Hydraulic Analysis on Stream-Aquifer Interaction by Storage Function Models

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Summary. To improve the river environment in an urbanized basin, it is important to restore the hydrologic relationships between streams and aquifers. In this paper, the dynamic interaction between them, the so-called "bank storage effect," is analyzed based on hydraulic models of a stream-aquifer system. In particular, linear and nonlinear storage function models are used in order to express the stream flow. It is shown that bank storage by the aquifer fulfills the functions to control the fluctuation of the stream flow.

Key words. stream-aquifer interaction, bank storage effect, river environment, groundwater discharge, storage function model

INTRODUCTION

In the natural hydrologic cycle, surface and subsurface water in a watershed are closely related and interact with each other. However, their relationships are affected by human activities. For instance, as the impervious area of a basin spreads due to urbanization, rainfall recharge into unconfined aquifers decreases and consequently the stream flood hazard increases. By renovating channels for flood control, natural streams are reconstructed into artificial channels. As a result, water exchange between stream and aquifer may be impacted and altered. It is well known that small streams in a city become drainage channels in rainy days and run dry in non-rainy days. In order to improve such a river environment, it is necessary to recover the natural water cycle in the watershed by recognizing the hydrologic connection between streams and aquifers.

Although the relationship between a stream and an aquifer has been investigated from various angles [1][2][3], most of the researches dealt with merely the response of an aquifer to fluctuation in the stream stage. At alluvial plains, however, the relationship between the two is interactive and the water exchange between them depends on their relative hydraulic state. Therefore, it is necessary that the stream-aquifer interaction is evaluated by solving two governing equations of the stream flow and the groundwater simultaneously. The purpose of this paper is to clarify a potential role that the aquifer plays for regulating the stream flow, the so-called "bank storage effect" [4], by expressing the stream flow in the storage function model.

FUNDAMENTAL EQUATIONS

Stream Flow. Let us consider a combined system of a stream channel and an unconfined aquifer as shown in Fig. 1. For simplicity, it is assumed that the channel has width B, slope I_O and straight reach length L, and the aquifer is of semi-infinite lateral extent on a horizontal base. Now, an inflow rate $I(t)$ at the upstream end of the channel reach, we seek the effect of the aquifer on the outflow rate $O(t)$ at the downstream end. At time $t = 0$, it is also assumed that $I(t)$ is equal to $O(t)$ and the stream stage is in equilibrium with the water-table in the aquifer. For storage volume in the reach, $S(t)$, and groundwater discharge from both sides of the aquifer into the channel, $Q_r(t)$, the fundamental equations of the stream flow take the following form:

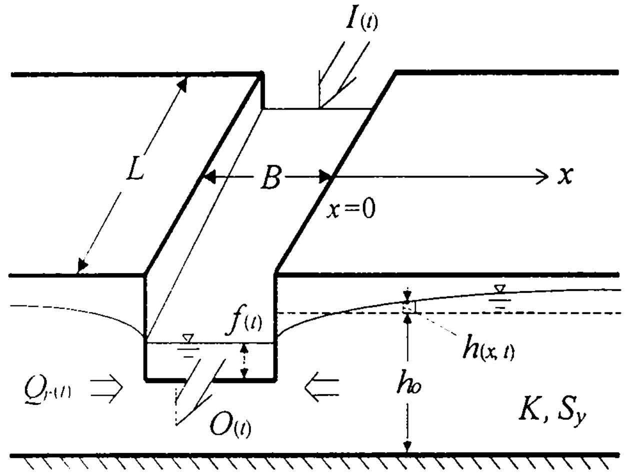

Fig. 1. Schematic of stream-aquifer system.

$$\frac{dS(t)}{dt} = I(t) - O(t) + Q_r(t) \qquad (1)$$

$$S(t) = kO(t)^p \qquad (2)$$

where k and p are coefficients of the storage function. $p = 1.0$ in the linear flood routing, $p = 0.6$ in the nonlinear flood routing based on the Manning formula. $S(t)$ can be expressed as $S(t) = Af(t) = BLf(t)$, where A is horizontal area in the reach and $f(t)$ is depth of the stream flow assumed to be uniform along the reach length.

Groundwater Flow. Generally speaking, the groundwater around the stream is three-dimensional flow with vertical velocity. However, in the case that the aquifer thickness is three times smaller than the channel width, the flow may be regarded as horizontal and that the Dupuit-Forchheimer assumption may hold [5]. Moreover, in the case that water-table fluctuation h is smaller than the average depth h_o, the Boussinesq equation of unconfined flow may be linearized. Based on these assumptions, the fundamental equation for one-dimensional flow toward the channel takes the following form:

$$\frac{\partial h(x,t)}{\partial t} = \frac{K h_o}{S_y}\frac{\partial^2 h(x,t)}{\partial x^2} + \frac{r_e(x,t)}{S_y} \qquad (3)$$

where K is the hydraulic conductivity of the aquifer, S_y is the specific yield of the unconfined aquifer, $r_e(x, t)$ is the recharge rate from above, x is horizontal coordinate measured in orthogonal direction from the channel, and $h(x, t)$ is the water-table elevation relative to the initial equilibrium elevation. The initial and boundary conditions are adopted as shown in Fig. 1, $h(x, 0) = 0$, $h(0, t) = f(t)$, $h(\infty, t) = 0$, and $r_e(x, t) = 0$. Thus, it can be shown that the variation of $h(x, t)$ due to $f(t)$ is formulated by the Duhamel theorem as:

$$h(x,t) = \int_0^t U(x,t-\tau)\frac{\partial f(\tau)}{\partial \tau}d\tau, \quad U(x,t-\tau) = \frac{2}{\sqrt{\pi}}\int_{\frac{x}{2\sqrt{\kappa(t-\tau)}}}^{\infty} e^{-\xi^2}d\xi = \mathrm{erfc}\left(\frac{x}{2\sqrt{\kappa(t-\tau)}}\right) \qquad (4)$$

where $\kappa = \frac{Kh_o}{S_y}$, and erfc(-) denotes the complementary error function. The groundwater discharge $Q_r(t)$ into the channel reach L from both sides of the aquifer can be obtained as follows:

$$Q_r(t) = 2 \times Lh_o \cdot K \frac{\partial h(x,t)}{\partial x}\bigg|_{x=0} = -2L\sqrt{\frac{S_Y K h_o}{\pi}} \cdot \int_0^t \frac{1}{\sqrt{t-\tau}} \frac{\partial f(\tau)}{\partial \tau} d\tau \tag{5}$$

From eqs.(1), (2), and (5), it is evident that both the stream flow and the groundwater discharge are interacts through the stream stage $f(t)$.

LINEAR STORAGE FUNCTION MODEL

For a linear storage function of the stream flow, $p = 1.0$ in eq. (2), Morel-Seytoux [6] obtained a closed-form solution of the interaction problem. Though it was a leading achievement, unfortunately the mathematical forms are slightly inadequate. Thus, we will rederive more accurate forms here.

Expressing the response of the channel depth $f(t)$ to the inflow $I(t)$ by the convolution integral with the response kernel $u(t)$, the outflow $O(t)$ is rewritten as follows.

$$O(t) = \frac{S(t)}{k} = \frac{A}{k} f(t) = \frac{A}{k} \int_0^t u(t-\tau) I(\tau) d\tau \tag{6}$$

The equation which $u(t)$ should satisfy becomes the following by substituting eqs. (2) and (5) into (1).

$$A\left(\frac{df(t)}{dt} + \frac{f(t)}{k}\right) = I(t) - 2L\sqrt{\frac{S_Y K h_o}{\pi}} \cdot \int_0^t \frac{1}{\sqrt{t-\tau}} \frac{\partial f(\tau)}{\partial \tau} d\tau \tag{7}$$

Eq. (7) is a linear integral differential equation that can be solved by the Laplace transform method. Expressing the Laplace transform of $f(t)$ by $\mathbf{L}\{f(t)\} = F(s)$, and taking $f(0) = 0$ into consideration, the Laplace transform of the above equation can be shown to be

$$F(s) = \frac{\mathbf{L}\{I(t)\}}{A\left(s + \frac{1}{k} + \frac{2L}{A}\sqrt{S_Y K h_o}\sqrt{s}\right)} \tag{8}$$

Since the Laplace transform of $f(t)$ in eq. (6) is $F(s) = \mathbf{L}\{I(t)\} \cdot \mathbf{L}\{u(t)\}$, as compared it with the above equation, $\mathbf{L}\{u(t)\}$ can be expressed as

$$\mathbf{L}\{u(t)\} = \frac{1}{A\left(s + \frac{1}{k} + 2\sqrt{\frac{S_Y K h_o}{B^2}}\sqrt{s}\right)} \tag{9}$$

$$u(t) = \mathbf{L}^{-1}\left\{\frac{1}{A\left(s + \frac{1}{k} + 2\sqrt{\frac{S_Y K h_o}{B^2}}\sqrt{s}\right)}\right\} = \mathbf{L}^{-1}\left\{\frac{1}{2A\sqrt{\frac{S_Y K h_o}{B^2} - \frac{1}{k}}}\left\{\frac{1}{\sqrt{s}+b} - \frac{1}{\sqrt{s}+a}\right\}\right\} \tag{10}$$

where $$a = \sqrt{\frac{S_y K h_o}{B^2}} + \sqrt{\frac{S_y K h_o}{B^2} - \frac{1}{k}}, \quad b = \sqrt{\frac{S_y K h_o}{B^2}} - \sqrt{\frac{S_y K h_o}{B^2} - \frac{1}{k}} \tag{11}$$

By using a table of the inverse Laplace transform [7], $u(t)$ is given as

$$u(t) = \frac{1}{2A\sqrt{\frac{S_y K h_o}{B^2} - \frac{1}{k}}} \left\{ a e^{a^2 t} \operatorname{erfc}\left(a\sqrt{t}\right) - b e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t}\right) \right\} \tag{12}$$

Consequently, by substituting eq. (12) into eq. (6), one can calculate the response of the outflow $O(t)$ to any fluctuation of the inflow $I(t)$. Assuming a combined system with a small stream and a highly permeable aquifer, let us evaluate the response kernel $u(t)$. Values of parameters are supposed to be B = 10 m and k = 1 hr for the channel, S_y = 0.2 and h_O = 10 m for the aquifer. For the aquifer hydraulic conductivity, two cases of K = 0 (without aquifer) and K = 50 m/hr are considered. Fig. 2 shows difference of $u(t)$ by the aquifer hydraulic conductivity K. According to this figure, it appears that the response kernel in case of K = 50 m/hr reduces more rapidly initially than in the case of K = 0. This reflects the initial impact of lateral flow to the aquifer on the attenuation of the inflow $I(t)$.

The linear response models as stated above is easily applicable to evaluate the bank storage effect. However, we should notice that the solution (12) is valid only when

$$\frac{S_y K h_o}{B^2} > \frac{1}{k} \tag{13}$$

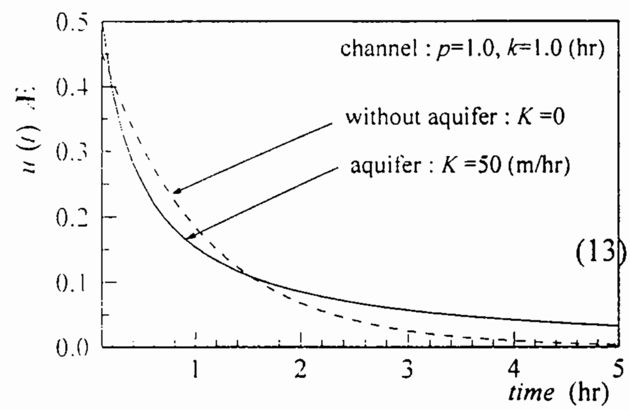

Fig. 2. Linear response function of outflow to inflow.

Since this condition may be very restrictive because it corresponds to the case of a channel with narrow width and an aquifer with high conductivity and large porosity, we cannot recognize eq. (12) to be a general solution of the bank storage problem. In addition, since this solution is based on the linear storage function, the stream stage fluctuates in proportion to a variation in the stream flow rate. Thus, it is possible that eq. (12) overestimates the exchange between the stream and the aquifer. We will consider a universal nonlinear model in the following section.

NONLINEAR STORAGE FUNCTION MODEL

Numerical Method. Applying Manning's formula to eq. (2), $p = 0.6$ and $k = n^{0.6} B^{0.4} I_o^{-0.3} L$, in which n is the channel roughness, are obtained [8]. In other words, since the storage function is nonlinear, it is difficult to obtain the analytical solution. Thus, we will attempt to obtain a numerical solution by linearizing the fundamental equation. By replacing of $y(t) = O(t)^p$, eq. (1) is rewritten as

$$k \frac{dy(t)}{dt} = I(t) - y(t)^{1/p} + Q_r(t) \tag{14}$$

Expanding the right-hand side in Taylor series about y_*, which is the value of y at $t^*=t-\Delta t$, in which Δt is chosen to be sufficiently small, and ignoring the higher-order terms,

$$\frac{dy(t)}{dt} - a\ y(t) = \frac{1}{k}\{I(t) - b + Q_r(t)\}, \quad a = -\frac{1}{kp} y_*^{\left(\frac{1}{p}-1\right)}, \quad b = \left(1-\frac{1}{p}\right) y_*^{\frac{1}{p}} \tag{15}$$

Noting that the left-hand side of eq. (15) is equal to $e^{at}\frac{d}{dt}\left(e^{-at}y(t)\right)$, then, by integrating in time, we have

$$y(t) = e^{at}y(0) + \int_0^t e^{a(t-\tau)}x(\tau)d\tau, \quad x(t) = \frac{1}{k}\{I(t) - b + Q_r(y(t))\} \tag{16}$$

$Q_r(y(t))$ can be rewritten by substituting $f(t) = \frac{k}{A}y(t)$ into eq.(5):

$$Q_r(t) = Q_r(y(t)) = -\frac{2Lk}{A}\sqrt{\frac{S_y K h_o}{\pi}} \cdot \int_0^t \frac{1}{\sqrt{t-\tau}} \frac{\partial y(\tau)}{\partial \tau} d\tau \tag{17}$$

Replacing time t in eq. (16) by discrete times i = 0, 1, 2,···, m,··· with increment T, the equation to be solved becomes

$$y_m = \phi^m y_0 + \sum_{i=1}^{m} \phi^{i-1}\gamma x_i, \qquad \phi = e^{aT},$$
$$\gamma = \frac{1}{a}\left(e^{aT}-1\right), \quad x_i = \frac{1}{k}\left(I_i - b + Q_{ri}\right) \tag{18}$$

Evaluation of Bank Storage Effect. As in the linear case, we evaluate the bank storage effect for a similar stream-aquifer system. For the aquifer it is assumed that S_y = 0.2, h_o = 10 m and K = 0, 1, 10 m/hr. For the channel, assuming p = 0.6 as mentioned earlier, B = 20 m, I_o = 1/1000, L = 4 km, and n = 0.03 in m-sec unit system, the value of k becomes k = 100 in m-hr unit system.

Fig. 3 shows differences of the outflow $O(t)$ for different aquifer hydraulic conductivity when the inflow I (t) is given by a leftmost curve in the figure. From the figure, it is recognized that in the case of higher conductivity, the peak of the curve $O(t)$ decreases and the tail of the curve becomes milder. This may be caused by increased exchange between the stream and the aquifer due to a higher conductivity (Fig. 4).

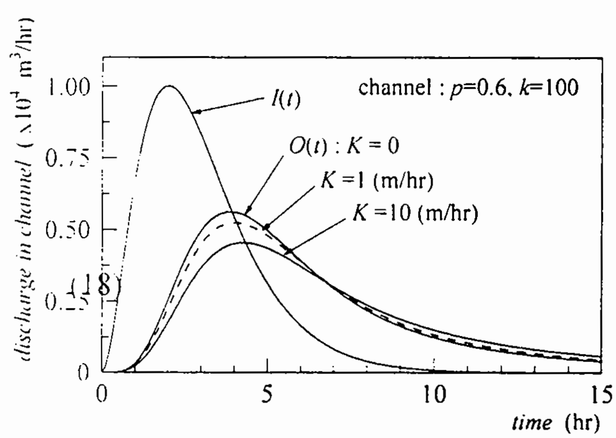

Fig. 3. Effect of aquifer hydraulic conductivity K on stream outflow rate in channel.

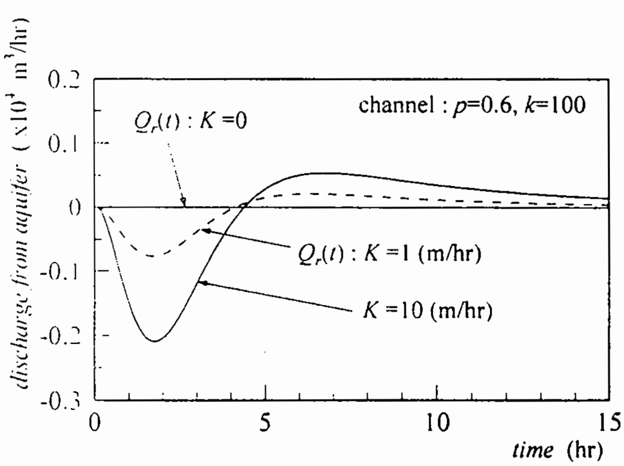

Fig. 4. Effect of aquifer hydraulic conductivity K on groundwater discharge from aquifer into channel.

According to Fig. 4, the groundwater discharge $Q_r(t)$ changes its flow direction from negative to positive in response to the fluctuation of the stream stage. In other words, this implies that in the combined system of the channel and the highly permeable aquifer, the latter may absorb the fluctuating inflow and regulate the stream outflow.

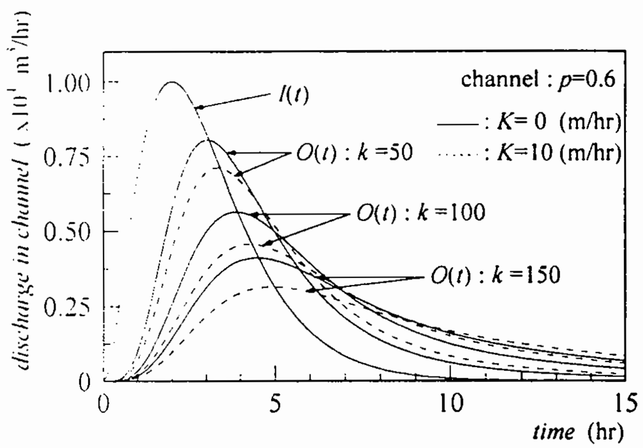

Fig. 5. Effect of channel roughness on stream outflow rate in two cases of aquifer hydraulic conductivity K.

Fig. 5 shows differences of the outflow $O(t)$ for different channel roughness in two cases of the aquifer hydraulic conductivity K. Assuming the Manning's roughness n = 0.01, 0.03, 0.06 for same shape of the channel, values of k are k = 50, 100, 150. According to the figure, as k becomes larger, $O(t)$ gets increasingly attenuated with the outflow regulated over a longer period of time. This effect by k is considered to be natural because of increase of friction resistance in the channel. In the channel with larger roughness, it is expected that the exchange with aquifer may become more active due to increased stage fluctuation. In the figure, however, differences of $O(t)$ are almost similar in case of various k. This implies that the amplified effect of the stream stage by the channel roughness does not significantly affect the evaluation of the bank storage effect.

CONCLUSION

To gain a basic understanding of a river environment in a watershed, relationships between streams and aquifers have been analyzed by using storage function models. It was shown that the existence of an aquifer with a high hydraulic conductivity fulfills the function of regulating the stream flow rate by the bank storage effect. Since this investigation merely dealt with an aspect of the behavior of the stream and the groundwater in a watershed, it is evident that additional work using different approaches is necessary.

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16. ABSTRACT
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