		number of sampling hours used to obtain daily composite sample for plant variance
o	i		variable used to indicate hours in which overflows occurred ($o_i = 1$) or did not occur ($o_i = 0$)
P( )	i,d,δ		probability density function for specified variable, rainfall, duration, interval, respectively
P	d		probability of occurrence of a specified duration
	wd		probability of occurrence of a specified duration being wet
	RD		probability that a specified number (RD) of samples are taken during wet hours over all durations from 1 to 24 hours
Pr	RD		probability that a specified number (RD) of samples are taken during wet hours over a day
Q	1,2,3,4,5		flow rate at specified location (see "C")
	d,w		flow rate during dry and wet hours, respectively
	i,k		flow rate during specified hour
	j		flow rate during specified day
	I		maximum hourly flow rate to treatment plant during storm event (interceptor capacity)

DEFINITIONS OF SYMBOLS (continued)

Symbol	Subscript	Superscript	Description
r^2			correlation coefficient squared = fraction of variability explained by correlation
RD			number of samples taken during wet hours for daily plant influent composite sample
s	i		variable used to indicate hours in which a plant influent sample is taken (s_i=1) or not taken (s_i=0) for the daily composite
STD			standard deviation of measurement error on daily composite plant influent sample (= σ_p)
t	d,w		length of dry and wet periods respectively during a day
V	1,2,3,4,5		total daily volume at specified location (see "C")
	d,w		total daily dry and wet volume, respectively
V{ }			variance of specified parameter
	1		variance of measurement error
	2		variance of averaging error
W			total load over day
α			fraction of day over which rainfall occurred
	min		minimum α analyzed in record
β			ratio of sewage flow rate (Q_2) to overall runoff coefficient (C)
	R		rate coefficient for exponential decay of runoff concentration during first flush
δ			interval between storm events
Δ			mean interval between storm events for total record
ε	ik,C_2k,Q_2k		fluctuation about hourly values for intensity, dry weather sewage concentration, and flow respectively

DEFINITIONS OF SYMBOLS (continued)

Symbol	Subscript	Superscript	Description
ε	C_{Pj}		fluctuation about daily plant value due to measurement error
	oj,rj		error in daily flow weighted and runoff overflow concentrations, respectively
	o,r	-	runoff overflow and bias, respectively, for period of record analyzed
	o,p,r		perturbation on overflow, plant and runoff concentrations, respectively
	i		hourly perturbation on dry weather sewage concentration or flow rate
σ	CR,CO		standard deviation of the errors (true - estimated) of the daily runoff and overflow concentrations for the period of record analyzed
	p		see "STD"
	CR,CO,p	2	variance of specified values
Σ			sum between specified limits
ν	R,O		coefficient of variation of the errors (true - estimated) of the daily runoff and overflow concentrations for the period of record analyzed